

DEFENSIVE MARKETING STRATEGIES

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## ABSTRACT

This paper analyzes how a firm should adjust its marketing expenditures and its price to defend its position in an existing market from attack by a competitive new product. Our focus is to provide usable managerial recommendations on the strategy of response. In particular we show that if products can be represented by their position in a multiattribute space, consumers are heterogeneous and maximize utility, and awareness advertising and distribution can be summarized by response functions, then for the profit maximizing firm,

- it is optimal to decrease awareness advertising,
- it is optimal to decrease the distribution budget unless the new product can be kept out of the market,
- a price increase may be optimal, and
- even under the optimal strategy, profits decrease as a result of the competitive new product.

Furthermore, if consumer tastes are uniformly distributed across the spectrum

- it is optimal to decrease price in regular markets,
- it is optimal (at the margin) to reposition by advertising, in the direction of the defending product's strength and
- it is optimal (at the margin) to improve product quality in the same direction.

In addition we provide practical procedures to estimate (1) the distribution of consumer tastes and (2) the position of the new product in perceptual space from sales data and knowledge of the percent of consumers who are aware of the new product and find it available. Competitive diagnostics, such as the 'angle of attack' are introduced to help the defensive manager.

## 1. PERSPECTIVE

Many new products are launched each year. While some of these products pioneer new markets, most are new brands launched into a market with an existing competitive structure. But there is no reason to expect the firms marketing the existing products to be passive. The new brand's sales come from the existing products' market shares or from foregone growth opportunities. In fact, for every new brand launched there are often four to five firms (or brands) who must actively defend their share of the market. This paper investigates how the firms marketing the existing brands should react to the launch of a competitive new brand. We will refer to this topic as defensive marketing strategy.

The offensive launch of a new brand has been well studied. See for example reviews by Pessemier (1977), Shocker and Srinivasan (1979), and Urban and Hauser (1980). Good strategies exist for brand positioning, the selection of brand features and price, the design of initial advertising strategies, and the selection of couponing, dealing, and sampling campaigns. But the analysis to support these decisions is often expensive and time consuming. (A typical positioning study takes 6 months and over \$100,000. [Urban and Hauser 1980, chapter 3]). While such defensive expenditures are justified for some markets and firms, most defensive actions demand a more rapid response with lower expenditures on market research. Indeed, defending firms must often routinely determine if any response is even necessary. Little or no practical analytic models exist to support such defensive reactions.

The competitive equilibrium of markets has also been well studied. See for example reviews by Lancaster (1979), Lane (1980), Scherer (1980), and Schmalensee (1980a, 1980b). This literature provides useful insights on how markets reach equilibrium, how market mechanisms lead to entry barriers, and how product differentiation affects market equilibrium. This body of research attempts to describe how markets behave and assess the social welfare implications of such behavior. This research does not attempt to prescribe how an existing firm - faced with a competitive new entrant - should readjust its price, advertising expenditures, channel expenditures and product quality to optimally defend its profit.

We are fortunate to have a breadth of related concepts in marketing and economics to draw upon. However, we will find that the special nature of defensive strategy will require the development of additional new theory. Ultimately, researchers will develop a portfolio of methods to address the full complexity of defensive strategies. We choose a more selective focus by addressing an important subclass of problems that are at the heart of many defensive strategies. Future research can then address broader classes of defensive strategies.

### *Problem Definition*

*Timely response.* We are concerned with firms who wish to begin planning soon after a competitive new brand is launched. This means that we must identify strategies well before the new brand reaches its equilibrium share. Thus we will require not only its current sales but have estimates of the percent of consumers who are aware of it and the (weighted) percent of retailers who carry the product.

*Prior information.* We address how to estimate a new product's position in the market (in a multi-attribute space) from sales, awareness, and availability data, but we assume that the defending firm already knows the positions of existing products prior to the launch of the competitive new product. This is not an unreasonable assumption because such perceptual maps are usually developed for a product launch and should be available from data collected when the (now)defending product was itself a new product. We assume of course that the maps have been updated with our procedure each time a new competitive product was launched. (This assumption can be relaxed if the firm is willing to invest in a positioning study while analyzing defensive strategies.)

*Defensive actions.* Our focus is on price, advertising expenditures (broken down by spending on awareness and on repositioning), distribution expenditures, and product improvement expenditures. We do not address detailed allocation decisions such as the advertising media decision or timing decisions; we assume that once the level of an advertising or distribution budget is set that standard normative marketing techniques are used to allocate within this budget. See Aaker and Myers (1975), Blattberg (1980), Stern and El-Ansary (1977), and Kotler (1980). This paper does not explicitly address the counter-launch of a "me-too" or "second-but-better" new product as a defensive strategy. Our analyses enable the firm to evaluate non-counter-launch strategies. Once such strategies are identified they can be compared to the potential profit stream from a counter-launch strategy.

Finally, we analyze in detail how one firm reacts to the new competitive entry. We do not analyze how other firms react to our defense. Once we fully understand the former problem, future research can use this understanding to address the second problem. Furthermore, there is some evidence in the economics literature that the Nash equilibrium solutions for sequential entry with foresight are consistent with our results. See Lane (1980, tables 1 and 2). However, these specific economic models do not include marketing variables and make simplifying uniformity assumptions that abstract away phenomena of interest to marketing scientists.

### *Research Philosophy*

Defensive strategy is a complex phenomena, and particularly for industrial products

advertising response functions and distribution response functions<sup>1</sup> are rarely known with certainty. While it is possible to hypothesize response functions and competitive counter-response to derive the optimal defensive strategy, such a research philosophy (which is valid for many marketing problems) does not reflect the current state of our understanding of defensive strategies. On the other hand, basic models of consumer response to product positioning and marketing expenditures are well documented in marketing and in economics.

Faced with little previous research on defensive strategies and a rich literature on consumer response we adopt a research philosophy that is common in the physical sciences and economics but less common in marketing.<sup>2</sup> We begin with an accepted model of consumer behavior and mathematically derive (1) methods to estimate the parameters that adapt the model to specific situations and (2) implications from the model that deductively identify qualitative guidelines for defensive strategy. For example, we prove a theorem that states that the optimal defensive strategy requires less money to be spent on distribution incentives unless the defending firm can spend enough to prevent entry of the new product. Such a result is robust with respect to the details of the distribution response function and is thus valuable to the manager who seeks insight into defensive strategies. Quantitative results are also derived but they depend on the accurate measurement of various response functions.

Such theorems naturally require assumptions, some of which abstract the world into a mathematical model that identifies structure within the complexities of reality. We recognize the "insofar as the propositions of mathematics refer to reality they are not certain and insofar as they are certain they do not refer to reality" (Einstein 1922). The strength of mathematical abstraction is that it allows us to seek the dominant forces of defensive strategy. Once these are understood, future research can modify our assumptions, find their limits, relax them, or add second order forces to more fully understand defensive strategies.

As an analogy, consider the assumption made in Newtonian mechanics that the effect of gravitational force on matter is independent of velocity. It is clearly false in the extreme case of special relativity when velocities approach that of light. Even in more mundane situations, other phenomena such as friction, which is velocity

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<sup>1</sup> A distribution response function gives product availability as a function of dollar expenditures for channel incentives. Advertising response functions have related definitions. For example, see Little (1979).

<sup>2</sup> This type of research is less common but not unknown. In 1964, Davis and Farley developed a mathematical theory of salesforce compensation. This theory was tested empirically and in 1980 Srinivasan synthesized the theory and empirical tests into a comprehensive theory. Other examples include Kalish and Lilien (1980) who show that federal subsidies for durable goods should be monotonically decreasing in time, and Blattberg, Buesing, Peacock, and Sen (1978) who theoretically identify the deal prone consumer.

dependent, affects the motion of falling objects. Nonetheless, by studying the simple case, stripped of its complexity, physicists can explain the tides and the motions of planets. Indeed, Einstein's special relativity may not have been derived without first knowing Newton's "laws."

Defensive strategy is not as grand as that of celestial mechanics but mathematical modeling can be used as it is used in the physical sciences. As in the physical sciences, we seek an interpretation of behavior in terms of the structure of a system. "Having studied the properties of a system, we construct in our mind's eye a model of the system... [We] predict the properties of such an ideal system. If many of the predicted properties are in agreement with the observed properties, the model is a good one. If none, or only a few, of the predicted properties are in agreement with the observed properties, the model is poor. This, ideal model of the system may be altered or replaced by a different model until its predictions are satisfactory." [Castellan 1971, p.50]

The remainder of this paper is deductive from a simple model of consumer response. We attempt to make our assumptions clear. Insofar as our model accurately abstracts reality, the results are true. But we caution the manager to examine his situation and compare it to our model before using our results. We expect future research to examine and overcome the limitations of our analysis both empirically and theoretically. We hope that "a very simple model can lead to a rich set of implications." [Sen 1980, p. S18].

We begin with the consumer model.

## 2. CONSUMER MODEL

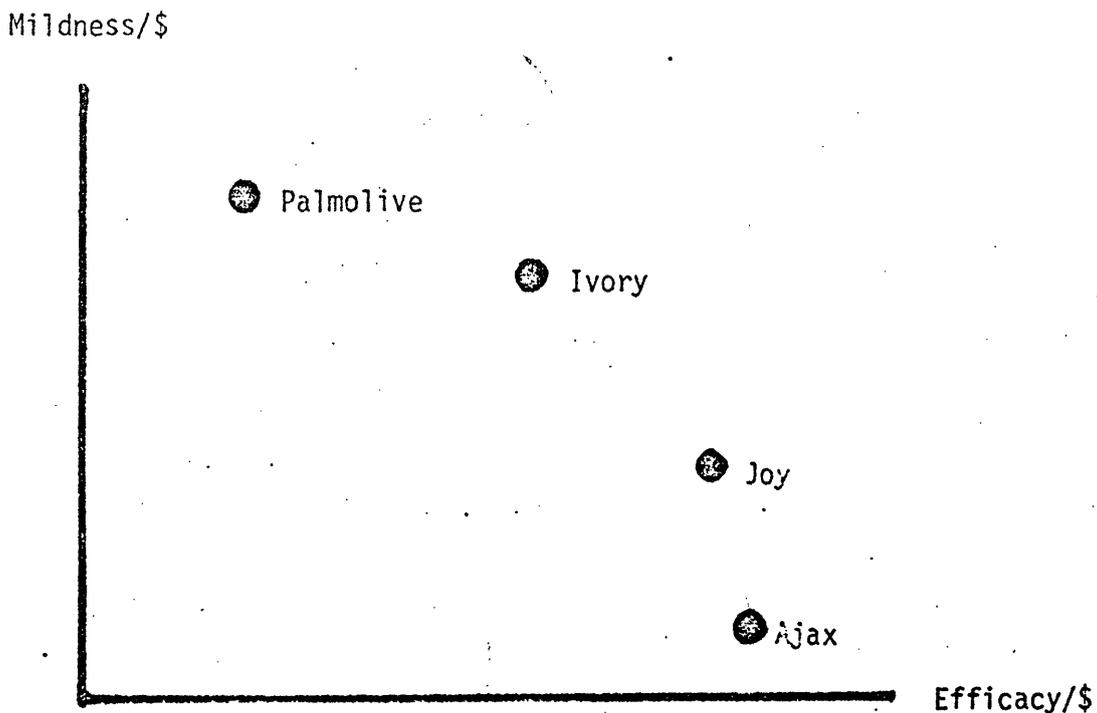
Our managerial interest is at the level of market response. Thus our primary concern in modeling consumers is to predict how many consumers will purchase our product, and our competitors' products, under alternative defensive marketing strategies. However, to promote the evolution of defensive analysis we build up market response as resulting from the response of individual consumers to market forces. Although defensive models may ultimately incorporate complex micro-models such as those reviewed by Bettman (1979), we begin with several simplifying assumptions such as utility maximization.

A further requirement of a defensive model is that it be sensitive to how a new product differentially impacts each existing product. Thus our consumer model can not be based on the axiom of proportional absorption, also referred to as the constant ratio rule of Luce's axiom, that is imbedded in many marketing models.

Finally, we require a model that incorporates important components of consumer behavior, but is not too complex to be inestimable from available data.

*Assumptions*

We assume: (1) Existing products can be represented by their position in a multi-attribute space such as that shown in figure 1. The position of the brand represents the amount of attributes that can be obtained by spending one dollar on that brand. (2) Each consumer chooses the product that maximizes his utility, and (3) The utility of the product category is a concave function of a summary measure linear in the product attributes, (or some linear transformation of the product attributes). This last assumption allows us to represent the brand choice decision with a linear utility function, however, we do not require the actual utility function to be linear. (For a detailed discussion of assumption 3, see Shugan and Hauser, 1981.)



*Figure 1: Hypothetical Perceptual Map of Four Liquid Dish Washing Detergents*

Assumption 1, representation by a product position, is commonly accepted in marketing. See Green and Wind (1975), Johnson (1970), Kotler (1980), Pessemier (1977), and Urban (1975). Scaling by price comes from a budget constraint applied to the bundle of consumer purchases and from separability conditions that allow us to model behavior in one market (say liquid dish washing detergents) as independent of another market (say deodorants). This implicit assumption is discussed in Hauser and Simmie (1981), Horsky, Sen and Shugan (1981), Keon (1980), Lancaster (1971, chapter 8), Ratchford (1975), Shugan and Hauser (1981) and Srinivasan (1980a).

Assumption 2, utility maximization, is reasonable for a market level model. At the individual level stochasticity (Bass 1974, Massy, Montgomery and Morrison 1970), situational variation, and measurement error make it nearly impossible to predict behavior with certainty. At the market level, utility maximization by a group of heterogeneous consumers appears to describe and predict sufficiently well to identify dominant market forces. For example, see Givon and Horsky (1978), Green and Rao (1972), Green and Srinivasan (1976), Jain, et.al. (1979), Pekelman and Sen (1979), Shocker and Srinivasan (1979), Wind and Spitz (1976), and Wittink and Montgomery (1979). Utility maximization is particularly reasonable when we define utility on a perceptual space because perceptions are influenced by both product characteristics and psycho-social cues such as advertising image, and hence already incorporate some of the effects due to information processing.

Assumption 3, linearity is a simplification to obtain analytic results. With linearity we sacrifice generality but obtain a manageable model of market behavior. In many cases the linearity assumption can be viewed as an approximation to the tangent of each consumer's utility function in the neighborhood of his chosen product. Furthermore, Green and Devita (1975) show that linear preference functions are good approximations, Einhorn (1970) and Einhorn and Kleinmuntz (1979) present evidence based on human information processing that justifies a linear approximation. In our theorems we treat the utility function as linear in the product attributes, however it is a simple matter to modify these theorems for any utility function that is linear in its 'taste' parameters. Thus, our analysis can incorporate much of the same class of non-linear utility functions discussed in McFadden (1973).

We make a final assumption to simplify exposition. We limit ourselves to two product attributes to provide pictorial representations of the attribute space.

*Notation*

By assumption, products are represented by their attributes. Let  $x_{1j}$  be the amount of attribute 1 obtainable from one unit of brand  $j$ . Define  $x_{2j}$  to be the amount of attribute 2 obtainable from one unit of brand  $j$ . (Note:  $\infty > x_{ij} > 0$  for  $i = 1, 2$ .) Let  $p_j$  be the current price of brand  $j$ . Let  $\tilde{u}_j$  be the utility that a randomly selected consumer places on purchasing brand  $j$ . Note that  $\tilde{u}_j$  is a random variable due to consumer heterogeneity. If every consumer is aware of each brand and finds it available, all brands will be in everyone's choice set, otherwise choice sets will vary. Let  $A$  be the set of all brands, let  $A_\ell$ , a subset of  $A$ , be the set of brands called choice set  $\ell$ , and let  $S_\ell$  be the probability that a randomly chosen consumer will select from choice set  $\ell$  for  $\ell = 1, 2, \dots, L$ . See Silk and Urban (1978) for empirical evidence on the variation of choice sets.

*Mathematical Derivation*

We first compute the probability,  $m_j$ , that a randomly chosen consumer purchases brand  $j$ . In marketing terms,  $m_j$  is the market share of product  $j$ . Applying assumption 2, we obtain equation (1).

$$m_j = \text{Prob} [\tilde{u}_j > \tilde{u}_i \text{ for all } i] \quad (1)$$

where  $\text{Prob} [\cdot]$  is a probability function

In addition, define  $m_{j|\ell} = \text{Prob} [\tilde{u}_j > \tilde{u}_i \text{ for all } i \text{ in } A_\ell]$ . In marketing terminology,

$m_{j|\ell}$  is the market share of product  $j$  among those customers

who evoke  $A_\ell$ . Now, equation (1) and assumption 3 imply equation (2).

Now, equation (1) and assumption 3 imply equation (2).

$$m_{j|\ell} = \text{Prob} [\tilde{w}_1 x_{1j} + \tilde{w}_2 x_{2j} / p_j > (\tilde{w}_1 x_{1i} + \tilde{w}_2 x_{2i}) / p_i \text{ for all } i \in A_\ell] \quad (2)$$

where  $i \in A_\ell$  denotes all products contained in choice set  $\ell$

Here  $w_1$  and  $w_2$  are relative "weights" a randomly selected consumer places on attributes 1 and 2 respectively. Suitable algebraic manipulation of equation (2) yields equation (3).

$$m_{j|\ell} = \text{Prob} [(x_{1j}/p_j - x_{1i}/p_i) > (\tilde{w}_2/\tilde{w}_1) (x_{2i}/p_i - x_{2j}/p_j) \text{ for all } i \in A_\ell] \quad (3)$$

Moreover, equation (3) is equivalent to equation (4).

$$m_{j|\ell} = \text{Prob} [\tilde{w}_2/\tilde{w}_1 > r_{ij} \text{ for all } i \in A_\ell^+] \cdot \text{Prob} [\tilde{w}_2/\tilde{w}_1 < r_{ij} \text{ for all } i \in A_\ell^{''}] \quad (4)$$

$$\text{where } r_{ij} = (x_{1j}/p_j - x_{1i}/p_i) / (x_{2i}/p_i - x_{2j}/p_j)$$

$$A_\ell^+ = \{i | i \in A_\ell \text{ and } x_{2i}/p_i > x_{2j}/p_j\}$$

$$A_\ell^{''} = \{i | i \in A_\ell \text{ and } x_{2i}/p_i < x_{2j}/p_j\}$$

Note that equation (4) illustrates that  $\tilde{w}_2/\tilde{w}_1$  is a sufficient statistic for computing choice probabilities and that  $r_{ij} = r_{ji}$ . Now, consider figure 2. Basically, 'Joy' will be chosen if the angle of the indifference curve defined by  $(w_2, w_1)$  lies between the angle of the line connecting 'Ivory' and 'Joy' and the line connecting 'Joy' and 'Ajax'. Note that consumers will only choose those brands, called *efficient brands*, that are not dominated on both dimensions by another product in the evoked set.

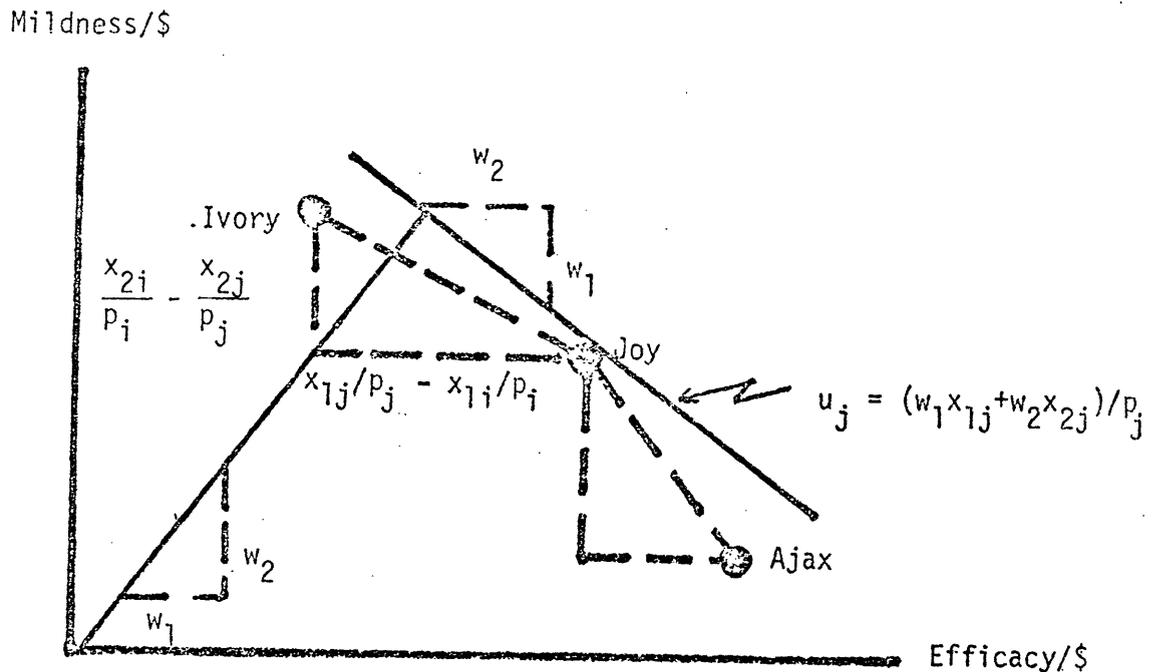


Figure 2. Geometric Illustration of the Relationships in Equation 3 for a Three Product Market

We simplify equation (4) by defining  $\tilde{\alpha} = \tan^{-1} (\tilde{w}_2/\tilde{w}_1)$  where  $\tilde{\alpha}$  is a random variable (derived from  $\tilde{w}_1$  and  $\tilde{w}_2$ ) and represents a measure of consumer preference. Moreover, we introduce the convention of numbering brands such that  $x_{2j}/p_j > x_{2i}/p_j$  if  $j > i$ . This will assure that numbers increase counter-clockwise for efficient brands in A. For the extreme points let  $r_{ij} = 0$  and let  $r_{j\infty} = \infty$ . We can now simplify equation (4) and obtain equation (5).

$$m_{j|k} = \text{Prob} \left[ \underset{\text{over } k}{\text{minimum}} \{r_{kj}\} > \tan \alpha > \underset{\text{over } h}{\text{maximum}} \{r_{h,j}\} \text{ for all } k > j > h \text{ in } A_j \right] \quad (5)$$

We now introduce consumer heterogeneity in the form of a variation in tastes across the consumer population. Since each consumer's utility function is defined by the angle,  $\tilde{\alpha}$ , we introduce a distribution,  $f(\alpha)$ , on the angles. One hypothetical distribution is shown in figure 3. As shown, small angles,  $\alpha \rightarrow 0$ , imply that consumers are more concerned with attribute 1, e.g. efficacy, than with attribute 2, e.g. mildness; large angles,  $\alpha \rightarrow 90^\circ$ , imply a greater emphasis on attribute 2.

Finally, we introduce the definition of *adjacency*. In words, a lower (upper) adjacent brand is simply the next efficient brand in  $A_\ell$  as we proceed clockwise (counter-clockwise) around the boundary. Mathematically, a brand,  $j-$ , is lower adjacent to brand  $j$  if (1)  $j-<j$ , (2)  $j->i$  for all  $i<j$  where both  $i$  and  $j$  are contained in  $A_\ell$ , and (3) both brands are efficient. Define the upper adjacent brand,  $j+$ , similarly. Note that  $j-$  and  $j+$  depend on the choice set  $A_\ell$ . With these definitions  $m_{j|\ell}$  is simply computed by equation (6)

$$m_{j|\ell} = \int_{\alpha_{jj-}}^{\alpha_{jj+}} f(\alpha) d\alpha \quad (6)$$

Here  $\alpha_{ij} = \tan^{-1} r_{ij}$ . Thus,  $m_{j|\ell}$  is the shaded area in figure 3. Finally, if the choice sets vary, the total market share,  $m_j$ , of brand  $j$  is given by equation (7).

$$m_j = \sum_{\ell=1}^L m_{j|\ell} S_\ell \quad (7)$$

An interesting property of equation 7 is that a brand can be inefficient on  $A$ , but have non-zero total market share if it is efficient on some subset with non-zero selection probability,  $S_\ell$ .

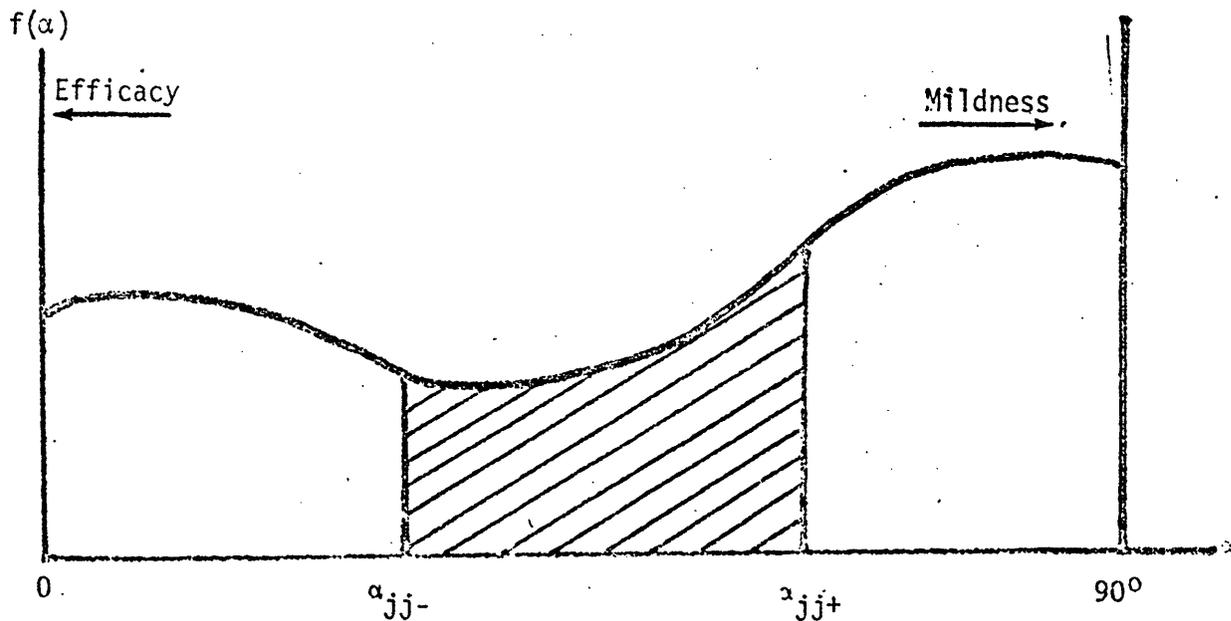


Figure 3: Hypothetical distribution of Tastes with Respect to Efficacy and Mildness. The shaded region represents those consumers who will choose 'Joy'.

As an illustration for  $A = \{\text{Ajax } (j=1), \text{Joy } (j=3), \text{Ivory } (j=4)\}$  suppose that the positions of the three brands are given by  $(x_{1j}/p_j, x_{2j}/p_j) = (5,1), (2,4.6),$  and  $(1,5)$  for  $j = 1,3,4$ , respectively. We first compute  $r_{ij}$ , finding  $r_{13} = 3/3.6 = .83$  and  $r_{34} = 1/(.4) = 2.5$ . Finding the angle whose tangent is  $r_{ij}$ , gives us  $\alpha_{01} = 0^\circ$ ,  $\alpha_{13} = 40^\circ$ ,  $\alpha_{34} = 68^\circ$ , and  $\alpha_{4\infty} = 90^\circ$ . For this example 'Ajax' is lower adjacent to 'Joy' and 'Ivory' is upper adjacent to 'Joy'. See figure 4.

If tastes are uniformly distributed over  $\tilde{\alpha}$ , then  $f(\alpha) = (1/90^\circ) d\alpha$  and  $m_{j|k} = (\alpha_{jj+} - \alpha_{jj-})/90^\circ$ . For our example if everyone evokes choice set A, market shares are .44 for 'Ajax', .31 for 'Joy', and .25 for 'Ivory'.

Suppose a new brand, 'Attack', is introduced at  $(3,4)$  and suppose it is in everyone's choice set. Since 'Attack' is positioned between 'Ajax' and 'Joy' we number it  $j = 2$ . See figure 4. We compute  $r_{12} = 0.67$ ,  $r_{23} = 1.67$ ,  $\alpha_{12} = 34^\circ$ , and  $\alpha_{23} = 59^\circ$ . The new market shares are .38 for 'Ajax', .27 for 'Attack', .08 for 'Joy', and .26 for 'Ivory'. Thus 'Attack' draws its share dominantly from 'Joy', somewhat from 'Ajax', and not at all from 'Ivory'. 'Joy' will definitely need a good defensive strategy!

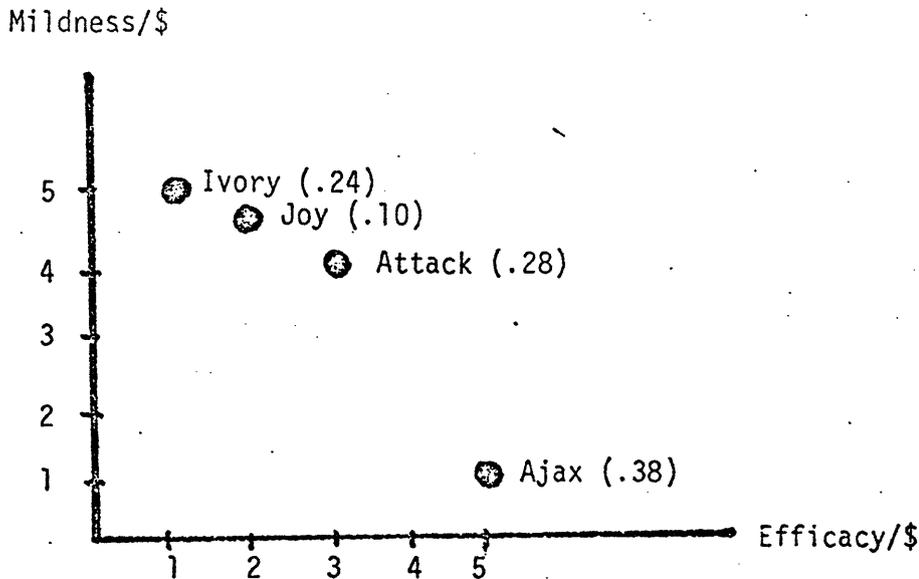


Figure 4. Hypothetical Market After the Entry of the New Product 'Attack' (Numbers in Parentheses Indicate New Market Shares.)

Clearly the threat posed by the new brand depends upon how it is positioned with respect to the defending firm's brand. We formalize this notion in section 6. However, to use the consumer model for forecasting consumer response we must be able to identify the taste distribution,  $f(\alpha)$ . Section 4 derives estimation procedures for  $f(\alpha)$ . For specific defensive tactics we must be able to identify the position of the new brand. Section 5 derives a Bayesian estimation procedure that enables us to update a defensive manager's prior beliefs about the new competitive brand's position and provides techniques to estimate the impact of the new brand on evoked sets. The updating procedure requires only sales data which is corrected for awareness and availability.

We turn now to the analysis of general strategies for defensive pricing, advertising, distribution incentives, and product improvement. The theorems in the following section are based on the consumer model derived above. For analytic simplicity, unless otherwise noted, we base our derivations on the full product set, A. Extension to variation in evoked sets is discussed in section 5.

### 3. STRATEGIC IMPLICATIONS

We examine in turn pricing, distribution, advertising, and product improvement.

#### *Pricing Strategy*

For simplicity we assume that advertising and distribution strategies are fixed. We relax this assumption in the following sections.

Based on the consumer model, equation (8) shows profits before entry of the new brand. We simplify the exposition by suppressing fixed costs which, in no way, affect our analysis.

$$\Pi_b(p) = (p - c_b)N_b \int_{\alpha_{jj-}}^{\alpha_{jj+}} f(\alpha) d\alpha \quad (8)$$

Here  $N_b$  is the number of consumers,  $c_b$  is our per unit costs before any competitive entry,  $p$  is our price (We are brand  $j$ .) and  $\Pi_b(p)$  is our before entry profit given price  $p$ . Note  $\alpha_{jj+}$ ,  $\alpha_{jj-}$ , and  $c_b$  are functions of price.

The new brand can enter the market in a number of ways. (1) If it is inefficient, it does not affect our profits. (2) If it exactly matches one of our competitors, our profit is unchanged. (3) If the new brand is not adjacent in the new market, our profits are unchanged. Conditions (1) - (3) are based on the recognition that none of our consumers are lost to the new brand.

Defensive strategy only becomes important under condition (4)  $j_+ > n > j_-$ . Since the problem is symmetric with respect to  $j_+ > n > j_-$  and  $j_- > n > j_+$ , we analyze the former case where the new product becomes efficient and upper adjacent.

In case (4), our profits after the launch of the competitive new brand, with price  $p$ , are given by equation (9).

$$\Pi_a(p) = (p - c_a) N_a \int_{\alpha_{jj-}}^{\alpha_{jn}} f(\alpha) d\alpha \quad (9)$$

Here,  $N_a$  and  $c_a$  are the market volume and our per unit costs after the competitive entry, respectively. Note that our per unit cost may change because of a change in sales volume.

At this point we introduce the function  $Z(p)$ , given by equation (10), which represents lost potential.

$$Z(p) = (p - c_a) N_a \int_{\alpha_{jn}}^{\alpha_{jj+}} f(\alpha) d\alpha \quad (10)$$

Note that  $c_a$  is a function of  $p$  because  $c_a$  may vary with sales volume. Moreover, if  $p - c_a$  is positive,  $Z(p)$  is non-negative for all  $f(\alpha)$  and positive for any  $f(\alpha)$  that is not identically zero in the range  $\alpha_{jn}$  to  $\alpha_{jj+}$ . If we lose no customers, the new brand is no threat, thus we are only concerned with the case of  $f(\alpha)$  not identically zero in the range  $\alpha_{jn}$  to  $\alpha_{jj+}$ . Call this case for an adjacent product, *competitive entry*.

Our first result formalizes the intuitive feeling that we can not be better off after the new product attacks our market.

*Theorem 1: If total market size does not increase, optimal defensive profits must decrease if the new product is competitive, regardless of the defensive price.*

*Proof:* Let  $p^0$  be our optimal price before entry and let  $p^*$  be our optimal price after entry.

$$\text{Let } M_a(p) = \int_{\alpha_{jj-}}^{\alpha_{jn}} f(\alpha) d\alpha \text{ and } M_b(p) = \int_{\alpha_{jj-}}^{\alpha_{jj+}} f(\alpha) d\alpha. \text{ Now } \alpha_{jj+} > \alpha_{jn}$$

because the new entry is competitive. Hence  $M_b(p^*) > M_a(p^*)$ . It is easy to show that  $M_b(p)$  is a decreasing function of price for a competitive entry. See Lemma 3 in the appendix. By assumption  $N_b \geq N_a$ , hence  $N_b M_b(p^*) > N_a M_a(p^*)$ . Since  $M_b(p^*)$  is decreasing in price, there exists a  $p^1 > p^*$  such that  $N_b M_b(p^1) = N_a M_a(p^*)$ .

Since cost is a function of volume  $c_b(p^1) = c_a(p^*)$ . But since  $p^1 > p^*$ ,  $(p^1 - c_b) N_b M_b(p^1) > (p^* - c_a) N_a M_a(p^*)$ , hence  $\Pi_b(p^1) > \Pi_a(p^*)$ . By the definition of optimal,  $\Pi_b(p^0) > \Pi_b(p^1)$ , hence  $\Pi_b(p^0) > \Pi_a(p^*)$ .

Theorem 1 illustrates the limits of defensive strategy. Unless the market is increasing at a rapid rate, the competitive entry will lower our profit even with the best defensive price. Even if the market is growing, as long as our growth is not due to the new brand, it is easy to show that the competitor decreases potential profits.

A key assumption in theorem 1 is that the defending firm is acting optimally before the new brand enters. However, there exist cases where a new brand awakens a "sleepy" market, the defending firm responds with an active defensive (heavy advertising and lower price), and finds itself with more sales and greater profits. A well known case of this phenomenon is the reaction by Tylenol to a competitive threat by Datril. Before Datril entered the market actively, Tylenol was a little known, highly priced alternative to aspirin. Tylenol is now the market leader in analgesics. Theorem 1 implies that Tylenol could have done at least as well had it moved optimally before Datril entered.

Theorem 1 states that no pricing strategy can regain the before-entry profit. Nonetheless, optimal defensive pricing is important. Defensive profits with an optimal defensive price may still be significantly greater than defensive profits with the wrong defensive price. Intuitively, consumer package goods managers expect a strong defense to require a price reduction. Theorem 2 shows that a price reduction is not always optimal.

*Theorem 2: There exist distributions of consumer tastes for which the optimal defensive price requires a price increase.*

*Proof (by counterexample):* Suppose  $f(\alpha)$  is discrete taking on values only at  $f(14^0) = 1/27$ ,  $f(18.5^0) = 15/27$ ,  $f(33.7^0) = 10/27$ , and  $f(84^0) = 1/27$ . Suppose we are positioned at  $(x_{1j}/p, x_{2j}/p) = (11/p, 50/p)$  and our two adjacent competitors are positioned at (1,60) and (21, 20). If  $c_b = \$.9/\text{unit}$  and  $N_b = 270,000$  our profit at various price levels is given in table 1. By inspection, the optimal price is the highest price, \$1.00, that captures both segments 2 and 3. Suppose now that the new brand enters at (16,40) and that  $c_a = c_b$  and  $N_a = N_b$ . By inspection of table 1, the optimal price is the highest price, \$1.03, that captures segment 3. Since the optimal price after entry exceeds that before entry, we have generated an example and proved existence.

TABLE 1: EXAMPLE TO DEMONSTRATE A CASE OF THE OPTIMAL DEFENSIVE PRICE REQUIRING A PRICE INCREASE (<sup>o</sup> indicates optimal price before competitive entry, \* indicates optimal price after entry).

PRICE	BEFORE ENTRY		AFTER ENTRY	
	VOLUME	PROFIT	VOLUME	PROFIT
under .90		< 0		< 0
.90	260,000	0	260,000	0
.91	250,000	2,500	250,000	2,500
.92	250,000	5,000	250,000	5,000
.93	250,000	7,500	250,000	7,500
.94	250,000	10,000	100,000	4,000
.95	250,000	12,500	100,000	5,000
.96	250,000	15,000	100,000	6,000
.97	250,000	17,500	100,000	7,000
.98	250,000	20,000	100,000	8,000
.99	250,000	22,000	100,000	9,000
1.00 <sup>o</sup>	250,000 <sup>o</sup>	25,000 <sup>o</sup>	100,000	10,000
1.01	100,000	11,000	100,000	11,000
1.02	100,000	12,000	100,000	12,000
1.03*	100,000	13,000	100,000*	13,000*
1.04	100,000	14,000	0	0
1.05	100,000	15,000	0	0
1.06	100,000	16,000	0	0
1.07	100,000	17,000	0	0
1.08	100,000	18,000	0	0
1.09	0	0	0	0
over 1.09	0	0	0	0

With careful inspection, the example used to illustrate theorem 2 is quite intuitive. The key idea in the example is that there are two dominant segments in the market. Thirty-seven percent (10/27) have a slight preference for attribute 1,  $w_2/w_1 = \tan\alpha = 2/3$ ; fifty-five percent (15/27) have a stronger preference for attribute 1,  $w_2/w_1 = 1/3$ . Before the competitive entry, we were selling to both markets. Segment 3 clearly preferred our brand. However, we were forced to lower our price to compete for segment 2. It was profitable to do so because of its large size. When the new brand entered, targeted

directly at segment 2 we could no longer compete profitably for segment 2. However, since we are still well positioned for segment 3 and they are willing to pay more for our brand, we raise our price to its new optimal profit level. In the social welfare sense, consumers must pay a price for variety. (In table 1, the price increase is only 3¢, but it is possible to generate examples where the price increase is very large.)

Theorem 2 clearly illustrates that in the case of a highly segmented market, it may be optimal to raise our price after the competitor enters. But not all markets are so extreme. The following theorem illustrates that a price decrease is optimal if taste segmentation is less extreme. In particular, we investigate the market forces present when preferences are uniformly distributed. This line of reasoning is not unlike that of Lancaster (1980) who assumes a different form of uniformity to study competition and product variety.<sup>3</sup> As Lancaster (1980, p. 583) states: Uniformity provides "a background of regularity against which variations in other features of the system can be studied."

We introduce an empirically testable market condition which we call *regularity*. Let  $\theta_j$  be the angle of a ray connecting product  $j$  to the origin,  $\theta_j = \tan^{-1}(x_{2j}/x_{1j})$ . Then a market is said to be regular if product  $j$ 's angle,  $\theta_j$ , lies between the angles,  $\alpha_{jj-}$  and  $\alpha_{jj+}$ , which define the limits of the tastes of product  $j$ 's consumers. Regularity is a reasonable condition which we expect many markets to satisfy, however, as intuitive as regularity seems it is not guaranteed. We present the following theorem for regular markets and then discuss its extension to irregular markets.

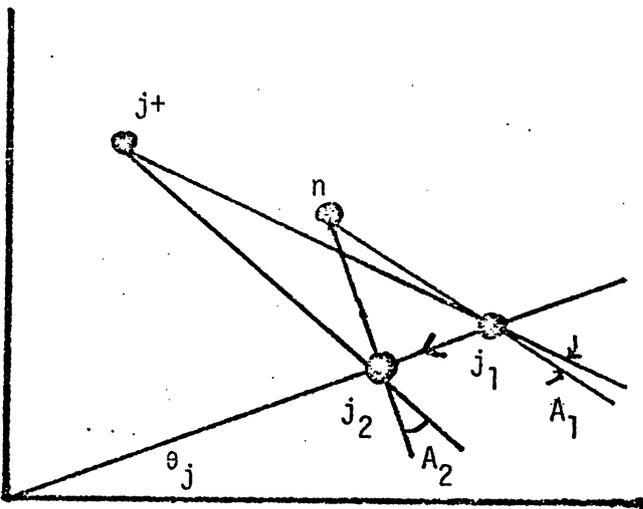
*Theorem 3, Defensive Pricing: If consumer tastes are uniformly distributed and if there are constant returns to scale, then the optimal defensive price strategy in a regular market is to decrease price.*

*Proof:* By assumption,  $c_a = c_b = c$ . Thus  $\pi_b(p) = [\pi_a(p) + Z(p)] (N_b/N_a)$  for all  $p$ . Since  $N_b/N_a$  is assumed independent of  $p$ ,  $\pi'_b(p) = [\pi'_a(p) + Z'(p)] (N_b/N_a)$ , where the prime (') denotes the partial derivative with respect to  $p$ . Since  $p^0$  is the before entry optimal price,  $\pi'_b(p^0) = 0$  and hence  $\pi'_a(p^0) = -Z'(p^0)$ . Thus  $\pi_a(p^0)$  is decreasing in  $p$  at  $p^0$  if  $Z(p^0)$  is increasing in  $p$ . Thus to show that a price decrease increases profit we must show that  $Z'(p^0) > 0$ . We show that this leads to a global maximum by showing that  $\pi_a(p)$  is unimodal under the conditions of the theorem.

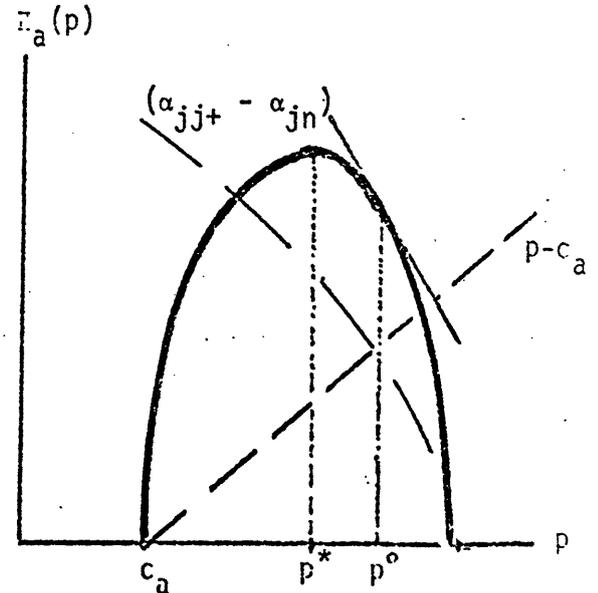
<sup>3</sup> Lancaster's uniformity assumption is more complex than an assumption of uniform tastes. However, to derive his results he also assumes a form of uniform tastes. See also analyses by Lane (1980).

(1) *Proof of  $Z'(p^0) > 0$ .* Since  $f(\alpha)$  is uniform,  $Z(p)$  is given by  $Z(p) = (p-c_a)(N_a/90) (\alpha_{jj+} - \alpha_{jn})$ . Taking derivatives with the product rule gives us  $Z'(p) = K(p-c_a)' (\alpha_{jj+} - \alpha_{jn}) + K (\alpha_{jj+} - \alpha_{jn})$  where  $K = N_a/90$  is a positive constant. Since  $\alpha_{jj+} - \alpha_{jn}$  is positive when  $n$  and  $j+$  are efficient and  $(p^0-c)$  is positive (or else  $p^0$  would not be optimal),  $Z'(p^0)$  is positive if  $(\alpha_{jj+}' - \alpha_{jn}')$  is non-negative at  $p^0$ . The proof of this fact is complex and is given in the appendix as Lemma 1. However, this can be seen intuitively in figure 5a. As we have drawn it, the angle  $A = (\alpha_{jj+} - \alpha_{jn})$  increases as we increase the price. (A price increase moves the point  $j$  toward the origin along the ray,  $\theta_j = \tan^{-1}(x_{2j}/x_{1j})$ ) As Lemma 1 shows, the condition  $\alpha_{jj+} > \alpha_{jn}$  is sufficient for  $A' \geq 0$ .  $\alpha_{jj+} > \alpha_{jn}$  is true in a regular market.

attribute 2/\$



a) First order conditions



b) Second order conditions

Figure 5: *Intuitive Visualization of the Proof of Defensive Pricing for Uniform Tastes: a)  $A = \alpha_{jj+} - \alpha_{jn}$  increases as price increases, b)  $\pi_a(p)$  is unimodal.*

(2) *Proof of  $\Pi_a(p)$  unimodal:* If  $\Pi_a''(p)$  is negative,  $\Pi_a(p)$  is unimodal, where the double prime (") denotes the second partial derivative. Using the product rule  $\Pi_a''(p) = K [2(\alpha_{jn}' - \alpha_{jj-}') + (p-c)(\alpha_{jn}'' - \alpha_{jj-}'')]$ . The first term is proportional to the first derivative of sales. As shown formally in Lemma 2 in the appendix, sales are indeed decreasing in price. Since  $(p-c_a)$  is non-negative for all  $p \geq c$ , the result follows if  $\alpha_{jn}'' - \alpha_{jj-}''$  is non-positive. [ $p_0 > c$  and  $\Pi_a(p) \leq 0$  for  $p < c$ , therefore we need only examine  $p \geq c_a$ .] The result that  $\alpha_{jn}'' - \alpha_{jj-}'' \leq 0$  for  $\theta_n \geq \alpha_{nj}$  and  $\theta_j \leq \alpha_{jj-}$ , which are true in regular markets is straightforward but algebraically tedious. See Lemma 4. Since conditions (1) and (2) are true, the defensive pricing theorem follows.

Theorem 3 provides us with useful insight on defensive pricing; insight that with experience can evolve into very usable "rules of thumb". For example, when some managers find the market share of their brand decreasing they quickly increase the price in the hopes of increasing lost revenue. Historically, transit managers fall into this class. (Witness the recent fare increases in both Boston and Chicago.) Other managers believe an aggressive price decrease is necessary to regain lost share. This strategy is common in package goods. Theorem 3 shows that if consumer tastes are uniform, the price decrease is likely to be optimal in terms of profit. The result itself may not surprise the aggressive package goods managers, but the general applicability of the result to regular markets is interesting.

*Irregular Markets.* Regularity is sufficient to prove theorem 3, however regularity is not necessary. For example, we only used the condition,  $\alpha_{jj+} \geq \theta_j$ , to prove that profits are decreasing in price at  $p^0$ . We only used the conditions,  $\theta_n \geq \alpha_{nj}$  and  $\theta_j \geq \alpha_{jj-}$ , to prove global optimality. Even these conditions were not necessary.<sup>4</sup> Thus it is possible, indeed probable, that a price decrease will also be optimal in irregular markets. We have not been able to develop a general proof, but in 1,700,000 randomly generated irregular markets we have found that profits are decreasing in price in all tested markets.

*Non-uniform Taste Distributions.* Even with uniformly distributed tastes the proof of theorem 3 is complex. But like regularity, uniformly distributed tastes is sufficient but not necessary. We leave it to future researchers to

<sup>4</sup> E.G.,  $Z'(p^0)$  contains two terms, the first is always positive; only the second negative when  $\alpha_{jj+} < \theta_j$ . Similarly only one term of  $\Pi_a''(p)$  changes sign when the conditions are violated and  $\Pi_a''(p) < 0$  is sufficient, not necessary, for  $\Pi_a(p)$  to be unimodal. Even these conditions can be relaxed.

identify necessary and sufficient conditions for a price decrease, but we suspect it is true for some general class of unimodal distributions.

*Summary.* Together theorems 2 and 3 provide the manager with guidelines to consider in making defensive pricing decisions. Our proven results are very specific, but we can generalize with the following propositions which are stated in the form of managerial guidelines:

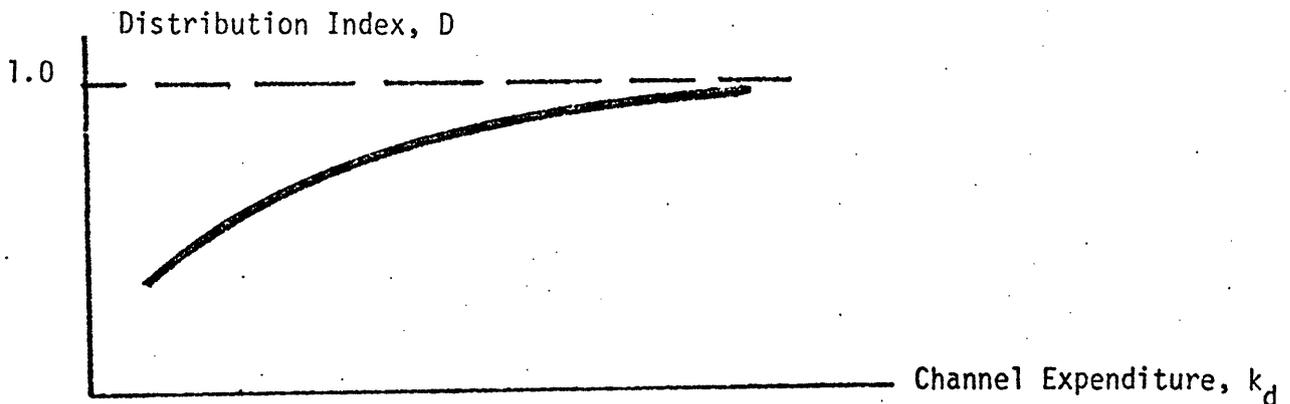
- If the market is highly segmented and the competitor is attacking one of your segments, examine the situation carefully for a price *increase* may be optimal.
- If the market is not segmented and consumer tastes are near uniform, a price *decrease* usually leads to optimal profits.

Finally, it is possible to show that if a competitive brand leaves the market and consumer tastes are near uniform, a price increase usually leads to optimal profits.

*Distribution Strategy*

One strategy to combat a new entrant is for the defending firm to exercise its power in the channel of distribution. Channel power is a complex phenomena, but to understand defensive distribution strategy we can begin by summarizing the phenomena with response functions. See Little (1979) for a discussion of the power and limitations of response analysis.

In response analysis we assume that sales are proportional to a distribution index,  $D$ . The index is in turn a function of the effort in dollars,  $k_d$ , that we allocate to distribution. In other words,  $k_d$  summarizes spending on inventory, transportation, channel incentives, salesforce, etc.  $D(k_d)$  is the result of spending  $k_d$  dollars optimally in the channel. Because we are dealing with distribution it is useful to think of  $D$  as a form of brand availability such that sales under perfect distribution is given by the sales that would be achieved if the product were available in all retail outlets. In this case,  $D(k_d \rightarrow \infty) = 1$  and  $0 \leq D(k_d) \leq 1$  for any finite expenditure,  $k_d$ . Finally, we assume that over the relevant range of analysis,  $D$  is a marginally decreasing function of channel support. An example response function is given in figure 6.



*Figure 6: An Example Distribution Response Function Exhibiting Decreasing Marginal Returns Over the Relevant Range*

Under the conditions of response analysis, profit after competitive entry is given by:

$$\pi_a(p, k_d) = (p - c_a) N_a M_a(p) D - k_d$$

where

$$M_a(p) = \int_{\alpha_{jj-}}^{\alpha_{nj}} f(\alpha) d\alpha \tag{11}$$

That is,  $M_a(p)$  is the potential market share of product  $j$  as a function of price. We begin by examining the interrelationship between defensive pricing and distribution strategies.

*Theorem 4, Price Decoupling: Based on the assumption of response analysis, the optimal defensive pricing strategy is independent of the optimal defensive distribution strategy, but the optimal distribution strategy depends on the optimal defensive price.*

*Proof:* Differentiating  $\pi_a(p, k_d)$  with respect to  $p$  and setting the derivative to zero yields  $[M(p) + (p-c_a)dM_a(p)/dp] N_a D = 0$ . Since  $N_a, D > 0$  for the optimal price, the solution,  $p^*$ , of the first order conditions is independent of  $k_d$ .

Differentiating  $\pi_a(p, k_d)$  with respect to  $k_d$  and setting the derivative to zero yields  $(p-c_a)N_a M_a(p)(dD/dk_d) = 1$ . The solution to this equation,  $k_d^*$ , is clearly a function of  $p$ .

*Theorem 5, Defensive Distribution Strategy: Based on the assumption of response analysis, the optimal defensive distribution strategy is to decrease spending on distribution.*

*Proof:* Differentiating  $\pi_a(p, k_d)$  with respect to  $k_d$  and recognizing that  $\pi_a(p) = (p-c_a)N_a M_a(p)$  as given by equations 8 and 10 yields the first order condition for the optimal  $k_d^*$  as  $\pi_a(p^*) [dD(k_d^*)/dk_d] = 1$ . Similarly we can show that the first order condition for the optimal before entry spending,  $k_d^0$ , is

$\pi_b(p^0)[dD(k_d^0)/dk_d] = 1$ . Since  $\pi_b(p^0) > \pi_a(p^*)$  by Theorem 1, we have  $dD(k_d^*)/dk_d > dD(k_d^0)/dk_d$ . But  $dD(k_d)/dk_d$  is decreasing in  $k_d$ , hence  $k_d^* < k_d^0$ .

*Theorem 6, Pre-emptive Distribution: If market volume does not increase, optimal defensive profits must decrease if a new brand enters competitively, regardless of the defensive price and distribution strategy. However, profit might be maintained, under certain conditions, if the new brand is prevented from entering the market.*

*Proof:* Define  $\Pi_b(p, k_d)$  analogously to equation 11. Recognize that  $\Pi_a(p, k_d) = \Pi_a(p)D(k_d) - k_d$  where  $\Pi_a(p)$  is defined by equation 9. Then  $\Pi_b(p^0, k_d^0) \geq \Pi_b(p^0, k_d^*) = \Pi_b(p^0)D(k_d^*) - k_d^* > \Pi_a(p^*)D(k_d^*) - k_d^* = \Pi_a(p^*, k_d^*)$ . The first step is by the definition of optimality; the second step is by expansion; the third step is by Theorem 1, and the fourth step is by definition. The last statement of the theorem follows from the recognition that it is potentially possible to construct situations where the cost of preventing entry is smaller than  $\Pi_b(p^0, k_d^0) - \Pi_a(p^*, k_d^*)$ .

Together theorems 4, 5, and 6 provide the defensive manager with valuable insights on how much of his budget to allocate to distribution. If he can prevent competitive entry by dominating the channel *and it is legal to do so*, this may be his best strategy. However, in cases when it is illegal to prevent entry, the optimal profit strategy is to *decrease* spending on distribution.

Although decreasing distribution expenditures appears counterintuitive for an active defense, it does make good economic sense. The market's potential profitability and rate of return decreases as the new product brings additional competition. Because the market is less profitable, the mathematics tells us that we should spend less of our resources in this market. Instead, we should divert our resources to more profitable ventures. If demand shrinks fast enough, the reward for investment in the mature product decreases until new product development becomes a more attractive alternative.

Just the opposite would be true if a competitor dropped out of the market; we would fight to get a share of its customers. Witness the active competition between the Chicago Tribune and Chicago Sun Times when the Chicago Daily News folded and the active competition by the Boston newspapers when the Record-American folded.

Theorem 4 tells us that price strategy is independent of distribution strategy but not vice versa. Thus the results of theorems 2 and 3 still hold. If preferences are uniformly distributed the defensive manager should decrease price and decrease distribution spending. In addition, these results imply that a defensive manager should first set price before other defensive variables.

Finally, we note that the tactics to implement a distribution strategy include details not addressed by response analysis. Nonetheless, although the details may vary, the basic strategy, as summarized by total spending, is to decrease the distribution budget when defending against a new competitive brand.

*Advertising Strategy*

There are two components to advertising strategy. One goal of advertising is to entice consumers into introducing our brand into their evoked sets. We will refer to this form of advertising, as *awareness advertising*, and handle it with response analysis much like we handled distribution. (Sales are proportional to the number of consumers aware of our product.) Another goal of advertising is to reposition our brand. For example, if we are under attack by a new liquid dishwashing detergent stressing mildness, we may wish to advertise to increase the perceived mildness (or efficacy) of our brand. In this form of advertising we invest advertising dollars to increase  $x_{1j}$  or  $x_{2j}$ . Since repositioning advertising affects  $\alpha_{nj}$  and  $\alpha_{jj}$ , it is more complex. In the following analysis we consider expenditures on awareness advertising,  $k_a$ , and repositioning,  $k_b$ , separately. In other words, our results enable the defensive manager to decide both on the overall advertising budget and on how to allocate it among awareness and repositioning.

We begin by considering awareness advertising. Let  $A$ , a function of  $k_a$ , be the probability a consumer is aware of our brand. Brands must continue spending in order to maintain awareness levels. Equation (11) expresses our profit after the new brand enters.

$$\pi_a(p, k_d, k_a) = (p - c_a) N_a M_a(p) A D - k_a - k_d \quad (12)$$

$M_a(p)$  was defined in equation 11.

*Theorem 7, Defensive Strategy for Awareness Advertising: The optimal defensive strategy for advertising includes decreasing the budget for awareness advertising.*

*Proof:* Differentiating  $\pi_a(p^*, k_d^*, k_a)$  with respect to  $k_a$  yields the equation  $[\pi_a(p^*, k_d^*) + k_d^*] [dA(k_a^*)/dk_a] = 1$ . Similarly we get  $[\pi_b(p^0, k_d^0) + k_d^0] [dA(k_a^0)/dk_a] = 1$ . The result follows analogously to the proof of theorem 5 since  $\pi_b(p^0, k_d^0) > \pi_a(p^*, k_d^*)$ ,  $k_d^0 > k_d^*$ , and  $A$  is marginally decreasing in  $k_a$ .

Theorem 7 is not surprising because awareness advertising is modeled with a response function that has properties similar to the response function for distribution. Thus all of our comments (and an analogy to the pre-emptive distribution theorem) apply to awareness advertising.

Repositioning advertising is more complex. First, we consider advertising that can increase consumers' perception of  $x_{1j}$ . Using a form of response analysis we assume that if  $k_r$  dollars are spent we can achieve a new position ( $x_{1j}(k_r)/p_j$ ,  $x_{2j}/p_j$ ) in perceptual space. We further assume that the repositioning function,  $x_{1j}(k_r)$ , is increasing in  $k_r$  but marginally decreasing (concave). If the latter were not true, the optimal  $k_r$  might be infinite. Profit is then given by:

$$\Pi_a(p, k_d, k_a, k_r) = (p - c_a) N_a M_a(p, k_r) A D - k_d - k_a - k_r \quad (13)$$

Here  $x_{1j}$ , and hence  $M_a(p, k_r)$ , is now an explicit function of  $k_r$ .

To select the optimal repositioning strategy we examine the first order condition implied by equation (13), it is given in equation (14).

$$dM_a(p^*, k_r^*)/dk_r = 1/[(p^* - c_a) N_a D(k_d^*) A(k_a^*)] \quad (14)$$

Even if we simplify  $M_a(p^*, k_r^*)$  with uniform tastes, equation 14 is quite complex. The optimal repositioning strategy depends on the optimal price, optimal distribution strategy, and optimal advertising awareness strategy.

Fortunately we can obtain some insight into the solution of equation 13 by examining the marginal forces affecting the optimal spending,  $k_r^0$ , before the new brand entered. While this may not guarantee a global optimal, it does suggest a directionality for improving profit. Define a conditional defensive profit function,  $\Pi_a(k_r|o)$ , given by equation (15).

$$\Pi_a(k_r|o) = (p^0 - c_b^0) N_b M_b(p^0, k_r) D(k_d^0) A(k_a^0) - k_d^0 - k_a^0 - k_r \quad (15)$$

The conditional defensive profit function can be interpreted as the profit we can obtain after competitive entry if we are only allowed to reposition. Then from equation (14) and equation (15) we find profit before entry is given by equation (16).

$$\Pi_b(k_r) = \Pi_a(k_r|o) + G \int_{\alpha_{jn}}^{\alpha_{jj+}} f(\alpha) d\alpha \quad (16)$$

where  $G = (p^0 - c_b^0) N_b D(k_d^0) A(k_a^0)$  is a positive constant independent of repositioning spending,  $k_r$ .

Theorem 8, Repositioning by Advertising: Suppose consumer tastes are uniformly distributed and the new competitive brand attacks along attribute 2 (i.e., an upper adjacent attack), then at the margin if repositioning is possible,

- (a) profits are increasing in repositioning spending along attribute 1 (i.e., away from the attack);
- (b) profits are increasing in repositioning spending along attribute 2 (i.e., toward the attack) if and only if  $(x_{1j}/p - x_{1n}/p_n)(1 + r_{nj}^{-2}) < (x_{1j}/p - x_{1j+}/p_+)(1 + r_{jj+}^{-2})$ .

where  $p_n$  and  $p_+$  are the prices of the new and upper adjacent brands, respectively.

Symmetric conditions hold for an attack along attribute 1.

*Proof:* Let  $z_j = x_{1j}/p$  and let  $y_j = x_{2j}/p$ . Let  $k_{ri}$  be repositioning spending along  $x_{ij}$ , for  $i = 1, 2$ . Differentiating equation 16 with respect to  $k_{ri}$  yields  $\Pi_b'(k_r^0) = \Pi_a'(k_r^0|0) + (G/9Q)(\alpha'_{jj+} - \alpha'_{jn}) = 0$  where (') denotes the derivative with respect to  $k_{ri}$ . Because  $G > 0$ ,  $\Pi_a'(k_r^0|0)$  follows if  $\alpha'_{jj+} < \alpha'_{jn}$ . Since  $z_j$  is proportional to  $x_{1j}$  and  $x'_{1j} > 0$ , part (a) follows if  $(d\alpha_{jj+}/dz_j) < (d\alpha_{jn}/dz_j)$ . Similarly, part (b) follows if  $(d\alpha_{jj+}/dy_j) < (d\alpha_{jn}/dy_j)$ .

*Part (a):*

Redefine (') to denote differentiation with respect to  $z_j$ . Then we must show  $\alpha'_{jj+} < \alpha'_{jn}$ . Since  $r_{jj+} = (z_j - z_{j+})/(y_{j+} - y_j)$ ,  $r'_{jj+} = (y_{j+} - y_j)^{-1}$ . Similarly  $r'_{jn} = (y_n - y_j)^{-1}$ . Hence  $0 < r'_{jj+} < r'_{jn}$ . Now  $r_{jj+} > r_{jn}$ , hence  $0 < (1 + r_{jj+}^2)^{-1} < (1 + r_{jn}^2)^{-1}$ . Putting these results together yields  $r'_{jj+} (1 + r_{jj+}^2)^{-1} < r'_{jn} (1 + r_{jn}^2)^{-1}$ . Finally for  $\alpha = \tan^{-1} r$ ,  $\alpha' = r'(1 + r^2)^{-1}$ . Thus  $\alpha'_{jj+} < \alpha'_{jn}$  and the result follows.

*Part (b):*

Redefine (') to denote differentiation with respect to  $y_j$ . We derive conditions for  $\alpha'_{jj+} < \alpha'_{jn}$ . As shown above this is equivalent to  $r'_{jj+} (1 + r_{jj+}^2)^{-1} < r'_{jn} (1 + r_{jn}^2)^{-1}$ . Since  $r_{jj+} = (z_j - z_{j+})/(y_{j+} - y_j)$ ,  $r'_{jj+} = (z_j - z_{j+})/(y_{j+} - y_j)^2 = r_{jj+}^2 / (z_j - z_{j+})$ . Similarly  $r'_{jn} = r_{jn}^2 / (z_j - z_n)$ . Substituting yields the condition in part (b). Finally, the last statement of the theorem follows by symmetry.

The results of theorem 8 can be stated more simply. Part (a) says reposition to your strength. I.e., the competitive new brand is attacking you on one flank, attribute 2, but you are still positioned better along your strength under this attack, attribute 1. Theorem 8 says that if you can move along attribute 1, do so.

Part (b) says that repositioning to your competitor's strength, attribute 2, is not automatic. The testable condition given in theorem 8 must be checked. (Section 5 of this paper provides a practical procedure to estimate the new brand's position which is the data necessary to check the condition of Theorem 8.) Movement along attribute 2 is more complex because there are two conflicting effects. First, marginal returns may be low in the direction of competitive strength. Second, we seek to regain lost sales by attacking our competitor's strength. The first effect will dominate when our competitor is very strong, i.e.,  $r_{jj+} \gg r_{jn}$ . In this case, Theorem 8b directs us not to counterattack on the competitor's strength. The second effect will dominate when the competitor attacks strongly on  $x_{1j}$  as well as  $x_{2j}$ , i.e. when  $x_{1j}/p \cong x_{1n}/p_n$ . In this case, Theorem 8b directs us to counterattack on our competitor's strength in addition to moving to our strength.

Theorem 8 is a conditional result at the old optimal. But coupled with theorem 7 it provides very usable insight into defensive advertising strategy. Theorem 7 says *decrease awareness advertising*. Theorem 8 says that at least in the case of uniform consumer tastes, marginal gains are possible if we *increase repositioning advertising along our strength*. Together these results suggest that the defensive manager (facing uniformly distributed tastes) should reallocate advertising from an awareness function to a repositioning function. For example, he might want to select copy that stresses "more effective in cleaning hard-to-clean dishes" over copy that simply gets attention for his brand.

#### *Product Improvement*

The last component of defensive strategy is whether the defending manager should invest money to improve his physical product. We consider the case where the manager has the option to make improvements in small increments. We assume that he has the option to increase consumer perceptions of his brand by modifying his brand to improve its position. In doing so he incurs increased production cost,  $c$ . In other words,  $x_{1j}$  (or  $x_{2j}$ ) is an increasing function of  $c$ . Let  $c_b$  be the optimal production cost before competitive entry and let  $c_a$  be the optimal production cost after entry.

As in advertising repositioning we examine conditional defensive profit to gain some insight on physical product improvement. We define a conditional defensive profit function,  $\Pi_a(c|0)$  for production cost similar to equation 15. We can then show by substitution that the before entry optimal profit as a function of production cost is given by:

$$\pi_b(c) = \pi_a(c|o) + H(p^0 - c) \int_{\alpha_{jn}}^{\alpha_{jj+}} f(\alpha) d\alpha \quad (17)$$

where  $H = N_b D(k_d^0) A(k_a^0)$  is a positive constant independent of production cost.

*Theorem 9, Defensive Product Improvement:* Suppose that consumer tastes are uniformly distributed and the competitive new brand attacks along attribute 2 (i.e., an upper adjacent attack), then at the margin if product improvement is possible,

(a) Profits are increasing in improvements in attribute 1;

(b) Profits are increasing in improvements in attribute 2 if

$$(x_{1j}/p - x_{1n}/p_n) (1 + r_{nj}^{-2}) < (x_{1j}/p - x_{1j+}/p_+) (1 + r_{jj+}^{-2}).$$

Symmetric conditions hold for an attack along attribute 1.

*Proof:* Differentiating equation 16 with respect to  $c$  yields

$$\pi_b'(c_b) = \pi_a'(c_b|o) + H[(p^0 - c_b) (\alpha_{jj+}' - \alpha_{jn}') - (\alpha_{jj+} - \alpha_{jn})] = 0 \text{ at } c_b.$$

Since  $H > 0$ , part a,  $\pi_a'(c_b|o) > 0$  follows if the term in brackets is negative.

Since  $(\alpha_{jj+} - \alpha_{jn}) > 0$  by the definition of competitive and since  $(p^0 - c_b) \geq 0$

at the before-entry optimal, the term in brackets is negative if  $(\alpha_{jj+}' - \alpha_{jn}')$

is negative at  $c_b$ . In the proof to theorem 8 we showed that  $d(\alpha_{jj+} - \alpha_{jn})/dz_j < 0$

for  $z_j = x_{1j}/p$ . Since  $x_{1j}$  is increasing in  $c$ , part (a) follows. As in theorem 8,

part b follows from algebraically simplifying the condition that  $d\alpha_{jj+}/dx_{1j} <$

$d\alpha_{jn}/dx_{1j}$ . The last statement is obvious by symmetry.

Since Theorem 9 is so similar to Theorem 8 we do not discuss it in detail.

Related comments apply.

### Summary

Our goal in this section was to provide insight on defensive strategy, i.e., rules of thumb that rely on believable abstractions that model major components of market response. Toward this end we searched for directional guidelines that help the manager understand qualitatively how to modify his marketing expenditures in response to a competitive new product. In many situations, especially where data is hard to obtain or extremely noisy, such qualitative results may be more

usable than specific quantitative results. If good data is available the qualitative results may help the manager understand and accept more specific optimization results.

In particular our theorems show in general that:

- distribution expenditures should be decreased, unless the new brand can be prevented from entering the market,
- awareness advertising should be decreased, and
- profit is always decreased by a competitive new brand.

More specifically, if consumer tastes are uniformly distributed,

- price should be decreased in regular markets,
- (at the margin) advertising for repositioning should be increased in the direction of the defending brand's strength,
- (at the margin) the brand should be improved in the direction of the defending brand's strength.

We have also shown that there exist highly segmented taste distributions for which a price increase may be optimal.

We caution the manager that like any mathematical scientific theory, the above results are based on assumptions. Our results are only true to the extent that the market which the defensive manager faces can be approximated by our model. Since our model is based on a previously tested model of consumer behavior, we believe there will be many situations that can be approximated by our model. At the very least we believe that theorems 1 through 9 provide the foundations of a theory of defensive strategy that can be subjected to empirical testing and theoretical modification. In the long run it will be the interplay of empirics and theory that will provide greater understanding of defensive strategy.

#### 4. ESTIMATION OF THE CONSUMER TASTE DISTRIBUTION

The defensive marketing strategy theorems depend on the new product's position,  $(x_{1n}/p_n, x_{2n}/p_n)$  and on the distribution of consumer tastes,  $f(\alpha)$ . We now provide a methodology to estimate this information from readily obtainable data.

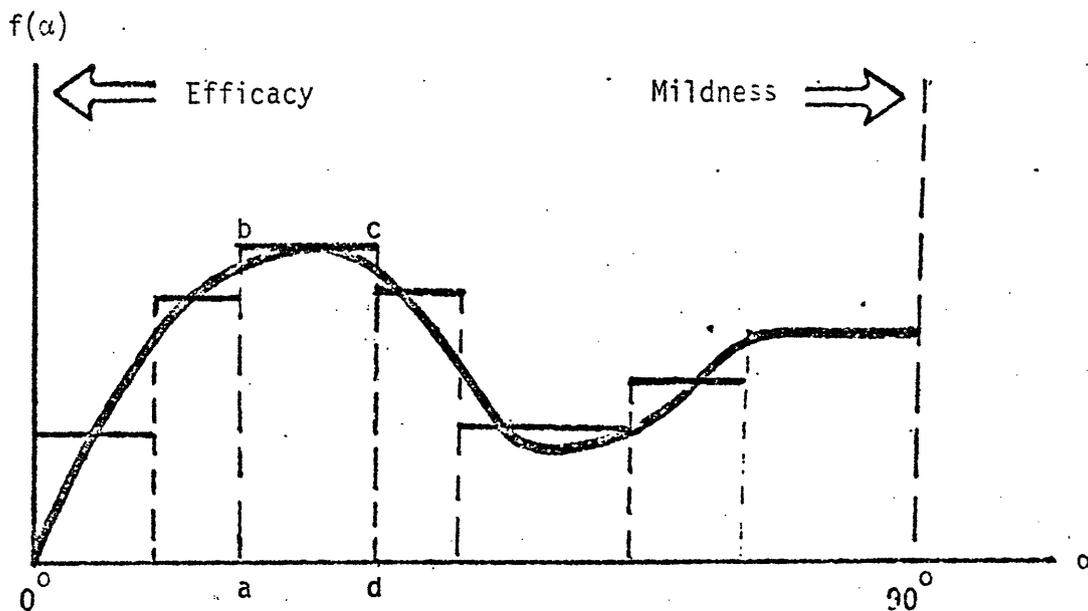
We begin with a technique to estimate consumer tastes when everyone has the same evoked set. We then extend this technique to the case where evoked sets vary.

*Homogenous Evoked Sets*

We return to the notation of section 2. Let  $m_{j|\ell} = \text{Prob} [\alpha_{jj-} < \tilde{\alpha} \leq \tilde{\alpha}_{jj+}]$  for  $j \in A_\ell$  and let  $M_{j|\ell}$  be observed market share of product  $j$  for consumers who evoke  $A_\ell$ . For this subsection we are only concerned with consumers who evoke  $A_\ell$ , thus for notational simplicity, drop the argument  $\ell$ . Our problem is then to select  $f(\alpha)$  such that  $m_j \equiv M_j$  for all  $j \in A_\ell$ .

One solution is to select a parameterized family of functions, say a Beta distribution,  $f_\beta(\alpha|\sigma, \delta)$ , and select  $\sigma$  and  $\delta$  with maximum likelihood techniques. While tempting, most common distributions are not appropriate for defensive strategy. For example, the directionality of defensive pricing strategy is dependent on whether  $f(\alpha)$  is bimodal (counter-example in theorem 2) or unimodal (see theorems 2 and 3). The beta distribution is limited to unimodal or bimodal at the extremes.

An alternate solution is illustrated in figure 7. We approximate  $f(\alpha)$  with a series of uniform distributions. This procedure can approximate any  $f(\alpha)$  and, in the limit, as the number of line segments gets large, the procedure converges to the true  $f(\alpha)$ . The problem now becomes how to select the endpoints and the heights of the uniform distribution.



*Figure 7: Approximating the Consumer Taste Distribution with a Series of Uniform Distributions.*

If our approximation is a good one, the area under the approximate curve should be close to the area under the actual curve. In other words, if we calculate the area under the actual curve between any two angles, that area should roughly equal the area under the curve formed by the uniform approximation. Now, if we take the area between the lower and upper adjacent angles for any brand, that area should equal the brand's market share. For example, in figure 7, if "a" and "d" are the lower and upper adjacent angles for the brand, the area of the rectangle a-b-c-d should equal the market share for the brand. Moreover, if we choose the end points to be the adjacent angles, the brand market shares become the estimates of the respective areas. Then the area of the  $j^{\text{th}}$  rectangle is  $m_j$  and the height,  $h_j$ , of the  $j^{\text{th}}$  uniform distribution becomes  $m_j/(\alpha_{jj+} - \alpha_{jj-})$ . Given this approximation, we exactly satisfy  $m_j = M_j$ . Figure 9 illustrates this approximation for a six brand market. It turns out that our approximation has a number of useful properties, such as being a maximum entropy prior, but for our purposes it provides one approximation to  $f(\alpha)$  that distinguishes among alternative defensive pricing strategies.

Summarizing  $\hat{f}(\alpha)$  estimates  $f(\alpha)$  and is given by equation (18).

$$\hat{f}(\alpha) = (\alpha_{jj+} - \alpha) M_j / (\alpha_{jj+} - \alpha_{jj-}) \text{ where } \alpha_{jj+} > \alpha > \alpha_{jj-} \quad (18)$$

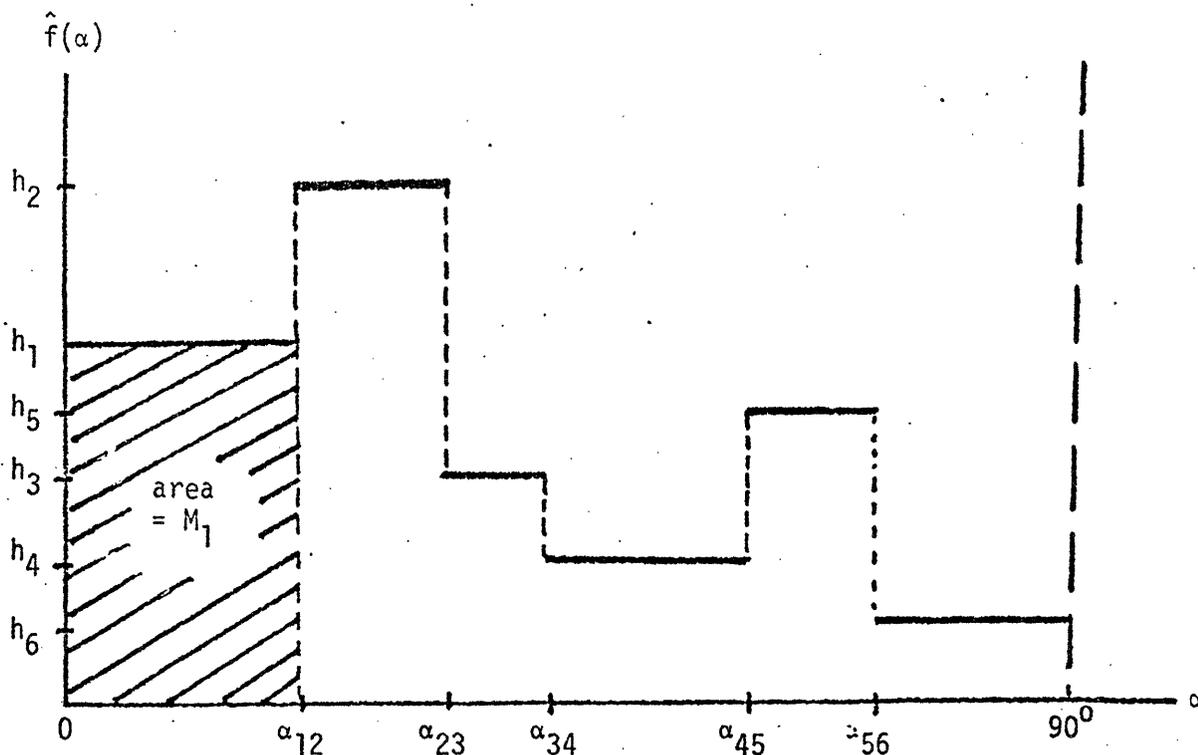


Figure 8: Estimation of Consumer Tastes For A Hypothetical Six Product Market

*Heterogeneous Evoked Sets*

Extension to heterogeneous evoked sets is deceptively simple. Since  $\alpha_{jj-}$  and  $\alpha_{jj+}$  are dependent on the evoked set,  $A_\ell$ , we can obtain  $\hat{f}(\alpha)$  by a weighted sum of  $f(\alpha)$  given  $A_\ell$ , denoted  $\hat{f}(\alpha|A_\ell)$ . That is,

$$\hat{f}(\alpha) = \sum_{\ell=1}^L S_\ell \hat{f}(\alpha|A_\ell) \quad (19)$$

where  $\hat{f}(\alpha)$  estimates  $f(\alpha)$ ,  $\hat{f}(\alpha|A_\ell)$  estimates  $f(\alpha|A_\ell)$  and  $S_\ell$  is the proportion of consumers who evoke  $A_\ell$ . In general, equation (19) will provide an excellent approximation to  $f(\alpha)$ . If there are  $N$  brands in the market there will be  $2^N$  choice sets and multiple brands within each choice set. Redundancy reduces the number of independent line segments to  $N(N-1)/2$ , but this is still a large number. For example,  $f(\alpha)$  for a 10 brand market will be approximated by 45 line segments. Estimates of  $S_\ell$  are discussed in the next section.

5. ESTIMATION OF THE NEW PRODUCT'S POSITION

Many of the theorems in section 3 are quite general depending only on the new brand being competitive. But, besides knowing how to respond (e.g., decrease distribution) most defending managers want to know how strongly to respond. The magnitude of response depends on the new brand's position. For example, intuitively, the closer the new brand is to our market, the more we should be concerned with an effective defense.

In this section we provide a general maximum likelihood procedure for estimating the new brand's position as well as a very practical Bayesian estimation procedure. We close with a technique to estimate the probability that the new product enters each evoked set.

*Maximum Likelihood Estimates*

Suppose the new brand,  $n$ , enters the evoked set  $A_\ell^*$  with the corresponding probability,  $S_\ell^*$ . ( $A_\ell^* = A_\ell \cup \{n\}$ .) Then the market share of the new brand as well as after-entry market shares for previous brands can be computed with the consumer model in section (2) if the new brand's position,  $(x_{1n}/p_n, x_{2n}/p_n)$ , is known. Thus, given the new brand's position, we can obtain equations (20) and (21).

$$m_j^* = \sum_{\ell=1}^L m_{j|\ell} (S_\ell - S_\ell^*) + \sum_{\ell=1}^L m_{j|\ell}^* S_\ell^* \quad (20)$$

$$m_n^* = \sum_{\ell=1}^L m_{n|\ell}^* S_\ell^* \quad (21)$$

where  $m_j^*$  = the post entry market share for brand j given the new brand enters at  $\underline{x}_n$ .

$m_{j|\ell}^*$  = the post entry market share for brand j among those with evoked set  $A_\ell$  given the new brand is positioned at  $\underline{x}_n$ .

$m_n|\ell^*$  = the market share of the new brand given  $\underline{x}_n$  and  $A_\ell$ .

If we assume consumers are drawn at random to form an estimation sample, the likelihood function,  $L(\underline{x}_n)$ , is given by equation (22).

$$L(\underline{x}_n) = \sum_{j=1}^J M_j \log m_j^*(\underline{x}_n) + M_n \log m_n^*(\underline{x}_n) \quad (22)$$

where  $L(\underline{x}_n)$  = the likelihood function evaluated at  $\underline{x}_n$

$m_j^*(\underline{x}_n)$  =  $m_j^*$  evaluated at  $\underline{x}_n$ .

$m_n^*(\underline{x}_n)$  =  $m_n^*$  evaluated at  $\underline{x}_n$ .

The maximum likelihood estimator of the new brand's position is then the value of  $\underline{x}_n$  that maximizes  $L(\underline{x}_n)$ .

Equation 22 is easy to derive but difficult to use. The key terms,  $m_j(\underline{x}_n)$  and  $m_n^*(\underline{x}_n)$ , are highly non-linear in  $\underline{x}_n$  even for a uniform distribution. Thus the optimization implied by equation 22 is soluble in theory, but extremely difficult in practice.<sup>5</sup> Fortunately, there is a more practical technique for obtaining estimates of  $\underline{x}_n$ .

#### *Bayes Estimates*

In discussing defensive strategies with product managers in both consumer and industrial products we discovered that most defending managers have reasonable hypotheses about how the competitive new brand is positioned. For example, when Colgate-Palmolive launched *Dermassage* liquid dishwashing detergent with the advertising message: "Dermassage actually improves dry, irritated detergent hands and cuts even the toughest grease", the experienced brand managers at Proctor and Gamble, Lever Bros. and Purex could be expected to make informed estimates of *Dermassage's* position in perceptual space. The existence of such prior estimates suggest a Bayesian solution.

Suppose that the defending manager provides a prior estimate,  $f_x(\underline{x}_n)$ , of the distribution of the new product's position,  $\tilde{\underline{x}}_n$ . (Note that  $\tilde{\underline{x}}_n$  denotes  $\underline{x}_n$  expressed as a random variable.) For tractability we discretize the prior distribution. We then use the consumer model in section 2 to derive  $m_j^*[\underline{x}_n(\beta)]$  and  $m_n^*[\underline{x}_n(\beta)]$  for each potential new product position, where  $\beta$  indexes the discretized new brand positions.

<sup>5</sup> If  $L(\underline{x}_n)$  is reasonably smooth, gradient search or grid search might be feasible.

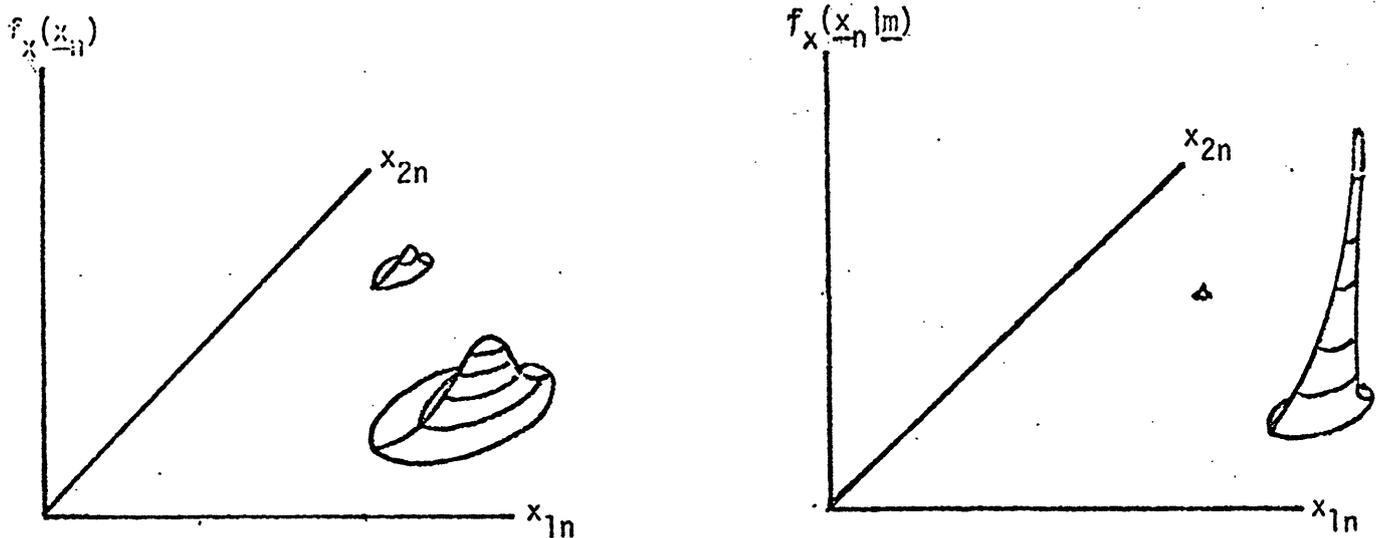
See equations (20) and (21). If consumers are drawn at random for the estimation sample, then the Bayesian posterior distribution,  $f_x(x_n | M)$  is given by:

$$f_x(x_n | M) = f_x[x_n(\beta)] K(M | m_\beta) / \sum_\gamma f_x[x_n(\gamma)] K(M | m_\gamma) \quad (23)$$

where  $K(M | m_\beta) = \{m_n[x_n(\beta)]\}^{M_n n} \cdot \prod_{j=1}^J \{m_j[x_n(\beta)]\}^{M_j n}$

$M$  is the vector of post entry market shares,  $K(M | m_\beta)$  is the kernel of the sampling distribution for  $n$  consumers drawn at random from a population defined by the multinomial probabilities, and  $m_\beta = \{m_n[x_n(\beta)]$  and  $m_j[x_n(\beta)]$  for all  $j$ . The use of equation (23) to update a managerial prior is shown in figure 9.

Equation (23) looks complicated but it is relatively easy to use. First, the manager specifies his prior beliefs about the new brand's position in the form of a discrete set of points  $x_n(\beta)$ , and the probabilities,  $f_x[x_n(\beta)]$ , that each point



a) Bayesian Prior

b) Updated Posterior

Figure 9 : Bayesian Updating Procedure for a Hypothetical Market. (The Bayesian prior (a) is based on managerial judgment. The Bayesian posterior (b) is the result of using observed sales data to update managerial judgment. See equation 23.)

is the new position. For example, he may believe that the points, (3.5, 4.0), (4.0, 3.5), (4.0, 4.0), are the potential, equally likely, positions of the new brand, Dermassage.<sup>6</sup>

<sup>6</sup> This hypothetical example does not necessarily reflect true market shares and market positions.

As shown in figure 10, the manager has a general idea of where Dermassage is positioned but does not know the exact position. We first use equations (20) and (21) to compute the predicted market shares,  $m_j[x_n(\beta)]$ , for each of the three potential new brand positions. For this example, assume that tastes are uniform and everyone evokes all brands. This case is shown in table 2. Now suppose that we sample 50 consumers and find that, in our sample, the observed market shares are Ajax (20%), Dermassage (30%), Joy (20%), and Ivory (30%). We use equation (23) to compute the kernel of the sampling distribution, e.g.,  $K(\underline{M} | \underline{m}_{\beta=1}) = (.30)^{10} (.10)^{15} (.30)^{10} (.30)^{15}$  for  $\beta=1$ . We then compute the posterior probabilities as given in table 2.

As we have chosen the data, the market shares from 50 consumers clearly identify (4.0, 4.0) as the most likely position for Dermassage. The point, (4.0, 3.5), still has an 11% chance of being the actual position; the point, (3.5, 4.0), is all but ruled out. Not all applications will be so dramatic in identifying the new brand's position but all applications will follow the same conceptual framework.

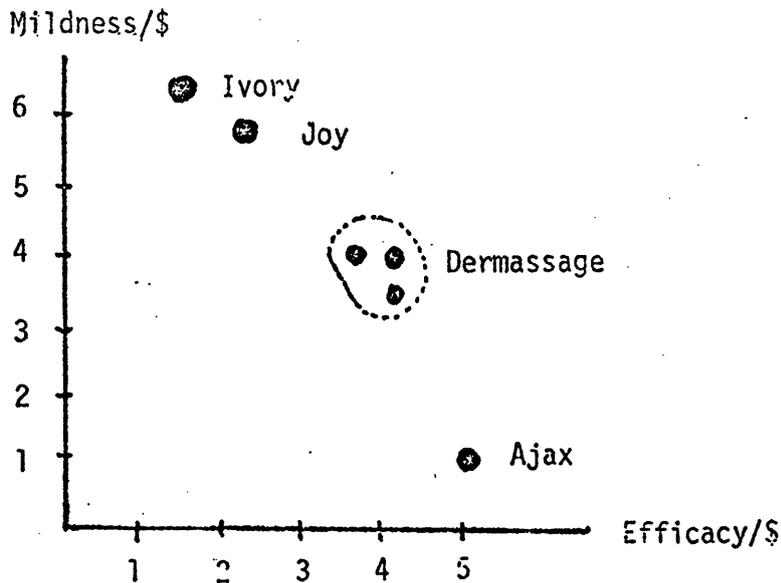


Figure 10: Example Managerial Priors for the New Product's Position (Three equally likely points for Dermassage.)

TABLE 2: CALCULATIONS FOR BAYESIAN UPDATING PROCEDURE

	$\beta=1$	$\beta=2$	$\beta=3$	Observed
Position of Dermassage	(3.5,4.0)	(4.0,3.5)	(4.0,4.0)	Share
Market Shares				
Ajax	.30	.24	.20	.20
Dermassage	.10	.19	.30	.30
Joy	.30	.27	.20	.20
Ivory	.30	.30	.30	.30
Prior Probability	.33	.33	.33	-
Posterior Probability	<.01	.11	.88	-

*Evoked Set Probabilities*

Equation 20 depends on the evoked set probabilities,  $S_{\ell}$  and  $S_{\ell}^*$ . In Section 2 we assume that  $S_{\ell}$  is known prior to the new product's launch. If the manager collects data on evoked sets after the launch of the new brand, equation 20 can be used directly.

In many cases evoked set information will be too expensive to collect after the new brand is launched. Equation 20 can still be used if we assume either that (1) the new brand enters evoked sets randomly or (2) the new brand enters evoked sets in proportion to the probability that it would be selected if it were in that evoked set.

Case 1 is based on the assumption that awareness and availability are independent of preference. In case 1, equation (20) and (21) reduce to equations (24) and (25), respectively.

$$m_j^*(x_n) = \sum_{\ell} [(1-W)m_{j|\ell} + Wm_{j|\ell}^*] S_{\ell} \quad (24)$$

$$m_n^*(x_n) = W \sum_{\ell} m_{n|\ell}^* S_{\ell} \quad (25)$$

where  $W$  is the aggregate percent of consumers who are aware of the new brand and find it available.

Case 2 is based on the assumption that evoking is functionally dependent on preference. In other words, case 2 assumes that consumers are more likely to become aware of brands that closely match their preferences. In case 2, equations (20) and (21) reduce to equations (26) and (27), respectively.

$$m_j^*(x_n) = \sum_{\ell} m_{j|\ell} S_{\ell} - [W \sum_{\ell} (m_{j|\ell} - m_{j|\ell}^*) m_{n|\ell}^* S_{\ell}] / [\sum_{\ell} m_{n|\ell}^* S_{\ell}] \quad (26)$$

$$m_n^*(x_n) = W \sum_{\ell} (m_{n|\ell}^*)^2 S_{\ell} / \sum_{\ell} m_{n|\ell}^* S_{\ell} \quad (27)$$

The derivations of equations (26) and (27) are given in lemma 5 in the appendix.

Equations (24)-(27) are useful computational results since together they provide flexibility in modeling how a new brand enters evoked sets. Most importantly each can be applied if we know only the aggregate percent of consumers who evoke the new brand.

*Summary*

This section has provided a practical means to estimate the new brand's position and its impact on sales. The only data on the new market that are required are (1) sales of each brand and (2) aggregate evoking of the new brand.<sup>7</sup>

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<sup>7</sup> We assume of course that the old product positions, the old evoked set probabilities, and the taste distribution are known.

## 6. DIAGNOSTICS ON COMPETITIVE RESPONSE

Defensive strategy is how to react to a competitive new product. However, the concepts developed in sections 2 and 3 are useful for representing competitive threats and, in particular, for representing how our competitors might react to the new brand. In this section we briefly outline these concepts. We leave full equilibrium analysis to future research.<sup>8</sup>

### *Angle of Attack*

The general equilibrium for a market depends on the cost structures faced by each firm as well as the distribution of consumer tastes. However, in talking with managers we have found it useful to represent the competitive threats posed by each firm as that portion of the taste spectrum that that firm is capturing. In particular we define the "angle of attack",  $\Lambda_j$ , to represent the average tastes of product  $j$ 's consumers, i.e.:

$$\Lambda_j = (\alpha_{jj+} + \alpha_{jj-})/2 \quad (28)$$

With this definition we should be most concerned with products that have angles of attack that are close to ours. In defensive strategy formulation we believe it is useful to superimpose the angle of attack of the new product on the preference distribution to determine how competitive the new product is and to determine which of our competitors is most likely to engage in an active defense.

A related concept is the "strength of attack",  $\phi_j$ , which indicates how product  $j$  is penetrating its portion of the taste spectrum. We define the strength of attack as:

$$\phi_j = \sqrt{x_{1j}^2 + x_{2j}^2} / p_j \quad (29)$$

where  $\phi_j$  is to be compared to the average over all efficient products.

### *Price Diagnostics*

Theorems 2 and 3 tell us how to react to a competitive new product, but they can just as easily be applied to predict how our competitors should react to the new product if they are maximizing profit. Beyond that we can provide

<sup>8</sup> Existing models of equilibrium analysis assume symmetries in both cost functions and taste distributions. The resulting model places products uniformly about the spectrum. See Lancaster (1980). For defensive strategies we need to know more about the dynamics of the market and the cost functions of existing firms. Symmetric markets are powerful economic tools but are not sufficient to address specific marketing problems in defensive strategy.

(1) an upper bound on competitors' reactions by assuming they price to remain efficient and (2) a lower bound by assuming that they price to avoid a suicidal price war with the new product. These upper and lower bounds are illustrated in figure 11.

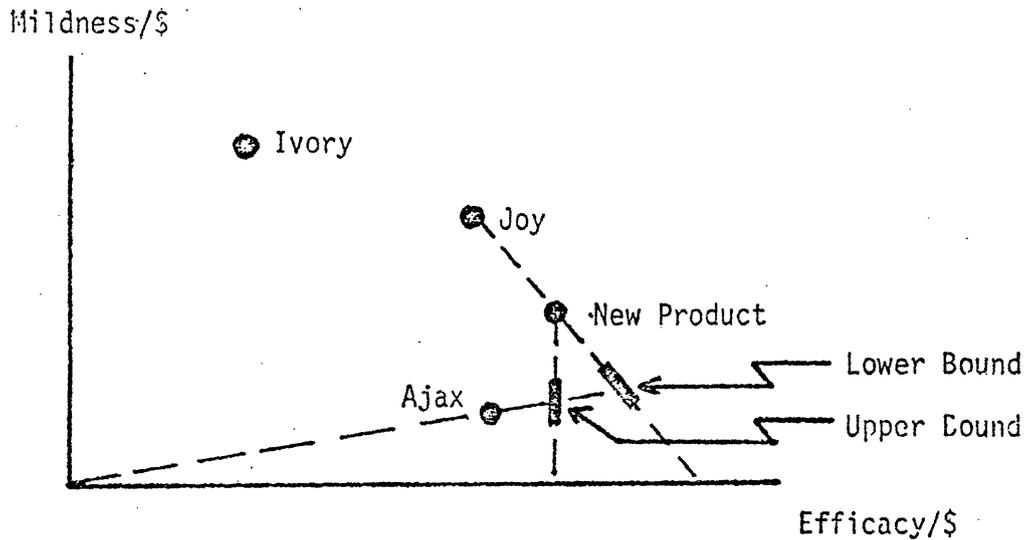


Figure 11: Upper and Lower Bounds on Competitive Price Responses to the New Product

## 7. NORMATIVE ANALYSIS

The directional insights of theorems 1 through 9 are generally applicable because they do not depend upon the specific details of the awareness, repositioning, distribution, and production cost response functions. In the few cases where the manager is fortunate enough to have well specified response functions from previous econometric or experimental marketing analysis, he can go beyond qualitative insights to specify quantitative levels of the marketing mix budgets. In particular, he can select  $p^*$ ,  $c_a^*$ ,  $k_a^*$ ,  $k_d^*$ , and  $k_r^*$  to maximize profit as defined by equation 13 in section 3. If the appropriate second order conditions are satisfied, then the optimal levels can be obtained (in theory) by solving the following five simultaneous differential equations, (30) through (34).

$$p^* = c^* E_p^* / (E_p^* - 1) \quad (30)$$

$$c^* = p^* E_c^* / (E_c^* + 1) \quad (31)$$

$$k_a^* = R^* E_a^* \quad (32)$$

$$k_r^* = R^* E_r^* \quad (33)$$

$$k_d^* = R^* E_d^* \quad (34)$$

where  $R^* = (p^* - c^*) M(p^*, c^*, k_r^*) A^* D^*$  is the net revenue before marketing costs at the optimal and  $E_p^*, E_a^*, E_d^*, E_r^*$  are the elasticities of demand with respect to price, cost, awareness, advertising, repositioning advertising, and distribution respectively.<sup>9</sup>

Equations 25 are complex because each elasticity is a function of the marketing mix variables. Furthermore, the measurements necessary to obtain accurate response functions may be difficult and expensive -- but they are feasible. See Bass (1980), Little (1979), and Parsons and Schultz (1976). A practical measurement and optimization model to set the specific levels of the defensive marketing mix has yet to be developed, but with the advent of improved marketing data such as that based on universal product codes (UPC) and instrumented markets, such models are likely to be developed in the next few years. In the meantime the qualitative results of theorems 1 through 9 help marketing scientists to better understand the optimization structures from which to develop such normative models.

#### 8. SUMMARY AND FUTURE DIRECTIONS

Among the key results of this paper are the nine theorems which investigate defensive market strategy. These theorems are the logical consequences of a consumer model based on the assumptions that consumers are heterogeneous and choose within a product category by maximizing a weighted sum of perceived product attributes. Since this consumer model is based on empirical marketing research we feel that it is a good starting point with which to analyze defensive marketing strategies.

We feel that the nine theorems provide usable managerial guidelines. When the appropriate data is available normative optimization models (equations 30-34) may be the best way to proceed, but, by their very nature, defensive marketing strategies are often made quickly and without extensive data collection. The nine theorems tell the manager that as the result of a competitive entrant (1) the defender's profit will decrease, (2) if entry cannot be prevented, budgets for distribution and awareness advertising should be decreased, and (3) the defender should carefully compare the competitors angle of attack to his position and the distribution of consumer tastes. If tastes are segmented and if the competitors clearly out-position his product in one of his consumer segments, a price increase may be optimal. If consumer tastes are uniformly distributed across the spectrum, the defender should decrease price,

<sup>9</sup> For example,  $E_a^* = (k_a^*/\text{demand})[\partial(\text{demand})/\partial k_a^*]$  where demand equals  $M(p^*, c^*, k_r^*) A(k_a^*)D(k_d^*)$ . All elasticities are defined to be positive.

reposition by advertising to his strength, and improve product quality in the same direction. Sections 4, 5 and 6 provide practical procedures to identify the consumer taste distribution, the new brand's position and its entry into evoked sets. This data allows us to decide when each theorem is appropriate. Because these results are robust with respect to the details of the model they are good managerial rules of thumb even when the data is extremely noisy.

We do not mean to imply that the theorems replace empirical modeling. We do mean to suggest that the theorems are a good beginning to guide the development of empirical models and to encourage the development of a generalizable managerial theory of how to respond to competitive new products.

#### *Future Directions*

There are many theoretical extensions that may be possible based on the nine defensive theorems. For example, a price decrease is optimal in regular markets with uniformly distributed tastes. We suspect that there are conditions under which a price decrease is optimal for many unimodal distributions of taste. Another extension might investigate the conditions under which the repositioning and product improvement theorems lead to global solutions. Other theoretical extensions might investigate market equilibrium, interrelationships among products in a product line, and the relationship of defensive strategy to marketing strategy in general.

Our theorems are based on mathematical deduction from one accepted consumer model. Empirical tests of our theorems, perhaps through practical normative models based on a good measurement system for the optimization equations in section 7, will determine how well that consumer model captures the essence of consumer response to an active defense. Such empirical tests should lead to new insights and improved understanding of defensive marketing strategies.

APPENDIX

This appendix provides the formal lemmas necessary for the proofs to theorems 1 and 3.

*Lemma 1:* The angle,  $A = \alpha_{jj+} - \alpha_{jn}$ , is non-decreasing in price for  $\alpha_{jj+} \geq \theta_j$ .

*Proof:* Lemma 1 is a problem in analytical geometry. We begin by simplifying notation. Examine figure 5. Let a be the line segment connecting points j+ and n, let b be the line segment connecting points j+ and j. Consider the triangle formed by points n, j, and j+, and let A, B, C be angles opposite a, b, and c respectively. Note that C is obtuse when n, j, and j+ are all efficient. By the law of cosines  $a^2 = b^2 + c^2 - 2bc \cos A$ . Since a is independent of p,  $a' = 0$ . By implicit differentiation  $0 = 2bb' + 2cc' - 2(bc' + c'b)\cos A - 2bc(\cos A)'$ . Simplifying and recognizing  $\cos A = (b^2 + c^2 - a^2)/2bc$ , we get  $2b^2c^2(\cos A)' = (c^2 + a^2 - b^2)bc' + (b^2 + a^2 - c^2)cb'$ . Since C is obtuse,  $c^2 + a^2 - b^2 > 0$  and  $b^2 + a^2 - c^2 < 0$ .

We now further simplify notation. Let  $z_i = x_{1i}/p_i$  and  $y_i = x_{2i}/p_i$  for  $i = j, n, j+$ . Note that both  $z_j$  and  $y_j$  are decreasing in  $p_j$  while  $z_n, y_n, z_{j+}, y_{j+}$  are independent of  $p_j$ . Temporarily redefine (') to denote differentiation with respect to  $z_j$  recognizing  $y_j = z_j \tan \theta_j$ . After simplification we get  $c' = (2/c)(y_{j+} - y_j)(\tan \alpha_{jj+} - \tan \theta_j)$  and  $b' = (2/b)(y_n - y_j)(\tan \alpha_{nj} - \tan \theta_j)$ . By assumption,  $\alpha_{jj+} \geq \theta_j$ , and since j+ is upper adjacent,  $y_{j+} \geq y_j$ . Thus  $c' \geq 0$ . Similarly  $b' \geq 0$  if  $\alpha_{nj} \leq \theta_j$ .

Suppose  $\alpha_{nj} \leq \theta_j$ . Then  $b' \leq 0$  and  $(b^2 + a^2 + c^2)cb' \geq 0$  since  $b^2 + a^2 - c^2 < 0$ . Thus both terms on the right hand side of the implicit equation for  $(\cos A)'$  are positive, hence  $(\cos A)' \geq 0$ . Now suppose  $\alpha_{nj} > \theta_j$ .  $(\cos A)'$  is still positive if  $(c^2 + a^2 - b^2)(b/c) \geq (b^2 + a^2 - c^2)(c/b)$  since  $(y_{j+} - y_j) \geq (y_n - y_j)$  and  $\tan \alpha_{jj+} \geq \tan \alpha_{nj}$ . This is easily shown by expansion and collecting terms. Thus  $(\cos A)' \geq 0$  for all  $\alpha_{nj}$ .

Finally, since  $\cos A$  is decreasing in A and  $(\cos A)' \geq 0$ , we have shown  $dA/dz_j \leq 0$ . But the derivative is with respect to  $z_j$  and  $z_j$  is decreasing in  $p_j$ . Thus we have the result that  $dA/dp_j \geq 0$ .

*Lemma 2:* Sales under uniformly distributed tastes are decreasing in price.

*Proof:* Sales under the condition of uniform tastes is given by  $N_a(a_{nj} - a_{jj-})$ . The result holds if  $\alpha'_{nj} - \alpha'_{jj-} < 0$  where the derivative is with respect to price.

By direct computation  $\alpha'_{nj} = p_n(x_{2j}x_{1n} - x_{1j}x_{2n})/[p_jx_{2n} - p_nx_{2j}]^2 + (p_nx_{1j} - p_jx_{1n})^2]$ .

Since the denominator is positive the result holds if  $x_{2n}x_{1n} \leq x_{1j}x_{2n}$ . Since  $n$  is now upper adjacent to  $j$ ,  $x_{2j} \leq x_{2n}$  and  $x_{1n} \leq x_{1j}$  where at least one inequality is strict. Thus  $\alpha'_{nj} < 0$ . By symmetry  $\alpha'_{jj-} > 0$ . Thus  $\alpha'_{nj} - \alpha'_{jj-} < 0$ .

*Lemma 3:  $M_b(p)$  is a non-increasing function of price.  $M_b(p)$  is a decreasing function of price for  $f(\alpha_{jj+})$  or  $f(\alpha_{jj-}) > 0$ .*

*Proof:*  $M_b(p) = \int_{\alpha_{jj-}}^{\alpha_{jj+}} f(\alpha) d\alpha$ . Let  $(\cdot)$  denote the derivative with respect to price.

$M_b'(p) = f(\alpha_{jj+}) \alpha'_{jj+} - f(\alpha_{jj-}) \alpha'_{jj-}$ . Following lemma 2, it is clear that  $\alpha'_{jj+} > 0$ , and  $\alpha'_{jj-} < 0$ . Since  $f(\alpha) \geq 0$ , it follows that  $M_b'(p) \leq 0$ . If  $f(\alpha_{jj+})$  or  $f(\alpha_{jj-}) > 0$ ,  $M_b'(p) < 0$ .

*Lemma 4:  $\alpha''_{jn} - \alpha''_{jj-}$  is non-positive if  $\theta_{j-} \leq \alpha_{jj-}$ .*

*Proof:* By direct computation

$$\alpha''_{nj} = - \left\{ \frac{p_n(x_{2j}x_{1n} - x_{1j}x_{2n})}{[(p_jx_{2n} - p_nx_{2j})^2 + (p_nx_{1j} - p_jx_{1n})^2]^2} \right\} \left\{ \begin{matrix} 2x_{2n}(p_jx_{2n} - p_nx_{2j}) - \\ 2x_{1n}(p_nx_{1j} - p_jx_{1n}) \end{matrix} \right\}$$

As shown in the proof of Lemma 2, the first term is negative. Thus the result holds if the second term is non-positive. Using the notation of Lemma 1, this condition reduces to  $z^2 - z_jz_n \leq y_jy_n - y_n^2$  after expansion and simplification. But this condition is simply  $y_n/z_n \leq (z_n - z_j)/(y_j - y_n)$  or  $\tan \theta_n \geq \tan \alpha_{nj}$ . By symmetry we show  $\alpha''_{jj-} \geq 0$  if  $\tan \theta_{j-} \leq \tan \alpha_{jj-}$ . Since  $\alpha''_{nj} \leq 0$  and  $\alpha''_{jj-} \geq 0$  under the conditions of the lemma, the result follows.

*Lemma 5: If evoking is proportional to the probability that the new product will be chosen if it is in the evoked set, then the forecast market shares are given by equation 27 in the text.*

*Proof:* By equation 21,  $m_n^*(x_n) = \sum_{\ell} m_{n|\ell}^* S_{\ell} = (\sum_{\ell} m_{n|\ell}^* S_{\ell}^*) (\sum_k m_{n|\ell}^* S_k^*) /$

$(\sum_k m_{n|k}^* S_k^*) = \sum_{\ell} \sum_k m_{n|\ell}^* m_{n|k}^* S_{\ell}^* S_k^* (S_k^* S_{\ell}^*)^{-1} / (\sum_k m_{n|k}^* S_k^*)$ . By assumption,  $S_k^*$  is proportional to  $S_k m_{n|k}^*$ . Substituting and rearranging terms yields  $m_n^*(x_n) =$

$\sum_{\ell} \sum_k m_{n|\ell}^* m_{n|k}^* S_{\ell}^* m_{n|\ell}^* S_k^* S_k^{-1} m_{n|k}^* / (\sum_k m_{n|k}^* S_k^*)$ . Rearranging terms yields

$m_n^*(x_n) = (\sum_k S_k^*) (\sum_{\ell} m_{n|\ell}^* S_{\ell}^*) / (\sum_k m_{n|k}^* S_k^*)$ . Recognizing  $W = \sum_k S_k^*$  yields the equation for  $m_n^*(x_n)$  in the text. Using this result and similar substitutions yields equation 25.

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