A MULTIPLE-ITEM INVENTORY MODEL WITH A JOB COMPLETION CRITERION

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1. INTRODUCTION

Many types of office and manufacturing equipment require servicing by specially-trained service representatives, who are called to the operating site when a piece of equipment malfunctions or fails. These service representatives must strive to make an on-site diagnosis and repair of the failed equipment. An inability to do this may be very costly due to the prolongation of the downtime for the equipment. In some instances the repair entails just the readjustment or reconfiguration of the equipment. In other instances the repair requires the replacement of failed components. When a component inventory is not kept at the equipment site, the service representative must carry his/her own inventory of spare components. Since there may be thousands of components for each type of equipment serviced by a service representative, it is not possible for the service representative to carry a spare for all components. Furthermore, the service representative must have spares for all of the failed components in order to complete the repair. This suggests an interesting inventory problem, that being the determination of the optimal mix of components to be carried by a service representative in order to achieve the desired job completion rate.

In a recent note [5], Smith, Chambers and Shlifer present their model and analysis for this problem. They assume that the service representative has an opportunity to restock between repair jobs; consequently the inventory mix problem is a one-period problem, analogous to the classical "newsboy problem" (e.g. [3], pp. 388-394). They also assume that failures of distinct component types are independent and that at most one unit of each component type may be required in a repair. If a service representative is unable to complete a repair due to not having stocked a failed component, the service representative must return to a central supply depot to procure the required components. Smith, et al assign a penalty cost to such occurances to represent the additional cost and inconvenience of this service failure. They then seek to find the mix of components that minimizes the expected annual inventory-holding and penalty costs. To do this, they prove a theorem which establishes that one of a set of n+1 stocking policies, is optimal, where n is the number of component types. The determination of the optimal stocking policy requires the evaluation of each of these n+1 policies.

This note presents an alternative model for this inventory problem. An office equipment manufacturer posed to me a question nearly identical to that addressed by Smith, et al. The major distinction of this model over that of Smith, et al is that the office equipment manufacturer was unwilling to assign a penalty cost to the failure to complete a repair on the first visit by the service representative. Rather the office equipment manufacturer desired to know the stocking policy that would guarantee a specified job completion rate with the minimum inventory holding cost. In the next section I present the model that was developed independently of the work by Smith, et al. As will be seen, this new model does not, by any means, dominate that of Smith, et al; rather it provides additional insight into the problem structure and solution.

2. MODEL FORMULATION

We make the same assumptions as those of Smith, et al. Namely, the service representative can restock between repair visits, components fail independently, and at most one unit of each component type may be needed for a repair. We assume there are n components with p_i, $i=1,2,\ldots,n$, being the probability that component i has failed and needs to be replaced. We define h_i, $i=1,2,\ldots,n$, to be the annual holding cost for a unit of component i and we let α represent the desired completion rate $(0 \leq \alpha \leq 1)$. For x_i, $i=1,2,\ldots,n$, being a zero-one variable to denote the stockage of component i, we formulate the decision problem as the following mathematical program:

$$\min \sum_{i=1}^{n} h_i x_i \tag{1}$$

$$\text{subject to} \quad \prod_{i=1}^{n} (1-p_i)^{1-x_i} \geq \alpha \tag{2}$$

$$x_i = 0,1 \qquad i=1,2,\ldots,n \tag{3}$$

The interpretation of (1) - (3) is to minimize inventory holding cost subject to a constraint on the job completion rate. To understand (2), note that a repair cannot be completed only when a component fails that has not been included in the service representative's inventory. Thus the probability of completing a repair is the probability of having no components fail that are not stocked. But this is given by the left-hand-side of (2) which is equivalent to $\prod_{i \notin S} (1-p_i)$ for S being the index set of components stocked in inventory. (That is, $S = \{i | x_i = 1\}$.)

We reexpress (2) by first taking the logarithm of each side of (2) to obtain

$$\sum_{i=1}^{n} (1-x_i) \log(1-p_i) \geq \log(\alpha) \quad , \tag{2a}$$

and then rearranging to get

$$\sum_{i=1}^{n} [-\log(1-p_i)]x_i \geq \beta \tag{2b}$$

where $\beta = \log(\alpha) - \sum_{i=1}^{n} \log(1-p_i)$. By substituting (2b) for (2), we transform (1) - (3) into a binary knapsack problem [4]. This knapsack problem may be solved optimally by several techniques (e.g. [1], [4]). Alternatively for large values of n we may consider a heuristic procedure, such as a greedy procedure [2] which has been found to be extremely effective in general. The implementation of a greedy procedure, which was recommended to the office equipment manufacturer, results in ranking the components in nondecreasing order according to the ratio $h_i/[-\log(1-p_i)]$. In comparison, the fundamental result of Smith, et al [5] for their problem is to rank the components in nondecreasing order according to the ratio h_i/p_i. But these rankings are nearly identical for small values of p_i, since $\log(1+x) \cong x$ for small x.

3. DISCUSSION

This note has shown how to formulate a single-period, multiple-item inventory problem with a job completion criterion as a binary knapsack problem. This model may be contrasted with that of Smith, et al [5] for a similar problem. The prime advantage of the approach given here is that it does not require the specification of a penalty cost for being unable to complete a repair job. The attraction of the Smith, et al model is that its optimal solution procedure may be easier than the optimal solution of our model; the maximum effort required by the algorithm given by Smith, et al is bounded by polynomial number of steps, whereas there is no known polynomial algorithm for the binary knapsack problem. However, there do exist very efficient and effective heuristic procedures for the binary knapsack problem, as well as optimization procedures [1] capable of solving very large problems (i.e. n=10000) in a few seconds of computer time.

We make two additional comments concerning the formulation given by (1) - (3). First, we get a completely comparable problem if we desire to maximize the job completion rate subject to a budget constraint on total inventory. Second, in many instances, there may be a space restriction on the total inventory stocked. For instance, the service representative may be limited to what can be carried in an attache case or in the trunk of a car. Here we need only augment (1) - (3) with a linear constraint modeling this restriction. The solution of this augmented problem, however, is more complex since we now have a two-dimensional knapsack problem.

References

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