EXPECTATION FORMATION IN BEHAVIORAL SIMULATION MODELS

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ABSTRACT

The nature and rationality of expectations are hotly debated in economics and management science. Expectations are usually portrayed in behavioral simulation models and system dynamics as adaptive learning processes. This paper presents a behavioral model of trend expectation formation. The model assumes expectations about the growth rate of a quantity are formed adaptively from the recent growth rate of the input variable itself. The model is then tested directly against actual forecasts in two quite different domains: short-term expectations of inflation and long-term energy demand forecasts. In both cases the model replicates the evolution of the expectations quite well over extended time periods. The results support the use of adaptive expectations and trend extrapolation. The results also suggest the presence of additional judgmental heuristics which can have dynamic and policy significance. In particular, there seem to be substantial conservatisms in both inflation expectations and energy demand forecasts: forecasters systematically underestimate the growth rate of the input. Such conservatisms are consistent with the empirical literature on judgment and decisionmaking. The results show it is possible to test the expectation formation processes assumed in behavioral simulation models; implications for use of adaptive expectations in behavioral models when empirical data are unavailable are also discussed.

Expectation Formation in Behavioral Simulation Models

The nature and rationality of expectations are hotly debated in economics and management science. Expectation formation is particularly important in behavioral simulation models, a class which includes most system dynamics models.[1] Expectations are usually modeled in system dynamics as adaptive learning processes (e.g. Holt et al. 1960, Forrester 1961, Cyert and March 1963, Mass 1975, Lyneis 1980, Meadows 1970, Low 1974, Sterman and Richardson 1985). Adaptive expectations are common in economic models as well, for example Irving Fisher's (1930) theory of interest rates, Nerlove's (1958) cobweb model (Arrow and Nerlove 1958), Friedman's (1957) permanent income hypothesis, Ando and Modigliani's (1963) lifecycle hypothesis of saving, and Eckstein's (1981) theory of "core inflation".[2] For example, a firm's expectation of the order rate for its product is often assumed to adjust over time to the actual order stream. Many times, the adjustment is assumed to be first-order information smoothing, though more complex patterns of adjustment may be chosen (e.g. Weymar 1968). Single exponential smoothing has been shown to outperform many other forecasting methods over longer time horizons (in the M-competition, a forecasting contest involving over four hundred time series: Makridakis et al. 1982, Makridakis et al. 1984, Carbone and Makridakis 1986).

However, sometimes expectations respond not just to the past history of the variable but to its past growth rate as well. For example, the past values and past trend in orders may be used to estimate the likely future order rate. Growth expectations in behavioral simulation and system dynamics are often modeled with the TREND function (Richmond 1977, Richardson and Pugh 1981). The TREND function is a set of differential equations which represent the formation of expectations about the current rate of growth in a given variable. But TREND is not just a clever way to calculate the rate of growth of a variable. As the input to decision rules in models, TREND represents a behavioral theory of how people form expectations, and takes into account the time required for people to collect and analyze data, and to react to changes in the growth rate.

Despite the prevalence of adaptive expectations and the widespread use of the TREND function in system dynamics modeling, there have been few direct tests of the appropriateness of these assumptions (exceptions include Sterman 1986b and Hines 1986). Because expectations are a matter of the first importance in modeling, it is surprising that the system dynamics community has not tested these assumptions more formally. This paper explores the structure and behavioral assumptions of the TREND function. The TREND function is then tested against actual expectations data in two quite different domains: short-term expectations of inflation and long-term energy demand forecasts.

Portraying Growth Expectations

The causal structure of the TREND function is given in figure 1. The TREND function can be thought of as an information processing procedure which takes as input a variable (including its past values) and produces as output a judgment of the current trend in the input variable:

$$\mathrm{TREND}_t = f(\mathrm{INPUT}_T) \qquad T \in (t_0, t). \tag{1}$$

The expected growth rate TREND is a state variable whose derivative is:

$$(d/dt)\mathrm{TREND}_t = (\mathrm{ITREND}_t - \mathrm{TREND}_t)/\tau_{PT} \tag{2}$$

where

TREND	=	expected trend in input variable (1/years)
ITREND	=	indicated trend in input variable (1/years)
τ_{PT}	=	time to perceive trend (years)
INPUT	=	input variable (input units).

The value of TREND is the expected rate of change in the input variable, expressed as a fraction of the input variable per time unit. It is assumed that the trend perceived and acted upon by decisionmakers adjusts adaptively to the trend indicated by the most recently available data, given by ITREND. First-order information smoothing is assumed.

The lag in the adjustment of the perceived trend represents the time required for a change in the indicated trend to be recognized and accepted by decisionmakers. The delay in the acceptance of a new trend as an operational input is often significant. The adjustment lag depends not only on the time required for individual decisionmakers to recognize the change, but on organizational inertia: a new trend may have to become part of the "conventional wisdom" before some are willing to act.

$$ITREND_t = [(PPC_t - RC_t)/RC_t]/\tau_{HRC} \quad (3)$$

$$(d/dt)PPC_t = (INPUT_t - PPC_t)/\tau_{PPC} \quad (4)$$

where

ITREND	=	indicated trend (1/years)
PPC	=	perceived present condition (input units)
RC	=	reference condition (input units)
τ_{HRC}	=	time horizon for reference condition (years).
τ_{PPC}	=	time to perceive present condition (years).
INPUT	=	input to trend function (input units)

The indicated trend is given by the difference between the perceived present condition of the input and its average value over some historical horizon (the reference condition), expressed as a fraction of the reference condition and annualized by the time horizon between the perceived present condition and the reference condition. The indicated trend depends not on the true value of the input variable but on the perceived present condition, which is an exponential smooth of the raw input. The smoothing represents two factors. First, assessing current status takes time. There is an inevitable delay in measuring the input variable and disseminating information about its recent values. In the case of corporate and aggregate economic data, the data collection and reporting lag may range from several weeks to a year. In the case of demographic, resource, or environmental data, the delays may be even longer. Second, even if the raw data were available immediately, smoothing is desirable to filter out high frequency noise in the raw values. Such noise arises from both the processes themselves, from measurement error, and from subsequent revisions in the reported data. The extent of noise in one common economic variable is shown in figure 2, the rate of inflation in the U.S. consumer price index (CPI). The CPI is reported monthly. Between 1947 and 1986 the standard deviation of inflation from month to month is 111% of its mean value, clearly showing the need to filter.

The reference condition RC is also a state variable:

$$(d/dt)RC_t = (PPC_t - RC_t)/\tau_{HRC}. \quad (5)$$

The reference condition of the input reflects the value of the input at some time in the past. The time horizon for the reference condition τ_{HRC} determines the relevant historical period considered in the forecasting process. Equivalently, $1/\tau_{HRC}$ is the rate at which old information is discounted. The reference condition is computed by smoothing the perceived present condition. Note that the reference condition is based on the perceived present condition rather than the input variable itself. The raw values are not available to decisionmakers – in most cases, only averaged data, reported after a significant collection delay, are available.

The judgment of the trend in a variable is often subjective, and strongly conditioned by experience. Thus, the time horizon for establishing the reference condition may reflect the memory and experience of individual decisionmakers. For example, managers whose professional experience was shaped by the high-growth decades of the 1950s and 60s may continue to forecast high growth despite the low actual growth rates of the 1970s and 80s. Their judgment may reflect a belief that the past "few years" are an aberration and the economy will soon resume the growth rate that characterized the past. In such a case, perceived trends may change only as fast as management turns over and is replaced.

The appendix describes the transient behavior of the TREND function and shows it produces unbiased estimates in the steady state of exponential input growth.

Example I: Inflationary Expectations

Twice a year since 1946 the Philadelphia-based financial columnist Joseph Livingston has conducted a survey of academic, business, and government economists. One of the survey questions solicits forecasts of the Consumer Price Index 6 and 12 months ahead (Carlson 1977). The imputed inflation forecasts have been extensively analyzed in the economics literature.[3] Figure 3 compares the 6-month and 12-month forecasts to the actual inflation rate for the corresponding period. Actual inflation was quite volatile from the end of World War II through the Korean war. Inflation was low during the late 1950s and early 60s. Between the mid 60s and 1980, inflation generally accelerated, and fluctuated substantially over the business cycle. Since 1981 inflation has fallen dramatically. Comparing the forecasts against the actual outcome highlights:

1. *Bias:* the forecasters consistently underpredict inflation, particularly during the 1960s and 1970s, when inflation accelerated.
2. *Phase shift:* the peak (trough) of the expected inflation rate lags the peak (trough) of the actual inflation rate. Forecasters consistently missed the turning points in inflation caused by the business cycle.
3. *Attenuation:* the actual rate of inflation fluctuates significantly over the business cycle, particularly in the 1970s and 1980s. The amplitude of the forecasts is substantially less than that of actual inflation.

The bias suggests a steady-state error as inflation accelerated in the 1960s and 1970s. The phase shift and attenuation are characteristic of simple linear filters such as exponential smoothing.

In most modeling situations actual expectations data are unavailable and the analyst must use prior estimates of the parameters. To test this procedure, the analysis of the Livingston data was performed entirely with prior estimates of the parameters of the TREND function. (A formal parameter estimation procedure is used in the second example, involving energy demand forecasts.) The values chosen to model the 6 month forecasts are τ_{PPC}=2, τ_{HRC}=12, and τ_{PT}=2 months. The Livingston forecasts are dated June and December of each year. Carlson (1977) shows that due to lags in reporting the CPI and in the time required to conduct the survey the Livingston panel make their forecasts knowing the CPI only through April and October, respectively. Thus there is a two-month delay in perceiving the current value of the index. The prior value of τ_{HRC}=12 months was selected as follows. The raw inflation data (figure 2) are dominated by high-frequency (monthly) noise. Six and 12 month forecasts should not be overly sensitive to monthly changes in inflation that may be revised or reversed next month. For professional reasons (consistency) and cognitive reasons (minimizing dissonance) forecasters are unlikely to revise their expectations dramatically from month to month despite the volatility of the monthly data. A smoothing time of 12 months attenuates 97% of the month-to-month noise yet will pass through 63% of a change in trend within one year (Forrester 1961, 417). The value of τ_{PT}=2 months implies respondents' beliefs adjust nearly completely to a change in the indicated trend within 6 months.[4] One would expect τ_{HRC} and τ_{PT} to be slightly longer for the 12-month forecasts.[5]

Figure 4 shows the simulation results.[6] The TREND function reproduces the bias, attenuation, and phase shift apparent in the actual forecasts. But the simulated forecasts are high on average compared to the Livingston data. In fact the TREND function yields a better forecast than the Livingston panel! The mean absolute error (MAE) between simulated and actual forecasts is .014, and the root mean square error (RMSE) .0195 (table 1). The Theil inequality statistics (Theil 1966, Sterman 1984) were used to decompose the mean square error (MSE) into three components: bias, unequal variation, and unequal covariation between the simulated and actual forecasts. Fully forty percent of the MSE is caused by bias. The remainder is due to unequal covariation, meaning 60% of the MSE is unsystematic. The unequal variation term is virtually zero (the two series have equal variances).

Two interpretations of the bias may be offered. First, the actual forecasting process used by the Livingston panel may be more sophisticated than the univariate TREND function. Other economic variables may be considered. Likely candidates include money supply growth, government budget deficit, and the unemployment rate, as assumed in Caskey (1985). In addition, different information processing routines may be used. Caskey assumes (on faith and without test) the Livingston forecasters follow Bayes' rule when updating their forecasts. However, such a theory must explain why more sophisticated information processing and use of economic theory produces results decidedly inferior to univariate trend extrapolation.

Alternatively, the Livingston panel may suffer from two common and closely related judgmental errors identified in behavioral decision theory.

1. *Anchoring and adjustment:* People often judge an unknown quantity by first recalling a known reference point and then using additional cues to adjust the base by some 'stretch factor'. The advantage of the anchoring and adjustment strategy is its simplicity and intuitive appeal. The disadvantage is the common tendency to underpredict — to revise prior judgments too little when faced with new data. Judgments are often unintentionally anchored to reference points that are implicit (such as even odds in a bet, or the axis of a graph). Judgments exhibit anchoring even when the irrelevance of the anchor to the judgmental task at hand is made salient to the subjects (Tversky and Kahneman 1974, Hogarth 1980).

2. *Conservatism:* BDT research supports the existence of judgmental conservatism in which subjects fail to follow Bayes theorem when updating beliefs (see references above). Typically, each new observation results in one-fifth to one-half of the adjustment indicated by Bayes rule (Edwards 1968).

Though the two judgmental errors are closely related, they have distinct dynamic consequences. A conservative panel which updates its beliefs insufficiently would have longer time constants τ_{HRC} and τ_{PT} compared to a Bayesian panel. Lengthening the time constants does lower the forecasts produced by the model by slowing the rise of expectations during the inflationary 60s and 70s. But it also introduces more phase lag, causing the peaks of the simulated forecasts to lag the peaks of the actual forecasts, and it reduces the variance of the simulated forecasts below that of the actual forecasts.[7] These considerations suggest the problem is not statistical conservatism but the presence of an anchor which biases the forecast downward from the values indicated by extrapolation of the recent inflation rate.

The anchoring and adjustment strategy can be modeled as follows: suppose the Livingston panel forms inflationary expectations as:

$$_t\pi_{t+h} = (1-\alpha)*\mathrm{TREND}_t + \alpha*\mathrm{ANCHOR}_t \qquad (6)$$

where

$_t\pi_{t+h}$	=	expectation at time t of average (exponential) inflation rate between t and t+h (1/years)
h	=	forecast horizon (years)
TREND	=	expected inflation rate generated by TREND function (1/years)

ANCHOR = underlying anchor on inflation forecast (1/years)
α = weight on anchor (dimensionless).

In equation 6, the simulated Livingston forecast is a weighted average of the TREND function with an anchor. The parameters of the TREND function are the same as those used in figure 4. The anchor can be thought of as an underlying reference point which the panel uses, consciously or unconsciously, when forecasting.

The simplest assumption is the 'fixed-anchor' model in which ANCHOR= 0. Zero price change is a natural choice for the anchor: zero change is the simplest naive model; further, plots of the past rate of inflation are likely to show the x-axis at π=0, possibly leading to an unconscious bias towards the axis.[8] Equation 6 then reduces to:

$$ {}_t\pi_{t+h} = (1-\alpha)*\text{TREND}_t \qquad (7)$$

which implies forecasters will always underpredict the magnitude of inflation. Figure 5 shows the fit of the 'fixed-anchor' model using a weight of .20. The fit is improved substantially compared to the 'no-anchor' model. The MAE falls by 29%, the RMSE by 21%. The Theil statistics show the bias is reduced to 8% of the MSE, with the bulk of the remaining error caused by unequal covariation.

The anchoring and adjustment model fits the forecasts well. But clearly, if $\pi>0$ for extended periods, forecasters should learn to expect continuing inflation and adjust ${}_t\pi_{t+h}$ upward, as argued in Jacobs and Jones 1980. The fact that the 'no-anchor' model is generally high between 1947 and 1983 suggests the panel's judgments were biased by a feeling that the underlying inflation rate was lower than the actual rate of inflation. But the underestimation by the 'no-anchor' model after 1983 suggests the anchor had risen during the high-inflation 70s causing the panel to continue to forecast high inflation in the mid 80s.

In the 'sea-anchor' model the anchor is specified by the TREND function, but with much longer parameters:

$$\text{ANCHOR}_t = \text{TREND}^A(\tau^A{}_{PPC}\tau^A{}_{HRC}\tau^A{}_{PT}) \qquad (8)$$

where

TREND^A = TREND function for formation of anchor

τ^A_{PPC} = Time to perceive present condition for anchor (years)
τ^A_{HRC} = Time horizon for reference condition for anchor (years)
τ^A_{PT} = Time to perceive trend for anchor (years).

The anchor should respond to changes in the underlying inflation trend but not to the business cycle swings in inflation. The parameters τ^A were chosen to reflect the long-term nature of the anchor: $\tau^A_{PPC} = 1$ year, $\tau^A_{HRC} = 10$ years, and $\tau^A_{PT} = 3$ years. These values are long enough to significantly attenuate a signal characteristic of the 3 - 7 year business cycle (Forrester 1961, 417). The initial value of the anchor was set to -.03%/year, implying the panel's judgments were initially biased towards mild deflation (many economists, recalling the deflation of the Great Depression and the recession and falling prices that followed World War I, worried that the U.S. would return to depression after World War II). The weight on the anchor was set to .25. Figure 6 compares the simulated and actual forecasts and figure 7 shows the components of the simulated forecast. Note that the anchor reduces the forecasts until 1983, when inflation falls substantially. The anchor then keeps the simulated forecast high, improving the fit between 1983 and 1985.

The 'sea-anchor' model is theoretically more satisfying and also more robust, as it allows for learning: if inflation remains steady the model will eventually produce unbiased forecasts (as seems to have occurred between 1958 and 1965). Similarly, in a hyperinflation the fixed-anchor model would seriously underpredict inflation, while the sea-anchor model would 'learn' to expect hyperinflation. The sea-anchor model reduces the MAE by another 11%. The MSE is still primarily unsystematic.

Example II: Energy Demand Forecasts

Since 1973 estimates of future energy consumption in the United States have fallen dramatically (figure 8). The drop in forecasts coincided with a marked slowdown in the growth of actual consumption. Forecasts made as recently as 1974 projected consumption in 1985 to be near 130 quadrillion BTUs (quads). Actual energy consumption in 1985 was less than 74 quads. In like fashion, forecasts of consumption in 2000 have fallen by nearly a factor of two since 1973. The large errors and seemingly reactive nature of the forecasts suggest trend extrapolation may have been used in many of the forecasts. Trend extrapolation, however, seems naive to many observers, who point out that energy demand forecasts are often the result of extensive studies involving detailed, multidisciplinary analysis.[9] How can trend extrapolation be used to proxy such complex and subtle judgments?

The TREND function provides the expectation of the growth rate in the input variable at the current moment in time. To produce a forecast of the input's value at some point in the future one must assume some degree of persistence. For example, one might assume that the current fractional growth rate in the input will continue throughout the forecast horizon. Alternatively, one might assume that the rate gradually approaches some more fundamental reference, that growth will be linear rather than exponential, or that the variable itself asymptotically approaches some limit.

The model of energy consumption used here assumes continued exponential growth in primary energy consumption at the currently perceived rate:

$$ {}_{t}FC_{FY} = PPC_t*(1+TREND_t*\tau_{PPC})*\exp[TREND_t*(FY-t)] \quad (9) $$

where

${}_{t}FC_{FY}$	=	Forecast made in year t of Consumption in Forecast Year (Quads/year)
FY	=	Forecast Year (year)
PPC	=	Perceived Present Condition (consumption) (Quads/year)
TREND	=	Expected Trend in Consumption (1/years)
τ_{PPC}	=	Time to Perceive Present Condition (years).

Note that equation 9 assumes forecasters recognize that it takes time to perceive the input and that they adjust for the growth they believe has occurred between the time consumption was measured and the present. In consequence, eq. 9 produces accurate forecasts in the steady state of exponential growth of actual consumption (Sterman 1986b).

Note that all parameters yield the same result in the steady state. Thus to estimate the model the actual growth rate of the input variable must vary significantly. Fortunately, the energy consumption and forecast data span a period which includes major changes in patterns of energy use, first accelerating up to 1973 and rapidly decelerating thereafter.[10]

The model is nonlinear, and the parameters were estimated with a multivariate hillclimbing program.[11] The mean absolute error (MAE) between the actual and simulated forecasts was chosen as the criterion of fit to be minimized in estimating the parameters:

$$MAE = MAE(\tau_{PT},\tau_{PPC},\tau_{HRC}) = (1/N) \sum_{t=1947}^{1985} \sum_{i} \left| {}_{t}HFC_{FY,i} - {}_{t}FC_{FY} \right| \qquad (10)$$

where

N	=	total forecasts available for forecast horizon FY
${}_{t}HFC_{FY}$	=	historical forecast in year t of consumption in forecast year (quads/year)
${}_{t}FC_{FY}$	=	simulated forecast in year t of consumption in forecast year (quads/year)
i	=	index of historical forecasts (${}_{t}HFC_{FY}$) made in year t.

To guard against the possibility of finding only local minima, the hillclimbing procedure was run from a variety of initial parameter values.

Table 2 presents the optimal parameter estimates for each forecast horizon. Note that the minimum possible error is greater than zero because there are often several different forecasts for each year. The MAE is compared against the mean absolute deviation of the historical forecasts. The mean absolute deviation (MAD) is computed exactly as in equation 10 but replacing the simulated forecast with the *median* of the historical forecasts for each year. Since the median minimizes absolute deviation, the MAD of the historical forecasts is the lower bound on the MAEs reported in the table.

The optimal parameters for 1980 and 1985 produce MAEs quite close to the lower bound. As a percentage of the mean historical forecasts, the increase in MAE over the MAD is just 5 and 2 percent for 1980 and 1985, respectively. Figures 9, 10, and 11 compare the simulated and actual forecasts for each forecast horizon using the optimal parameters. The simulated forecasts for 1980 are somewhat low before 1965 but are a good fit after that date. The simulated forecasts for 1985 are an excellent fit.

However, table 2 and figure 11 show the optimal parameters for the year 2000 forecasts to be implausibly short, particularly τ_{PT} and τ_{PPC}. The sum of the three parameters for the year 2000 forecasts is substantially less than the sum for the 1980 and 1985 forecasts. It is implausible for forecasters to be more responsive to short-term variations in energy growth rates when projecting consumption to the distant horizon of 2000 compared to the much nearer horizon of 1985. Further, the short delays in assessing current consumption and reacting to changes in the growth rate mean the simulated forecast is far too volatile, swinging wildly in response to business cycle fluctuations in energy consumption. Finally, the simulated forecast is biased upward, reaching a peak of over 250 quads in 1969. Attempting to solve the problem by setting the parameters to the optimal values for 1985 results in the forecasts shown in figure 12. Here the extreme volatility of the forecasts is reduced, but the forecasts are consistently too high, reaching a peak of 225 quads in 1971. The MAE is 33 quads, double the mean absolute deviation of the historical forecasts.

The overestimation of the year 2000 forecasts is curious in light of the unbiased 1985 forecasts. The forecasting procedure in equation 9 presumes a continuation of exponential growth at the currently perceived rate throughout the forecast horizon. For forecasts of consumption over shorter horizons such as 1980 and 1985 the assumption of uniform exponential growth is clearly more likely to be valid than for forecasts over an additional 15 years. As with inflation expectations two interpretations can be offered. First, it may be that the forecasters, through complex reasoning and application of economic theory, recognized that continued exponential growth at historical rates was unlikely over such an extended time frame and adjusted the assumed growth rate downward, particularly in the later years. An alternative interpretation grounded in behavioral decision theory would suggest a downward bias introduced as exponential growth projects energy consumption progressively farther from its current level.

Biases in judgment and forecasting, particularly in forecasting exponential growth, are well known and amply documented elsewhere (see Armstrong 1985, Phillips and Edwards 1966, Wagenaar and Timmers 1979, Tversky and Kahneman 1974).

To test for the presence of bias equation 9 was modified to assume a linear rather than exponential extrapolation of current energy consumption growth:

$$ {}_{t}FC_{FY} = PPC_t*(1+TREND_t*\tau_{PPC})*[1+TREND_t*(FY-t)]. \qquad (9') $$

The linear extrapolation does not necessarily mean forecasters believe energy growth to be a linear process. A more likely interpretation is simply that they expect the fractional rate of growth of consumption to decline in the future, resulting in a roughly linear path. The optimal parameters for the revised model are also presented in table 2. The linear model generates parameters which are similar to those for the shorter forecast horizons. The MAE is 19.9 quads, an increase over the MAD of 3 percent of the mean forecast. Figure 13 shows the revised model virtually eliminates the bias and captures the decline in the forecasts quite well.

Are forecasters also biased for the nearer horizons of 1980 and 1985? The answer seems to be no. Sterman 1986b shows that the linear model performs significantly worse than the exponential model for the 1980 and 1985 horizons, consistent with the hypothesis that the forecasters assume the fractional growth rate to diminish as forecasted consumption becomes larger.

Interpreting the Results

The results demonstrate that univariate trend extrapolation is an adequate model of actual expectation formation processes in two quite different domains. Past inflation explains short-run forecasts of inflation. Past growth in energy consumption can explain the history of energy demand forecasts, for three distinct forecast horizons.

Do the results imply that forecasts are actually made by trend extrapolation or only that they can be mimicked by trend extrapolation? Consider the inflation forecasts. The univariate trend model does not prove that inflationary expectations are in fact purely adaptive or that past inflation is the only input to the forecast. But the excellent correspondence between the simulated and actual forecasts shows that other variables have only a weak effect on the forecast. Such a conclusion is consistent with behavioral decision theory. People prefer relatively certain information over uncertain, noisy information. The future values of potentially relevant variables such as monetary policy, unemployment, economic growth, exchange rates, and budget deficits, are themselves highly uncertain and difficult to forecast. There is substantial disagreement among economists about the nature of the relationships between these variables and the rate of inflation. Further, people are incapable of correctly deducing the consequences of intricate dynamic systems such as the economy and tend instead to process information with simple, incomplete, and erroneous mental models (Sterman 1986a). Cues other than inflation itself are therefore likely to be heavily discounted in the forecasting process.[12]

Decision aids such as econometric models do not resolve the problem since the modeler's judgment is always needed to specify the model structure and the future values of the exogenous variables. In fact, the forecasts of most econometric models are heavily 'add-factored' or adjusted ad hoc by the modelers (Sterman 1985b). The practice is defended by the modelbuilders since they are able to bring their expert knowledge and intuition to bear, overcoming limitations of the models and taking the latest data into account. But experience and expertise are not proof against error. It has been frequently shown that experts are prone to many of the same judgmental biases observed in the public at large (Tversky and Kahneman 1974, Kahneman, Slovic, and Tversky 1982). Indeed, Caskey (1985) shows the Livingston and DRI forecasts of inflation are virtually identical.

The dominance of trend extrapolation over formal models is also apparent for the energy demand projections. As noted, forecasts of energy consumption have been made with a wide range of techniques and models. Many of these models are quite complex and do not appear to be simple univariate extrapolations. Yet regardless of the level of sophistication, each model relies upon exogenous variables or parameters, and for at least some of these there will be no strong theory to guide the forecaster in estimating their future values. To illustrate, the univariate model used here could be improved by using a model that determines energy consumption in terms of more fundamental economic forces. Two such models are:

$$\ln(CONS_t) = \ln(EGR_t)+\ln(GNP_t) \tag{11}$$

$$\ln(CONS_t) = a_1+a_2\ln(GNP_t)+a_3\ln(P_t) \tag{12}$$

where

CONS	=	energy consumption (quads/year)
EGR	=	energy/GNP ratio (quads/\$)
GNP	=	real GNP (\$/year)

P	=	average real energy price ($/BTU)
a_1, a_2, a_3	=	regression coefficients.

The model in (11) posits energy demand as a function of GNP and the energy/GNP ratio. The model in (12) allows the energy/GNP ratio to vary with energy prices by defining energy consumption in terms of standard income and price elasticities. Such models are easily estimated and utilize more economic theory than the simple univariate trend forecast used in the simulations. But one must still forecast the exogenous variables. Trend extrapolation is likely to be a dominant input to the forecasts of those exogenous variables. Elaborating the model of energy consumption does not remove the need for trend extrapolation at some level. Indeed, many of the studies whose forecasts are reported in figure 8 relied on large, complex, and costly models. Yet, in all these models there are exogenous variables which must be forecast. Whether these are GNP and the energy/GNP ratio, population growth and assumed energy per capita, or population growth, assumed future technical progress, and assumed future energy prices, there is always at least one such exogenous variable for which theory provides no strong guidance. Such inputs serve as free parameters which can be used to manipulate the forecasts to be consistent with the conventional wisdom of the time. The correspondence of the simulated and actual forecasts suggests trend extrapolation acts as a strong constraint upon choice of these "fudge factors".

Conclusion

Proper representation of the expectation formation process is of paramount importance in behavioral simulation. This paper shows it is possible to test the expectation formation process assumed in system dynamics models. The results support the use of adaptive expectations and trend extrapolation. The results also suggest that such empirical analysis can reveal the presence of additional judgmental heuristics which can have dynamic and policy significance.

Short-term inflation forecasts spanning nearly forty years are explained well by extrapolation of past price trends. However, the expected inflation rate appears to be anchored towards an underlying expectation of secular inflation, resulting in underprediction and attenuation of the business cycle peaks in actual inflation. Similarly, energy demand forecasts made between the late 1950s and early 1980s were examined. Forecasts of consumption in 1980 and 1985 seem to be simple exponential extrapolations of past energy consumption itself. But for forecasts made during the same period to the more distant horizon of the year 2000, the results strongly suggest a substantial downward bias. In particular, forecasters projected growth that is roughly linear rather than exponential.

Detailed examination of the methodology and behind-the-scenes reasoning of the individual forecasters would be required to determine if the errors resulted from explicit calculation or from inadvertent psychological biases.

The results, particularly the discovery of biases and conservatisms in the forecasting process, have clear significance for research in system dynamics and policy modeling. The implications of the judgmental biases for policy modeling of energy and macroeconomic dynamics are obvious. Equally important, strong empirical support for these biases is found in the literature on individual decisionmaking behavior. Analysts should and can take advantage of this literature in the formulation and testing of models of aggregate behavior. Further development of the relationships between behavioral decision theory and system dynamics would appear to be a fruitful direction for research.

In terms of modeling practice, two approaches to parameter estimation were used. For energy demand, a formal estimation procedure was used. The estimated parameters do not seem unreasonable given known lags in the reporting of energy consumption data, the need to filter out much of the business-cycle variation in energy consumption, and knowledge of the transient response of the TREND function. For the inflation forecasts, a-priori parameters were used and lead to excellent agreement between model and data. While parameters should always be estimated if data exist, the results suggest modelers can be reasonably confident using a-priori estimates of the parameters when actual expectations data are unavailable, as is usually the case in behavioral modeling.

The complexity of human systems is steadily growing, while our cognitive capabilities and limitations remain unchanged. Complex models may be useful, even necessary, for policy design and evaluation, for representing and reconciling alternative viewpoints, or for developing theoretical understanding. Unfortunately, models are all too often used to forecast and react to difficulty rather than as a laboratory for the design of robust and resilient systems. The results presented here call into question the wisdom of the forecasting orientation, and particularly the utility of large, complex models for *forecasting*. The cost and effort required to use such models for forecasting has not proven to be commensurate with their forecast accuracy when compared to far simpler and less expensive methods. It is hoped the research reported here will foster a much-needed redirection of modeling away from the prediction of events and towards the design of robust systems which are less likely to generate problematic behavior.

Appendix: Behavior of the TREND function

To be a reasonable model of growth expectation formation, TREND should produce, in the steady state, an accurate estimate of the growth rate in the input variable. That is, if

$$INPUT_t = INPUT_{to}*\exp(g*(t-t_o)) \qquad (13)$$

then $\lim_{t \to \infty} TREND_t = g.$

The proof relies on the fact that the steady-state response of a first-order exponential smoothing process to exponential growth is exponential growth at the same rate as the input. In the steady state, however, the smoothed variable lags behind the input by a constant fraction of the smoothed value. The equation for a first-order smoothing process is:

$$(dy/dt) = y' = (x-y)/AT \qquad (14)$$

where AT = the adjustment time of the process. The steady-state solution of equation 14, for the case of an exponentially growing input ($x = x_0\exp(g(t-t_0))$, x_0 and t_0 = initial values, g = growth rate), can be found in most introductory differential equations texts:

$$y = x/(1+gAT). \qquad (15)$$

That is, the smoothed variable lags the input with a steady-state error proportional to both the growth rate of the input and the average lag between input and output. The solution can be verified by substitution in the differential equation (14).

In the TREND function, PPC is a smooth of INPUT, so in the steady state, PPC will be growing exponentially at rate g. Since RC is a smooth of PPC, it will also be growing at rate g. Therefore, RC'/RC = g. But by equation (5)

$$RC' = (PPC-RC)/\tau_{HRC} \qquad (16)$$

so

$$g = [(PPC-RC)/\tau_{HRC}]/RC = ITREND. \qquad (17)$$

Since TREND is a smooth of ITREND, TREND = ITREND = g in the steady state. Thus, in the steady state, TREND yields an unbiased estimate of the exponential growth rate in the input variable.

During transients, of course, TREND will differ from the true growth rate of the input.[13] To illustrate the transient response, figure 14 shows the adjustment of the expected trend to an exponentially growing input for various values of the three parameters τ_{PT}, τ_{PPC}, and τ_{HRC}. In the example the input grows at 5 percent/year, starting from a stationary equilibrium. The true growth rate thus follows a step input from 0 to 5 percent. In all cases the response of TREND is s-shaped. The expected trend smoothly approaches the true trend from below, without overshoot. The parameters τ_{PT}, τ_{PPC}, and τ_{HRC} control the mean and shape of the distributed lag response of TREND to a change in the input's growth rate.

NOTES

[1] Behavioral simulation models are a class of dynamic models which share the following characteristics:

(i) A descriptive rather than normative representation of human behavior. Decisionmaking behavior is portrayed in terms of the heuristics and routines used by the actors in the system rather than as the behavior which maximizes utility.

(ii) The limitations of human cognitive capabilities are explicitly accounted for in modeling behavior.

(iii) The availability and quality of information is explicitly treated including possible bias, misinterpretation, distortion, and delay.

(iv) The physical and institutional structure of the system is explicit, including organizational design such as task and goal segmentation, the stock and flow networks that characterize the physical processes under study, and lags between action and response.

(v) A disequilibrium treatment is adopted, focussing on the feedback processes which cause adjustments in the face of various external disturbances.

See e.g. Simon 1982, Cyert and March 1963, Nelson and Winter 1982, Forrester 1961, Morecroft 1983, 1985, Sterman 1985, Sterman and Richardson 1985.

[2] See e.g. Mincer 1969. Adaptive expectations contrast against rational expectations in which expectations are assumed to be based on a true model of the system. See Muth 1961, Lucas 1976, Begg 1982. For critiques of rational expectations see Simon 1979, 1978, Klamer 1983, Shaw 1984 (especially Chapter 10).

[3] Caskey 1985, Peek and Wilcox 1984, Hafer and Resler 1982, Bomberger and Frazer 1981, Jacobs and Jones 1980, Pearce 1979, Mullineaux 1978, Pesando 1975. Tim Schiller of the Federal Reserve Bank of Philadelphia supplied me with the Livingston data. Livingston asks the panel to forecast the level of the CPI six and twelve months ahead. Following

Carlson (1977), I assume the panel have available to them only the level of the CPI two months prior to the date of the survey, and thus I treat the data as 8 and 14 month forecasts. The imputed inflation forecast is computed as:

$${}_t\pi_{t+h} = \ln({}_{t-2}CPI_{t+h}/CPI_{t-2})/[(h+2)/12] \qquad h = 6, 12 \text{ months.}$$

Note that this yields the continuous compounding growth rate, corresponding to the output of the TREND function. The actual inflation outcomes are computed by substituting the actual value of the CPI at t+h for the forecast. Note also that the revised CPI data are used in the analysis while the panel used the unrevised data.

4 After three time-constants a first-order smoothing process has reached 95% of its final value. The 97% attenuation is calculated by noting that the period of the highest frequency signal in a monthly time-series is 2 months.

5 But only slightly. The six month forecast determines the inflation path for the first half of the annual forecasts. It is unlikely that forecasters will project a radically different inflation rate for the second half of the forecast year. In fact, the 6 and 12 month forecasts are quite similar. Longer time constants for the 12 month forecast, by placing less weight on recent data, imply forecasters expect inflation to regress towards its long-term average more completely after a year than 6 months.

6 The model is formulated in continuous time. It is simulated by Euler integration with a time step dt = 1 month, the reporting interval for inflation. The energy forecasts are simulated with a time step of .125 years, small enough so integration error is not significant.

7 Note that the argument above cannot be used to establish statistically whether the panel follows Bayes' Rule, or to estimate which parameters of the TREND function correspond to a Bayesian strategy. To do so would require knowledge of the panel's prior beliefs about the

inflation process, a subject for investigation at the micro-level of the individual panel members. The argument shows only that a statistically conservative panel would have longer time constants than a Bayesian panel with the same beliefs. The panel may in fact be conservative but such conservatism does not explain why the TREND function produces forecasts which have less bias than the panel's.

8 The graphical argument for ANCHOR=0 depends on whether the panel members plot past inflation before making their forecasts, and suggests examination of the panel's forecasting process at the micro-level would be a fruitful field study. Most graphs show the x-axis at zero rather than at other plausible locations such as the average inflation rate over the sample.

9 E.g. DOE 1983 and the studies cited in figure 8.

10 In all cases, the simulations begin in 1947 with an assumed initial growth rate of 2%/year. Given the parameter values reported below, the simulated forecasts are virtually independent of the initial growth rate by the late 1950s, when the actual forecast data begin. Energy consumption grew at 2.1%/year between 1930 and 1945 (Schurr and Netschert 1960, p. 35). The input to the TREND function is the actual consumption of primary energy in the United States (DOE 1978 and various issues of the DOE Monthly Energy Review). The actual forecast data were acquired by digitizing the data shown in figure 8, using a Macintosh computer with digitizing pad (Sterman 1986b).

11 The data and hillclimbing computer program are available from the author upon request.

12 This argument conflicts with the rational expectations position that economic agents act on the basis of (or *as if* they had) expectations which optimally process all available information. Proponents of rational expectations argue that people cannot be systematically wrong about the

future or fooled by government actions, while adaptive expectations are myopic and reactive. The empirical evidence shows that people are in fact often systematically wrong and that they fail to use information optimally. The adaptive model proposed here does not rule out learning from experience. But such learning takes time. A proper model of expectations should focus on the procedures by which people select and filter cues, process that information, and revise their beliefs if feedback on outcomes becomes available (Simon 1984).

13 The initial values of the state variables are computed so that the TREND function is initialized in steady-state with respect to an assumed initial growth rate:

$$PPC_{to} = INPUT_{to}/(1+TREND_{to}*TPPC)$$

$$RC_{to}= PPC_{to}/(1+TREND_{to}*THRC)$$

These initial conditions avoid unwanted transients in the adjustment of TREND to the actual growth of the input.

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Figure 1

Causal Structure of the TREND Function

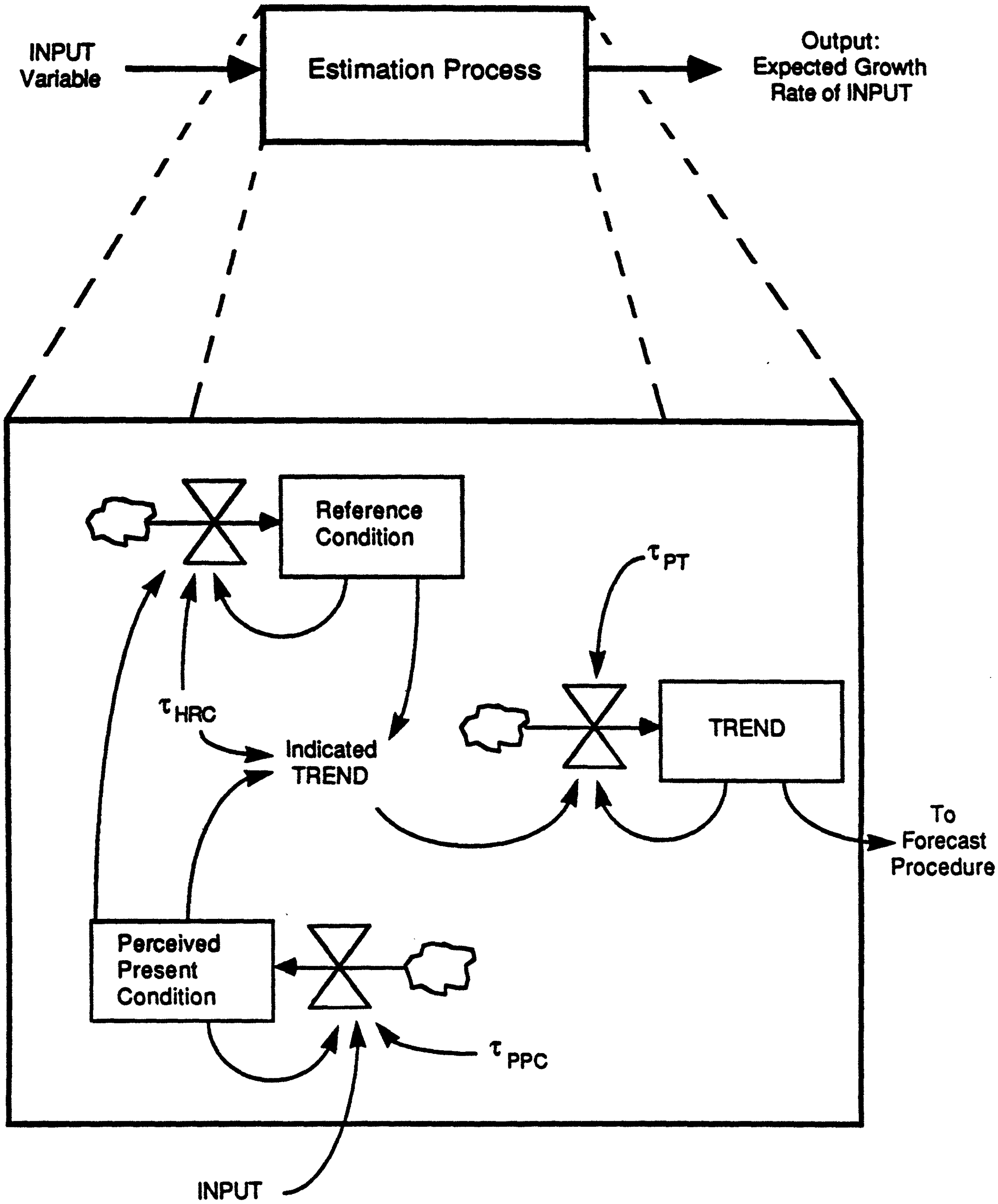


Figure 2: Consumer price inflation in the United States, 1947 to 1985
(Monthly data at annual rates).

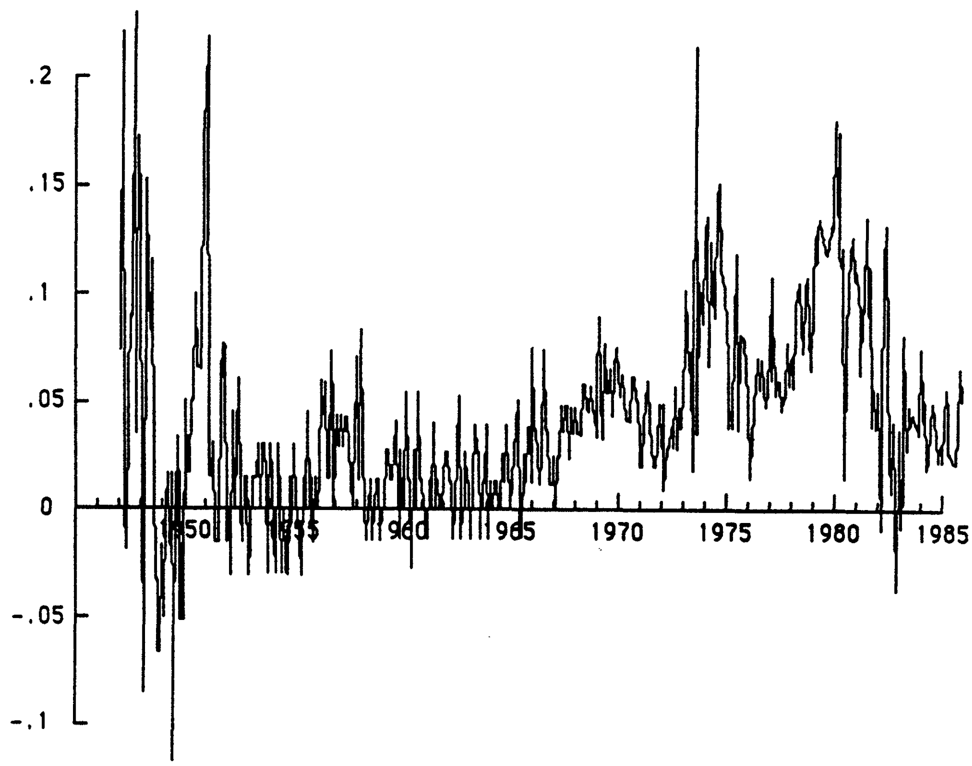
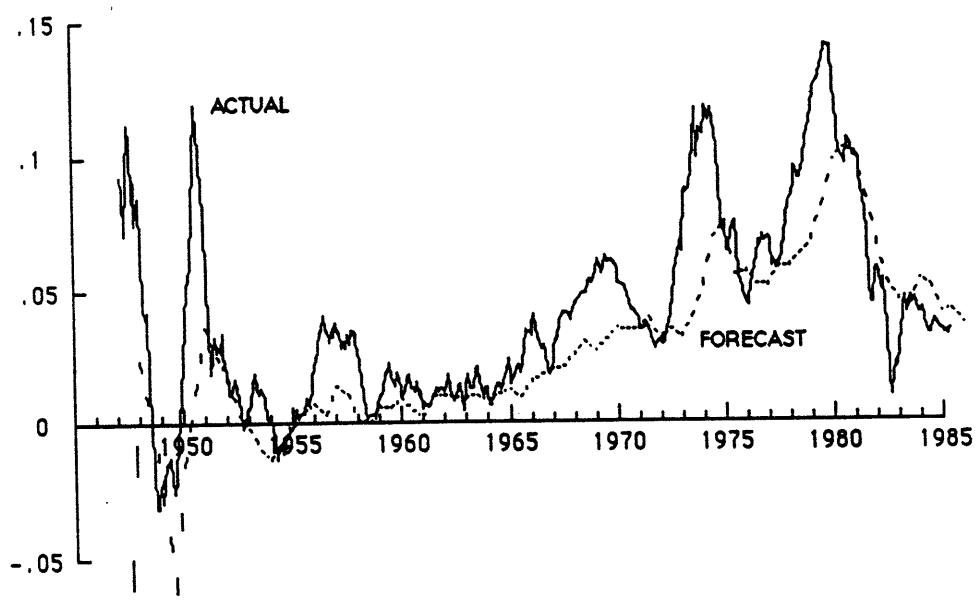


Figure 3: The Livingston panel's 6 and 12 month inflation forecasts, compared to actual inflation, 1946.6-1985.12



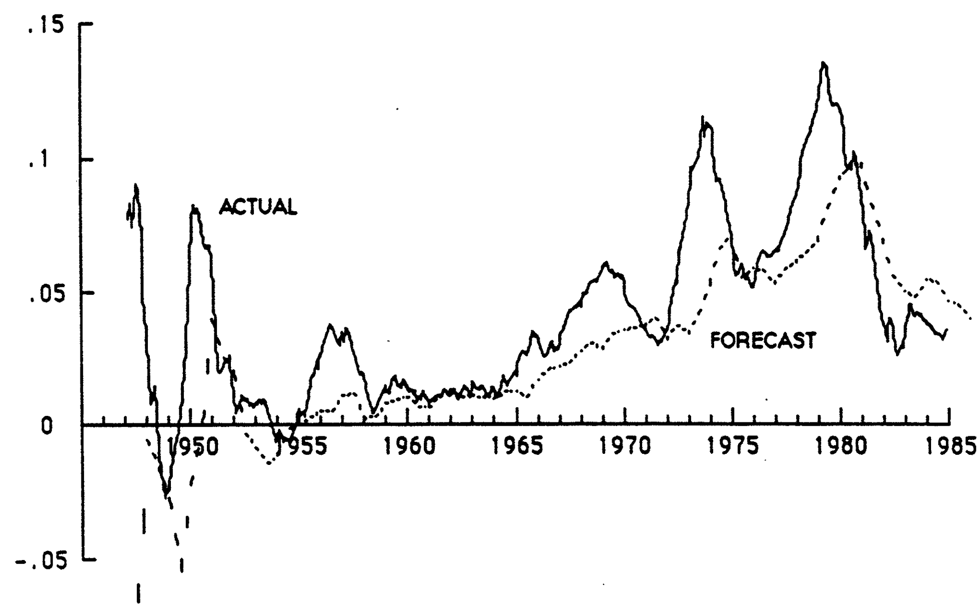


Figure 4: Simulated Livingston forecasts: 'No-anchor' model

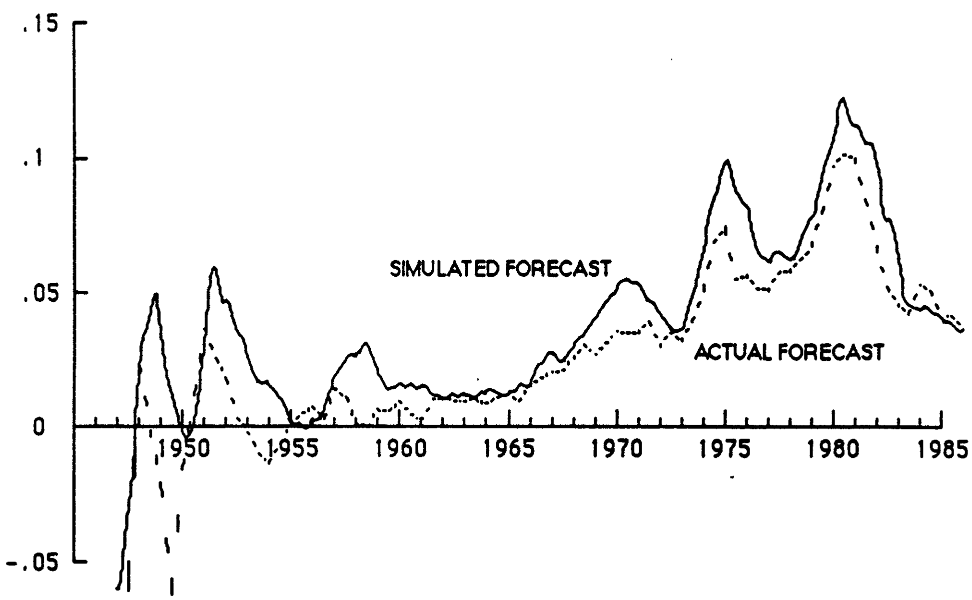


Figure 5: Simulated Livingston forecasts: 'Fixed-anchor' model

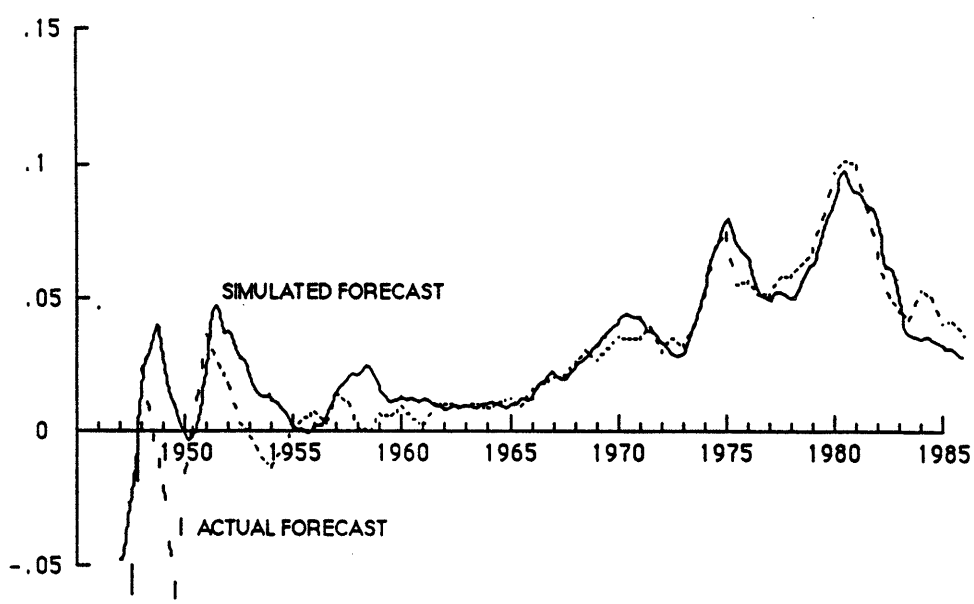


Figure 6: Simulated Livingston forecasts: 'Sea-anchor' model

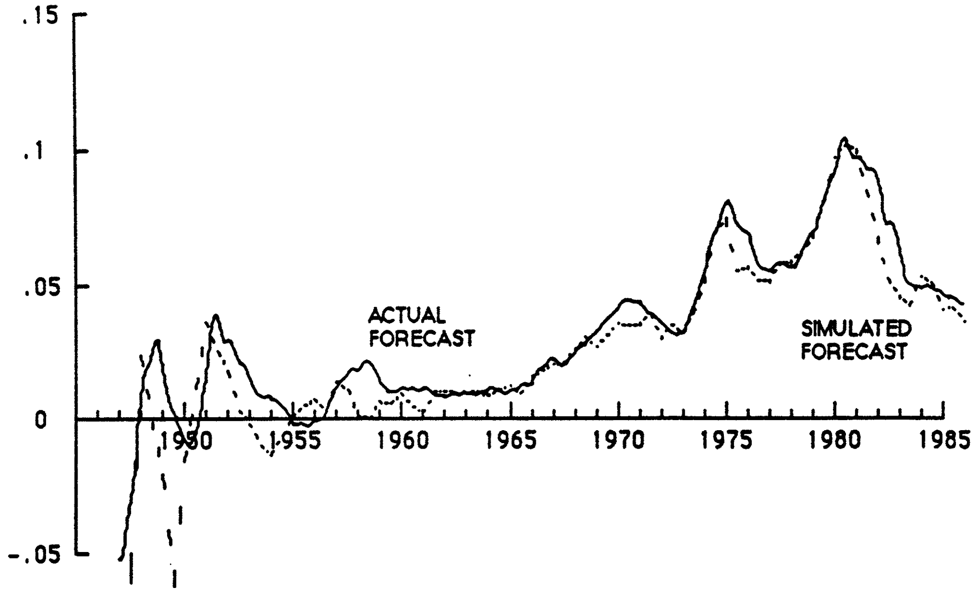


Figure 7: Components of the 'Sea-anchor' forecast

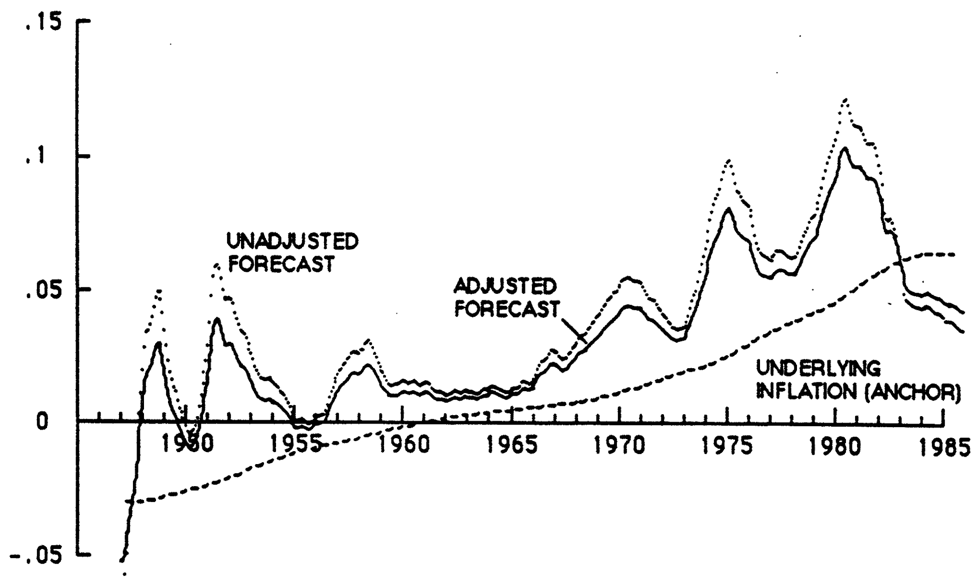


Table 1

Error analysis of simulated inflation forecasts

Model	MAE (1/years)	RMSE (1/years)	MSE $(1/\text{years})^2$	U^M	U^S (dimensionless)	U^C (dimensionless)	R
No-anchor	.0140	.0195	3.81E-4	.40	.00	.60	.88
Fixed-anchor	.0099	.0155	2.41E-4	.08	.15	.77	.88
Sea-anchor	.0088	.0138	1.92E-4	.16	.03	.81	.91

MAE = Mean Absolute Error

(R)MSE = (Root) Mean Square Error

U^M = Fraction of MSE due to bias

U^S = Fraction of MSE due to unequal variance

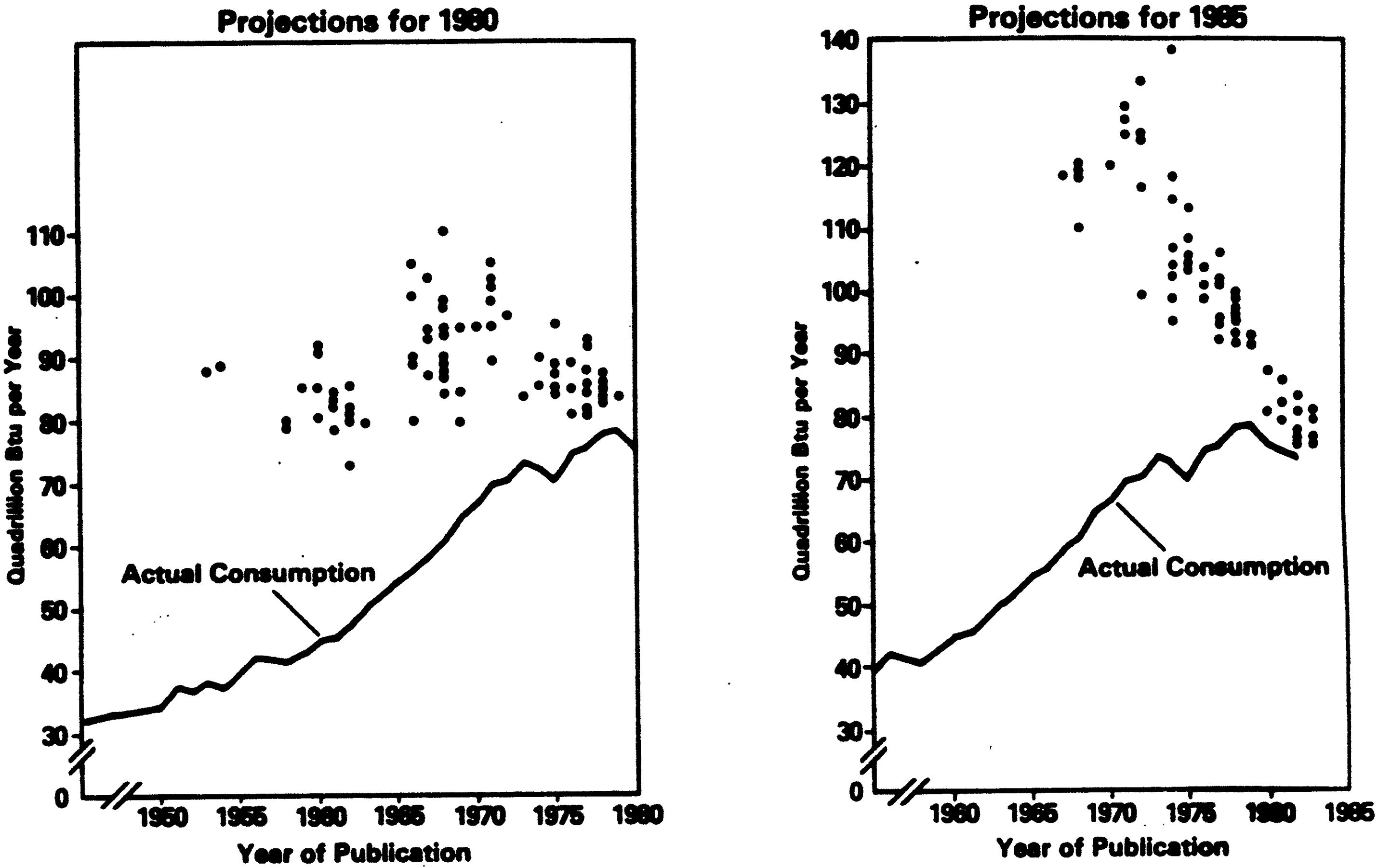
U^C = Fraction of MSE due to unequal covariance

R = Correlation coefficient between simulated and actual forecasts

Figure 8a. Source: DOE 1983, p. 7-9.

PROJECTIONS FOR U.S. PRIMARY ENERGY CONSUMPTION FOR 1980 AND 1985 VERSUS REAL PRIMARY ENERGY CONSUMPTION

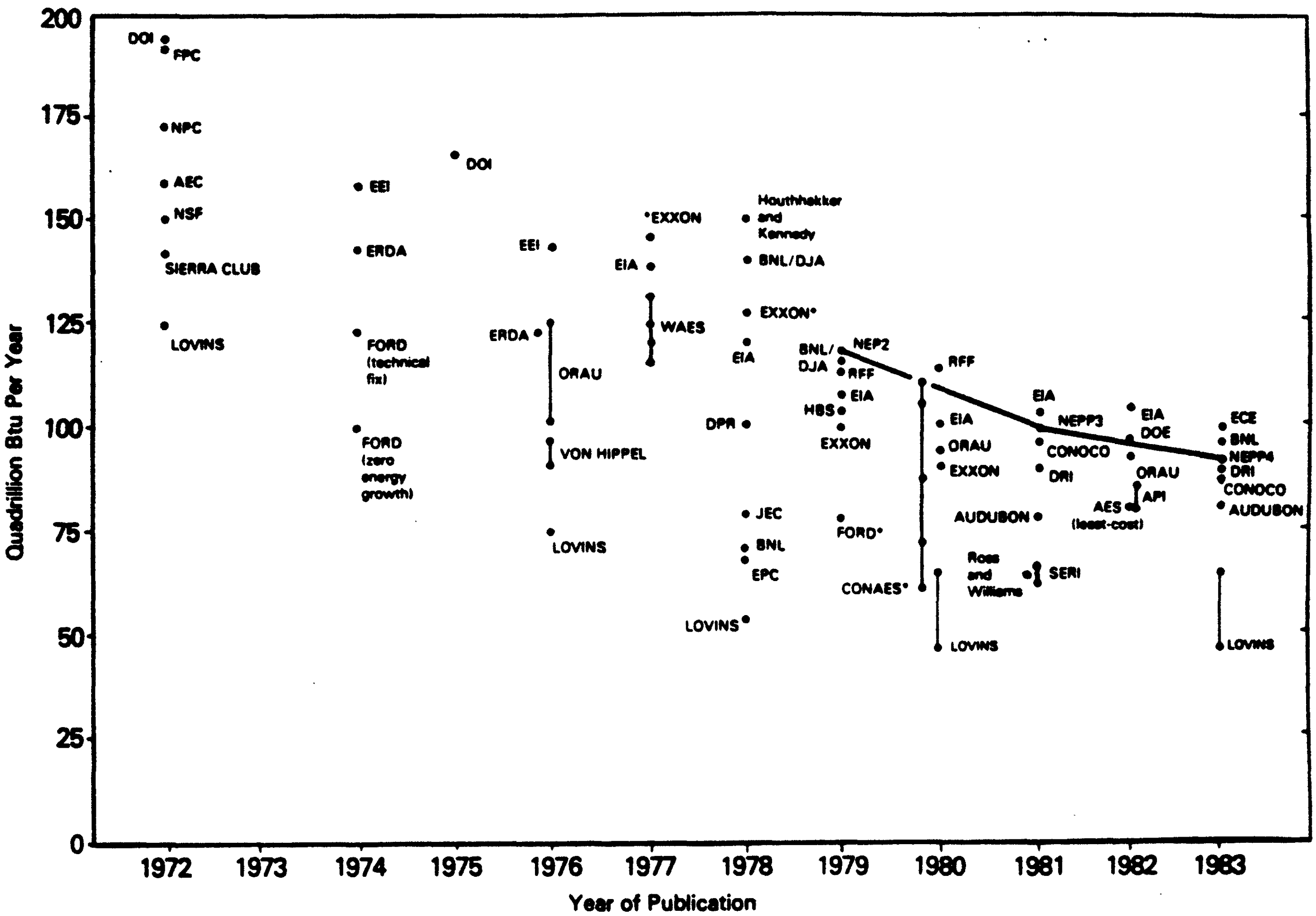
(Quadrillion Btu per Year)



Sources: Committee on Interior and Insular Affairs (1972); National Science Foundation (1972); Joint Economic Committee (1970); Committee on Science and Astronautics (1973); Committee on Energy and Natural Resources (1978); Ascher (1978); Energy Information Administration (1977-1982); Office of Policy, Department of Energy (1979-1983).

Figure 8b. Source: DOE 1983, p. 7-10.

PROJECTIONS OF U.S. PRIMARY ENERGY CONSUMPTION FOR THE YEAR 2000



Sources: Oak Ridge Associated Universities(1982) and; Department of Interior(1972); National Petroleum Council(1972); Domestic Policy Review of Solar Energy, U.S. DOE(1978); Energy Information Administration(1979-1981); Office of Policy, U.S. DOE(1979-1983); Lovins(1981); American Petroleum Institute(1982); Audubon(1983); Beltramo and Manne(1983).

*Estimates derived from the study (a year 2000 number was not reported).

Table 2

Optimal parameter estimates: energy demand forecasts

Forecast Horizon	τ_{PT}	τ_{PPC} (years)	τ_{HRC}	MAE (Quads)	MAD[a]
1980	2.7	1.3	2.7	7.2	3.4
1985	1.2	2.4	4.0	5.7	4.1
2000 model 1 [b]	0.2	0.1	5.0	23.9	16.7
2000 model 1 [b]	1.2	2.4	4.0	33.3	16.7
2000 model 2 [c]	2.0	1.7	2.2	19.9	16.7

a MAE: Mean Absolute Error between simulated and actual forecasts.

MAD: Mean Absolute Deviation between forecasts and median forecasts for each year.

b Model 1: Exponential extrapolation of expected growth rate (equation 9).

c Model 2: Linear extrapolation of expected growth rate (equation 9').

Figure 9: Simulated and actual forecasts of US primary energy consumption in 1980 (Quads/year)

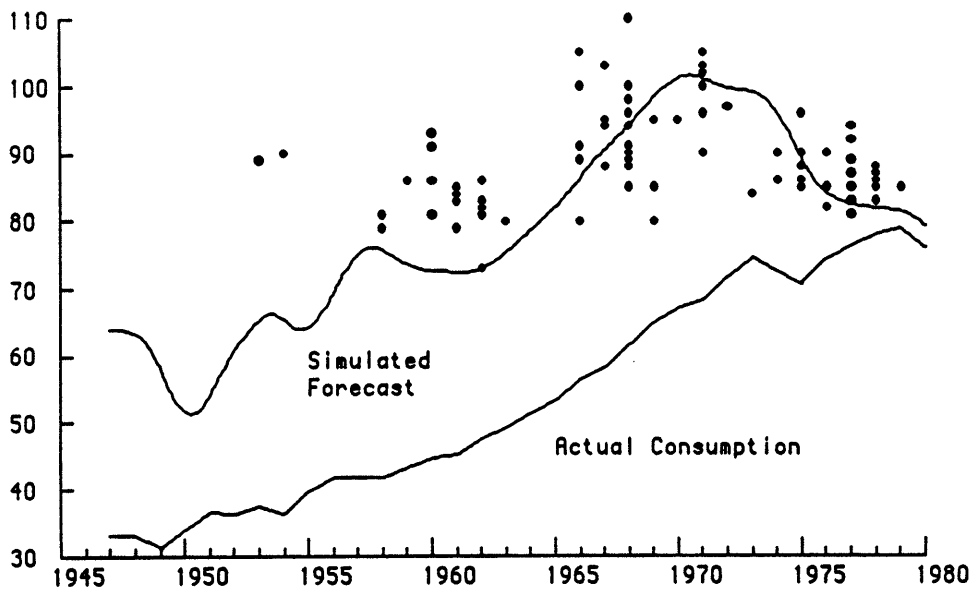


Figure 10: Simulated and actual forecasts of US primary energy consumption in 1985 (Quads/year)

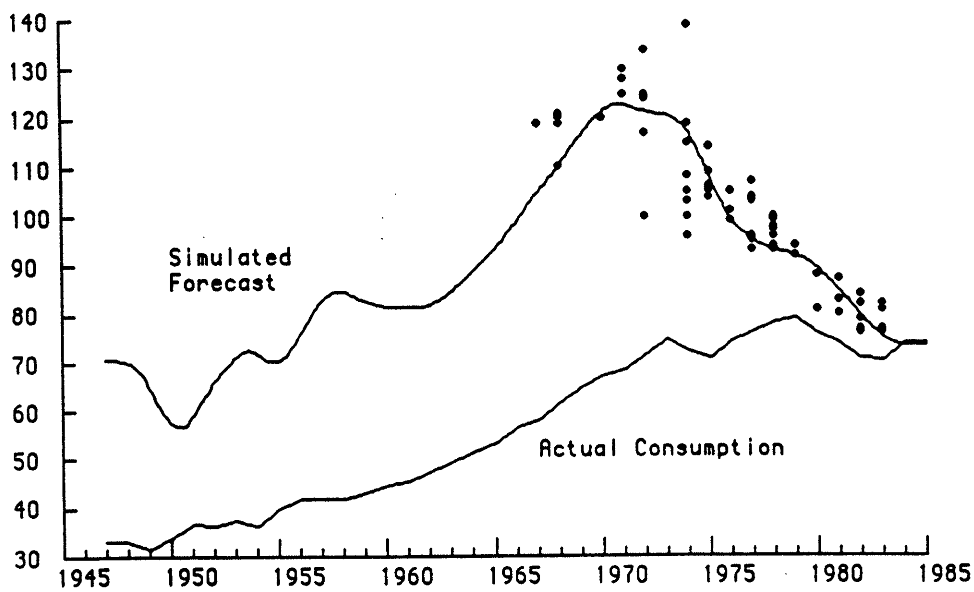


Figure 11: Simulated and actual forecasts of US primary energy consumption in 2000, exponential extrapolation (Quads/year)

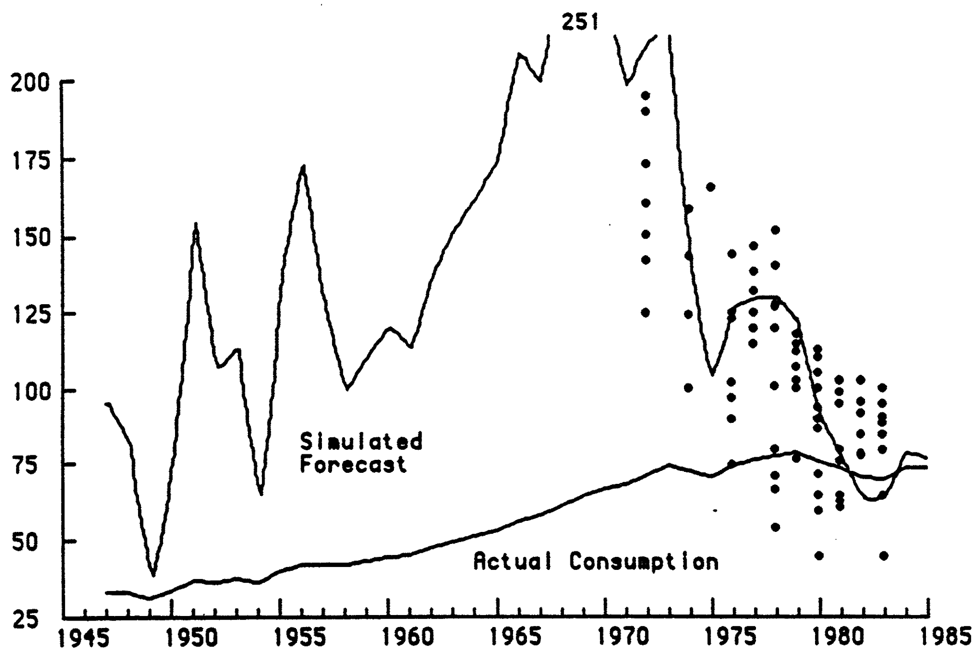


Figure 12: Simulated and actual forecasts of US primary energy consumption in 2000, exponential extrapolation with optimal parameters for 1985 (Quads/year)

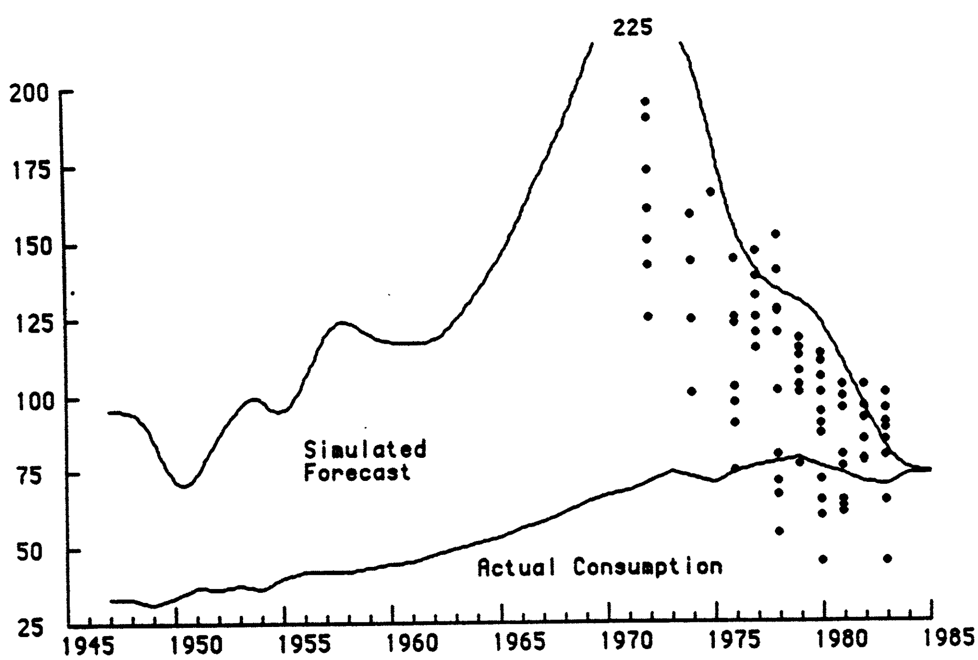


Figure 13: Simulated and actual forecasts of US primary energy consumption in 2000, linear extrapolation (Quads/year)

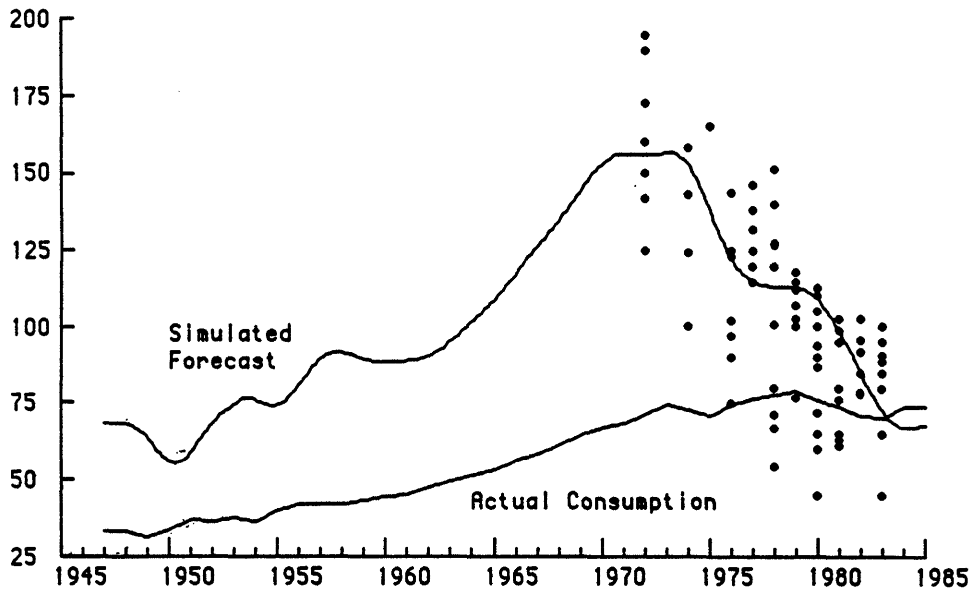
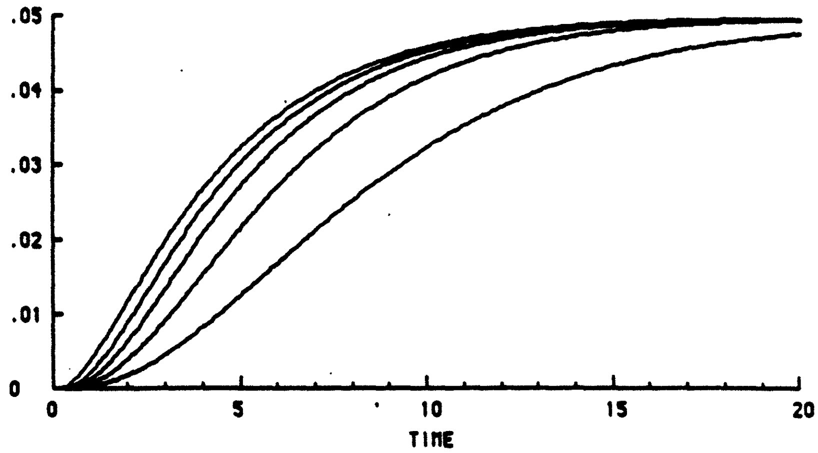
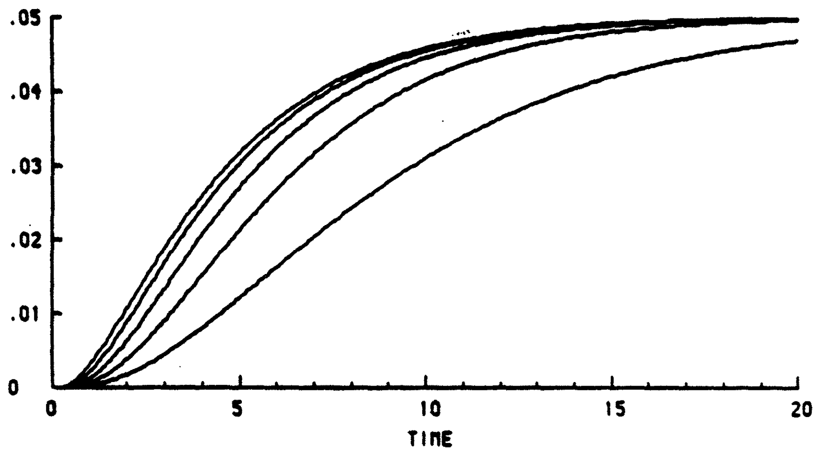


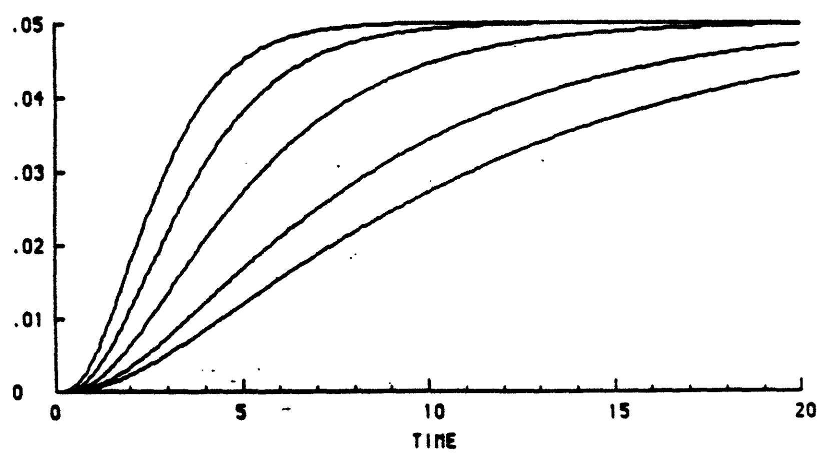
Figure 14: Response of TREND function to exponential growth in input of 5%/year. Shown for various values of the parameters τ_{PPC} (Time to Perceive Present Condition), τ_{HRC} (Time Horizon for Reference Condition), and τ_{PT} (Time to Perceive Trend).



τ_{PT}=1, τ_{HRC}=5 years.
From left to right:
τ_{PPC}=.125, .5, 1, 2, 5.



τ_{PPC}=1, τ_{HRC}=5 years.
From left to right:
τ_{PT}=.125, .5, 1, 2, 5.



τ_{PPC}=1, τ_{PT}=1 year.
From left to right:
τ_{HRC}=1, 2, 4, 8, 12.