Discounting Rules for Risky Assets

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I. INTRODUCTION

We still do not understand the role of taxes in determining optimal capital structure, if there is an optimal capital structure. Therefore, we have no general rule for calculating discount rates for capital investments which are partly debt-financed. The only bulletproof rules apply to two special cases. First, we know that risk-free, after-corporate-tax nominal cash flows should be discounted at the after-corporate-tax risk free interest rate. Second, we know that projects that exactly duplicate the firm's existing assets, both in risk and financing, are correctly valued by discounting at the firm's weighted average cost of capital.

The discounting rules for these two special cases work regardless of "right" theory of debt and taxes. For example, Ruback (1986) shows that the discount rate for risk-free flows can be derived as a special case of the adjusted discount rate formula derived by Modigliani and Miller (MM) in 1963 and also as a special case of Myers's adjusted present value method (1974), which as originally presented adopted MM's assumptions about the value of corporate interest tax shields. But the same discounting rule also follows from Miller's 1977 "Debt and Taxes" paper, because in that model the opportunity cost of equity investment in a risk-free asset is the after-tax risk-free rate. Ruback proves these discounting rules by arguing that any

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stream of risk-free future cash inflows can be "zeroed out" by a borrowing plan under which after-tax debt service is matched to the penny to the cash inflows. (Cash outflows can be zeroed out by a matched lending plan.) Since debt service can be covered exactly, the initial amount borrowed under the plan can be money in the bank at "time zero," which needless to say is not difficult to value.[1]

We set out to find a discounting rule which could be used to value any risky cash flow stream. We failed. But we did find a rule which guarantees a project value under any equilibrium theory of debt and taxes, so long as the corporation adheres to a specific financing policy for the project. We do not claim that this financing policy is optimal, only that it is feasible. If there is a different optimal policy, <u>and if the manager knows what that policy is</u>, project value can exceed our guaranteed value. For managers who share our ignorance of optimal capital structure, however, the guaranteed value should be helpful as a lower bound.

Our discounting rule does not require exotic ingredients -- only the risk-free interest rate, the marginal corporate tax rate, a risk measure or measures for the stream, and the expected rate of return on a reference portfolio of traded securities. If a one-factor capital asset pricing model is assumed, as we do for convenience in most of this paper, then the risk measure is the asset beta and the reference portfolio is the market.

Our rule for calculating the discount rate for a risky project is:

$$r^* = r_f (1-T_c)(1-\beta) + \beta r_m \quad (1)$$

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1 Franks and Hodges (1978) first used this argument to value financial leases.

where $r_f(1-T_c)$ is the nominal Treasury rate, after taxes at the marginal corporate rate, T_c, r_m is the expected rate of return on the market, and β is the "asset beta" of the cash flow. The asset beta is the beta of a direct equity claim on the cash flow, that is, the beta the cash flow would have if it were traded as an all-equity financed mini-firm. We assume this beta is known.

The intuition behind this cost of capital rule is straightforward. The right cost of capital for a risky project is its opportunity cost, which is the expected rate of return on a capital market investment with identical risk. A firm could use investments in T-bills and the market portfolio to form a replicating portfolio with the same risk as the project. The replicating portfolio is constructed by investing $1-\beta$ percent of its funds in the T-bills, with an after-corporate-tax return of $r_f(1-T_c)$, and investing β percent of its funds in the market, with an expected return of r_m. (This replicating strategy assumes that a corporation does not pay taxes on its investment in the market portfolio.) The replicating portfolio has the same beta as the risky project and provides an after-corporate tax return of r^*. The after-tax opportunity cost of investing in the risky project is therefore given by equation (1), and that rate, r^*, should be used to value the project.

Our discount rate rule can also be interpreted as a weighted average cost of capital for a project:

$$WACC = r_D\,(1-T_c)\,\frac{D}{V} + r_E\,\frac{E}{V} \qquad (2)$$

This project weighted average cost of capital can be used to value a project as long as the debt and equity rates of return and weights are for the project. Our rule simply assigns specific values to the components of WACC: the debt ratio, D/V, is set equal to 1-β; the equity ratio, E/V, is set equal to β. With these weights, if the debt is riskless (so that $r_d = r_f$), the equity has a beta of one and $r_E = r_m$.

The next section presents the discounting rule, proves it gives a guaranteed value, and discusses practical application and underlying assumptions.

II. A DISCOUNTING RULE FOR RISKY CASH FLOWS.

The discount rate we propose is a weighted average of the after-corporate tax risk-free interest rate and the expected rate of return on a reference portfolio of risky securities. The weight on the reference portfolio's return is the cash flow's risk relative to the reference portfolio.

The only requirement for the reference portfolio is that it can be levered or unlevered to match its risk level to the risk of the cash flows. Under the capital asset pricing model, or any single-factor model, the natural reference portfolio is the market portfolio, and the risk measure is beta. The beta of an equity investment in a cash flow can always be made equal to one, the market beta, by levering or unlevering. For now we take the market as the reference portfolio. But it is important to emphasize that the only aspect of the Capital Asset Pricing Model that we depend on is that beta is the correct measure of risk. We make no specific assumptions about the intercept and slope of the security market line. We use the market as a reference portfolio because it is actively traded, and is likely to be fairly priced, and because its expected return should be easier to estimate than expected returns on other equity portfolios or specific common stocks. We also assume that the firm has sufficient taxable income, either from the cash flow being valued or from other corporate assets, that it can always use interest tax shields immediately when interest is paid. We assume that it could borrow $(1-\beta)$ of the cash flow's value over any <u>short</u> period at the risk-free interest rate. If β exceeds one, this amounts to lending $(\beta-1)$ times the cash flow's value at the risk-free rate.

Finally, we assume that capital markets are complete enough to support value additivity. We ignore transaction costs or other market imperfections.

Consider an asset generating a single cash flow $\tilde{X}$, with expectation $X = E(X)$, to be received next period. $\tilde{X}$ is net of corporate taxes. However, these taxes do not reflect any interest tax shields on debt associated with, or supported by $\tilde{X}$. In other words, the corporate tax paid on $\tilde{X}$ is calculated assuming all-equity financing.

We will now give two proofs that discounting X at r^* gives a lower bound to its market value. The first proof is quick and simple. The second is longer but more informative.

<u>First Proof</u>

We calculate V, the market value of $\tilde{X}$, as if the asset generating $\tilde{X}$ were traded as a separately financed mini-firm. Given value additivity, V is also the project's contribution to its parent firm's value. We can think of adding the mini-firm's value to the left-hand side of the parent's balance sheet and it's debt and equity values to the right-hand side of the parent's balance sheet.

Suppose the firm "finances" the project with $D = (1-\beta)V$ dollars of debt. That is, it accepts $D = (1-\beta)V$ as its capital structure policy for the asset generating $\tilde{X}$. The mini-firm's initial market value balance sheet is:

ASSETS	LIABILITIES
$V = V(\tilde{X},D)$	$D = (1-\beta)V$
	$E = \beta V$
V	V

Note that V may depend on debt policy. We do not assume that borrowing $(1-\beta)V$ is the best policy, only that it is a feasible policy. We do assume, provisionally, that the beta of $V(\tilde{X},D)$ does not depend on D.[2]

The beta of the equity claim on $\tilde{X}$ is one. Since the beta of the portfolio of D and E equals the asset beta, and since $\beta_D = 0$,

$$\beta = \beta_D \frac{D}{V} + \beta_E \frac{E}{V} = \beta_E \frac{E}{V} \quad . \tag{3}$$

Rearranging (2), and substituting the values for the project's debt ($(1-\beta)V$) and equity (βV), proves that:

$$\beta_E = \beta(1 + \frac{D}{E}) = \beta(1 + \frac{1-\beta}{\beta}) = 1 \quad .$$

Thus r_E, the expected rate of return investors would demand on the equity, equals r_m, the expected rate of return on the reference (market) portfolio.

The expected portfolio rate of return on the debt and equity claims on $\tilde{X}$ is weighted average of r_f, the risk-free-rate, and r_m, the expected equity return. The weights are the financing proportions D/V and E/V. This return comes as a cash payout, which in total is the cash flow $\tilde{X}$ plus the interest tax shield $T_c r_f D$. The expected return per dollar invested is therefore

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2 This is not always right, because the interest tax shield $r_f T_c D$ is a safe nominal flow. Later in the paper we consider the error this provisional assumption may introduce.

$(X + T_c r_f D)/V$. The two expressions for expected return are equal.

$$1 + r_f \left(\frac{D}{V}\right) + r_m \left(\frac{E}{V}\right) = \frac{X + r_f T_c D}{V}$$

$$1 + r_f (1-T_c) \left(\frac{D}{V}\right) + r_m\left(\frac{E}{V}\right) = X/V$$

Since D/V = 1 - β and E/V = β, the left hand side is just 1 + r*:

$$1 + r^* = X/V$$

$$V = \frac{X}{1 + r^*} \qquad (4)$$

In application, equation (4) is the starting point, not the end result. The firm forecasts $\tilde{X}$, discounts it at r* to obtain V, and then issues debt of (1-β)V. Our proof shows that the actual market value of $\tilde{X}$ (or of the debt plus the residual equity claim on $\tilde{X}$) is in fact V under the assumed financing policy.

<u>Second Proof</u>

In the first proof, we never identified the market value of an <u>unlevered</u> claim on $\tilde{X}$. Now we introduce a security market line <u>for equities</u> under different assumptions about debt and taxes. Let T_{pe} and T_{pd} be effective personal tax rates on equity and interest income, respectively. Let r_{fe} be the expected rate of return demanded by investors in risk-free (zero-beta) equities.

If $T_{pe} = T_{pd}$, the MM (1963) case, then $r_{fe} = r_f$. But if the two personal tax rates are not equal, the after personal tax rates on safe debt and safe equity[3] must be the same:

$$r_{fe} (1-T_{pe}) = r_f (1-T_{pd}). \qquad (5)$$

Thus in Miller's (1977) model, where $T_{pe}= 0$ and the marginal investor's T_{pd} equals the corporate rate, $r_{fe}= r_f (1-T_c)$.

We do not know r_f, r_e or the personal tax rates of the relevant marginal investors. We assume the firm knows r_f and r_m, but not the intercept or slope of the security market line because r_{fe} is unknown:

$$r (\beta) = r_{fe} + \beta(r_m - r_{fe}). \qquad (6)$$

Figure 1 shows three possible lines: first, the "MM" line with $r_{fe} = r_f$, which is the same as the original capital asset pricing model's line; second, the "Miller line" with $T_{pe} = 0$ and $r_{fe} = r_f(1-T_c)$; and finally an intermediate case. Obviously the expected return depends on the line assumed, unless it happens that $\beta = 1$. For illustration we have marked three possible values at $\beta = 0.5$.

The MM line implies a strong tax advantage to corporate borrowing, the intermediate line a weaker advantage, and the Miller line no advantage at all. We do not know which line is right. But the value of a future cash flow does not depend on the line so long as the firm adheres to the debt policy underlying our discounting rule.

Given some security market line, and thus some discount rate r for an unlevered equity claim on $\tilde{X}$, market value can be calculated by adjusted

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3 "Safe equity" refers to a stock or equity portfolio which has only diversifiable risk. A well-diversified investor would regard the after-tax payoffs of safe equity and Treasury bills as perfect substitutes.

present value (APV) as the sum of the base case value plus the value of the interest tax shields:

$$V = APV = \frac{X}{1+r} + \frac{T^* r_f(1-\beta)\ APV}{1+r}, \qquad (7)$$

where $(1-\beta)APV = (1-\beta)V$ is the debt issued against $\tilde{X}$; r is the discount rate for an all equity claim to the cash flow; and $T^*r_f\ (1-\beta)APV$ is the net interest tax shield when personal as well as corporate taxes are considered. We continue our provisional assumption that interest tax shields are just as risky as the cash flow X, and thus discount both terms in equation (7) at r.

When the firm switches debt for equity, and pays an additional dollar of interest, the corporate tax shield is T_c, or $T_c(1-T_{pe})$ after equity investors' taxes. At the same time the switch subjects one dollar of investment income to tax at T_{pd} rather than T_{pe}, at a cost to investors of $T_{pd} - T_{pe}$. The net tax gain after all taxes is $T_c(1-T_{pe}) - T_{pe} + T_{pd}$. To express this as a before-personal-tax amount, we "gross it up" by dividing through by $1-T_{pe}$:

$$T^* = T_c - \frac{(T_{pd} - T_{pe})}{1 - T_{pe}} \qquad (8)$$

This obvious special cases are "MM", where $T_{pd} = T_{pe} = 0$ and $T^* = T_c$, and "Miller" with $T_{pe} = 0$, $T_{pd} = T_c$, and $T^* = 0$.[4]

Equation (7) boils down to

$$APV = \frac{X}{1 + r - T^*r_f\ (1-\beta)},$$

4 In a Miller equilibrium with $T_{pe} > 0$, $(1-T_{pd}) = (1-T_c)(1-T_{pe})$, which also gives $T^*=0$.

so the APV calculation implicitly discounts at the rate $r - T^* r_f(1-\beta)$. Thus we must show that:

$$r - T^* r_f(1-\beta) = (1-\beta) r_f (1-T_c) + \beta r_m = r^* \quad .$$

Substituting for r from (7) and simplifying leaves:

$$\left(\frac{1-T_{pd}}{1-T_{pe}}\right) - T^* = (1-T_c) \; .$$

Substitute for T^* from equation (8) and start cancelling: all the tax rates offset and the equality is shown.

Numerical Example

Suppose we observe $r_f = .10$ and $r_m = .20$. The corporate tax rate is $T_c = .5$. The cash flow's expected value is 100 an its beta is 0.5. Our discounting rule gives $r^* = (1 - .5)(.10)(1-.5) + (.5)(.20) = .125$ and a value $V = 100/1.125 = 88.89$.

Table 1 shows that exactly the same APV is obtained under three different assumptions about debt and taxes and the security market line.

The calculations in Table 1 clarify why our discounting rule works under any equilibrium model of debt and taxes. If we move from Case 1 (MM) to Case 2 (Miller), the cash flow X loses value because T^* drops from .50 to zero. But it also gains value because r, the all-equity opportunity cost of capital, falls from .15 to .125. The loss and gain exactly offset. Given r_f, r_m and T_c, and given our proposed financing policy, calculated value can never be increased by assuming a higher value for T^* because a consistent assumption about the security market line requires increasing r to offset the tax gain.[5]

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[5] This is not a standard comparative static analysis of the marginal properties of an equilibrium. Instead we start with the observed rates, r_f and r_m, which could be generated by any of a large number of equilibria. We then ask whether project value depends on what the true equilibrium is.

TABLE 1

Calculating adjusted present value under different assumptions about debt and taxes - numerical example.

Assumptions and Notation

$r_f = .10$	Treasury bill rate
$r_m = .20$	Expected market return
$T_c = .50$	Corporate tax rate
$X = 100$	Expected after-tax cash flow after one period
$\beta = .5$	Beta of unlevered claim on cash flow
$r = r_{fe} + \beta(r_m - r_{fe})$	Security market line
$r_{fe} = r_f \dfrac{(1-T_{pd})}{(1-T_{pe})}$	Expected return on zero-beta equity investment
$T^* = T_c - \dfrac{(T_{pd} - T_{pe})}{1 - T_{pe}}$	Net tax gain from corporate interest payment of $1.00

Case 1 (MM)

$Tp_d = T_{pe},\ r_{fe} = r_f,\ r = r_f + \beta(r_m - r_f)$

$r = .10 + .5(.20 - .10) = .15$

$T^* = T_c = .50$

$$APV = \frac{100}{1.15} + \frac{.5(.10)(1-.5)APV}{1.15} = 88.89$$

Case 2 (Miller)

$T_{pd} = T_c,\ T_{pe} = 0,\ r_{fe} = r_f(1-T_c),\ r = r_f(1-T_c) + \beta(r_m - r_f(1-T_c))$

$r = .10(1 - .5) + .5\ (20 - .10(1 - .5)) = .125$

$T^* = 0$

$$APV = \frac{100}{1.125} + \frac{0\ (.10)(1 - .5)APV}{1.125} = 88.89$$

TABLE 1, Continued

Case 3 (Intermediate)

$$T_{pe} = .1,\ T_{pd} = .3,\ r_{fe} = .10\ \left(\frac{1 - .3}{1 - .1}\right) = .0778$$

$$r = .0778 + .5\ (\ .20 - .0778) = .1389$$

$$T^{*} = .5 - \left(\frac{.3 - .1}{1 - .1}\right) = .2778$$

$$APV = \frac{100}{1.1389} + \frac{.2778\ (.10)(.5)\ APV}{1.1389} = 88.89$$

General Discounting Rule

$$r^{*} = (1 - \beta)\ r_f\ (1-T_c) + \beta r_m$$

$$= (1 - .5)\ .10\ (1 - .5) + .5\ (.20) = .125$$

$$V = \frac{X}{1 + r^{*}}$$

$$= \frac{100}{1.125} = 88.89$$

The table also shows why our proposed rule may understate the cash flow's actual value. Its value could be increased in cases 1 and 3 by borrowing more than 50 percent of its value. In general our discounting rule will understate value _if_ there are significant tax advantages to corporate debt ($T^{*}>0$), _if_ agency, moral hazard, or bankruptcy costs do not prevent borrowing more than $(1-\beta)V$, and _if_ managers act to lever up beyond $(1-\beta)V$. However, our rule guarantees a project value to a manager who is uncertain about "debt and taxes," who worries about the cost of financial distress which may be encountered at debt levels above $(1-\beta)V$, or who has trouble convincing a conservative organization to lever up aggressively.

A Qualification

So far we have assumed that the risk of the total cash payout to debt and equity combined does not depend on the debt amount. This is not always right, because the corporate interest tax shield $T_c r_f D$ is a safe nominal flow, received when interest is paid next period. The overall beta of debt and equity is thus reduced by borrowing _whenever interest tax shields contribute to firm value_. If they do not contribute, the overall beta is unchanged by borrowing despite the addition of the safe interest tax shields.

Consider the beta of investing in the total cash payout to debt and equity investors. It depends on the covariance of the return on this investment with the market return, $\tilde{r}_m$, that is:

$$COV[(\tilde{X} + r_f T_c D)/V,\ \tilde{r}_m] = COV(\tilde{X},\ \tilde{r}_m)/V.$$

The safe tax shield $V_f T_c D$ affects this covariance only as it affects V.

In an MM would, as D increases, V increases and the covariance and beta fall. In a Miller equilibrium, V does not depend on D, and the covariance and beta are therefore constant too.

If Miller is right, our discounting rule (Eq. 11) gives exactly the right answer given the financing policy of $D = (1-\beta)V$. But if MM are right, our rule understates project value, because the equity beta is less than one when $D = (1-\beta)V$.

If we knew that MM were right, this problem would be fixed by slightly modifying the assumed financing policy to put more weight on $r_f(1-T_c)$, the after-tax risk free rate, and less on r_m, the expected market return. We now work through the modification to see how much difference this modification might make.

Safe nominal flows are valued by discounting at the after-tax risk free rate. Thus the interest tax shield's present value is:

$$\frac{T_c r_f D}{1+r_f(1-T_c)} = yD \tag{9}$$

Suppose the firm "cashes in" this present value by borrowing an additional amount yD, generating this market value balance sheet:

ASSETS	LIABILITIES
$V - yD$	$D = (1-\beta)(V-yD)+yD$
yD	E
$V = V(\tilde{X},D)$	V

The debt weight works out to be $(1-\beta)/(1-\beta y)$:

$$D = (1-\beta)(V-yD) + yD = (1-\beta)V + \beta yD$$
$$= \frac{(1-\beta)V}{1-\beta y}$$

The equity weight is:

$$1 - \frac{1-\beta}{1-\beta y} = \frac{\beta(1-y)}{1-\beta y},$$

and the debt-equity ratio is $D/E = (1-\beta)/\beta(1-y)$. The revised discount rate is:

$$r^* = \left(\frac{1-\beta}{1-\beta y}\right) r_f \, (1-T_c) + \frac{\beta(1-y)}{1-\beta y} r_m \qquad (10)$$

Now we show that β_E, the beta of the equity claim, is again one despite the addition of the safe asset yD to the left-hand side of the balance sheet. Systematic risk is the same on both sides,

$$\beta(V-yD) + \beta_D yD = \beta_D D + \beta_E E$$

and since $\beta_D = 0$,

$$\beta_E = \beta\left(1+ \frac{D(1-y)}{E}\right)$$

$$= \beta\left(1+ \frac{(1-\beta)(1-y)}{\beta(1-y)}\right) = 1.$$

Since $\beta_E = 1$, r_E must equal r_m.

We need not repeat the proof that discounting at r^* correctly values $\tilde{X}$ under the revised financing policy, because the proofs follow exactly as given above. However, discounting at equation (10)'s r^* values $\tilde{X}$ a bit more generously, because equation (10)'s discount rate is lower.

The adjustment of weights in equation (10) is probably not an important practical refinement. For example, under the assumptions of Table 1, the weight on the after-tax risk free rate would change from $1-\beta = .5$ to:

$$\frac{(1-\beta)}{1-\beta y} = \frac{.5}{1-.5\left(\frac{.5(.10)}{1+.10(1-.5)}\right)} = .512.$$

The discount rate changes from $r^* = .125$ in Table 1 to:

$$r^* = .512\ (.10)(1-.5) + .488(.20) = .123.$$

Thus our discounting rule, Eg. (1), is not entirely insulated from the debate about taxes and optimal capital structure. The rule will overstate the correct discount rate when there is a tax advantage to corporate borrowing. We believe the overstatement is minor - note that an estimate of r_m could easily be a full percentage point off target. Of course a manager who believed that there is a tax advantage to corporate borrowing would calculate r^* by equation (10), taking the chance of using a discount rate that is slightly too low.

Discounting over t Periods.

Moving from one to t-period discounting is easy once the t-period financing policy is specified. Our discounting rule can be applied period by period if debt is adjusted to the rule's specified fraction of market value at the start of each period.

Consider a cash flow to be received at t. Then at the start of t-2, say, the market value balance sheet will be:

ASSETS	LIABILITIES
$V_{t-2}(\tilde{X}_t, D_{t-2}, \tilde{D}_{t-1})$	$D = W_D V$ $E = W_E V$
V	V

where $W_D + W_E = 1$, and W_D equals either $(1-\beta)$.

We assume that an <u>unlevered</u> equity claim $\tilde{X}_t$ can be properly valued by discounting at a constant risk adjusted rate. That in turn means that the ingredients of our discount rate r^* (i.e., β, r_f, and r_m) are also constant,[6] and that equation (1) generates the same r^* for each future period.

Think of how the value of an <u>unlevered</u> claim on $\tilde{X}_t$ is determined at t-2. It is:

$$V^o_{t-2} = \frac{E_{t-2}(\tilde{V}^o_{t-1})}{1 + r} = \frac{E_{t-2}(\tilde{X}_t/(1+r))}{1 + r}$$

$$= \frac{E_{t-2}(\tilde{X}_t)}{(1 + r)^2}$$

where V^o indicates the unlevered value. In other words, the unlevered value

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6 Three conditions are usually considered necessary for discounting a cash flow at a constant risk-adjusted rate:

1. A known, constant beta for the an all-equity claim on the cash flow;
2. A known, constant market risk premium;
3. A known, constant Treasury bill rate.

Condition 1 implies that uncertainty is resolved at a constant rate over time. It also implies that the "detrended" stream of project cash flows would follow a multiplicative random walk. ("Detrended" cash flows are expressed as percentages of their ex ante expectations.) See Myers and Turnbull (1977) and Fama (1977).

of $\tilde{X}_t$ at t-2 is the expectation of its uncertain value at t-1, which in turn is linked to the expectation of $\tilde{X}_t$ given information available at t-2.

The value of $\tilde{X}_t$ at t-1 under our assumed financing policy is proportional to V^o_{t-1}:

$$\tilde{V}_{t-1} = \frac{\tilde{E}_{t-1}(\tilde{X}_t)}{1 + r^*} = \tilde{V}^o_{t-1} \left(\frac{1 + r}{1 + r^*}\right)$$

Given this proportional link, the "asset beta" of $\tilde{V}_{t-1}$ as viewed from t-2 is identical to the beta of $\tilde{V}^o_{t-1}$ viewed from the same point. We can therefore treat $\tilde{V}_{t-1}$ as if it were a cash payoff to investment at t-2. The cash payoff is discounted at r^*.

$$V_{t-2} = \frac{E_{t-2}(\tilde{V}_{t-1})}{1 + r^*} .$$

Since $E_{t-2}(\tilde{V}_{t-1}) = \dfrac{E_{t-2}(\tilde{X}_t)}{1 + r^*}$,

$$V_{t-2} = \frac{E_{t-2}(\tilde{X}_t)}{(1 + r^*)^2} .$$

The argument obviously repeats for t-3: $\tilde{V}_{t-2}$, as viewed from t-3, is proportional to $\tilde{V}^o_{t-2}$:

$$\tilde{V}_{t-2} = \frac{\tilde{E}_{t-2}(\tilde{X}_t)}{(1 + r^*)^2} = \tilde{V}^o_{t-2} \left(\frac{1 + r}{1 + r^*}\right)^2$$

Since $\tilde{V}_{t-2}$ and $\tilde{V}^o_{t-2}$ are proportional, claims on them again have the same beta. We can treat $\tilde{V}_{t-2}$ as if it were an end-of-period cash payoff and again apply our discounting rule. In general,[7]

$$V_{j<t} = \frac{E_j(\tilde{X}_t)}{(1 + r^*)^j} \qquad (11)$$

III. SUMMARY

This paper develops a rule for calculating a discount rate to value risky projects. The rule assumes that asset risk can be measured by a single index (e.g., beta), but makes no other assumptions about specific form of the asset pricing model. The rule works for all equilibrium theories of debt and taxes. The rule works because it treats all projects as combinations of two assets: Treasury bills and the market portfolio. We know how to value each of these assets under any theory of debt and taxes and under any assumption about the slope and intercept of the market line for equity securities.

Given the corporate tax rate, the interest rate on Treasury bills, and the expected rate of return on the market, we can calculate the cost of capital for a feasible financing strategy. The firm finances the project

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[7] The r^* used in equation (11) could come either from equation (1) or equation (10). Using the latter treats each period's interest tax shield as a safe, nominal flow to be received at the end of that period. However, interest tax shields in <u>subsequent</u> periods are not know, since debt levels will be adjusted to <u>ex post</u> changes in the cash flow's market value. For example, the firm at t-2 would view the interest tax shield $T_c r_f D_{t-2}$ as a safe nominal flow to be received at t-1. But the interest tax shield to be received at t is, when viewed from t-2, a random variable proportional to V_{t-1}, that is $T_c r_f W_D V_{t-1}$. The beta of a claim on this final tax shield held from t-2 to t-1 is the same as the beta of an unlevered claim on X_t. The value of this claim is included in V_{t-1}, and therefore in V_{t-2} when $E_{t-2}(V_{t-1})$ is discounted by r^*.

Our treatment of interest tax shields associated with future debt levels is consistent with Miles and Ezzell (1980).

with equity and debt in the proportions β and (1-β). Value increasing projects could be completely financed using this strategy. The weighted average cost of financing this project provides a discount rate that values the project correctly.

Of course, other financing strategies are possible. If the firm knew the correct theory of debt and taxes, it could probably come up with a financing strategy that resulted in a lower cost of capital than our rule provides. Conversely, a different strategy could be worse than our rule, and result in a higher cost of capital. Our contribution is to provide a method for valuing risky projects that works for a variety of different theories of debt and taxes and involves a financing strategy that is feasible. We can guarantee a project value not withstanding our ignorance about optimal capital structure.

REFERENCES

Fama, Eugene F. 1977. "Risk-adjusted Discount Rates and Capital Budgeting Under Uncertainty". Journal of Financial Economics 5, pp. 3-24.

Hodges, Julian R. and Stewart D. Hodges. 1978. "Valuation of Financial Lease Contracts: A Note." Journal of Finance 33, pp. 657-669.

Miles, J. and R. Ezzell. 1980. "The Weighted Average Cost of Capital, Perfect Capital Markets and Project Life: A Clarification". Journal of Financial and Quantitative Analysis 15, pp. 719-730.

Miller, Merton. 1977. "Debt and Taxes". Journal of Finance 32, 261-275.

Modigliani, F and M. Miller. 1963. "Taxes and the Cost of Capital: A Correction". American Economic Review 53, pp. 433-442.

Myers, Stewart C. 1974. "Interactions of Corporate Financing and Investment Decisions: Implications for Capital Budgeting". Journal of Finance 29, pp. 1-25.

Myers, Stewart C. and Stuart M. Turnbull. 1977. "Capital Budgeting and the Capital Asset Pricing Model: Good News and Bad News". Journal of Finance 32, pp. 321-333.

Ruback, Richard S. 1986. "Calculating the Market Value of Riskless Cash Flows". Journal of Financial Economics 15, 323-339.

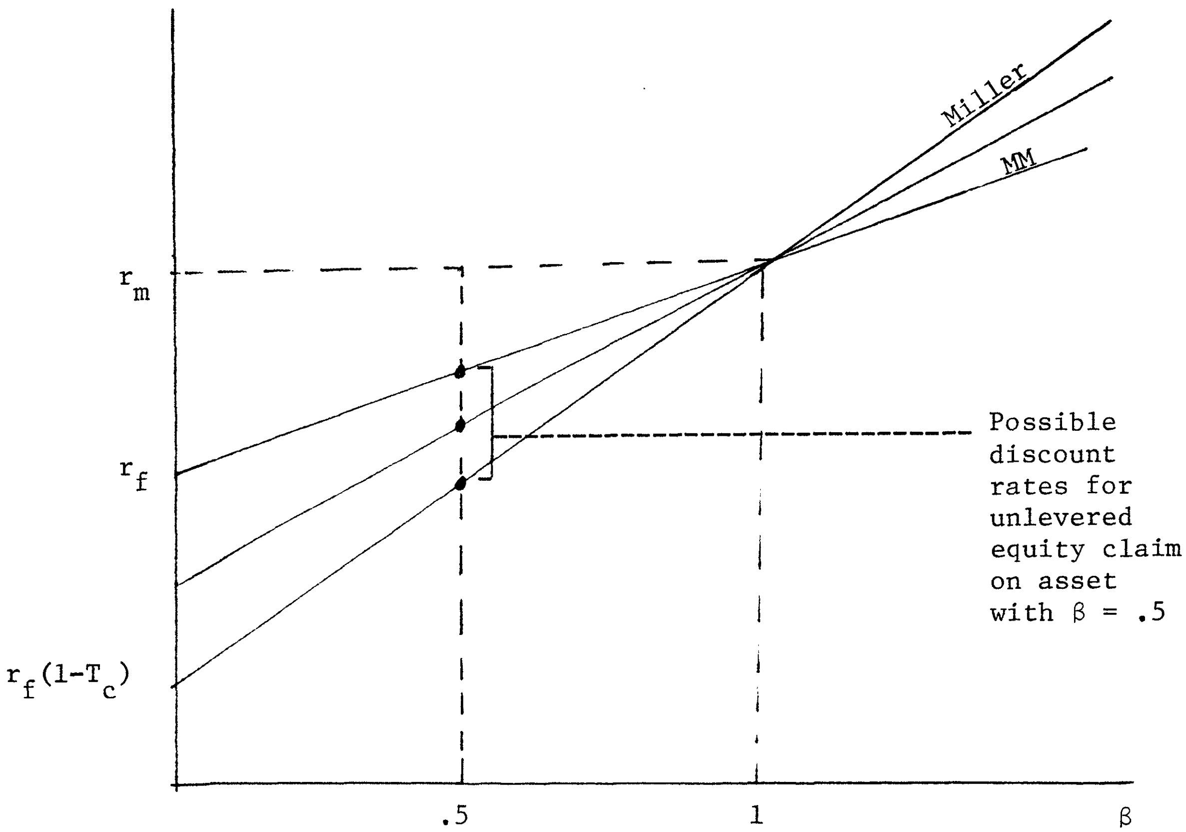

Figure 1

Security market lines implied by three theories of debt and taxes. For each case the intercept, r_{fe}, is given by $r_{fe}\ (1-T_{pe}) = r_f\ (1-T_{pd})$.