

ECONOMIC CAPACITY UTILIZATION AND PRODUCTIVITY
MEASUREMENT FOR MULTIPRODUCT FIRMS WITH MULTIPLE
QUASI-FIXED INPUTS

By

Ernst R. Berndt
and
Melvyn A. Fuss

Working Paper #3001-89-EF&A

April 1989

ECONOMIC CAPACITY UTILIZATION AND PRODUCTIVITY MEASUREMENT
FOR MULTIPRODUCT FIRMS WITH MULTIPLE QUASI-FIXED INPUTS

by

Ernst R. Berndt

Massachusetts Institute of Technology and NBER

and

Melvyn A. Fuss

University of Toronto and NBER

Research support from the National Science Foundation Grant #SES-8420937 is gratefully acknowledged. An earlier version of this paper was given at the December 1987 annual meetings of the American Economic Association in New York City.

April 1989

ABSTRACT

ECONOMIC CAPACITY UTILIZATION AND PRODUCTIVITY MEASUREMENT FOR MULTIPRODUCT FIRMS WITH MULTIPLE QUASI-FIXED INPUTS

by Ernst R. Berndt* and Melvyn A. Fuss**

In this paper we develop measures of economic capacity output and economic capacity utilization for firms producing multiple outputs and having one or more quasi-fixed inputs. Although we produce an impossibility theorem showing that based only on the assumption of cost minimization, the concept of capacity output is undefined whenever the number of outputs I exceeds the number of fixed inputs M , we are able to provide alternative constructive procedures for defining capacity output whenever $I \leq M$. We also propose a number of additional primal and dual measures of utilization of the variable and fixed inputs, including a multi-fixed input analog to Tobin's q . We relate these alternative utilization measures to one another, and show that unambiguous inequality relationships among them (relative to unity) can typically be specified a priori only under rather restrictive assumptions. We show that unless restrictive assumptions are made, the multi-fixed input analogs to Tobin's q have little informational content regarding incentives for net investment of any specific fixed input. Finally, we demonstrate the usefulness of the alternative utilization measures by showing how they can be incorporated to adjust traditional measures of multifactor productivity growth for variations in short-run utilization.

JEL Numbers: 226, 621, 641

*Massachusetts Institute of Technology and NBER

**University of Toronto and NBER

I. Introduction

Economists have long been interested in measuring the performance of an economy by comparing the level of current activity to some benchmark or capacity notion. In macroeconomics, for example, it has been customary for some time to compare the current unemployment rate to some non-zero "full employment" level of unemployment. In this paper we focus on another measure of economic performance, namely, the capacity utilization (CU) rate of a sector or of a firm, in which actual output is compared to potential or capacity output. CU measures are often used as regressors in equations explaining investment or inflation, and are also used to adjust multifactor productivity growth for variations over the business cycle. An innovation of this paper is that we consider CU in the multiproduct context.

One traditional measure of capacity output is based on engineering notions and refers to the maximum possible sustainable output. An alternative measure, put forth by Cassells [1937], Klein [1960] and Hickman [1964], has foundations in economic theory and is defined as that level of output corresponding to a tangency point between the short- and long-run average cost curve.¹ Economic measures of CU are then computed as the ratio of actual output y divided by economic capacity output y^* , that is, primal CU is computed as $CU_p \equiv y/y^*$.² Such primal CU measures can be less than, equal to, or greater than unity.

A dual measure of CU can also be computed, defined for a given level of output as total shadow costs C'' divided by total market costs C , i.e. $CU_d \equiv C''/C$. In turn, total shadow costs C'' are costs of variable inputs plus shadow rental costs of the quasi-fixed inputs, the latter defined as one-period

¹For a survey of this literature, see Berndt, Morrison and Wood [1983].

²For recent empirical implementations, see, for example, Berndt-Morrison [1981], Berndt-Hesse [1985], Berndt-Fuss [1986], Hulten [1986], Morrison [1986] and Slade [1986].

reductions in variable input costs given a small increase in the levels of each of the quasi-fixed inputs, and evaluated at their existing, short-run levels. Total market costs C are costs of variable inputs plus the market costs of the quasi-fixed inputs, where the latter are evaluated using ex ante or market rental prices. Like its primal CU_p counterpart, the dual CU_d measure can be greater than, equal to, or less than unity. Although both CU_p and CU_d are generally simultaneously either less than, equal to, or greater than unity, in general $CU_p \neq CU_d$.

Each of these traditional CU measures is based on the notion of an aggregate potential output without consideration of the fact that most firms produce multiple outputs. In the airline industry, for example, freight is typically distinguished from passenger output, and passenger-mile output is often disaggregated into first class, business and economy. In the rail industry, various types of freight output are usually distinguished, and in telecommunications a minimal breakdown typically consists of long-distance and local telephone calls. Within the automobile industry, output is usually disaggregated into size classes such as compact, mid-size and luxury. In brief, multi-product firms are very common.

One possible way to proceed with CU measurement is to acknowledge multi-product production technologies and attempt only to compute output-specific CU rates using engineering notions. However, a major problem emerges since output-specific CU rates cannot be uniquely defined in the traditional way whenever outputs are substitutable along a transformation frontier, for in such cases no unique individual potential output exists. An example is the computer assisted design and computer assisted manufacturing equipment (CAD/CAM) of modern assembly lines, which permits virtually instantaneous

changes in the composition of outputs. This fundamental barrier to the measurement of output-specific CU using engineering notions can be overcome through the use of the concept of economic capacity output.

The generalization of economic CU to multiproduct technologies is not a trivial one. Suppose, for example, that a firm initially is in full or long-run equilibrium and produces two outputs, y_1 and y_2 . Suddenly, demand for y_1 increases while that for y_2 decreases. Within this context, we may wish to examine questions such as the following: (i) What is the appropriate measure of output-specific CU? (ii) Under what conditions can an aggregate CU measure be written as a weighted average of output-specific CU_1 and CU_2 ratios, and what are the appropriate weights? (iii) When output-specific CU_i ratios are all greater (less) than unity, is it the case that one or more fixed inputs must also be over (under)-utilized? (iv) How does the ratio of the shadow value of a fixed input to its ex ante rental price relate to the notion of CU? (v) How should multifactor productivity growth measures in the multiproduct firm be adjusted for variations in product-specific CU_i or aggregate CU?

The key to understanding our extension of the measurement of CU to multiple output technologies is the concept of a reference output vector. The reference output vector y^* is the vector of outputs that a firm produces, or expects to produce, in long-run or full equilibrium. The vector y^* will be called the economic full capacity vector. In the case of profit maximizing behavior, y^* will be defined as the vector of outputs produced when all inputs, including quasi-fixed factors, are at their long-run levels. For cost minimizing behavior, it will be convenient to define y^* as the vector of output levels which would be produced in full equilibrium, conditional on the currently observed levels of the quasi-fixed inputs. In either case, a CU

vector could be computed, whose I elements represent product-specific economic capacity utilization rates, each calculated as $CU_i \equiv y_i^*/y_i$, $i = 1, \dots, I$, where y_i is the currently produced output level.

We begin in Section II with the conceptually rather simple case of profit-maximizing behavior. In Section III we develop the concept of multiple-output CU within the cost minimization framework, and consider three important cases involving (i) single output, single fixed input, (ii) single output, multiple fixed inputs, and (iii) multiple outputs and multiple fixed inputs. In Section IV we consider implications for the measurement of multifactor productivity measurement, while in Section V we suggest extensions and briefly comment on issues involved in empirical implementation.

II. Multiple Output Capacity Utilization under Profit Maximization

In this section we present a brief discussion of the one period profit maximization case, for in this situation the determination of a reference, or capacity, output vector is straightforward. This case also serves as a useful introduction to the more complex cost-minimization examples considered below. Denote the technology set T and the price set P as:

Technology: $(y, v, z) \in T$ where $y \equiv$ output vector

$v \equiv$ variable input vector

$z \equiv$ quasi-fixed input vector

Prices: $(p, w, q) \in P$ where $p \equiv$ output price vector

$w \equiv$ variable input price vector

$q \equiv$ ex ante quasi-fixed

input (rental) price vector.

Under the assumption of static profit maximization, the ex ante choice facing the firm is to choose that set of y, v, z such that expected variable

profits (expected revenue minus expected variable input costs -- in the ex ante case, all expected input costs) are maximized. This optimization results in a profit function of the form

$$\Pi^*[p^e, w^e, q^e] = \Pi^*[p^e, w^e, z^*(p^e, w^e, q^e)]. \quad (2.1)$$

Using Hotelling's Lemma we obtain the economic capacity supply of the i^{th} output y_i^* as

$$y_i^* = \frac{\partial \Pi^*}{\partial p_i^e} = \text{economic capacity supply of } i^{\text{th}} \text{ output.} \quad (2.2)$$

Note that $y^* = (y_1^* \dots y_n^*)$ satisfies the definition of economic capacity output (the reference vector) since it is the vector of outputs the firm expects to produce in full equilibrium.

The ex post problem facing the firm is to maximize Π over y and variable input quantities, where

$$\Pi = \Pi[p, w, z^*(p^e, w^e, q^e)]. \quad (2.3)$$

Hence the ex post problem involves optimization subject to the constraint that the vector of quasi-fixed inputs, chosen on the basis of expected prices, is now pre-determined.

The actual or ex post profit-maximizing supply of the i^{th} output is

$$y_i = \frac{\partial \Pi}{\partial p_i} \quad (2.4)$$

We now define output-specific capacity utilization ratios CU_i as

$$CU_i \equiv y_i / y_i^* = \frac{\partial \Pi}{\partial p_i} \div \frac{\partial \Pi^*}{\partial p_i^e} \quad (2.5)$$

Note that CU_i can be greater than, equal to, or less than unity. For example, when $CU_i < 1$, the ex post profit maximizing output is less than the ex ante output level that was expected to be produced, and hence the capacity of

output i is said to be underutilized. Note that whenever expectations are realized, it should be the case that $CU_i = 1$ for all outputs i .

Although the CU notion based on profit maximization is relatively straightforward in theory, empirical implementation presents difficult problems, due to the reliance on unrealized expectations. To understand these problems better, we now consider development of the reference vector y^* derived from the assumption of cost minimization rather than profit maximization.

III. Capacity Utilization from the Vantage of the Dual Cost Function

It is useful to begin by distinguishing retrospective from current-valued notions of the reference vector y^* and CU. At the beginning of time period t , the firm inherits stocks of m quasi-fixed factors, factors acquired in previous time periods when the prevailing input price and output demand expectations may have differed from those realized at time t . Suppose it were possible to recover input price expectations from historical data. One could then employ economic theory and assume that the firm's previous fixed input purchases constituted optimal solutions to the problem of attaining equality between expected shadow prices (in turn a function of expected input prices and the expected level of outputs) and the expected rental prices.

Working backwards using observed histories on fixed input accumulations and the recovered expectations data, one could employ the expected shadow value framework to obtain an output vector at time t consistent with these histories and recovered expectations. Call that output vector a reference vector y^* . Actual outputs could then be divided by reference outputs, and deviations from unity in the resulting output-specific CU rates could then be

interpreted as being attributable to differences between expectations and realizations.

While in principle it would be possible to compute such retrospective CU measures, practical problems involved in recovering price expectations data would be considerable, and thus we shall not pursue this retrospective notion of y^* and CU any further here. Note that such a consideration suggests we should abandon CU measures based on profit maximization, for as discussed in Section II above, such a retrospective notion would need to underly empirical implementation of CU measures based on profit maximization.

An alternative framework for defining y^* and CU based more on ex post factor price realizations is, however, conceptually attractive and empirically implementable. Specifically, at the beginning of time period t , let the firm again inherit stocks of the m quasi-fixed factors. Instead of evaluating the input flows from these m factors using expected rental prices from past time periods, the firm now values these input flows using current ex ante rental prices. Along with current realized prices of variable inputs and stocks of the m quasi-fixed inputs, the firm then solves for that reference vector y^* such that for each fixed input the current exogenous ex ante rental price equals its shadow value (in turn a function of current ex ante input prices, not past expected prices).

Each element of realized y could then be compared with the corresponding element of y^* , and output-specific CU rates could then be computed. Deviations from unity in these CU rates could of course be consistent with short-run profit maximization, but such deviations would also imply incentives for changing stocks of the fixed factors, and thus only when all output-specific CU rates were unity would the firm be in full equilibrium. We now develop in more detail this current-valued notion of CU.

Given exogenous w, y , and q , the firm's short-run variable cost function is written as

$$C^S(w, y, z). \quad (3.1)$$

Fixed costs, evaluated at the current ex ante rental prices, equal

$$q \cdot z. \quad (3.2)$$

When the firm is in full equilibrium, minimized short- and long-run total costs must be equal, and quasi-fixed inputs are at their full equilibrium levels such that current shadow values equal current ex ante rental prices.

At this full equilibrium point,

$$C^*(w, q, y) = C^S[w, y, z(w, q, y)] + q \cdot z(w, q, y). \quad (3.3)$$

We now consider several apparently different notions of capacity output. Our first definition of the reference vector y^* is that set of outputs at which current shadow values equal current ex ante rental prices for each fixed input, i.e.,

$$\frac{\partial C^S}{\partial z_m} + q_m = 0, \quad m=1, \dots, M. \quad (3.4)$$

This notion is derived by differentiating (3.3) with respect to the current ex ante prices of the fixed inputs, employing Shephard's Lemma, and rearranging:

$$\begin{aligned} \frac{\partial C^*}{\partial q_k} &= \sum_{m=1}^M \frac{\partial C^S}{\partial z_m} \cdot \frac{\partial z_m}{\partial q_k} + z_k^S + \sum_{m=1}^M q_m \cdot \frac{\partial z_m}{\partial q_k} \\ \Rightarrow z_k^* &= z_k^S + \sum_{m=1}^M \frac{\partial z_m}{\partial q_k} \cdot \left[\frac{\partial C^S}{\partial z_m} + q_m \right], \quad k = 1, \dots, M. \end{aligned} \quad (3.5)$$

From (3.5) we see that in general temporary and full-equilibrium demand levels for the fixed inputs are equal only when shadow values equal ex ante rental prices for each fixed factor. Further, when $z_k^* - z_k^S \neq 0$, the difference

is a function of the substitutability relationships among the fixed factors and the gap between shadow values and ex ante rental prices.

A second alternative definition of the reference vector y^* is that set of outputs at which, for each output y_i , short-run or temporary marginal cost (SRMC_i) equals long-run or full equilibrium marginal cost (LRMC_i). As one might expect, this definition is very closely related to the shadow value criterion. To see this, differentiate (3.3) with respect to the y_i and then rearrange, obtaining

$$\begin{aligned} \frac{\partial C^*}{\partial y_i} &= \frac{\partial C^S}{\partial y_i} + \sum_{m=1}^M \left[\frac{\partial C^S}{\partial z_m} \cdot \frac{\partial z_m}{\partial y_i} + q_m \cdot \frac{\partial z_m}{\partial y_i} \right] \\ &= \frac{\partial C^S}{\partial y_i} + \sum_{m=1}^M \frac{\partial z_m}{\partial y_i} \left[\frac{\partial C^S}{\partial z_m} + q_m \right], \quad i = 1, \dots, I. \end{aligned} \quad (3.6)$$

From (3.6) it is clear that in general SRMC_i = LRMC_i if and only if, for each fixed input, shadow values equal ex ante rental prices.³ Hence the marginal cost and shadow value criteria are closely related. Notice also that when LRMC_i - SRMC_i ≠ 0, the difference is a function of the input-output coefficients for the fixed factors and the gap between shadow values and ex ante rental prices.

A third possible criterion for defining the reference vector y^* is that set of outputs at which short-run or temporary equilibrium demands for each of the J variable inputs equal long-run or full equilibrium demands. This too is closely related to the shadow value criterion. To see this, differentiate (3.3) with respect to prices of the variable inputs, use Shephard's Lemma, and then rearrange, obtaining

³For a special case of this result, see Berndt, Fuss and Waverman [1980], pp. 34-35; also see Fuss [1987].

$$\frac{\partial C^*}{\partial w_j} = \frac{\partial C^S}{\partial w_j} + \sum_{m=1}^M \left[\frac{\partial C^S}{\partial z_m} \cdot \frac{\partial z_m}{\partial w_j} + q_m \cdot \frac{\partial z_m}{\partial w_j} \right]$$

$$\Rightarrow v_j^* = v_j^S + \sum_{m=1}^M \frac{\partial z_m}{\partial w_j} \left[\frac{\partial C^S}{\partial z_m} + q_m \right], \quad j = 1, \dots, J. \quad (3.7)$$

Equation (3.7) implies that differences between short- and long-run demands for variable inputs in general equal zero if and only if shadow values of each of the M quasi-fixed inputs equal their ex ante rental prices. When, however, $v_j^* \neq v_j^S$, the difference is a function of the substitutability relationships between fixed and variable inputs and the gap between shadow values and ex ante rental prices.

The above discussion highlights the importance of the shadow value relationships and the potential equivalence of defining the reference vector y^* in four different ways: (i) in terms of shadow values equalling ex ante rental prices, (ii) in terms of fixed input levels, $z_m^* = z_m^S$, (iii) in terms of short- and long-run marginal costs, $LRMC_i = SRMC_i$, and (iv) in terms of variable input demands, $v_j^* = v_j^S$. Before exploring these alternative reference vector notions further, however, we must first establish certain definitions.

Given the reference vector y^* , output-specific CU rates are defined as

$$CU_i \equiv y_i / y_i^*, \quad i=1, \dots, I, \quad (3.8)$$

and input-specific utilization rates for the fixed factors FI_m are defined as

$$FI_m \equiv z_m^* / z_m^S, \quad m=1, \dots, M. \quad (3.9)$$

On the dual side, we now define a CU measure based on short- and long-run marginal costs as

$$MC_i \equiv SRMC_i / LRMC_i, \quad i=1, \dots, I. \quad (3.10)$$

Further, following the Abel [1979] and Hayashi [1982] interpretation of Tobin's q , for each fixed input we take the ratio of the current shadow value to the current ex ante rental price, and denote this shadow value ratio

$$SH_m \equiv \frac{-\partial C^S / \partial z_m}{q_m}, \quad m=1, \dots, M. \quad (3.11)$$

Another measure of utilization, referred to in the Introduction as a dual capacity utilization measure CU_d , compares total shadow costs of production at observed input quantities with a total cost measure at full equilibrium that values fixed inputs at their ex ante rental prices, where both measures are evaluated at the same level of output quantities. Specifically, this total cost measure is now defined as

$$TC \equiv \left(\sum_{j=1}^J w_j v_j^S + \sum_{m=1}^M SH_m q_m z_m^S \right) / \left(\sum_{j=1}^J w_j v_j^* + \sum_{m=1}^M q_m z_m^* \right) \quad (3.12)$$

Finally, we define a variable input utilization ratio as

$$VI_j \equiv v_j / v_j^*, \quad j=1, \dots, J, \quad (3.13)$$

where v_j^* is defined in (3.7).

In order to explore further the relationships among these various primal and dual measures of utilization, we now consider in greater detail three special cases: (i) $I = M = 1$ (one output, one fixed input); (ii) $I = 1, M > 1$ (one output, multiple fixed inputs); and (iii) $I > 1, M > 1$ (multiple outputs, multiple fixed inputs). We begin with the traditional case where $I = M = 1$.

III.A y^* and the Measurement of Utilization with One Output, One Fixed Input

Economic measures of y^* and CU in the case of a single output and a single fixed factor have previously been considered by, among others, Cassels [1937], Klein [1960], Hickman [1964], Berndt-Morrison [1981], Berndt-Hesse

[1985], Berndt-Fuss [1986] and Hulten [1986]. We now briefly review this literature, and relate it to the development presented above.

In the case of a single output, the firm's short- and long-run average total cost curves (SRAC, LRAC) are of course uniquely defined. According to the Wong-Viner envelope theorem, the firm's LRAC curve is the envelope of tangencies with the SRAC curves, where each of the SRAC are indexed by the level of the single quasi-fixed input (usually, capital, denoted K), as in Figure 1.

{Insert Figure 1 at this point}

Traditionally, in this context the firm's capacity level of output y^* has been defined as that level of output at which the SRAC curve is tangent to the LRAC curve, and CU is then defined as in (3.8), i.e. $CU \equiv y/y^*$. For example, as shown in Figure 1, given capital stock K_1 , the $SRAC_1$ curve is tangent to the LRAC curve at capacity output level y_1^* . If actual output were y_0 where $y_0 < y_1^*$, then $CU_0 \equiv (y_0/y_1^*) < 1$. If actual output were y_1 where $y_1 > y_1^*$, then $CU_1 \equiv (y_1/y_1^*) > 1$. In an analogous manner, capacity output levels y_2^* , y_3^* , y_4^* and y_5^* correspond with the appropriate SRAC curves where $K_5 > K_4 > K_3 > K_2 > K_1$. Note that at the minimum point of the LRAC curve, returns to scale are constant; at this point, the capacity output level y_3^* is also the minimum cost point on the $SRAC_3$ curve.

Several other important points merit special attention. First, note that when $y > y^*$, $CU > 1$ and since $SRMC > LRMC$, MC in (3.10) > 1 . From (3.6), $(\partial C^S/\partial K) + \alpha_K < 0$, implying that $SH_K > 1$ and $TC > 1$ (see (3.11) and

(3.13)).⁴ This in turn implies, using (3.5), that $FI_K > 1$. Hence, whenever $y > y^*$, CU , MC , FI_K , SH_K and TC will also each be greater than unity. Similar reasoning can be employed to establish that whenever y is less than (equal to) y^* , CU , MC , FI_K , SH_K and TC will also each be less than (equal to) unity. Moreover, in the special case of long-run constant returns to scale, it can be shown that $CU = FI_K$.⁵

In terms of the VI_j measure of utilization (3.13), note from (3.7) and (3.13) that when $I=M=1$ and there is only one variable input, VI_j will always be on the same side of unity as CU , MC , FI_K and SH_K ; if, for example, the fixed capital input is heavily utilized such that $z_K^* > z_K$ implying that $FI_K > 1$, then current levels of the variable input v_1 are greater than the full equilibrium level v_1^* , indicating $VI_1 > 1$. Further, in the more general case when $J > 1$, VI_j will always be on the same side of these other utilization measures provided that j is substitutable with the single fixed input z_K .

Finally it is worth noting that deviations from unity in these various primal and dual utilization measures can of course be entirely consistent with short-run profit maximization, even for competitive firms. In order to maximize long-run profits, however, firms encountering values of these utilization measures that differ from unity face incentives to change their levels of K , in particular, increasing K when the measures are larger than unity and decreasing K when they are less than unity.

⁴This assumes that $\partial z_m / \partial y_i$ (in this context, $\partial K / \partial y$) is positive, which occurs as long as K is not an inferior input.

⁵Under constant returns to scale, $\partial \ln K / \partial \ln y = 1$, which implies that $\partial K / \partial y = K/y$ equals, say, μ , which is constant as long as input prices are fixed. Given y , it follows that $K^* = \mu y$. Alternatively, given K , $K^S = \mu y^*$. Taking ratios, we see that $K^*/K^S = y/y^*$, or $FI_K = CU$. Note, however, that unless there are no variable inputs, $SH_K \neq MC$, even with constant returns to scale. Rather, SH_K here equals $1 - (C/q_K \cdot K^*) \cdot (1 - MC)$.

III.B y* and Measures of Utilization with One Output, Multiple Fixed Inputs

We now consider a slightly more general case, one in which there is still only one output, but where now there are multiple fixed inputs. Although this situation has not received as much attention as the $I = M = 1$ case, some attention has been focused on the case of $M > 1$. Wildasin [1984], for example, considered Tobin's q measures in a theoretical model with multiple fixed inputs, Berndt [1980] computed capacity output in a model with capital and nonproduction labor as fixed inputs, Epstein-Denny [1983] treated labor and capital as fixed inputs in their study of U.S. manufacturing, Schankerman-Nadiri [1986] examined shadow value and ex ante rental price relationships both conceptually and empirically for two types of capital (research and development and equipment-structures capital) in the U.S. Bell System, and Morrison [1988a] developed and implemented empirically a model in which CU and other utilization measures are computed for U.S. and Japanese manufacturing industries, where capital and certain labor inputs are treated as quasi-fixed.

Since we are still dealing with a single output, in the $I = 1, M > 1$ case the short- and long-run average cost curves are well-defined. However, the various SRAC curves shown in Figure 1 can no longer be indexed by a single fixed factor. In particular, since the point of tangency of each SRAC curve with the common LRAC curve by definition represents a point of long-run equilibrium where total costs are minimized, as long as fixed inputs are not inferior, it follows that successive SRAC curves from $SRAC_1$ to $SRAC_5$ correspond with larger levels of each of the fixed inputs. This implies that the cost curves in Figure 1 should now be envisaged as being indexed by a vector whose

elements are monotonically increasing functions of y . Note that this rules out the case where, for example, on the $SRAC_4$ curve there is less of the z_1 fixed input and more of the z_2 fixed input than on the $SRAC_3$ curve.

As in the case of the single fixed factor, one can simply define y^* as that level of output at which the SRAC curve is tangent to the LRAC curve. By (3.3) and (3.5), at this level of output it is of course the case that for each fixed input, shadow values equal ex ante rental prices. This suggests, then, that in this $I=1, M>1$ case one simply compute y^* using (3.5).

Since there are M such shadow value relationships in (3.5), and since any of them could be solved for that level of y such that $-\partial C^S/\partial z_m = q_m$, this raises the issue of which one of the M should be employed, and whether the M solutions are identical. Fortunately, it can easily be shown that each yields the same value of y^* .⁶ This implies that in the single output case, one need only choose any one of the M shadow value relationships in (3.4) and solve it for y^* .

It is useful here to examine relationships among the various utilization measures discussed earlier in this section. Suppose $y > y^*$, i.e. $CU > 1$. From Figure 1 and use of the Le Chatelier principle, it is clear that at this level of output, $SRMC > LRMC$. By (3.6), this implies that

⁶To see this, assume that given a particular set of q_m and v_j , the firm is initially in full or long-run equilibrium such that the capacity output level y^* is produced, and shadow values equal ex ante market rentals for each of the M fixed inputs. Now let there be a small change in the price of a variable input. For each of the M shadow value equations, solve for that level of y such that the shadow value equals the ex ante rental price, and denote that level of y as y_m^* . The proof now follows by contradiction: Let the y_m^* differ. However, given that the optimal combination of fixed and variable inputs following the change in v_j produces a unique level of output, it must be the case that $y_m^* = y^*$, i.e., the single-valued output of the production process, embodied in the restricted cost function, ensures that y_m^* is unique.

$$\sum_{m=1}^M \frac{\partial z_m}{\partial y} \left[\frac{\partial C^S}{\partial z} + q_m \right] < 0 .$$

Assuming that fixed inputs are not inferior ($\partial z_m / \partial y > 0$), if $CU > 1$, this implies that for at least one fixed input, it must be the case that $SH_m > 1$. Note that $CU > 1$ does not necessarily imply, however, that all $SH_m > 1$. On the other hand, if all $SH_m > 1$, then by (3.6) it must be the case that $SRMC > LRMC$ (i.e., that $MC > 1$), and that $y > y^*$.

A number of other propositions can now be stated, providing additional insights into relationships among the various utilization measures. Among these propositions are the following:

Proposition 1: For the case of $I = 1$, $M > 1$, each of the CU , MC and TC utilization measures will always simultaneously be either less than, equal to, or greater than unity.

Proof: If $CU > 1$, then the firm is operating at a point where $LRMC > SRMC$, which implies by (3.10) that $MC > 1$. Further, $MC > 1$ implies that $TC > 1$. Analogous arguments apply for CU less than or equal to unity.

Proposition 2: For the case of $I = 1$, $M > 1$, if $SH_m > 1$ for all m , this does not provide sufficient information to determine whether for any individual fixed input k , $FI_k > 1$.

Proof: As seen in equation (3.7) this occurs because of the differing possible substitutability and complementarity relationships involving fixed and variable inputs.

Corollary: If the set of fixed inputs is homothetically separable from each of the variable inputs so that an aggregate measure of the fixed input exists, and if this aggregate is substitutable

with each v_j , then $SH_m > 1$ for all fixed inputs would be sufficient to ensure that for any k , $FI_k > 1$.

Proof: To see this, note that when the fixed inputs are substitutable with each variable input, $\partial z_m / \partial w_j > 0$ for all m and j . Together with the fact that the term in square brackets in (3.7) is negative, this implies that $VI_j > 1$ ($v_j^* < v_j^S$), which in turn implies that $FI_k = z_k^* / z_k^S > 1$.

Comment: An implication is that if "Tobin's q " measures for all fixed inputs were greater than unity, in general one could still not state that for any individual fixed input m , the firm could reduce costs further by purchasing additional units of z_m .

However, if an aggregate of all the fixed inputs exists and if it is substitutable with each variable input, then $SH_k > 1$ for all k implies $FI_m > 1$ for all m .

Proposition 3: For the case of $I = 1$, $M > 1$, if $SH_k > 1$ for only one k , and if for all other fixed inputs $SH_\ell = 1$, $\ell \neq k$, then $FI_k > 1$.

Proof: This follows directly from (3.5).

Comment: This case is admittedly likely to be rare, but could occur if $(M-1)$ fixed inputs were homothetically separable from the k^{th} fixed input.

Proposition 4: For the case of $I = 1$, $M > 1$, $CU > 1$ implies that at least one $SH_k > 1$, but it does not imply that at least one $FI_m > 1$.

Proof: The first result follows directly from (3.6). To see the second result, consider (3.5) in the case of two fixed inputs:

$$z_1^* = z_1^S + \frac{\partial z_1}{\partial q_1} \cdot \left[\frac{\partial c^S}{\partial z_1} + q_1 \right] + \frac{\partial z_1}{\partial q_2} \cdot \left[\frac{\partial c^S}{\partial z_2} + q_2 \right]. \quad (3.14)$$

The derivative before the first square brackets must be negative. Suppose that the derivative before the second is positive. If $SH_1 > 1$, then the term in the first brackets is negative. But even here, one cannot sign the difference between z_1^* and z_1^S unless one has additional information concerning the magnitude of the negative term in the second square brackets. Hence one cannot assess whether FI_1 is greater than, equal to, or less than one.

We now turn to our most general case, in which there are multiple fixed inputs and multiple outputs.

III.C Measures of Utilization with Multiple Outputs, Multiple Fixed Inputs

For the case of multiple outputs average cost is no longer well-defined. The Viner-Wong envelope condition can no longer be used to determine capacity output, but as we demonstrate below, a related measure based on the concept of the average incremental cost (AIC) curve can be used to demonstrate the notion of economic capacity output geometrically.

Equations (3.5), (3.6) and (3.7) are valid in the multiple output - multiple input case, and these can be used to determine the reference vectors of outputs y^* and fixed inputs z^* . This implies that the six indicators of economic utilization -- CU_i , FI_m , MC_i , SH_m , TC and VI_j -- are appropriate measures in the context of multiple outputs and multiple inputs.

An important issue arises in this most general case, however, which was not present in the single-output context. In particular, a possibility now arises that elements of y^* are indeterminate in certain cases. Indeterminacy is an important issue, for if elements of y^* cannot be determined in the cost minimization context, then CU measures cannot be constructed.

This indeterminacy problem is easily illustrated in the two output, one fixed input case. Recall that long run or full equilibrium is determined by the single shadow price equation $SH = 1$. However, this equation cannot be solved for both economic capacity outputs y_1^* and y_2^* ; rather, only a locus $h(y_1^*, y_2^*) = 0$ can be determined. A unique solution can therefore only be obtained by incorporating additional demand information; for example, the long-run profit maximizing conditions $MR_i = LRMC_i$ would be sufficient to obtain unique solutions for y_1^* and y_2^* .⁷

The indeterminacy problem for the reference output vector will occur whenever $M < I$. It obviously constrains severely our ability to obtain the vector y^* from considerations of cost minimization alone.

Another important issue in the multiple output context concerns use of average cost. As noted earlier, a geometric interpretation of multiple output capacity utilization can be derived in spite of the non-existence of average cost. In particular, following Baumol, Panzar and Willig [1982], define the average incremental cost curve for output i as

$$AIC_i \equiv \frac{C(\hat{y}, y_i) - C(\hat{y}, 0)}{y_i}, \quad i = 1, \dots, I. \quad (3.15)$$

⁷Alternatively, one could assume perfect competition and specify a variable profit function; unique solutions for the y_i^* could be obtained by solving shadow value equations for y_i^* analogous to (3.4) based on the variable profit function. In this case and with decreasing returns to scale, the output vector would be uniquely determined even if $M < I$.

where

$$\hat{y} \equiv (y_1, \dots, y_{i-1}, y_{i+1}, \dots, y_I).$$

Consider the condition such that the short-run and long-run, or full equilibrium incremental cost curves AIC_i^S and AIC_i^* are tangent to one another,

$$\frac{\partial AIC_i^S}{\partial y_i} = \frac{\partial AIC_i^*}{\partial y_i} \quad (3.16)$$

It can be easily shown that (3.16) is satisfied when $SRMC_i = LRMC_i$. But from (3.6) $SRMC_i = LRMC_i$ ($MC_i = 1$) is precisely the condition defining capacity output y_i^* . However, this condition, while necessary for long-run or full equilibrium, is not sufficient unless either all $SRMC_i = LRMC_i$, $i=1, \dots, I$, or there is non-joint production. The problem is that a subset of the MC_i could be equal to unity without all $SH_m = 1$, $m = 1, \dots, M$. Thus the envelope condition between AIC_i^S and AIC_i^* does provide the appropriate geometric interpretation of y_i^* when all other outputs are fixed at their economic capacity levels. Figures 2(a) and 2(b) demonstrate this latter situation for the two output case.

{Insert Figures 2(a) and 2(b) near this point}

As expected, assuming that the reference vector y^* is determinate, relationships among the various utilization measures become more complex when there are both multiple products and multiple fixed inputs. Although we expect, for example, that if $CU_i > 1$ for all $i = 1, \dots, I$, then $TC > 1$ and $SH_k > 1$ for at least one k , it is not necessarily the case that the corresponding $MC_i > 1$ for all $i = 1, \dots, I$, nor is it necessarily the case that at least one $FI_m > 1$. Further, we conjecture that an aggregate measure of CU will exist if

and only if a consistent aggregate index of output exists, i.e. if and only if the outputs are separable from the fixed and variable inputs.

IV. Implications for Computing CU-Adjusted Multifactor Productivity Growth

As was noted in the Introduction, one important use of CU measures is to adjust multifactor productivity (MFP) growth for cyclical variations in CU.⁸ We now consider how the various utilization measures developed in this paper might be appropriately employed to measure MFP in the multiproduct context with multiple fixed inputs as well.

In the case of a single output, no fixed inputs and a constant returns to scale production technology, one can follow Robert M. Solow [1957] and Dale W. Jorgenson and Zvi Griliches [1967] and interpret the rate of MFP growth as the outward shift in the production function due to technical change, and calculate this as

$$\dot{A}_0/A_0 \equiv \dot{y}/y - \sum_{j=1}^J \bar{s}_j \cdot (\dot{v}_j/v_j) \quad (4.1)$$

where s_j is the cost share of the j^{th} (variable) input,

$$s_j \equiv w_j v_j / \sum_{j=1}^J w_j v_j \quad (4.2)$$

⁸We limit our discussion here to multifactor productivity growth and do not consider single-input measures of productivity such as average labor productivity y/L , for single-input productivity measures do not appear to be useful in the multiproduct context. In theory, one might be able to compute meaningful input-specific average incremental product measures for each output, but we do not examine such concepts here.

the superscript \cdot denotes a time derivative (in discrete form, usually a first difference), and the superscript $\bar{}$ refers to the arithmetic mean of the cost share in the two adjacent time periods. Note that under constant returns to scale and cost minimization, cost shares correspond to logarithmic marginal products. Hence from (4.1) it is clear that MFP growth corresponds to growth in output minus growth in aggregate input, where the measure of aggregate input weights the growth rates of each of the inputs by their logarithmic marginal product. In terms of average cost, note that this measure of MFP growth corresponds to a downward shift in the cost function, and not a movement along it.

Makoto Ohta [1974] has shown that if one maintains the single output, no fixed input and cost minimization assumptions but allows for non-constant returns to scale, then to measure only the fruits of technical progress and not include the effects of scale economies in an MFP measure, one must adapt (4.1) to

$$\dot{A}_1/A_1 \equiv \bar{g} \cdot (\dot{y}/y) - \sum_{j=1}^J \bar{s}_j \cdot (\dot{v}_j/v_j), \quad (4.3)$$

where \bar{g} is the arithmetic mean of the elasticity of total costs with respect to output in the two adjacent time periods. Note that when returns to scale are increasing, constant, or decreasing, \bar{g} is less than, equal to, or greater than one, respectively. With increasing returns to scale, for example, \bar{g} is less than unity, implying that growth in output is diminished before growth in aggregate input is subtracted, resulting in lower MFP growth than would occur were scale economies not taken into account. Further, the MFP growth measure in (4.3) corresponds to a shift in the cost function, not a movement along it.

The multiproduct analog to (4.3) has been employed in studies by, among others, Caves, Christensen and Swanson [1980] and Denny, Fuss and Waverman [1981], and is simply computed as

$$\dot{A}_2/A_2 \equiv \sum_{i=1}^I \bar{g}_i \cdot (\dot{y}_i/y_i) - \sum_{j=1}^J \bar{s}_j \cdot (\dot{v}_j/v_j), \quad (4.4)$$

where g_i is the elasticity of total cost with respect to output i , i.e. $g_i \equiv \partial \ln C / \partial \ln y_i$. Notice that MFP growth represents the shift in the multiproduct cost function due to disembodied technical progress. It is worth remarking that as long as one interprets (4.4) as a discrete approximation to a total derivative of the logarithm of a multiproduct cost function with respect to time, then measurement of MFP growth using (4.4) is appropriate even if the outputs are not separable from the inputs; in the nonseparable case, however, the first term on the right-hand side of (4.4) can no longer be interpreted as growth in aggregate output, nor can the second term be viewed as growth in aggregate input.⁹

The above results are based on the assumption of instantaneous adjustment of all inputs to their full equilibrium values, i.e. that there are no fixed inputs in the short run, and that each of the utilization measures considered in Section III of this paper are always equal to unity. As has been emphasized by, among others, Berndt-Fuss [1986], Hulten [1986] and Morrison [1985,1986,1988], however, once one allows for the possibility of fixed inputs, in order to measure MFP growth properly, one must not only distinguish short-run from full equilibrium cost functions, but one must also

⁹For further discussion, see Caves, Christensen and Swanson [1981, pp. 168-170].

separate movements along a short-run cost function from shifts in that function. In essence, these authors suggest a procedure by which the s_j weights in (4.4) are replaced with shadow value weights based on SH_m from (3.11), thereby accounting properly for divergences from full equilibrium.

As we shall see, in fact there are a substantial number of alternative ways of adjusting MFP growth for cyclical variations in utilization, each of them firmly grounded in economic theory, not just the one procedure emphasized by Berndt-Fuss, Hulten and Morrison. We now examine three alternative procedures in detail, and comment more briefly on several other possibilities; all these possibilities employ the various utilization measures developed earlier in this paper. In order to keep our discussion here simple, however, we will ignore issues concerning nonstatic expectations and discrepancies between expectations and realizations of output demand and prices of fixed inputs, and will simply assume that realized prizes reflect fulfilled expectations; issues of nonstatic expectations are discussed elsewhere in more detail, such as in Berndt-Fuss [1986] and Morrison [1985b,1986]. Further, to keep our notation simple, hereafter we dispense with the superscript S that denoted short-run observations on variables.

Bearing in mind that MFP growth corresponds to shifts in cost functions rather than movements along them, we now consider two generic procedures for transforming temporary equilibrium observations into full equilibrium notional observations: (i) adjusting the observed quantities of fixed and variable inputs or outputs to notional full equilibrium levels, given ex ante rental prices; or (ii) adjusting the observed cost share weights of the inputs and the cost elasticity weights of the outputs to account for divergences from full equilibrium, given quantities of the fixed factors. We begin with (i).

One way of adjusting MFP measures to account for variations in utilization is to compute, for the two time periods under consideration, what demands for variable and fixed inputs would have been in the two time periods had full equilibrium occurred, given observed output quantities and the prices of the variable and fixed inputs.¹⁰ This yields an MFP equation, analogous to (4.4), with the J variable inputs now explicitly distinguished from the M fixed inputs, and where observed input levels are adjusted by the VI_j and FI_m short-run utilization measures. This MFP growth measure is computed as

$$\begin{aligned} \dot{A}_3/A_3 &\equiv \sum_{i=1}^I \bar{g}_i^* \cdot (\dot{y}_i/y_i) - \sum_{j=1}^J \bar{s}_j^* \cdot (\dot{v}_j^*/v_j^*) - \sum_{m=1}^M \bar{s}_m^* \cdot (\dot{z}_m^*/z_m^*) \\ &= \sum_{i=1}^I \bar{g}_i^* \cdot (\dot{y}_i/y_i) - \sum_{j=1}^J \bar{s}_j^* \cdot (v_j^*/VI_j) / (v_j/VI_j) - \sum_{m=1}^M \bar{s}_m^* \cdot (FI_m^* \cdot z_m^*) / (FI_m \cdot z_m) \end{aligned} \quad (4.5)$$

where the equilibrium cost shares are $s_j^* \equiv w_j v_j^*/C^*$, $j = 1, \dots, J$, and $s_m^* \equiv q_m z_m^*/C^*$, $m = 1, \dots, M$, total costs in full equilibrium are $C^* \equiv \sum_j w_j v_j^* + \sum_m q_m z_m^*$, g_i^* is the elasticity of total costs (3.3) with respect to output i evaluated at v_j^* and z_m^* , and, based on (3.9) and (3.13), $z_m^* = FI_m Z_m$ and $v_j^* = v_j/VI_j$.

An alternative approach to MFP measurement adjusted for short-run utilization effects is based on a full equilibrium notional quantity adjustment that focuses on the full equilibrium quantities of the outputs

¹⁰Note that this procedure is not the same as that used by Jorgenson-Griliches [1967], in which traditional measures of capital quantity input are multiplied by estimated rates of capital utilization using an electricity consumption adjustment, and the measure of q_K is based on an ex post rather than ex ante rate of return. For further discussion of this point, see Berndt-Fuss [1986, fn. 10, p. 15 and p. 27].

(capacity outputs) and variable inputs, rather than on the equilibrium quantities of the fixed inputs.¹¹ Obviously, such a procedure is operative only if the concept of capacity output is meaningful; as discussed earlier, under the assumption of cost minimization, the capacity output vector is uniquely defined only if $M > I$. Assuming that the $M > I$ condition is satisfied, one can compute for the two time periods under consideration, what the quantities of capacity outputs and variable inputs would have been in the two time periods, given observed quantities of the fixed inputs and prices of the variable inputs. This forms the basis of an MFP equation, analogous to (4.5), where observed output and variable input levels are adjusted by the CU_i and VI_j short-run utilization measures:

$$\begin{aligned} \dot{A}_4/A_4 &\equiv \sum_{i=1}^I \bar{g}_i' \cdot (\dot{y}_i'/y_i') - \sum_{j=1}^J \bar{s}_j' \cdot (\dot{v}_j'/v_j') - \sum_{m=1}^M \bar{s}_m' \cdot (\dot{z}_m'/z_m') & (4.6) \\ &= \sum_{i=1}^I \bar{g}_i' \cdot (\dot{y}_i'/y_i') \cdot (CU_i'/CU_i) - \sum_{j=1}^J \bar{s}_j' \cdot (v_j'/VI_j)/(v_j'/VI_j) - \sum_{m=1}^M \bar{s}_m' \cdot (\dot{z}_m'/z_m') \end{aligned}$$

where the full equilibrium cost shares $s_j' \equiv w_j v_j'/C'$, $j = 1, \dots, J$, and $s_m' \equiv q_m z_m'/C'$, $m = 1, \dots, M$. Total costs in full equilibrium at capacity output production levels are $C' \equiv \sum_j w_j v_j' + \sum_m q_m z_m'$, and g_i' is the elasticity of total costs (3.3) with respect to output i evaluated at the capacity output levels. Note that since output production in (4.6) is at capacity output

¹¹To the best of our knowledge, the first empirical MFP study employing this procedure is that by Berndt [1980], who calculated capacity output and equilibrium variable inputs in the context of a single output, two fixed input framework under the assumption of constant returns to scale; extensions to the single-output nonconstant returns to scale case have been calculated by Morrison [1985b, 1986].

levels y_i^1 while that in (4.5) is at observed output levels y_i , the levels of variable input demands v_j^1 at full equilibrium in (4.6) will differ from the v_j^* in the full equilibrium of (4.5), and $C^* \neq C^1$. In general, $\dot{A}_3/A_3 \neq \dot{A}_4/A_4$. Only in the case of constant returns to scale will $\dot{A}_3/A_3 = \dot{A}_4/A_4$. In this case, for $\alpha > 0$, if $y_i = \alpha y_i^1$, then $v_j^* = \alpha v_j^1$, $z_m^* = \alpha z_m^1$, $C^* = \alpha C^1$, and $s_j^* = s_j^1$.

The next procedure for adjusting MFP growth for variations in utilization involves altering the weights applied to the observed growth rates, rather than using the computed growth rates of notional full equilibrium quantities. As was noted earlier, one can define a shadow cost function where quasi-fixed inputs are evaluated at their shadow rather than ex ante rental prices; specifically, using (3.11), define total shadow costs as

$$C'' \equiv \sum_j w_j v_j + \sum_m S H_m q_m z_m. \quad (4.7)$$

In the context of a single-output constant returns to scale technology, Berndt-Fuss [1986], Hulten [1986] and Morrison [1986] have shown that MFP growth (corresponding with a downward shift in the minimum point of the short-run average total cost function) in the presence of fixed inputs can be measured properly by employing observed quantities of output and the fixed and variable inputs, but replacing cost shares based on ex ante rental prices with cost shares based on shadow values of the fixed inputs. A generalization of this approach to the case of nonconstant returns to scale has been developed and implemented by Morrison [1986, 1988b]. In our multiproduct context, we can write this measure of MFP growth adjusted for utilization changes as

$$\dot{A}_5/A_5 \equiv \sum_{i=1}^I \bar{g}_i'' \cdot (\dot{y}_i/y_i) - \sum_{j=1}^J \bar{s}_j'' \cdot (\dot{v}_j/v_j) - \sum_{m=1}^M \bar{s}_m'' \cdot (\dot{z}_m/z_m) \quad (4.8)$$

where the $s^{\prime\prime}$ input cost shares are defined in terms of cost shares in total shadow costs, i.e., $s_j^{\prime\prime} \equiv w_j v_j / C^{\prime\prime}$ for the J variable inputs, $s_m^{\prime\prime} \equiv SH_m q_m z_m / C^{\prime\prime}$ for the M fixed inputs, total shadow costs $C^{\prime\prime}$ are defined in (4.7) (and, from (3.12), can also be written as $C^{\prime\prime} = TC \cdot C^*$), and $g_i^{\prime\prime}$ is the elasticity of the shadow cost function (4.7) with respect to output i , evaluated at the observed output levels.

To interpret (4.8), consider the case of one output and one fixed input. If a firm is operating to the right of the tangency point between its short- and long-run average cost functions, then the utilization rate of the single fixed input $FI_1 > 1$, its shadow value is greater than the ex ante rental price implying $SH_1 > 1$, and thus in (4.8) any growth in the fixed input between the two time periods under consideration receives a larger share weight (reflecting high utilization) than would be the case were the ex ante price weight to be used instead. This larger weight reflects, of course, the higher short-run marginal product of the fixed input.

In general \dot{A}_5/A_5 will not be equal to the other two capacity utilization-adjusted measures of MFP growth. However, conditional on a constant returns to scale technology, the three measures will be equal. This can be seen most easily by comparing \dot{A}_3/A_3 and \dot{A}_5/A_5 . Both measures are based on data corresponding to cost-minimizing choices of inputs needed to produce the output vector y . The input mixes differ because the prices of quasi-fixed inputs used in the calculations are not the same (for \dot{A}_5/A_5 the shadow prices of quasi-fixed factors are assumed to be operative). As long as the input requirement frontier shifts uniformly between the two periods (the

case of constant returns to scale), the two measures of MFP growth will be equal.¹²

The above three utilization-adjusted MFP growth equations demonstrate that the measures of capacity utilization in a multiproduct context with multiple fixed inputs developed in Section III of this paper have useful applications. While all three of the MFP growth measures considered in Section IV are primal in the sense that they compute MFP growth as basically growth in output quantity minus growth in input quantity, we could instead have developed dual MFP growth measures computed essentially as growth in costs minus growth in input prices; in fact, a dual MFP growth measure adjusted for utilization based on evaluation of costs at notional capacity output and variable input values, analogous to (4.6), has been empirically implemented in the single output case by Fuss and Waverman [1986,1989]. Another set of measures could also have been developed where output costs are replaced by output prices; in order to reflect only shifts in production and dual cost functions, additional specific assumptions would need to be made concerning the existence of economic profits, the nature of any imperfect competition and the implied markups over marginal cost, and the scale economies of production. Although each of these alternative measures of utilization-adjusted MFP growth would be of interest, we leave their development and further discussion to another paper, for our point here has

¹²This result assumes that the production function is neoclassical in the sense that quasi-fixed factors are never idle, so that underutilization refers to a use of the quasi-fixed factors that is technically efficient but not cost-minimizing at current input prices. For an alternative technology, such as the putty-clay form, the shadow price of underutilized fixed factors would be zero, and \dot{A}_5/A_5 would not be an appropriate measure. The other two measures would also not be appropriate, but could more easily be adapted to this situation.

simply been to demonstrate that the various utilization measures developed in this paper are useful in the context of MFP growth measurement.

V. Concluding Remarks

Economic measures of capacity output and capacity utilization are attractive because their interpretation is clear. Heretofore, economic measures of capacity output and capacity utilization have been developed only in the context of single output firms. Our purpose in this paper has been to develop measures of capacity output and capacity utilization for firms producing multiple outputs and having one or more quasi-fixed inputs. In our development, we have presented an "impossibility theorem" (based on cost minimization only, the notion of a capacity output reference vector is nonexistent if $I > M$, i.e. if the number of outputs exceeds the number of fixed inputs), but we have also provided constructive definitions of capacity output and utilization whenever $I \leq M$. In particular, we have proposed a variety of output-specific and fixed input-specific primal and dual measures of utilization. We have related these measures to one another, and have emphasized that inequality relationships (relative to unity) among the utilization measures can typically be specified a priori only under rather restrictive conditions. We have shown that analogs to Tobin's q in the context of multiple quasi-fixed inputs have little informational content regarding incentives for investment, unless additional restrictive assumptions are employed. Finally, we have shown how these utilization measures can usefully be employed to adjust traditional measures of multifactor productivity (MFP) growth for cyclical changes in utilization, even in the $I >$

M case when capacity output and capacity utilization are undefined.¹³

In our judgment, it would be particularly useful if future research in this area focussed on issues of empirical implementation. We expect that a considerable amount can be learned from comparing the various utilization measures empirically, as well as from examining empirically the effects of alternative utilization adjustments to traditionally measured MFP growth.¹⁴

¹³Recall that \dot{A}_4/A_4 is undefined when y_1^* cannot be determined. This problem does not arise with \dot{A}_3/A_3 and \dot{A}_5/A_5 .

¹⁴A recent effort in this direction is that by Conrad and Unger [1989], who consider the three alternative utilization adjustments to MFP growth presented in Section IV of this paper in the context of a model with one output, one fixed input, and constant returns to scale. Since constant returns to scale was assumed, the three adjustments yielded the same measure of adjusted MFP growth (see our discussion at the end of Section IV).

VI. REFERENCES

- Abel, Andrew B. [1979], Investment and the Value of Capital, New York: Garland Publishing Co. (Ph.D. dissertation, Massachusetts Institute of Technology, Department of Economics, 1978.)
- Baumol, William J., John C. Panzar and Robert D. Willig [1982], Contestable Markets and The Theory of Industry Structure, New York: Harcourt Brace Jovanovich, Inc.
- Berndt, Ernst R. [1980], "Energy Price Increases and the Productivity Slowdown in United States Manufacturing," in The Decline in Productivity Growth: Proceedings of a Conference Held in June 1980, Boston: The Federal Reserve Bank of Boston, June, pp. 60-89.
- Berndt, Ernst R. and Melvyn A. Fuss [1986], "Productivity Measurement with Adjustments for Variations in Capacity Utilization, and Other Forms of Temporary Equilibrium," Journal of Econometrics, Vol. 33, No. 1/2, October/November 1986, pp. 7-29.
- Berndt, Ernst R., Melvyn A. Fuss and Leonard Waverman [1980], "Empirical Analyses of Dynamic Adjustment Models of the Demand for Energy in U.S. Manufacturing Industries, 1947-1974," Final Report to the Electric Power Research Institute, Project 683-1, Palo Alto, CA: EPRI EA-1613, November.
- Berndt, Ernst R. and Dieter M. Hesse [1985], "Measuring and Assessing Capacity Utilization in the Manufacturing Sector of Nine OECD Countries," European Economic Review, Vol. 30, No. 5, October, pp. 961-989.
- Berndt, Ernst R. and Catherine J. Morrison [1981], "Capacity Utilization: Underlying Economic Theory and an Alternative Approach," American Economic Review, Vol. 71, No. 2, May, pp. 48-52.
- Berndt, Ernst R., Catherine J. Morrison, and David O. Wood [1983], "The Modeling, Interpretation and Measurement of Capacity Utilization," Technical Note, Bureau of Census, U.S. Department of Commerce, Contract Requisition 63-3-2, May.
- Cassels, J. M. [1937], "Excess Capacity and Monopolistic Competition," Quarterly Journal of Economics, Vol. 51, pp. 426-443.
- Caves, Douglas W., Laurits R. Christensen and Joseph A. Swanson [1980], "Productivity in U.S. Railroads, 1951-1974," The Bell Journal of Economics, Vol. 11, No. 1, Spring, pp. 166-181.
- Conrad, Klaus and Ralph Unger [1989], "Productivity Gaps and Capacity Utilization in the Manufacturing Sectors of Five OECD Countries," Journal of Productivity Analysis, forthcoming.
- Denny, Michael, Melvyn Fuss and Leonard Waverman [1981], "The Measurement and Interpretation of Total Factor Productivity in Regulated Industries,

- with an Application to Canadian Telecommunications," ch. 8 in Thomas G. Cowing and Rodney E. Stevenson, eds., Productivity Measurement in Regulated Industries, New York: Academic Press, pp. 179-218.
- Epstein, Larry G. and Michael G. S. Denny [1983], "The Multivariate Flexible Accelerator Model: Its Empirical Restrictions and an Application to U.S. Manufacturing," Econometrica, Vol. 51, No. 3, May, pp. 647-674.
- Fuss, Melvyn A. [1987], "Production and Cost Functions." in J. Eatwell, M. Milgate and P. Newman, eds., The New Palgrave: A Dictionary of Economic Theory and Doctrine, New York: MacMillan, Vol. 3, pp. 995-1000.
- Fuss, Melvyn A. and Leonard Waverman [1989], "Productivity Growth in the Automobile Industry 1970-1984: A Comparison of Canada, Japan and the United States," in Charles R. Hulten, ed., Productivity Growth in the United States and Japan, National Bureau of Economic Research Studies in Income and Wealth, Chicago: University of Chicago Press, forthcoming.
- Fuss, Melvyn A. and Leonard Waverman [1986], "The Extent and Sources of Cost and Efficiency Differences Between US and Japanese Automobile Producers," National Bureau of Economic Research Working Paper No. 1849, March.
- Hayashi, Fumio [1982], "Tobin's Marginal q and Average q : A Neoclassical Interpretation," Econometrica, Vol. 50, pp. 213-224.
- Hickman, Bert G. [1964], "On A New Method of Capacity Estimation," Journal of the American Statistical Association, Vol. 59, No. 306, pp. 529-549.
- Hulten, Charles R. [1986], "Productivity Change, Capacity Utilization, and the Sources of Efficiency Growth," Journal of Econometrics, Vol. 33, No. 1/2, October/November 1986, pp. 31-50.
- Jorgenson, Dale W. and Zvi Griliches [1967], "The Explanation of Productivity Change," Review of Economic Studies, Vol. 34, pp. 249-282.
- Klein, Lawrence R. [1960], "Some Theoretical Issues in the Measurement of Capacity," Econometrica, Vol. 28, pp. 272-286.
- Morrison, Catherine J. [1988a], "Quasi-Fixed Inputs in U.S. and Japanese Manufacturing: A Generalized Leontief Restricted Cost Function Approach," Review of Economics and Statistics, Vol. 70, No. 2, May, pp. 275-287.
- Morrison, Catherine J. [1988b], "A Framework for Adjusting Productivity Growth Measures: Accounting for Subequilibrium, Returns to Scale and Markups," Cambridge, MA: National Bureau of Economic Research, draft manuscript, August.
- Morrison, Catherine J. [1986], "Productivity Measurement with Non-Static Expectations and Varying Capacity Utilization: An Integrated Approach," Journal of Econometrics, Vol. 33, No. 1/2, October/November 1986, pp. 51-74.

- Morrison, Catherine J. [1985a], "Primal and Dual Measures of Economic Capacity Utilization: An Application to Productivity Measurement in the U.S. Automobile Industry," Journal of Business and Economic Statistics, Vol. 3, No. 4, October, pp. 312-324.
- Morrison, Catherine J. [1985b], "On the Economic Interpretation and Measurement of Optimal Capacity Utilization with Anticipatory Expectations," Review of Economic Studies, Vol. 52, pp. 295-310.
- Morrison, Catherine J. and Ernst R. Berndt [1981], "Short Run Labor Productivity in a Dynamic Model," Journal of Econometrics, Vol. 16, pp. 339-365.
- Ohta, Makoto [1974], "A Note on the Duality Between Production and Cost Functions: Rate of Returns to Scale and Rate of Technical Progress," Economic Studies Quarterly, Vol. 25, No. 1, pp. 63-65.
- Schankerman, Mark and M. Ishaq Nadiri [1986], "A Test of Static Equilibrium Models and Rates of Return to Quasi-Fixed Factors, with an Application to the Bell System," Journal of Econometrics, Vol. 33, No. 1/2, October/November 1986, pp. 97-118.
- Slade, Margaret E. [1986], "Total-Factor Productivity Measurement when Equilibrium is Temporary: A Monte Carlo Assessment," Journal of Econometrics, Vol. 33, No. 1/2, October/November 1986, pp. 75-95.
- Solow, Robert M. [1957], "Technical Change and the Aggregate Production Function," Review of Economics and Statistics, Vol. 39, No. 5, August, pp. 312-320.
- Wildasin, David E. [1984], "The q Theory of Investment with Many Capital Goods," American Economic Review, Vol. 74, No. 1, March, pp. 203-210.

FIGURE 1



