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**Overcoming Collusion: Using a Supervisor
to Costlessly Resolve Moral Hazard**

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Using A Supervisor To Costlessly Resolve Moral Hazard

A principal, who can not directly observe an agent's effort, may induce effort by rewarding the agent for output. However, first best profits are generally unobtainable because rewarding the agent for output requires an inefficient allocation of risk when the agent is risk averse and output is a noisy measure of effort. Under reasonable conditions, we offer a costless solution to this moral hazard problem. In particular, we contemplate situations in which an agent first decides upon an effort level, a supervisor observes this effort level, and the principal and supervisor then both observe a noisy signal of the agent's effort (output). When the agent and supervisor are unable to contract upon the agent's effort level, our proposed mechanism will yield first best output and profits. To achieve this outcome we solve several problems. First, it is a dominant strategy for the supervisor to truthfully report the agent's effort level despite opportunities for mis-reporting and side-contracting on the supervisor's report. Second, the agent is induced to select the first best effort level. Third, the contracts do not expose either the supervisor or the agent to risk. Finally, in equilibrium, neither the agent nor the supervisor receive rents.

In order to ensure that truth-telling is a dominant strategy for the supervisor we must prevent both extortion and bribery. Extortion may occur if the supervisor can credibly threaten to select a contract which (implicitly) under-reports the performance of the agent. In return for forbearance from under-reporting, the supervisor may demand a portion of the additional salary

that an honest report would pay the upstream agent. We overcome opportunities for extortion by ensuring that threats not to accurately select the appropriate contract are imperfect. The agent can ignore the supervisor's demand, knowing that the supervisor acting in his own best interests will prefer not to under-report.

Bribery may arise in the form of a side-contract under which the supervisor over-reports the agent's effort level, in return for a share of the additional compensation that results. Bribery is averted by making selection of a misleading contract too costly to the supervisor. The maximum bribe that the agent would be willing to pay, in order to inflate reports of his effort level, is equal to the increase in compensation that the duplicity would yield. We show that it is possible to design a menu of contracts for which the potential gains from duplicity are less than the cost to the supervisor of over-reporting. As with other information-revealing mechanisms that rely on selection from a menu of contracts, each of the contracts vary in the extent to which they contain fixed and variable (output-based) components. When the supervisor observes that the agent has exerted a higher level of effort, the supervisor expects output to increase and prefers to choose a contract with a larger variable component. We exploit this relationship and under reasonable conditions demonstrate the existence of a menu of contracts for which truth-telling is a dominant strategy for the supervisor, despite opportunities for side-contracting.

When truth-telling is a dominant strategy for the supervisor, the principal assumes the supervisor's information structure, and learns the agent's effort level directly. In such

circumstances, it is trivial to design a wage schedule that induces the agent to exert the first best effort level and neither transfers rent to the agent nor exposes the agent to risk. Existence of a menu of contracts that does not expose the supervisor to risk is less obvious. The supervisor must choose a contract before observing output, otherwise his choice will not depend upon his private information. Does inducing the supervisor to report truthfully using output based contracts simply transfer the moral hazard problem from the agent to the supervisor? No, because the tension between allocating risk inefficiently and inducing effort no longer exists (the agent is paid directly for his effort using the supervisor's truthful report). Therefore, the slope of the contract chosen by the supervisor on the equilibrium path may be set at zero, so that in equilibrium, the supervisor is not exposed to any uncertainty.

Finally, we ensure that the menu of contracts does not transfer rent to the supervisor. Motivating the supervisor to respond truthfully gives rise to what appears to be an adverse selection problem - the supervisor has private information about the state of productivity. In a true adverse selection problem, the principal would transfer rent to the supervisor in high productivity states in order to ensure participation in low productivity states. However, the problem of inducing the supervisor to respond truthfully does not correspond fully to a true adverse selection problem, because the productivity state is determined not by an act of nature, but rather by the level of effort exerted by the agent. Given that we assumed that side-contracting on the agent's effort level was not possible, our design of the agent's wage schedule ensures that the productivity state follows the equilibrium path. As a result, neither the principal nor the supervisor are concerned with the possibility of rent transfers to or from the supervisor

at productivity levels off the equilibrium path.

The paper can be distinguished from the previous literature in at least three respects. First, we do not assume that the supervisor is benevolent. If a supervisor does not act in his own best interest and is willing to truthfully reveal his private information to the principal, then no contracting mechanism is required for the principal to assume the (benevolent) supervisor's information structure.¹ Kofman and Lawarrée (1993a) demonstrate that a benevolent supervisor's information need not always be perfect. They present a model which explains the use of external auditors as a mechanism for monitoring possible collusion between internal auditors and internal agents. To the extent that the services of external auditors are costly, the principal may prefer to specify random rather than universal external audits. In a subsequent paper (Kofman and Lawarrée, 1993b), the authors observe that even if there is only a small probability that the supervisor is benevolent then it is profitable to incorporate his report. Our proposed solution does not rely upon the presence of a benevolent external agent. Rather, we assume that the supervisor will mis-report the agent's effort whenever it is in the supervisor's interests to do so.

Second, the supervisor is able to make a false or inaccurate report. When a supervisor can only report hard information he faces just two options: truthfully reveal private information or reveal nothing. In such circumstances extortion is not possible and the supervisor can be employed as a whistle-blower (Kofman and Lawarrée, 1993b). The agent is ordered to report

¹This is equivalent to assuming that the supervisor is risk neutral.

his private information (explicitly or implicitly) to the principal. If the agent misleads the principal, the supervisor reveals that the agent is cheating, in which case the agent pays a fine to the supervisor (thwarting bribery). In equilibrium, the supervisor is never used because the agent reveals his information truthfully. Although the supervisor need not always be able to identify when the agent is cheating, if the supervisor's information is occasionally incorrect, the agent must be compensated for assuming risk and the supervisor will occasionally receive rents. By allowing the supervisor to report soft information, we introduce the possibility of extortion and focus on contracts under which neither extortion nor bribery are profitable.

Third, we assume that it is not possible for the agent and the supervisor to collude upon the agent's choice of effort. It is well-established that collusion may be beneficial if privately informed agents can write risk-sharing contracts that the less well-informed principal is unable to write (Varian, 1990; Itoh, 1993). As long as the agents observe each other's efforts (if any) and are capable of contracting on those effort levels, then side-contracting between the agents may yield improved co-ordination and risk-sharing, relaxing the agents' participation constraints. By restricting attention in the present paper to situations in which the agent and supervisor are unable to collude upon the agent's effort, we exclude the possibility that allowing collusion is beneficial.

We prove existence of a menu of contracts from which a self-interested supervisor's choice will accurately reveal an agent's effort level. When combined with an appropriate wage schedule for the agent, the resulting reward scheme implements the first best outcome and yields

first best profits. This is demonstrated with an example in which the agent has constant absolute risk aversion and the noise is normal. These assumptions are relaxed in the Appendix.

I. Model and Analysis

We consider a three-layer hierarchy that includes a risk neutral principal, a risk averse supervisor (S) and a risk averse agent (A). In the first period, A selects an action (a) from $\{a_1, \dots, a_n\} \subset \mathfrak{X}$, which yields expected gross profits $\pi(a)$ in the second period. A 's choice is observed by S but not by the principal. The principal may communicate with S between periods, but, to the extent that this communication influences A 's rewards, we have to worry about collusion between A and S . Although the potential gains from collusion may be divided in many ways, without loss of generality we assume that after S has observed a , A makes a take-it-or-leave-it offer to S , specifying that S will receive a "bribe" b in exchange for sending a specific message to the principal. We assume that this side-contract is enforceable.

A 's cost of a_i equals c_i - we delete dominated actions and order (a_i, c_i) such that both are increasing in i . We assume that the outside options available to both S and A are equal to zero and, for the moment, define their utility functions by:

$$U_S = 1 - \exp[-\rho(v + \delta b)] \tag{1}$$

$$U_A = U(w - \delta b - c) \tag{2}$$

where: $\rho > 0$, $U' > 0$, $U'' < 0$, $U(0) = 0$

v = the wages paid by the principal to the supervisor
 w = the wages paid by the principal to the agent
 δ = 1 if S chooses A 's preferred contract and 0 otherwise

Gross profits are a noisy function of a :

$$\pi = f(a) + z, \quad z \sim N(0, \sigma^2) \quad (3)$$

Recall that the a 's are labelled such that f is increasing in a . We assume that profit can not be discarded by either the agent or the supervisor. Label a_j as the first best effort level² and, for a given a_j , define the *linear reward scheme* $R_j(\epsilon, \alpha, \beta)$ by:

$$\begin{aligned}
 w^j &= c_j \\
 w^i &= c_i - \epsilon, \quad \epsilon \in \mathfrak{R}_+, \quad i \neq j \\
 v^i &= \alpha^i \pi + \beta^i, \quad \alpha^i \in \mathfrak{R}, \quad \beta^i \in \mathfrak{R}
 \end{aligned} \quad (4)$$

The scheme lets S select a contract $v^j(\pi) \in \{v^1(\pi), \dots, v^n(\pi)\}$, such that if S selects $v^j(\pi)$, A is paid w^j .³ Given this contracting scheme the sequence of actions is:

1.1	1.2	1.3	1.4
A selects a which S observes	A offers to pay b to S if S selects $v^j(\pi)$	S selects $v^j(\pi)$ and is paid b if $v^j(\pi) = v^j(\pi)$	π is realized, w^j and $v^j(\pi)$ are paid

²Such that: $j \in \operatorname{argmax}_i f(a_i) - c_i$.

³Superscripts refer to contract choices by S , while subscripts denote the action chosen by A .

When A chooses a_i and offers bribes b_i^k and S chooses v^k , the certainty equivalent of S 's utility is equal to :

$$\alpha^k E[\pi(a_i)] + \beta^k + b_i^k \delta - \frac{\rho \alpha^{k^2} \sigma^2}{2} \quad (5)$$

The maximum bribe that A will offer S to over-report his effort level is equal to the additional income that the over-reporting will yield. Using b_i^ℓ to denote the maximum bribe that A will offer S for implicitly reporting a_ℓ when a_i was chosen, we can write:

$$\begin{aligned} b_i^\ell &= c_\ell - c_i, \quad i \neq j, \ell \neq j \\ b_i^j &= c_j - c_i + \epsilon \\ b_j^\ell &= c_\ell - c_j - \epsilon \end{aligned} \quad (6)$$

We begin by considering bribery and extortion in single steps (over-reporting and under-reporting by a single effort level). In order for truth-telling to be a dominant strategy, our reward scheme must necessarily satisfy the following conditions:

No bribe: A cannot bribe S into choosing v^{j+1} rather than v^j when $a = a_i$ (NB)

No Extortion: S prefers v^{j+1} to v^j when $a = a_{i+1}$.⁴ (NE)

Formally, (NB) and (NE) are true if:

⁴Note that truth-telling precludes extortion (see earlier discussion).

$$\alpha^i f(a_i) + \beta^i - \frac{\rho \alpha^{i^2} \sigma^2}{2} = \alpha^{i+1} f(a_i) + \beta^{i+1} + b_i^{i+1} - \frac{\rho \alpha^{i+1^2} \sigma^2}{2} + e \quad (7)$$

$$\alpha^i f(a_{i+1}) + \beta^i - \frac{\rho \alpha^i \sigma^2}{2} + e = \alpha^{i+1} f(a_{i+1}) + \beta^{i+1} - \frac{\rho \alpha^{i+1^2} \sigma^2}{2} \quad (8)$$

where the positive e 's give us dominant strategy implementation. Rearranging yields:

$$\alpha^{i+1} = \alpha^i + \frac{2e + b_i^{i+1}}{f(a_{i+1}) - f(a_i)} \quad (9)$$

$$\beta^{i+1} = \beta^i - f(a_{i+1}) \frac{2e + b_i^{i+1}}{f(a_{i+1}) - f(a_i)} + \frac{\rho \sigma^2}{2} (\alpha^{i+1^2} - \alpha^{i^2}) + e \quad (10)$$

To ensure that S assumes no risk in equilibrium, we set $\alpha^j = 0$. Given $\alpha^j = 0$, S 's participation constraint is binding when $\beta^j = 0$. Substituting yields:

$$\alpha^{j+1} = \frac{2e + b_i^{i+1}}{f(a_{j+1}) - f(a_j)} \quad (11)$$

$$\beta^{j+1} = -f(a_{j+1}) \frac{2e + b_i^{i+1}}{f(a_{j+1}) - f(a_j)} + \frac{\rho \alpha^{j+1^2} \sigma^2}{2} + e \quad (12)$$

Having found α^{j+1} and β^{j+1} , it is trivial to solve for α^{j+2} and β^{j+2} . Indeed, iterative application of this procedure will yield α^i and $\beta^i \forall i \neq j$. A contradiction argument reveals that $\alpha^i > \alpha^{i-1}$ and $\beta^i < \beta^{i-1} \forall i$. We must now worry about bribery and extortion in steps greater than one.

Lemma 1

- a. If A cannot bribe S to choose v^{j+1} when $a = a_i$, then A cannot bribe S to choose v^{j+m} when $a = a_i \forall i, m > 1$.
- b. If S prefers v^{j+1} over v^j when $a = a_{i+1}$, then S also prefers v^{j+1} over v^{j-m} when $a = a_{i+1} \forall i, m > 1$.

Proof:

- a. S cannot be bribed into selecting v^{j+m} when $a = a_i$ iff:

$$\alpha^i f(a_i) + \beta^i - \frac{\rho \alpha^{i^2} \sigma^2}{2} \geq \alpha^{i+m} f(a_i) + \beta^{i+m} + b_i^{i+m} - \frac{\rho \alpha^{i+m^2} \sigma^2}{2} \quad (13)$$

We can rewrite β^{i+m} and α^{i+m} as:

$$\beta^{i+m} = \beta^i - \sum_{t=i}^{i+m-1} f(a_t) \frac{2e + b_t^{t+1}}{f(a_{t+1}) - f(a_t)} - me - b_i^{i+m} + \frac{\rho \sigma^2}{2} (\alpha^{i+m^2} - \alpha^{i^2}) \quad (14)$$

$$\alpha^{i+m} = \alpha^i + \sum_{t=i}^{i+m-1} \frac{2e + b_t^{t+1}}{f(a_{t+1}) - f(a_t)} \quad (15)$$

Substituting:

$$me + \sum_{t=i+1}^{i+m-1} \frac{2e + b_t^{t+1}}{f(a_{t+1}) - f(a_t)} [f(a_t) - f(a_i)] \geq 0 \quad (16)$$

- b. S prefers v^{j+1} over v^{j-m} when $a = a_{i+1}$ iff:

$$\alpha^{i-m} f(a_{i+1}) + \beta^{i-m} - \frac{\rho \alpha^{i-m^2} \sigma^2}{2} \leq \alpha^{i+1} f(a_{i+1}) + \beta^{i+1} - \frac{\rho \alpha^{i+1^2} \sigma^2}{2} \quad (17)$$

Rewriting β^{i+m} and α^{i+m} :

$$\beta^{i+1} = \beta^{i-m} - \sum_{t=i-m}^i f(a_{t+1}) \frac{2e + b_t^{t+1}}{f(a_{t+1}) - f(a_t)} + me + \frac{\rho \sigma^2}{2} (\alpha^{i+1^2} - \alpha^{i-m^2}) \quad (18)$$

$$\alpha^{i+1} = \alpha^{i-m} + \sum_{t=i-m}^i \frac{2e + b_t^{t+1}}{f(a_{t+1}) - f(a_t)} \quad (19)$$

Substituting:

$$me + \sum_{t=i-m}^{i-1} \frac{2e + b_t^{t+1}}{f(a_{t+1}) - f(a_t)} [f(a_{i+1}) - f(a_{t+1})] \geq 0 \quad (20)$$

Q.E.D. ■

We have shown that if the supervisor prefers to report truthfully rather than accept the maximum feasible bribe to over-report by one *step*, then bribery in steps greater than one is also unprofitable. We have also established an analogous result for extortion. Therefore, ensuring that truth-telling is a dominant strategy requires satisfaction of just $n-1$ bribery and $n-1$ extortion constraints. We begin by setting the slope and intercept of the contract on the equilibrium path and then iteratively use Equations (9) and (10) to specify the remaining contracts. By setting α^j equal to zero, in equilibrium, no rent is lost from exposing the (risk averse) supervisor to uncertainty. Given $\alpha^j = 0$, we can set $\beta^j = 0$, so that, in equilibrium, the principal retains all of the rents and earns first best profits.

Proposition 1:

$\forall j \exists (\epsilon, \alpha, \beta): R_j(\epsilon, \alpha, \beta)$ can dominant-strategy-implement a_j in a collusion-proof, subgame perfect equilibrium and earn the principal first best profits.

Proof:

Obvious by construction and application of Lemma 1.

Q.E.D. ■

It should be clear that our proposed reward scheme is not unique. The bribery and extortion constraints need not be binding while we are also not limited to linear contracts. Other sets of linear functions can achieve the first best outcome as can other classes of functions.

II. Discussion and Conclusions

We have demonstrated the existence of a contracting mechanism through which a principal can costlessly overcome moral hazard by exploiting the knowledge of a privately informed supervisor. Rewarding the agent using the supervisor's report allows the principal to separate the otherwise inter-twined tasks of revealing and inducing effort. Because the agent's incentives to exert effort depend only upon the supervisor's incentives to report honestly and not upon the supervisor's equilibrium contract, we neither reward the supervisor for revealing his private information nor expose him to risk. In equilibrium the supervisor does nothing. However, his anticipated reaction when the agent deviates from the equilibrium path and the subsequent punishment imposed upon the agent are sufficient to deter deviations by the agent.⁵

⁵In this respect, the mechanism is somewhat analogous to the *whistle-blower* mechanism proposed by Kofman and Lawarrée (1993b).

Control of the equilibrium path (through the efforts of the agent) is necessary to enforce truth-telling without transferring risk or rent to the supervisor. In problems of adverse selection, the equilibrium path is controlled by nature rather than the principal, so the proposed mechanism does not offer a costless solution to adverse selection. The principal may also lose control of the equilibrium path when the supervisor and agent are able to contract on the agent's effort. We assume that the agent and supervisor can contract upon just the supervisor's report and not upon the agent's effort level. Although this assumption is common in the literature, some readers may find it troublesome. Enforceability of side-contracts generally pre-supposes repeated interaction, the indivisibility of which argues for either all side contracts being enforceable or none being so. We note, however, that enforceability does not necessarily imply feasibility. For example, a specific supervisor may be chosen from a pool of candidates only after the agent has selected an effort level. Even when enforceability is ensured by the anticipation of future selections from the same pool, side-contracting may not be feasible before a specific selection has been made.

We also restrict the agent to choosing from a finite action space. Infinite action spaces result in well-known problems for menu selection mechanisms and we can at most approximate collusion-proof implementation. We can, however, extend the analysis to the case of a productive supervisor, in which the output observed by the principal is a noisy function of the sequential efforts of the agent and the supervisor. By offering the supervisor an appropriate menu of contracts we are able to induce the supervisor to implicitly reveal the efforts of the agent. Although we do not eliminate the moral hazard problem for the supervisor, we can

overcome free-riding and costlessly achieve the same outcome as when the principal is able to observe the agent's effort directly.

The promise of achieving first best profits suggests that it should be possible to find examples of incentives systems in practice that exploit the essence of the scheme that we are proposing. Indeed, profit centers exhibit some similar features to the proposed scheme. By way of illustration, consider a firm in which department *A* supplies profit center *S* with inputs. In deciding how much of *A*'s inputs to purchase, *S* determines the extent to which its own performance will depend upon that of *A*. If *A* has performed poorly, reliance upon *A* will prevent *S* from achieving its profit goals. In this manner, *S*'s purchasing decision can reveal private information about *A*'s performance, which may explain why many firms use internal demand as a basis for rewarding internal suppliers. In support of this example, we note that a frequently cited advantage of profit centers is that they exploit the superior local knowledge of center managers (Eccles, 1983; Grabski, 1985).

Appendix 1

Relaxing The Assumptions of Constant Absolute Risk Aversion And Normality

Redefine the supervisor's utility function as:

$$U_S = W(v + \delta b) \quad (\text{A1})$$

where: $W' > 0$, $W'' < 0$, $W(0) = 0$

We require that U has a third derivative which is continuous and bounded. We further allow the error term, z , to be distributed such that $E(z)=0$ and the third absolute central moment of the distribution is of smaller order than the variance (σ^2). Using a Taylor series expansion (Pratt, 1964) for small values of σ^2 we can make the following approximation of the certainty equivalent of S 's expected utility when A chooses a_i and offers bribes b_i^k and S chooses v^k :

$$E W(\alpha^k (f(a_i) + z) + \beta^k + b_i^k \delta) - W\left(\alpha^k f(a_i) + \beta^k + b_i^k \delta - \frac{\sigma^2 \alpha^{k^2}}{2} \frac{W''}{W'}\right) \quad (\text{A2})$$

For notational convenience, we define y_i^k as the negative of the absolute risk aversion:

$$y_{i,b}^k = \frac{W''}{W'} \Bigg|_{\alpha^k f(a_i) + \beta^k + b_i^k} \quad (\text{A3})$$

$$y_i^k = \frac{W''}{W'} \Bigg|_{\alpha^k f(a_i) + \beta^k}$$

Substituting yields the following sufficient condition for Equation 16:

$$\sum_{t=i+1}^{i+m-1} (\alpha^{t+1} - \alpha^t) (f(a_t) - f(a_i)) + m e + \sum_{t=i+1}^{i+m-1} \frac{\alpha^{t^2} \sigma^2 (y_i^t - y_{t-1,b}^t)}{2} \quad (\text{A4})$$

$$+ \frac{\alpha^{i+m^2} \sigma^2 (y_{i,b}^{i+m} - y_{i+m-1,b}^{i+m})}{2} \geq 0$$

Similarly, we can derive the following sufficient condition for Equation 20:

$$\begin{aligned}
& me + \sum_{t=i-m}^{i-1} (f(a_{i+1}) - f(a_{t+1}))(\alpha^{t+1} - \alpha^t) + \\
& \sum_{t=i-m+1}^i \frac{\sigma^2 \alpha^{t^2}}{2} (y_t^i - y_{t+1}^i) + \frac{\sigma^2 \alpha^{i-m^2}}{2} (y_{i+1}^{i-m} - y_{i-m+1}^{i-m}) \geq 0
\end{aligned} \tag{A5}$$

As long as σ^2 is small and the coefficient of absolute risk aversion does not change too rapidly so that Equations A4 and A5 are satisfied, both Lemma 1 and Proposition 1 will continue to hold.

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