THE DESIGN OF A SMALL INTERCEPTOR ROCKET SYSTEM

by

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Larry D. Brock

Submitted to the Department of Aeronautics and Astronautics on May 23, 1961 in partial fulfillment of the requirements for the degrees of Bachelor of Science and Master of Science.

ABSTRACT

This thesis gives the preliminary design of an orbit-to-orbit interceptor rocket system. The system is part of an anti-missile defense system that uses a submarine as a forward base. The early stages of the flight of an enemy missile are tracked by radar from the submarine. The trajectory of the enemy missile is predicted and an instrumented package is placed into a trajectory coincident with the target complex. Some discrimination technique identifies the targets from among the probable decoys and tankage fragments. It is then the responsibility of the interceptor system to destroy these targets.

The operation of the interceptor system is divided into three sections: (1) the tracking phase, (2) the computation phase, and (3) the launch and guidance phase. During the tracking phase the target is tracked by radar for 20 seconds. During the computation phase this data is smoothed by least squares correlation techniques. From the tracking data and the capabilities of the anti-missile rocket, a launch direction is calculated. The rocket is positioned and then fired along this direction. During burning, corrections are made in the rocket's heading by command guidance. It then coasts free fall to the target.

An outline of the design is given for each part of the system. The system is analyzed and the accuracy requirements are determined. It is found that a successful interception can be accomplished and

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that the required accuracies are reasonable. Some of the more important conclusions are: (1) the variation in gravity between the vehicle and target cannot be neglected; (2) accuracies of 10% on the burning time and 2% on the final velocity are required for the rocket; (3) guidance is needed only during burning and no second stage is needed to make corrections at the end of flight; (4) angular rate and position gyros in the rocket can be eliminated by simulating the motion of the rocket in the vehicle.

Thesis Supervisor: H. Guyford Stever

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OBJECT

The object of this thesis is to form the preliminary design of an interceptor rocket system that is capable of achieving a successful interception with a minimum of required weight. It is submitted that this objective of accuracy plus minimum weight is adequately fulfilled by a one-stage rocket that is controlled only during burning, after which it is left to coast in free fall to the target.

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CHAPTER I

INTRODUCTION

The subject of this thesis is an anti-missile interceptor system, which, in turn, forms a part of a submarine-based anti-missile defense project. The over-all project is described in this chapter in order to provide the background against which the interceptor system can be intelligently discussed. Once the over-all project has been given, the remainder of the chapter is concerned with certain general characteristics of the interceptor system itself, namely, its relationship to the entire project, its mission requirements, and the factors affecting its design. The operation of the proposed system is then briefly summarized preparatory to the detailed discussions in the following chapters.

1.1 A Description of the Over-all Project

The object of the entire study is to investigate the feasibility of using a submarine as a forward base for an anti-missile defense system. The possibility was first suggested by the M.I.T. Research Laboratory of Electronics.[1] As the defense system was originally conceived, the submarine would be stationed near the enemy coast.

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[1] MIT-RLE, Internal Report No. 18.

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From this vantage point, long range radar carried on board the submarine would be capable of observing and tracking a threatening missile soon after burnout. From the tracking information, the trajectory of the enemy missile would be predicted and an anti-missile launched to intercept and destroy the enemy missile.

However, certain difficulties were encountered in this originally proposed system. Study results of a radar that could be mounted on a submarine indicated that the tracking of an object with the reflective area of a warhead would not be possible. The only object that could be tracked would be the missile tankage before it was exploded. Since the separation velocity between the warhead and tankage could not be determined, the trajectory of the warhead could not be predicted accurately enough for a successful interception.

Because of this flaw in the original proposal, a change in emphasis was made. The primary advantage of the originally conceived system was the circumvention of the necessity for discriminating between the warhead and any decoys that might be traveling with it. It was hoped that the target could be tracked and intercepted before the cloud of decoys had grown to sufficient size to require discrimination. However, since the objective of avoiding discrimination appeared impossible to attain, it was decided to determine if a forward-based system could be used to advantage in the discrimination problem itself. The advantage of a forward-base is that more time could be used in the discrimination process than in a system that must accomplish this process in a few seconds at the terminal end of a hostile trajectory.

With this change in emphasis the operation of the system was modified. The system would again track the tankage and predict its trajectory. Instead of then launching an anti-missile so as just to intercept the path of the enemy trajectory, it would place a vehicle in an orbit coincident with the target complex, as shown in Fig. 1.1. From the vantage point of a near orbit, it would use some discrimination technique to eliminate most of the particles in the cloud as not being the warhead. It is proposed to destroy the remaining particles by small auxiliary rockets.

Two discrimination techniques were suggested.[2,3] One would use infrared techniques and the other would use a radar method. To use the infrared method the vehicle would be placed in a trajectory slightly below the target complex. All of the particles in the cloud would be observed using a very high quality optical infrared system. From studies that have been made on the dynamics of tankage fragments,[4] it is agreed that the tumbling rate of tank-

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[2] MIT Inst. Lab. Report R-280

[3] MIT Inst. Lab. Report R-321

[4] Bendix BPO 867-3, Vol. 2

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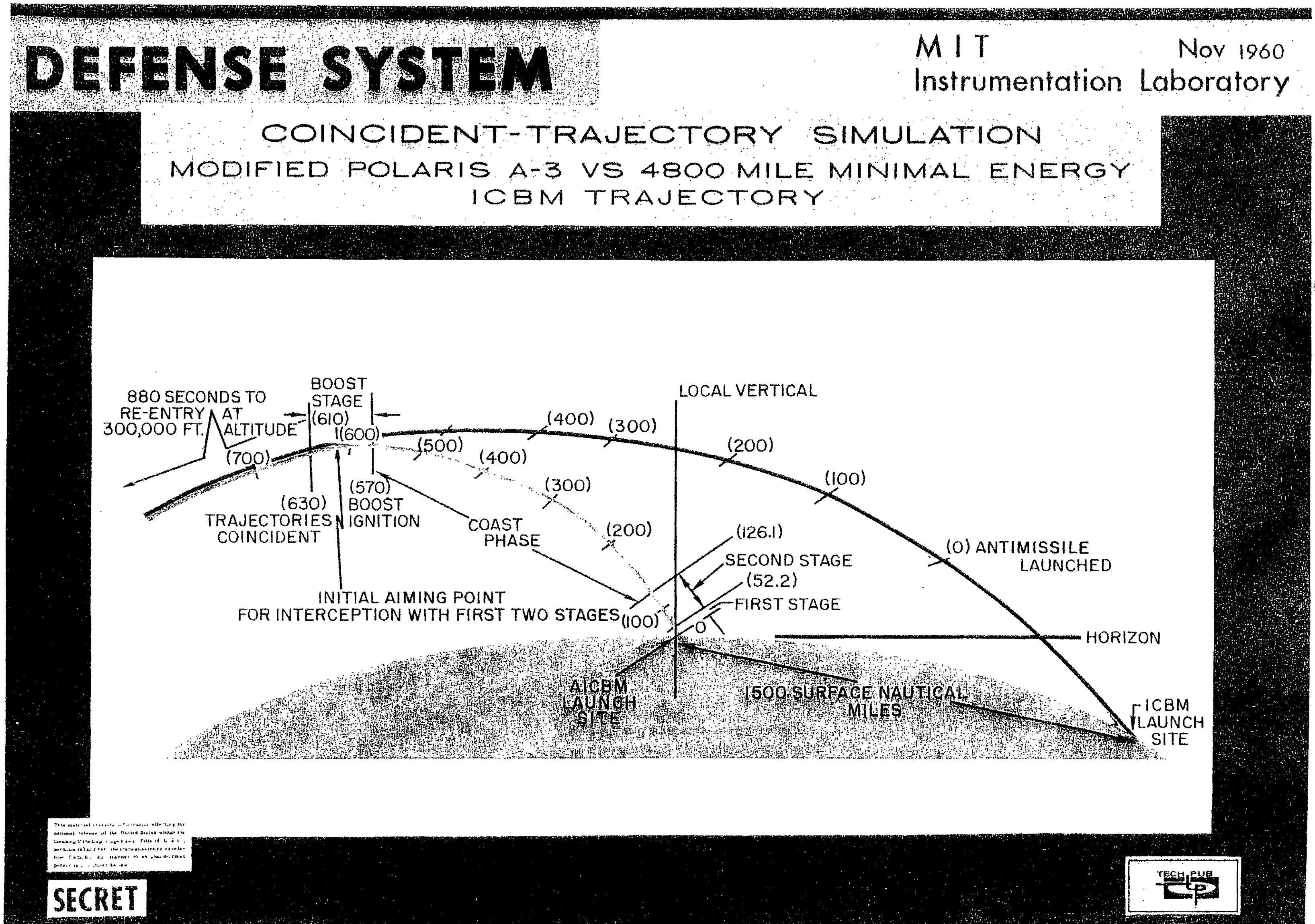

Fig. 1.1

Enemy and Defense Missile Trajectories

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age fragment would be an order of magnitude larger than that of the warhead. Since most of the objects in the cloud would be tankage fragments, they could be eliminated by observing the frequency of their infrared emission.

The radar technique that was suggested would require that the vehicle be placed as nearly as possible in the same trajectory that the tankage would have had if it had not been exploded. For the purposes of explaining the radar discrimination method, it is assumed that the distance of a particle from the point where the tankage would have been is approximately:

$$R = \dot{R}(t - t_o)$$

where $\dot{R}$ is the range rate and t_o is the time the particle left the tankage. It is then seen that the quantity $R/\dot{R}$, measured for a particular particle from the point where the tankage would have been, gives an indication of the time since that particle left the tankage. It is assumed that the warhead is separated from the tankage soon after burnout and that decoys may be ejected. When the warhead and tankage are at a safe distance, the tankage is exploded to provide more decoys. With the assumption that the warhead was one of the first particles separated from the tankage, most of the objects in the cloud can again be eliminated as not being the warhead by measuring their range-over-range rate. This range-over-range rate is complicated by the variations in the gravity field, but the basic principle is still the same.

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1.2 Description of the Interceptor Rocket System

The interceptor rocket system, which is the subject of this thesis, is responsible for the final destruction of the targets. The operation of the system begins after the targets have been identified by other parts of the over-all system. It first tracks the targets, then launches a rocket to destroy each target. The parts of the system include the actual rockets, a launch mechanism, radar, computer, and inertial reference equipment. (The radar, computer, and inertial reference equipment are used for other functions in the over-all system.) It is assumed, for the purposes of the design to follow, that there be 6 targets that need to be destroyed.

1.3 Design Considerations

One of the most stringent requirements imposed on the design of the interceptor rocket system is the weight limitations. In the over-all system, two stages of the anti-missile are used to place the vehicle in a trajectory that intercepts or is tangent to the enemy trajectory. At the point where the vehicle comes in contact with the enemy trajectory, a booster stage is used to put the vehicle in the coincident trajectory. Achieving this coincident trajectory requires a very high propulsive capability. In a sample problem that was simulated on a digital computer, the ratio of payload weight to initial weight of .011 was required. In other words, for every added pound in the vehicle, 98 pounds would

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have to be added to the initial weight of the anti-missile. The interceptor rocket itself has a payload ratio of .31. If 6 rockets are carried, then for every pound that is added to the payload of each of the small rockets, 1740 pounds are added to the weight of the missile. Since it is proposed to carry several of these missiles in a submarine, it is desirable to make the interceptor system as simple and light as possible.

1.4 Forming the Basic Design

In forming the basic design, the two necessary components of the rocket, i.e. the warhead and propulsive unit are first considered. It is then determined what minimum equipment must be added to complete a successful interception.

The weight requirements demand that the warhead have as high an energy concentration as possible. This would call for some type of small nuclear weapon. The weight of the propulsive unit depends entirely on the weight to be accelerated and the velocity requirement. The velocity requirement, in turn, depends on the time of flight desired. Since the miss distance depends on flight time and the size of the warhead on miss distance, there is a relation between the weight of the warhead and the weight of the propulsive unit. Because of the highly classified nature of the data on nuclear weapons, no attempt is made here to optimize this weight trade-off. For the design purposes of this thesis it is assumed that the warhead weighs 50 pounds and has a destructive radius of 200 feet.

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The rocket must be launched with an angular accuracy of one milliraidan. Since an unguided rocket could not be launched this accurately, some type of guidance equipment must be added. Two guidance techniques are considered here. The first method would use a small second stage that would employ some sensing device (infrared, radar, etc.) to home in on the target at the end of flight. The second method would control the rocket only during burning by command guidance from the vehicle. These two methods are now compared to see which would be best in this application.

The primary disadvantage of the first system is its weight. Each rocket would be required to carry a sensing device plus the associated instrumentation and control system. An additional propulsive unit would be needed to make the necessary correction at the end of flight. There would also be a problem of target identification. Some assurance would be needed that the second stage homed in on the correct particle. Furthermore, for the homing operations, the attitude of the rocket must be controlled. This would require inertial reference equipment and a reaction wheel or gas jet attitude control system on each rocket. The first system is more accurate but requires a considerable amount of extra equipment.

The second system, on the other hand, would require almost no extra equipment. All that would be needed to control the rocket by command from the vehicle would be a radio receiver, control vanes

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in the rocket nozzle and the associated servos and electronics. The command guidance would automatically stabilize the rocket in pitch and yaw. The rocket would also have to be stabilized in roll. This would require one small gyro. The rocket coasts in free fall to its target after burnout so no second stage rocket or attitude control devices are needed. Since there is no corrective thrust at the end of flight, the position of the target has to be known very precisely relative to the vehicle, the launch direction has to be calculated exactly, the burning characteristics of the rocket have to be very near the design values, and the command guidance has to be accurate. But if it is possible to achieve these needed accuracies, the command guidance system would be the more desirable of the two since it would be the lightest and least complicated. The command guidance is then the one chosen for the interceptor system designed in this thesis. The errors are analyzed to see if it indeed is capable of performing a successful interception.

1.5 The Operation of the Interceptor Rocket System

The operation of the proposed interceptor system is divided into three phases: (1) the tracking phase, (2) the computation phase, and (3) the launch and guidance phase. The sequence of phases is shown graphically in Fig. 1.2. During the first phase a specified target is tracked for 20 seconds, with the position data taken at one-second intervals. The position data is referred

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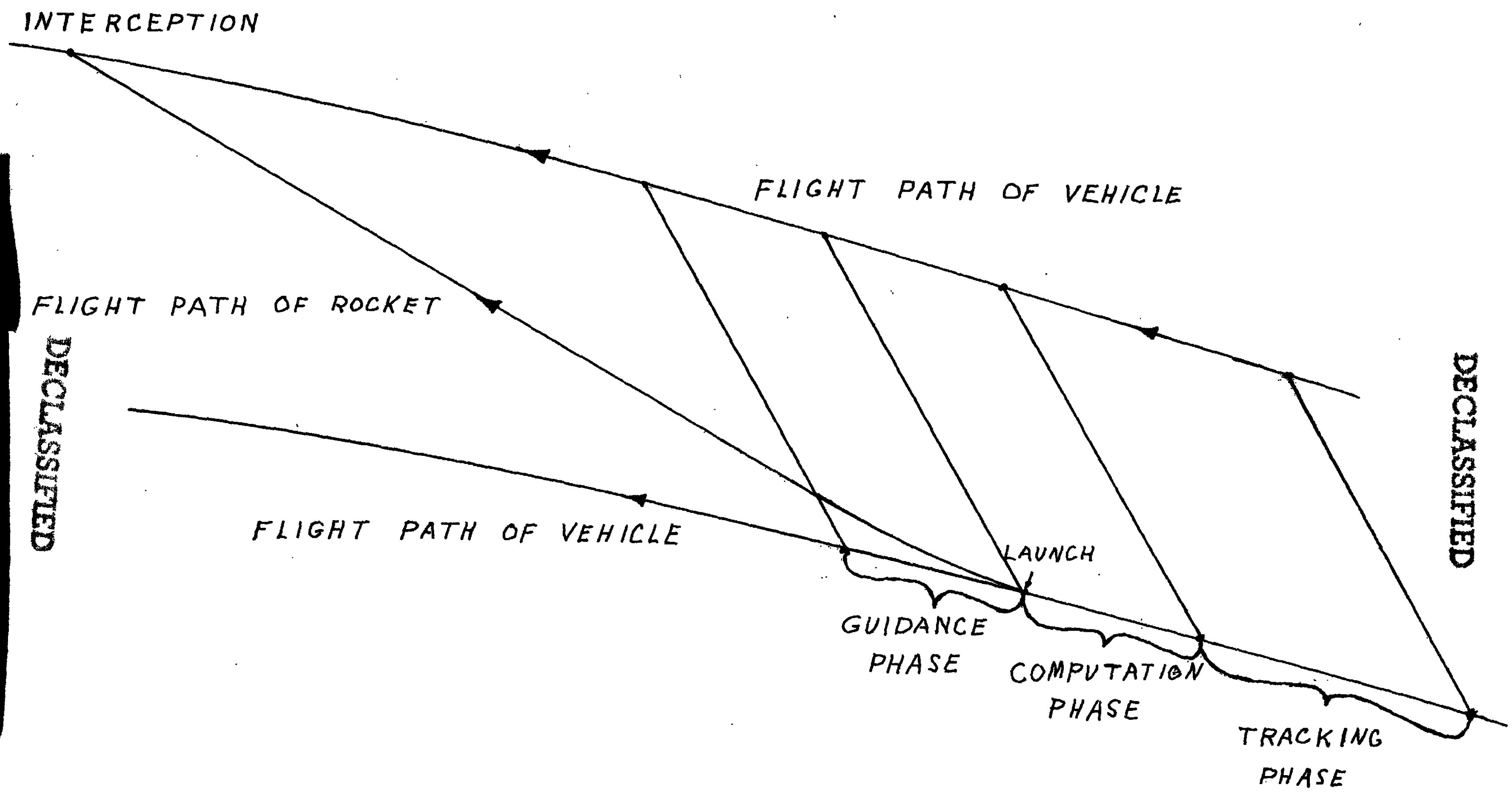

Fig. 1.2

Sequence of Phases

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to an arbitrary non-rotating co-ordinate system fixed in the vehicle. During the computation phase the tracking data is used to predict the target trajectory. From the known characteristics of the interceptor rocket and its time of launch, the position of the rocket is known as a function of time and launch direction. At the instant of interception the rocket and target will be at the same position. By using the equations giving the positions of the rocket and target as functions of time, the flight time and launch direction can be found. Launch equipment on board the vehicle place the rocket in the proper orientation for firing. At a specified time the rocket is fired. During burning the rocket is tracked by the radar. If the rocket deviates from the desired direction, a command is sent to actuate the control vanes bringing the rocket back to the planned path. After burnout the system begins the same process for the next target. The most distant target is intercepted first to keep interference from the exploding warhead at a minimum. A functional diagram of the operation of the system during the tracking and computation phases is shown in Fig. 1.3 and during the guidance phase in Fig. 1.4.

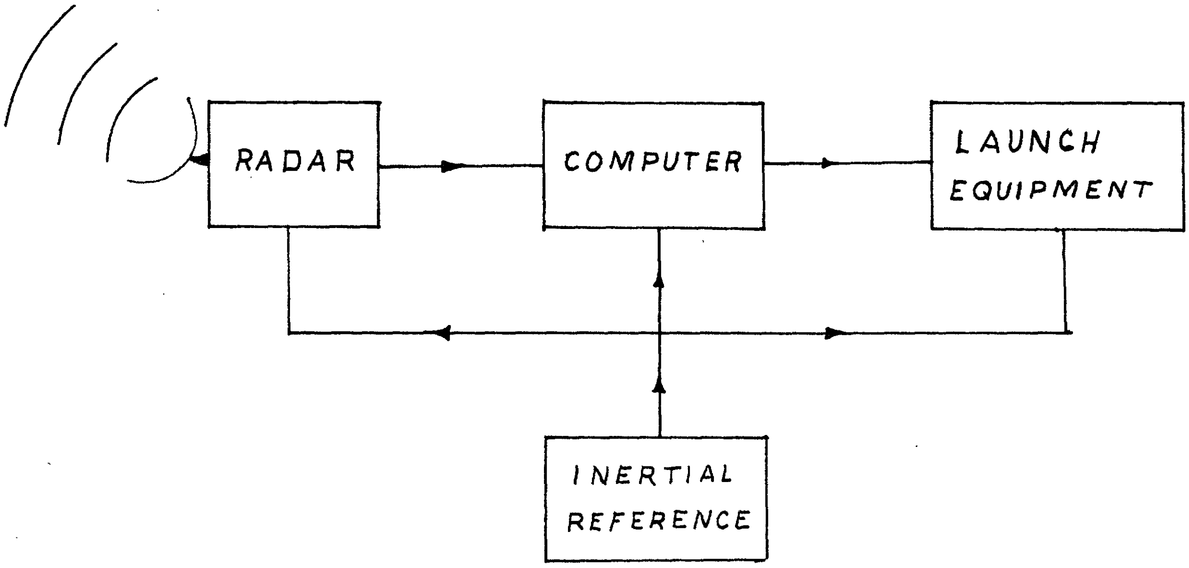

Fig. 1.3

Operation of the System During the Tracking and Computation Phase

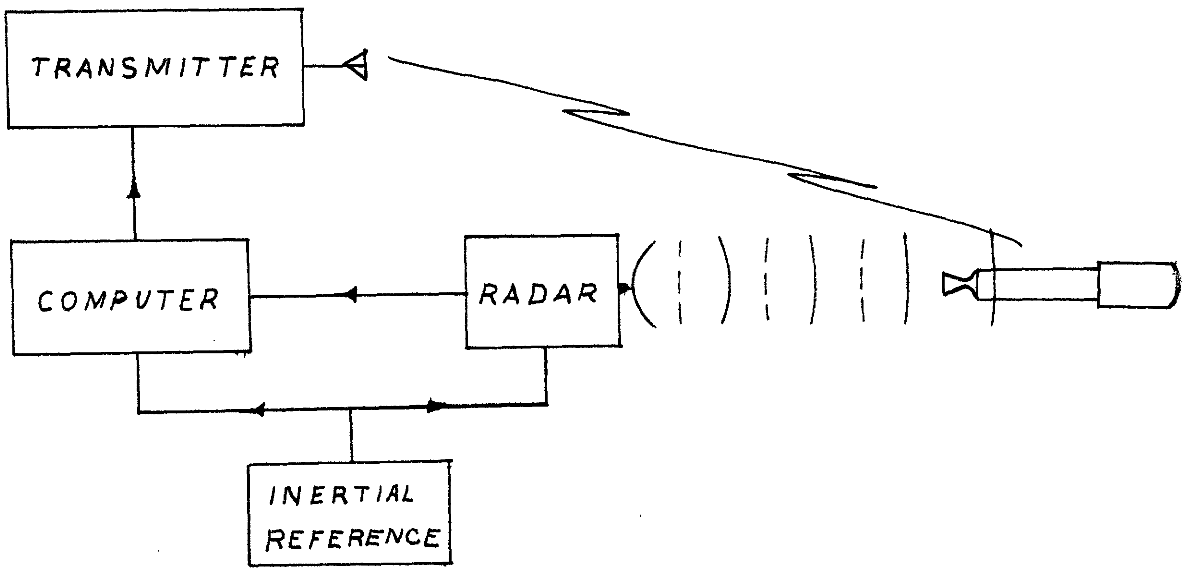

Fig. 1.4

Operation of the System During the Guidance Phase

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1.6 Summary of Results

The results of the study that follows show that the proposed rocket interceptor system is feasible. It is found that the accuracy required to predict the position of the target could be achieved by tracking the target for 20 seconds and then smoothing the tracking data by the least squares method. The position of the target as a function of time is approximated by the Taylor series. The results indicate that the third term, caused by the variation in the gravity field, is needed to achieve the desired accuracy. The fourth and subsequent terms are negligible. The design characteristics required for the rocket itself are reasonable. The rocket could be built with the present state of the art. A command guidance system that only controls the rocket during burning is sufficient to bring the rocket to within the desired distance from the target. A second stage is not needed to make corrections at the end of flight. These results are shown by an analog computer simulation of the guidance system.

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CHAPTER II

DERIVATION OF EQUATIONS FOR THE TRACKING AND COMPUTATION PHASE

The equations needed in the tracking and computation phase are derived in this chapter. The discussion is developed as follows: (1) The equations of motion of the target relative to a co-ordinate system centered in the tracking vehicle are formulated and analyzed using the Taylor series; (2) The method of least squares is adapted for use in smoothing the radar data; (3) The launch direction is then calculated; (4) Finally, two sources of error in the launch direction vehicle are investigated.

2.1 The Equations of Motion

As stated above, the Taylor series will be used to describe the motion of the target relative to a vehicle-centered, non-rotating, arbitrary co-ordinate system. The motion will be described in the non-orthogonal directions x, y, z, shown in Fig. 2.1.

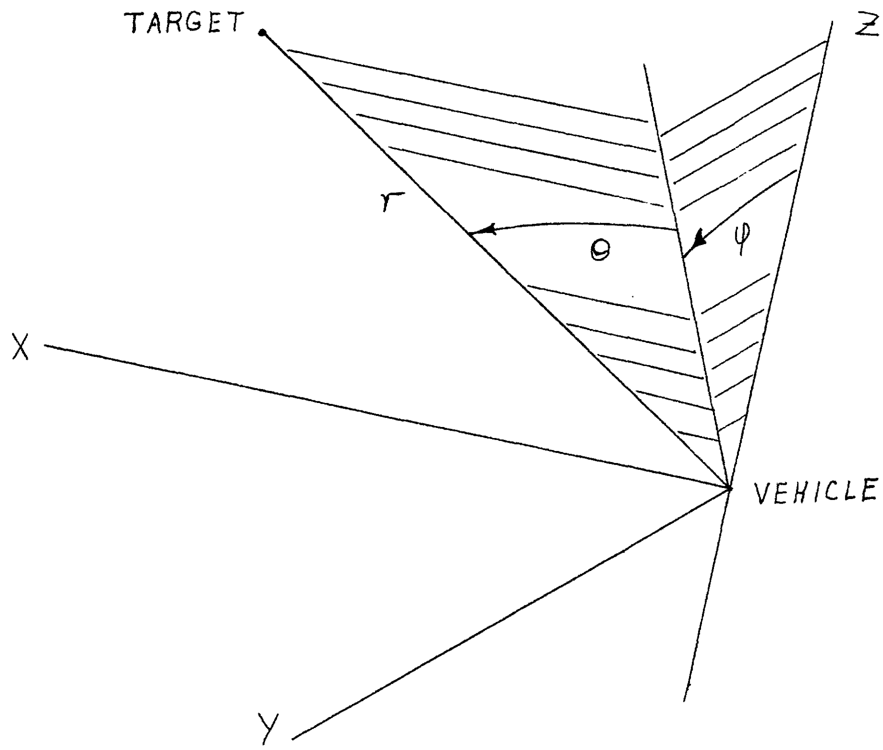

Fig. 2.1

Co-ordinate System Used in Developing the Equations of Motion

It is assumed that the coordinates of the target as a function of time can be written in the following form:

$$X = X_0 + \dot{X}_0 (t - t_0) + \frac{\ddot{X}_0}{2} (t - t_0)^2 + \frac{\dddot{X}_0}{6} (t - t_0)^3 + \cdots$$

$$Y = Y_0 + \dot{Y}_0 (t - t_0) + \frac{\ddot{Y}_0}{2} (t - t_0)^2 + \frac{\dddot{Y}_0}{6} (t - t_0)^3 + \cdots \qquad (2.1)$$

$$r = r_0 + \dot{r}_0 (t - t_0) + \frac{\ddot{r}_0}{2} (t - t_0)^2 + \frac{\dddot{r}_0}{6} (t - t_0)^3 + \cdots$$

where $(X_0, Y_0, r_0, \dot{X}_0, \cdots)$ are constants evaluated at time $t = t_0$.

2.2 The Effect of the Variation of the Gravity Field

To determine the nature of equations (2.1) the constants involved are evaluated. The constants in the first two terms ($X_o, Y_o, \Gamma_o, \dot{X}_o, \dot{Y}_o, \dot{\Gamma}_o$) represent the target's position and velocity at time $t = t_o$. These constants can be derived easily from the radar data. The constants in the third terms represent the acceleration of the target relative to the vehicle. In the non-rotating co-ordinate system shown in Fig. 2.1 the only accelerations will be those caused by the gradient of the gravity field. In other words, because both target and vehicle are in free fall, the only acceleration of the target relative to the vehicle will be that caused by the difference in the pull of gravity on the target and vehicle. The acceleration of gravity is:

$$\bar{G} = \frac{K}{R^2} \bar{1}_G \tag{2.2}$$

where: $\bar{G}$ is the gravitational acceleration vector defined positively down

K is the gravitational parameter of the earth

R is the magnitude of the vector from the center of the earth to the particle.

$\bar{1}_G$ is a unit vector in the $\bar{G}$ direction, which is the negative $\bar{R}$ direction.

The change in $\bar{G}$ between the vehicle and target is given approximately by:

$$\Delta \bar{G} = \frac{KR^2 \Delta \bar{I}_G - 2KR\Delta R \bar{I}_G}{R^4} \tag{2.3}$$

$$= \frac{K}{R^2} \Delta \bar{I}_G - 2\frac{K}{R^3} \Delta R \bar{I}_G$$

The geometric relationship of $\Delta \bar{G}$ is shown in Fig. 2.2 to Fig. 2.6.

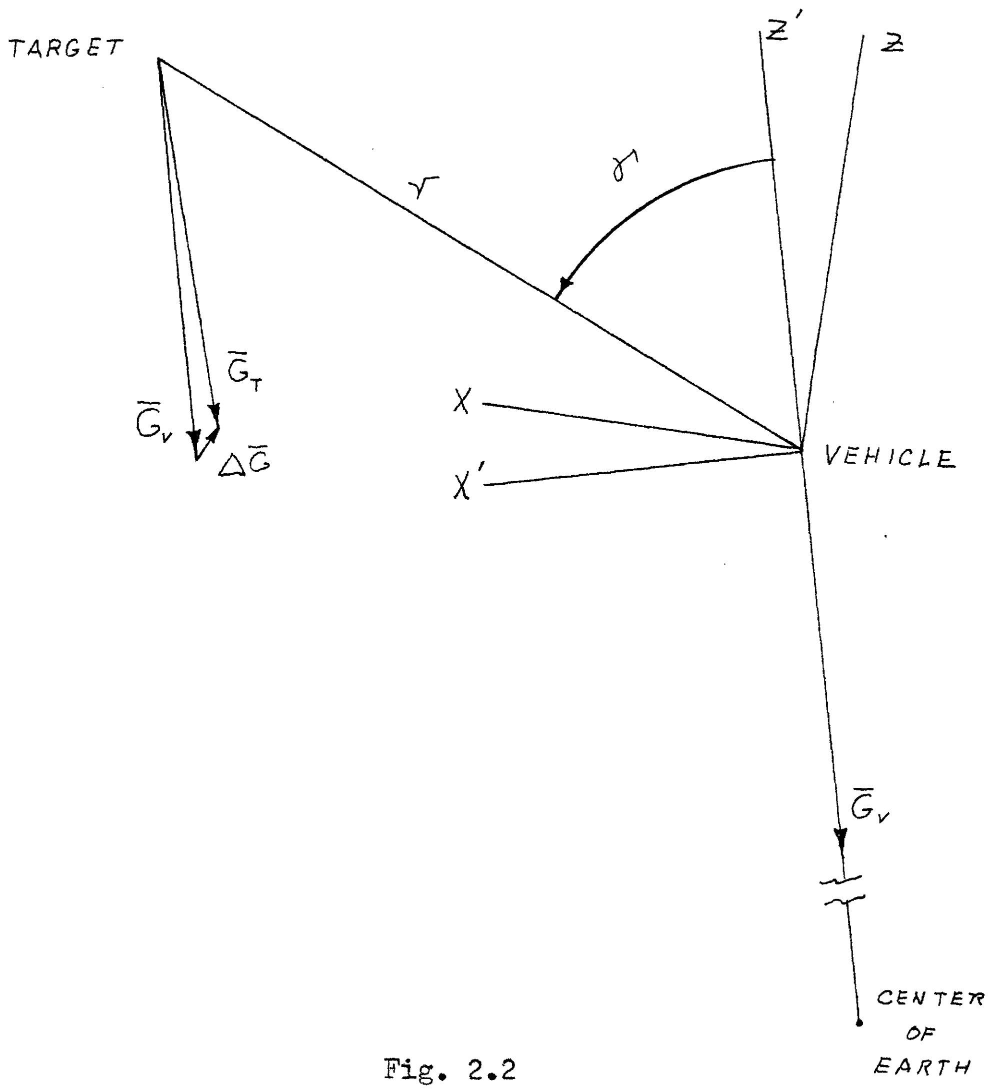

Fig. 2.2

The Variation in the Gravity Field

Since the purpose of this development is to determine the nature of the constants in equations (2.1), there will be no loss in generality if equation (2.3) is restricted to the plane of the trajectory. Also, for this development, define the x, z plane of Fig. 2.1 as the plane of the trajectory. Define rotating co-ordinates (x', z') with z' along $\overline{G}$, as shown in Fig. 2.3.

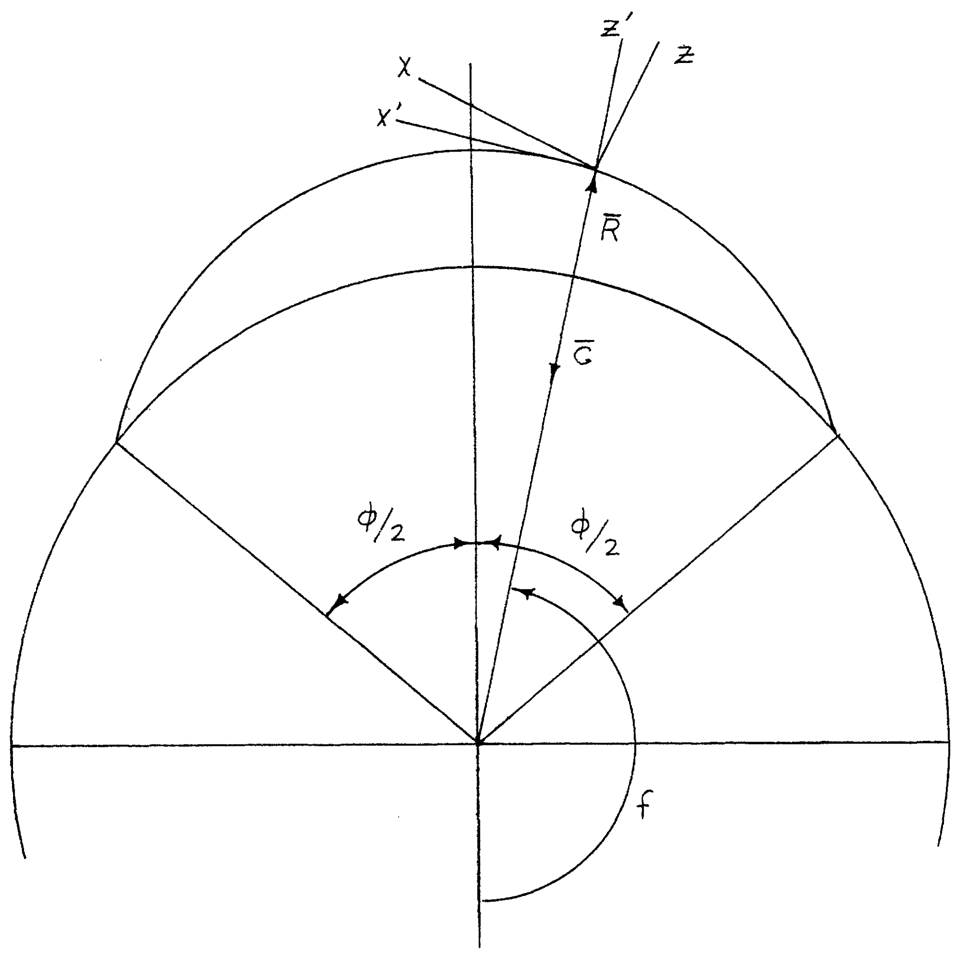

Fig. 2.3

Relationship of Co-ordinates to the Trajectory

Then:

$$\Delta G = r \cos \gamma \tag{2.4}$$

$$\Delta \bar{I}_G = - \frac{r \sin \gamma}{R} \bar{I}_{x'} \tag{2.5}$$

as shown in Fig. 2.4

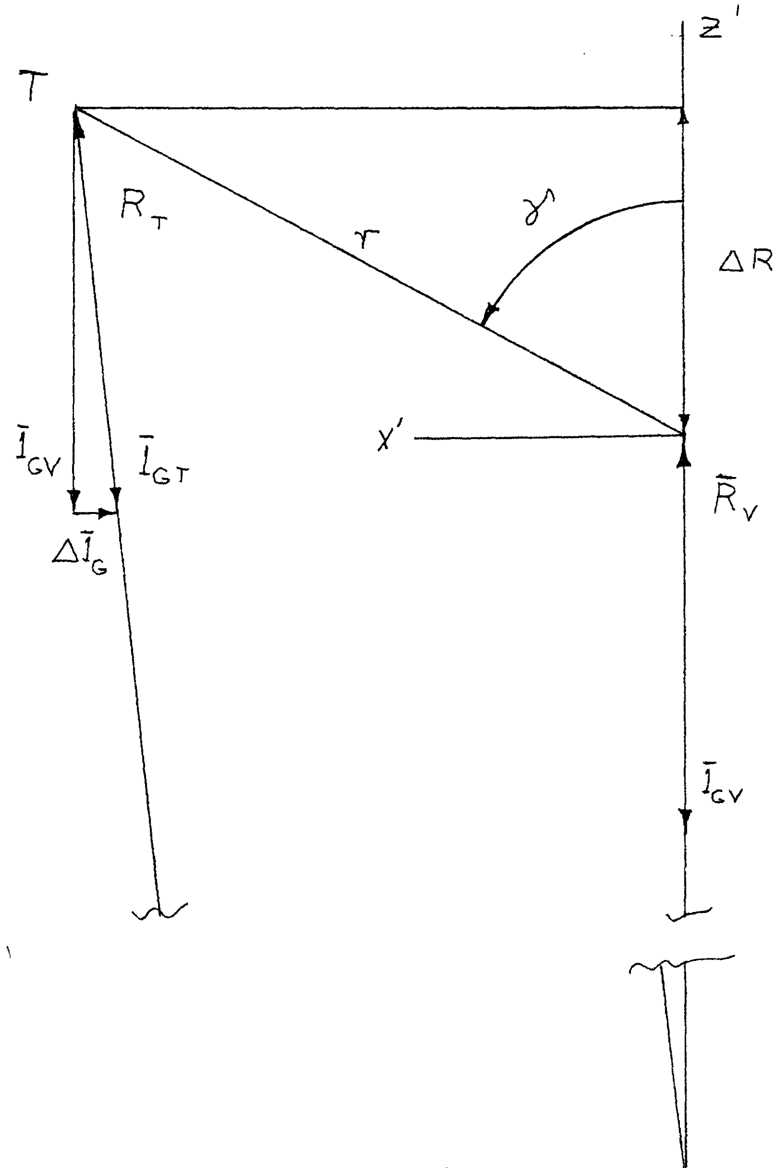

Fig. 2.4

Definition of ΔR and $\Delta \bar{I}_G$

Equation (2.3) is then:

$$\Delta \overline{G} = -\frac{Kr \sin\gamma}{R^3}\overline{I}_{x'} + \frac{2Kr\cos\gamma}{R^3}\overline{I}_{z'} \qquad (2.6)$$

where:

$\overline{I}_x$' and $\overline{I}_z$' one unit vectors in the x' and z' directions.

$\overline{I}_z$' is equal to minus $\overline{I}_g$.

The geometric relationship of $\Delta\overline{G}$ is shown in Fig. 2.5.

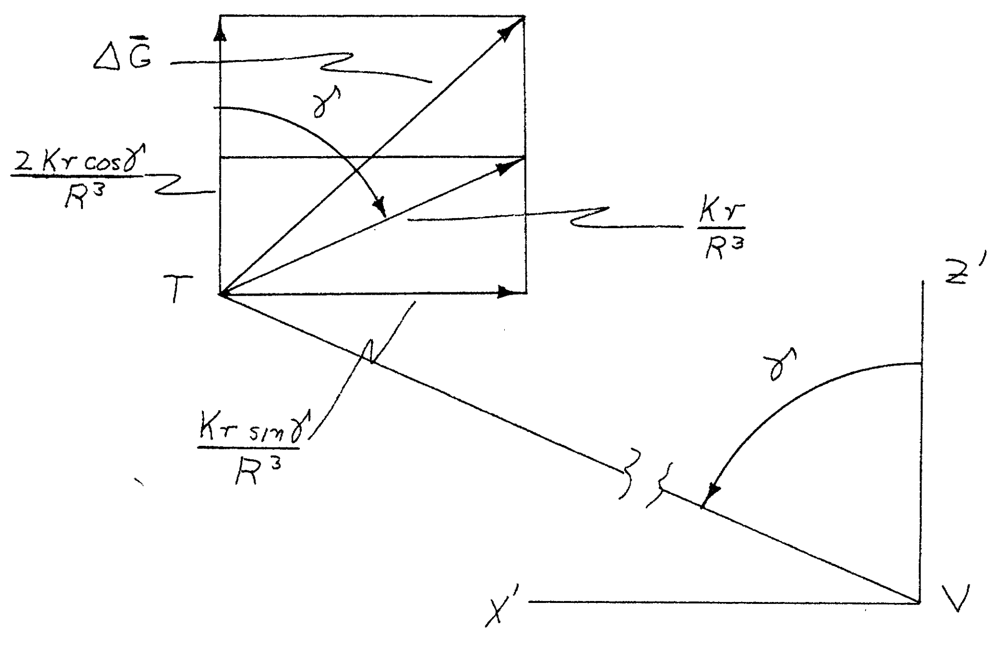

Fig. 2.5

Geometric Relationship of $\Delta\overline{G}$

To provide some conception of the size of these acceleration terms, they will now be evaluated for a typical situation. It is

[redacted]

assumed that a typical trajectory has a 6000 n. mi. optimum range. The other parameters can be calculated on the basis of this assumption:

$$\phi = \text{range angle} = \frac{6000}{60} = 100^\circ$$

$$e = \frac{1 - \sin\frac{\phi}{2}}{\cos\frac{\phi}{2}} = .364$$

$$p = R_E \left(1 + e \cos\left(180 - \frac{\phi}{2}\right)\right) = 1.60 \times 10^7 \text{ ft}$$

$$R_{max} = \frac{p}{1-e} = 2.51 \times 10^7 \text{ ft},$$

$$K = 1.41 \times 10^{16} \text{ ft}^3/\text{sec}^2$$

If a maximum separation velocity of 200 ft/sec is assumed between target and tankage and if it is also assumed that interception takes place approximately 1000 seconds after separation, then:

$$r = 200{,}000 \text{ ft}$$

Thus the maximum value ΔG will be:

$$\Delta G = \frac{2Kr}{R^3}$$

$$= \frac{(2)(1.41 \times 10^{16})(2.0 \times 10^5)}{(2.51 \times 10^7)^3} = .367 \text{ ft/sec}^2$$

Equations (2.1) must predict the position of the target for approximately 60 seconds. The magnitude of the third terms could then be:

[redacted]

$$\left|\frac{\ddot{r}_0}{2} t^2\right|_{max} = \frac{|\Delta G|_{max}}{2} t^2$$

$$= \frac{.357}{2}(60)^2 = 645 \text{ ft} \tag{2.7}$$

which would definitely be significant. The third terms in the expansion are therefore needed.

The constants for the fourth terms in the expansion represent the rate of change of acceleration with respect to inertial space. This will be the rate of change with respect to the rotating (primed) co-ordinate system plus the acceleration times the angular rate of the primed co-ordinates.

If: $$\Delta\bar{G} = -\frac{Kr}{R^3}(\sin\gamma\,\bar{I}_{y'} - 2\cos\gamma\,\bar{I}_{z'}) \tag{2.8}$$

Then: $$\left.\frac{d\Delta\bar{G}}{dt}\right|_I = \left.\frac{d\Delta\bar{G}}{dt}\right|_{x',z'} + \bar{\omega}\times\Delta\bar{G} \tag{2.9}$$

$$\left.\frac{d\Delta\bar{G}}{dt}\right|_{x',z'} = -\left(\frac{KR^3\dot{r} - 3KR^2r\dot{R}}{R^6}\right)(\sin\gamma\,\bar{I}_{x'} - 2\cos\gamma\,\bar{I}_{z'}) - \frac{Kr\dot{\gamma}}{R^3}(\cos\gamma\,\bar{I}_{y'} + 2\sin\gamma\,\bar{I}_{z'}) \tag{2.10}$$

The angular rate of the primed co-ordinates is the rate of change of f, which is given by:

$$\omega = \dot{f} = \frac{\sqrt{Kp}}{R^2} \tag{2.11}$$

and is perpendicular to $\Delta\bar{G}$.

Thus: $$\bar{\omega}\times\Delta\bar{G} = -\frac{Kr\dot{f}}{R^3}(\cos\gamma\,\bar{I}_{x'} + 2\sin\gamma\,\bar{I}_{z'}) \tag{2.12}$$

Then substituting equations (2.10) and (2.12) into equation (2.9):

$$\left.\frac{d\Delta\bar{G}}{dt}\right|_{I} = -\left(\frac{K\dot{r}}{R^3} - \frac{3Kr\dot{R}}{R^4}\right)(\sin\gamma\,\bar{I}_{y'} - 2\cos\bar{I}_{z'})$$

$$- \frac{K\dot{r}}{R^3}(\dot{\gamma} + \dot{f})(\cos\gamma\,\bar{I}_{x'} + \sin\gamma\,\bar{I}_{z'}) \tag{2.13}$$

The magnitude of each part of equation (2.12) is now estimated to determine the importance of the fourth terms of equations (2.1):

$$\left|\frac{d\Delta\bar{G}}{dt}\right| \leq \left|\left(\frac{K\dot{r}}{R^3} - \frac{3Kr\dot{R}}{R^4}\right)(\sin\gamma\,\bar{I}_{x'} - 2\cos\gamma\,\bar{I}_{z'})\right|$$

$$+ \left|\frac{Kr}{R^3}(\dot{\gamma} + \dot{f})(\cos\gamma\,\bar{I}_{x'} + 2\sin\gamma\,\bar{I}_{z'})\right| \tag{2.14}$$

$$< \left|2\left(\frac{K\dot{r}}{R^3} - \frac{3Kr\dot{R}}{R^4}\right)\right| + \left|\frac{2Kr}{R^3}(\dot{\gamma} + \dot{f})\right|$$

With $\dot{r}$ = 200 ft/sec and $\dot{R}$ = 4.14 x 10^3 ft/sec, the first term of equation (2.14) is:

$$2\left(\frac{(1.41\times10^{16})(200)}{(2.51\times10^{7})} - \frac{(3)(1.41\times10^{16})(2.0\times10^{3})(4.14\times10^{3})}{(2.51\times10^{7})^4}\right) \tag{2.15}$$

$$= (3.75\times10^{-4} - 1.77\times10^{-4}) = 1.98\times10^{-4}\ \text{ft/sec}^3$$

The components for the second term of equation (2.14) will be:

$$\dot{f} = \frac{\sqrt{Kp}}{R^2} = \frac{\sqrt{(1.41\times10^{16})(1.60\times10^{7})}}{(2.51\times10^{7})^2} = .755\times10^{-3}\ \text{rad/sec} \tag{2.16}$$

The $\dot{\gamma}$ term results from the component of velocity of the vehicle perpendicular to the line of sight between the target and vehicle. If it is assumed that the target was ejected radically from the center, the only perpendicular component of velocity is caused by transverse acceleration. As can be seen from Fig. 2.5, this transverse acceleration is greatest when γ is approximately 30° and is less than $\Delta \bar{G}$ max. Then the transverse velocity is less than:

$$v_T < \Delta G\, t = \frac{2K\dot{r}}{R^3}\frac{t^2}{2}$$

$$t = 1000 \text{ sec} \tag{2.17}$$

$$v_T < \frac{(2)(1.41 \times 10^{16})(200)}{(2.51 \times 10^7)^3}\frac{(1000)^2}{2} = 187 \text{ ft/sec}$$

$$\dot{\gamma} = \frac{1.87}{(2.0 \times 10^5)} = .934 \times 10^{-3} \text{ rad/sec} \tag{2.18}$$

The components for the second term of equation (2.14) is:

$$\frac{2Kr}{R^3}(\dot{\gamma} + \dot{f}) = \frac{(2)(1.41 \times 10^{16})(2.0 \times 10^5)}{(2.51 \times 10^7)^3}(.755 + .934) \times 10^{-3} \tag{2.19}$$

$$= 6.04 \times 10^{-4} \text{ ft/sec}^3$$

The fourth terms of equations (2.1) after 60 sec will be proportional to:

$$\left|\frac{d\Delta G}{dt}\right|\frac{t^3}{6} = (1.98 \times 10^{-4} + 6.04 \times 10^{-4})\frac{(60)^3}{6} \tag{2.20}$$

$$= 2.9 \text{ ft.}$$

Thus it is seen that the fourth terms of equations (2.1) are negligible and that the first three terms of the Taylor expansion is all that is needed.

In practice, the acceleration terms would be determined by the radar data and are not derived by the relations given above. This eliminates the need for knowing the orientation of the co-ordinate system with respect to gravity.

2.3 The Method of Least Squares

The radar on board the vehicle will track the target in the arbitrary non-rotating co-ordinate system in the spherical co-ordinates (r, θ, ψ). The information is then transformed into (x, y, r) by:

$$
\begin{aligned}
x &= r \sin \theta \\
y &= r \cos \theta \sin \psi \\
r &= r
\end{aligned}
\qquad (2.21)
$$

If the measurements made by the radar were exact, the constants $x_0, y_0, r_0, \dot{x}_0, \dot{y}_0, \cdots, \ddot{r}_0$ could be solved from equations (2.1) with three position fixes. Since the radar data will have randum uncertainties, redundant measurements are made. From these data "best values" ($\hat{x}_0, \hat{y}_0, \hat{r}_0, \hat{\dot{x}}_0, \hat{\dot{y}}_0, \cdots, \hat{\ddot{r}}_0$) are found by the least squared error method.

These "best values" are found by minimizing the sum of the squared errors. These errors are the difference between the measured value at a certain time and the value predicted by equations (2.1) using the "best value" constants. Taking the r

equation as an example, the squared error would be:

$$E = \epsilon^2 = (r_1 - \bar{r}_1)^2 + (r_2 - \bar{r}_2)^2 + \cdots + (r_n - \bar{r}_n)^2 \quad (2.22)$$

Where n is the number of measurements made and:

$$\begin{aligned} \bar{r}_1 &= \hat{r}_0 + \hat{\dot{r}}_0 (t_1 - t_0) + \frac{\hat{\ddot{r}}_0}{2} (t_1 - t_0)^2 \\ \bar{r}_2 &= \hat{r}_0 + \hat{\dot{r}}_0 (t_2 - t_0) + \frac{\hat{\ddot{r}}_0}{2} (t_2 - t_0)^2 \\ &\vdots \\ \bar{r}_n &= \hat{r}_0 + \hat{\dot{r}}_0 (t_n - t_0) + \frac{\hat{\ddot{r}}_0}{2} (t_n - t_0)^2 \end{aligned} \quad (2.23)$$

The mean squared error will than be:

$$\begin{aligned} E = &\left[r_1 - \left(\hat{r}_0 + \hat{\dot{r}}_0 (t_1 - t_0) + \frac{\hat{\ddot{r}}_0}{2}(t_1 - t_0)^2\right)\right]^2 \\ &+ \left[r_2 - \left(\hat{r}_0 + \hat{\dot{r}}_0 (t_2 - t_0) + \frac{\hat{\ddot{r}}_0}{2}(t_2 - t_0)^2\right)\right]^2 \\ &\cdots + \left[r_n - \left(\hat{r}_0 + \hat{\dot{r}}_0 (t_n - t_0) + \frac{\hat{\ddot{r}}_0}{2}(t_n - t_0)^2\right)\right]^2 \end{aligned} \quad (2.24)$$

To minimize this mean squared error the partial derivatives with respect to the three constants ($\hat{r}_0, \hat{\dot{r}}_0, \hat{\ddot{r}}_0$) are set equal to zero:

$$\begin{aligned} \frac{\partial E}{\partial \hat{r}_0} = &\ 2\left[r_1 - \left(\hat{r}_0 + \hat{\dot{r}}_0 (t_1 - t_0) + \frac{\hat{\ddot{r}}_0}{2}(t_1 - t_0)\right)\right] \\ &+ 2\left[r_2 - \left(\hat{r}_0 + \hat{\dot{r}}_0 (t_2 - t_0) + \frac{\hat{\ddot{r}}_0}{2}(t_2 - t_0)\right)\right] \\ &\cdots + 2\left[r_n - \left(\hat{r}_0 + \hat{\dot{r}}_0 (t_n - t_0) + \frac{\hat{\ddot{r}}_0}{2}(t_n - t_0)\right)\right] = 0 \end{aligned}$$

$$\frac{\partial E}{\partial \hat{\dot{r}}_0} = 2\left[r_1 - \left(\hat{r}_0 + \hat{\dot{r}}_0 (t_1 - t_0) + \frac{\hat{\ddot{r}}_0}{2}(t_1 - t_0)^2\right)\right](t_1 - t_0)$$
$$+ 2\left[r_2 - \left(\hat{r}_0 + \hat{\dot{r}}_0 (t_2 - t_0) + \frac{\hat{\ddot{r}}_0}{2}(t_2 - t_0)^2\right)\right](t_2 - t_0)$$
$$\ldots + 2\left[r_n - \left(\hat{r}_0 + \hat{\dot{r}}_0 (t_n - t_0) + \frac{\hat{\ddot{r}}_0}{2}(t_n - t_0)^2\right)\right](t_n - t_0) = 0 \quad (2.25)$$

$$\frac{\partial E}{\partial \hat{\ddot{r}}_0} = 2\left[r_1 - \left(\hat{r}_0 + \hat{\dot{r}}_0 (t_1 - t_0) + \frac{\hat{\ddot{r}}_0}{2}(t_1 - t_0)^2\right)\right]\frac{(t_1 - t_0)^2}{2}$$
$$+ 2\left[r_2 - \left(\hat{r}_0 + \hat{\dot{r}}_0 (t_2 - t_0) + \frac{\hat{\ddot{r}}_0}{2}(t_2 - t_0)^2\right)\right]\frac{(t_2 - t_0)^2}{2}$$
$$\ldots + 2\left[r_n - \left(\hat{r}_0 + \hat{\dot{r}}_0 (t_n - t_0) + \frac{\hat{\ddot{r}}_0}{2}(t_n - t_0)^2\right)\right]\frac{(t_n - t_0)^2}{2}$$

This results in three simultaneous equations for the constants:

$$n\hat{r}_0 + \sum_{i=1}^{n}(t_i - t_0)\hat{\dot{r}}_0 + \sum_{i=1}^{n}\frac{(t_i - t_0)^2}{2}\hat{\ddot{r}}_0 = \sum_{i=1}^{n} r_i$$
$$\sum_{i=1}^{n}(t_i - t_0)\hat{r}_0 + \sum_{i=1}^{n}(t_i - t_0)^2\hat{\dot{r}}_0 + \sum_{i=1}^{n}\frac{(t_i - t_0)^3}{2}\hat{\ddot{r}}_0 = \sum_{i=1}^{n}(t_i - t_0) r_i \quad (2.26)$$
$$\sum_{i=1}^{n}\frac{(t_i - t_0)^2}{2}\hat{r}_0 + \sum_{i=1}^{n}\frac{(t_i - t_0)^3}{2}\hat{\dot{r}}_0 + \sum_{i=1}^{n}\frac{(t_i - t_0)^4}{4}\hat{\ddot{r}}_0 = \sum_{i=1}^{n}\frac{(t_i - t_0)^2}{2} r_i$$

The notation can be made more concise by using matrix notation.

If a matrix A is defined as:

$$A = \begin{bmatrix} 1 & (t_1 - t_0) & \frac{(t_1 - t_0)^2}{2} \\ 1 & (t_2 - t_0) & \frac{(t_2 - t_0)^2}{2} \\ \vdots & \vdots & \vdots \\ 1 & (t_n - t_0) & \frac{(t_n - t_0)^2}{2} \end{bmatrix} \quad (2.27)$$

Equations (2.26) can be rewritten as:

$$A_T A R_o = A_T R \qquad (2.28)$$

where R is a column matrix consisting of the measured values:

$$R = \begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_n \end{bmatrix} \qquad (2.29)$$

and R_o is a column matrix consisting of the desired "best values":

$$R_o = \begin{bmatrix} \hat{r}_o \\ \hat{\dot{r}}_o \\ \hat{\ddot{r}}_o \end{bmatrix} \qquad (2.30)$$

If:

$$C = A^{-1} A_T^{-1} A_T \qquad (2.31)$$

then the desired values will be

$$R_o = C R \qquad (2.32)$$

Also, for the other two components:

$$X_o = C X \qquad Y_o = C Y \qquad (2.33)$$

where:

$$X_o = \begin{bmatrix} \hat{x}_o \\ \hat{\dot{x}}_o \\ \hat{\ddot{x}}_o \end{bmatrix} \qquad Y_o = \begin{bmatrix} \hat{y}_o \\ \hat{\dot{y}}_o \\ \hat{\ddot{y}}_o \end{bmatrix} \qquad (2.34$$

and:

$$X = \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_n \end{bmatrix} \qquad Y = \begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_n \end{bmatrix} \tag{2.35}$$

The matrix A and thus C are precalculated constants that depend only on the number of fixes and the time between fixes. The time between fixes is picked as one second. The number of fixes will depend on the accuracy required. It is desired that the position of the target be known within 200 feet after 60 seconds. This will require that the velocity constants be known to within 3 ft/sec. Because of the nature of the radar, the angular measurements will be the most critical. If the distance from the vehicle to the target is approximately 200,000 feet, the required accuracy for the angular rate is 1.5×10^{-5} rad/sec. The number of fixes needed at a rate of one per second is approximately 20. This will improve the accuracy by approximately:

$$\sigma_n \cong \frac{\sigma}{\sqrt{n}} \tag{2.36}$$

where σ is the deviation of one measurement, σ_n is the deviation of "best value" if there are n measurements, and n is the number of measurements. With an assumed accuracy for the radar of .001 radian, the accuracy of the "best values" will be:

$$\sigma_n \cong \frac{.001}{\sqrt{20}} = 2.2 \times 10^{-4} \text{ rad.} \tag{2.37}$$

With a tracking time of 20 seconds, this standard deviation will give an angular rate deviation of:

$$\frac{\sigma_m}{t} = \frac{2.2 \times 10^{-4}}{20} = 1.1 \times 10^{-5} \ rad/sec \qquad (2.38)$$

which is seen to be within the required tolerance.

2.4 The Calculation of the Launch Direction

At the end of the tracking period, all necessary information is available for the calculation of the launch direction. The launch direction is calculated by deriving the equations for the position of the target and for the attacking rocket as a function of time, with time $t = 0$ at the time the rocket stops burning. The position of the target and rocket are then set equal to obtain the time of flight and launch conditions.

All time intervals in the operation of the system until rocket burnout are constant and are determined by design considerations. After the system receives a command to destroy a target, it tracks the target for 20 seconds as described above. After this, there is a time period in which the computer solves for the launch conditions and the vehicle prepares to launch the rocket. Then at a given time from the initiation of tracking the rocket is launched. Since the rocket is designed to burn for a definite period, the time of rocket burnout is also fixed relative to the initiation of tracking. The position of the target relative to the vehicle is known at rocket burnout.

The position of the target as a function of time after burnout is given by equations of the same form as equations (2.1). If time t_o is picked as the burnout time, the constants for equations (2.1) will be given by equations (2.32) and (2.33). The elements of the A matrix will then be defined by the time interval between the beginning of tracking and burnout, and the interval between fixes as described in the previous paragraph. If time $t = t_o$ is arbitrarily set equal to zero, the equations for the position of the target will be:

$$X_T = \hat{X}_o + \hat{\dot{X}}_o t + \hat{\ddot{X}}_o \frac{t^2}{2} \qquad (2.39)$$

$$Y_T = \hat{Y}_o + \hat{\dot{Y}}_o t + \hat{\ddot{Y}}_o \frac{t^2}{2}$$

$$r_T = \hat{r}_o + \hat{\dot{r}}_o t + \hat{\ddot{r}}_o \frac{t^2}{2}$$

The position of the rocket at burnout is determined by the design of the rocket and the launch direction. It is assumed that the rocket accelerates approximately in a straight line. Then the velocity of the rocket in the $\bar{r}$ direction at burnout is:

$$\dot{r}_{oR} = \int_0^{t_b} a(t)\, dt \qquad (2.40)$$

and the position is:

$$r_{oR} = \int_0^{t_b} \int_0^{t} a(\tau)\, d\tau\, dt \qquad (2.41)$$

The acceleration as a function of time can be found experimentally through static firings of the rocket. The accuracy requirements for these constants and the design of the rockets will be discussed later. The initial conditions in the other two directions are:

$$
\begin{aligned}
X_{oR} &= r_{oR} \sin \theta_L \\
\dot{X}_{oR} &= \dot{r}_{oR} \sin \theta_L \\
Y_{oR} &= r_{oR} \cos \theta_L \sin \varphi_L \\
\dot{Y}_{oR} &= \dot{r}_{oR} \cos \theta_L \sin \varphi_L
\end{aligned}
\tag{2.42}
$$

where θ_L and φ_L are the launch angles these relationships are shown in Fig. 2.6.

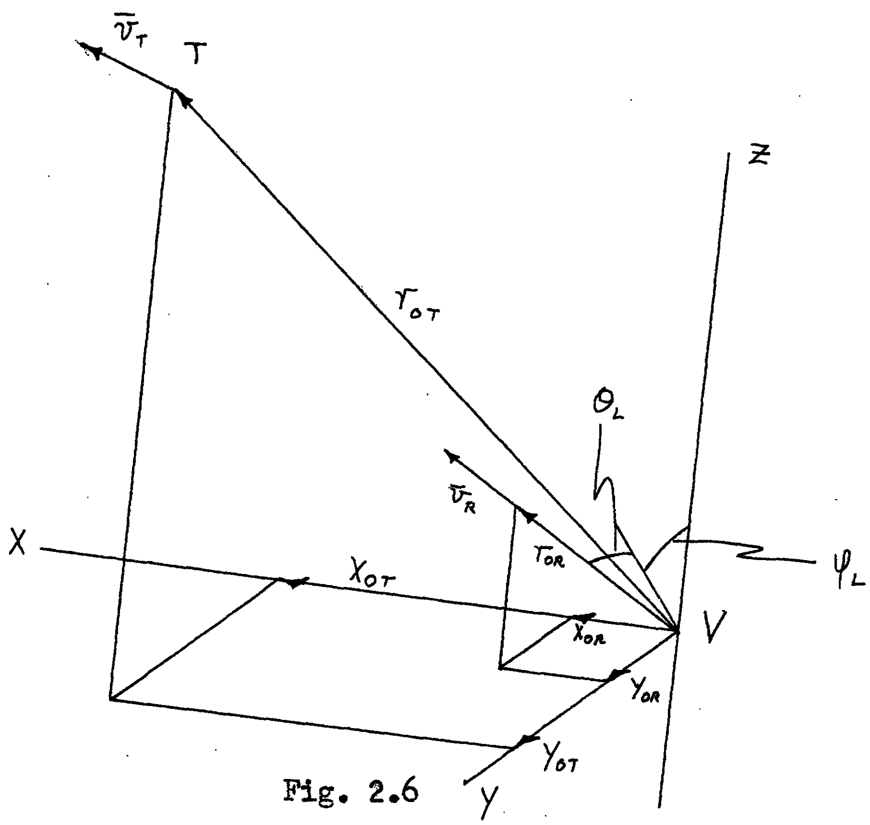

Fig. 2.6

Geometrical Relationships at the Time of Launch

After burnout the only accelerations of the rocket are those due to the difference in the gravity field. They are of the same nature as the acceleration acting on the target as described in the first section of this chapter. The acceleration of the rocket is proportional to:

$$\ddot{r}_R \sim \frac{K r}{R^3} \tag{2.43}$$

where $r = r_{oR} + \dot{r}_{oR} t$. Since the acceleration caused by gravity is itself a first order effect, any first order effects on it can be neglected as second order effects. Thus r_{oR} can be neglected compared to $\dot{r}_{oR} t$ since r_{oR} represents only about 10% of the total flight path. The acceleration is then:

$$\ddot{r}_R \sim \frac{K \dot{r}_{oR} t}{R^3} \tag{2.44}$$

and the displacement at the end of flight is:

$$\Delta r_R \sim \frac{K \dot{r}_{oR}}{R^3} \frac{t_f^3}{6} \tag{2.45}$$

where t_f is the time of flight. The acceleration of the target is:

$$\ddot{r}_T \sim \frac{K r_{oT}}{R^3} \tag{2.46}$$

and the displacement is:

$$\Delta r_T \sim \frac{K r_{oT}}{R^3} \frac{t_f^2}{2} \tag{2.47}$$

Since $\dot{r}_{oR}\, t$ is approximately r_{oR}, the displacement of the rocket is:

$$\Delta r_R \sim \frac{K(\dot{r}_o t_f)}{3R^3}\,\frac{t_f^2}{2} \cong \frac{K r_{oT}}{3R^3}\,\frac{t_f^2}{2} \sim \frac{\Delta r_T}{3} \tag{2.48}$$

Thus the rocket acceleration is approximately one-third the target acceleration as derived from the radar data.

The equations of motion for the rocket are:

$$\begin{aligned} X_R &= (r_{oR} + \dot{r}_{oR}\, t)\sin\theta_L + \frac{\hat{\ddot{X}}_o}{3}\,\frac{t^2}{2} \\ Y_R &= (r_{oR} + \dot{r}_{oR}\, t)\cos\theta_L \sin\varphi_L + \frac{\hat{\ddot{Y}}_o}{3}\,\frac{t^2}{2} \\ r_R &= r_{oR} + \dot{r}_{oR}\, t + \frac{\hat{\ddot{r}}_o}{3}\,\frac{t^2}{2} \end{aligned} \tag{2.49}$$

To find the launch conditions, set equations (2.39) equal to equations (2.49). The time of flight is solved from the r equations:

$$r_T = \hat{r}_o + \hat{\dot{r}}_o\, t + \hat{\ddot{r}}_o\,\frac{t^2}{2} = r_R = r_{oR} + \dot{r}_{oR}\, t + \frac{\hat{\ddot{r}}_o}{3}\,\frac{t^2}{2} \tag{2.50}$$

$$\frac{\hat{\ddot{r}}_o}{3}\, t^2 + \left(\hat{\dot{r}}_o - \dot{r}_{oR}\right) t + \left(\hat{r}_o - r_{oR}\right) = 0 \tag{2.51}$$

The solution of this quadradic involves the difference of large numbers and it also involves a square root which makes its solution on the computor more difficult. Since the accelerations are small an iterative solution is more accurate.

A first approximation for the time of flight is,

$$t_{f_1} = \frac{\hat{r}_o - r_{oR}}{\dot{r}_{oR} - \hat{\dot{r}}_o} \tag{2.52}$$

Then a second approximation is given by:

$$\left[\frac{1}{3}\hat{\ddot{r}}_o \, t_{f_1} + \left(\hat{\dot{r}}_o - \dot{r}_{oR}\right)\right] t_{f_2} = -\left(\hat{r}_o - r_{oR}\right) \tag{2.53}$$

Substituting equation (2.46) for t, gives:

$$t_{f_2} = \frac{\hat{r}_o - r_{oR}}{\left(\dot{r}_{oR} - \hat{\dot{r}}_o\right) - \frac{\hat{\ddot{r}}_o}{3}\left(\frac{\hat{r}_o - r_{oR}}{\dot{r}_{oR} - \hat{\dot{r}}_o}\right)} \tag{2.54}$$

To confirm the accuracy of this approximation by a typical example:

$$t_{f_1} = \frac{200,000}{5000} = 40 \text{ sec} \tag{2.55}$$

$$t_{f_2} = \frac{200,000}{5000 - \frac{.18}{3}\,40} = 40.019209 \text{ sec} \tag{2.56}$$

The next approximation would be:

$$t_{f_3} = \frac{200,000}{5000 - \frac{.18}{3}\,40.02} = 40.019218 \text{ sec} \tag{2.57}$$

The error would be .9 x 10^{-5} which would be very much less than the errors in the other numbers in the equation.

With the time of flight known, the launch direction can be solved from the other two equations. From the x equation:

$$X_T = \hat{X}_o + \hat{\dot{X}}_o t_f + \hat{\ddot{X}}_o \frac{t_f^2}{2} = X_R = \left(r_{oR} + \dot{r}_{oR} t_f\right)\sin\theta_L + \frac{\hat{\ddot{X}}_o}{6} t_f^2 \tag{2.58}$$

$$\theta_L = \sin^{-1} \frac{\hat{y}_0 + \hat{\dot{x}}_0 t_f + \hat{\ddot{y}}_0 \frac{t_f^2}{2}}{r_{0R} + \dot{r}_{0R} t_f} \tag{2.59}$$

From the y equation:

$$y_T = \hat{y}_0 + \hat{\dot{y}}_0 t_f + \hat{\ddot{y}}_0 \frac{t_f^2}{2} = y_R = (r_{0R} + \dot{r}_{0R}) \cos\theta_L \sin\varphi_L + \frac{\hat{\ddot{y}}_0}{3} \frac{t_f^2}{2} \tag{2.60}$$

$$\varphi_L = \sin^{-1} \frac{\hat{y}_0 + \hat{\dot{y}}_0 t_f + \hat{\ddot{y}}_0 \frac{t_f^2}{2}}{(r_{0R} + \dot{r}_{0R} t_f) \cos\theta_L} \tag{2.61}$$

2.5 Error Calculations

Two different sources of error are investigated here. These are the error in position caused by an error in burning time and the error in position caused by an error in final velocity.

One of the most likely sources of error is an error in burning time: Burning time is dependent on the initial temperature of the propellant which is difficult to control. If the actual initial temperature differs from the design value, the fuel will burn at some rate other than the design rate, but since it is likely that all the fuel will be burned, the rocket will reach approximately the same final velocity. Thus the primary effect of an erroneous burning rate will be an error in the initial position of the rocket (r_{R0}) and in the burning time (t_b).

To determine the effect of an erroneous burning time, the error caused by a 10% slower burning time is calculated. It would take approximately a 50°F error in initial temperature to cause an error this large in the burning time. Thus it is probable that the burning time can be held well within this 10% tolerance. In these calculations the primed quantities are the actual values and the unprimed quantities are the design values. Also, the gravity accelerations can be neglected in these calculations as second order effects and the acceleration of the rocket can be assumed constant.

For this example, let:

$$\begin{aligned} t_b &= 10 \text{ sec.} \\ t_b' &= 11 \text{ sec} \\ \hat{r}_o &= 175{,}000 \text{ ft} \\ \hat{\dot{r}}_o &= 200 \text{ ft/sec} \\ \dot{r}_{oR} &= 5000 \text{ ft/sec} \end{aligned} \tag{2.62}$$

Since the acceleration of the rocket is assumed constant, the design acceleration is:

$$a = \frac{\dot{r}_{oR}}{t_b} = \frac{5000}{10} = 500 \text{ ft/sec}^2 \tag{2.63}$$

and the actual value is:

$$a' = \frac{\dot{r}_{oR}}{t_b'} = \frac{5000}{11} = 454.55 \text{ ft/sec}^2 \tag{2.64}$$

The design value for the initial position of the rocket is:

$$r_{OR} = \frac{a t^2}{2} = \frac{500 (10)^2}{2} = 25,000 \text{ ft} \qquad (2.65)$$

and the actual value is:

$$r_{OR}' = \frac{a' t'^2}{2} = \frac{454.56 (11)^2}{2} = 27,500.27 \text{ ft} \qquad (2.66)$$

The effects of these errors combine to produce an error in the calculation time of flight. The calculated value is:

$$t_f = \frac{\hat{r}_o - r_{OR}}{\dot{r}_{OR} - \hat{\dot{r}}_o} = \frac{175,000 - 25,000}{5000 - 200} = 31,250 \text{ sec} \qquad (2.67)$$

The actual value for the time of flight is:

$$t_f' = \frac{\hat{r}_o - r_{OR}'}{\dot{r}_{OR} - \hat{\dot{r}}_o} = \frac{175,000 - 27,500.27}{5000 - 200} = 30.742 \text{ sec} \qquad (2.68)$$

The miss distance can be found by refering to Fig. 2.7.

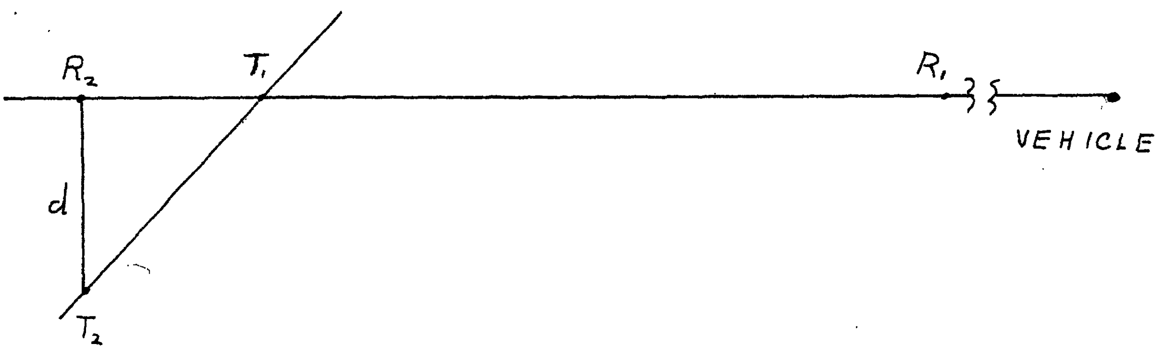

Fig. 2.7

Miss Distance

At the calculated interception time the target will be at position T_1, at the intersection of the target and rocket flight paths. Although the rocket was launched at the correct time, it accelerated slower than planned so it will be at the position R_1 at the calculated interception time. At some time later the rocket and target will be at positions R_2 and T_2, respectively, which is approximately their closest miss distance. The time interval between the calculated interception time and the time when the rocket and target are at positions R_2 and T_2 is the difference between the actual and designed rocket burning time plus the difference in time of flight from burnout as shown in the following equations:

$$\delta t = (t_b' - t_b) + (t_f' - t_f) \tag{2.69}$$

$$= (11 - 10) + (30.742 - 31.250) = 1 - .521$$

$$= .479 \text{ sec.}$$

Thus the difference between the position of the target and rocket at the calculated interception time is:

$$R_1R_2 - T_1R_2 = \dot{r}_{oR}\,\delta t - \hat{\dot{r}}_G\,\delta t \tag{2.70}$$

$$= (5000 - 200)(.479 \text{ sec})$$

$$= 2300 \text{ ft.}$$

At a time interval δt later, the rocket and target pass at approximately the closest miss distance. If the velocity in the transverse direction is 187 ft/sec (See equation 2.17) the minimum miss distance is:

$$d = R_2 T_2 = v_T \delta t$$

$$\cong (187)(.479) = 90 \text{ ft} \qquad (2.71)$$

If the weapon is detonated at the calculated interception time, the rocket will be well outside the planned tolerance. However, the weapon could be detonated within the required accuracy by a proximity fuse. This possibility will not be considered further here. An alternate means of detonating the weapon could be by command from the vehicle. The vehicle would recalculate the interception time by using the actual velocity and position of the rocket as measured by the radar. The radar will measure the velocity and position of the rocket at approximately five seconds after the planned burn out, at this time the rocket will have burned out. The values of velocity and position are inserted back into equation (2.54) to determine the new interception time. This time is relayed to the rocket, and a clock detonates the weapon at the proper time.

The other source of error considered is an error in final velocity. This error can be caused by chunks of fuel breaking off, by erosion, by residual fuel, etc. The effect of a final velocity error on miss distance are readily calculated.

If a 2% error in final velocity is taken as an example, the error in time of interception will be:

$$\delta t = t_f' - t_f = \frac{\hat{r}_o - r_{oR}}{\dot{r}_{oR}' - \hat{\dot{r}}_o} - \frac{\hat{r}_o - r_{oR}}{\dot{r}_{oR} - \hat{\dot{r}}_o} \tag{2.72}$$

$$= \frac{175{,}000 - 25{,}000}{4900 - 200} - 31{,}250$$

$$= 31{,}915 - 31{,}250 = .665 \text{ sec}$$

The minimum miss distance is then:

$$d = v_T \, \delta t = (187)(.665) = 124 \text{ ft} \tag{2.73}$$

Thus it is seen that with reasonable tolerances of 10% on burning time and 2% on final velocity, a launch direction can be calculated that will bring the rocket to within the required distance from the target. It remains to be shown (Chapter IV) that the rocket can actually be launched along this direction.

CHAPTER III

PHYSICAL DESIGN OF THE ROCKET

In this chapter a rough estimate is made of the important characteristics of the actual rocket. A velocity impulse is chosen to satisfy the requirements of the over-all system. Estimates are made of the weight of the payload, structure, and other equipment, and a fuel is chosen. A burning time is selected that fulfills the time requirements of the control system. All of this material is combined to determine the mass ratio of the rocket, the configuration of the propellant grain, the characteristics of the nozzle, etc. From these are found the weight, dymanic characteristics, acceleration, etc. of the rocket.

3.1 Design of the Rocket Motor

In designing the rocket the weight of the warhead is assumed to be 50 pounds. The actual design of the warhead would involve highly classified information and also would be beyond the scope of this thesis. If the actual weight of the warhead should differ from the assumed weight, all other weights and design parameters can be changed by a proportional amount. It is also assumed that the control system weighs 10 pounds and that the structure accounts for 15 per cent of the total weight. It is desired that the maximum time of flight be approximately 40 seconds. Since the target could be as much as 200,000 feet away, the desired velocity impulse is 5000 feet per second.

The fuel chosen is ammonium perchlorate ovidizer with polybutadience fuel binder and aluminum additive. The characteristics of the fuel are:

I_{sp}	@1000 psi	250 sec (at sea level)
Burning rate @1000 psi		.467 in/sec
Burning exponent, n		.236
Density		.063 lb/ft^3

Since the specific impulse at sea level is expected to be improved to 260 seconds in the near future, this number will be used in the design. The ideal exhaust velocity at sea level can then be determined from these fuel characteristics:

$$v_{2(SL)} = I_{sp}\, g$$
$$= (260 \text{ sec})(32.2 \text{ ft/sec}^2)$$
$$= 8380 \text{ ft/sec}$$

From this ideal velocity the characteristic parameter of the fuel, arbitrarily called K, can be found. The parameter K is involved with the burning temperature of the fuel. The equation is:

$$v_{2(SL)} = \sqrt{K\left[1 - \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}\right]} \qquad (3.2)$$

where:

$$K = \frac{2gk}{k-1} R T_c$$

k = Specific heat ratio (assumed to be 1.25)

P_1 = 1000 psi chamber pressure

P_2 = 14.7 psi exit pressure

then:

$$8380 \text{ ft/sec} = \sqrt{K\left[1-\left(\frac{14.7}{1000}\right)^{\frac{1.25-1}{1.25}}\right]}$$

thus:

$$K = \frac{(8380)^2}{.57} = 1.23 \times 10^8 \ ft^2/sec^2 \qquad (3.3)$$

For convenience in the manufacture and handling of the rocket, the exit of the rocket nozzle is assumed to have approximately the same diameter as the case. With this assumption and anticipating the size of the case and throat area from the results of the design to follow, the ratio of exit area to throat area is approximately 54. The pressure ratio can be found from the relation:

$$\frac{A_t}{A_2} = \left(\frac{k+1}{2}\right)^{\frac{1}{k-1}}\left(\frac{P_2}{P_1}\right)^{\frac{1}{k}}\sqrt{\frac{k+1}{k-1}\left[1-\left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}\right]} \qquad (3.4)$$

Solving this equation using the graphs in reference (   ), the pressure ratio will be:

$$\frac{P_1}{P_2} = 800 \qquad (3.5)$$

thus the exit pressure will be:

$$P_2 = \frac{1000}{800} = 1.25 \text{ psi}$$

Using equation (3.2) the ideal exhaust velocity in a vacuum can be derived:

$$v_{2(VAC)} = \sqrt{\frac{(8380)^2}{.57}\left[1-\left(\frac{1.25}{1000}\right)^{\frac{1.25-1}{1.25}}\right]} \qquad (3.6)$$

$$= 8380\sqrt{\frac{.83}{.57}}$$

$$= 10{,}050 \text{ ft/sec}$$

The effective exhaust velocity is:

$$c = v_{2(VAC)} + \frac{(P_2 - P_a)A_e g}{\dot{W}} \qquad (3.7)$$

where:

$P_a = 0$ atmospheric pressure

$A_e = 38.6 \text{ in}^2$ exit area

$\dot{W}$ = mass flow rate, is anticipated to be approximately 4.2 lb/sec

thus:

$$c = 10{,}050 + \frac{(1.52)(38.6)(32.2)}{4.2}$$

$$= 10{,}050 + 370 = 10{,}420 \text{ ft/sec}$$

The specific impulse in vacuum is then:

$$I_{sp} = \frac{c}{g} \qquad (3.8)$$

$$= \frac{10{,}420}{32.2} = 324 \text{ sec}$$

In vacuum there is no loss due to drag. Also, relative to the co-ordinate system used, the loss due to gravity can be neglected in calculating the performance of the rocket. Thus the actual velocity impulse achieved by the rocket is very nearly the ideal velocity impulse. The mass ratio can then be calculated by:

$$V_o = c \ln M R \tag{3.9}$$

$$\ln MR = \frac{V_o}{c} = \frac{5000}{10,420} = .480$$

$$MR = e^{.480} = 1.61$$

The mass ratio is the total weight over the burnout weight. Thus the weight of the propellant can be found in terms of the total weight by:

$$MR = \frac{W_T}{W_{BO}} = \frac{W_T}{W_T - W_{BO}} \tag{3.10}$$

thus:

$$W_p = W_T \left(1 - \frac{1}{MR}\right) \tag{3.11}$$

The total weight of the rocket is then:

$$W_{TOTAL} = W_{WARHEAD} + W_{CONTROL\ SYSTEM} + W_{STRUCTURE} + W_{PROPELLANT} \tag{3.12}$$

Substituting the values:

$$W_T = 50 \text{ lb} + 10 \text{ lb} + .15\ W_T \text{lb} + W_T \left(1 - \frac{1}{MR} \right) \text{lb}$$

$$(1 - .15 - .38)\ W_T = 60 \text{ lb}$$

$$W_T = 128 \text{ lb}$$

From the total weight, the weight of the propellant is found to be:

$$W_p = .15\ W_T = 48\ \text{lb} \tag{3.13}$$

and the weight of the structure is approximately:

$$W_s = W_T\left(1 - \frac{1}{MR}\right) \cong 20\ \text{lb} \tag{3.14}$$

With a 10-second burning time the mass flow rate is:

$$\dot{W} = \frac{W_P}{t_b} = \frac{48}{10} = 4.8\ \text{lb/sec} \tag{3.15}$$

With a burning rate of .467 in per second and a density of .063 pound per cubic inch, the burning area is:

$$\begin{aligned} \dot{W} &= \rho\ BR\ A_B \\ A_B &= \frac{\dot{W}}{\rho\ BR} \\ &= \frac{4.8}{(.063)(.467)} = 163\ \text{in}^2 \end{aligned} \tag{3.16}$$

If the rocket is seven inches in diameter, a cone 14.4 inches high will have the proper surface area. The cone is doubled back on itself twice to conserve space as shown in Fig. 3.1. The total volume is:

$$V = \frac{W}{\rho} = \frac{48}{.063} = 760\ \text{in}^3 \tag{3.17}$$

Using this volume, the length of the propellant grain from the base of the cone on one end to the base of the cone on the other end is:

$$l_p = \frac{V}{\pi r^2} = \frac{760}{38.6} = 19.7\ \text{in} \tag{3.18}$$

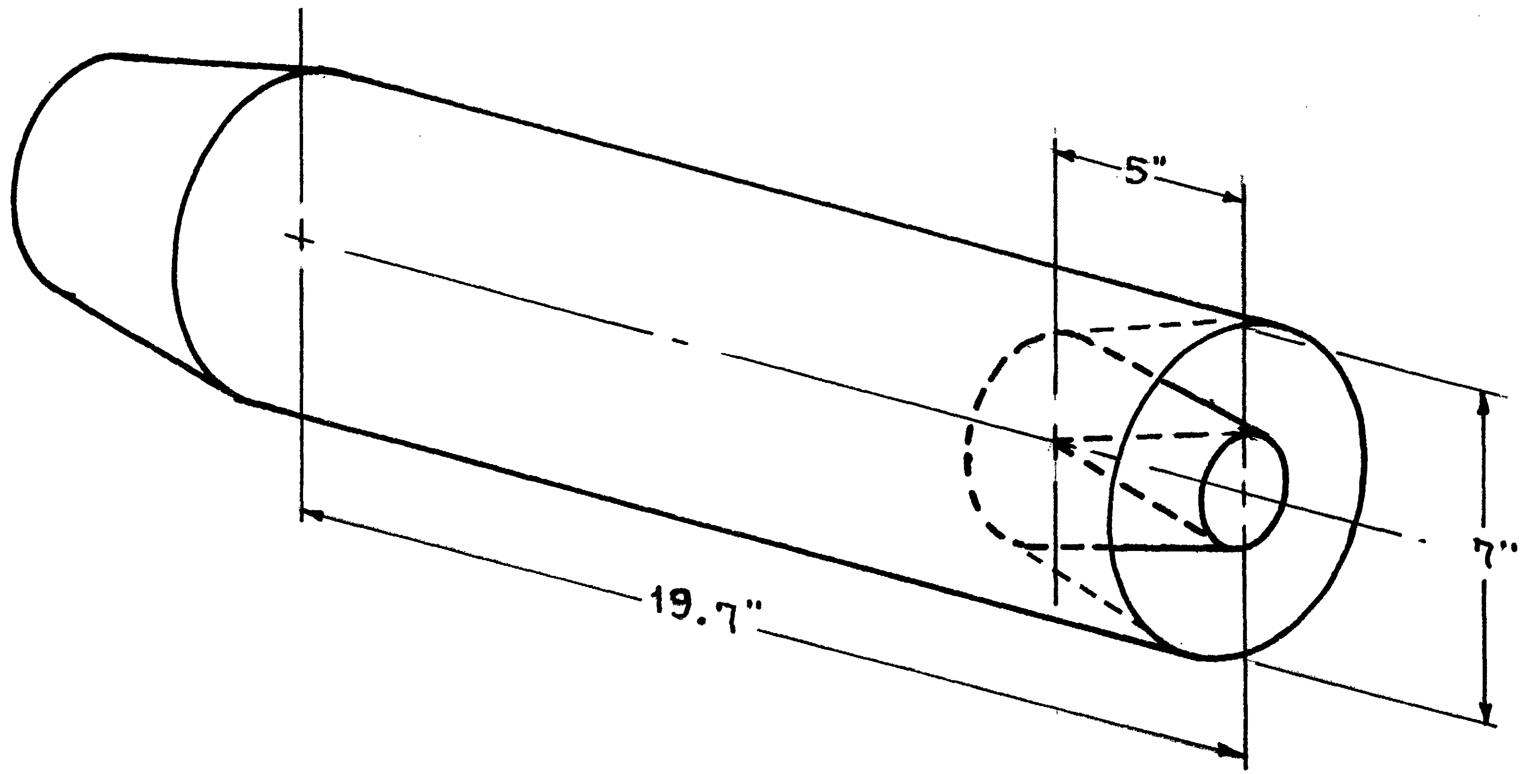

FIGURE 3.1 GRAIN CONFIGURATION.

The height of the cone is 4.5 inches giving a total length for the propellant grain of approximately 24 inches.

To determine if the estimate made for the weight of the structure is approximately correct, the weight of the walls of the rocket case are calculated. From this the other weights involved are estimated:

The thickness of the walls are:

$$t_w = \frac{\text{safety factor x press. x diameter}}{\text{stress}} \tag{3.19}$$

With a safety factor of two and a maximum tensile strength of 130,000 psi the thickness is:

$$t_w = \frac{(2)(1000)(7)}{(130,000)(2)} \tag{3.20}$$

$$= 0.054 \text{ in.}$$

With a density of the steel of 0.29 lb/in^3 the weight of the cylindrical portion of the case is:

$$W_c = t_w \pi D l \rho \tag{3.21}$$

$$= (0.054)(\pi)(7)(24)(0.29)$$

$$= 8.2 \text{ lb}$$

If the ends are considered spherical and with twice the thickness of the wall they will weigh;

$$W_E = 4 \pi R^2 t_F \rho$$

$$= 4 \pi (3.5)^2 (.1)(.29) \tag{3.22}$$

$$= 4.5 \text{ lb}$$

By comparing these weights with the weight of similar rockets, the nozzle and other parts should weigh approximately 7 pounds. The total weight is then:

$$W_s = 8.2 + 4.5 + 7 = 19.7 \text{ lb} \tag{3.23}$$

Thus the structure could probably be built within the original weight estimate of 20 pounds.

The throat area is calculated by determining the mass flow rate in the critical section by:

$$\dot{W} = \frac{A_t v_t}{V_t} \tag{3.24}$$

$$= A_t P_1 g \frac{k\sqrt{[2/k+1]^{k+1/k-1}}}{\sqrt{g k R T_1}}$$

From equations (3.2) and (3.3):

$$\sqrt{g k R T} = \sqrt{\frac{(k-1) K}{2}} \tag{3.25}$$

$$= \sqrt{\frac{(1.25-1)}{2}\frac{(8380)^2}{.54}}$$

$$= 4040 \text{ ft/sec}$$

Thus:

$$A_t = \frac{\dot{w}\sqrt{g k R T}}{P_1 g k\sqrt{[2/k+1]^{k+1/k-1}}} \tag{3.26}$$

$$= \frac{(4.8)(4040)}{(1000)(32.2)(.7356)}$$

$$= .82 \text{ in}^2$$

The area ratio is then:

$$\frac{A_2}{A_t} = \frac{38.6}{.82} = 47 \qquad (3.27)$$

The nozzle shape can thus be modified slightly to conform with the original area ratio of 54.

3.2 Dynamic Charteristics of the Rocket

A rough estimate of the dynamic characteristics of the rocket is necessary in order to design the control system for the rocket. This will include the moment of inertia and acceleration as a function of time. (In actual practice these parameters would be determined by experiment for the final system design.) The moment of inertia is determined by calculating the moment of inertia of each component about its center of mass and then combining them by the parallel axis theorem to find the total moment of inertia about the center of mass of the rocket.

The moment of inertia of each part is:

a. Propellant - If the propellant is assumed to be a cylinder:

$$I_p(t) = \frac{m(t)}{12}\left(3R^2 + \ell(t)^2\right) \qquad (3.27)$$

where:

m (t) is the mass as a function of time, i.e.,

$$m(t) = m_o - \dot{w}t = 48 - 4.8\,t \qquad (3.28)$$

and l (t) is the length of the propellant as a function of time, i.e.,

$$l\,(t) = l_o - lt = 20 - 2t$$

thus:

$$I_p = \frac{48 - 4.8t}{12}\left(3\,(3.5)^2 + (20 - 2t)^2\right) \qquad (3.30)$$

$$= (4.0 - 0.4t)(436.6 - 80t + 4t^2)$$

$$= 1750 - 497t + 48t^2 - 1.6t^3 \ \text{lb in}^2$$

b. Warhead - If the warhead is assumed to be a sphere with a radius of 6 inches:

$$I_w = \frac{2mR^2}{5} = \frac{2(50)(6)^2}{5} \qquad (3.31)$$

$$= 750 \ \text{lb/in}^2$$

c. Structure - The moment of inertia of the structure is assumed to be:

$$I_s = m\left(\frac{l}{2}\right)^2 = 19(10)^2 \qquad (3.32)$$

$$= 1900 \ \text{lb in}^2$$

d. Control System - The moment of inertia of the control system about its own center of mass is neglected.

The moment of inertia of the entire vehicle is calculated using the weight and balance diagram in Fig. 3.2. The center of mass travels 1.55 inches during burning, thus its rate of travel is .155 inch per second. The moment of inertia of each part about the center of mass of the system is:

$$I = I_{cm} + M(d \pm .155\,t)^2 \tag{3.33}$$

where d is the distance from the center of mass of the part to the original center of mass of the system. This may also be a function of the time as in the case of the propellant.

The total moment of inertia is then:

$$I_T = I_{P\,cm} + I_{W\,cm} + I_{S\,cm} + M_p((5.5 - 2t) + .155\,t)^2 + M_w(13 - .155\,t)^2 + M_s(9 + .155\,t)^2 + M_{cs}(26 + .155\,t)^2 \tag{3.34}$$

$$I = 1750 - 497\,t + 48t^2 - 1.6t^3 + 750 + 1900 + (48 - 4.8t)(5.5 - 1.85t)^2 + 50(13 - .155\,t)^2 + 19(9 + .155\,t)^2 + 10(26 + .155\,t)^2$$

$$I = 22{,}480 - 1692\,t + 310\,t^2 - 17.8\,t^3 \quad lb\ in^2$$

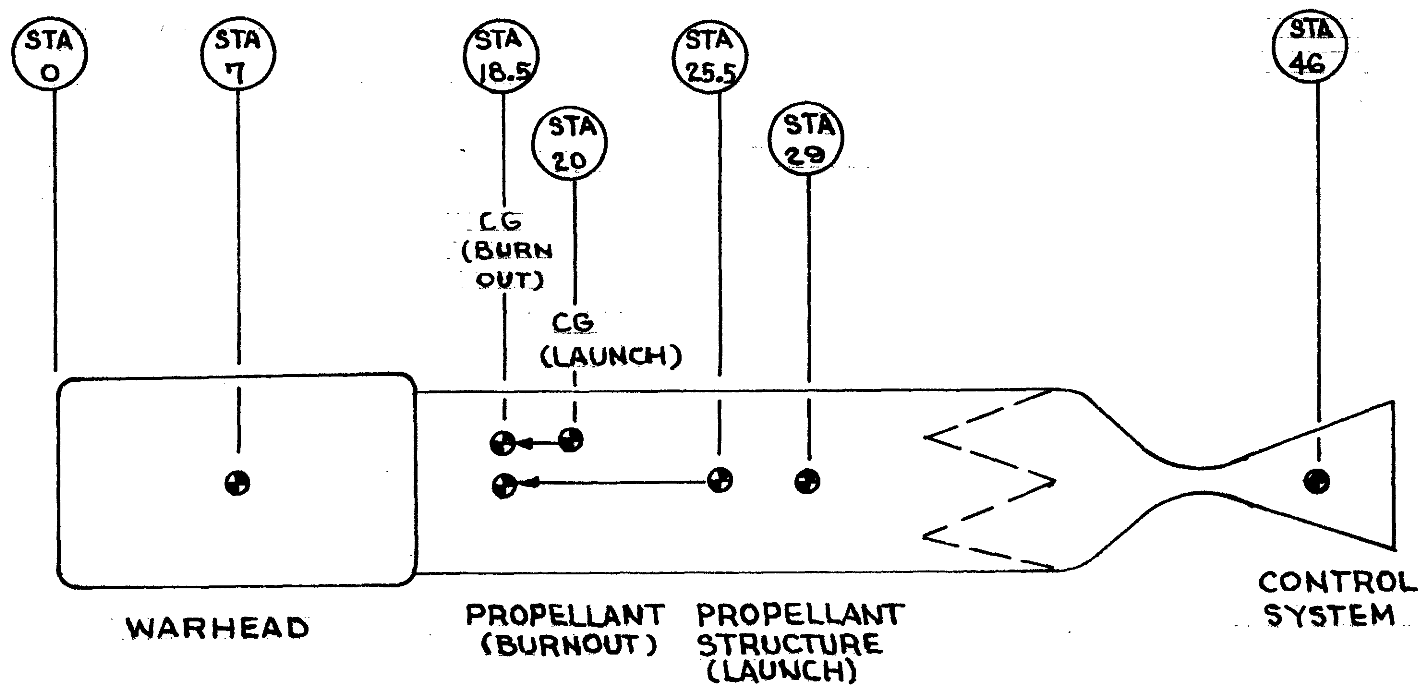

FIGURE 3.2 WEIGHT AND BALANCE DIAGRAM

The angular acceleration of the vehicle as a function of the force applied at the rocket nozzle is needed for the design of the control system. This angular acceleration is given by:

$$\ddot{\gamma} = \frac{F_T h g}{I_T} \tag{3.35}$$

where:

h is the distance from the central vane to the center of mass:

$$h = (h_o + \dot{h} t) = (26 + .155 t) \tag{3.36}$$

F_T is the transverse force acting at the rocket nozzle due to a deflection of the control vane.

The angular acceleration is then:

$$\ddot{\gamma} = \frac{F_T(26 + .15t)(32.2)(12)}{22,480 - 1692t + 310t^2 - 17.8t^3} \tag{3.37}$$

A graph of the transfer function between force and angular acceleration is shown in Fig. 3.3.

The acceleration of the rocket as a function of time is also needed for the design of the control system. The thrust is given by:

$$F = \dot{W} I_{sp} = (4.8)(324)$$

$$= 1550 \ lb_f \tag{3.38}$$

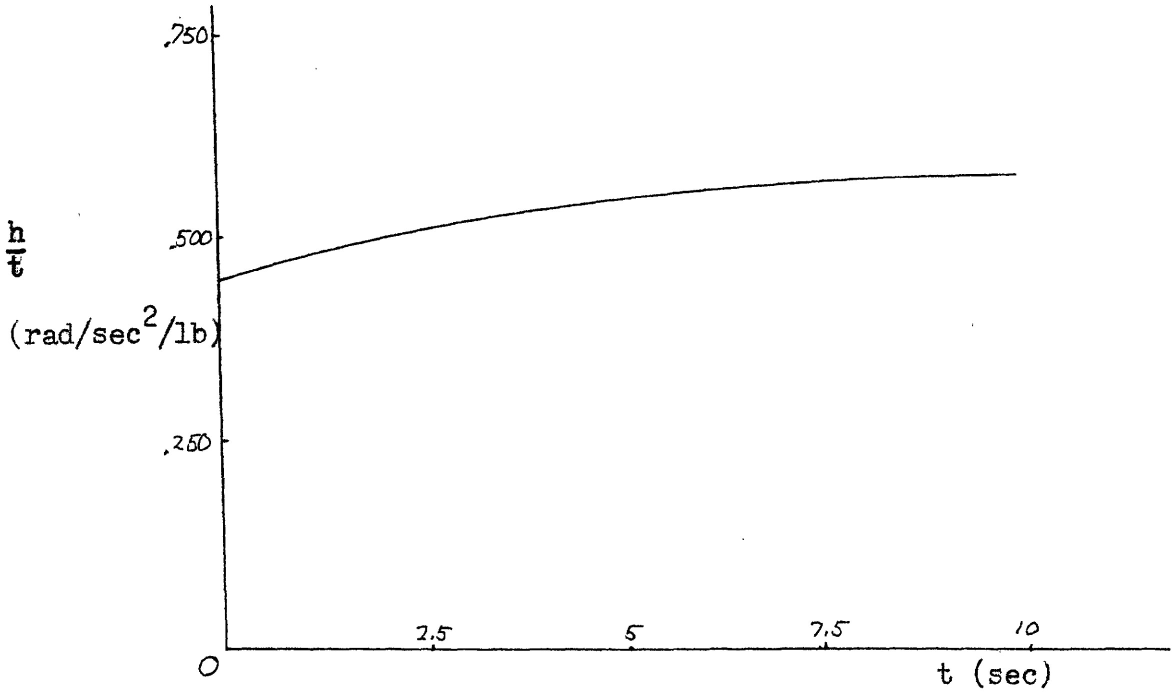

Fig. 3.3

Angular Acceleration as a Function

of Time and Applied Force

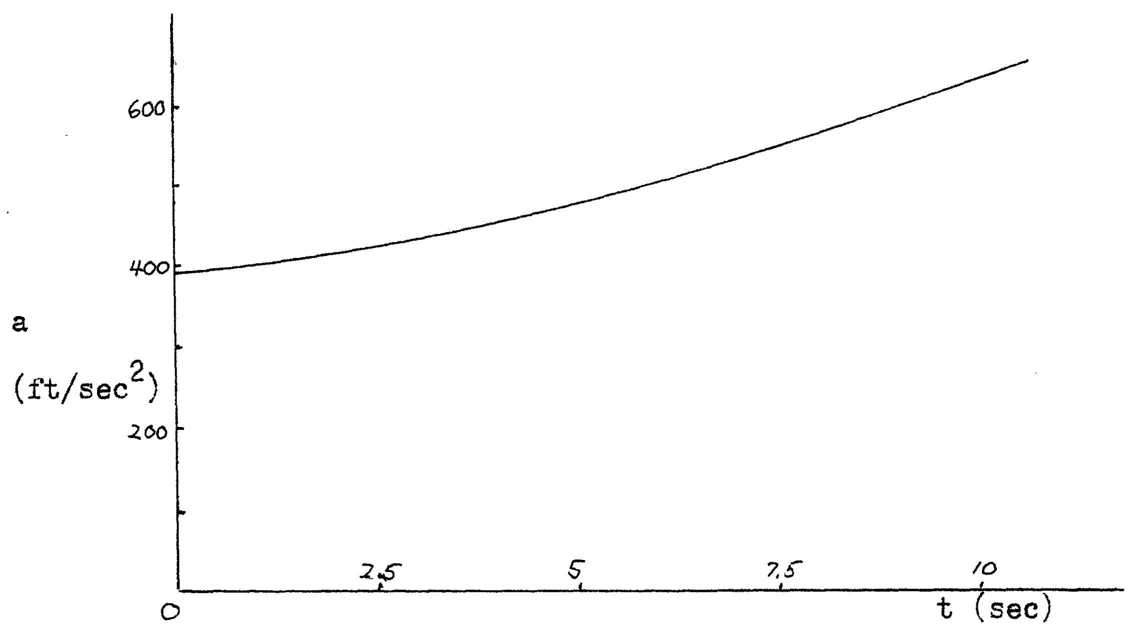

Fig. 3.4

Acceleration as a Function of Time

The mass of the rocket is:

$$m = (m_0 - \dot{w}t) \tag{3.39}$$

$$= (128 - 4.8t) \text{ lb}$$

Thus the acceleration is then:

$$a(t) = \frac{Fg}{m} = \frac{(1550)(32.2)}{128 - 4.8t} \tag{3.40}$$

The acceleration as a function of time is shown in Fig. 3.4.

3.3 Control Vanes

The position of the rocket is controlled by four small vanes located at the end of the rocket nozzle. The size of the vanes needed are determined here.

The density of the jet exhaust at the position of the vanes can be found by:

$$\dot{W} = \rho_2 A_2 V_2 \tag{3.41}$$

$$\rho_2 = \frac{\dot{W}}{A_2 V_2} = \frac{(4.80)}{(.27)(10,050)}$$

$$= 1.78 \times 10^{-3} \text{ lb/ft}^3$$

Since the vanes are in supersonic flow the change in the coefficient of lift for a change in angle of attack is approximately two. In other words:

$$\frac{\partial C_L}{\partial \alpha} \cong 2 \tag{3.42}$$

Thus the transverse force is:

$$F_T = \frac{\partial C_L}{\partial \alpha} \alpha \frac{\rho_2}{2} V_2^2 A_v \tag{3.43}$$

The maximum angular acceleration demanded by the control system is 3.58 rad/sec^2, at time t = 0. From equation (3.37):

$$\frac{h}{I} = .448 \text{ rad/sec}^2 \text{ per } lb_f \tag{3.44}$$

Thus the transverse force needed is:

$$F_T = \frac{\ddot{\gamma}}{\frac{h}{I}} = \frac{3.58}{.448} \tag{3.45}$$

$$= 8.0 \; lb_f$$

If it is required that the maximum valve for the angle of attack, α, be approximately .075 radian, the area required will be:

$$A_v = \frac{F g}{\frac{\partial C_L}{\partial \alpha} \alpha \frac{\rho_2}{2} V^2} = \frac{8.0}{2\,(.075)\left(\frac{1.78 \times 10^{-3}}{2}\right)(1.01 \times 10^8)}$$

$$= 1.9 \times 10^{-2} \text{ ft}^2 = 2.8 \text{ in}^2$$

Thus with two vanes used in each direction, the area of each vane is 1.4 square inches.

CHAPTER IV

THE LAUNCH AND GUIDANCE SYSTEMS

This chapter opens with a brief description of a proposed launch mechanism for the interceptor system. The description is necessarily brief since the design of the launch mechanism depends to a great extent on the design of the entire vehicle, which is not the subject of this thesis. The bulk of the chapter is thus devoted to a detailed discussion of the guidance system design and to an analysis of the results of an analog computer simulation of this guidance system.

4.1 The Launch System

For the reason noted above, only a brief outline of a possible launch mechanism will now be presented. In the launch mechanism proposed, rockets are carried much like shells in a revolver. Since the co-ordinate system used to develop the equations for launch direction (Fig. 2.1) is arbitrary, the co-ordinate system can be chosen so that the x axis is approximately along the axis of the "revolver." In other words, the co-ordinate system will be approximately coincident with a co-ordinate fixed in the vehicle. The rocket to be fired is rotated to the proper position determined by the angle φ and is then elevated to the angle θ. These angular rotations are shown in Fig. 4.1.

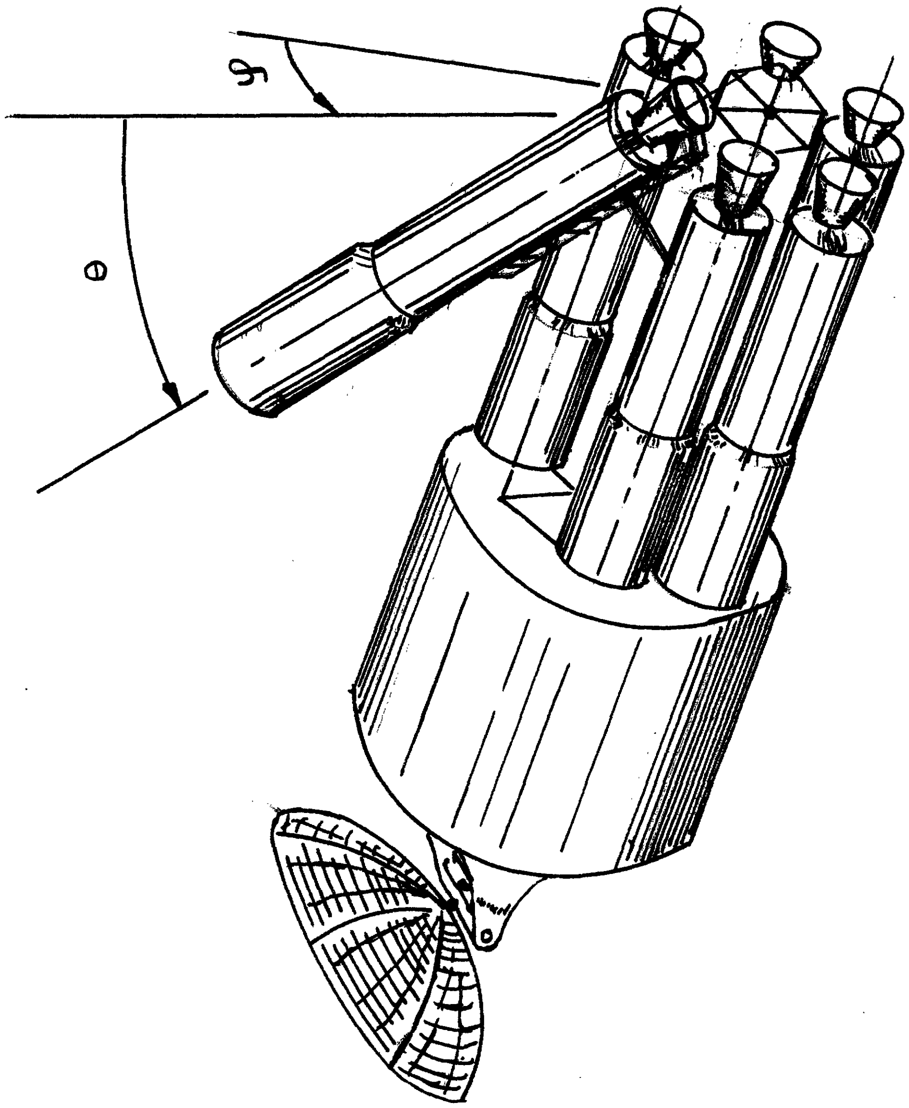

FIGURE 4.1 EXTERNAL CONFIGURATION OF THE VEHICLE.

From the nature of the co-ordinate system chosen, it is apparent that the radar data and thus the launch direction are actually relative to a co-ordinate system determined by the inertial reference equipment. The attitude of the vehicle is also controlled relative to the inertial reference equipment. This attitude control is performed by an attitude control system which controls the vehicle's attitude in such a way that the vehicle co-ordinates are approximately along the co-ordinates determined by the inertial reference. The approximation, of course, is only as good as the attitude control system itself. The actual launch direction will therefore be in error at least as much as the error in the vehicle's attitude. Also, additional errors will probably be caused by the faulty separation of the rockets from the launch mechanism. The purpose of the guidance system, to be discussed later in this chapter, will be to correct these errors in launch direction.

The launching of the rocket puts added loads on the attitude control system. When the rockets are rotated in a certain direction, the vehicle tends to rotate in the opposite direction by the law of conservation of angular momentum. The attitude control system must be able to compensate for these rotations. When the rocket is actually launched, a torque is generated by friction between the rocket and launch mechanism and a second torque is caused by the rocket exhaust hitting the vehicle. The torque caused by the friction tending to pull the vehicle with the rocket is in the opposite direction to the torque caused by the rocket exhaust pushing the vehicle away from the rocket. It may therefore be possible to adjust the friction so that the torques approximately counteract each other, greatly reducing the energy requirements of attitude control system. The actual design would have to be determined by experiment on the actual equipment.

4.2 The Guidance System

The rocket is controlled during burning by command from the vehicle. The actual position of the rocket is measured by the radar and compared with the desired position determined by the calculated launch direction. From the resulting error, a command signal is computed by the guidance system. The command signal is then sent to the rocket to bring it back to the proper path.

To simplify the design and analysis of the guidance system, the motion of the rocket is restricted to one plane as shown in Fig. 4.2. (The design of the system for the perpendicular plane would be the same.) The x and z axis are the same as the x and z axis in Fig. 2.1.

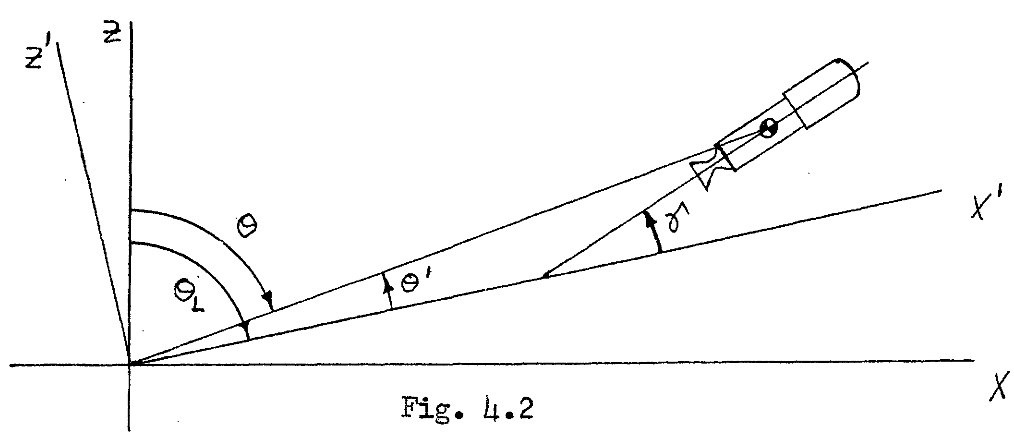

Fig. 4.2

Guidance Co-ordinate Systems

A primed co-ordinate system is defined so that the x^1 axis is along the desired launch direction defined by the angle θ_L. The object is then to null the quantities z^1 and $\dot{z}^1$ before burnout. In terms of the quantities measured by the radar:

$$z' = r \sin \theta' \tag{4.1}$$

$$\dot{z}' = \dot{r} \sin \theta' + r \cos \theta' \dot{\theta}' \tag{4.2}$$

where:

$$\theta' = \theta_L - \theta \tag{4.3}$$

and where θ and r are measured by the radar and θ_L is the calculated launch direction. Since the error is small, small angle approximations can be made. Thus:

$$z' = r\theta' \tag{4.4}$$

$$z' = \dot{r}\theta' + r\dot{\theta}' \tag{4.5}$$

The rocket is controlled by vanes in the rocket nozzle. A deflection of the control vanes produces a torque on the rocket which is proportional to the angular acceleration of the rocket. The equation for the angular motion is:

$$\ddot{\gamma} = \frac{F_T h}{I} \tag{4.6}$$

where:

F_T is the transverse force acting at the rocket nozzle due to a deflection of a control surface and is proportional to a command from the control system. $\frac{h}{I}$ is the relation between force and angular acceleration and is given by Fig. 3.3. The angular position of the rocket is proportional to the linear acceleration in the z^1 direction. The equation of motion for the rocket in linear motion is:

$$\ddot{z}' = \frac{F}{m} \sin \gamma \cong \frac{F}{m} \gamma \qquad (4.7)$$

where:

$\frac{F}{m}$ is the acceleration of the rocket given by Fig. 3.4. γ is the angular position of the rocket relative to the desired direction. Hence, the linear position is proportional to the fourth integral of the signal needed by the control vanes. (The dynamics of the control vanes themselves are neglected.)

Normally, a second-order guidance system is formed by using the position error and velocity feedback. In such a system, a signal proportional to a commanded angular position is sent to the rocket. The rocket then uses a second-order position control system, composed of a position and rate gyro, to derive a signal for the control vanes.

The rocket design proposed in this thesis, however, eliminates the need for a position and a rate gyro located in the rocket itself, with a consequent reduction in rocket weight. Because of the definite design characteristics and burning time of the rocket, it is possible to simulate the rocket's motion in the control system of the vehicle. The control system will then need the initial angular position ($\hat{\gamma}_o$) and rate ($\dot{\hat{\gamma}}_o$) of the rocket as initial conditions in the simulation. It is assumed that at the time the control system begins to operate, the rocket is leaving the vehicle in a straight line. Then $\dot{\hat{\gamma}}_o$ is equal to zero and $\hat{\gamma}_o$ is the error angle θ_0^1 measured by the radar. The command signal sent to the rocket from the vehicle would then be directly proportional to the deflection of the central vane. The only equipment needed by the rocket itself is a radio receiver and a servo to position the control vanes relative to the command signal. The control system is shown in Fig. 4.3.

4.3 Design of the Guidance System

The guidance system divides naturally into two sections, (1) an inner loop that controls the angular position of the rocket and (2) an outer loop that controls the linear position of the rocket. The inner, angular position loop is normally contained in the rocket itself, but, as seen from the previous sections, the operation of this loop is simulated in the vehicle. The dynamics of these two loops will interact, but, to simplify the design process, the interactions are neglected and the two loops are analyzed separately. First a natural frequency is picked for the outer loop to satisfy the required solution time. A natural frequency is then picked for the inner loop to minimize the effects of interactions. The natural frequency and damping ratio are used to determine the parameters for the inner loop.

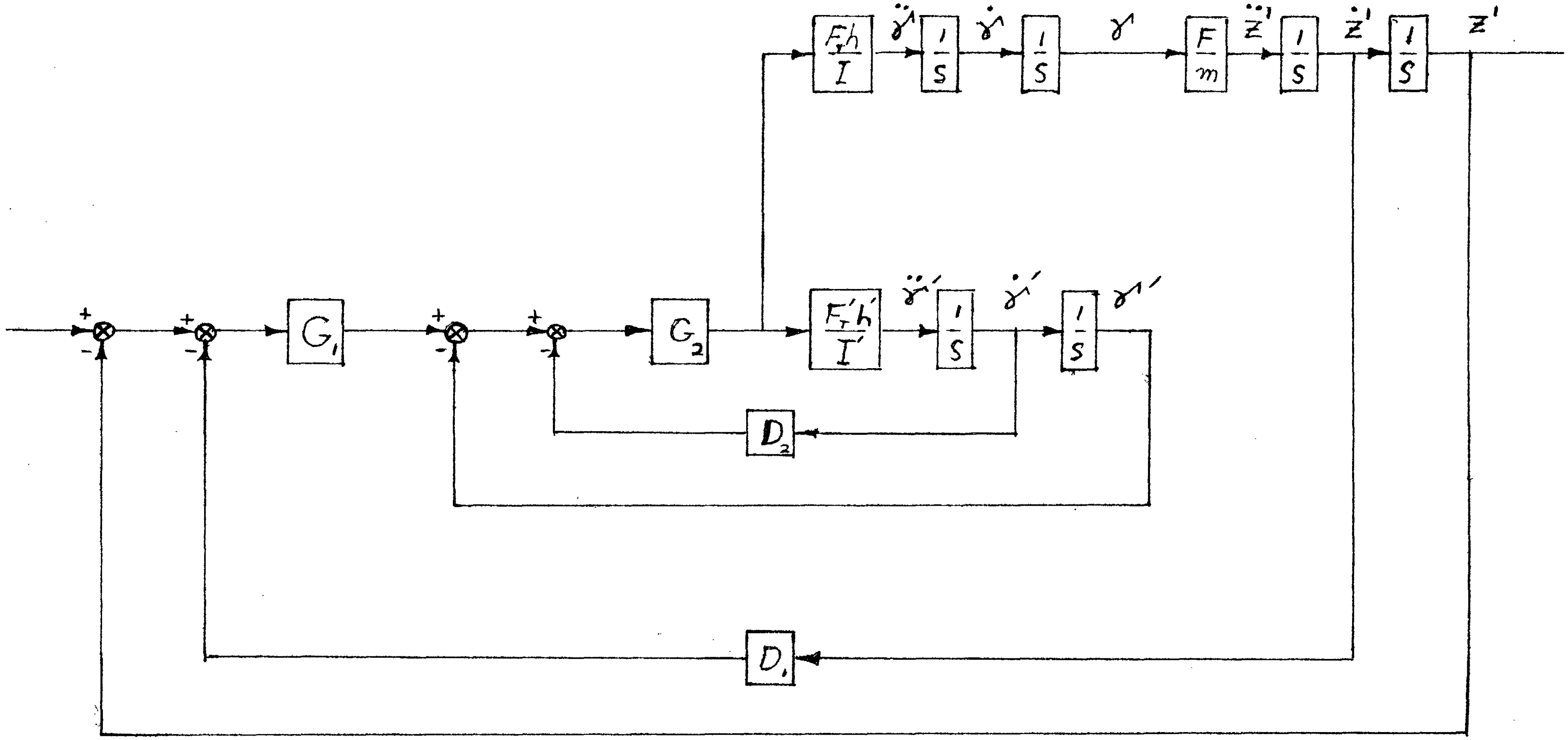

Fig. 4.3

The Control System

The parameters are then determined for the outer loop using the natural frequency for the outer loop, the damping ratio, and the parameters previously determined for the inner loop. The guidance system is then analyzed as a whole, using root locus methods, to determine the effects of the interactions.

The solution time of the system is dominated by the outer loop. If it is required that the final value be approximately 5% of the initial value, the envelope of the response should be:

$$e^{-\sigma t} = .05 \qquad (4.8)$$

Thus:

$$e^{\sigma t} = 20 \qquad (4.9)$$

$$\sigma t = 3.0$$

with t = 10 sec.

$$\sigma = .30 \qquad (4.10)$$

The damping ratio (ζ) is chosen to be .707.

Then:

$$\sigma = \zeta \omega_n \qquad (4.11)$$

$$\omega_n = \frac{\sigma}{\zeta} = \frac{.300}{.707} = .424 \text{ rad/sec}$$

Thus the natural frequency of the outer loop should be .424 rad/sec. To avoid interaction between the two loops, the natural frequency of the inner loop should be 10 times that of the outer loop or 4.24 rad/sec.

To arrive at an estimate for the values of the parameters in the inner loop, the inner loop is simplified by ignoring the effects of the simulation as shown in Fig. 4.4.

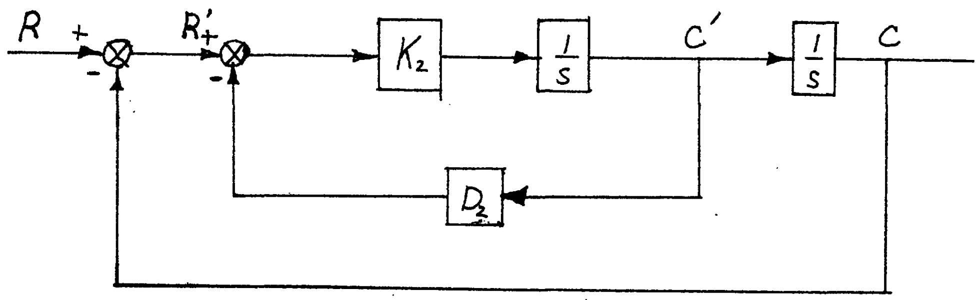

Fig. 4.4

The Simplified Loop

From this figure:

$$\frac{C'}{R'} = \frac{\frac{K_2}{S}}{1 + \frac{D_2 K_2}{S}} = \frac{K_2}{S + D_2 K_2} \qquad (4.12)$$

and:

$$\frac{C}{R} = \frac{\frac{1}{S}\,\frac{K_2}{S + D_2 K_2}}{1 + \frac{1}{S}\,\frac{K_2}{S + D_2 K_2}} \qquad (4.13)$$

$$= \frac{K_2}{S^2 + K_2 D_2 S + K_2}$$

In terms of natural frequency and damping ratio, the transfer equation for a second-order system is:

$$\frac{C}{R} = \frac{G\omega_n^2}{S^2 + 2\zeta\omega_n S + \omega_n^2} \qquad (4.14)$$

Thus the parameters for the inner loop are:

$$G = 1$$

$$K_2 = \omega_n^2 = (4.24)^2 = 17.9 \text{ rad/sec}^2 \text{ per rad} \quad (4.15)$$

$$D_2 = \frac{2\zeta\omega_n}{K_2} = \frac{(2)(.707)(4.24)}{17.9}$$

$$= .332 \text{ rad/sec}^2 \text{ per rad/sec}$$

The simplified outer loop is shown in Fig. 4.5.

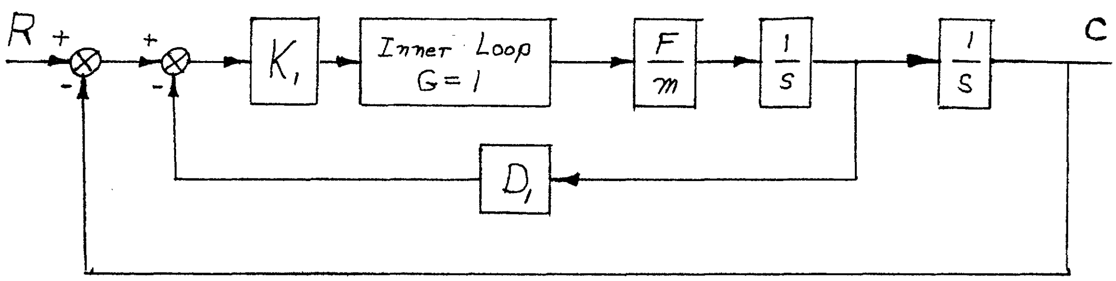

Fig. 4.5

Simplified Outer Loop

In this analysis the assumption is made that the inner loop has a gain of one and that the effect of its dynamics on the dynamics of the outer loop is negligible. Thus the transfer function of the outer loop is approximately:

$$\frac{C}{R} = \frac{K_1 \frac{F}{m}}{s^2 + K_1 \frac{F}{m} D_1 s + K_1 \frac{F}{m}} \quad (4.16)$$

From equation (4.14) the parameters for the outer loop are:

$$G = 1$$

$$K_1 \frac{E}{m} = \omega_n^2 = (.424)^2 = .180 \text{ ft/sec}^2 \text{ per ft} \quad (4.17)$$

$$D_1 = \frac{2\zeta\omega_n}{K_1 \frac{E}{m}} = \frac{(2)(.707)(.424)}{(.180)}$$

$$= 3.22 \text{ ft/sec}^2 \text{ per ft/sec}$$

The average value for the transfer between acceleration and angular position ($\frac{E}{m}$) from Fig. 3.4 is approximately 482 ft/sec^2 per rad. Then K_1 is .374 x 10^{-3} rad. per ft.

The actual inner loop, including the effects of simulation as shown in Fig. 4.3, can be rearranged as shown in Fig. 4.6.

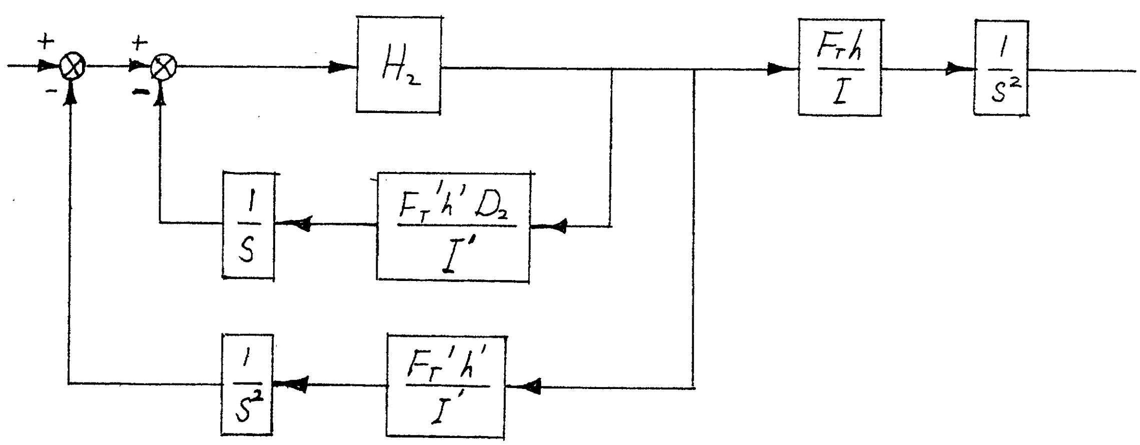

Fig. 4.6

The Actual Inner Loop

In this arrangement:

$$K_2 = H_2 \frac{F_T h}{I} \quad (4.18)$$

The term $\frac{F_T h}{I}$ is a function of time and is shown in Fig. 3.3. The parameter H_2 is chosen so that K_2 will have the design value (equations (4.15)) when $\frac{F_T h}{I}$ is average. Thus:

$$H_2 = \frac{K_2}{\frac{F_T h}{I}_{avg}} = \frac{17.9 \text{ rad/sec}^2 \text{ per rad}}{.550 \text{ rad/sec}^2 \text{ per lb}} \tag{4.19}$$

$$= 32.6 \text{ lb per rad}$$

The term $\frac{F_T' h'}{I'}$ is the simulation of $\frac{F_T h}{I}$. It is found that a linear approximation to $\frac{F_T h}{I}$ is sufficient for the desired accuracies. From Fig. 4.6 the transfer function is seen to be:

$$\frac{C}{R} = \frac{H_2 s^2}{s^2 + \frac{H_2 F_T' h' D_2}{I'} s + \frac{H_2 F_T' h'}{I'}} \frac{\frac{F_T h}{I}}{s^2} \tag{4.20}$$

The natural frequency and damping ratio of the inner loop are the same as those chosen for the simplified inner loop. See equations (4.14). The gain of the loop is:

$$G = \frac{\frac{F_T h}{I}}{\frac{F_T' h'}{I'}} \tag{4.21}$$

The gain is very nearly one. The only variation in the gain from one is caused by the inaccuracy of the simulation. These errors are considered in section 4.5 of this chapter. In the development to follow, the gain is assumed to be one.

A root locus is made for this system to determine more exactly its dynamic characteristics. A root locus, shown in Fig. 4.8, is first drawn for the damping portion of the outer loop, shown in Fig. 4.7.

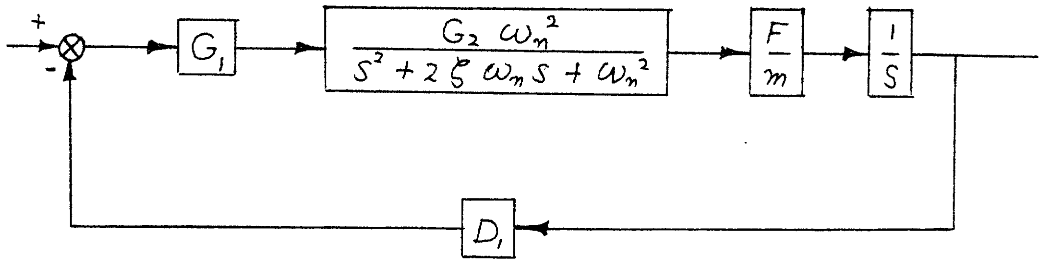

Fig. 4.7

Damping Portion of the Outer Loop

The closed loop poles of this damping loop are then the open loop poles of the whole system. (See Fig. 4.9). The final root locus, i.e., that for the entire system, is shown in Fig. 4.10.

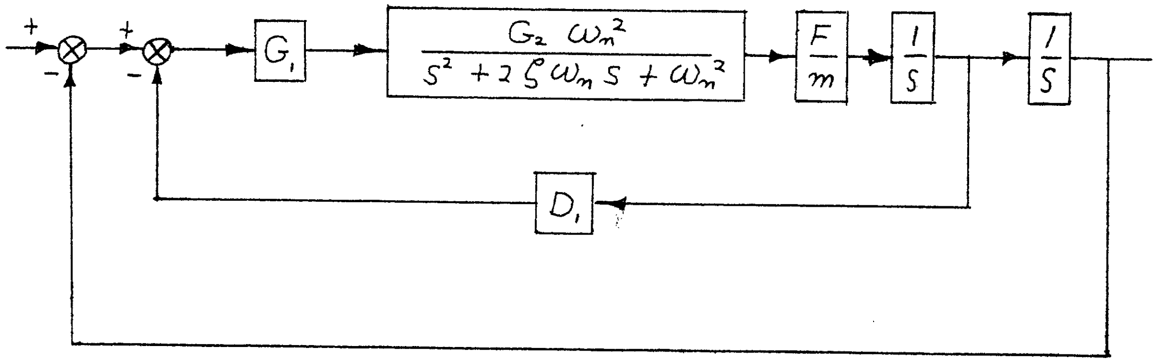

Fig. 4.9

The Entire System

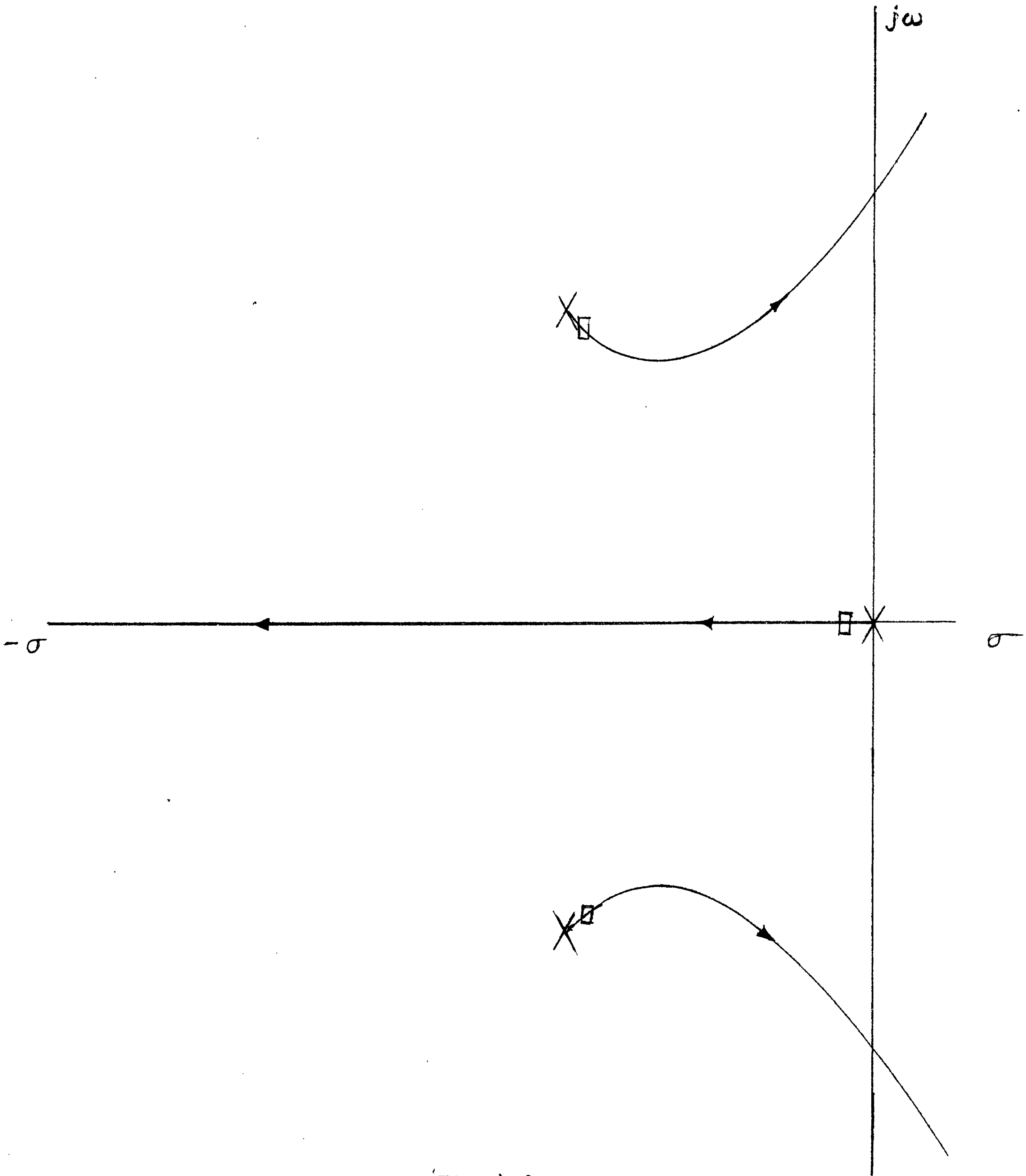

Fig. 4.8

Root Locus of Damping Portion of Outer Loop

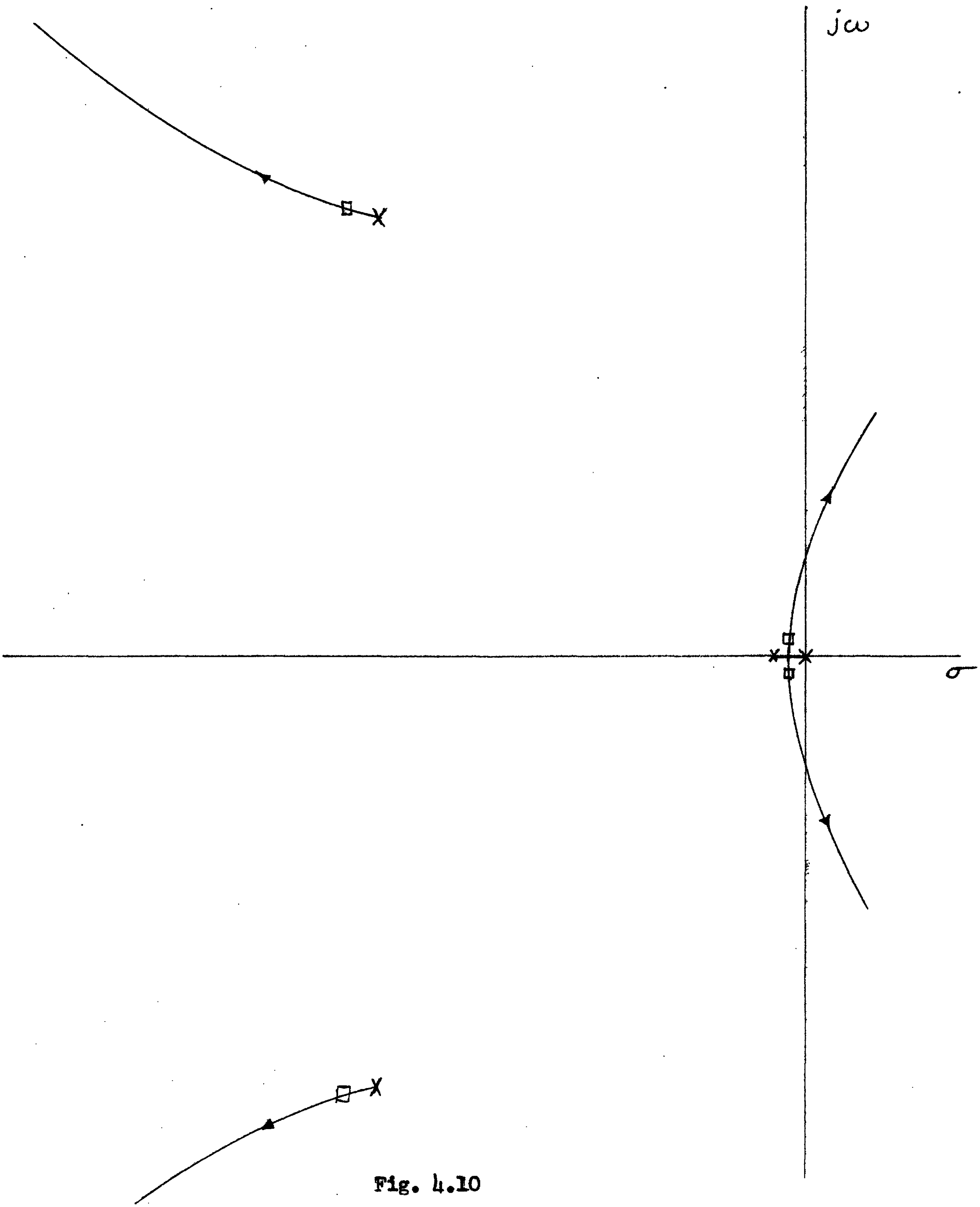

Fig. 4.10

Root Locus of Entire System

From the root locus it is seen that the interactions of the two loops have little effect on the characteristics. The parameters derived using the simplified systems are thus retained as the final values.

4.4 Simulation of the Guidance System

To determine its actual dynamic characteristics, the system was simulated on a PACE analog computer. Fig. 4.11 shows the computer diagram which is a very close representation of the actual system shown in Fig. 4.3. The time varying numbers $\frac{F_I h}{I}$ and $\frac{F}{m}$ are approximated by linear functions. The control system begins one second after launch.

The launch equipment will be able to launch the missile with an angular accuracy of at least 5.7° (.1 radian). Thus, this angular accuracy is taken as an upper limit for the initial error in the launch direction. The initial conditions for the guidance can be found by inserting the initial error angle and range in equations (4.4) and (4.5). The initial values are then:

$$Z_o' = r_o \theta_o' = (160)(.1) = 16 \text{ ft} \tag{4.22}$$

$$\dot{Z}_o' = \dot{r}_o \theta_o' + r_o \dot{\theta}_o' = (320)(.1) + (160)(0) = 32 \text{ ft/sec} \tag{4.23}$$

and:

$$\gamma_o = \theta_o = .1 \text{ rad} \tag{4.24}$$

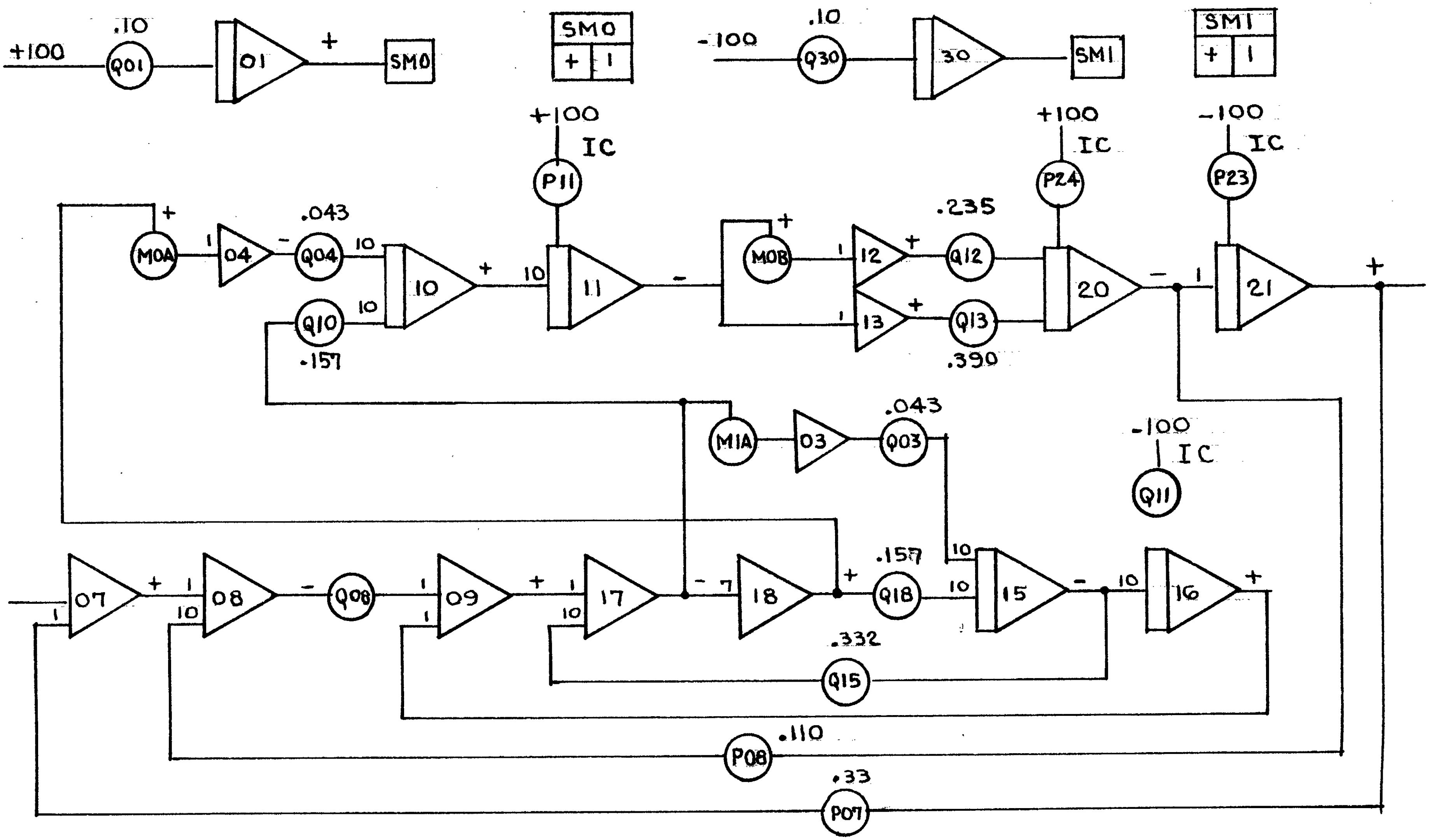

FIGURE 4.11 PACE ANALOG COMPUTER DIAGRAM.

Fig. 4.12 shows the results of the simulation with these initial conditions. The final value of z^1 was found to be less than 5 feet and the final value of $\dot{z}^1$ was less than 2 feet per second. With a maximum flight time of 40 seconds, the final position error caused by guidance uncertainty is less than 85 feet.

4.5 Error Considerations

The effect of errors in the important parameters of the system were also determined by simulation on the analog computer. The error in burnout position and velocity caused by a plus or minus 10% error in burning time, gain of the control system, moment of inertia, and acceleration were simulated. The error was determined for a 10% difference between the actual initial angular position and the initial position used by the control system to simulate the motion of the rocket. The angular velocity data from the radar will probably be the most inaccurate information used by the control system. The effects are shown of a plus or minus 20% error in the magnitude of the velocity feedback. The results are shown in Table 4.1.

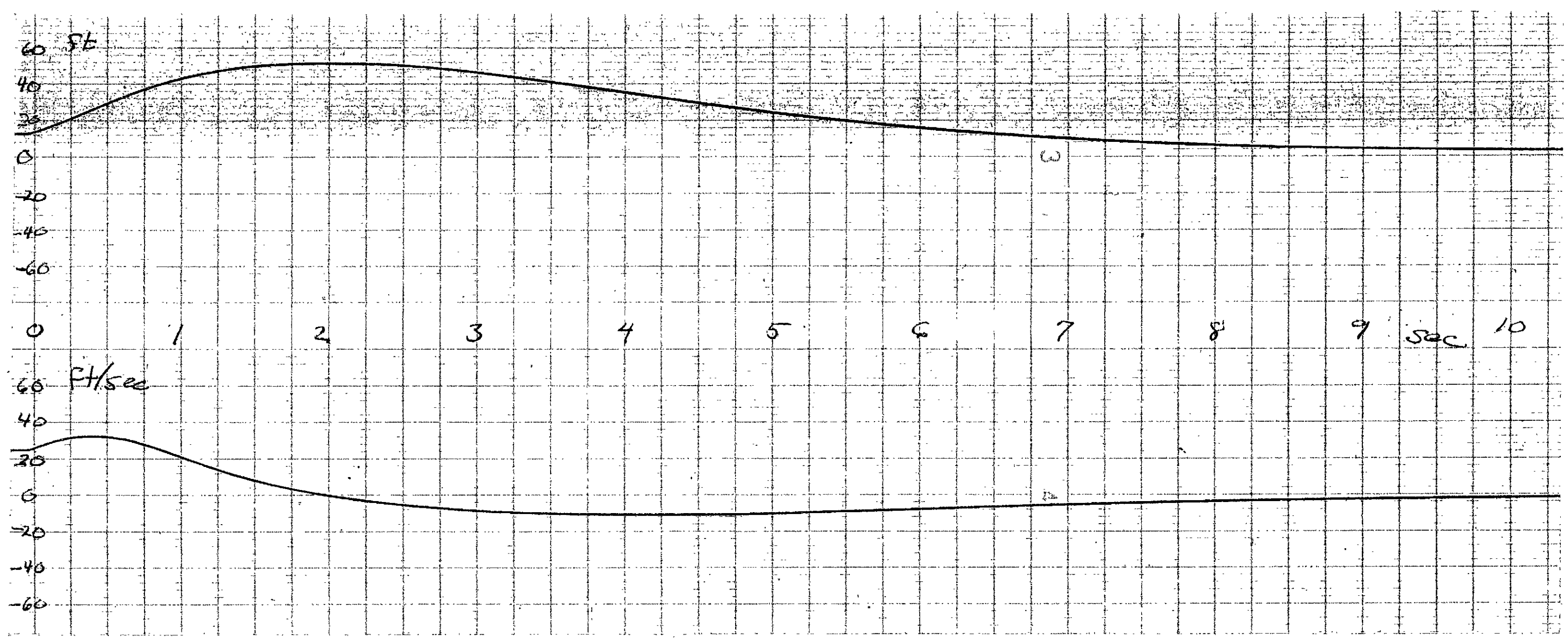

Fig. 4.12

z^1 and $\dot{z}^1$

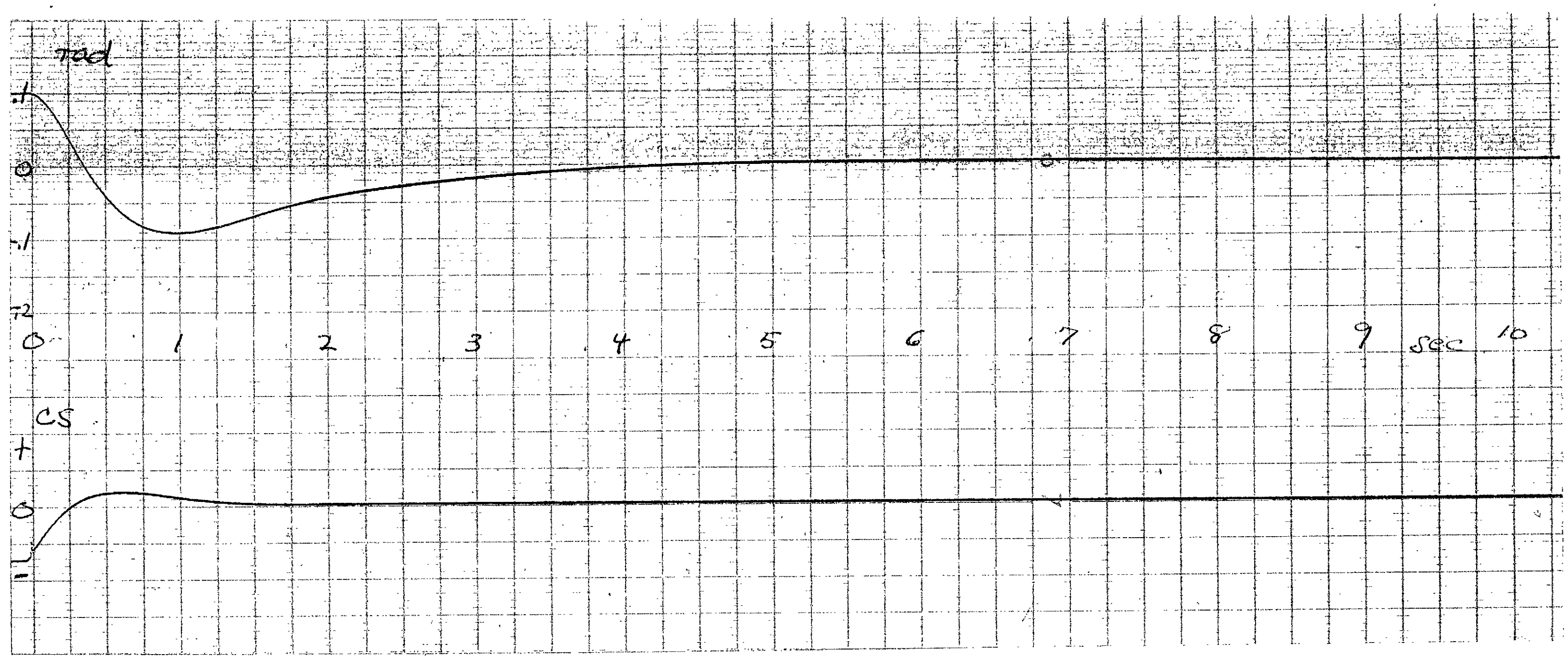

Fig. 4.12 (cont.)

γ and the Command Signal to the Control Vanes

Table 4.1

Effect of Parameter Errors

Parameter	Per cent Error	Error at Burnout	
		Position (feet)	Velocity (ft/sec)
Burning Time	+10% -10%	+15 -12	~ 0 ~ 0
Gain	+10% -10%	+8 -14	~ 0 +.3
Moment of Inertia	+10% -10%	-18 +30	+1.0 -1.0
Acceleration	+10% -10%	-6 +4	+0.3 -0.6
Initial Condition	+10% -10%	+18 -12	+2.0 -2.0
Velocity Feedback	+20% -20%	+4 -10	-2.0 +2.0

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