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Modeling the Diffusion of Financial Innovations

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Abstract

Once a new financial product is introduced into the market, new producers are likely to enter as more users adopt the product. Because of the relatively simple process of user adoption in financial services, it is likely that strong system interdependencies will take place, and that new producers will enter the market influenced not only by the rate of producer adoption but also by the rate of user adoption. A model is developed to take into account such interdependencies. The model is helpful in determining user diffusion curves based on producer diffusion data, and it has good descriptive and forecasting capabilities when applied to two financial products.

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The purpose of this chapter is to study patterns of diffusion of financial innovations. In particular, we are concerned with developing a model to explain and predict the adoption of a financial innovation by producers and users within one country. Such a model could be a useful tool for financial product developers to determine sales volumes that could be expected over time, and to estimate the number of competitors likely to enter markets for new products. In order to assess such diffusion patterns, we utilize and extend diffusion models and the diffusion modeling methodology proposed by Mansfield (1961), Bass (1969), especially Mahajan and Schoeman (1977), and others.

Although diffusion models have been widely used for industrial and consumer products, they have been seldom used for financial services. Molyneux and Shamroukh (1996: 505) point out that the strategic aspects of innovation adoption by banks need to be included into our general understanding of financial innovation because "...the decision to adopt a new financial product by one bank and the timing of adoption are likely to have an impact on other banks' adoption decisions."

Moreover, these models are largely based on the existence within populations of potential adopters of "innovators" and "imitators" which, in the first case, acquire products influenced by external sources of information and, in the second instance, adopt products influenced by previous adopters. Thus, fundamentally, the dynamics of most diffusion models are driven by the intrinsic characteristics of individuals within given populations of potential adopters. The effects of user adoption dynamics upon producer adoption dynamics are rarely considered. Clark and Staunton (1989:125) indicate that such linkage must be established more clearly in diffusion modeling because one should expect that profit expectations, as seen through the lens of demand growth or decay, are the key motives for producers to adopt a given practice:

Particular attention needs to be given to examining the variability of the supply side and to the consequences for the diffusion process. For example, the case study of the fast-food sector indicated that McDonald's undertook much of the development of the innovations in technology and organization within the firm, and that example poses the issue of the relations between users and suppliers. Economists have emphasized the significance to the supply side of future expectations of profits and of the role of profits in creating "corridors" down which innovations are likely to flow (Metcalfe, 1981). Moreover, the supply side takes widely-varying forms. One particular form which is briefly noted is the tendency for the supplier to benefit from the learning experiences of the users.

Freeman (1995: 481), in a recent and comprehensive review of the literature on technological innovation, addresses the same issue and criticizes diffusion theory for overlooking user-producer interactions:

> One of the major problems in diffusion research is to take into account the supply side as well as the demand side. As Gibbons and Metcalfe (1986) particularly show, the interaction between supply and demand results in the evolution both of new and improved products and of new design configurations. Although empirical research has amply demonstrated the role of suppliers in improving the product, diversifying new models, enlarging the market, promoting applications research, training potential users, and coping with institutional barriers, much diffusion research (despite the good advice of Metcalfe, 1988 and other researchers) continues to neglect the supply side and treat diffusion as a demand phenomenon.

Many diffusion theorists have recognized this weakness also. Mahajan et al. (1990) stress the need to incorporate explicitly product growth rates into models of diffusion among producers. In a recent review of diffusion modeling in marketing these authors make reference to the Bass' (1969) model (which is one of the most frequently cited diffusion models in the marketing literature) and posit questions as suggestions for further research (Mahajan et al., 1990: 21):

> How do the number of competitors and the rivalry among them influence the growth of a product category? Does the growth affect the entry/exit patterns of

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competitors? Answers to these questions are within the domain of the diffusion modeling framework and provide a linkage with the strategic planning literature. Theoretical and empirical work on these questions will enhance the utility of diffusion models.

In financial services it is extremely important to establish such user-producer interrelationships because products are very easy to imitate. Thus, transient monopolies are not allowed to innovators by virtue of product complexity or legal protection, but by pioneering advantages. It is possible that a bandwagon of producers, uninhibited by cost or complexity considerations (as would if imitating industrial products), will enter the market when demand starts rising. Various factors affect rates of adoption. First, many innovations require changes in laws and regulation to take place. These changes in regulation relax constraints and allow firms to enter new markets. Second, technical complexity and firm level capability required to bring about many innovations are often small. Thus, imitation by producers is easy. Third, the decision to adopt by a user often imposes little or no burden upon the customer, and, as a result, attributes associated with many financial innovations suggest rapid diffusion rates and an almost unimpaired reciprocal transfer of information between producers and users[1]. These factors result in small barriers to entry and imply that diffusion curves would exhibit high imitation rates both among users and among producers, and that there should be system interdependencies much more readily observable. Therefore, it doesn't seem realistic just to look either at suppliers or users in financial innovation. It is, apparently, more sensible to attempt developing models that relate users and producers directly.

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[1] The five attributes of innovations that regulate diffusion rates are (Rogers, 1971): 1) The relative advantage of the product with respect to extant practice, 2) Compatibility, 3) Complexity, 4) Trialability, and 5) Observability. In many financial innovations many of these attributes tend to favor a rapid diffusion rate. In addition, many decisions to adopt are individual, influenced through mass media, and promoted, directly or indirectly, by regulatory changes.

To shed some light into these issues, in what follows we first review pertinent literature and develop a mathematical model. The model is tested in a subsequent section where we find that it has good descriptive capabilities and also provides reasonable parameters to estimate a user diffusion curve. At the end we summarize and conclude.

Literature review and model development.

Diffusion models are generally based upon the premise that an innovation is a piece of information that spreads within social groups. Such phenomenon can be represented using mathematical functions (like, for instance, the logistic growth curve (Griliches, 1957)). Mansfield (1961) proposed a method to explain such diffusion patterns in terms of a deterministic model where the equation for the cumulative proportion of total adopters has the form:

$$(*) \quad F(t) = \frac{\exp[bt + c_0]}{b + \exp[bt + c_0]}$$

This model presumes that diffusion will be primarily an imitation phenomenon, and that the rate of imitation is governed by only one parameter (parameter b in equation *). This model has shown good descriptive capabilities when tested with data from industrial and administrative innovations (Mansfield, 1961; Teece, 1980).

Later, Bass (1967) introduced a model which explains diffusion not only in terms of the rate of imitation among adopters but also innovation. That is, an innovation will be initially adopted by entitites (within a given social group) which are influenced by external signals and , subsequently, by adopters who imitate other adopters. Bass' model subsumes Mansfield's one. It contains two parameters in which one, a in equation **, governs the

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rate of "innovation", and the other, b in the same equation, governs the rate of imitation (in a manner analogous to Mansfield's model).

$$(**) \qquad F(t) = \frac{\overline{F}\left(1 - \exp\left[-(a+k)t\right]\right)}{1 + \frac{k}{a}\exp\left[-(a+k)t\right]} \qquad\qquad where\ b = \frac{k}{\overline{F}}$$

k is a constant.

Mahajan and Schoeman (1977) have suggested a fundamental diffusion model to represent time patterns of diffusion processes. In their model the rate of diffusion is directly proportional to the proportion of potential adopters available at that time. Or (Mahajan and Schoeman, 1977:13), "...in other words, as the cumulative proportion of adopters approaches its ceiling, say $\overline{F}$, the rate of diffusion decreases proportionately.". The authors express such situation as follows.

$$(1) \qquad f(t) = \frac{dF(t)}{dt} \propto \left(\overline{F} - F(t)\right)$$

Where:

$F(t)$ = cumulative proportion of adopters at time t,

$\overline{F}$ = ceiling on the cumulative proportion of adopters,

$f(t)$ = proportion of adopters at time t.[2]

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[2]Notice that $F(t)$ is the cumulative sum of entrants at time t, i.e.,

$$F(t) = \Sigma f(t)$$

hence,

$$f(t) = \frac{dF(t)}{dt}$$

in other words, the rate of change of the cumulative proportion of adopters with respect to time is proportional to the remaining potential adopters. Thus, the constant of proportionality provides us with an equality. Such "constant" is a function of time:

$$(2) \qquad \frac{dF(t)}{dt} = g(t)\left(\overline{F} - F(t)\right)$$

Different authors provide different specifications for this diffusion coefficient *g(t)*. As Mahajan and Peterson (1978) indicate, some specify it as a function of time (Dobb, 1956) but most authors represent *g(t)* as a function of the number of previous adopters (e.g. Fourt et al., 1960; Mahajan and Haynes, 1977). This means that *g(t)* is a function (usually linear for parsimony) of *F(t)*:

$$(3) \qquad g(t) = a + b \cdot F(t)$$

where *a* and *b* are positive constants. From here it is possible to obtain a basic formula to represent diffusion patterns as generally portrayed in many diffusion studies. This formula is given by Mahajan and Schoeman (1977) as:

$$(4) \qquad \frac{dF(t)}{dt} = \left(a + bF(t)\right) \cdot \left(\overline{F} - F(t)\right)$$

From this expression we get an equation for the cumulative proportion of adopters at time *t* (Mahajan and Schoeman, 1977):

$$(5) \qquad F(t) = \frac{\overline{F} - \dfrac{a\left(\overline{F} - F_0\right)}{\left(a + bF_0\right)} \exp\left[-(a + b\overline{F})(t - t_0)\right]}{1 + \dfrac{b\left(\overline{F} - F_0\right)}{\left(a + bF_0\right)} \exp\left[-(a + b\overline{F})(t - t_0)\right]}$$

Using different sets of assumptions it is now possible to derive a variety of models commonly used in the diffusion literature like Mansfield's imitation model and Bass' model (Mahajan and Schoeman, 1977). For example, Mansfield's model is derived by setting $a = 0$, that is basing diffusion solely on imitation[3] to obtain:

$$(6) \qquad F(t) = \frac{\exp\left[bt + c_0\right]}{b + \exp\left[bt + c_0\right]}$$

which is the equation used by Mansfield (1961).

Proposed model

The above models are based on the premise that the "constant" of proportionality is a function of the number of previous adopters (producers or users, but not both). For instance, banks that adopt a particular practice, say credit cards, would do so either as innovators (pioneers) or imitating other banks which have entered the market, but not responding to changes in the number of credit card users. In contrast, in order to start exploring the interrelationships of producer adoption with user adoption, we will assume here that $g(t)$ is a function of the number of users, and, thus, the rate of adoption among

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[3]Mansfield (1961) explored the rate of imitation in technical change.

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producers will be influenced not only by bank imitation and innovation but by the rate to which the product is adopted downstream by its users. Thus, if we let $H(t)$ be the cumulative proportion of users adopting the product at time t, our basic equation becomes:

$$(7) \qquad \frac{dF(t)}{dt} = (a + bH(t)) \cdot (\overline{F} - F(t))$$

As we mentioned before, Mahajan and Schoeman have derived a generalized form for $H(t)$. Hence, $H(t)$ is (Mahajan and Schoeman, 1977):

$$(8) \qquad H(t) = \frac{\overline{H} - \dfrac{c(\overline{H} - H_0)}{(c + dH_0)} \exp\left[-(c + d\overline{H})(t - t_0)\right]}{1 + \dfrac{d(\overline{H} - H_0)}{(c + dH_0)} \exp\left[-(c + d\overline{H})(t - t_0)\right]}$$

Where

$H_0 = H(t = t_0)$

c = index of uninfluenced user adoption

d = index of influenced user adoption.

Using this generalized form for $H(t)$ we will assume that the number of producers who enter the market is influenced by the number of users who adopt the product over

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time. The number of adoptions by users, however, is not influenced by the number of producers[4], but by the rate of user innovation and imitation.

Thus we obtain:

$$(9) \quad \frac{dF(t)}{dt} = \left\{ a + b \left[\frac{\overline{H} - \dfrac{c(\overline{H} - H_0)}{(c + dH_0)} \exp\left[-\left(c + d\overline{H}\right)\left(t - t_0\right)\right]}{1 + \dfrac{d(\overline{H} - H_0)}{(c + dH_0)} \exp\left[-\left(c + d\overline{H}\right)\left(t - t_0\right)\right]} \right] \right\} \cdot \left(\overline{F} - F(t)\right)$$

From here we use elementary integration to derive an expression for *F(t)* (See Appendix 1 for details):

$$(10) \quad F(t) = \overline{F} - 1 \, / \, \exp \left\{ \begin{array}{l} \left[a\left(t - t_0\right) + \dfrac{b\overline{H}}{\left(c + dH\right)} \ln\left\{ \exp\left[\left(c + dH\right)\left(t - t_0\right)\right] + \dfrac{d\left(\overline{H} - H_0\right)}{\left(c + dH_0\right)} \right\} + \right. \\[2em] \dfrac{bc}{d\left(c + d\overline{H}\right)} \ln\left\{ 1 + \dfrac{d\left(\overline{H} - H_0\right)}{\left(c + dH_0\right)} \exp\left[-\left(c + d\overline{H}\right)\left(t - t_0\right)\right] \right\} - \\[2em] \left. \dfrac{b}{d} \ln\left\{ 1 + \dfrac{d\left(\overline{H} - H_0\right)}{\left(c + dH_0\right)} \right\} - \ln\left(\overline{F} - F_0\right) \right] \end{array} \right\}$$

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[4]Jain et al. (1991) developed a generalized model with supply restrictions. Their model, however, introduces supply restrictions by splitting the user diffusion curve into two stages. Thus, in the first stage products "diffuse" into a pool of waiting applicants and, in a second stage, become adopters. With this time delay forced by the inventory of waiting applicants, the authors are able to depict user diffusion curves under supply restrictions quite accurately.

This expression can be simplified. In doing so, however, we also want to reexpress it in terms of absolute cumulative number of adopters and not in terms of proportions of adopters (as it currently stands). Thus, if we define for the producers:

$P(t)$ = Cumulative number of adopters as a function of time

M = Total potential number of adopters and,

$\overline{P}$ = Ceiling on the number of producers adopting the product or process.

$P_0 = P(t = t_0)$

Then,

$$F(t) = \frac{P(t)}{M}, \quad \overline{F} = \frac{\overline{P}}{M}, \quad F_0 = \frac{P_0}{M}$$

In words, the cumulative proportion of adopters at time t is going to be equal to the total number of adopters at time t divided by the total potential number of adopters. The ceiling on the proportion of adopters (i.e., the expected proportion of all potential adopters to adopt) will be equal to the ceiling on the total number of adopters divided by the total potential number of adopters.

Similarly, for the users of the product (produced by the producers), we define

$U(t)$ = Cumulative number of users adopting the product at time t

N = Total number of potential users

$\overline{H}$ = Ceiling on the number of users.

$H_0 = H(t = t_0)$

Then,

$$H(t) = \frac{U(t)}{N}, \quad \overline{H} = \frac{\overline{U}}{N}, \quad F_0 = \frac{U_0}{N}$$

Substituting these expressions into (10) we get:

(11)
$$\frac{P(t)}{M} = \frac{\overline{P}}{M} - 1 \Big/ \exp\left\{ \begin{array}{l} \left[a\left(t - t_0\right) + \frac{b\overline{U}}{N\left(c + d\frac{\overline{U}}{N}\right)} \ln\left\{ \exp\left[\left(c + d\frac{\overline{U}}{N}\right)\left(t - t_0\right)\right] + \frac{d\left(\frac{\overline{U}}{N} - \frac{U_0}{N}\right)}{\left(c + d\frac{U_0}{N}\right)} \right\} \right] + \\[4ex] \frac{bcN}{d\left(cN + d\overline{U}\right)} \ln\left\{ 1 + \frac{d\left(\overline{U} - U_0\right)}{\left(cN + dU_0\right)} \exp\left[-\left(\frac{cN + d\overline{U}}{N}\right)\left(t - t_0\right)\right] \right\} - \\[4ex] \frac{b}{d} \ln\left\{ 1 + \frac{d\left(\overline{U} - U_0\right)}{\left(cN + dU_0\right)} \right\} - \ln\left(\frac{\overline{P}}{M} - \frac{P_0}{M}\right) \end{array} \right\}$$

This is the general expression for P(t). If we assume that the total potential number of users of the product equals the ceiling on the number of users (i.e., that the ceiling on the number of adopters is equal to the total number of potential adopters), and that there are no adoptions by producers or consumers at time equal to zero, then
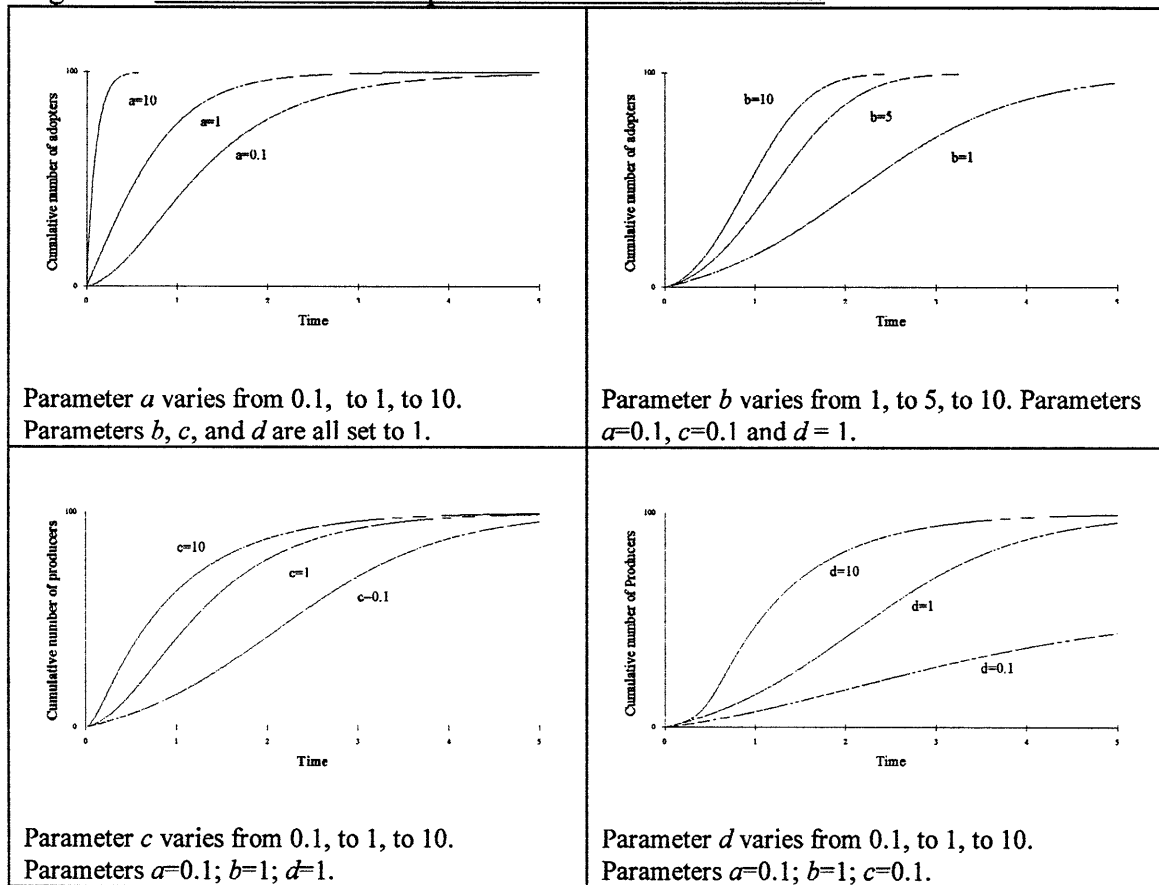
$$U_0 = P_0 = 0$$

$$\overline{U} = N$$

and letting, for simplicity, $T = (t - t_0)$, we obtain (details in Appendix 2):

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$$(12) \qquad P(t) = \bar{P} \cdot \left\{ 1 - \frac{(c+d)^{\left(\frac{b}{d}\right)} \exp\left[\frac{bc - ad}{d} \cdot T\right]}{\left[d + c \cdot \exp(c+d)T\right]^{\left(\frac{b}{d}\right)}} \right\}$$

In this expression, a can be interpreted as an index of uninfluenced adoption by producers (i.e., adoption by innovators), b can be interpreted as an index of influenced adoption by producers (i.e., producers who adopt imitating other producers); c is an index of uninfluenced adoption by users and d is an index of influenced adoption by users. Hence, we can see in the expression that the total number of producers adopting at time t is going to be a fraction of the total number of potential adopters. As time passes this fraction increases influenced by a combination of a, b, c, and d. Moreover, we could expect parameters c and d to behave well when used in a model to describe the time pattern of diffusion of the innovation among the users via the fundamental model (or its variations) of equation (8). Notice that, with the above assumptions, the model has four parameters and one term, the expected ceiling on the number of adopters, which has to be provided by the analyst or which, alternatively, could also be treated as a parameter in a regression analysis. Notice that the link with the user diffusion curve is given via two parameters (c and d). Thus, the model can be used with externally estimated parameters or it could be used to estimate such parameters for characterization of a plausible demand curve. Figure 1 illustrates the effects of variations in each parameter on the diffusion curve when other parameters are held constant. In Appendix 3 we characterize the equation and compare it to the standard model of Equation 5.

Figure 1. Incremental effect of parameters on diffusion curves.

Parameter *a* varies from 0.1, to 1, to 10. Parameters *b*, *c*, and *d* are all set to 1.	Parameter *b* varies from 1, to 5, to 10. Parameters *a*=0.1, *c*=0.1 and *d* = 1.
Parameter *c* varies from 0.1, to 1, to 10. Parameters *a*=0.1; *b*=1; *d*=1.	Parameter *d* varies from 0.1, to 1, to 10. Parameters *a*=0.1; *b*=1; *c*=0.1.

A pilot study

Mutual Funds

We used this model to study the diffusion of mutual funds in Costa Rica. Data for number of producers and number of users were gathered from the country's regulatory commission. All funds must first obtain permission to operate from this commission and have to report fairly detailed data about their operation on a regular basis. Given the data

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available, we measured producer diffusion using the number of funds in the market and user diffusion using number of investors. Our data did not permit us to assess repeated purchases (i.e., same customer holding more than one account) hence we assumed all investors were different.

Figure 2 displays the total number of shareholder accounts over time. The data show an initial small peak before the curve starts shooting upward. This corresponds to a producer (the pioneer) which entered the market and then progressively abandoned it. The number of customers in the market again increases after a a second producer enters the market. Figure 3 displays total number of producers.
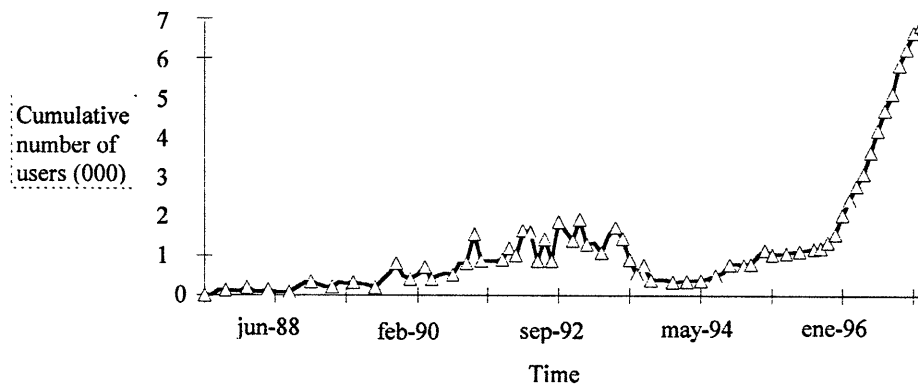

Figure 2. <u>Total number of shareholder accounts for mutual funds in Costa Rica.</u>

We provided an estimate on the ceiling of the number of producers. To do so, we asked 6 industry experts to provide an estimate of number of funds in the long run. All provided numbers that ranged between one hundred and two hundred different funds with accounts per fund that ranged from 250 to 1000. The rationale was in most cases that practically all banks would eventually have to enter this market and, in fact, many had already been registered but were not operating at the time of this study.

30

25

20

Total number of
producers   15

10

5

0

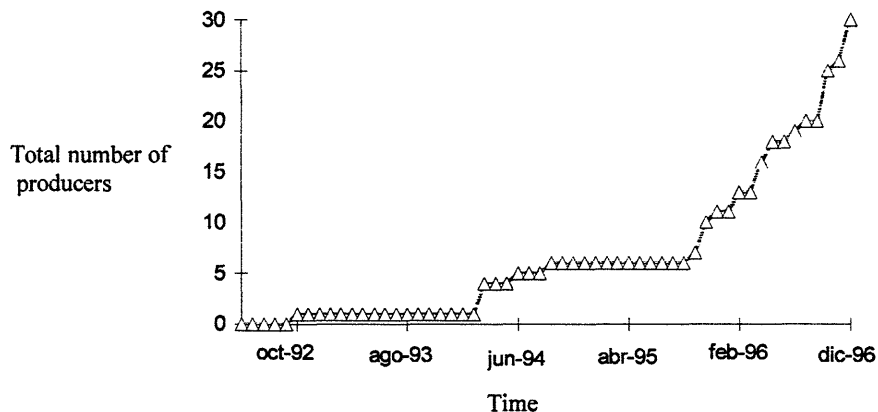oct-92   ago-93   jun-94   abr-95   feb-96   dic-96

Time

Figure 3. Total number of funds in Costa Rica

We also used comparative U. S. data to estimate a likely number of funds. In the
U. S. the number of shareholder accounts per fund has averaged 21,157 (standard
deviation 1,.084)[5]. Using these data and normalizing to total population we estimated a
peak of 163 funds in the country and, given the exploratory characteristics of this pilot
study, used this number to estimate parameters[6]. Notice, however, that the average
accounts per fund in Costa Rica at the time of the study was only 235[7].

The parameters of the model for the mutual fund data were estimated using a non-
linear least squares procedure with a Levenberg-Marquardt algorithm. We rescaled the
time axis to avoid overflowing (due to too large exponents). The parameter estimates a,
c, and d are quite well estimated with relatively small standard errors. Parameter b shows
a relatively high standard error. Considering the small number of data points and also the
particularities of the data set (particularly the producer who leaves the market early after

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[5]In 1995 there were 5,761 funds operating in the U. S. The average is computed using available data from
years 1980, 1985, 1989-95.
[6]Parameters appear to be fairly stable to variations in the ceiling of total Producers.
[7]Using the ceiling as a parameter in a non-linear regression we did not attain convergence after 1000
iterations, however the ceiling (with a very large standard error) appears to stabilize around 148.

entry, creating a somewhat bimodal curve), the result seems to be plausible although not entirely satisfactory. The estimates (standard error) are: a =0.01898 (0.00226); b = 6.6098 (9.5), c = 0.00003187 (0.0000156); d = 11.748 (1.66); and the R-squared statistic is 0.976. The parameter estimates are positive (as expected). Imitation effects, as represented by parameters *b* and *d* seem to exert the greatest influence upon this fit. Figure 4 shows that with these parameters the model appears to closely fit the data. This close correspondence indicates that the model is successful in, at least, providing a good descriptive functional form of innovation diffusion among producers.
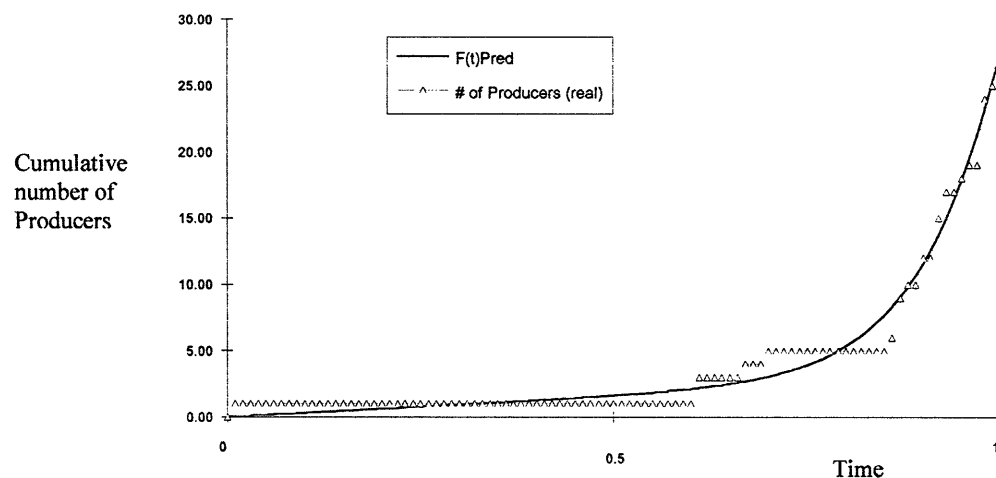

Figure 4. <u>Actual number of producers and predicted number of producers for data on mutual funds.</u>

Comparison to simpler model

We compared the fit obtained with the proposed model with an alternative model, namely the one described by the generalized standard model of diffusion (equation (8)). Parameter estimates for this model were obtained using also a non-linear least squares

procedure with a Levenberg-Marquardt algorithm. Two parameters were estimated, parameter a, which indicates innovation by producers and parameter b which indicates imitation. The ceiling provided was again the estimated figure of 163. Figure 5 depicts the fitted model against the real data. The model also shows a close correspondence to the data.
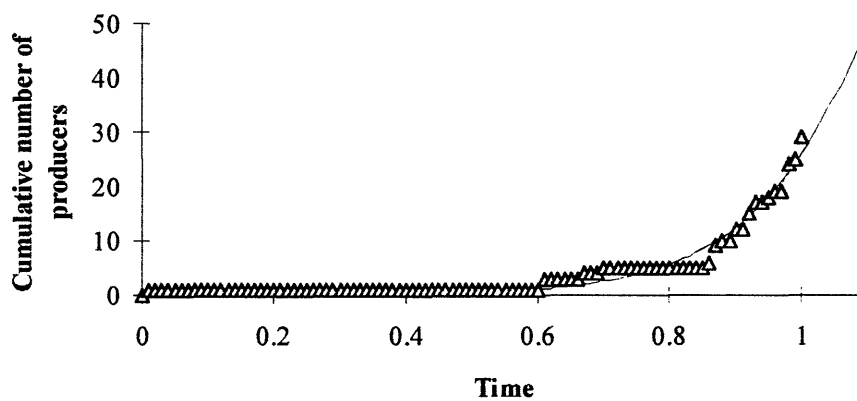

Figure 5. <u>Predicted producers versus real producers. Mutual funds.</u>

Parameter estimates (standard errors) for this model were a=0.000396 (9.6e-5) and b = 8.28 (0.293). Convergence was attained more easily and the fit was good although, at least using the R-squared statistic as a comparison measure, not as good as the proposed model (R-squared = 0.955 for the more parsimonious model vs. 0.976 for the proposed model). The disadvantage of this model is that it doesn't provide at least an approximate parameter to estimate a user diffusion curve.

Forecasting of demand

We then used the parameter estimates obtained for the proposed model as inputs for the generalized model (equation 8) in order to generate, with data from producer diffusion, a demand curve. The demand curve thus generated was compared to the actual data. Notice that for such exercise we needed to provide a ceiling in the number of adopters. Figure 6 is a plot for different ceilings. We can see that the data appear to fit a curve whose ceiling is given by 30,000 users using the parameter estimates of the proposed model (i.e., c = 0.00003187 and d = 11.748). These results would point to about 127 funds in the long run with the present average of 235 customers per fund.
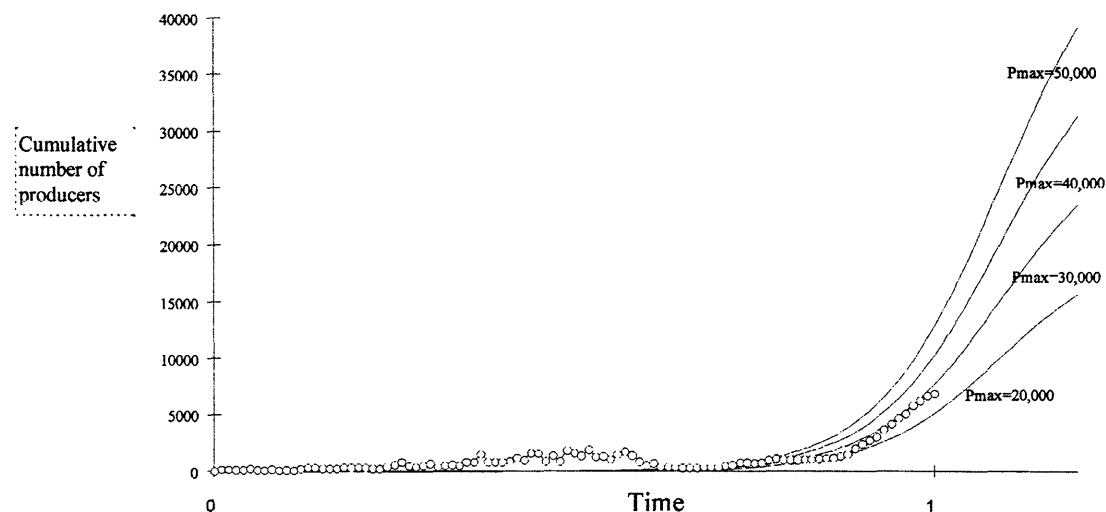

Figure 6. Comparison of real data on number of mutual fund users with predicted models for different ceilings on the number of users.

We then compared parameter estimates with the estimates generated using a parsimonious model solely based on imitation and innovation among users (Equation 8).

Using similar statistical methodologies we fitted equation 8 to our data and obtained

parameter estimates for the user diffusion curve.

Table 1. Parameter estimates of user diffusion curve using
the parsimonious model of Equation (8) for various ceilings.

Ceiling	c	d	R2
40000	6.631857e-5	10.32675	76.17%
30000	6.338549e-5	10.84625	75.97%
20000	5.0570957e-5	11.58276	75.55%
10000	1.937082e-5	13.99623	73.78%

These parameters are shown in Table 1 for different ceilings. We can see that our

estimation of b= 11.75 is not too far off the mark and provides the basis for a useful

forecasting of the demand curve based only on the producer diffusion curve. In fact, as

shown in figure 7, both estimates provide very similar user diffusion curves, although the

parameters obtained with the proposed model tend to overestimate the dependent

variable[8].

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[8]Where the fraction of the ceiling of adopters is given by the final equation

$$\frac{\exp(11.748)t - 1}{\exp(11.748)t + (11.748 / 3.187e - 5)}$$
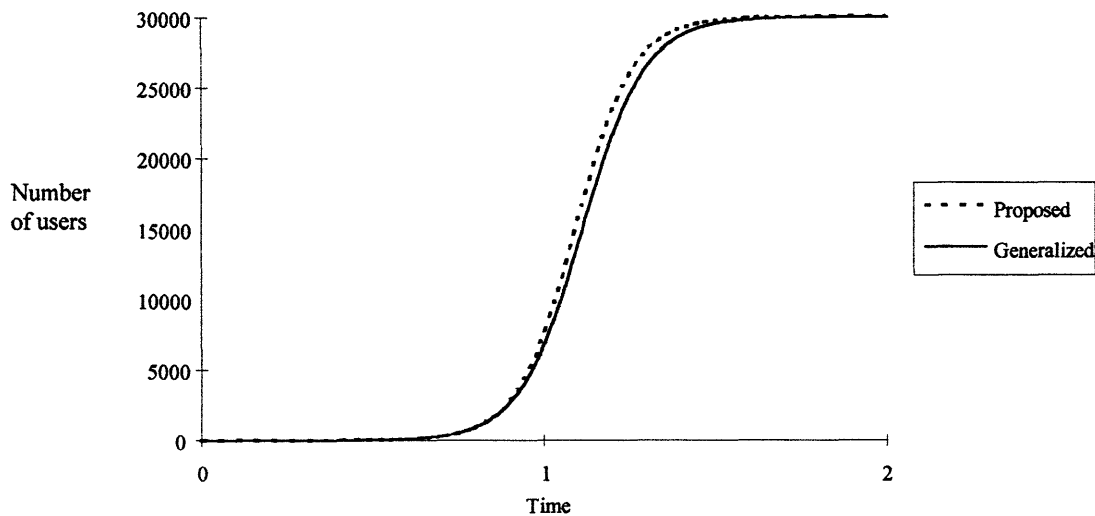
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Figure 7. Estimates of user diffusion curve using parameters from proposed model (dotted curve) and from model based on Equation (8) for mutual funds.

Credit Cards

Further empirical testing was carried on using data about credit card diffusion in Costa Rica. Data were gathered from informants, industry experts, and archival data. Archival data was obtained from periodicals, books, papers and monographs. We assembled a database which provided a fairly accurate longitudinal history of this product line. We measured producer diffusion using the number of credit card operators in the market and user diffusion using number of customers. Our data did not enable us to assess repeated purchases.

Figure 8 displays the total number (in thousands) of credit cards in circulation over time. The data show that the industry starts booming several years after the first company entered the market, roughly at the time of entry of the second producer. We used as our

estimate of the ceiling of producers the total number of commercial banks in the country (28 at the time of the study).

Figure 8. Total number of credit cards in circulation.

The parameters of the model for the credit card data were estimated using also a non-linear least squares procedure with a Levengerg-Marquardt algorithm. We had to rescale the time axis to avoid overflowing (due to large exponents). In fact, this appeared to be a serious drawback of this model. Too few data points or inadequate starting values made convergence very difficult. We had monthly data for the number of producers (that is we could pinpoint the month and year of entry with accuracy) but only yearly data for the user side. Difficulties experienced in fitting the regression model with yearly data made us approximate the user data month by month. The parameter estimates were then quite well estimated with relatively small standard errors. The estimates (standard errors) for the model were: a = 0.05 (0.00548); b = 4.23 (1.25); c = 0.00170 (0.000324); d = 3.37 (0.31). Figure 9 shows that these parameters render a predicted producer diffusion curve that appears to have close correspondence with the actual data.
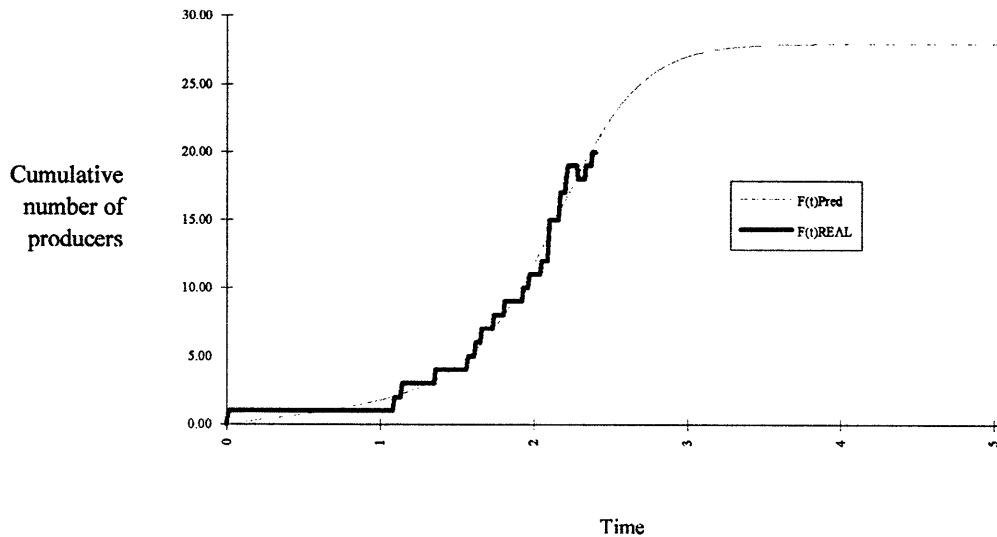
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Figure 9. Actual number of producers and predicted number of producers for credit cards.

Comparison to simpler model

For comparison purposes we also fitted the generalized standard model (equation 8) to our credit card data using similar non-linear regression procedures. Parameter estimates (standard errors) for a ceiling of 28 producers were: $a = 7.73e-3$ ($5.95e-4$); $b = 2.809$ (0.0475). Again we found that this model permitted a much easier application of our non-linear regression procedure. The model, as can be observed in figure 10, provides a very good approximation to the real data on credit card producer diffusion.
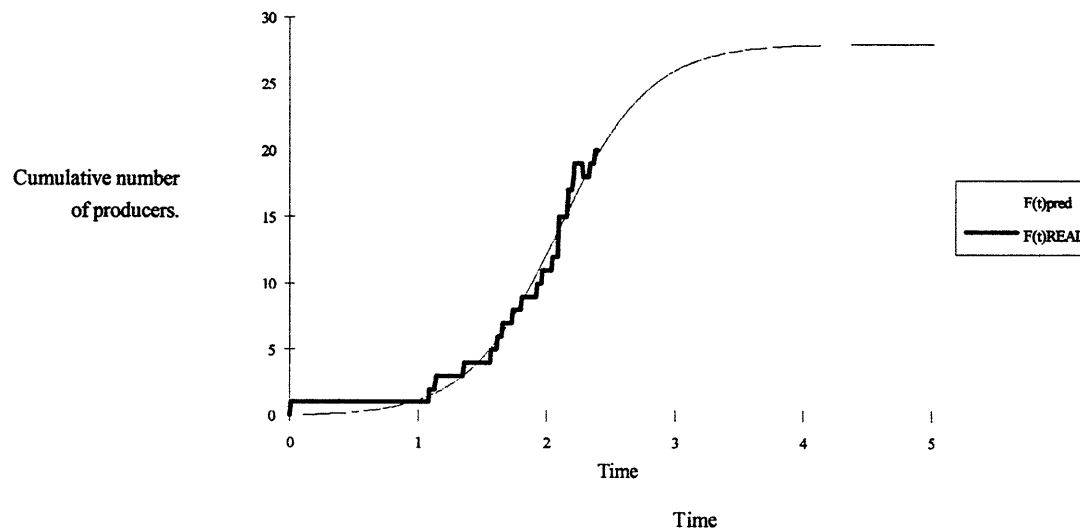
Cumulative number
of producers.

Figure 10. Predicter producers versus real producers using simpler model.
Credit Cards.

Forecasting of demand.

The parameter estimates generated with the proposed model were therupon utilized as

inputs for the generalized model. With this we were trying to generate a user diffusion

curve based on parameters generated with producer data. In this exercise we again

needed to provide a ceiling on the number of users. We, hence, calculated fits for different

ceilings and visually determined the approximate one predicted by the model. Figure 11

depicts actual user data with different forecasted curves for various ceilings on the number of adopters.
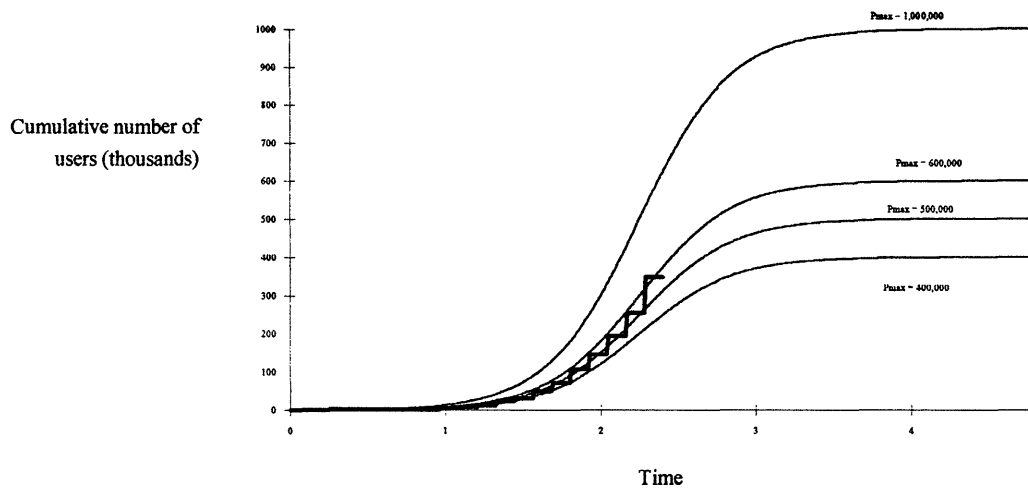
Figure 11. Comparison of real data on number of credit card users with predicted models for different ceilings on the number of users.

We can see that the data appear to render a very close correspondence to a curve whose ceiling is in the vicinity of 600,000 users. The parameter estimates obtained with producer data only are not very much different from parameter estimates obtained using a more parsimonious model based only on imitation and innovation among users (Equation 8). Table 2 shows parameter estimates for user diffusion curves generated with this simpler model.

Table 2. Parameter estimates of user diffusion curve using the parsimonious model of Equation (8) for various ceilings. Credit Cards.

Ceiling	c	d
1,000,000	9.89e-4	3.15
750,000	9.07e-4	3.42
600,000	7.22e-4	3.73
550,000	6.23e-4	3.89
450,000	3.66e-4	4.38
350,000	1.07e-4	5.35

We again see that our estimation of a= 1.70e-3 and b= 3.37 gives a very good approximation and provides the basis for a useful forecasting of the demand curve based only on the producer diffusion curve. In Figure 12 we can observe that the proposed model exhibits a very close similarity to a diffusion curved derived using the generalized model for a customer ceiling of 550,000 cards.
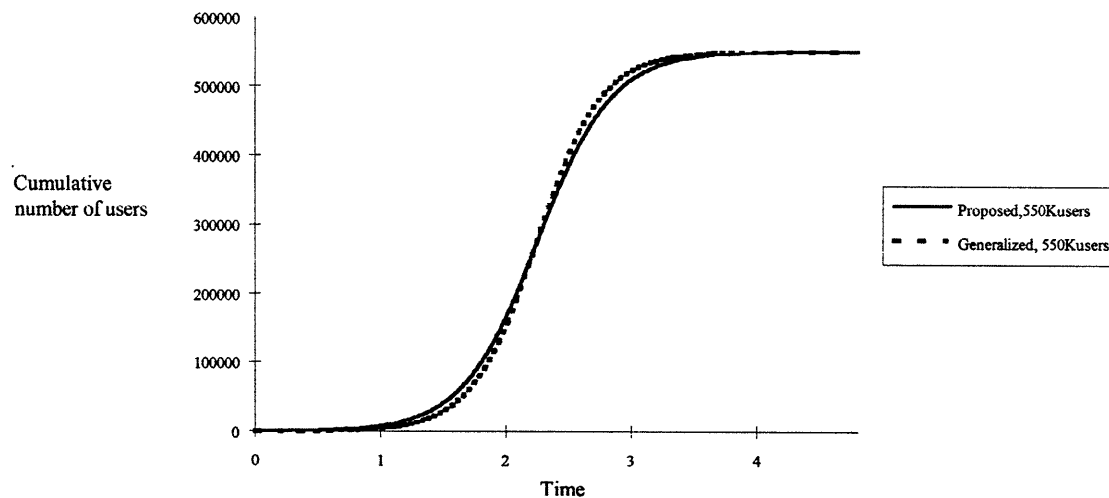

Figure 12. Estimates of user diffuion curve using parameters from proposed model (dotted curve) and from model based on Equation (8) for credit cards.

Pension funds

We sought to repeat the empirical exercise by fitting data on pension funds. We had a longitudinal data set with yearly data for the number of users and producers. Only 9 companies had entered the market at the time of the study. We purposefully then limited the number of points to a total of 8 (years from 1988 to 1996). When using such a discrete time scale, the model started exhibiting serious drawbacks. The lack of "degrees of freedom" made convergence impossible despite attempting fits using several different starting values. Though such behavior was predictable (given that we had so few datapoints per parameter to be estimated), the issue illustrated the limitations of this model and, perhaps, the limitations of diffusion modeling in general. Such limitation stems from the fact that for the model to work we need a large amount of datapoints, and when enough points become available the forecasting exercise is not useful anymore. As Mahajan et al. (1990:9) indicate "by the time sufficient observations have developed for reliable estimation, it is too late to use the estimates for forecasting purposes."

Ex ante, however, we did not expect the model to work very well in this case. Adoption decisions by users are, in the case of pension funds, much different from the adoption decisions faced by users of credit cards or mutual funds. In these two products the decision to launch by a producer soon results in a unilateral decision to adopt (or not) by an individual user. Our model does seem to respond to more automatic, simple decisions to adopt by users; decisions which are not mediated by many institutional considerations. In the case of pension funds, however, such might not be the case. There is an institutional framework at the nation's level. State-owned funds, however inefficient, still require compulsory contributions which result in automatic deductions from people's paychecks. Moreover, banks or financial organizations that wish to engage in such

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dealings require a long term financial and institutional commitment. Furthermore, pension funds are not primarily marketed to the individual, but to employers. Hence, although we have data in terms of total individual users, such data do not necessarily accurately reflect adoption decisions by users, who might have gotten enrolled as part of a corporate decision and not as a result of their own individual will.

We must note, however, that a simpler model (equation 8) with this small dataset did converge, and it rendered parameter estimates that adequately described user and producer diffusion curves. This indicates that the diffusion model here proposed, and in general all diffusion models, should be chosen according to the circumstances. Parker (1994) shows that different models behave better under different circumstances (i.e., pre-launch, post-launch when few data are available, post-launch with many data available). This hints at the possibility of our proposed model being useful within a somewhat narrow arrangement of circumstances, particularly in post-launch situations with a moderate amount of data, or in pre-launch situations where many data about similar products are available.

CONCLUSION

We developed a model to represent patterns of financial innovation diffusion. The model is derived from prior work in the theory of technological diffusion. A generalized formula derived by Mahajan and Schoeman (1977) is extended to take into account both demand and supply considerations. The resultant model provides good descriptive properties and fits well with available empirical data for two financial innovations. Parameter estimates

are used to estimate plausible demand curves using the generalized formula proposed by Mahajan and Schoeman (1977). The estimated curves reproduce, based on data about number of producers and without any information about users, diffusion curves that closely match curves generated using data on number of users with other usually accepted model of diffusion. One of the principal uses of this model could be to provide reasonable parameters to estimate a demand curve based on the rate of producer adoption of the product at issue or other similar products.

The model and approach taken here are more exploratory than normative. Because financial products are relatively easy and inexpensive to imitate, it is apparent that adoption diffusion curves by producers will not be driven by the relative cost of imitation but by the success and the rate of diffusion of the product among users. Hence, a model that somehow relates both user and producer diffusion dynamics is both more realistic and could be useful.

Though our main interest here is in financial innovations, the model could also be applied to other types of products. Perhaps such applications with larger data sets could unveil limitations of this model. We tried to establish whether parameters remained stable for different starting values of the nonlinear algorithm, and they did so. Our data however, are clearly representative of an emerging market within one country. Such data only provided few data points turning our empirical test into a forecasting exercise. The use of the model requires judicious determination of likely ceilings in the number of adopters. An evident weakness of this model is that an overestimation of total number of producers likely to enter the market will render an overestimation of the demand curve. Another evident weakness of this model is its instability when only few datapoints are available. Testing model properties retrospectively, with larger sets of data is necessary to evaluate more conclusively the applicability and soundness of this model.

References

Bass, F. M. (1969). "A new product growth model for consumer durables."
Management Science, Vol. 15, 215-227.

Clark, P. and N. Staunton (1989). *Innovation in technology and organization.*
London: Routledge.

Dobb, S. C. (1956). "Testing message diffusion in harmonic logistic curves."
Psychometrica, Vol. 21, June, 191-205.

Fourt, L. A. and J. W. Woodlock (1960). "Early prediction of market success for new
grocery products." *Journal of Marketing*, Vol. 25, October, 31-38.

Freeman, C. (1994). "The economics of technical change." *Cambridge Journal of
Economics*, 18, 463-514.

Gibbons, M. and J. S. Metcalfe (1986). "Technological variety and the process of
competition." in Arcangeli, F. , P. David, and G. Dosi, eds., 1994, *Frontiers on
Innovation Diffusion*, Report of the Venice Conference on Innovation Diffusion,
Venice: Oxford University Press.

Griliches, Z. (1957). "Hybrid corn: An exploration in the economics of technological
change." *Econometrica*, Vol. 25, No. 4, October, 501-522.

Jain, Dipak, V. Mahajan and E. Muller (1991). "Innovation diffusion in the presence of
supply restrictions." *Marketing Science*, Vol. 10, No. 1, Winter.

Mahajan, V. and R. A. Peterson (1978). "Innovation diffusion in a dynamic potential
adopter population." *Management Science*, Vol. 24, No. 15, November, 1589-
1597.

Mahajan, V. and M. E. F. Schoeman (1977). "Generalized model for the time pattern of
the diffusion process." *IEEE Transactions on Engineering Management*, Vol.
EM-24, February, 12-18.

Mahajan, V. and K. E. Haynes (1977). "Modeling the diffusion of public policy
innovations among the U. S. States." *Socio-economic Planning Sciences*, Vol. 11
October, 259-263.

Mahajan, V., E. Muller, F. M. Bass (1990). "New product diffusion models in marketing: A review and directions for research." *Journal of Marketing*, Vol. 54, 1-26.

Mansfield, E. (1961). "Technical change and the rate of imitation." *Econometrica*, Vol. 29, 741-766.

Metcalfe, J. S. (1981). "Impulse and diffusion in the strategy of technological change." *Futures*, Vol. 13, No. 5, 347-359.

Metcalfe, J. S. (1988). "The diffusion of innovation: An interpretative survey." In Dosi, G., C. Freeman, R. Nelson, G. Silverberg, and L. Soete, eds., 1988, *Technical Change and Economic Theory*, New York: Columbia University Press.

Molyneux, P. and N. Shamroukh (1996). "Diffusion of financial innovations: The case of junk bonds and note issuance facilities." *Journal of Money, Credit, and Banking*, Vol. 28, No. 3, August, Part 2, 502-522.

Parker, P. M. (1994). "Aggregate diffusion forecasting models in marketing: A critical review." *International Journal of Forecasting*, 10, 353-380.

Teece, D. J. (1980). "The diffusion of an administrative innovation." *Management Science*, Vol. 26, No. 5, May, 464-470.

Appendix 1

Mahajan and Schoeman have derived a generalized form for $H(t)$ (Mahajan et al., 1977):

$$H(t) = \frac{\overline{H} - \frac{c(\overline{H} - H0)}{(c + dH0)} \exp\left[-(c + d\overline{H})(t - t0)\right]}{1 + \frac{d(H - H0)}{(c + dH0)} \exp\left[-(c + d\overline{H})(t - t0)\right]}$$

Where

$$H0 = H\left(t = t0\right)$$

and, c = index of uninfluenced user adoption and d= index of influenced user adoption.

Thus we obtain:

$$\frac{dF(t)}{dt} = \left\{ a + b\left[\frac{\overline{H} - \frac{c(H - H0)}{(c + dH0)} \exp\left[-\left(c + d\overline{H}\right)\left(t - t0\right)\right]}{1 + \frac{d(H - H0)}{(c + dH0)} \exp\left[-\left(c + d\overline{H}\right)\left(t - t0\right)\right]} \right] \right\} \cdot \left(F - F(t)\right)$$

Integrating both sides of the above equation,

$$-\ln\left(F - F(t)\right) + c0 = \int a \cdot dt + b\int \left[\frac{\overline{H} - \frac{c\left(\overline{H} - H0\right)}{\left(c + dH0\right)} \exp\left[-\left(c + d\overline{H}\right)\left(t - t0\right)\right]}{1 - \frac{d\left(\overline{H} - H0\right)}{\left(c + dH0\right)} \exp\left[-\left(c + d\overline{H}\right)\left(t - t0\right)\right]} \right] dt$$

We use partial fractions to integrate the second term on the right-hand side of the equation:

This term can be expanded as

$$b\int \frac{H\,dt}{1+\dfrac{d\left(\overline{H}-H0\right)}{\left(c+dH0\right)}\exp\left[-\left(c+d\overline{H}\right)(t-t0)\right]} - \frac{bc\left(\overline{H}-H0\right)}{c+dH0}\int \frac{\exp\left[-\left(c+d\overline{H}\right)(t-t0)\right]\,dt}{1+\dfrac{d\left(\overline{H}-H0\right)}{\left(c+dH0\right)}\exp\left[-\left(c+d\overline{H}\right)(t-t0)\right]}$$

Let's deal first with the first term of the above equation. We multiply and divide by

$$\exp\left[\left(c+d\overline{H}\right)(t-t0)\right]$$

and obtain, using substitution, the following expression for the first integral:

$$b\int \frac{H\,dt}{1+\dfrac{d\left(\overline{H}-H0\right)}{\left(c+dH0\right)}\exp\left[-\left(c+d\overline{H}\right)(t-t0)\right]} = \frac{b\overline{H}}{\left(c+d\overline{H}\right)}\ln\left\{\exp\left[\left(c+d\overline{H}\right)(t-t0)\right]+\frac{d\left(\overline{H}-H0\right)}{\left(c+dH0\right)}\right\}+c0$$

For the second term of expression we use substitution,

$$v = 1+\frac{d\left(\overline{H}-H0\right)}{\left(c+dH0\right)}\exp\left[-\left(c+dH\right)(t-t0)\right]$$

$$dv = \frac{d\left(\overline{H}-H0\right)}{\left(c+dH0\right)}\exp\left[-\left(c+d\overline{H}\right)(t-t0)\right]\left[-\left(c+d\overline{H}\right)\right]dt$$

and the integral becomes

$$\frac{bc\left(\overline{H}-H_0\right)}{\left(c+dH_0\right)}\int \frac{\left(c+dH_0\right)}{d\left(\overline{H}-H_0\right)(-1)\left(c+d\overline{H}\right)}\frac{dv}{v}$$

which is equal to,

$$\frac{bc}{d\left(c + d\overline{H}\right)} \ln\left\{1 + \frac{d\left(\overline{H} - H_0\right)}{\left(c + dH_0\right)} \exp\left[-\left(c + d\overline{H}\right)\left(t - t_0\right)\right]\right\} + c_0$$

We thus obtain the following expression,

$$-\ln\left(F - F(t)\right) + c_0 = at + \frac{b\overline{H}}{\left(c + d\overline{H}\right)} \ln\left\{\exp\left[\left(c + d\overline{H}\right)\left(t - t_0\right)\right] + \frac{d\left(\overline{H} - H_0\right)}{\left(c + dH_0\right)}\right\} +$$

$$\frac{bc}{d\left(c + d\overline{H}\right)} \ln\left\{1 + \frac{d\left(H - H_0\right)}{\left(c + dH_0\right)} \exp\left[-\left(c + d\overline{H}\right)\left(t - t_0\right)\right]\right\}$$

We can now determine the constant of integration by assuming that

$$F\left(t = t_0\right) = F_0$$

then,

$$c_0 = at_0 + \frac{b\overline{H}}{\left(c + d\overline{H}\right)} \ln\left\{1 + \frac{d\left(\overline{H} - H_0\right)}{\left(c + dH_0\right)}\right\} + \frac{bc}{d\left(c + d\overline{H}\right)} \ln\left[1 + \frac{d\left(\overline{H} - H_0\right)}{\left(c + dH_0\right)}\right] +$$

$$\ln\left(\overline{F} - F_0\right)$$

$$c_0 = at_0 + \left[\frac{b\overline{H}}{\left(c + d\overline{H}\right)} + \frac{bc}{d\left(c + d\overline{H}\right)}\right] \ln\left\{1 + \frac{d\left(H - H_0\right)}{\left(c + dH_0\right)}\right\} + \ln\left(\overline{F} - F_0\right)$$

$$c_0 = at_0 + \frac{b}{d} \ln\left\{1 + \frac{d\left(\overline{H} - H_0\right)}{\left(c + dH_0\right)}\right\} + \ln\left(\overline{F} - F_0\right)$$

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Substituting into equation we obtain an expression for F(t):

$$F(t) = F - 1 / \exp\left\{ \begin{array}{l} \left[a\left(t - t_0\right) + \dfrac{b\overline{H}}{\left(c + d\overline{H}\right)} \ln\left\{ \exp\left[\left(c + d\overline{II}\right)\left(t - t_0\right)\right] + \dfrac{d\left(\overline{II} - II_0\right)}{\left(c + dII_0\right)} \right\} + \right] \\[3em] \left[\dfrac{bc}{d\left(c + dH\right)} \ln\left\{ 1 + \dfrac{d\left(H - H_0\right)}{\left(c + dH_0\right)} \exp\left[-\left(c + dH\right)\left(t - t_0\right)\right]\right\} - \right] \\[3em] \left[\dfrac{b}{d} \ln\left\{ 1 + \dfrac{d\left(\overline{H} - H_0\right)}{\left(c + dH_0\right)} \right\} - \ln\left(\overline{F} - F_0\right) \right] \end{array} \right\}$$

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Appendix 2

$$\frac{P(t)}{M} = \frac{\overline{P}}{M} - 1 \Big/ \exp\left\{ \begin{array}{l} a\left(t - t_0\right) + \dfrac{b}{(c+d)} \ln\left\{ \exp\left[(c+d)\left(t - t_0\right)\right] + \dfrac{d}{c} \right\} + \\[2em] \dfrac{bc}{d(c+d)} \ln\left\{ 1 + \dfrac{d}{c} \exp\left[-(c+d)\left(t - t_0\right)\right] \right\} - \\[2em] \dfrac{b}{d} \ln\left\{ \dfrac{c+d}{c} \right\} - \ln\left(\dfrac{\overline{P}}{M}\right) \end{array} \right\}$$

Let, for simplicity,

$$T = \left(t - t_0\right)$$

Then we get

$$\frac{P(t)}{M} = \frac{\overline{P}}{M} - \frac{\left(\dfrac{c+d}{c}\right)^{\frac{b(d+c)}{d(c+d)}} \cdot \dfrac{\overline{P}}{M}}{\exp(at) \cdot \left[\dfrac{d}{c} + \exp(c+d)T\right]^{\left(\frac{b}{c+d}\right)} \cdot \left[1 + \dfrac{d}{c\left[\exp(c+d)T\right]}\right]^{\frac{bc}{d(c+d)}}}$$

Therefore,

$$\frac{P(t)}{M} = \frac{\overline{P}}{M}\left\{ 1 - \frac{(c+d)^{\frac{b}{d}} \cdot}{\left[\exp(at)\right] \cdot \left[\dfrac{\left(\frac{b}{d}\right)}{c}\right]\left[\dfrac{d + c\exp(c+d)T}{c}\right]^{\left(\frac{b}{c+d}\right)} \cdot \left[\dfrac{d + c \cdot \exp(c+d)T}{c \cdot \exp(c+d)T}\right]^{\left(\frac{bc}{d(c+d)}\right)}} \right\}$$

So we get,

$$P(t) = \overline{P}\left\{1 - \frac{(c+d)^{\frac{b}{d}} \cdot (c)^{\left(\frac{b}{c+d}\right)} \cdot (c)^{\left(\frac{bc}{d(c+d)}\right)} \cdot \left[\exp(c+d)T\right]^{\left(\frac{bc}{d(c+d)}\right)}}{\left[\exp(at)\right] \cdot (c)^{\left(\frac{b}{d}\right)} \cdot \left[d + c \cdot \exp(c+d)T\right]^{\left(\frac{db+bc}{d(c+d)}\right)}}\right\}$$

and,

$$P(t) = \overline{P} \cdot \left\{1 - \frac{(c+d)^{\left(\frac{b}{d}\right)} \cdot \left[\exp\left(\frac{bcT}{d}\right)\right]}{\left[\exp(aT)\right] \cdot \left[d + c \cdot \exp(c+d)T\right]^{\left(\frac{b}{d}\right)}}\right\}$$

$\therefore$

$$P(t) = \overline{P} \cdot \left\{1 - \frac{(c+d)^{\left(\frac{b}{d}\right)} \exp\left[\frac{bc - ad}{d} \cdot T\right]}{\left[d + c \cdot \exp(c+d)T\right]^{\left(\frac{b}{d}\right)}}\right\}$$

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Appendix 3

To better understand the difference between the proposed model (Equation 12) and the more standard model based solely on imitation and innovation amongst producers (Equation 5), we characterized Equation 12 utilizing the parameters obtained from the mutual funds data. The expressions for the first and second derivative of the cumulative proportion of adopters are graphed and displayed in Figure App3-1. We observe, as expected that the the noncumulative proportion of adopters peaks once at a point in time that corresponds to the inflection point of the diffusion sigmoid. Differences with the standard model can be also observed in Figure App3-2, using the parameters obtained for each of them with our mutual fund data. We can see that both models do not exhibit any apparent relevant difference during early stages of the diffusion process. There is an initial short period of time in which the difference between the standard and the proposed model is positive (indicating a slightly faster rate of producer entry according to the standard model), but the rate of adoption by producers appears to increase faster than in the standard model thereafter, presumably because of the combined influence (in the case of our proposed model) of adoption rates by users and also producers. The proposed model peaks a bit earlier than the other model. Soon after reaching its ceiling the difference between the standard and the proposed model is positive (indicating less entries being predicted at this stage by the latter than by the former). In all, the effect of user adoptions appears to exert an influence in the rate of adoption by producers. Such influence is reflected in the form of a faster rate of entry up to the inflection point, and a slower rate of entry thereafter.

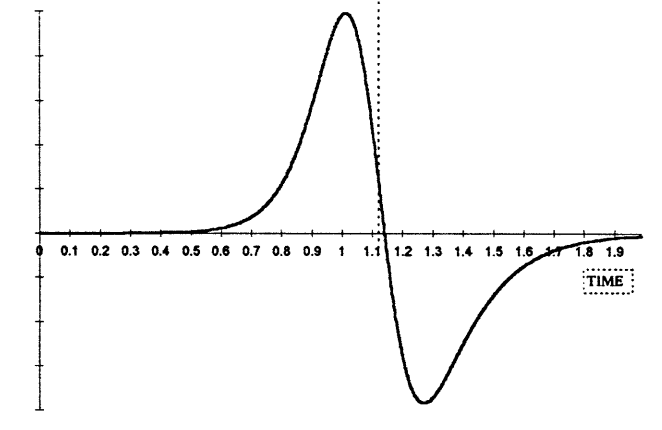# Figure App3-1 Analytical properties of the proposed model.

Cumulative
Proportion of
Adopters.
F(t)

TIME
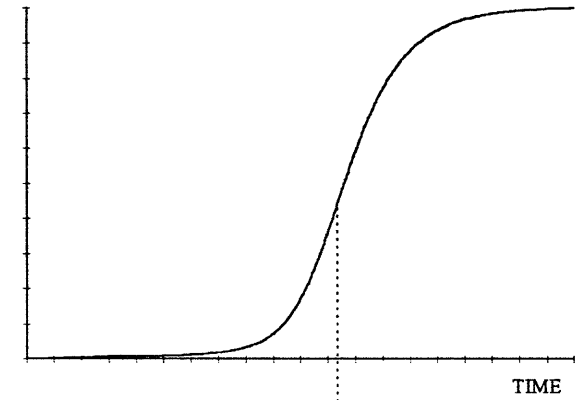
Noncumulative
Proportion of
Adopters, f(t),
$$\left(\frac{dF(t)}{dt}\right)$$

0   0.5   1   1.5   TIME   2
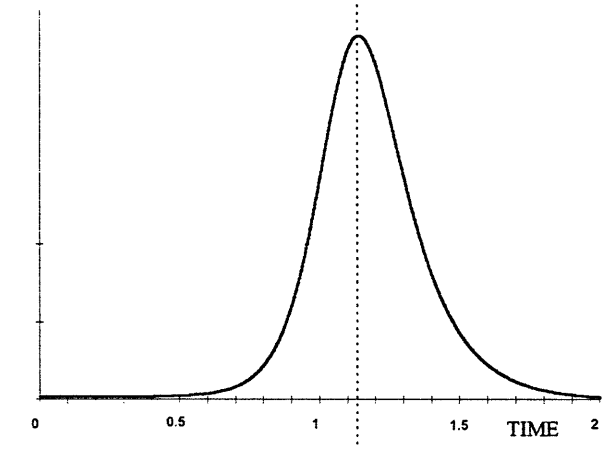
Rate of change
in the
Noncumulative
Proportion of
Adopters.
$$\left(\frac{df(t)}{dt}\right)$$

0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1 1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8 1.9
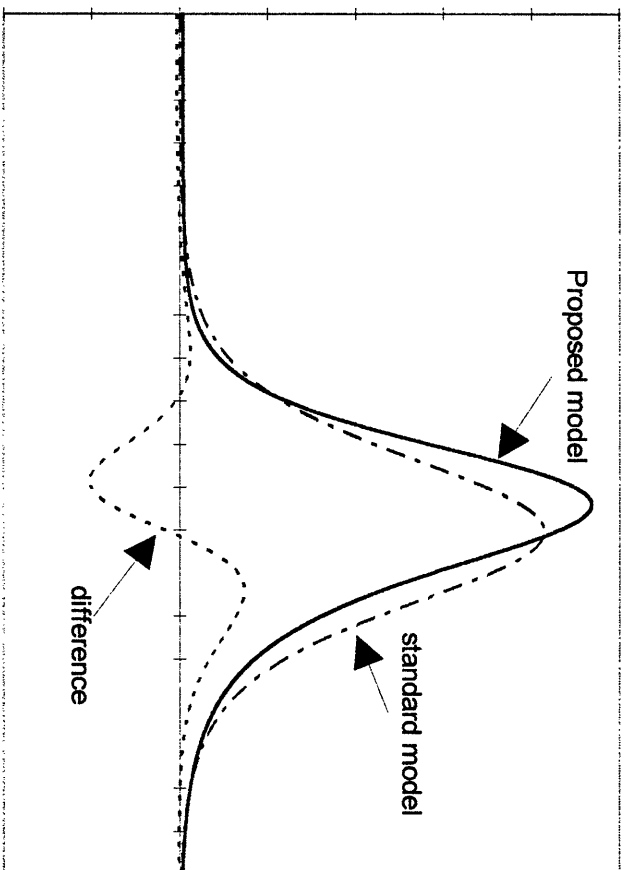
TIME

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Figure App3-2: Difference between the proposed model and the standard model

Model (expressed in terms of proportions):

$$F(t) = 1 - \frac{(c+d)^{\frac{b}{d}} \exp\left[\frac{bc-ad}{d}\right]t}{\left[d+c\exp(c+d)t\right]^{\frac{b}{d}}}$$

First derivative:

$$f(t) = \frac{dF(t)}{dt} = (c+d)^{\frac{b}{d}} \exp\left(\frac{bc-ad}{d}\right)t \left\{ \frac{c(a+b)\exp(c+d)t - bc + ad)}{(d+c\exp(c+d)t)^{\frac{b}{d}+1}} \right\}$$

Second derivative:

$$
\left\{ \left[d + c \exp(c+d)t \right] t \right]^{\frac{b}{d}+2} \right\} \frac{df(t)}{dt} =
$$

$$
(c+d)^{\frac{b}{d}} \left\{ \left[\left[c(a+b) \left[\frac{bc - ad + cd + d^2}{d} \right] \exp\left(\frac{bc - ad + cd + d^2}{d} \right) - (bc - ad)\left(\frac{bc - ad}{d} \right) \exp\left(\frac{bc - ad}{d} \right) t \right] t \right] \left(d + c \exp(c+d)t \right) \right\} -
$$

$$
c(c+d)\left(\exp(c+d)t \right)\left(\frac{b}{d} + 1 \right) \left[c(a+b) \exp\left(\frac{bc - ad + cd + d^2}{d} \right) t - (bc - ad)\exp\left(\frac{bc - ad}{d} \right) t \right]
$$

Difference with standard model given by:

(a+bF(t))(1-F(t)) - (a+bH(t))(1-F(t)) = b(F(t) - H(t)) (1 - F(t))

$$b\left\{\left[\frac{1-\exp\bigl(-(a+b)\bigr)t}{1+\dfrac{b}{a}\exp\bigl(-(a+b)t\bigr)} - \frac{1-\exp\bigl(-(c+d)t\bigr)}{1+\dfrac{d}{c}\exp\bigl(-(c+d)t\bigr)}\right] \cdot \left[1 - \frac{1-\exp\bigl(-(a+b)t\bigr)}{1+\dfrac{b}{a}\exp\bigl(-(a+b)t\bigr)}\right]\right\}$$