A METHODOLOGY FOR THE ASSESSMENT OF THE PROLIFERATION RESISTANCE OF NUCLEAR POWER SYSTEMS

Topical Report*

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PRINCIPLES OF MULTIATTRIBUTE DECISION ANALYSIS

A.1 General Remarks

The purpose of this Appendix is to present a brief summary of the elements of multiattribute decision analysis. Multiattribute decision analysis addresses decision problems which involve simultaneous satisfaction of several objectives which often are conflicting. In particular, this theory is designed to help a decision maker (or decision unit) make a choice among a set of prespecified alternatives, where the consequences of choosing a particular alternative can be expressed in terms of the levels that a number of "indices of value" or"attributes" attain. We can divide these decision problems into two categories; namely, those that involve decisions under certainty and those that involve decisions under uncertainty. The former are those for which the consequences of each alternative are well-defined; that is, the outcome of a particular course of action can be predetermined. The latter are those for which the consequences of some alternatives are uncertain; that is, the outcome of a particular course of action cannot be deterministically predetermined. What it is known, however, is the probability with which each possible outcome will obtain.

The theory of multiattribute decision analysis is developed by R.L. Keeney and H. Raiffa in Ref [6]. This Appendix is liberally adapted from their work. The certainty problem is described in section A.2 while the uncertainty problem is described in section A.3.

A.2. Multiattribute Preferences Under Certainty: Value Function

Decision analysis under certainty addresses the problem of establishing the relevant preferences of the decision maker for each possible outcome. Since each alternative course of action is uniquely related to an outcome, a preference structure over the outcomes implies a preference structure over the alternatives.

Some symbolism will be helpful at this point. We denote an alternative by α and the set of all possible alternatives by A. With each α we associate the n indices of value or attributes $X_1(\alpha)$, $X_2(\alpha)$, . . . $X_n(\alpha)$. As explained in Chapter III each <u>attribute</u> X_i refers to a general property of α (e.g. cost, development time) and is associated with an <u>evaluator</u> x_i which measures this attribute (e.g. dollars, years). These n attributes constitute, therefore, a mapping of A into a n-dimensional space which we call <u>evaluation space</u>. It is noteworthy that given a point $(x_1, \ldots, x_n)$ in the evaluation space, the magnitudes of x_i and x_j for $i \neq j$ cannot be compared since they are usually expressed in different units (e.g. dollars, years, radiation units). There is a need therefore for the specification of an index that combines $X_1(\alpha)$, ..., $X_n(\alpha)$ into a scalar index of preferability or value. Alternatively stated, it is adequate to specify a scalar-valued function v defined on the evaluation space with the property that

$$v(x_1,x_2, \ldots x_n) \geq v(x_1',x_2', \ldots, x_n') \Leftrightarrow (x_1, \ldots,x_n) \succsim (x_1,\ldots,x_n)$$

where the symbol $\succsim$ reads "preferred or indifferent to".

We refer to the function v as a <u>value-function</u>. Other names used in the literature are: ordinal utility function, preference-function or worth function.

Given the value function v, the problem reduces into the one of ordering the α's in A, in a descending order of values v.

A.2.1. Dominance and the Efficient Frontier.

For convenience in the following we assume that preferences increase in each x_i.

We say that $\underline{x}'$ dominates $\underline{x}''$ whenever

(a) $x_i' \geq x_i'$ all i A.1

(b) $x_i' > x_i''$ for some i A.2

If $\underline{x}'$ dominates $\underline{x}''$ then obviously α' is preferred to α" since α' is at least as good as α" for every evaluator [see Eq. A.1] and strictly better for at least one [see Eq. A.2].

Let R be the set of all points in the n-dimensional evaluation space that corresponds to all alternatives α in A. We call the set of points in R that are not dominated, the <u>efficient frontier</u> of R. It is also known as "Pareto optimal set". The efficient frontier is illustrated for a 2-dimensional case in Figure A.1 with the heavy line. It is noteworthy that each point inside R is dominated by at least one point in the efficient frontier.

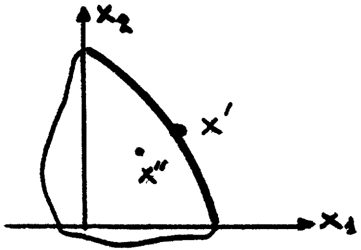


Figure A.1 Efficient frontier in a 2-dimensional case.

The determination of the efficient frontier of the problem is the formal expression for the screening process that was mentioned in Section IV.6. The result of this procedure is the identification of the alternatives that are to be ranked using the value function .

A.2.2. <u>Preference structure, Indifference Surfaces and Value Function</u>.

In a formal approach to the construction of the value function it is assumed that in the opinion of the decision maker, any two points $\underline{x}'$ and $\underline{x}''$ are <u>comparable</u> in the sense that one, and only one of the following holds:

(a) $\underline{x}'$ is indifferent to $\underline{x}''$ $(\underline{x}' \sim \underline{x}'')$
(b) $\underline{x}'$ is preferred to $\underline{x}''$ $(\underline{x}' > \underline{x}'')$
(c) $\underline{x}'$ is less preferred than $\underline{x}''$ $(\underline{x}' < \underline{x}'')$

All three relations (a), (b) and (c) are assumed transitive.

A *preference structure* is then defined on the evaluation space if any two points are comparable and no intransitivities exist.

For each point $\underline{x}$, all the points that are indifferent to it, define an *indifference surface*. Once defined, the indifference surfaces can be ranked in order of increasing preferences. The "optimum" alternative α^* is then the one that corresponds to the point $\underline{x}^*$ of the efficient frontier that belongs to the indifference surface of the highest value.

A function v, which associates a real number $v(\underline{x})$ to each point $\underline{x}$ in an evaluation space, is said to be a *value* function representing the decision maker's preference structure, provided that

$$\begin{aligned} \underline{x}' \sim \underline{x}'' &\Leftrightarrow v(\underline{x}') = v(\underline{x}'') \\ \underline{x}' \succ \underline{x}'' &\Leftrightarrow v(\underline{x}') > v(\underline{x}'') \end{aligned} \qquad \text{A.4}$$

If v is a value function reflecting the decision maker's preferences, then his problem can be put into the format of the standard optimization problem: find $\alpha \varepsilon A$ to maximize $v[X(\alpha)]$.

Given a value function $v(\underline{x})$ the indifference surfaces are defined and, therefore, the preference structure in the evaluation space is uniquely defined. The converse, however, is not true: a preference structure does not uniquely specify a value function.

The value functions v_1 and v_2 are strategically equivalent, written $v_1 \sim v_2$, if v_1 and v_2 have the same indifference surfaces and induced preference structure. It can be shown that if $T(\cdot)$ is any strictly monotonically increasing real-valued function (of a real variable) and if $v_2(\underline{x}) = T[v_1(\underline{x})]$, then it is immaterial whether we choose $\alpha\varepsilon A$ to maximize v_1 or v_2. In other words v_1 and v_2 are strategically equivalent.

For example, if all x_i are positive and

$$v_1(\underline{x}) = \sum_i k_i x_i \qquad k_i > 0 \quad \text{all } i$$

then

$$v_2(\underline{x}) = \sqrt{\sum_i k_i x_i}$$

and $$v_3(\underline{x}) = \log(\sum_i k_i x_i)$$

would be strategically equivalent to v_1. All these functions are representations of the same preference structure. Indeed for operational purposes, given v we will want to choose T such that the value function T(v) is easy to manipulate mathematically.

From the above discussion it follows that the whole problem is equivalent to the one of defining the indifference surfaces in the evaluation space. Keeney and Raiffa [6] present procedures for the systematic definition of the indifference surfaces by "asking" the decision maker to define points on these surfaces. In the 2-dimensional case for example, one such procedure consists in asking the decision maker to assess, starting from a point $\underline{x}'(x_1', x_2')$, how great a change in x_2 would compensate for a given change in x_1, and thus, producing a new point $\underline{x}''$ on the indifference curve through $\underline{x}'$. In the limit, for small changes, this procedure results in the definition of the marginal rate of substitution of x_1 for x_2 at $\underline{x}'$. This procedure can be generalized for the multi-dimensional case. The difficulty of the assessment increases, however, with the dimensionality (number of attributes of the problem).

A.2.3 Property Identification

The definition of the indifference surfaces and, therefore, of $v(\underline{x})$ becomes easier if general properties of $v(\underline{x})$ are known beforehand. Thus, it is advantageous to first consider such general properties as representation, monotonicity, and concavity. Keeney and Raiffa present a number of representation theorems (mainly from measurement theory) that break down the assessment of the value function into component parts. These theorems are presented in terms of properties of the preference structure induced by the decision maker in the evaluation space. Basically, all the simplifications are based on the preference-independence property that might exist among various subsets of attributes.

Definition. The set of attributes Y is preferentially independent of the complementary set Z if and only if the conditional preference structure in the y-space given $\underline{z}'$, does not depend on $\underline{z}'$. An important result can be cast in the form of the following theorem.

Theorem 1. If the set $Y = \{x_1, \ldots, x_s\}$ is preferentially indendendent of the complementary set $Z = \{x_{s+1}, \ldots, x_n\}$ then

$$v(\underline{y},\underline{z}) = f(v_y(\underline{y}), x_{s+1}, \ldots, x_n). \qquad \text{A.5}$$

In other words the value function $v_y(\underline{y})$ can be constructed in the y-space without worrying about the exact value of $\underline{z}$. Then the value function $v(\underline{y},\underline{z})$ depends on $\underline{y}$ only through the aggregator $v_y(\underline{y})$. If in addition the set z is preferentially independent of Y, the value function has the form

$$v(\underline{y},\underline{z}) = f[v_y(\underline{y}), v_z(\underline{z})]. \qquad \text{A.6}$$

Another important representation theorem states that

Theorem 2. If Y, Z are subsets of the set S of attributes such that

$$Y \cup Z \neq S \text{ and } Y \cap Z \neq \phi$$

and Y and Z are preferentially independent of their respective complements, then the sets

(i) $Y \cup Z$

(ii) $Y \cap Z$

(iii) $Y-Z$ and $Z-Y$

(iv) $(Y-Z) \cup (Z-Y)$

are each preferentially independent of their respective complements. The simplest representation of a value function occurs whenever the attributes are mutually preferentially independent. Definition: The attributes $x_1, \ldots, x_n$ are mutually preferentially independent if every subset y of those attributes is preferentially independent of its complementary set of attributes.

Theorem 3. Given attributes $x_1, \ldots, x_n$ $n \geq 3$ an additive value function

$$v(x_1, x_2, \ldots, x_n) = \sum_i \lambda_i v_i(x_i)$$

(where v_i is a value function over X_i scaled from 0 to 1 and $\sum_{i=1}^{n} \lambda_i = 1$, $\lambda_i > 0$ all i) exists if and only if the attributes are mutually preferentially independent.

From theorem 2 above it follows that: If every pair of attributes is preferentially independent of its complementary set, then the attributes are mutually preferentially independent. The existence of preferentially independent sets of attributes results, therefore, in a significant reduction of the complexity of the problem. Thus during the property identification phase of the value function assessment we seek to identify preferentially independent subsets of attributes. Of course, in practice, it would not be reasonable to check directly for all possible preferential independence conditions. The nature of the problem, however, usually suggests groups for which the preference independence conditions should be checked. One general guideline is to divide the set of attributes into natural groups of attributes; i.e., attributes measured in the same or similar units. For our problem such groups could be monetary-attributes, time-attributes, difficulty-attributes, etc. Another possible method is to try to identify preferentially independent sets of attributes starting with sets that correspond to higher levels in the objective hierarchy (see Chapter III). Then, this procedure is repeated within each of the sets defined in the previous step, etc., until the lowest level objectives have been reached.

An example of property identification procedure is given in Appendix B. Examples of value function assessments are given in Appendices B and E.

A.3. Multiattribute Preferences Under Uncertainty: Utility Function.

Decision analysis under uncertainty addresses the problem of establishing the relevant preferences of a decision maker under uncertainty. In particular, since now each alternative is not associated with a unique outcome, but rather with a probability distribution over the outcomes, decision analysis under uncertainty consists in establishing the preferences of the decision maker over probability distributions.

Using the symbolism of the previous section where x_i designates a specific label of X_i, our task is to assess a *utility function* $u(\underline{x}) = u(x_1, x_2, \ldots, x_n)$ over the n attributes. The utility function u has the characteristic property that, given two probability distributions A and B over the multiattribute consequences $\tilde{\underline{x}}^{(1)}$, *probability distribution A is at least as desirable as B if and only if*

$$E_A[u\ (\tilde{\underline{x}})] > E_B[u\ (\tilde{\underline{x}})] \qquad \text{A.9}$$

where E_A and E_B are the usual expectation operators taken with respect to distribution measures A and B, respectively. This asserts that expected utility is the appropriate criterion to use in choosing among alternatives.

As a special degenerate case of Eq.A.9 we conclude that outcome $\underline{x}^A$ is *at least as desirable* as $\underline{x}^B$ *if and only if*

$$u(\underline{x}^A) \geq u(\underline{x}^B). \qquad \text{A.10}$$

This means that a utility function is also a value function (2). The reverse is not true, however.

A.3.1 Property Identification

The assessment of a utility function $u(\underline{x})$ includes the assessment of the preferences of the decision maker over lotteries involving the x's; i.e., over risky options yielding payoffs in terms of the x's. The direct assessment of u(x) becomes more and more difficult as the dimension of x and the number of possible x-outcomes increases. This assessment can be facilitated, however, if some information about the functional form of u(x) is available.

The basic approach utilized by Keeney and Raiffa in Ref [6] is: (1) to postulate various sets of assumptions about the basic preference attitudes of the decision maker, and (2) to derive functional forms of the multiattribute utility function consistent with these assumptions. In practice, this means that it must first be verified whether some of the assumptions are valid for the particular problem at hand; then a utility function consistent with the varified assumptions must be assessed. Ideally, a representation of the utility function is sought such that

$$u(x_1, x_2, \ldots, x_n) = f[f_1(x_1), f_2(x_2), \ldots, f_n(x_n)] \quad \text{A.11}$$

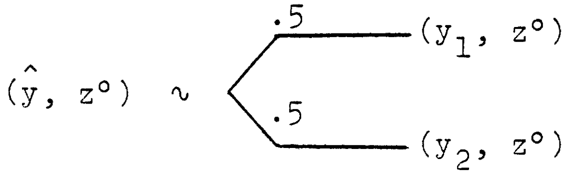
where f_i is a function of attribute X_i only, for i = 1,2, ..., n, and where f has a simple form, an additive or multiplicative form, for example. When this is possible, the assessment of u can be greatly simplified.

The fundamental concept of multiattribute utility theory upon which the various utility representations are based, is that of utility independence. Its role in multiattribute utility theory is similar to that of probabilistic independence in multivariate probability theory.

Let Y and Z denote two subsets of attributes.

Definition. Y is utility independent of Z if conditional preferences for lotteries on Y given $\underline{z}$ do not depend on the particular level of $\underline{z}$.

For example, let Y and Z contain only one attribute each. Furthermore, let us suppose that the decision maker asserted that he is indifferent between a certain option yielding $(\hat{y}, z^o)$ and a risky option yielding (y_1, z^o) with 50% chance and (y_2, z^o) also with 50% chance; i.e.,



If now the decision maker asserts that the $\hat{y}$ value does not change when we shift the z-value from z^o to another level, say z', and, in general, if he asserts that the $\hat{y}$ value depends only on y_1, y_2, and the associated probabilities and this is true for any fixed y_1, y_2 then, we say that the attribute $\underline{Y}$ is utility independent of the attribute Z.

If Y is utility independent of Z and Z is utility independent of Y then we say that Y and Z are mutually utility indepdendent.

Keeney and Raiffa present in Ref. [6] numerous simplifications of the form of the utility function that result from various degrees of utility independence among the attributes of a particular problem. The simplification that is of interest to our work is the one that involves the use of certainty equivalents.

A.3.2 Use of Certainty Equivalents

As stated in section IV.5.2, the certainty equivalent of a single attribute Y is the value $\hat{y}$ which, in the opinion of the decision maker, is equivalent to the uncertain option $\tilde{y}$. In a multiattribute decision problem each alternative is associated with an uncertain outcome which is characterized by a multivariate random veriable $\tilde{x}$. The certainty equivalent $\underline{\tilde{x}}$ would be the solution of the equation.

$$u(\underline{\hat{x}}) = E[u(\underline{\tilde{x}})]$$

where the expectation E is taken with repsect to the joint measure of x. Such an assessment requires the prior knowledge of the multiattribute utility function u(x). Nevertheless, the certainty equivalent $\underline{\hat{x}}$ can be easily assessed in cases that are formally described in the following theorem.

<u>Theorem</u>. The certainty equivalent $\underline{\hat{x}}$ for a lottery $\tilde{x}$ is given by

$$\hat{\underline{x}} = (\hat{x}_1, \hat{x}_2, \ldots, \hat{x}_n)$$

where $\hat{x}_i$ (i=1,2,...n) is the certainty equivalent for the one-dimensional variable $\tilde{x}_1$, calculated using the marginal probability distribution on $\tilde{x}_i$, provided that the attributes x_i are: (a) mutually utility independent, and (b) probabilistically independent.

In other words, if the preferences of the decision maker for lotteries involving one attribute and the probability distribution over this attribute do not depend on the levels of the other attributes, and if this is true for each and every attribute, then we can approach the decision problem as follows. First, n one-dimensional utility functions $u_i(x_i)$ (i=1,2,...n) are assessed. Next, for each alternative the n certainty equivalents $\hat{x}_i$(i = 1,2,...,n) are assessed using the appropriate marginal probability distributions. In this manner, the uncertain outcome of each alternative is replaced by a certain outcome; namely $\hat{\underline{x}} = (\hat{x}_1, \hat{x}_2, \ldots, \hat{x}_n)$, and the decision problem has been reduced into one under certainty. If a value function is defined over the $\underline{x}$'s, the ranking of of the alternatives can be achieved in terms of this value function and of the certainty equivalents. Of course, all the qualitative arguments using dominance and extended dominance (see Section IV.6.) are also valid.

APPENDIX B

INHERENT DIFFICULTY IN THE CONVERSION OF NUCLEAR MATERIAL TO WEAPONS - USABLE FORM

B.1 General Remarks

The purpose of this Appendix is to develop a scale for the attribute: inherent difficulty. As discussed in Section III.3.3, this attribute provides a measure of the degree of difficulty of the proliferation effort due to problems encountered in the conversion of fuel cycle materials to weapons usable form. Since a conventional measure for the degree of difficulty, e.g., cost or time, does not exist, this attribute needs to be decomposed into measurable sub-attributes.

B.2 Decomposition into measurable attributes

The fissile material contained in nuclear fuel may not be directly usable in nuclear explosives. In most cases, it must be "purified" to a certain degree by removing various kinds of unwanted material. In general, this "purification" involves chemical separation of different elements and/or isotopic separation of the fissile from the non-fissile uranium isotopes. Thus, we can say that the difficulty in nuclear material conversion is reduced, if the difficulty involved in the chemical and/or isotopic separation of the material is reduced. Therefore, the inherent difficulty can be decomposed into two components.

(1) Difficulty of chemical separation

(2) Difficulty of isotopic separation.

A potential proliferator using either of these techniques is faced with difficulties due partly to problems present in every industrial process and stemming from the associated scientific and technological complexity, and partly to problems stemming from the unique nuclear nature of these processes. A logical measure of the former problem is the availability of relevant information or "know-how" in the country in question, and of the latter, the radioactivity and criticality problems potentially present in the processes. We can, therefore, say that the difficulty of the chemical or the isotopic separation can be measured by the following three attributes.

(1) Status of information

(2) Degree of radioactivity

(3) Criticality problems

The inherent difficulty is thus decomposed into six sub-attributes (see figure B.1) which are discussed in the following subsections.

<u>Status of information</u>: The status of information refers to the existence and availability of the necessary "know-how"

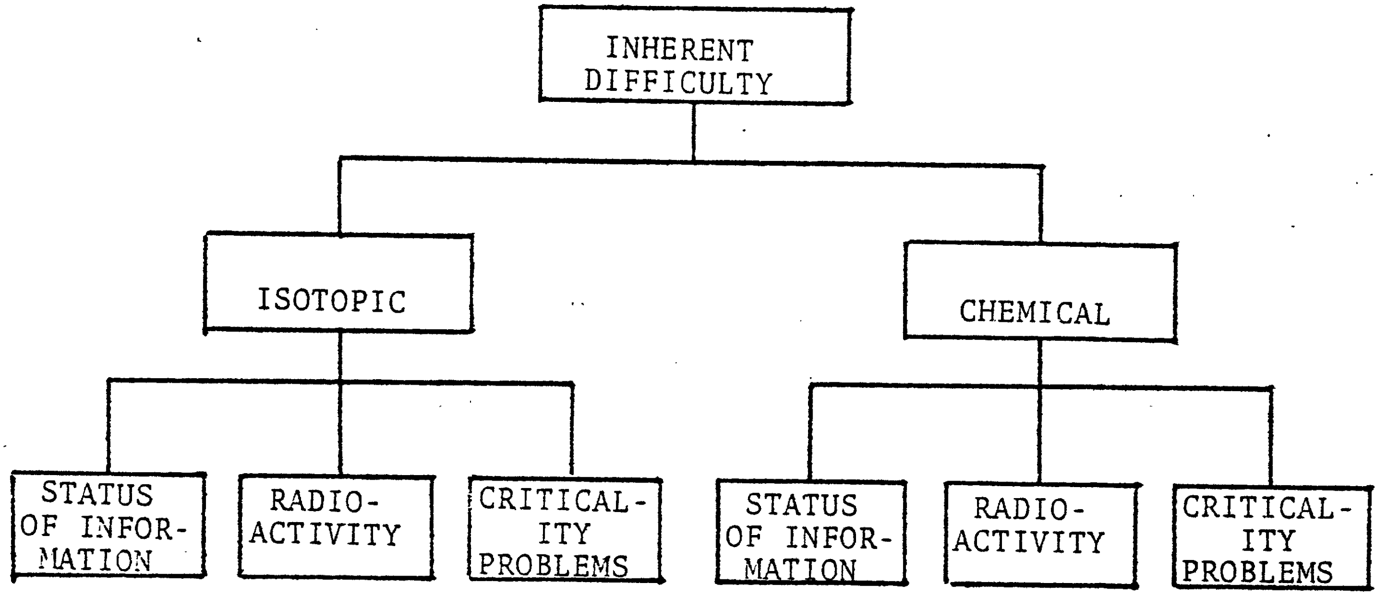


Figure B.1. Decomposition of the attribute inherent difficulty.

TABLE B.1

States of Information. Science and Technology Levels: 1≡Known; 2≡Readily Available; 3≡Unknown and/or classified.

STATES OF INFORMATION	SCIENCE	TECHNOLOGY
A	1	1
B	1	2
C	1	3
D	2	1
E	2	2
F	2	3
G	3	1
H	3	2
I	3	3

for the process in question. This information must be acquired by the prospective proliferator in order to succeed in separating the fissile material from the fuel. Information can be acquired by developing indigenous expertise and/or employing "foreign" experts.

There are two kinds of information pertaining to any industrial process:
(1) Scientific information dealing with the basic principles (physical laws and theories) on which the process is based; and (2) Technological information dealing with the implementation of the theoretical principles into an actual production process.

The status of each of these two kinds of information can be characterized by one of the following three levels.

Level 1: KNOWN

Level 2: READILY AVAILABLE

Level 3: UNKNOWN AND/OR CLASSIFIED.

Known scientific information means that the basic scientific principles and descriptions of the process are well understood in the country in question. This implies the existence of research center(s) and/or universities with active research in the relevant area, as well as the existence of small laboratories.

Known technological information means that the process is demonstrated in the country on larger than laboratory scale.

This assumes the existence of at least a pilot plant for chemical or isotopic separation of fissile material.

Readily available scientific information refers to information that exists in the open literature and to information that can be acquired by training scientific personnel in advanced countries (universities and/or government laboratories).

Readily available technological information refers to processes that have been developed and are used by technologically advanced countries. These countries are, furthermore, willing to transfer the pertinent technological know-how in the form of aid or trade.

Unknown and/or classified scientific information refers to information related to processes that have been either developed by advanced countries but kept classified, or that have been proposed based on general physical principles, but, at the moment, lack the necessary scientific research and development which are required to demonstrate feasibility.

Unknown and/or classified technological information refers to processes that have not been proved yet on a large scale, or that have been kept classified.

The combination of the three levels for the scientific and technological information result in 9 possible states for the attribute: status of information. These 9 states are tabulated in TABLE B.1. Some examples of how these states can be used to characterize the status of information in a particular country follow.

For Japan the status of information for chemical separation of Pu from spent fuel is A. This means that relevant processes are well understood and demonstrated in the country.

For Brazil the status of information for chemical separation of Pu from spent fuel (in the present "state of the world") is B. This means that the scientific "know-how" exists in the country and the technology is readily available (for example, can be bought).

For Nigeria the status of information for chemical separation of Pu from spent fuel (in the present "state of the world") is E. This means that neither the scientific nor the technological "know-how" exist in the country, but they can be acquired.

For most countries the status of information relevant to isotopic enrichment by diffusion is either F or E.

States G and H, and in general states that have the scientific information in a "higher" level than the technological information, correspond to situations in which a country has relevant industrial activities but it has not developed the aspect of the technology that can be used in the fissile material separation. For example, a country might have a strong laser-related industry and yet the information concerning laser enrichment could be classified or unknown.

Scale of status of information. As discussed in section III.2, the decomposition of a sub-objective stops whenever an operational measure of effectiveness of this sub-objective exists. Furthermore, we saw in the previous subsection that the status of information can be in any of nine possible states. Therefore, the first step in the development of a scale for the status of information would be to order these nine states in terms of increasing difficulty. We can then think of the status of information as a discrete variable i that can take nine values $(i=1,2,\ldots,9)$, i.e., we can think of a mapping $d(X)=i$ of the nine states $(X=A,\ldots,I)$ to the nine integers $(i=1,\ldots,9)$. Thus a state X would be more difficult than Y, if and only if $d(X) > d(Y)$. Of course, this scaling represents only an ordinal ordering of the states in terms of increasing difficulty and not a cardinal ordering. If, for example $d(X) = 6$ and $d(Y) = 3$, we know that Y represents an easier state than X but we don't know how much easier. A cardinal ordering of the states will result from the assessment of the preferences of the proliferator about the various states. The generation of the ordinal scale for the status of information is presented in Section B.4 while the assessment of a cardinal scale is in Section B.6.

Radioactivity. The second measure of the inherent difficulty in the separation of the fissile material is the radioactivity of the materials involved in the separation process. This activity measured 1 meter from the material can be anywhere from less than 10 rad/hr (cold) up to 10^6 rads/hr (very hot). Obviously the higher the radioactivity the more difficult the separation process.

Criticality Problems. The third measure of the inherent difficulty is the potential for criticality accidents during the separation of the fissile material. The extent to which such problems exist depends on the particular material and on the size of the facility in use.

For the purposes of this analysis two values of this attribute have been assumed: (1) High criticality problems; and (2) Low criticality problems.

B.3 Index of Inherent Difficulty

The attribute: inherent difficulty can be decomposed (as seen in the previous section) into six measurable subattributes. A value function assessed on these six subattributes can serve as a subjective index for the inherent difficulty. This index can then be used either in assessing a value function over the five attributes of the proliferation resistance (see Appendix E) or for simple intercomparisons of two pathways.

In principle, this procedure could present two problems: First, the use of an aggregate index for inherent difficulty assumes the existence of preferential independence (See Appendix A) among the set Z of the inherent difficulty sub-attributes and the remaining 4 attributes of the proliferation resistance. Secondly, even if preferential independence exists, a particular value of this index is not necessarily intuitively meaningful, and thus may not be useful for tradeoffs with other attributes. For the present application the first problem is not very serious. From preliminary assessments it seems very probable that the set of the inherent difficulty sub-attributes is preferentially independent of the other proliferation resistance attributes. Even if it turns out that this is not always true, ranges of the attributes for which preferential independence exists can be found, and the problem can be solved repeatedly in each of these ranges. The second problem, however, might present serious difficulties. This is because it is highly improbable that statements of the sort: "How much, in terms of attribute x, is a change in inherent difficulty from .5 to .6 worth?" will be meaningful to the decision maker. The .5 and .6 values are meaningful only up to monotone transformations. This is not due to the lack of operational procedures for the structure of isopreference curves between the aggregate attribute of inherent difficulty and any other attributes; these do exist, however, tradeoffs between inherent difficulty and other attributes might not be meaningful to a decision maker even though the inherent difficulty is precisely defined in terms of the six attributes. This could happen if the decision maker, whose preferences about time, money and difficulty must be assessed, is an individual who lacks the requisite technical background. It follows, therefore, that there is a need of a measure of the difficulty that will make sense to the rather non-technically minded decision maker. Such a measure could be the probability of successful completion of the task, conditional on the absence of outside intervention. Such a probability measure can be developed by technical experts combining the inherent difficulty index with the difficulty in the weapon design and fabrication.

In the remaining of this Appendix we demonstrate how a value function can be assessed over the sub-attributes of inherent difficulty. Furthermore, we use this value function to cardinally rank a number of proliferation pathways in terms of decreasing inherent difficulty. This ranking is then compared with an ordinal ranking provided by SAI[2].

B.4 Questionnaire for Development of an Ordinal Scale for the Status of Information for Isotopic Separation

The 9 states of information characterizing the isotopic separation of weapons material are given in TABLE B.1. We want to rank these states in order of increasing difficulty. In other words, we want to generate a correspondence between the 9 states and the nine integers 1,2,...9 where 1 corresponds to state A, 9 to state I and if i>j, the state that corresponds to i represents a more difficult situation than the state that corresponds to j.

For each of the following pairs of states of information, indicate the state that in your opinion represents the lesser difficulty, and hence, is more preferred. For example, if you think that:

(a) H represents a less difficult state than F, then d(H)<d(F)

(b) H represents an equally difficult state as F, then d(H)=d(F)

(c) H represents a more difficult state than F, then d(H)>d(F)

Please compare:

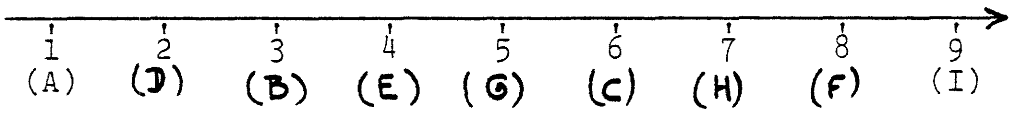
			> = < ?	
Q.1	H versus F	d(H)	<	d(F)
Q.2	H versus C	d(H)	>	d(C)
Q.3	G versus F	d(G)	<	d(F)
Q.4	G versus E	d(G)	>	d(E)
Q.5	G versus C	d(G)	<	d(C)
Q.6	G versus B	d(G)	>	d(B)
Q.7	E versus C	d(E)	<	d(C)
Q.8	D versus C	d(D)	<	d(C)
Q.9	D versus B	d(D)	<	d(B)

To order the 9 states of information in terms of decreasing difficulty $\binom{9}{2}=36$ comparisons are required. From the definition of the states, however, the relations between the elements of 27 of those pairs are uniquely defined. The remaining 9 are determined by answering Questions Q.1 through Q.9. A sample response and the resultant ordering are given below. (The obvious relations of the states with I and A are omitted.)

$H < G$
Q.1? $H > F$ ✓
$H < E$
$H < D$
Q.2? $H < C$ ✓
$H < B$
Q.3? $G > F$ ✓
Q.4? $G < E$ ✓
$G < D$
Q.5? $G > C$ ✓
Q.6? $G < B$ ✓

$F < E$
$F < D$
$F < C$
$F < B$
$E < D$
Q.7? $E > C$ ✓
$E < B$
Q.8? $D > C$ ✓
Q.9? $D > B$ ✓
$C < B$

Resulting ordering:



B.5 Questionnaire for Development of an Ordinal Scale for the Status of Information for Chemical Separation

The 9 states of information characterizing the chemical separation of weapons material are given in TABLE B.1. We want to rank these states in order of increasing difficulty. In other words, we want to generate a correspondence between the 9 states and the nine integers 1,2,...9 where 1 corresponds to state A, 9 to state I and if i>j, the state that corresponds to i represents a more difficult situation than the state that corresponds to j.

For each of the following pairs of states of information, indicate the state that in your opinion represents the lesser difficulty, and hence, is more preferred. For example, if you think that:

(a) H represents a less difficult state than F, then d(H)<d(F)

(b) H represents an equally difficult state as F, then d(H)=d(F)

(c) H represents a more difficult state than F, then d(H)>d(F)

Please compare:

			> = < ?	
Q.1	H versus F	d(H)	<	d(F)
Q.2	H versus C	d(H)	>	d(C)
Q.3	G versus F	d(G)	<	d(F)
Q.4	G versus E	d(G)	>	d(E)
Q.5	G versus C	d(G)	<	d(C)
Q.6	G versus B	d(G)	>	d(B)
Q.7	E versus C	d(E)	<	d(C)
Q.8	D versus C	d(D)	<	d(C)
Q.9	D versus B	d(D)	<	d(B)

To order the 9 states of information in terms of decreasing difficulty $\binom{9}{2}=36$ comparisons are required. From the definition of the states, however, the relations between the elements of 27 of those pairs are uniquely defined. The remaining 9 are determined by answering Questions Q.1 through Q.9. A sample response and the resultant ordering are given below. (The obvious relations of the states with I and A are omitted.)

$H < G$
Q.1? $H > F$ ✓
$H < E$
$H < D$
Q.2? $H < C$ ✓
$H < B$
Q.3? $G > F$ ✓
Q.4? $G < E$ ✓
$G < D$
Q.5? $G > C$ ✓
Q.6? $G < B$ ✓

$F < E$
$F < D$
$F < C$
$F < B$
$E < D$
Q.7? $E > C$ ✓
$E < B$
Q.8? $D > C$ ✓
Q.9? $D > B$ ✓
$C < B$

Resulting ordering:

1	2	3	4	5	6	7	8	9
(A)	(D)	(B)	(E)	(G)	(C)	(H)	(F)	(I)

B.6 Testing for Preferential Independence for the Inherent-Difficulty Attributes

We consider the LWR-Denatured Thorium cycle with reactors only allowed to operate in a country of Type B (See Section IV.2). The nuclear weapons aspiration is 10 weapons of military quality in one year (a_2).

The following questions consider tradeoffs between the cost attribute and the sub-attributes of inherent difficulty. The levels of the remaining attributes (Development Time, Warning Period, and Weapons Material) will be held constant at pre-specified values. Thus, an alternative will be denoted by

$$\{x, z_1, z_2, \ldots z_6\}$$

where

x : denotes the cost

z_1: the status of information for chemical separation

z_2: the radioactivity level for chemical separation

z_3: the level of the criticality problems for chemical separation

z_4: the status of information for isotopic separation

z_5: the radioactivity level for isotopic separation

z_6: the level of the criticality problems for isotopic separation

For the following questions, the attributes: Development Time (x_1), Warning Period (x_2) and Weapons Material (x_4) have the following values:

x_1 = 4 years

x_2 = 10%

x_4 = H.E. Uranium-233

We consider a pathway that consists in seizing the spent fuel, separating chemically the Uranium from the Thorium and Pu, and then, enriching the fuel in U-233.

For an all-covert mode of operation the inherent difficulty attributes have the following values: z_1=B, z_2=10^6 rad/hr, z_3=HIGH, z_4=C, z_5=10^2 rad/hr, z_6=HIGH, and the cost of this operation is \$100 million. The pertinent questions which test for preferential independence and sample responses are given below.

Define the amount of money for which you would be indifferent between the following alternatives.

1. $\{100\$M, B, 10^{6}, \text{HIGH}, \underline{C}, 10^{2}, \text{HIGH}\} \sim \{200, B, 10^{6}, \text{HIGH}, \underline{A}, 10^{2}, \text{HIGH}\}$
2. $\{100\$M, B, 10^{6}, \text{HIGH}, C, \underline{10^{2}}, \text{HIGH}\} \sim \{110, B, 10^{6}, \text{HIGH}, C, \underline{0}, \text{HIGH}\}$
3. $\{100\$M, B, 10^{6}, \text{HIGH}, C, 10^{2}, \underline{\text{HIGH}}\} \sim \{105, B, 10^{6}, \text{HIGH}, C, 10^{2}, \underline{\text{LOW}}\}$
4. $\{100\$M, \underline{B}, 10^{6}, \text{HIGH}, C, 10^{2}, \text{HIGH}\} \sim \{120, \underline{A}, 10^{6}, \text{HIGH}, C, 10^{2}, \text{HIGH}\}$
5. $\{100\$M, B, \underline{10^{6}}, \text{HIGH}, C, 10^{2}, \text{HIGH}\} \sim \{140, B, \underline{0}, \text{HIGH}, C, 10^{2}, \text{HIGH}\}$
6. $\{100\$M, B, 10^{6}, \underline{\text{HIGH}}, C, 10^{2}, \text{HIGH}\} \sim \{105, B, 10^{6}, \underline{\text{LOW}}, C, 10^{2}, \text{HIGH}\}$

Let us call the pathway we are examining pathway I. We now consider a variation of this pathway: Pathway II has exactly the same values for the attributes Development Time, Warning Period, Weapons Material, and Cost as pathway I but now reprocessing of the fuel is not necessary before enrichment. (This corresponds to using the fresh, denatured fuel as source material.)

For pathway II please answer the following questions.

7. $\{100\$M, \text{—}, \text{—}, \text{—}, \underline{C}, 10^{2}, \text{HIGH}\} \sim \{200, \text{—}, \text{—}, \text{—}, \underline{A}, 10^{2}, \text{HIGH}\}$
8. $\{100\$M, \text{—}, \text{—}, \text{—}, C, \underline{10^{2}}, \text{HIGH}\} \sim \{110, \text{—}, \text{—}, \text{—}, C, \underline{0}, \text{HIGH}\}$
9. $\{100\$M, \text{—}, \text{—}, \text{—}, C, 10^{2}, \underline{\text{HIGH}}\} \sim \{105, \text{—}, \text{—}, \text{—}, C, 10^{2}, \underline{\text{LOW}}\}$

Finally, we consider a third variation of pathway I, pathway III, which has exactly the same values as pathway I, for the attributes Development Time, Warning Period, Weapons Material, and Cost but now enrichment of the material is not necessary. In this case, we assume that after reprocessing the material is exchanged with enriched fuel without having to do the enrichment ourselves.

For pathway III please answer the following questions.

10. $\{100\$M, \underline{B}, 10^{6}, \text{HIGH}, \text{—}, \text{—}, \text{—},\} \sim \{120, \underline{A}, 10^{6}, \text{HIGH}, \text{—}, \text{—}, \text{—}\}$
11. $\{100\$M, B, \underline{10^{6}}, \text{HIGH}, \text{—}, \text{—}, \text{—},\} \sim \{140, B, \underline{0}, \text{HIGH}, \text{—}, \text{—}, \text{—}\}$
12. $\{100\$M, B, 10^{6}, \underline{\text{HIGH}}, \text{—}, \text{—}, \text{—},\} \sim \{105, B, 10^{6}, \underline{\text{LOW}}, \text{—}, \text{—}, \text{—}\}$

Are your answers in questions #1 and #7 the same?

1. ☑ YES : Is it always true that the amount of money you would be willing to spend in order to achieve a certain reduction in the difficulty involved with the status of information of the isotopic separation of the weapons material does not depend on the level of the difficulty associated with the chemical separation?

    1.1 ☑ YES ⇒ Cost & status of information Preferentially Independent (P.I.) of the chemical difficulty

    1.2 ☐ NO ⇒ Go to 2.1.

2. ☐ NO : Were you aware that these questions involved the same tradeoff between cost and status of information for isotopic separation but at different levels of difficulty for the chemical separation?

    2.1 ☐ YES ⇒ Explain in which way tradeoffs between money and status of information depend on the level of difficulty of the chemical separation.

    2.2 ☐ NO ⇒ Do you still feel that the value of going from C to A in questions #1 and #7 is different?

        2.2.1 ☐ YES ⇒ Go to 2.1.

        2.2.2 ☐ NO ⇒ Go to 1.

Are your answers in questions #2 and #8 the same?

1. ☑ YES : Is it always true that the amount of money you would be willing to spend in order to achieve a certain reduction in the difficulty involved with the radioactivity of the isotopic separation of the weapons material <u>does not</u> depend on the level of the difficulty associated with the chemical separation?

    1.1 ☑ YES ⇒ Cost & radioactivity Preferentially Independent (P.I.) of the chemical difficulty

    1.2 ☐ NO ⇒ Go to 2.1.

2. ☐ NO : Were you aware that these questions involved the same tradeoff between cost and radioactivity for isotopic separation but at different levels of difficulty for the chemical separation?

    2.1 ☐ YES ⇒ Explain in which way tradeoffs between money and radioactivity depend on the level of difficulty of the chemical separation.

    2.2 ☐ NO ⇒ Do you still feel that the value of going from 10^2 to 0 in questions #2 and #8 is different?

        2.2.1 ☐ YES ⇒ Go to 2.1.

        2.2.2 ☐ NO ⇒ Go to 1.

Are your answers in questions #3 and #9 the same?

1. ☑ YES: Is it always true that the amount of money you would be willing to spend in order to achieve a certain reduction in the difficulty involved with the criticality problems of the isotopic separation of the weapons material <u>does</u> <u>not</u> depend on the level of the difficulty associated with the chemical separation?

   1.1 ☑ YES ⇒ Cost & criticality problems Preferentially Independent (P.I.) of the chemical difficulty

   1.2 ☐ NO ⇒ Go to 2.1.

2. ☐ NO: Were you aware that these questions involved the same tradeoff between cost and criticality problems for isotopic separation but at different levels of difficulty for the chemical separation?

   2.1 ☐ YES ⇒ Explain in which way tradeoffs between money and criticality problems depend on the level of difficulty of the chemical separation.

   2.2 ☐ NO ⇒ Do you still feel that the value of going from HIGH to LOW in questions #3 and #9 is different?

      2.2.1 ☐ YES ⇒ Go to 2.1.

      2.2.2 ☐ NO ⇒ Go to 1.

Are your answers in questions #4 and #10 the same?

1. ☑ YES: Is it always true that the amount of money you would be willing to spend in order to achieve a certain reduction in the difficulty involved with the status of information of the chemical separation of the weapons material <u>does</u> <u>not</u> depend on the level of the difficulty associated with the isotopic separation?

    1.1 ☑ YES ⇒ Cost & status of information Preferentially Independent (P.I.) of the isotopic difficulty

    1.2 ☐ NO ⇒ Go to 2.1.

2. ☐ NO: Were you aware that these questions involved the same tradeoff between cost and status of information for chemical separation but at different levels of difficulty for the isotopic separation?

    2.1 ☐ YES ⇒ Explain in which way tradeoffs between money and status of information depend on the level of difficulty of the isotopic separation.

    2.2 ☐ NO ⇒ Do you still feel that the value of going from B to A in questions #4 and #10 is different?

        2.2.1 ☐ YES ⇒ Go to 2.1.

        2.2.2 ☐ NO ⇒ Go to 1.

Are your answers in questions #5 and #11 the same?

1. ☑ YES: Is it always true that the amount of money you would be willing to spend in order to achieve a certain reduction in the difficulty involved with the radioactivity of the chemical separation of the weapons material <u>does not</u> depend on the level of the difficulty associated with the isotopic separation?

    1.1 ☑ YES ⇒ Cost & radioactivity Preferentially Independent (P.I.) of the isotopic difficulty

    1.2 ☐ NO ⇒ Go to 2.1.

2. ☐ NO: Were you aware that these questions involved the same tradeoff between cost and radioactivity for chemical separation but at different levels of difficulty for the isotopic separation?

    2.1 ☐ YES ⇒ Explain in which way tradeoffs between money and radioactivity depend on the level of difficulty of the isotopic separation.

    2.2 ☐ NO ⇒ Do you still feel that the value of going from 10^6 to 0 in questions #5 and #12 is different?

        2.2.1 ☐ YES ⇒ Go to 2.1.

        2.2.2 ☐ NO ⇒ Go to 1.

Are your answers in questions #6 and #12 the same?

1. ☑ YES: Is it always true that the amount of money you would be willing to spend in order to achieve a certain reduction in the difficulty involved with the criticality problems of the chemical separation of the weapons material <u>does</u> <u>not</u> depend on the level of the difficulty associated with the isotopic separation?

    1.1 ☑ YES ⇒ Cost & criticality problems Preferentially Independent (P.I.) of the isotopic difficulty

    1.2 ☐ NO ⇒ Go to 2.1.

2. ☐ NO: Were you aware that these questions involved the same tradeoff between cost and criticality problems for chemical separation but at different levels of difficulty for the isotopic separation?

    2.1 ☐ YES ⇒ Explain in which way tradeoffs between money and criticality problems depend on the level of difficulty of the isotopic separation.

    2.2 ☐ NO ⇒ Do you still feel that the value of going from HIGH to LOW in questions #6 and #12 is different?

        2.2.1 ☐ YES ⇒ Go to 2.1.

        2.2.2 ☐ NO ⇒ Go to 1.

If I were to change the levels of the attributes: Development Time, Warning Period, and Weapons Material from the values they had before to:

Development Time x_1 = 2 years
Warning Period x_2 = 1%
Weapons Material x_4 = H.E. Uranium-235

would your questions 1 to 3 change?

<u>Question 1</u>: 1. ☐ YES: Explain why you feel that the value of going from C to A in the status of information for isotopic separation is different.

2. ☑ NO: Would it be correct to say that the additional amount of <u>money</u> you would pay for a particular change in the <u>status of information for the isotopic</u> separation depends only on the initial level of cost and on the initial and final states of the information and on nothing else?

2.1 ☑ YES ⇒ <u>Cost</u> & <u>Status of Information for isotopic separation</u> P.I.

2.2 ☐ NO ⇒ Elaborate.

Question 2: 1. ☐ YES : Explain why you feel that the value of reducing by 10^2 rad/hr the radioactivity level in the isotopic separation is different under the present circumstances:

2. ☑ NO : Would it be correct to say that the additional amount of money you would pay for a particular reduction in the radioactivity level of the isotopic separation depends only on the initial level of cost and on the initial and final levels of the radioactivity and on nothing else?

2.1 ☑ YES ⇒ Cost & Radioactivity for isotopic separation P.I.

2.2 ☐ NO ⇒ Elaborate.

Question 3: 1. ☐ YES: Explain why you feel that the value of reducing the criticality problems in the isotopic separation is different now.

2. ☑ NO: Would it be correct to say that the additional amount of money you would pay for the reduction of the criticality problem depends only on the initial level of cost and on nothing else?

2.1 ☑ YES ⇒ Cost & Criticality Problem for isotopic separation P.I.

2.2 ☐ NO ⇒ Elaborate.

With the new levels of the attributes x_1, x_2, x_4 ($x_1=2$ years, $x_2=1\%$, x_4=H.E. Uranium-235), would your answers to questions 4 to 6 change?

Question 4: 1. ☐ YES: Explain why you feel that the value of going from B to A in the status of information for chemical separation is now different.

2. ☑ NO: Would it be correct to say that the additional amount of money you would pay for a particular change in the status of information for the chemical separation depends only on the initial level of cost and on the initial and final states of the information and on nothing else?

2.1 ☑ YES ⇒ Cost & Status of Information for Chemical Separation P.I.

2.2 ☐ NO ⇒ Elaborate.

Question 5: 1. ☐ YES: Explain why you feel that the value of reducing by 10^6 rad/hr the radioactivity level in the chemical separation is different under the present circumstances.

2. ☑ NO: Would it be correct to say that the additional amount of money you would pay for particular reduction in the radioactivity level of the chemical separation depends only on the initial level of cost and on the initial and final levels of the radioactivity and on nothing else:

2.1 ☑ YES ⇒ Cost & Radioactivity of Chemical Separation P.I.

2.2 ☐ NO ⇒ Elaborate.

Question 6: 1. ☐ YES: Explain why you feel that the value of reducing the criticality problems in the chemical separation is different now.

2. ☑ NO: Would it be correct to say that the additional amount of money you would pay for the reduction of the criticality problems in the chemical separation depends only on the initial level of cost and on nothing else?

2.1 ☑ YES ⇒ Cost & Criticality Problem: for chemical separation P.I.

2.2 ☐ NO ⇒ Elaborate.

B.7 Value Function Assessment over the Inherent-Difficulty Attributes

The answers to the questions of the previous section indicate that the set of inherent-difficulty attributes is mutually preferentially independent (see Appendix A, Sec. A.2.3). It follows, therefore, that a value function defined over these six attributes will be of the additive form (see Sec. A.2.3), namely

$$v(z) = \sum_{i=1}^{6} \lambda_i v_i(z_i) \qquad \text{B.1}$$

In this section we present the assessment of the component value functions $v_i(z_i)$ (sections B.7.1 to B.7.6) and of the weighting coefficients λ_i (section B.7.7).

B.7.1 Component Value Function For Radioactivity In Chemical Separation

The purpose of this section is to assess a value function for the attribute "radioactivity" for chemical separation. The range of this attribute is from 0 rad/hr up to 10^6 rad/hr. Thus, the use of a logarithmic scale seems appropriate.

We first define certain properties of the function.

A. MONOTONICITY If r represents a level of radioactivity is it always true that

$$r \geq r' \text{ implies } v(r) \leq v(r') \text{ ?}$$

1. ☑ YES ⇒ The function is monotonic.
2. ☐ NO ⇒ Describe form of function (Establish regions of monotonicity).

B. CONVEXITY AND CONCAVITY

We can determine the shape of the value functions if the following questions are answered.

For the following pairs of changes in radioactivity establish the one that corresponds to a larger increment in the difficulty.

Q.1.	(1 - 10)	$<$	$(10 - 10^2)$
Q.2.	$(10 - 10^2)$	$<$	$(10^2 - 10^3)$
Q.3.	$(10^2 - 10^3)$	$\sim$	$(10^3 - 10^4)$
Q.4.	$(10^3 - 10^4)$	$>$	$(10^4 - 10^5)$
Q.5.	$(10^4 - 10^5)$	$>$	$(10^5 - 10^6)$

Monotonic: Convex ☐ , Concave ☐ , S-Shaped ☑
Non Monotonic: Shape

C. MIDVALUE SPLITTING TECHNIQUE

We set the value of 1 rad/hr of radioactivity at zero and the value of 10^6 rad/hr at -1. i.e.

$$v(1)=0 \quad \text{and} \quad v(10^6)=-1$$

We want now to establish the levels of radioactivity that have values of -.50, -.75, -.25. To do so, we use the midvalue splitting technique. According to this technique, you are asked to establish the level of radioactivity $r_{.5}$ for which you think that the difficulty in going from a zero level to $r_{.5}$ is equivalent to the difficulty in going from $r_{.5}$ to 10^6 rad/hr i.e.

Q.1. $\{10^6 \to r_{.5}\} \sim \{r_{.5} \to 1\}$ $\boxed{r_{.5} = 10^3 \text{ rad/hr}}$ *

Since $v(r_{.5})-v(10^6)=v(1)-v(r_{.5})$ $v(r_{.5})=-.5$

Similarly we establish the -.75 and -.25 values by answering the following questions.

Q.2. $\{10^6 \to r_{.75}\} \sim \{r_{.75} \to r_{.5}\}$ $\boxed{r_{.75} = 5\times 10^3 \text{ rad/hr}}$

$v(r_{.75})=-.75$

Q.3. $\{r_{.5} \to r_{.25}\} \sim \{r_{.25} \to 0\}$ $\boxed{r_{.25} = 2\times 10^2 \text{ rad/hr}}$

$v(r_{.25})=-.25$

<u>Result</u>: S-shaped like normal cumulative function (see Figure B.2)

Radio-activity (Log.)	0	1	2	3	4	5	6
Value	0	-.02	-.16	-.50	-.84	-.96	-1.

*Boxed relations represent responses of the decision maker.

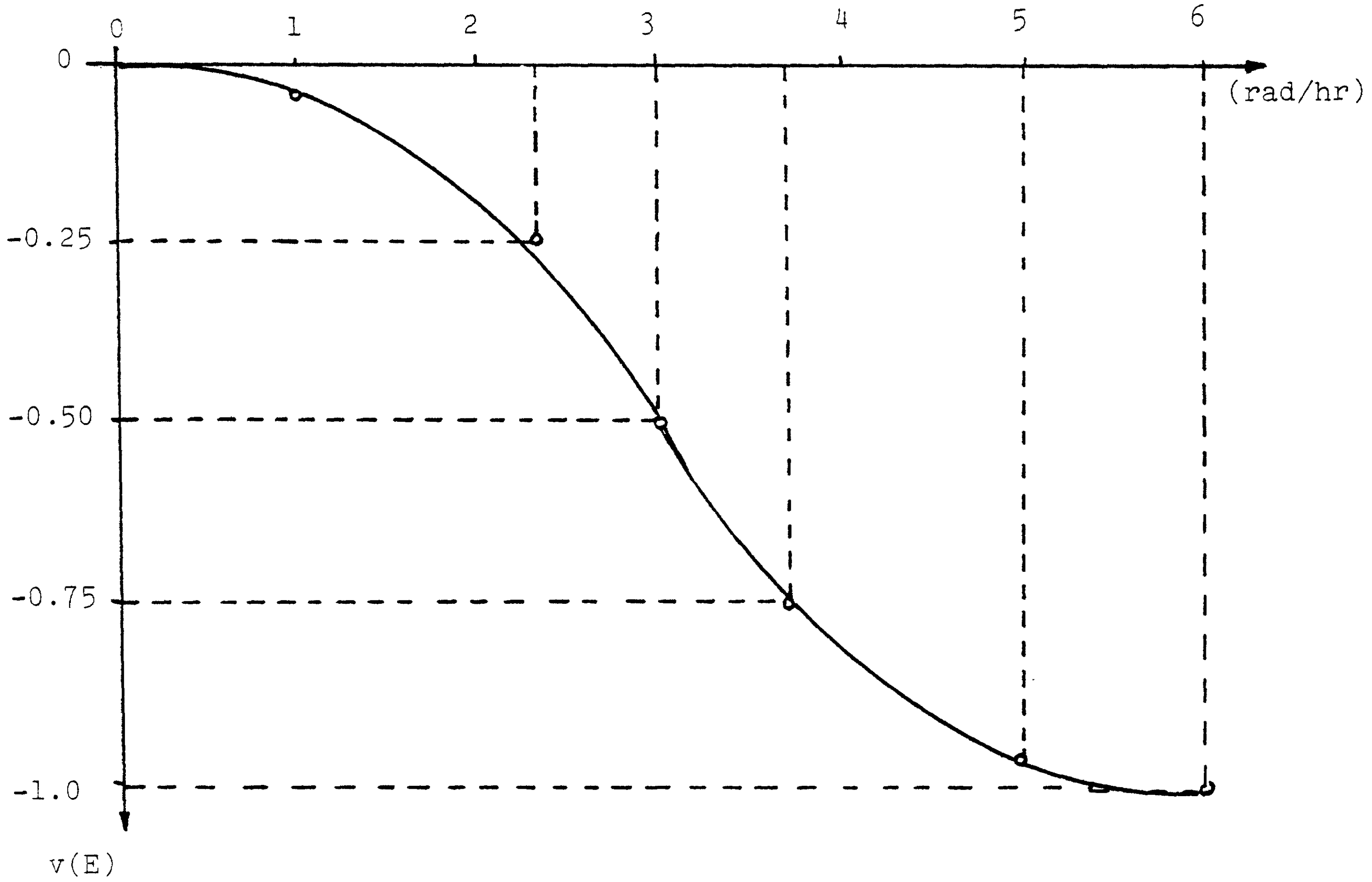


Figure B.2. Component Value Function for Radioactivity Level of Chemical Separation.

B.7.2. Component Value Function For Status Of Information For Chemical Separation

The purpose of this section is to assess a value function for the attribute "status of information" for chemical separation. This attribute can take 9 discrete values (1 to 9) which have been already ordered in an ordinal sense. The assessment of a value function will provide a cardinal ordering for these values.

We first define certain properties of the function.

A. MONOTONICITY. From the definition of the ordinal scale for the status of information it follows that the value function is monotonic i.e.

$$v(X)>v(Y) \text{ if and only if } d(X)<d(Y)$$

where $d(X)$ is the integer (1-9) corresponding to state X(A-I).

B. CONVEXITY AND CONCAVITY

We consider changes in the status of information and denote them by $(i-1 \to i)$ meaning that the status of information has changed from the state corresponding to $(i-1)$ to the one corresponding to (i). Then by comparing pairs of such changes we want to establish which one involves the highest change in the inherent difficulty.

For example, if

$(i-1 \to i) < (i \to i+1)$ then $v(i)-v(i-1) < v(i+1)-v(i)$

please answer the following questions:

Q.1. $(1 \to 2) < (2 \to 3)$
Q.2. $(2 \to 3) < (3 \to 4)$
Q.3. $(3 \to 4) < (4 \to 5)$
Q.4. $(4 \to 5) \sim (5 \to 6)$
Q.5. $(5 \to 6) > (6 \to 7)$
Q.6. $(6 \to 7) > (7 \to 8)$
Q.7. $(7 \to 8) > (8 \to 9)$

MONOTONIC: CONVEX ☐ CONCAVE ☐ S-SHAPED ☑

NONMONOTONIC: SHAPE

C. MIDVALUE SPLITTING TECHNIQUE

We set the value of 1 (or A) at 0 and the value of 9 (or I) at -1. Thus

$$v(1)=0 \quad \text{and } v(9)=-1$$

We now want to establish a value i of the status of information such that the decrease in difficulty in going from 9 to i is the same in going from i to 1.

Q.1. $\{9\to i\} \sim \{i \to 1\}$

or equivalently define a state X such that

$\{I\to X\} \sim \{X\to A\}$

$\boxed{\begin{array}{c} 5<i<6 \\ G \succ X \succ C \\ x=5.5 \end{array}}$ *

Since the status of information can take only integer values it might not be possible to identify an i (X) that satisfies Q.1. In that case we can define a noninteger value x ($i<x<i+1$) and artificially put $v(X)=-.50$. After establishing the value functions we can go back and check if the values v(i) and v(i+1) agree with the preferences of the assessor.

In the same way we establish the -.25 and -.75 points.

Q.2. $\{9\to \overset{?}{y}\}$ $\{\overset{?}{y}\to 1\}$

or equivalently define a state Y such that

$\{I\to Y\}$ $\{Y\to X\}$

$v(Y)=-.75$

$\boxed{\begin{array}{c} 4<y<5 \\ E \succ Y \succ G \\ y=4.5 \end{array}}$

Q.3. $\{i\to \overset{?}{z}\}$ $\{\overset{?}{z}\to 1\}$

or equivalently define a state Z such that

$\{X\to Z\}$ $\{Z\to A\}$

$v(Z)=-.25$

$\boxed{\begin{array}{c} 6<z<7 \\ C \succ Z \succ H \\ z=6.5 \end{array}}$

Result: S-Shaped like normal cumulative function (see Figure B.3)

Status of Information	A	D	B	E	G	C	H	F	I
Value	0	-.01	-.05	-.16	-.37	-.67	-.84	-.95	-.99

*Boxed relations represent responses of the decision maker.

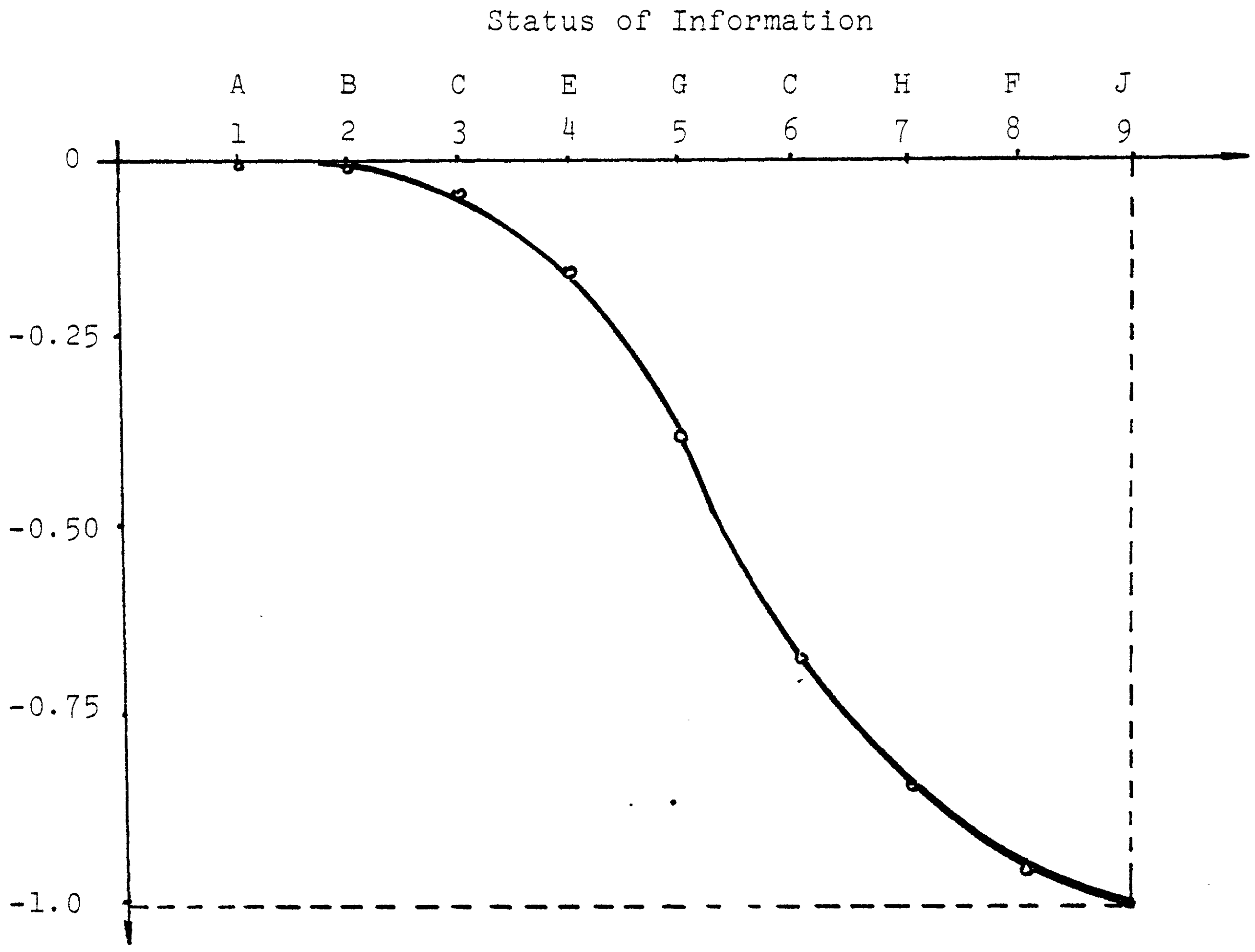


Figure B.3. Component Value Functions for Status of Information for Chemical Separation

B.7.3 Component Value Function For Criticality Problems For Chemical Separation

The purpose of this section is to assess a value function for the attribute "criticality problems" for chemical separation. Since this attribute can take only two values (HIGH & LOW), we simply assign the value of zero to the low level, and the value of minus unity to the HIGH level. Thus

$$v(\text{LOW}) = 0 \qquad v(\text{HIGH}) = -1$$

B.7.4. Component Value Function For Radioactivity For Isotopic Separation

The purpose of this section is to assess a value function for the attribute "radioactivity" for isotopic separation. The range of this attribute is from 0 rad/hr up to 10^6 rad/hr. Thus, the use of a logarithmic scale seems appropriate.

We first define certain properties of the function.

A. MONOTONICITY If r represents a level of radioactivity is it always true that

$r \geq r'$ implies $v(r) \leq v(r')$?

1. ☑ YES: The function is monotonic.
2. ☐ NO: Describe form of function (Establish regions of monotonicity).

B. CONVEXITY AND CONCAVITY

We can determine the shape of the value functions if the following questions are answered.

For the following pairs of changes in radioactivity establish the one that corresponds to a larger increment in the difficulty.

Q.1.	$(1 - 10)$	$\prec$	$(10 - 10^2)$
Q.2.	$(10 - 10^2)$	$\prec$	$(10^2 - 10^3)$
Q.3.	$(10^2 - 10^3)$	$\sim$	$(10^3 - 10^4)$
Q.4.	$(10^3 - 10^4)$	$\succ$	$(10^4 - 10^5)$
Q.5.	$(10^4 - 10^5)$	$\succ$	$(10^5 - 10^6)$

MONOTONIC: CONVEX ☐ CONCAVE ☐ S-SHAPED ☑

C. MIDVALUE SPLITTING TECHNIQUE

We set the value of 1 rad/hr of radioactivity at zero and the value of 10^6 rad/hr at -1. i.e.

$$v(1)=0 \quad \text{and} \quad v(10^6)=-1$$

We want now to establish the levels of radioactivity that have values of -.50, -.75, -.25. To do so, we use the midvalue splitting technique. According to this technique, you are asked to establish the level of radioactivity $r_{.5}$ for which you think that the difficulty in going from a zero level to $r_{.5}$ is equivalent to the difficulty in going from $r_{.5}$ to 10^6 rad/hr i.e.

Q.1. $$\{10^6 \rightarrow r_{.5}\} \sim \{r_{.5} \rightarrow 1\} \qquad \boxed{r_{.5} = 10^3 \text{ rad/hr}}\,^{*}$$

Since $v(r_{.5})-v(10^6)=v(1)-v(r_{.5}) \quad v(r_{.5})=-.5$

Similarly we establish the -.75 and -.25 values by answering the following questions.

Q.2. $$\{10^6 \rightarrow r_{.75}\} \sim \{r_{.75} \rightarrow r_{.5}\} \qquad \boxed{r_{.75} = 5\times10^3 \text{ rad/hr}}$$

$v(r_{.75})=-.75$

Q.3. $$\{r_{.5} \rightarrow r_{.25}\} \sim \{r_{.25} \rightarrow 0\} \qquad \boxed{r_{.25} = 2\times10^2 \text{ rad/hr}}$$

$v(r_{.25})=-.25$

Result: S-shaped like normal cumulative function (see Figure B.2)

Radio-activity (Log.)	0	1	2	3	4	5	6
Value	0	-.02	-.16	-.50	-.84	-.96	-1.

*Boxed relations represent responses of the decision maker.

B.7.5 Component Value Function For Status Of Information For Isotopic Separation

The purpose of this section is to assess a value function for the attribute "status of information" for isotopic separation. This attribute can take 9 discrete values (1 to 9) which have been already ordered in an ordinal sense. The assessment of a value function will provide a cardinal ordering for these values.

We first define certain properties of the function.

A. MONOTONICITY. From the definition of the ordinal scale for the status of information it follows that the value function is monotonic i.e.

$v(X)>v(Y)$ if and only if $d(X)<d(Y)$

where $d(X)$ is the integer (1-9) corresponding to state X(A-I).

B. CONVEXITY AND CONCAVITY

We consider changes in the status of information and denote them by $(i-1 \to i)$ meaning that the status of information has changed from the state corresponding to $(i-1)$ to the one corresponding to (i). Then by comparing pairs of such changes we want to establish which one involves the highest change in the inherent difficulty.

For example, if

$(i-1 \to i) < (i \to i+1)$ then $v(i)-v(i-1) < v(i+1)-v(i)$

please answer the following questions:

Q.1. $(1 \to 2) < (2 \to 3)$

Q.2. $(2 \to 3) < (3 \to 4)$

Q.3. $(3 \to 4) < (4 \to 5)$

Q.4. $(4 \to 5) \sim (5 \to 6)$

Q.5. $(5 \to 6) > (6 \to 7)$

Q.6. $(6 \to 7) > (7 \to 8)$

Q.7. $(7 \to 8) > (8 \to 9)$

MONOTONIC: CONVEX ☐ CONCAVE ☐ S-SHAPED ☑

NONMONOTONIC: SHAPE

C. MIDVALUE SPLITTING TECHNIQUE

We set the value of 1 (or A) at 0 and the value of 9 (or I) at -1. Thus

$$v(1)=0 \quad \text{and } v(9)=-1$$

We now want to establish a value i of the status of information such that the decrease in difficulty in going from 9 to i is the same in going from i to 1.

Q.1. $\{9 \to i\} \sim \{i \to 1\}$

or equivalently define a state X such that

$\{I \to X\} \sim \{X \to A\}$

$\boxed{\begin{array}{c} 5<i<6 \\ G \succ X \succ C \\ x=5.5 \end{array}}$ *

Since the status of information can take only integer values it might not be possible to identify an i (X) that satisfies Q.1. In that case we can define a noninteger value x $(i<x<i+1)$ and artificially put $v(X)=-.50$. After establishing the value functions we can go back and check if the values $v(i)$ and $v(i+1)$ agree with the preferences of the assessor.

In the same way we establish the $-.25$ and $-.75$ points.

Q.2. $\{9 \to \overset{?}{y}\} \sim \{\overset{?}{y} \to 1\}$

or equivalently define a state Y such that

$\{I \to Y\} \sim \{Y \to X\}$

$v(Y)=-.75$

$\boxed{\begin{array}{c} 4<y<5 \\ E \succ Y \succ G \\ y=4.5 \end{array}}$

Q.3. $\{i \to \overset{?}{z}\} \sim \{\overset{?}{z} \to 1\}$

or equivalently define a state Z such that

$\{X \to Z\} \sim \{Z \to A\}$

$v(Z)=-.25$

$\boxed{\begin{array}{c} 6<z<7 \\ C \succ Z \succ G \\ z=6.5 \end{array}}$

Result: S-Shaped like normal cumulative function (see Figure B.3)

Status of Information	A	D	B	E	G	C	H	F	I
Value	0	-.01	-.05	-.16	-.37	-.67	-.84	-.95	-.99

*Boxed relations represent responses of the decision maker.

B.7.6. Component Value Function For Criticality Problems For Isotopic Separation

The purpose of this section is to assess a value function for the attribute "criticality problems" for isotopic separation. Since this attribute can take only two values (HIGH & LOW), we simply assign the value of zero to the low level, and the value of minus unity to the HIGH level. Thus

$$v(\text{LOW}) = 0 \qquad v(\text{HIGH}) = -1$$

B.7.7 Assessment Of Weighting Coefficients (λ's)

The answers to the following questions are needed for the assessment of the weighting coefficients.

Q.1. If all the attributes were at their lowest level (z_i^o, i=1,. . . 6) and you had a choice of "pushing" only one up to its highest value (see Figure B.4) which one would you choose?

Answer: z_4: "isotopic" status of information.

Q.2. Assess the level of status of information for isotopic separation for which you are indifferent between the following two alternatives.

$$\{z^*_1=A,\ z_2^o,\ z_3^o,\ z_4^o,\ z_5^o,\ z_6^o\} \sim \{z_1^o,\ z_2^o,\ z_3^o,\ z_4,\ z_5^o,\ z_6^o\}$$

Answer: $Z_4 = E$

Q.3. Assess the level of z_4 so that you are indifferent between the following alternatives.

$$\{z_1^o,\ z^*_2=1,\ z_3^o,\ z_4^o,\ z_5^o,\ z_6^o\} \sim \{z_1^o,\ z_2^o,\ z_3^o,\ z_4,\ z_5^o,\ z_6^o\}$$

Answer: $z_4 = C$

Q.4. Assess the level of z_4 so that you are indifferent between

$$\{z_1^o,\ z_2^o,\ z_3^*=\text{LOW},\ z_4^o,\ z_5^o,\ z_6^o\} \sim \{z_1^o,\ z_2^o,\ z_3^o,\ z_4,\ z_5^o,\ z_6^o\}$$

Answer:

Q.5. Assess the level of z_4 so that you are indifferent between

$$\{z_1^o,\ z_2^o,\ z_3^o,\ z_4^*=A,\ z_5^o,\ z_6^o\} \sim \{z_1^o,\ z_2^o,\ z_3^o,\ z_4,\ z_5^o,\ z_6^o\}$$

Answer: Not Applicable ($Z_4 = A$)

Q.6. Assess the level of z_4 so that you are indifferent between

$$\{z_1^o,\ z_2^o,\ z_3^o,\ z_4^o,\ z_5^*=1,\ z_6^o\} \sim \{z_1^o,\ z_2^o,\ z_3^o,\ z_4,\ z_5^o,\ z_6^o\}$$

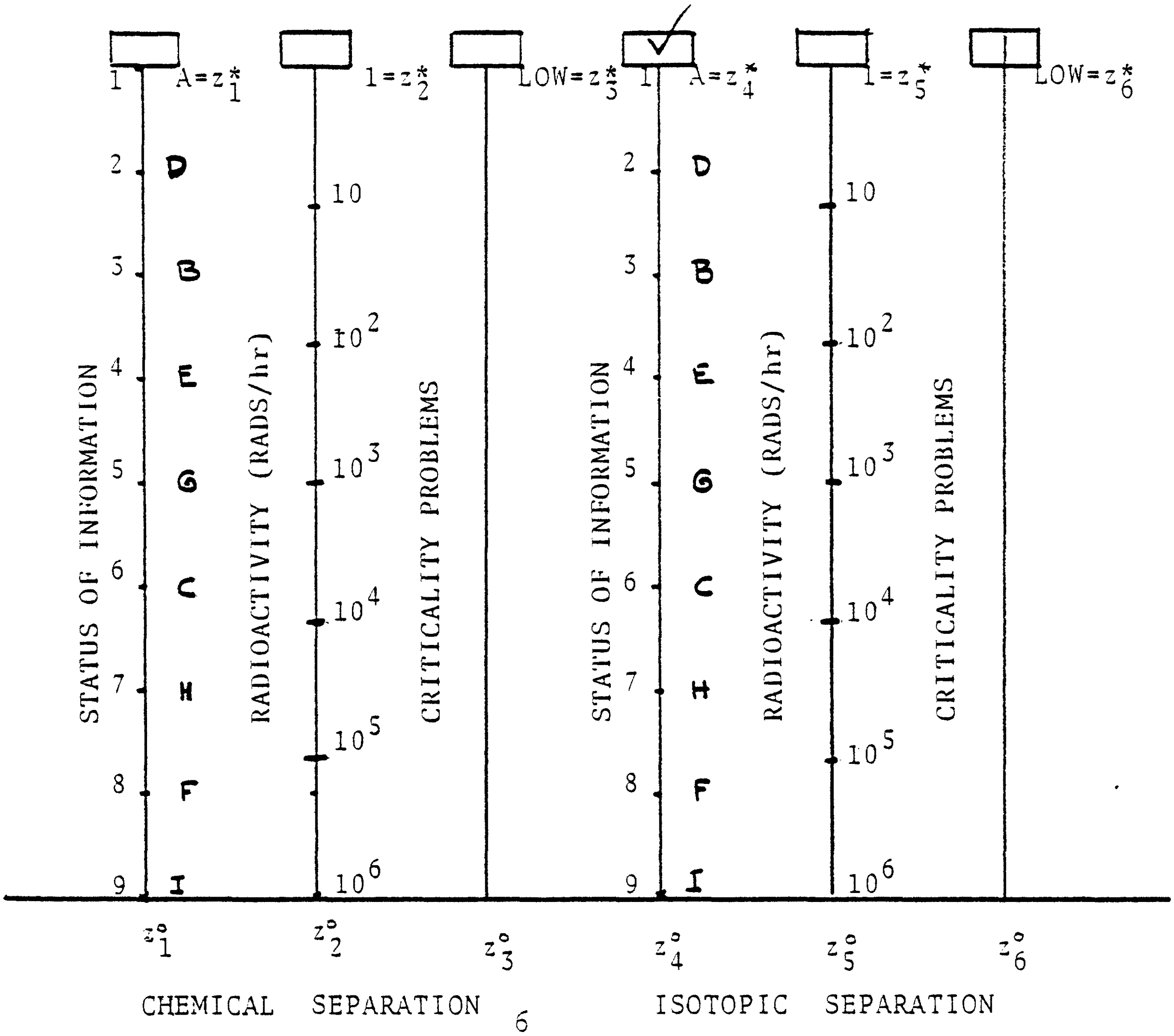
Answer: $Z_4 = C$

Q.7. Assess the level of z_4 so that you are indifferent between

$$\{z_1^o,\ z_2^o,\ z_3^o,\ z_4^o,\ z_5^o,\ z_6^*=\text{LOW}\} \sim \{z_1^o,\ z_2^o,\ z_3^o,\ z_4,\ z_5^o,\ z_6^o\}$$

Answer: $Z_4 = H$

Figure B.4



P.I. $v(z_1, z_2, \ldots, z_6) = \sum_{i=1}^{6} \lambda_i v_i(z_i)$

$$\left.\begin{array}{ll} \text{Q.2} & \lambda_1 = .84\lambda_4 \\ \text{Q.3} & \lambda_2 = .37\lambda_4 \\ \text{Q.4} & \lambda_3 = .05\lambda_4 \\ \text{Q.6} & \lambda_5 = .37\lambda_4 \\ \text{Q.7} & \lambda_6 = .16\lambda_4 \end{array}\right\} \Longrightarrow \begin{array}{l} \lambda_1 = .30 \\ \lambda_2 = .13 \\ \lambda_3 = .02 \\ \lambda_4 = .36 \\ \lambda_5 = .13 \\ \lambda_6 = .06 \end{array}$$

$$\sum_{i=1}^{6} \lambda_i = 1$$

Using the developed value function, we assessed the value of the inherent difficulty of various proliferation pathways considered by SAI.[2] The pathways are given in the Table on the following page,reproduced from an SAI working paper.[2] The assessment of the values of the attribute "scores" for these pathways is given in Table B.2. The values of the inherent difficulty are also given in the last column of the same table. The ordinal ordering provided by SAI is given in the table of page this ordering is compared in Table B.3 with the cardinal ordering resulting from the value assessment. The two assessments are in very good agreement.

TABLE B.2

"Scores" of Inherent Difficulty Attributes for Various Proliferation Pathways

COUNTRY B

SYSTEM	CHEMICAL			ISOTOPIC			INHERENT DIFFICULTY
	Status of Info.	Radio Activity	Critical. Probl.	Status of Info.	Radio Activity	Critical. Probl.	
1	A	1	LOW	N.A.	N.A.	N.A.	0
2							
3	B	10^4	LOW	N.A.	N.A.	N.A.	-.12
4(*)	E	10^4	LOW	N.A.	N.A.	N.A.	-.16
5	B	10^5	LOW	N.A.	N.A.	N.A.	-.14
6							
7	N.A.	N.A.	N.A.	F	1	HIGH	-.40
8	B	10^2	LOW	F	10^2	HIGH	-.46
9	B	10^6	LOW	F	10^2	HIGH	-.57
10	B	10^6	LOW	N.A.	N.A.	N.A.	-.15
11	A	10^5	LOW	N.A.	N.A.	N.A.	-.12
12	A	10^4	LOW	N.A.	N.A.	N.A.	-.11

(*) Case No. 4 corresponds to a less developed country

N.A. ≡ Not applicable

DEFINITION OF CASES FOR PRELIMINARY ASSESSMENT

	1	2	3	4	5	6	7	8	9	10	11	12
SYSTEM												
Fuel Cycle	LWR + PU Recycle		LWR+Pu Recycle (pre-Irr. MOX)		LWR Once-Thru			HWR Denatured Th Fuel			LMFBR Coprocessed Recycle Spiked	
Configuration: Full Cycle	●	●									●	●
Reactor Only			●	●	●	●	●	●	●	●		
Institutional: Era	← Today →		← 1980 →		1980	Today	1980	← 1990 →			← 2000 →	
U.S. Domestic Safeguards		●										
Verified Fuel Storage			●	●	●		●	●	●	●	●	●
Path Characterization (Scenario)												
Proliferator: National	NNWS2		NNWS2	NNWS3	← NNWS2 →			← NNWS2 →		NNWS3	← NNWS1 →	
Subnational		●										
Weapon Objectives: 5-10/yr-type C												
1-10 ASAP-type C	●		●	●	●	●	●	●	●	●	●	●
1-3 Demos - type A		●										
Action Modes: All-covert						●						
Covert-Overt	●	●	●	●	●		●	●	●	●	●	●
Proliferation Path												
Starting Material:	PuO_2		Pre-Irr MOX		Spent Fuel		Fresh Fuel	Fresh Fuel	Spent Fuel		MOX	MOX
Diversion Point:	Repro. Plant		FF-Reactor		Reactor SF Storage		FF-Reactor	FF-Reactor	Reactor SF Storage		Repro	FF-Reactor
Weapons Usable Material:	Pu Metal	PuO_2	Pu Metal		Pu Metal		HEU-235	HEU-233	HEU-233	Pu	Pu	Pu
Type of Path: Direct		●										
Conversion	●											
Reprocessing			●	●	●	●		●	●	●	●	●
Enrichment							●	●	●			

* FF - Fresh Fuel
SF - Spent Fuel

RANK ORDERING OF SYSTEMS ACCORDING TO INHERENT DIFFICULTY OF PROLIFERATION PROCESSES*
(Based on Dominant Processes Involved in Proliferation Path)

Rank Ordering of Dominant Processes (Listed in order of decreasing difficulty)	Cases	System**
Hot separation of U-Pu-Th or U-Th <u>plus</u> U-233 enrichment	8,9	HWR Denatured Th Fuel
U-235 enrichment	7	
Hot Separation - Spent Denatured Th Fuel	10	HWR - Denatured Th Fuel (with reduced weapons objectives)
Hot Separation - Spent U-Pu Fuel	5,6	LWR - Once thru
Hot Separation - Spiked or pre-irradiated fuel	3,4 11,12	LWR + Pu Recycle (Pre-irradiated MOX Fuel) LMFBR Coprocessed Recycle
←Cold Conversion $PuO_2 \rightarrow$ Pu metal	1	LWR + Pu Recycle

* This ranking does not reflect differences in proliferator capabilities or in whether commercial non-reactor fuel cycle facilities are located in the proliferating country.

** Systems rank ordered by placement in category corresponding to associated case involving least difficult dominant process.

TABLE B.3

Ordering of Pathways in Terms of Decreasing Inherent Difficulty

Less difficult ⟶

"OUR" ORDERING	SAI ORDERING
#9 (-.57)	8,9
#8 (-.46)	7
#7 (-.40)	10
#4 (-.16)	5,6
#10 (-.15)	
#5 (-.14)	3,4
#3, 11 (-.12)	
#12 (-.11)	11,12
#1 (0)	1

APPENDIX C

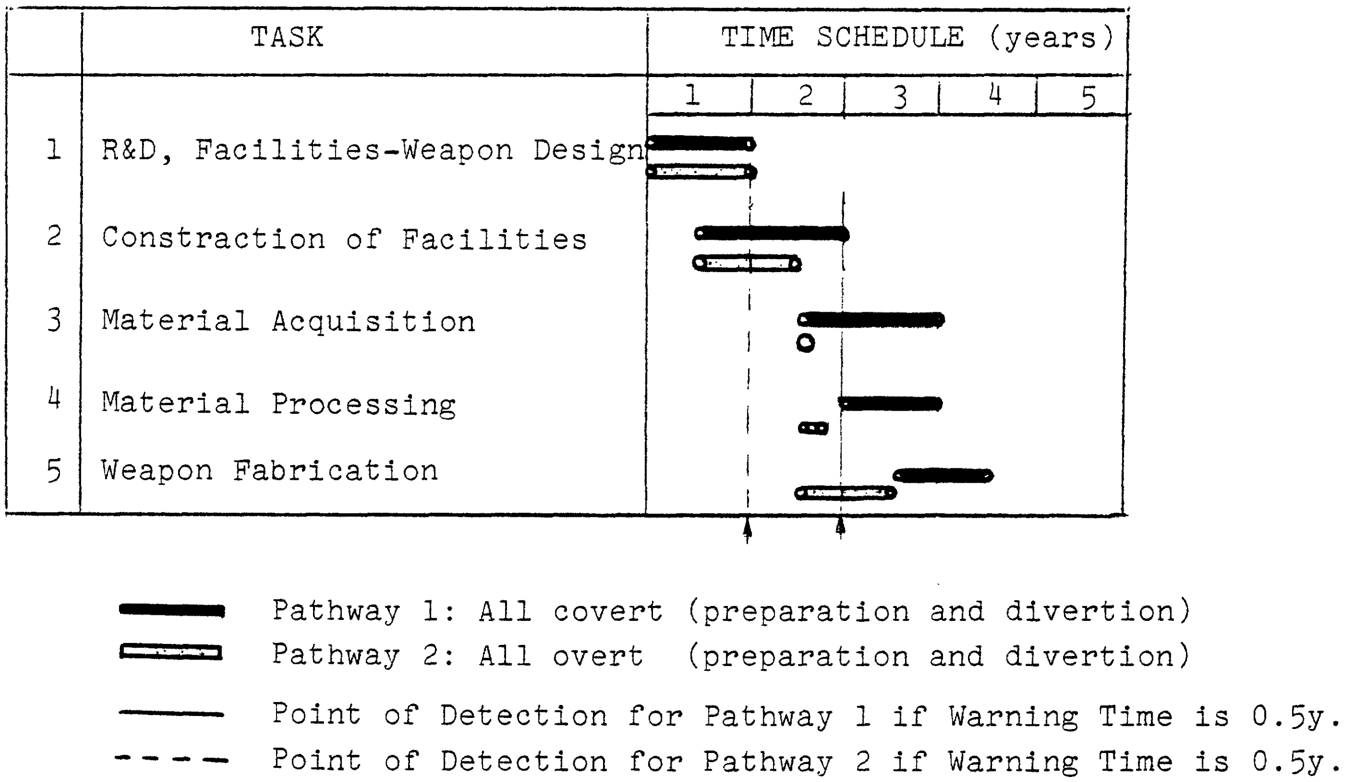
ON THE ATTRIBUTE "WARNING PERIOD"

C.1 Definition of Warning Period

One important factor that affects the choice of a particular proliferation pathway by a would-be proliferator, and hence the resistance of an alternative system, is the likelihood that the proliferation effort will be impeded by detection and subsequent "inside"(3) or "outside" intervention. To address this aspect of the problem, we initially considered as an attribute the warning time, proposed by SAI [1] and defined as "the time from detection of an ongoing proliferation effort to the completion of the first explosive." The conventional wisdom concerning the importance of this attribute is that the more time available for intervention, the higher the likelihood that the proliferation effort will be aborted. However, it was soon recognized that there are two fundamental deficiencies inherent in the conventional definition of this attribute. First, it could not be used for comparison purposes, in the sense that two different pathways with the same warning times are not necessarily equivalent as far as the interruptability of the proliferation is concerned. Second, even for a single pathway, the impact of the availability of a given warning time on the interruptability of the proliferation effort is not always clear. To illustrate these two points we consider a scenario involving two proliferation pathways with time-schedules as shown in Figure C.1. Pathway 1 corresponds to an all-covert proliferation effort, while pathway 2 corresponds to an all-overt effort. From an examination of the two time-schedules it is clear that the same value of the warning time has a different importance for each pathway. For example, for pathway 1 a warning time of a half year means (see Figure C.1) that the detection takes place after subtasks 1 and 2 have been completed and after 1/3 of the fuel has been clandestinely diverted. The same warning time (0.5 year) for pathway 2 means that the detection takes place when almost 50% of the facilities are yet to be constructed and when no nuclear fuel has been diverted. Obviously, the 0.5 year of warning time does not have the same "value" for these two pathways.

The second problem with the warning time concerns the meaning of its absolute value, even for a single pathway. For example, if the detection of a proliferation effort, scheduled as shown in Figure C.1 for pathway 1, takes place one and a half years into the effort, we would say that there is one year of warning time before the construction of the first weapon. This of course assumes that the proliferator will continue operating according to the schedule of pathway 1. Most likely, however, the proliferator will not follow the

Figure C.1



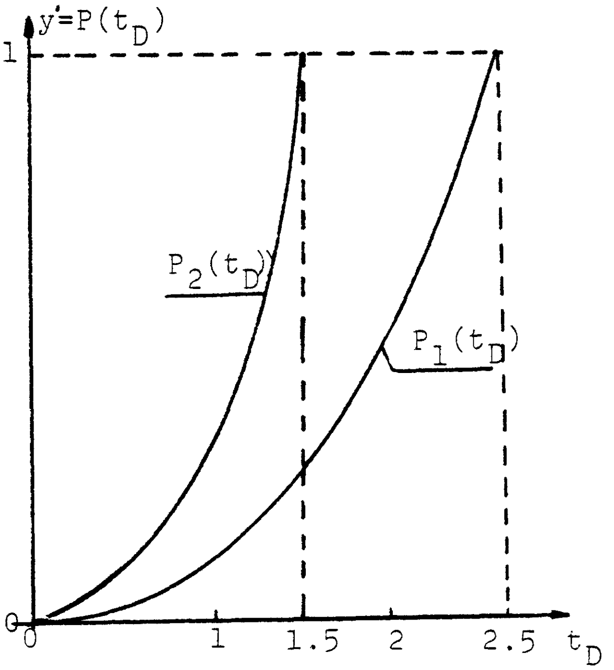


Figure C.2. Fraction of What has been completed as a function of time.

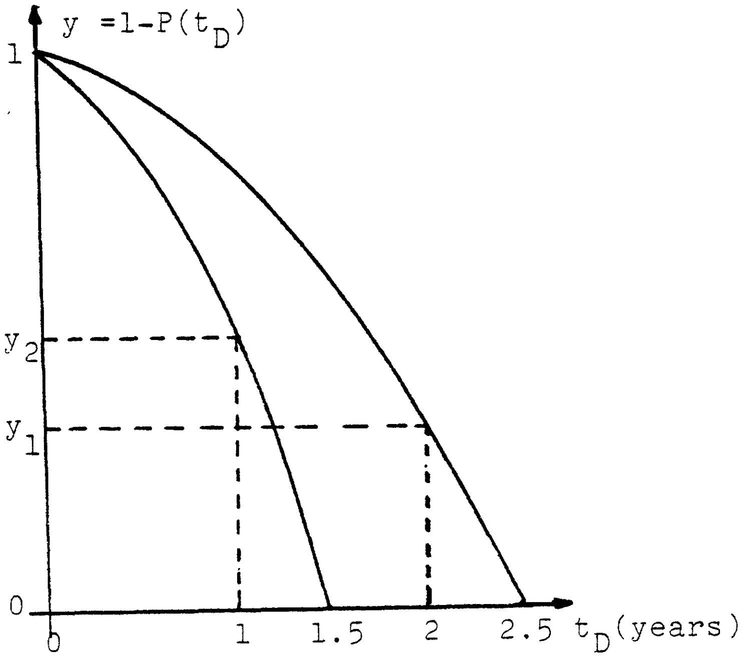


Figure C.3. Fraction of What remains to be done as a function of time. For 0.5 year of warning time $y_2 > y_1$.

original schedule. His reaction can be anywhere between aborting the effort to an all-out attempt to finish. Somewhere in-between, we might have an initial, temporary slowdown, with the intent to resume the effort at a later more convenient time, when the "political dust will have settled."

From the above discussion we conclude that the time remaining, according to the initial scenario, from detection up to completion of the first explosive is not always a useful evaluator of a pathway. We also conclude that the importance of detecting a proliferation effort at a particular moment depends on what remains to be done rather than on how much time was initially allocated to this remaining task. We can, however, use the fraction of the proliferation effort that remains to be completed at the moment of detection as an evaluator of the vulnerability of the effort to intervention.

We denote the fraction of the effort to be completed at the moment of detection by y, and call it warning period. A warning period of 0% means that the effort was undetected and a warning period of 100% means that the effort is detected right at the beginning. Furthermore, two proliferation pathways, for a given system and country, having the same warning period y are equivalent as far as their vulnerability to outside intervention is concerned.

To connect the warning period y with the time-to-detection, t_D, i.e., the time at which the detection takes place as measured from the beginning of the proliferation effort, we can establish a "production" function

$$y' = P(t_D) \qquad \text{C.1}$$

where y' gives the fraction of the effort that has been completed by the time of detection. Of course, we have that

$$P(0) = 0$$
$$P(T) = 1$$

and

$$y \equiv 1-y' = 1-P(t_D), \qquad \text{C.2}$$

where T is the weapons development time. The function $y' = P(t_D)$ is schematically presented in Figure C.2 for the two pathways considered above, while the function $y = 1-P(t_D)$ for the same pathways is shown in Figure C.3.

If a production function is established, then for a given warning period y we can determine the time-to-detection (see Figure C.3), and then, from the time-schedule (see Figure C.1), we know the exact state of the proliferation effort at the moment of detection.

In a more detailed analysis, the attribute warning period could be decomposed into a number of sub-attributes to give it a better operational meaning. For example, the proliferation effort can be thought of as requiring a certain amount of labor (measured in man-hours or man-years) and a certain amount of capital. The required labor could be further decomposed into various types of labor (scientific, skilled, unskilled). Similarly, the required amount of capital can be decomposed into capital for equipment and materials that can be acquired inside the boundaries of the proliferating country, and in capital for equipment and materials that must be imported. At any instant of the proliferation effort the fractions of the various kinds of labor and capital that have been already committed are known (through time schedules of the form shown in Figure C.1). If now we assign an importance coefficient to each of these sub-attributes, the fraction y' of the proliferation effort that has been completed at each instant of time is equal to the weighted sum of the completed fractions of the various subattributes. In symbols,

$$y'(t_D) = \sum_i \lambda_i y_i'(t_D) \qquad \text{C.3}$$

where $y_i'(t_D)$ denotes the fraction of the i-th sub-attribute committed up to time t_D and λ_i its importance coefficient. Seen from another point of view $y'(t_D)$ in Eq. C.3 gives the value of the work completed at time t_D.

For the purposes of this research, however, it was assumed that the decision maker(s) has an intimate knowledge of the time-schedule of each pathway as well as of the corresponding production function and hence, he (they) can completely and unambiguously understand the importance of a particular warning period (as it is expressed in dimensionless percentage form) with regard to what remains to be done at the moment of detection. The meaning of the warning period is further examined in the following subsection.

C.1.1 Importance of Warning Period (4)

In trying to establish the contribution of the warning period to the resistance of a pathway, it is necessary to assess the preferences of the potential proliferator concerning various values of this attribute. The first characteristic that we should establish is whether the direction of the preference is always the same over the range of the attribute. In other words, is a shorter warning period always preferred to a larger one or is the opposite true? Intuitively, it would seem that a short warning period would be more preferred to a larger one. But this attitude is based on the assumption that upon detection the effort is going to continue, and thus, the "closer" one is to the end the better. On the other hand, one could argue that if upon detection the effort is stopped permanently, then the closer one was to the end the more he has committed, and therefore, the higher the "loss" in aborting the effort. Thus, one would prefer being "caught"

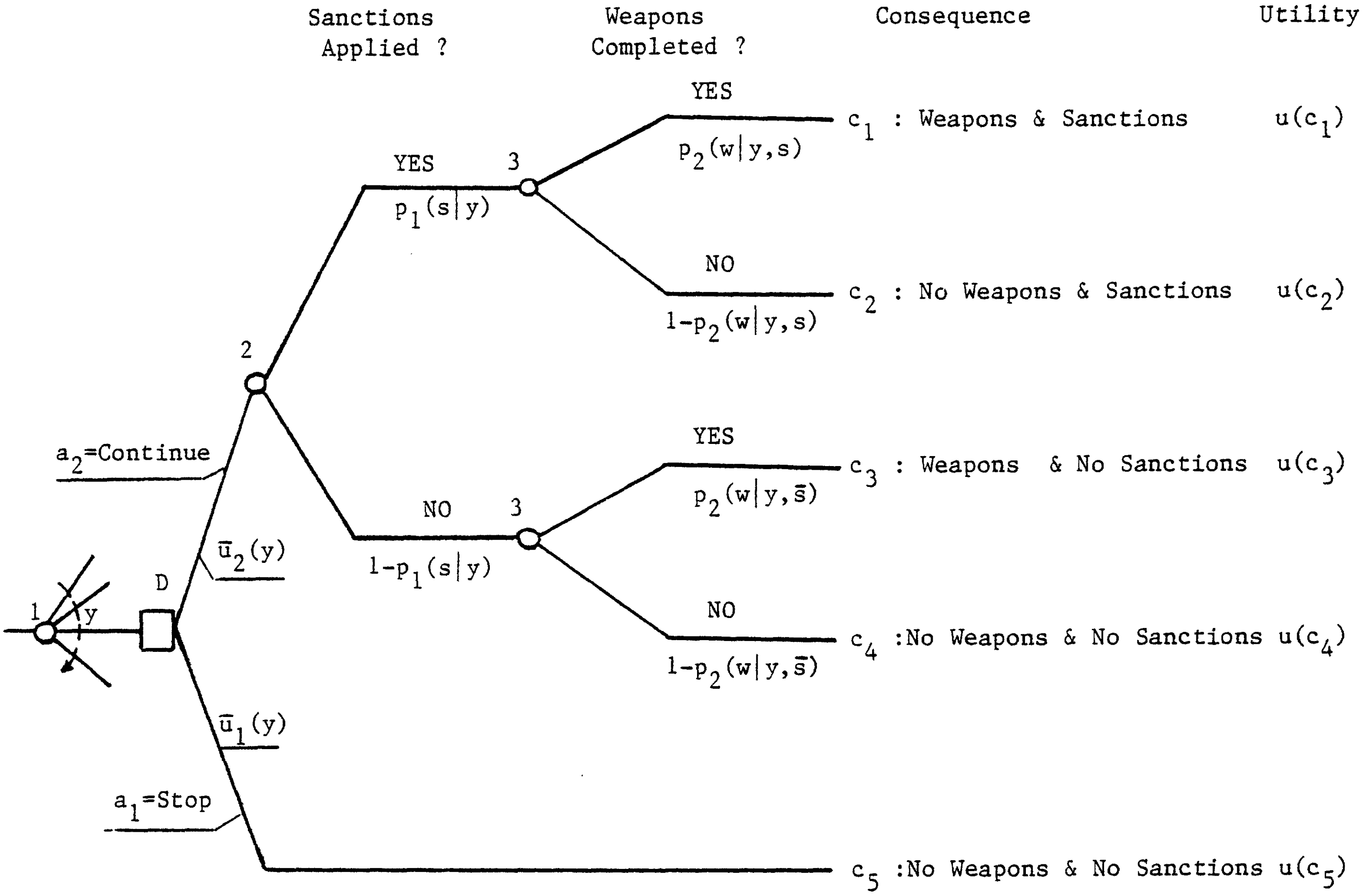


Figure C.4. Decision Tree for the Determination of the Utility of the Warning Period y.

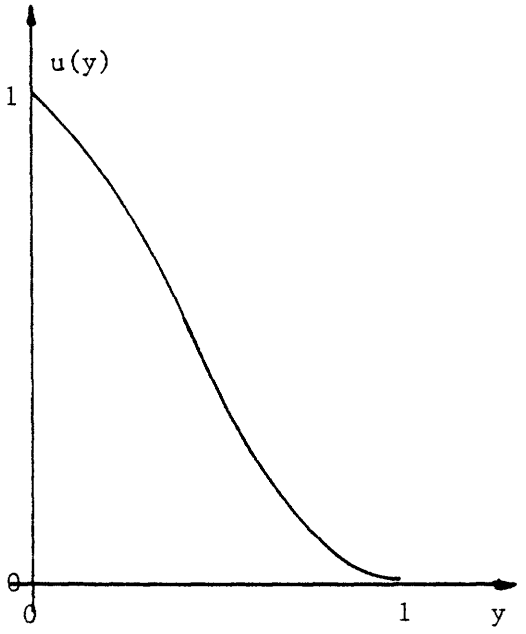


Figure C.5. Shape of Utility Function for Warning Period for "Low" Sanctions.

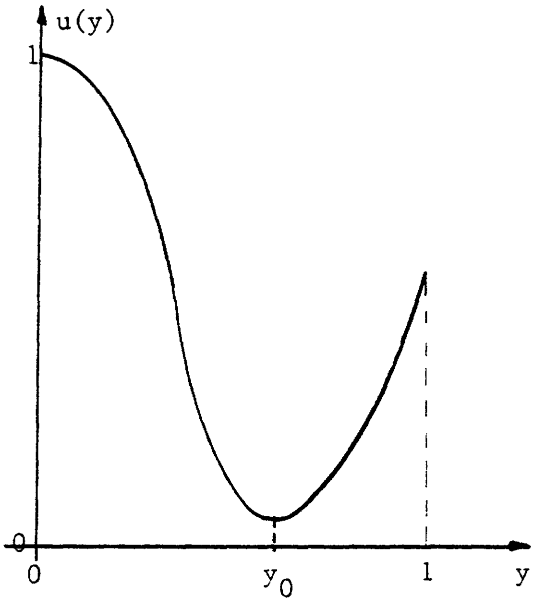


Figure C.6. Shape of Utility Function for Warning Period for "High" Sanctions.

before committing significant resources, which implies that a longer warning period is better than a shorter one. From the above discussion, it follows that what is of paramount importance for the establishment of a preference order for the warning period is the probability that "the effort will be stopped upon detection." This probability depends, in turn, on the severity of the sanctions and on the probability that the sanctions will be applied. We can understand this situation better with the help of the decision tree shown in Figure C.4.

The tree starts at the chance node 1 depicting the randomness of the detection moment. Let y be the value of a particular "warning period." The proliferator knows that his efforts have been detected, and he knows that sanctions might be applied. There are potentially two courses of action(5): (1) the proliferator can stop the effort (alternative α_1) resulting in consequence c_5 = No weapons - No sanctions, after having committed enough "resources" (political, economic, etc.) for completing 1-y of the total task; and (2) he might respond with a crash-effort (crisis response) trying to complete his objective (alternative α_2). If he follows the second alternative he will encounter chance node No.2, at which the sanctions might be applied with probability $p_1(s|y)$ or not applied with probability $1-p_1(s|y)$. If the sanctions are applied, then the proliferator will encounter chance node No.3 for achieving the objective with probability $p_2(w|y,s)$ and not achieving it with probability $1-p_2(w|y,s)$. If the sanctions are not applied then again the weapons objective might be achieved, with probability $p_2(w|y,\bar{s})$ or not achieved, with probability $1-p_2(w|y,\bar{s})$.

The y dependence in $p_1(s|y)$ allows for the potential dependence of international and regional responses on the amount of the remaining effort. This dependence may be important, if only in the eyes of the proliferator. For example, there is a widespread belief that if y represents only a part of the arsenal (meaning that some weapons have been already acquired), then the response of the international community may be muted, since the proliferator will have joined the club of the weapon-states. Furthermore, possible regional adversaries may not respond either, in fear of the existing weapons arsenal. The historical record lends support to this conjecture. The y and s dependence of p_2 means that the probability of achieving the objective depends both on what remains to be done and on whether sanctions have been applied.

The consequences at the end of the tree are now expressed in a form such that the decision maker can easily access their utility. If this is done, and we "fold back the tree" by calculating the expected utility before each chance node, we can calculate the expected utility for the two alternatives, α_1 and α_2. The proliferator will choose the alternative with the highest expected utility, and this is the utility u(y) of the "warning period" y. Thus,

$$u(y) = \max\ [u(\alpha_1), u(\alpha_2)]$$

If this procedure is repeated for all y's the utility function u(y) can be assessed. Since the utility of the consequences c_1 to c_4 is independent of y, what is needed is the assessment of $u(c_5/y)$ as well as the assessment of the functional dependence of p_1 and p_2 on y. Such a procedure could be tedious however, and a decision maker might be able to assess u(y) directly keeping the above analysis implicitly in mind. In principle, this requires that the nature of the sanctions as well as the likelihood of their application be well known. This is the rationale for including potential institutional constraints in the definition of an alternative system.

At the present state of the non-proliferation art, the nature of the sanctions (let alone their applicability) is not well defined. They are, however, of paramount importance in assessing the utility of the warning period. This became evident when two "decision makers" were asked to assess their utility for the various warning times (from the proliferator's point of view). The question of the sanctions was left vague. Both agreed that the absolute value of the warning time was not of great importance; what mattered was the fraction of the task that remained to be done. The first assessed his utility function as monotonically decreasing with y; i.e., the smaller the y the better (see Figure C.5). The second decision maker felt that there was a particular value y_o (slightly before the acquisition of the first weapon) such that for $y>y_o$ the larger the y the better, while for $y \leq y_o$ the smaller the y the better (see Figure C.6).

The basis for this difference of opinion hinges on the assumed viability of sanctions. That is, the first decision maker had implicitly assumed that sanctions would be of little importance to the proliferator even if applied, and/or that the probability of applying them is so small that the proliferator would continue his efforts even if he were detected. The second decision maker, however, implicitly assumed that the sanctions were so severe and their probability of implementation so high that detection was in his mind equivalent to aborting the effort.

Although the exact form of the sanctions and the conditions for their application are not well defined at present, we can nevertheless include them parametrically. Thus, we define three levels of sanctions (High, Medium, Low) and two levels for the probability of applying them. Specifying a combination of sanction level and application probability will simplify the direct assessment of the utility of the "warning period." In this way, the impact of the sanctions and their likelihood on the resistance of the various pathways can be established. In the demonstration of the methodology presented in Chapter IV, however, we considered only the "low-sanction" case.

C.2 Uncertainty Assessment for Warning Period

A proliferation effort can be detected at any instant of its duration. The time-to-detection can, therefore, vary from zero up to a value equal to the weapons development time. To quantify the uncertainty of the time-to-detection, we will treat it as a random variable. Since the warning period is a function of the time-to-detection, it is also a random variable. The probability distribution of this variable reflects the detectability of the proliferation effort and depends on the alternative system, on the country, on the weapons aspiration and, of course, on the pathway.

For a given proliferation pathway it is easier to assess the uncertainty in the time-to-detection t_D than in the warning period y. The latter can, however, be determined from the former, since the two random variables are functionally related. That is, the probability that the warning period will be less than y_o is equal fo the probability that the time-to-detection will be greater than t_{Do}, where t_{Do} is determined by y_o and the production function $P(t_{Do})$ such

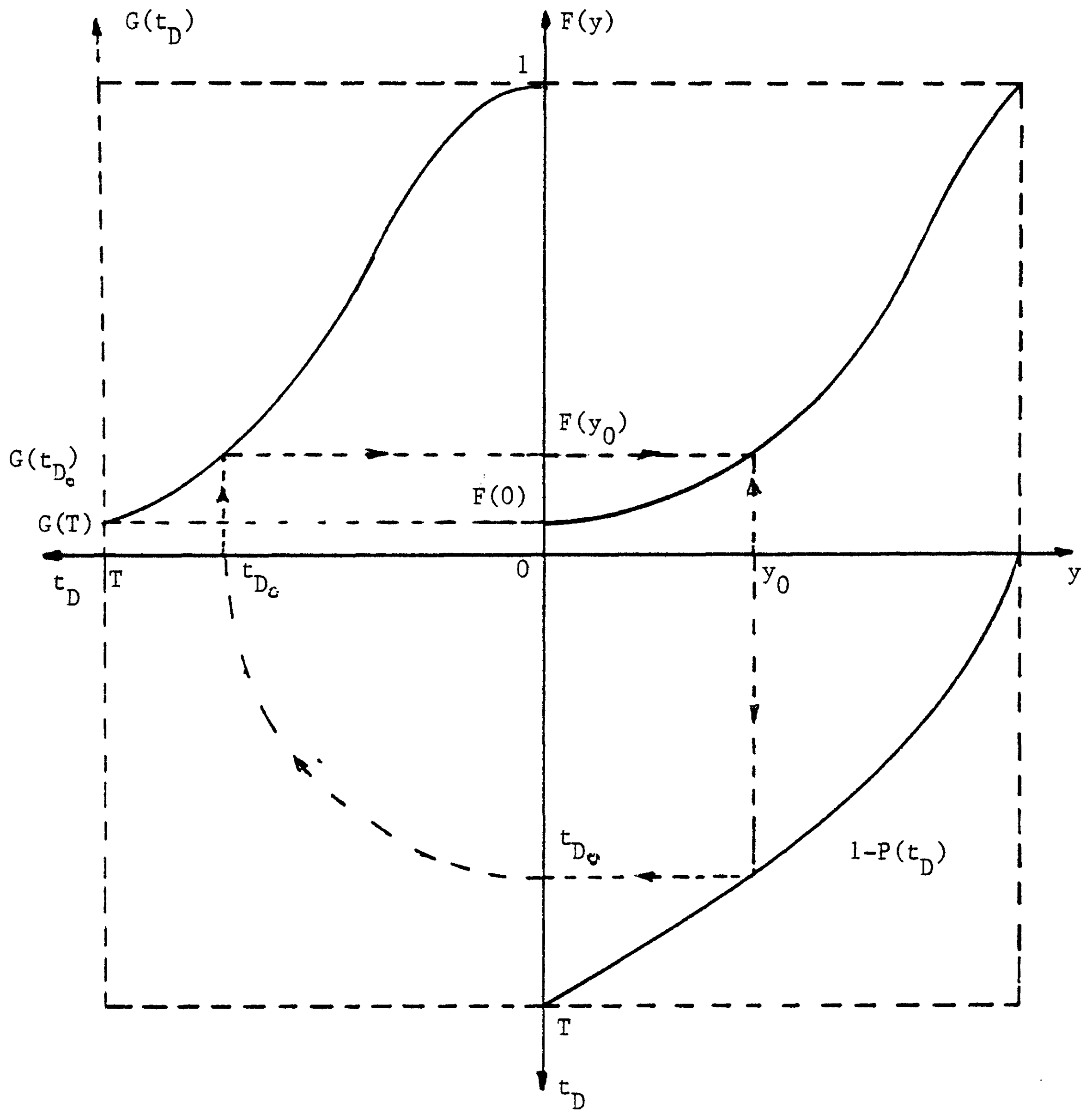


Figure C.7. Graphical Determination of Cumulative Distribution Function F(y) of Warning Period.

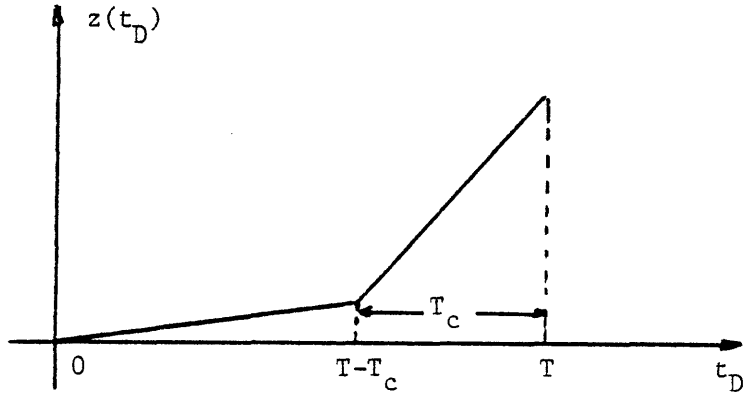


Figure C.8. Probability of detection at t_D conditional on not have been detected by t_D. Diversion of Nuclear Fuel Starts at $t=T-T_c$. T_c : Duration of Diversion.

that

$$y_o = 1 - P(t_{Do}) \qquad C.4$$

In symbols we have that

$$F(y) \equiv Pr\{y \leq y_o\} = G(t_{Do}) \equiv Pr\{t_D > t_{Do}\} \qquad C.5$$

This relation is depicted graphically in Figure C.7. There are two ways in which $G(t_D)$ can be assessed:

(1) The decision maker can directly assess the probability distribution function, $G(t_D)$, of t_D by determining the probability that t_D will be greater than t for various t's. Usually the assessment of 4 or 5 percentiles suffices for the determination of $G(t_D)$;

(2) The decision maker chooses a mathematical model which he feels best describes the way the detection probability changes with time, and then he assesses enough percentiles (usually 1 or 2) for the determination of the constants in the model.

An example of the second approach follows.

The proliferation effort is divided into two periods:

(1) Period from time zero up to the moment the diversion of fissile material begins.

(2) Period from the start of diversion up to the completion of the objective.

It is assumed that during each period the conditional probability that the proliferation will be detected between t and t+dt, <u>given</u> that it has not been detected up to t, is increasing linearly with time (see Figure C.8). Then if z(t) denotes this conditional detection rate we have that

$$z(t_D) = \begin{cases} k_1 t_D & \text{if } 0 \leq t_D \leq T-T_c \\ (k_1-k_2)T_1 + k_2 t_D & \text{if } T-T_c \leq t_D \leq T \end{cases} \qquad \text{C.6}$$

where $T_1 = T-T_c$ and where T_c is the completion time, i.e., the time from the start of diversion of nuclear fuel up to the completion of the first explosive. It can be shown that $G(t_D)$ and $z(t_D)$ are related as follows

$$G(t_D) = \exp[-\int_o^{t_D} z(t_D')dt_D'] \qquad \text{C.7}$$

By virtue of Eqs. C.5 and C.6 it follows that

$$G(t_D) = \begin{cases} \exp\left[- \frac{k_1 t_D^2}{2}\right] & \text{if } 0 \leq t_D < T-T_c \\ \exp\left[- \frac{k_2 t_D^2}{2} + (k_2-k_1)(T-T_c)t_D - \frac{(k_2-k_1)(T-T_c)^2}{2}\right] & \text{if } T-T_c \leq t_D \end{cases}$$

$$\text{C.8}$$

It is noteworthy that $G(T) \neq 0$ and hence, there is a finite probability that the proliferation effort will be undetected. For a particular proliferation scenario (system-country-aspiration-pathway) the constants k_1, k_2 can be determined if the decision maker assesses the probability that the effort will be undetected for a given period of time during each of the proliferation phases. For example, if he assesses that the probability of not being detected after one year in the preparation phase, is equal to A, then

$$A = G(1)$$

and from Eq. C.8 it follows that

$$k_1 = -2\ln(A)$$

The assessed values of the constants k_1, k_2, for the various pathways considered in the example of Chapter IV, are given in Tables C.1 and C.2. The probability distribution F(y) for the warning period was derived by assuming a linear production function, i.e., that $y' = t_D/T$.

C.3 Utility Assessment

As discussed in Chapter IV and in Appendix A, choice under uncertainty can be guided by introducing the concept of utility. In this section we present a short introduction to the basic fundamentals of utility theory excerpted from [6] and discuss the idea of certainty equivalent. We also provide a systematic procedure for the assessment of the utility function for the attribute: warning period.

C.3.1 The Concept of Utility

Suppose for a moment that the warning period could take only discrete values labeled $y_1, y_2, \ldots, y_i, \ldots y_n$.

TABLE C.1

Probability of detecting the proliferation effort by t_D.

Preparation phase. $(\Pr\{t_D \leq t\}, \quad 0<t<T-T_c)$

t_D (years)	Covert		Overt	
	COUNTRY B $k_1=.15y^{-2}$	COUNTRY C $k_1=.30y^{-2}$	COUNTRY B $k_1=.45y^{-2}$	COUNTRY C $k_1=1.0y^{-2}$
1	7%	14%	20%	39%
2	26%	45%	59%	86%
3	50%	74%	87%	99%
4	70%	81%	97%	99.97%

TABLE C.2

Probability of detection the proliferation effort by t'_D

Diversion phase $(\Pr\{t'_D \leq t\}, \quad 0\leq t \leq T_c)$

	COVERT					OVERT	
	COUNTRY B		COUNTRY C			COUNTRY B	COUNTRY C
t_D years	LWR-U235 or LWR-Pu Recycle $k_2=.70y^{-2}$	LWR-Th $k_2=1.40y^{-2}$	LWR-U235 LWR-Pu Recycle $k_2=1.40y^{-2}$	LWR-Th $k_2=1.83y^{-2}$	t_D mths	$k_2=58y^{-2}$	$k_2=102y^{-2}$
1	30%	50%	50%	60%	1	18%	30%
2	75%	94%	94%	97%	2	59%	76%
3	96%	99.9%	99.9%	99.97%	3	96%	96%
4	99.6%	99.999%	99.999%	∿100%	4	99.65%	99.65%

Furthermore, suppose that the labeling is such that y_1 corresponds to a warning period of 100%, y_n to a warning period of 0% and that y_1 is less preferred than y_2, which is less preferred than y_3, and so on. In symbols we assume that

$$y_1 \prec y_2 \prec y_3 \prec \ldots \prec y_n \qquad \text{C.9}$$

where $y_i \prec y_j$ means that y_i is less preferred than y_j. Now suppose that a decision maker is asked to state his preference between pathways 1 and 2 where

1. Pathway 1 will result in warning period y_i with probability p_i for i=1, 2, ..., n. Of course, $p_i \geq 0$ all i, and $\sum_i p_i = 1$.

2. Pathway 2 will result in warning period y_i with probability p_i'', for i=1,2...n. Again $p_i'' \geq 0$ all i, and $\sum_i p_i''=1$. Next, suppose that the decision maker asserts that, for each i, he is indifferent between the following two options:

Certainty Options - Warning period y_i.

Risky Option - Warning period y_n (0%: the best warning period) with probability u_i and y_1 (100%: the worst warning period) with the complementary probability $1-u_i$.

Furthermore, the decision maker is consistent in that he assigns $u_n=1$ and $u_1=0$, and the u's are such that

$$u_1 < u_2 < \ldots < u_n \qquad \text{C.10}$$

comparing (C.9) and (C.10) we can see that the u's can be thought of as a numerical scaling of the y's.

We can now define the expected value $\bar{u}$ of the u's as

$$\bar{u} = \sum_i p_i u_i$$

where p_i is the probability of y_i and u_i the <u>utility</u> of y_i. The fundamental result of utility theory is that the expected value of the u's can also be used to numerically scale probability distributions over the y's. To illustrate the reasoning let us reconsider the choice between pathway 1 (which results in y_i with probability p_i') and pathway 2 (which results in y_i with p_i''). If we associate to each y_i its scaled u_i value then the expected u-scores for pathways 1 and 2, which we label by $\bar{u}'$ and $\bar{u}''$ are

$$\bar{u}' = \sum_i p_i' u_i$$

and C.11

$$\bar{u}'' = \sum_i p_i'' u_i$$

There are compelling reasons for the decision maker to rank order pathways 1 and 2 in terms of the magnitudes of $\bar{u}'$ and $\bar{u}''$. The argument briefly is as follows. Consider pathway 1. It results with probability p_i in warning period y_i. But y_i is considered by the decision maker as indifferent to a u_i chance at y_n and a complementary chance at y_1. So, in effect, pathway 1 is equivalent to giving the decision maker a $\bar{u}'$ chance at y_n and a complementary chance at y_1. This completes the argument, which rests heavily on the substitution of the risky option (y_n with u_i, y_1 with $1-u_,$) for each y_i.

Of course, this line of thought can easily be estended to cases for which the warning period is not a discrete variable but a continuous one, namely when it can take any value between 0% and 100%. In that case, instead of discrete probabilities, we have a probability density function p(y) which gives the probability that the warning period will take a value in a small (infinitesimal) interval around y. Similarly, instead of discrete values u_i we have a function u(y) that gives the utility of each y (see figure C.5). If such a utility function is assessed, we can rank order pathways 3 and 4 in Figure IV.4 with respect to the warning period, by calculating the expected utility of this attribute for each pathway. In this case, the expected utilities are given by the integrals (extensions of Eq. C.11 for the continuous case)

$$\bar{u}_3 \equiv \int_0^1 p_3(y)\ u(y)dy$$

C.12

$$\bar{u}_4 \equiv \int_0^1 p_4(y)\ u(y)dy$$

and $p_3(y)$ and $p_4(y)$ denote the probability density functions for pathways 3 and 4, respectively. The pathway with the higher expected utility is the more preferred as far as the warning period is concerned.

It is noteworthy that the utility approach is very general, and that it includes as special cases situations such as: (a) the decision maker feels that the only matter of importance is not to be detected, and if detected he is indifferent to when detection occurs. This implies that the utility function has the form

$$u(y) = \begin{cases} 0 \text{ if } y \neq 0 \\ 1 \text{ if } y = 0 \end{cases} \qquad \text{C.13}$$

and hence, the expected utility $\bar{u}$ is equal to $F(0)$, namely the probability the the warning period will be equal to zero; (b) the decision maker feels that the utility of the warning period decreases linearly with its value, i.e.,

$$u(y) = 1 - y$$

This implies that $\bar{u} = 1 - \bar{y}$ and, hence, that the shorter the expected value of the warning period the more preferred the pathway.

From the above discussion, it follows that if the utility function of the warning period is assessed, then for each pathway the expected utility could be calculated and used as the "score" for the attribute warning period in Tables IV.1 to IV.12. Although such an approach would provide us with a means of comparing two pathways <u>with respect to</u> the warning period it is not very useful when composite comparisons must be made. For example, a pathway with an expected utility of warning period equal to 0.5 is preferred to one with 0.4. However, this 0.1 difference in expected utility might not be very meaningful when compared to a reduction of 1 year in development time. To remedy this situation, we can use the concept of certainty equivalent.

C.3.2 Certainty Equivalent

As already stated, the key idea of the utility theory is the idea of substitution, namely the idea that a decision maker is indifferent between a certainty option y_i and a risky option yielding 0% with a certain probability and 100% with the complementary probability. We can generalize this idea as follows.

Let p(y) be the probability density function describing the uncertainty about the warning period of a given pathway, u(y) the utility function, $\bar{u}$ the expected utility, i.e.,

$$\bar{u} = \int_0^1 p(y)\, u(y) dy \qquad \text{C.14}$$

and $\hat{y}$ the value of y that satisfies the relation.

$$u(\hat{y}) = \bar{u} \qquad C.15$$

Then, the decision maker should be indifferent between a certainty option yielding $\hat{y}$ and the risky option of the pathway for which the value of y is uncertain because these two options have the same expected utility. The value $\hat{y}$ is called the <u>certainty equivalent</u> of the risky option characterized by p(y).

If the certainty equivalent of the warning period is calculated for all the pathways, we could replace each pathway that includes uncertainty about the warning period by a pathway that yields a warning period $\hat{y}$ for certain. Of course, the values of the other attributes remain unchanged. Such pathways are equivalent for decision-making purposes. It follows, therefore, that we can fill in the columns of warning period in Tables IV.1 to IV.12 with the corresponding certainty equivalents. Comparisons between pathways are once more meaningful. For example, in pathway 2 and 3 Table IV.1, we can compare the reduction of the development time from 2 to 1.5 years with the increase of the warning period from 3% to 6%.

The details of the assessment of the utility function for the warning period and the calculation of the certainty equivalents are given in the following section.

C.4 Utility Assessment of Warning Period

In assessing the utility function of the warning period for the paradigm of Chapter IV, the following procedure was followed.

C.4.1 Identification of the Relevant Qualitative Characteristics

Two characteristics of the preferences of the decision maker that are first assessed are those of utility independence and monotonicity; i.e., we first examine whether the preferences towards various uncertain values of warning period depend on the level of the other attributes(6), and whether the preference is constantly decreasing or increasing with the warning period. Both these properties must hold for the certainty equivalent technique (see Section C.3.2) to be applicable.

As discussed in Section IV.6, we can divide the various proliferation pathways into two categories, namely those involving chemical and isotopic separation of the weapons material, respectively. Since pathways involving the former differ significantly from those involving the latter, in all the attributes, it is reasonable to check whether the preferences of the decision maker towards the warning period change as we move from one category to the other. If the attitude does not change, then we can proceed in assessing the utility function without considering the level of the other attributes. If, however, the preferences of the proliferator do depend on the level of the other attributes we try to establish whether the utility independence property holds within each category of pathways and if not, on which attributes it depends and how.

From the analysis done up to date and the discussion with various experts it appears reasonable that in assessing the utility function of the warning period, two regions should be distinguished.

Region I: This region covers the proliferation effort from its start up to the beginning of the production of the first weapon. If y_1 denotes the fraction of the work remaining to be done - at the moment of detection - up to the beginning of the production of weapons, then

$y_1 = 1.0$ means"nothing has been done", and
$y_1 = 0.0$ means"production of first weapon has just begun".

Region II: This region covers the proliferation effort from the start of the weapons production to the completion of the arsenal. If y_2 denotes the fraction of the arsenal not completed upon detection, then

$y_2 = 1.0$ means"production of first weapon has just begun" and
$y_2 = 0.0$ means"the arsenal is completed".

Thus, two utility functions, one for each region of the warning period, are first assessed and then these individual assessments are combined to cover the whole range of the warning period.

In what follows an alternative pathway will be denoted by (x_1,x_2,x_3,x_4,x_5), where the x_i's denote the attributes weapon-development time, warning period, inherent difficulty, weapons material and monetary cost, respectively. Whenever there is no ambiguity about the levels of the other attributes we will denote a pathway as $(\ldots x_2 \ldots)$.

C.4.2 Utility assessment for Region I

First the utility of the various levels of warning period corresponding to detection prior to the construction of the first weapon is assessed. The assessment is done for country B having aspiration α_2.

C.4.2.1 Checking for utility independence

Keeping the levels of the attributes x_1,x_3,x_4,x_5 constant, the decision maker is asked in each of the following questions the alternative that he prefers. Each question involves a choice between a risky option (a lottery yielding one of two possible values each with a probability .5) and a certainty option.

For questions Q.1 through Q.5 it is assumed that the attributes have the following values:

x_1=2.5 years, x_3=(M/-), x_4 =R.G-Pu x_5= \$20 million

Question	Sample Response PREFER
Q.1 $L=\left\langle\begin{array}{l}.5\ (\ldots,1.0,\ldots)\\ .5\ (\ldots,0.0,\ldots)\end{array}\right.$ or T=(...,0.02,...) for sure	T
Q.2 $L=\left\langle\begin{array}{l}.5\ (\ldots,1.0,\ldots)\\ .5\ (\ldots,0.0,\ldots)\end{array}\right.$ or S=(...,0.95,...) for sure	L
Q.3 $L=\left\langle\begin{array}{l}.5\ (\ldots,1.0,\ldots)\\ .5\ (\ldots,0.0,\ldots)\end{array}\right.$ or R=(...,0.50,...) for sure	L
Q.4 $L=\left\langle\begin{array}{l}.5\ (\ldots,1.0,\ldots)\\ .5\ (\ldots,0.0,\ldots)\end{array}\right.$ or Q=(...,0.10,...) for sure	L

Finally, we ask the decision maker to determine the level of the warning period $y_{.5}$ for which he is indifferent be-between

$$\text{Q.5}\quad L=\left\langle\begin{array}{l}.5\ (2.5,1.0,(M/-),\ \text{RG-Pu},20)\\ .5\ (2.5,0.0,(M/-),\ \text{RG-Pu},20)\end{array}\right.$$

and I=(2.5,...,(M/-), RG-Pu,20)

$y_{.5}$ = 0.05

We now consider a pathway that requires spent fuel reprocessing and uranium enrichment. For this pathway the attributes have the following values

x_1=6 years, x_3=(M/H), x_4=HE-U233, x_5=\$300 million

These values are kept constant for the following questions.

PREFER

Q.6 L= .5 (...,1.0,...)
.5 (...,0.0,...)
or T=(...,0.01,...) for sure — T

Q.7 L= .5 (...,1.0,...)
.5 (...,0.0,...)
or S=(...,0.98,...) for sure — L

Q.8 L= .5 (...,1.0,...)
.5 (...,0.0,...)
or R=(...,0.50,...) for sure — L

Q.9 L= .5 (...,1.0,...)
.5 (...,0.0,...)
or Q=(...,0.08,...) for sure — L

Finally, we ask the decision maker to determine the level of the warning period $y_{.5}$ for which is is indifferent between

Q.10 L= .5 (6,1.0,(M/H),HE-U233,300)
.5 (6,0.0,(M/H),HE-U233,300)
and I=(6,...,(M/H),RG-Pu,20)
$y_{.5}$ = 0.05

If the answer to questions Q.5 and Q.10 are the same we ask the decision maker whether he feels that this is accidental or whether the value $y_{.5}$ does not depend on the levels of the other four attributes. If the answer is that $y_{.5}$ does not depend on the level of the other attributes, then we ask if he feels that preferences under uncertainty for the warning period depend on the levels of the other attributes in any way. If the answer is no then utility independence has been established. If the answer to any of the above questions is yes, then we try to identify regions (subspaces) of the attributes for which the utility independence property holds. The whole analysis is then repeated for each such subspace.

C.4.2.2 Checking for monotonicity

The question of monotonicity was discussed at length at the end of Section C.1 (see also Figure C.5). For the purposes of this assessment it was assumed that the conditions that assure monotonicity are satisfied.

C.4.2.3 Attitude Towards Risk

The following questions are designed for the assessment of the decision maker's attitude towards risk. In other words, we want to establish whether the decision maker is risk averse, risk prone or risk neutral(7). Operationally, this means that for a monotonically decreasing utility function its shape will be convex, concave or linear, respectively. Since utility independence has been established, the questions refer only to the warning period.

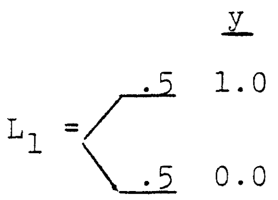
		y		PREFER L	PREFER Y	INDIFFERENT
L_1 =	.5	0.20	or y=0.10 for sure	✓		
	.5	0.0				
L_2 =	.5	0.30	or y=0.20 for sure	✓		
	.5	0.10				
L_3	.5	0.40	or y=0.30 for sure	✓		
	.5	0.20				
L_4 =	.5	0.50	or y=0.40 for sure	✓		
	.5	0.30				
L_5 =	.5	0.60	or y=0.50 for sure	✓		
	.5	0.40				
L_6 =	.5	0.70	or y=0.60 for sure	✓		
	.5	0.50				
L_7 =	.5	0.80	or y=0.70 for sure	✓		
	.5	0.60				
L_8 =	.5	0.90	or y=0.80 for sure	✓		
	.5	0.70				

ATTITUDE TOWARDS RISK: Risk Prone

C.4.2.4. Specification of Quantitative Restrictions.

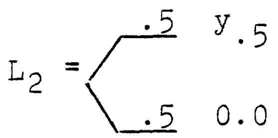
In this part of the assessment we specify several points of the utility function. These points along with the properties identified thus far, provide a very good basis for our analytical definition of the utility function.

The decision maker is asked to specify the levels of warning period that establish indifference in the following cases.



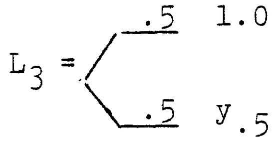
and $y_{.5}$ for sure $y_{.5}$= 0.05

where $u(1)\equiv 0$, $u(0)\equiv 1$ and $u(y_{.5}) = .5$



and $y_{.75}$ for sure $y_{.75}$= 0.02

where $u(y_{.75}) = .75$



and $y_{.25}$ for sure $y_{.25}$= 0.10

where $u(y_{.25}) = .25$

The implications of these assessments are shown schematically in Figure C.9.

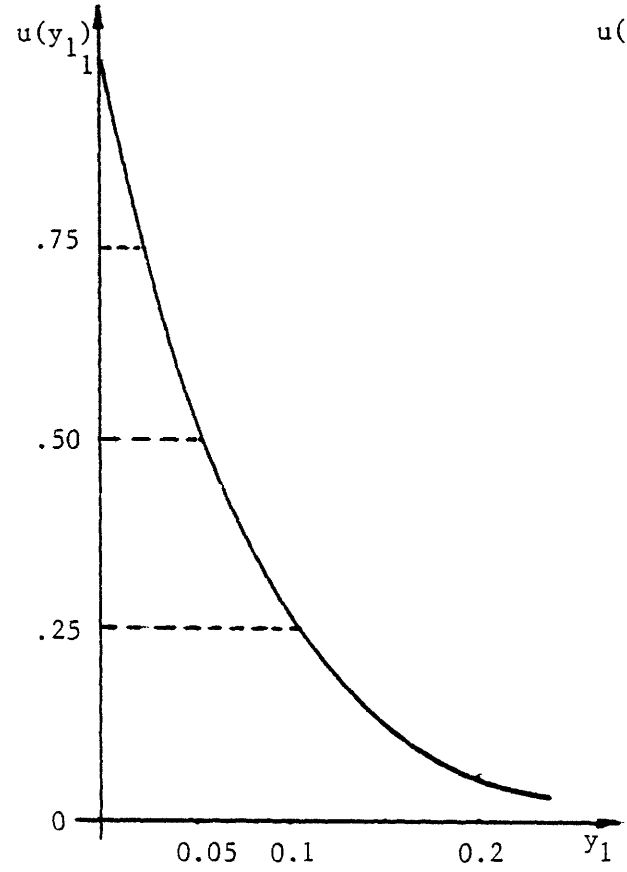


Figure C.9. Utility Function for Region I.

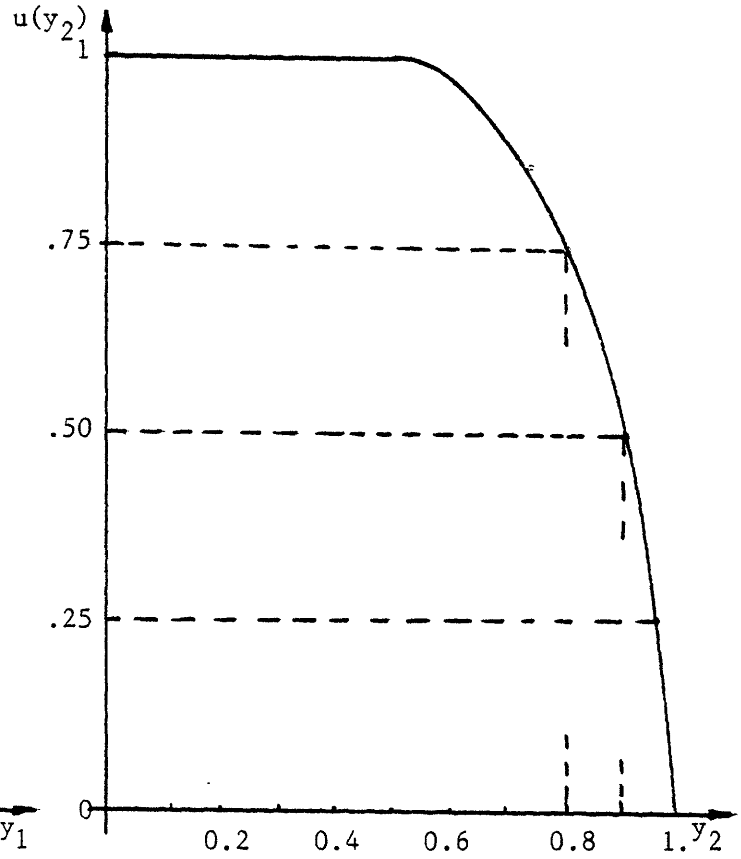


Figure C.10. Utility Function for Region II.

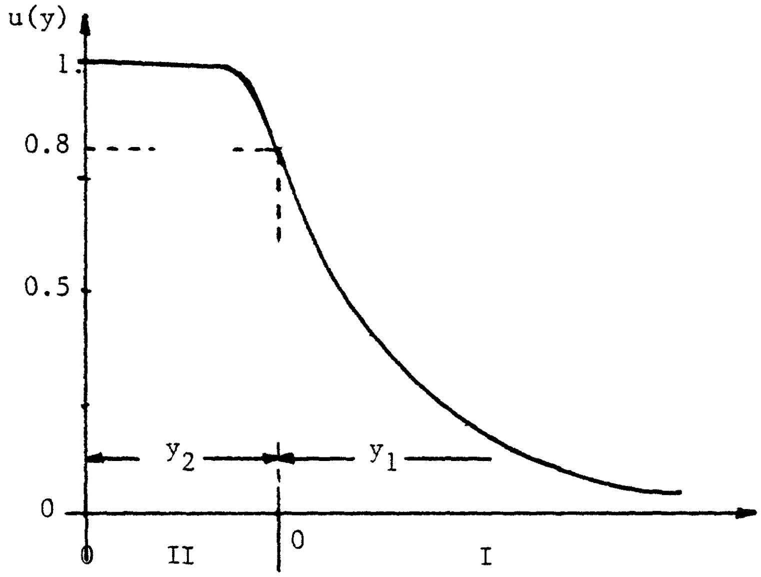


Figure C.11. Renormalized Utility Function for the two Regions of Warning Period.

C.4.3 Utility Assessment for Region II

The assessment of the utility function for region II is completely analogous to that for region I. Similar questions were asked and utility-independence and monotonicity have been established. The risk attitude and the quantitative restrictions, however, were different and are presented in the following two subsections respectively.

C.4.3.1 Utility independence holds

C.4.3.2 Utility functions monotonically decreasing

C.4.3.3 Attitude towards risk

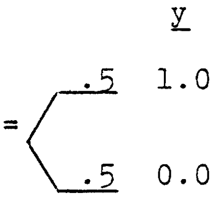
		y		PREFER L	PREFER Y	INDIFFERENT
L_1 =	.5 .5	0.20 0.0	or y=0.10 for sure		✓	
L_2 =	.5 .5	0.30 0.10	or y=0.20 for sure		✓	
L_3	.5 .5	0.40 0.20	or y=0.30 for sure		✓	
L_4 =	.5 .5	0.50 0.30	or y=0.40 for sure		✓	
L_5 =	.5 .5	0.60 0.40	or y=0.50 for sure		✓	
L_6 =	.5 .5	0.70 0.50	or y=0.60 for sure		✓	
L_7 =	.5 .5	0.80 0.60	or y=0.70 for sure		✓	
L_8 =	.5 .5	0.90 0.70	or y=0.80 for sure		✓	

ATTITUDE TOWARDS RISK: Risk Averse

C.4.3.4. Specification of Quantitative Restrictions.

The decision maker is asked to specify the levels of warning period that establish indifference in the following cases.

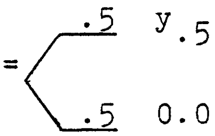
$L_1 =$



and $y_{.5}$ for sure $y_{.5} =$ 0.90

where $u(1) \equiv 0$, $u(0) \equiv 1$ and $u(y_{.5}) = .5$

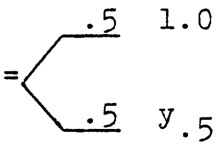
$L_2 =$



and $y_{.75}$ for sure $y_{.75} =$ 0.95

where $u(y_{.75}) = .75$

$L_3 =$



and $y_{.25}$ for sure $y_{.25} =$ 0.80

where $u(y_{.25}) = .25$

The implications of these assessments are shown schematically in Figure C.10.

C.4.4. Renormalization of Utility Function.

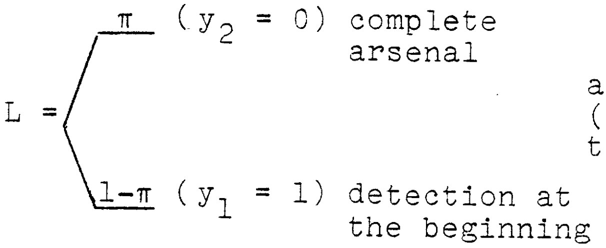
Once the utility function is determined for each region of the warning period, y, a renormalization that will produce a utility function over the whole range of y can be performed as follows:

(1) The utility of $y_2=0$ is set equal to 1.
($y_2=0$ means that the arsenal is completed without detection)

(2) The utility of $y_1=1$ is set equal to 0.
($y_1=1$ means detection at the very beginning of the effort)

(3) We set $u_1(0) = u_2(1)$
(since $y_1=0$ & $y_2=1$ denote the same warning period, i.e. beginning of weapons production).

(4) The decision maker is asked to specify the probability π for which he would be indifferent between



and $I=(y_2 = 1)$ for sure (starting weapons production)

The utility function form that complies with the assessed qualitative and quantitative restrictions as well as with conditions (1) to (4) above is the following

$$u(y)=\begin{cases} 1-\left(\frac{y_2}{\alpha}\right)^{\beta} & \text{for } 0\leq y_2 < 1 \\ e^{-\gamma y_1-\delta} & \text{for } 0\leq y_1 < 1 \text{ .} \end{cases}$$

The parameters $\alpha,\beta,\gamma,\delta$ can be evaluated as follows. Conditions (3) and (4) above, provide two relations involving $\alpha,\beta,\gamma,\delta$. Two more relations can be provided by using the assessed mid-value points i.e. the points y_2' and y_1' for which

$$u(y_2') = \frac{u(y_2=1) + u(y_2=0)}{2}$$

and

$$u(y_1') = \frac{u(y_1=1) + u(y_1=0)}{2}$$

These two equations along with the relations resulting from (3)&(4) above i.e.

$$1-\left(\frac{1}{\alpha}\right)^{\beta} = \pi$$

$$e^{-\delta} = \pi \quad ,$$

provide a system of four equations that can be solved for $\alpha,\beta,\gamma,\delta$. The numberical assessments for the paradigm of this work are given in Table C.3 and in Figure C.11.

TABLE C.3

PARAMETER VALUES OF THE UTILITY FUNCTION FOR ASPIRATIONS α_1 and α_2

	ASPIRATION	
	α_1(*)	α_2
α	N.A.	1.28
β	N.A.	6.58
γ	6.93	13.86
δ	0	0.22

(*) For aspiration α_1, region II does not exist

APPENDIX D

COMPLETENESS AND NONREDUNDANCY OF THE PROLIFERATION RESISTANCE ATTRIBUTES

D.1 General Remarks

In this Appendix we compare the set of the proliferation resistance attributes that we developed in Chapter III against other sets of attributes that have been proposed by various parties. After the definition of each proposed attribute we comment briefly on the relation of the attribute to the set of attributes proposed in this study.

D.2 List of Attributes

D.2.1 Proposed by R. Rochlin, Non-Proliferation Bureau, U.S. Arms Control and Disarmament Agency (ACDA)

1. IAEA warning time: time between IAEA announcement of an illegal act and availability of first explosive.

Comment. This attribute is included in the warning period (#5). It pertains to a particular aspect of an alternative system, i.e. the one in which the IAEA will be responsible for confirming the detection of an ongoing proliferation. A given set of institutional arrangements (the ones presently existing or others) will affect the probability that the detection (or, if necessary, the confirmation of the detection) of the proliferating activity will take place at a given instant, and hence will affect the probability that the detection will take place when a certain proportion of the task has yet to be completed.

2. Dedicated action time: the period during which a would-be proliferator could--as a result of appropriate intelligence--be caught in a compromising position prior to the availability of first explosive.

Comment. This attribute is included in the warning period (#5). If it is assumed that a detection can take place only during a part of the development time or, in general, that the probability of detecting an illegal action is higher during a particular period of the development time, then the length of this period affects the probability distribution of the warning period and hence its expected utility. Thus, the length of the dedicated action period is included in the assessment of the probability distribution of the warning period.

3. Cost: Direct cost of nuclear explosive program.

--Resources at risk if the explosives program results in the loss of power reactor operations.

Comment. Same as #2 monetary cost. In particular, components 2.1 and 2.3.

4. Complexity: technical failure modes; observables that could permit detection of clandestine activities.

Comment. Technical failure modes affect the degree of inherent difficulty in the weapons material procurement (#3) and in the fabrication of the weapons (#4). The "observables" affect the probability distribution of the warning period (#5).

5. Military value: production rate of nuclear explosives after the first weapon, weapon usability: yield, yield uncertainty, weight, etc.

Comment. According to the problem-structure proposed in this methodology, the production rate of nuclear explosives, and to a certain extent their usability, are part of the nuclear weapons aspiration and hence, constant over the various pathways. However, the degree of difficulty with which the postulated rate of production and usability are achieved varies from pathway to pathway. This difficulty is measured by the type of fissile material used. A finer distinction of the weapons' usability within the general categorization defined in the weapons aspiration is certainly possible. This might involve a more detailed consideration of the isotopic concentration of the fissile materia. It is felt, however, that such a distinction is not necessary for the present level of analysis.

D.2.2 Proposed by H. Rowen, School of Business, Stanford University

6. Accessibility to explosive materials measured in: (6.1) resource inputs; (6.2) facilities; (6.3) people; (6.4) money.

Comment. (6.1), (6.2), (6.4) and (6.3) are included in the monetary cost (#1). (6.2) and (6.3) affect the inherent difficulty of fissile material procurement (#3, See also Appendix B).

7. The time from a safeguarded state to the possession of various numbers of weapons.

Comment. For a given nuclear weapon aspiration this is the weapon development time (#1).

8. The time from various decisions to the possession of weapons.

Comment. These are the various components of the weapon-development time.

9. The time for converting signals of illicit acts into usable "warning."

Comment. This time affects the probability distribution of the warning period (#5). This is because the probability of a particular warning period depends on the probability that a signal will be generated at a particular instant and on the time necessary for converting this signal into usable "warning."

10. Various response or action times: these are the times required for various governments, in the region and outside, to take serious possible actions in response to signals of dangerous moves.

Comment. These times will affect the relative "value" of the various warning periods for the proliferator. Thus, such times will affect the preferences of the proliferator about the various warning periods and hence, they are implicitly accounted for. For more details, refer to Appendix C.

11. Estimates of material stocks and flows, including the number and characteristics of weapons that might be produced.

Comment. The number and general characteristics of weapons are constant for all pathways since they are part of the nuclear-weapons aspiration. Therefore, the material stocks and flows influence the rate at which material must be diverted to meet a particular time constraint and these two (rate and duration of diversion) influence in turn the probability distributions of the warning period and the length of the development time.

12. Risks, dangers, and technical uncertainties associated with various programs to acquire weapons.

Comment. These factors are included in the inherent difficulty of the fissile material procurement (#3) and the difficulty in the weapons design and fabrication (#4).

13. The assured "legal" constraints. These will define the legitimacy of various activities, facilities, and materials of various kinds: criticality experiments, research reactors, spent fuel stocks, fresh fuel stocks, hot cells, etc. These constraints need to be defined in terms of their universality; i.e. whether different activities will be permitted in different countries.

14. Characteristics including scale of possible covert or ambiguous (i.e. those with civil and military functions) facilities.

Comment on 13 and 14. In the proposed structure of the problem, the assessment of the proliferation resistance of a particular pathway conditional on a combination of an alternative system and a specific country assumes specification of the institutional constraints, including, what it is legal and what it is not. Because of differences among various countries a particular pathway of a given alternative system may be characterized by completely different values of the five attributes, and hence may represent different degrees of resistance to different countries. Furthermore, differences in resistance may arise not only from possible differences in attribute values, but also from differences in preferences and value trade-offs among these attributes for different countries.

D.2.3 Proposed by T. Greenwood, Department of Political Science, M.I.T., and Office of Science and Technology Policy (OSTP)

Greenwood proposes the formation of four clusters of nuclear fuel cycles by comparing them to a "benchmark" case, using as criteria:

15. Cost.

16. Difficulty.

17. Time.

18. Warning time.

Comment. These four criteria are in effect four of our five attributes.

D.2.4 Compiled by Science Applications, Inc. (SAI)

19. Direct cost of weapon/arsenal.

Comment. Contained in monetary cost (#2).

20. Indirect costs.

Comment. Contained in monetary cost (#2).

21. Political costs.

Comment. These costs depend on whether the proliferation effort has succeeded or not. If it has succeeded the cost will be incurred via sanctions and similar hostile reactions.

Basically, this response follows from the fact that nuclear weapons have been acquired, and not on the particular pathway through which they were acquired. If the effort has failed, the political cost might include the resources already committed. This is reflected in the utility of various warning periods (see Appendix C).

22. Resources at risk.

23. Economic risk.

Comment for 22 and 23. Same as for #21. They also affect the cost (#2).

24. Time from decision.

Comment. Included in weapon development time (#1).

25. IAEA response time.

Comment. See #4 of this list.

26. Dedicated response time.

Comment. Affect probability distribution and utility of warning period (#5, Appendix C).

27. Time from material acquisition.

Comment. Affects probability distribution of warning period (#5).

28. Warning time.

Comment. See warning period (#5).

29. <u>Time to detection</u>.

<u>Comment</u>. See warning period (#5).

30. <u>Lead time</u>.

<u>Comment</u>. See warning period (#5).

31. <u>Susceptibility to international controls</u>.

<u>Comment</u>. Affects the probability distribution of warning period, i.e., the more controls the larger the expected warning period.

32. <u>Interruptability</u>.

<u>Comment</u>. It is reflected in the probability distribution and utility of warning period.

33. <u>Sanctionability</u>.

<u>Comment</u>. This is part of the institutional arrangements. For a given system it has the same effect on all pathways.

34. <u>Sensitive activities</u>.

<u>Comment</u>. See comments on #13 and #14 of this list.

35. <u>Institutional arrangements</u>.

<u>Comment</u>. The institutional arrangements are part of our alternative system definition. The values of the attributes for the pathways and the preferences about them may have a strong dependence on the institutional agreements.

36. <u>Non-proliferation standard agreements</u>

<u>Comment</u>. See #35 of this list.

37. <u>Precedent for future technologies</u>: As a first approach to the problem we will examine only the nuclear technologies that can be significantly deployed within the next 30 years (1980-2010). For a later time period (i.e. 2010-2050), the analysis can be repeated with systems that may be available at that time. It is noteworthy that the analysis of systems for the first time period might yield different results if repeated for another time-period. This is basically due to changes in the technological and economic status of various countries.

38. <u>Legal starting point</u>: Affects the probability distribution of the warning period (#5).

39. <u>Probability of detection</u>:

<u>Comment</u>. Affects the probability distribution of warning period.

40. <u>Probability of success</u>

<u>Comment</u>. Implicitly included in the inherent difficulty of the fissile material procurement (#3) and in the difficulty in the design and fabrication of the weapons (#4).

41. Key steps to produce weapon material

Comment. Included in the inherent difficulty (#3)

42. Facilities required per reactor

Comment. Affects the inherent difficulty (#3)

43. Amount of material in fuel cycle

Comment. Given that the material is sufficient for achieving the weapons objective, the amount of fissile material will affect the probability distribution of the warning period in the following two ways: (a) for a fixed diversion period, the rate of diversion (and hence, the probability of detection) is lower for larger amounts of material in fuel cycle; (b) for fixed rate of diversion (fraction of nuclear fuel) the lower the amount of fissile material in the cycle the longer the diversion period and hence the higher the detection probability.

44. Material unattractiveness

Comment. Attribute #4, weapons material

45. Material accessibility

Comment. Included in the inherent difficulty and partly in the probability distribution of the warning period.

46. Material modifiability

Comment. Included in the inherent difficulty.

47. Safeguardability

Comment. Affects the probability distribution of the warning period (#5)

48. Rate of clandestine diversion

Comment. Affects the probability distribution of the warning period (#5). See also #43 of this list.

49. Difficulty of material acquisition.

Comment. This is the inherent difficulty of fissile material procurement (#3).

50. Protectability

Comment. Included in the inherent difficulty and in the probability distribution of the warning period, i.e. the better the protection the higher the probability of detection.

51. Weapons usability

Comment. Attribute (#4) and nuclear weapon aspiration.

52. Detectability

Comment. Included in probability distribution of warning period (#5).

53. Ease of Circumvention

Comment. See #52 of this list

54. Complexity

Comment. Included in the inherent difficulty (#3).

55. Facility modifiability

Comment. Included in the inherent difficulty (#3) and in the probability distribution of the warning period (#5), i.e., the greater the need for new facilities, the higher the detection probability. It also affects the cost and time attributes.

56. Need for sensitive technology

Comment. Affects the inherent difficulty (#3)

57. Military value

Comment. See #51 of this list

58. Likelihood of detection

Comment. See #52 of this list.

59. Visibility

Comment. See #58 and 52 of this list.

60. Activity risk

Comment. Included in inherent difficulty of fissile material procurement and difficulty in weapons design and fabrication.

61. <u>Specialized skills and knowledge</u>

<u>Comment</u>. Included in inherent difficulty.

APPENDIX E

VALUE FUNCTION ASSESSMENT OVER THE PROLIFERATION-RESISTANCE ATTRIBUTES

E.1. General Remarks.

In this Appendix we present an example of a value function assessment over the proliferation resistance attributes (see Chapter III). The purpose of this function is to provide a numerical measure of the relative attractiveness of the various proliferation pathways to the would-be proliferator. It is assumed(8) that the attributes are mutually preferential independent (see Appendix A) and hence, that the value function has the additive form

$$v(x_1,x_2,x_3,x_4,x_5) = \sum_{i=1}^{5} \lambda_i v_i(x).$$

Since the attribute inherent difficulty is expressed in terms of six sub-attributes (see Appendix B) the above equation becomes

$$v(x_1,x_2,x_{31},x_{32},\ldots,x_{36},x_4,x_5) = \lambda_1 v_1(x_1) + \lambda_2 v_2(x_2) + \sum_{j=1}^{6} \lambda_{3j} v_{3j}(x_{3j}) + \lambda_4 v_4(x_4) + \lambda_5 v_5(x_5) \qquad \text{E.1}$$

where $$\lambda_1 + \lambda_2 \sum_{j=1}^{6} \lambda_{3j} + \lambda_4 + \lambda_5 = 1 \qquad \text{E.1a}$$

The procedure consists of two steps. First, the component value functions v_i are assessed in sections E.2 to E.6, respectively. Next, the weighting coefficients λ_i's are assessed in section E.7. In section E.8 we discuss the results of this quantitative analysis, and compare them with the results of the qualitative analysis of section IV.6. Finally, we present some concluding remarks in section E.9.

E.2. Component Value Function for Weapon-Development Time.

The purpose of this section is to assess a value function for the attribute: weapon-development time. Four separate assessments are made corresponding to the following four sets of conditions.

(1) Weapons aspiration a_1 & "Business as usual" environment.

(2) Weapons aspiration a_1 & "Crisis" environment.

(3) Weapons aspiration a_2 & "Business as usual" environment.

(4) Weapons aspiration a_2 & "Crisis" environment.

The assessment was made for a country of type B (see section IV.2). Examination of Tables IV.1 to IV.6

reveals that the minimum development time is 1 year and the maximum 6 years. The range of this attribute was, therefore,

$$0 \le x_1 \le 6. \qquad \text{E.2}$$

First,the properties affecting the "shape" of the value function were explored.

A. Monotonicity. For all four sets of conditions it was determined that the value function was monotonic. This follows from the fact that shorter weapon development times were always preferred to longer ones. Mathematically this means that

$$x_1' \le x_1'' \text{ implies } v_1(x_1') \ge v_1(x_1''). \qquad \text{E.3}$$

for all x_1's.

B. Convexity and Concavity. The shape of the value function can be determined if the following questions are answered. "For each of the following pairs of changes in the weapon development time establish the one that corresponds to a larger change in the value of this attribute."

Q.1 (0 to 1) $\succ$ (1 to 2)
Q.2 (1 to 2) $\succ$ (2 to 3)
Q.3 (2 to 3) $\succ$ (3 to 4)

Q.4 (3 to 4) $\succ$ (4 to 5)

Q.5 (4 to 5) $\succ$ (5 to 6)

The same questions were repeated for all four sets of conditions, and in all cases it was determined that the shape of the value function was concave (see Figure E.1).

C. Numerical Assessment of Value Function -- Midvalue Splitting Technique. We set the value of 0 years of development time at zero, and the value of 6 years of development time at -1, i.e.

$$v_1(0) = 0 \quad \text{and} \quad v_1(6) = -1 \qquad \text{E.4}$$

The midvalue splitting technique consists in assessing -- by relevant questions -- the levels of the development time that have values -.50, -.75, -.25. This is done by considering two particular levels of the attribute, and then asking the decision maker to identify a third level that divides this interval into two intervals of "equal value". For example, if $x_{.5}$ is the level of development time for which the decision maker feels that the reduction in the value associated with going from 0 to $x_{.5}$ years of development time is equal to the reduction in the value associated with going from $x_{.5}$ to 6 years, then we have that

$$v_1(0) - v_1(x_{.5}) = v_1(x_{.5}) - v_1(6) \qquad \text{E.5}$$

and by virtue of Eq. E.4 it follows that

$$v_1(x_{.5}) = -.50. \qquad \text{E.6}$$

Similarly, we can ask the decision maker to assess the $x_{.25}$ and $x_{.75}$ levels. Of course, more points can be established if we continue subdividing the value intervals. Once a set of points is obtained, a mathematical curve can be fitted through these points. Then, the values of the $x_{.25}$, $x_{.50}$, and $x_{.75}$ points are calculated from the mathematical expression for the curve and checked against the initial assessments. If gross descrepencies exist, the decision maker is asked to reconsider any assessments that constitute logical inconsistencies, a new curve is produced, and so on, until a mathematical form is found that adequately represents the preferences of the decision maker.

In our example, the $x_{.50}$, $x_{.25}$, and $x_{.75}$ points were assessed four times; once for each set of conditions. The mathematical form of the function that best approximates these assessments for all four cases is

$$v_1(x_1) = \exp[-\beta x_1] - 1 \qquad \text{E.7}$$

The levels of the midvalue points and the corresponding

TABLE E.1

Initial -.25, -.50, and -.75 value assessments and final values for the weapon development time under various conditions

VALUE	DEVELOPMENT TIME (Years)			
	Aspiration a_1		Aspiration a_2	
	"Business as usual"	"Crisis"	"Business as usual"	"Crisis"
0	0	0	0	0
-.25	0.50	0.33	1.25	0.75
-.50	1.50	1.00	3.00	2.00
-.75	3.00	1.66	5.50	3.00
-.1	6.00	6.00	6.00	6.00
β	0.49	0.83	0.23	0.35

value of β for each set of conditions are given in Table E.1. The resulting value functions are given in Figure E.1.

E.3. Component Value Function for Warning Period

In Appendix C, we assessed a utility function for the attribute : warning period. Since a utility function is also a value function we will use the utility function assessed in Appendix C as the component value function(9) for the warning period. These functions are repeated here for convenience.

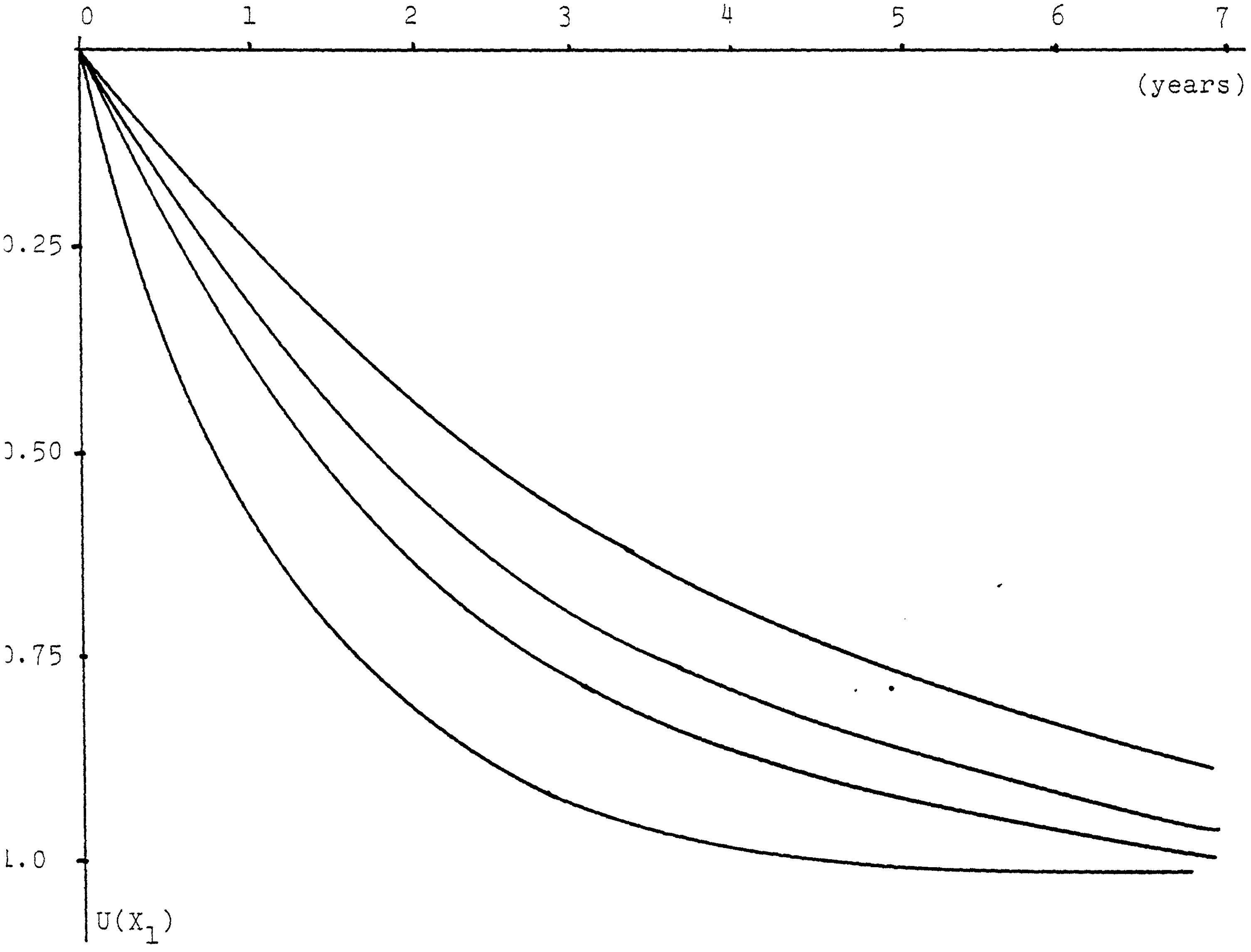


Figure E.1. Component Value Function for Development Time. Country B.

1. "Business as usual" Environment & a_1-Aspiration
2. "Crisis" Environment & a_1-Aspiration
3. "Business as usual" Environment & a_2-Aspiration
4. "Crisis" Environment & a_2-Aspiration

(1) Aspiration a_1: Value function $v_2(x_2) = e^{-\gamma x_2} - 1$ E.8

where $\gamma = 6.93$

(2) Aspiration a_2: Value function $v_2(x_2) = \begin{cases} -\alpha(x_2)^{\beta} & \text{E.9a} \\ e^{-\gamma x_2 - \delta} - 1 & \text{E.9b} \end{cases}$

where $\alpha = 1.28$, $\beta = 6.58$,

$\gamma = 13.86$, $\delta = .22$

where Eq.9a corresponds to the warning period before completion of the arsenal, given that fabrication of the first explosive has begun, and Eq.9b corresponds to the warning period before the fabrication of first weapon. (See also Appendix C, Section C.4.4.)

The decision makers involved with this assessment asserted that the same value function holds for both a "business as usual" environment and a "crisis" environment. It should be noted, however, that this does not mean that the contribution of a particular warning period to the value of a pathway is the same under a "business as usual" and a "crisis" environment. This contribution is given by $\lambda_2 v_2(x_2)$ (see Eq. E.1). Hence, although $v_2(x_2)$ is the same for both environments the weighting coefficient changes. (See section E.7.)

E.4 Component Value Function for Inherent Difficulty.

As discussed in Appendix B, in order to assess a value function over the attribute inherent difficulty, we need to decompose it into six sub-attributes. The component value functions corresponding to these sub-attributes have been assessed in Appendix B. The same functions will be used here.

E.5. Component Value Function for Weapons Material.

As discussed in Section III 3.4, this attribute can be quantized in four distinct levels corresponding to the nature of the fissile material: (a) Reactor Grade Plutonium; (b) Weapons Grade Plutonium; (c) Highly enriched U-233; and (d) Highly enriched U-235. The relative values of these four levels were directly assessed by the decision makers, and are given in Table E.2 for the various sets of conditions.

E.6. Component Value Function for Monetary Cost.

In this section we assess the component value function for the attribute: monetary cost. Four separate assessments were made as in the case of attribute: development time. (See section E.2.) The range of

TABLE E.2

Value of the various weapons materials

Weapons Material	VALUE		
	a_1	$a_2^{(1)}$	$a_2^{(2)}$
U-235	0	0	0
U-233	-.25	-.15	-.10
WG-Pu	-.50	-.30	-.20
RG-Pu	-1.00	-1.00	-1.00

(1) Difficulty associated with the construction of a_2 weapons with RG-Pu small; and (2) Difficulty associated with the construction of a_2 weapons with RG-Pu large.

this attribute expressed in millions of dollars is

$$0 \leq x_5 < 700 \qquad \text{E.10}$$

First the properties affecting the shape of the value functions were determined.

A. Monotonicity. The decision makers asserted that lower costs are always preferred to higher costs. Monotonicity, i.e.,

$$x_5' \leq x_5'' \text{ always implying } v_5(x_5') \geq v_5(x_5'')$$

was thus established.

B. Convexity and Concavity. The decision makers, by answering the following questions under the four sets of conditions, established the shape of the component value functions.

"Compare the change in value associated with the following pairs of cost increases." (Cost in $M.)

Q.1	(0 to 100)	$<$	(100 to 200)
Q.2	(100 to 200)	$<$	(200 to 300)
Q.3	(200 to 300)	$<$	(300 to 400)
Q.4	(300 to 400)	$<$	(400 to 500)
Q.5	(400 to 500)	$<$	(500 to 600)
Q.6	(500 to 600)	$<$	(600 to 700)

It was determined that the value function is convex for all four sets of conditions (see Figure E.2).

C. Numerical Assessment of Value Function - Midvalue Splitting Technique. We set the value of zero cost at zero and the value of 700 $M at -1; i.e.,

$$v_5(0) = 0 \quad \text{and} \quad v_5(700) = -1 \qquad \text{E.11}$$

Next, the levels of the cost having -.25, -.50, and -.75

values were assessed using the midvalue splitting technique as was done for the development time attribute (see Section E.2.C). The assessments of these levels are shown in Table E.3. The curves fitted to these

TABLE E.3

Cost levles of -.25, -.50 and -.75 values for various sets of conditions

VALUE	COST ($M)			
	ASPIRATION a_1		ASPIRATION a_2	
	"Business as usual"	"Crisis"	"Business as usual"	"Crisis"
0	0	0	0	0
-.25	150	200	250	350
-.50	250	300	400	500
-.75	350	375	575	625
-1.00	700(1)	700(2)	700	700
α	2.6×10^{-4}	1×10^{-5}	1.96×10^{-4}	3.48×10^{-6}
β	1.37	1.90	1.30	1.91

(1) For aspiration a_1 and a "business as usual" environment, it was assumed that $v_5(x_5) = -1$ for $x_5 \geq 415\$M$.

(2) For aspiration a_1 and "crisis" environment it was assumed that $v_5(x_5) = -1$ for $x_5 \geq 430\$M$.

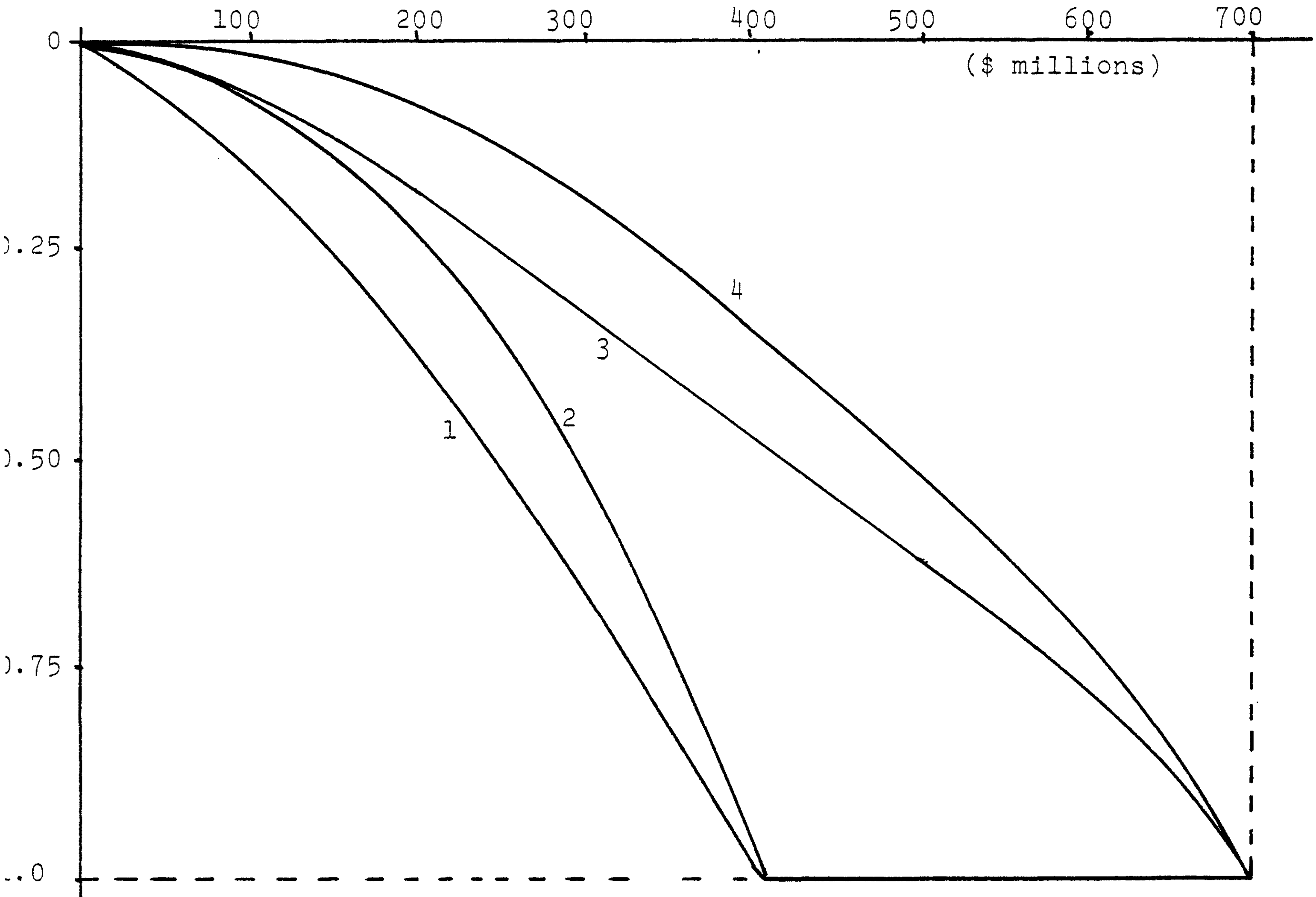


Figure E.2. Component Value Function for Cost.
Country B.

1. "Business as usual" environment & a_1-aspiration
2. "Crisis" environment & a_1-aspiration
3. "Business as usual" environment & a_2-aspiration
4. "Crisis" environment & a_2-aspiration

points have the form

$$v_5(x_5) = -\alpha(x_5)^{\beta} \qquad \text{E.12}$$

The values of the parameters α, β for the four sets of conditions are given in Table E.3 and the corresponding functions are shown in Figure E.2.

E.7. Assessment of the Weighting Coefficients.

In this section we present a method for assessing the weighting coefficients (λ's) in Eq.E.1. The basic idea is to ask the decision maker to identify several pairs of pathways that have the same resistance, and therefore the same value. For each pair ($\underline{x}'$,$\underline{x}''$) of pathways that have the same resistance, it follows that

$$v(\underline{x}') = v(\underline{x}'') \qquad \text{E.13}$$

Replacing $v(\underline{x})$ in Eq. E.13 with the expression given in Eq.E.1, we obtain one equation relating the λ's. The objective is to establish enough pairs of equally preferred pathways that will yield, via E.13, a sufficient number of equations that can be solved for the λ's. Of course, this is an evolutionary procedure. Once, the first set of λ's is obtained, more pairs of equally preferred pairs of pathways are established, and their value -- using the derived λ's -- can be calculated to check for consistency. If there are major differences between the preference relations directly assessed by the decision maker and those suggested by the calculated $v(\underline{x})$, then the λ's are reestimated using another set of indifference assessments. This procedure is repeated until a set of λ's is evaluated that the decision maker feels adequately represents his preferences.

As discussed in Section E.1, there are 10 weighting coefficients that must be calculated. However, relations among six of these, namely among those corresponding to the inherent difficulty attributes, are already available. (See Appendix B.) We can therefore express any five of the inherent-difficulty λ's in terms of the sixth and thus, we have reduced the 10 unknowns to 5. If we establish four relations of the form of Eq.E.13, then these four together with Eq.E.1a will provide a set of five equations that can be solved for the five unknowns. This procedure is now presented for various sets of conditions.

E.7.1. Case-Study I: Country B, Aspiration a_1, "Business as Usual" Environment.

In the questions that follow, a proliferation pathway is denoted by the values of the five attributes as $(x_1,x_2,\underline{x}_3,x_4,x_5)$. The attribute of inherent difficulty, whenever included in a trade off, will be represented by one of the sub-attributes, namely the one that changes. In all other instances the common value of the inherent difficulty will be denoted by a ---.

The first question involved trade-offs between the warning period and the status of information for isotopic enrichment. The following pathway (I) is considered:

I = (6 years, 0%, C, U-235, 350$M)

i.e., a proliferation pathway is considered that: (a) will take 6 years; (b) will not be detected (0%); (c) is characterized by a level C status of information for isotopic enrichment (there is no radioactivity or criticality problems involved); (d) requires the enrichment of uranium in U-235; and (e) will cost 350$M. Then, the decision maker is asked to assess the status of information for pathway II given by

II = (6 years, 30%, ?, U-235, 350$M)

such that he would be indifferent between I and II (I $\sim$ II). Since such questions might not be easy to answer directly,(10) a step-by-step procedure, including a series of questions as given below, can be used.

We ask the decision maker to compare the following pairs of pathways. In practice, each pair is suggested by the answer to the preceding question.

Pathway I		Pathway II	Answer
(6, 0%, C, U-235, 350)	?	(6, 30%, A, U-235, 350)	I < II
(6, 0%, C, U-235, 350)	?	(6, 30%, G, U-235, 350)	I > II
(6, 0%, C, U-235, 350)	?	(6, 30%, D, U-235, 350)	I < II
(6, 0%, C, U-235, 350)	?	(6, 30%, B, U-235, 350)	I > II

The answer to the last question was that I is slightly preferred to II. Since there is no level of the status of information between D and B, in order to increase the value of pathway II to make it equally preferred to pathway I, we started decreasing the level of the warning period. Finally indifference was achieved for the following pair.

$$(6,\ 0\%,\ C,\ U\text{-}235,\ 350) \sim (6,\ 25\%,\ B,\ U\text{-}235,\ 350)$$

Expressing the values of these two pathways in terms of Eq.E.1 we get

$$\lambda_2 V_2(0) + \lambda_{34} v_{34}(C) = \lambda_2 v_2(.25) + \lambda_{34} v_{34}(B)$$

or that

$$\lambda_2 = \frac{v_{34}(B) - v_{34}(C)}{v_2(0) - v_2(.25)} \lambda_{34} \qquad \text{E.14}$$

Similarly we established the following indifferences:(11)

$$(1,\ 30\%,\ C,\ U\text{-}235,\ 350) \sim (6,\ \overset{?}{7}\%,\ G,\ U\text{-}235,\ 350)$$

or that

$$\lambda_1 = \frac{v_2(.3) - v_2(.07)}{v_1(6) = v_1(1)}\,\lambda_2; \qquad \text{E.15}$$

$$(6,\ 30\%,\ C,\ U\text{-}235,\ 10) \sim (6,\ \overset{?}{5}\%,\ C,\ U\text{-}235,\ 350)$$

or that

$$\lambda_5 = \frac{v_2(.3) - v_2(.05)}{v_5(350) - v_5(10)}\,\lambda_2; \qquad \text{E.16}$$

and

$$(3,\ 30\%,\ C,\ U\text{-}235,\ 350) \sim (3,\ \overset{?}{15}\%,\ C,\ RG\text{-}Pu,\ 350)$$

or that

$$\lambda_4 = \frac{v_2(.3) - v_2(.15)}{v_4(RG\text{-}Pu) - v_4(U\text{-}235)}\,\lambda_2. \qquad \text{E.17}$$

Since the component value functions for the various attributes are known, equations E.14-to-E.17 provide four numerical relations for the λ's. These relations when combined with Eq.E.1a and the known ratios of the inherent-difficulty λ's yield the numerical values presented in Table E.4.

E.7.2. Case Study II: Country B, Aspiration a_2, "Crisis" Environment.

Under these conditions, we established by preliminary questions that the most important attribute was the development time. Thus, the questions in this case involved a large change in one of the attributes and the decision maker had to adjust the level of development time that achieved indifference. The following four pairs of equally resistant pathways were established.

$$(6,\ 30\%,\ \text{A},\ \text{U-235},\ 350) \overset{?}{\sim} (1,\ 30\%,\ \text{C},\ \text{U-235},\ 350)$$

$$(6,\ 0\%,\ -,\ \text{R.G-Pu},\ 20) \overset{?}{\sim} (2,\ 30\%,\ -,\ \text{R.G-Pu},\ 20)$$

$$(6,\ 30\%,\ -,\ \text{R.G-Pu},\ 10) \overset{?}{\sim} (3,\ 30\%,\ -,\ \text{R.G-Pu-}350)$$

$$(6,\ 30\%,\ -,\ \text{U-235},\ 350) \overset{?}{\sim} (4,\ 30\%,\ -,\ \text{R.G-Pu},\ 350)$$

The resulting λ's are given in Table E.4.

E.7.3. Case Study III: Country B, Aspiration a, "Business as Usual" Environment. Small Difficulty Associated with RG-Pu as Weapons Material.

Under these conditions it was established by preliminary questions that the most important attribute (i.e. the one that the proliferator would try to reduce first)(12) is the status of information for the isotopic enrichment. Thus, the questions initially involved a large change in the level of an attribute and then, the decision maker was asked to compensate this change by adjusting the level of the status of information for isotopic enrichment. Here, as in Case I, in order to accommodate for the relative inflexibility associated with the discrete nature of the levels characterizing the status of information we had to vary more than one attributes to achieve indifference. The following four pairs of equally resistant pathways were established.

$$(6,\ \overset{?}{20\%},\ \overset{?}{\text{A}},\ \text{U-235},\ 580) \sim (6,\ 0\%,\ \text{C},\ \text{U-235},\ 580)$$

$$(\overset{?}{2},\ 20\%,\ \text{C},\ \text{U-235},\ 580) \sim (6,\ 20\%,\ \text{G},\ \text{U-235},\ 580)$$

$$(3,\ 20\%,\ -,\ \text{R.G-Pu},100) \sim (3,\ \overset{?}{5\%},\ -,\ \text{R.G-Pu},500)$$

$$(4,\ 20\%,\ -,\ \text{R.G-Pu},100) \sim (4,\ 20\%,\ -,\ \text{U-235},\ \overset{?}{600})$$

The resulting λ's are given in Table E.4.

E.7.4 Case Study IV: Country B, Aspiration a_2, "Business as Usual" Environment. Large Difficulty Associated with R.G-Pu as Weapons Material

Under these conditions it was established that the most important attribute(12) is the weapons material. The main difference between this case and the previous one, lies in the shifting of the relative weight among the attributes that evaluate the technical difficulty of the proliferation effort. Thus, in this case the following pairs of equally resistant pathways were established.

$$(6\text{y},\ 20\%,\ \text{C},\ \text{U-235},\ 580) \sim (6\text{y},\ \overset{?}{0\%},\ \overset{?}{\text{A}},\ \text{R.G-Pu},\ 580)$$

$$(6\text{y},\ 20\%,\ \overset{?}{\text{A}},\ \text{U-235},\ 580) \sim (6\text{y},\ 0\%,\ \text{G},\ \text{U-235},\ 580)$$

$$(\overset{?}{2},\ 20\%,\ \text{C},\ \text{U-235},\ 580) \sim (6\text{y},\ 20\%,\ \text{G},\ \text{U-235},\ 580)$$

$$(5,\ 20\%,\ -,\ \text{R.G-Pu},\ 100) \sim (5,\ \overset{?}{5\%},\ -,\ \text{R.G-Pu},\ 500)$$

The resulting λ's are given in Table E.4.

E.8. Resistance Value of Pathways.

Before calculating the "values" of the various pathways using the results of Sections E.2 to E.7 in Eq.E.1, the decision maker was presented with Table E.4 to determine whether the calculated λ's were in agreement with his preferences. Each λ_i is a measure of the importance of the corresponding attribute with respect to the others. If we consider the most resistant pathway, i.e., the pathway that has all the attributes at their lowest level, and therefore, has a value of -1 (see section E.1), then the λ's give the fractions of the total resistance attributable to the corresponding attributes.

We note, for example, that for case I (i.e., aspiration a_1 and "business as usual" environment), if a pathway requires both chemical and isotopic separation and if all the attributes have their worst possible values the contribution of the inherent difficulty to the overall resistance of this pathway is 58%(13) of the total and thus, it is by far the major contributor to the resistance. The inherent difficulty remains the major contributor to the most resistant pathways that involve only chemical or only isotopic separation

TABLE E.4

WEIGHTING COEFFICIENTS (λ's) OF ADDITIVE VALUE FUNCTION FOR FOUR CASE STUDIES

ATTRIBUTE			λ	CASE I	CASE II	CASE III	CASE IV
DEVELOPMENT TIME			λ_1	.13	.31	.15	.13
WARNING PERIOD			λ_2	.15	.07	.17	.14
INHERENT DIFFICULTY	CHEMICAL	Status of Information	λ_{3j}	.18	.17	.16	.13
		Radioactivity	λ_{32}	.08	.08	.07	.06
		Criticality	λ_{33}	.01	.01	.01	.01
	ISOTOPIC	Status of Information	λ_{34}	.20	.20	.19	.15
		Radioactivity	λ_{35}	.08	.08	.07	.06
		Criticality	λ_{36}	.03	.03	.03	.03
WEAPONS MATERIAL			λ_4	.03	.01	.06	.21
COST			λ_5	.11	.04	.08	.08

of the fissile material. Thus, the most resistant pathway involving only chemical separation has a value of -.69 [-1.-(-20-.08-.03)] and the contribution of the inherent difficulty is 35% (= $\frac{.27}{.69}$). Of course, this does not mean that for <u>any</u> pathway the inherent difficulty will always be the major contributor. The contributions of the attribute will also depend on their level. For instance, a pathway having all the attributes except the weapons material in their "best" levels will have a resistance that is wholly attributable to the weapons material. The decision maker agreed with these remarks, and said that the λ's in Table E.4 generally express his feelings about the relative importance of the various attributes. Continuing in this fashion, cases I and II were compared next. In this case, the decision maker said that for a crisis environment the inherent difficulty remained an important contributor, but now an equally if not more important factor was the development time. In a "crisis" environment the importance of warning period, cost, and weapons material was judged to be marginal. These attitudes are expressed by the λ's calculated for case II. Finally comparing cases III and IV, (Sec. E.7.3 & E.7.4) the decision maker said that he felt that the major difference lay in the change of the difficulty associated with R.G-Pu.

He said that for case IV this difficulty is of the same order as the difficulty associated with the procurement of the fissile material. This attitude is reflected in the λ's for case IV.

Next, the assessed component value functions and the weighting coefficients were used along with Eq. E.1 to evaluate the relative resistance value of the various pathways of the three systems considered in Chapter IV. The scores of the attributes for the pathways are given for a country of type B in Tables IV.1 to IV.6. The corresponding scores for the inherent difficulty attributes are given in Tables E.5 to E.7.(14) The resistance value of each pathway was calculated for the four sets of conditions presented in section E.2, and the results are presented in Tables E.8 and E.10.

Table E.8 gives the values of the pathways for the three systems for country B, having aspiration a_1 under two environments: "business as usual" and "crisis". The value of system IV (independent pathway) is also given in this table as the 10th pathway for each system. From the values of the various pathways in Table E.8, we conclude that the least resistant pathways of systems I, II and III for a "business as usual" environment are no. 2, no. 2, and no. 6, respectively. The corres-

TABLE E.5

"Scores" of Inherent Difficulty Attributes

System: LWR-Once Through- Reactors Only- Light Sanctions

Country : B

N.W. Aspiration : a_1 or a_2

Pathway		Chemical			Isotopic		
No.	Descript.	Status of Info.	Radio activ. (rad/hr)	Crit. Prob.	Status of Info.	Radio activ. (rad/hr)	Crit. Prob.
1	C-C-SF	B	10^5	L	N.A.*	N.A.	N.A.
2	C-O-SF	B	10^5	L	N.A.	N.A.	N.A.
3	O-O-SF	B	10^5	L	N.A.	N.A.	N.A.
4	C-C-FF	N.A.	N.A.	N.A.	C	O	L
5	C-O-FF	N.A.	N.A.	N.A.	C	O	L
6	O-O-FF	N.A.	N.A.	N.A.	C	O	L
7	I	B	10^4	L	N.A.	N.A.	N.A.
8							
9							
10							

* Not Applicable

TABLE E.6.

"Scores" of Inherent Difficulty Attributes

System: LWR-Denatured Thorium- Reactors only- Light Sanctions

Country: B

N.W. Aspiration: a_1 or a_2

Pathway		Chemical			Isotopic		
No.	Descript.	Status of Info.	Radio activ. (rad/hr)	Crit. Prob.	Status of Info.	Radio activ. (rad/hr)	Crit. Prob.
1	C-C-SF	B	10^6	L	N.A.*	N.A.	N.A.
2	C-O-SF	B	10^6	L	N.A.	N.A.	N.A.
3	O-O-SF	B	10^6	L	N.A.	N.A.	N.A.
4	C-C-SF	B	10^6	L	C	10^2	L
5	C-O-SF	B	10^6	L	C	10^2	L
6	O-O-SF	B	10^6	L	C	10^2	L
7	C-C-FF	B	10^2	L	C	10^2	L
8	C-O-FF	B	10^2	L	C	10^2	L
9	O-O-FF	B	10^2	L	C	10^2	L
10	I	B	10^4	L	N.A.	N.A.	N.A.

* Not Applicable

TABLE E.7

"Scores" of Inherent Difficulty Attributes

System: LWR- Pu- Recycle - Reactors Only (Pre-Irr.MOX)-Light Sanction

Country : B

N.W. Aspiration : a_1 or a_2

Pathways		Chemical			Isotopic		
No	Descript.	Status of Info.	Radio activ. (rad/hr)	Crit. Prob.	Status of Info.	Radio activ. (rad/hr)	Crit. Prob.
1	C-C-SF	B	10^5	L	N.A.*	N.A.	N.A.
2	C-O-SF	B	10^5	L	N.A.	N.A.	N.A.
3	O-O-SF	B	10^5	L	N.A.	N.A.	N.A.
4	C-C-FF	B	10^4	L	N.A.	N.A.	N.A.
5	C-O-FF	B	10^4	L	N.A.	N.A.	N.A.
6	O-O-FF	B	10^4	L	N.A.	N.A.	N.A.
7	C-C-FF	B	10^4	L	C	O	L
8	C-O-FF	B	10^4	L	C	O	L
9	O-O-FF	B	10^4	L	C	O	L
10	I	B	10^4	L	N.A.	N.A.	N.A.

* Not Applicable

TABLE E.8

RELATIVE RESISTANCE VALUE OF VARIOUS PATH WAYS OF SYSTEMS I TO III

(see Tables IV.1 to IV.3)

PATHWAY No.	COUNTRY B : ASPIRATION a_1					
	"Business As Usual"			"Crisis"		
	SYSTEM I	SYSTEM II	SYSTEM III	SYSTEM I	SYSTEM II	SYSTEM III
1	-.235	-.238	-.235	-.363	-.337	-.363
2	-.226*	-.229*	-.226	-.360	-.363	-.360
3	-.237	-.240	-.237	-.340*	-.343*	-.340
4	-.424	-.431	-.196	-.507	-.566	-.316
5	-.418	-.429	-.191	-.505	-.566	-.314
6	-.448	-.461	-.179*	-.511	-.576	-.271*
7	---	-.371	-.512	---	-.501	-.588
8	---	-.369	-.510	---	-.500	-.586
9	---	-.411	-.531	---	-.510	-.589
System IV	-.203	-.203	-.203	-.380	-.380	-.380

TABLE E.9

ORDERING OF SYSTEMS IN TERMS OF DECREASING PROLIFERATION RESISTANCE

Country B, Aspiration a_1

	BUSINESS AS USUAL		CRISIS	
	VALUE	ORDER	VALUE	ORDER
SYSTEM I	-.226	2	-.340	3
SYSTEM II	-.229	1(∼2)	-.343	2(∼3)
SYSTEM III	-.179	4	-.271	4
SYSTEM IV	-.203	3	-.380	1

ponding pathways for the "crisis" environment are: no. 3, no. 3 and no. 6. These results are in exact agreement with the results drawn from the "qualitative" examination of the pathways using the ideas of dominance and extended dominance (see section IV.6, Tables IV.13 and IV.15). The resulting "ordering" of the systems shown in Table E.9 is, of course, exactly the same as that obtained after the analysis of section IV.7(15) (see Tables IV.17 and IV.19).

Table E.10 gives the resistance values of the various pathways of the three systems, for country B, having aspiration a_2, operating under a "business as usual" environment and for two different assumptions:
(a) Small difficulty associated with the construction of a_2 weapons with Reactor-Grade Plutonium; and
(b) Large difficulty. For the former, the least resistant pathways for systems I, II and III are, respectively, no. 2, 3 and 6. These results are not in agreement with the results obtained from the "qualitative" analysis presented in section IV.6. According to that analysis the corresponding least resistant pathways are no. 2, 2, and 5 (see Table IV.13). The first discrepency occurs with system II. The decision maker was confronted with this inconsistency, and after careful examination of pathways 2 and 3 (see Table IV.4), he asserted that

3 is less resistant than 2. The rationale was the following. In going from 2 to 3 we experience a decrease in the resistance because of the decrease in the development time, and increases in the resistance because of the increases in the warning period and cost. From the quantitative assessment it follows the difference in the resistance value of \$30 million and \$50 million is negligible ($\sim$.001). The decision maker agreed that that is exactly how he feels. For country B and a_2-weapons this difference in cost should not be of any importance. Then the question is whether the reduction of one year (2.5 to 1.5) in the development time is worth more or less than a decrease by .01 (.03 to .02) in the warning period. After some thought he said that he would prefer the reduction in the development time. This attitude was consistent with the quantitative assessment and thus, pathway no. 3 is less resistant than pathway no. 2. For similar reasons he asserted that for system III (see Table IV.5) pathway no. 6 is less resistant than pathway no. 5.

From Table E.10 we conclude that for "large R.G-Pu difficulty" the least resistant pathways for systems I, II and III are no. 5, 8, and 6, respectively. From Table IV. 11 we see that the result of the corresponding qualitative analysis was that the least resistant

TABLE E.10

RELATIVE RESISTANCE VALUES OF VARIOUS PATHWAYS OF SYSTEM I AND III FOR COUNTRY B AND ASPIRATION a_2

(See Tables IV.4 to IV.6)

PATHWAY No.	SMALL RG-Pu DIFFICULTY			LARGE RG-Pu DIFFICULTY		
	SYSTEM I	SYSTEM II	SYSTEM III	SYSTEM I	SYSTEM II	SYSTEM III
1	-.294	-.341	-.294	-.408	-.447	-.408
2	-.258*	-.271	-.258	-.378	-.390	-.378
3	-.261	-.264*	-.261	-.380	-.383	-.380
4	-.428	-.451	-.214	-.359	-.338	-.341
5	-.417	-.438	-.219	-.349*	-.377	-.345
6	-.427	-.451	-.210*	-.360	-.387	-.337*
7	---	-.392	-.521	---	-.338	-.441
8	---	-.384	-.508	---	-.332*	-.429
9	---	-.397	-.512	---	-.342	-.434
SYSTEM IV	-.228	-.228	-.228	-.229	-.229	-.229

TABLE E.11
ORDERING OF SYSTEMS IN TERMS OF DECREASING PROLIFERATION RESISTANCE

Country B, Aspiration a_2

	Small RG-Pu Difficulty		Large RG-Pu Difficulty	
	VALUE	ORDER	VALUE	ORDER
SYSTEM I	-.258	2	-.349	1
SYSTEM II	-.264	1(∿2)	-.332	3
SYSTEM III	-.210	4	-.337	2
SYSTEM IV	-.228	3	-.229	4

pathways are no. 5, 5, and 8. Examination of pathways 8 and 5 of system II (see Table IV.5) revealed that the difference in the resistance of these two pathways consists of the difference in the inherent difficulty associated with the chemical separation and the difference in the cost. Pathway no. 5 requires "hot" chemistry while pathway no. 8 requires almost "cold" chemistry (see also Table E.6). When the decision maker compared this difference in the inherent difficulty with the $60 million difference in the cost, he asserted that the former resistance is of greater value than the latter, and that he should have chosen pathway no. 8 in his "qualitative" analysis. For system III (see Table IV.6), the decision maker admitted that he had chosen pathway no. 8 by excluding all pathways that had R.G-Pu as weapons material. After examining the implications of the quantitative assessment he asserted that, although the difficulty associated with R.G-Pu in pathway no. 6 is greater than the difficulty associated with the uranium enrichment of pathway no. 8, the difference in the values of the other attributes is so overwhelmingly in favor of no. 6 that once more he agreed with his quantitative assessment.

Based on these least resistant pathways the ordering of the systems is as shown in Table E.11.

The ordering for "small R.G-Pu difficulty" is identical to the one obtained from the qualitative analysis in spite of the difference in the least resistant pathways (see Table IV.18). For "large R.G-Pu difficulty" the ordering resulting from the quantitative analysis differs from that resulting from the qualitative analysis, the main source of the difference being the different pathway representing system III.

E.9. Concluding Remarks.

In this Appendix we have presented a demonstration of the application of the quantitative techniques of multi-attribute value theory in the assessment of the proliferation resistance of an alternative system in a given country having a particular nuclear weapons aspiration. Such quantitative analyses should provide useful insight into the factors that affect the differential proliferation resistance of alternative systems, as well as a means of checking qualitative analyses for consistency. Care must be taken, however, in interpreting the results of such an analysis. The resulting "composite" measures should be viewed critically and always in conjunction with a qualitative analysis. The resistance value of an alternative system is a useful tool in comparing systems among themselves and not as an end in itself. Finally, the existence of preferential independence must be verified before an additive form for value function is used for a particular subspace of the attribute space.

NOTES ON THE APPENDICES

1. The symbolism $\tilde{x}_i$ indicates that x_i is a random variable; $\underline{\tilde{x}}$ denotes a multivariate random variable.

2. Another way of looking at this is that certainty is a special case of uncertainty.

3. For example, in a situation where a weapons program is being carried out in a democracy, without the knowledge of groups inside and outside the government who would oppose such an effort.

4. This subsection requires a familiarity with the concept of utility. A short introduction to utility theory is presented in Section C.2.1.

5. Others can be easily added in a more detailed model.

6. For a detailed definition of utility independence see Appendix A.

7. For a complete definition of these terms see [6].

8. This assumption was verified for our example by a procedure similar to the one described in Appendix B

for the inherent difficulty attributes.

9. Since choice under certainty (where value functions are used) is a special case of choice under uncertainty (where utility functions are used), a utility function is always a value function. This is true because $x' \precsim x''$ if and only if $u(x') \leq u(x'')$. See also Appendix A.

10. Here we have an additional difficulty stemming from the descrete nature of the levels of the status of information.

11. The question mark indicates the attribute that was adjusted to achieve indifference.

12. It is reminded that the most important attribute is the attribute that the decision maker feels contributes the most to the resistance of a pathway.

13. To be more precise, this contribution is $.58/.97 = .60$, since such a pathway involving enrichment will have as weapons material U-235, and thus a total value of -.97. $(\lambda_{36}u_{36}(U235) = 0)$

14. It is assumed that the inherent difficulty depends only on the type of pathway and not on the aspiration. Thus, the inherent difficulty of pathways No. 1 in Table IV.1 and No. 4 in Table IV.4, etc. is the same.

15. In a complete quantitative analysis, a value function expressing the point of view of the international community should be assessed and the systems would be ordered in terms of the values of their least resistant pathways obtained using this value function and not the proliferator's value function. For the demonstration purpose of this report, however, this was not necessary. For this particular example the order would not have changed regardless of the form of the new value function because of dominance considerations.