A Mathematical Programming/Economic Equilibrium Model for the Quantitative Analysis of the Stability of Japan's Energy System

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ABSTRACT

Japan's energy supply-demand system is fully dependent on the import of primary energy resources from foreign countries. So the availability of primary energy, including crude oil and coal, is a very important factor for the stability of our energy system. In order to measure our energy system's stability under an uncertain future availability of energy resources, we built a mathematical programming / economic equilibrium model based upon linear programming techniques. In the model analysis uncertain future availability of primary energy resources is expressed as random variables with a given probability distribution, and the economic equilibrium point is obtained by iterative convergent computation.

From our numerical results we know an optimal energy supply-demand structure with equilibrium prices of primary energy resources at the future target year, and obtain supply stability and instability probabilities of our energy system. Furthermore, applicability of decomposition techniques to our energy model analysis and necessary and sufficient conditions for the stability of our energy system are discussed.

During the years following the Arab oil embargo of 1973, there have been many energy policy debates throughout the world, including Japan. Energy policy debates concern various technical, environmental, social, economical, political and even military problems. Energy policy modeling efforts have increased due to not only the necessity of such interdisciplinary research, but also the greater availability of high speed computers. Since Hoffman [1973] proposed energy network systems analyses for supply-demand energy problems, various systems analysis approaches have been developed. (See e.g. Charpentier [1974] and Manne, et al [1979] for energy models. Also see Shapiro [1975, 1977], Oyama [1980, 1980a, 1983b], Modiano and Shapiro [1980], and Shapiro and White [1982].) We investigated the Japanese electric power system (see Energy Study Group [1979], Saito and Oyama [1980]) to see what our energy supply and demand situation will be like in the year 2000.

Many economic equilibrium models have also been developed which use linear and nonlinear programming techniques. (See e.g. Kennedy [1974], Hogan [1975], Griffin [1977], Hogan and Weyant [1980], Daniel and Goldberg [1981]. See also Takayama and Judge [1971] for price and resource allocation models.) Shapiro [1978] discusses decomposition techniques to show the relationship between linear programming and econometric components of energy planning models. Furthermore, Shapiro [1978] presents the interpretation of the Kuhn-Tucker optimality conditions as an economic equilibrium point for certain mathematical programming models.

However, in most of these modeling studies, future energy demands and the availability of primary energy resources are given exogenously. We believe that our future energy demand and the availability of primary energy resources should be uncertain.

In our analysis, we express the structure of our energy supply and demand system as a network as does Hoffman [1973]. We then formulate a linear programming economic equilibrium problem in which the supply availabilities of primary energy resources, such as crude oil and coal, are defined as random variables.

This paper proposes an iterative convergent procedure to compute an economic equilibrium point for our energy model. The equilibrium point indicates an energy supply and demand structure for the future target year. Our mathematical programming / economic equilibrium (MP/EE) model is a mathematical programming optimization model to find an economic equilibrium point as an optimal solution of a linear programming problem.

We define the stability probability of our energy system as the probability that the given linear programming model is feasible under the "randomized" supply availability constraints. In addition to these stability probabilities of our energy system at the target year, distributions of prices and demands for crude oil and coal are obtained from the economic equilibrium solutions of our energy model. We can also evaluate our energy conservation policy in various demand sectors by combining shadow price analysis with the probabilistic approach.

In section 1 we explain the current energy situation in Japan. In Section 2 we outline our MP/EE energy model including the formulation of our energy model and the computational procedure to solve it. Numerical results are given in Section 3 and their theoretical analysis is described in Section 4. Finally, in Section 5, we summarize our model and comment on our approach.

1. Introduction

Japan imports almost 90% of her total primary energy. Major imported energy resources are fossil fuels such as crude oil, coal and liquefied natural gas (LNG). These primary energy resources are mostly imported from Middle Eastern countries and the South Pacific region, which contain many politically and economically unstable countries. Considering the difficulties which arose during the oil embargo of 1973-'74, it is probable that we will not always be able to obtain an adequate amount of primary energy resources. The oil embargo had a very serious impact, and created confusion in our country's economic and engineering system. Therefore, whether or not we can obtain enough primary energy resources to meet future expected demand concerns us greatly.

The oil embargo in 1973 induced a large crude oil price increase from $2.51/bbl in 1972 to $10.79/bbl in 1974, and it had serious effects on the world economy and energy supply-demand system. After this "first oil crisis" our primary energy consumption has stopped increasing as rapidly. Oil imports decreased by 4.4% from 1973 to 1974, and 4.8% from 1974 to 1975.

The "second oil crisis", which resulted from the Iranian revolution in 1978, also caused a second large price increase. Crude oil prices increased from $13.77/bbl in 1978 to $32.97/bbl in 1980. Our crude oil imports decreased by 10.7% from 1979 to 1980. Total primary energy consumption also decreased by 3.4% during the same period. Through these two "oil crises", energy conservation has prevailed in our industrial demand sector. Our energy system has structurally changed to a system that does not depend heavily on crude oil.

It is probable that we will have another "oil crisis" induced by a disruption in oil supplies due to some unexpected events in oil exporting countries. Therefore, it is important to quantitatively evaluate our energy system's stability under various levels of primary energy supply constraints. The "stability" of our energy system is fully dependent upon the possibility of importing crude oil and coal. Our energy system can be determined to be "stable" or "unstable", corresponding to whether or not we can have a sufficient supply of primary energy resources to meet our future energy demand. By defining the "supply stability probability" of our energy system as the probability that our mathematical programming energy model has a feasible solution, the "supply stability" of our energy system can be quantitatively investigated.

2. MP/EE Energy Model

2.1 Energy System and Linear Programming Model

Our energy system, which involves energy flow from various supply regions to final demand sectors, is illustrated as a network system in Figure 1. The supply sector consists of seven divisions, including five supply regions, domestic production, and stockpile·transfer. Four kinds of primary energy of hydro-nuclear, crude oil, coal, and LNG-natural gas are transformed into petroleum products, coal products, and secondary energy of electricity and city gas. The final demand sector consists of four categories: industry, residential·commercial, transportation, and stockpile·transfer.

A feasible energy flow in the network has to satisfy the future energy demand under various supply constraints, and physical and engineering constraints. The energy flows on the arcs of the network correspond to unknown variables of the model, and network constraints are linear equalities and inequalities using those variables. Thus the problem of finding an equilibrium energy flow can be formulated as a linear programming problem.

The goal is to obtain a desirable feasible energy flow corresponding to an economic equilibrium point in our mathematical programming energy model. A desirable energy flow in the network energy system of Figure 1 can be determined as a flow attaining a maximum economic surplus criterion; i.e., minimizing supply cost less demand cost. One can obtain an optimal energy flow by solving linear programming problems iteratively, until satisfying a convergence criterion.

2.2 Structure of the MP/EE Energy Model

In the MP/EE energy model there are five kinds of endogenous variables $\{x_i, y_j, z_k, w_l, d^i{}_j\}$. Four kinds of them $\{x_i, y_j, z_k, w_l\}$ correspond to energy flows, as in the network of Figure 1, while the remaining endogenous variables $\{d^i{}_j\}$ indicate "flexible" demands for imported primary energies of crude oil and coal.

x_i, $i \varepsilon M = \{1, \ldots, 17\}$: primary energy transported from the supply regions and the stockpile·transfer node

y_j, $j \varepsilon N = \{1, \ldots, 14\}$: primary energy transformed into petroleum products, coal products, or secondary energy; primary energy directly consumed in demand sector

z_k, $k \varepsilon K = \{1, \ldots, 28\}$: petroleum and coal products transformed into secondary energy or consumed in demand sector

w_l, $l \varepsilon L = \{1, \ldots, 5\}$: secondary energy consumed in final demand sector

$d^i{}_j$, $i \varepsilon I = \{R, L\}$, $j \varepsilon J = \{\pm 1, \pm 2, \ldots, \pm 6\}$

: variables indicating the perturbation of primary energy resources' (R:crude oil L:coal) demand from their standard demands

Using the above variables, constraints in the linear programming model are expressed as follows:

(1) Availability Constraints of Primary Energy Resources

In the energy network of Figure 1, primary energy resources enter the system through supply nodes. The amount of primary energy at each supply node has an upper bound determined by the physical, economical or, sometimes, political situations in the supply regions. The physical availability of each primary energy resource from each supply region is given as follows:

$$x_i \leq b_i, \qquad i \varepsilon M. \qquad (1)$$

(2) Flow Conservation Constraints

The set of nodes in the network is divided into three groups ——— supply nodes corresponding to supply regions, demand nodes indicating demand sectors from p=13 to p=16, and remaining intermediate nodes. At each intermediate node $p \varepsilon \{1, \ldots, 12\}$, in-flow has to be equal to out-flow. Therefore,

$$\sum_{i \varepsilon M_p} x_i = \sum_{j \varepsilon N_p} y_j \qquad p=1,2,3 \qquad (2)$$

$$\sum_{j \varepsilon N_p} y_j = \sum_{k \varepsilon K_p} z_k \qquad p=4,\ldots,10 \qquad (3)$$

$$\sum_{j \varepsilon N_p} y_j + \sum_{k \varepsilon K_p} z_k = \sum_{l \varepsilon L_p} w_l \qquad p=11,12 \qquad (4)$$

(3) Upper and Lower Bounding Capacity Constraints

Upper and lower bounds are given for the variables indicating production of petroleum and coal products $\{z_k, k\varepsilon K'\subset K\}$, and consumption of electricity and city gas $\{w_l, l\varepsilon L\}$. These constraints are written as follows:

$$LB^z{}_k \le z_k \le UB^z{}_k, \qquad k\varepsilon K'\subset K \qquad (5)$$

$$LB^w{}_l \le w_l \le UB^w{}_l, \qquad l\varepsilon L \qquad (6)$$

where $LB^z{}_k$, $UB^z{}_k$, $LB^w{}_l$ and $UB^w{}_l$ are lower and upper bounds of $\{z_k\}$ and $\{w_l\}$, respectively. K' is a proper subset of K, hence constraints (5) are given for some variables of $\{z_k\}$.

(4) Yield Constraints of Petroleum Products

Refinery systems have their own physical and engineering restrictions regarding the yields of petroleum products. Each petroleum product has both lower and upper production bounds.

$$Y^L{}_j C \le y_j \le Y^U{}_j C \qquad 3\le j\le 6 \qquad (7)$$

$$C = x_1 + x_2 + x_4 + x_7 + x_{10} + x_{12} + x_{15} - y_2$$

where $Y^L{}_j$ and $Y^U{}_j$ are the lower and upper bounds of the yield of petroleum product indicated by y_j, and C is the total crude oil entering the refineries.

(5) Demand Requirement Constraints for Imported Primary Energy Resources

Imports of primary energy resources are restricted by the following constraint:

$$\sum_{i\varepsilon M_k} x_i \ge D^k{}_0 + \sum_{j\varepsilon J^+} d^k{}_j - \sum_{j\varepsilon J^-} d^k{}_j \qquad k\varepsilon\{R,L\} \qquad (8)$$

where J^+ and J^- indicate sets of positive and negative indices of $J=\{\pm1, \pm2, ..., \pm6\}$, respectively. The left side of the above inequality expresses the flow of each primary energy resource from each supply region to Japan, while the right side expresses the perturbed demand of each primary energy resource from the standard demand. The variable $\{d^k_j\}$, with the superscript k deleted, is illustrated in Figure 2, where the demand quantity D_0 corresponds to the standard demand D^k_0 in the constraint (8). Each variable d^k_j has an upper bound corresponding to the interval in Figure 2.

$$0 \leq d^k_j \leq \Delta^k_j, \qquad k \varepsilon \{R, L\},\ j \varepsilon J. \tag{9}$$

(6) Final Energy Demand Requirement Constraints

In the four demand nodes corresponding to industry, residential·commercial, transportation and stockpile·transfer, the following final energy demand requirement constraints have to be satisfied:

$$\sum_{j \varepsilon N_p} y_j + \sum_{k \varepsilon K_p} z_k + \sum_{l \varepsilon L_p} w_l \geq D_p \qquad p=13,...,16 \tag{10}$$

(7) Objective Function

The total supply cost of the energy system in Figure 1 is defined as the sum of fuel costs and transformation costs. In this model, we define the fuel cost to be the cost of obtaining the resource in each supply region and transporting it to Japan, i.e., the CIF cost. The transformation cost is defined to be the cost for transforming primary energy resources into petroleum and coal products, electricity, and city gas, and consists mainly of the capital cost necessary for energy transformation. Let the fuel cost per thermal unit (10^{10} kcal) be c_i, $i \varepsilon M$, and let the transformation cost per unit kcal of petroleum and coal products be d_j, $j \varepsilon N$. The transformation cost per kcal of secondary energy is given by e_l, $l \varepsilon L$. Then the total energy supply cost can be written as follows:

$$\sum_{i \varepsilon M} c_i x_i + \sum_{j \varepsilon N} d_j y_j + \sum_{l \varepsilon L} e_l w_l . \tag{11}$$

However, the demand cost of the energy system, defined as a total cost for meeting a forecast energy demand, is given as an approximation to the area under the nonincreasing demand curve. By using a step function in Figure 2, the cost is

$$\int_{0}^{Q} g(q) dq = \sum_{j \varepsilon J_Q} P_j d_j \tag{12}$$

where

$$Q = Q_0 + \sum_{j \varepsilon J_Q} d_j , \tag{13}$$

and J_Q is a set of indices {j} corresponding to the intervals contained in the range from 0 to Q, and P_j is a commodity price corresponding to the demand in the j-th interval. Subtracting from (12) the constant demand cost corresponding to the integral of the demand curve from 0 to Q_0 in Figure 2, we obtain the following sum:

$$\int_{Q_0}^{Q} f(q)dq = \sum_{j\varepsilon J_Q} sgn(j)P_j d_j \qquad (14)$$

where sgn(j) indicates the sign of index j ($j \neq 0$), i.e.,

$$sgn(j) = 1 \qquad \text{if } j>0$$
$$= -1 \qquad \text{if } j<0.$$

Thus adding (12) for each $i \varepsilon I = \{R,L\}$, our objective function can be given as follows:

$$\text{Minimize} \quad \sum_{i\varepsilon M} c_i x_i + \sum_{j\varepsilon N} d_j y_j + \sum_{l\varepsilon L} e_l w_l - \sum_{i\varepsilon I}\sum_{j\varepsilon J} sgn(j)P^i{}_j d^i{}_j. \qquad (15)$$

The negative of the above objective function can be interpreted as maximizing the demand cost less supply cost, while meeting the future energy demand requirements. Hence the optimization problem corresponds to finding the economic equilibrium point maximizing economic surplus.

The probabilistic aspects of our energy model are as follows: many energy supplying countries are somewhat politically and economically unstable. Hence, we assume that supply availabilities of the primary energy resources which correspond to the right hand side values b_i in (1) are random variables. Suppose that the upper bound of the total availability of some primary energy resources from the overseas supply region to Japan follow beta distributions whose upper and lower bounds are denoted by b_M and b_m, and whose parameters are integers p and q, respectively. Then the random variable b has the following probability density function:

$$f(b) = \frac{(b-b_m)^{p-1}(b_M-b)^{q-1}}{K}, \qquad b_m \leq b \leq b_M \qquad (16)$$

where K is a constant. When parameters p and q are integers, K is given by

$$K = \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)}$$

$$= \frac{(p-1)!(q-1)!}{(p+q-1)!}(b_M-b_m)^{p+q-1} \qquad (17)$$

where $\Gamma(\cdot)$ is a Gamma function

$$\Gamma(p) = \int_0^\infty e^{-x}x^{p-1}dx.$$

Suppose parameters p and q satisfy p>1 and q>1, then the random variable b has the following mean μ, variance σ^2, and mode m.

$$\mu = \frac{b_m q + b_M p}{p + q} \tag{18a}$$

$$\sigma^2 = \frac{pq(b_M - b_m)^2}{(p+q)^2(p+q-1)^2} \tag{18b}$$

$$m = \frac{b_m(q-1) + b_M(p-1)}{p+q-2} . \tag{18c}$$

2.3 Computational Method

Our energy model described in the previous section can be formulated in a vector-matrix form as follows:

$$\text{Minimize} \quad cx - pd \tag{19}$$

$$\text{subject to} \quad A_1 x \leq b_1 \tag{20a}$$

$$A_2 x = b_2 \tag{20b}$$

$$A_3 x \geq b_3 + Kd \tag{20c}$$

$$d \leq b_4 \tag{20d}$$

$$x,\ d \geq 0 \tag{20e}$$

where the unknown variable vectors x and d consist of the variables $\{x_i, y_j, z_k, w_l\}$ and $\{d^i{}_j\}$, respectively. Here, p and d are price and commodity vectors, whose elements are given by $P^i{}_j$ and $\mathrm{sgn}(j)d^i{}_j$ in (14), respectively. Constraints (20a), (20b) and (20c) are the resource availability constraints, balancing equations and demand requirement constraints, respectively. (20d) indicates the bounding constraints in (9) for the demand variables $\{d^k{}_j\}$.

We define our resource supply cost minimization submodel as follows:

$$\text{Minimize} \quad c^s x_s \tag{21}$$

$$\text{subject to} \quad A^s{}_1 x_s \leq b^s{}_1 \tag{22a}$$

$$A^s{}_2 x_s = b^s{}_2 \tag{22b}$$

$$A^s{}_3 x_s \geq b^s{}_3 \tag{22c}$$

$$x_s \geq 0 \tag{22d}$$

where $A^s{}_i$, $b^s{}_i$ for $i=1,2,3$, c^s and x_s are, respectively, submatrices and subvectors of the corresponding A_i, b_i for $i=1,2,3$, c and x in the original linear programming problem given by (19)-(20). The submatrices and subvectors described above imply that our resource supply submodel consists of the components related to crude oil and coal only, rather than the whole supply model.

Let the shadow price, i.e., the optimal dual solution, for the demand requirement constraint (22c) of primary energy resource $i\varepsilon I$ be $\pi^*{}_i$. Then the equilibrating condition for our MP/EE energy model can be written as follows:

$$\pi^*{}_i = g_i(D^i{}_0 + \sum_{j\varepsilon J^+} d^{i*}{}_j - \sum_{j\varepsilon J^-} d^{i*}{}_j) \tag{23}$$

where g_i indicates the i-th component of the demand function $g(q)$, corresponding to the primary energy resource $i\varepsilon I$. $D^i{}_0$ and

d^{i*}_j indicate the standard energy demand for the resource $i \varepsilon I$ and an optimal solution for the demand variable d^i_j, respectively.

The optimization problem given by (19)-(20) is an economic surplus maximization problem. A general market equilibrium problem cannot always be transformed into an economic surplus maximization problem. In order for the transformation to be possible, the demand function g(q) needs to be integrable (see e.g. Hurwicz [1971]). Therefore, the Jacobian matrix of the demand function has to be symmetric, i.e. the cross price elasticities between two different commodities must be symmetric. In our energy model, cross price elasticities between different commodities were assumed to be zero, and thus our Jacobian matrix is symmetric.

Let us look at a computational procedure for obtaining an economic equilibrium point in our energy model. Firstly, a sequence of random numbers with a beta distribution (beta random numbers) are generated. Two sequences of beta random numbers are generated simultaneously, and each pair of these numbers is assigned to the corresponding right side in the constraints given by (1). Then the linear programming economic equilibrium model is solved to obtain an optimal energy flow meeting future energy demand. The computational method in our MP/EE model analysis is presented in the flow chart of Figure 3. Solving our MP/EE energy model iteratively is a principal part of our analysis. The details of the solution algorithm are given in Figure 4.

The correcting process at the t-th iteration, given primary energy prices p_t and supply costs c^s, is written as follows:

$$p_{t+1} = \frac{1}{2}(\pi^*_t + p_t) \qquad (24)$$

$$c^s_{t+1} = c^s + \Delta c^s_t \qquad (25)$$

where

$$\Delta c^s_t = \lambda(\pi^*_t - c^s), \qquad 0 \leq \lambda \leq 1. \qquad (26)$$

The convergence of this iterative computation is attained when the shadow price of each primary energy resource of crude oil and coal equals that obtained from the approximate demand curve corresponding to optimal commodity demand.

3. Numerical Results

3.1 Assumptions and Input Data

We define the year 1983 as the base year, and then look at the year 1990 as our future target. Firstly, we assume that average annual growth rates of final energy demand between the base year and the target year are 2.0%, 3.0%, 2.0% and 2.0% for industry, residential·commercial, transportation and stockpile·transfer, respectively. Final energy demands in 1983 and 1990 are given in Table I. Supplies of primary energy resources in the base year and the target year are shown in Table II. In the Table, Other Middle East Region denotes the oil-exporting Middle

East countries excluding Saudi Arabia, i.e., Iran, Iraq, Bahrein, Kuwait, Neutral Region, Qatar, Oman and the United Arab Emirates. The South Region consists mainly of Southern Pacific countries such as Indonesia, Brunei and Australia. The Other Region for crude oil includes African oil-exporting countries such as Algeria and Nigeria. The coal-exporting Other Region includes South Africa, China, and Soviet Union.

Upper bound availabilities of primary energy resources in 1990 are estimated as follows. The upper bound for crude oil import from the Middle East is based upon an average annual increase of 4.0% between the base year 1983 and the target year 1990. In estimating upper bounds for coal import from the Southern Region, North-South America and Other Region, an average annual increase 4.0% is assumed. Estimates for supplies of LNG from the Other Middle East Region, crude oil and LNG from the North-South America Region, crude oil from the Other Region, domestic crude oil, coal, natural gas and stockpile·transfer are all based on the average annual increase rates 2.0 - 4.0% from 1983 to 1990. The upper bound availability of hydro-nuclear power is estimated according to an average annual increase of 4.0 - 5.0 % from the base year.

CIF prices of primary energy resources from various supply regions in 1983 and their estimates for the year 1990 are given in Table III. Crude oil prices in 1983 indicate 'average' prices in oil-exporting countries in the region. For example, the crude oil price in Saudi Arabia is that of Arabian light, and the oil price in Other Middle East Region is based on the United Arab Emirates Murban. Prices in the Southern Region, North-South America Region and Other Region are those of Indonesian Sumatra Light, Mexican Isthmus and Algerian Sahara Blend, respectively. Crude oil price estimates for the target year 1990 are obtained from 1983 data by assuming an average annual price increase as 4.0%, except that the increase rate is 5.0% for Other Middle East Region.

Coal prices in 1983 are the weighted mean of steam coal and material coal from each supply region. Coal prices in the South Region and North-South America Region are based on those of Australian and the United States, respectively. The Other Region's coal price is the weighted mean of South African, Chinese and Russian coals. Estimates for future coal prices in 1990 are based on an average annual increase of 5.0% from 1983.

Natural gas and LNG prices in 1983 are those of Abu Dhabi LNG for the Middle East Region, Brunei and Indonesian for the South Region, and Alaskan for North-South America Region. LNG price estimates for the target year 1990 are obtained from 1983 data by assuming an average annual increase of 4.0%.

Transformation costs for petroleum products, coal products, and secondary energy (electricity and city gas) are given in Table IV. Costs for fuel oil, kerosine·gas oil, and gasoline·naphtha are weighted means minus fuel costs. The other petroleum products' transformation cost is basically the LPG price, and the coke transformation cost is the coke price minus the material coal price. Electricity transformation costs for both industry and transportation are the capital costs in the electricity rate for industry, and those for residential·commercial are also the capital costs in the rate for residences. City gas transformation costs are the capital costs in the industrial and residential·commercial city gas rates.

Upper and lower bounds for constraints (5) and (6) with respect to variables z_k and w_l, are presented in Tables V and VI. Lower bounds for petroleum and coal products are either 0.0 for those whose consumption is relatively small, or the amount of the base year's consumption when they are large. The upper bound is either the consumption of the base year or a 50% increase added when they are relatively small. The average annual increase 4.0% is assumed from 1983 to 1990 for those whose consumption is large. Lower bounds for electricity and city gas are the consumption in the base year. Upper bounds are obtained from the base year's consumption by assuming an average annual increase of 5.0%.

The upper and lower bounds for petroleum products' yields given in Table VII are based on the assumptions that demand for light petroleum products such as kerosine, gas oil, gasoline and naphtha will increase in the future, while demand for heavy petroleum products such as heavy fuel oil will decrease.

The main sources of Japanese energy data used in our model analysis are Energy Statistics [1985], Handbook of Electric Power Industry [1985], Industrial Statistics Table [1984], and

Petroleum Statistics [1984].

Parameters b_m and b_M for the beta distribution are the minimum and the maximum, respectively, indicating extreme estimates for the future availability of crude oil and coal. Parameters p and q are determined so that mean values are nearly equal to the expected future availability of these resources. These parameters are presented in Table VIII.

Beta random numbers are generated by applying the inverse transformation method to uniformly distributed random numbers. Random numbers following uniform distribution between 0 and 1 are generated by using the square method. (For more information on random number generation, see e.g. Fishman [1973], Bratley, et al [1983].)

Approximate demand curves for imported crude oil and coal are based upon their own and cross price elasticity data. The price elasticity ε_{ij}, $i,j \in \{R$:crude oil, L:coal$\}$, represents the decrease (%) of commodity i's demand corresponding to a unit % of commodity j's price increase. According to the translog model analysis in Oyama [1983c], own and cross price elasticities of primary energy resources in 1980 are $\varepsilon_{RR}=-0.07$, $\varepsilon_{RL}=0.04$, $\varepsilon_{LR}=0.69$, $\varepsilon_{LL}=-0.74$. Hence from the above data on ε_{ij}'s we can say that in Japan crude oil is rather price insensitive compared with coal, and these resources are substitutes each other from the positivity of ε_{RL} and ε_{LR}.

3.2 Supply Stability Probability and Equilibrium Prices

We wrote a FORTRAN computer program to analyze our model. The program consists of nearly 3800 statements, most of which (around 80%) comprise the product form simplex method for solving the linear programming problem. Others relate to random number generation, iterative procedures, and output formatting for figures, histgrams and so on.

The linear programming MP/EE model contains 121 variables (including 17 x_i's, 14 y_j's, 28 z_k's, 5 w_l's, 24 d^i_j's and 33 slack variables) and 51 constraints (excluding bounding constraints). An optimal solution for each iteration is obtained within a second of CPU time on the IBM 3033 computer system, requiring about 140 pivots if we start from phase 1 of the simplex technique. In order to obtain an economic equilibrium point for each pair of resource availability constraints, it is necessary to solve 4 - 6 linear programming problems alternating between the MP/EE energy model and the supply submodel problems. Since the latter model is rather simple it can be solved quickly. Hence, solving the MP/EE energy model takes up most of the CPU time. We generated 250 pairs of beta random numbers. So, a total of 250 cases of these economic equilibrium problems are solved in about 12 CPU minutes by the IBM 3033 system.

As shown in Figure 3, we solved our MP/EE model for N(=250) cases. For some combinations of beta random numbers the model may be infeasible since either crude oil or coal supplies may be insufficient to meet our future final energy demand.

Figure 5 shows model feasibility results for each pair of beta random numbers. In Figure 5, the vertical coordinate indicates the availability of imported coal, while the horizontal coordinate indicates the availability of imported crude oil. For each combination of these energy resources' availability, an F or I indicates whether the model is feasible or infeasible.

Let the number of infeasible cases among total N cases be N_I. Then we define the "supply stability probability" (P_S) of our energy system by the ratio of the feasible cases to the total number of cases.

$$P_S = \frac{N - N_I}{N} . \qquad (27)$$

Our energy system can be understood to be "stable" with the probability P_S and "unstable" with the probability $1-P_S$. We call P_S and $1-P_S$ as <u>stability</u> and <u>instability</u> <u>probabilities</u> of our energy system, respectively.

Our numerical experiments show that the stability and instability probabilities of Japan's energy system in the target year 1990 can be presented as follows:

$$P_S = \frac{205}{250} = 0.82, \qquad 1-P_S = \frac{45}{250} = 0.18. \qquad (28)$$

We should consider the instability probability as an implication that an extremely difficult situation may occur with the probability of 0.18 unless our energy system is structurally changed. An infeasibility result may be changed into a "feasible" case by adding more infrastructure to our energy system, or by transforming our energy system into a more flexible one so that it can meet variable final energy demands by promoting substitution among primary and secondary energy resources.

Figure 6 is obtained by combining the feasibility results in Figure 5 with the equilibrium prices' results of imported crude oil and coal. The figure first divides the whole region into feasible and infeasible areas. Then the feasible region is divided into nine parts depending on the crude oil and coal equilibrium prices P_R and P_L. Note that this division of the feasible region is not very accurate, since the equilibrium price of each primary energy resource can vary depending on the supply availability of the other. However, this partitioning helps us to know approximately how much each energy resource's equilibrium price will be changed by the degrees of supply availability.

4. Theoretical Analysis

4.1 Application of Decomposition Techniques

Decomposition techniques, which were originally proposed for solving multi-stage block diagonal linear programming problems, have recently been applied to energy policy analysis in various revised forms. Shapiro [1978] has given an idea for using decomposition techniques to combine econometric forecasting submodels with a linear programming optimization submodel, creating a complete energy planning model. Shapiro and White [1982] have applied decomposition techniques to the constructive integration and optimization of coal supply and demand models. The decomposition technique is not an efficient way to solve the energy model, but its interpretation and economic implication are significant in many model analyses. In our MP/EE model analysis, we show the possibility of applying the decomposition techniques to solve a market equilibrium problem.

We can rewrite our MP/EE problem (19)-(20) as follows:

$$\text{Minimize} \quad cx - pd \tag{29}$$

$$\text{subject to} \quad Ax - Kd \geq q_1 \tag{30a}$$

$$Bx \geq q_2 \tag{30b}$$

$$-Id \geq -q_3 \tag{30c}$$

$$x,\ d \geq 0 \tag{30d}$$

We define polyhedra X and D (assumed bounded) as follows:

$$X = \{\ x \mid Bx \geq q_2,\ x \geq 0\ \} \tag{31}$$

$$D = \{\ d \mid -Id \geq -q_3,\ d \geq 0\ \} \tag{32}$$

Let x^i, $i \in I = \{1,2,...,n_x\}$ and d^j, $j \in J = \{1,2,...,n_d\}$ be extreme points of the above polyhedra X and D, respectively, and n_x and n_d are numbers of extreme points in X and D, respectively. Then, the price directive master problem for (29)-(30) can be written as follows:

$$\text{Minimize} \quad \sum_{i \in I} c\lambda_i x^i - \sum_{j \in J} p\mu_j d^j \tag{33}$$

subject to $\sum_{i \varepsilon I} A\lambda_i x^i - \sum_{j \varepsilon J} K\mu_j d^j \geq q_1$ (34a)

$$\sum_{i \varepsilon I} \lambda_i = 1 \quad (34b)$$

$$\sum_{j \varepsilon J} \mu_j = 1 \quad (34c)$$

$$\lambda_i,\ \mu_j \geq 0 \qquad i \varepsilon I,\ j \varepsilon J. \quad (34d)$$

Denoting the dual variables (shadow prices) for the above constraints (34a)-(34c) by π, π_x and π_d, respectively, dual problem to the above problem can be written as:

Maximize $\pi q_1 + \pi_x + \pi_d$ (35)

subject to $\pi_x + \pi A x^i \leq c x^i \qquad i \varepsilon I$ (36a)

$$\pi_d - \pi K d^j \leq -p d^j \qquad j \varepsilon J \quad (36b)$$

$$\pi \geq 0. \quad (36c)$$

If we start from $|I|=|J|=0$ in (35)-(36), then at each iteration we add new extreme points x^i, d^j in the following way: using the optimal dual solution π (initially $\pi=\pi_x=\pi_d=0$), we solve the following problems:

$$z_x = \min.(c - \pi A)x \quad (37a)$$

subject to $x \varepsilon X$. (37b)

$$z_d = \min.(-p + \pi K)d \quad (38a)$$

subject to $d \varepsilon D$. (38b)

Suppose $z_x \geq \pi_x$ and $z_d \geq \pi_d$; then the solution (π, π_x, π_d) is optimal in the dual problem (35)-(36). If not, i.e., $z_x < \pi_x$ or $z_d < \pi_d$, then add a new extreme point solution to generate a new column in the form

$$\begin{pmatrix} cx^r \\ Ax^r \\ 1 \\ 0 \end{pmatrix} \quad \text{or} \quad \begin{pmatrix} -pd^r \\ Kd^r \\ 0 \\ 1 \end{pmatrix},$$

where x^r and d^r are extreme point solutions to (37) and (38), respectively. Then, we return to the problem of the form (33)-(34). Thus, an optimal solution to the original problem (29)-(30) can be obtained by using an optimal solution λ^*_i and μ^*_j for (33)-(34) as follows:

$$x^* = \sum_{i \varepsilon I} \lambda^*_i x^i \tag{39}$$

$$d^* = \sum_{j \varepsilon J} \mu^*_j d^j \tag{40}$$

Now, the key to solving our MP/EE problem using the decomposition technique is that the problem (38a)-(38b) is very easy to solve since the polyhedron D is just a hypercube. Hence, one can solve the problem analytically, without applying simplex methods. This problem is still easy to solve even when the matrix K contains both nonzero own and cross price elasticities, and also when demand variable vector d is given many elements by defining more endogenous demand variables. Incidentally, applying the resource directive master problem approach in the decomposition technique does not help much, since we cannot obtain a simple, analytically solvable linear programming problem as in (38a)-(38b).

4.2 Analysis on Instability Probability

In our MP/EE model analysis, the instability probability of our energy system was defined to be the probability that the energy model results are infeasible. As we can see in Figure 6, the dividing line between feasible and infeasible regions may be expressed by a straight line. From our numerical results, the equation of the dividing line in Figure 6 can be approximated as follows:

$$u + v = 3.16\times10^5 \tag{41}$$

where u and v indicate the supply availabilities of imported crude oil and coal, respectively. We can expect that the point (u,v) is feasible if $u+v\geq3.16\times10^5$. It is infeasible otherwise. However, the above equation given by (41) cannot be an 'exact' dividing line since neither of these two variables can be close to zero. This is because of various lower bounding constraints on some of the decision variables in our energy model.

Let X and Y be random variables following beta distributions with $a_1\leq X\leq b_1$ and $a_2\leq Y\leq b_2$, respectively. Then we can illustrate an "approximate" infeasible region as in a shaded area in Figure 7. Suppose the equation of the boundary line between feasible and infeasible regions in Figure 7 is expressed by

$$pX + qY = r \;, \qquad p,\ q,\ r\text{: constants.} \tag{42}$$

Then the infeasible region of the shaded area is the set of points (X,Y) satisfying

$$pX + qY \leq r \tag{43a}$$

$$p>0,\ q>0,\ a_1\leq X\leq b_1,\ a_2\leq Y\leq b_2. \tag{43b}$$

The infeasible region given above can be transformed into the case of the standard beta distribution, with random numbers x and y distributed between 0 and 1. Therefore the instability probability for our energy system is approximated as follows:

$$\frac{1}{B(p,q)B(r,s)} \iint_{\substack{0\le x\le 1\\ 0\le y\le 1}}^{ax+by\le c} x^{p-1}(1-x)^{q-1}y^{r-1}(1-y)^{s-1}dxdy$$

$$= \int_0^1 x^{p-1}(1-x)^{q-1} \int_0^{(c-ax)/b} y^{r-1}(1-y)^{s-1}dydx$$

$$= \int_0^1 x^{p-1}(1-x)^{q-1}F(x)dx \qquad (44)$$

The last expression in the above formula can be given by a numerical integral since we have a detailed table of values of beta functions.

Now let us look at our instability probability more closely. The necessary and sufficient condition for the linear program problem (19)-(20) to be feasible are given as follows by using Farkas' lemma (see e.g. Shapiro [1979], p28) in a slightly modified form.

$$z_y \le 0 \qquad (45)$$

where

$$z_y = \max.\{ -y_1b_1 + y_2b_2 + y_3b_3 - y_4b_4 \} \qquad (46a)$$

$$\text{subject to} \quad -y_1A_1 + y_2A_2 + y_3A_3 \le 0 \qquad (46b)$$

$$-y_3K - y_4I \le 0 \qquad (46c)$$

$$y_1,\ y_3,\ y_4 \ge 0 \qquad (46d)$$

If $z_y \leq 0$ for all the random variable components of b_1, then the model given by (19)-(20) can always be feasible, i.e., $P_s = 1.0$.

We can give a sufficient condition for our MP/EE model (19)-(20) to be feasible. Since the general linear programming problem's objective function is convex and nondecreasing with respect to the right hand side values, the problem (46a)-(46d) should have the greatest lower bound of the objective function values, corresponding to the inf.(infimum) of the vector b_1. Let

$$b^0_1 = \text{inf. } b_1 \tag{47}$$

i.e., $b^0_1 \leq b_1$ for all random variable components of a vector b_1 and no b_1' such that $b_1' \leq b^0_1$ satisfies $b_1' \leq b_1$ for all random variable components of b_1. Let z^0_y be defined by

$$z^0_y = \max \{ -y_1 b^0_1 + y_2 b_2 + y_3 b_3 - y_4 b_4 \} \tag{48a}$$

$$\text{subject to} \quad -y_1 A_1 + y_2 A_2 + y_3 A_3 \leq 0 \tag{48b}$$

$$-y_3 K - y_4 I \leq 0 \tag{48c}$$

$$y_1, y_3, y_4 \geq 0 \tag{48d}$$

Then, if $z^0_y \leq 0$, our MP/EE model is feasible for any random variable vector b_1. Note that the above sufficient condition guarantees the feasibility of the MP/EE model by solving a single linear programming problem, while the necessary and sufficient condition given by (45)-(46) for the model's feasibility requires that (45) holds for all possible b_1.

5. Summary

In this analysis, we have investigated the effects of primary energy resources' supply constraints on Japan's energy system. It is generally true that when a primary energy resource's availability is high, its commodity price goes down and vice versa. However, since we have considered two kinds of primary energy resources simultaneously, and furthermore their price elasticities are very different, the price effects of resources supply constraints are a little more complex to analyse. Through our MP/EE model we obtain an economic equilibrium point for each energy resource as shown in Figure 6. However, the splitting of the feasible region into smaller regions based on commodity prices, as in the figure, is complicated because the equilibrium points may be dependent on the computational method and its convergence criteria.

Our computational technique is fundamentally similar to the PIES model (see e.g. Hogan [1975], and Hogan, et al [1978]; also Ahn [1979], and Ahn and Hogan [1982] for convergence arguments for special cases) in its main framework, except that the iterative procedures and some assumptions about supply and demand functions are different. In the PIES model, the demand function was assumed to be continuously differentiable, and the (inverse) supply mapping was a point-to-set mapping, while we assumed for our model that both supply and demand functions were point-to-set mappings. The assumptions of the PIES model guarantee both the existence and the uniqueness of an equilibrium point, while our MP/EE model assumes only the existence of an equilibrium solution.

Our iterative computational method worked very well, and we could obtain an equilibrium point after several iterations for all feasible cases. Although the convergence proof for our computational method is not given in this paper, we believe the convergence is guaranteed by showing the fact that the shadow price of the demand requirement constraint (25c) is expressed by the approximate demand function value corresponding to the optimal resource demand. We are presently working on this proof.

Approximating a demand function is another problem. In this paper we assumed the existence of nonzero own price elasticities for primary energy resources only, neglecting cross price elasticities between two distinct primary energy resources. Both own and cross price elasticities can be simultaneously considered in our model analysis by incorporating this information into the matrix K of (20c). The consideration of nonzero cross price elasticities does not make solving the problem more difficult, but rather changes the problem formulation slightly by adding more nonzero elements in the coefficient matrix. Applying decomposition techniques should also be very effective in solving our MP/EE model in this case.

The stability probability was defined to be the probability that the MP/EE model was feasible. We tried a single sequence of random numbers as our import supply availability, and then obtained the stability and instability probabilities of our energy system. We know that if the substitutability between crude oil and coal increases, then the stability probability will also increase, since there are more ways to meet the forecast energy demand.

We can conclude that the Japanese energy system needs to be more flexible, so that it can structurally adjust variations of primary energy supply availability. For example, the Japanese cement industry changed almost totally from fuel oil to coal in one year (from 1979 to 1980). In another good example, our power industry is introducing mixed fuel thermal power plants consuming fuel oil, coal and LNG.

If our energy system were well organized to consume more coal, it will greatly heighten the stability probability of the primary energy supply, and also lower the total energy system cost. We would not have to depend so heavily on crude oil, which has higher supply uncertainty and instability. We must note that coal transportation and storage infrastructure and environmental countermeasures for SO_x and NO_x emissions and burned ashes are very important in the case where we consume a great amount of coal.

Thus, the substitutability among primary energy resources is a very important factor in our energy system's stability. In order to further elucidate the relationship between the stability probability and the energy resources substitutability, we need more numerical experiments, trying different values for lower and upper bounds of certain energy flows, and varying the yields and efficiencies of petroleum and coal products.

The model described in Section 2 is a single period static optimization model. Letting a part of the right hand side in the linear programming model be a random variable, we could apply probabilistic and stochastic analyses, obtaining supply stability and instability probabilities. We can add dynamic analysis by increasing the number of periods and estimating the stability probability in the more distant future. In this case, the following difficulties occur: uncertainty with respect to future primary energy prices and supply availability, subsequent variations of final energy forecast and optimal solutions, justification of probability distribution, and availability and reliability of data. Obtaining the large scale structure of the linear programming model and computational techniques necessary to obtain an economic equilibrium point efficiently will be another difficulty. Therefore we believe two or three stages, representing the next 10 - 15 years will be the largest time span we can deal with reasonably.

We believe that the approach introduced in this paper can be useful to quantitatively analyse the energy system stability of countries like Japan which depend heavily on imported primary energy resources. We are considering further modification of our energy systems approach by incorporating dynamic terms and more modeling of national economic structures.

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TABLE I. Final Energy Demand

(10^{10} kcal)

Sectors	1983	1990
Industry	211858	243358
Residential･Commercial	99465	122329
Transportation	56869	65324
Stockpile･Transfer	45844	52660

TABLE III. Energy Prices by Supply Region

(10^{6} yen/10^{10} kcal)

Energy	Supply Region	1983 Data	1990 Price
Crude Oil	Saudi Arabia	45.996	60.488
	Other Middle East	47.437	66.749
	South Region	48.964	64.433
	North-South America	47.398	62.373
	Other Region	50.729	66.756
Coal	South Region	22.712	31.958
	North-South America	19.184	26.994
	Other Region	17.761	24.992
Natural Gas LNG	Middle East	48.072	63.259
	South Region	45.219	59.505
	North-South America	46.042	60.588

TABLE II. Primary Energy Resources by Supply Region

(10^{10} kcal)

Energy	Supply Region	1983 Data	1990 Supply Availability
Crude Oil	Saudi Arabia	59904	74100
	Other Middle East	91649	113368
	South Region	38882	48096
	North-South America	9196	11375
	Other Region	13213	16344
	Domestic	447	588
	Stockpile・Transfer	28531	37545
Coal	South Region	26890	35385
	North-South America	21312	28045
	Other Region	8834	11625
	Domestic	11173	12834
	Stockpile・Transfer	7197	8851
Natural Gas LNG	Middle East	2411	2965
	South Region	23655	29093
	North-South America	1389	1708
	Domestic	2154	2474
	Stockpile・Transfer	1335	1642

TABLE IV. Transformation Costs of Secondary Energy Resources

(10^{10} yen/10^{10} kcal)

Energy Resources	Transformation costs
Fuel Oil	2.01
Kerosine·Gas Oil	32.96
Naphtha·Gasoline	49.85
Other Petroleum Products	24.85
Coke	19.29
Coke Gas·Blast Furnace Gas	31.39
Electricity (Industry, Transport)	215.52
Electricity (Residential)	284.42
City Gas (Industry)	71.95
City Gas (Residential)	106.57

TABLE VI. Upper and Lower Bounds for Secondary Energy

(10^{10} kcal)

Secondary Energy		Lower Bound	Upper Bound
Energy	Use		
Electricity	Industry	101,500	120,500
	Residential	60,000	70,000
	Transportation	4,000	5,500
City Gas	Industry	2,400	3,500
	Residential	9,500	13,500

TABLE V. Upper and Lower Bounds for Petroleum and Coal Products

(10^{10} kcal)

Products		Lower Bound	Upper Bound
Energy	Use		
Crude Oil	Electricity	0.0	18,600
	Stockpile	32,000	41,000
Fuel Oil	Electricity	32,000	41,200
	Industry	37,000	49,400
	Residential	0.0	12,600
	Transportation	0.0	7,000
Kerosine Gas Oil	Industry	0.0	12,000
	Residential	25,000	33,000
	Transportation	13,500	17,600
Naphtha	Industry	22,000	29,000
	City Gas	0.0	1,400
Gasoline	Transportation	35,000	41,500
Petroleum Products	Electricity	2,600	3,500
	Industry	12,500	16,500
	Residential	0.0	9,500
	Transportation	2,000	3,500
	City Gas	2,000	2,900
Coke	Industry	32,000	42,500
	Residential	0.0	50
	Stockpile	1,300	1,800
	City Gas	2,000	2,900
Coke Gas	Electricity	4,200	5,600
	Industry	10,300	14,500

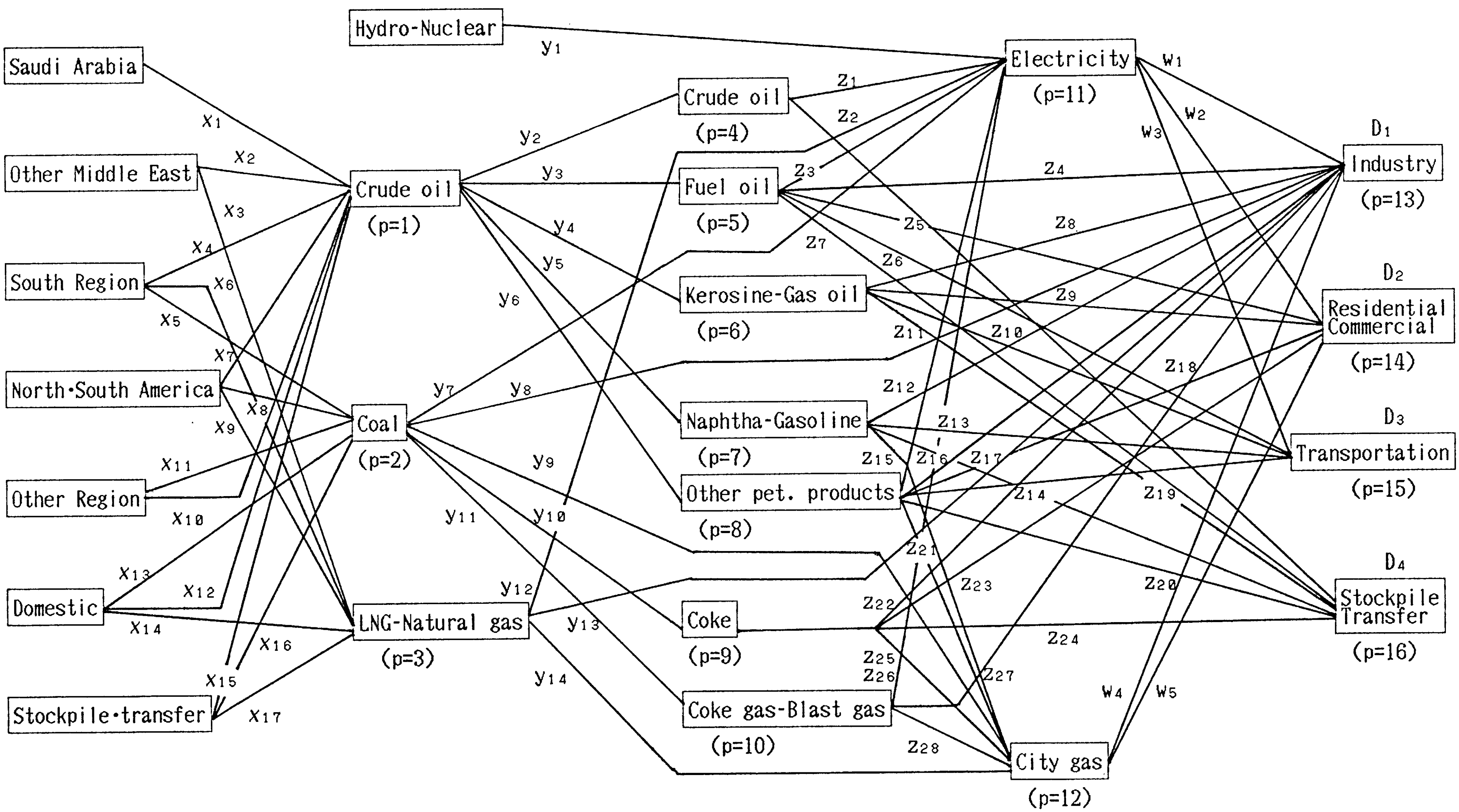
TABLE VII. Upper and Lower Bounds for Petroleum Products Yields

Petroleum Products	Lower Bounds	Upper Bounds
Fuel Oil	0.40	0.55
Kerosine·Gas Oil	0.10	0.30
Naphtha·Gasoline	0.20	0.30
Other Petroleum Products	0.0	0.15

TABLE VIII. Parameters for Beta Distribution

Energy Resource	Parameters			
	b_m	b_M	p	q
Crude Oil	200,000	300,000	4.0	2.0
Coal	45,000	135,000	2.0	4.0

Figure 1. Energy system and endogenous variables



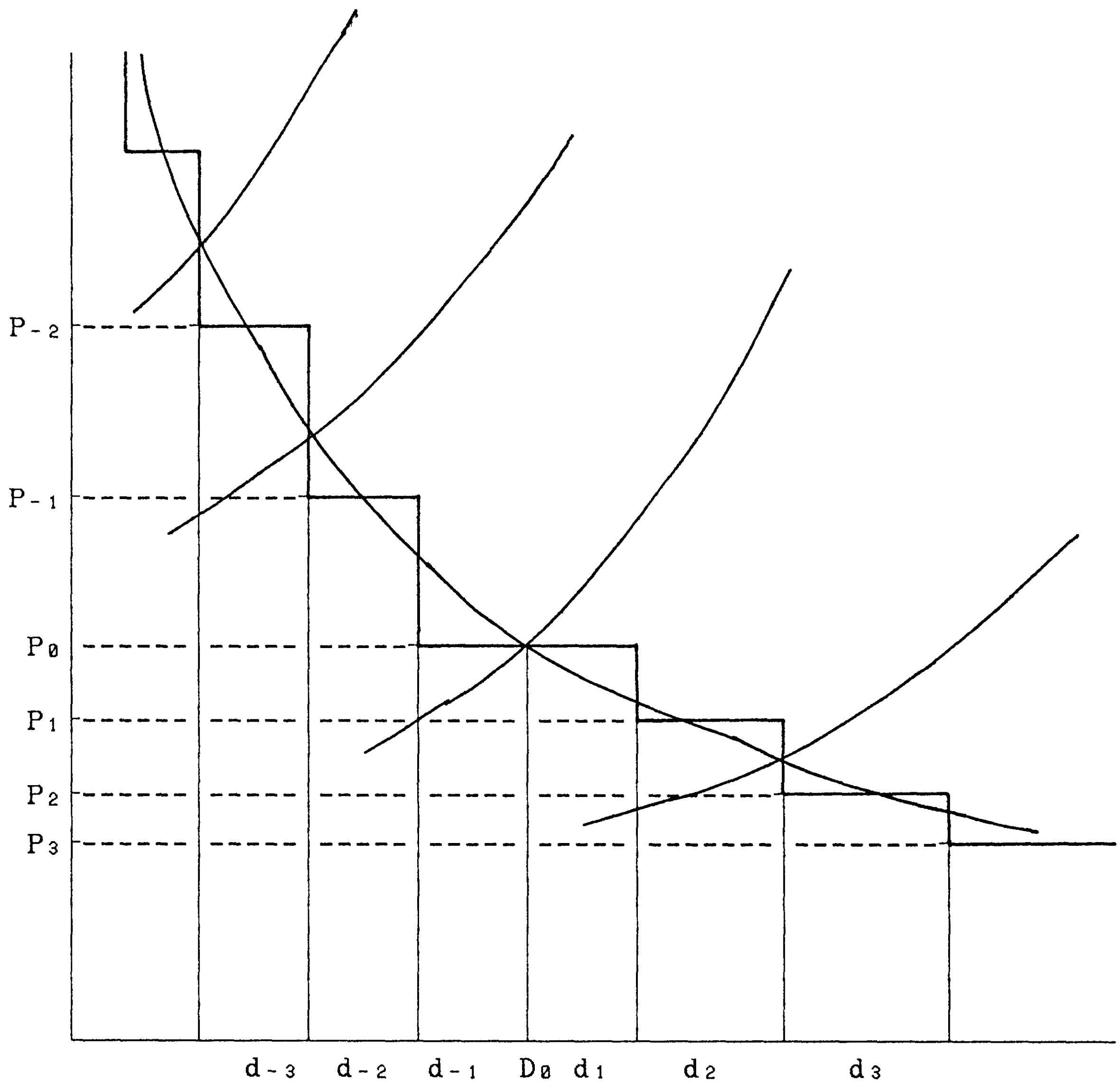


Figure 2. Approximate demand function and supply functions

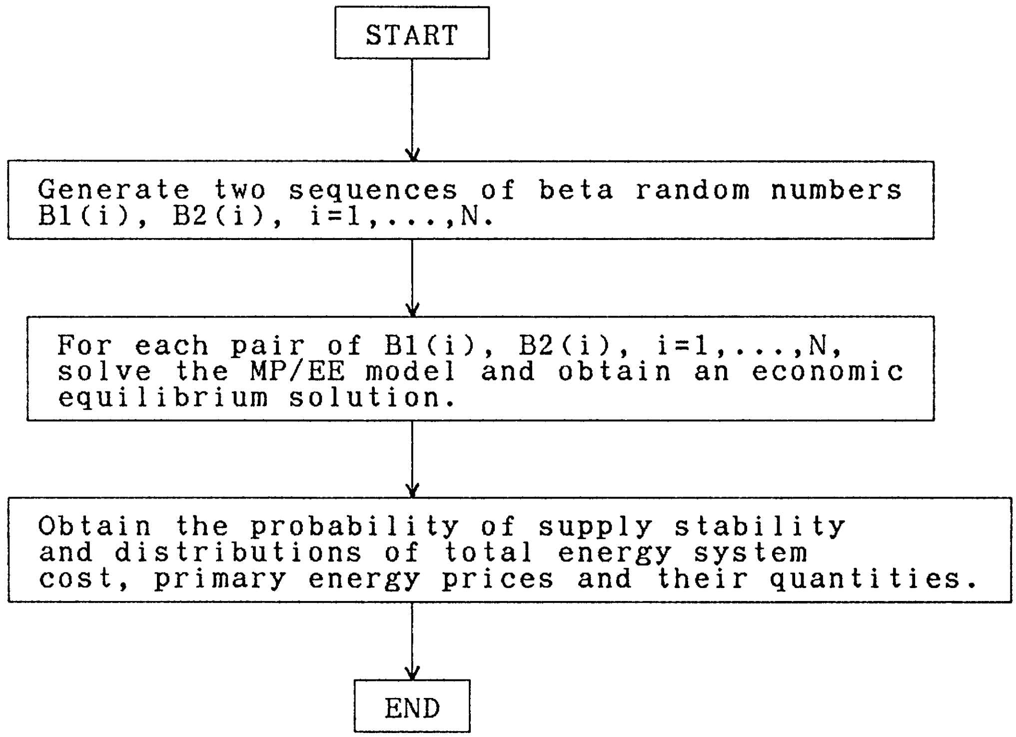


Figure 3. Computational procedure for the model analysis

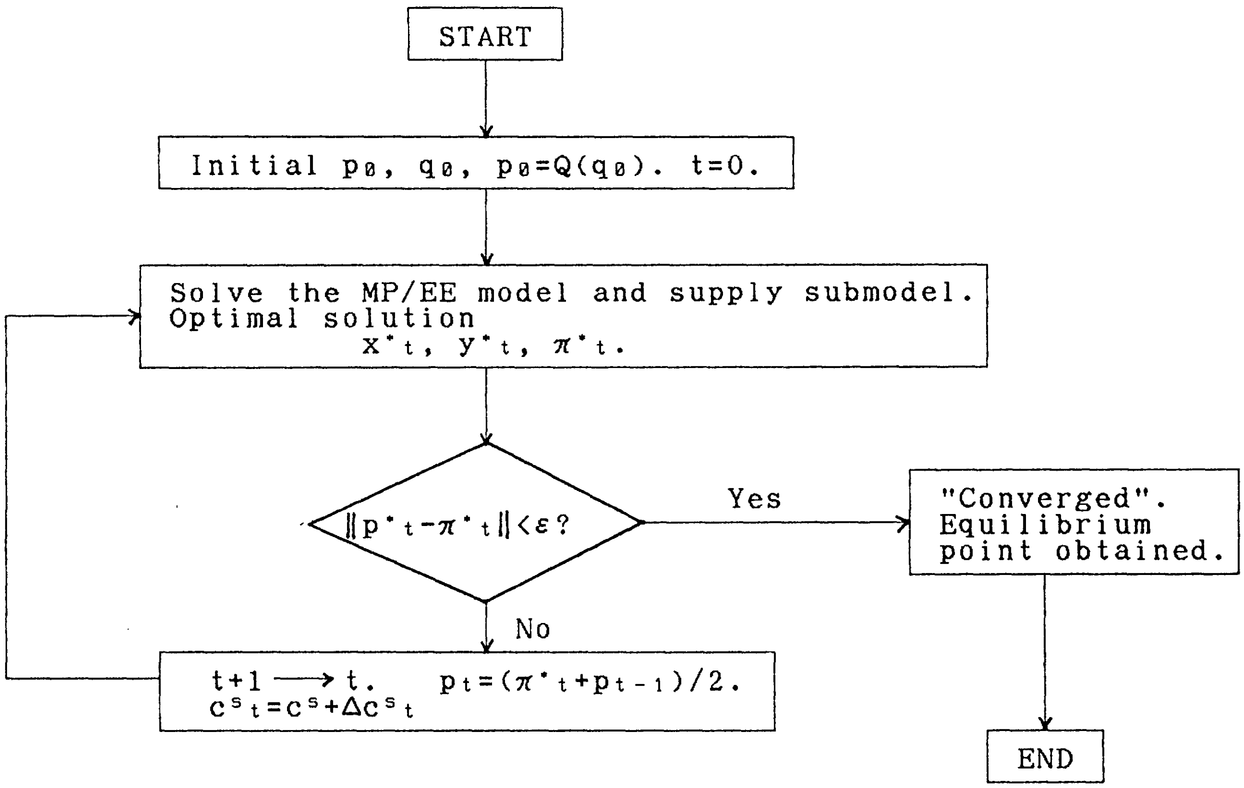


Figure 4. Algorithm for solving the MP/EE model

Figure 5. Model feasibility results (N=250)

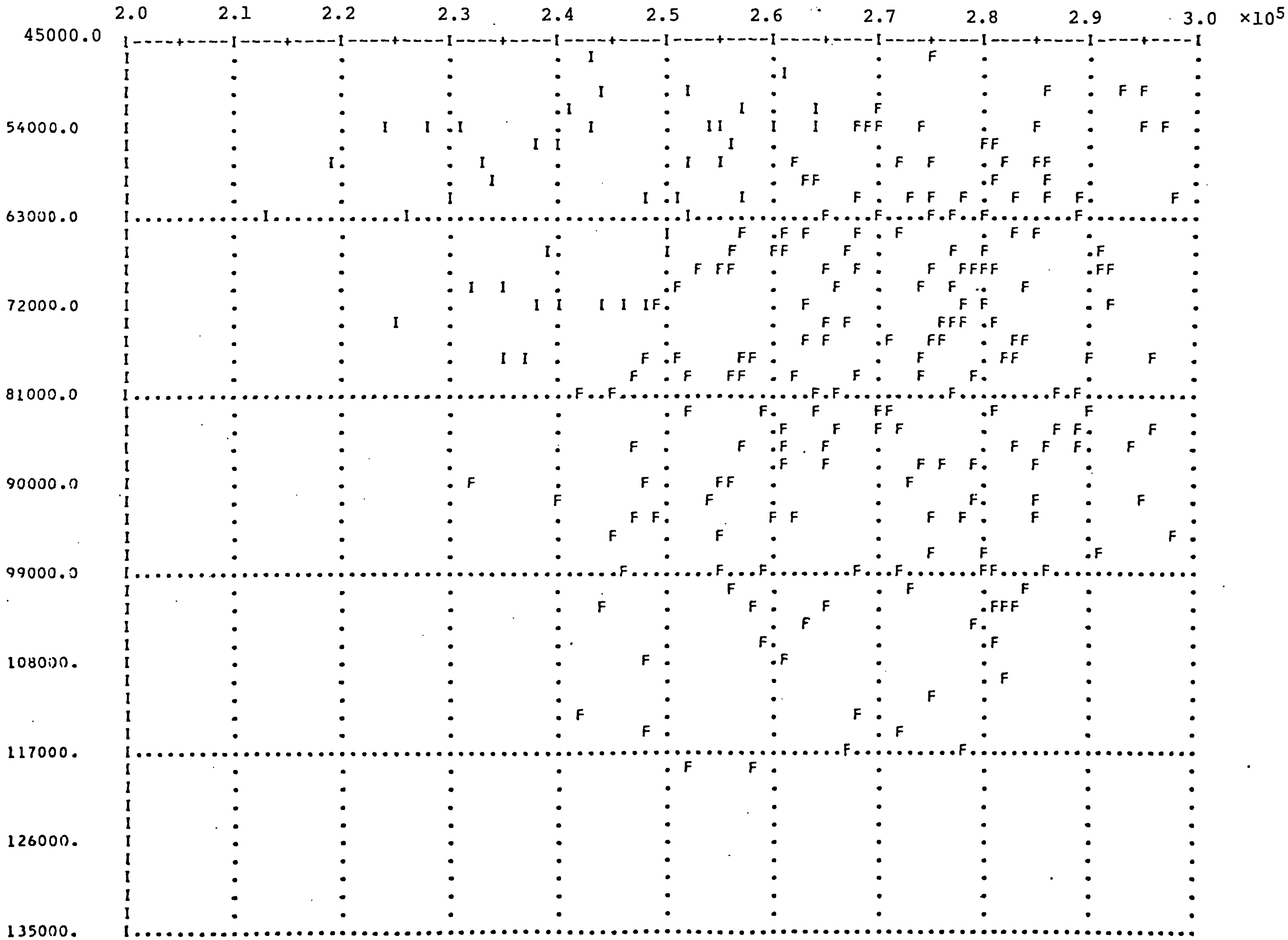


Figure 6. Distribution of model feasibility results and equilibrium results

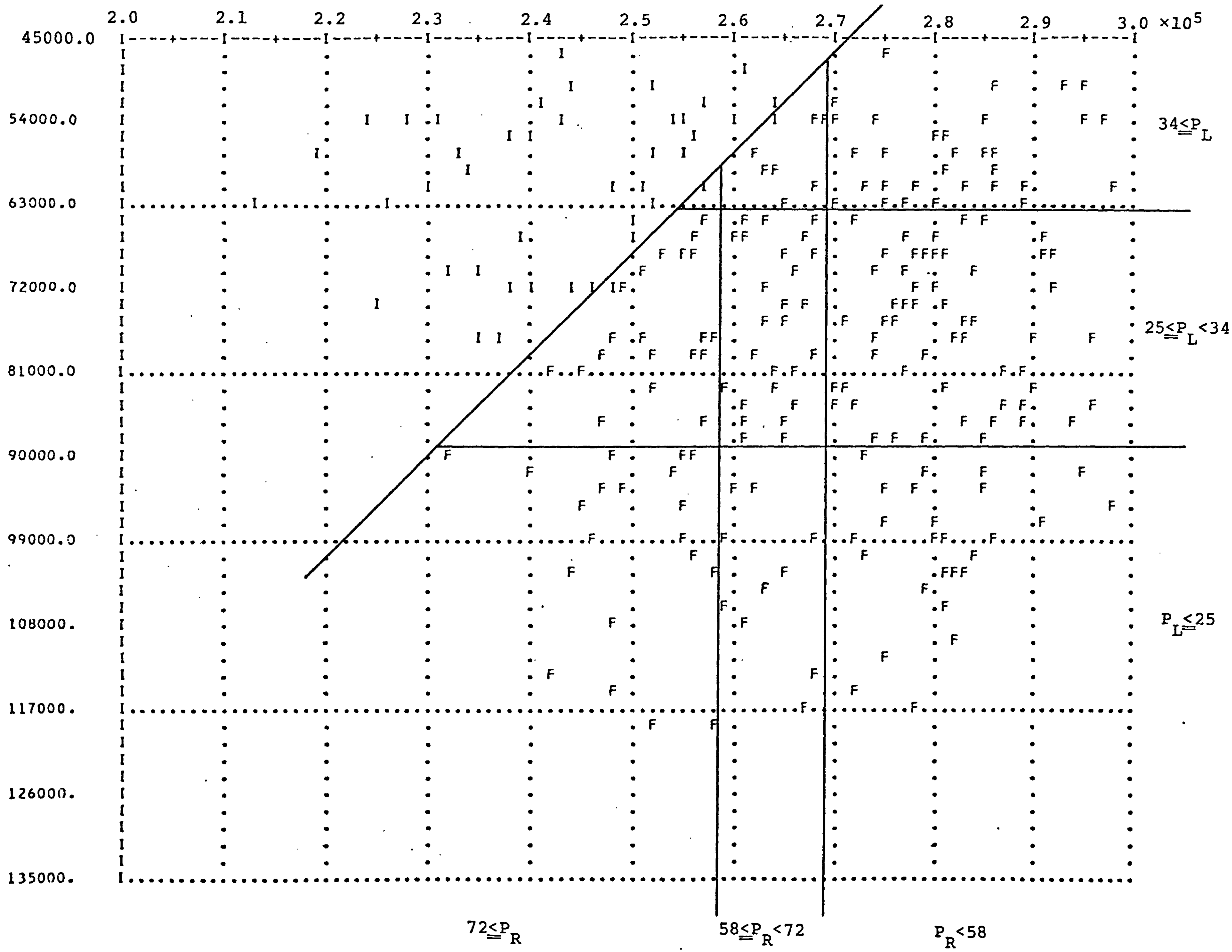
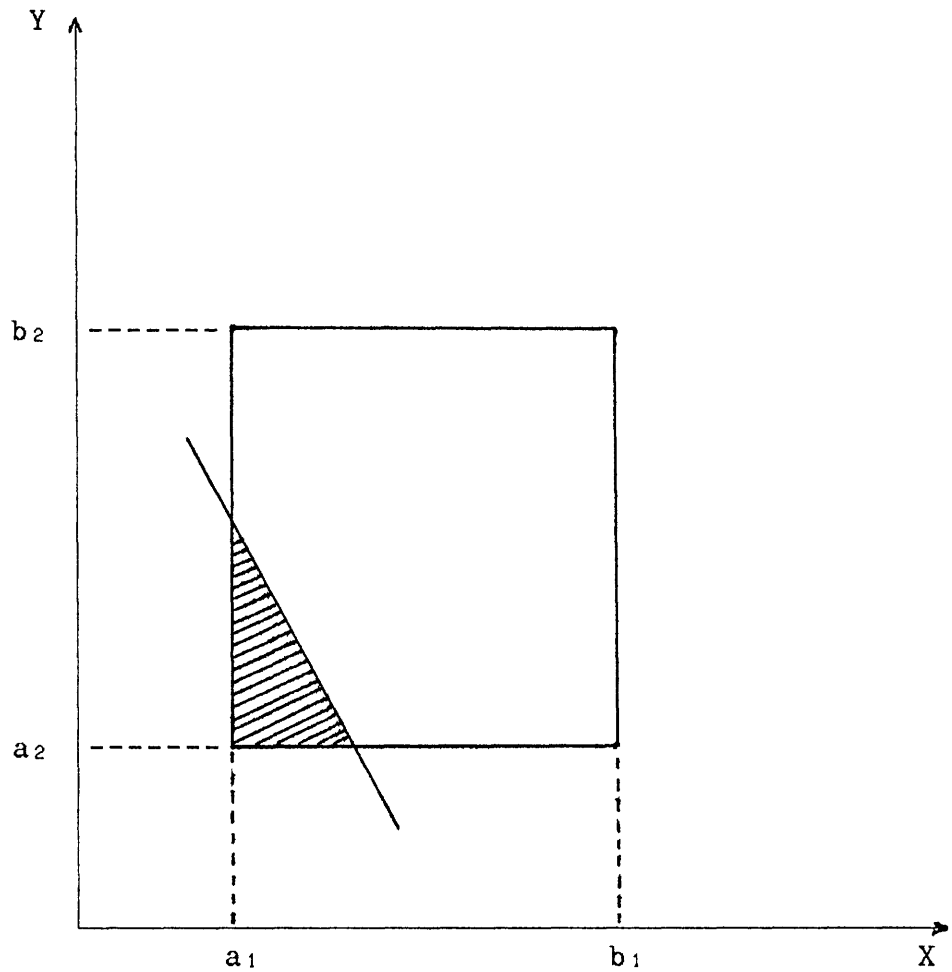


Figure 7. Illustration of an infeasible region (general case)



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