ALLOCATION OF NATURAL GAS IN TIMES OF SHORTAGE: A MATHEMATICAL PROGRAMMING MODEL OF THE PRODUCTION, TRANSMISSION, AND DEMAND FOR NATURAL GAS UNDER FEDERAL POWER COMMISSION REGULATION

by

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Abstract

The purpose of this study was to develop a model of the production, transmission, and demand for natural gas which would represent the effects of Federal Power Commission regulation in a realistic manner and which could be used as a tool for analysis of alternative FPC policies regarding producers, interstate pipelines, and the allocation of natural gas in times of shortage. This thesis describes the development of GASNET, a mathematical programming model of the U.S. natural gas system, from its theoretical formulation to its implementation on the SESAME mathematical programming system to forecasts of supply and demand for natural gas in 1980. It also describes the development of a modified linearization algorithm which is shown to converge to a Kuhn-Tucker point of the non-convex, non-linear program which represents the natural gas system under FPC regulation.

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Chapter 1 Introduction

1.1 The Purpose of this Study

In 1972, MIT was awarded a National Science Foundation grant to study the U.S. natural gas system. A research team of faculty and graduate students in the Alfred P. Sloan School of Management, organized and directed by Professors P. W. MacAvoy and R. S. Pindyck, focused on the development of an econometric model of the supply and demand for natural gas in the United States (1973, 1974). In 1974, as part of a continuation grant from the NSF, an extension study was funded to analyze more closely the interestate pipeline transmission system as it relates to the markets for natural gas. Once this study had begun, it became apparent that new approaches to the traditional transshipment problem would have to be evolved to analyze the natural gas pipeline network under Federal Power Commission regulation. In addition, it also became clear that one of the vital problems the FPC would soon have to begin dealing with was allocation of natural gas in times of shortage, because there was little doubt that the shortage was here. Thus the purpose of the study became to develop a model of the production, transmission, and demand for natural gas which would represent the effects of FPC regulation in a realistic manner and which could be used as a tool for analysis of alternative FPC policies regarding natural gas producers (wellhead prices), interstate pipelines (rates of return), and the allocation of natural gas in times of shortage.

1.2 The Natural Gas System of the United States

Getting natural gas to final consumers in the U.S. is a complex, multi-level process involving thousands of small and hundreds of large private companies, a large number of publically owned utilities, as well as various state and local regulatory agencies and the U.S. Federal Power Commission (FPC). The process begins with private exploration of onshore lands and offshore waters for geological signs indicative of the potential of underground petroleum deposits. While such exploration has been traditionally centered in the Southwest U.S., it is now increasingly moving to more remote areas and deeper waters as is evidenced by recent activity in Alaska and the Canadian Arctic and the U.S. Outer Continental Shelf (OCS). The promise of the OCS has resulted not only in increased activity in the traditional Gulf Coast areas (off Southern Louisiana and Texas), but also in strong interest along the U.S. East Coast and the Gulf of Alaska.

The Oil and Gas Journal (1974) reported that more than 75% of onshore exploration and exploratory well drilling is done by independent oil companies and not by the "majors" such as Shell and Humble. On the other hand, the higher costs of offshore exploration and development have resulted in the opposite phenomenon on the OCS. According to Merrill, Lynch, Pierce, Fenner, and Smith (1972), the majors accounted for 88.6% of all leased OCS acreage in 1972 with the independents and others dividing the remaining 11.4%.

If petroleum is found via an exploratory well, additional wells will be drilled to determine the extent of the field and its economic merits. The results of these drillings will be used to decide whether the field is actually worth the effort and expense of drilling and completing production wells. If the producer has found natural gas in either non-associated or oil-associated form, he will soon begin to contract with interstate and intrastate pipeline companies for the sale of his gas. Sales to interstate pipelines, however, fall under the jurisdiction of the Federal Power Commission which regulates the wellhead prices for which the gas may be sold.

In the 1950's, the FPC attempted to regulate each producer separately using the traditional "cost of service" methods. Finding that trying to regulate over 5,000 producers individually was impossible, it has moved to increasingly more simplified schemes. From the regional price ceilings of the 1960's, it has most recently shifted to a nationwide wellhead price ceiling. A recent FPC decision (1974) set the national ceiling price at 50¢ per thousand cubic feet (Mcf) of gas subject to yearly incremental increases of 1¢/Mcf.

In contrast, the intrastate pipelines of the Southwest have been willing to pay much more for natural gas than the interstate pipelines are allowed to offer. Thus contracts have been made for gas at over $1.50 per thousand cubic feet, or three times the national rate. As a result the interstate pipeline companies have been very hard pressed to get new contracts for natural gas as their old contracts run out.

The interstate pipeline companies, in negotiating their purchases of natural gas, have usually attempted to get long term (20-year) contracts. Thus even when renegotiation of price is included in the contract, its long term has tended to keep prices low independently of FPC price ceilings.

The pipeline companies themselves are regulated by the FPC on a rate of return basis. The companies and FPC staff make estimates of operating costs, expected demands, value of capital plant, depreciation, and the cost of capital as inputs for the FPC's determination of a rate of return on each company's capital "rate base" which is non-confiscatory and "fair and reasonable." Based on this rate of return, a price schedule is agreed upon by the pipeline company and the FPC, or in case of no agreement, by arbitration or the courts. It is this price schedule that the company must abide by; its actual rate of return may be either higher or lower than the rate determined to be fair by the FPC depending on the actual costs and demands experienced by the pipeline company. If its actual return is higher than the allowed rate, however, the FPC could, as a result of later hearings, reduce the company's rate of return or rate base.

The interstate pipeline companies sell some gas directly to final users but most goes to intrastate natural gas distribution companies which are regulated by the various state utility commissions. It is these utilities which distribute most of the natural gas to residential, commercial, and industrial customers.

In recent years the Congress has come under increasing pressure to deregulate natural gas producers (but not pipelines). In a recent FPC opinion (1974), Chairman Nassikas urged Congress to do so. On the other hand, consumer groups and leaders such as Ralph Nader and Senator Philip Hart have consistently opposed deregulation and call instead for tougher price controls (1974).

It is extremely difficult to deny, however, that the artificially low price of natural gas throughout the 1960's has brought about a condition where demand for natural gas by residential, commercial, and industrial consumers has far exceeded the supply producers have been willing or able to make available for sale. Thus "interruptible" service in off-peak hours to industrial customers has disappeared, acceptance of new requests for natural gas service to residences in the North Central and Northeast has been cut back or stopped altogether, and even curtailments of previously contracted gas have occurred and are expected to increase in coming years (see Table 1.1).

The Federal Energy Administration is currently involved in the analysis of various allocation options for natural gas given the certainty of continuing supply shortages. While the FEA may be able to enforce surcharges on natural gas sales to reduce demand or decide who will get gas and who will not, it is actually up to Congress and the courts to decide whether or not to deregulate natural gas producers.

TABLE 1.1 Curtailments of Natural Gas on Firm Contracts

	Quantity (BCF)	% Firm Curtailment
April - July 1973 (actual)	515.4	8.8
August - October 1973 (estimate)	450.9	10.4
November - March 1974 (estimate)	679.7	7.4

Source: Federal Power Commission, Natural Gas Supplies of the Interstate Pipeline Companies, 1973.

1.3 The Mathematical Framework for this Model

The basic purpose of this study has been to develop and analyze a mathematical model of the supply, distribution, and demand for natural gas. In times of shortage, the policy maker would like to be able to determine a means for allocating his scarce resources in the most optimal way for those involved. Thus he often finds that a mathematical programming model offers him a framework for the analysis of a variety of possible options which satisfy a set of resource constraints and allows him to determine which one offers the most in terms of whatever his objective is.

This study lends itself to a mathematical programming approach. In the case of the problem at hand, the pipeline companies find themselves facing a certain supply situation which amounts to a limit on the amount of gas which will be made available to them at any given price. They have a similar demand situation. In this case, they know that they are required to deliver a certain quantity of gas to their customers at the contracted price and that at any given price there will be an upper limit on the demand for gas at that price. In addition, the pipelines themselves have certain physical limits on the quantity of gas which can be delivered in a given time, and there are flow requirements which provide for the conservation of mass. Finally, there are the FPC rate of return constraints which limit the amount pipelines may charge for their gas. In the face of all of these constraints, the company will try to operate his pipeline to maximize profits. In the larger picture, therefore, the model to be analyzed in this study has been chosen to maximize aggregate pipeline profits subject to the constraints described above.

But what if all of these constraints cannot be met? In actual fact, it appears that this is the situation at present: wellhead prices are too low to clear the markets for natural gas, hence demands are not being met. The model must reflect this possibility and do something about fairly allocating those excess demands. The model is, in fact, constructed to do just that.

Unfortunately the model which has been developed for these purposes is not simple. The constraints and the objective function are both non-linear. A basic result has ben achieved, however, in that an algorithm has been developed which can be proved to converge to a Kuhn-Tucker point of the non-linear program. Whether this point is an optimal solution is not yet known.

Because of the complexity and size of the model, a complete set of tests was not possible. Instead a simpler model was used to generate comparative results for the late 1960's and early 1970's, and forecasts for regional supplies, demands, prices and flows for 1980.

The development of this model and its results will be the major tasks of Chapters 3 through 7 of this thesis. The next chapter will consist of a survey of the literature with the intention of providing a context for the present study.

Chapter 2 Survey of the Literature

2.1 Introduction

This chapter provides a general survey of other efforts in the area of systems research, economics, and modeling in energy in order to define the context for the present study. The survey will cover several different topics, including the economics of energy regulation, energy supply models, energy demand models, pipeline models, supply and demand models, and other energy related studies. In some instances methodology which has been exploited for economic analyses of commodities other than energy will be described where that methodology proves useful in energy models as well.

2.2. The Economics of Energy Regulation

Energy regulation in the United States falls into several distinct categories. There is regulation at the Federal and sub-Federal levels. There is regulation in the producing, distributing, and consuming sectors. There is separate regulation of gas, oil, and electricity. The two types of regulation of most concern to us in this study are field price regulation of interstate natural gas sales and rate of return and price regulation of the interstate pipeline companies, both of which are responsibilities of the U.S.

Federal Power Commission. For a discussion of Federal regulation in the electricity industry, one might look at Breyer and MacAvoy (1974). An economic analysis of the effects of U.S. crude oil import quotas and oil production restrictions (prorationing) is contained in Erickson (1970).

Wellisz (1963) analyzes rate of return regulation as applied by the Federal Power Commission to interstate natural gas pipeline companies. In particular the author looks at the effects that the FPC formulas for cost allocations between jurisdictional and non-jurisdictional and between peak load and non-peak load sales have on the efficient allocation of resources. The conclusion he draws, based upon analyses of consumer's surplus, is that the limitation of pipeline profits to a "fair return" does not necessarily lead to opportunity cost pricing, i.e., efficient resource allocation. With this type of regulation, peak load customers will tend to obtain service at prices lower than opportunity costs and off-peak customers at rates higher than opportunity costs. At lower prices demand for peak load capacity will increase, allowing the pipelines to build up the capital plant on which they earn their rate of return. Thus pipelines will tend to overexpand and load factor (ratio of average to peak load sales) will decrease, resulting in a less efficient allocation of resources than would have been the case under opportunity cost pricing.

In their book, Energy Regulation by the Federal Power Commission (1974), Breyer and MacAvoy look at some of the same issues. What do the historical data reveal about pipeline regulation as practiced by the FPC?

An analysis of the rates of return allowed various pipelines during the 1960's showed that they were several percentage points higher than the opportunity cost of capital of the energy utility industry as a whole. In some cases, allowed returns were higher than earned returns. Thus, in effect, there was no regulation at all.

In addition, Brayer and MacAvoy point out that FPC price structures were often chosen so that prices charged by different pipelines were approximately the same in areas where more than one pipeline delivered gas and based on cost of service only where the pipeline had no competition.

In effect, Breyer and MacAvoy conclude, the FPC did not produce benefits to consumers in the way of lower prices or improved quality of operations. It appears that rates of return and prices were chosen so that pipeline behavior was altered very little by the regulatory process. One might wonder then whether the $6 million per year spent by the FPC on pipeline regulation produced any significant benefits at all.

Averch and Johnson, in their classic paper (1962), show how rate of return regulation changes the behavior of the firm with respect to the factors of production. The optimal mix of capital and labor (or fuel) is altered in such situations from what it would be in a competitive opportunity cost setting. Under regulation the firm tends to become more capital intensive, building up its rate base so that its total profits are higher. Since that paper was written a flurry of articles have appeared in the literature analyzing further the "Averch-Johnson effect" or trying to show its existence in one of the many regulated industries in the U.S. Courville (1974) and Spann (1974) examine the evidence for the electric utilities industry both confirming the existence of the "Averch-Johnson effect."

One of the reasons for describing briefly these economic issues is that the scope of the present study does not deal directly with any one of them from a theoretical point of view. This study has had as its purpose the modeling and analysis of the natural gas system and, in particular, the pipeline transmission network, in a way that could be useful to energy policy making bodies. Thus a more pragmatic approach has been taken, one which attempts to model the situation as it exists, but with enough generality so that alternative policies could be analyzed as well as the continuation of present ones.

It is important to recognize, however, that there are unresolved economic issues involved in regulation of private companies and that there do exist papers in the literature which address these questions.

The papers described to this point have dealt with the regulation of the interstate pipeline companies. Probably a much more important effect has been produced by the Federal Power Commission through its ceiling price regulation of producers of gas for the interstate pipelines. The facts that the Federal Power Commission didn't want that job in the first place, and that the Chairman of the FPC (1974) has been publicly calling on Congress to deregulate the natural gas producers, is ample evidence that much has gone amiss in this field, too. Breyer and MacAvoy (1974) discuss the major historical, legal, and economic issues involved in wellhead price regulation. Their conclusion is that it was ill advised from the start and has grown steadily worse to the point of having created a natural gas shortage. MacAvoy goes so far as to entitle an article in The Journal of Law and Economics, "The Regulation Induced Shortage of Natural Gas" (1971).

Much of the literature investigating the regulation of natural gas producers has centered on attempts to quantify the negative effects of that regulation through the use of econometrically estimated models of the supply of natural gas. In his Journal of Law and Economics paper, for example, MacAvoy describes two simple econometric models of supply which attempt to show the effects of the introduction of price controls in the early 1960's. He does this by estimating a supply model during the pre-regulation 1950's, simulating this model over the 1960's, and comparing the results to actual prices and supplies over the 1960's. The results show what prices would have been necessary in order to achieve a level of new supplies sufficient to support the actual production levels of the 1960's at the historical level of the reserve additions to production ratio of the 1950's. Simulations showed that prices would have continued to rise as they had been doing during the 1950's when monopsony pipeline positions were eroding. Instead, the prices under regulation stayed constant in dollar terms and actually decreased in real terms, thus creating a low investment potential for exploration for natural gas and a resulting shortage in the market for reserves.

The next section discusses more fully the development of other models of the supply process for natural gas.

2.3 Natural Gas Supply Models

There have been several attempts to model the complicated processes involved in the exploration, discovery, development, and production of natural gas reserves since the study of F. M. Fisher in the early 1960's (1964). This early study, though hampered by lack of sufficient data, did provide a good framework for several later analyses of petroleum supply. The approach taken was to divide the process of exploration and development into three phases: investment in well drilling, probability of an exploratory well discovering gas and/or oil, and the size distribution associated with such a succesful discovery.

Erickson and Spann (1971) took this approach in a Bell Journal of Economics and Management Science article. In this article the authors first present a theoretical analysis of joint costs of new supplies for the oil and gas industries. They show how the regulation of new contract prices for natural gas creates a negative effect on supply which is exacerbated by the economics of joint exploration for oil and gas.

Next they develop and econometrically estimate a supply model with the same general structure as Fisher's model: wildcat drilling activity, success ratios, and discovery sizes for oil and gas.

The results from this model show that the effect of regulation in the 1960's was to reduce exploration for gas significantly. Using estimated supply equations for the pre-regulation era, the authors show that the predicted levels of discoveries for the regulated period are much higher than actual levels. This indicates the stifling effect of wellhead price regulation on new discoveries. Finally, the authors attempt to estimate the price increase necessary to clear the market for reserves in the Gulf Coast area. Based on an estimate of 0.5 for the elasticity of supply and a supply shortage of 10 - 25%, the percentage increase in gas price needed to clear markets is estimated to be between 20 and 50%. Thus an increase in the Louisiana wellhead price of from 20¢ to 30¢ would have been needed to clear markets in 1970. It is interesting that in this article (published prior to the increases in foreign oil prices) the authors also suggested that U.S. oil prices were too high and should be lowered.

The MacAvoy-Pindyck model (1974) of the U.S. natural gas supply system is similar to the Erickson and Spann model. It is differentiated from that and other supply models in that it includes a demand and regional distribution model as well. It will be discussed more fully in a subsequent section.

While working with the Federal Power Commission, J.D. Khazzoom (1971) developed an econometric model of natural gas supply in the U.S. This is a simple two equation model: one for new discoveries and one for extensions and revisions. These three quantities comprise additions to reserves. In his model these dependent variables are estimated to be simple functions of lagged prices and lagged dependent variables with no other explanatory variables. He uses the model to analyze the short and long term price response of the dependent variables. The author finds that new discoveries can be modelled fairly well with this approach but not extensions and revisions. Thus he ignores extensions and revisions when using his model to simulate future reserve additions and instead concentrates on new discoveries. These forecasts are made on the basis of assumed levels of exogenous oil prices and are parameterized by ceiling prices on new contracts. Khazzoom analyzes the results of these simulations in terms of immediate and delayed response of new discoveries to ceiling price increases. The results indicate reductions in the level of new discoveries with present price levels and even with small increases in the real price of natural gas. To reach the level of production necessary to meet expected future demands, larger price increases are necessary. The author closes his article with a discussion of the special significance of new discoveries in the Louisiana offshore area, since the ratio of new discoveries to production has been very low and decreasing there. He suggests that reducing drilling risk in the Gulf of Mexico by altering lease bonus payment schedules may be one way to provide greater incentive for exploration in this vital natural gas area.

Challa and Subrahmanyam (1974) discuss the exploratory process for gas and oil in terms of the expected benefits and risks to the gas and oil producer. Taking a utility maximization approach, the authors develop a framework for analysis in which the basic elements are the risks and expectations involved in the exploration process. In testing their theoretical results, the authors use the gas supply models of Fisher, Erickson and Spann, and MacAvoy and Pindyck in modified form. As in these models, the three sections of the model are (1) drilling of wells, (2) success ratios, and (3) average discovery size of successful wells. The first of these sections -- total wells -- is modelled as a function of expected revenues, energy price ratios, average total costs, depletion factors, interest rate, and risk. The second section is either a binomial or trinomial logit model which is used to simulate the decision to drill for gas or oil as well as their respective success ratios. The explanatory variables here are the same as for the first equation. In this case two equations are necessary in order to determine the breakdown of wells drilled into successful oil or gas, and unsuccessful dry holes. Finally, the size equations are modelled individually for associated and non-associated discoveries. The same basic variables apply here also. The results of these estimations are considerably less promising for the latter equations than for wells and success ratios, reflecting the great variability in discovery size in the late 1960's and early 1970's.

In a Society of Petroleum Engineers paper (1974), Hardy and Neill describe the American Gas Association's Total Energy Resource Analysis (TERA) model. As is the case for the MacAvoy-Pindyck model, TERA is more than just a supply model. Investment in well drilling is modelled as a function of a profitability index of price divided by costs. The authors feel this gives a more representative view of actual investment behavior and is inflation proof as well. Success ratios are econometrically estimated as functions of previous success ratios. Thus, trends in success ratios are determined. Discovery sizes are judgmentally estimated based on historical trends.

J. G. Debanne (1971) uses a completely different model for investment, one which computes the quantity of investment necessary to discover new reserves in a partially depleted basin. This simple model is used in conjunction with the productive capacity relations in his model to simulate the behavior pattern wherein producers try to maximize their reserves subject to maintaining a non-decreasing level of cash flow. As in other cases described above, this model includes distribution and demand submodels which will be described in subsequent sections.

Kaufman and Bradley (1971) discuss an approach to the exploration process for new reservoirs in existing basins. The model uses the results of previous drilling efforts in the basin to estimate the probability of discovering a new reservoir of a given size on the next drilling effort. This model is based on three basic assumptions: (1) there are a finite number of fields to be discovered in a basin, (2) searching for them is a process of sampling without replacement from a list of prospects, and (3) discoveries are proportional to field size, hence larger fields tend to be discovered first. These assumptions lead to quite complex functional distributions from which to generate the probabilities and sizes of discoveries. Thus this approach trades off simplicity and ease of estimation for a more realistic probabilistic model of the exploration and discovery process.

The models described above concern the supply of reserves of natural gas. Several of these models also consider production out of reserves. In the MacAvoy-Pindyck model production of natural gas is modelled as a function of reserves level and the logarithm of the price of gas. Thus it attempts to explain two features of production: (1) dependence on reserve level, and (2) increasing average costs of production. The TERA model allows for no short run price elasticity in its production equations. In this model the FPC National Deliverability Schedule (1973) is used to estimate how much production will be available from the discoveries made in each previous year. The estimated potential production from each district is the total quantity available for production from each of these previous discoveries as estimated from historical deliverability patterns. In Debanne's model of continental energy supply and demand, the productive capacity in each region is limited by the level of reserves through a "Reserve Life Index" (RLI) which estimates how many years of production is actually available in the ground. In his model production can only increase by increasing reserves levels through investments in new well drilling.

2.4 Energy Demand Models

Demand for natural gas actually occurs at three levels: demand by the interstate pipelines for gas from the wellhead, demand by local distributors for gas from the pipelines, and final demand for consumption by residences, businesses, and industry. These demands occur sequentially with the final demand for consumption signally the wholesale and then pipeline demands.

In addition, natural gas has traditionally been sold on long term contracts at both the wellhead and wholesale levels. These contracts were usually based on twenty years of service. Recently, however, the gas shortage has forced both pipelines and distributors into the positon that any gas on any length contract is better than none at all.

At the retail level residential and commercial customers can be expected to remain gas customers for as long as their gas-burning equipment lasts. When it needs replacement, they can choose to remain gas customers or switch to another fuel. Industrial customers generally have more latitude in switching fuels unless they are using gas as an ingredient in the process of manufacturing their product (ammonia, for example).

Any realistic demand model for gas must be able to incorporate this long term feature. Two general models to be discussed here do so though in somewhat different ways.

Balestra (1967) in a formulation for residential gas demand, discusses an econometric model for "new demand." New demand can be defined with the help of Figure 2.1. In the left-hand figure demand, Q_{t-1}, in the year t-1 is divided into three parts: (1) that fraction of demand in t-1 associated with customers who need to replace their gas-burning equipment and decide to switch to oil or electricity (cQ_{t-1}), (2) those who in similar circumstances decide to stay with gas (dQ_{t-1}), and (3) continuing demand ($(1 - c - d)Q_{t-1}$). In the right-hand figure we see that demand associated with those who switched is missing, while there has been some growth in demand of old (replacement and continuing) customers and new customers (Q'). Total new demand, δQ_t (the shaded regions), is given by

$$\begin{aligned}\delta Q_t &= Q_t - (1-c-d)Q_{t-1} \\ &= \Delta Q_t + (c+d)Q_{t-1} \\ &= \Delta Q_t + rQ_{t-1}\end{aligned}$$

where $r = c + d$ is the total appliance depreciation rate for gas-burning equipment. The quantity dQ_{t-1} is included in "new demand"

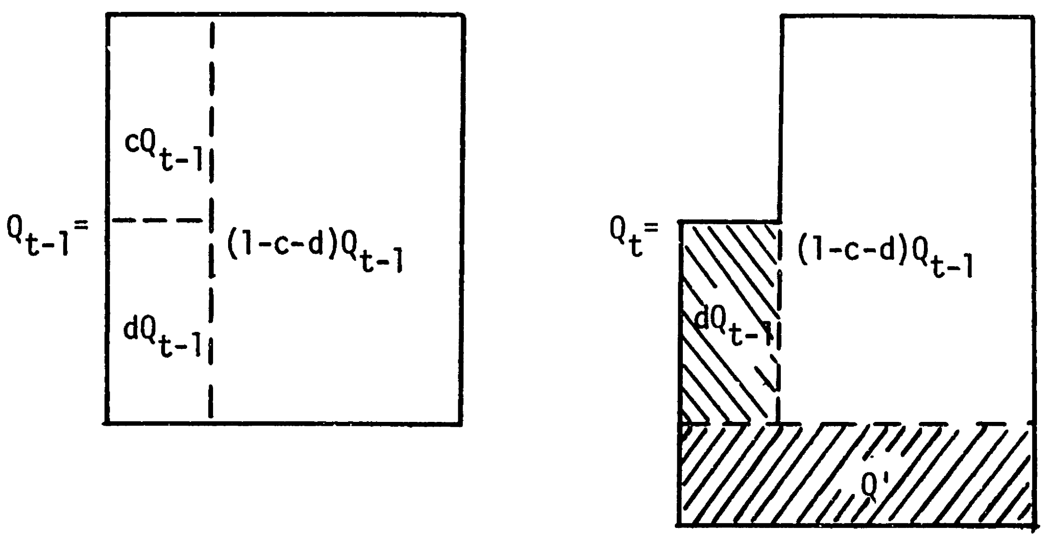

Figure 2.1 "New Demand" in the Balestra Formulation

because the customers who decide to replace their gas equipment could have switched to another fuel. Thus they are in the same category as new customers choosing equipment for the first time. Note that δQ_t is independent of the fraction of those who decide to switch fuels when replacing obsolete equipment. In Balestra's formulation new demand is estimated as a function of gas price, changes in income, and changes in population, since it is felt that gas price will affect new customer purchases and old customer replacement decisions while changes in population and income will influence the consumption of continuing customers. The difficulty in Balestra's formulation lies in estimating the depreciation rate, r, since there is no data available on stocks and depreciation rates of gas-burning appliances. Balestra could not get a reasonable result in estimating an equation for r. In the MacAvoy-Pindyck study (1973) an estimate of 0.07 was used for both residential and industrial sales, but even there a positive and statistically significant result could only be obtained for the West region of the country. Part of the difficulty may be that different regions of the country use various gas-burning appliances to different degrees. Thus depreciation rates may not be the same from region to region and cross-sectional time series regressions which attempt to force a common depreciation rate simply may not work.

The demand model of the MacAvoy-Pindyck (1974) study closely follows Balestra's formulation. Rather than running a cross-sectional time series regression over the whole country, however, the MIT group divides the U.S. into five separate wholesale market regions. In each of these regions cross-sectional time series regressions are performed on Balestra-type "new demand" functions with gas price, alternative fuel price, and economic growth variables as explanatory variables. Equations are estimated separately for residential-commercial and industrial demand in each region.

Another tack is taken by Houthakker, Verleger, and Sheehan (1974) in their analysis of demand for gasoline and residential electricity. In this paper a different type of dynamic model is formulated. "Desire demand," Q^*, is the quantity of some good which would be demanded if only its price and the consumer's income were considered. In this approach it can be expressed as

$$Q^* = \gamma P^{\alpha} I^{\beta} \qquad 2.4.1$$

where γ is a proportionality constant and α and β are price and income elasticities. Because of inertia in the system, buyers don't often achieve their desired demand levels. For example, the price of some fuel might increase significantly reducing its demand, but because a new burner for a different fuel would require a significant investment, this reduction in demand would be less than the desire reduction. We could model the process of change toward desired demand as

$$\frac{Q_t}{Q_{t-1}} = \left(\frac{Q^*}{Q_{t-1}}\right)^{\lambda} , \quad 0 \leq \lambda \leq 1 \qquad 2.4.2$$

where λ might be called the "drag coefficient." $\lambda = 0$ would correspond to no movement toward desired demand (Q_t = constant), while $\lambda = 1$ would be perfect response of consumption to desired demand.

Using 2.4.1 to eliminate Q^* from 2.4.2, we get

$$Q_t = (\gamma P^{\alpha} I^{\beta})^{\lambda} (Q_{t-1})^{1-\lambda} \qquad 2.4.3$$

or in logarithmic form

$$\log Q_t = \lambda \log\gamma + \lambda\alpha \log P + \lambda\beta \log I + (1-\lambda)\log Q_{t-1} \qquad 2.4.4$$

The coefficients may be interpreted as follows: at equilibrium when t becomes very large, the price and income elasticities are α and β respectively. These are the long run elasticities for the process. In the short run, the corresponding elasticities are $\lambda\alpha$ and $\lambda\beta$ respectively, where again λ measures the rate of response to desired demand.

The Houthakker approach has not been used for natural gas (so far as I know), but it would be very interesting to see how results from such a study would compare with results using the Balestra approach. A relationship between λ and r, for example, might be discovered or proved. Estimating λ in equations of the form 2.4.4 over cross-sectional and time series data might present a problem similar to that discussed in the Balestra formulation for estimating r, namely λ may actually be quite different for different regions of the country where different consumption patterns prevail.

A third demand formulation is presented in the Federal Energy Administration's description of the Data Resources, Inc. Energy Demand Model (EDM) (1974). This model is based on a logit approach, wherein total energy and shares won by each fuel are estimated separately and then combined to come up with total demand for each fuel. Because it deals with the entire energy system, EDM is beyond the scope of this particular study, though it might figure in an extension of this work to include other fuel supply and demand.

In the computational aspect of the study described in this thesis, the Balestra-type demand formulation has been used. Though there are certain difficulties inherent in using a linear formulation estimated over cross-sectional data (elasticities for larger states are underestimated with the reverse true for smaller states), it does offer convenience as a linear function. The model is general enough to handle any non-increasing demand function, however, so that the power function form for Q_t implied by 2.4.4 could also have been used.

2.5 Pipeline System Models

Until recently with the introduction of liquified natural gas transmission by tanker, the only way gas producers could get their product to final consumers was by pipeline. Pipeline systems are used to gather gas from many different wells into a single large pipeline, to transmit gas over large distances from the Southwest to the North and the West, and to distribute the gas to millions of local customers. Many different kinds of problems present themselves to the producers, transmission companies, and distribution companies. Producers must worry about the rates of extraction of high pressure gas from various wells; they must keep pressures at appropriate levels for optimal retrieval efficiency. Distributors have to worry about meeting hourly-changing demand patterns. Interstate pipelines are interested in tradeoffs between investment in greater peak load transmission capacity and greater storage capacity near demand centers. All of them must concern themselves with long range planning and extensions of their networks.

Thus there is a range of problems here which can be roughly characterized by the time intervals involved in the decision making process. In the case of distribution company operations, the time unit is the hour; at the other extreme, long range planning may be concerned with a period of up to 20 years.

In an American Society of Mechanical Engineering paper, Wienecke (1972) describes an optimization model used by the Colorado

Interstate Corp., a natural gas pipeline company, to control dispatching of gas on a short term basis (hourly). The model is dynamic, since dispatching of gas to meet changing demands causes changes in the pressure of gas at various points in the network. Thus the quantity flows and pressures, compressor power needed, and cost associated with it, are all time-varying quantities which must obey certain differential equations (conservation of mass and momentum) and other empirical relations (compressor cost versus pressure and quantity throughput). The algorithm used to optimize this system was first developed by a team at the University of California at Berkeley (Hax, 1967). Some parts of this work were based on earlier AGA pipeline models (Halbert, Distefano, 1970).

At the other end of the pipe, Rothfarb, Frank, et al. (1970), discuss development of computerized algorithms for the solution of pipeline planning and design problems for offshore natural gas lines. The three problems analyzed in the study were: (1) selection of optimal pipe diameters; (2) selection of minimum cost networks given gas field locations and flow requirements; and (3) optimal expansion of existing pipeline networks to include newly discovered gas fields.

To design the basic structure of the pipeline system, the authors have developed a heuristic tree-generating algorithm (a tree is a connected network with no loops) which computes a low-cost (though not necessarily minimum cost) network designed to satisfy the company's requirements for moving gas from offshore fields to onshore natural gas liquids extraction plants or major transmission pipelines. Given this basic network, a second algorithm is called to optimize the pipeline design, i.e., to compute optimal pipeline diameters. The authors show that for a given network structure, this algorithm converges to the globally optimal design.

Moving from the realm of company related problems to national policy, Waverman (1973) describes the results of a model he built to simulate the economic effects of the Canadian Government's decision to build the trans-Canadian pipeline. The purpose of the pipeline was to deliver gas from gas fields in Alberta to Eastern Canada rather than exporting that gas to the U.S. West Coast and importing gas from the Gulf Coast Region for Eastern Canadian demand. Using the basic framework of Debanne's continental oil and gas model (1971b), Waverman proceeds to show that the Canadians' nationalistic rationalizations for the pipeline resulted in a several hundred million dollar loss in potential sales to the U.S. and in the costs of construction of the pipeline itself.

Lawrence (1973), in his Ph. D. thesis, constructs a simple model of the gas transmission system based on Bureau of Mines (1950-1972) estimates of interstate movements of natural gas. This model does not pretend to mimic the actual pipeline network, but at a more aggregated level does attempt to simulate the flows and costs actually prevailing in the late 1960's and early 1970's.

In Debanne's model of continental oil and gas supply and demand (1971a), a schematic representation of the pipeline network is made by connecting various supply and demand centers and approximating throughput and costs by assuming optimal pipe design factors. In this model transportation costs include operating and fixed costs and costs of expansion. A special version of the Ford-Fulkerson out-of-kilter algorithm which handles these non-linear cost functions is described. The model is built in such a way that new pipelines and loopings for expansion of existing lines can be added when needed to satisfy increasing demand. Thus this model can be used for long range studies of optimal investment strategies for the system as a whole.

In the MacAvoy-Pindyck study (1974), the pipeline network is modelled indirectly by a regional distribution matrix which allocates fixed percentages of each production district's output to each demand district based on historical (1971-1972) flow patterns. Though this I/O table approach accurately describes what occurred in the early 1970's at an aggregated level, it is not capable of simulating changing pipeline patterns and is thus a limited tool for allocation purposes.

The pipeline model we have constructed for this study is intended to be an accurate representation of the actual interstate network as it looks on a yearly basis. We have aimed the study at the effects that Federal regulation of the industry have on the system as a whole. Consequently it is intended to be useful as an aid to economic policy making rather than as an engineering tool.

2.6 Equilibrium and Disequilibrium Supply and Demand Models

Several of the models discussed above are actually parts of larger models consisting of supply, distribution, and demand submodels. The way in which these sectors are connected and the levels of disaggregation are, however, very different from one study to the next.

In the MacAvoy-Pindyck model (1974), for example, the regionalized supply model is connected to the separately regionalized demand model through a static input/output coefficient matrix and associated average markups from wellhead to wholesale prices. The model is driven by a set of wellhead prices which determine investment in new drilling, additions to reserves, and production. The production and wellhead prices are combined with "price markups" to generate the wholesale prices which determine demands. The input/output matrix is used to compute regional supplies available for consumption which are then compared with the calculated demands to determine whether there is excess supply or demand. Thus, in effect, the supply side of the model influences the demand side, but not vice versa. MacAvoy and Pindyck feel that this is representative of the present situation in which demands far exceed the supplies available to satisfy them. The problem with the methodology is that forecasts sometimes indicate excess supplies in some consumption regions and excess demands in others. The model does not have the flexibility to balance these off and thus is not very useful as an allocation tool.

In the AGA TERA model demand prices are computed simply by inflating previous year prices. These prices are then used to calculate total energy demand and energy market shares for each fuel. The supply and pricing submodels are used to determine whether a supply-demand balance can be achieved. If so, an iterative procedure is used to determine quantity and price levels. If not, then supplemental sources of fuel are called for and (unspecified) allocations performed.

Unfortunately, Hardy and Neill's (1974) description of the TERA model is not explicit enough with respect to the supply-demand balancing and allocations submodel to analyze or compare it with other models. The impression one gets is that much of the model is judgmentally rather than econometrically estimated and that to some degree the complexity of the industry requires such an approach. On the other hand, with respect to those parts of TERA which are the most difficult to model (the supply of new discoveries, for example) it does not seem to offer any particular advantages or insights over any of the other models examined.

In three separate papers (1971a, 1971b, 1973), Debanne describes the development of a complex model of the North American energy system. The model has expanded from a single commodity oil model to a multi-commodity oil, gas, nuclear, and hydro power model. The model determines yearly productive capacity for oil and gas as a fraction of total reserves. New reserves are added through a behavioral investment model based on the assumption that producers maximize reserves subject to a non-decreasing cash flow from year to year. Oil and gas are assumed to flow to markets in pipelines or oil tankers which are optimally costed and sized according to engineering formulae. Cost of service includes both variable and fixed transmission costs. For each transmission link these costs are non-linear functions of the throughput in that line.

A variant of the Ford Fulkerson out-of-kilter algorithm is used to allocate gas and oil in a least cost manner from supply to demand regions. Because unit costs are not constant, an iterative procedure is described in which an assumed initial flow is used to compute average transportation costs in each link. These are then used to compute a second set of flows which are used to recompute costs and a new set of optimal flows. This procedure is continued until a stable set of flows and costs is obtained. During this algorithm wellhead prices are adjusted so that at the final iteration all demands are met without excess supplies being produced. These wellhead prices are then used to compute reserve additions and, therefore, total productive capacity for the next year's run.

In this model the price of oil in the price leading region is used as the base price against which all other prices equilibrate. Because of severe regulation of interstate gas production and flows, this is an unlikely scenario. In fact, when Debanne actually implements the two commodity gas and oil model, he finds that his wellhead price adjustment mechanism doesn't work and that he must substitute another ad hoc method. The investment model also doesn't work too well when dealing with oil and gas separately. In spite of these difficulties, the model handles the problem of pipeline expansion and non-linear total costs in a very ingenious and probably realistic manner.

The next set of papers concern a particular approach to problems involving spatially separated supply and demand centers: spatial equilibrium models. The first major breakthrough in the analysis of these models came in an article by Samuelson (1952) where he showed that the basic set of economic relations which must be satisfied in order that trade take place between spatially separated areas was equivalent to a maximization problem.

In a spatial equilibrium problem at least one region can produce more than it can consume and vice versa for another. If transportation costs are low enough both regions may be able to benefit from trade.

This is illustrated for a two region economy in Figure 2.2. At equilibrium, region 1 in the figure produces a quantity X_{12} more than it can consume at price P_1, while at price $P_2 = P_1 + t_{12}$, region 2 would consume X_{12} more than it can produce. Thus P_1,P_2 and X_{12} are equilibrium prices and flows for this problem. Note that the singly hatched regions are the net increase in producers plus consumers surplus in each region as a result of the trade. The doubly hatched triangles are constructed to be of the same area as the single hatched ones.

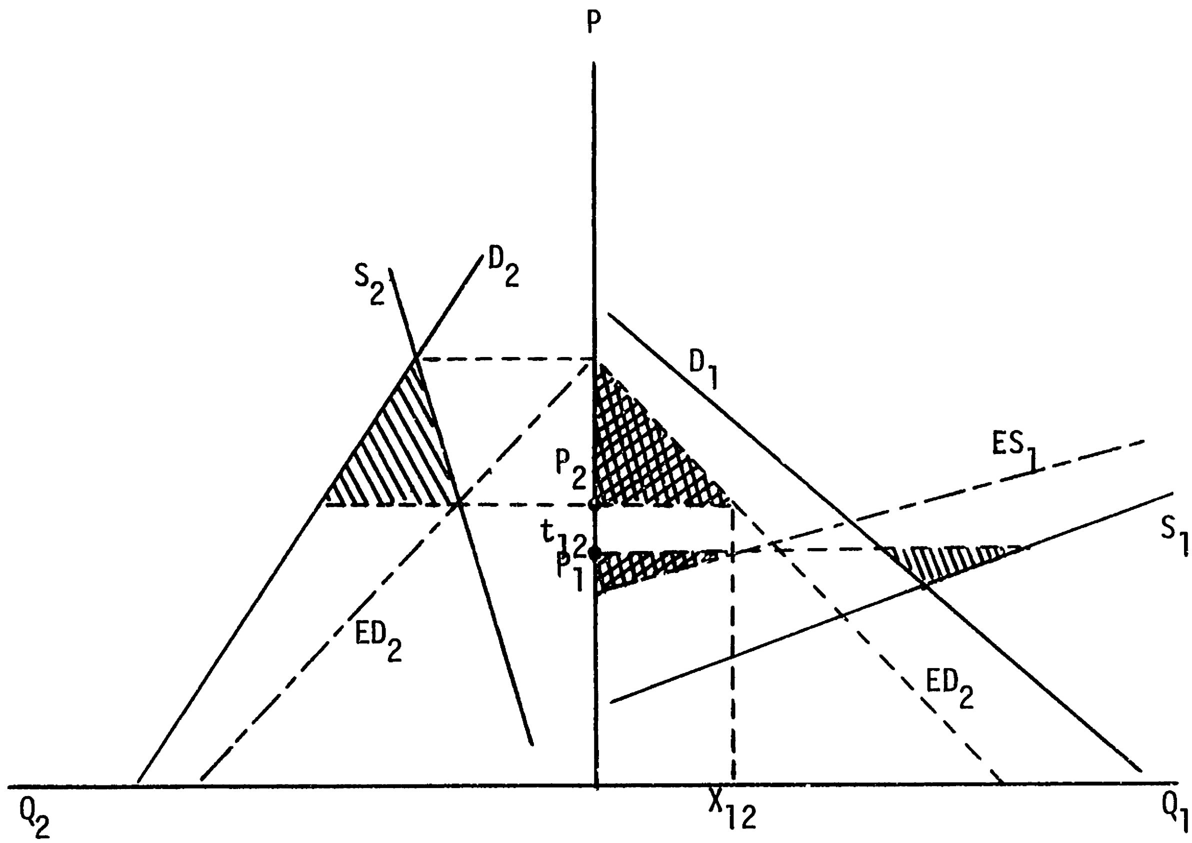

Figure 2.2 Spatial Equilibrium between Two Regions

Samuelson showed that the sum of these areas is equal to

$$\sum_j \int_0^{Y_j} [P_j^D(Q) - P_j^S(Q)]dQ - \sum_{ij} t_{ij}X_{ij} \qquad 2.6.1$$

for the general n-region case. In this expression $P_j^D(Q)$ and $P_j^S(Q)$ are the demand and supply functions for region j in the price form ; X_{ij} is the quantity flow from region i to region j; and Y_j is the absolute value of the excess supply or excess demand in region j.

Note that if X_{12} were increased slightly, the area between ED_2 and ES_1 would increase, but that the rectangular area $t_{12}X_{12}$ would increase more. Thus the integral would decrease. On the other hand, if X_{12} were decreased, the area between the curves would decrease more than the rectangle $t_{12}X_{12}$. Thus the integral would also decrease. Thus at the equilibrium point where $P_2 = P_1 + t_{12}$ the integral is maximized.

Samuelson shows that the first order necessary conditions for the maximization of 2.6.1 subject to $X_{ij} \geq 0$ are

$$\sum_i X_{ij} - \sum_k X_{jk} = D_j(P_j) - S_j(P_j) \qquad 2.6.2a$$

$$P_j - P_i \leq t_{ij} \qquad 2.6.2b$$

$$(P_j - P_i - t_{ij})X_{ij} = 0 \qquad 2.6.2c$$

$$P_i, P_j \geq 0 \qquad 2.6.2d$$

where $D_j(\cdot)$ and $S_j(\cdot)$ are the inverses of $P_j^D(\cdot)$ and $P_j^S(\cdot)$ respectively. Note that if $X_{ij} \rightarrow 0$ then X_{ji} must equal 0 and vice versa as long as t_{ij} and t_{ji} are greater than 0.

These relations are the economic conditions necessary for equilibrium: (a) total supply (domestic plus imported) equals demand; (b) the price in the receiving region cannot be higher than the price in the sending region plus transportation costs; (c) if it is less, then no shipment will occur ($X_{ij} = 0$) since it will then be unprofitable; and (d) prices are greater than 0.

Takayama and Judge (1964) show that Samuelson's formulation can be simplified by writing it in terms of prices rather than quantities. If the supply and demand functions $P^S(Q)$ and $P^D(Q)$ can be inverted, then the double cross hatched areas in Figure 2 will be integrals of these new functions with respect to P. Now the problem can be stated as minimize the sum of those integrals with the restriction implied by 2.6.2b, i.e.,

$$\text{minimize} \quad \sum_j \int_0^{P_j} [S_j(P) - D_j(P)]dP \qquad 2.6.3a$$

$$\text{subject to} \quad P_j - P_i \leq t_{ij} \qquad 2.6.3b$$

Note that in this formulation if P_j is strictly less than $P_i + t_{ij}$, the area in both triangles will increase. Thus at equilibrium, the area will be minimized when $P_j - P_i = t_{ij}$. Also if any inequality in the set 2.6.3b is non-binding, then the associated dual variable U_{ij} is 0. But by writing the Kuhn-Tucker conditions for 2.6.3 one can see they are identical to the set 2.6.2 if $U_{ij} = X_{ij}$. Hence the dual variable for 2.6.3 correspond to flows.

Thus there are two formulations of the spatial equilibrium problem, one in quantity and the other in price variables. When supply and demand relations are linear, the problem becomes a diagonal quadratic program. The authors show how the results can be generalized to multicommodity flows when it is assumed that for products i and j the relationship

$$\frac{\partial Q_i}{\partial P_j} = \frac{\partial Q_j}{\partial P_i} \qquad 2.6.4$$

holds for Q either supply or demand. Erickson and Spann (1971) show that 2.6.4 are necessary conditions for profit maximization in an industry with joint costs (such as the oil and gas industry). A similar proof follows for maximization of utility of the consumer on the demand side. However, just because microeconomic theory says that these relationships hold does not mean that they do in actuality. Also econometric estimates of supply or demand usually do not require that they hold.

In a rather unnecessarily elaborate paper Takayama and Woodland (1970) provide a rigorous proof of the equivalence of the quantity and price formulation of the spatial equilibrium problem. They do so by showing that the Kuhn-Tucker optimality conditions for the two problems are of identical form when a correspondence is made between quantity and price variables and Lagrange multipliers for the two sets of conditions.

In this formulation multicommodity flows and demand equations are restricted so that price cross-derivatives between goods must be equal, i.e. 2.6.4 must hold; with linear supply and demand functions the matrix of quadratic terms in the objective function must by symmetric and also positive semi-definite.

For invertible and integrable non-linear demand and supply functions, the authors show that similar duality results apply if, in addition, the integrated functions are concave.

In another paper Lee and Seaver (1971) model equilibrium in spatially separated markets as a set of simultaneous equations by simply writing the economic conditions necessary for such an equilibrium. Whereas Samuelson has shown that this can be converted into an optimization problem, the authors of this paper take another approach. Since to solve the quadratic program implied by spatial equilibrium with linear supply and demand requires (in Wolfe's approach) the reconversion of the problem back into its original form, why bother converting in the first place? Instead, they suggest, simply solve the set of equations using Wolfe's modified simplex algorithm right from the beginning. This leads to an interesting possibility for multicommodity spatial equilibrium problems where price cross-elasticities for different products are not symmetric. Even though the problem cannot be expressed as a quadratic program, it might be directly solvable by Wolfe's method. The problem is that one can't use results from convex programming to show existence of a minimum, but perhaps it can be shown in another way.

The spatial equilibrium problem as given in 2.6.2 but with linear supply and demand equations can also be expressed as a linear complementarity problem. In general form this problem can be written:

Find W, Z in R^n such that

$$W = g + MZ \qquad 2.6.5a$$

$$W \cdot Z = 0 \qquad 2.6.5b$$

$$\text{for} \quad W \geq 0 \text{ and } Z \geq 0 \qquad 2.6.5c$$

The correspondence between variables 2.6.2 and 2.6.5 would be

$$W_{ij} \leftrightarrow -(P_j - P_i - t_{ij})$$

$$Z_{ij} \leftrightarrow X_{ij}$$

These problems have been solved by pivoting techniques different from the simplex method. Major results have been obtained by Cottle and Dantzig (1968), Dantzig and Cottle (1967), Eaves (1969), and Lemke (1965, 1970).

A separate but related technique has been developed by Scarf (1973) for finding fixed points of point-to-set mappings. Given a vector of prices p, the spatial equilibrium problem reduces to a simple transportation problem. If we let the optimal shadow prices of the solution to this transportation problem be given by $\pi = f(p)$, then the solution to the spatial equilibrium problem is a set of prices p^* such that

$$p^* = f(p^*) \qquad 2.6.6$$

i.e. a fixed point of the mapping f. Scarf's algorithm can be used to solve such a problem if the number of prices is not too large.

Perhaps the most important reminder in Lee and Seaver's paper is that supply and demand equations cannot be estimated correctly without taking each other into account. That is, since equilibria are determined by the intersection of supply and demand curves (in the simple one region case) or by sets of simultaneous equations in more complex cases, supply and demand curves must be simultaneously estimated also. In Figure 2.3 this is demonstrated. The three

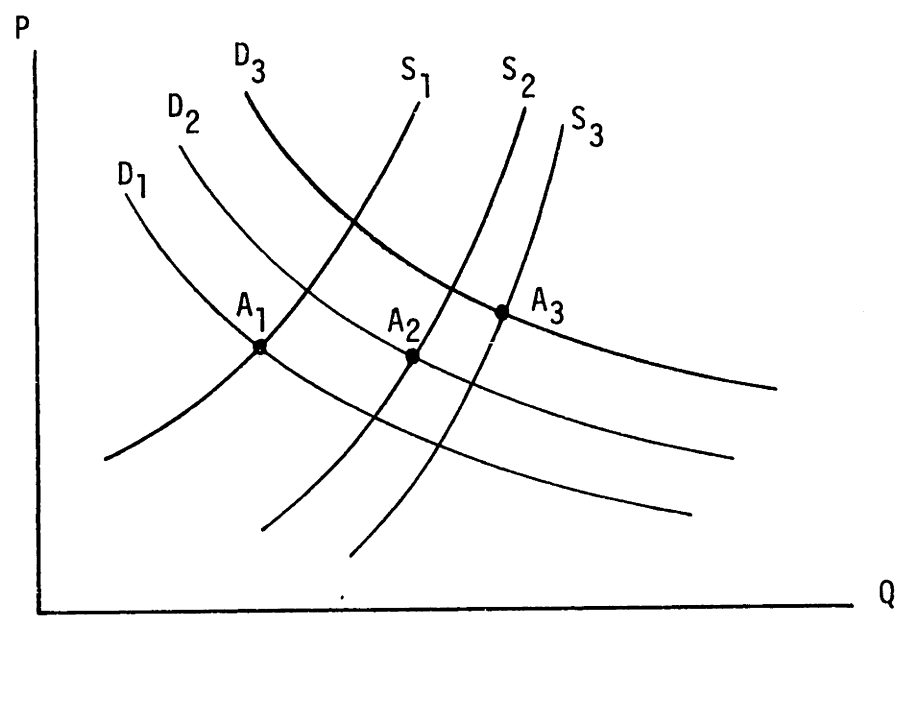

Figure 2.3 Changes in Equilibrium Points due to Shifts in Supply and Demand

points A_1, A_2, A_3 are observed equilibria at different times. When trying to estimate a demand equation, we cannot tell how much of the change in the equilibrium point was a result of changes in exogenous demand variables and how much was due to changes in exogenous supply variables. Solving the general linear supply and demand functions, assuming supply equals demand, we can determine price as a function of exogenous supply and demand variables. Thus one way to estimate the equations simultaneously is to first estimate P as a function of exogenous supply and demand variables and then use the resulting estimate $\hat{P}$ of P as the explanatory price variable in the supply and demand regressions. This is called two-stage least squares. Note that it is not sufficient to use only exogenous supply variables in the first stage procedure except when demand is greater than supply. In this case there is no equilibrium and the points A_1, A_2, and A_3 are only restricted to be on the supply curves, but not the demand curves. In addition of course, demand is unobservable in this case.

Hall, Heady, *et al.*, present a model of the U.S. agricultural sector (1975) wherein primary resources are converted into intermediate and then desired (final) products for final consumption in ten spatially separated markets constituting the continental U.S. In the model, product transformation is carried out by a linear activity analysis in which resources used up and products made are both proportional to the activity involved in the transformation. These activities are limited by the price independent quantities of available resources. The model assumes that all intermediate products are used up in making desired end products, the demand for which is given by a linear function of the various product prices. Production is assumed to occur without profit, i.e. product prices can only cover the imputed resource costs, activity costs, and imputed intermediate product costs incurred. Intermediate or final goods can be transported from one area to another with a constant unit transportation charge. Given these constraints, the model is set up to maximize the quadratic objective function representing producers' revenues minus costs.

The problem as set up is large ($m + n = 3524$) and requires 9100 iterations of a special quadratic programming algorithm developed by the authors to reach a solution. The authors acknowledge both numeric difficulties and excessive computer time and point out that others have shown how this problem now can be cut in half to yield the same answers.

In particular, Takayama and Judge (1970) in yet another article on the equivalence of the quantity and price formulations of the spatial equilibrium problem, show that the Hall and Heady price <u>and</u> quantity model is a correct representation for the spatial equilibrium problem with more than one commodity even when 2.6.4 is not satisfied. They suggest, however, as Lee and Seaver do in another context, that the problem 2.6.2 be solved by Wolfe's modified simplex method for quadratic programs, without ever actually expressing it as a quadratic program. They do not prove, however, that such a procedure will yield the correct answer.

Most of the papers dealing with spatial equilibrium models have applied their analysis to agricultural products. Lawrence (1973) in his Ph.D. dissertation applied this type of analysis to natural gas. In doing so, however, he neglected the fact that natural gas is a regulated industry at both the wellhead and wholesale levels and that competitive equilibrium models simply do not apply.

The Federal Energy Administration (Hogan, 1974) also applies a version of multi-commodity spatial equilibrium with activity analysis in the production sector. Rather than using a Wolfe's algorithm approach, the FEA developed an iterative procedure wherein subproblems in a Samuelson-type quantity formulation are successively solved by separable linear programming. The kth subproblem is derived from the (k-1)st by using its optimal shadow prices to update the demand functions in the kth iteration. When at two successive iterations the shadow prices are equal to within a given tolerance, then they are assumed to be the optimal prices for the problem.

The use of this algorithm is only required when the cross-derivate conditions 2.6.4 are not satisfied. This is, however, the usual case for econometrically estimated demand functions. As mentioned previously for supply functions, Erickson and Spann (1971) have shown that 2.6.4 should hold if producers are operating to maximize profits when producing oil and gas. When estimating supply functions over large geographical areas, however, there is no guarantee that 2.6.4 will hold.

As in Lawrence's thesis, the FEA model does not incorporate into its model the complications that regulation causes for natural gas. None of the models presented above includes explicit recognition of the influence of the FPC in its pricing submodels. The Debanne model is the only one which takes account of fixed as well as variable costs in pipeline operations, but even there the formulation is not sufficient for a realistic model of the regulated natural gas industry.

The purpose of this thesis is to provide a more realistic model than has been attempted heretofore, by bringing explicitly into the formulation both wellhead price and rate of return regulation. In addition, a normative approach to the problem of allocating these costs regionally is suggested, though other means of handling this problem are also mentioned. The yearly time frame of the data used in this model precludes its being useful, as is, in the analysis of the peak load versus off-peak pricing problem. The present version of the model is also sectorally aggregated at the demand level. Thus it does not now deal with the problem of allocations to different demand sectors in times of shortage, a very important issue. Both of these shortcomings could probably be overcome, but we have not tried to do so in this model. This model's main purpose concerns the regional allocation of natural gas in times of shortage in an industry under regulatory control.

2.7 Other Energy Models

Because of the great importance of energy to the economies of the nations of the world many studies have been done or are currently underway focusing on one or more energy-related problems. Carter (1974) discusses the use of input-output analysis to energy problems. Her concern is with the effects that changes in the prices or quantities of energy supplies will have on the other sectors of the economy. Finon (1974) describes the development of a model of the French energy industry for use in long range planning. The model minimizes total present value costs of developing, producing, distributing, and consuming energy in France subject to constraints on plant capacity, consumer needs, and pollution levels.

Devine and Lesso (1972), on the other hand, look at the industry problem of optimally locating oil and gas producing platforms in the Gulf of Mexico. Here a two-stage algorithm is described which first allocates wells to platforms, then places platforms at the optimal location for that allocation, then reallocates wells to platforms given their new positions, and so forth, until a stable optimal pattern is found.

A more extensive compilation of energy-related optimization techniques up to 1972 is presented by Dougherty (1972). In his survey, the author evaluates these techniques in terms of their proved, probable, or possible usefulness to the energy industry.

The major subject headings in which he places his 112 listings are Drilling, Development and Production, Stimulation, Lease Bidding, Reservoir Simulation, Exploration Planning and Risk, and Natural Gas Systems. Most of the models have been constructed for industry use, though a few are aimed at policy analysis. (Debanne is listed, for example.)

In 1974 Charpentier (1974) compiled a review of energy models for the International Institute for Applied Systems Analysis. Complementing the previous reference, this list contains 81 models, most of which deal with some aspect of energy/economics rather than engineering. His classification includes six categories: models applied to one type of fuel (A, national; B, international), more than one type of fuel (C;D), and linkage between energy and the general economy (E;F). Each model is described in a standard form including the following data: the model's subject and goal, the system described, the area modelled in time and space, modelling techniques, input data, output data, and observations.

The original journal references for the models are also listed for the convenience of the reader.

Shapiro (1975) gives a survey of Operations Research methods in three areas of strategic planning in energy systems: (1) energy system equilibrium analysis, (2) research and development planning models for new energy technologies, and (3) emergency oil stockpiling and allocation models.

Table 2.1 presents a summary of the articles described in this survey.

As extensive as this review and particularly the previous three surveys are, they are none the less not exhaustive. More than anything else, however, its purpose has been to set the stage, to provide the context, for the model to be presented next.

TABLE 2.1 Survey Summary

Category	Reference
Economics of Energy Regulation	Breyer and MacAvoy (1974)
	Erickson (1970)
	Wellisz (1963)
	Averch and Johnson (1962)
	Courville (1974)
	Spann (1974)
	MacAvoy (1971)
Natural Gas Supply Models	Fisher (1964)
	Erickson and Spann (1971)
	MacAvoy and Pindyck (1974)
	Khazzoom (1971)
	Challa and Subrahmanyam (1974)
	Hardy and Neill (1974)
	Kaufman and Bradley (1971)
Energy Demand Models	Balestra (1967)
	MacAvoy and Pindyck (1974)
	Houthakker, Verleger, and Sheehan (1974)
	FEA (1974)

TABLE 2.1 (continued)

Category	Reference
Pipeline System Models	Wienecke (1972)
	Hax (1967)
	Rothfarb, Frank, *et al*. (1970)
	Waverman (1973)
	Debanne (1971a,b)
	Lawrence (1973)
	MacAvoy and Pindyck (1974)
Supply and Demand Models	MacAvoy and Pindyck (1974)
	Hardy and Neill (1974)
	Debanne (1971a, 1971b, 1973)
Spatial Equilibrium Models	Samuelson (1952)
	Takayama and Judge (1964, 1970)
	Takayama and Woodland (1970)
	Lee and Seaver (1971)
	Hall, Heady, *et al*. (1975)
	Lawrence (1973)
	Hogan (1974)

TABLE 2.1 (continued)

Category	Reference
Alternative Solution Methods for Spatial Equilibrium Models	Cottle and Dantzig (1968)
	Dantzig and Cottle (1967)
	Eaves (1969)
	Lemke (1970)
	Scarf (1973)
Other Energy Models	Carter (1974)
	Finon (1974)
	Devine and Lesso (1972)
Compilations of Energy Studies	Dougherty (1972)
	Charpentier (1974)
	Shapiro (1975)

Chapter 3 GASNET: A Mathematical Programming Model of the U. S. Natural Gas System

3.1 Purpose

As stated in the introduction, the purpose of this study was to develop a model of the production, transmission, and demand for natural gas which represented the effects of FPC regulation in a realistic manner, a model which could be used as a tool for the analysis of alternative FPC policies regarding natural gas producers, interstate pipeline companies, and the allocation of natural gas in times of shortage.

3.2 Overview of the Model

GASNET, the model which will be described in this chapter, is a natural outgrowth of the MacAvoy-Pindyck MIT Natural Gas Policy Model (1973, 1974). As such it builds upon much of what was developed during the course of that study. In particular, the MacAvoy-Pindyck econometric models of production and demand were the basis on which the production and demand functions for this model were constructed.

At the same time GASNET has been developed in such a way that it can be used in conjunction with other production and demand models. Thus it is not restricted to the particular formulations of the MIT Policy Model.

The generality of GASNET is a function of its modular structure. As a mathematical program it consists of an objective function and four separate sets of constraints, corresponding to the following sub-models:

1. production out of reserves under wellhead price regulation;
2. transmission via the interstate pipeline network;
3. pricing of wholesale gas by interstate pipelines under FPC regulation;
4. wholesale demand for natural gas.

The objective function of this mathematical programming model is chosen to satisfy two modelling needs:

1. a description of the behavior of the pipelines under regulatory constraint;
2. a means to allocate natural gas efficiently and equitably during times of shortage.

The general form of GASNET is shown in Figure 3.1. In this figure rectangular blocks represent submodels, circles represent prices and other economic influences, and the triangle represents Federal regulation. Solid arrows stand for direction of gas flows. Thus production out of reserves is sold to pipeline companies who transmit it to wholesale customers around the country. (In this model we neglect the final distribution to retail customers. As discussed in Chapter 7, this simplification might conceivably lead to distortions in the estimation of the model.)

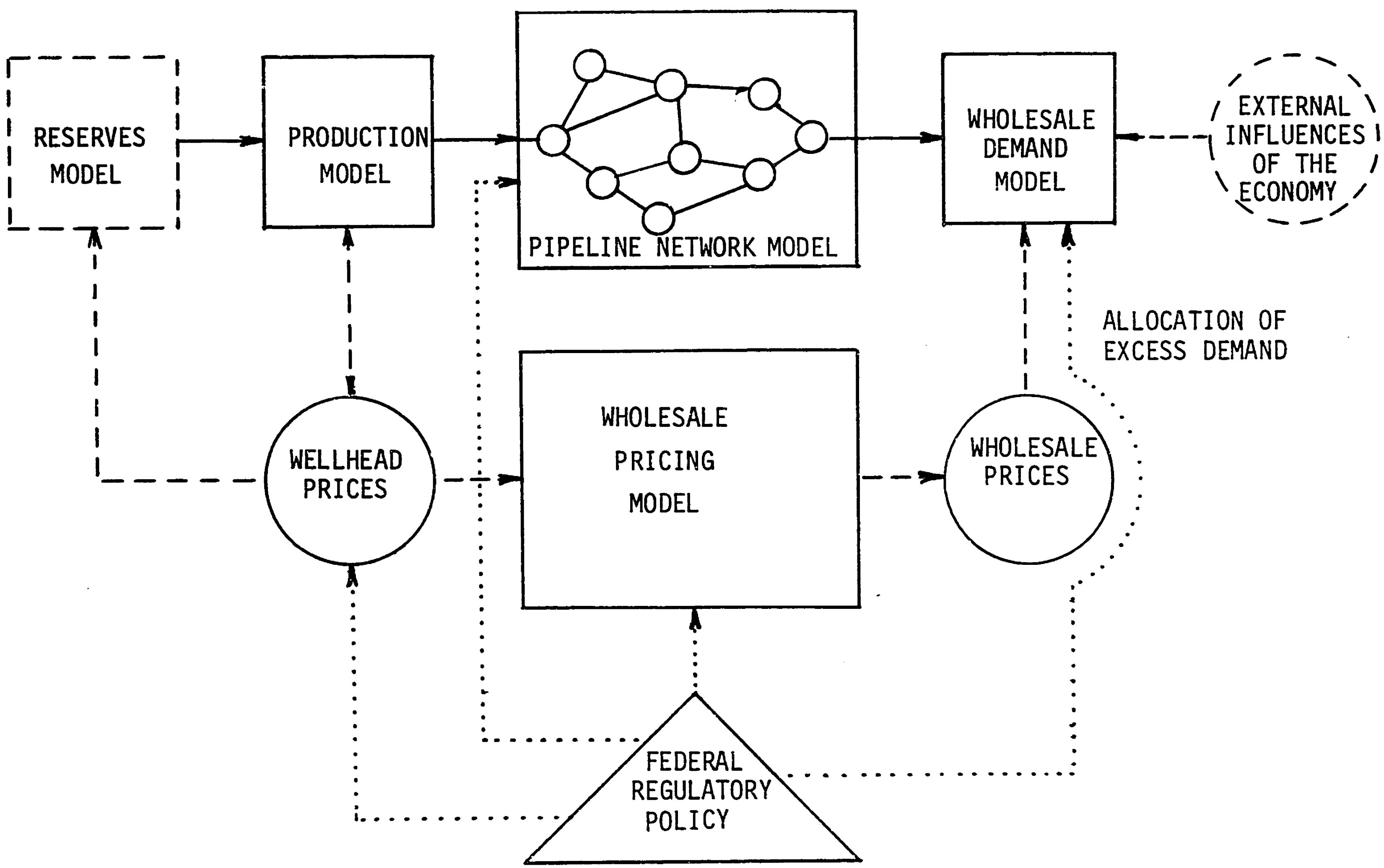

Figure 3.1 The Structure of GASNET

The dashed arrows represent influences and informational inputs. Wellhead prices influence level of reserves and production and are inputs to the wholesale pricing submodel. Wholesale prices and external influences of the economy (growth factors, other price levels) affect wholesale demand.

Dotted lines represent regulatory control. These extend to wellhead prices, wholesale prices, pipeline certification for new or expanded service, and to the allocation of excess demand when demands for gas cannot be fully satisfied.

The block labelled Reserves is dashed to indicate that it is an external given of this model. GASNET as currently operating does not attempt to predict levels of new discoveries and other additions to reserves as they respond to changes in the price of gas and other external variables. With a few minor modifications, however, the reserves sub-model of the MacAvoy-Pindyck Policy Model or other supply models could be easily adjoined. The details of such a reserves model will not be discussed here as they are outside the scope of this study. References and brief discussions of such models are given in Chapter 2, Section 2.3.

In the following sections each of the four basic submodels contained in GASNET will be discussed. We will also consider the changes or additions required to represent the various types of Federal regulation.

3.3 The Gas Production Submodel

The function of the production submodel within the whole model is to predict the level of natural gas produced during a particular year as a function of the wellhead price and the level of reserves available for future production. The relevant price for this model is the wellhead price on new contracts since most of the gas sold at the wellhead to interstate pipeline companies is sold on long term contracts at a relatively fixed price. Thus any marginal production resulting from changes in the marginal cost of production or in the amount that the pipelines are willing to pay will be reflected in the new contract price and only indirectly in the average price of gas.

The fact that producers sell gas on long term contracts is also a major reason for the inclusion of the reserves level in the production equation. Gas sold to the pipeline companies for production several years from now belongs to the pipeline company as "dedicated reserves" and is not available for sale to another pipeline company no matter how much he is willing to pay. The reserves level, therefore, sets a limit on the amount of new contract gas which can be produced in a given year.

Given detailed historical marginal cost curves for each producer in a given production region as a function of each producer's reserves, one could construct by superposition a family of regional new supply curves parameterized by the total reserves level, R (see Figure 3.2).

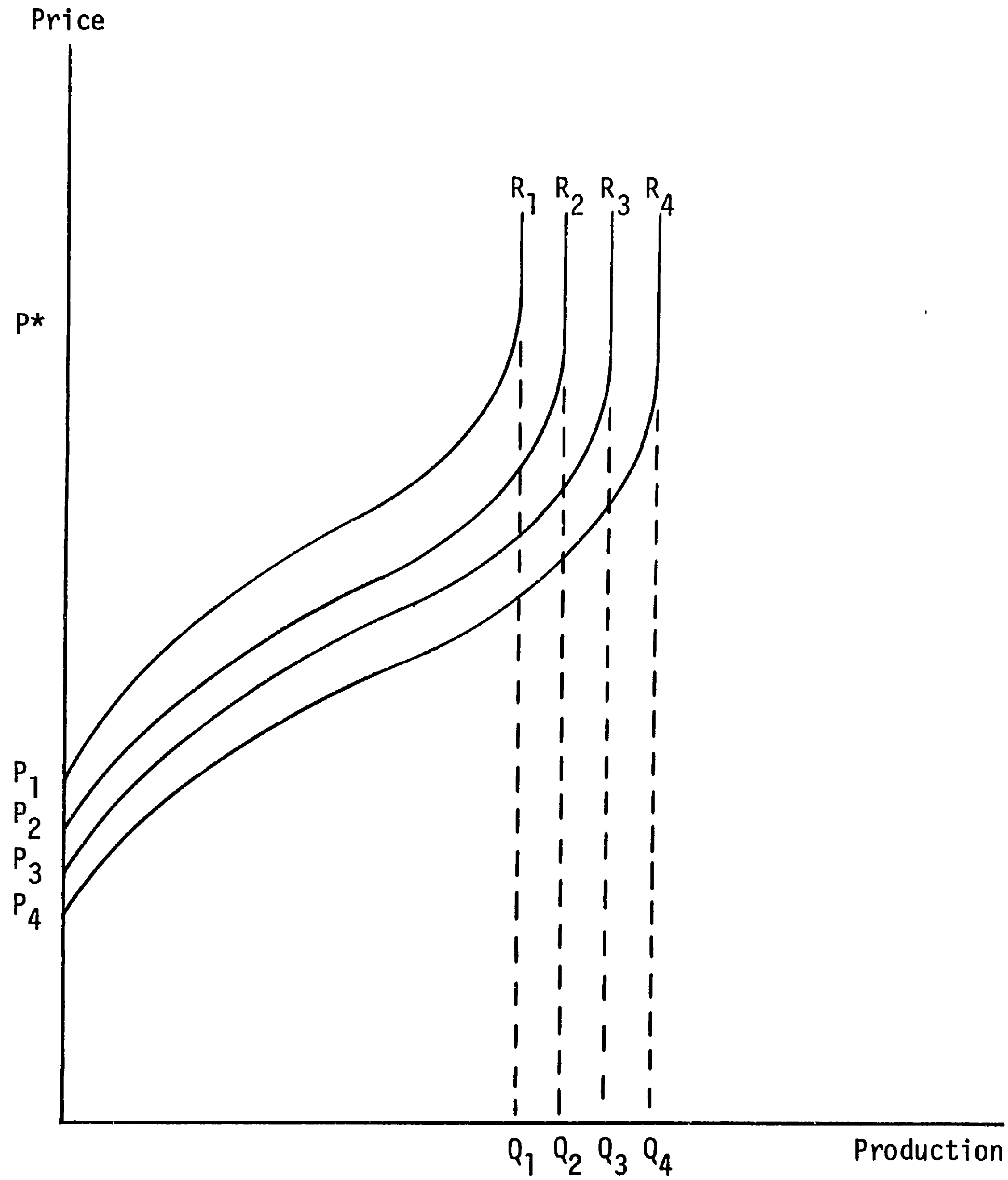

Figure 3.2 Short-run Supply (Production out of Reserves) Parameterized by Reserves Level (R)

For each curve, C_i, in such a set there may be a minimum price P_i below which there will be no new production. Above this price the supply curve is relatively elastic until the effects of the finite reserves level, R_i, is felt. At this point the curve begins to turn up more and more sharply to a maximum new production allowable, Q_i, as determined by the reserves level, R_i, and other characteristics of the individual reservoirs from which the gas is produced. Prices above P* produce no new gas since production is already at its maximum level.

In the past, state conservation boards in states such as Texas and Louisiana have kept allowed production at 60 - 70% of capacity for purposes of efficient long term development of oil and gas reservoirs. In recent years because of the high price of foreign oil and the shortage of natural gas, production allowables are generally near 100%. Thus in the 1970s production levels in the short run are expected to be relatively price insensitive.

In the long run, however, the production of gas depends strongly on the prices of gas and oil through investment in new well drilling leading to new discoveries and additions to reserves. Because it takes some time before additions to reserves are actually proved, a several year lag structure is called for in the model. A T year lag structure is built into the production model by using the reserves level as of T years ago as an explanatory variable. As discussed previously, GASNET does not include a long run supply (reserves) model, whose function would be to predict changes in reserves level as wellhead prices change. An extension of the model to include this process is discussed in Section 7.2.

In GASNET the production out of reserves submodel consists of a set of econometrically estimated equations, one for each production district in the model. These districts are listed in Table 3.1 and shown on the map in Figure 3.2.

TABLE 3.1 Production Districts in GASNET

Region	Districts
West	California; Colorado-Utah; Montana-North Dakota; New Mexico (San Juan Basin); Wyoming.
Hugoton-Anadarko	Kansas; Oklahoma; Texas Railroad Commission District # 12.
Appalachia	Michigan; Ohio; Pennsylvania; West Virginia-Kentucky.
East Texas-Louisiana North	Arkansas; Louisiana North; Mississippi; Texas RRC 5, 6, 7B, 9.
Texas Gulf Coast*	Texas RRC 1, 2, 3, 4.
Louisiana South**	Louisiana South Onshore, Louisiana Offshore.
Permian Basin**	New Mexico (Permian Basin), Texas RRC 7C, 8, 8A.
Imports	Canada (Alberta); Mexico (via Texas).

* Includes Offshore Texas Gas Fields.

** Estimated as unified regions.

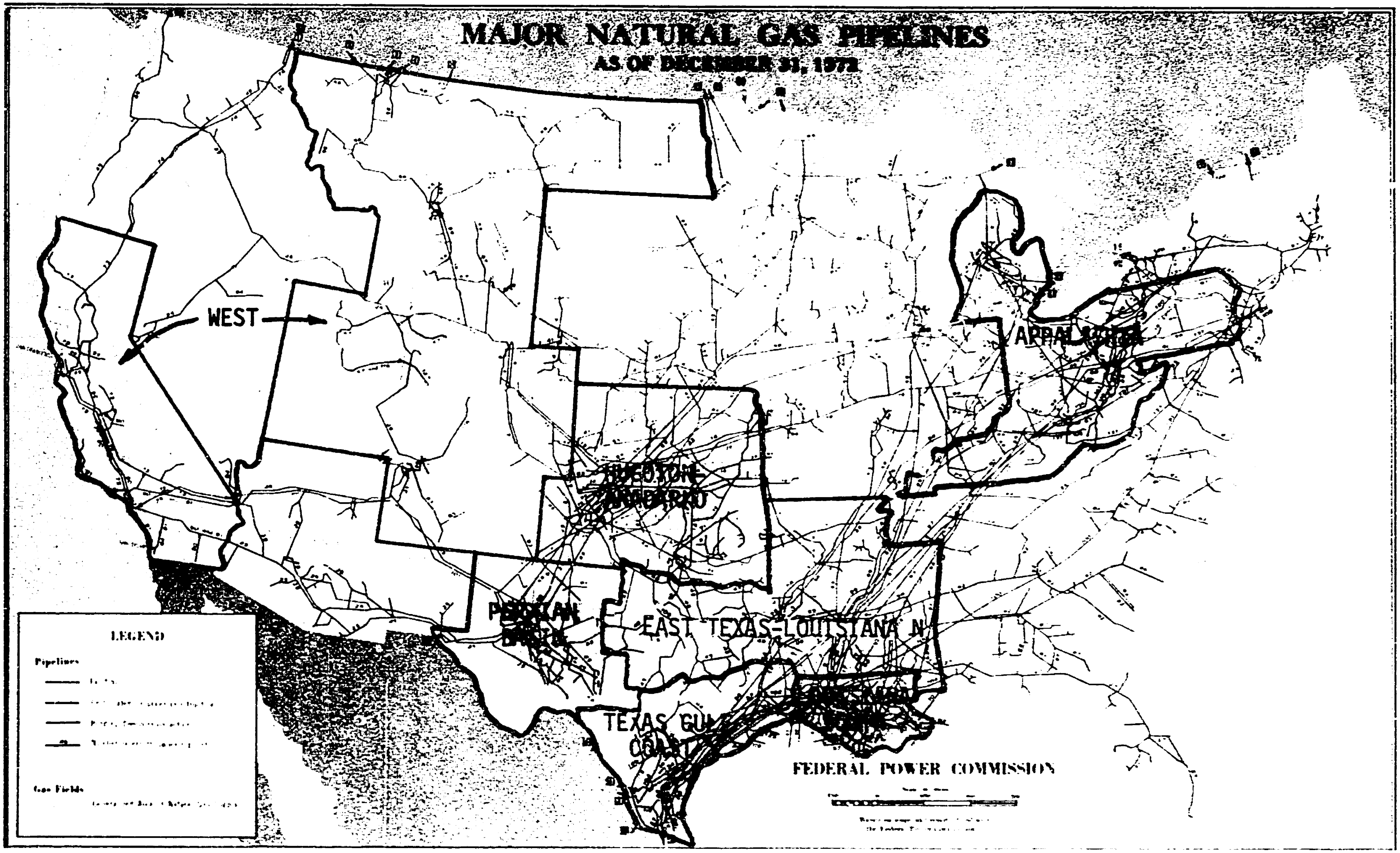

Figure 3.3 US Natural Gas Production Regions

The general form of the short-run supply (production) function to be estimated for each district is

$$Q_t^S = S(P_t^S; R_{t-T}) \qquad 3.3.1$$

where Q_t^S and P_t^S are the production and wellhead price at time t, respectively, and R_{t-T} is the reserves level lagged T years from the present. Because the number of new contracts signed each year is small, particularly in the smaller production districts, new contract data was difficult to use in estimating these equations. Therefore, the quantities and prices used include not only new contract sales, but sales on old contracts as well.

The econometric estimates of these production equations are discussed in detail in Section 4.2.

3.4 The Wholesale Demand Submodel

In GASNET the driving force for gas production is the wholesale demand for gas by customers of the pipeline companies. In the actual system, of course, these customers are most often not final consumers but rather local distributors of gas to residential, commercial, and industrial users. This final level of distribution for retail demand has been ignored in order to keep the model from becoming too complex. Because each state has a different public utilities commission to regulate such gas utilities, the price markups from wholesale to retail sales and relative prices of industrial and non-industrial gas would be different from state to state. As discussed in Section 7.6, this could result in distortions in the cross-sectional estimations of demand used in this model. Because an immense amount of data collection and modelling effort would have been required to model the system at this level, we decided to keep the demand model at the wholesale level and accept those possible distortions. Section 7.7 discusses an extension of the model to the retail level.

For the demand submodel the country was divided into five large demand areas: the Northeast, Southeast, North Central, South Central, and West. These areas were in turn divided into demand regions which consisted of the continental states. In three cases -- New England, Maryland-Deleware-D.C., and Montana-North Dakota -- states were aggregated into multi-state demand regions. Thus there are a total of forty-one demand regions within the five large demand areas (see Table 3.2 and Fig. 3.3).

This two level division was chosen in order that demand equations could be estimated separately for areas of the country with widely different weather patterns, industrial development, and end use patterns. At the same time the effects on demand of variations within each of the large, relatively homogeneous demand areas could be determined through the use of the cross-sectional time series estimation techniques described in more detail in Section 4.2.

TABLE 3.2 Natural Gas Demand Regions

Region	States (or Subregions)
Northeast	Greater Maryland (Delaware, D. C., Maryland); New England (Connecticut, Maine, Massachusetts, New Hampshire, Rhode Island, Vermont); New Jersey; New York; Ohio; Pennsylvania; Virginia, West Virginia.
Southeast	Alabama; Florida; Georgia; Kentucky; North Carolina; South Carolina; Tennessee.
North Central	Illinois; Indiana; Iowa; Michigan; Minnesota; Missouri; Nebraska; South Dakota; Wisconsin.
West	Arizona; California; Colorado; Idaho; Montana-North Dakota; Nevada; New Mexico; Oregon; Utah; Washington; Wyoming.

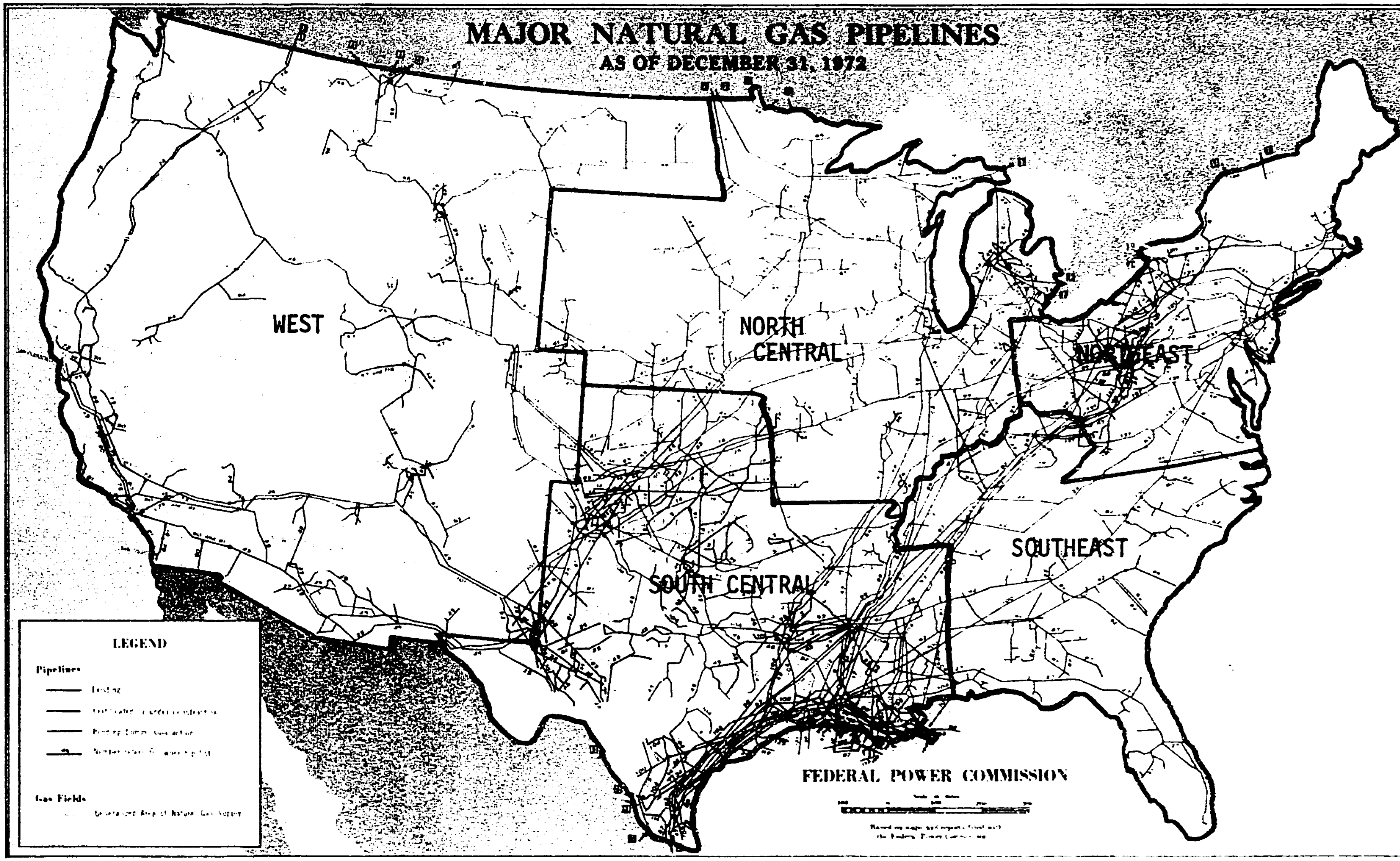

Figure 3.4 US Natural Gas Demand Regions

Even though we are modelling wholesale demand for gas, which is undifferentiated for the most part with respect to end use, we have divided demand into two components: industrial and residential-commercial. Thus though the model does not explicitly include the distribution of gas to retail consumers, it approximates it to some degree by estimating separate equations for industrial and non-industrial demand. We believe this approach represents the actual system better than would a single demand equation for each district.

Following a formulation by Balestra (1967), the demand equations for both industrial and residential commercial gas take the general functional form

$$\delta Q_t \quad = \quad D(P_t^d;P_t^{alt},g_{t-T}) \qquad 3.4.1$$

where P_t^d is the wholesale demand price of gas in the year t, P_t^{alt} the price of an alternative fuel (oil or coal as applicable), and g_{t-T} a lagged economic growth variable. For the industrial equation g_{t-T} can be either capital investment in manufacturing with T = 1 or value added in manufacturing in the current year (T = 0). For non-industrial demand g_{t-T} is either the state population or total state personal income in the current year.

δQ_t is an expression which represents the "new demand" for gas. As discussed by Balestra δQ_t consists of two parts: (1) demand from new customers and increases in the demand of old customers who do not have to replace gas burning equipment in year t, and (2) demand of old customers who choose to replace obsolete gas burning appliances with new gas appliances rather than switch to another fuel. If a fraction, r, of last year's demand came from customers who make decisions this year about which fuel they are going to use, then the amount of "new gas" consumed this year is actually greater than $\Delta Q_t = Q_t - Q_{t-1}$. It is rather

$$\delta Q_t = Q_t - (1-r)Q_{t-1} = \Delta Q_t + rQ_{t-1} . \qquad 3.4.1$$

That is, "new gas" is the difference between total gas consumed this year and that fraction of last year's consumption which comes from owners who do not have to replace their equipment this year. The fraction r is related to the expected life of gas burning equipment and has been estimated in the MacAvoy-Pindyck study to be about 7%.

To be consistent, the price, P_t^d, used in these equations should be the average new contract price. Because of the extreme expense in time and effort which would have been required to extract these prices from FPC Form 2 reports of the pipeline companies, we decided instead to use the somewhat more easily determined average prices on all contracts.

Since gas flows in the network model include both new and contract gas, the equations we estimated for industrial and non-industrial demand were combined into a total demand equation for each demand region. These equations create a dynamic link from one time stage of the model to the next through the Balestra formulation of new gas demand. The general equation form for each region's demand is then

$$Q_t^d = (1-r)Q_{t-1}^d + D(P_t^d;P_t^{alt},g_{t-T}^{ind},g_t^{res}) \qquad 3.4.3$$

The exact form of the function, D, and the estimations using generalized least squares on cross-sectional time series data are discussed in detail in Section 4.2.2.

3.5 The Pipeline Network Submodel

3.5.1 The Gas Pipeline System as a Network

The composite of the major U. S. natural gas pipelines is itself a highly complex network consisting of many thousands of miles of pipe of various diameters, compressor stations to boost the pressure of gas in the pipe, and thousands of branches, loopings, and junctions within and between the different companies' lines (see Figure 3.4). In the supply regions these various sub-networks connect up to the actual sources of supply, gas and oil wells, through another network of gathering lines sometimes owned by the producer and sometimes by other pipeline companies. At the other end, the demand regions are usually serviced by intrastate distribution companies with ultracomplex networks. These companies purchase their gas from the interstate pipelines for the most part. To make matters even more complicated, however, some of the interstate gas pipelines also

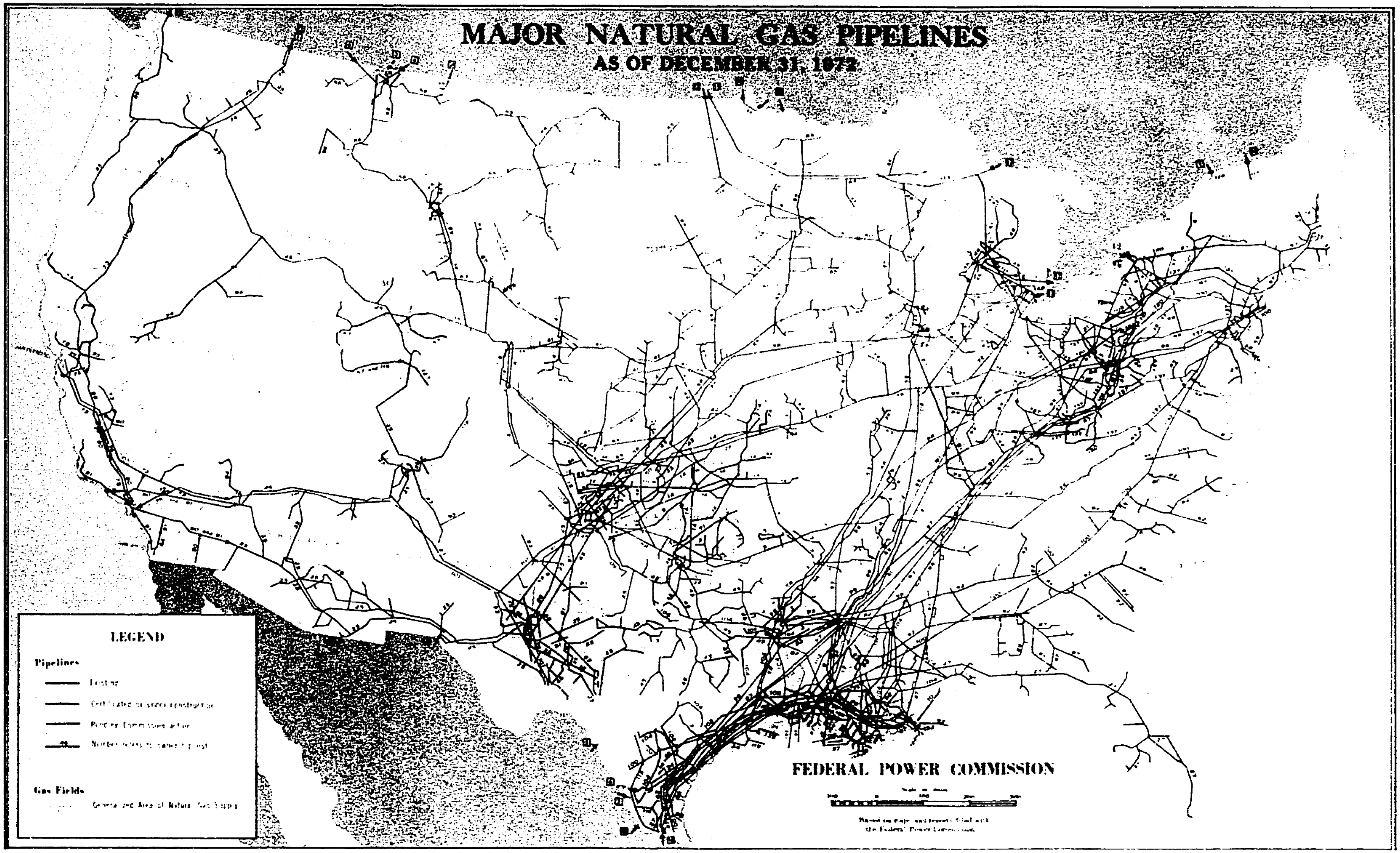

Figure 3.5 The US Natural Gas Pipeline Network

engage in distribution at the retail level, some store gas underground near demand areas for use during periods of peak load demand, and some, in addition, serve as both gas and electricity utilities, burning their own gas to produce electricity.

While it is theoretically possible to model this entire network, it is clearly desirable to try to simplify it enough to be computationally tractable while yet maintaining its basic integrity. Because the purpose of this study has been to construct a model to be used to analyze large scale movements of natural gas from supply to demand regions, the time scale of its operation can be fairly long, yearly movements rather than daily or hourly ones. Thus the dynamics of startup and shutdown and optimal dynamic dispatching of gas under swiftly changing demands, which would be extremely important concerns for a distributor of natural gas as well as for the gas gatherer in the field, are complications which we do not deal with in this model. The model is greatly simplified by dealing exclusively with the major gas transmission companies and their connections. Thus the network can be reduced to (1) a set of supply areas from which the gas flows into the various pipelines, (2) the pipelines themselves with their interconnections within and among each other, and (3) the various regions of the country into which gas flows as a result of wholesale demand.

As is always the case in a model of this type, other simplifications must be made due to the quality of the data. In this particular case most of the relevant data can be found for individual states, but not at any lower level of disaggregation. The one exception is in the area of supply region data which is tabulated for all FPC pricing districts within the major producing states. In this model sales and purchases by each company take place at only one location for each state crossed by its pipeline. This assumption results in a tremendous simplification not only in the network structure but also in the data gathering, handling, and processing as well, while yet maintaining the basic integrity of the individual pipelines and their interrelationships. While distortions will most certainly occur in states like California and Texas which have more than one population center served by different supply areas, the model is flexible enough to allow others who would like to develop the data to a finer level of aggregation to easily alter it to accommodate the additional information. See Section 7.6 for an extension in that direction.

3.5.2 The Network Model

The network model is exemplified in Figure 3.6. Here company A purchases gas from producers in Texas Railroad Commission Districts 2, 3, and 4 (Texas Gulf Coast), and Louisiana North, picks up some more in Mississippi as purchases from pipeline company B, and sells at wholesale in Texas, Louisiana, Mississippi, and Tennessee. A similar story could be told for pipeline company B.

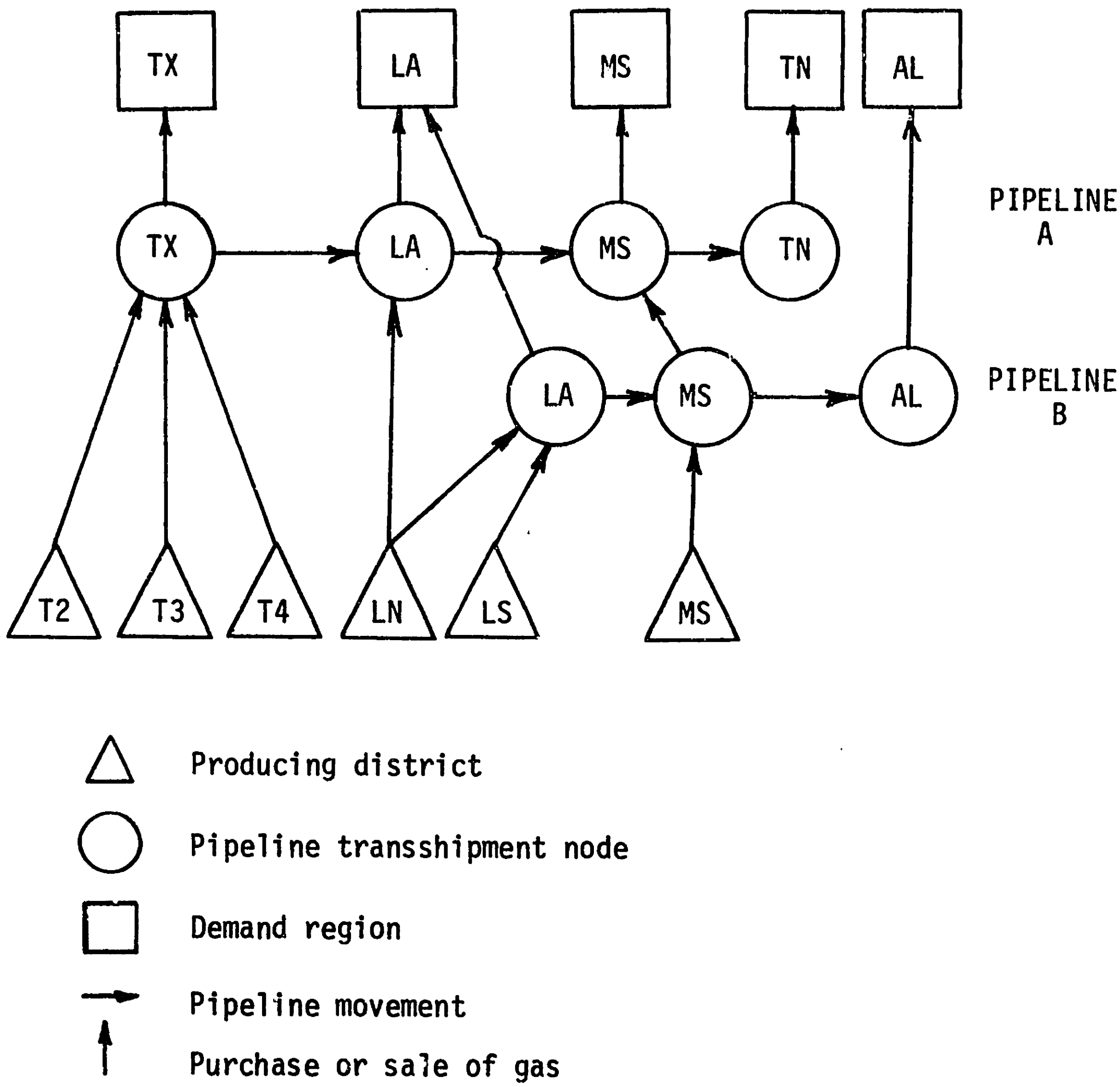

Figure 3.6 Hypothetical Two-company Subnetwork

Using the 1968 FPC map of Principal Natural Gas Pipelines in the U. S., we constructed a model of the interstate pipeline network which includes about one hundred pipeline companies. The companies included are listed in Appendix A. In this model there are about 35 triangular nodes representing supply regions in the U. S. and Canada, about 50 square demand nodes representing the continental United States, and about 350 circular transshipment nodes representing the interconnection points along the pipelines themselves. The number of interconnections between these nodes is approximately 1200. Some of the pipelines existing in 1966 have consolidated into larger companies since then. The model includes all such companies. Thus for any particular year the model is actually somewhat smaller than these numbers indicate.

3.5.3 The Mathematics of the Network

Basically there are three types of mathematical relations corresponding to the pipeline network model: (1) those which define the network connections through conservation of mass equations, (2) those which describe the capacity constraints of each separate pipeline link, and (3) those which relate the gas flowing into and out of the pipe to supplies and demands throughout the country.

The conservation equations can be written as

$$\sum_j \varepsilon_{ijk} X^t_{ijk} + \sum_m X^c_{mik} + \sum_n X^s_{ni} = \sum_l X^t_{ikl} + \sum_q X^c_{iqk} + X^d_{ik} \qquad 3.5.1$$

for all combinations of pipelines, i, and states, k.

In this equation the following symbols are used:

ε_{ijk} the fraction of gas leaving state j on pipeline i which arrives in state k

X^t_{kjk} the amount of gas leaving state j for state k on pipeline i

X^c_{mik} sales by pipeline m to pipeline i in state k

X^s_{ni} purchases by pipeline i from supply region n (within state k)

X^d_{ik} wholesale sales by pipeline i in state k.

In this equation, which is of course the conservation of mass equation for pipeline i in state k (assuming constant pressure and temperature gas), the summations are taken over the appropriate subsets of the index sets which correspond to connections which actually exist.

The second set of relations are simply

$$X^t_{ijk} \leq b_{ijk} \tag{3.5.2}$$

where b_{ijk} is the capacity of the pipeline section (s) between states j and k on pipeline i.

The third set of relations are the requirements of supply and demand, i.e.

$$\sum_i X^d_{ik} \geq D_k(P^d_k;\alpha_k) \qquad 3.5.3$$

and

$$\sum_i X^s_{ni} \leq S_n(P^s_n;\beta_n) \qquad 3.5.4$$

where α_k and β_n are sets of parameters corresponding to variables other than the price of gas. The relations merely require that the quantity of gas delivered to state k be at least equal to the demand at the wholesale price, P^d_k, and that the amount sold in supply region n be not more than the available supply at the wellhead price, P^s_n. Thus 3.5.3 and 3.5.4 are the supply and demand balance relations for wholesale and wellhead sales. The inequalities are generally not necessary but are useful for computational simplification when LP codes are used.

In some particular problems it may be necessary to modify these relations to account for the possibility that demand is greater than available supply. This will be discussed in Section 3.7.2.

The efficiency parameters ε_{ijk} and the capacities b_{ijk} of the network model have been estimated using historical data from the pipeline companies and the Federal Power Commission. The details of the procedures we used are discussed in Section 4.2.

3.6 Pipeline Pricing Submodels

3.6.1 Introduction

The models described in Sections 3.3 - 3.4 define the relationships of production to the wellhead price and demand to the wholesale price of gas. The pipeline model in Section 3.5 contains the conservation of mass equations for each pipeline node, the capacities of the pipeline areas, and the supply and demand requirements of the pipeline companies. The only remaining conditions which are needed in order to have a completely defined system are those which specify how the pipeline companies decide upon the prices they will charge in various demand regions as a function of the pipelines' costs and consumer demands. In an unregulated system these price specifications would be completely under the control of the pipeline companies who would choose their prices to maximize some company objective (profits, for example). If the system is competitive, microeconomic theory says that marginal cost pricing should prevail, since those who charge higher prices will lose their customers to those who charge less. If the pipeline company has few or no competitors, it may choose to set prices high enough to collect monopoly profits.

If, on the other hand, the pipelines are regulated by a state or Federal agency they will not have complete control over their pricing policies. In perhaps the most frequently used regulatory mechanism for public utilities, the prices charged by the regulated company are chosen so that its expected net profit after taxes does not exceed a specified rate of return on its undepreciated capital stock. This rate of return should be chosen so that pipelines are able to cover their capital costs without making excess profits.

In the following sections mathematical models of each of these three types of pricing mechanisms are presented.

3.6.2 The Unregulated Competitive Pricing Model

The economic conditions for equilibrium in an unregulated competitive industry are that marginal cost pricing shall prevail, that no excess profits shall be earned, and that no sales will be made at a loss. If pipeline i buys a certain quantity, X_{ijk}, of gas from producing region j at a price, P_{ij}, equal to the marginal cost of production of that gas, and transports it to region k at a costant unit cost t_{ijk}, then its selling price, P_{ik}, will be determined by the following set of relations:

$$P_{ik} - P_{ij} \leq t_{ijk} \qquad 3.6.1a$$

$$X_{ijk}(P_{ik} - P_{ij} - t_{ijk}) = 0 \qquad 3.6.1b$$

The first set assures that no excess profits will be acquired by the pipeline. Its price must be less than or equal to the marginal cost of providing the gas. The second assures that either marginal cost pricing occurs or else no sale will be made; that is, the company will not sell at a price less than its marginal cost.

In the case of limited capacities, such as exist in natural gas pipelines, these equations must be modified to include the possibility of economic rents, r_{ijk}. Suppose that a pipeline company were delivering gas to region k at full capacity b_{ijk}, but that the demand for gas in region k was greater than b_{ijk}. If there were no other source of gas then no amount of price increase could bring forth more gas. If another source of more expensive gas existed, then region k could get more gas at the marginal cost of this second source. Since all gas would then sell at this higher price (which is the marginal cost of another unit of gas) the economic rent

$$r_{ijk} = P_{ik} - P_{ij} - t_{ijk}$$

would be earned by the more efficient pipeline on each unit of its sales. Since these rents can only occur when the pipeline's capacity b_{ijk} is fully utilized, the complete set of equations describing this situation is given by:

$$X_{ijk} \leq b_{ijk} \qquad 3.6.2a$$

$$r_{ijk}(X_{ijk} - b_{ijk}) = 0 \qquad 3.6.2b$$

$$P_{ik} - P_{ij} - r_{ijk} \leq t_{ijk} \qquad 3.6.2c$$

$$X_{ijk}(P_{ik} - P_{ij} - r_{ijk} - t_{ijk}) = 0 \qquad 3.6.2d$$

Note that if $b_{ijk} > 0$ and $r_{ijk} > 0$, then $X_{ijk} = b_{ijk} > 0$, which implies that $P_{ik} - P_{ij} - r_{ijk} = t_{ijk}$, i.e. marginal cost pricing occurs when pipelines operate at full capacity as well as at only partial capacity.

In the case of constant unit transportation costs, marginal cost pricing will only reimburse the pipeline company its direct operating expenses. The only means for it to recover its capital costs, i.e. to pay its stock and bond holders, is for it to earn economic rents on the constrained finite capacity of its pipe. The most efficient pipelines may make more in rents than they must pay to their debt and equity holders (and to the U. S. government in the form of taxes) thus earning an "excess profit." Other less efficient lines, by not earning via rents a sufficient revenue to pay their capital costs, will sooner or later succumb to their competition.

Incorporating these relationships with the production, demand, and pipeline network submodels, the complete model of competitive equilibrium in the natural gas industry can be written as follows in terms of the variables and symbols described in Table 3.3.

$$\sum_i X^d_{ij} \geq D_j(P^d_j) \quad , \quad j\varepsilon D \qquad 3.6.3a$$

$$\sum_i X^s_{ki} \leq S_k(P^s_k) \quad , \quad k\varepsilon S \qquad 3.6.3b$$

$$\sum_j \varepsilon_{ijk} X^t_{ijk} + \sum_m X^c_{mik} + \sum X^s_{ni} =$$

$$\sum_l X^t_{ikl} + \sum_p X^c_{ipk} + X^d_{ik} \qquad i\varepsilon C;\ k\varepsilon T;\ n\varepsilon S_k \qquad 3.6.3c$$

$$X^t_{ijk} \leq b_{ijk} \qquad i\varepsilon C;\ j,\ k\varepsilon T \qquad 3.6.3d$$

$$r_{ijk}(X^t_{ijk} - b_{ijk}) = 0 \qquad i\varepsilon C;\ j,\ k\varepsilon T \qquad 3.6.3e$$

$$P^d_k - P^t_{ik} \leq 0 \qquad i\varepsilon C;\ k\varepsilon D \qquad 3.6.3f$$

$$X^d_{ik}(P^d_k - P^t_{ik}) = 0 \qquad i\varepsilon C;\ k\varepsilon D \qquad 3.6.3g$$

$$\varepsilon_{ijk}P^t_{ik} - P^t_{ij} - r_{ijk} \leq \tfrac{1}{2}(1 + \varepsilon_{ijk})t_{ijk} \qquad i\varepsilon C;\ j,\ k\varepsilon T \qquad 3.6.3h$$

$$X^t_{ijk}(\varepsilon_{ijk}P^t_{ik} - P^t_{ij} - r_{ijk} - \tfrac{1}{2}(1 + \varepsilon_{ijk})t_{ijk}) = 0 \qquad i\varepsilon C;\ j,k\varepsilon T \qquad 3.6.3i$$

$$P^t_{ik} - P^s_{ni} \leq 0 \qquad i\varepsilon C;\ k\varepsilon T;\ n\varepsilon S_k \qquad 3.6.3j$$

$$X^s_{ni}(P^t_{ik} - P^s_{ni}) = 0 \qquad i\varepsilon C;\ k\varepsilon T;\ n\varepsilon S_k \qquad 3.6.3k$$

$$P^t_{jk} - P^t_{ik} \leq m_{ijk} \qquad i,\ j\varepsilon C;\ k\varepsilon T \qquad 3.6.3l$$

$$X^c_{ijk}(P^t_{jk} - P^t_{ik} - m_{ijk}) = 0 \qquad i,\ j\varepsilon C;\ k\varepsilon T \qquad 3.6.3m$$

TABLE 3.3 Variables, Parameters, Functions and Index Sets in the Competitive Model.

Variables	
X^d_j	Sales to demand region j by pipeline i.
P^d_j	Wholesale price of gas in demand region j.
X^s_{ki}	Wellhead purchases by pipeline i in supply district k.
P^s_k	Wellhead price of gas in supply region k.
X^t_{ijk}	Gas flow from node j toward node k in pipeline i.
X^c_{mik}	Intercompany sale of gas by pipeline m to pipeline i in state k.
r_{ijk}	Economic rent on section (j, k) of pipeline i.
P^t_{ik}	Imputed price of gas at transshipment node k of pipeline i.
Parameters	
b_{ijk}	Maximum throughout capacity of section (j, k) of pipeline i.
t_{ijk}	Unit variable cost of transmission on section (j, k) of pipeline i.
m_{ijk}	Intercompany markup on gas sold by company i to company j in state k.
ε_{ijk}	Fraction of gas not used as pipeline fuel or lost in transmission on section (j, k) of pipeline i.
Functions	
$D_k(\cdot)$	Demand for gas in region k.
$S_j(\cdot)$	Short-run supply of gas in district j.
Index Sets	
D	Demand regions.
S	Supply districts.
S_k	Supply districts in demand region k.
C	Pipeline companies.
T	Transshipment nodes.

In 3.6.3 all variables are constrained to be non-negative, except for the imputed prices, P^t_{ik}.

The complexity of this set of equations and inequalities is due to the fact that several different types of sales (wellhead, wholesale, intercompany, intracompany) are described in different equations but have the same general form.

In 3.6.3h the right hand side term is $\frac{1}{2}(1 + \varepsilon_{ijk})t_{ijk}$ rather than simply t_{ijk}. The more complex expression is needed to account for the fact that not all gas leaving node j arrives at node k. The average quantity of gas flowing over the section (j, k) on pipeline i is given by $\frac{1}{2}(1 + \varepsilon_{ijk})X^t_{ijk}$. Thus the total cost of moving this gas is $\frac{1}{2}(1+\varepsilon_{ihk})t_{ijk}X^t_{ijk}$. The imputed cost of the gas at node j is $P^t_{ij}X^t_{ijk}$, and if $X_{ijk} = b_{ijk}$ there is an additional economic rent of $r_{ijk}X^t_{ijk}$. Thus the imputed price times quantity of gas received at node k is given by

$$P^t_{ik}(\varepsilon_{ijk}X^t_{ijk}) = (P^t_{ik} + r_{ijk})X^t_{ijk} + \tfrac{1}{2}(1 + \varepsilon_{ijk})t_{ijk}X^t_{ijk} \qquad 3.6.4$$

This equation is equivalent to 3.6.3i for all values of X^t_{ijk}. When $X^t_{ijk} > 0$ this relation reduces to the equality aspect of 3.6.3h.

Note that the throughput factors ε_{ijk} also appear in the mass balance equations 3.6.3c. Thus even though there are losses in the lines $(1 - \varepsilon_{ijk})$, there is conservation of mass at transshipment nodes T.

As Samuelson (1952) first noticed, for a simpler version of this problem, the relationships defining this model can be recast as an equavalent mathematical programming problem greatly simplified in form. In addition important inferences can be made about the existence and uniqueness of solutions to the original problem because of the special form of the simplified, equivalent problem.

Takayama and Judge (1964) took Samuelson's discovery one step further by showing that the equivalent problem could be expressed as two dual problems. In the Samuelson formulation, quantities were primal variables and Lagrange dual variables could be interpreted as prices. Takayama and Judge showed a yet simpler problem where prices were primal variables and quantities dual.

We can extend these "spatial equilibrium models" for simple transportation problems to the more complicated transshipment model defined by 3.6.3 by noticing that this set of constraints is equivalent to the Kuhn-Tucker conditions for the following convex program in the variables P^d_j, P^s_n, P^t_{ik}, and r_{ijk}, with Lagrange multipliers X^t_{ijk}, X^c_{mik}, X^d_{ij}, and X^s_{ni} :

$$\underset{P,R \geq 0}{\text{minimize}} \sum_{k \varepsilon S} \int_0^{P^s_k} S_k(p')dp' - \sum_{j \varepsilon D} \int_0^{P^d_j} D_j(p'')dp'' + \sum_T b_{ijk} r_{ijk} \qquad 3.6.5a$$

subject to:

(dual)	(constraint)		
X^d_{ik}	$P^d_k - P^t_{ik} \leq 0$	$i\varepsilon C;\ k\varepsilon D$	3.6.5b
X^t_{ijk}	$\varepsilon_{ijk}P^t_{ik} - P^t_{ij} - r_{ijk} \leq t^*_{ijk}$	$i\varepsilon C;\ j,\ k\varepsilon T$	3.6.5c
X^c_{ijk}	$P^t_{jk} - P^t_{ik} \leq m_{ijk}$	$i\varepsilon C;\ j,\ k\varepsilon T$	3.6.5d
X^s_{ji}	$P^t_{ik} - P^s_j \leq 0$	$i\varepsilon C;\ j\varepsilon S_k$	3.6.5e

where

$$t^*_{ijk} = \tfrac{1}{2}(1 + \varepsilon_{ijk})t_{ijk} .$$

Under the usual assumptions of an increasing differentiable and integrable supply function and a decreasing differentiable and integrable demand function, the objective function for this problem has a non-positive second derivative for each price variable and is linear in r_{ijk}; this is therefore a convex program with linear constraints. In addition all prices and rents are non-negative and can be limited by some very large number N without altering the solution to the problem. This is so since the objective function is an increasing function of all these variables except P^t_{ik} which doesn't enter the objective function, but is bounded by some P^s_n. Thus an equivalent problem with compact feasible region could be generated. For such programs the Kuhn-Tucker conditions are known to be both necessary and sufficient. Therefore since there exists a feasible solution to this convex program (namely $P = r = 0$), there is an optimal solution. This solution is also a Kuhn-Tucker point for 3.6.5, i.e. a solution to the competitive equilibrium model 3.6.5, since 3.6.5 is a convex program with linear constraints.

If the supply and demand functions $S_k(\cdot)$ and $D_i(\cdot)$ are <u>linear</u> functions of the wellhead and wholesale prices respectively, then the mathematical program simplifies further to a <u>quadratic program</u> with only diagonal elements in the matrix Q of quadratic terms. The objective function in this case where

$$D_i(P_i^d) \quad = \quad a_i - b_i P_i^d \qquad 3.6.6$$

and

$$S_j(P_j^s) \quad = \quad g_j + h_j P_j^s \qquad 3.6.7$$

with a_i, b_i, and $h_j \geq 0$ is given by

$$Z \quad = \quad \sum_S (g_j P_j^s + \tfrac{1}{2} h_j P_j^{s2}) - \sum_D (a_i P_i^d - \tfrac{1}{2} b_i P_i^{d2}) + \sum_T b_{ijk} r_{ijk} \qquad 3.6.8$$

If at least one of b_i or h_j is strictly greater than 0, then the objective function is quadratic, with Q diagonal and positive semi-definite. Thus if the feasible set is non-empty and compact then there is a global minimum to the problem, which is the equilibrium set of prices and flows for the economic conditions set forth in 3.6.3. If $b_i = h_j = 0$ then, of course, the problem reduces to the dual form of a transshipment problem with capacity constraints.

If the supply and/or demand functions are non-linear but can be approximated by piecewise linear functions, then the problem can again be cast as a diagonal quadratic program. Using a delta-type approximation one can express a given price, P, as

$$P = \sum_{k=1}^{n} (P_k - P_{k-1})\lambda_k = \sum_{k=1}^{n} \Delta P_k \lambda_k \qquad 3.6.9$$

where $\{P_i\}_{i=0,\ldots,n}$ is a set of points partitioning the maximum domain $[P_o, P_n]$ of P and λ_k are continuous variables in the domain [0,1] (see Figure 3.7). The increasing function, $S(\cdot)$ can be approximated by

$$S(P) \simeq \sum_{k=1}^{n} (S(P_k) - S(P_{k-1}))\lambda_k \qquad 3.6.10a$$

$$\simeq \sum_{k=1}^{n} (S_k - S_{k-1})\lambda_k \cdot \qquad 3.6.10b$$

The objective function 3.6.5a contains the integral of the increasing function S(P). This can be approximated as

$$\int_0^P S(p')dp' \simeq \sum_{k=1}^{n} [S_{k-1} + \tfrac{1}{2} S_k \lambda_k]\lambda_k (P_k - P_{k-1}) \qquad 3.6.11$$

The reason for this is easy to see: since the integral is to be minimized, the best way to do so is to include the smallest quadrilaterals in Figure 3.5 first. Thus if at equilibrium $P_{k-1} \leq P < P_k$, then $\lambda_m = 1$ for $m \leq k-1$ and $\lambda_m = 0$ for $m > k$. Each term in the approximating expression for the area under the curve S(P) represents the area of the corresponding shaded quadrilaterals

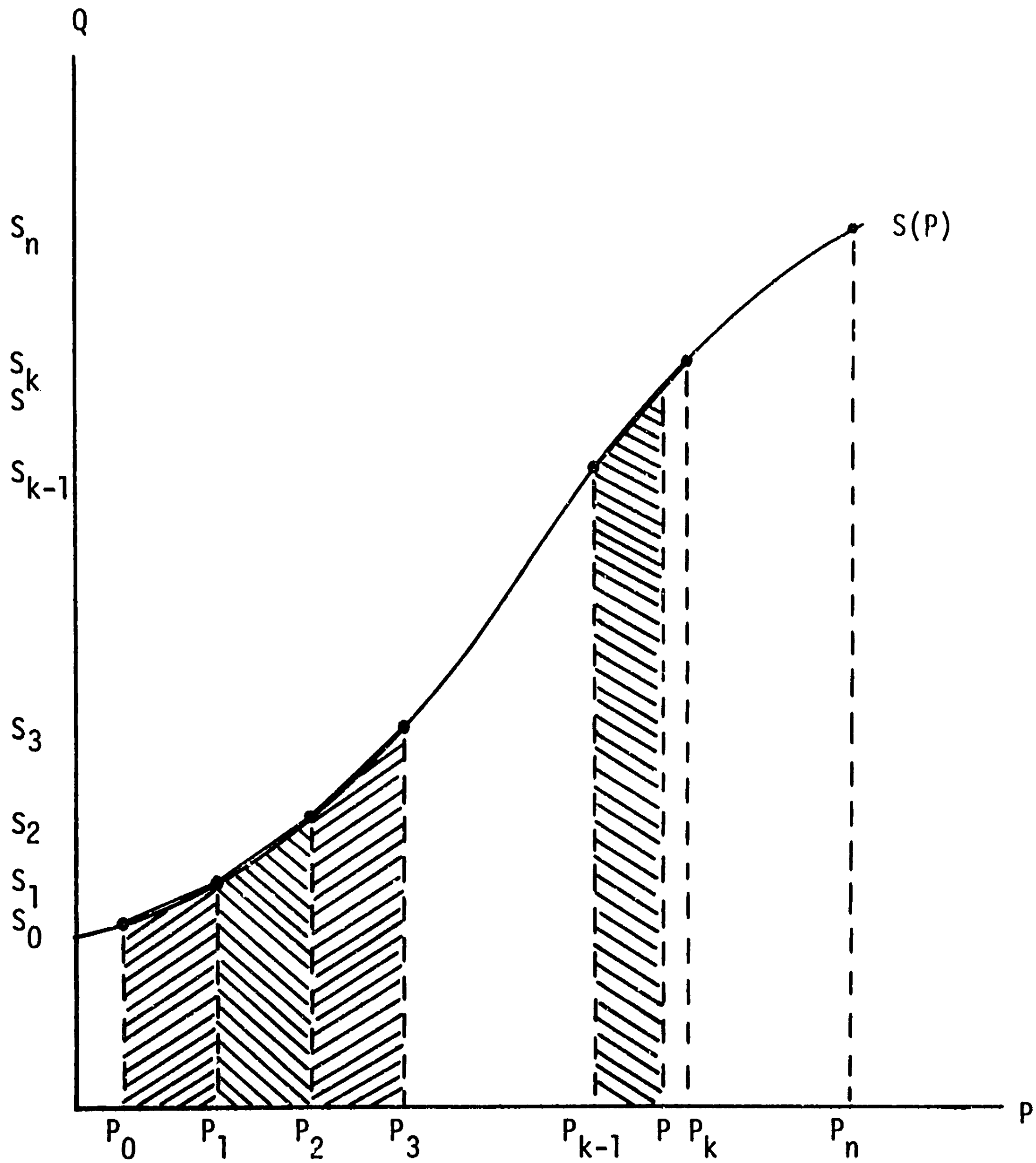

Figure 3.7 Piecewise Linear Approximation of Supply Function S(P)

for $m \leq k-1$, that shaded portion of the k^{th} quadrilaterial with base $[P_{k-1},(1-\lambda_k)P_{k-1} + \lambda_k P_k]$ for $m = k$, and 0 for $m > k$.

A similar analysis can be made for the decreasing demand function $D(\cdot)$. The result is that 3.6.3 and 3.6.5 can always be cast as diagonal quadratic programs.

It is interesting when using such approximations to look at the new Kuhn-Tucker conditions derived from the approximating problem. In this case there are n variables (λ_i, $i=1,\ldots,n$) corresponding to the single price P. Thus for each approximated price in the objective function, there will be n conditions where there was only one before.

We need only look at the terms involving λ_k, for this analysis, i.e.

$$\min_{\lambda \geq 0} \quad \sum_{k=1}^{n} [S_{k-1} + \tfrac{1}{2}(S_k - S_{k-1})\lambda_k]\lambda_k \Delta P_k \qquad 3.6.12a$$

$$X_i^s : \quad P_i^t - \sum_{k=1}^{n} \Delta P_k \lambda_k \leq C_i \text{ (constant)} \qquad 3.6.12c$$

$$n_k : \quad \lambda_k \leq 1 \qquad 3.6.12c$$

where X_i^s and n_k are Lagrange multipliers corresponding to 3.6.12b and 3.6.12c. The Lagrangian expression corresponding to the variable λ_k is given by

$$(S_{k-1} + (S_k - S_{k-1})\lambda_k)\Delta P_k - \sum_i X_i^s \Delta P_k + n_k \geq 0 \ . \qquad 3.6.13$$

Dividing 3.6.13 by $\Delta P_k > 0$ and redefining $n_k \rightarrow \frac{n_k}{\Delta P_k}$, it becomes

$$\sum_i X_i^s \leq S_{k-1}(1-\lambda_k) + S_k\lambda_k + n_k \qquad k = 1,n \qquad 3.6.14$$

If at equilibrium $P_{m-1} \leq P < P_m$, then $\lambda_1,\ldots,\lambda_{m-1} = 1$,

and $\lambda_{m+1},\ldots,\lambda_n = 0$. Also when $\lambda_k = 1$, n_k can be different

from 0. Thus

$$\sum_i X_i^s \leq S_k + n_k \qquad 1 \leq k \leq m-1 \qquad 3.6.15a$$

$$\sum_i X_i^s \leq S_{k-1} \qquad m+1 \leq k \leq n \qquad 3.6.15b$$

and

$$\sum_i X_i^s \leq S_{k-1}(1-\lambda_k) + S_k\lambda_k \text{ for } k=m \text{ and } 0 \leq \lambda_k < 1 \qquad 3.6.15c$$

Thus since n_k is only restricted to be non-negative, 3.6.15a will always be satisfied and since $S_{k-1} > S_m$ so will 3.6.15b. The m^{th} constraint corresponds to the single constraint in the original problem, where the right hand side is a linear approximation to the supply function at point P.

This result shows that any spatial equilibrium model can be expressed as a diagonal quadratic program by piecewise linearizing the supply and demand functions. Such a model is superior to a simple linearization of the objective function since the resulting program would correspond to a model with step-function approximations to supplies and demands rather than piecewise linear approximations.

3.6.3 The Unregulated Monopolistic Pricing Model

In the entire United States there are approximately 80 interstate pipeline companies. Some demand regions are serviced by several of these pipelines, others by only one or two. If these pipelines were unregulated, attempts at profit maximization might well result in monopoly pricing in wholesale markets.

A model for this type of behavior is relatively simple to construct. Pricing, purchases, flows, and sales will be determined completely by a maximum profits objective subject to the demand and supply relations, transportation costs and capacities, and conservation equations. Specifically, the model is

$$\max_{P,X\geq 0} \sum_{D} P_i^d D_i(P_i^d) - \sum_{S} P_j^s S_j(P_j^s) - \sum_{T} t^*_{ijk} X^t_{ijk} \qquad 3.6.16a$$

$$\text{s.t.} \quad \sum_i X^d_{ij} \geq D_j(P_j^d) \qquad j\varepsilon D \qquad 3.6.16b$$

$$\sum_i X^s_{ki} \leq S_k(P_k^s) \qquad k\varepsilon S \qquad 3.6.16c$$

$$\sum_j \varepsilon_{ijk} X^t_{ijk} + \sum_m X^c_{mik} + \sum_n X^s_{nk} = \sum_l X^t_{ikl} + \sum_q X^c_{iqk} + X^d_{ik} \qquad 3.6.16d$$

$$i\varepsilon C;\ k\varepsilon T$$

$$x_{ijk} \leq b_{ijk} \qquad i\varepsilon C;\ j,k\varepsilon T \qquad 3.6.16e$$

where

$$t^*_{ijk} = \tfrac{1}{2}(1 + \varepsilon_{ijk})t_{ijk} \ .$$

The tacit assumption in this formulation is that the supply and demand relations will be met exactly since the demand and supply functions are used in the objective function rather than the amounts sold at the wellhead, $\sum_i X^s_{ki}$, and in the demand regions, $\sum_i X^d_{ij}$. If the equalities do not hold, however, the corresponding price will be zero and the objective function will still measure profits correctly.

If $D_i(\cdot)$ and $S_i(\cdot)$ are linear functions of price then this is again a diagonal quadratic program with linear constraints. In this case, however, both prices and quantities are primal variables. If $D_i(\cdot)$ and/or $S_i(\cdot)$ are non-linear but can be adequately approximated by piecewise linear functions, the form of the constraints and objective function allow a standard separable linear programming approach, and as will be shown in the next section, a diagonal quadratic approach as well.

3.6.4 A Model of Pipelines under Regulation

The interstate natural gas pipeline companies are regulated by the Federal Power Commission. This regulation takes the form of both price regulation and approval or disapproval of applications for new pipeline construction, servicing new customers, and discontinuing service to others. In this section regulation of pricing is of concern.

The general purpose of the Federal Power Commission with respect to price regulation of interstate pipeline sales is to keep consumer prices low and fair while still allowing a reasonable rate of return on the pipeline's capital. Such a rate of return would have to be at least equal to the company's cost of capital in order that the regulation be non-confiscatory. As Breyer and MacAvoy (1974) point out, the process of actually determining both the dollar value of the company's capital plant and a "reasonable" rate of return is extremely complicated both economically and legally. In Section 4.2.5 the procedure used in this study to estimate the pipeline companies' cost of capital is described in greater detail. For the present, however, it will be assumed that the value of the pipelines' capital plant and a rate of return for each company is given.

The Federal Power Commission must also determine a pricing structure which distributes the costs of producing and transporting gas fairly to widely separated consuming areas. A marginal cost pricing policy such as described in Section 3.6.2 tends to favor the demand centers close to the producing regions in the Southwest with lower prices at the expense of the more distant demand centers in the North Central and Northeast. A nationwide wholesale price would tend to have the opposite effect. The FPC attempts to reach a compromise between these two extremes.

In the model presented here the fixed transportation costs of each pipeline link are allocated on the basis of the capital costs of the lines servicing (flowing into) each demand region. Alternatively a fraction, f, of the capital costs of lines flowing in, and 1-f of the capital costs of lines flowing out, could be allocated to each state along the pipeline. While the most economically efficient procedure might require f = 1, states nearer the major gas producing areas benefit greatly from the economies of scale in pipeline capacity resulting from demand farther along the pipe. Thus some value less than f = 1 may be more equitable.

The FPC attempts to limit the pipeline's revenues to a level which covers its supply cost, operating and maintenance expense, depreciation, interest on debt, taxes, and a fair return on equity capital.

Symbolically,

$$PQ \leq CQ + (O + M)Q + DK + iK^d + T + rK^e \qquad 3.6.17$$

where P is the average price charged by the pipeline, and C is the average price it paid for quantity Q of gas; O is unit operating and M unit maintenance expense; D is depreciation rate on initial plant investment K; i, the interest rate on that portion of K financed by debt; T, state and Federal income taxes; and r, the approved rate of return on equity capital K^e.

If t is the combined state and Federal tax rate then since taxes are only paid on gross profits, we may write

$$T = t(PQ - CQ - O - M - DK - iK^d) . \qquad 3.6.18$$

Thus

$$(P - C - O - M)Q \leq DK + iK^d + \frac{r}{1-t} K^e \quad 3.6.19a$$

$$\leq (D + \frac{1}{1-t} [(1-t)i\frac{K^d}{K} + r\frac{K^e}{K}])K \quad 3.6.19b$$

Defining, R, the average rate of return on undepreciated capital, $\overline{K}$, as

$$R = [(1-t)iK^d + rK^e]/ \overline{K} \quad 3.6.20$$

and the fraction, U, of original investment still on the books (undepreciated) as

$$U = \overline{K} / K \quad 3.6.21$$

equation 3.6.19b may be rewritten as

$$(P - C - O - M)Q \leq (\frac{R}{1-t} U + D)K \ . \quad 3.6.22$$

Now instead of using this expression in the aggregate, we want to allocate these costs over the various areas of the pipeline. Using the method described above with f = 1 we get

$$P^t_{ij}[\sum_k X^t_{ijk} + \sum_m X^c_{imj} + X^d_{ij}] \quad \text{(revenues)}$$

$$- (\sum_n P^t_{in} X^t_{inj} + \sum_l P^t_{lj} X^c_{lij} + \sum p^s_{gi} X^s_{gi}) \quad \text{(gas cost)}$$

$$- \sum_n \tfrac{1}{2}(1+\varepsilon_{inj})t_{inj}X^t_{inj} \quad \text{(variable transportation costs)}$$

$$\leq (\frac{R_i}{1-t} U_i + D_i)\sum_n N_{inj} \quad \text{(fixed costs and taxes)} \quad 3.6.23$$

$$i\varepsilon C;\ j\varepsilon T$$

where R_i, U_i, and D_i correspond to R, U, and D for pipeline i, and N_{inj} is that portion of pipeline i's capital plant invested in the pipeline section (n,j). The summation over n of N_{inj} corresponds to f = 1, i.e. all those pipeline arcs flowing into node j.

Using the mass balance equation 3.6.3c the relation 3.6.23 may be written as

$$\sum_n [\varepsilon_{inj}P^t_{ij} - P^t_{ij} - \tfrac{1}{2}(1 + \varepsilon_{inj})t_{inj}]X^t_{inj}$$
$$+ \sum_l [P^t_{ij} - P^t_{lj} - m_{lij}]X^c_{ij} + \sum_g [P^t_{ij} - P^s_{gi}]X^s_{gi}$$
$$\leq (\frac{1}{1-T} R_iU_i + D_i) \sum_n N_{inj} \qquad i\varepsilon C;\ j\varepsilon T \qquad 3.6.24$$

Notice that the expressions in brackets are familiar from the unregulated competitive model. If these expressions were not summed over n, l, and g respectively and included rents r_{inj}, they would correspond to 3.6.3h - 3.6.3m, which determine marginal cost pricing in the unregulated model. In 3.6.24, however, the bracketed expressions are not restricted to be non-positive. Instead the weighted sum of these terms is restricted to be less than the right hand side expression corresponding to fixed costs and taxes. This is average cost rather than marginal cost pricing.

One more expression is needed to fully define the regulated pricing model: an expression for the demand price P^d_j. In the competitive model all sellers must sell at the same price (equations 3.6.3f and 3.6.3g) if they are going to sell anything at all. The present model recognizes that in the case of demand regions which are not localized at a point, there may be price variations between different pipelines. Even when such price differentials are small, our econometric estimations use average prices within the region as input data. The relations used in this model provide that the demand price used in the demand function $D_i(\cdot)$ is given by the average price of all gas sold in that region weighted by the quantity sold by each pipeline. Again using the conservation of mass equation 3.6.3c, we have

$$\sum_i (P_j^d - P_{ij}^t)X_{ij}^d = 0 \qquad j \in D \qquad 3.6.25$$

As in 3.6.24, this equation has a strong resemblance to the relations 3.6.3f and 3.6.3g in the unregulated competitive model. Prices may vary from the average in this case, however, whereas they must all be equal in the unregulated model.

A nearly complete pipeline model can now be expressed by the relations 3.6.3a - 3.6.3d, 3.6.24, and 3.6.25. What is needed to complement the rate of return regulation of the FPC is an overall objective for the system given these constraints. With pipeline companies as decision makers one would expect the maximization of profits to be this objective. In the case of a nationalized system, the objective might be the maximization of consumers' surplus. In either case the increasing (decreasing) nature of the supply (demand) functions with price result in a concave objective funtion (to be maximized).

The entire model is reproduced below:

$$\underset{X,P\geq 0}{\text{maximize}} \quad \sum_i f_i(P_i^d) - \sum_k g_k(P_k^s)$$

$$- [\sum_T \tfrac{1}{2}(1 + \varepsilon_{ijk})t_{ijk}X_{ijk}^t + \sum_{(i,j,k)} m_{ijk}X_{ijk}^c] \qquad 3.5.26a$$

such that

$$\sum_i X_{ij}^d \geq D_j(P_j^d) \qquad j\varepsilon D \qquad 3.6.26b$$

$$\sum_i X_{ki}^s \leq S_k(P_k^s) \qquad k\varepsilon S \qquad 3.6.26c$$

$$\sum_j \varepsilon_{ijk}X_{ijk}^t + \sum_m X_{mik}^c + \sum_n X_{nk}^s = \sum_l X_{ik}^s + \sum_p X_{ipk}^c + X_{ik}^d$$

$$i\varepsilon C;\ k\varepsilon T \qquad 3.6.26d$$

$$X_{ijk} \leq b_{ijk} \qquad i\varepsilon C;\ j,k\varepsilon T \qquad 3.6.26e$$

$$\sum_i (P_j^d - P_{ij}^t)X_{ij}^d = 0 \qquad j\varepsilon D \qquad 3.6.26f$$

$$\sum_n [\varepsilon_{inj}P_{ij}^t - P_{in}^t - \tfrac{1}{2}(1 + \varepsilon_{inj})t_{inj}]X_{inj}^t + \sum_l [P_{ij}^t - P_{lj}^t - m_{lij}]X_{lij}^c$$

$$+ \sum_g [P_{ij}^t - P_{gi}^s]X_{gi}^s \leq (\tfrac{1}{1-t}R_iU_i + D_i)\sum_n N_{inj} \qquad 3.6.26g$$

$$i\varepsilon C,\ j\varepsilon T$$

where $f_i(P_i^d) = P_iD_i(P_i^d)$ in the case of profit maximization and $\int_0^{P_i^d} D_i(P)dP$ in the case of maximization of consumer surplus with similar definitions for $g_k(P_k^s)$.

These constraints are not linear. The case of non-linear demand and/or supply functions can easily be handled by piecewise linear approximations, because each is a separable function.

In Section 3.6.3 a means for approximating the integral of an increasing function as a quadratic function with no cross terms was described. A similar procedure can be used in the case of a function of the form PS(P) where S is an increasing function of P (see Figure 3.8).

The expression PS(P) can be approximated by the sum of the various rectangles in the figure. This can be expressed as

$$PS(P) \quad \simeq \quad \sum_{j=1}^{n} [\Delta S_k \cdot P_{k-1} \lambda_k + \Delta P_k \cdot S_{k-1} \lambda_k + \Delta S_k \Delta P_k \lambda_k{}^2] \qquad 3.6.53$$

where λ_k is in the range [0,1], and where (P_0,S_0) is the point of intersection of the curse with either the ordinate or the abscissa. Thus the maximum profits objective function can also be approximated as a quadratic expression if supply and demand curves are approximated by piecewise continuous curves.

The pricing equations 3.6.26f - 3.6.26g are nonlinear, non-convex, non-concave sums of cross products of prices and flows. The feasible region for this nonlinear program is, therefore, not convex. Because of this complication one cannot automatically assume that a solution to this problem is necessarily a Kuhn-Tucker point. In Chapter 5, however, an algorithm is developed which is shown to converge to a Kuhn-Tucker point for this problem. Whether or not this point is a solution is not yet known.

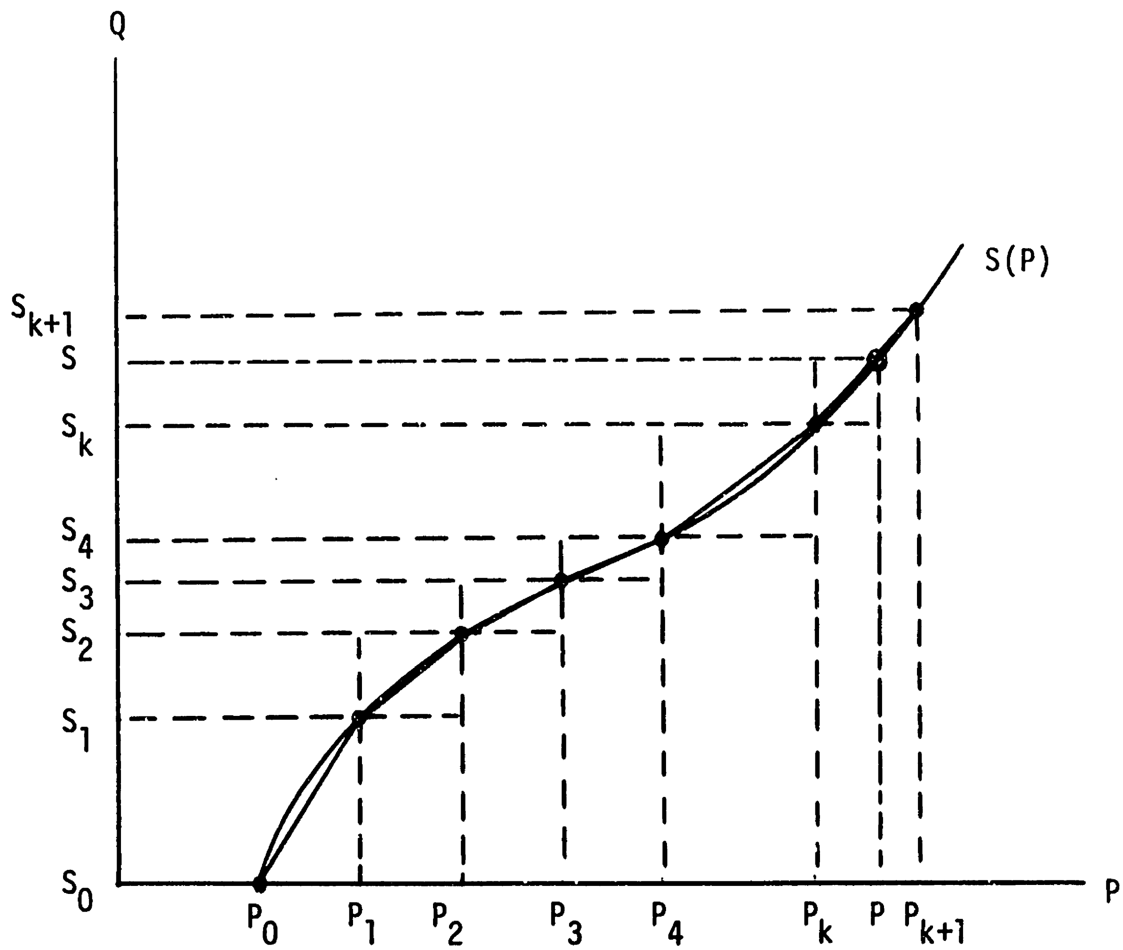

Figure 3.8 Approximating PS(P) as a Quadratic Function

3.7 Regulation of the Natural Gas Producers

3.7.1 Wellhead Price Constraints

The models presented in Section 3.6 can be modified very simply to account for regulation of natural gas producers. The most important aspect of this regulation for the purposes of this study is the FPC ceiling prices on wellhead sales of natural gas. All models need only be expanded by the inclusion of relations of the types

$$P_j^S \leq P_j^C \qquad j \varepsilon S \qquad 3.7.1$$

where P_j^C are the FPC imposed ceiling prices on gas and P_j^S is the new contract wellhead price of gas. If P_j^S is average wellhead price a somewhat more complex expression results since average price includes both new and old contracts. One way of modelling this is to use the historic ratio of new to old sales and average the previous years average price with the new ceiling price using that ratio. One would then redefine P_j^C to be this new "average ceiling price."

In the case of the unregulated industry model such constraints result in no theoretical difficulties in achieving an equilibrium since restrictions on wellhead prices will be compensated for by higher pipeline rents. In the case of the regulated model, however, rents cannot be collected. By keeping prices low enough at the wellhead the model may well be infeasible, i.e. lower prices at the wholesale level result in a demand which simply can't be met at the low wellhead prices. This appears in fact to be the situation in the United States in the 1970s when the low wellhead prices of the 1960s created an artificially high demand for gas which producers simply could not satisfy as they reached the limits of their productive capacity. Thus the regulated industry model must be modified to allow for the possibility of unsatisfied demand.

3.7.2 Excess Demand Model

Because price ceilings on wellhead sales can result in an infeasible regulated industry model (3.6.26), it must be modified to be of any analytical use.

In place of 3.6.26b write

$$\sum_i X_{ij}^d + E_j^d \quad \geq \quad D_j(P_j^d) \qquad j \varepsilon D \qquad 3.7.2$$

where E_j^d is the level of excess demand in region j. Since E_j^d is a non-negative variable this means that 3.7.2 is no longer a binding constraint, in the sense that E_j^d can become as great as is necessary to allow the inequality to hold.

The objective of any governmental policy, however, would be to minimize this excess demand. Thus it appears reasonable to apply a penalty to any solution of the modified model which has positive excess demands. This is done by modifying the objective function 3.6.26a as follows:

$$\max_{X,P,E\geq 0} \quad \sum_i f_i(P_i^d) - \sum_k g_k(P_k^s) - \sum_i W_i E_i^d \qquad 3.7.3$$

where W_i are positive weights corresponding to each demand region i. The next question is: if there is going to be excess demand how can it be allocated fairly and efficiently to the various demand regions?

The answer chosen for this model is that the weights should be chosen to minimize the sum of the fractional excess demands in each state, i.e.

$$W_i \quad = \quad N \;/\; D_i(P_i^d) \qquad 3.7.4$$

where N is some large positive number. To keep this new term in the objective function linear, we decided to use estimated prices for these general demand functions, $D(\cdot)$. Previous year prices would be good proxies, for example. In the case of linear demand functions of the form

$$D_i(P_i^d) \quad = \quad a_i - b_i P_i^d \qquad 3.7.5$$

the expressions

$$W_i \quad = \quad N \;/\; a_i \qquad 3.7.6$$

were used. In the model results discussed in Chapter 6, N was chosen to be 1000000 so that the reduction of excess demand would be of higher priority than maximization of profits.

Chapter 4 Specification of the Model

4.1 Introduction

In order to apply the general models described in Chapter 3, specific values must be supplied for each of the models' parameters. In the case of the production and demand models these values have been determined by the use of a relatively sophisticated econometric technique, namely a generalized least squares method devised by Pindyck (1974) for applications to cross sectional time series data.

The pipeline and pricing models, on the other hand, have required a more disaggregated approach, one which uses the statistics of each of the pipeline companies individually and the network model as a whole.

These specifications are discussed in greater detail in the following sections.

4.2 Econometric Estimation of the Production and Demand Submodels

4.2.1 Generalized Least Squares Estimation for Cross-Sectional Time Series

In order to build a forecasting model for natural gas, we have attempted to estimate equations for the production and demand submodels based on trends and regional variations in the price of gas and other explanatory variables. To do so we used a special version of the Generalized Least Squares (GLS) estimation method devised by Pindyck (1974) for cross-sectional time series models.

In contrast to the Ordinary Least Squares (OLS) method, this version of GLS takes account of unexplained trends in the data from period to period (serial correlation) and regional variations in the standard deviations of the estimates (heteroskedasticity).

The first stage of the GLS procedure is exactly the same as that for OLS: given a set of cross-sectional times series data X_{ijt} corresponding to values of the i^{th} explanatory variable for district j in period t, and given the values Y_{jt} of the dependent variable, we may estimate the coefficients, β, of the regression equation (in matrix form)

$$\tilde{Y} = X\beta + \tilde{U} \qquad 4.2.1$$

by

$$\hat{\beta} = (X'X)^{-1}X'\tilde{Y} \qquad 4.2.2$$

where X is a matrix in which each column contains the values of a particular explanatory variable for all periods and cross-sections. In this linear model written in the expanded form

$$\tilde{Y}_{jt} = X_{jt}\beta + \tilde{U}_{jt} \qquad 4.2.3$$

it is assumed that the "error terms" $\tilde{U}_{jt}$ are independently and identically distributed, uncorrelated random variables. In cross-sectional time series data, however, it is very likely that the $\tilde{U}_{jt}$ are neither identically distributed nor uncorrelated.

To correct the estimates, $\hat{\beta}$, for serial correlation, a second regression is performed using results from 4.2.1.

The equation 4.2.3 is transformed into

$$\tilde{Y}_{jt} - \hat{\rho}_j\tilde{Y}_{j,t-1} = (X_{jt} - \hat{\rho}X_{j,t-1}) + \tilde{V}_{jt} \qquad 4.2.4$$

where $\hat{\rho}_j$ is the estimated correlation coefficient between the vectors $\tilde{U}_{jt}$ and $\tilde{U}_{j,t-1}$, i.e.

$$\hat{\rho}_j = \frac{\frac{1}{n-1}\sum_{t=1}^{n}(U_{jt} - \bar{U}_j)(U_{j,t+1} - \bar{U}_j)}{\frac{1}{n-k}\sum_{t=1}^{n}(U_{jt} - \bar{U}_j)^2} \qquad 4.2.5$$

where n is the number of time periods in the data, k the number of explanatory variables, and a bar above a variable indicates its mean. The estimator $\hat{\beta}_1$ from this regression has been corrected for serial correlation.

The results from 4.2.4 are then used to compute a final estimate for β corrected for both serial correlation and heteroskedasticity. The standard deviations σ_j of the error terms $\tilde{V}_{jt}$ in 4.2.4 are estimated by

$$\hat{\sigma}_j = \left(\frac{1}{n-k-1}\sum_{t=2}^{n}(V_{jt} - \bar{V}_j)^2 / (1 - \hat{\rho}_j^{\,2})\right)^{\frac{1}{2}} \qquad 4.2.6$$

and used to transform 4.2.4 into

$$\frac{\tilde{Y}_{jy} - \hat{\rho}_j\tilde{Y}_{j,t-1}}{\hat{\sigma}_j} = \left(\frac{X_{jt} - \hat{\rho}X_{j,t-1}}{\hat{\sigma}_j}\right)\beta + \tilde{W}_{jt} \qquad 4.2.7$$

In this regression the $\tilde{W}_{jt}$ should be uncorrelated and identically and independently distributed random variables.

The resulting estimator $\hat{\beta}_2$ has been corrected for both serial correlation and heteroskedasticity in the cross-sectional time series data.

4.2.2 The Econometric Results for the Production Submodel

A variant of the GLS method described above was used to estimate the coefficients of the production equations. In order to account for the fact that supply and demand equations are interrelated in a set of simultaneous equations, instrumental variables were used to correct for this simultaneity.

Two groups of estimates were made for the production model: the MIT Natural Gas Policy Model estimates were based on the equation form

$$Q \quad = \quad a_0 + a_1 \log P + a_2 Y_{-T} \qquad 4.2.8$$

where Q is production, P is wellhead price, Y_{-T} is reserves lagged T years, and a_i i = 0,1,2 are coefficients; for the present study the linear form

$$Q \quad = \quad a_0 + a_1 P + a_2 Y_{-T} \qquad 4.2.9$$

was used.

The results of the logarithmic and linear forms appear in Tables 4.1 and 4.2 respectively.

The dependent and independent variables correspond to the doubly transformed variables in 4.2.7 and the coefficients to $\hat{\beta}_2$. The numbers in parentheses below the coefficients are the corresponding t-statistics.

The variables in these equations are:

QG production of gas in millions of cubic feet;

PG wellhead prices of gas in cents per thousand cubic feet;

YG gas reserves in millions of cubic feet;

LS district dummy for Louisiana South onshore, defined as 1 for LaS and 0 for all other districts;

LN district dummy for Louisiana North;

MT district dummy for Montana;

WY district dummy for Wyoming.

TABLE 4.1 Logarithmic Production out of Reserves Equations

1. Permian Basin

$$QG = -\ 6447760.0 + 1856730.0 \log(PG) + 0.1226\ YG_{t-2}$$
$$(-2.35) \quad (1.67) \quad (5.24)$$

$R^2 = 0.925$ F = 67.7 S.E. = 1.42×10^5 D.W.(0) = 1.98

LHS Mean = 1.736×10^6

2. Gulf Coast and Mid-Continent

$$QG = -\ 169422.0 + 5881360.0\ LS + 340752.0 \log(PG) + 0.02638\ YG_{t-1}$$
$$(-0.352) \quad (6.95) \quad (2.00) \quad (6.78)$$

$R^2 = 0.906$ F = 193.7 S.E. = 0.727 D.W.(7) = 0.9

Mean of QG before transformation = 1404680.0 LHS Mean = 2.655

3. Remaining Continental Production

$$QG = -\ 9424.0 + 23034.0 \log(PG) + 0.05999\ YG_{t-1}$$
$$(-0.22) \quad (1.65) \quad (29.23)$$

$R^2 = 0.968$ F = 1174.2 S.E. = 0.785 D.W.(9) = 1.00

LHS Mean = 5.21

4. Offshore Louisiana

$$QG = -\ 7347493.0 + 2340900.0 \log(PG) + 0.1166\ YG_{t-3}$$
$$(4.50) \quad (5.10) \quad (32.23)$$

$R^2 = 0.993$ F = 727.9 S.E. = 89800.0 D.W.(0) - 2.34

LHS Mean = 1361000.0

TABLE 4.2 Linear Production out of Reserves Equations

5. West

$$QG = -\ 207008 + 13532.6\ PG + 0.0562\ YG_{-1} - 16712\ MT + 78072\ WY$$

(-7.60) (13.26) (20.29) (-0.59) (4.35)

$R^2 = 0.971$ F = 234.8 SE = 38100 LHS Mean = 360154

6. Hugoton-Anadarko

$$QG = -\ 1914200 + 16393\ PG + 0.0386\ YG_{-1}$$

(-10.66) (15.80) (7.053)

$R^2 = 0.934$ F = 149.6 SE = 79500 LHS Mean = 1168820

7. Appalachia

$$QG = -\ 39726.1 + 0.0945\ YG_{-1}$$

(-6.01) (37.66)

$R^2 = 0.990$ F = 1418 SE = 10900 LHS Mean = 186900

8. Mid- and East Texas, Louisiana North, and Mississippi

$$QG = -\ 120963 + 7598\ PG + 0.0695\ YG_{-1} + 210201\ LN$$

(-1.19) (1.48) (12.25) (13.03)

$F^2 = 0.953$ F = 243.8 SE = 38900 LHS Mean = 298401

9. Texas Gulf Coast

$$QG = -\ 541057 + 50643\ PG + 0.0408\ YG_{-1}$$

(-1.45) (2.34) (10.40)

$R^2 = 0.840$ F = 55.1 SE = 16100 LHS Mean = 1220880

10. South Louisiana (Inc. Offshore)

$$QG = -\ 49288200 + 1482130\ PG + 0.3238\ YT_{-1}$$

(-4.52) (3.14) (5.80)

$R^2 = 0.888$ F = 19.7 SE = 61600 LHS Mean = 5658520

FIGURE 4.2 (continued)

10. Permian Basin

$$QC = -\ 6126050 + 422115\ QC + 0.0546\ YT_{-1}$$

(-5.23) (6.00) (2.87)

R^2 = 0.907 F = 24.2 SE = 178000 LHS Mean = 2032510

4.2.3 The Econometric Results for the Demand Submodel

The demand submodel for natural gas was estimated by the same GLS technique used for the production submodel. We divided demand into three categories: (a) residential and commercial, (b) industrial, and (c) lease and plant fuel.

We considered new residential and commercial demand to be a function of gas price, an alternate fuel price, and either population or personal income (see Table 4.3 for definition of variables). The equations were estimated in the Balestra form with linear explanatory variables; for example:

$$\delta TRCS = a_0 - a_1 PGW + a_2 PALT + a_3 \delta NN \qquad 4.2.10$$

where population, YY, could be used instead of NN, and where

$$\delta X = X_t - (1-d)X_{t-1}$$

with d = 0.07, for any variable X.

New industrial demand is given by the similar expression

$$\delta TINS = a_0 - a_1 PGW + a_2 PALT + a_3 CAP \ . \qquad 4.2.11$$

Here VAM may replace CAP.

We model lease and plant fuel demand as a constant fraction of production for each state, since the level of such sales is dependent on the level of petroleum operations. The resulting estimates are used by simply reducing the production in district j

TABLE 4.3 Variables in the Demand Model

AL, AR, AZ, CA, ..., WY	Dummy variables for the 41 demand states (conforming to the postal code except for NE = New England).
CAP	New capital expeditures ($\$10^6$).
LPFS	Lease and plant fuel.
NN	Population (1000's).
PALT	Price of alternative fuels ($'s per Mcf - energy - equivalent).
PFOIL	Average wholesale price of #2 fuel oil (cents per gallon).
PGW	Wholesale natural gas price (cents per Mcf).
POIL	Average wholesale price of fuel oil paid by electric power companies ($'s per Mcf - energy - equivalent).
QG	Production of natural gas (MMcf).
TINS	Industrial demand for natural gas (MMcf).
TRCS	Residential and commercial demand for natural gas (MMcf).
VAM	Value added in manufacturing ($\$10^5$).
YY	Personal income by state ($\$10^6$).

available for sale to pipelines by the fraction f_j; i.e. if $\overline{Q}_j$ is available production,

$$\overline{Q}_j = (1 - f_j)Q_j \qquad 4.2.12$$

The econometric estimates for demand are given in Table 4.4.

4.2.4 Discussion of Results

In the production submodel, total gas production out of reserves was first estimated as a linear combination of the logarithm of the price and lagged reserves. In the case of equations 2 and 3 the GLS procedure discussed in Section 4.2.2 was used. In the case of the Permian and offshore Louisiana equations only the simpler serial correlation correction was necessary since these were single districts, and not cross sections. These two cases were handled differently from the others because of their peculiar natures: offshore Louisiana leasing procedures and the Permian Basin's high production and great areal extent. Note that Louisiana South (onshore) is included in the Gulf Coast and Mid-continent equation (2) but has a large dummy variable to account for its very high productivity.

One might also notice that in the case of the Permian Basis and the "Remaining Continental Production" districts the log-price term is not very significant as demonstrated by t-statistics of 1.65 and 1.67. In addition the elasticities of supply implied by equations 1 - 4 are probably too high. For equations of this type

TABLE 4.4 Demand Equations

(a) Residential and Commercial Demand for Gas

1. Northeast

$$\delta TRCS^{*} = -\ 2163.7 - 899.85PGW + 3294.2PFOIL + 49.99\delta NN$$
$$(-0.13)\quad(-3.95)\quad(2.11)\quad(8.32)$$

R^2 = 0.425 F = 12.8 SE = 0.732 LHS Mean = 1.72

Mean of LHS Before Transformation = 27285
Serial Correlation Coefficient ρ = 0.2263

2. North Central

$$\delta TRCS = 50342 - 2597.5PGW + 113048PALT + 74.77\delta NN$$
$$(3.88)\quad(-4.33)\quad(3.80)\quad(7.63)$$

$$+\ 28453IL + 8040IO + 12304WI$$
$$(2.37)\quad(2.71)\quad I3.31)$$

R^2 = 0.307 F = 4.13 SE = 0.626 LHS Mean = 1.49

Mean of LHS Before Transformation = 26603.
Serial Correlation Coefficient, ρ = - 0.0525

3. Southeast

$$\delta TRCS = 4497.6 - 758.7PGW + 2350.1PFOIL + 2.327\delta YY$$
$$(0.41)\quad(-2.84)\quad(1.88)\quad(1.88)$$

$$-\ 8245.1FL + 6821.9GA + 8363KY - 3304.9SC$$
$$(-3.61)\quad(3.63)\quad(3.71)\quad(-2.10)$$

R^2 = 0.561 F = 7.47 SE = 0.586 LHS Mean = 1.21

Mean of LHS Before Transformation = 7660.9
Serial Correlation Coefficient, ρ = - 0.2121

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* Note that $\delta X = X = (1-\alpha)X_{-1}$ for any variable X, where α, the depreciation rate for gas appliances, is assumed to be 0.07.

4. South Central

$$\delta TRCS = \underset{(1.38)}{5245.1} - \underset{(-2.69)}{5415.1PGW} + \underset{(2.31)}{9521.5PFOIL} + \underset{(1.24)}{8682.2KS} + \underset{(1.48)}{11473MS}$$

$R^2 = 0.132$ F = 1.41 SE = 0.576 LHS Mean = 0.465

Mean of LHS Before Transformation = 12453.8
Serial Correlation Coefficient, $\rho = 0.1869$

5. West

$$\delta TRCS = \underset{(2.38)}{8083.3} - \underset{(-5.60)}{3.184PGW} + \underset{(1.76)}{403.2PFOIL} + \underset{(6.71)}{19.74\delta NN} + \underset{(5.87)}{50137CA} + \underset{(4.27)}{3443NV}$$

$R^2 = 0.570$ F = 18.83 SE = 0.634 LHS Mean = 1.67

LHS Mean Before Transformation = 13408
Serial Correlation Coefficient, $\rho = -0.7000$

(b) Industrial Demand for Gas

6. Northeast

$$\delta TINS = \underset{(4.29)}{28500} - \underset{(-3.51)}{621.4PGW} + \underset{(1.67)}{13330PALT} + \underset{(2.93)}{8.888CAP_{-1}} + \underset{(6.04)}{31248OH} + \underset{(3.24)}{17912PA}$$

$R^2 = 0.581$ F = 13.88 SE = 0.576 LHS Mean = 1.38

LHS Mean Before Transformation = 18095
Serial Correlation Coefficient, $\rho = -0.1283$

7. North Central

$$\delta TINS = \underset{(2.93)}{40300.1} - \underset{(-3.39)}{2091.7PGW} + \underset{(3.45)}{95838PALT} + \underset{(10.90)}{2.81VAM} + \underset{(5.68)}{14094IO}$$

$$+ \underset{(3.08)}{8467.8MN} + \underset{(4.42)}{16984WI}$$

$R^2 = 0.745$ F = 27.3 SE = 0.536 LHS Mean = 1.44

LHS Mean Before Transformation = 23496
Serial Correlation Coefficient, $\rho = -0.0227$

8. <u>Southeast</u>

$$\delta TINS = \underset{(3.37)}{-\ 14693} + \underset{(-2.62)}{1228.9PGW} + \underset{(2.62)}{61242PALT} + \underset{(1.77)}{22.255CAP_{-1}} + \underset{(2.55)}{36100.6NC}$$

$R^2 = 0.583$ F = 15.39 SE = 0.665 LHS Mean = 1.19

LHS Mean Before Transformation = 15695
Serial Correlation Coefficient, $\rho = 0.2673$

9. <u>South Central</u>

$$\delta TINS = \underset{(2.21)}{138716} - \underset{(-2.77)}{7043.8PGW} + \underset{(1.99)}{116247POIL} + \underset{(5.79)}{167.12CAP_{-1}} + \underset{(3.51)}{60703.6LA}$$

$R^2 = 0.663$ F = 15.22 SE = 0.473 LHS Mean = 1.31

LHS Mean Before Transformation = 9440.8
Serial Correlation Coefficient, $\rho = -\ 0.2125$

10. <u>West</u>

$$\delta TINS = \underset{(5.47)}{8020.2} - \underset{(6.22)}{416.5PGW} + \underset{(5.02)}{46256PCOAL} + \underset{(3.79)}{7.63CAP_{-1}}$$

$$+ \underset{(8.82)}{71010CA} - \underset{(-2.00)}{2931.3MT}$$

$R^2 = 0.520$ F = 15.4 SE = 0.694 LHS Mean = 1.66

LHS Mean Before Transformation = 13249
Serial Correlation Coefficient, $\rho = -0.8225$

(c) <u>Demand for Gas as Lease and Plant Fuel (OLS)</u>

$$11.\ LPFS = (\underset{(2.30)}{0.036AR} + \underset{(30.9)}{0.134CA} + \underset{(1.16)}{0.029CO} + \underset{(8.22)}{0.025KS} + \underset{(103.3)}{0.036LA}$$

$$+ \underset{(3.00)}{0.055MS} + \underset{(17.48)}{0.042NM} + \underset{(1.26)}{0.048OH} + \underset{(36.25)}{0.058OK} + \underset{(0.81)}{0.027PA}$$

$$+ \underset{(284.1)}{0.096TX} + \underset{(0.62)}{0.036UT} + \underset{(7.44)}{0.059WY})\ QG$$

$R^2 = 0.999$ F = 6265 SE = 6000 LHS Mean = 102360 the price elasticity of supply is given by

$$e = \frac{\partial QG/QG}{\partial PG/PG} = \frac{a_1}{QG} \qquad 4.2.13$$

where a_1 is the coefficient of the log of the price. In Table 4.5 we give these elasticities for various districts assuming 1972 production levels. In the case of the linear production equations 5 - 11, the price elasticity of supply is given by

$$e = \frac{a_1 \cdot PG}{QG} \quad . \qquad 4.2.14$$

The general tendency is for the elasticities in the linear case to be greater than their counterparts in the logarithmic production equations (see Table 4.6). From the equations defining these elasticities it is also apparent that higher prices tend to worsen the distortion toward high elasticities in the linear case, but not in the logarithmic case. Finally, as production increases, the price elasticity of supply decreases in both models. Thus as production goes up price becomes less of a factor in bringing production out (in the short-run) and a greater factor as production goes down.

The "reserves elasticity of supply" is in both sets of equations given by

$$e = a_2 YG_{-t}/QG \qquad 4.2.15$$

where a_2 is the coefficient on lagged reserves YG_{-t}. These elasticities are presented in Table 4.6 for various districts.

TABLE 4.5 Average Price Elasticity of Supply

by Production Region (1972)

Region	Logarithmic Price Model	Linear Price Model
California	0.037	0.703
Colorado-Utah	0.139	1.252
Kansas	0.398	2.522
Louisiana North	0.043	0.220
Louisiana South Onshore	0.100	5.249
Louisiana South Offshore	2.367	5.249
Montana	--	4.234
Mississippi	0.140	0.914
NM (Northwest)	0.045	0.358
Permian Basin	0.900	3.200
Oklahoma	0.227	1.785
Texas RRC #1	2.619	0.958
#2	0.479	1.128
#3	0.237	0.595
#4	0.231	0.533
#6	0.051	0.234
#9	0.117	0.631
#10	0.304	2.288
Wyoming	0.079	0.698

Source: Computed from regression results of the production nodes and sales data from Federal Power Commission Form 2 reports.

TABLE 4.6 Average Reserves Elasticity of Supply by Production Region, 1962-1972

Region	Logarithmic Price Model	Linear Price Model
California	0.710	0.665
Colorado-Utah	1.071	1.003
Kansas	0.461	0.675
Louisiana North	0.501	0.790
Louisiana South, Onshore	0.432	4.415
Louisiana South, Offshore	1.600	4.415
Montana	--	1.404
Mississippi	0.602	0.948
New Mexico Northwest	1.106	1.036
Permian Basin	1.606	0.715
Oklahoma	0.320	0.468
Texas RRC #1	0.462	1.657
#2	0.420	1.105
#3	0.462	1.218
#4	0.541	1.424
#6	0.873	1.376
#9	0.694	1.093
#10	0.280	0.410
Wyoming	0.797	0.747

Source: Computed from regression results of the production models and sales data from Federal Commission Form 2 reports.

The demand equations have all been estimated as linear functions of price. In the MacAvoy-Pindyck study lagged price was used in the residential/commercial equations because it was felt that there might be a delay factor in consumers' decisions about equipment replacement or initial purchase of gas equipment. Such a formulation would not allow one to calculate equilibrium prices using GASNET, since supply and demand prices would be out of phase. Since these equilibrium prices are of interest in this study, the MacAvoy-Pindyck equations were reestimated with current year prices with the results listed as equations 1 - 11 in Table 4.4.

The price elasticity of demand for these equations is given by

$$e = \frac{a_1 PGW}{\delta Q} \qquad 4.2.16$$

where δQ is either δTINS or δTRCS as appropriate. One difficulty with this cross-sectional formulation is that whereas price is relatively homogeneous across several states within a demand region, new demand (δQ) varies greatly according to the size of the state. Thus the larger states tend to have lower and smaller states larger elasticities than is probably the case. Table 4.7 gives a sample of elasticities in 1972 for various demand regions.

From the results it is apparent that the South Central region is being modelled as extremely price sensitive. This means that small price increases could create large decreases in demand in these states. In fact, however, these states have experienced the largest

TABLE 4.7 Average Price Elasticity of Demand, 1962-1972

Region	State	R/C Elasticity	Industrial Elasticity
Northeast	New England	-2.721	-1.879
	New Jersey	-1.879	-1.298
	New York	-0.896	-0.619
	Pennsylvania	-1.155	-0.797
	Ohio	-0.673	-0.464
	Md-Del-DC	-3.281	-2.265
	Virginia	-4.528	-3.127
	West Virginia	-5.927	-4.093
North Central	Illinois	-0.904	-0.728
	Indiana	-6.682	-5.380
	Michigan	-2.050	-1.651
	Wisconsin	-6.249	-5.032
	Iowa	-6.405	-5.158
	Minnesota	-5.728	-4.613
	Missouri	-5.728	-2.707
	Nebraska	-9.287	-7.478
	South Dakota	-48.126	-38.755
Southeast	Florida	-12.546	-20.321
	Georgia	-2.441	-3.953
	North Carolina	-4.411	-7.145
	South Carolina	-7.569	-12.259

TABLE 4.7 (continued)

Region	State	R/C Elasticity	Industrial Elasticity
	Alabama	-3.325	-5.386
	Kentucky	-2.325	-3.767
	Tennessee	-3.082	-4.992
South Central	Kansas	-12.631	-16.430
	Arkansas	-16.564	-21.547
	Oklahoma	-14.143	-18.397
	Texas	-4.720	-6.140
	Mississippi	-33.582	-43.682
	Louisiana	-8.996	-11.701
West	Arizona	-1.792	-2.344
	Colorado	-0.548	-0.717
	Idaho	-5.852	-7.655
	Nevada	-5.688	-7.441
	New Mexico	-1.869	-2.446
	Utah	-1.867	-2.443
	Wyoming	-1.761	-2.303
	California	-0.124	-0.162
	Oregon	-3.017	-3.946
	Washington	-1.367	-1.788
	Montana-N. Dakota	-1.780	-3.383

Source: Compiled from regression results of the demand model and sales data from Federal Power Commission Form 2 reports.

increases in price due to the fact that intrastate gas prices have risen so steeply in the last few years. The model as presently formulated does not represent the dichotomy between intrastate and interstate supply or demand. An extension of the model to include such a division is described in Chapter 7.

The final demand equation is for lease and plant fuel sales in the gas producing states. The equation merely postulates that in each state a constant fraction of total production goes to lease and plant fuel sales, the fractions being different for each state. As the t-statistics indicate, most (but not all) of the estimated coefficients are statistically significant.

4.3 Pipeline Network and Pricing Submodel Parameters

4.3.1 Natural Gas Sales, Purchases and Flow Patterns, 1966-1972

Each year every major interstate pipeline company must file a report which contains among other things a complete listing of the pipeline's sales and purchases. These are the annual FPC Form 2 reports. In the data acquisition and organizational stage of this project we recorded the price and quantity of every sale made during the years 1962-1972 by an interstate pipeline company whose report was available from the FPC. Each of these transactions concerned either a sale to an intrastate utility company, a sale to another FPC regulated interstate pipeline company, or a direct sale to industrial or residential customers. Sales by each pipeline to intrastate utilities were totaled by state of sale since these were clearly intended for final demand to consumers within that state. Sales to other interstate pipelines were recorded separately, since that gas may have travelled out of state via the buying company's pipeline.

Though purchases from producers was also available in the Form 2's, it was more convenient to use the FPC's annual summary of such purchases in Sales by Producers of Natural Gas to Interstate Pipeline Companies. The table we used lists purchases by pipeline company and production district. Since this publication is not available prior to 1966, and since the latest volume at the time of the data collection stage of this project was for 1972, the study has focused on the period 1966-1972 for its parameter estimations.

The data described above constitutes the sales and purchase patterns of the pipeline companies from 1966 to 1972. Using this data in conjunction with the pipeline network model, we estimated the yearly flow of gas through each section of the network during the period 1966-1972. We did this by solving the set of conservation of mass equations for each transshipment node of the network, 352 in all. The equations were of the form:

$$\sum_j X^t_{ijk} + \sum_m X^c_{mik} + \sum_n X^s_{ni} - \sum_l X^t_{ikl} - \sum_q X^c_{iqk} - X^d_{ik} = 0 \qquad 4.3.1$$

$$i \epsilon C; \; k \epsilon T$$

In this equation only the transshipment flows X^t_{ijk} are unknown. The equations may therefore be written in the matrix form

$$A_i X^t_i = b_i \qquad 4.3.2$$

for each pipeline company, i, where A_i is the node-arc incidence matrix, X^t_i are the flows in company i's pipeline areas, and b_i includes the known sales and purchases X^c, X^s, and X^d.

For most pipeline companies the number of arcs (unknown X^t's) is equal to the number of nodes plus one, since their networks are tree structures with no loops. For each of these companies an additional node was created at a point near the midpoint of the network to function as a balancing node for any discrepancy between total gas input and total gas output in the line (see Figure 4.1). Such a discrepancy could be the result of a number of factors: gas used as pipeline fuel, differences between gas delivered to and withdrawn from storage, unexplained losses, as well as errors in the data. In four cases pipelines had a loop in their networks. Thus they each had two less nodes than flows and the set of conservation of mass equations plus balance equation was not sufficient to fully define the flow pattern in the network. For these cases another equation was added which made the system 4.3.2 determinate by restricting two flows within the loop to be in a fixed ratio. For example, in Figure 4.2 a new equation $X_{34} - rX_{35} = 0$ might have been added. These ratios, r, were not chosen arbitrarily but were based on pipeline capacities as estimated from the 1968 FPC Map of Interstate Natural Gas Pipeline Companies in the U. S.

Because there were about eighty such systems of equations (one for each company included in the model) we decided to solve the entire set of equations once for each year rather than solving for each of the companies separately. We solved the system of equations as a linear program with a null (all zero) objective function for each year 1966-1972. Since there was only one feasible solution to the system of equations the form of the objective function didn't matter.

A sample of the computed flow values is given in Table 4.8. The first column contains the pipeline arc code name. To the left of the decimal point is the company code number, the next set of

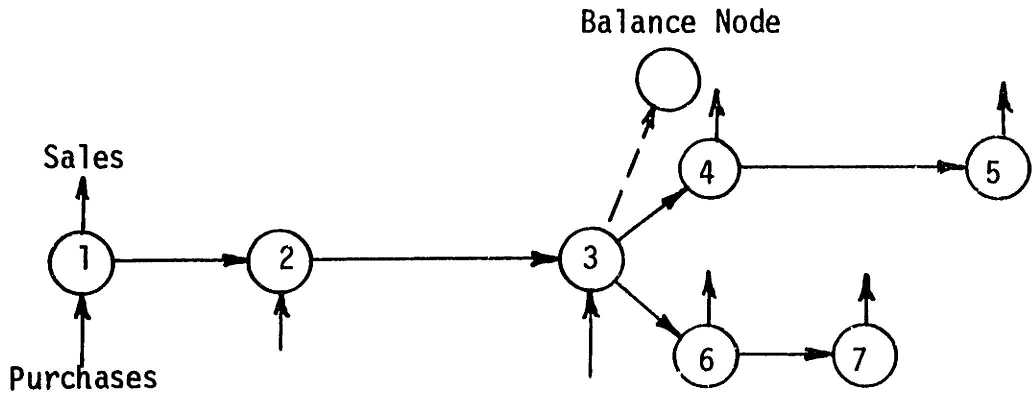

Figure 4.1 Balance Node for a Pipeline Subnetwork

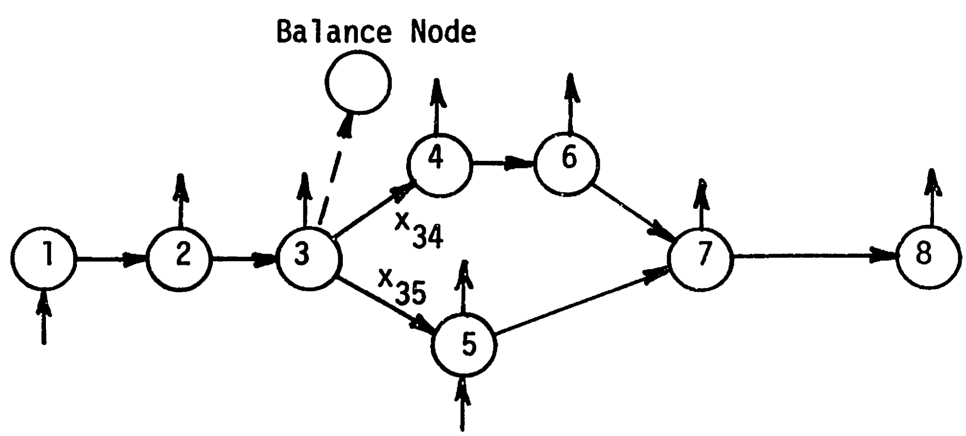

Figure 4.2 Pipeline with a Loop

TABLE 4.8 Pipeline Flows (MMcf) during 1966-1972 for a Small Sample of Pipeline Arcs

Arc Codes	1966	1967	1968	1969	1970	1971	1972
5.1801	11801	13098	11831	12913	10815	6740	10479
5.1833	1911	1865	1661	1648	2105	1949	1688
5.1800	-398	-1079	-1645	-52	-78	-95	54
6.2325	102810	114871	120974	125622	137863	135676	133663
6.2527	99918	109194	116369	121866	132115	132261	130575
6.4749	73867	81206	86526	88583	97310	97157	95215
6.4922	60657	66782	70015	70684	76636	76707	77415
6.2300	-1753	-44	1084	546	495	1065	3612
7.3415	13194	13457	13494	13457	13457	0	0
7.1514	452793	404680	416865	484929	547319	0	0
7.1438	381104	350468	354123	407283	457954	0	0
7.3827	197806	196437	202008	214201	219794	0	0
7.3638	13064	6335	6370	6335	6335	0	0
7.1500	-28670	42881	45772	42373	44814	0	0

Note: Pipeline flows for company number 7 are 0 after 1970 because it was absorbed into company number 149 in 1971.

Source: Computed using data from Federal Power Commission Form 2 reports.

two digits is the transmitting state's number, the final two, that of the receiving state. Those codes with 00 as the last two digits are the discrepancy values which balance the input and output for each company. In most cases these values are positive as one would expect since gas is used as pipeline fuel. In the cases where these numbers are negative, there are either unexplained data errors or there has been a much greater quantity of gas taken out of storage than put back for that year. Figure 4.3 shows the simple network model and 1972 flows for company number 6 (Algonquin).

In general the flow values for most companies increased from 1966 to 1970. From 1970 to 1972 these values began to peak and actually decrease. This is indicative of the pipelines' inability to get enough gas to satisfy demand in the last part of this period. It also shows why it is not a good idea to estimate consumer demand in 1972 by the quantity actually supplied by the pipeline. By this time there was already excess demand for gas in the U. S.

4.3.2 Pipeline Capacities

The quantity of gas which can move along a pipeline from one node to the next is subject to an upper limit. This upper limit, or capacity, is determined to a large extent by the physical characteristics of the pipe itself and especially by the horsepower available in the lines' gas compressors.

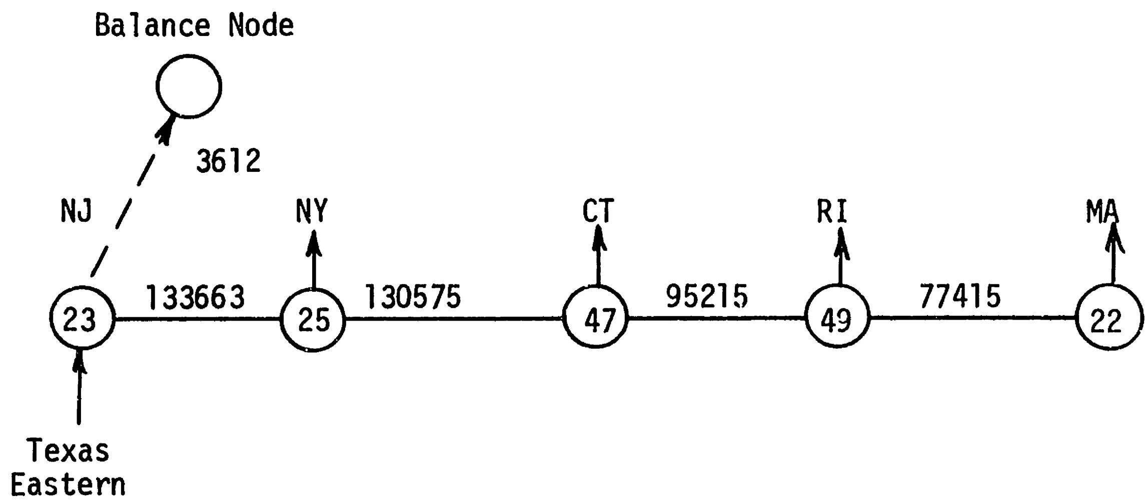

Figure 4.3 Pipeline Network Model and Flows for Algonquin in 1972

Though some of this data is available in the FPC Form 2 reports, it is not available at the level of disaggregation which makes it possible to estimate the capacities for individual pipeline arcs. Thus an alternative approach was taken which used the flow data described above and FPC (1973) estimates of capacity utilization (load factor). Load factor is defined as the ratio of average daily throughput to peak daily throughput for the year. These estimates indicated an average load factor of 0.82 in 1966 to 0.89 in 1970 for a sample of 16 pipelines. Analysis of the flow data in Table 4.8 seems to indicate a decrease in load factor after 1970 or 1971. For the pipeline model, capacity constraints in each arc were estimated to be the computed throughputs divided by the average U. S. capacity utilization factor. This method tends to underestimate the capacity of currently underutilized pipelines and overestimate the capacity of those which have load factors higher than the average. Because the FPC's figures only go to 1970, we had to make additional estimates of the load factor in 1971 and 1972 when it was apparently decreasing. These load factors were chosen to be 0.88 and 0.87 so that the actual capacities in 1971 and 1972 were approximately the same as those in 1970.

4.3.3 Throughput Efficiency Factors

The throughput efficiency factors, ε_{ijk}, were estimated using the annual FPC Statistics of Interstate Pipeline Companies (1966-1972). Total quantities of gas used as pipeline fuel or unaccountable lost in transmission were allocated to the various segments of the company's pipeline as follows: the total flow of gas in each arc was multiplied by the approximate pipeline arc mileage as determined by measuring the arc on the 1968 FPC pipeline map. The quantity of gas used or lost in each arc was then estimated to be the total quantity used or lost in the whole network multiplied by the fraction of the companies "quantity-miles" associated with that arc. Dividing this by the quantity of gas flowing in the arc resulted in a fraction lost and subtracting this from 1 gave the throughput efficiency factor ε_{ijk}.

In symbols

$$\varepsilon_{ijk} = 1 - \frac{M_{ijk}X^t_{ijk}}{\sum_{(j,k)} M_{ijk}X^t_{ijk}} \cdot L_i/X^t_{ijk} \qquad 4.3.3$$

where M_{ijk} is mileage, X^t_{ijk} is the throughput in arc (j,k), and L_i is the total quantity used or lost by company i. Here we have assumed that all parts of the pipeline network are equivalent in terms of pipeline fuel usage and losses. For long pipelines this may not be the case. For those it might have been better to estimate geographical factors which could differentiably weight different arcs to account for greater or lesser amounts of fuel used by the compressors in different terrains. Such factors might be quite difficult to actually estimate, however, so the simpler procedure was used.

4.3.4 Variable Transmission Costs

The variable cost of sending a unit of gas from j to k on pipeline i was estimated assuming that such unit costs were constant, i.e. that there were no economies of scale in operating costs. These unit transmission costs could be estimated, therefore, by the same method used to estimate the throughput efficiency factors. Total operating, maintenance, and labor costs associated with operations and maintenance were extracted from FPC Statistics of the Interstate Pipeline Companies for each company in each year 1966-1972. These expenses were then allocated to the various arcs in each pipeline's network just as losses were in the previous section. The unit cost t_{ijk} was thus estimated to be:

$$t_{ijk} = \frac{M_{ijk}X^t_{ijk}}{\sum_{(j,k)} M_{ijk}X^t_{ijk}} \cdot C_i/X^t_{ijk} \qquad 4.3.4$$

where C_i is total operating and maintenance expense for company i.

In this case cost per Mcf-mile was assumed to be uniform throughout the company's network. The comments in the previous section regarding geographical inhomogeneities apply to variable transmission costs as well.

4.3.5 Fixed Costs and Income Taxes

4.3.5.1 Capital Rate Base

In rate of return regulation the revenues which a company can make are limited to the coverage of its operating costs, depreciation, expense, income taxes, interest, and a rate of return on that portion of its capital plant which has not yet been written off.

The matter of how to measure the rate base, i.e., the capital value of a given plant (e.g. purchase cost or replacement value) and which items should be included and which not is historically one of vigorous controversy. For this model we have used the purchase cost of the company's transmission plant as reported in the FPC annual Statistics of Interstate Pipeline Companies. We have excluded capital investment in real estate, e.g. administrative buildings and equipment other than that actually used in gas transmission. We have also excluded the value of distribution plants for those interstate pipeline companies which have them. Sales by these companies to retail consumers are modelled as though they were made to a local utility in order to keep the demand model consistent at the wholesale level.

4.3.5.2 Depreciation Expense

In this model depreciation costs have been included as expenses just as they are in FPC accounting procedures. The actual depreciation practices of the interstate pipeline companies as they existed in 1969 have been reported by the FPC (1969). Though these practices vary from company to company, most pipelines depreciate their capital plans at around 3 percent per year (see Table 4.9).

TABLE 4.9 Depreciation Rates for a Sample of Eight Interstate Pipeline Companies (1969)

Company	Depreciation Rate	Accumulated Provisions for Depreciation
Alabama-Tennessee	3.39%	35.14%
Algonquin	3.01	27.06
Michigan-Wisconsin	3.56	29.90
Arkansas-Louisiana	2.76	31.85
Natural Gas PL of Am.	3.42	33.53
El Paso	3.35	38.84
Tenneco	3.09	33.59
United Gas PL	3.07	46.67

Source: Federal Power Commission Depreciation Practices of the Natural Gas Companies.

4.3.5.3 Interest

Each pipeline has a different ratio of debt and equity holdings. Debt cost (interest) is an expense for which full allowance is made in computing allowed revenues. To estimate the debt costs of pipelines, a fraction of their total capital plant in transmission equal to the ratio of debt to debt plus equity is multiplied by the "risk-free rate" (RFR) of BAA bonds as reported for the years 1966-1967 in Moody's Public Utilities Manual (1966-1972). These interest rates are listed in Table 4.10.

TABLE 4.10 BAA Bond Rates, 1966-1972

1966	5.665
1967	6.228
1968	6.936
1969	7.813
1970	9.105
1971	8.563
1972	8.155

Source: Moody's Public Utilities Manual (annual).

4.3.5.4 Income Taxes

Income taxes are calculated as a straight percentage of total revenues minus expenses, where depreciation and interest are included as expenses. Thus taxes paid, T, can be expressed as

$$T \quad = \quad t\,(R - OM - d\cdot K - RFR\cdot K^{d}) \qquad 4.3.5$$

In this equation t is the Federal plus state income tax rate, R is pipeline revenues, OM is operating and maintenance expense (including salaries for O&M), d is depreciation rate, K is total capital plant in transmission, RFR is the risk free (BAA) bond rate, and K^d is the fraction of capital plant financed by debt. If T is computed to be negative then this means that the taxable income of the pipeline is less than zero, in which case taxes paid, T, is set equal to zero.

4.3.5.5 Return on Equity

The FPC also allows the interstate pipeline companies a certain rate of return on <u>undepreciated</u> (equity) capital. Because pipeline companies are involved in buying and selling of pipeline sections, investing in new pipeline construction, phasing out old pipe, as well as depreciating currently operating pipe for tax purposes, it is difficult to estimate precisely from published statistics the level of undepreciated (equity) capital transmission plant for each company. As reported in Table 4.9, however, the FPC did publish estimates of accumulated provisions for depreciation in 1969.

In the model described here we have used these values to calculate undepreciated capital for each company under the assumption that for most companies the fraction of undepreciated capital varies only slowly with time.

The rate of return which the FPC allows on equity is always a matter of controversy. The lowest rate the FPC could legally allow is the actual cost of equity capital for the company. Evidence presented by Breyer and MacAvoy (1974) seems to indicate that the FPC has allowed rates that are actually several percentage points higher than the companies' opportunity cost of capital, that is, what stockholders could make in alternate enterprises with the same level of risk.

For this study estimates of the pipeline companies' actual equity costs were made on the basis of regression results obtained by Merrill, Lynch, Pierce, Fenner and Smith (1973). In their analysis, MLPFS computed the alpha and beta coefficients in the equation

$$r^{eq} = \alpha + \beta r^{m} \qquad 4.3.6$$

for all stocks on the New York Stock Exchange. Here r^{eq} is the return for the particular stock in question, and r^{m} is the average return on the market. Using values of r^{m} obtained from Standard and Poors Analysts Handbook (1973) and the α and β from MLPFS, the cost of equity capital was calculated for each pipeline company for each year 1966-1972 (see Table 4.11). Total equity costs C^{eq} could be

TABLE 4.11 Cost of Equity Capital for a Sample of U. S. Interstate Pipelines

Company	1966	1967	1968	1969	1970	1971	1972
Ark-La	0.1378	0.1258	0.1313	0.1269	0.1100	0.1156	0.1244
NGP Am	0.1043	0.0953	0.0994	0.0961	0.0833	0.0875	0.0942
El Paso	0.1391	0.0270	0.1325	0.1281	0.1110	0.1166	0.1256
Lone Star	0.1185	0.1082	0.1129	0.1091	0.0946	0.0994	0.1070
Texas East.	0.1146	0.1047	0.1092	0.1056	0.0915	0.0961	0.1035
Columbia	0.0940	0.0858	0.0896	0.0866	0.0750	0.0788	0.0849
Transco	0.1133	0.1035	0.1080	0.1044	0.0905	0.0905	0.1035
Return on Market (S&P)	0.1288	0.1176	0.1227	0.1186	0.1028	0.1080	0.1163

Source: Computed using data from the following sources: Merrill, Lynch, Pierce, Fenner, and Smith; Standard and Poors; Moody's Public Utilities Manual.

computed as

$$C^{eq} = r^{eq}(1 - d^{acc})K \qquad 4.3.7$$

where d^{acc} is accumulated provision for depreciation and K is total capital plant in transmission.

4.3.6 Price Schedules

Using the cost and flow values discussed above, we can calculate the gross revenues that a pipeline will have to make in order to cover its operating expenses, fixed costs, and taxes. In equation form using the variables defined previously and letting C be the total cost of gas,

$$\begin{aligned} R \;=\; & C + OM + d\cdot K + RFR\cdot K^{d} + t(R - C - OM - d\cdot K - RFR\cdot K^{d}) \\ & + r^{eq}(1 - d^{acc})K^{eq} \end{aligned} \qquad 4.3.8$$

Rearranging and solving for R one finds

$$R = C + OM + dK + RFR\cdot K^{d} + \frac{1}{1-t}\, r^{eq}(1-d^{acc})K^{eq} \qquad 4.3.9a$$

$$= C + OM + d\cdot K + \frac{1}{1-t}\left[\frac{(1-t)RFR}{1-d^{acc}} \cdot \frac{D}{D+E} + r^{eq}\cdot \frac{E}{D+E}\right]\overline{K} \qquad 4.3.9b$$

where D is total debt, E is equity, and $\overline{K}$ is total undepreciated capital plant in transmission ($\overline{K} = (1-d^{acc})K$). The expression in brackets is the average cost of capital, R, consisting of a weighted sum of debt and equity cost of capital. One might notice that the state plus Federal tax rate is another form of price control. The higher the tax rate, the higher the price must be for the pipeline company to cover its capital costs and operating expenses. If a pipeline company sells a quantity, Q, of gas, then the average price it must charge to cover its variable and fixed costs will be given by R/Q where R is given by 4.3.9b. If the pipeline were to charge all of its customers this same price it could be argued that customers close to the sources of supply would be discriminated against. On the other hand another strategy might be to charge each customer the actual cost of delivering the gas to him, i.e. the average gas cost plus the cost of transmission. This might seem to discriminate against the consumers farthest from the source of supply. In this model, a customer in state j pays only those variable and fixed costs associated with moving gas to state j. He does not pay part of the variable or fixed costs associated with movements further downstream. We have decided to use the latter strategy because it appears to be more economically efficient.

To determine how much of the fixed costs should be allocated to each arc (i,j,k) we examined the actual costs involved in constructing a pipeline as reported by the Federal Power Commission (1965). Though the actual costs are several years out of date, a very important relationship demonstrating the economies of scale involved in the construction of larger diameter pipelines can be gleaned from this data. In Figure 4.4 construction costs are plotted

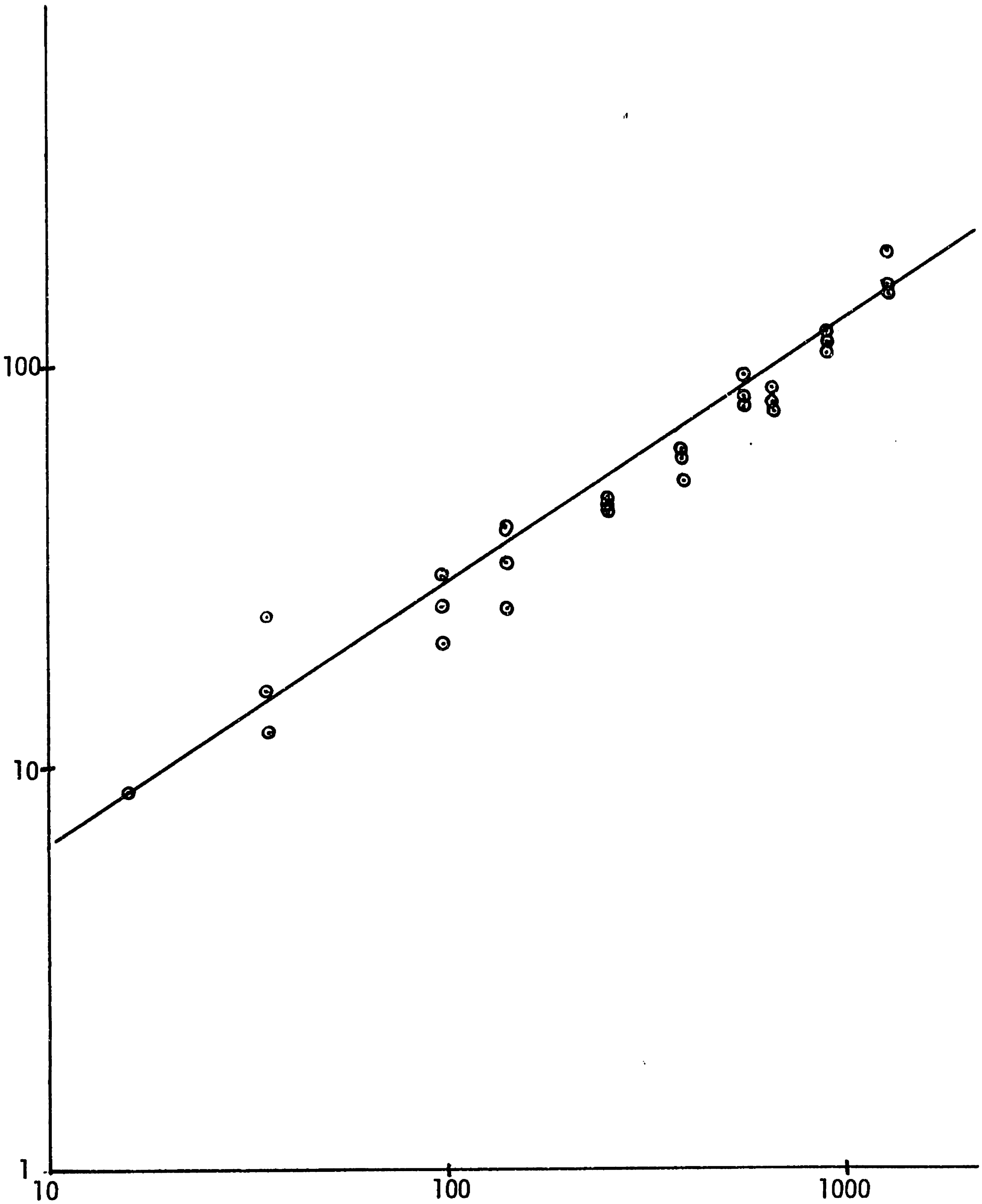

Figure 4.4 Construction Costs ($1000's) vs. Pipe Diameter^2 (Sq. in.)

against diameter squared for pipelines constructed in 1962, 1963, and 1964. An ordinary least squares analysis of this data produced the result

$$\log C = 3.1206 + 0.6589 \log D^2 \qquad 4.3.10$$

where C is construction cost per mile and D is pipe diameter. Since the capacity of a given pipe is proportional to its cross-sectional area and thus the diameter squared, the immediate inference is that the fixed costs associated with a given pipeline section are approximately proportional to the 0.66 power of its capacity. Using the estimates of capacity, b_{ijk}, which have been developed and described in Section 4.3.2, fixed costs can thus be allocated to the various sections of a company's system by use of the formula

$$N_{ink} = \frac{b_{ijk}^{0.66}}{\sum_{(j,k)} b_{ijk}^{0.66}} \cdot N_i \qquad 4.3.11$$

where

$$N_i = d \cdot K + \frac{R}{1-t} \overline{K} \ . \qquad 4.3.12$$

This method of allocating fixed cost around the various demand regions of the U. S. appears to be more economically efficient than having it equally shared (by a homogeneous price structure) or distributed rather arbitrarily according to some zone plan such as has been attempted by the FPC (see Breyer and MacAvoy, 1974).

It does have the major disadvantage, however, that two pipelines serving the same set of customers will not necessarily have the same price. In real life, the company with the lower price would have a decided advantage in such a circumstance. In this model the demand would be determined by the average price of gas delivered to the state, and not by the marginal costs of the next deliverable Mcf. Thus it does not mirror reality too precisely on this point. It is interesting to note, as Breyer and MacAvoy have pointed out, that the FPC has bent its own rules on the allocation of costs in order that no such advantageous competitive position should be allowed in markets where there is more than one interstate pipeline.

Chapter 5 Algorithms for the Solution of the Unregulated and Regulated Industry Models

5.1 Quadratic Programming Algorithms for the Unregulated Competitive Model

5.1.1 Introduction

The model of the unregulated competitive industry developed for this study has been presented in Section 3.6.2. In the price formulation with prices and rents as activities, and quantity flows and sales as dual variables, the model can be schematized as the quadratic program

$$\text{minimize} \quad cp + p'Qp \qquad \text{5.1.1a}$$

$$p \geq 0$$

$$\text{subject to} \quad x: \quad Ap \leq b \ . \qquad \text{5.1.1b}$$

where Q is the diagonal, positive semi-definite matrix of quadratic terms, where a prime indicates transpose, and where x is the vector of dual variables associated with row 5.1.1b.

In this section two algorithms will be discussed which can be used to solve a problem of this type. The first is the familiar algorithm of Wolfe (1959) which can be used to solve quadratic programs in which Q is symmetric and positive semi-definite. The second method was first developed by Dantzig (1963) and most recently modified and implemented by Orchard-Hayes (1975) at the National Bureau of Economic Research/Computer Research Center. This second algorithm is seemingly more restrictive than the first, requiring that Q be diagonal with all non-negative and at least one positive element. Since any symmetric positive semi-definite matrix can be diagonalized by a similarity transformation, however, its scope is actually just as great theoretically. As a practical matter, finding the proper transform may not be an easy task.

5.1.2 Wolfe's Algorithm

The first step in Wolfe's algorithm is to write the Kuhn-Tucker conditions for the problem, since Wolfe has shown that a Kuhn-Tucker point which is feasible for 5.1.1 is also a solution to the quadratic program.

The K-T conditions are

$$p'Q + c + xA \geq 0 \qquad 5.1.2a$$

$$x(Ap - b) = 0 \qquad 5.1.2b$$

$$(p'Q + c + xA)p = 0 \qquad 5.1.2c$$

along with the primal constraint set 5.1.1b.

If these equations are rewritten as equalities with explicit slacks, s and t, the set appears as follows:

$$Ap + s = b \qquad 5.1.3a$$

$$Qp + A'x' - t = -c \qquad 5.1.3b$$

$$xs = 0 \qquad 5.1.3c$$

$$pt = 0 \qquad 5.1.3d$$

$$p,x,s,t \geq 0 \quad . \qquad 5.1.3e$$

Since x, s, p, and t are all non-negative, 5.1.3c,d actually require at least one of the pair (x_i, s_i) and one of (p_j, t_j) must be zero for each i and j. Equations 5.1.3a,b (along with 5.1.3e) on the other hand are linear equations in x, s, p, and t. This suggests that a modified linear programming approach which always keeps at least one of each of the pairs (x,s) and (p,t) at zero value might be used to solve this problem. Wolfe's algorithm does just that. It poses the problem as a phase 1 linear program on the constraints 5.1.3a,b, and e with the additional "basis entry criterion" that if x_i is in the basis then s_i cannot enter and vice versa, with the same condition holding for the pair p_j and t_j. Wolfe demonstrates that this algorithm will converge to an optimal solution if Q is symmetric and positive semi-definite.

5.1.3 The DIAGQLP Procedure of SESAME

For problems where Q can be made diagonal with all non-negative elements, the algorithm of Dantzig (1963) as implemented by Orchard-Hayes (1975) on the National Bureau of Economic Research's SESAME mathematical programming system can be used. In this scheme a basic feasible solution to the constraints $Ax \leq b$ is first obtained by using a standard phase 1 linear program. A solution to the quadratic program is then obtained by a feasible direction method using a modified simplex algorithm.

As in the usual simplex method, DIAGQLP has criteria for choosing a new vector to enter the basis and one to leave. If the current set of basic variables is $\{u_j\}$ then

$$\sum_j A_{ij} u_j \quad = \quad b_i \qquad 5.1.4$$

and the objective function z is equal to

$$z \quad = \quad \sum_j (c_j u_j + q_j u_j^2) \qquad 5.1.5$$

where q_j is the jjth element of the diagonal matrix Q.

If some currently nonbasic variable, p_k, is selected to enter the basis then it will enter at some new non-zero value θ.

Let $\overline{A}_{jk}$ be the representation of A_{ik}, the coefficient column for the variable p_k, in terms of the current basis. Then the new solution is

$$\sum_j A_{ij} [u_j - \theta \overline{A}_{jk}] + A_{ik}\theta \quad = \quad b_i \qquad 5.1.6$$

with corresponding value of z,

$$z' = \sum_j [c_j(u_j - \theta\bar{A}_{jk}) + q_j(u_j - \theta\bar{A}_{jk})^2] + c_k\theta + q_k\theta^2 \quad 5.1.7$$

Thus the change in the objective function is

$$\Delta z = [q_k + \sum_d \bar{A}^2_{jk}q_d]\theta^2 - [2 \sum_j u_j\bar{A}_{jk}q_j + \sum c_j\bar{A}_{jk} - c_k]\theta \quad 5.1.8a$$

$$= V_{2k}\theta^2 - 2V_{ik}\theta + d_k\theta \quad 5.1.8b$$

where d_k is the usual reduced cost $c_k - \sum_j c_j \cdot \bar{A}_{jk}$. For the new vector to give an improvement with $\theta > 0$, the modified reduced cost d'_k must satisfy

$$d'_k = d_k - 2V_{ik} < 0 \; . \quad 5.1.9$$

The nonbasic vector p_k with the smallest value of d'_k is selected to enter the basis.

The vector which leaves the basis is determined by choosing θ to achieve the maximum decrease in the objective function while yet maintaining feasibility. Two conditions are thus imposed on θ:

$$\theta \leq \theta^\circ = \min_j [u_j/\bar{A}_{jk}] \quad 5.1.10$$

(so that the new u_j's are all feasible) and

$$\Delta z(\theta) < 0 \; . \quad 5.1.11$$

The latter condition is satisfied for the values

$$0 \quad < \quad \theta \quad < \quad \frac{2V_{1k-dk}}{V_{2k}} \qquad 5.1.12$$

The minimum of $\Delta z(\theta)$ is found by setting its derivative equal to Q. The resulting value, θ^*, is at the midpoint of the range 5.1.12.

$$\theta^* \quad = \quad \frac{2V_{1k-dk}}{2V_{2k}} \qquad 5.1.13$$

(See Figure 5.1.) If θ° is less than $2\theta^*$ then the vector which satisfies

$$\theta^\circ \quad = \quad \min_{d} \left[\frac{u_j}{B_{jk}} \right] \qquad 5.1.14$$

is removed from the basis. If θ° is greater than or equal to $2\theta^*$ then bringing this vector into the basis would actually increase the objective function, possibly resulting in non-convergence of the algorithm. In this case the variable p_k is not brought into the basis but is instead fixed at the new value $\overline{p}_k + \theta^*$ for a maximal decrease in the objective function. If there still exist such fixed vectors when the algorithm can find no more vectors to bring into the basis, a special procedure is initiated which brings as many of these fixed vectors into the set of basic variables as possible. When all the modified reduced costs are non-negative, Dantzig shows that an optimal solution has been found.

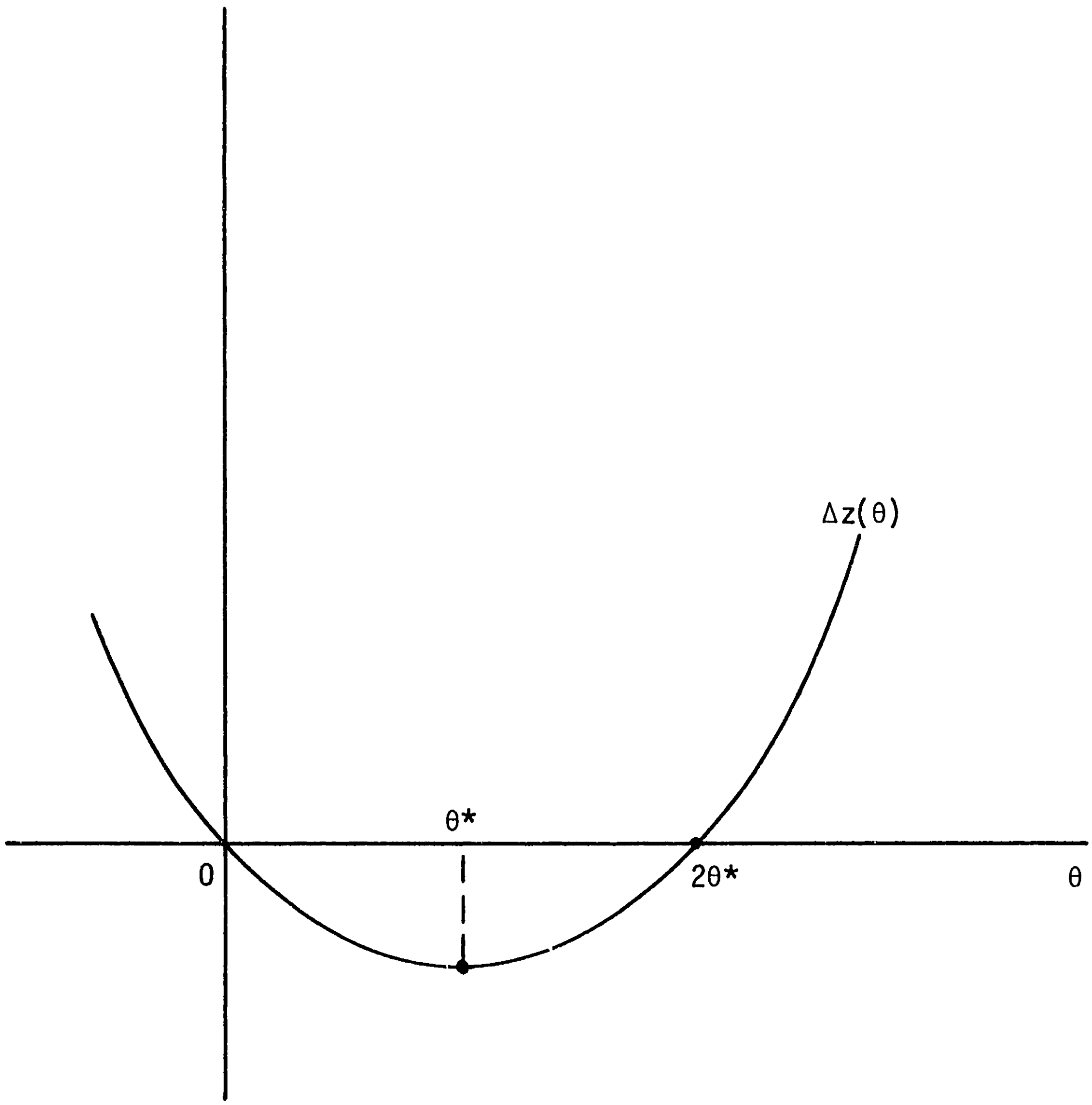

Figure 5.1 The Minimum of Δz(θ).

5.1.4 Computational Experience with DIAGQLP

Using the IBM 370/168 computer at Cornell University operating under VM, the DIAGQLP procedure has been used to solve the unregulated competitive model described in Section 3.6.2 for the years 1966 to 1972. This model, which has 1361 rows and 1401 variables, was solved first for 1972 in 1069 iterations. The optimal basis for 1972 was then used to initiate the 1971 problem, which had both a different right hand side and a different objective function. This basis was initially infeasible, but became feasible and then optimal in 504 iterations. This optimal basis was then used as a starting basis for the 1970 model which was solved in 405 iterations. Table 5.1 shows the results of these computer runs in terms of iterations required to solve the quadratic program. It shows that a considerable amount of computational effort was saved by using the previous optimal basis to start each new problem. Even when an all slack basis was used for the 1972 case, only 1069 iterations were required for a problem with 1361 rows. The diagonal quadratic programming algorithm of Orchard-Hayes thus appears to be relatively efficient for the solution of the competitive spatial equilibrium problem in the dual form.

TABLE 5.1 Number of Iterations to Solve the Quadratic Program

Year	Initial Basis	No. of Iterations
1972	all slack	1069
1971	1972 optimal	504
1970	1971 optimal	405
1969	1970 optimal	443
1968	1969 optimal	292
1967	1968 optimal	408
1966	1967 optimal	632

Number of rows: 1361

Number of variables: 1401

5.2 Nonlinear Programming Algorithm for the Regulated Industry Model

5.2.1 Introduction

The model of the regulated industry described in Section 3.6.4 poses serious difficulties for theoretical analysis. The program has a convex (quadratic) objective function but the constraints do not define a convex feasible region. In particular the non-linear constraints which represent FPC rate of return regulation and cost allocation are neither convex nor concave and the conservation of mass constraints are equalities. The results which have been obtained for convex programs (convex objectives over convex feasible regions), regarding the relations between Kuhn-Tucker points and solutions to the convex program do not necessarily apply to the problem presented here.

In this section three methods are presented which might conceivably be used in attempts to solve the problem. The first two are methods which have been used to solve many nonlinear programs. The third method is a new modified linearization method for programs of the special type of interest in this study.

An important result is obtained in this study: this new algorithm converges to a Kuhn-Tucker point of the nonlinear nonconvex program. Whether this Kuhn-Tucker point is a solution is not yet known.

5.2.2 Rosen's Partitioning Method

The following method for non-linear programming problems was developed by Rosen (1963) and used as part of a complex multi-refinery optimization model. The non-linear constraints of the Rosen model, though representing a very different process (achievement of required octane levels), turn out to be almost identical to those in this study (representing rate of return limitations).

The model can be written as follows:

$$\text{minimize} \quad \sum_{i=1}^{n} c_i(y)x_i + c_0(y) \qquad 5.2.1a$$

$$\text{subject to} \quad A_i(y)x_i \geq b_i(y) \qquad i = i,\ldots n \qquad 5.2.1b$$

where $c_i(y)$ and $b_i(y)$ are vector functions of the vector y and A_i is a matrix whose components are each functions of y, and which explicitly includes the non-negativity constraints on x_i, which are vector variables. Given a specific value of y, the problem reduces to a set of non-interacting linear programs. The Rosen partition method is a procedure for determining a sequence of y's such that the limit of the sequence will be an optimal value for y for the problem 5.2.1.

Lasdon (1970) shows how this procedure is developed and gives a proof of its convergence when c_i and A_i are constant and $b_i(.)$ are convex differentiable functions.

The regulated model of section 3.6.4 is a special case of 5.2.1 with a single block of constraints, constants $c \geq 0$ and $b \geq 0$, linear functions $A(\cdot)$ and quadratic (diagonal) $c_0(y)$. With a change in the direction of the inequalities it can be written as

$$\text{minimize} \quad cx + c_0(y) \qquad 5.2.2a$$

$$\text{subject to} \quad A(y)x \leq b \qquad 5.2.2b$$

A variation of Rosen's approach can be used to generate an algorithm which could be used to try to solve this non-linear program. Because the feasible set is not convex, however, this method will not necessarily be guaranteed to reach an optimal solution. This is also the case for Rosen's refinery problem, where no proof of convergence has been given.

Suppose that a set of values, y_0, is given such that there is a feasible solution to $A(y_0)x \leq b$. Then the matrix A (which has more rows than columns) can always be broken into two parts, an invertible matrix A_1 and the remainder A_2. If A_1 is chosen to be an optimal basis of the problem 5.2.2 with $y = y_0$, then an equivalent problem would be

$$\text{minimize} \quad cx + c_0(y_0) \qquad 5.2.3a$$

$$\text{subject to} \quad A_1(y_0)x = b_1 \qquad 5.2.3b$$

$$A_2(y_0)x \leq b_2 \qquad 5.2.3c$$

Here A_1 is related to the basis B in the usual notation as follows:

$$A = \begin{bmatrix} B & 0 \\ 0 & -I \end{bmatrix} \qquad 5.2.4$$

where B is an invertible subset of A corresponding to positive values for x and the negative identity matrix corresponds to non-negativity constraints and thus those components of x which are 0 in this basis. Since A_1 is invertible we may write

$$x = A_1^{-1}(y_0)b_1 \qquad 5.2.5$$

and

$$cx + c_0(y_0) = cA_1^{-1}(y_0)b_1 + c_0(y_0) \qquad 5.2.6a$$

$$= W_1 b_1 + C_0(y_0) \qquad 5.2.6b$$

where W_1 are the shadow prices for the relations 5.2.3b.

A "coordinating" problem for the y's could thus be written as

$$\text{minimize} \quad c_0(y) + CA_1^{-1}(y)b_1 \qquad 5.2.7a$$

$$\text{subject to} \quad A_2(y)\, A_1^{-1}(y)b_1 \leq b_2 \qquad 5.2.7b$$

These constraints will be generally non-linear, non-convex, and non-concave as a result of the inversion of A_1 (y). If the second term in 5.2.7a and the constraints 5.2.7b are linearized around $y = y_0$, the resulting program will be quadratic with linear constraints.

Since y_0 is a feasible vector for 5.2.7b the solution to this quadratic program will produce a lower value for the objective function than would y_0. Whether it would result in an overall improvement in the unlinearized objective function 5.2.7a is not known. If it did then using y_1 to relinearize 5.2.2, solving the resulting linear program, using the results to form a new coordinating problem, and so forth, may eventually yield a solution.

Because of the non-convex nature of the constraints $A_i(y)x \leq b_i$, one cannot say whether this method will converge or not for problems of the type of interest here. Thus unfortunately it is a bit difficult to assess the meaning of the answer one would get by applying this method. When $A(\cdot)$ and $C(\cdot)$ are constants, each iteration produces a feasible solution with non-increasing objective function. Thus computations may be terminated at any point with an objective function no higher than all previous ones. With the non-convex problem of interest here, this may not be so. Yet Rosen, _et al_. (1963) have used this method to generate "solutions" to problems "isomorphic" to the regulated network problem of section 3.6.4.

Because of the uncertainties involved in this method, this algorithm was not used to analyze the regulated model of section 3.6.4.

5.2.3 The Method of Approximate Programming (MAP)

In the Griffith and Steward (1961) method of approximate programming, the nonlinear relations are first linearized around an arbitrary starting point, x_0. The linearized problem, now a linear program, is then solved under additional conditions that limit the amount that each variable can move away from x_0. This new solution is then used as the new point about which the problem is relinearized and again solved. This procedure is continued until two consecutive solutions satisfy the convergence inertia of the problem.

Symbolically the non-linear problem

$$\text{minimize} \quad M(x) \qquad 5.2.20a$$

$$\text{subject to} \quad g(x) \leq 0 \qquad 5.2.20b$$

is approximated by

$$\text{minimize} \quad [\nabla M(x^k)]x \qquad 5.2.21a$$

$$\text{subject to} \quad g(x^k) + \nabla g(x^k)(x-x^k) \leq 0 \qquad 5.2.21b$$

$$\text{and} \quad |x_i - x_i^k| < \delta_i^k \qquad 5.2.21c$$

where x^k is the point about which the non-linear functions M and g are linearized and where δ_i^k are small numbers limiting the steps which x_i may take. If the solution criteria

$$|M(x^k) - M(x^{k+1})| \quad < \quad \tau_0 \qquad 5.2.22a$$

$$g_j(x^{k+1}) \quad \leq \quad \tau_2 \qquad 5.2.22b$$

are achieved by the solution x^{k+1}, where $\tau_i > 0$ are preset tolerances, then the problem is solved -- a local minimum has been found. If not then the nonlinear functions are again linearized around x^{k+1}, the procedure continuing one more step.

According to Jacoby et al. (1972), no proof of convergence has been obtained for this algorithm, even in the case of convex functions. In spite of this, the method has been used for the solution of a wide variety of convex and non-convex problems. The art of the method lies in choosing the step bounds δ^k. If the step size is not right for the problem, slow convergence may result. No general strategies regarding the selection of these step sizes have been developed.

5.2.4 A Modified Linearization Algorithm for Programs with Quadratic Cross-Product Constraints

In section 3.6.4 a model of the regulated natural gas industry was presented which consisted of the minimization of a quadratic function of wellhead and wholesale prices and natural gas flows over a constraint set containing both linear and quadratic cross-product constraints. In matrix form this model could be written in terms of a price vector, p, and flow vector, x, as

$$\text{minimize} \quad p'Qp + ep + cx \qquad 5.2.23a$$

$$\text{subject to} \quad Ax + Bp \leq b \qquad 5.2.23b$$

$$(p'E_i - a_i)x \leq d_i \qquad i = 1,\ldots n \qquad 5.2.23c$$

$$p,\ x \geq 0 \qquad 5.2.23d$$

where Q is a diagonal matrix with nonnegative elements; where A, B, and E_i are constant matrices; and where a_i, b, c, d, and e are vector constants with a_i, c $\geq$ 0.

If the constraints 5.2.23c are linearized around the point $(p_k,\ x_k)$ they become

$$p_k'E_ix_k + (p-p_k)'E_ix_k + p_k'E_i(x-x_k) - a_ix \leq d_i \qquad 5.2.24$$

This can be written as

$$(p_k'E_i - a_i)x + (p'E_i - a_i)x_k \leq d_i + p_k'E_ix_k - a_ix_k \qquad 5.2.25$$

which, if (p_k, x_k) is a feasible vector for the constraints 5.2.23c implies that

$$(p_k'E_i - a_i)x + (p'E_i - a_i)x_k \leq 2d_i \qquad 5.2.26$$

Now suppose that (p_k, x_k) is an optimal solution to the nonlinear problem 5.2.23. Then it is clear from the symmetry of 5.2.26 that (p_k, x_k) is also a feasible vector for these constraints. This suggests that a possible approach to solving 5.2.23 would be to choose a point (p_k, x_k), substitute the linear constraints 5.2.26 for the nonlinear constraints 5.2.23c and solve the resulting problem for a new point (p_{k+1}, x_{k+1}). If $(p_{k+1}, x_{k+1}) = (p_k, x_k)$ then this point would be a feasible point for the nonlinear program 5.2.23 and a potential solution. If $(p_{k+1}, x_{k+1}) \neq (p_k, x_k)$, then one might use the new point to relinearize the constraints 5.2.23c and solve the program again. The resulting solution (p_{k+2}, x_{k+2}) would then be tested for equality with (p_{k+1}, x_{k+1}) and so forth.

As it stands, there is no reason to believe that this algorithm would converge because there is no incentive for (p_{k+1}, x_{k+1}) to equal (p_k, x_k). We could improve the algorithm by including a term in the objective function which attempts to provide this incentive. The k^{th} iteration of the algorithm would then be the quadratic program:

$$\text{minimize} \quad p'Qp + ep + cx + N_k(|p-p_k|^2 + |x-x_k|^2) \qquad 5.2.27a$$

$$\text{subject to} \quad Ax + Bp \leq b \qquad 5.2.27b$$

$$(p_k'E_i - a_i)x + (p'E_i - a_i)x_k \leq d_i \qquad 5.2.27c$$

$$i = 1, \ldots n$$

$$p, x \geq 0 \qquad 5.2.27d$$

where N_k is a positive constant and all other symbols have been previously defined.

The symmetry of the linearized constraints 5.2.27c leads to a very interesting property: if (p_k, x_k) is optimal for the $(k-1)^{st}$ subproblem of the algorithm; i.e., the quadratic program with constraints linearized around (p_{k-1}, x_{k-1}), then (p_k, x_k) will be a feasible vector for the $(k+1)^{st}$ subproblem. This can be seen as follows: the constraints of the $(k+1)^{st}$ subproblem are:

$$Ax + Bp \leq b \qquad 5.2.28a$$

$$(p_{k+1}'E_i - a_i)x + (p'E_i - a_i)x_{k+1} \leq 2d_i \qquad 5.2.28b$$

$$p, x \geq 0 \qquad 5.2.28c$$

where (p_{k+1}, x_{k+1}) is the optimal solution to the k^{th} subproblem. Now since (p_k, x_k) is feasible in the $(k-1)^{st}$ subproblem it satisfies 5.2.28a and 5.2.28c. In addition since it is optimal in the $(k-1)^{st}$ subproblem, it is used as the point of linearization for the k^{th} subproblem which is solved by (p_{k+1}, x_{k+1}). Thus it must be true that

$$(p_k'E_i - a_i)X_{k+1} + (P_{k+1}'E_i - a_i)x_k \leq 2d_i . \qquad 5.2.29$$

By interchanging the terms on the left-hand side and comparing with 5.2.28b, one can see that (p_k, x_k) satisfies 5.2.28b. Thus it is a feasible vector for the $(k+1)^{st}$ subproblem.

The implication of this property is that if there exist feasible vectors (p_1, x_1) and (p_2, x_2) to the subproblems linearized around a starting point (p_0, x_0) and the generated point (p_1, x_1) respectively, then all further subproblems will have nonempty feasible regions. Thus if one starts with an appropriate point (p_0, x_0), this procedure will generate a sequence of points (p_k, x_k) which are optimal solutions to the sequence of quadratic subproblems 5.2.27.

The important questions now are: will this algorithm converge? That is, will the vectors (p_k, x_k) become arbitrarily close to some limiting point (p^*, x^*) as k becomes large? And if it does exist, what is the significance of the point (p^*, x^*)?

To answer the first question the global convergence theorem for descent algorithms (Luenberger, 1973) will be invoked in a slightly generalized form. First let us define some terms needed to describe the theorem. In the following an <u>algorithm</u> A is defined to be a point-to-set mapping on a space X that assigns to every point $x \varepsilon X$ a subset of X. Operated iteratively the algorithm will generate a sequence of points $\{X_k\}$ such that $x_{k-1} \varepsilon A(x_k)$.

A solution set B for the algorithm A is defined as a subset of X given by the nature of the problem under study. For example if we are trying to find a fixed point of a mapping A then we might define a solution set to consist of all points y such that if $y \epsilon A(x)$ then $|y-x|^2 < \delta$ for some predetermined $\delta > 0$.

A descent function Z of A and B is a continuous real valued function on X with the properties that

(1) if $x \notin B$ and $y \epsilon A(x)$, then $Z(y) < Z(x)$, and

(2) if $x \epsilon B$ and $y \epsilon A(x)$ then $Z(y) \leq Z(x)$.

A descent function sequence $\{Z_k\}$ of A and B on a closed subset $T \subseteq X$ is defined to be a sequence of continuous real valued functions Z_k such that

(1) $\{Z_k\}$ is uniformly convergent to a continuous real valued function Z on T;

(2) if $x_k \notin B$, $x_k \epsilon T$, and $y \epsilon A(x_k)$, then $Z_k(y) < Z_{k-1}(x_k)$ for $k=1,\ldots$;

(3) if $x_k \to x \notin B$, $x_k \epsilon T$, $x_{k+1} \epsilon A(x_k)$ and $y \epsilon A(x)$ then $Z(y) < Z(x)$;

(4) if $x \epsilon B$ and $y \epsilon A(x)$, then $Z(y) \leq Z(x)$.

The algorithm A is said to be closed at $x \epsilon X$ if the assumptions

(1) $x_k \to x$ for $x_k \epsilon X$,

(2) $y_k \to y$ for $y_k \epsilon A(x_k)$, imply that

(3) $y \epsilon A(x)$.

A is said to be closed on the set X if it is closed at each point of X.

The statement of the theorem is as follows.

Generalized Global Convergence Theorem:

Let A be an algorithm on X, and suppose that, given x_0, the sequence $\{x_k\}_{k=0}^{\infty}$ is generated satisfying $x_{k+1} \in A(x_k)$. Let a solution set $B \subset X$ be given and suppose

(1) all points x_k are contained in a compact set $T \subset X$;

(2) there exists a descent function sequence $\{X_k\}$, k=1,2,... of A and B on the set T;

(3) the mapping A is closed at points outside B;

then the limit of any convergent subsequence of $\{x_k\}$ is a solution.

The proof of this theorem follows the steps of Luenberger (1973).

Proof: Suppose the convergent subsequence $\{x_k\}$, $k \in S$ converges to the limit x. First we show that the sequence $Z_{k-1}(X_k)$ converges to Z(x) on the subsequence $\{x_k\}$, $k \in S$. Using the triangle inequality we have

$$|Z_{k-1}(x_k) - Z(x)| = |Z_{k-1}(x_k) - Z(x_k) + Z(x_k) - Z(x)| \quad 5.2.30a$$

$$\leq |Z_{k-1}(x_k) - Z(x_k)| + |Z(x_k) - Z(x)| \quad 5.2.30b$$

By the uniform convergence of $\{Z_k\}$ to Z for each $\varepsilon_1 > 0$ there exists an integer N_1 such that for $k > N_1$

$$|Z_k(x') - Z(x')| < \varepsilon_1 \text{ for all } x' \in T \quad 5.2.31$$

By continuity of Z for each $\varepsilon_2 > 0$, there exists an integer N_2 such that for $k > N_2$, $k \in S$

$$|Z(x_k) - Z(x)| < \varepsilon_2 \quad . \quad 5.2.32$$

Thus for $k>N = Max(N_1-1,N_2)$, $k\varepsilon S$ we have

$$|Z_{k-1}(x_k) - Z(x)| \quad < \quad \varepsilon_1 + \varepsilon_2 \qquad 5.2.33$$

i.e. $Z_{k-1}(x_k) \rightarrow Z(x)$ on the subsequence $k\varepsilon S$.

Now we wish to show that $Z_{k-1}(x_k) \rightarrow Z(x)$ on the entire sequence. First by monotonicity of $Z_{k-1}(x_k)$

$$Z_{k-1}(x_k) \quad \geq \quad Z(x); \qquad 5.2.34$$

otherwise $Z_{k-1}(x_k)$ could not converge to $Z(x)$ on the subsequence $k\varepsilon S$ because it would continue to get smaller by the monotonicity of $Z_{k-1}(x_k)$. Now by monotonicity of $Z_{k-1}(x_k)$ and convergence of $Z_k(x_k)$ to $Z(x)$, we may write

$$Z_{k-1}(x_k) \quad \geq \quad Z_{m-1}(x_m) \quad \geq \quad Z_{n-1}(x_n) \quad \geq \quad Z(x) \qquad 5.2.35$$

where $k<m<n$ and k and $n\varepsilon S$ and $m\notin S$. That is, since $Z_{k-1}(x_k)$ converges to $Z(x)$ on the subsequence S, there will always be an $n>m$ such that $n\varepsilon S$ if $m\notin S$. By taking the limit as k and n go to infinity on the subsequence S we get

$$Z(x) \quad \geq \quad \lim_{m\rightarrow\infty} Z_{m-1}(x_m) \quad \geq \quad Z(x) \qquad 5.2.36$$

Thus $Z_{k-1}(x_k) \rightarrow Z(x)$ on the entire sequence $k = 1,2,\ldots.$

The remainder of the proof follows the line of reasoning in Luenberger. To show that x is a solution assume it is not. Consider the subsequence $\{x_{k+1}\}$, $k\varepsilon S$. Since all members of the sequence are contained in the compact set T, there is a $\bar{S}\subset S$ such that $\{x_{k+1}\}$, $k\varepsilon\bar{S}$ converges to some limit $\bar{x}$. We thus have $x_k \to x$, $k\varepsilon\bar{S}$, and $x_{k+1}\varepsilon A(x_k)$ with $x_{k+1} \to \bar{x}$, $k\varepsilon\bar{S}$. But since A is closed at x it follows that $\bar{x}\varepsilon A(x)$. From the previous paragraph $Z_k(x_{k+1}) \to Z(\bar{x})$ on the subsequence $k\varepsilon\bar{S}$. But also $Z_{k-1}(x_k) \to Z(x)$ on the entire sequence k = 1,2,.... Thus $Z(\bar{x}) = Z(x)$. But this violates condition (3) of the definition of a descent function sequence which requires that $Z(\bar{x}) < Z(x)$. Thus the assumption that x is not a solution is contradicted. QED.

For the particular problem of interest here, the mapping A takes the point (p_k,x_k) into the solution set of problem 5.2.27, the solution set B is given by the set $\{(p,x)\}$ such that

$$|p-\bar{p}|^2 + |x-\bar{x}|^2 < \delta \qquad 5.2.37$$

where $(p,x)\,\varepsilon\, A(\bar{p},\bar{x})$ and δ is some small positive number, and the Z_k are chosen to be the continuous functions:

$$Z_k(p,k) = \frac{1}{N_k}(p'Qp+ep+cx) + |p-p_k|^2 + |x-x_k|^2 \qquad 5.2.38$$

where $(p_k,x_k)\varepsilon A(p_{k-1},x_{k-1})$ k=1,2,... and (p_0,x_0) is arbitrary.

In using this theorem we shall first show that the set $\{Z_k\}$ is a descent function sequence. Then we shall show that A is point-to-point and continuous and, therefore, closed. Finally we shall show that the feasible set of each iteration in the algorithm is compact. With these three conditions satisfied the generalized global convergence theorem holds and the limit of any generated sequence $\{p_k, x_k\}$ $k=1,2,\ldots$ is a solution to the problem.

To see that $\{Z_k\}$, $k=1,2,\ldots$ is a descent function sequence consider the following: if (p_{k+1}, x_{k+1}) is optimal at stage k, then since (p_{k-1}, x_{k-1}) is feasible at this stage, and since Z_k is proportional to the objective function in 5.2.27,

$$Z_k(p_{k+1}, x_{k+1}) \leq Z_k(p_{k-1}, x_{k-1}) \ . \qquad 5.2.39$$

Using the definition of Z_k we have

$$Z_k(p_{k-1}, x_{k-1}) = \frac{1}{N_k}[p'_{k-1}Qp_{k-1} + ep_{k-1} + cx_{k-1}] + |p_{k-1} - p_k|^2 + |x_{k-1} - x_k|^2 \qquad 5.2.40$$

But

$$Z_{k-1}(p_k, x_k) = \frac{1}{N_{k-1}}[p'_kQp_k + ep_k + cx_k] + |p_k - p_{k-1}|^2 + |x_k - x_{k-1}|^2 \qquad 5.2.41$$

Combining 5.2.39 - 5.2.41 we see that $Z_k(p_{k+1}, x_{k+1}) < Z_{k-1}(p_k, x_k)$ if and only if

$$N_k > N_{k-1}\, f_k/f_{k-1} \qquad 5.2.42$$

where each f_k (assumed to be nonzero) is defined by

$$f_k = p_k'Qp_k + cp_k + ex_k \text{ for any } k = 1,2,\ldots. \qquad 5.2.43$$

If we choose

$$N_k = C_k N_{k-1} |f_k/f_{k-1}| \qquad 5.2.44$$

where $C_k > 1$ and $\prod_{k=1}^{\infty} C_k < \infty$,

then the sequence

$$\{Z_k(p_{k-1},x_{k-1})\} \quad k = 1,2,\ldots \qquad 5.2.45$$

will be monotonically decreasing. Thus conditons (2) and (4) in the definition of a descent function sequence are satisfied.

To see that the sequence is uniformly convergent (condition (1)), consider the sequence $(p_k,x_k) \to (p^*,x^*)$. Then define $Z(p,x)$ to be

$$Z(p,x) = \frac{1}{N^*}[p'Qp + ep + cx] + |p-p^*|^2 + |x-x^*|^2 \qquad 5.2.46$$

where N^* is the limit of N_k on the subsequence S such that $(p_k,x_k) \to (p^*,x^*)$.

We can show that $N^* < \infty$ exists since we may write it as

$$N^* = \lim_{\substack{k\to\infty \\ k\varepsilon S}} N_k = N_0 \prod_{k=1}^{\infty} C_k \cdot \lim_{\substack{k\to\infty \\ k\varepsilon S}} {}^{f_k}/_{f_0} \qquad 5.2.47$$

But $\prod_{k=1}^{\infty} C_k < \infty$ and since f_k is a continuous function of (p_k,x_k) bounded on the compact set T, $\lim_{\substack{k\to\infty \\ k\varepsilon S}} {}^{f_k}/_{f_0} < \infty$ exists. Therefore $N^* < \infty$.

An example of a sequence C_k with the property that $\prod_{k=1}^{\infty} C_k < \infty$ is $C_k = \exp\{k^{-2}\}$. Here $\prod_{k=1}^{\infty} \exp\{k^{-2}\} = \exp\{\sum_{k=1}^{\infty} k^{-2}\}$. Since the summation is bounded by

$$1 + \int_1^{\infty} x^{-2}dx \quad = \quad 1 + (-\tfrac{1}{x})_1^{\infty} \quad = \quad 2 \qquad 5.2.48$$

we have that N* exists and is less than $e^2 N_0|f^*/f_0|$.

We have also

$$Z(p,x) - Z_k(p,x) \quad = \quad (\frac{1}{N^*} - \frac{1}{N_k})(p'Qp + ep + cx)$$

$$+ |p-p^*|^2 + |x-x^*|^2 \qquad 5.2.49a$$

$$= \quad \frac{N_k - N^*}{N^* N_k} \; (pQp + ep + cx)$$

$$+ (p^*-p_k)(p^*+p_k-2p) + (x^*-x_k)(x^*+x_k-2x)$$

5.2.49b

If we again assume (p,x) are contained in a compact set then all the expressions in 5.2.49 are bounded. Thus the entire expression is bounded independently of (p,x). Since $N_k \rightarrow N^*$ and $x_k \rightarrow x^*$ on the subsequence S, $Z_k(p,x) \rightarrow Z(p,x)$ uniformly for all (p,x).

Condition (3) of the definition of a descent function sequence is proved as follows: assume $\{p_k, x_k\}$ is a convergent sequence. Then there exists an integer N such that for k>N we have

$$|(p_k,x_k) - (p_{k-1},x_{k-1})| \leq |(p_k,x_k) - (p^*,x^*)| + |(p^*,x^*) - (p_{k-1},x_{k-1})| \qquad 5.2.50a$$

$$< \varepsilon + \varepsilon \qquad k > N \qquad 5.2.50b$$

Squaring we get

$$|p_k - p_{k-1}|^2 + |x_k - x_{k-1}|^2 < 4\varepsilon^2 \qquad 5.2.51$$

Choosing ε so that $\varepsilon < \sqrt{\delta/4}$, (p_k, x_k) becomes a solution to the problem since the convergence criterion has been met. Therefore a convergent sequence always results in a solution. Condition (3) is satisfied by logical default, i.e. it is true because the condition which could make it false never occurs. Thus the final condition for the existance of a descent function sequence has been satisfied.

We now show that the algorithm A is closed since it is actually a point-to-point and continuous mapping. It is point-to-point since 5.2.27 has a strictly convex (diagonal positive definite) objective frunction with linear constraints at each iteration. Thus at each iteration the optimal solution is unique.

Next we show that A is continuous. Consider the linear constraint set of 5.2.27 and for simplicity let one of them be given by

$$ay \leq b \qquad 5.2.52$$

where y is (p,x), a is a vector, and b a constant. Now a and b are actually functions of the linearization point x_k. By moving this point slightly the constraint set becomes

$$[a(x_k) + J^a(x_k)\cdot dx_k]\, x \leq b(x_k) + \nabla b(x_k)\cdot dx_k \quad . \qquad 5.2.53$$

But since $a(x_k)$ and $b(x_k)$ are linear functions of x_k, the Jacobian $J^a(x_k)$ and gradient $\nabla b(x_k)$ are constants. Thus the hyperplane bounding the half-space implied by this constraint is rotated and translated by the movement in x_k.

Consider a point Z on the original hyperplane and another point $\overline{Z}$ the intersection of an arbitrary ray from the original hyperplane to the second hyperplane. We may write the inner product

$$a(\overline{Z} - Z) = |a|\ |\overline{Z} - Z|\sin\theta \qquad 5.2.54$$

where θ is the angle of the ray to the normal from the original hyperplane. Thus

$$|\overline{Z} - Z| = \frac{a(\overline{Z} - Z)}{|a|\sin\theta} \qquad 5.2.55a$$

$$= \frac{a\overline{Z} - b}{|a|\sin\theta} \qquad 5.2.55b$$

$$= \frac{\nabla b \cdot dx_k - J^a \cdot dx_k}{|a| \sin\theta} \qquad 5.2.55c$$

since $az = b$ and $(a + J^a \cdot dx_k)\bar{Z} = b + \nabla b \cdot dx_k$.

Thus the distance $|\bar{Z} - Z|$ is proportional to $|dx_k|$. This means that a small change in the point x_k changes each point on the boundary of the feasible region by a proportionally small amount. Because the feasible region is bounded there exists a δ such that $|\bar{Z} - Z| < \delta$ for all points Z and corresponding points $\bar{Z}$. We can make δ as small as we wish by decreasing $|dx_k|$ since all distances $|\bar{Z} - Z|$ are proportional to $|dx_k|$.

Next consider the objective function. The equipotentials of this function are concentric ellipsoids about a center determined by completing the square in 5.2.27a. This point is easily seen to be a linear function, $q(x_k)$ where x_k is the linearization point. Thus if x_k changes, the center changes again proportionally to $|dx_k|$. Suppose the optimal point is an interior point of the feasible region. Then it is easy to see that this optimal point is the center of the set of ellipsoids since this minimizes the objective in the absence of constraints. Since this center is a linear function of x_k if $x_k \to x$, then the center $w_k \to w$ where $w = q(x)$. Thus the algorithm would be continuous in this case.

If the center of the ellipsoids is outside the feasible region then the optimal point is on its boundary. For sufficiently small changes in x_k, the boundary shifts, and shifts in the center of the ellipsoid still keep this center outside the feasible region. Thus for small enough changes in x_k the optimal point will still be on the boundary.

Now we want to show that if x_k is changed slightly the change in the optimal points y_k will be of the same order. We do this in two steps. First we change the value for x_k in the constraints 5.2.27c only and find the new optimal point. Since the change in x_k is of order $|dx_k|$ the distance from the optimal point y_k in the direction of the gradient of the objective function to the new hyperplane is of the order $|dx_k|$. Thus the new optimal point y' is within distance of order $|dx_k|$ of the old optimal point.

If the optimal point y_k is on an edge or corner of the feasible region, the new optimal point may not be on the ray in the direction of the gradient ∇f, but whatever the direction it will still be only a distance proportional to $|dx_k|$ from y_k as has been shown above. Figure 5.2 shows this graphically for two dimensions. Note that since the objective function is strictly convex y_k and y' are unique.

Now we change the objective function to allow for the change in x_k and optimize it over the new constraints. The change in x_k causes a change of order $|dx_k|$ in the center of the equipotential ellipsoids. This change consists in general of a translation dw_k and rotation $d\alpha$ both of order $|dx_k|$. Thus the total shift in the

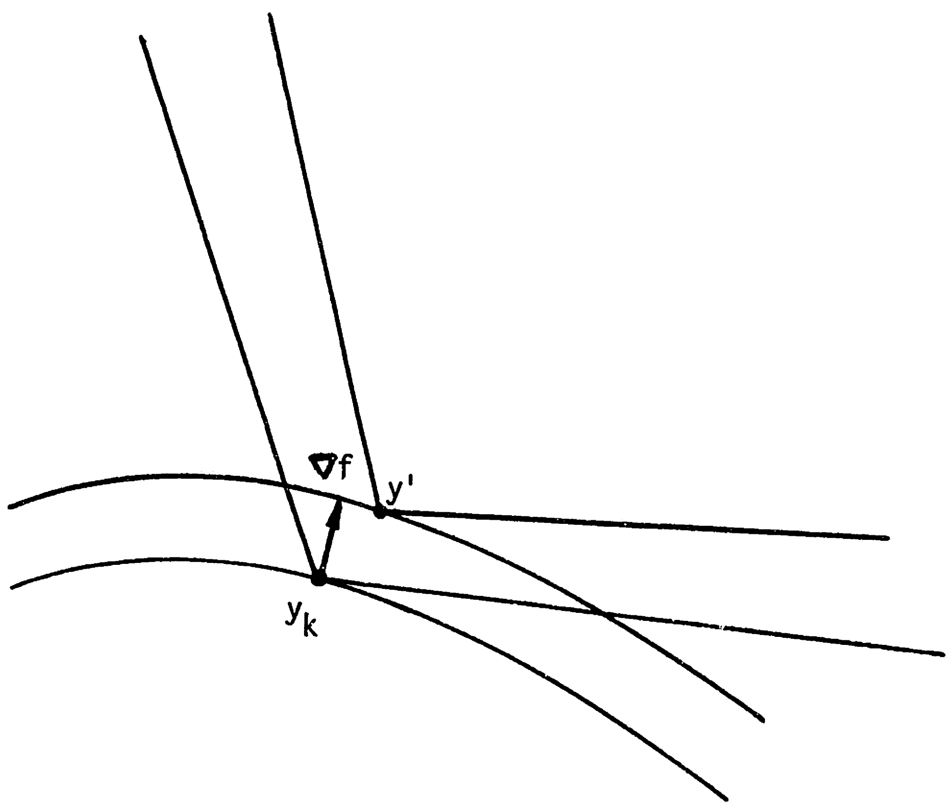

Figure 5.2 Change in Optimal Point y_k when on Edge or Corner of Feasible Region

optimal point will be given to first order by the sum of two components

$$|y'' - y'| = (c_1/\sin\theta)|dx_k| + C_2R|dx_k| \qquad 5.2.58$$

where C_1 and C_2 are proportionality constants, θ is the angle between the translation of the center point and the gradient ∇f at y', and R is the distance from the center to y' (see Figure 5.3).

If y' is on a corner of edge either the optimal point will move as above or else it will stay at the point y'.

The total change in the optimal point due to both shifts can be limited by the triangle inequality:

$$|y - y''| \leq |y - y'| + |y' - y''| \qquad 5.2.59$$

Since both of the right hand side terms are of order $|dx_k|$ so is $|y - y''|$. Thus as $|dx_k| \to 0$, $y'' \to y$ and A is continuous at the point x_k outside the feasible region. Since x_k is any point outside the feasible region, A is continuous for all such points. We have shown that A is continuous at all points x_k inside or outside the feasible region. Since it is point-to-point and continuous it is closed.

Note that the results would not follow if the equipotentials of the objective function were not strictly convex since then the points y_k, y', and y'' would not have to be unique, but could rather be one of an infinite set of points on a line contained in the boundary of the feasible region. Since the mathematical program

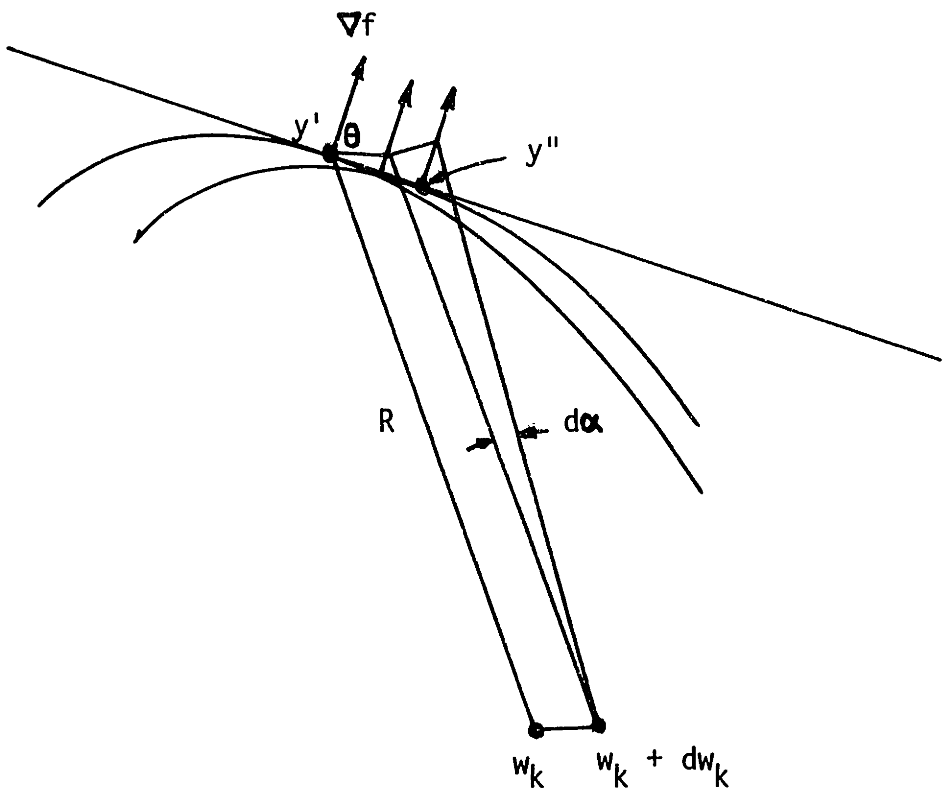

Figure 5.3 Shift in Optimal Point due to Change in Objective Function

would find all these points to be equivalent, an infinitesimal change in x_k could result in a finite change in y_k.

Earlier it was assumed that (p,x) is contained in a compact set. This can be shown as follows: first, all flows x are subject to capacity constraints. Thus x is restricted from above by upper bounds and from below by non-negativity constraints. Prices, p, are constrained by both rate of return regulation and wellhead price ceilings. Thus p is also constrained by upper and lower bounds. Since all constraints are of the less than or equal to variety the feasible set is closed. Being closed and bounded it is compact.

Having proved that the algorithm A with solution set B and descent function sequence $\{Z_k\}$ satisfy the conditions of the generalized global convergence theorem, we may conclude that the limit (p^*,x^*) of any convergent subsequence generated by this algorithm is a solution as long as $f_k \neq 0$.

What is the significance of such a point (p^*,x^*)? From the definition of the solution set and the discussion above, (p^*,x^*) is the solution to the quadratic program

minimize $\quad p'Qp + ep + cx + N^*[|x-x^*|^2 + |p-p^*|^2] \qquad$ 5.2.60a

subject to $\quad Ax + Bp \leq b \qquad$ 5.2.60b

$(p^{*\prime}E_i - a_i)x + (p'E_i - a_i)x^* \leq 2d_i \qquad$ 5.2.60c

$p, x \geq 0 \qquad$ 5.2.60d

where N^* is the limit of N_k as $k \to \infty$.

Writing the Kuhn-Tucker conditions at the point (p^*,x^*) yields

$$2(p^*)'Q + e + gB + \sum_i h_i x^{*\prime} E_i{}' \geq 0 \qquad 5.2.61a$$

$$c + gA + \sum_i h_i[p^{*\prime}E_i - a_i] \geq 0 \qquad 5.2.61b$$

$$g[Ax^* + Bp^* - b] = 0 \qquad 5.2.61c$$

$$\sum_i h_i[(p^{*\prime}E_i - a_i)x^* + (p^{*\prime}E_i - a_i)x^* - 2d_i] = 0 \qquad 5.2.61d$$

where g and h_i are dual variables associated with the linear and linearized constraints respectively. Thus the complete set of optimality conditions for this linear program consists of the constraint set and the Kuhn-Tucker conditions. But these are precisely identical with the Kuhn-Tucker conditions for the original nonlinear problem, if the linearized constraint (5.2.60c) and complementarity slackness equations (5.2.61d) are divided by 2. Thus the significance of (p^*,x^*) is that it is a Kuhn-Tucker point of the nonlinear program 5.2.23.

Whether this Kuhn-Tucker point can be shown to be a solution to the problem 5.2.23 is not known at this time. Hopefully we will be able to do some further theoretical work on this point in the near future.

5.2.5 Implementation of the Algorithm

At each iteration of this algorithm, the constraints 5.2.27c and objective function 5.2.27a change as the nonlinear program is relinearized (in the special sense of section 5.2.4) around the new point (p_k, x_k). Thus coefficients in the constraint rows, the right-hand side vector and the objective function all change. The general form for these coefficients is given by the following reexpression of problem 5.2.27:

$$\text{minimize} \quad p'(Q + N_k I)p + [e - 2N_k p_k']p + N_k x'x$$

$$+ [c - 2N_k x_k']x \qquad 5.2.52a$$

$$\text{subject to} \quad Ax + Bp \leq b \qquad 5.2.51b$$

$$[p'_k E_i - a_i]x + p'[E_i x_k] \leq [2d_i + a_i x_k] \qquad 5.2.52c$$

$$p, x \geq 0 \qquad 5.2.52d$$

Note that all of the coefficients which must be updated (the bracketed expressions) are linear combinations of the optimal solution vector (p_k, x_k) of the previous subproblem. Thus one way to implement the updating process would be to adjoin to the model a "free" row for each such linear expression. That is, a new set of rows could be included as alternate objective functions whose values would be computed but which would not actually be used in finding the solution to the problem. These rows would be of the form:

$$2N_k p + q = e' \tag{5.2.53a}$$

$$2N_k x + z = c \tag{5.2.53b}$$

$$E_i' p - r_i = a_i \tag{5.2.53c}$$

$$E_i x - S_i = 0 \tag{5.2.53d}$$

$$a_i x - t_i = -2d_i \tag{5.2.53e}$$

where q, z, r_i, S_i, and t_i are unconstrained variables which will have the appropriate values for the new coefficients for iteration k+1 when the optimal solution to iteration k is found. The updating would actually be performed by reading these values from the solution at iteration k and writing them in a new matrix generator for iteration k+1. Alternately in the system used in this study, the NBER SESAME mathematical programming system, the model can be set up with names in place of coefficient values. Before beginning the simplex or modified simplex routine, one calls another routine which fixes the values of these coefficients by associating their names with the names and values in an external data set created by the user. Thus, at the conclusion of iteration k, the names and values of q, z, r_i, S_i, and t_i are written into a data set which is then used by the FIXVALUE routine in SESAME to set up the $(k+1)^{st}$ subproblem.

The drawback to this procedure in this form is that the number of constraint rows is increased greatly. In the problem of interest here, there were over 2,000 such free rows resulting in a total problem size of 3,275 rows. Even though free rows do not constrain the problem, they are a tremendous burden to carry through hundreds of iterations when they are actually only needed when the optimal values for x and p are known. In SESAME this difficulty is partially mollified by an option called NOFREE which sets up the problem with only the objective function as a free row. When the optimal solution is found, the basis is saved and the problem set up again without NOFREE. The basis then solves this problem at the first iteration and calculates the new coefficients at the same time. Another approach might be to set up another model only containing the free rows. Once an optimal basis is found in subproblem k, that basis could be used as a starting basis for the model of free rows, solving it at the first iteration since there would actually be no real constraints. The data handling would be more difficult with this approach, but it would keep down the size of the original problem. A third approach would be to do the data manipulations external to the mathematical programming system. This also has the disadvantage that one would have to set up the equivalent expressions for the new coefficients each time the algorithm comes to a new iteration, a task equivalent to setting up a new matrix generator.

In this study the first approach was used. Although the algorithm seemed to be working (i.e., leading to decreasing objective functions at each stage), the size of the model led to numerical and computational difficulties which could not be overcome. Although the source of these difficulties was not absolutely identified, it was suspected that the fact that SESAME's arithmetic was done in 1 - 1/2 precision rather than double-precision may have led to the numerical difficulties with large problems. The same problem occurred when a separable linear approximation to the quadratic objective function was used.

Thus the algorithm was abandoned for this problem. In its place a simpler procedure was used as an approximation to the above algorithm, which will be described in the next section. It is hoped, however, that the modified linearization algorithm developed during the course of this study will be tested in the future on a smaller problem and/or on another mathematical programming system.

5.2.6 Approximate Solution to the Nonlinear Model Using Historical Flow and Pricing Patterns

In Chapter 1 the method by which the Federal Power Commission determines pipeline price structures was described. Estimates of demands, value of capital plant, and cost of capital based on historical values are made by the company and FPC staff and prices chosen so as to give the pipeline the opportunity to make a set rate of return on its capital plant. The nonlinear model 5.2.23 does not need the demand estimates since it determines demands as part of the solution. An approximation to this model which would simulate to some extent the FPC procedure would be to estimate the upcoming year's pipeline flows and demands, x, on the basis of historical values and trends and to "linearize" 2.2.23c by using the estimated value $\bar{x}$ in place of x. Thus, the constraint set would be linearized and the quadratic program solvable by either Dantzig's or Wolfe's method. If it turned out that $\bar{x}$ was very different from the computed values x, then a better estimate of $\bar{x}$ should be provided. On the other hand, if the computed solution x^* was equal to $\bar{x}$, this does not mean that x^* is a solution of the original problem since the Kuhn-Tucker conditions at x^* for this "linearized" version of the problem are not the same as those for the nonlinear problem at x^*.

For purposes of this study, a new model was set up which used this simplified procedure. $\bar{x}$ was approximated as x_{t-1}, the optimal solution or actual flows and demands for the previous year. The model was successfully run for several different values of exogenous parameters, such as the economy-wide growth rate and policy variables (pipeline rate of return, wellhead prices, pipeline growth rate) in simulations over the period 1971 to 1980. The results from this model are presented in Chapter 6.

The performance of the SESAME system in these computations was mixed. When starting a given problem with an all slack basis, the system found an optimal solution to the separable linear programming approximation of the quadratic program in between 1200 and 1400 iterations. The model size is 825 rows and 1500 columns (including 700 λ-type variables). Including the input and output manipulations, the program solves this problem in between 85 and 108 CPU seconds on an IBM 370/168 operating under VM. If one can start a given problem with an advanced basis (a previously stored optimal basis), the time can be reduced to 20 to 50 CPU seconds and 30 to 300 iterations. We could not always use this approach, however, while doing separable linear programming on the SESAME system. Thus, the reduction in computation time from 85 - 108 to 20 - 50 CPU seconds occurred sporadically rather than consistently.

With some modifications the input and output programs used in conjunction with this model could be altered to allow testing with other mathematical programming systems such as MPSX. It is hoped that such testing will be possible in the future.

Chapter 6 Results from the Models

6.1 Introduction

In this chapter we present and examine the results from the models of the natural gas system under alternative regulatory policies. In the first section we give an historical survey of natural gas production, consumption, and prices. Next we present the results of the models of an unregulated and regulated industry for comparison with the historical data. Following this we describe the forecasting model developed for this study in greater detail along with a set of forecasts for 1975 and 1980. We follow an analysis of these forecasts on an aggregate level with a more detailed regional analysis. Finally a comparison of these results with those of the MacAvoy-Pindyck study concludes the chapter.

6.2 Historical Background

Table 6.1 gives the historical levels of supply and consumption in billions of cubic feet and wellhead and wholesale prices in dollars per thousand cubic feet for the years 1966-1974. U.S. supply consists of production from the lower 48 states and the offshore areas of California and in the Gulf of Mexico plus imports of natural gas from Canada and Mexico. (The small amount of gas imported as

TABLE 6.1

Historical Supply and Consumption of Natural Gas in the U.S.

SUPPLY

Year	U.S. Prod. (BCF)	Imports (BCF)	Total (BCF)	Average Wholesale Price ($/MCF)
1966	17,203	463	17,666	.171
1967	18,083	547	18,630	.182
1968	18,981	625	19,606	.175
1969	20,368	682	21,050	.178
1970	21,559	768	22,341	.182
1971	21,704	870	22,574	.194
1972	22,143	938	23,081	.218
1973	22,132	1,032	23,164	.234
1974	21,143	940	22,083	.267

CONSUMPTION (Excluding losses and pipeline fuel)

Year	Wholesale (BCF)	Lease & Plant Fuel (BCF)	Total (BCF)	Average Wholesale Price ($/MCF)
1966	16,426	1,000*	17,426	.324
1967	17,258	1,100*	18,358	.316
1968	18,573	1,205	19,778	.323
1969	19,686	1,308	20,994	.323
1970	20,554	1,361	21,915	.339
1971	20,358	1,372	21,730	.389
1972	20,700	1,408	22,108	.410*
1973	20,845	1,400*	22,245	NA
1974	20,380	1,400*	21,780	NA

* Estimated

Source: U.S. Bureau of Mines Mineral Industry Surveys, FPC Form 2 Reports, AGA Gas Facts.

LNG is not included nor is the steadily increasing production of gas in Alaska. These will be considered to be supplemental sources in the remainder of this analysis.) U.S. production increased steadily from 17.2 trillion cubic feet (TCF) in 1966 to 22.1 TCF in 1972. In 1973 production was about the same level as in 1972 but then dropped by about 4.5% in 1974. Imports also increased steadily from 463 billion cubic feet (BCF) in 1966 to 1032 BCF in 1973, only to fall by 8.9% to 940 BCF in 1974. Total supply then increased from 17.7 TCF in 1966 to 23.2 TCF in 1973, dropping by 4.7% to 22.1 TCF in 1974. During the 1966-1970 period the average wellhead price of gas fluctuated between \$.171 and \$.182 per thousand cubic feet (MCF). Then as FPC price ceilings on new contracts for natural gas began to rise, the average wellhead price steadily increased to \$.267/MCF in 1974.

Consumption of natural gas is divided into two categories: wholesale and lease and plant fuel. Wholesale consumption is actually a misnomer since direct (retail) sales to industrial and other users by interstate pipelines is also included in this category. It basically includes all sales by inter- and intra-state pipelines to utilities and retail customers. It also includes final sales by producers other than for use on leases or as gasoline plant fuel which is included in a separate category. "Wholesale" consumption increased steadily from 16.4 TCF in 1966 to 20.6 TCF in 1970. These sales have remained relatively stable at this level since then. Lease and plant fuel usage has grown from about 1.0 TCF to about 1.4 TCF from 1966 to 1972. Total consumption thus increased from 17.4 TCF in 1966 to 21.9 in 1970 and has remained fairly stable since then. The wholesale price of gas fluctuated between 31.6¢/MCF and 33.9¢/MCF in the period 1966-1970, increasing sharply to 38.9¢/MCF in 1971 and about 41¢/MCF in 1972, reflecting pass through of higher average wellhead prices.

The discrepancy between total supply and total consumption is due in main to two things: (1) use of natural gas as pipeline fuel and (2) differences in the amounts deposited and withdrawn from storage. In addition there is a certain level of unexplained losses in transmission and storage.

The most interesting and important statistics in Table 6.1 are the average wellhead prices. When most other prices were increasing in the 1960's, the price of natural gas stayed virtually constant. As a result, new drilling activity fell off, resulting in lower reserve additions to production ratios until in the 1970's production out of reserves stopped increasing and began to decline. During the 1970's the FPC began to loosen price ceilings, recognizing the deleterious effect of the low price on new exploratory drilling and acknowledging the market distortions caused by the artificially low gas price relative to the price of oil. Whereas at first the emphasis was on stimulating the search for new supplies through higher prices, the higher ceilings now seem to be aimed at reducing demand toward equilibrium levels. The effects of various potential FPC strategies with respect to wellhead price ceilings will be discussed below.

Table 6.2 presents the estimated levels of fixed costs and taxes paid by the interstate pipeline companies from 1966 to 1972 along with the corresponding average cost of capital. The Federal Power Commission determines a rate of return for each pipeline company based on its estimated cost of equity capital, debt-equity ratio, and general performance, among other things. Thus, in addition to determining wellhead price ceilings on new contracts, the Federal Power Commission also influences pricing at the wholesale level through its control of the pipeline's rate of return. From the table it is apparent that as sales have gone up, so have fixed costs and taxes, from $1,298 million in 1966 to $1,732 million in 1972, while the average cost of capital has stayed remarkably stable, fluctuating only between 6.46 and 6.84%.

How the effective rate of return granted the pipelines by the FPC compares to these costs of capital will be discussed also in the sections below.

TABLE 6.2 Pipeline Fixed Costs (Including Federal Income Tax) and Average Cost of Capital

Year	Fixed Costs & Taxes Million $	Average Cost of Capital (%)
1966	1298.4	6.68
1967	1363.3	6.46
1968	1533.0	6.69
1969	1726.1	6.81
1970	1776.1	6.61
1971	1798.4	6.69
1972	1731.8	6.84

Notes: Fixed costs plus taxes includes depreciation expense, interest, dividends, and taxes. See Section 4.3.6 for more detail.

Source: Moody's Public Utilities Manual; Standard and Poors Analysts Handbook, 1973; Merrill, Lynch, Pierce, Fenner, and Smith; FPC Statistics of Interstate Pipeline Companies.

6.3 Model Results for the Historical Period 1966-1974

6.3.1 Unregulated Competitive Industry Model Results

The competitive industry model is characterized by short run marginal cost pricing by the transmission companies except when constraints on the capacity of the pipelines drives the price up (economic rents). In this model fixed costs are considered sunk so that only variable operating costs are included in the transmission costs. Because of this, the only way for the pipeline companies in this model to recover their fixed costs is through economic rents.

Table 6.3 displays the aggregated results of the competitive model for the years 1966-1972. The supply totals include imports from Canada and Mexico at their historical levels. Consumption includes lease and plant fuel as well as residential, commercial, and industrial sales, but does not include use as pipeline fuel or losses. It is also assumed that there was no net increase or decrease in underground storage in these years. The table shows that the unregulated competitive model agrees very closely with historical levels of supply, consumption, and wellhead price, but is consistently low by 10-14¢/MCF in average wholesale price.

This trend indicates that the linear supply functions described in Chapter 4 operate well for the period 1966-1972 when predicted wellhead prices are very close to historical levels. It also seems

TABLE 6.3 COMPETITIVE INDUSTRY MODEL RESULTS

TOTAL U.S. SUPPLY

Year	Quantity (TCF)	Price ($/MCF)	$Q_{mod}-Q_{hist}$ (TCF)	$P_{mod}-P_{hist}$ ($/MCF)
1966	17.6	.172	-0.1	+.001
1967	18.7	.175	+0.1	-.007
1968	19.6	.176	-0.0	+.001
1969	20.7	.180	-0.4	+.002
1970	21.9	.189	-0.4	+.007
1971	23.0	.196	+0.4	+.002
1972	22.5	.201	-0.6	-.017

TOTAL U.S. CONSUMPTION

Year	Quantity (TCF)	Price ($/MCF)	$Q_{mod}-Q_{hist}$ (TCF)	$P_{mod}-P_{hist}$ ($/MCF)
1966	17.2	.220	-0.2	-.104
1967	18.5	.218	+0.1	-.098
1968	19.5	.220	-0.3	-.103
1969	20.6	.225	-0.4	-.098
1970	21.7	.235	-0.2	-.104
1971	22.7	.251	+1.0	-.138
1972	22.3	.295	+0.2	-.115

Source: Competitive Industry Model calculations.

to indicate that the price elasticity of demand in this regime might be generally quite low since large differences between predicted and actual wellhead price change the level of consumption very little. It may also be that the demand equations underestimate actual demand at a given price.

The results from this model imply an average transmission cost (wholesale minus wellhead price) of between 5 and 9¢/MCF. Historical values run about 10¢/MCF higher. This would indicate that the pipeline companies are not able to recover their fixed costs in this model. In Table 6.4 pipeline rents generated from constrained capacity are compared with actual pipeline fixed costs (including taxes). In the years 1966-1971 only between 10.6 and 16.7% of the pipelines' fixed costs were recovered. Only in 1972 when many pipelines in the model reached throughput capacity did economic rents reach 50% of their fixed costs.

These results seem to imply that the capital intensive gas transmission companies could not survive in conditions of intensely competitive (short-run marginal cost pricing) behavior. Marginal cost pricing would keep revenues below the level necessary for the less efficient pipelines to pay off their debt and equity holders. Thus it appears that regulation might be a necessity in this industry not just to keep those companies enjoying monopoly positions from charging monopoly prices, but also to keep those companies in competitive regions from destroying each other through intensive competition.

TABLE 6.4 Competitive Equilibrium Model Results --

Pipeline Rents

Year	Pipeline Fixed Costs & Taxes	Rents	Rents/ Fixed Costs
1966	1298.4	217.0	.167
1967	1363.3	143.9	.106
1968	1533.0	169.9	.111
1969	1726.1	204.3	.118
1970	1776.6	207.3	.117
1971	1798.4	263.7	.147
1972	1731.8	894.4	.516

Source: Competitive industry model calculations.

On the other hand the actual behavior of pipelines without regulation would probably not result in short-run marginal cost pricing. Because the fixed costs of pipelines are so high, the "demand charge" would probably be used to cover their overhead costs and a "commodity charge" to cover operating expenses. A demand charge is a fixed yearly charge for having service, while a commodity charge is a unit charge on the quantity actually purchased. The model for this type of pricing behavior is non-linear because of the fixed costs and would have to be handled in a manner similar to that used for the regulated industry model described in Chapter 3.

As a final remark regarding the unregulated competitive model, notice that explicit inclusion of price ceilings in the model was unnecessary since the predicted equilibrium prices are very near the actual levels experienced in each of the years 1966-1972. In particular, one might conclude that in a truly competitive market with unregulated pipelines that ceiling price regulation would also be unnecessary. It could also be, however, that the results have been biased toward their historical levels simply by the way in which they were estimated. That is, the estimates were made over this same period and not over a set of data having a much wider range of prices and quantities, and may be biased toward the same values by having a reduced applicability outside the historical range. In either case, the difficulties faced by the pipelines continue to exist, thus calling for an analysis of the system with FPC pipeline regulation.

6.3.2 Regulated Industry Model Results

In Chapter 3 we described a model of the regulated industry in which the maximum wholesale price a company could charge in a given state was determined by the fixed and variable costs of all that company's pipeline links leading into that state weighted by the expected throughput in each link. For the results which follow these costs and throughputs were taken to be the estimated historical costs and throughputs for each year as described in Chapter 4.

The model operates by maximizing aggregate pipeline profits minus a weighted sum of excess demands, under the pipeline and wellhead price ceilings and constraints of the network.

Table 6.5 presents results from the model for 1971 to 1975. In a sense the years 1973-1975 are forecasts since most of the data base does not go beyond 1972.

On the supply side, the model computed totals of between 20.5 TCF in 1971 and 22.8 TCF in 1975 at prices running from $.174/MCF in 1971 to $.283 in 1975. Compared to historical values, the model results are generally somewhat low in both quantity and price.

On the demand side, consumption was also generally lower than historical values. The comparisons for 1973-74 may not be very accurate due to the fact that the 73-74 historical consumption data came from a different source (Bureau of Mines) than the earlier data which was estimated using FPC Form 2's. It is clear, however, that wholesale prices are still quite a bit lower in the model

TABLE 6.5 Comparison of Regulated Industry Model Results with Historical Values of Supply and Consumption

	Historical		Model Results	
Year	Supply (TCF)	Wholesale Price ($/MCF)	Supply (TCF)	Wholesale Price ($/MCF)
1971	22.6	.194	20.5	.174
1972	23.1	.218	20.7	.198
1973	23.2	.234	22.3	.197
1974	22.1	.267	22.5	.252
1975	NA	NA	22.8	.283

Year	Consum.	Wholesale Price	Consum.	Wholesale Price	Exc. Demand
1971	21.7	.389	20.3	.327	1.3
1972	22.1	.410*	20.4	.342	0.7
1973	22.2	NA	21.9	.348	0.1
1974	21.8	NA	22.1	.393	0.1
1975	NA	NA	22.4	.426	0.1

* Estimated

Notes: Assumed 8% pipeling rate of return (ROR); 3% exogenous demand growth after 1972; 15% roll-in factor on average wellhead price after 1973; 1972 pipeline capacities for all years after 1972.

Source: FPC Form 2 Reports, Bureau of Mines Minerals Yearbooks, Foster Associates, and model results.

results than in reality. This may indicate that the 8% rate of return used in these calculations is less than the actual average value being allowed by the FPC, and that the 3% annual increase in exogenous demand (demand due to factors other than gas price) is too low in this period. Finally, the excess demand predicted for the period is relatively small for all the years except 1971 when the wholesale price estimate was particularly low. This would also lend credence to the possibility that a 3% exogenous demand growth rate is too low.

In Table 6.6 the average gross pipeline profit per MCF is presented for three cases: historical, unregulated competitive industry model, and regulated industry model with 8% rate of return. The actual values have been between 13.1¢ and 18.7¢ per MCF. The competitive model predicts between 4.1 and 9.2¢. As discussed previously, these gross profits will not cover capital costs.

The regulated model allows the pipeline between 13.6 and 15.1¢/MCF, assuming 8% rate of return. For the years 1971 and 1972 this is about 20% lower than the historical level. This may indicate, therefore, that the pipeline's actual rate of return in nearer to 10% than to the 8% assumed here, or that the actual costs of the pipelines are higher than those used in these computations.

Perhaps the most important factor, however, in the model's prediction of lower than historical costs is that the system is optimized without constraining each company to satisfy its old contracts each year. That is, relatively inefficient companies

TABLE 6.6 Gross Pipeline Profits ($/MCF Sales)

	Historical	Competitive Model	Reg. Model 8% ROR
1966	.151	.044	--
1967	.131	.041	--
1968	.150	.043	--
1969	.145	.044	--
1970	.153	.044	--
1971	.187	.052	.151
1972	.178	.092	.141
1973	NA	--	.147
1974	NA	--	.136
1975	NA	--	.138

Source: Form 2 Reports; GASNET calculations; competitive industry model calculations.

are getting less gas due to the optimization of the model than would be the case in reality. An improvement could be made by putting lower limits on purchases and sales of each pipeline, based on previous year quantities. Because a majority of the old contracts will have expired by 1980, however, we decided to leave these constraints out of the forecasting model.

Table 6.7 displays the results of the model for 1972 for several different rates of return. The row labelled "Cost of Capital" presents the results when the company's estimated costs of capital were used. In the next two cases 8% and 12% were applied directly across the board as the FPC rates of return. In the last case, the rate of return restrictions were lifted entirely, allowing pipelines to maximize profits monopolistically. (In cases where there are more than one company the model assumes they will collude to maximize total profits.) The results show that total quantities and prices vary little as functions of pipeline rate of return. The 50% increase in ROR from 8% to 12%, for example, resulted in only a 3%/MCF increase in average wholesale price with a reduction of only 0.3 TCF in consumption. Monopoly behavior, however, produced a huge jump in wholesale prices and large corresponding drop in consumption and supply. It should be recognized, however, that such a large price increase would likely exceed the range over which the demand and supply functions are valid. Thus the actual values would probably be different from those predicted here.

TABLE 6.7 Sensitivity of 1972 Model Results to Pipeline Rate of Return

	Supply* (TCF)	Whd. Price ($/MCF)	Consump. (TCF)	Wholesale Price ($/MCF)
Cost of Capital	20.0	.186	19.8	.406
8%	19.9	.187	19.4	.415
12%	19.6	.186	19.4	.444
Monopoly	15.7	.158	15.6	1.145
Historical Values	23.0	.218	22.1	.410

Note: These results were generated using a forerunner of GASNET. In this old model the excess demand variables were not included in the demand relations and are in the allocation model described earlier. For all cases but MONOPOLY, no feasible solution was found. In those cases the demands were manually adjusted downward until feasibility was reached. This heuristic procedure results in prices which are higher and quantities lower than in the analagous runs using the allocation model.

Source: GASNET calculations.

In conclusion, the model used here to describe the regulated natural gas industry tends to predict reasonable estimages for natural gas supply, consumption, and wellhead prices, but at 8% pipeline rate of return, predicts wholesale prices lower than those actually existing in 1971-1974. We believe that the model tends to be more efficient than the actual system since it does not require each company to meet old sales contracts. For forecasting purposes, however, this is not so much of a problem, since most of the old contracts will have expired.

6.4 Forecasting Supply and Demand in 1980

For the purpose of forecasting supply and demand for the period 1975-1980, we used the regulated industry model described in the previous section as a regulatory policy model. In this model three exogenous growth assumptions and three policy variables are independently fixed at the beginning of the routine, the forecast year chosen, and the model run.

The three growth parameters which may be set are (1) exogenous demand growth rate in the price-independent parts of the demand equation (i.e., alternative fuel price, population growth, capital investment, etc.); (2) the reserves decline rate (based on reserve additions to production ratios of less than unity); and (3) pipeline capital plant growth rate. The latter parameter should be negative in periods of no construction since capital rate base continues to depreciate at about 3% per year.

The three policy variables are wellhead price ceilings on new contracts for natural gas, pipeline rate of return, and pipeline capacity growth rate.

While the model could be modified to allow individual pipeline additions, the present version applies capacity increases across the board.

Table 6.8 presents a computer printout of a typical console session with the model. By typing the command "gasnet" at the computer console, the policy analyst gains access to the model. The program automatically requests values for the six parameters described above. The procedure then sets up an input data base to the model, fixes the coefficients in the optimization routine, solves the problem, and prints out the results of the run in terms of regional supplies, demands, excess demands, and prices. The procedure takes between 20 and 120 CPU seconds on the IBM 360/168 operating under VM at Cornell University, depending on whether it can use previously produced solutions as starting values for the routine.

Eight forecasts for 1980 made using this model are presented in Table 6.9. The first five (cases 1 - 5) use the same price dependent production functions as were in all previous results. A comparison of the first two forecasts indicates how important the exogenous demand growth rate is in this model. The parameters

```
gasnet
EXECUTION BEGINS...
TYPE EXOGENOUS DEMAND GROWTH RATE, RESERVES DECLINE RATE,
AND PIPELINE CAPACITY GROWTH RATE ON CONSECUTIVE LINES
.0.05
.0.00
.0.00
TYPE YEAR OF FORECAST
.80
%EXECUTION BEGINS...
TYPE YEAR OF INTEREST, NATIONAL CEILING IN 1974, AND
YEARLY CEILING PRICE INCREASE ON CONSECUTIVE LINES
.80
.0.50
.0.01
EXECUTIVE BEGINS...
TYPE RATE OF RETURN, PIPELINE CAPACITY GROWTH RATE,
AND CAPITAL PLANT GROWTH RATE ON CONSECUTIVE LINES
.0.08
.0.00
.0.00
TYPE YEAR OF FORECAST
.80
%%
SESAME V9.2

SESAME COMMAND:
SESAME COMMAND:
SESAME COMMAND:  %%%%
SESAME COMMAND:
SESAME COMMAND:
SESAME COMMAND:  %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
SESAME COMMAND:
SESAME COMMAND:
SESAME COMMAND:
SOLUTION

        OPTIMAL SOLUTION AT ITERATION NUMBER 1235

        ...NAME...     ...ACTIVITY...       DEFINED AS

        FUNCTIONAL        2974764.9         OBJ
        RESTRAINTS                          RHS
        BOUNDS....                          UPPER

SESAME COMMAND:
SESAME COMMAND:  PRT FILE 0598  TO  XXY    COPY 01 NOHOLD
EXECUTION BEGINS...
GO TO TOP OF NEXT PAGE AND TYPE YEAR OF INTEREST
```

TABLE 6.8 (Continued)

80

SUPPLY REGIONS

DISTRICT	YEAR	QUANTITY BCF/YEAR	PRICE $/MCF
WEST			
	1980	1723.	0.431
HUG.ANAD			
	1980	3135.	0.409
APPALACH			
	1980	371.	0.458
E.TX-LN			
	1980	1281.	0.427
TX. GULF			
	1980	5714.	0.428
LA.SOUTH			
	1980	11030.	0.435
PERMIAN			
	1980	3007	0.423
IMPORTS			
	1980	930.	0.0
US SUPP			
	1980	27191.	0.435

DEMAND REGIONS

DISTRICT	YEAR	QUANTITY BCF/YEAR	PRICE $/MCF	EXC.DEMD BCF/YEAR
WEST				
	1980	4135.	0.516	315.
N. EAST				
	1980	4002.	0.754	1632.
S. EAST				
	1980	2164.	0.663	52.
S. CENT				
	1980	10177.	0.433	0.
N. CENT				
	1980	4637.	0.666	1164.
L&P FUEL				
	1980	1556.	0.0	0.
US CONS				
	1980	26672.	0.561	3162.

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TABLE 6.9 GASNET Forecasts -- 1980

Case	Average Wellhead Price ($/MCF)	Total Supply (TCF)	Average Wholesale Price ($/MCF)	Total Consump. (TCF)	Excess Demand (TCF)
1	0.427	25.1	0.549	24.6	0.8
2	0.435	27.2	0.561	26.7	3.2
3	0.433	26.8	0.562	26.1	3.6
4	0.433	26.7	0.598	26.0	3.2
5	0.610	29.7	0.738	28.8	0.3
6	0.435	21.6	0.594	21.1	7.9
7	0.673	21.6	0.823	21.1	4.4
8	1.252	21.6	1.394	21.2	0.6

Case Definitions:	1	2	3	4	5	6	7	8
Exogenous								
Demand Growth Rate (%)	3	5	5	5	5	5	5	5
Reserves Decline Rate (%)	0	0	5	5	5	0	0	0
PL Capital Plant Growth (%)	0	0	0	0	3	0	0	0
PL Capacity Growth Rate (%)	0	0	0	0	3	0	0	0
PL ROR (%)	8	8	8	12	8	8	8	12
National Ceiling Price ($/MCF)	.50	.50	.50	.50	.75	.50	.75	2.00
Yearly Increment	.01	.01	.01	.01	.05	.01	.05	.00

Cases 1 - 5 use econometrically estimated linear supply functions

Cases 6-8 assume constant production at 1974 levels.

Source: GASNET calculations.

in common for these two forecasts are reserves decline rate (0%), pipeline capital growth rate (0%), pipeline capacity growth rate (0%), pipeline rate of return (8%), and ceiling price (50¢ and 1¢ yearly increase). In changing the exogenous demand growth from 3% to 5%, however, excess demand in 1980 goes from 0.8 to 3.2 trillion cubic feet. Both supply and consumption increase 2.1 TCF with average prices up over 1¢/MCF.

These two forecasts assume that additions to gas reserves keep up with production during the period 1975-1980. If, as is more likely, additions are less than production, say at about half its level, then the reserves level will decrease by about 5% per year. This change in assumptions (Case 3) leads to an increase in the estimate of excess demand from 3.2 to 3.6 TCF in 1980. Actual supply falls about 0.4 TCF as does consumption. Thus total demand stays about the same, but there is simply less supply to fill it.

In Case 4 an increase in pipeline rate of return reduces supply by 0.1 TCF, demand by 0.5 TCF, and excess demand by 0.4 TCF. It also increases average wholesale price by 3.6¢/MCF, which is, of course, why the demand was reduced.

In Case 5 pipeline capital and capacity are allowed to grow by 3% per annum, ROR is set at 8%, and the ceiling price of gas is increased to 75¢ in 1974 with 5¢/year increases. Supply jumps by 3.0 TCF as wellhead prices increase by 40% over the previous case. Consumption also increases, but total demand decreases slightly so that excess demand drops to 0.3 TCF.

Thus, given the 5% growth rate in exogenous demand, it appears that a 75¢/MCF wellhead price ceiling in 1974 with 5¢ yearly increases will just about clear markets in 1980. Unfortunately, however, this is not a very likely outcome. The supply levels predicted by this model, from 25.1 - 29.7 TCF, are simply much higher than the producers of gas are able to produce efficiently from the existing reservoirs in the lower 48 states and offshore.

Table 6.10 presents a variety of forecasts made by government agencies, consulting groups, and the MacAvoy-Pindyck group at MIT. All except MacAvoy-Pindyck predict supply levels in the neighborhood of 19-21 TCF for 1980. Because these estimates have been made on the basis of engineering and reservoir studies of maximum efficient producing rates, it is unlikely that they are too far removed from the realities of the production situation. The problem with the MacAvoy-Pindyck supply model (and the one used in this study) seem to be that they are far too price responsive in the short run. The econometric estimates were made in a period in which relatively high growth was correlated with small price changes in large producing areas such as Louisiana South and the Permian Basin (see Section 4.2). The period of the late 1970's, however, is one of low growth and large price increases. Thus the applicability of these supply functions is in doubt for forecasting in the 1970's.

For this reason three more simulations were run with the assumption that production stayed constant at 1974 levels (see Table 6.9). The three scenarios were 8% ROR with 50¢ ceiling in

TABLE 6.10 Supply Forecasts -- 1980

Lower 48 Plus Imports at 1974 Levels

Forecast	Quantity (BCF)
Foster Associates[1]	20,129
M/P Phased Deregulation[2]	34,044
Stronger Regulation[2]	28,626
TERA Phased Deregulation[3]	19,240
Continued Regulation[3]	18,440
Project Independence ($11/oil)[4]	21,640
FPC-Bureau of Natural Gas[4]	20,540

Sources: (1) Foster Associates (1974)

(2) Lloyd (1975)

(3) Hardy and Neill (1974)

(4) Neri (1975a)

1974 and 1¢ annual increments, 8% ROR with 75¢ and 5¢ increments, and a deregulation scenario of 12% ROR with $2.00/MCF on new contracts in 1974 and thereafter. The results were as expected: the excess demand for gas is likely to be much greater than predicted using the price-sensitive short-run supply functions. In the continued regulation case (50¢ in 1974), it reaches a staggering 7.9 TCF in 1980. In the phased deregulation (75¢ with 5¢ increments) it is still 4.4 TCF. Only in the complete deregulation case ($2.00 in 1974) does excess demand approach zero, becoming only 0.6 TCF in 1980.

The price to pay for this movement toward equilibrium is very high, however, a rise from about 45¢ in 1974 to about $1.40 in 1980 wholesale prices. Wellhead prices advance at the same time from 27¢ to $1.25 per MCF. Supply stays constant while demand is simply slashed by 25% through the giant price increases. While these price increases are very large, it must be kept in mind that they still represent a cost to the consumer in many regions far less than the corresponding cost of fuel oil. Even today, new intrastate gas contracts are being made at prices in excess of $1.50/MCF. If oil prices stay high, the demand for gas at nearly any price will probably continue. Thus it may be that the consumer will have to pay more for gas now because of the distortions in the market caused by the artificially low prices he paid in the 1960's.

6.5 Regional Allocation of Excess Demand

Up to now the forecasts for 1980 have been discussed only in terms of aggregate U.S. quantities and prices. In this section we discuss the results of using GASNET to allocate excess demand to the various demand regions. Table 6.11 lists the results from four 1980 forecasts using the econometrically estimated price responsive supply functions.

TABLE 6.11 Regional Excess Demands 1980 in BCF

Region	Case 2		Case 3		Case 4		Case 5	
	Q	%	Q	%	Q	%	Q	%
West	315	7.1	576	13.0	554	12.6	232	5.3
Northeast	1632	29.0	1783	31.7	1621	29.3	88	1.7
Southeast	52	2.8	52	2.3	17	0.8	0	0.0
South Central	0	0.0	0	0.0	0	0.0	0	0.0
North Central	1164	20.1	1221	21.1	1051	18.7	0	0.0
Total U.S.	3162	10.6	3632	12.2	3242	11.1	320	1.1

Note: Cases 2 - 5 are defined in Table 6.9.

Source: GASNET calculations.

With this supply function, the Northeast and North Central regions experience the bulk of the excess demand. The Northeast region is short about 30% and the North Central about 20% in all but the phased deregulation scenario (Case 5) where almost all excess demand disappears.

In Table 6.12 the same calculations are made for the three forecasts using price unresponsive supply (constant production at 1974 levels).

TABLE 6.12 Regional Excess Demands 1980 in BCF

Region	Case 6		Case 7		Case 8	
	Q	%	Q	%	Q	%
West	793	17.9	642	15.0	436	11.0
Northeast	1433	26.8	1006	19.8	136	3.2
Southeast	52	2.3	0	0.0	0	0.0
South Central	4335	44.2	2309	25.6	0	0.0
North Central	1271	22.0	458	9.2	0	0.0
Total U.S.	7884	27.2	4415	17.3	622	2.8

Note: Cases 6 - 8 are defined in Table 6.9.

Source: GASNET calculations.

In this case excess demand is much greater for both continued regulation and phased deregulation (Cases 6 and 8) and is reduced to almost zero with complete immediate deregulation (Case 8). The distribution of the excess demand among the regions is different from the previous results mainly in the sudden appearance of a huge amount of excess demand in the South Central region. The reason for this is that sales in this region are relatively unprofitable to the pipelines in comparison with sales in the East and North. Thus, when supplies are extremely limited, they would much prefer to ship gas to these markets rather than selling it in the producing areas.

Most of this effect is due to the structure of the model itself, since GASNET only allows pipelines to earn profits on sales to utilities in states receiving gas along a pipeline from another state. Thus end-of-the-line states in producing areas are priced quite low, raising demand and at the same time lowering the incentive to sell to them in times of shortage. Because the model does not explicitly include intrastate pipelines and the intrastate markets for natural gase, it clearly does not reflect the actual situation. At present, prices paid for intrastate gas in Texas have passed $1.50/MCF. A more realistic model must be able to include the effects of the competition between interstate and intrastate pipelines for scarce gas (see Chapter 7).

The weights presently in use for excess demand in the objective function are also biasing factors. The idea was to weight excess demands by the inverse of total demand so that the relative fraction of excess demand would be the same state to state. Since actual level of demand is not known until the forecast is made, a proxy is used for the weight, namely the exogenous demands a_j in the demand expression $D_j = a_j - b_j p_j$. The ratios of the a_j's are not the same as the ratios of D_j's. Thus, some bias may also occur because of this.

6.6 Comparison with Results from the MacAvoy-Pindyck Model

In this section we make a comparison of the results from GASNET and the MacAvoy-Pindyck model (Lloyd, 1975). Table 6.13 gives the results of several forecasts for 1980 from the MacAvoy-Pindyck model and GASNET.

In Table 6.13 National, Case 3, and Case 6 should be considered approximately comparable as should Dereg, Case 5, and Case 7. Case 8, immediate deregulation, stands alone. Looking at the results for continued regulation, we see that the MacAvoy-Pindyck model predicts levels of supply and demand much higher than either GASNET A or GASNET B. One of the reasons for this difference is that prices, income, and industrial growth variables are increased by an inflation index in the M-P model but not in GASNET A or B, since we felt that

TABLE 6.13 Comparison of 1980 forecasts

Model	Forecast	Supply	Whd. Price	Consumption	Wholesale Price	Excess Demand
	Case	TCF	$/MCF	TCF	$/MCF	TCF
M-P	National	30.2	0.497	30.2	0.727	8.3
	Dereg	34.0	0.696	34.0	0.883	0.1
GASNET A	Case 3	26.8	0.433	26.1	0.562	3.6
	Case 5	29.7	0.610	28.8	0.738	0.3
GASNET B	Case 7	21.6	0.435	21.1	0.594	7.9
	Case 7	21.6	0.673	21.1	0.823	4.4
	Case 8	21.6	1.252	21.1	1.394	0.6

National is the current FPC policy modified to allow 2¢/Mcf yearly increases.

Dereg is a 25¢ jump in ceiling price in 1974 with 7¢/Mcf yearly increases.

GASNET A uses price sensitive linear supply functions.

GASNET B uses constant 1974 production levels.

Case 3 - Case 8 are defined in Tables 6.11 and 6.12.

Source: Lloyd (1975) and GASNET calculations.

the high inflation rates of the 1970's tended to bias the linear demand relations highly upward. That is, if inflation is included in price-related variables, the forecast of demand will be higher than actual demand by an amount equal to the rate of inflation. This factor alone adds about 2.5 TCF to M-P demand levels. In addition, the huge increase in oil prices in the 1970's compared with the alternate fuel price elasticities in the M-P demand equations, probably produces a much higher level of forecast demand than is likely to be the case. Oil prices have moved beyond the range of their applicability to the M-P demand equations. While the same is true for the GASNET demand relations (since they are only slight modifications of the M-P demand formulation), the 5% exogenous demand yearly growth factor used in the GASNET A and B forecasts is lower than that implied by the much higher oil prices in the M-P demand model. As a result, GASNET A and B may underestimate actual demand, while M-P certainly overestimates it.

The higher supply levels in the M-P forecasts in comparison with GASNET A are the result of slightly higher wellhead price scenarios with yearly increments of 2¢ and 7¢, rather than 1¢ and 5¢ in GASNET A.

In contrast to both of these models, GASNET B assumes a constant level of production at 1974 levels throughout the 1970's. Considering that production has been approximately constant from 19 -1974, and that discoveries have falled off greatly in the last several years, it is not an unlikely scenario. It may even be optimistic.

Thus, although there is no econometric model to back it up, it is a relevant scenario to consider.

In comparison with M-P and GASNET A, the results for GASNET B have one very important difference. The policy of phased deregulation (Case 7) does not lead to market clearing prices in 1980. This is in direct conflict with the M-P conclusion. In GASNET B supply and consumption remain at 1974 levels of 21.6 and 21.1 TCF respectively, while total excess demand is still 4.4 TCF or 17.3% of total demand in 1980. Only with immediate deregulation in 1974, where new contract prices are allowed to rise to equilibrium levels (assumed to be no higher than $2.00/MCF), does the market approach clearing in 1980. The average wellhead price for this scenario is nearly twice as high in 1980 as in the M-P case, reflecting the present trend in intrastate gas prices to approach the marginal price of imported oil.

In conclusion, the results of the present study indicate that only the immediate deregulation of natural gas wellhead prices could bring about a condition approaching market equilibrium by 1980. It would do so not by increasing supply, since short-run supply elasticities are believed to be very low or zero during the 1970's. Instead, market clearing is accomplished by cutting back excess demand through large price increases.

In the longer run, it can be expected that deregulation of natural gas wellhead prices will result in increased exploration for gas and, hopefully, increased discoveries. However, it may also be that the majority of the major gas fields in the U.S. have already been discovered, and that high levels of new reserves are simply not there to be found. In that case, conservation and efficiency in the use of natural gas may become more important than ever before.

Chapter 7 Limitations and Extensions of the Model

7.1 Introduction

In this section we present a brief discussion of the most important limitations of GASNET. As will be seen, each limitation implies a corresponding extension. With each of these extensions we discuss an approach which might be taken and the problems which are likely to be encountered.

7.2 Reserves Model

At the present time GASNET does not explicitly forecast levels of discoveries and other reserve additions through investment in new well drilling. To be complete such a model is needed. GASNET is structured so that such a reserves model could be easily appended as a linked but separate program wherein lagged reserves levels help determine current production and equilibrium (or disequilibrium) wellhead prices influence investment activity. Thus the two models could be solved alternately rather than simultaneously in a forecasting run.

7.3 Peak Versus Off-Peak Loading

At the present time, GASNET is based on annual data compiled from various government sources. Thus it does not deal directly with the problems associated with peak loading during the winter months. This means that allocations in time, i.e., problems of depositing and withdrawing gas from storage, have been completely neglected. In addition, FPC policies regarding the allocation of fixed and variable costs to peak and off-peak customers cannot be analyzed with the present model.

The model could, theoretically, be extended by putting it on a quarterly rather than annual basis. This could be accomplished by multiplying the size of the model by four, each variable would have a quarter associated with it, adding storage nodes and intertemporal conservation equations, and optimizing the expanded model year by year. The difficulties in this approach would be, first of all, getting quarterly data to fully specify the model, and, second, computing the solution to a model with four times as many variables and constraints.

The former problem may have a solution in some work presently being done by the Federal Energy Administration in which the American Gas Association has supplied estimates of production and consumption by quarter for the last 1960's and early 1970's (Neri, 1975b). In addition, more would have to be known about peak pipeline capacities, storage capacities, and costs.

The second problem could conceivably be handled by some kind of decomposition algorithm since the model would only be connected by a relatively few intertemporal conservation of gas equations. It would still, however, mean at least four times the computer expense per year of simulation.

7.4 Sectoral Allocation

In the present version of the model, total demand in each state is computed rather than demand for each separate consuming sector. This limitation can be easily remedied, however, by the following extension: in place of a single demand node (and demand relation) for each state, include one for each consuming sector in each state. A priority allocation plan during times of shortage can be modelled by appropriately weighting the excess demands in each sector so that excess demand will be minimized in the highest priority sector first.

The present model could be easily modified to allow such an allocation schedule between residential/commercial and industrial sales since demand in these sectors has already been estimated separately. It was merely to keep the model simple that we did not already install this capability.

7.5 Interstate versus Intrastate Markets

GASNET does not model the present day competition between interstate and intrastate sales realistically. Because of the gulf between prices in these two markets in the producing states, correct modelling of this scene is very important.

An extension of the model which would handle this problem would include: first, separate equations for intrastate and interstate production containing both interstate and intrastate prices (perhaps a logic formulation would be appropriate for this aspect of the problem); secondly, it should include a simple representation of the major intrastate pipeline networks involved in transmission within the producing states, including costs of transmission, capacities, and pricing mechanisms. Whether such data can be gotten from the pipeline companies or state agencies is not presently known.

7.6 Interstate Pipeline Modelling Improvements

Better representation of the interstate pipelines within producing states is also needed. When the network was originally being modelled, a major simplification was made to accommodate a particular coding method: pipelines purchasing gas within a state were assumed to get it at only one central location within the state and made sales only at one central location in any state. In some states, such as California, Texas and Louisiana, this simplification makes the model quite unrealistic. Now, a more efficient coding system makes a more accurate model possible. Again, there are data problems associated with intrastate demand areas. If these could be solved, the model could be easily extended to include these improvements.

In addition, better estimates of pipeline capacity could be made if more information about the size and number of pipes and horsepower installed in each arc were known.

7.7 Distribution by Local Utilities

At present, the model only includes sales at the wholesale level. The entire level of distribution via local utilities is neglected. Because these utilities are regulated by different agencies from state to state, there may be significant differences in the costs associated with the distribution of gas. Thus the demand analysis used in this study may not be an adequate representation of the actual scene. Even within a state, sales to different sectors are made at different prices, a fact which the present analysis was not able to capture.

An extension of the model to include the effects of this level of distribution could be made rather simply if the data were available to do so. Estimates of variable and fixed costs associated with retail utilities in each state could be made on an aggregate basis and averaged over the state. These costs, along with capacity estimates, could then be allocated to demand arcs representing intrastate retail distribution. The basic structure of the model would be the same. Arcs leading from pipeline to demand nodes which presently have zero costs would now bear positive costs and prices would be retail rather than wholesale at the final demand nodes.

The difficulties with this extension are the same as those in previous cases: does the data exist, and, if so, can it be gotten and used for this extension? If the various state commissions do have data similar to that of the FPC, then the problem would be soluble, though it might be very expensive and time consuming to deal with 50 agencies rather than one.

7.8 Multiperiod Optimization

Pipelines, producers, and regulatory agencies are interested in longer-term as well as short-term problems. The present model does not deal explicitly with long range planning of future pipeline capacity and thus is limited in its usefulness. A possible extension would be to increase the size of the model to include such a possibility. Again, the difficulty would be in problem size: a 20-year model with 16,000 rows would be simply insoluble with present mathematical programming systems. Some type of decomposition approach would have to be taken.

7.9 Competitive Fuels

In the current model only natural gas is included. It is clearly a much better approach to include competitive fuels within the model rather than simply assuming the behavior of demand and prices for these fuels as exogenously determined inputs. An extended model should include a demand submodel which shows the interaction between the two (or more) competitive fuels. It should also include a supply model which takes into account the joint costs of producing oil and gas from the same geological area. A further complication involves the generation of electricity using gas or oil as a fuel when that electricity is used for house or water heating in competition with oil and gas. The demand for oil as an input to the production of gasoli makes the problem even more complex.

A reasonable extension would be one which didn't try to include all these problems at once, but rather moved step by step toward greater inclusiveness. The next step for this model would be to include supply and production of oil and demand for fuel oil. The same basic formulation could be used with a parallel transmission model for crude oil and fuel oil. An activity analysis for the production of fuel oil from crude would be necessary as well. The pricing mechanisms would be different for this parallel model since oil is not regulated in the same way that gas is. In addition, an import sector would have to be included. All of these extensions are possible within the general framework of the model. To actually do the work involved in setting up such a model would require a great deal of time and money.

7.10 Nonlinear Programming Algorithms for the Regulated Industry Model

At present we do not know if the modified linearization algorithm described in Section 5.2.4 converges to an optimal solution for the model of the regulated industry. Further theoretical and experimental research needs to be done on this and other algorithms possibly useful in solving this problem.

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Appendix

Pipeline Companies Represented in this Study

Below are listed the 80 pipeline companies represented in this study with the states in which they operate. All of these companies were included in the analysis of the competitive industry model. Because we wanted to simplify GASNET, the model of the regulated industry, many of the smaller companies and all those which were no longer in operation in 1972 were excluded. Those 35 which were included in GASNET are marked with an asterisk.

1. Alabama-Tennessee Natural Gas Company

    Alabama, Mississippi, Tennessee

2. Algonquin Gas Transmission Company*

    Connecticut, Massachusetts, New Jersey, New York, Rhode Island

3. Arkansas-Louisiana Gas Company*

    Arkansas, Kansas, Louisiana, Missouri, Oklahoma, Texas

4. Arkansas Oklahoma Gas Company

    Arkansas, Oklahoma

5. Atlantic Seabord Corporation (Now part of Columbia Gas Transmission)

    Maryland, Virginia, West Virginia

6. Carnegie Natural Gas Company

    Pennsylvania, West Virginia

7. Cascade Natural Gas Corporation

    Colorado, Oregon, Utah, Washington

8. Cimarron Transmission Company

    Oklahoma

9. Cities Service Gas Company*

    Kansas, Missouri, Nebraska, Oklahoma, Texas

10. Colorado Interstate Gas Company*

    Colorado, Kansas, Oklahoma, Texas, Wyoming

11. Columbia Gas Transmission*

    Kentucky, Louisiana, Maryland, Mississippi, New Jersey, New York, Ohio, Pennsylvania, Tennessee, Texas, Virginia, West Virginia

12. Consolidated Gas Supply Corporation*

    New York, Pennsylvania, Virginia, West Virginia

13. Cumberland and Allegheny (now part of Columbia Gas Transmission)

    Maryland, West Virginia

14. East Tennessee Natural Gas Company

    Tennessee, Virginia

15. Eastern Shore Natural Gas Company*

    Delaware, Maryland, Pennsylvania

16. El Paso Natural Gas Company*

    Arizona, Colorado, Idaho, New Mexico, Oregon, Texas, Utah, Washington, Wyoming

17. Equitable Gas Company*

    Pennsylvania, West Virginia

18. Florida Gas Transmission Company*

    Alabama, Florida, Louisiana, Mississippi, Texas

19. Gas Transport, Inc.

    Ohio, West Virginia

20. Grand Valley Transmission Company

    Utah

21. Granite State Gas Transmission, Inc.*

    New Hampshire

22. Great Lakes Transmission Company*

    Michigan, Minnesota, Wisconsin

23. Home Gas Company (now part of Columbia Gas Transmission)

    New York

24. Humble Gas Transmission Company (now Mid Louisiana Gas Company)

    Louisiana, Mississippi

25. Inland Gas Company

    Kentucky

26. Interstate Power Company

    Illinois, Iowa

27. Iowa-Illinois Gas and Electric Company

    Illinois, Iowa

28. Iowa Public Service Company

    Iowa, Nebraska, South Dakota

29. Iroquois Gas Corporation

    New York

30. Jupiter Corporation

    Louisiana, Texas

31. Kansas-Nebraska Natural Gas Company, Inc.*

    Colorado, Kansas, Nebraska, Wyoming

32. Kentucky Gas Transmission Corporation (now part of Columbia Gas Transmission)

    Kentucky

33. Kentucky-West Virginia Gas Company

    Kentucky

34. Lake Shore Pipeline Company

    Ohio, Pennsylvania

35. Lone Star Gas Company

    Oklahoma, Texas

36. Louisiana-Nevada Transit Co.

    Arkansas, Louisiana

37. Manufacturer's Light and Heat Company (now part of Columbia Gas Transmission)

    Pennsylvania, West Virginia

38. McCulloch Interstate Gas Corporation

    Wyoming

39. Michigan Gas Storage Company

    Michigan

40. Michigan-Wisconsin Pipe Line Company*

    Illinois, Indiana, Iowa, Kansas, Kentucky, Louisiana, Michigan, Mississippi, Missouri, Ohio, Oklahoma, Tennessee, Texas, Wisconsin

41. Midwestern Gas Transmission Company*

    Illinois, Indiana, Kentucky, Minnesota, Tennessee, Wisconsin

42. Mississippi River Transmission Corporation*

    Arkansas, Illinois, Louisiana, Missouri, Texas

43. Montana-Dakota Utilities*

    Montana, North Dakota, South Dakota, Wyoming

44. Mountain Fuel Supply Company*

    Colorado, Utah, Wyoming

45. Natural Gas Pipeline Company of America*

    Arkansas, Illinois, Iowa, Kansas, Missouri, Nebraska, New Mexico, Oklahoma, Texas

46. North Penn Gas Company, Inc.

    Pennsylvania

47. Northern Natural Gas Company*

    Illinois, Iowa, Kansas, Michigan, Minnesota, Nebraska, New Mexico, Oklahoma, South Dakota, Texas, Wisconsin

48. Northern Utilities, Inc.

    Wyoming

49. Ohio Guel Gas Company (now part of Columbia Gas Transmission)

    Ohio

50. Ohio River Pipe Line Corporation

    Indiana, Kentucky

51. Oklahoma Natural Gas Gathering Corporation

    Oklahoma

52. Orange and Rockland Utilities, Inc.

New York

53. Pacific Gas Transmission Company*

California, Idaho, Oregon, Washington

54. Panhandle Eastern Pipe Line Company*

Illinois, Indiana, Kansas, Michigan, Missouri, Ohio, Oklahoma, Texas

55. Pennsylvania Gas Company

New York, Pennsylvania

56. Shenandoah Gas Company

Virginia, West Virginia

57. South Georgia Natural Gas Company

Alabama, Florida, Georgia

58. South Texas Natural Gas Gathering Company*

Texas

59. Southern Natural Gas Company*

Alabama, Georgia, Louisiana, Mississippi, South Carolina, Texas

60. Southwest Gas Corporation*

Arizona, California, Nevada

61. Sylvania Corporation

Pennsylvania

62. Tennessee Gas Pipeline Corporation (Tenneco)*

Alabama, Arkansas, Connecticut, Kentucky, Louisiana, Massachusetts, Mississippi, New Hampshire, New Jersey, New York, Ohio, Pennsylvania, Rhode Island, Tennessee, Texas

63. Tennessee Natural Gas Lines, Inc.*

Tennessee

64. Tensas Gas Gathering Corporation

Louisiana, Mississippi

65. Texas Eastern Transmission Corporation*

Arkansas, Indiana, Kentucky, Louisiana, Mississippi, Ohio, Tennessee, Texas

66. Texas Gas Pipe Line Corporation

Louisiana, Texas

67. Texas Gas Transmission Corporation*

Arkansas, Indiana, Kentucky, Louisiana, Mississippi, Ohio, Tennessee, Texas

68. Transcontinental Gas Pipe Line Corporation*

Alabama, Georgia, Louisiana, Maryland, Mississippi, New Jersey, New York, North Carolina, Pennsylvania, South Carolina, Texas, Virginia

69. Transwestern Pipeline Company*

Arizona, New Mexico, Oklahoma, Texas

70. Trunkline Gas Company*

Arkansas, Illinois, Indiana, Kentucky, Louisiana, Mississippi, Tennessee, Texas

71. Union Gas System, Inc.

Kansas, Oklahoma

72. United Fuel Gas Company (now part of Columbia Gas Transmission)

Kentucky, Ohio, West Virginia

73. United Gas Pipe Line Company*

    Alabama, Florida, Louisiana, Mississippi, Texas

74. United Natural Gas Company*

    Pennsylvania

75. Valley Gas Company

    Rhode Island

76. Valley Gas Transmission, Inc.

    Texas

77. Washington Gas Light Company*

    D. C., Maryland, Virginia

78. West Texas Gas Gathering Co.

    Texas

79. Western Gas Interstate Co.

    New Mexico, Oklahoma, Texas

80. Zenith Natural Gas Company

    Kansas

BIOGRAPHY

Robert Eugene Brooks received his A.B. degree in Arts and Science at the University of California, Berkeley, in June 1968 and his M.A. degree in physics at the University of Texas, Austin, in August 1972.

He has taught classes in applied mathematics and is currently a Program Advisor with Campus-Free College, Boston, Massachusetts.

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He has written two papers for presentation at the 1974 and 1975 National Conferences of the American Institute of Chemical Engineers: "Decision Analysis in Hazardous Material Transportation," with Ashok S. Kalelkar and Lawrence J. Partridge (1974), and "Use of Multidimensional Utility Functions in Decision Analysis in Hazardous Material Transportation: Applicability and Limitations" with Ashok S. Kalelkar (1975).

He is a student of the philosopher L. Ron Hubbard, has completed courses in Communcation and Study and is a ministerial student of the Church of Scientology.

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