archive

PRICING POLICIES FOR A TWO-PART EXHAUSTIBLE RESOURCE
CARTEL: THE CASE OF OPEC*

Esteban Hnyilicza
Robert S. Pindyck

MITEL76-008WP

April 1976

Massachusetts Institute of Technology
Cambridge, Massachusetts U.S.A.

ABSTRACT

This paper examines pricing policies for OPEC under the assumption that the cartel is composed of a block of <u>spender</u> countries with large cash needs and a block of <u>saver</u> countries with little immediate need for cash and a lower rate of discount. The decision problem for the two-part cartel is embodied in a game-theoretic framework and the optimal bargaining solution is computed using results from the theory of cooperative games developed by Nash. The set of feasible bargaining points -- and the corresponding Nash solution -- is computed under two assumptions on the behavior of output shares: that they are subject to choice and that they are fixed at historical values. Our results suggest that for fixed output shares, there is little room for bargaining and the price path approximates the optimal monopoly price path. If the shares are subject to control, optimal paths depend significantly on the relative bargaining power of each block.

E. Hnyilicza
R.S. Pindyck
April   1976

PRICING POLICIES FOR A TWO-PART EXHAUSTIBLE RESOURCE CARTEL: THE CASE OF OPEC

1. Introduction

Recent studies of optimal pricing policies for OPEC have treated the cartel as a unified group of countries that all have the same objectives, so that the behavior of the cartel is that of a pure monopolist. Pindyck [13], for example, developed an optimal pricing model in which OPEC, facing a net demand for oil that is the difference between a dynamic total demand function and a dynamic supply function for "competitive fringe" countries, and subject to production costs that rise as reserves are depleted, sets price over time to maximize its sum of discounted profits.[1] While studies such as this provide a useful first approximation to cartel behavior in that they describe how pricing policies depend on the inherent dynamics of reserve depletion and short-term lag adjustments, they do not account for the fact that many cartels are composed of producers with different objectives and   different degrees of bargaining power. Cartel "policy" in fact represents a negotiated agreement that reflects the different interests of the member producers.

OPEC is a good example of a two-part cartel. It consists of one group of saver countries (Saudi Arabia, Libya, Iraq, Abu Dhabi, Bahrain, Kuwait, and Qatar) that have little immediate need for cash and would thus use a low rate of discount in computing a sum of discounted profits, and a second group of spender countries (Iran, Venezuela, Indonesia, Algeria, Nigeria, and Ecuador)

---

[1]Optimal pricing models for OPEC have also been constructed recently by Cremer and Weitzman [2] and Kalymon [7]. Those models differ from that of Pindyck in that they do not account for short-term adjustment lags. Non-optimizing simulation models of OPEC behavior have also been constructed; see for example Blitzer et al. [1].

with large cash needs and a higher rate of discount. These groups also happen to differ with respect to the proven reserves available to be depleted over time; saver countries as a group have considerably greater proven reserves (a 1975 reserve-production ratio of 57) than do the spender countries (a 1975 R-P ratio of 28).[2] As we will see in this paper, the differences in discount rates and reserves will reinforce each other in terms of creating differences in desired policies for each group. Actual cartel policy depends on an agreement between the two groups that reflects both differences in objectives and in bargaining power.

Our approach in this paper is to seek a bargaining solution for the two-part cartel based on the theory of <u>cooperative</u> games developed by Nash.[3] To this end, we find optimal trajectories <u>for both price and the ratio of output shares</u>, assuming the cartel maximizes a <u>weighted sum</u> of the objectives (sums of discounted profits) for each of the two groups of countries. By repeatedly changing the weights, resolving and recomputing optimal sums of discounted profits for each group, we compute an <u>efficient (Pareto-optimal) frontier</u> in the space of realized objectives for the two groups of countries. Next, that set of weights that corresponds to a Nash cooperative solution is found. This corresponds to the bargaining solution, and gives us the optimal trajectories for price and market shares.

---

[2]Source: <u>Oil and Gas Journal</u>, December 1975.

[3]As we will explain shortly, the Nash <u>cooperative</u> solution should not be confused with the Nash solution for a non-cooperative game; the latter has received much more attention by economists.

The plan of this paper is as follows. In the next section we review
the optimal monopoly pricing model for OPEC developed by Pindyck, together
with the price trajectories implied by that model. This is necessary
because we use a slightly modified version of that model to calculate p
policies for OPEC as a two-part cartel in this paper, and also because we
wish to examine the extent to which pricing policies for a two-part cartel
would differ from those of a monopolistic cartel. The third section pre-
sents our framework of analysis for the two-part cartel; there we explain
the meaning of the Nash cooperative solution, and describe in detail how
the solution is obtained. Section 4 contains our empirical results, and
describes the characteristics of optimal prices and output shares for the
two-part cartel. Finally, we offer a summary and some concluding remarks.

2. Optimal Policies for a Monopolistic Cartel

The optimal pricing policies obtained by Pindyck for a monopolistic oil
cartel are based on a highly simplified dynamic model of the world oil market.
We review that model and its implications here since it provides the basis for
our description of the two-part cartel.[4]

The world oil market is described by the following equations, all of which
are parameterized to be consistent with the reserve, production, and elasticity
estimates of the OECD [11], and with average elasticity estimates obtained from
aggregate time series data:

---

[4]For a more detailed discussion of the monopolistic model and its implications
regarding the profits that OPEC can expect to accrue if it remains a cohesive
cartel, see Pindyck [13].

$$TD_t = 1.0 - .13P_t + .87TD_{t-1} + 2.3(1.015)^t \tag{1}$$

$$S_t = (1.1 + .10P_t) \cdot (1.02)^{-CS/7} + .75S_{t-1} \tag{2}$$

$$CS_t = CS_{t-1} + S_t \tag{3}$$

$$D_t = TD_t - S_t \tag{4}$$

$$R_t = R_{t-1} - D_t \tag{5}$$

where TD_t = total demand for oil (billions of barrels per year)

D_t = demand for cartel oil (bb/yr)

S_t = supply of competitive fringe (bb/yr)

CS_t = cumulative supply of competitive fringe (billions of barrels)

R_t = reserves of cartel (billions of barrels)

P_t = price of oil ($ per barrel), in real terms.

The demand equation (1) is based on a total demand of 18 billion barrels per year at a price of $6 per barrel, and at that price the short-run and long-run price elasticities are .04 and .33 respectively (with a Koyck adjustment), while at a $12 price the elasticities are .09 and .90 respectively. The last term in the equation provides an autonomous rate of growth in demand of 1.5% per year, corresponding to a long-run income elasticity of 0.5 and a 3% real rate of growth in income. Equation (2) determines supply for the competitive fringe, and is based on a level of 6.5 billion barrels per year at a $6 price. The short-run and long-run price elasticities are .09 and .35 respectively at the $6 price, and .16 and .52 respectively at a $12 price. Depletion of competitive fringe reserves pushes the supply function to the left over time. After

a cumulative production of 210 billions barrels (e.g. 7bb/yr. for 30 years) supply would fall (assuming a fixed price) to 55% of its original value.[5]

The objective of the monopolistic cartel is to maximize the sum of discounted profits:

$$\text{Max} \atop \{P_t\} \quad W = \sum_{t=1}^{N} \frac{1}{(1+\delta)^t} [P_t - 250/R_t]D_t .$$ (6)

Here the average cost of production for the cartel rises hyperbolically as R_t goes to 0. The initial reserve level is taken to be 500 billion barrels, and initial average cost is 50¢ per barrel. The planning horizon N is chosen to be large enough to approximate the infinite-horizon problem.[6]

Since average costs become infinite as R_t approaches 0, a resource exhaustion constraint need not be introduced explicitly, so that equations (1) to (6) represent a classical, unconstrained, discrete-time optimal control problem, and numerical solutions are easily obtained.[7] Optimal price trajectories for discount rates of .02, .05, and .10 are shown graphically in Figure 1, and prices, total demand, OPEC production, OPEC reserves, and discounted profits are given for δ = .05 in Table 1.

Observe that the optimal monopoly price is $13 to $14 in the first year (1975), declines over the next 5 years to around $10, and then rises slowly. This price pattern is a characteristic result of incorporating adjustment lags in the model –

---

[5] There is no fixed upper bound on cumulative production by competitive fringe countries; there is always some price at which additional supplies would be forthcoming. For example, after 210 billion barrels have been produced, a price of $18.50 would be needed to maintain production at 6.5 bb/yr.

[6] N=40 years usually provides a close enough approximation.

[7] Solutions were obtained using a general-purpose nonlinear optimal control algorithm developed by Hnyilicza [3]. In calculating optimal price policies the initial conditions were TD_0 = 18.0, S_0 6.5, CS_0 = 0, and R_0 = 500. Price trajectories were calculated over a 40 year horizon.
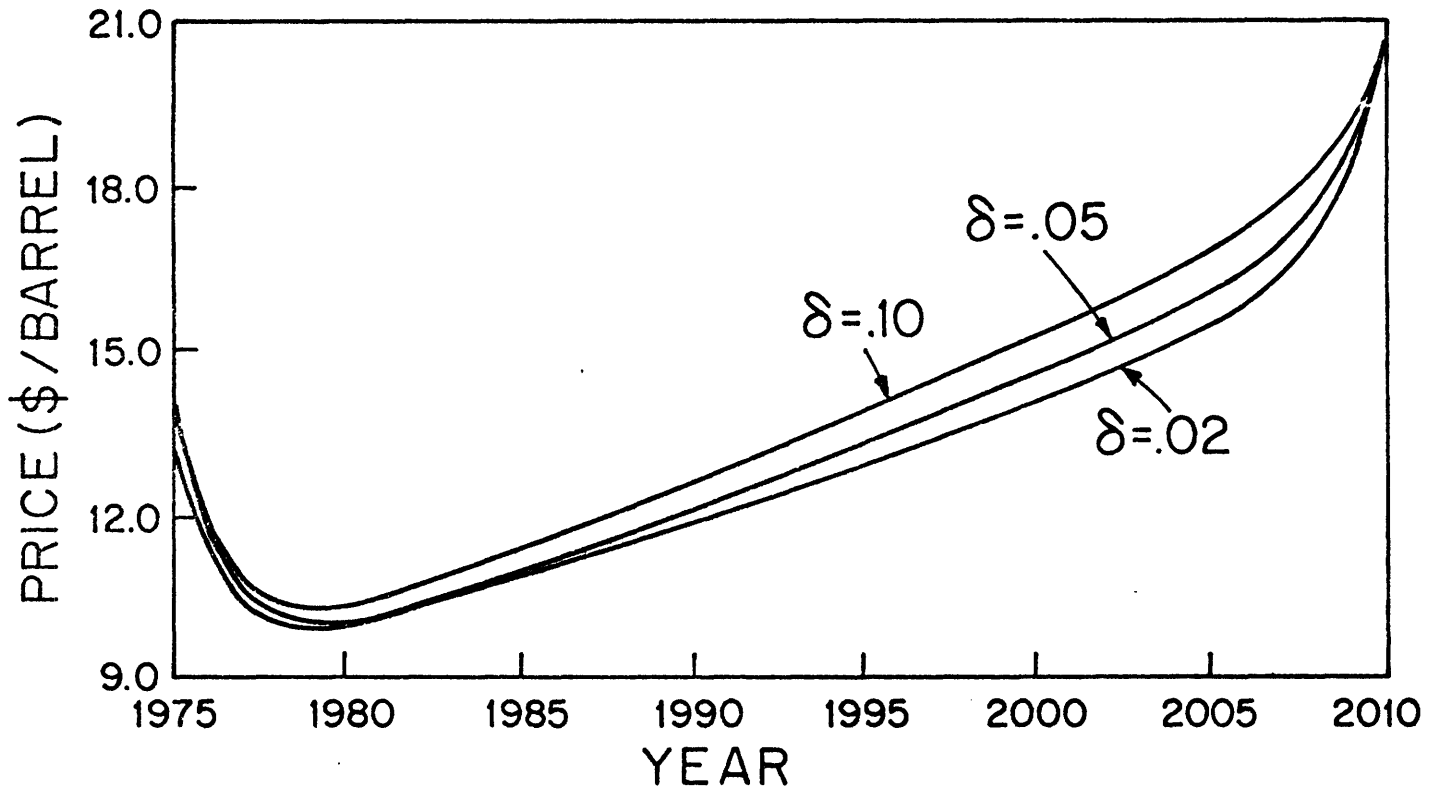
Optimal Price Trajectories for Monopolistic Cartel

Table 1

Monopolistic Cartel: Solution for $\delta = .05$

t	P	TD	D	R	Π_d
1975	13.24	17.24	9.94	488.5	126.5
1976	11.19	16.88	9.23	478.6	93.8
1977	10.26	16.72	8.94	469.3	78.9
1978	9.90	16.66	8.87	460.4	71.7
1979	9.82	16.66	8.91	451.5	67.9
1980	9.88	16.69	9.00	442.6	65.7
1985	10.84	16.96	9.67	396.3	60.6
1990	11.98	17.32	10.40	346.5	56.3
1995	13.18	17.74	11.15	293.0	51.8
2000	14.46	18.22	11.91	235.7	47.1
2005	15.92	18.75	12.66	174.6	42.5
2010	20.29	18.67	12.55	110.5	41.0

it is optimal for OPEC to charge a higher price initially, taking advantage of the fact that net demand can adjust only slowly. Of course, these results are dependent on the particular model and parameter values described above. However, changing the model's parameters has only a small effect on the numerical results. For example, if the elasticities (short- and long-term) of total demand are doubled, optimal prices decrease by less than 20%. Doubling the elasticity of competitive supply results in a decrease in price of about 10%. Replacing the total demand and competitive supply equations with isoelastic equations (using the $6 elasticities from the linear equations) results in price trajectories that are within 15% of those reported in Table 1. Finally, doubling or halving initial OPEC production costs, or changing the initial level of OPEC reserves from 500 billion barrels to 800 billion barrels has little effect (less than 10%) on the optimal monopolistic price trajectories.

3. The Two-Part Cartel: Framework of Analysis

We model the two-part cartel as consisting of a group of saver countries that have the objective

$$\text{Max } W_1 = \sum_{t=1}^{N} \frac{1}{(1+\delta_1)^t} [P_t - m_1/R_t^1] \, D_t^1 \qquad (7)$$

and a group of spender countries with the objective

$$\text{Max } W_2 = \sum_{t=1}^{N} \frac{1}{(1+\delta_2)^t} [P_t - m_2/R_t^2] \, D_t^2 \; . \qquad (8)$$

Here δ_1 is assumed to be smaller than δ_2.[8] D_t^1 and D_t^2 are the production

levels of each group of countries, and are determined by a division of total

cartel production:

$$D_t^1 = \beta_t D_t \tag{9a}$$

$$D_t^2 = (1-\beta_t)D_t \tag{9b}$$

with $0 \le \beta_t \le 1$. The depletion of the reserve levels for each group of countries

is accounted for by the equations

$$R_t^1 = R_{t-1}^1 - D_t^1 \tag{10a}$$

$$R_t^2 = R_{t-1}^2 - D_t^2 \tag{10b}$$

These equations, together with equations (1) to (4) from the previous

section, comprise our model of a two-part cartel in the world oil market. We

must now determine how the two groups of countries can cooperate to set price -

and divide up output - in an "optimal" manner. Suppose a cooperative agreement

is worked out whereby price and output shares are set to maximize a <u>weighted</u> <u>sum</u>

of the objectives of each group of countries:

$$\begin{array}{c} \text{Max } W = \alpha W_1 + (1-\alpha)W_2 \ , \quad 0 \le \beta_t \le 1 \\ \{P_t\} \\ \{\beta_t\} \end{array} \tag{11}$$

---

[8] If both groups of countries could borrow and lend in a perfect capital market
there would be no reason for the discount rates to differ. They cannot do this,
in part because of moral hazard, so that the discount rate reflects the <u>use</u>
value of the exhaustible resource (in terms of the return on domestic investment)
rather than its exchange value.

Note that this optimization problem of equations (1) to (4) and (9) to (11)

involves two control variables, price and the output share ratio β_t, and both

of these variables can vary over time.

By varying α between 0 and 1 and solving the resulting set of parametric

optimization problems, we obtain the Pareto-optimal frontier in the space of

realized outcomes (W_1, W_2), as in Figure 2. Clearly each point on the frontier

corresponds to a different trade-off between the relative objectives of the

two groups of countries. Note that the frontier need not touch the W_1 or W_2

axes – when $\alpha = 1$, for example, no weight is assigned to W_2, but the policy

that maximizes W_1 might result in a W_2 greater than zero.

Determining the value of α that is most likely to prevail as a result of a

negotiated agreement between the two groups of countries requires the solution of

a cooperative two-person game,[9] i.e. requires a theory of bargaining. An extremely

general and robust theory of bargaining was put forth by Nash in 1953 [10] two years

after he developed his better known non-cooperative theory. We describe the Nash

solution concept briefly below.

Since each of the two parties in a bargaining game attempt to move along

the set of bargaining outcomes in opposite directions, the problem is to determine

a meaningful measure of bargaining power for the two parties. Nash's approach was

to introduce the notion of a threat point, i.e. the outcome that would result if

---

[9] The most familiar solution concepts for two-person games correspond to the class
of non-cooperative strategies in which it is assumed that the players do not
have the ability to communicate among themselves or to coordinate the selection
of a joint strategy. The classical minimax solution to the two-person zero-sum
game proposed by Von Neumann and Morgenstern belongs to this category. Similarly,
the notion of a Nash equilibrium strategy is a non-cooperative solution which can
be viewed as a direct generalization of the minimax concept. However, from the
standpoint of our two part cartel model for OPEC, it seems unrealistic to assume
that each of the two blocks of countries will be unaware of the decisions and
capabilities of the other and that there will be no collusion between the two
groups. It is more likely that both groups of countries will attempt to coordinate
their strategies so as to increase the net gains accruing to each.

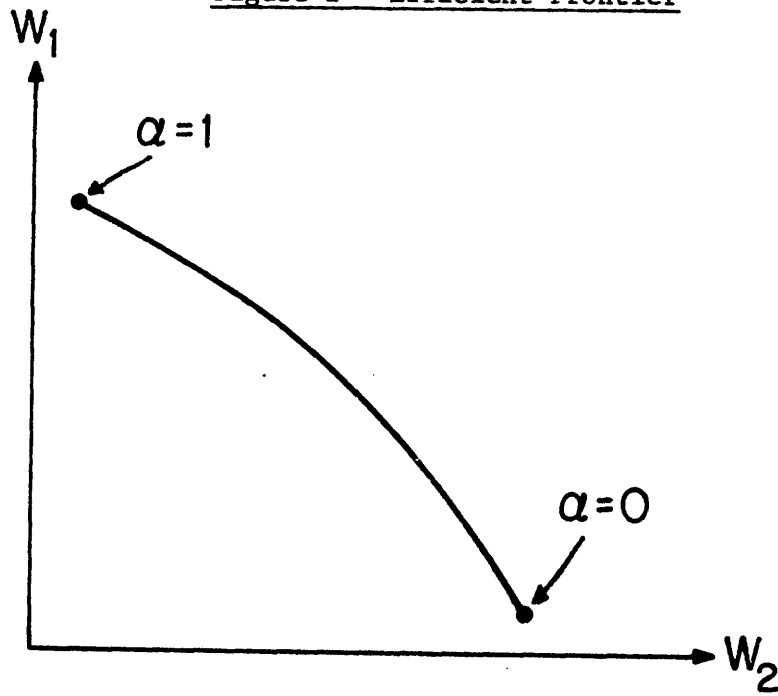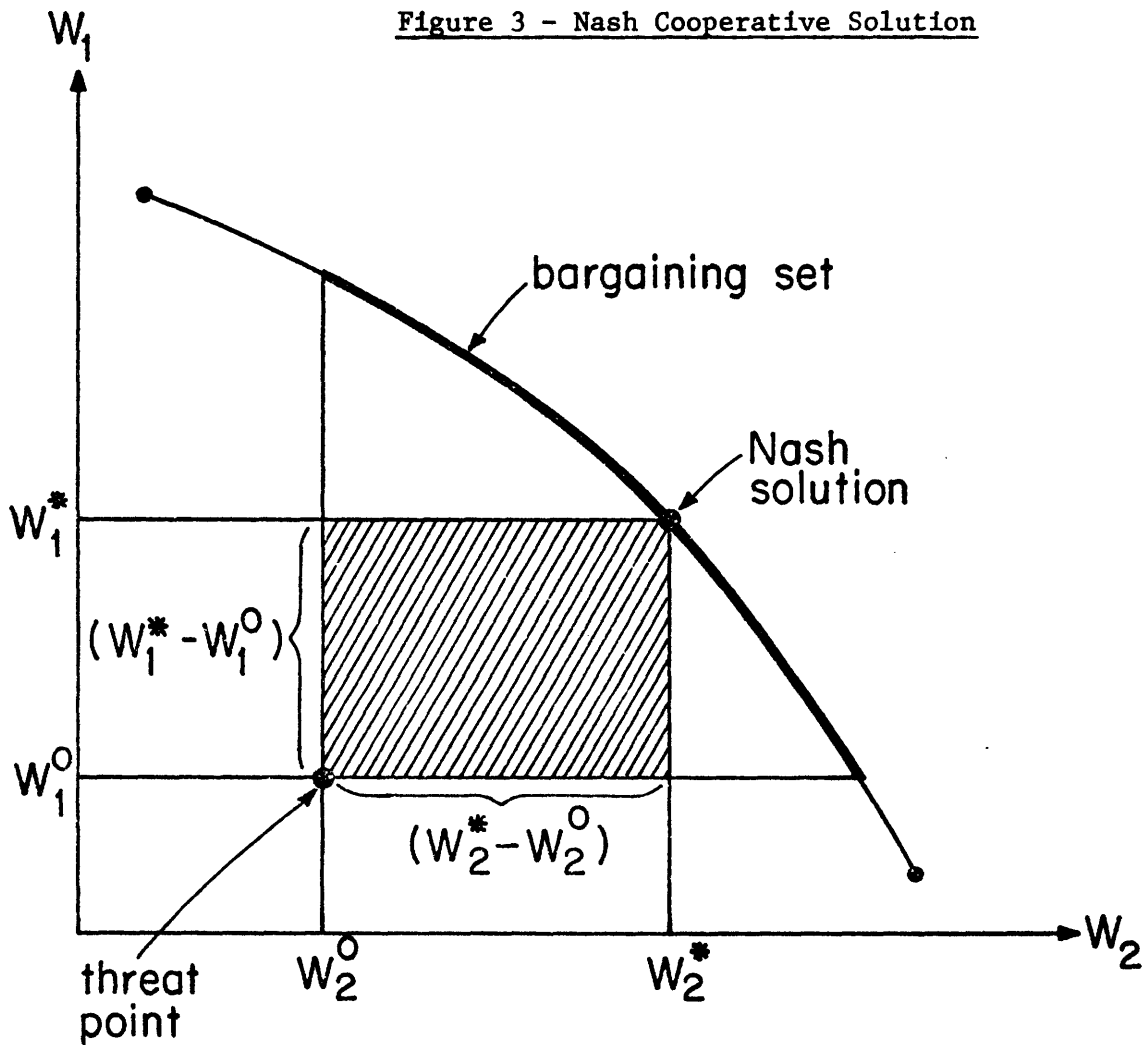## Figure 2 - Efficient Frontier

W_1

$\alpha = 1$

$\alpha = 0$

W_2

Figure 3 - Nash Cooperative Solution

W_1

bargaining set

Nash solution

W_1^*

$(W_1^* - W_1^O)$

W_1^O

$(W_2^* - W_2^O)$

threat point

W_2^O

W_2^*

W_2

negotiations were to break down and non-cooperative behavior were to ensue. In

Figure 3 the threat point is given by (W_1^o, W_2^o), and it might correspond

to the solution of a Nash <u>non-cooperative</u> game, or any other non-cooperative

game.[10] Obviously the bargaining set is bounded by W_1^o and W_2^o, since it would

be irrational for either party to accept a payoff less than that resulting

from non-cooperative behavior. In broad terms, Nash's solution is based on

the premise that the relevant measure of "relative power" which determines the

outcome of the bargaining process is given by the relative utilities at the

status quo or point of no agreement. This is plausible, since the reason

each party is willing to bargain is that it expects to accrue a payoff over

and above the payoff attained at the threat point. It seems reasonable that

both parties should be willing to accept a division of the net incremental

gains in a proportion directly related to the losses incurred by <u>not</u> making

an agreement.[11]

Nash demonstrated that his proposed solution to the bargaining problem is

in fact the <u>only</u> solution (W_1^*, W_2^*) that satisfies axioms of rationality, feas-

ibility, Pareto optimality, independence of irrelevant alternatives, symmetry,

---

[10] Nash also developed a solution concept for the case when the choice of the
threat point itself enters endogenously into the bargaining process. We
will not be concerned with this case here.

[11] An assumption for the Nash cooperative solution to hold is that both parties
must have the ability to make binding agreements, i.e. either a bargain is
agreed to, in which case both parties are committed to the strategy agreed
to, or else the outcome that will occur will be the threat point.

and independence with respect to linear transformations of the set of payoffs.[12]

Furthermore, that solution is such that $(W_1^* - W_1^o)(W_2^* - W_2^o)$ is maximized.[13] This last result makes the actual computation of the Nash solution straightforward; the Nash solution for α is simply that point on the Pareto-optimal surface for which the area of the shaded rectangle in Figure 3 is maximized.[14]

Of course it might be reasonable to expect bargaining solutions to prevail other than the Nash solution; for example, the two parties might behave in a less sophisticated manner and simply divide the sum of discounted profits in a ratio equal to that which prevailed in the preceeding two or three years. We study the sensitivity of the Nash solution by comparing it to solutions corresponding to other points on the Pareto-optimal frontier.

---

[12]Let S be the set of feasible outcomes and let $(W_1^*, W_2^*) = \phi(S, W_1^o, W_2^o)$ be the bargaining solution. Then by "rationality" we mean that $(W_1^*, W_2^*) \geq (W_1^o, W_2^o)$, by "feasibility" we mean that $(W_1^*, W_2^*) \epsilon S$, by "Pareto-optimality" we mean that if $(W_1, W_2) \epsilon S$ and $(W_1, W_2) \geq (W_1^*, W_2^*)$, then $(W_1, W_2) = (W_1^*, W_2^*)$, by "independence of irrelevant alternatives" we mean that if $(W_1^*, W_2^*) \epsilon T \subset S$ and $(W_1^*, W_2^*) = \phi(S, W_1^o, W_2^o)$, then $(W_1^*, W_2^*) = \phi(T, W_1^o, W_2^o)$, and by "independence of linear transformations" we mean that if T is obtained from S by the linear transformation $W_1' = \alpha_1 W_1 + \beta_1$ and $W_2' = \alpha_2 W_2 + \beta_2$, and if $\phi(S, W_1^o, W_2^o) = (W_1^*, W_2^*)$, then $\phi(T, \alpha_1 W_1^o + \beta_1, \alpha_2 W_2^o + \beta_2) = (\alpha_1 W_1^* + \beta_1, \alpha_2 W_2^* + \beta_2)$, and by "symmetry" we mean that if S is such that $(W_1, W_2) \epsilon S \leftrightarrow (W_2, W_1) \epsilon S$, and $W_1^o = W_2^o$, then $W_1^* = W_2^*$. These axioms are quite general, and could apply to many bargaining situations.

[13]For a simple proof of this, and further discussion of Nash cooperative behavior, see Owen [12]. Further discussion is also provided by Luce and Raiffa [8], and Harsanyi [3].

[14]Our choice of a monetary utility measure, i.e. the sum of discounted profits for each group of countries, represents no loss of generality.

Note that for any value of α, the optimal path for β_t will follow a "bang-bang" solution. In particular, the optimal β_t will remain at zero for some time (until spender country reserves are depleted) and then jump to 1 (where it will remain until saver country reserves are depleted).[15] It may not be realistic to expect the two groups of countries to agree to this allocation of output even if it is optimal, since the temptation to cheat would be considerable (and saver countries would have to risk the possibility of the cartel breaking up before they even begin to deplete their reserves). Instead we might expect the two groups to divide output in proportion to historical production levels, and simply optimize with respect to price. This would result in a different Pareto-optimal frontier, one that is closer to the origin. We also compute solutions for this case of a fixed and constant β_t. As we will see, under this constraint there is little room for disagreement over price policy for the two groups of countries.

---

[15] To see why the solution is of this form, write the Hamiltonian for the optimal control problem:

$$H = \frac{\alpha}{(1+\delta_1)^t} [P_t - m_1/R_t^1]D_t\beta_t + \frac{(1-\alpha)}{(1+\delta_2)^t} [P_t - m_2/R_t^2]D_t(1-\beta_t) - \lambda_{1t}D_t\beta_t - \lambda_{2t}D_t(1-\beta_t) +$$

$$\lambda_{3t}S_t + \lambda_{4t}g(TD_t, S_t, CS_t, P_t)$$

where $\lambda_1, \lambda_2, \lambda_3$, and λ_4 are co-state variables, and g is a function determining net demand D. Since this Hamiltonian is a linear function of β_t, it is maximized by β_t equal to 0 or 1 (depending on whether the sum of all those terms multiplying β_t is negative or positive). Since $\delta_2 > \delta_1$, the second term in H will decrease more rapidly as t increases than will the first term. For small values of α we would thus expect β_t to first be 0, and later switch to 1. When α is large (greater than .6), β_t will switch more than once.

4. Optimal Policies for the Two-Part Cartel

Optimal policies for the two-part cartel are obtained by solving the optimization problem defined by equations (1)-(4) and (9)-(11), with W_1 and W_2 defined by equations (7) and (8). As we explained above, the efficient frontier (defining the locus of possible bargaining solutions) is obtained by repeatedly resolving this optimization problem for different values of α.[16] For every value of α the optimal solution calls for a division of output (β_t) that assigns zero production to saver countries for part of the time, and zero output to spender countries for the rest of the time. Since this may be a politically infeasible solution – although it is economically optimal – we also compute an efficient frontier under the constraint that β_t is fixed and constant at its average historical value. We set β_t equal to 0.65 in computing this second frontier.

In order to find the Nash solution on an efficient frontier it is first necessary to obtain the "threat point," i.e. the values of W_1 and W_2 that would result from non-cooperative behavior. Ideally this should be the solution to a dynamic duppóly problem, since presumably the countries in each group would continue to set price and output in a unified manner, but the two groups would compete with each other. However, the solution to a dynamic duopoly problem depends on the particular behavioral assumptions that are made, e.g. Nash (non-cooperative) behavior, Stackelberg, etc. In addition, even using the simplest assumptions obtaining numerical solutions for this nonlinear and non-quadratic dynamic game problem would be computationally difficult. Therefore we compute the threat point taking completely competitive behavior as the alternative to a negotiated agreement.

---

[16] A time horizon of 40 years is used throughout; this provides a reasonably close approximation to the infinite-horizon problem. The initial reserve level for the saver countries is 355 billion barrels, and for the spender countries is 145 billion barrels. Initial production cost is $.50 per barrel for both groups.
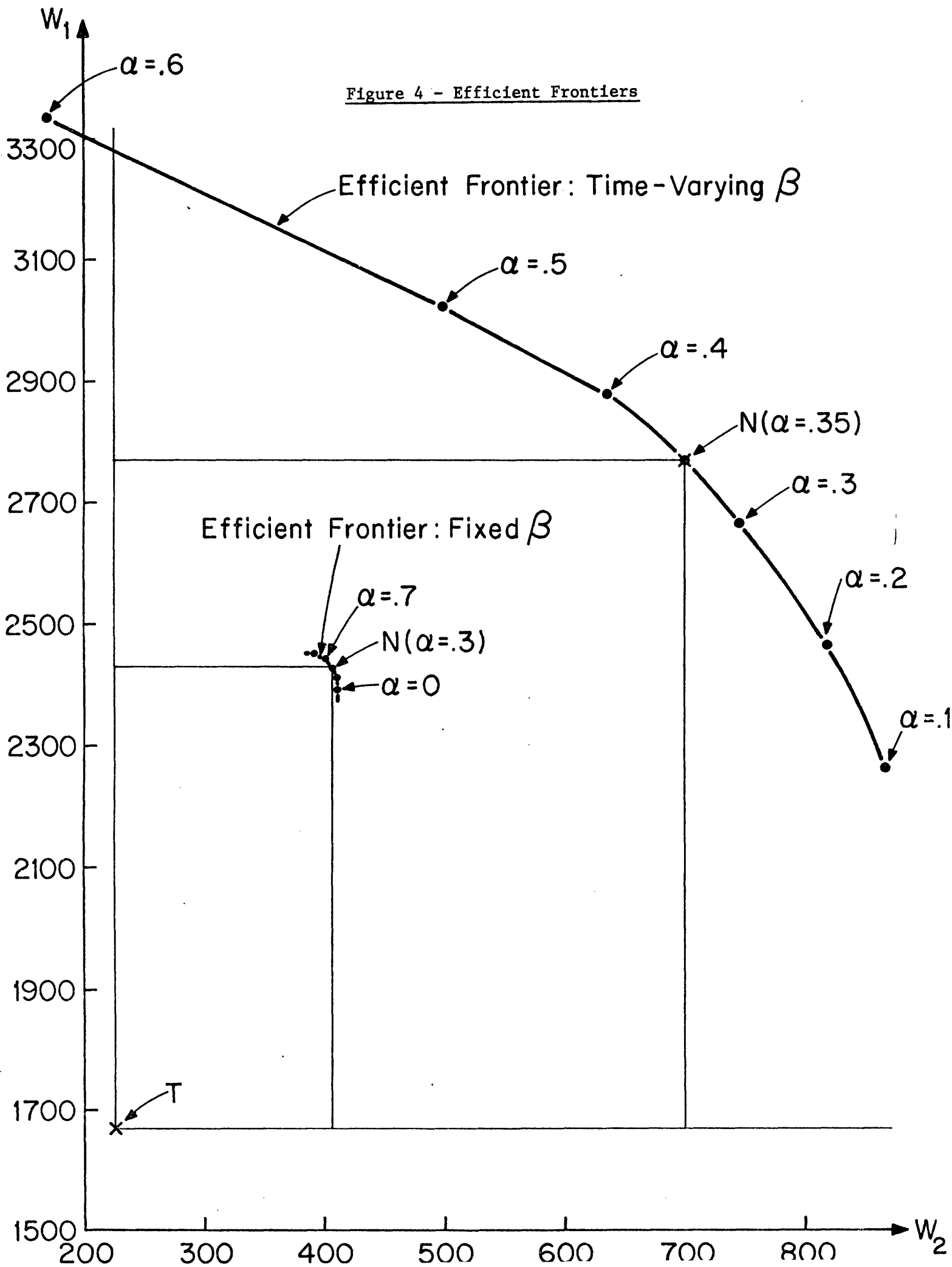
Figure 4 - Efficient Frontiers

TABLE 2

Time-Varying β, Nash Solution (α = .35)

t	β	P	TD	D_1	D_2	R_1	R_2
1975	0	14.85	17.03	0	9.57	355.0	133.5
1976	0	12.32	16.55	0	8.66	355.0	123.9
1977	0	11.17	16.31	0	8.27	355.0	115.3
1978	0	10.71	16.20	0	8.13	355.0	107.0
1979	0	10.57	16.16	0	8.13	355.0	98.9
1980	0	10.55	16.17	0	8.20	355.0	90.7
1981	0	10.53	16.21	0	8.35	355.0	82.5
1982	0	10.33	16.32	0	8.59	355.0	74.2
1983	0	9.72	16.52	0	8.98	355.0	65.6
1984	0	8.18	16.94	0	9.71	355.0	56.6
1985	0	4.65	17.80	0	11.12	355.0	46.9
1986	1	13.62	17.43	10.48	0	355.0	35.8
1987	1	12.66	17.27	10.23	0	344.5	35.8
1988	1	12.26	17.22	10.18	0	334.3	35.8
1989	1	12.15	17.23	10.23	0	324.1	35.8
1990	1	12.19	17.28	10.34	0	313.9	35.8
1995	1	13.10	17.73	11.15	0	260.6	35.8
2000	1	14.25	18.31	12.04	0	203.1	35.8
2005	1	15.63	18.92	12.91	0	141.1	35.8
2010	1	20.34	18.82	12.75	0	75.7	35.8

W_1 = 2778 $\qquad\qquad$ W_2 = 698

In computing the competitive solution we assume that all producers maximize profits taking price as given, and that they all use the same 5% discount rate in determining output levels.[17] The competitive price begins at $3.06 and rises slowly to $12.60 after 38 years, at which point exhaustion occurs. Total discounted profits to saver countries (discounted at 2%) is $1668 billion, and to spender countries (discounted at 10%) is $223 billion.

The efficient frontiers for both the time-varying β and fixed β cases are shown in Figure 4 together with the threat point. For each frontier the Nash solution (W_1^*, W_2^*) is found by maximizing $(W_1 - W_1^o)(W_2 - W_2^o)$, where (W_1^o, W_2^o) is the threat point, i.e. by finding the rectangle between the threat point and the frontier that has the largest area. The Nash solutions are shown in the figure; for β varying the solution corresponds to $\alpha = .35$, and for β fixed it corresponds to $\alpha = .30$. For both solutions the values of price, output shares, and other relevant variables are given in Tables 2 and 3.

Observe in Table 2 that when output shares can be chosen freely, the Nash solution calls for β_t switching once from 0 to 1. For the first 11 years β is 0 and only the spender countries produce, and after that time – at which point the spender countries have exhausted most of their reserves – β is 1 and only the saver countries produce. This is optimal since production by spender countries

---

[17]Competitive producers maximizing their sum of discounted profits must set output balancing profits this year against profits in later years. The resulting market price will satisfy the difference equation

$$P_t = (1+\delta)P_{t-1} - \delta C(R_{t-1})$$

where C is production cost as a function of the reserve level. In addition, the initial price must be such that market demand and supply are equal at every point in time, and, as price rises over time, resource exhaustion occurs at the same time that net demand goes to zero. This is shown in Pindyck [13]. Computing the optimal price trajectory for the competitive case is thus straightforward; pick an initial P_0 and solve over time the above equation together with equations (1) to (5). Repeat this for different values of P_0 until D_t and R_t become zero simultaneously.

is discounted more heavily. With saver country oil losing its value less
rapidly, it is better held in the ground while spender country oil is
produced first.[18]

The optimal price trajectory is quite different from that in the mono-
polistic case. Initially, while spender countries are producing, the price
starts at $14.85 and falls slowly to about $10 after 8 years. This pattern
is similar to that in the monopolistic case, and takes advantage of the lag
adjustments to price changes in total demand and competitive supply. In the
next three years, however, price drops to $4.65, and then in 1986, when β_t
switches to 1 and saver countries begin producing, the price jumps back up
to nearly $14. This pattern is optimal because it permits demand to expand
again so that saver countries can charge a high price (again taking advantage
of lag adjustments) when they start producing in 1986. During the remaining
25 or 30 years the price pattern is again similar to the monopolistic case;
price drops slightly for five years, and then begins a slow, steady increase.

Although this price pattern – a sharp decline in price just before saver
countries begin producing, and then a sharp increase afterwards – will hold for
different values of α , the point at which β switches from 0 to 1 will vary
considerably as α is varied. Optimal price trajectories for three values of
α (.1, .3, and .5) are shown in Figure 5. For α equal to .1, β switches in

---

[18]This is true for values of α less than .6. For larger values of α (i.e.
assigning a larger weight to spender country profits) β will begin with
a value of 1, switch to 0 after 2 or 3 years and remain at 0 for several
years, and then switch back to 1, remaining there for the rest of the
planning horizon. This double switching is a result of the discrete-time
nature of our problem, and permits saver countries to receive still larger
profits at the expense of spender countries.

1991,[19] for α equal to .3, β switches in 1987, and for α equal to .5, β

switches in 1982. Thus when β is free to vary, the optimal policy depends

highly on α , i.e. on the negotiated agreement between the two groups of

countries.

When β is fixed and only price can be chosen optimally, the resulting

solution is close to the monopoly solution. There is a considerable welfare

loss, however, to both groups of countries. As can be seen in Figure 4, the

efficient frontier is much closer to the threat point when β is fixed. The

Nash solution in this case corresponds to α equal to .3, and the resulting

trajectories for price and other variables are given in Table 3. By comparing

these results with those in Table 1, we see that the solution is very close to

the monopoly solution with a 5% discount rate. Since the monopoly price tra-

jectory is not very sensitive to the discount rate (see Figure 1), there is

little that can be done without adjusting output shares to better satisfy the

different objectives of the two groups of countries. As a result, there is

very little to bargain over when β is fixed. As can be seen in Figure 6,

different values of α result in optimal price trajectories that are almost

exactly the same, i.e. the price trajectory that is optimal for saver countries

is almost optimal for spender countries. This is reflected in the narrow range

of the efficient frontier in Figure 4.

---

[19] Here almost all of the weight in the objective function goes to the profits of
spender countries, so the pricing policy for the first 13 years is close to what
would be optimal for a monopolistic cartel consisting only of the spender countries.
Prices rise steply from 1980 to 1988 because spender country reserves are rela-
tively small. Even after price drops to about $2 in 1990, saver countries must
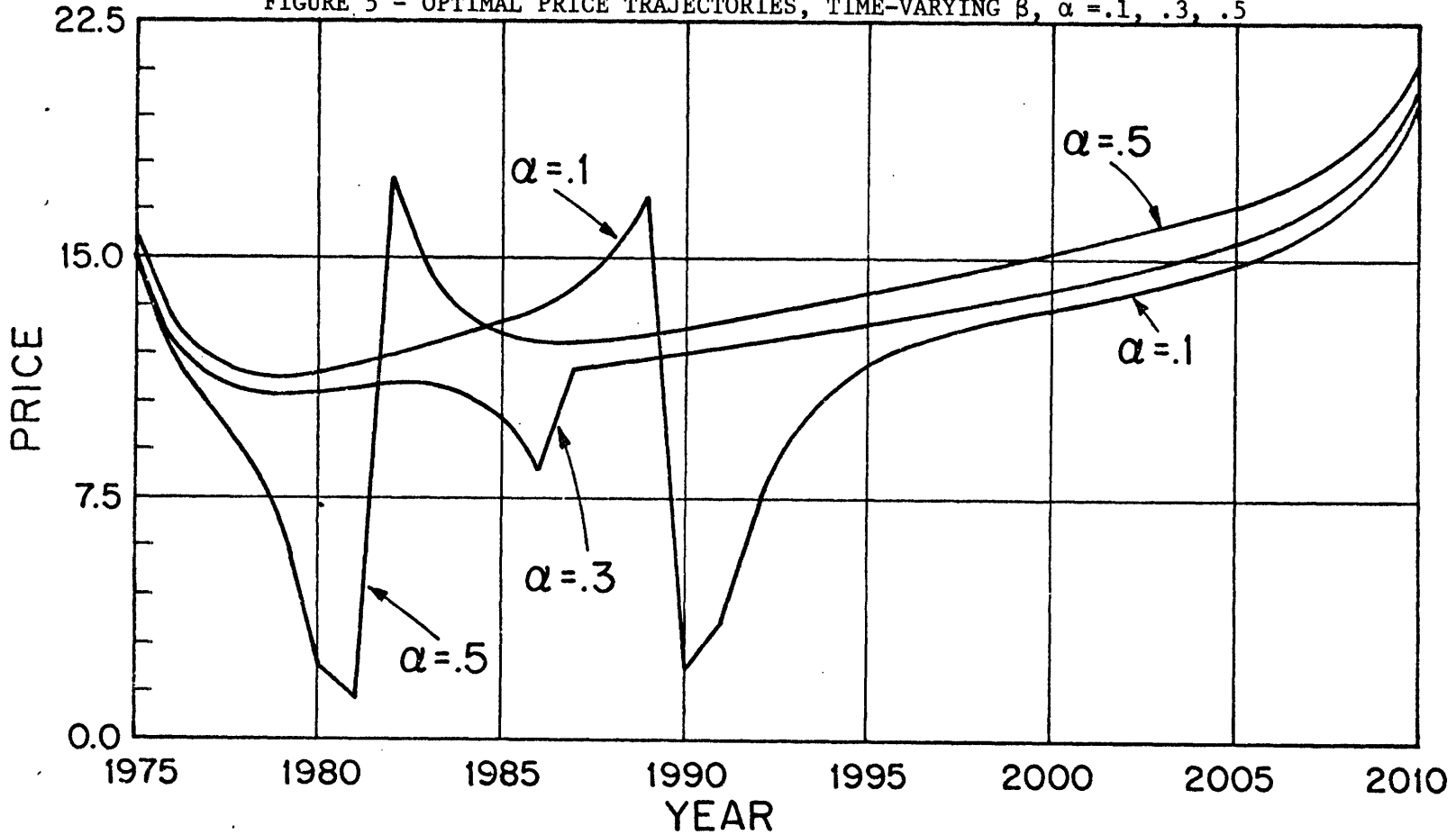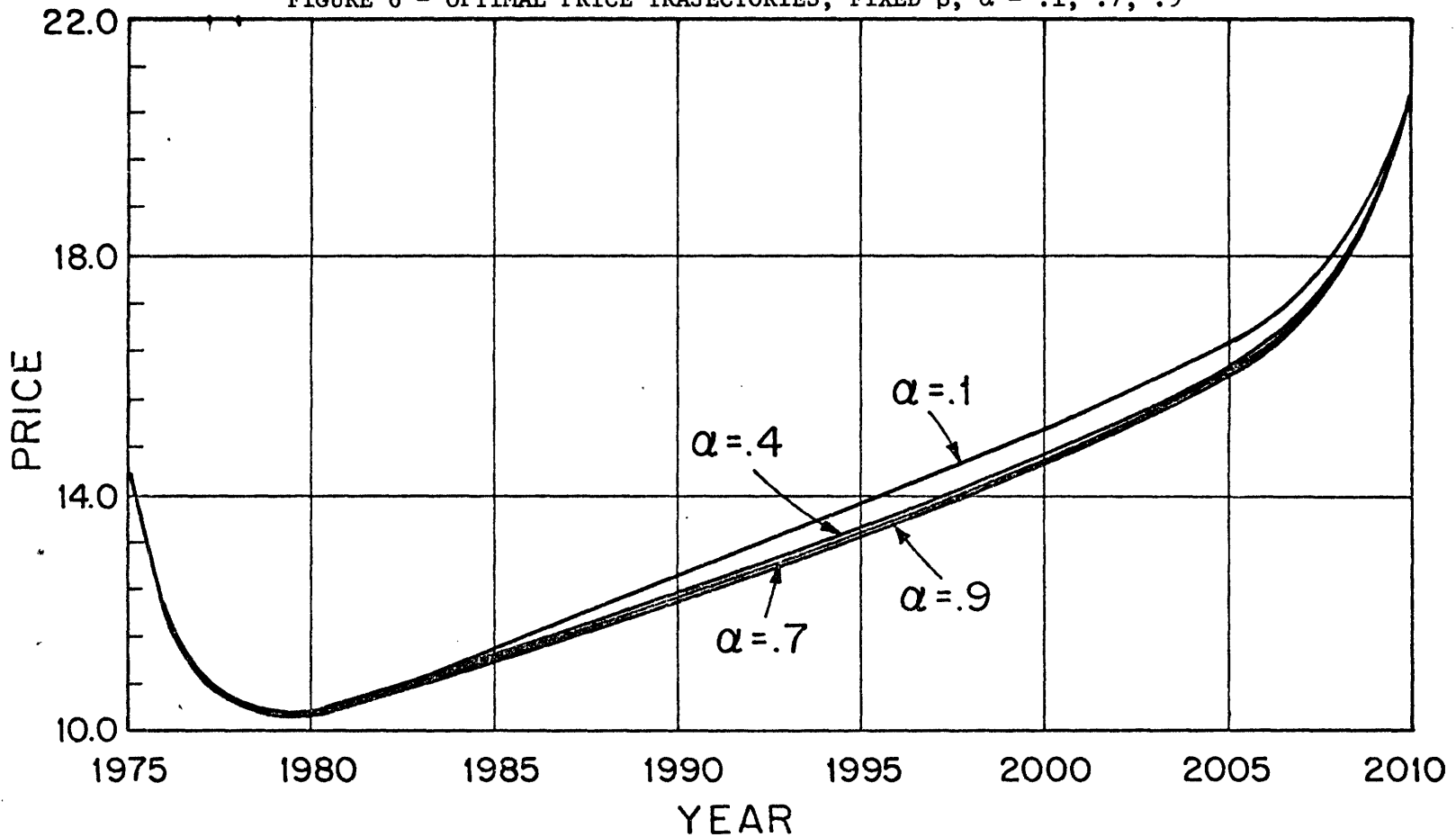raise price more gradually in order to allow demand to increase again.

FIGURE 5 - OPTIMAL PRICE TRAJECTORIES, TIME-VARYING β, α =.1, .3, .5

FIGURE 6 - OPTIMAL PRICE TRAJECTORIES, FIXED β, α = .1, .7, .9

TABLE 3

Fixed β, Nash Solution (α = .3)

t	P	TD	D_1	D_2	R_1	R_2
1975	14.39	17.09	6.27	3.38	347.5	141.0
1976	11.99	16.64	5.77	3.08	341.2	137.6
1977	10.88	16.44	5.49	2.96	335.5	134.5
1978	10.42	16.35	5.42	2.92	330.0	131.5
1979	10.30	16.33	5.44	2.93	324.6	128.6
1980	10.35	16.34	5.49	2.95	319.1	125.7
1981	10.48	16.37	5.56	2.99	313.6	122.7
1982	10.65	16.41	5.64	3.04	308.1	119.7
1983	10.85	16.45	5.77	3.09	302.4	116.7
1984	11.06	16.51	5.83	3.13	296.7	113.6
1985	11.28	16.56	5.92	3.19	290.8	110.4
1990	12.38	16.91	6.41	3.45	260.1	93.9
1995	13.52	17.36	6.94	3.74	227.0	76.1
2000	14.72	17.89	7.50	4.03	191.1	56.8
2005	16.12	18.47	8.04	4.33	152.5	36.0
2010	20.61	18.40	7.98	4.29	111.7	14.0

$W_1 = 2439$ $W_2 = 407$

5. Conclusions

We have seen that most of the policy negotiations confronting a two-part oil cartel will be over the allotment of output shares over time, since pricing strategy follows almost directly from output strategy. In addition, the output strategy that is optimal is a drastic one – saver countries must produce nothing for the first 10 or 12 years. If OPEC members find this policy unacceptable (as indeed we would expect them to) and instead hold output shares fixed, the losses will be significant, particularly for spender countries. The Nash solution for fixed output shares results in total discounted profits of $2439 billion and $407 billion for saver and spender countries respectively, as compared to $2778 billion and $698 billion for the Nash solution with time-varying output shares.

When output shares are open for policy discussion, OPEC members will have a lot to argue about, and any resulting optimal policy will depend considerably on the relative bargaining power of the two groups of countries. On the other hand when output shares are fixed, there is very little to discuss, and policy formulation could almost be left to the computer.[20]

While it is hard to imagine OPEC members agreeing to the kinds of on-off policies obtained in this paper, there is likely to be some flexibility in adjusting output shares. A "compromise" policy might be adopted whereby saver countries initially cut back production more than spender countries, but then expand production after 10 or 15 years, either with agreed-upon cutbacks by

---

[20]So far OPEC policy formulation has not been relegated to a computer. For a discussion of the ways by which OPEC arrives at a policy, see Mikdashi [9].

spender countries (who by then may have exhausted a significant fraction of their reserves) or with a drop in price. In fact, such a compromise policy may be exactly what we are observing now. Saudi Arabia, Iraq, and the other saver countries have been absorbing most of the cuts in production, while Iran, Indonesia, and the other spender countries are maintaining production levels that are much closer to full capacity. Recent OPEC prices would also be quite consistent with such a compromise policy; recall from Tables 2 and 3 that in the Nash solution price falls from about $14 to about $10 during the first five years for both the fixed β and time-varying β cases. If such a compromise policy has been or is about to be adopted, we would expect price to drop somewhat in the few years before Saudi Arabia and the other saver countries increase their production shares.

To what extent is a two-part cartel model such as ours, as opposed to a simpler monopolistic model, needed to understand OPEC policy? We have seen that if output shares are fixed the two-part cartel will choose the same pricing policy as the monopolistic cartel. Output allocation, however, is likely to be an important aspect of OPEC policy, particularly in the future as the supply of oil from competitive fringe countries increases and OPEC is forced to cut back production further. Recognizing that the cartel consists of producers with somewhat different interests will be essential in predicting its response to these future cutbacks.

REFERENCES

1. Blitzer, C., A. Meeraus, and A. Stoutjeskijk, "A Dynamic Model of OPEC Trade and Production," _Journal of Development Economics_, Fall, 1975.

2. Cremer, J., and M. Weitzman, "OPEC and the Monopoly Price of World Oil," _European Economic Review_, this issue.

3. Harsanyi, J. C., "Approaches to the Bargaining Problem Before and After the Theory of Games," _Econometrica_, Vol. 24, April 1956.

4. Harsanyi, J. C., "N-Person Cooperative Games," _International Economic Review_, Vol. 4, 1963, pp 194-220.

5. Hnyilieza, E., "OPCON: A Program for Optimal Control of Nonlinear Systems," May 1975.

6. Hotelling, H., "The Economics of Exhaustible Resources," _Journal of Political Economy_, April 1931.

7. Kalymon, B. A., "Economic Incentives in OPEC Oil Pricing Policy," _Journal of Development Economics_, Vol. 2, 1975, pp 337-362.

8. Luce, R. D. and H. Raiffa, _Games and Decisions_, John Wiley & Sons, New York, 1957.

9. Mikdashi, Z., "The OPEC Process," _Daedalus_, Fall 1975.

10. Nash, J. F., "Two-Person Cooperative Games," _Econometrica_, Vol. 21, January 1953, pp 128-140.

11. Organization for Economic Cooperation and Development, _Energy Prospects to 1985_, Paris, 1974.

12. Owen, G., _Game Theory_, W. B. Saunders Co., Philadelphia, 1968.

13. Pindyck, R. S., "Gains to Producers from the Cartelization of Exhaustible Resources," M.I.T. Sloan School Working Paper, May 1976.