Design and Analysis of a Bulk Note Feeding Device

By

Henry John Dotterer

SUBMITTED TO THE DEPARTMENT OF MECHANICAL ENGINEERING IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF

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June, 1991

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Abstract

A new technology in the field of bill handling has been investigated. Eleven methods of counting and separating cash **in bulk** from stacks are proposed. These are critiqued, and the design judged most promising is tested in prototype. That device utilizes a rubber contact wheel spun about two parallel axes to contact, buckle, count and lift bills. It descends into a stack as it cycles, forming a second stack (of counted notes) above itself. The prototype's performance is examined analytically to aid in optimization; the relationships predicted between parameters are found to be consistent with intuition. Also, pertinent forces are predicted within an order of magnitude. Furthermore, a software simulation of the prototype is provided, for use in investigating the effects of different device parameters. Final recommendations are then made, regarding the advancement of bill feeding technology.

Thesis Supervisor: Harry West
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I. Introduction

In May of 1989, Omron Tateisi Corporation of Japan joined with Professor Harry West of MIT's mechanical engineering department for the purpose of establishing an intercultural technical exchange. Omron is a large company with a diversity of technical strengths, producing products for use in the fields of manufacturing, laboratory measurement, money handling, and automation. On several projects dealing with Omron's money handling technology, Harry West and his students at MIT[1] have provided design and analysis work, while Omron has provided hands-on and fabrication skills.

Omron's current money machine, the ABIO (Automatic Bill In/Out machine), uses an ingenious but complex system of belt drives, solenoid diverters, stack containers, and bill feeders. The technology in the ABIO, like in other money machines in use, is based upon the manipulation of bills individually. Over the past 20 years, Omron's technology has evolved toward the goal of achieving optimum reliability and speed, with minimum space and cost.

The ABIO currently uses three bill feeding devices to send cash to a user. These are necessary to service the stacks of 1,000, 5,000 and 10,000 yen notes. Utilizing rollers to separate bills from the stack, they feed bills individually into a belt drive system. Omron wishes to make the process of bill feeding in its ABIO more efficient.

It is believed that improvement in bill handling efficiency will not come from the manipulation of bills individually, but from the handling of bills in stacks. Furthermore, the cost and space required for a machine might be reduced by using one feeder to service all three of the machine's money stacks. (Figure I-1 compares the two methods.) Hence, Omron and MIT this year began a joint project to develop a technology for the handling of bills in bulk, by an end effector on a robotic manipulator which could move this device to each of the stacks.

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[1] Members of the project are Professor Harry West from MIT's Mechanical Engineering Department; Ross Levinsky, a second year masters student of the same department; Tad Snow, a design consultant***; Mr. Ryuchi Onamoto; Mr. Ichiro Kubo; and Mr. Sugitate.***

Several methods of developing such technology have been suggested, and a prototype stack handling unit has been fabricated. This thesis includes a discussion of the methods investigated, documentation and evaluation of the prototype built, and a theoretical analysis of the factors influencing the device's performance. Further work is necessary before bulk bill handling will be realized; recommendations are made.

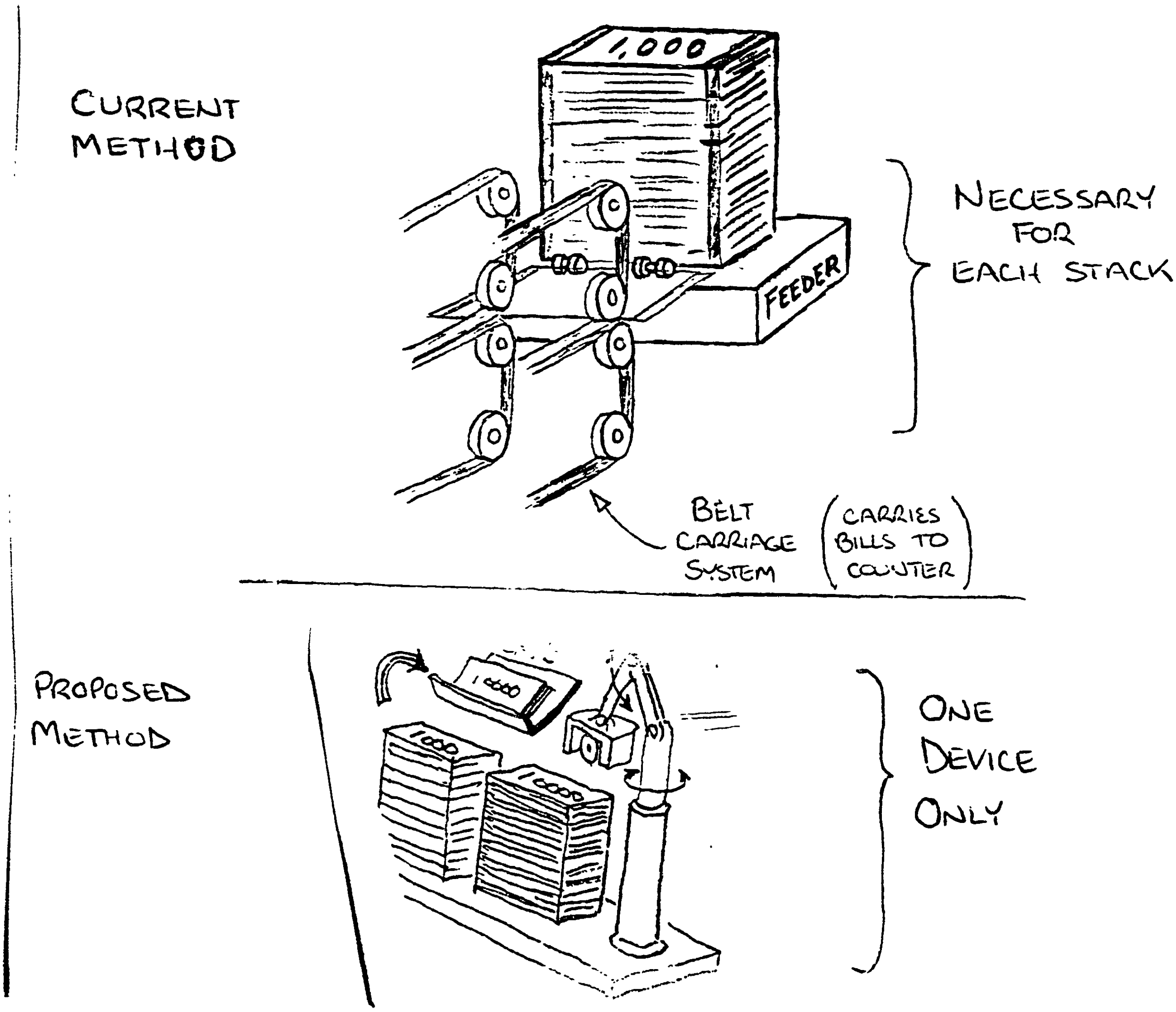


Figure I-1: Bill Feeding Technologies

II. Designs Proposed

Many forms of stacked bill counting were considered before a prototype was designed. This section contains 11 methods proposed by engineers at Omron and at MIT.

II-1. Type A. Buckling of Bills

Four methods of counting bills by buckling were considered, each involving the deformation of bills from the top of a stack through the use of rollers. In these methods, portions of bills are separated from large stacks into a stack of known number to be grasped and transported.

Method 1. Single-Sided, Dual Roller

To anchor the stacked bills, a stabilizer is placed at some distance along the bills' length, extending across its width. A dual roller is located to one side of this stabilizer, consisting of one to three small "contact wheels" driven at some chosen velocity, connected to a larger "plate roller", driven independently. The contact rollers extend across the stack's width. This roller arrangement is used to contact, buckle and lift bills individually as shown schematically in figure II-1.

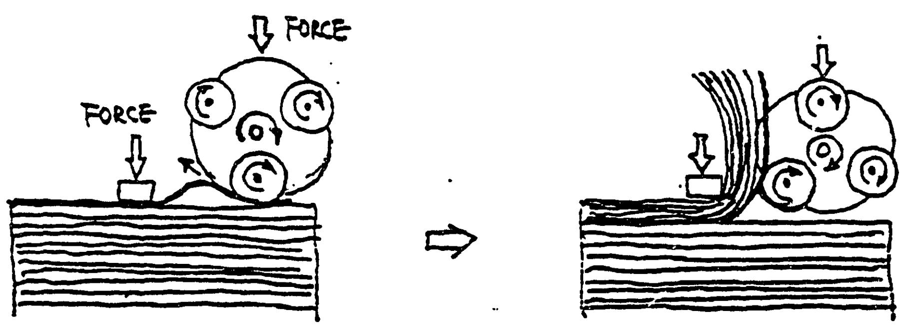


Figure II-1: Method 1

Starting flat in the stack, bills collect above the roller arrangement as they are counted. Each note is counted as its buckled portion obscures an optical sensor while being transported upward. A high friction wheel contact surface is used to insure reliable transportation of bills. In this way, a stack of bills of a desired number is collected. This design was eventually accepted, and is discussed in section III.

Method 2. Double-Sided Buckling, Single Roller

In this method, shown in figure II-2, bills are anchored by a central stabilizer and buckled on both sides by contact wheels. As in method 1, the bills are counted by optical sensors as they are lifted above the rollers. By counting bills on both sides of the stabilizer, the possibility of miscounting is diminished. In this arrangement, the feeding mechanism is connected to the stack container. The mobile manipulator consist of only a stabilizer and a grasping mechanism. Because there would have to be one feeder arrangement for each stack, the rollers in the three stacks could operate simultaneously to prepare the appropriate number of bills before the manipulator arrived. Although two more feeders and three complex containers would be necessary for this method, this method might be preferable to method one because of the decreased weight and simplicity of the manipulator, and the potentially higher bill delivery speed.

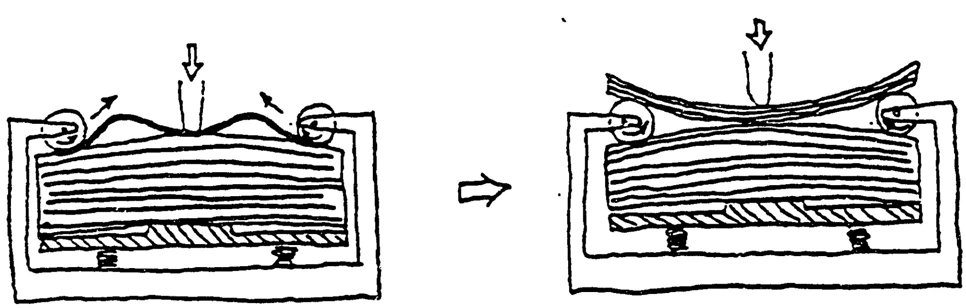


Figure II-2: Method 2

Method 3. Single Buckle; Dual Roller

The third method is similar to the second, but does not require the feeder to be connected to the container. As figure II-3 shows, two contact wheels are counter-rotated to produce a buckle between them. This buckle pushes past a flexible film, and is counted as it does so. From then on it is isolated from the stack by the film. After the buckle passes the film, the rollers continue to rotate, and carry the two edges of the bill upward, so that the note finally rests atop a portion of the container. The wheels, rotating continuously, start the next cycle. In this way, a stack is formed atop the container, and may be carried off.

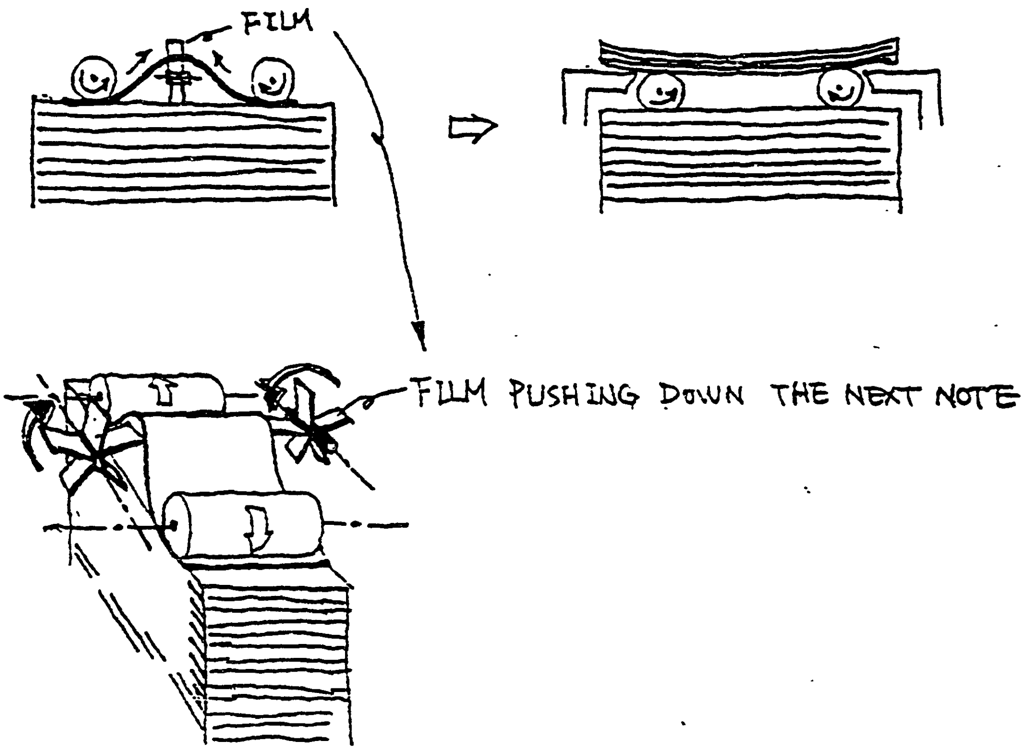


Figure II-3: Method 3

Method 4. Roller, Shaft Assist

In this method, a roller is again used to buckle a bill. After the bill has been buckled some amount, a shaft or shafts move into the gap created between the bill and the stack. These shafts are then raised, assisting in the lifting of both the buckled bill and the counted stack, which the note would otherwise have to support. Figure II-4 depicts this method.

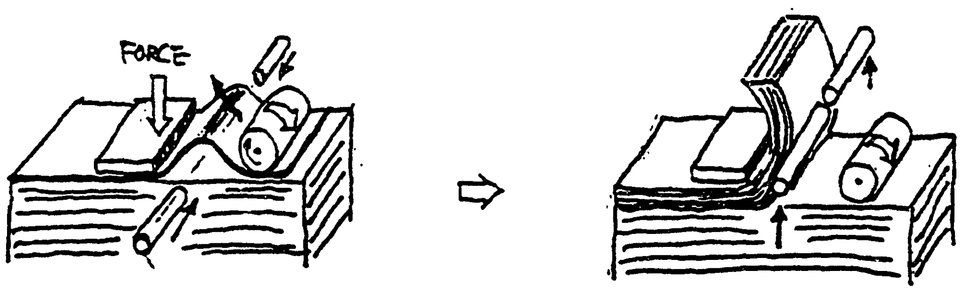


Figure II-4: Method 4

Type A methods were judged to have the highest possibility of success, partially because they are most similar in design to the note feeders which Omron has already used successfully.

II-2. Type B. Utilization of Stacked Bills' Energy

The second type of bulk counting method investigated involves the use of the bill or stack's internal energy of deformation. In these methods, bills are initially deformed some amount, and then are allowed to return to their lowest energy state one by one. They are counted and collected as they do so. Three forms of internal energy counters were proposed.

Method 5. Prefolding, Roller

In this method, sketched in figure II-5, bills initially flat in a stack are folded over before being returned (individually, and in controlled and measured quantity) to their original configuration. This method makes use of not only the springiness of the stack, but also gravity, since bills roll downward as the roller seperates them from the stack.

This method doesn't allow for a large stack. Omron's stacks often contain 1000 to 2000 notes. For this reason, this method would be useful to Omron only if some strategy were developed for preparing the notes in stacks manageable by this proposed device.

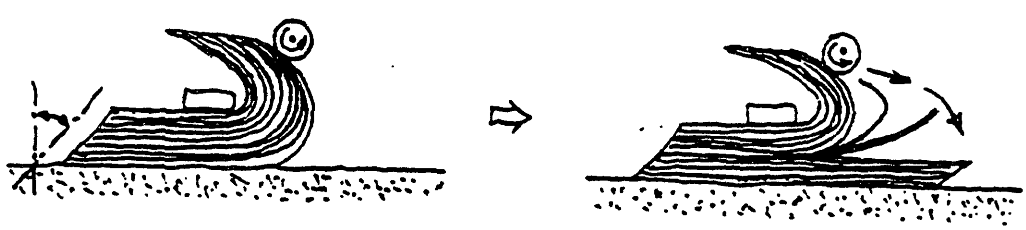


Figure II-5: Method 5

Method 6. Prefolding, Air

Like method 5, this method would utilize the energy of folded bills, and the action of gravity. It would be less dependent on these, however, than a jet (or several jets) of air used to peel and unfold the bills individually from the folded stack. Figure II-6 shows the device.

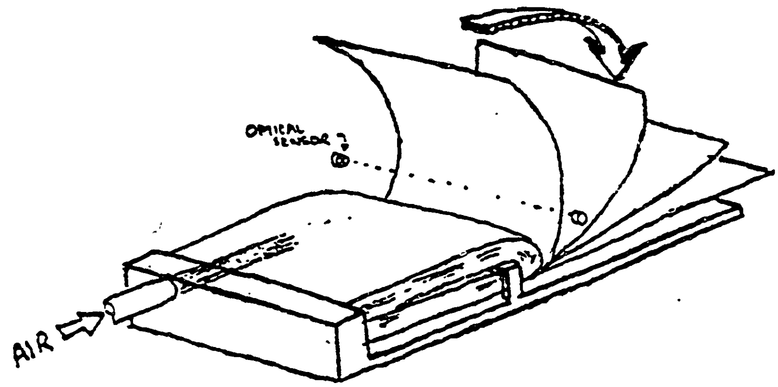


Figure II-6: Method 5

The air jet creates a low pressure space above the top bill, drawing it up, out from under a detente/container (into which it has been preloaded.) The same jet, once the lip of the bill has been sufficiently raised, directs the bill rapidly back from the folded stack, simultaneously creating a low pressure space for the next bill. In this way, the bills are continuosly lifted and straightened. An optical sensor is used to count each bill as it passes.

In this method, it might be difficult to keep bills from converging as they are accelarated from the stack. In other words, as a bill is lifted, it obscures the air jet as it is accelerated. Hence, the bill in front of it is no longer being pushed. Bill convergence might be a problem. Additionally, seperating bills using a low pressure space is risky, since holes or tears in the top bill allow the second bill in a stack to be acted on as well. There are, however, existing ATM machines (ie. Diebold) which utilize low pressure feeders reliably.

Methods 5 and 6 rely on folded stacks for their operation. This means that there must be a background operation in which bills are prepared. Also, the number of bills that can be counted is limited by the way a folded stack's dimensions and bills' shape depends on the number of bills in the stack. For these reasons, methods 5 and 76have been judged to have a lower possibility of success than other methods.

Method 7. "Knife" Seperation

This method makes use of a stack's weight and energy from bending, but does not require bills to be prepared before the arrival of the counter. A two pronged knife is used to lift, count, collect and grasp stacked bills. The action of this device is best described as occuring in four stages, as shown in figure II-7a.

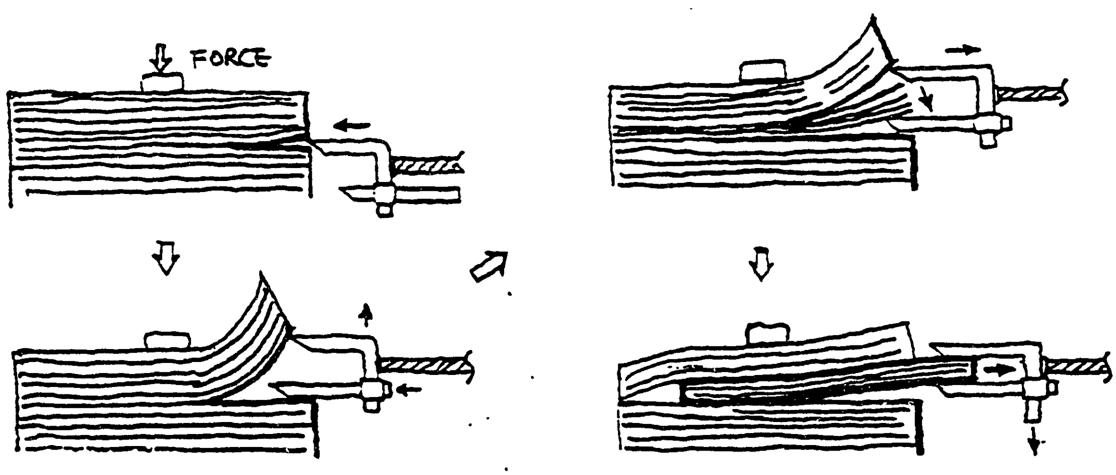


Figure II-7a: Method 7

In stage one, the lower edge of the knife is retracted, so that the top edge extends further forward. This upper edge is inserted into the stack at some distance below a stabilizer located atop the stack. In stage 2, the entire knife assembly is raised, as the lower edge of the knife is extended forward- in this position, the bills are exerting a force against the top knife, since they are bent and lifted. Next, the entire knife assembly is slowly pulled from the stack. in this stage, bills fall one at a time onto the surface of the lower knife, being counted by an optical sensor as they descend. In this way, a stack of known number is collected between the top and bottom knifes. In the final stage, the two knifes would close on and remove the counted stack.

This method would require relatively simple manipulator motions, and could conceivably be built inexpensively. It does not seem to be a particularly promising method, however. In an idealized case, using well formed bills, this method might be very valuable. Omron's machines, however, are designed to manipulate worn and folded bills. Method 7 is not well suited for handling these bills. Two types of deformation will negatively affect the reliability of this method.

Consider a stack of worn bills - the edges are not perfectly even, of course. In stage 3 of its operation, method 7 might fail by allowing a short bill to fall simultaneously with a large one just below it, since the edge of the short bill would not be caught. The probability of this occuring can be lessened, though, by increased bending of the bills, since this tends to make the higher bills jut out further.

See figure II-7b for a side view of a worn stack. It is clear that a knife being inserted into the stack might crumple a bill while dividing the stack. This problem might be avoided through the use of prongs, but there will always be a chance for failure during insertion.

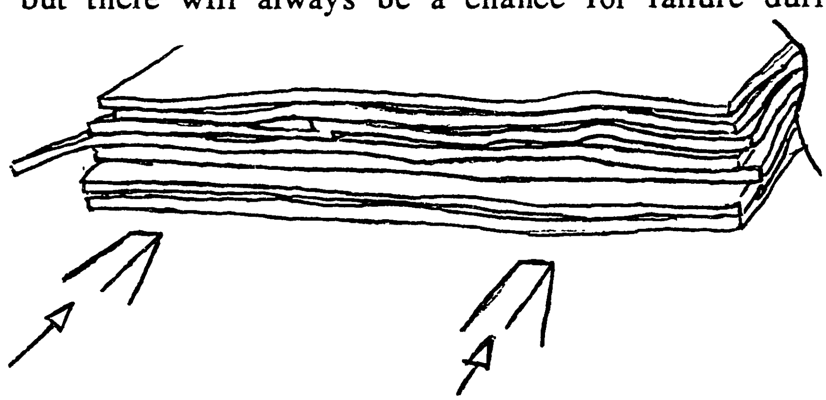


Figure II-7b: Worn Stack, Side View

Another problem would be removing the stack without pulling the bills above and below the counted stack out by the friction forces between bills. For these reasons, method 8 was not pursued.

II-3. Type C. Rotation of Bills

Method 8. Spinning

Methods were examined in which rather than being bent or buckled, bills are spun with respect to a stack (and of course counted), forming a new stack of known number. Figure II-8a shows a device employing this method. Bills are rotated atop the stack by a drive device designed to take just one bill. These bills are fed through the device, eventually stopped by a detente. In this method, bills are counted as they rotate past an optical sensor, and collect at the detente to be grasped and transported.

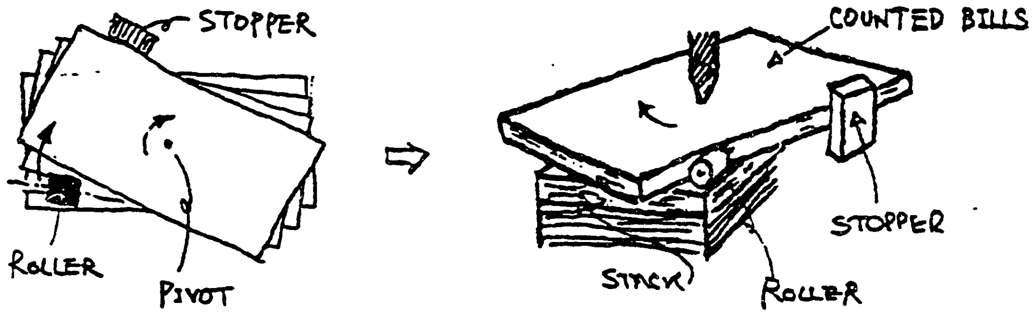


Figure II-8a: Method 8

Alternatively, bills could be counted by a device resting atop the stack which registered small changes in stack height; Omron has proposed an LED and photodetector arrangement, as shown in figure II-8b.

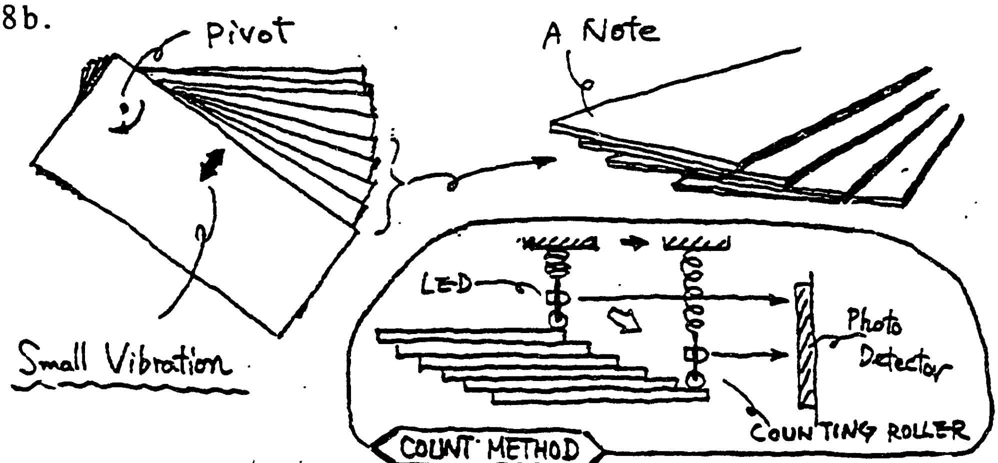


Figure II-8b: Alternative Counting Method

This method could utilize technology which Omron has already developed; the current Omron ABIO[1] machine contains a drive device which passes only one bill., although its application to bill rotation has not been investigated.

Unfortunately, this method requires a relatively large amount of space, and the ATM market is size competitive. Also, it seems, through rudimentary experimentation, that there is a high chance of turning two bills simultaneously, since there is nothing to stabilize the second bill. For these reasons, this method has not been examined further.

II-4. Type D. Vacuum

Method 9. Suction

This method involves the raising of bills through suction, and is shown in figure II-9. Utilizing a low pressure tube of sorts, bills are individually counted as they are lifted by a stream of air entering the tube. Since only the outermost area of each bill need be lifted, this method of counting might prove quite fast, although the number of bills it could lift would be limited by its width. Notice that the outer edge of the tube must be made in some way to be independent of the inner portion, since as the height of the outside of the stack decreases, the height of the inner stays constant. This method is less appealing than many of the others presented, because of its limitations and reliance on air, which Omron has little experience dealing with.

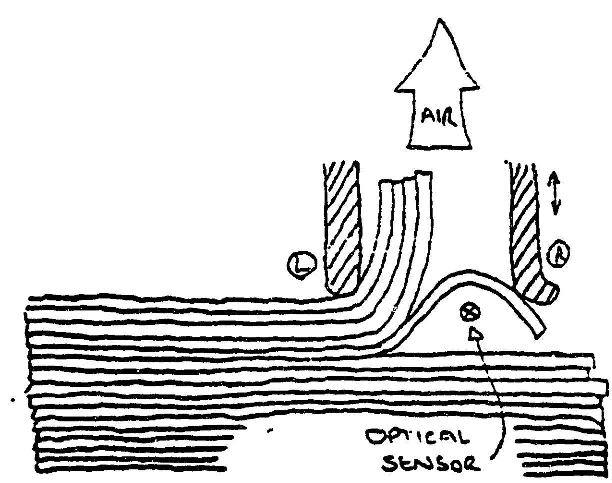


Figure II-9: Method 9

II-5. Type E. Background Preparation

A moment's consideration reveals that the majority of a high speed banking machine's time will be spent in waiting; a customer or teller may need several seconds of input time to clarify his transaction and account information, while the actual transaction takes just a fraction of that time. During this time, the machine is largely idle. It has therefore been proposed that this time be used to perform "background" operations, preparing the machine to most efficiently handle the next transaction. The last two methods proposed and presented here deal with possible idle time preparation which would speed critical counting time.

Method 10. Layering

Figure II-10 shows bills which have been staggered at definite intervals, and overlapped during background time. They are held in place by a pair of belts located above and below their surfaces, similar to the belt transport system currently used in Omron's machines. They are ready to be grasped by a mobile manipulator. This manipulator would travel just the right distance to collect the number of bills it desired, relying on an established distance per bill count ratio. As it did so, the manipulator would cause the bills to be compacted into a single stack of desired number. In this way a large number of bills could be collected very quickly.

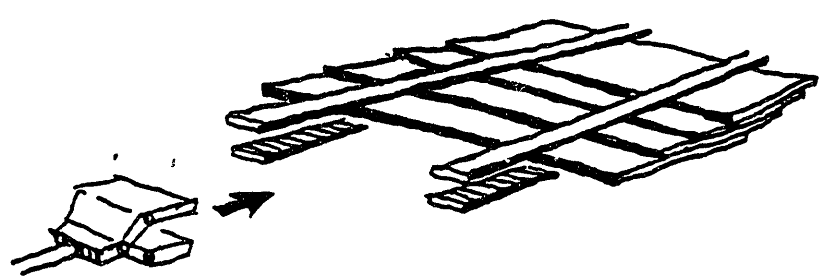


Figure II-10: Method 10

Method 11. Spiraling

This method is similar to method 10, but utilizes an alternative note configuration to speed counting time. Rather than being overlapped, bills are angularly staggered; each bill is spun some angular distance from the bill below it. Figure II-11 shows the resulting stack configuration. As it approaches the stack, the manipulator rotates a distance corresponding to the number of bills desired. It then descends and grasps the proper number of bills, compacting them into an aligned stack as it does so.

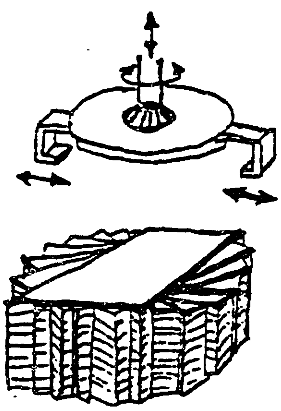


Figure II-11: Method 11

The number of bills that can be collected in this method in a single pass of the manipulator is limited by the angular spacing of the bills. For example, if the spacing between bills is 30 degrees, the maximum number of bills that could be collected in a pass is 12 (or, 360 / 30). The stack will be prepared for additional passes immediately, however, since the configuration continues to the bottom of the stack.

Methods 10 and 11 were rejected due to the added space they would require for operation.

II-6. Design Considerations

There are several general alterations that could be made which might make the methods presented above more feasible. One such alteration is inversion. In many of the methods bills are transported against gravity- it might be useful to invert each mechanism, using gravity to advantage rather than disadvantage. In the case of the buckling methods (1-4), an increased number of bills could then be counted continuously, since the counted stack would no longer have to be supported by each bill during buckling. Also, jets (or blasts) of air could be used to assist in transporting bills in the desired direction. Another addition many of the methods could make use of are air assist streams. Also, any of the buckling devices, might be found to be more effective when the contact wheel is shifted angularly with respect to the stack. In this way, some of the irregularity problems presented by common bill folds might be avoided. Figure II-12 shows this concept.

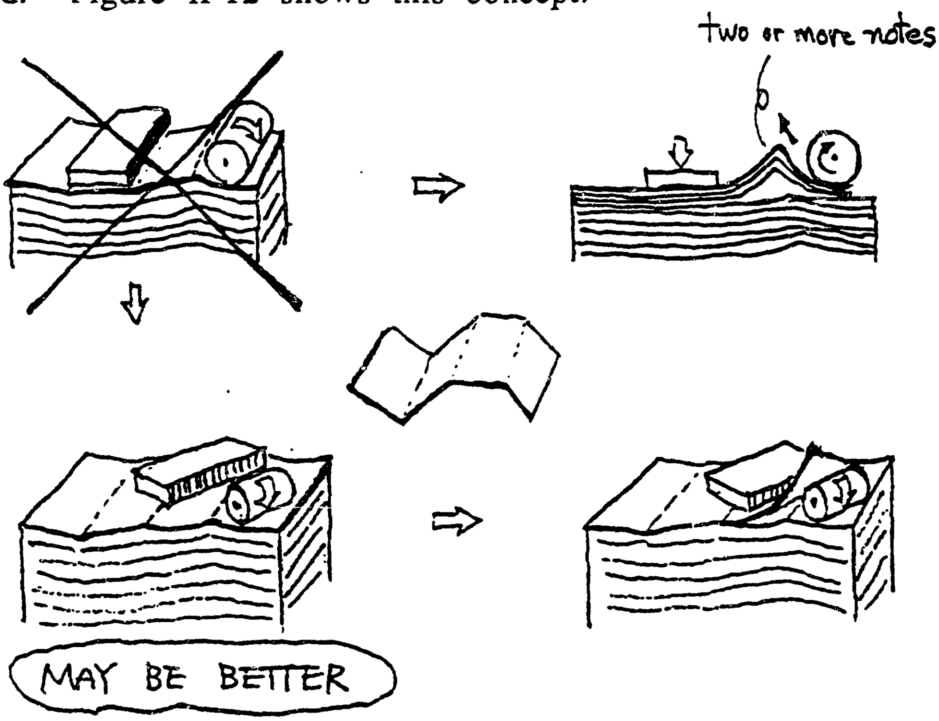


Figure II-12: Nonaligned Buckling

Moreover, many of the concepts presented above could be used simultaneously. For instance, method 1 (single side, dual roller) could be used on both sides of a bill and could be built into a stack's container, thereby incorporating method 2 (single roller, dual side). Also, the shafts suggested in method 4 could be used to assist in many of the other methods (ie. method 1, 2, 3, 5, etc.) Likewise, many other methods could be combined. Not only were the basic methods considered, but also combinations of methods. This being the case, it was desirable to design a prototype which might be able to test several of the most promising methods rather than just one. This was accomplished, and is discussed in section III.

III. The Prototype

A prototype has been designed and built for the purpose of testing the feasibility of method 1. This prototype is sketched in figure III-1. Appendix E contains mechical drawings of the prototype and its parts.

In this device, a contact wheel is rotated about its own axis and the plate roller's axis by belts driven by independent motors. The stabilizer holds the stack in place as notes are buckled, and its force comes from a spring attaching it to an unpright slider rail. The motor/rollers are also attached to this rail, and are allowed to slide down as it descend into the stack. A spring applying lift to the motor/roller cluster lowers the wheel contact force to a desireable level. Counted bills are captured between the stabilizer and the rollers.

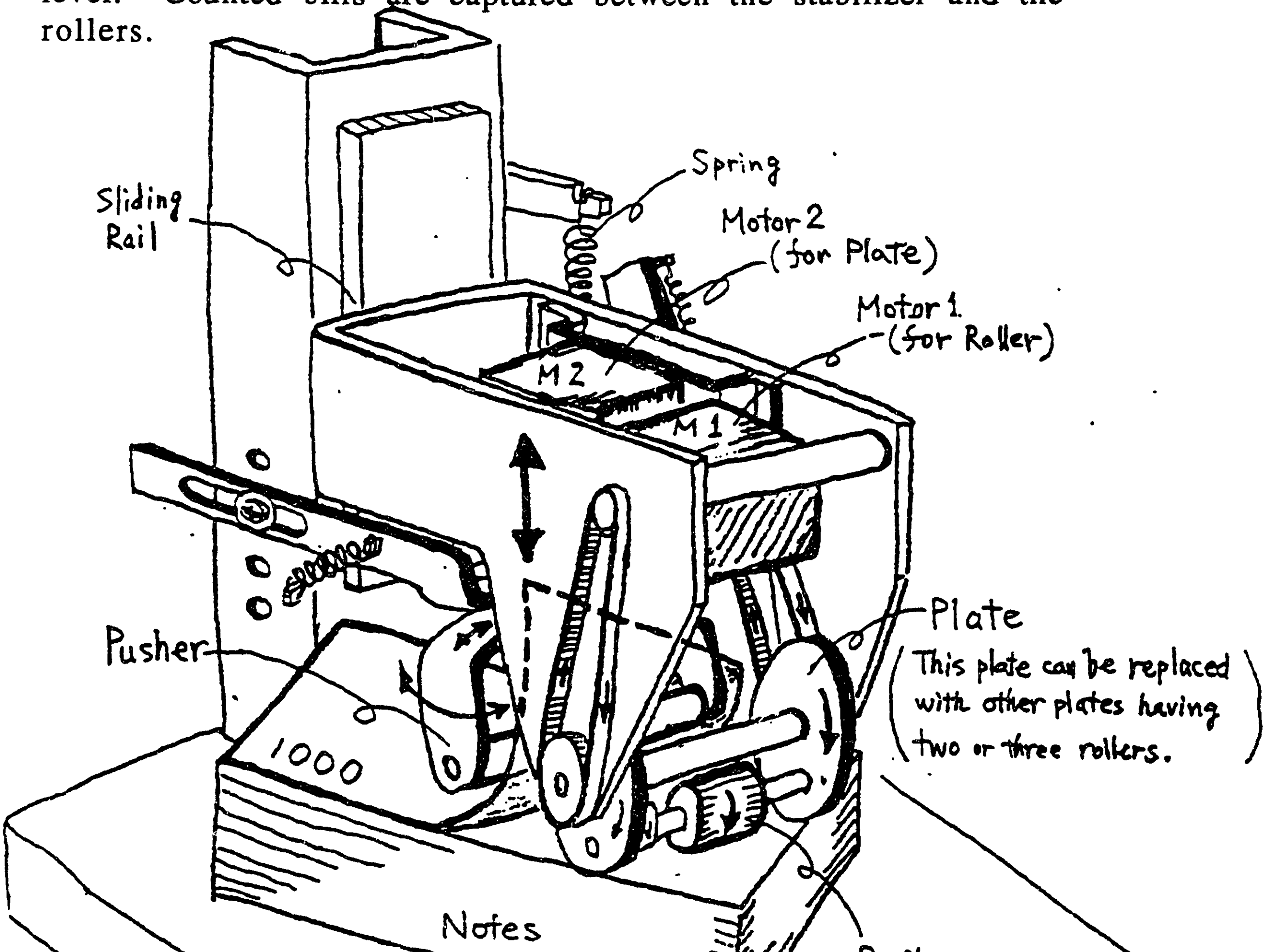


Figure III-1: The Prototype, Sketched

Trial runs of the prototype have revealed a design problem. As greater numbers of bills are counted, the force from the counted stack tends to cause deformed bills to fold rather than be flipped upward. This problem is drawn in figure III-2. To avoid this problem, the device shown in figure III-3 was built. In this design, the roller arrangement has an added "seperator" system. Three lever shaped pieces are rotated along with the contact wheels, and are kept parallel to the uncounted stack by belts. These pieces, spaced between each contact wheel, serve the dual purpose of helping to lift buckled bills and keeping the counted stack from folding notes as they are deformed.

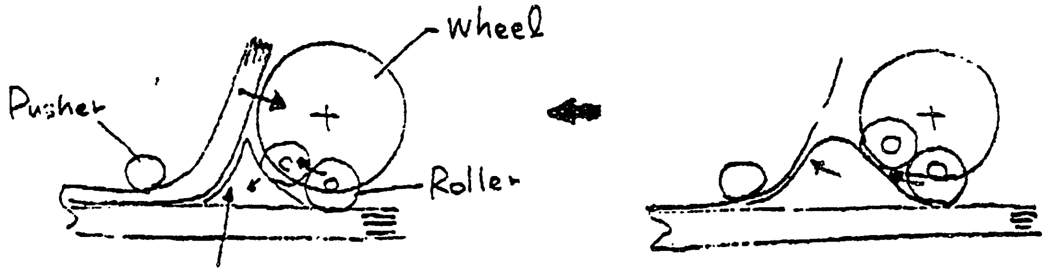


Figure III-2: The Folding Problem

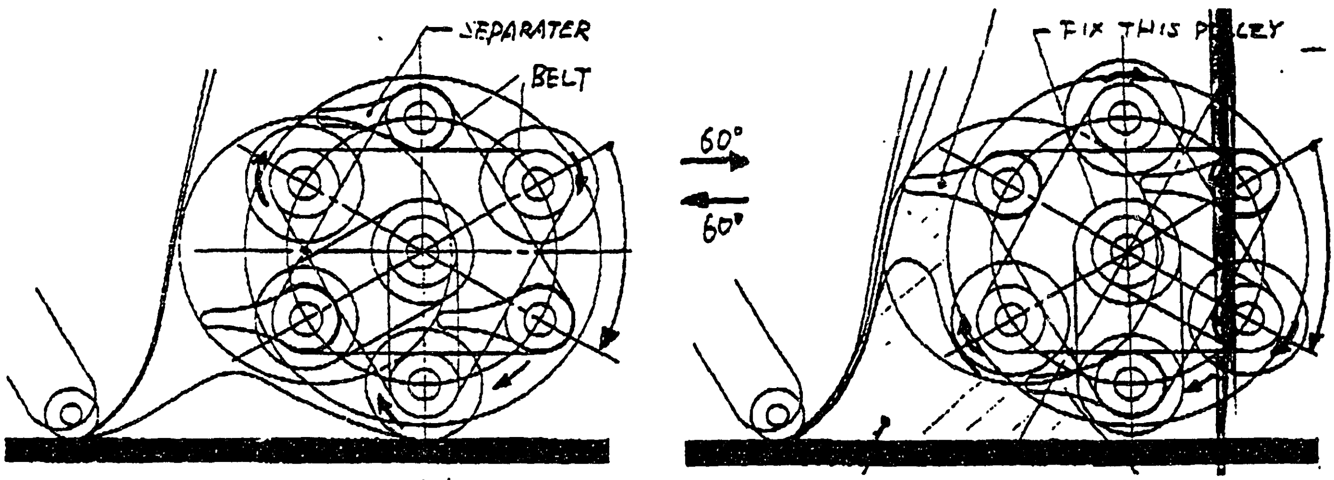


Figure III-3: Spacer Design

The prototype was designed in such a way that several of the parameters are variable, so that different design configurations could be experimented with. Many device parameters can be varied. The stabilizer to contact point distance can be changed by sliding the stabilizer support back towards the sliding rail. The wheel force can be varied by changing springs, the relative roller and wheel speeds can be set by controlling the two motors. The sizes of the plate and contact wheel (or wheels) is flexible, as parts can be substituted. Also, the device's orientation with regard to the stack is variable (both transversely and angularly), as the stack can be moved.

Because the prototype was designed to be flexible, it is more complex than it ultimately would be. For example, rather than having two variable speed motors, which are necessary to test for the optimum relative speeds, the final model would have a single constant speed motor with a belt transmission arrangement to drive the two wheels at the proper speeds. The stabilizer and rollers would no longer need to be independent, since the proper stabilizer/contact distance would be known. Because of these changes, the device would be lighter, cheaper and simpler than the prototype. A conceptual "market-ready" device is sketched in figure III-4. Notice that this end effector is capable of clamping onto and carrying the bills it has counted.

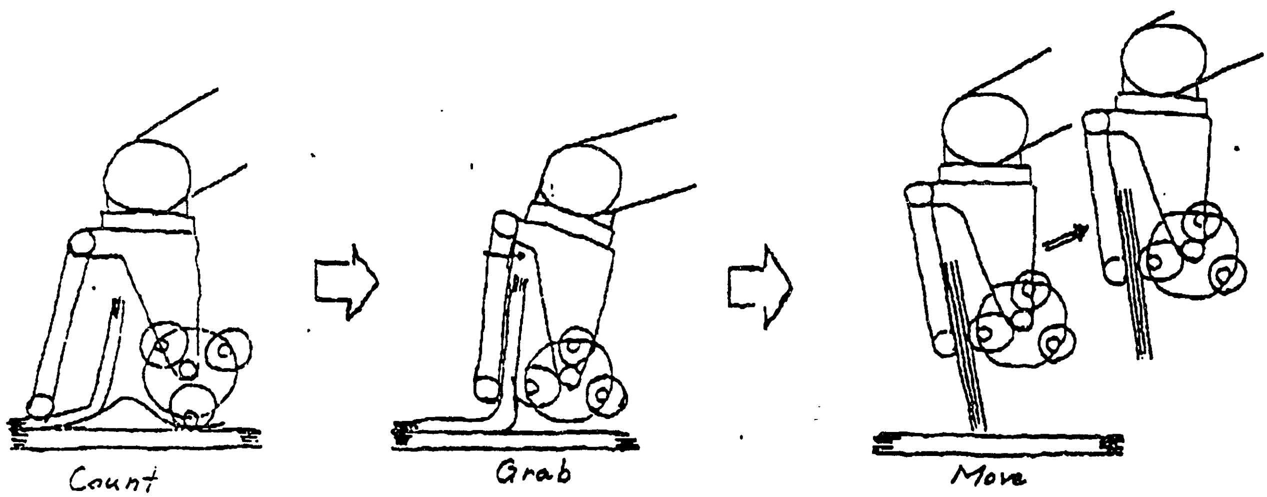


Figure III-4: Conceptual Final Model

IV. Analysis

During the time the stack counting device detailed in section III was being built, an analysis was being performed. It was desired to develop equations which could be used to provide a more thorough understanding of the device for the purpose of performance optimization.

Many of the device's parameters are difficult to account for numerically. For instance, exact friction coefficient between bills and with the contact wheel will vary significantly with the age of each bill, environmental factors, and many other considerations. Likewise, bill modulus of elasticity, the effects of air dynamics, and other crucial factors are difficult to quantify; at best, ranges can be specified to predict these factors. Deriving exact numerical analysis is therefore unreasonable. Unfortunately, the many factors contributing to the device's performance make it difficult to optimize through trial and error. The purpose of performing analysis of the counting device, then, was to show "cause/effect" relationships which would help in correcting problems and optimizing the machine.

An engineer is often posed with performance problems at the prototype stage. In systems where there are few factors affecting performance, trial and error may prove sufficient for correction and optimization. But in a device such as the bill counter, in which wheel and plate radius, stabilizer to wheel distance, wheel force, wheel speeds, unsupported bill length, etc., are all variable, some direction (ie. conceptual model) must be provided to aid in the tweaking of the system. It is for this reason that analysis of the counter was undertaken.

The analysis is broken down into two regions - the "buckling" region and the "lifting" region of the cycle. The buckling region is the part of the cycle in which the contact wheel applies force to the stack, and the top bill is initially deformed. The lifting region is the part of the cycle in which the contact wheel, not in contact with the stack, lifts and manipulates the top bill. Equations have been formulated which characterize the dynamics of the bill in each of these areas. Rough numerical solutions have been provided and compared with rudimentary experimental data. They prove to provide an adequate model of the behavior of the device as a function of its parameters.

IV-1. Analysis of Buckling Region

As might be expected, the primary concern in the buckling region of the counting device's cycle is that one and only one bill be deformed. Analytically, the horizontal force on the top bill must be above the buckling force, but the horizontal force on the second bill must be below the buckling force. Since the force on each bill is related to the wheel (normal) force, we can find a range for the force from the wheel on top of the stack.

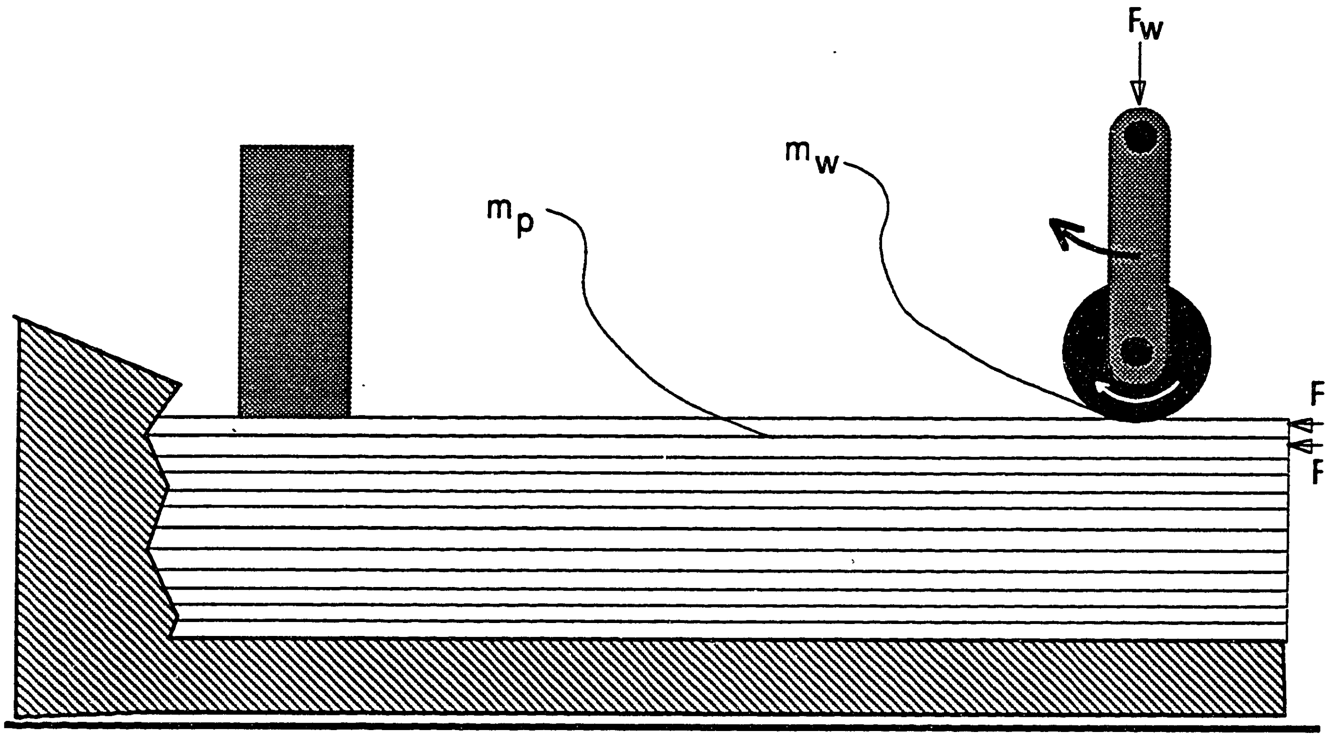


Figure IV-1: Counter in Buckling Region of Cycle

From the drawing,

$$F_1 > F_b > F_2$$

The force on the top bill should be higher than the buckling force, F_b, and the force on the second bill should be lower than the buckling force. We can rearrange this equation as:

$$F_w\,(m_w - m_{p,max}) > F_b > F_w\,(m_{p,max} - m_{p,min})$$

where m_1 refers to $(m_w - m_p)$ and m_2 refers to $(m_{p,max} - m_{p,min})$. Then:

$$\frac{1}{F_w m_1} < \frac{1}{F_b} < \frac{1}{F_w m_2} \qquad \text{or,}$$

$$\frac{F_b}{m_1} < F_w < \frac{F_b}{m_2} \qquad \text{(equation 1)}$$

Now, to get an estimate of F_w, we just say it is halfway between the two limits.

$$F_w = \frac{F_b(m_2 + m_1)}{2m_2m_1} \qquad \text{(equation 2)}$$

Buckling force, F_b is (see Appendix A):

$$F_b = \frac{3.3Ebh^3(m_1+m_2)}{L^2m_2m_1}$$

b is bill width
h is bill thickness
L is length of bill between wheel and stopper
Modulus of Elasticity, E, is (see Appendix B):

$$E = \frac{6.875*10^{-3}L_e^4}{dbh^3}$$

L_e is length of unsupported bill in deflection experiment.
d is deflection in unsupported bill

Substituting the values for E and F_b into equation (2), we get:

$$F_w = \left(\frac{11.3*10^{-3}(m_1+m_2)}{L^2m_1m_2}\right)\left(\frac{L_e^4}{d}\right)$$

For convenience, this could be written as:

$$F_w = \frac{Kf}{L^2}$$

in which

$$K = \left(\frac{11.3*10^{-3}(m_1+m_2)}{m_1 m_2}\right)$$

$$K = \left(\frac{11.3*10^{-3}(m_w - m_{p,min})}{(m_w - m_{p,max})*(m_{p,max} - m_{p,min})}\right)$$

and

f = bill floppiness, or $\frac{L_e^4}{d}$ (from cantilever test)

for an old bill, $f = 3.05 * 10^{-3}$

for a new bill, $f = 4.15 * 10^{-4}$

L_e is in meters.

This analysis shows that in order to lift a single bill properly, we should be within the range $\frac{F_b}{m_1} < F_w < \frac{F_b}{m_2}$ (equation 1); using typical friction coefficients, F_w is between 1.5 and 10 times the buckling force of one bill. This will not be numerically accurate, as many assumptions have been made.

For simplification, the stack and wheel are modelled as incompressible, although in operation the stack is actually depressed at the point of wheel contact. This suggests that our estimate of the necessary wheel force will be high, because the necessary buckling force will be lower since the bills are effectively "pre-buckled".

Additional considerations, however, tend to raise the upper limit on the wheel force. It is necessary for all the force exerted on the second bill to be carried by the friction between the wheel and the top bill. Therefore the maximum possible horizontal force exerted on the second bill is limited by this friction force. As you go down several bills into the stack, it is unlikely that the bill will buckle. Also, when two bills are buckled together, in the deformed state they have stored energy which allows them to push against the wheel with a certain force. The wheel therefore pushes against the first bill with a resultant force equal to twice the force from one bill. This will be on a surface with a high friction coefficient, m_W. The second bill will only have a normal force equal to what one bill can exert. This will be over a surface with a low friction coefficient, m_P. Therefore, it is likely that the second bill will drop, even if it buckles originally (see figure IV-2.) At the high speeds at which the device oerates, air and bill mass effects must be examined. In short, this analysis of wheel force should give only a rough numerical estimate of the ideal force, although it accurately presents the relationship between system variables and performance.

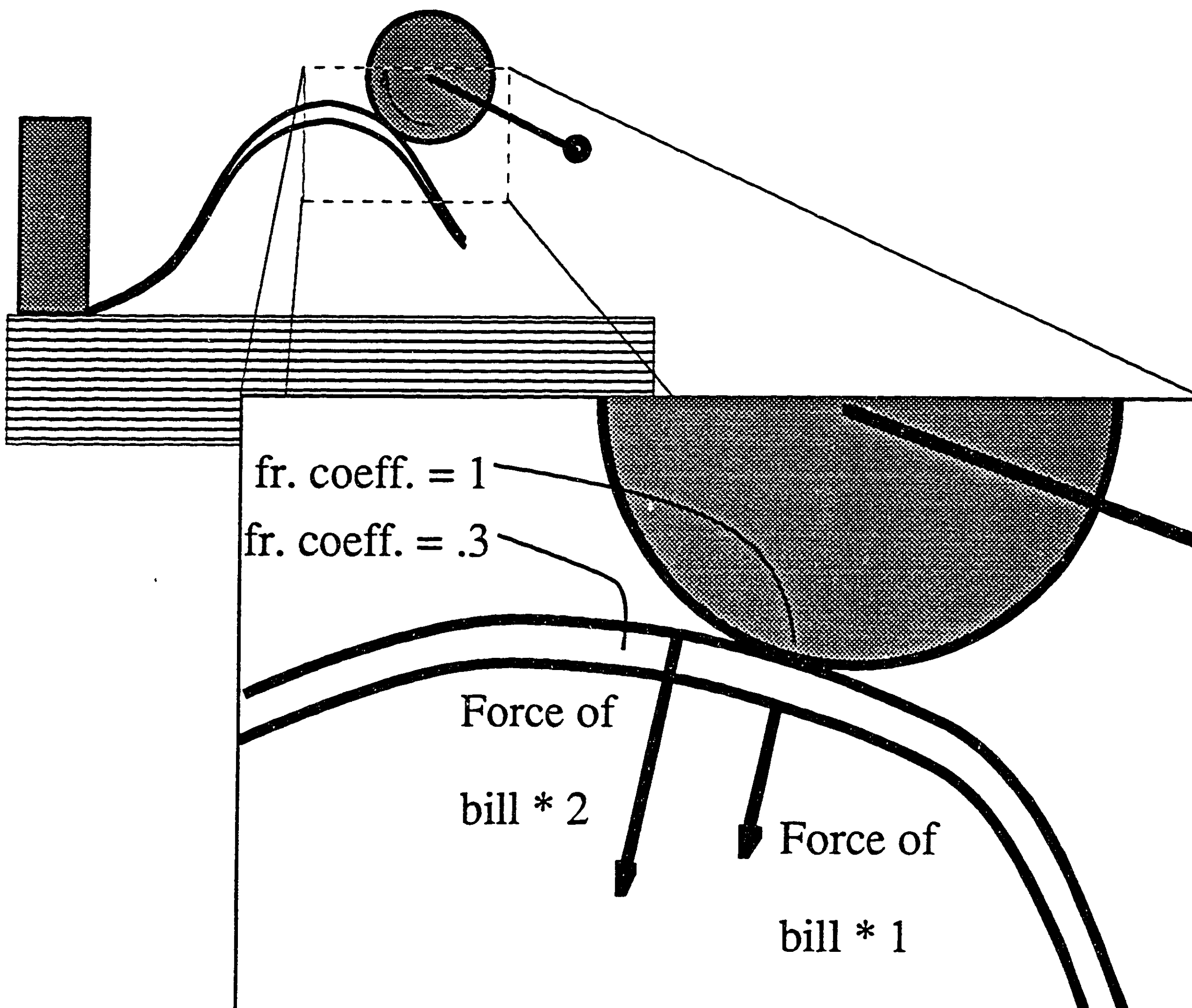


Figure IV-2: Statics of Dual Buckling

IV-2 Analysis of Lift Region

In order for the bill counter to operate reliably and efficiently, it is necessary that buckled bills not be allowed to fall back to the supply stack while being lifted. So it is necessary to determine the forces being applied to a note at the surface where it touches the contact wheel; to insure against "fall-back" failure, the forces at this surface should be such that slip does not occur.

The forces at the wheel/bill interface are a function of only the properties of the bill (ie. its "springiness") and the bill's inertia, since there is no support which would allow the contact wheel to exert force. In other words, the force exerted by the wheel on the bill is reactionary, and is related to the mechanical properties and inertia of the bill. Therefore, in order to find the forces at the contact area, we must first determine those mechanical factors.

Since the bill is being accelerated, its inertia will have the effect of increasing the contact force between the bill and roller. At high speeds, this force will be substantial. As the speed of operation decreases,. however, the effect of inertia diminishes and effectively goes to zero at low speeds. Since our purpose is to determine the conditions necessary to insure against slip, the beneficial effects of inertia (at high speed) need not be considered. Hence, only the bill's relevant mechanical properties need be found.

Figure IV-3 shows how the direction and magnitude of the contact forces vary with the bill's shape. Each of the bills in the figure has the same length. Minimum energy principles suggest that the buckled portion of the bill will be sinusoidal in shape. For a given wheel position, as the amplitude of this sinusoid increase the force normal to the wheel increases and the tangential force decreases. Notice that in the case of bill one, there will be a high tangential force and low normal force. As the bill progresses to stages 2 and 3, the normal force increases and the tangential force decreases. It is clear, then, that the forces are dependant not only on the springiness of the note, but also on its shape. Therefore, it is necessary that the shape of the bill be known. In section IV-2a, equations have been formulated which relate geometric parameters of the counting device to deformed bill shape.

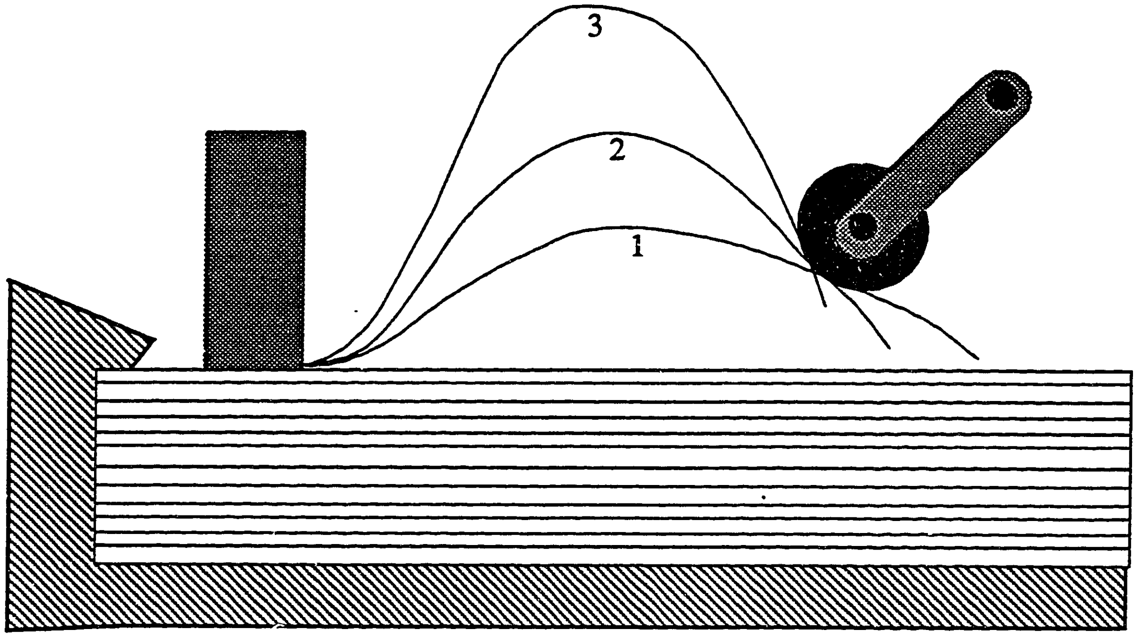


Figure IV-3: Wheel Forces vs Shape, Trends

IV-2a. Determining Shape as a Function of Wheel Geometry

Minimum energy principles suggest that the buckled portion of the bill will be sinusoidal in shape. Since the bill will be acted upon uniformly across its width, it is acceptable to consider the bill's shape in only two dimensions. For shape calculations, the bill is modelled as a sinusoid in two dimensions, having no thickness. We therefore assume that

$$y = A \sin (kx)$$

Where y is the height of a point on the bill (looking at the bill from its side), and x is the corresponding distance from an origin. A and k represent the sinusoid's amplitude and period, respectively.

For the purposes of simulation, it is useful to place the origin of the coordinate axes at the line of contact between the stabilizer edge and bill to be buckled. With this origin, we must use

$$y = A (1 - \cos (kx))$$

which has the same shape. The task now is to fit this curve to the boundary conditions imposed by the contact wheel and stabilizer.

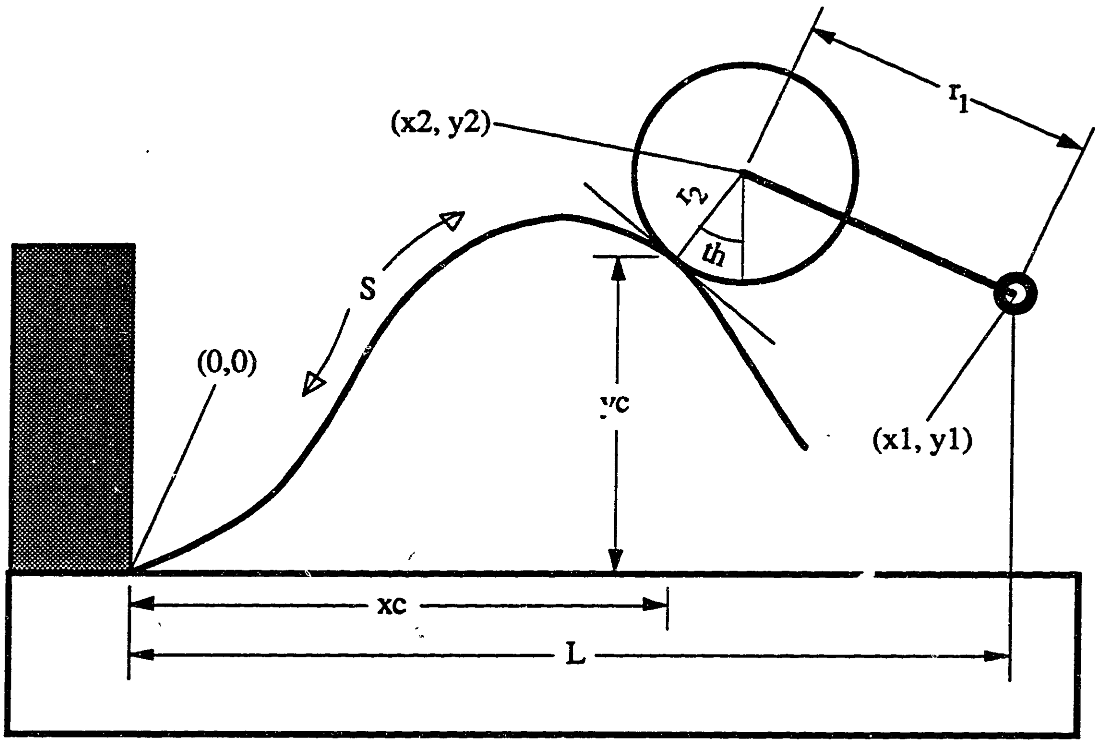


Figure IV-4: Notation Used for Shape Analysis

Figure IV-4 is used to depict the mathematical symbols used in the analysis performed in this section (th refers to θ.) Several equations can be formed to relate the geometric parameters.

First, since it is assumed that there is no slip, it is clear that the length, S, is equal to the length of the original portion of the bill to be buckled, L, plus the length of bill turned out by the wheel. We write,

$$S = L + r_2\,\theta$$

We can also find S from integrating for length of the bill, which we have modelled as a sine curve. This yields:

$$S = \int_0^{x_c} \sqrt{1 + A^2 k^2 \sin^2(kx)}\,dx$$

Furthermore, we know that the slope of the curve at any point is

$$y'(x) = Ak\sin(kx)$$

So at the contact wheel, the slope must be

$$y'(x_c) = \frac{y_2 - y_c}{x_2 - x_c} = -\tan\theta$$

Some discussion of the physical characteristics of the lifting region is now necessary. It should be noted that the angle θ at which the bill contacts the wheel is not known. Although the point of original contact between the bill and wheel can be traced to any wheel geometry, and the bill does not slipped, the bill will not be contacting the wheel at the original point. This is because the curved bill tends to "roll" against the contact wheel some distance downward as it is lifted. This phenomenon is best understood by visualizing a contact wheel rotating at some velocity, attached to a plate which has stopped moving. (Figure IV-5). As the bill is turned out, the absolute angle of contact with the wheel increases, although with respect to the original contact point, the bill is rolling downward. Equations have not been formulated to quantify this roll directly, so θ must be found through the use of our equations.

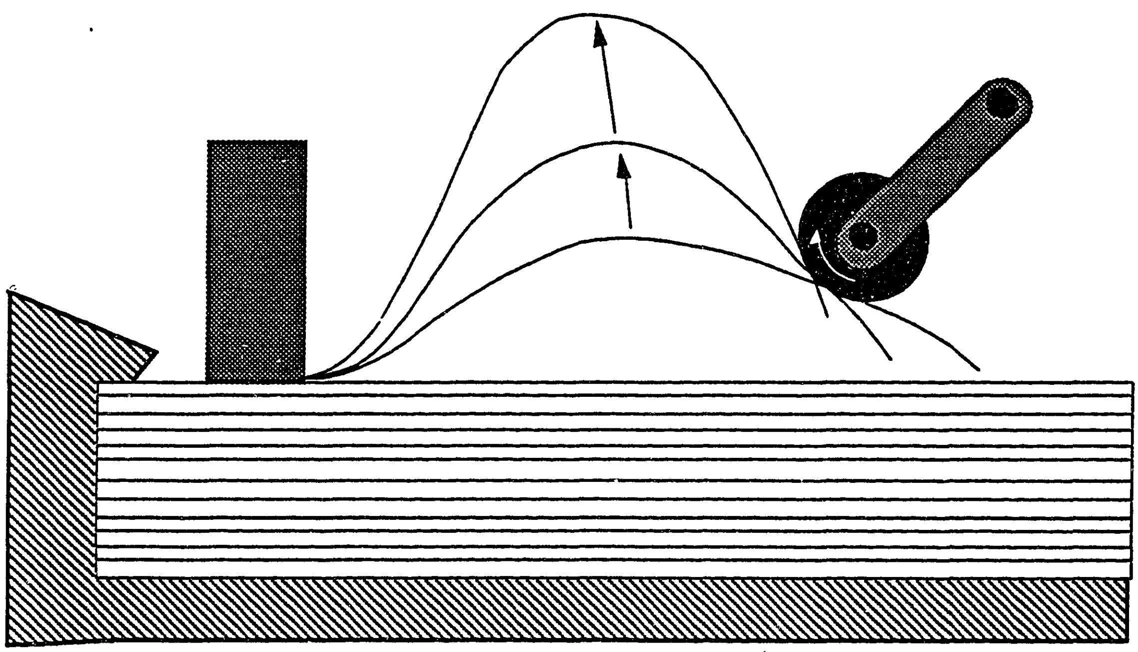


Figure IV-5: Bill Roll

We therefore have 3 unknowns, θ, A, and k. Using boundary conditions (ie. $Y(x_c) = y_c$) and the equations shown above, these variables can be found numerically, as explained in section V.

IV-2b. Analyzing Forces at Determined Bill Shape

Having characterized the geometry of the rollers and bill, the next step in analysis of the lift region is to determine the resultant forces at the contact wheel, in order to confirm that the bill will not fall back to the uncounted stack.

Considering friction, we can say,

$$F_n \mu_w >= F_t$$

where F_n is the normal force and F_t is the tangential force being exerted by the contact wheel on the deformed bill. (See figure IV-6.) This is our primary condition- if the addend of the coefficient of friction and the normal force on the bill is at least as great as the tangential force, the bill will not slip and the machine will lift the bill reliably.

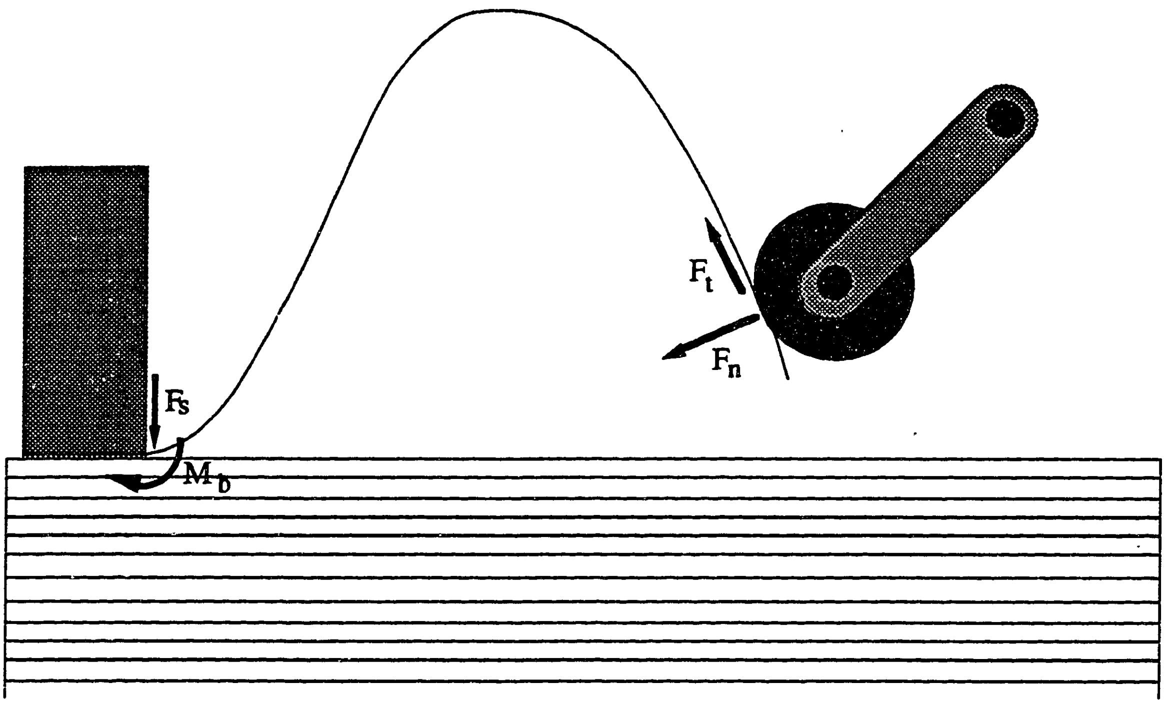


Figure IV-6: Contact Wheel Forces

Using beam theory, we can make ballpark estimates of the tangential and normal forces between a deformed bill and the contact roller, as a function of shape. Principles of beam theory hold that the moments and forces exerted on a bill in static equilibrium sum to zero. (As explained above, for purposes of this analysis, the note is modelled as static.[1]) Since there are only two points of interaction with the bill, the stabilizer edge and the contact wheel surface, we know that the moments and directional forces at these two points must sum to zero. We can state mathematically:

$$M_b + M_n + M_t = 0 \qquad \text{(eq. 1)}$$

Where M_b is the moment on the bill at the base of the stabilizer, an d M_n and M_t are the moments resulting from the normal wheel force

[1] The analysis we are deriving here will be used to characterize the necessary geometrical conditions to insure reliable lifting of bills. By neglecting the effects of inertia, the model we develop will tend to be a bit conservative; in other words it is possible that when our model predicts the machine will not operate reliably, it actually will because of the added forces resulting from bill inertia. But a good and safe estimation will be derived.

and the tangential wheel force, respectively. M_b, from beam equations, is:

$$M_b = EI\frac{d^2y}{d^2x} \qquad \text{(eq. 2)}$$

I and E represent moment of inertia and modulus of elasticity of the bill. M_n and M_t can be found by multiplying the pertinent force at the contact wheel by the normal distance (d_n or d_t) to a line containing the force's vector. Mathematically,

$$M_n = d_nF_n \qquad \text{(eqs. 3)}$$

and

$$M_t = d_tF_t$$

In equation 2, $\frac{d^2y}{d^2x}$ represents the slope of the bill at some value x along its length. Since we have defined y = A(1 - cos (kx)),

$$\frac{d^2y}{d^2x} = Ak^2\sin(kx) = Ak^2 \qquad \text{(eq. 4)}$$

Substituting equations 2, 3, and 4 into 1, we get:

$$EIAk^2 = d_nF_n + d_tF_t \qquad \text{(eq. 5)}$$

The shear force across a beam is the third derivative of the shape,

$$\text{shear} = \frac{d^3y}{d^3x} = Ak^3\cos(kx) = 0 \quad \text{(at } x=0\text{)}$$

Since the only other forces on the bill in the y direction are the components of the normal and tangential forces, we know:

$$F_n\cos(\theta) + F_t\sin(\theta) = 0$$

or,

$$F_n\cos(\theta) = -F_t\sin(\theta)$$

and,

$$F_n = -F_t\tan(\theta) \qquad \text{(eq. 6)}$$

Equations 5 and 6 form two equations for the unknowns, F_t and F_n. They can be solved to yield:

$$F_t = \frac{EIAk^2}{d_t - d_n\tan(\theta)} \qquad \text{(eq. 7)}$$

$$F_n = \frac{EIAk^2}{d_n - \dfrac{d_t}{\tan(\theta)}} \qquad \text{(eq. 8)}$$

But d_t and d_n must be found. Consider figure IV-7.

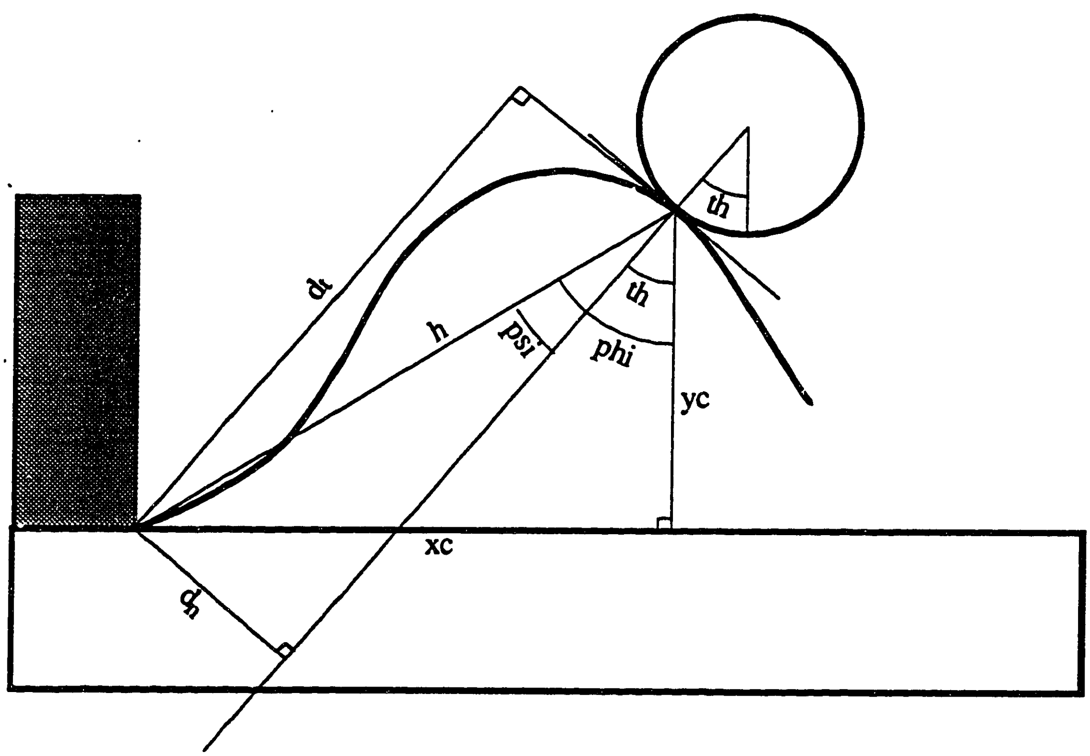


Figure IV-7: Moment Distance Calculation Notation

It is clear that d_t, d_n and h form a right triangle. Therefore,

$$d_n^2 + d_t^2 = h^2$$

and so,

$$d_t = \sqrt{h^2 - d_n^2} \qquad \text{(eq. 9)}$$

Now, we must find d_n.

Since,

$$\sin(psi) = \frac{d_n}{h}$$

$$d_n = h\sin(psi) \qquad \text{(eq. 10)}$$

But,

$$psi = phi - th \qquad \text{(eq. 11)}$$

Th is known from the solution obtained in section ***, and phi is simply:

$$phi = \arctan(\frac{xc}{yc})$$

Therefore, from equations 10 and 11, dn is known to be:

$$d_n = h\sin(\arctan(\frac{xc}{yc}) - th)$$

where h is just

$$h = \sqrt{xc^2 + yc^2}$$

This gives that

$$d_n = \sqrt{xc^2 + yc^2}\ \sin(\arctan(\frac{xc}{yc}) - th) \qquad \text{(eq. 12)}$$

From equations 6, 7, 9 and 12, the desired forces F_t and F_n can be found. If $F_n\mu >= F_t$ at all times during the lift cycle, the note will be raised properly.

V. Simulation

A program has been developed to allow the simulation of the stacked bill counting device with various parameter values. Written in UNIX C for the Xwindows environment, this program graphically simulates the performance of the bill counting unit, allowing the user to specify roller radii, roller/stabilizer distance, relative roller speeds, wheel contact friction, stabilizer/wheel distance, etc. A bill is graphically lifted and manipulated by the wheel/plate arrangement. The user is told when a bill has passed across the roller entirely, and also is warned when a bill has presumably slipped. The code is shown in appendix D.

Figure V-1 shows a flow chart of the simulation routine. The program begins by assigning values to each of the physical parameters of the device. It then presents a menu so that the user may experiment with different values for the roller radii, the buckling distance, the wheel speeds, etc. In this way, the parameters for the simulation are set. After initialization has been completed, the stack and wheels are drawn in a pop-up window. The program then begins stepping through the action of the device.

First, the plate roller (large radius) is rotated one degree, and the contact roller is rotated an amount proportional to its speed relative to the plate. Next, calculations are made so that the bill may be drawn. This is done numerically, and involves iterations for several variables.

Initially, a theta (angle of contact) is assumed. On the first pass, the theta is estimated by a no slip, no roll criterion. In other words, theta is set equal to the angle which the point of original contact has rotated to. This angle represents the upper bound for theta. Having assumed this theta, the x and y coordinates of the point of contact are easily found.

The method then proceeds as follows: k is set at $\frac{2\pi}{x_c}$, the highest possible k (at most, the bill will form a single cycle of a sine wave.) The corresponding a is then found by solving for the known contact point, y_c, using $y(x) = A\,(1 - \cos(kx))$. These two values are then used to see if the length criterion, developed in section IV-2a, is met.

$$\left(\int_0^{x_c} \sqrt{1 + A^2k^2\sin^2(kx)}\, dx = L + r_2\, \theta^{2} \right).$$ If this condition is not met, k is lowered and the process is repeated, until the proper a and k are found for this theta.

After a and k have been found using this method, k is checked against an alternate criterion, developed as follows. Since we have equations for y and y', both in terms of nonlinear functions of k multiplied by A, we can divide one over the other and plug in known values for y and y' to find a nonlinear relationship which k must satisfy. This equation is:

$$\frac{y'(x)}{y(x)} = \frac{Ak\sin(kx)}{A(1 - \cos(kx))} = \frac{k\sin(kx)}{(1 - \cos(kx))}.$$

And at $x = x_c$, it is:

$$\frac{y'(x)}{y_c} = \frac{k\sin(kx_c)}{(1 - \cos(kx_c))}.$$

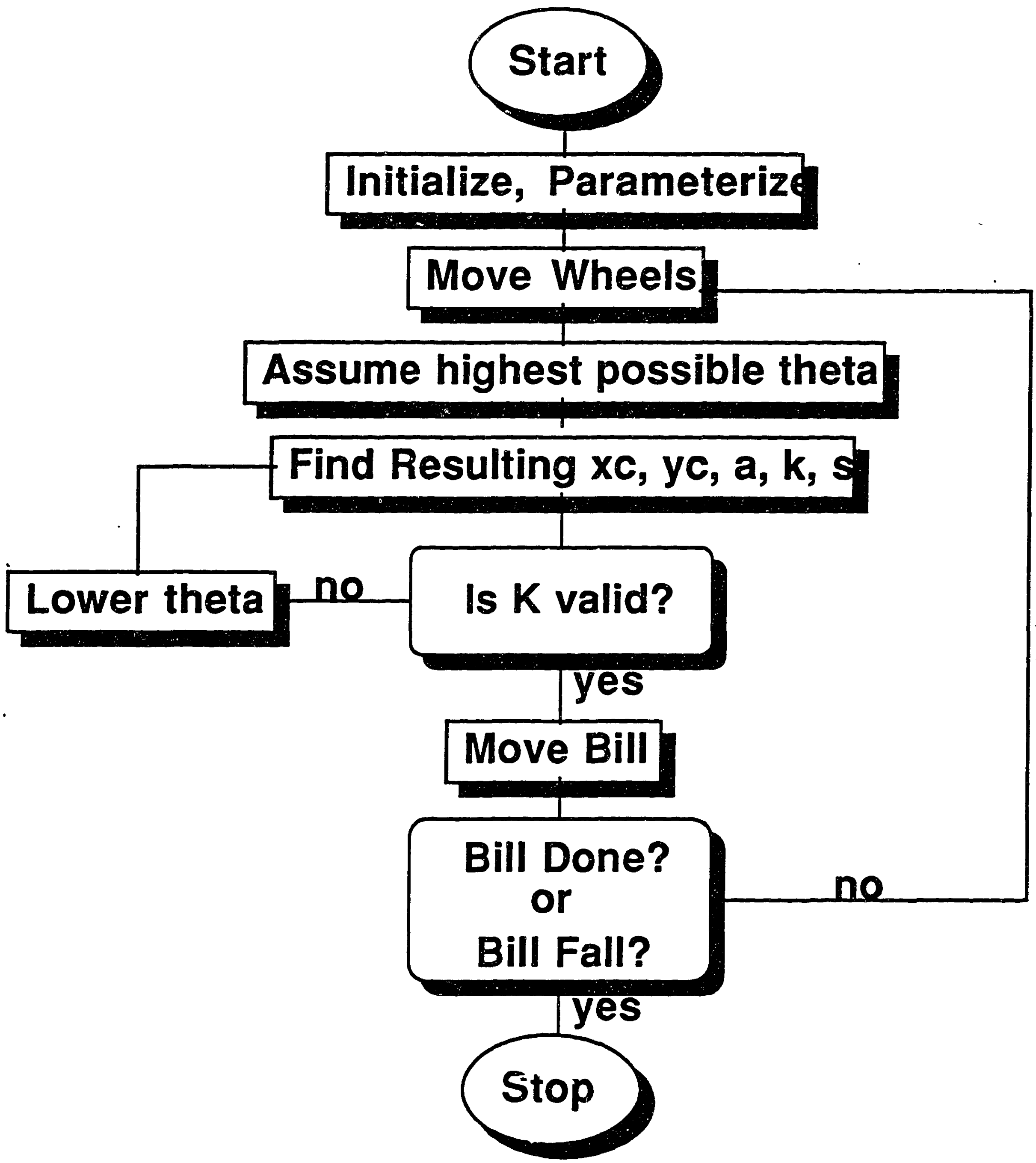
The k determined above is tested using this equation, and the assumed y_c and $y'(x_c)$. If k does not meet this relationship within a range consistent with our level of exactness, then the assumed theta was not correct, and must be stepped down. This process is repeated until all conditions are met.

Once the conditions have been met, the bill is moved, and it's resulting properties are tested. If the entire length of the bill is found to have been turned through by the wheel, a message appears, the bill freezes, and the wheels continue on to finish their cycle. If the bill is found to be under a condition of slip, again a message appears, but in this case the bill turns red and the program stops. If the bill is being safely lifted, however, the program continues on to the next angle of the plate roller.

---

[2] It is important to note that the θ used here refers to the angle through which the bill has been turned out. This is not equal to the angle which the contact roller has turned absolutely, but is the amount it has rotated with respect to its own axis. In other words, it is the angle of absolute rotation of the wheel minus the angle of rotation of the roller.

Simulation Routine Flowchart



VI. Recommendations

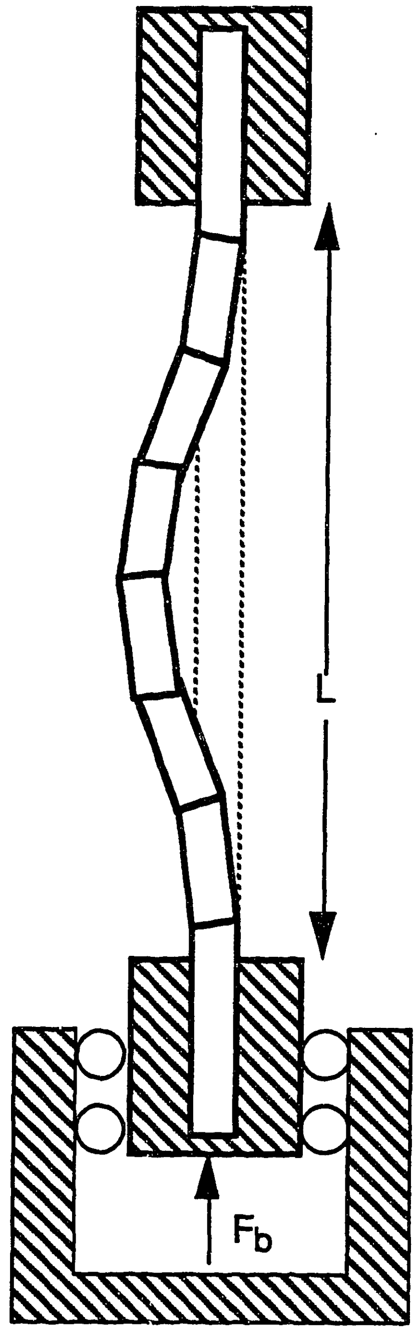
Due to time constraints, many simplifying assumptions have been made in the analysis of the prototype in this paper. To more accurately model the device's operation, a finite element analysis should be done on the bill. This is being done, currently.

The prototype, although it has not been optimized, has proven the method of stacked bill counting chosen to be feasible. Further product development is necessary, however, before it is clear whether or not the method proposed is preferable to the current one. This is also being done.

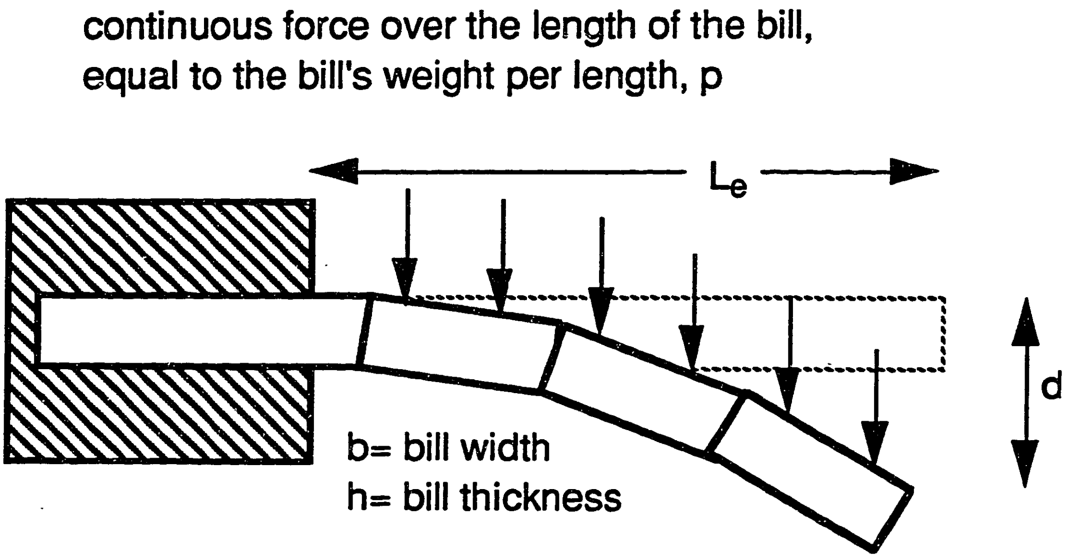
Appendix A. Buckling Force

b= bill width
h= bill thickness

$$F_b = \frac{39.5\ Ebh^3}{L^2}$$



Appendix B. Modulus of Elasticity



$$E = \frac{4pL_e^4}{dbh^3}$$

p = weight per length.
for a bill, which weighs about 1.1 grams, or .0011 kg, it is .158kg/m.

Appendix C. Simulation Code

```
# include <stdio.h>
# include <math.h>
# include <X11/Xlib.h>
# include <X11/Xutil.h>
# include <X11/X.h>
# include <X11/keysym.h>

main (argc,argv)
int argc;
char **argv;
{Display *display;
Window window;
GC gc;
int screen;
unsigned long gcvaluesmask;
XGCValues gcvalues;
XSegment stack[5];   /* doesn't do anything */
XSizeHints hints;
XColor Green,DarkSlateGrey;
Colormap cmap;
unsigned long foreground, background;
XWindowAttributes window_attributes;

int curvex, curvey, oldcurvex, oldcurvey, gr, lb = 400, sk, z;
double fr, am, du, dl, d2, dy, olda, oldk, kval, ktest, ksign, koldsign, a, s,
       stest, oldsign, sign, oldth, low, dt, dn, xcr, ycr;
double r = .045, om = 0, xc, oldxc, yc, th, i, j, an1, an2, an3, an4;
double loldy = 0, loldx = 0, ldy =0, ldx = 0, li, lds, lstest, lx, ly;
double k, dum, miss[100], kv[100], av[100], missv, h, ft, fn;
double EI = 112600;
int wn = 1, rl=75, r2=40, L=350, x0 = 100, y0 = 300, y1, x1, ppm = 1, lc = 1,
    x, y, oldx, oldy;
int x2, y2, y3, x3, y4, y5, x4, x5, oldx2, oldy2, oldx3, oldy3, oldy4,
    oldy5, oldx4, oldx5;

int  factor=257;    /* Color factor */
int fail=0;         /* Colors or Black,White */
int pointcounter;
float  inputval;  /* Used in changing menu values */

/* Screen */
display = XOpenDisplay("");
screen = DefaultScreen (display);
cmap = XDefaultColormap(display,screen);

DarkSlateGrey.red = 47*factor;
DarkSlateGrey.green = 79*factor;
DarkSlateGrey.blue = 79*factor;

/* Defined color above, now try to allocate it and check if its there
   Note that XAllocColor will try to give you a color as close to the
   specified colors as possible. If fail=1 white and black pixels need
   to be used */

if (XAllocColor(display,cmap,&DarkSlateGrey)==0) fail=1;

Green.red = 35*factor;
Green.green = 142*factor;
Green.blue = 35*factor;

if (XAllocColor(display,cmap,&Green)==0) fail=1;
```

```
if (fail==1){
  /* No color all3ocated, use black/white */
  background = BlackPixel(display,screen);
  foreground = WhitePixel(display,screen);
}
else{
  background = DarkSlateGrey.pixel;
  foreground = Green.pixel;
}

/* Define window size and position */
hints.x = 0; hints.y = 0;
hints.width = 600;
hints.height = 450;
hints.flags = USPosition | USSize;

/* Window creation -- Only write to after mapping !! */
window = XCreateSimpleWindow(display,DefaultRootWindow(display),hints.x,
                            hints.y,hints.width,hints.height,5,
                            foreground,background);
XSetStandardProperties(display, window,"Yen","Yen",None,argv,argc, &hints);

/* Graphic Context */
gcvaluesmask = GCLineWidth;
gcvalues.line_width = 0;
gc = XCreateGC (display, window,gcvaluesmask,&gcvalues);
XSetBackground (display, gc, background);
XSetForeground  (display, gc, foreground);

/* Input spec */
XSelectInput (display, window, ButtonPressMask | KeyPressMask | ExposureMask);

/* Map window to screen */
XMapRaised (display, window);

/* Flush buffer explicitly */
XFlush(display);


Menudisplay:

inputval = 0;
printf ("    Current variable values. If you wish to change anything, type the vari
printf("\n\n");
printf ("%s%d\n","1. wn  - number of wheels = ",wn);
printf("%s%d\n","2. lc  - trace on or off = ",lc);
printf("%s%6.4f\n","3. om - relative speed of wheel to plate = ",om);
printf ("%s%d\n","4. L - distance between pin and wheel = ",(L/3));
printf ("%s%d\n","5. r2 - radius of wheel = ",(r2/3));
printf ("%s%d\n","6. r1 - radius of plate = ",(r1/3));
printf ("%s%d\n","7. lb - unsupported bill length = ",(lb/3));

scanf ("%f", &inputval);
if (inputval == 1.0)
  {printf("Enter new number of wheels.\n");
   scanf("%f",&inputval);
   wn = inputval;
   goto Menudisplay; }
else if (inputval == 2.0)
  {printf("Trace on (1) or trace off (0).\n");
   scanf("%d",&inputval);
   lc = inputval;
   goto Menudisplay; }
else if (inputval == 3.0)
  {printf("Enter ratio of wheel drive speed to plate drive speed.\n");
   scanf("%f",&inputval);
```

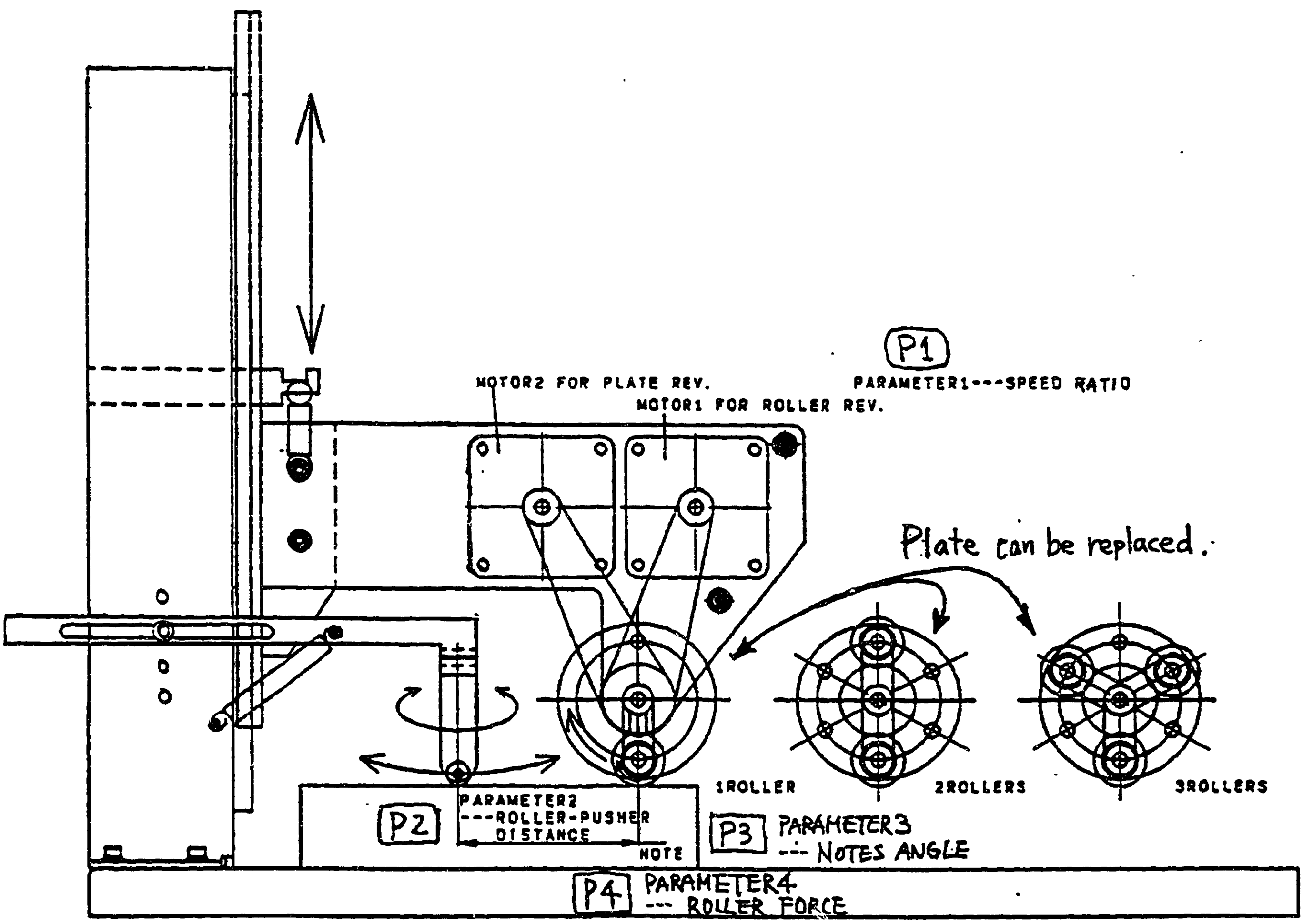
```
    om = inputval;
    goto Menudisplay; }
else if (inputval == 4.0)
   {printf("Enter distance between pin and roller, mm.\n");
    scanf("%f",&inputval);
    L = inputval * 3;
    goto Menudisplay; }
else if (inputval == 5.0)
   {printf("Enter radius of wheel, mm.\n");
    scanf("%f",&inputval);
    r2 = inputval * 3;
    goto Menudisplay; }
else if (inputval == 6.0)
   {printf("Enter radius of plate, mm.\n");
    scanf("%f",&inputval);
    r1 = inputval * 3;
    goto Menudisplay; }
else if (inputval == 7.0)
   {printf("Enter unsupported length, mm.\n");
    scanf("%f",&inputval);
    lb = inputval * 3;
    goto Menudisplay; }
else if (inputval == 0.0)
    goto Mainprogram;
goto Menudisplay;


Mainprogram:

if (lb > 480) {printf("The yen has a maximum length of 160 mm.  Please correct lb."
if (L > 480) {printf("The yen has a maximum length of 160 mm.  Please correct lb.")
if (L > lb) {printf("Unsupported length must be at least as high as roller to stabi


XClearWindow(display,window);
XGetWindowAttributes (display, window, &window_attributes);

Graphicsdisplay:

XDrawRectangle (display,window,gc,x0-50,y0-100,50,100); /* stabilizer */
XDrawRectangle (display,window,gc,50,300,50+lb,80); /* stack */
XDrawRectangle (display,window,gc,50,310,50+lb,60);
XDrawRectangle (display,window,gc,50,320,50+lb,40);
XDrawRectangle (display,window,gc,50,330,50+lb,20);
XDrawRectangle (display,window,gc,50,340,50+lb,0);

XFlush(display);

y1 = y0 - (r1 + r2);
x1 = x0 + L;

for(z=0;z<=1;z++)
  {
sk = 0;
for(i=10;i<=360;i++)
{
an1 = (3.14159/180)*i;
an2 = an1 * (om + 1);

x2 = x1 - r1 * sin(an1);
y2 = y1 + r1 * cos(an1);
```

```
XSetForeground (display, gc, foreground);
XDrawArc (display,window,gc,x2-r2,y2-r2,r2*2,r2*2,0,360*64);
XDrawLine (display,window,gc,x1,y1,x2,y2);
XDrawLine (display,window,gc,x2,y2,x3,y3);
XFlush(display);

/* erase circle and centerlines */
XSetForeground (display,gc,background);
XDrawArc (display,window,gc,oldx2-r2,oldy2-r2,r2*2,r2*2,0,360*64);
XDrawLine (display,window,gc,x1,y1,oldx2,oldy2);
XDrawLine (display,window,gc,oldx2,oldy2,oldx3,oldy3);
XFlush(display);

/* follow locus of contact point */
if (lc == 1)
{
XSetForeground (display, gc, WhitePixel(display,screen));
XDrawPoint (display,window,gc,oldx3,oldy3);
XFlush(display);}

/* draw */
XSetForeground (display, gc, foreground);
XDrawArc (display,window,gc,x2-r2,y2-r2,r2*2,r2*2,0,360*64);
XDrawLine (display,window,gc,x1,y1,x2,y2);
XDrawLine (display,window,gc,x2,y2,x3,y3);
XFlush(display);

XFlush(display);

oldx2 = x2;
oldy2 = y2;
oldx3 = x3;
oldy3 = y3;

if (sk == 1) goto skip;

/***** and now,...........THE BILL SHAPER !!........... *******/
/* this part takes a starting theta, and slides down curve until conditions
   are satisfied. */

th = oldth + ((3.14159/180) * (om+1));

 try2:

/* find contact point conditions */
xc = x2 - r2 * sin(th);
yc = y2 + r2 * cos(th);
/* dy = (y2 - yc) / (x2 - xc); */
dy = tan (th);

s = L + r2 * (an2 - an1);
/* printf("length %f\n",(s/3)); */
if (s > lb) {printf("The bill has passed."); sk = 1; goto skip;}

k = 2 * 3.14159 / (xc - x0);

 akloop:

for (gr=0;gr<100;gr++)
{
a = (y0 - yc) / (1 - cos(k * (xc - x0)));

lstest = 0;
loldy = 0;
loldx = 0;
for (li=1; li<= 50; li++)
```

```
  {lx = (li / 50) * (xc - x0);
   ly = a * (1 - cos(k * lx));
   ldy = ly - loldy;
   ldx = lx - loldx;
   lds = sqrt ((ldy * ldy) + (ldx * ldx));
   lstest = lstest + lds;
   loldy = ly;
   loldx = lx;  }

miss[gr] = lstest - s;
av[gr] = a;
kv[gr] = k;

k = k - (.0314159 / (xc - x0));
}

low = 99999;
for(gr=0;gr<100;gr++)
{if (miss[gr]>0) missv = miss[gr]; else missv = -1 * miss[gr];
 if (missv < low) {k = kv[gr]; a = av[gr]; low = missv;}}

kval = (y0 - yc)/ dy;
ktest = (1 - cos (k * xc)) / (k * sin (k * xc));
/* printf("ktest= %f\n kval= %f\n",ktest,kval); */
if (ktest>kval) {th = th - (3.14159/180); goto try2;}

oldth = th;


xcr = (xc-x0)/3000; ycr = (y0-yc)/3000;
h = sqrt ((xcr * xcr) + (ycr * ycr));

dn = h * sin (atan (xcr/ycr) - th);
dt = sqrt ((h * h) - (dn * dn));

printf("h= %f\ndt= %f\ndn= %f\n",h,dt,dn);

ft = (EI * (a/3000)) / ((dn * sin (th)) - dt);
fn = tan (th) * ft;

printf("ft= %f, fn= %f\n",ft,fn);

XSetForeground (display, gc, background);
XFillRectangle (display,window,gc,x0+1,0,x2-r2,y0);
XFlush(display);

 skip:

XSetLineAttributes(display,gc,3,LineSolid,CapButt,JoinMiter);

/*** draw ***/
XSetForeground (display, gc, foreground);
oldcurvex = x0; oldcurvey = y0;
for (fr = x0; fr < xc; fr += ((xc - x0) / 20))
{ curvex = fr;
  curvey = y0 - (a * (1 - cos (k * (fr - x0))));
  XDrawLine (display,window,gc,oldcurvex,oldcurvey,curvex,curvey);
  oldcurvex = curvex;
  oldcurvey = curvey;}
XFlush(display);

if (sk == 1) {for(j=0;j<10000;j++){}}
XSetLineAttributes(display,gc,0,LineSolid,CapNotLast,JoinBevel);
}
}
```

```
if (1)
{XFlush(display);
 printf("Would you like a replay? 0 for yes, 1 for no.");
 XFlush(display);
 scanf("%f",&inputval);
 if (inputval == 0)
   {pointcounter = 0;
    XClearWindow(display,window);
    goto Graphicsdisplay;}
 else if (inputval==1) goto Menudisplay;}

goto Graphicsdisplay;
}
```

Appendix E. Prototype Specifications



DEVELOPMENT OF NOTE-HANDLING ENDEFFECTOR TECHNOLOGY STEP1

DESIGND BY Y. SUGITATE(OMRON CO.)

[SIDE VIEW of PROTOTYPE STEP1]