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REAL TIME PREDICTION OF MARINE VESSEL MOTIONS USING KALMAN FILTERING TECHNIQUES*

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Real Time Prediction of Marine Vessel Motions, Using Kalman Filtering Techniques

by Michael Stefanos Triantafyllou and Marc Bodson, *M.I.T.*

ABSTRACT

The feasibility of predicting the motions of a vessel in real time 2 to 10 seconds ahead in time is presented, using Kalman filtering techniques.

First, a simple outline of the theory of prediction is provided. Subsequently, the particular problems of modeling the ship dynamics are presented, such as the rational approximation of the frequency dependent quantities and the non-minimum phase characteristics involved.

The role and sensitivity of the Kalman filter to error in estimating various parameters is presented. In particular, the significant effect of the modal frequency of the sea spectrum is discussed.

By plotting the rms error versus prediction time it is demonstrated that the upper limit for good predictability is about 5 seconds for all motions, except for roll, for which it can extend up to 10 seconds. Simulations are presented for the case of predicting motions with very few measurements containing relatively large noise. The prediction time is reduced down to 2 seconds except for roll which can be still predicted up to 10 seconds ahead.

Using the experience gained in predicting ship motions, a discussion is provided for implementing a similar scheme for semi-submersibles.

INTRODUCTION

In this paper, the word prediction is used in its strict sense, i.e., at a specific instant of time we would like to predict the future behavior of a vessel for a few seconds, within some confidence bounds.

The ability to predict accurately the motions of a vessel can reduce significantly the probability of failure of operations in rough seas. The present study started as part of an effort to ensure safe landing of aircrafts on relatively small vessels [1], but the basic principles are the same for any offshore operation, such as cargo transfer in the open sea, structure installation and floating crane operation.

Accurate vessel motion models are necessary for good prediction, while the predictor should have a simple structure so as to be implemented easily and with small storage requirements. The noise in the measurements can cause significant errors, so special attention must be given to the treatment of the noise, while the number of measurements should be kept small. The Kalman filter is a powerful tool to achieve all these goals and in the present study it is demonstrated how its efficient use can lead to simple and effective predictive implementations.

LINEAR OPTIMAL PREDICTOR

In order to present the basic concepts involved in prediction, the following simple example will be used: Consider a dashpot-spring system excited by a force f(t) resulting in motion x(t) (Figure 1). By Newton's law:

$$b\dot{x} + kx = f(t) \quad \text{.............................(1)}$$

so the solution is:

$$x(t) = e^{-\frac{k}{b}(t-t_0)} x(t_0) + \int_{t_0}^{t} e^{-\frac{k}{b}(t-s)} f(s)ds \quad \text{..(2)}$$

References and illustrations at end of paper.

If f(t) is white noise, i.e. completely unpredictable, it is not difficult to see that the only predictable part is the first term in the right hand side so that if we denote by p(t) the prediction, then:

$$p(t+\tau) = e^{-\frac{k}{b}\tau} x(t) \qquad (3)$$

The fact that the prediction decays with time indicates the fact that the white noise influences more and more the future behavior of the system, so that our predicting ability is governed by the decaying, undriven system dynamics.

If the force is not white noise, but has a spectrum $S(\omega)$ then a transfer function $H(\omega)$ is found such that

$$S(\omega) = |H(\omega)|^2 \qquad (4)$$

This is called spectral factorization [2] and leads to a fictitious system with white noise as input and the force f(t) as output. For example, if the given spectrum is given as:

$$S(\omega) = \frac{a^2}{\omega^2 + a^2} \qquad (5)$$

where a is a constant, then the transfer function and its time domain representation are respectively:

$$H(\omega) = \frac{a}{a+i\omega} \qquad (6)$$

$$\dot{f}(t) = -af(t) + aw(t) \qquad (7)$$

where w(t) is white noise of unit intensity. Equations (1) and (7) can be put together to form a composite system driven by white noise. In matrix form it can be written as:

$$\begin{bmatrix} \dot{x} \\ \dot{f} \end{bmatrix} = \begin{bmatrix} -\frac{k}{b} & \frac{1}{b} \\ 0 & -a \end{bmatrix} \begin{bmatrix} x \\ f \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} w(t) \qquad (8)$$

or, in short:

$$\dot{\underline{x}} = A\,\underline{x} + Bw \qquad (9)$$

By defining the exponential of a matrix [10] a compact solution can be derived for equation (9) in the form:

$$\underline{x}(t) = e^{A(t-t_0)}\underline{x}(t_0) + \int_{t_0}^{t} e^{A(t-s)}Bw(s)ds \qquad (10)$$

Again, the predictable part is the first term in the right hand side so the optimal prediction is:

$$\underline{p}(t+\tau) = e^{A\tau}\,\underline{x}(t) \qquad (11)$$

Wiener was the first to derive expressions for the optimal linear predictor, but in the general case the predictor may be implemented using differentiators [2]. This is circumvented by using systems with rational transfer functions, i.e. in the form of equation (9). It also requires an approximate spectral factorization using a rational $H(\omega)$.

To summarize, it is important to realize that the prediction is based on both the dynamics of the system and the form of the spectrum of the driving forces.

For complex systems, a large number of first order differential equations in the form of the matrix equation (9) are required to describe the dynamics, so that when prediction is attempted we need to know the value of all the state variables $\underline{x}(t)$ [equation (11)]. This would limit seriously the applicability of the present theory, since it would require an excessive effort of measuring all these variables, and this in some cases is even absolutely impossible (such as in the case of the fictitous variables representing the force spectrum).

As a result, a Kalman filter is required, which can be driven by only a few measurements, while it can reconstruct all the remaining variables. Also, it can be used to reduce the effect of measurement noise by an optimum choice of its gains.

It should be noted, however, that a smaller number of measurements and the presence of noise reduce the time within which a good prediction can be made, as shown in the sequel.

EQUATIONS OF MOTION OF THE VESSEL

First, we will outline the modeling of the vessel motions which constitute a major part of the predictable model.

The theory of the linear motions of a slender vessel is well established and, with the exception of roll, can provide good estimates of the motions [3], [4]. By including the nonlinear roll damping in the form of an equivalent damping, good roll motion predictions were obtained [5]. Extensions of this theory for a number of offshore floating structures are available [6], [7].

The complex problem of wave induced motions can be described within linear theory as follows: The incident waves are diffracted by the structure, while waves are radiated by the vessel as it oscillates. The diffraction waves can be found by assuming the vessel motionless within the incident waves, while the radiation waves are found by oscillating the vessel in calm water [4]. The force caused by the incident waves is called the Froude Krylov force, and added to the diffraction force produces the exciting force. If the motion is denoted as

$$\underline{x}(t) = \underline{x}_o \, e^{i\omega_o t} \quad \text{(12)}$$

then the radiation force is

$$\underline{F}_r(t) = T(\omega_o) \, \underline{x}_o \, e^{i\omega_o t} \quad \text{(13)}$$

while the exciting force is proportional to the wave amplitude a:

$$\underline{F}_e(t) = F_o(\omega_o) \, a \, e^{i\omega_o t} \quad \text{(14)}$$

The frequency dependent constant $T(\omega_o)$ can be decomposed in the following form

$$T(\omega_o) = -\omega_o^2 A(\omega_o^2) + i\omega_o B(\omega_o) + C \quad \text{(15)}$$

so that by Newton's law if M is the mass matrix of the vessel, we obtain

$$[M + A(\omega_o)] \, \ddot{\underline{x}}(t) + B(\omega_o)\dot{\underline{x}}(t) + C\underline{x}(t) =$$
$$= F_o(\omega)\eta \quad \text{(16)}$$

where we recognize C as the hydrostatic matrix, η denotes the wave elevation and A, B are frequency dependent constants, which represent an added mass and a damping respectively. This equation is used extensively in hydrodynamics as the basic equation of motion, it should be noted though that it is a hybrid equation mixing time and frequency domains, so that it actually represents a higher order differential equation [8]. As a result, it requires special attention when modeling the ship motions.

Speed Effects

In case the vessel is moving forward with speed U the frequency at which the vessel is oscillating is the frequency of encounter ω_e, which for deep water is given as:

$$\omega_e = \omega + \frac{\omega^2}{g} U \cos\phi \quad \text{(17)}$$

where ω is the wave frequency, ϕ the angle of incidence and g the gravity constant.

The response $\underline{x}(t)$ and the force $F_e(t)$ change with frequency ω_e, it should be noted though that the constant F_o depends on ω and not on ω_e [3].

The matrices A, B change also with U in a parametric form, described in detail in [3], [9].

NON-MINIMUM PHASE EFFECTS

A simple way to describe the non-minimum phase effects is by considering a vessel in very long waves.

The heave motion is maximum when the amplitude is maximum, whereas the pitch motion is maximum when the slope is maximum, so that a 90° phase difference exists between heave and pitch at very small frequencies. This means that the transfer function between heave and pitch has zeros in the right half plane.

WAVE SPECTRUM

The waves in a specific location are composed of waves generated by the local wind, and waves generated by a distant storm, which are characterized as swell. The spectrum of the sea therefore contains two peaks, one at low frequencies (swell) and one at higher frequencies (local storm).

For the development of the models in the present study unidirectional seas were assumed, described by a Bretschneider spectrum:

$$S(\omega) = \frac{1.25}{4} H^2 \frac{\omega_m^4}{\omega^5} \exp\{-1.25 \frac{\omega_m^4}{\omega}\} \quad \text{(18)}$$

where ω_m is the peak frequency of the spectrum and H the significant wave height. Two such spectra can be combined, with different ω_m, to model both the swell and the local storm. If the vessel is moving with speed U then the spectrum seen from the vessel becomes

$$S(\omega) = \left[\frac{S(\omega)}{d\omega_e/d\omega}\right]_{\omega = f(\omega_e)} \quad \text{(19)}$$

where

$$f(\omega_e) = \frac{-1+\sqrt{1+4\omega_e \frac{U\cos\phi}{g}}}{2\frac{U}{g}\cos\phi} \quad \text{(20)}$$

RATIONAL APPROXIMATION

The equations of motion are frequency dependent while the sea spectrum has a sharply peaked form. In order to use the very powerful state space techniques we need to approximate the transfer function with rational functions (i.e. ratio of polynomials) of $i\omega$.

The approximation is described in more detail in [8], [9]. The basic features of the approximation are the following: Between heave and pitch we need to introduce non-minimum phase characteristics by including a zero in the right half plane.

The added mass and damping are frequency dependent, but are also related by the Kramers-Kroning relations, so they can be approximated as the real and imaginary parts of the same analytic function.

The exciting force amplitude depends on the wave frequency and not the frequency of encounter, so the approximation must be performed in powers of $i\omega$ and not $i\omega_e$, and then apply equation [20].

Finally, the sea spectrum factorization in terms of a rational function is obtained by using the following transfer function:

$$H(S) = \frac{\sqrt{S_o} \quad S^2}{[1+2\zeta\frac{S}{\omega_o} + (\frac{S}{\omega_o})^2]^2} \quad \text{at } S= i\omega \quad \text{.........(21)}$$

$$\text{where } \zeta = 0.707 \quad \text{.........(22)}$$

$$S_o = \frac{1.25}{4\omega_m} H\, B(\alpha) \quad \text{.........(23)}$$

$$\omega_o = \omega_o(\alpha) \quad \text{.........(24)}$$

$$\alpha = \frac{U}{g}\omega_m \cos\phi \quad \text{.........(25)}$$

$B(\alpha)$ and $\omega_o(\alpha)$ are functions of [9], so that the peak of [21] coincides with the peak of the Bretschneider spectrum.

Once the transfer functions have been approximated by national functions, a state space model is constructed in the form

$$\dot{\underline{x}} = A\,\underline{x} + BW_1 \quad \text{.........(26)}$$

$$\underline{y} = C\,\underline{x} + \underline{W}_2 \quad \text{.........(27)}$$

where $\underline{x}$ is the state vector, $\underline{y}$ are the measurements, A is the matrix of the system and sea dynamics, W_1 is white noise, B is a column vector and C the measurement matrix. The vector white noise $\underline{W}_2$ represents the noise in the measurements which is due primarily to vibrations of the vessel structure.

KALMAN FILTER

Let V_1, V_2 the intensities of W_1, $\underline{W}_2$ respectively. Then the Kalman filter can be used to reconstruct the state optimally, in the sense of minimizing the mean square error. The form of the filter is [10].

$$\mathring{\hat{\underline{x}}} = A\,\hat{\underline{x}} + K\,(\underline{y} - C\hat{\underline{x}}) \quad \text{.........(28)}$$

where $\hat{x}$ the estimate of the state, A and C the same matrices as in (26), (27) and K the gains of the filter, given by the equation (steady state filter).

$$K = Q\,C^T\,V_2^{-1} \quad \text{.........(29)}$$

where Q satisfies the equation

$$AQ + QA^T + BV_1B^T - QC^TV_2^{-1}CQ = 0 \quad \text{.........(30)}$$

For practical purposes we will assume that the user has access to software for Kalman filter design as in [11]. The interested reader can find a detailed outline of the Kalman filter design and properties in [10].

PREDICTOR

The optimal prediction is now obtained by propagating the state estimate in time, i.e. the prediction τ seconds ahead of the measurements y(t), denoted as $\hat{\underline{y}}(t)$, is given as:

$$\hat{\underline{y}}(t+\tau) = C\,\underline{Z}\,(t+\tau) \quad \text{.........(31)}$$

where

$$\underline{Z}(t+\tau) = e^{A\tau}\,\hat{\underline{x}}(t) \quad \text{.........(32)}$$

The implementation of the predictor therefore, includes the Kalman filter described by equation (28), which is driven by the noisy measurements $\underline{y}$ and provides an estimate of the state $\hat{\underline{x}}$; and the system described in (31), (32) which simply propagates in time the state estimate $\hat{\underline{x}}(t)$.

It should be noted that the error defined as

$$\underline{e}(t) = \underline{x}(t) - \underline{Z}(t) \quad \text{.........(33)}$$

is governed by the same differential equation as $\underline{x}(t)$. This can be seen by subtracting (32) from (26) resulting in

$$\mathring{\underline{e}} = \underline{A}e + B\underline{W}_1 \quad \text{.........(34)}$$

This indicates that for large prediction times, the error covariance equals the state covariance, i.e. 100% error occurs, as expected.

APPLICATION FOR A SHIP

State-space models for the motions of a small prismatic coefficient vessel were developed in [9]. To first order the heave and pitch motions are decoupled from the roll, sway and yaw motions so we consider the two sets of equations separately.

The prediction of heave and pitch involves a 15 order model, 6 states of which describe the sea. Figure 2 is a plot of the heave and pitch rms error as a percentage of the corresponding rms motion, versus prediction time. As expected for large prediction times, the rms error equals the rms motion (100% uncertainty). For 5 sec prediction the rms error is 25% for heave and 20% for pitch. The form of the curve indicates that heave and pitch are almost equally predictable.

The prediction of roll, sway and yaw involves a 16 order model, with 6 states describing the sea.

Figure 3 is a plot of the rms error for each of the three motions versus prediction time. It can be seen that sway and yaw are predictable for about 5 sec, while roll is distinctly more predictable, up to 10 sec.

These results assume perfect state reconstruction. Actually few measurements are available and they include noise. In order to have a feeling for the deterioration of the prediction time we consider the following rather extremely limiting case: Figure 4 is a simulation of the heave prediction using the heave, pitch model with only two measurements, heave and pitch motions, including large noise. Prediction starts at t = 40 sec so that all the Kalman filter transients have decayed, and it is seen that prediction time is restricted to about 2 sec.

Figure 5 is a simulation of the roll motion using the roll, sway, yaw model, again using measurements of the motions only, and with large noise. The prediction is quite good up to 10 sec ahead of time, while the phase prediction is even better, extending up to several cycles ahead. This contrasts sharply with sway (figure 5), whose prediction extends about 2 sec. ahead (i.e. similar to the heave-pitch case).

Again, these results are to demonstrate the prediction limits in the case of using a minimal number of noisy measurements. The inclusion of velocity measurements increases the prediction time, whose upper bound is provided by Figures 2 and 3.

The efficient form of the Kalman filter and the predictor allowed their implementation on a microcomputer. Special attention must be paid to the fact that roll is very lightly damped so that the discretization of the continuous equations must be checked for stability, as well as to the fact that sway and yaw have no restoring forces, while the excitation depends on the wave slope, so that pole-zero cancellations may occur if modeling is done improperly.

THE INFLUENCE OF THE SEA PARAMETERS

As indicated above, the accurate modeling of the sea is an essential part for good predictability of the motions. The essential feature of the sea spectrum is its relatively narrow, exponentially decaying peak. Also, the actual sea is directional, whereas in the present model, a uni-directional sea is assumed.

An investigation of the influence of the various parameters involved has been conducted, particularly for the performance of the Kalman filter, which is the basic component of the predictor. The main parameters considered are the significant wave height, the modal frequency, the heading of the waves and the double peaked spectrum.

The Kalman filter performance was relatively insensitive to errors in estimating the significant wave height and heading. The heading insensitivity is particularly important, because the performance of the predictor remains unaffected for directional seas.

The filter was particularly sensitive to errors in the modal frequency above ±20%. Also, if a single peak was assumed and the sea was double peaked larger errors resulted.

The modal frequency can be estimated by using the average up-crossing period, or by using an extended Kalman filter, which estimates the value of the modal frequency.

PREDICTION OF THE MOTIONS OF A SEMI-SUBMERSIBLE

Hydrodynamic theories have been developed to evaluate the motions of semi-submersibles [6]. The development of the equations of motion follows exactly the steps indicated in the case of the ship motions, except for the following simplifying effects: The diffraction and radiation effects are not frequency dependent, because the major part of the structure is sufficiently submerged, so that free surface effects are secondary within the frequencies of the wave spectrum.

The modeling of the semi-submersible motions is simpler, because the frequency dependence is restricted to the exciting force. Nonetheless, the non-minimum phase relation between heave and pitch described also for a ship and the heave force (and pitch moment) cancellation frequencies which are peculiar to semi-submersibles, are essential features for good motion predictability.

The heave force on a semi-submersible becomes close to zero when the force on the upper part of the pontoon equals the force on the lower part. This is possible because, although the wave pressure decays exponentially, the area in the upper part of the pontoon is reduced by the area of the surface piercing legs [6]. Similarly, for beam seas, the force on one pontoon cancels the force on the other pontoon at a specific frequency. The pitch moment presents similar concellation frequencies. Those frequencies correspond to zeros of the force transfer function and are essential parts of the modeling.

Considering the problem of predicting the relative motion between a ship and a semi-submersible, both models must be available, while it can be expected that because of the additional zeros of the transfer function of the semi-submersible, its pitch motion will be in opposite direction than the ship pitch motion for specific frequency bands.

CONCLUSIONS

For an efficient prediction of marine vessel motions it is important to model accurately the vessel dynamics and the sea spectrum.

The modeling of the vessel dynamics requires a rational approximation of the frequency dependent quantities, i.e. the added mass and damping matrices, and the exciting force vector; and a careful modeling of the non-minimum phase relation between heave and pitch.

The modeling of the sea involves the spectral factorization of an adequate sea spectrum and the good estimation of the modal frequency. In the case of a double peak spectrum, a model for the swell must be included.

For practical purposes, an upper bound of about 5 seconds for all motions, except for roll which can extend up to 10 sceonds must be considered. It was shown that with only few measurements and significant noise the motions are predictable 2 seconds ahead, with the exception of roll which was still predictable 10 seconds ahead.

The use of a Kalman filter is considered essential because it allows the use of straight-forward techniques, while it deals efficiently with noisy measurements. Also, only a relatively small number of measurements is essential. An increase in the number of available measurements increases the predictability of the motions up to the theoretical upper bound.

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APPENDIX

The ship models were derived using the transfer functions provided by the M.I.T. Sea-keeping program [12]. The characteristics of the vessel are:

Length = 529 ft.
Beam = 55 ft.
Draft = 18 ft.
Block coefficient = 0.461
Metacentric height = 4.16 ft.
Longitudinal Center of Gravity = 1.07 ft. AFT
Displacement = 6,800 ton

The heave-pitch model used to obtain the numerical results was derived for forward speed 21 ft./sec. and heading 0 degrees. The sway, roll, yaw model was derived for 15.5 ft./sec. and 45 degrees. The sea in both cases was described as sea state 5, fully developed seas (H = 10 ft., ω_M = 0.72 rad./sec.).

The matrices used for the numerical application for each of the two models are provided in Table 1.

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TABLE 1

Matrix A for heave and pitch (15 x 15)

0.	1.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
-1.491E+00	-1.726E+00	0.	1.491E+00	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	1.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	-1.491E+00	-1.726E+00	0.	1.491E+00	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	1.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	-1.491E+00	-1.726E+00	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	1.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	-1.213E+00	-5.350E-01	-9.452E+00	-2.190E+01	7.850E-01	0.	0.	0.	-5.027E-01
0.	0.	0.	0.	0.	0.	0.	0.	0.	1.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	8.818E-04	3.106E-04	-1.163E+00	-4.347E-01	-1.529E-03	0.	0.	0.	1.605E-02
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	1.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	-6.635E-01	-1.152E+00	6.635E-01	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	1.	0.
6.595E+02	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	-6.635E-01	-1.152E+00	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	9.728E-01	1.249E+00	-9.728E-01	4.402E-01	-1.058E+00

Matrix A for sway, roll, yaw (16 x 16)

0.	1.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
-1.136E+00	-1.507E+00	0.	1.136E+00	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	1.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	-1.136E+00	-1.507E+00	0.	1.136E+00	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	1.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	-1.136E+00	-1.507E+00	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	1.	0.	0.	0.	0.	0.	0.	0.	0.
0.	5.722E+01	0.	0.	0.	0.	-2.610E-01	-5.202E-01	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	1.	0.	0.	0.	0.	0.	0.
0.	6.859E+02	0.	0.	0.	0.	0.	0.	-4.575E-01	-6.696E-01	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	1.	0.	0.	0.	0.
0.	6.481E+03	0.	0.	0.	0.	0.	0.	0.	0.	-8.111E-01	-4.458E-01	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	2.423E-03	0.	0.	0.	-4.469E-06	0.	-8.343E-03	2.748E-01	1.097E-01	8.041E+00
0.	0.	0.	0.	0.	0.	3.218E-06	1.375E-06	0.	8.142E-06	-1.999E-08	-1.804E-07	4.343E-05	-1.961E-02	-2.344E-01	1.286E-02
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	1.	0.	0.
0.	0.	0.	0.	0.	0.	-4.057E-06	0.	0.	0.	1.299E-07	0.	-4.611E-04	-2.319E-02	4.892E-04	-3.242E-02

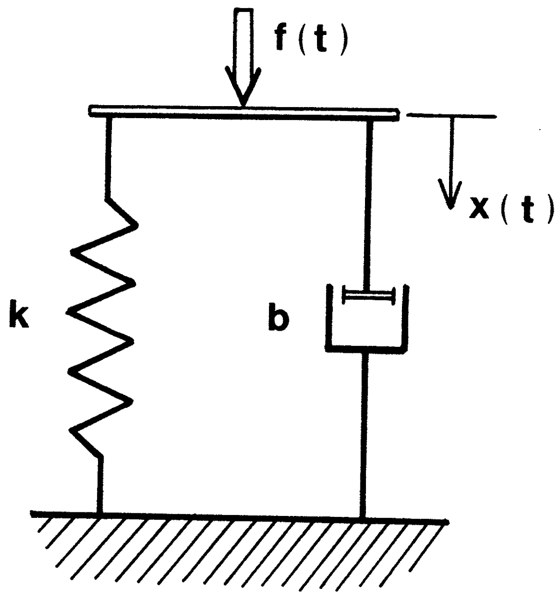

Fig. 1 — Damper — spring system

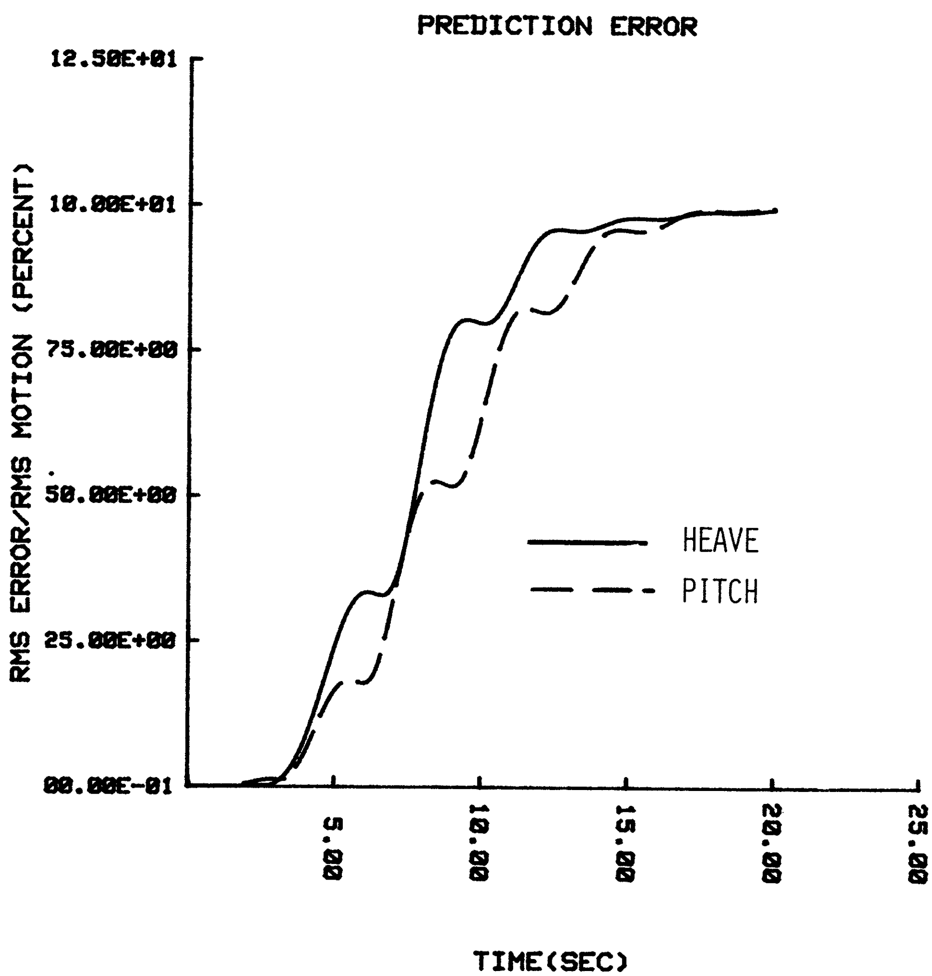

Fig. 2 — Heave and pitch rms error vs. prediction time

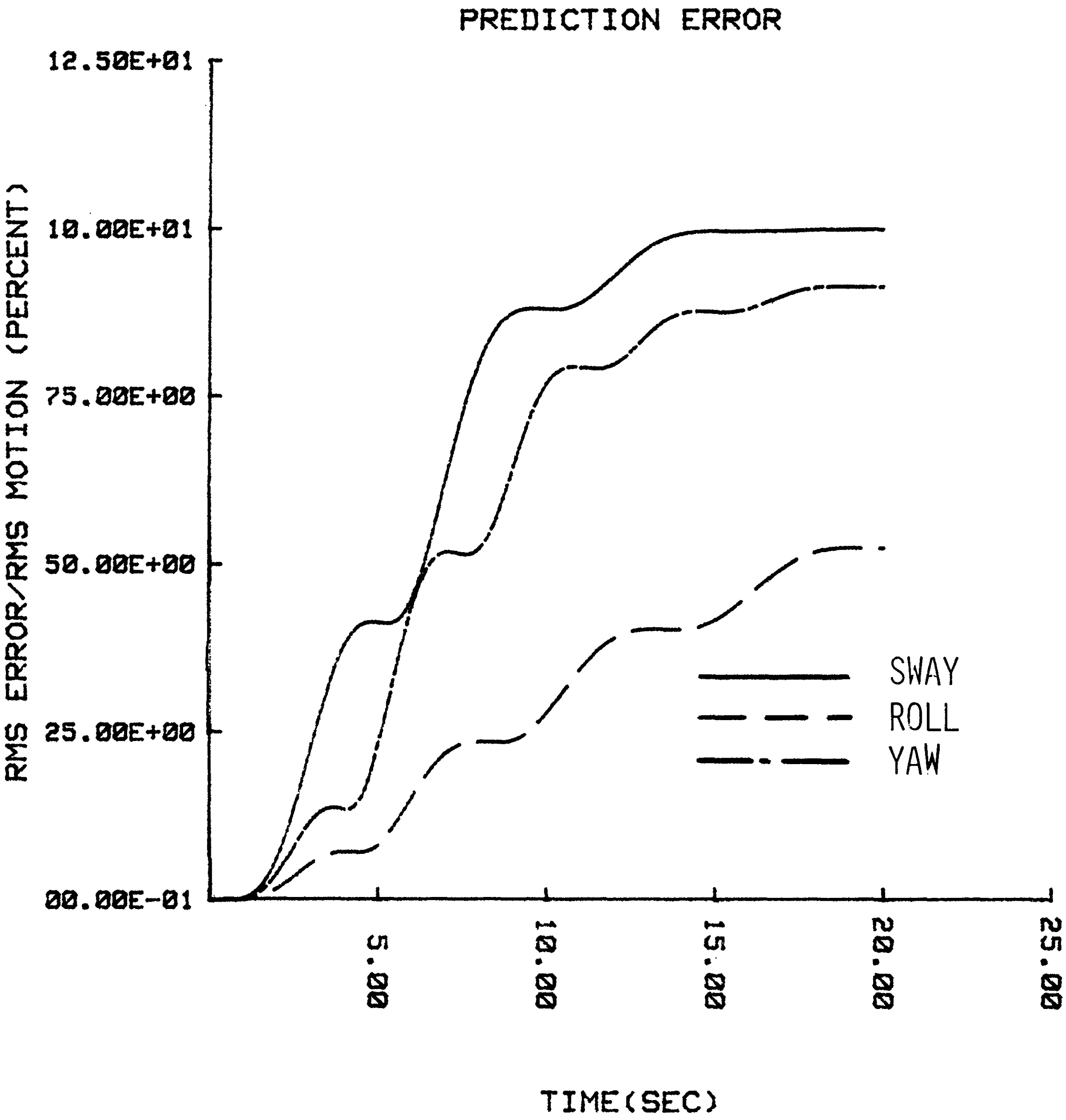

Fig. 3 — Sway, roll and yaw rms error vs. prediction time

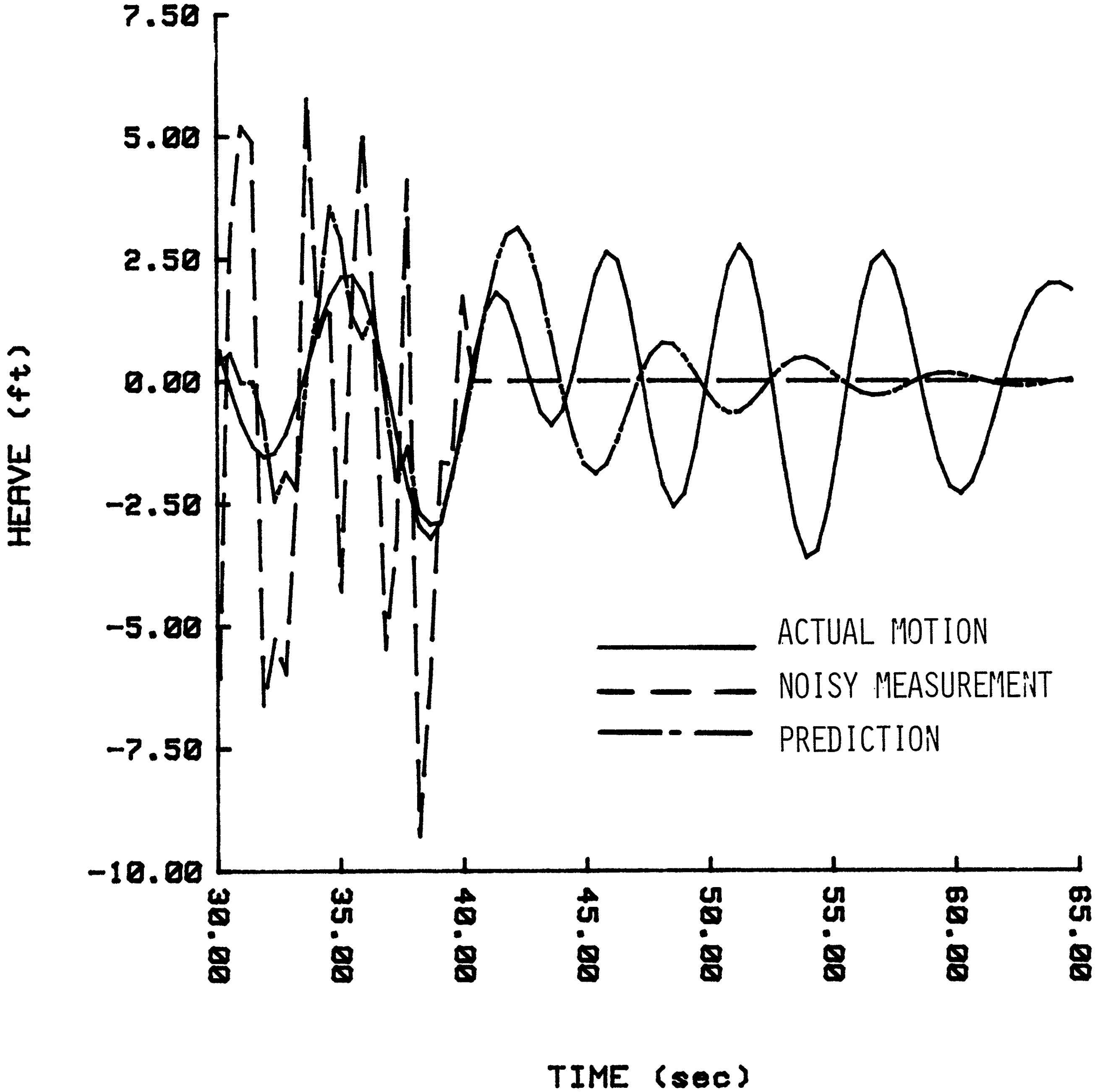

Fig. 4 — Heave simulation and prediction including the Kalman filter

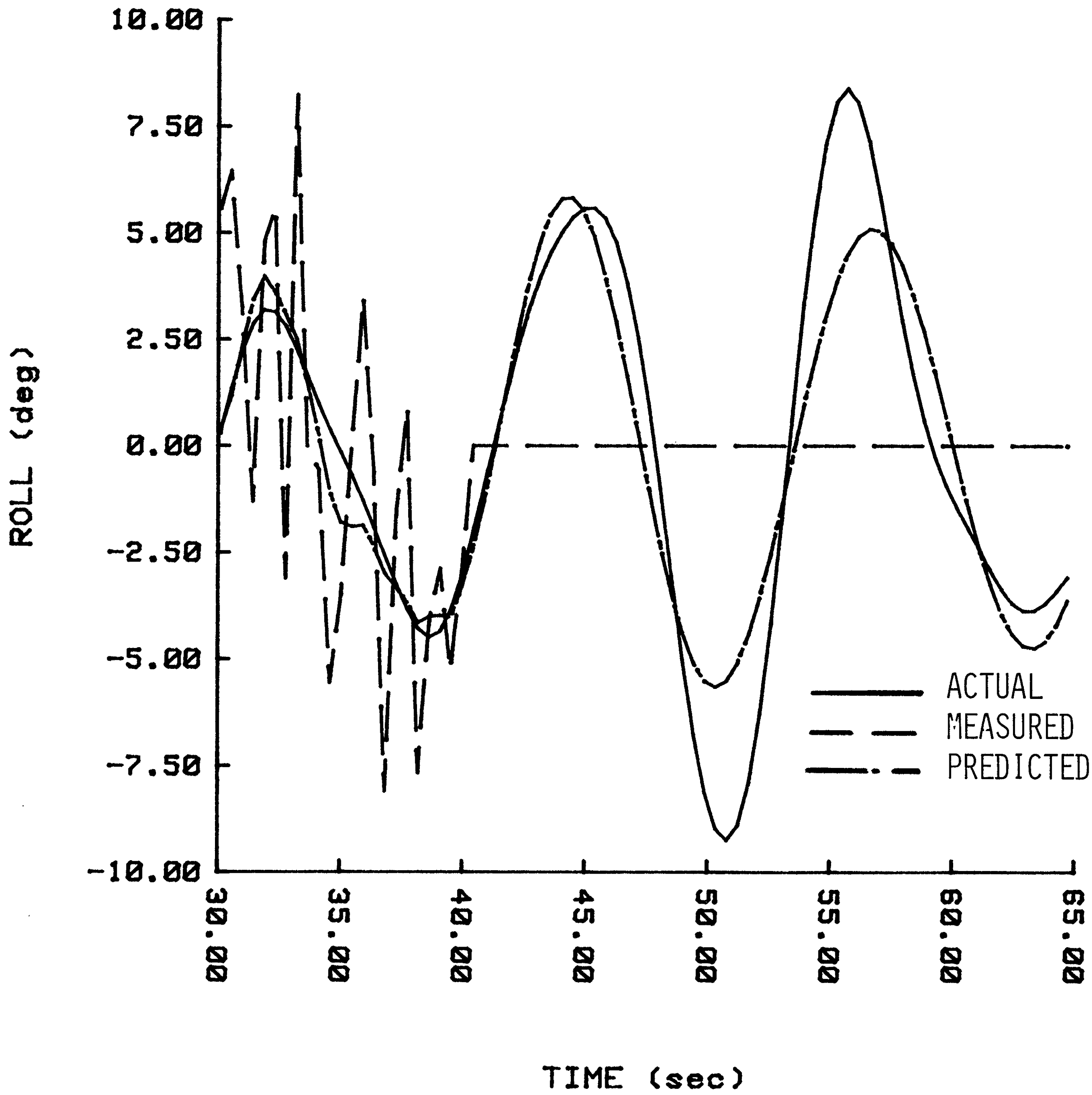

Fig. 5 — Roll simulation and prediction including the Kalman filter

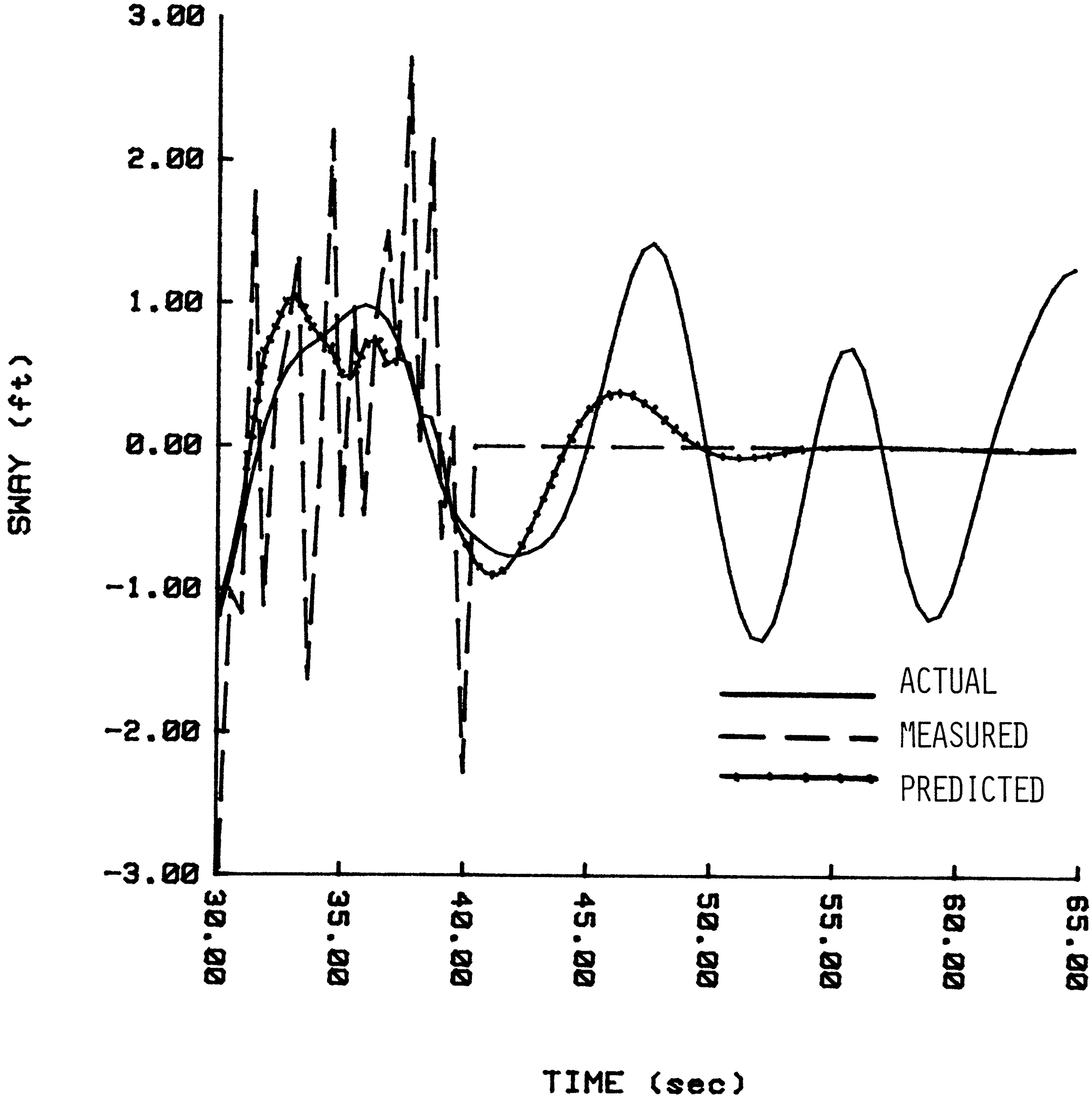

Fig. 6 — Sway simulation and prediction including the Kalman filter