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NAVIGATION OF A MANNED SATELLITE SUPPLY VEHICLE BACK TO EARTH

by

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Certified by

Thesis Supervisor

Chairman, Departmental Committee on Graduate Students

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May 25, 1959

Professor Leicester F. Hamilton
Secretary of the Faculty
Massachusetts Institute of Technology
Cambridge 39, Massachusetts

Dear Professor Hamilton:

In accordance with the regulations of the faculty, I hereby submit a thesis entitled Navigation of a Manned Satellite Supply Vehicle Back to Earth in partial fulfillment of the requirements for the degree of Master of Science.

James E. McCormick

John H. Axley

Forrest R. Hanvey

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ABSTRACT

The purpose of this thesis is to find a suitable system for furnishing all requisite navigational information to the pilot of a satellite supply vehicle during his return to earth. A desired method of return is presumed and the latitude of control available to the pilot, using this method, examined. From this, the requisite system specifications to guide him to a landing at a desired point are found. Various natural phenomena potentially capable of furnishing navigational information are examined for suitability. A navigation system is then designed, in functional form, which will give a maximum of flexibility in use and meet the specifications set.

Thesis Supervisor: Philip H. Whitaker

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The authors further express their appreciation to the United States Air Force and the Air Force Institute of Technology for the opportunity to do this graduate work.

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OBJECT

The object of this thesis is to investigate the navigational requirements of a satellite vehicle in returning from orbit to an earth landing, and to propose a navigation system which will meet these requirements.

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CHAPTER 1

INTRODUCTION

The problem of maintaining an earth satellite space station poses many challenging problems to both the astronaut and systems engineer. One of these problems, that of navigating a manned satellite supply vehicle back to earth, is taken as the subject of this thesis. A navigation system is proposed which the authors feel will provide all requisite navigational information to an acceptable degree of accuracy. It cannot be presumed that the system proposed is an optimum system or the only one which can do the job, but the authors believe that the system is achievable at the present state of the instrumentation art and in the light of the present knowledge of the earth's environment. The proposed system may be separated into two parts, (1) apparatus carried by the vehicle and (2) assisting apparatus outside of the vehicle. In the future, further information on geophysical phenomena may make one or more of the environmental properties considered by this thesis, but found to be unuseable, applicable and to the point for a navigation system.

The vehicle will carry a special purpose computer, an inertially stable platform which is monitored during portions of the flight by a pair of star tracking units, and the display and control mechanisms necessary for the accomplishment of practical navigation. Several external aids will be utilized to accomplish the navigation problem. An aircraft equiped with radar will track and supply information to the supply vehicle just prior to re-entry, and electromagnetic ground tracking and guidance will be utilized

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during the final portion of approach to a landing.

Much of the calculation of navigation information will be done on the ground from tracking information gathered in advance of the start of the return trip and the results will be utilized to initiate the return of the supply vehicle. Associated with such ground calculations and measurements there will be inherent errors which contribute to the total system error and must be considered in establishing system specifications. The proposed system is presented in this thesis in its functional form, and very little attempt will be made to set specifications of individual units.

The following method of approach is taken in this thesis to arrive at an acceptable navigation system design for the supply vehicle. First, certain assumptions are made regarding the satellite orbit from which descent is to be made and a desired method of descent is assumed. Secondly, properties of the earth's environment which may furnish navigational information are examined to find if they are usable. Thirdly, the descent trajectory is examined to find what requirements it imposes on the navigation system. Lastly, a system is designed which will meet these requirements.

Prior to the initiation of the return trip, the supply vehicle is assumed to be in a nearly circular orbit, around a spherical earth at an altitude between 300 and 500 miles. The navigation system proposed is extendable with modifications, to the noncircular orbit case, but the associated error analysis in order to set system specifications becomes lengthy and was not attempted for this thesis[1]. A short development of some of the equations for orbital departure from a noncircular orbit in the fashion assumed by this thesis is presented in Appendix B-11. Orbital perturbations due to the earth's oblateness are assumed calculable to a sufficient accuracy that they do not lead to serious navigation errors[2], and other nonspherical effects, such as height above sea level of the landing field, are assumed taken care of in the ground computation scheme prior to the return flight.

It is also assumed that prior to initiating return to earth the orbit of the supply vehicle is coincident with that of the satellite being supplied, of which there has been an extensive period of tracking from the ground, so that the supply vehicle's position is known to a high degree of accuracy while it is in orbit. Immediately after a given period of tracking from the ground the vehicle's position is known very accurately, and it is assumed that one half an orbital revolution later the satellite's position in space is known to within a box in space one mile long and four tenths of a mile high and wide. Two days later, however, this information has deteriorated to a box ten miles long and 4 miles high and wide, unless, of course, another tracking period has taken place during the two day period. These assumptions on the knowledge of the supply vehicle's position help to determine the system accuracy requirements and lead naturally to two situations of varying complexity. First, it might be imagined that the return trip would be initiated soon after a period of tracking from the ground, in which case the supply vehicle's position of re-entry would be based upon well known initial conditions. In general, however, the area within which descent from a given orbit must begin in order to land at a given landing field is restricted, and passage through the area by the supply vehicle may occur some time after the last opportunity to obtain ground tracking information. If one had to wait until the orbital plane itself became adjusted to just exactly the right conditions, it would require an inordinate amount of time to complete the average supply mission. Instead of waiting for perfect alignment, it is assumed that the return vehicle can shift the plane of its orbit slightly one way or the other by properly orienting the retro-rocket which initiates the return.[2] In this way the length of time one would have to wait to get into a favorable position is assumed to have been lowered to a maximum of two days after the last period of ground track.

A linear type error analysis shows that in either case the errors in knowledge of position and motion are too large to be allowed to proceed uncorrected during the re-entry period (see Appendix F). The envisioned navigation system is designed to eliminate most of the possible error caused solely by the long delay between last ground track and initiation of return to earth. The major portion of such error appears as a distance error along the track. A navigational fix is to be given to the supply vehicle by a radar equipped aircraft just prior to re-entry into the atmosphere to eliminate this error. Before discussing the system further, a glimpse at the trajectory assumed for the supply vehicle is in order.

The return trajectory (see Fig. I-1) starts when at a predetermined time (or position as interpreted at the ground tracking center) the supply vehicle fires a retro-rocket which imparts a slowing increment of velocity of sufficient magnitude to cause the resultant elliptical orbit, according to Kepler's Laws of motion, to have a perigee of approximately 300,000 feet. The point of firing of the first retro-thrust becomes the approximate apogee for that ellipse. Three-hundred-thousand feet is chosen arbitrarily as that altitude at which a vehicle just begins to sense a perceptable atmosphere above the earth. Actually, the atmosphere is continuous to far greater heights, but the errors involved in assuming that as the standard or average value of the beginning of the sensible atmosphere for re-entry purposes appear to be less than the errors from other sources; this arbitrary choice continues to be made in many theoretical studies. For more precise studies a more current research work on the atmosphere should be consulted.[3] Because of the above assumption it may be seen that, aside from some small effects due to the shape of the earth,[4] the vehicle may be assumed to follow essentially Keplerian Laws of motion during this portion of the return flight. Both the time to fire and the magnitude of the retro-thrust are precomputed values which come from the ground tracking station during the last tracking period prior to the retro-thrust.

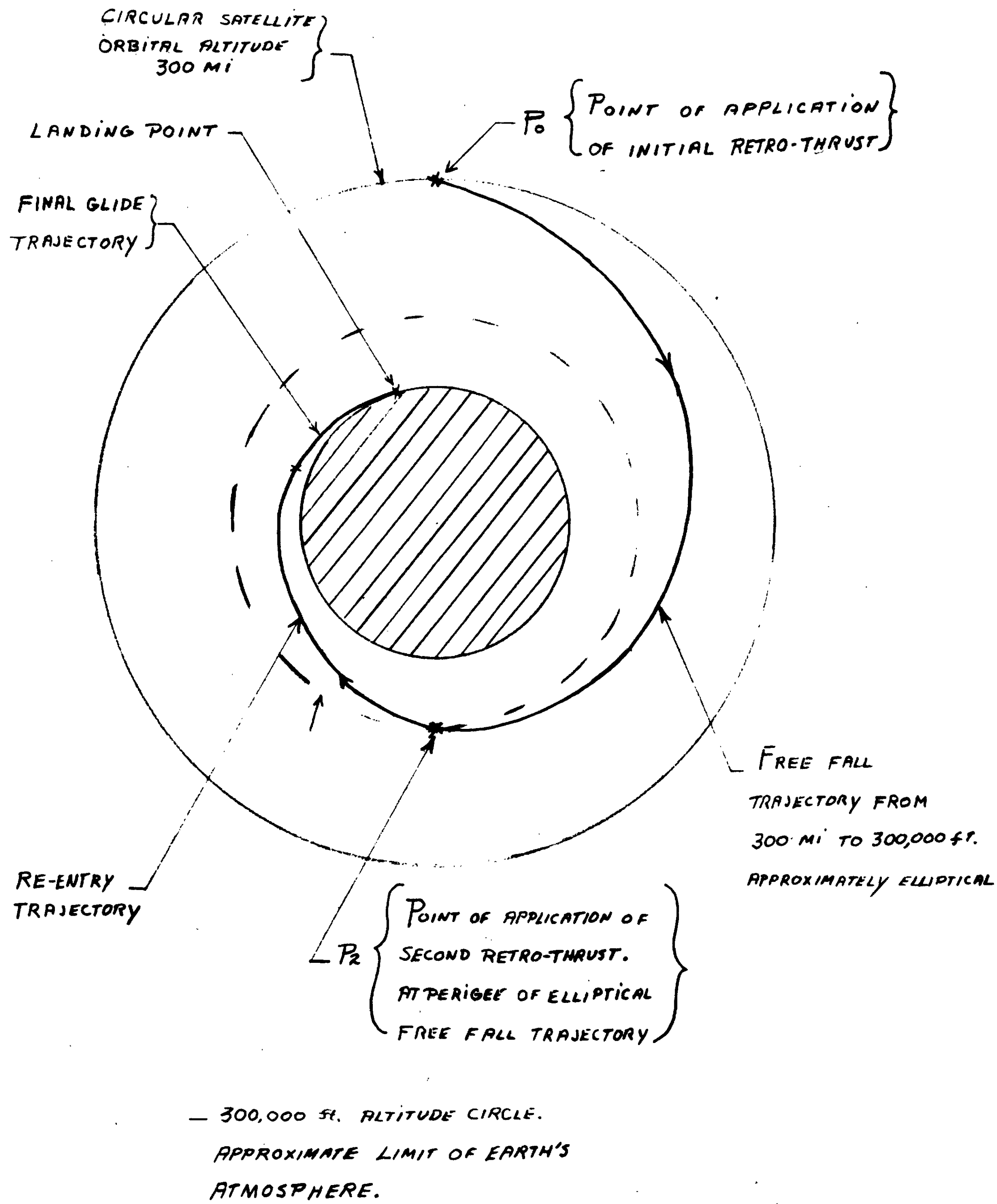
CIRCULAR SATELLITE ORBITAL ALTITUDE 300 MI
LANDING POINT
P_0 POINT OF APPLICATION OF INITIAL RETRO-THRUST
FINAL GLIDE TRAJECTORY
FREE FALL TRAJECTORY FROM 300 MI TO 300,000 ft. APPROXIMATELY ELLIPTICAL
RE-ENTRY TRAJECTORY
P_2 POINT OF APPLICATION OF SECOND RETRO-THRUST. AT PERIGEE OF ELLIPTICAL FREE FALL TRAJECTORY
— 300,000 ft. ALTITUDE CIRCLE. APPROXIMATE LIMIT OF EARTH'S ATMOSPHERE.

FIG 1-1

At the perigee height of approximately 300,000 feet a second retro-thrust is applied which slows the supply vehicle to some velocity less than the local circular orbital velocity. The supply vehicle thus begins re-entry at a small flight path angle below the local horizontal; it then follows a re-entry trajectory which amounts to a very long glide of approximately seven-thousand miles in range before dropping to an altitude of approximately 90,000 feet. The glide is assumed to be made at a nominal L/D of 1.0, although other values might also have been chosen. For our problem, at least two considerations restrict the range of L/D's at which the vehicle may fly. First of all, a condition of zero or negative lift is unacceptable. This is due to the fact that the inertial platform utilized for the navigation from three-hundred-thousand feet to ninety-thousand feet is stable and its navigation errors in indicating the vertical are oscillatory for any flight condition in which the net specific vertical force is positive,[5] which means essentially a positive lift in this case, and the errors diverge if the net specific vertical force is negative. The second consideration is one tied, not to the navigation system, but to the vehicle and occupant themselves. The maximum acceleration and heating which can be withstood are properties of the vehicle and occupant, and the vehicle is forced to follow a rather restricted type of trajectory through the atmosphere until altitudes and speeds are reached where these problems are no longer serious. This accounts for the so-called re-entry phase from 300,000 feet to approximately 90,000 feet. From 90,000 feet on until landing the vehicle is considered to be flying or gliding with a range of control over L/D with which to vary the point at which the vehicle lands. This control would be utilized to take out the effect of errors of the navigation system or control system, etc.

It is tacitly assumed throughout this thesis that it is desirable to navigate the vehicle to a specific and rather well defined landing area rather than just to a general area such as a large body of water.

In summary, certain assumptions have been made regarding the orbit of the satellite. A desired method of descent of the supply vehicle has also been assumed. With these assumptions the decent path was examined in detail and the effects of uncertainties in initial conditions upon that descent path established (see Appendix C and F). The next logical step is the investigation of factors which might provide navigational information.

CHAPTER 2

GEOPHYSICAL PROPERTIES

In this chapter the properties of the earth and its environment which may provide a source of navigational intelligence are examined for applicability and accessability.

In the past, a standard atmosphere based upon the fairly well known properties of the lower atmosphere has been defined, and an instrument used to measure some characteristic such as pressure or temperature of the atmosphere through which the vehicle was flying. The reading of the instrument when calibrated by an initial setting based upon known conditions would give an indication of the height difference between the reference and the local vehicle altitude. By thus providing altitude information, one of the three navigational coordinates was established to a fair degree of accuracy. Other geophysical quantities may be made to yield more than just height information.

A. Pressure

The most commonly used quantity for height measurements was the ambient air pressure, in the form of the well known altimeter. An extrapolation of such a measurement to very high altitudes has been envisioned,[6] but the densities involved at very high altitudes are so small, that the turbulence of ion motion and temperature gradients, etc., make the local variations of the ambient pressure of a single altitude as high as ten to one.[7] Because of this fact, it appears highly unlikely that a pressure measuring device could do a very good job of correlating high altitude pressure and the altitude itself. The measurement might be of some practical value for navigation if some sort of long term averaging of its readings took place. Since the supply vehicle return is a relatively short period flight, the ambient pressure is removed from consideration as a basic physical parameter upon which to base a portion of the present navigation system. This, of course, does not mean that conventional altimeters as a navigational assist for the low altitude portion of the trajectory are eliminated entirely; it simply means that over a major portion of the trajectory, where altitude information is important to the navigation problem, pressure measurements would not satisfactorily provide this information.

B. Temperature

High altitude temperature would seem to be a fairly good quantity to measure for an estimate of altitude. This is true, theoretically at least, because the fluctuations in temperature should not be a very high percentage of the ambient value.[8] By temperature it is meant the average kinetic energy of the surrounding gas particles or ions.

Several problems are associated with the use of temperature as a navigational parameter. Firstly, it would be fairly hard to devise an accurate and portable device which would measure only the kinetic energy of the particles and not any radiant energy incident upon the vehicle.[9] Secondly, the variation of temperature in the atmosphere is not well enough established to use it as a navigational parameter.[10] Thirdly, even the models of standard atmospheres present temperature as a piecewise linear variation with altitude and not as a continuous function.[11] Fourthly, the variation of temperature in the lower atmosphere may be a significant portion of the ambient value.[12] Because of these and other similar arguments, temperature is eliminated from serious consideration as a navigational parameter for the present system.

C. Earth's Electric Field

The earth is statically charged to an average value of around 400,000 coulombs.[13] As such, its surface represents the negative surface for an electrostatic field extending outward in space. The field has a gradient at the earth's surface of approximately 130 volts per meter, but this value is subject to wide local fluctuations in the vicinity of electrical storms and other disturbances. Presumably, the ionosphere presents a highly conducting shell which tends to keep the charge distribution balanced, and in so doing the motions of large numbers of ions from one spot to another represent large current flows in the atmosphere. Solar disturbances change the ion level in the ionosphere and its apparent height, etc. Consequently, the electric field must undergo wide fluctuations throughout the atmosphere. In the light of present knowledge alone, it does not appear that the earth's electric field offers any hope of accurate navigational information at high altitudes.[14]

D. Earth's Magnetic Field

The earth's magnetic field has been used as a navigational aid for quite sometime. Its properties at or near the surface are fairly well known and are mapped. The earth's magnetic field is essentially that of a uniformly magnetized sphere.[15] Deviations from this field are significant, but fairly small in percentage of the total field, and many of the variations can be predicted. On the surface of the earth the most commonly used quantity is the horizontal component of the earth's magnetic field. A better parameter to use if accuracy is desired is the magnitude of the total field strength. The possibility of its use as a navigational aid near the surface of the earth is receiving much attention at this time. The locus of points of equal field strength represents a surface which intersects the earth at approximately a magnetic latitude. It is now widely held that the sources of the earth's main field are inside the earth, although the possibility exists that as much as one percent of the field is due to external causes. Potential theory shows that if the magnetic field is known on a closed surface containing all of the sources of that magnetic field, then the magnetic field that exists outside the closed surface is also determined for every point in space. Since the earth's field is largely known on its surface it is possible theoretically to extrapolate the surface data upward with a high degree of precision. The local anomalies are smoothed appreciably as low as 100,000 feet, and the large regional anomalies are smoothed out at higher altitudes. The accuracy of the upward extrapolation cannot be any greater than the knowledge of the surface conditions, so that inaccuracies over some areas might be as large as several percent of the total field strength instead of the one percent which should be the maximum inaccuracy over the major portion of the earth. Theoretically then we have a potential source of navigational information in the form of the earth's magnetic field.[16]

Several things combine, however, to make this source unexploitable. The accuracy of the measuring instruments is not a limit, nor is their weight or complexity. Mounting of such instruments upon a re-entry vehicle so that they are free from local distrubances caused by the vehicle itself is a practical problem which has to be solved if magnetic information is to be of any use in a case such as the supply vehicle's return to earth. A present limit to usefulness is the lack of knowledge of the time variation of the surface field to sufficient accuracy to get extrapolated data of much use to a navigation system. Supposedly, this limitation can be eliminated with time. A more fundamental limitation for the present work is that the general configuration of the field may be known to a high degree of precision, but the regional anomalies conspire to give it a shape or spatial.functional relationship which is not easily designated mathematically, nor would a device utilizing this shape be easily instrumented or programmed. If only the main dipole portion of the field were used, the field could be easily described mathematically, but the results would only have good accuracy at very high altitudes, where the large regional anomalies had been essentially smoothed to an insignificant contribution. As a result of these present problems, the earth's magnetic field may be removed from the list of useable navigational aids for the problem at hand.

E. Other Properties

Although various other physical properties of the earth's environment may be of some application to specialized navigation problems, the present investigation narrows down to a consideration of three well known and established techniques, i.e. inertial navigation, stellar navigation, and navigation through use of man-made electromagnetic transmissions carrying navigational data of use to the vehicle. Since each of these techniques is used in part for the navigation system under consideration in this paper, they will not be discussed further at this point.

From the preceeding discussion it is seen that the earth's magnetic field, electrostatic field, atmospheric pressure, and temperature gradient are not useable as sources of navigational information with current equipment in the flight regime in which the proposed system is to operate.

CHAPTER 3

ESTIMATE OF THE REQUISITE SYSTEM CAPABILITIES

The flight trajectory over which the vehicle must operate is unusual when compared to conventional aircraft flights. This being the case, examination of the trajectory is necessary to determine what requirements it may place on the navigation system. A discussion of these requirements follows.

The over all system requirement in this case is to give the pilot and/or automatic control system enough information to allow the vehicle's guidance to a desired landing area. Although the control portion of the problem is not normally the navigation system's assignment, the navigation and control systems are interconnected for the current problem. This is due to a peculiarity of the re-entry trajectory, namely a very long distance of travel and a long time spent in re-entry. In the case of a ballistic missile entering the atmosphere at a sharp angle and plummeting to earth in essentially a straight line path, the drag (or ballistic coefficient) or the atmospheric density do not have to be known very closely in order to predict the point of impact fairly precisely. At the other extreme, for a vehicle re-entering at approximately zero angle with respect to the local horizontal and at a speed closely approaching local orbital velocity, the resultant trajectory is more like a slow spiral satellite decay path than a re-entry trajectory. It is this second type of re-entry path which is envisioned for the supply vehicle in order to limit heating and acceleration (see Appendix C).

Under the extended re-entry conditions assumed, a slight error in the estimate of the drag coefficient or the air density structure assumed could cause the range at which 90,000 feet altitude was obtained[*] to vary from the predicted value by a significant amount. A one percent variation would amount to about seventy miles of range. Such large magnitude errors would be wholly unacceptable, and it is at this point that a crucial assumption is made tieing the control system and the navigation system together. In order to achieve the desired level of guidance accuracy the navigation system is made a part of a flight path control system which operates during the re-entry phase, i.e. from 300,000 feet to 90,000 feet altitude. Prior to the time the supply vehicle leaves the earth, the best estimates of the vehicle's aerodynamic coefficients are made, and using an assumed atmospheric model a theoretical trajectory for re-entry is calculated. This trajectory must be close to the actual trajectory which would be followed by the vehicle in the absence of any flight path control system. The three identifying coordinates of a point on the theoretical flight path[**], for a number of uniformly spaced[***] points, are stored in a portion of the memory of the special purpose computer carried by the vehicle. A control variable is chosen from the two coordinate variables representing the vertical by choosing that coordinate which is expected to vary most rapidly with time during the re-entry. The computer is mechanized such that when it is given a particular value of the control variable it enters the stored trajectory and finds the two remaining coordinates associated with that particular point in the trajectory.

*Recall that 90,000 feet is the altitude at which a normal, gliding, controlled flight is assumed to start after the re-entry phase.

**For the system used in this thesis these would be latitude, longitude, and altitude.

***Uniformly spaced with respect to the control variable, i.e. either latitude or longitude whichever was expected to change most rapidly during the trajectory.

To see how the system is intended to work as a flight path controller, we take our supply vehicle up into orbit, start the return trip, and arrive at the re-entry phase. During the re-entry phase the navigation system will be continually supplying the three coordinates identifying the vehicle's position in space, neglecting navigational errors for the moment. Therefore, the measured value of the control variable is used to enter the computer, and the other measured coordinates compared to see if they coincide with the stored values. If they do not, errors are indicated by this comparison, and these signals can be used to force the vehicle to fly the preplanned re-entry trajectory. The resultant changes in acceleration and heating encountered must lie within tolerable limits, which they are sure to do if the original flight path estimate was accurate. This, or some comparable, type of flight path control is a necessity for this problem, in that it is the only way of keeping errors in aerodynamic estimates from becoming gross errors in the vehicle's landing point. The programmed trajectory need not be set into the computer before the vehicle takes off, for the information could be transferred to the supply vehicle from the ground during a tracking period.

It may be seen in Appendix C that a particular trajectory during re-entry was assumed for the purpose of analysis in this thesis. The L/D ratio was chosen as 1.0, and the re-entry angle as zero degrees. This is not a unique trajectory; it was chosen as a sample of the possible trajectories in that heating and accelerations experienced were not severe. Other trajectories could be used without affecting the results of this thesis. For further analysis it will be assumed that the control system is such that the trajectory actually flown from 300,000 feet to 90,000 feet essentially coincides with the programmed trajectory except for the navigation system errors. Thus, it becomes possible to state the required navigation system accuracy as if it were the entire flight path control system accuracy. It might be noted in passing that the control problem brought about in attempting to fly a given trajectory is by no means a trivial one. A change in angle of attack will accomplish both speed and height changes, but not one independently. It would seem desirable from a control standpoint to have an independent degree of control over velocity in the form of partially extended dive brakes which could be adjusted to give more or less drag, or some other control mechanism to change one variable independently. However, these are control system problems for any flight path control system, and as such will not be discussed further in this thesis.

One other control system problem which should be discussed is that of properly orienting the retro-thrust rockets so that the velocity increment applied to the vehicle is actually applied in a direction approximating the desired direction. Various errors or orbital perturbances arise from misalignment of this retro-thrust, and it was found that to keep these errors to a neglible value it is desirable to control the retro-thrust direction to within 1/4 degree of the correct value (see Appendix F). This sets one of the first specifications on the navigational system. It must be capable of giving reference directions to within 1/4 degree so that the control system can control the direction of retro-thrust. This means that if an inertial system were used alone (without extended monitoring) over a two day interval, it should not have a drift rate as high as .31 minutes of arc per hour; assuming perfect initial alignment. Such inertial systems are indeed possible today under very closely controlled laboratory conditions.[17] It is possible that this very low drift rate could more easily be attained in orbit than in the laboratory. This is due to the fact that drift rate is assumed to be due in large measure to mass unbalance effects, which would be entirely inoperative in a weightless orbital condition, However, it is not necessary to assume this small level of drift rate if we are willing to accept some monitoring instrumentation which keeps the inertial reference aligned over portions of the flight time. A star tracking system, for instance, would be capable of such monitoring action (see Appendix E). As it turns out, the system proposed does have a star tracking unit for other reasons besides this one, and it can be used for this job also. As yet, however, we have only established the 1/4 degree required reference accuracy, without establishing that either an inertial system or star tracker are essential. Indeed, the required accuracy might just as easily be obtained from a horizon scanner type mechanism, at least for the first retro-thrust, which would be applied at an altitude where such scanners are more effective than at lower altitudes.[18]

Besides knowing the direction reference for the two retro-thrust applications, it is extremely important that the magnitude of the retro-thrust be controlled to a high degree of precision. It will be assumed that the control system is capable of controlling the change in velocity due to the retro-thrust to within $\pm$ 1.0 foot/second of the desired value. Using this assumption and calculating the uncertainties in position due to velocity uncertainties it becomes evident that the magnitude of the velocity increment to be applied can be a precomputed quantity calculated at the ground tracking station and transferred to the vehicle during the last tracking interval (see Appendix F). This precomputation holds for the second retro-thrust also, and both quantities may be transferred to the vehicle at the same time.

The time to fire each retro-thrust must also be known at the supply vehicle. This sets the minor requirement that the supply vehicle have some sort of clock aboard to keep time as a reference, assuming that the firing of the first retro-thrust is not made during a period of radar (or radio) tracking (or communication) from the ground. With an accurate clock (see Appendix F) aboard, the problem of determining the firing time can become one of transmitting to the supply vehicle, during the last tracking period, the time to go until the first retro-thrust, and the time to wait until the first retro-thrust. A problem of resultant accuracy arises if this scheme is used per se, however. The errors committed at the first retro-thrust couple into the errors at the second, and the resultant uncertainty of position is large enough that the authors feel that another system requirement is set. This requirement is that the supply vehicle must get a navigational fix sometime during or just before the return flight in order to navigate properly to earth (see Appendix F). In essence this states that the initial knowledge of satellite position and motion is not good enough to allow accurate navigation to earth unless the trip is started immediately after a period of accurate ground tracking.

This requirement leads to the investigation of methods of supplying navigational fixes to the supply vehicle and of what happens to the information once it is obtained at the supply vehicle. It is impossible to consider this problem without presupposing the form that the vehicle's navigation system is going to take, and then examining methods of transferring information to that system. Before investigating this point, therefore, we shall look at the general problem of navigation and how it might be solved for our satellite supply vehicle, for up till now we have only eliminated certain navigational parameters without supplying workable substitutions.

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CHAPTER 4

SUPPLY VEHICLE NAVIGATION

The general problem of navigation in a three-dimensional space* involves first the identification of the three coordinate values which exist at a point in that space and the time derivatives of these coordinate values as the vehicle moves in that space, and second the utilization of this information for guidance to some other point. For practical navigation of surface vehicles, in the past the most general practice has been to assume altitude constant and thus reduce the navigation problem to two coordinates instead of three. Until recently, for airborne vehicles, the coordinate of altitude has been supplied independently in the form of a barometric measurement, and navigation done on the same basis as for surface vehicles.

The necessary navigational information can be gathered either from on-board measurements at the vehicle, or by electromagnetic transmission of signals and data to the vehicle, or by a combination of both. Other modes of information propogation may be utilized in special cases, such as sound in sonar systems, but only electromagnetic propagation need be considered in the case of the satellite supply mission. The electromagnetic transmissions may come to the vehicle in terms of natural phenomena such as light from the stars or reflected from the surface of the earth, or it may come in the form of man-made signals containing information of use to the craft.

*Neglect any relativity effects and apply only the concepts of Newtonian mechanics to describe space (for the problem at hand).

[redacted]

There are many schemes of navigation based solely on reception and/or generation of man-made electromagnetic signals with certain known properties which can be interpreted in terms of position and/or motion with respect to some reference such as the earth. Practical considerations such as power required and coverage of the earth's surface by a number of stations have led to the elimination of any of these solely man-made electromagnetic schemes. That is, it is considered impractical to build an entire network of ground tracking stations with sufficient coverage to give normal guidance throughout the return trajectory, etc. (see Appendix D).

Man-made electromagnetic transmissions, however, represent a very valuable assist to the general navigational problem, and can be utilized to advantage. In this light, a ground based tracking station for the satellite proper, if suitably oriented with respect to the desired landing area for the supply vehicle, could be made to track and supply control or command information to the supply vehicle in the terminal portion of the trajectory, i.e. the glide down from 90,000 feet. This is the conventional ILS or GCA type of aircraft control to landing, only on an extended range scale and possibly made automatic. This same ground station could transfer information to the supply vehicle while it was still in orbit during any period of tracking of the satellite, since satellite and supply vehicles are assumed coincident prior to the initiation of the supply vehicle return trip.

Natural electromagnetic radiations can be quite as useful for navigation as the man-made quantities. Celestial navigation is a practiced art (see Appendix E). There is, however, no practical way of measuring altitude by celestial means. Celestial navigation over the earth's surface requires an accurate knowledge of the local vertical direction or equivalently the local horizontal plane (see Appendix E). Upon examination we see that knowledge of the local altitude, the local vertical, earth rotational rate, time, initial position, and the ability to compare changes in local vertical direction, gives us all of the necessary information to calculate the vehicle's position by other than celestial means.

The fixed stars, however, are capable of providing us with one of the best inertial references known,* and as such can serve as a very valuable assist to inertial instrumentation. Use of star trackers as a navigational assist has been studied by several interested groups,[19] in which a gyro controlled inertial reference table is to be monitored by star tracking apparatus in order to eliminate uncertainties due to such long term causes as gyro drift, etc. Another very important use of such a monitoring system may be seen in that it can furnish a reference direction within the vehicle with respect to which the direction of the local vertical, as calculated on the ground by the tracking station, may be specified. To illustrate how this may be accomplished, note that a ground tracking station during a period of tracking may establish the position of the supply vehicle over the surface of the earth to a high degree of accuracy.[20] The position of an inertial platform within the vehicle is assumed to be monitored through tracking two stars, and it is also assumed that the ground tracking station has knowledge of which stars are being tracked. The ground station may then calculate the angles that the local vertical at the supply vehicle should make with the stars being tracked. This information may be transmitted to the supply vehicle, and the direction of the local vertical physically established, or equivalently, established in a computer coordinate system in which the navigational computations are made. Since contemporary designs of stellar monitored inertial systems claim tracking accuracies of better than fifteen seconds of arc,[21] this becomes a very precise method of establishing a vertical direction at the vehicle.

*According to the theory of relativity, all inertial systems are equivalent; therefore the "best" here is with reference to the ease of observations or calculations rather than any privileged position mechanically.

Another use of natural electromagnetic radiation would be the observation of the earth, instead of the stars, by a horizon scanning device. Such devices are being proposed at the present time, and accuracies of indication of the vertical are claimed up to .1 degree (or six minutes) of arc for altitudes of observation of approximately 300 miles.[22] The performance of the horizon scanner type device suffers as the altitude of observation is decreased from 300 miles, and at 300,000 feet it could hardly be expected to yield the same accuracy of vertical indication as at 300 miles.[23] Below 300,000 feet the dynamics of re-entry would combine with the dynamics of the horizon scanner, and the inherent capability of the horizon scanner to indicate the vertical at these altitudes would suffer sufficiently to eliminate use of the horizon scanner as more than a navigational aid for high altitude observations. However, if properly coupled to a gyro monitored inertial platform, the horizon scanner would appear to offer a means of monitoring the inertial platform through on-board observations. Two serious problems arise in the utilization of the horizon scanner in this way, however. The first problem is the degradation of performance of the horizon scanner as lower altitudes are reached. It might be argued that the horizon scanner would only monitor the inertial platform prior to starting the descent, and then the platform de-coupled from the scanner and navigation done utilizing only the inertial navigation system. The initial accuracies obtainable from the horizon scanner are not acceptable for this use (see Appendix F). That this is true will be seen under the discussion of the all-inertial navigation system which follows.

It remains to investigate navigation based upon on-board measurements not involving electromagnetic transmissions. As seen in Chapter 2, a number of environmental properties which might have been measured were eliminated from consideration as practical navigation aids for this problem. The remaining measurements of interest are those made by an all-inertial type navigation system containing a gyro monitored inertial reference.[24] This type of system utilizing Schuler tuning concepts for accurate operation[25] has been discussed in some detail elsewhere,[26] and this discussion will not be repeated here except to note difficulties of such a system which make it unacceptable by itself for the navigation of the supply vehicle to earth. Russell[27] shows that such a system, if used to calculate height variations, is divergent in the height coordinate, in that an error in height gives rise to a further error, etc. The calculation done by Russell was based upon a linear approximation to the gravity about an inertial point near the earth, and as such can not be applied directly to estimate the divergence in height information of the present system which flies 7,000 miles in the re-entry trajectory. Although the estimation of height divergence in this nonlinear problem is more involved than the simple linear analysis done by Russell, an estimate of the maximum altitude errors is possible through linear approximations of the assumed trajectory, and it is seen that this divergence of altitude information sets the most stringent requirement on the system yet encountered (see Appendix F). Accurate altitude information is necessary during the re-entry phase, for without it, the range errors when 90,000 feet altitude is reached become unacceptable.

Working back from the desired accuracy of navigation to system requirements forced by this height information divergence, we need to examine the re-entry trajectory. Since the re-entry trajectory from 300,000 feet down to 90,000 feet lasts for approximately 1,800 seconds, in order for an all-inertial system to be acceptable for this phase, the height information must diverge so slowly that it is not greatly in error at the end of this time. The rate of divergence is determined by the initial height error, the initial velocity error, and the error in identifying the vertical (when it is obtained by integrating accelerations due to aerodynamic forces). Examination of the re-entry phase shows that for this height error at 90,000 feet to be small enough to be acceptable, the vertical information must be initially accurate to approximately 6 seconds of arc, initial height error less than 40 feet, platform drift rate less than .24 minutes of arc per hour, and initial velocity error less than .1 foot/second (see Appendix F). These requirements are completely unrealistic for practical equipment to achieve using the best navigational information available from any conceivable source just prior to re-entry. These requirements eliminate the all-inertial system from consideration as a navigation scheme for the supply vehicle.

After having eliminated each of the considered systems in turn, if used by themselves, a hybrid system must be chosen which will accomplish the navigational job. More than one such system exists, but the authors have picked a single system which they feel will meet the navigational requirements. The system proposed will be described in the next chapter.

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CHAPTER 5

THE NAVIGATION SYSTEM

A. Discussion

Of the various navigational schemes investigated in this thesis (electromagnetic, magnetic, stellar, etc.) an inertial system, using gyroscopes as an inertial reference and accelerometers to indicate accelerations due to air loads on the vehicle, is considered to be the most desirable during the re-entry phase of the flight. During orbital descent, prior to re-entry, there exists no requirement for more accurate navigational information than may be predicted on the basis of elapsed time since firing of the initial retro-thrust. It follows that the system best suited to operation during the re-entry phase of flight is the system which should be used.

The desirability of an inertial system during this phase of flight is based on several factors. First, is the adaptability of such a system to utilization for any satellite orbit and landing point without necessitating the construction of an extensive ground control net. Secondly, it is not subject to external interference in the form of ionization of the atmosphere in the immediate vicinity of the vehicle as, electromagnetic systems may be, nor is it subject to bending of light rays due to shock waves and radiated heat, which could incapicitate a stellar navigation scheme.

The inertial system to be used consists of three single-degree of-freedom gyros whose input axes are mutually perpendicular, and three single-degree-of-freedom, floated, rotational, integrating accelerometers whose input axes are also mutually perpendicular; the gyros and accelerometers being fixed with respect to one

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another.* Were an attempt made to track the local vertical with one axis of the platform, the relatively high precessional rate required of the platform would cause the accuracy of the system to deteriorate, since the inertia of the platform would enter into the performance equation of the system. The relatively large horizontal velocity of the vehicle on re-entering the atmosphere dictates that, for best operation, the inertial "platform" should be non-rotational with respect to inertial space (as compared to a system in which one axis of the platform is made to track the local vertical direction).

In considering the type of computational system to be used in conjunction with the inertial platform, there are several considerations. First, storage of the precomputed re-entry trajectory, which the vehicle must follow, in digital form in the vehicle (see Appendix C) requires that a certain amount of digital memory and a means of access to it be present in the vehicle. Secondly, comparison of the accuracy requirements given in Appendix F with the available accuracies in analogue computer components would show that, at the present state of the art, analogue components of the highest quality would suffice but with little margin of safety.** These considerations would tend to indicate that use of a special purpose digital computer to perform the required computations. Use of a digital computer is further enhanced by the rapid solution time attainable in modern computers which, when coupled with the use of an inertially fixed gyro-accelerometer platform, makes the system response time practically instantaneous.

*The reader is referred to Sherman M. Fairchild Publication Fund Paper No. FF-16 by Walter Wrigley, Roger B. Woodbury, and John Hovorka for a discussion of this type of system.

**Refer to manufacturers listings. Listing in detail of the best current accuracies available in all applicable components is beyond the scope of this thesis.

Considerations thus far have led to the selection of an inertially fixed gyro and accelerometer platform whose acceleration outputs are processed in a digital computer to give the desired navigational information. Appendix F shows that initial alignment of the system, prior to re-entry, is required to a high degree of accuracy. Two means have been considered thus far in the thesis for obtaining an indication of the initial vertical direction at re-entry. These were the use of a horizon scanning device, and the use of a star tracker working the classic navigational problem in reverse to give the vertical direction rather than position. It was found that the required accuracy could not be attained using horizon scanner devices in current development. In the classical solution of the stellar navigation problem, knowledge of the direction of the local vertical (horizon), and the angles of the stars with respect to it, are used to determine position. In the system under discussion here, knowledge of position is used to determine the direction angles of the local vertical with respect to the stars. Three stars are required to give a unique solution to the problem but, with proper selection of the stars to be tracked, the possibility of selecting the wrong vertical indication of the two possible, when tracking only two stars, can be eliminated. Thus, tracking of two stars with an automatic tracker, plus knowledge of position, will suffice to give an accurate indication of the vertical direction. The use of a star tracker for this purpose has the additional advantage that an accurate directional reference in the horizontal is also obtained.

To obtain a sufficiently accurate position indication at the time of starting the inertial system, it was shown in Chapter 4 that this information must be obtained from an external source. The proposed method of obtaining this information is as follows:

(a) A conventional aircraft carrying a tracking radar is to remain in the vicinity of the perigee of the orbital descent. The radar must be capable of giving range and azimuth of the supply vehicle with respect to the aircraft.

(b) The aircraft must have a navigational system capable of giving its longitude and latitude and of indicating the vertical direction and the north direction, in order that range and azimuth of the supply vehicle from the aircraft can be resolved into relative position, which will suffice to determine the latitude and longitude of the supply vehicle. The system which fills all of these needs is an inertial navigation system. This system is conventional and will not be discussed in this thesis.

(c) The supply vehicle will contain a radar "beacon", i.e. radar receiver-transmitter, which will assist the radar aircraft in tracking and in acquisition (if the transmitter in the supply vehicle is keyed internally prior to receipt of keying signals from the aircraft radar) and also serve as a data link from the aircraft to the supply vehicle.

(d) From knowledge of the predicted velocity vector of the supply vehicle and of a pre-computed point for turning on the computer in the vehicle, a suitable computer in the aircraft can use tracking information to predict the closest approach of the supply vehicle to this point. The coordinates of this closest approach must be encoded in the aircraft and transmitted to the supply vehicle, through the data link, where they become the initial values of latitude and longitude for the vehicle digital computer.

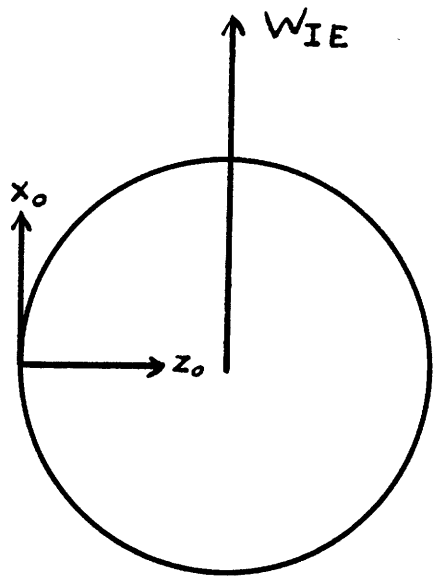
It is felt that using existing equipment, or adaptations thereof, the aircraft can determine the coordinates of the point at which the vehicles computer is started to the nearest mile, giving at most a 1' of arc error in initial alignment from this cause, but cannot determine the supply vehicle velocity to an accuracy better than that to which it can be predicted. This accuracy is unacceptable if the divergence of height computations discussed in Appendix F is to remain within reasonable bounds through 1,800 seconds of flight. Rather than consider extensive ground electronic installations as a means of overcoming this difficulty an alternative is proposed. The near-orbital velocity of the vehicle during the early portions of the re-entry cause its flight path to be practically horizontal (see Appendix C) with a resulting slow rate of change of height. A relatively large percentage error in prediction of the magnitude of the velocity at the beginning of re-entry represents, in the early stages of flight, only a corresponding percentage error in prediction of height rate since the height rate is simply the vertical component of the vehicle velocity. When, however an error in prediction of the direction of the velocity is made, the errors in actual height rate as compared to predicted height rate become serious. Indeed, the height rate error, due to this cause, may be considered as the total vehicle velocity times the sine of the error in prediction of the angle of re-entry. For the large (approx. 25,000 fps) vehicle velocities at re-entry this error becomes greater than the predicted height rate for even small errors in prediction of the angle. As a result, use of a predicted height for navigation is subject to large errors and is out of the question. The remaining alternative method of obtaining height information which will retain the flexibility of an all inertial system and yet not be subject to atmospheric uncertainties or divergence in computation is a radar height finder (see Appendix D).

Height information need only be measured after re-entry into the atmosphere. Therefore, the height finder radar need only operate up to heights of 57 miles (300,000 ft.). Propagation through the ionized atmosphere surrounding the vehicle is deemed possible (see Appendix D). The power requirements are not as large as those for a conventional radar, which would track an aircraft at these ranges, due to the very large target (the earth). The axis of the transmitting antenna can be maintained vertical through use of the inertial system.

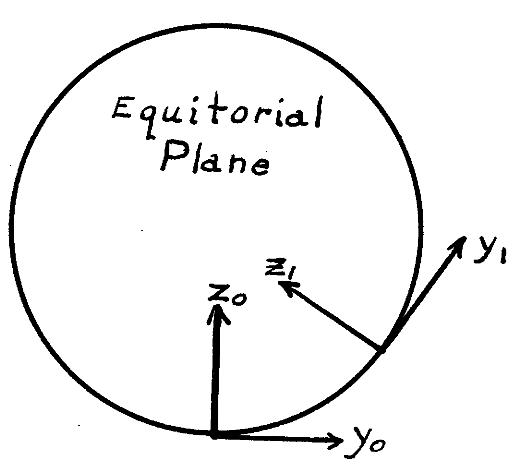
B. Design of the Navigation System

1. Derivation of the equations to be solved by the navigation computer:

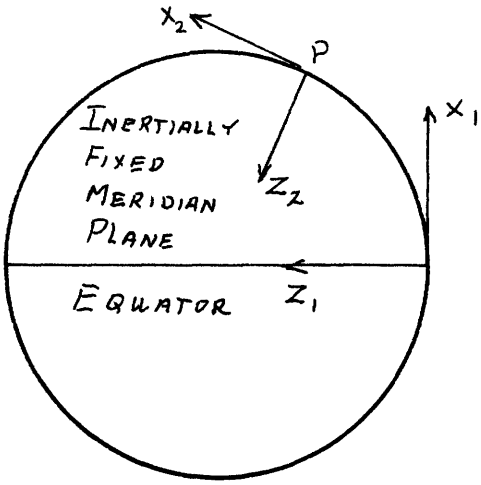
Define a set of inertially non-rotating orthogonal axes x_o, y_o, and z_o forming a right handed system with x_o parallel to the earth's rotational axis and with origin at some reference longitude, on the surface of a non-rotating earth, at the equator:



Define axes x_1, y_1, and z_1 such that the origin is at the intersection of the meridian, of an arbitrary point P, on the non-rotating earth's surface and the equator, with x_1 parallel to x_o and z_1 directed inward toward the center of the earth. This set of axes is also inertially non-rotating.



Define a third set of coordinates x_2 , y_2 , and z_2 with origin at P, z_2 directed inward toward the center of the earth, and y_2 parallel to y_1.



Since the fictional earth used to define these coordinate systems is inertially non-rotating, the latitude and longitude angles involved are celestial latitude (Lat_c) and celestial longitude (Lon_c).

Letting the letter V with appropriate coordinate subscripts stand for the component of velocity, with respect to inertial space, of the vehicle, lying in the direction of the indicated axis we get.

$$\frac{d\,Lat_c}{dt} = \frac{V_{x_2}}{R_E + h}$$

Where R_E is the radius of the earth and h is height above the earth's surface.

$$\frac{d\,Lon_c}{dt} = \frac{V_{y_2}}{(R_E + h)\cos Lat_c}$$

or

$$Lat_c = Lat = Lat_o + \int_o^t \frac{V_{x_2}}{R_E + h}\,dt \qquad (5\text{-}1)$$

$$Lon_c = Lon_{c_o} + \int_o^t \frac{V_{y_z}}{(R_E + h)\cos Lat}\,dt \qquad (5\text{-}2)$$

since $Lat_c = Lat$

Where Lat is latitude on the earth (rotating).

$$V_{x_2} = V_{x_1} \cos Lat + V_{z_1} \sin Lat$$

$$V_{y_2} = V_{y_1}$$

$$V_{z_2} = V_{z_1} \cos Lat - V_{x_1} \sin Lat$$

$$V_{x_1} = V_{x_o}$$

$$V_{y_1} = V_{y_o} \cos Lon_c + V_{z_o} \sin Lon_c$$

$$V_{z_1} = V_{z_o} \cos Lon_c - V_{y_o} \sin Lon_c$$

Giving:

$$V_{x_2} = V_{x_o} \cos Lat + V_{z_o} \cos Lon_c \sin Lat - V_{y_o} \sin Lon_c \sin Lat. \tag{5-3}$$

$$V_{y_2} = V_{y_o} \cos Lon_c + V_{z_o} \sin Lon_c \tag{5-4}$$

$$V_{z_2} = V_{z_o} \cos Lon_c \cos Lat - V_{y_o} \sin Lon_c \cos Lat - V_{x_o} \sin Lat. \tag{5-5}$$

Using vector notation:

$$\frac{d\overline{V}}{dt} = \overline{a} + \overline{G}$$

Where $\overline{a}$ is acceleration with respect to inertial space due to non-field forces, $\overline{G}$ is acceleration with respect to inertial space due to field forces. Note that the vector sum of the accelerometer acceleration indications will be $\overline{a}$.

$$V_{x_o}(t) = V_{x_o}(o) + \int_o^t a_{x_o}\, dt + \int_o^t G_{x_o}\, dt \qquad (5-6)$$

$$V_{y_o}(t) = V_{y_o}(o) + \int_o^t a_{y_o}\, dt + \int_o^t G_{y_o}\, dt \qquad (5-7)$$

$$V_{z_o}(t) = V_{z_o}(o) + \int_o^t a_{z_o}\, dt + \int_o^t G_{z_o}\, dt \qquad (5-8)$$

but

$$\overline{G} = \overline{1}_{z_2} G$$

where $G = \dfrac{E}{(R_E + h)^2}$ where E is the product of the earth's mass and the universal gravitational constant. An inverse square gravitational attraction law is assumed.

$$G_{x_1} = -G \sin Lat$$

$$G_{y_1} = 0$$

$$G_{z_1} = G \cos Lat$$

giving

$$G_{x_o} = -G \sin Lat \qquad (5-9)$$

$$G_{y_o} = -G \cos Lat \sin Lon_c \qquad (5-10)$$

$$G_{z_o} = G \cos Lat \cos Lon_c \qquad (5-11)$$

note that

$$Lon(t) = Lon_c - W_{IE}\, t \qquad (5-12)$$

Where Lon is earth longitude and W_{IE} is the angular velocity of the earth with respect to inertial space.

In order to furnish the pilot with latitude, longitude, and height indications it is necessary that the computer solve equations (5-1) through (5-12). Note here that with the use of integrating accelerometers, the integration of the terms

$$\int_{o}^{t} a_n \, dt$$

appearing in equations (5-6), (5-7), and (5-8) is already performed.

With height information obtained from a radar height finder, the solution of these equations gives all requisite navigational information.

C. Transitional Sequences

The proposed navigational system must operate in two widely differing flight regimes and as a result its functions differ from time to time during the course of the flight. In particular, during the orbital descent from the satellite orbit to atmospheric re-entry altitude the pilot of the vehicle has no requirement for accurate knowledge of his position or altitude since no control is available to him. He can, however, obtain a good indication of both his position and altitude simply through knowledge of the time interval which has elapsed since firing of his initial retro-thrust rocket if this is found advisable. It was pointed out previously that the orbital velocity of the satellite is very well known and also that the velocity increment given the supply vehicle during the initial retro-thrust period represents only a small portion of total velocity vector of the vehicle. Therefore, errors in the velocity increment do not appreciably deteriorate the accuracy of the knowledge of the vehicle's velocity vector. Thus, using the equations of orbital motion and the desired velocity of the vehicle immediately following application of the initial retro-thrust, its position and height at any time following, until application of the second retro-thrust, can be accurately computed in advance and tabulated. The ability to accurately predict any navigational information which is desired during the orbital descent relieves the navigation system from any requirements during this interval except for indication of the orientation of the velocity vector (total) during the firing of the initial retro-thrust, in order that it may be properly directed. This latter requirement necessitates that, some time prior to arrival at the precomputed position at which the first retro-rocket is to be fired, the pilot must acquire two designated stars (which must be visible at the firing point) with the star trackers (using the acquisition system provided). It should be noted here that the pilot is given no assistance in acquisition by the navigation system and must himself be capable of recognizing the desired stars from their positions in the stellar constellations. The orientation of the desired computer inertial reference frame with respect to these two stars can be precomputed. The angles so found will remain fixed since the stars themselves are inertial. The method of operation of the navigation system at this time is that the star trackers, in tracking the stars, furnish the reference orientation, the inertial system serving only to isolate the star trackers from base motion. Since the orbital velocity vector is well known as a function of position, the angles describing its predicted orientation with respect to the two stars at the desired firing position furnish the reference orientation required for firing the first retro-rocket.

During the orbital descent, at some convenient time, there must be a transition period during which the pilot must acquire, with the two star trackers, two new stars which will be visible at the desired firing point for the second retro-thrust. This transition is easily made, since the navigation system is not furnishing any data during this phase. It is only necessary for the pilot to cease tracking the previously used stars and acquire the new ones, leaving theinertial system free in the interim. Transients, occuring when again coupling the inertial system to the star trackers, will be very small; since no change in orientation of the inertial platform is required.

Prior to arrival at the pre-computed point for starting the inertial navigation system computer, the pilot must set into the computer the precomputed velocity components (in the computer inertial reference frame) associated with that point.

When track is acquired by the radar airplane, the computer in the aircraft can compute the point of closest approach of the supply vehicle to the desired computer starting point on the basis of the predicted velocity vector of the supply vehicle. The aircraft computer can also find the angles which the vertical will make with respect to the two stars at this closest approach. The equivalent of these angles must be transmitted to the supply vehicle as must the latitude and longitude of the starting point. Due to the short tracking time interval available for transmission of this data, the setting of this data into the computer must be automatic.

When this data is set into the computer it is ready to begin computation. At the computer starting point, the aircraft must transmit a starting pulse to put the inertial navigation system into operation.

The aircraft computer can also compute the point of closest approach of the supply vehicle to the precomputed point at which the initial velocity vector for re-entry into the atmosphere should be established (to conform with the predicted trajectory). At the time, equal to the calibrated burning time of the rocket (for the magnitude of the precomputed velocity increment to be applied), before arrival of the supply vehicle at the point of closest approach to the desired firing point, the radar aircraft must transmit a firing pulse to the supply vehicle.

The only remaining navigational transition is to conventional navigational aids on arriving near the landing field. This transition requires no setting of new initial values and may take place when the pilot sees that accelerations have dropped to an acceptable level and the signals from the navigational aids are being received.

D. Source of Externally Determined Data to be Furnished The Navigation System

Of some interest in understanding the manner in which the envisioned navigational system functions, is a sequential examination of the various operations which must be performed by the pilot or by some external system in order to furnish to the navigation system the necessary initial values of position, identification of stars, etc, as may be required. The sequence is not unique but may be considered representative.

Since this navigation system is to operate only during the return to the earth's surface, there are no requirements placed on it until the return is initiated. The first requirement is that it furnish indication of the velocity vector for orientational control during the firing of the initial retro-thrust. Prior to this time, the pilot must be furnished with the orientation angles describing the direction of the velocity vector, at the desired firing point, with respect to two stars. Some time prior to firing of the initial retro-thrust the two stars must be acquired by the star tracking system and the inertial system slaved to the star tracker. The interval of time need only be sufficiently long to allow the system to stabilize and the rocket to be oriented, but may be as long as desired.

There remains only to determine the instant at which the rocket is to be fired. Since the navigation system furnishes no positional information in this mode of operation, the firing instant must be determined as the end of a pre-computed time interval synchronized during the time of last track of the vehicle by a ground installation. For purposes of orienting the rocket for firing of the initial retro-thrust, the quantities which must be computed by a ground installation are:

a. Position at which the retro-thrust is to be fired.

b. The velocity increment which is to be given by the retro-thrust.

c. Angles describing the direction of travel at the firing point with respect to two stars.

d. The time interval between a synchronizing signal and the time at which retro-thrust is to be initiated.

It has been shown previously in this thesis that knowledge of position and height during the orbital flight following initial retro-thrust and preceeding the second retro-thrust can be adequately predicted. It is not envisioned that the pilot will have any requirement to know positional coordinates or height during this interval. If, however, this is found desirable, then a table of position and height versus time, or its electronic equivalent, must be precomputed and furnished to the pilot.

During this orbital period, at any convenient time, the pilot must ransfer the two star trackers to stars which will be visible at the time of the second retro-thrust. To accomplish this he must be furnished the angles describing the orientation of the inertial reference frame with respect to the two new stars.

Prior to starting the inertial guidance system computer the requisite initial conditions must be set into the computer. These quantities are:

a. Initial velocity with respect to inertial space. This must be expressed in component form in the inertial reference frame used. The initial velocity to be used in the computer is the predicted value at the preselected starting point for the computer. The velocity direction to be used in resolving into components is obtained from the precomputed azimuth direction, and the angle with respect to the horizontal at this point.

b. Orientation angles describing the direction of the vertical relative to the stars at the computer starting point (in the inertial reference frame). These angles must be computed by the tracking aircraft using knowledge of the predicted velocity components and the desired starting point, plus measured position, to give a predicted best starting point.

At the computer starting point, a starting signal must be sent to the supply vehicle by the tracking aircraft.

At the calibrated firing duration time of the retro-thrust rocket (for the velocity decrement to be applied) prior to arrival of the supply vehicle at the firing point, a firing signal must be sent to the vehicle by the tracking aircraft.

In resume: Prior to starting of the computer and subsequent firing of the second retro-thrust the pilot must know:

a. Angles describing the orientation of two stars with respect to the desired inertial reference frame.

b. Angles describing the computer starting point vertical in the inertial frame.

c. Velocity at computer starting time resolved into components in the inertial reference frame.

The supply vehicle must receive, from the tracking aircraft, signals to start the computer and to fire the second retro-thrust.

Prior to firing the second retro-thrust the pilot must know the pre-computed velocity increment to be given, with respect to inertial space.

The tracking aircraft must know the predicted velocity components of the supply vehicle at firing starting point in order to compute the best position for sending the firing signal for the second retro-thrust.

E. Functional Diagrams of the Proposed System

If the different modes of operation of the navigation system are considered chronologically as they occur during the flight, the first mode is one in which the system furnishes an indication of the direction of the satellite velocity vector with respect to two preselected stars.

The second mode of operation of the navigation system occurs when initial alignment of the inertial system is being obtained during track by the airborne radar.

The third mode of operation of the system is after the inertial system becomes operational and radar height information is being obtained.

List of Symbols: Mode I

W_{IV}	Angular velocity of the vehicle with respect to inertial space.
$(C)A_{st}$	Angular tracking error of star tracker.
$A_{[R, ST]}$	Angle between the star tracker and an arbitrary reference in the vehicle.
$A_{[R, Ro]}$	Angle between the rocket axis and the same arbitrary reference as for $A_{[R, st]}$.
$A_{[ST, Ro]}$	Angle between the star tracker axis and the rocket axis.

Mode II

$A_{[R, ST]}$	Angle between the space integrator and an arbitrary reference.
$A_{[R, ST]}$	Angle between the star tracker and the same arbitrary reference as for $A_{[R, SI]}$.
$A_{[RF, S]}$	Desired angle between the reference computation frame and the stars.

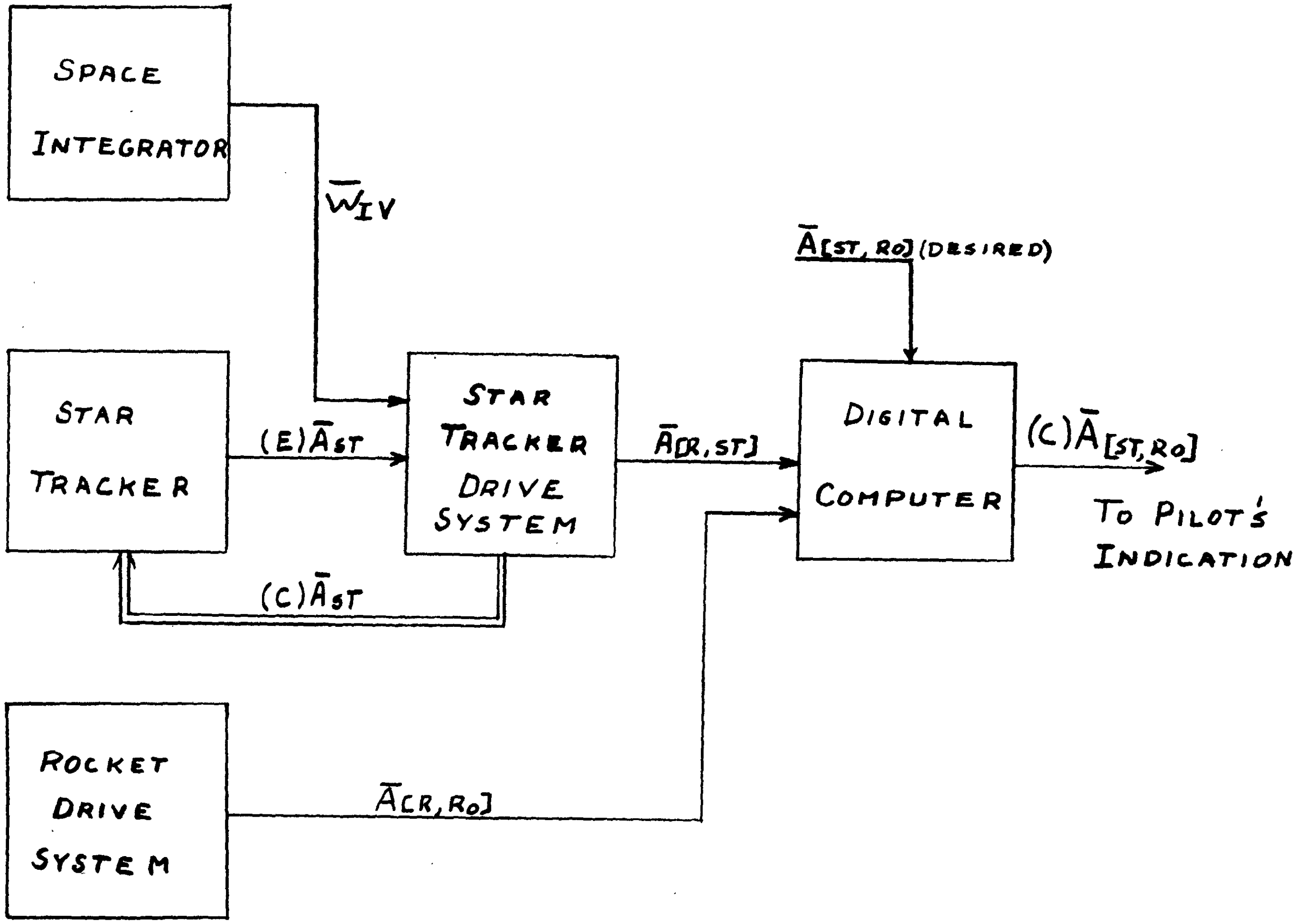


FIGURE 5-1: SYSTEM FUNCTIONAL DIAGRAM; MODE I

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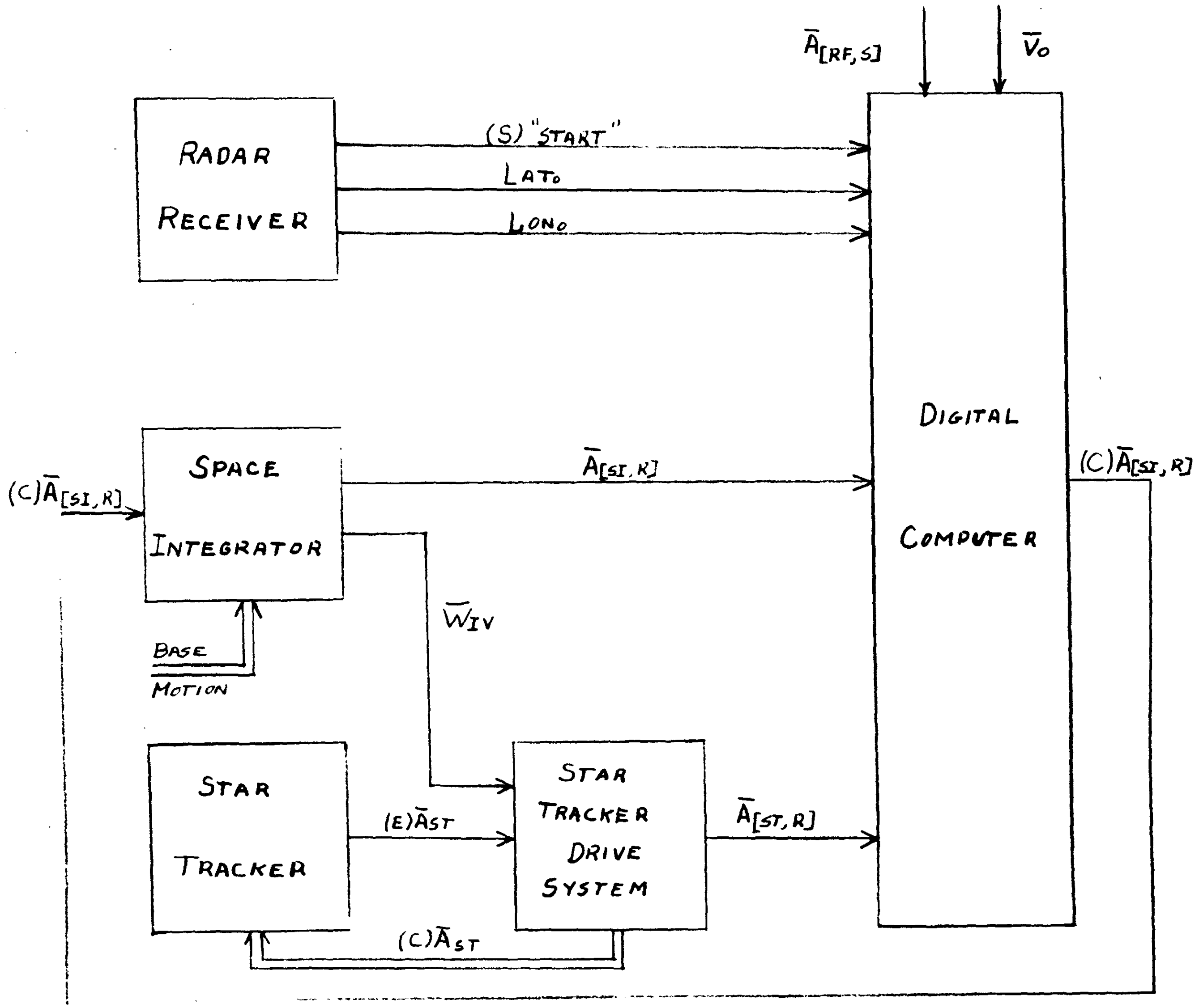

FIGURE 5-2: SYSTEM FUNCTIONAL DIAGRAM ; MODE II

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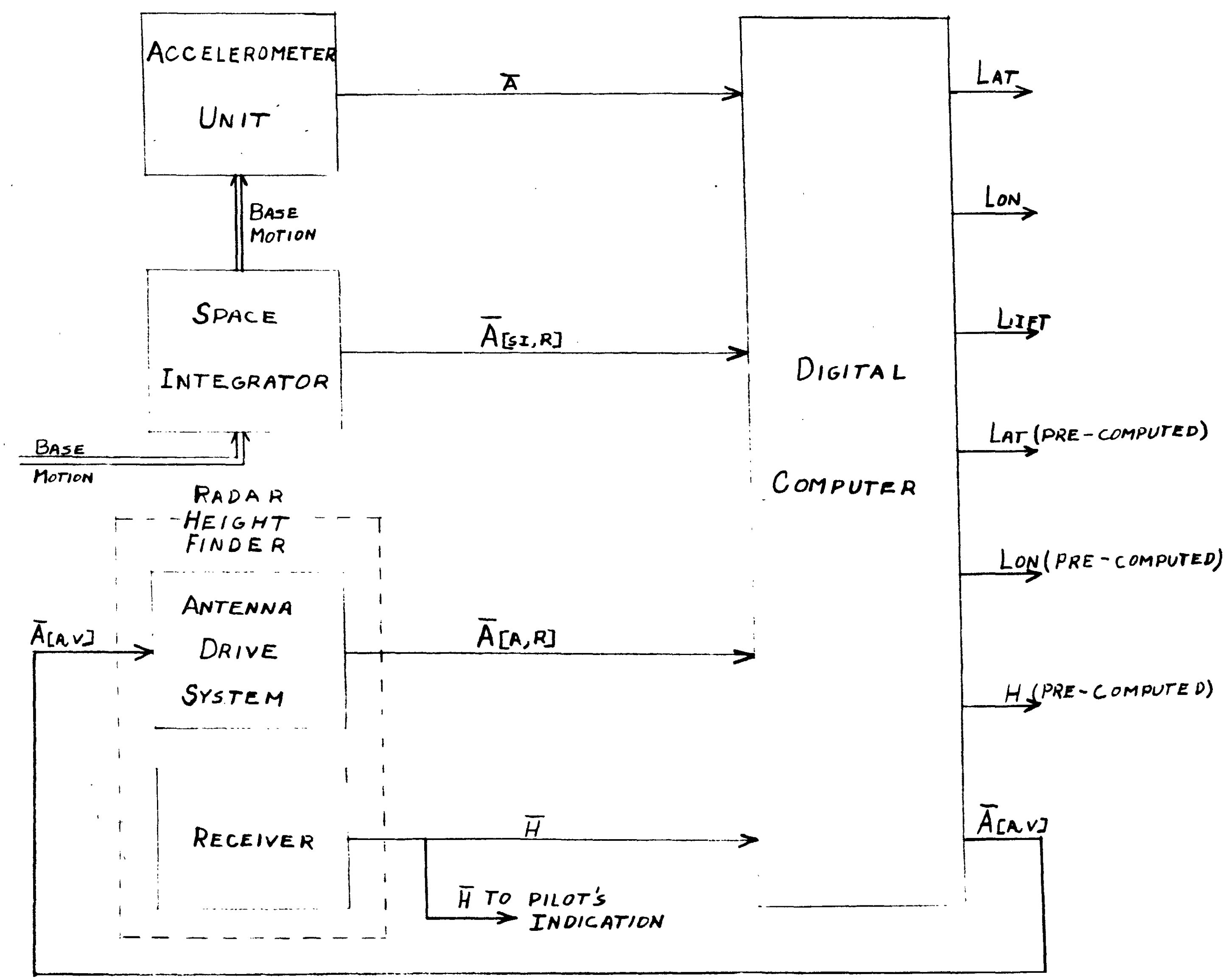

FIGURE 5-3: SYSTEM FUNCTIONAL DIAGRAM; MODE III

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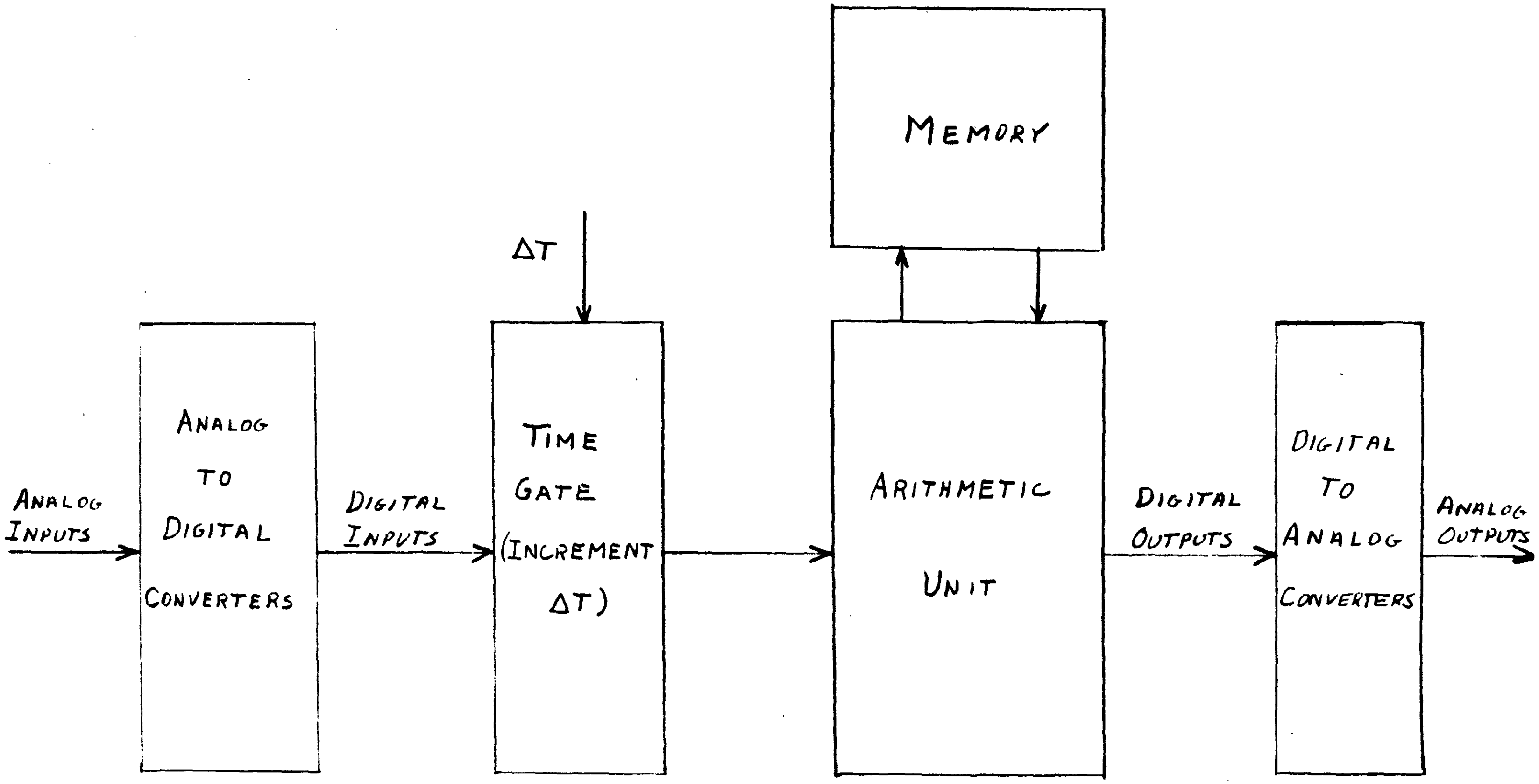

FIGURE 5-4: NAVIGATION SYSTEM DIGITAL COMPUTER

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Mode III

$A_{[R, SI]}$ Angle between the space integrator and an arbitrary reference in the vehicle.

$A_{[R, A]}$ Angle between the radar antenna axis and the same arbitrary reference as used for $A_{[R, SI]}$.

$A_{[A, V]}$ Angle between the antenna axis and the indicated vertical as found in the computer from $A_{[R, SI]}$ and $A_{[R, A]}$.

Not shown is the acquisitional device for the star tracker. It consists of a mechanical sight much like a pistol sight, pivoted in front of the pilot. Signal pick-offs on the axes of this sight are used to drive the automatic star trackers, one at a time, to coincide with the sighting axis of the acquisition device. The star trackers can then be switched to their automatic tracking mode.

The presumption that the system designed in this thesis should be capable of guiding the pilot to a landing at a specified point was chosen as the most demanding of various possibilities. Relaxation of this requirment in allowing landing anywhere within a specified area (as a desert, lake, or ocean) allows a corresponding relaxation in the accuracy requirements on the system. If the area allowed is sufficiently large, the relatively simpler method of obtaining a vertical direction indication through use of a horizon scanner, will suffice instead of the tracking radar - star tracker used. In addition, a radar fix can be done without obviating the need for a tracking aircraft and radar beacon in the vehicle. Although this thesis does not specifically discuss this navigation scheme, the method of approach to the requisite specifications remains unchanged and extension of the data given in Appendix F to this case is simple.

APPENDIX A

ADAPTATION OF KEPLER'S LAWS OF ORBITAL MOTION

The Keplerian equations of orbital motion may be found derived in most standard graduate physics tests.[28] The notation used varies greatly from one author to another; therefore, insofar as convenient, this thesis will adopt the system of nomenclature used in a study by the G. L. Martin Company of Baltimore, Md.[29] The study done by the Martin Company presents a far reaching analysis of many of the problems to be met in the design of satellite vehicle guidance systems, but the results of this study were largely not useable per se for this thesis, consequently certain useful expressions were derived by the authors and are presented in this appendix. No attempt will be made to reference most of the equations to other sources, for they may be easily derived by anyone desiring to do so. Only the elliptical equations of motion will be considered in this thesis, there being no need to consider the hyperbolic and parabolic cases.

A-1. List of Symbols

F	force of attraction of the earth on an orbital vehicle.
m	mass of the orbital vehicle.
M_e	mass of the earth.
r	radius from center of attraction to the orbital vehicle.
θ	angle the radius makes with a reference radius in the plane of motion, measured (+) in the direction of motion, measured from the radius to apogee as a

	reference radius.
h	altitude above the surface of the assumed spherical earth (also angular momentum).
R_e	radius of the earth (assumed spherical). 2.09029×10^7 feet (see Martin Co. Report).
r	$R_e + h$.
T	orbital period.
μ	gravitational constant $= 1.408142 \times 10^6 \ \frac{ft^3}{sec^2}$
A	a constant.
B	a constant.
C	a constant.
τ	angle of misalignment between velocity vector and retro-thrust direction.
γ_d	the angle deflection of the direction of motion due to a retro-thrust misalignment.
γ	flight path elevation angle (i.e. above the local horizontal).
e	eccentricity.
h	angular momentum of the vehicle divided by the mass (also used for altitude symbol, for there is little chance of confusion here).
V	velocity
V_c	circular satellite velocity at the altitude in question.
V_a & V_p	velocities at apogee and perigee, respectively.
a	semi-major diameter of an orbit.
b	semi-minor diameter of an orbit.

ΔV	increment of velocity change due to retro-thrust application, sometimes called the characteristic velocity of a maneuver.
h_a	apogee height.
h_p	perigee height.
ψ	angle of misalignment of the indicated vertical with the true vertical, in the plane of the velocity vector and the true vertical, measured from the true vertical to the indicated vertical, considered (+) clockwise.
s	(as a subscript) conditions at the satellite or in the initial orbital condition.

A-2. Equations of Motion

A-2a. $$\frac{1}{r} = \frac{\mu}{h^2} - A \cos \theta$$

A-2b. $$h = r V \cos \gamma = \text{constant}$$

A-2c. $$e = \frac{h^2 A}{\mu} = \frac{h_a - h_p}{2a} = \frac{h_a - h_p}{2 R_e + h_a + h_p}$$

A-2d. $$r = \frac{a(1 - e^2)}{1 - e \cos \theta}$$

A-2e. $$V = \sqrt{\mu} \sqrt{\frac{2}{r} - \frac{1}{a}} = \sqrt{\frac{\mu}{a}} \sqrt{\frac{2a - r}{r}} = \sqrt{\frac{\mu}{r}} \sqrt{\frac{2a - r}{a}}$$

A-2f. $$V_c = \sqrt{\frac{\mu}{r_c}}$$

Energy Equation K.E. + P.E. = Constant

A-2g. $$\frac{-\mu m}{2a} = \frac{-\mu m}{r} + \frac{V^2 m}{r} = \text{constant}$$

A-2h. $a = \dfrac{e}{A\,(1 - e^2)} = \dfrac{h^2}{\mu\,(1 - e^2)}$

A-2i. $A = \dfrac{e}{a\,(1 - e^2)}$

A-2j. $h = \sqrt{\mu\; a\,(1 - e^2)} = r\,V \cos\gamma$

A-2k. $\cos\gamma = \sqrt{\dfrac{\mu}{r}}\;\sqrt{\dfrac{2\,(1 - e^2)}{\dfrac{2a - r}{a}}}$

A-2l. $\tan\gamma = \dfrac{-\,e\sin\theta}{1 - e\cos\theta}$

A-2m. $\gamma = \dfrac{-\,e\sin\theta}{1 - e\cos\theta}$ for small ecc. i.e. $e < .1$

A-2n. $\cos\gamma_o = \dfrac{V_p\, r_p \cos\gamma_p}{V_o\, r_o}$

A-2o. $V_a = \sqrt{\dfrac{\mu}{a}\;\dfrac{(1 - e)}{(1 + e)}}$

A-2p. $V_p = \sqrt{\dfrac{\mu}{a}\;\dfrac{(1 + e)}{(1 - e)}}$

A-2q. $F = -\dfrac{\mu\, m}{r^2}$

APPENDIX B

ORBITAL ERROR ANALYSIS

B-1. Orbital Initial Conditions

For the purposes of the investigation of this thesis it is assumed that the satellite vehicle is initially in a nearly circular orbit, in which the apogee and perigee heights differ at most by 12 miles. This value is taken since it appears feasible, with present day equipment, to establish an orbit of such a low eccentricity, because the analysis of the return trajectory of the vehicle to earth is greatly simplified if this assumption is made, and because other investigators have set this condition in their analysis of other portions of the satellite supply mission (The Space Group of the M.I.T. Instrumentation Laboratory). It will also be assumed that the satellite vehicle's orbit lies somewhere between 300 and 500 miles in altitude. This assumption is based upon the short lifetime expectancy of lower circling satellites, and on the probable utility of satellites in an orbit low enough to maintain communications and have good visible information of the earth over which it passes.

When the above assumptions are made, the resulting satellite orbits have very small eccentricities. The maximum eccentricity is obtained by the orbit whose perigee is 288 miles and whose apogee is 300 miles in altitude. This maximum value of eccentricity is:

$$e_{max} = .0014108$$

The problem of return from a noncircular orbit becomes much more complicated for calculations in error analysis; however, the system proposed by this thesis for the return from the nearly circular orbit should also meet the requirements of the more general return problem. The only modifications necessary would be the form of the calculations carried out on the ground prior to the initiation of orbital departure, and an increase in the accuracy of the knowledge of the initial orbital conditions forced by the sensitivity of the correct value of the retro-thrust to be applied to the local flight path angle at which it is applied (see Appendix B-12c). Consequently, a short introduction to some of the equations for the general orbital departure is included in this appendix (see Appendix B-11).

The orbits of the satellite and supply vehicle will be assumed coincident prior to orbital departure by the supply vehicle. The position of the supply vehicle in space will therefore be considered to be known from ground tracking data to within a rectangular box 1 mile long and .4 miles high and wide for times within one half of an orbital period after the last period of ground tracking information and to within 10 miles by 4 x 4 miles for times up to two days after the last period of ground tracking. Proceeding from these assumptions is the further assumption that the velocity of the supply vehicle is known to ± 1.0 foot/sec at all times up to one-half an orbital period and to ± 1.15 feet/sec up to two days after the last period of ground tracking.

B-2. Velocity Variation in a Nearly Circular Orbit (Symbols Used are those Defined in Appendix A)

$$\frac{V_p - V_a}{V_c \text{ (at a)}} = \sqrt{\frac{1+e}{1-e}} - \sqrt{\frac{1-e}{1+e}}$$

for small eccentricities this becomes

$$\frac{V_p - V_a}{V_c} = (1+e) - (1-e) = 2e$$

The maximum fractional variation occurs for the lowest altitude orbit and the highest orbital velocity also occurs at the lowest orbit; therefore, the maximum variation in velocity occurs for the orbit where $h_p = 288$ miles and $h_a = 300$ miles.

$2\,e_{max} = .00282$

$V_p - V_{a\ (max)} = 70.5$ ft/sec (variation in velocity from perigee to apogee).

Thus, to establish the velocity of the supply vehicle to ± 1.0 foot/sec at all times, the position of the satellite apogee or perigee must be known to within approximately $\pm\, 2.5^\circ$ of central angle, θ. (The variation goes approximately as $70.5 \cos \theta_s$.)

B-3. Maximum Local Flight Path Angle for Nearly Circular Orbits[30]

$$\tan \gamma_{max} = \frac{e}{(1 - e)^{1/2}}$$

for small eccentricities, γ is small; therefore,

$$\gamma_{max} = \frac{e}{(1 - e)^{1/2}}$$

for the 300 mile orbit, $e = .00141$, and $\gamma_{max} = .08^\circ$

$$\gamma^\circ = \frac{- e \sin \theta}{57.3}^{\circ}$$

Knowledge of the central angle of apogee of the satellite orbit to $\pm\, 2.5^\circ$ connotates the continuous knowledge of γ to $\pm\, .2$ minutes of arc or better. Thus the initial value of γ upon which to base orbital departure computations will be known to at least this accuracy for the assumed nearly circular orbits. In all likelihood, the knowledge of this γ would be utilized by the computational scheme at the ground tracking station rather than assuming a perfectly circular orbit and allowing errors of re-entry angle due solely to this assumption of as much as .08° (or five minutes of arc). Starting re-entry at 25,000 ft/sec velocity with five minutes of arc error in re-entry angle leads to an initial height error rate of 36.4 ft/sec. With a completely inertial measurement system, this height error rate would be catastrophic to the knowledge of the height coordinate (see Appendix F-9). However, with height measurements supplied to the vehicle continuously, such small entrace angle variations should lead to insignificant errors in other variables.

B-4. Circular Satellite Velocity

The region of interest is for circular orbits from 300 to 500 miles in altitude.

$$V_c = \sqrt{\frac{\mu}{r_c}}$$

for convenient reference, this equation is plotted in Fig. B-1.

Also note,

$$\frac{\partial V_c}{\partial r_c} = -\frac{V_c}{2\, r_c}$$

this expression is plotted in Fig. B-2.

B-5. Increment of Velocity Required at Orbital Departure (To have a Perigee of 300,000 ft). (Initial Orbit Circular)

$$\Delta V_{required} = V_c \left[1 - \sqrt{\frac{r_p}{a}} \right]$$

Wher a and r_p refer to the resulting elliptical orbit. This expression is plotted vs. original altitude as Fig. B-3. Fig. B-4 shows the eccentricity of the resultant elliptical orbit from apogee to 300,000 ft. as a function of the original altitude.

B-6. Incremental Perigee Height Change for V Change

$$\frac{\partial \Delta V}{\partial a} = \frac{2\partial \Delta V}{\partial h_p} = -\frac{1}{2} V_c \sqrt{\frac{a}{2a - r_c}} \left(\frac{r_c}{a^2}\right)$$

therefore

$$\frac{\partial \Delta V}{\partial h_p} = -\frac{V_c}{4} \sqrt{\frac{a}{2a - r_c}} \left(\frac{r_c}{a^2}\right)$$

where a refers to the semi-major diameter of the ellipse after the retro-thrust.

$$\frac{\partial h_p}{\partial \Delta V} = -\left(\frac{a^2}{r_c}\right) \frac{4}{V_c} \sqrt{\frac{2a - r_c}{a}}$$

This equation is plotted vs. altitude in Fig. B-5.

B-7. Rate of Change of Altitude With Respect to Central Angle

$$\frac{\partial r}{\partial \theta} \Big/ a = -\frac{(1 - e^2)\, e \sin \theta}{(1 - e \cos \theta)^2}$$

For very small values of eccentricity $(e < .002)$

$$\frac{\partial r}{\partial \theta} \Big/ a = -e \sin \theta \quad \text{or} \quad \frac{\partial r}{\partial \theta} \cong -e\, a \sin \theta$$

For larger values of eccentricities with θ near perigee $(\theta = 180^\circ \pm \Delta \theta)$ making small angle approximations,

$$\frac{\partial r}{\partial \theta} \Big/ a = + \left[\left(\frac{1 - e}{1 + e}\right) e \right] (\theta - \pi) \qquad \text{(perigee)}$$

Near apogee

$$\frac{\partial r}{\partial \theta} \Big/ a = -\frac{(1 + e)}{(1 - e)}\, e\, \theta$$

These are plotted as Fig. B-6.

B-8. Rate of Change of Local Flight Path Angle (γ) with Respect to the Central Angle (θ):

$$\tan \gamma = \frac{-e \sin \theta}{1 - e \cos \theta} = \gamma \quad \text{for small eccentricities} \quad e < .1$$

since, $$\gamma_{max} = \frac{-e}{\sqrt{1 - e^2}} \quad \text{for small eccentricities} \quad e < .1$$

therefore, $$\frac{\partial \gamma}{\partial \theta} = \frac{e\,(e - \cos \theta)}{(1 - e \cos \theta)^2}$$

Near perigee, $$\frac{\partial \gamma}{\partial \theta} = \frac{e}{1 + e} \approx e$$

very approx. for $e < .1$

Near apogee, $$\frac{\partial \gamma}{\partial \theta} = \frac{-e}{1 - e} \approx -e$$

Thus, $\Delta \gamma = \pm e \Delta \theta$ approx. for $\begin{matrix} + \text{ perigee} \\ - \text{ apogee} \end{matrix}$ relationship

B-9. Effects of Misalignment of Retro-Thrust Direction

Let γ_d be the deflection of the local flight path direction.

Let τ be the misalignment angle between the direction of the retro-thrust and the velocity vector.

Then, approximately, $$\frac{\Delta V}{\sin \gamma_d} = \frac{V}{\sin \tau}$$

therefore; $$\gamma_d = \tau \frac{\Delta V}{V}$$

where, $$\frac{\Delta V}{V} = .028 \quad \text{for the 500 mile orbit}$$

$$\frac{\Delta V}{V} = .017 \quad \text{for the 300 mile orbit}$$

B-10. Change of Altitude Near Perigee as a Function of the Central Angle

$$r - r_p = a\,(1 - e^2)\left[\frac{1}{1 - e\cos\theta} - \frac{1}{1 + e}\right]$$

For $\quad e < .1 \qquad r - r_p \approx ae\left[\dfrac{1 + \cos\theta}{(1 + e)(1 - e\cos\theta)}\right]$

For $\quad 170^\circ < \theta < 190^\circ$

$$\frac{r - r_p}{a} \approx e\,\frac{1 + \cos\theta}{(1 + e)^2}$$

This is plotted for various values of e as Fig. 8-7.

B-11. General Equations for Departure from a Noncircular Satellite Orbit

Although this thesis is not directly associated with the problem of return from a noncircular orbit, the resultant navigation system proposed by this thesis appears ammenable to that problem as well as the purely circular case. An introduction to a few of the equations necessary to analyze such a problem now follows. Keplerian laws of motion are still assumed.

Let the subscript (s) denote the conditions associated with the original satellite orbit, and subscript (o) be associated with the initial conditions required of the orbiting vehicle in order to have a perigee of 300,000 feet, the desired re-entry altitude. Also, a_s, e_s, V_s, r_s, etc. are all assumed known as a function of time from ground tracking data.

From the energy equation the perigee velocity becomes:

B-11a. $$V_p^{\,2} = V_o^{\,2} + 2\mu\left(\frac{1}{r_p} - \frac{1}{r_o}\right)$$

From the constancy of angular momentum write:

B-11b. $$\cos \gamma_o = \frac{V_p r_p \cos \gamma_p}{V_o r_o} = \frac{V_p r_p}{V_o r_o}$$

Therefore, if V_o is the required velocity of orbital departure,

B-11c. $$V_o = \sqrt{\frac{2 \mu r_p \left(1 - \frac{r_p}{r_s}\right)}{r_s^2 \cos^2 \gamma_s - r_p^2}}$$

B-11d. $$V_s = \sqrt{\mu \left(\frac{2}{r_s} - \frac{1}{a_s}\right)}$$

Thus the increment of velocity to be applied:

B-11e. $$\Delta V = V_s - V_o = \sqrt{\mu \left(\frac{2}{r_s} - \frac{1}{a_s}\right)} - \sqrt{\frac{2\mu r_p \left(1 - \frac{r_p}{r_s}\right)}{r_s^2 \cos^2 \gamma_s - r_p^2}}$$

Substituting for r_s in V_s equation and $r_s^2 \cos^2 \gamma_s$ in the ΔV equation, we get;

B-11f. $$\Delta V = \sqrt{\frac{\mu}{a_s (1 - e_s^2)}} \left\{ \sqrt{1 - 2 e_s \cos \theta_s + e_s^2} - \sqrt{\frac{2 r_p \left[a_s (1 - e_s^2) - r_p (1 - e_s \cos \theta_s)\right]\left[1 - 2 e_s \cos \theta_s + e_s^2\right]}{a_s^2 \left(1 - 2 e_s^2 + e_s^4\right) - r_p^2 \left(1 - 2 e_s \cos \theta_s + e_s^2\right)}} \right\}$$

or

B-11g. $$\Delta V = V_s \left\{ 1 - \sqrt{\frac{2 r_p \left[a_s (1 - e_s^2) - r_p (1 - e_s \cos \theta_s)\right]}{a_s^2 \left[1 - 2 e_s^2 + e_s^4\right] - r_p^2 (1 - 2 e_s \cos \theta_s + e_s^2)}} \right\}$$

where V_s is given by

B-11h. $$V_s = \sqrt{\frac{\mu}{a_s (1 - e_s^2)} (1 - 2 e_s \cos \theta_s + e_s^2)}$$

Thus, the above equations could be solved as a function of θ_s for the increment of velocity to be applied, as long as the constants associated with the satellite orbit were known. The above equations specify the magnitude of the retro-thrust velocity change in order to have a perigee of 300,000 feet, but they do not give the range at which this perigee occurs.

For the time variation of θ_s we note that

B-11i. $$h_s = \text{constant} = r_s^2 \frac{d\theta_s}{dt}$$

B-11j. $$\theta_s = \int_{t_1}^{t_2} \frac{h_s}{r_s^2} dt$$

B-11k. $$T_s = \frac{2\pi a_s^{3/2}}{\sqrt{\mu}}$$ where T_s is the satellite orbital period.

B-11l. $$\frac{d\theta_s}{dt} = \frac{2\pi}{T_s} \frac{(1 - e_s \cos \theta_s)^2}{(1 - e_s^2)^{3/2}}$$

Therefore[31],

B-11m. $$\frac{e_s \sqrt{1 - e_s^2} \sin \theta_s}{1 - e_s \cos \theta_s} + \sin^{-1} \frac{(e - \cos \theta_s)}{(1 - e \cos \theta_s)} + \frac{\pi}{2} = \frac{\sqrt{\mu}}{a^{3/2}} t$$

What is the range at which re-entry occurs using the above equations? The range of concern is the range from the first retro-thrust to the resultant perigee as measured by the central angle traversed. Through manipulation, the equation of motion in the new orbit can be expressed as,

B-11n. $$r = \frac{\frac{h^2}{\mu}}{1 - \frac{h}{\mu}\sqrt{V_o^2 - \frac{2\mu}{r_o} + \frac{\mu^2}{h^2}} \cos \theta}$$

[redacted]

where the eccentricity is,

B-11o. $$e = \frac{h}{\mu}\sqrt{V_o^2 - \frac{2\mu}{r_o} + \frac{\mu^2}{h^2}}$$

Setting $r = r_o$ in this equation will evaluate the $\cos\theta_o$, where numerically θ_o is the angle to go till apogee if γ_o is (+), and/or numerically, θ_o is the angle to go till perigee if γ_o is (-).

Table for Range to Perigee

	$\cos\theta_o$ +	$\cos\theta_o$ -
γ_o -	$180^\circ - \theta_o$	$180^\circ - \theta_o$
γ_o +	$180 + \theta_o$	$180^\circ + \theta_o$

Thus, the sign of $\cos\theta_o$ determines θ_o as some angle between 0° and 180° for the purposes of this calculation. Then, the sign of γ_o determines whether you add or subtract this angle to/from 180° to get the angle till perigee.

The general error analysis which proceeds from the above equations is not a feasible study without the aid of high speed calculating machines. A check was made, however, on the equation for the sensitivity of perigee height to error increments in initial velocity.

B-11q.
$$r_p = \frac{-\mu r_o}{V_o^2 r_o - 2\mu} - \sqrt{\frac{\mu^2 r_o^2 + V_o^4 r_o^4 \cos^2\gamma_o - 2\mu V_o^2 r_o^3 \cos^2\gamma_o}{V_o^4 r_o^2 - 4\mu V_o^2 r_o + 4\mu^2}}$$

Then,

B-11r. $$\frac{\partial r_p}{\partial V_o} = \frac{2\mu V_o r_o^2}{V_o^4 r_o^2 - 4\mu V_o^2 r_o + 4\mu^2} - \left[\quad\right]$$

[redacted]

Where:

$$[\;] = \left[2 V_o r_o^2 \mu \sqrt{\frac{V_o^4 r_o^2 - 4\mu V_o^2 r_o + 4\mu^2}{\mu^2 + V_o^4 r_o^2 \cos^2\gamma_o - 2\mu V_o^2 r_o \cos^2\gamma_o}} \;\{\;\}\right]$$

and;

$$\{\;\} = \left\{ \frac{\cos^2\gamma_o (4\mu V_o^2 r_o - 2\mu^2 - V_o^4 r_o^2) - \mu V_o^2 r_o}{V_o^8 r_o^4 - 8\mu V_o^6 r_o^3 + 24\mu^2 V_o^4 r_o^2 - 16\mu^3 V_o^2 r_o + 16\mu^4} \right\}$$

In the case of the circular orbit, the equation for V_{req} becomes:

$$V_{o\,req} = \sqrt{\frac{2\mu r_p}{r_o(r_o + r_p)}} = \sqrt{\frac{2\mu}{r_o}}\sqrt{\frac{r_p}{r_o + r_p}}$$

This can be rearranged,

B-11t. $$r_p = \frac{V_o^2 r_o^2}{2\mu - V_o^2 r_o}$$

or

B-11u. $$\frac{\partial r_p}{\partial V_o} = \frac{4\mu V_o r_o^2}{V_o^4 r_o^2 - 4\mu V_o^2 r_o + 4\mu^2}$$

When calculated for the 300 mile circular orbit, this checks fairly well with the already calculated $1/\partial\Delta V/\partial h_p$ or the equivalent of $\frac{\partial h_p}{\partial \Delta V} = \frac{\partial h_p}{\partial V_o}$, etc.

Also,

B-11v.

$$\frac{\partial r_p}{\partial \gamma_o} = \sqrt{\frac{V_o^4 r_o^2 - 4\mu V_o^2 r_o + 4\mu^2}{\mu^2 r_o^2 + V_o^4 r_o^4 \cos^2\gamma_o - 2\mu V_o^2 r_o^3 \cos^2\gamma_o}} \cdot \left\{ \frac{(V_o^4 r_o^4 - 2\mu V_o^2 r_o^3)\cos\gamma_o \sin\gamma_o}{V_o^4 r_o^2 - 4\mu V_o^2 r_o + 4\mu^2} \right\}$$

It was not deemed profitable to carry the general case analysis any further; however, the next section takes one aspect of the general equation analysis and reduces it to a useful result for the nearly circular orbital case.

B-12. Rate of Change of Velocity Increment Required at Retro-Thrust with Respect to the Initial Flight Path Angle

The general equations above allow us to take,

B-12a. $$\frac{\partial \Delta V}{\partial \gamma_o} = - \frac{\sqrt{2\mu\, r_p \left(1 - \frac{r_p}{r_o}\right)}\sqrt{r_o^2 \cos^2 \gamma_o - r_p^2}\; r_o^2 \cos\gamma_o \sin\gamma_o}{\left(r_o^2 \cos^2 \gamma_o - r_p^2\right)^2}$$

For ellipses with small eccentricities this reduces to,

B-12b. $$\frac{\partial \Delta V}{\partial \gamma_o} \approx - \frac{\sqrt{2\mu\, r_p \left(1 - \frac{r_p}{r_o}\right)}}{\left(r_o^2 - r_p^2\right)^{3/2}}\, r_o^2 \sin \gamma_o$$

This approximation is closely valid for the 300 mile original orbital altitude, but the approximation becomes less valid as higher initial altitudes are considered.

For the case of the nearly circular orbit of altitude 300 miles this approximate equation may be written,

B-12c. $$\frac{\partial \Delta V}{\partial \gamma_o} \approx - V_{o\ \text{required}} \frac{r_o^2}{r_o^2 - r_p^2}\, \gamma_o$$

The V_o for the 300 mile orbit is approx. 24,650 ft/sec, and

$$\frac{r_o^2}{r_o^2 - r_p^2} \approx 9$$

Thus,

$$\frac{\partial \Delta V}{\partial \gamma_o} \approx - 22.2 \times 10^4\, \gamma_o$$ for the 300 mile circular orbit case.

The above equation, although not valid for highly eccentric orbits nor for large values of γ_o, shows the extreme sensitivity of the required retro-thrust magnitude to γ_o. This factor alone shows that it would be greatly preferable to initiate the return of the supply vehicle from a nearly circular orbit than from an eccentric one.

The $\frac{\partial \Delta V}{\partial \gamma_o}$ for the 300 mile orbit altitude considered above is decreased by almost a factor of two when the 500 mile orbit is considered. Thus the lower initial altitude orbit will place the most stringent requirements on the knowledge of γ_o. Note also that the γ_{max} is the highest for this lowest altitude orbit, which re-enforces the tendency for this orbit to set the most critical requirements upon a control or navigation system.

B-13. Oblateness Effects of Earth

The major effects of the earth's oblateness are the perturbances to the otherwise nearly elliptical orbit, and the geometric change of altitude due solely to the nearly circular orbit traversing a non-spherical earth and therefore coming closer at some times than others. The geometrical effects are mainly a function of latitude and will be assumed to be completely compensated in the ground computations in the system.

The orbital perturbances take three predominant forms, a. Regression of the Nodes, b. Precession of the Apsides, c. Changes in the Orbital Period. The approximate equations governing these perturbances are given in the Martin Company report, "Dynamical Analysis and Design Performance Requirements for Satellite Vehicle Guidance System," Chapter I. It will be assumed that these equations are accurate enough to allow no large errors due to inaccurate knowledge of these oblateness effects. If this is not so, then closer approximations must be obtained from some other source, so that the ground computation may be done with the closest tolerance possible. This assumption will allow the design of a system to proceed unimpeded by consideration of ground based calculation errors.

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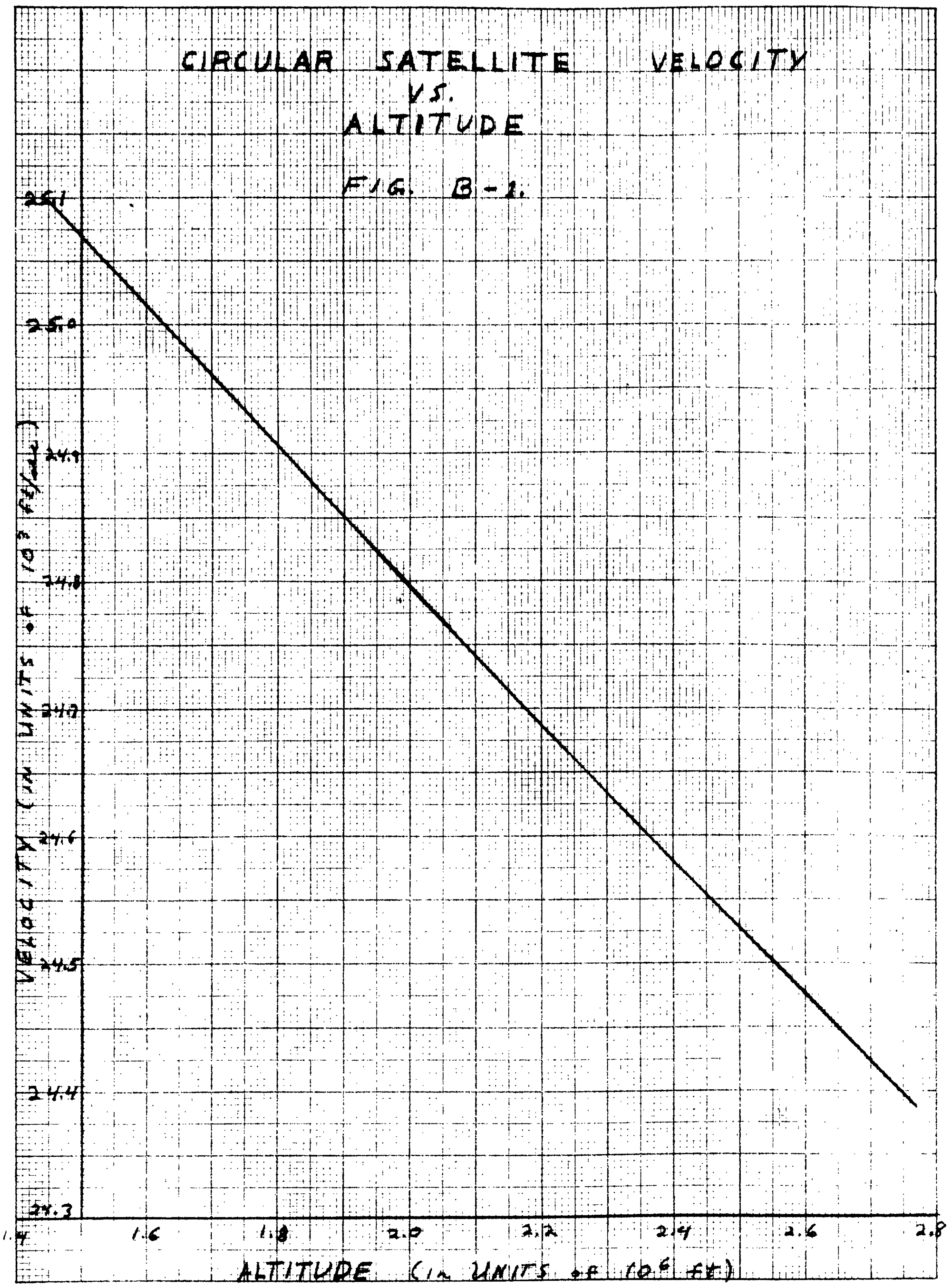

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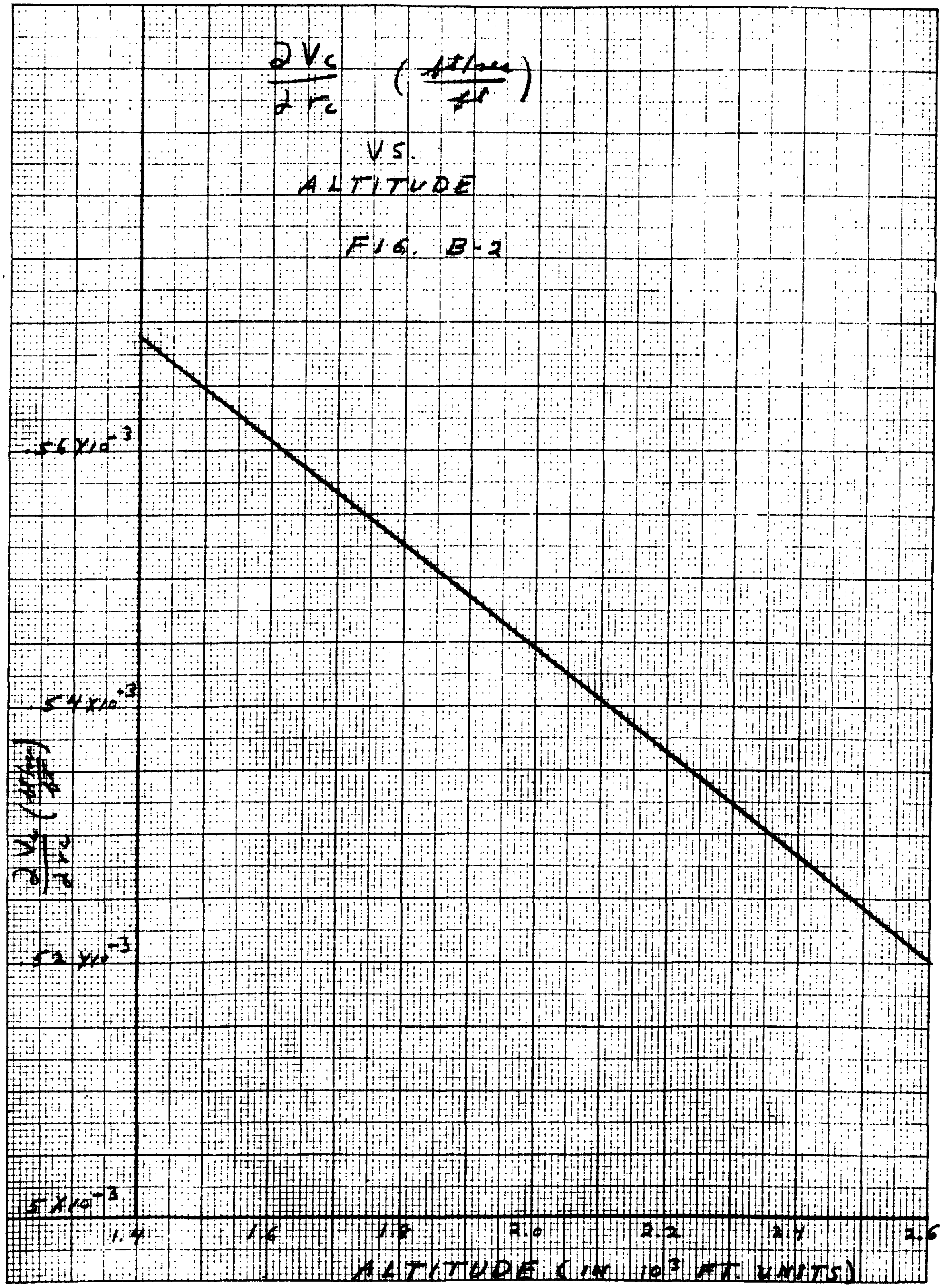

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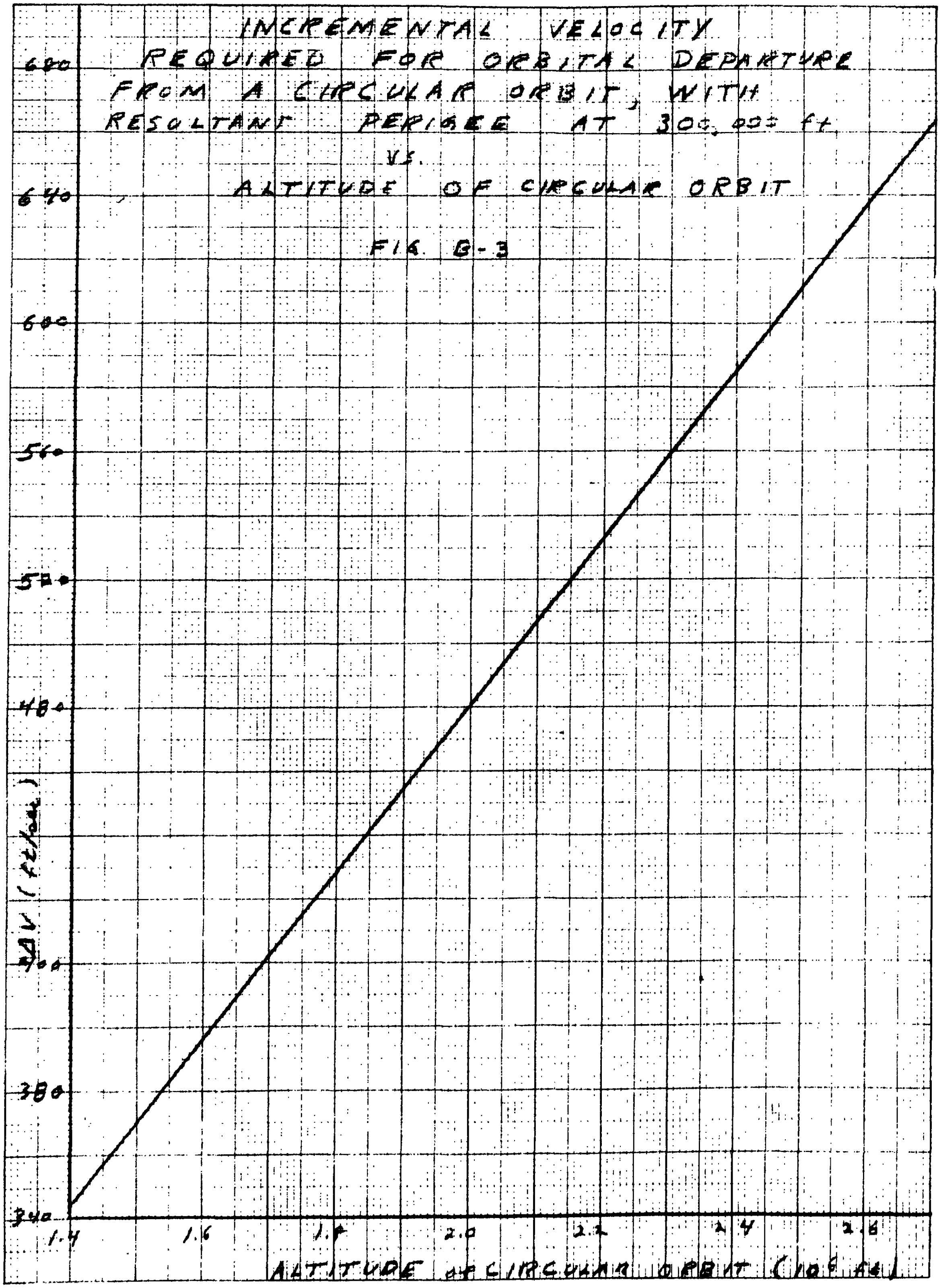

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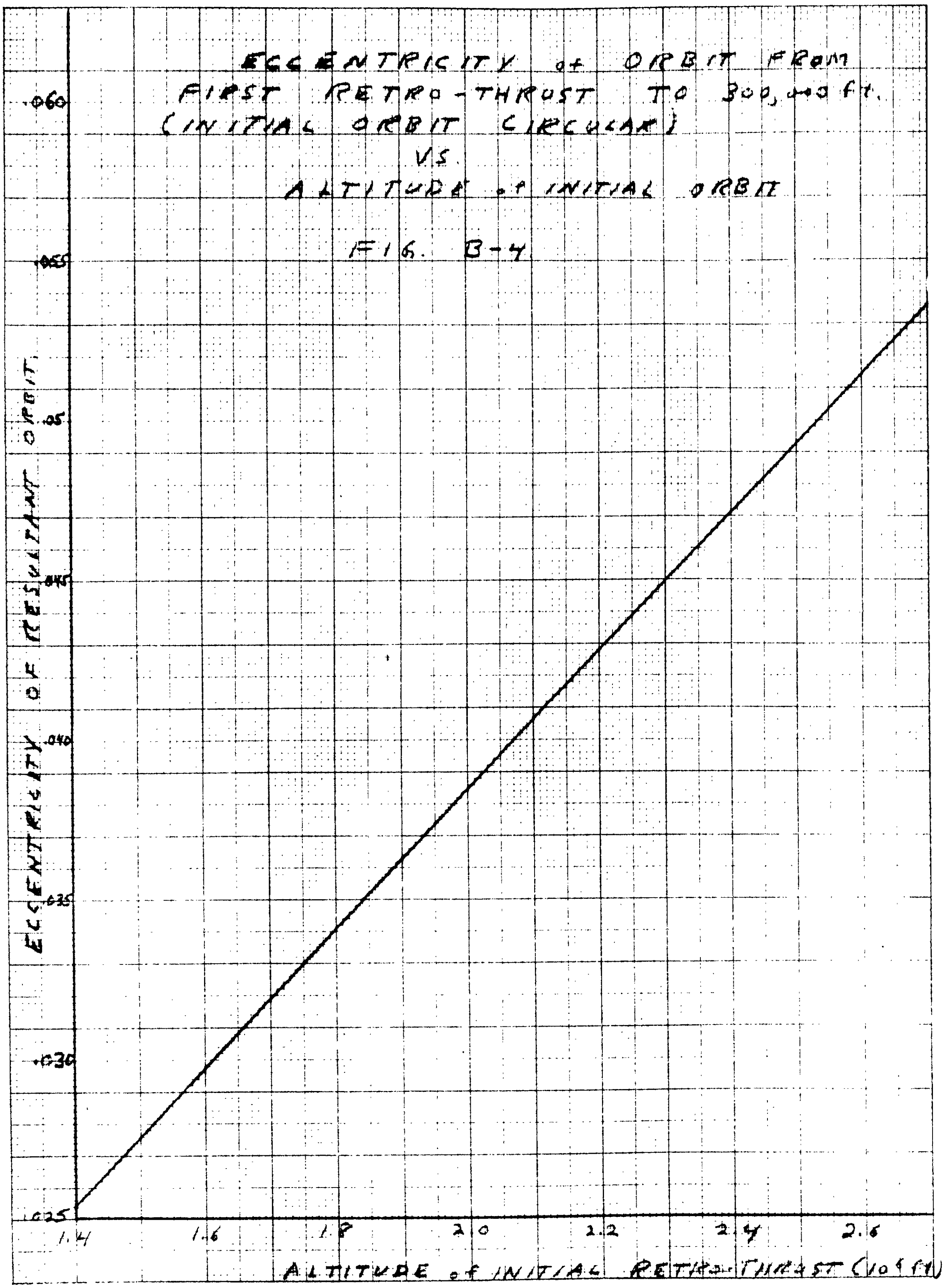

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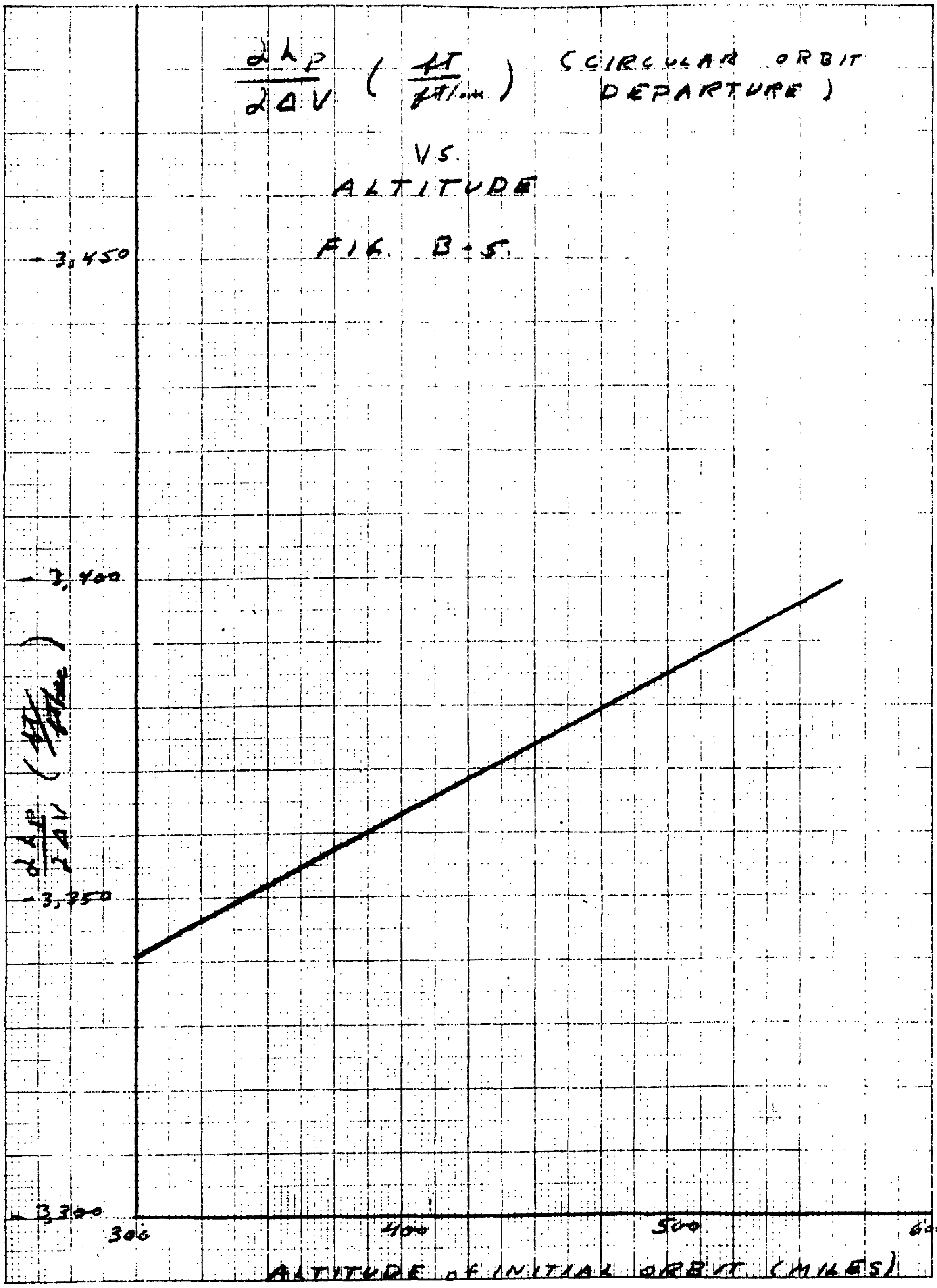

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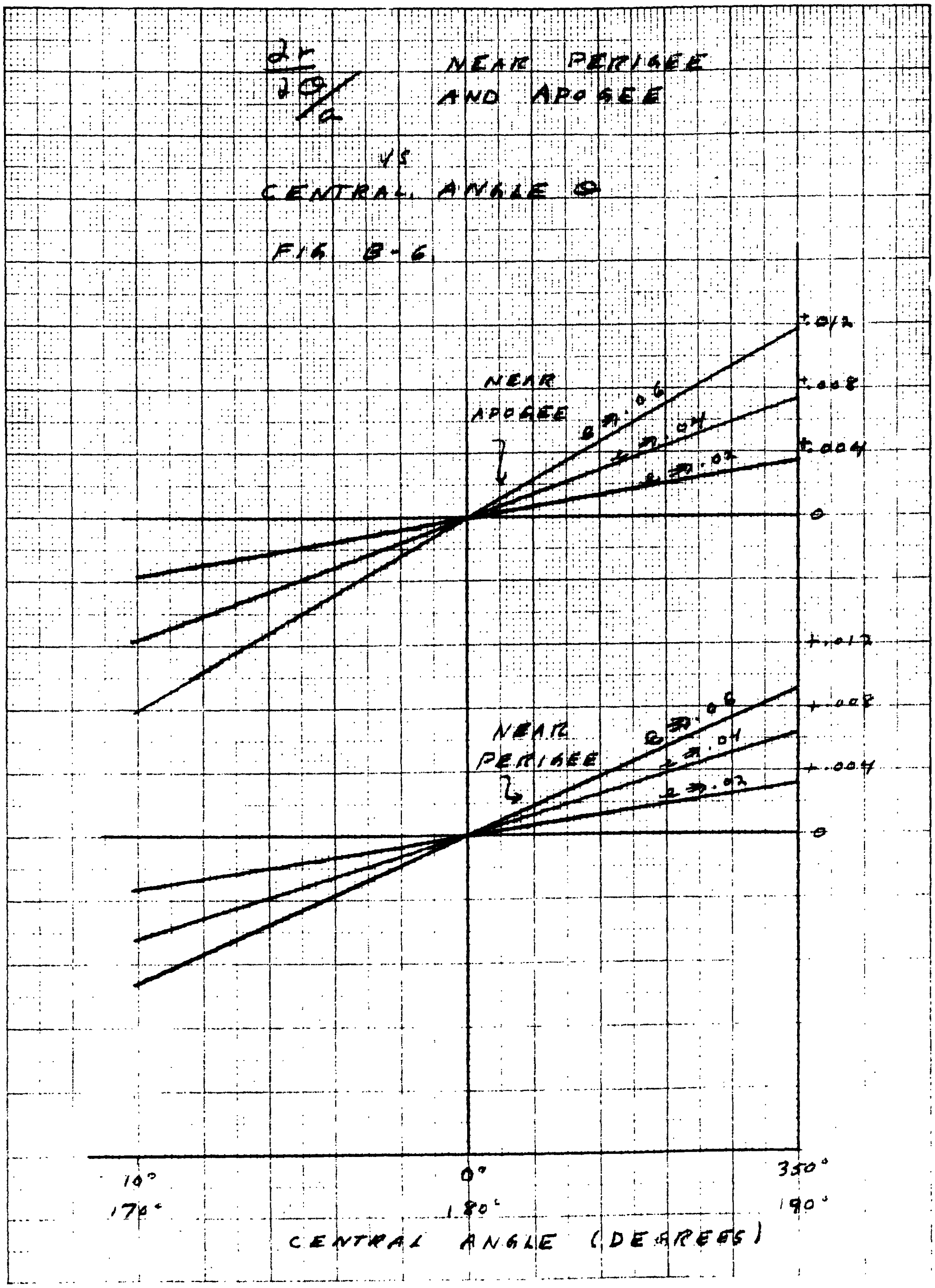

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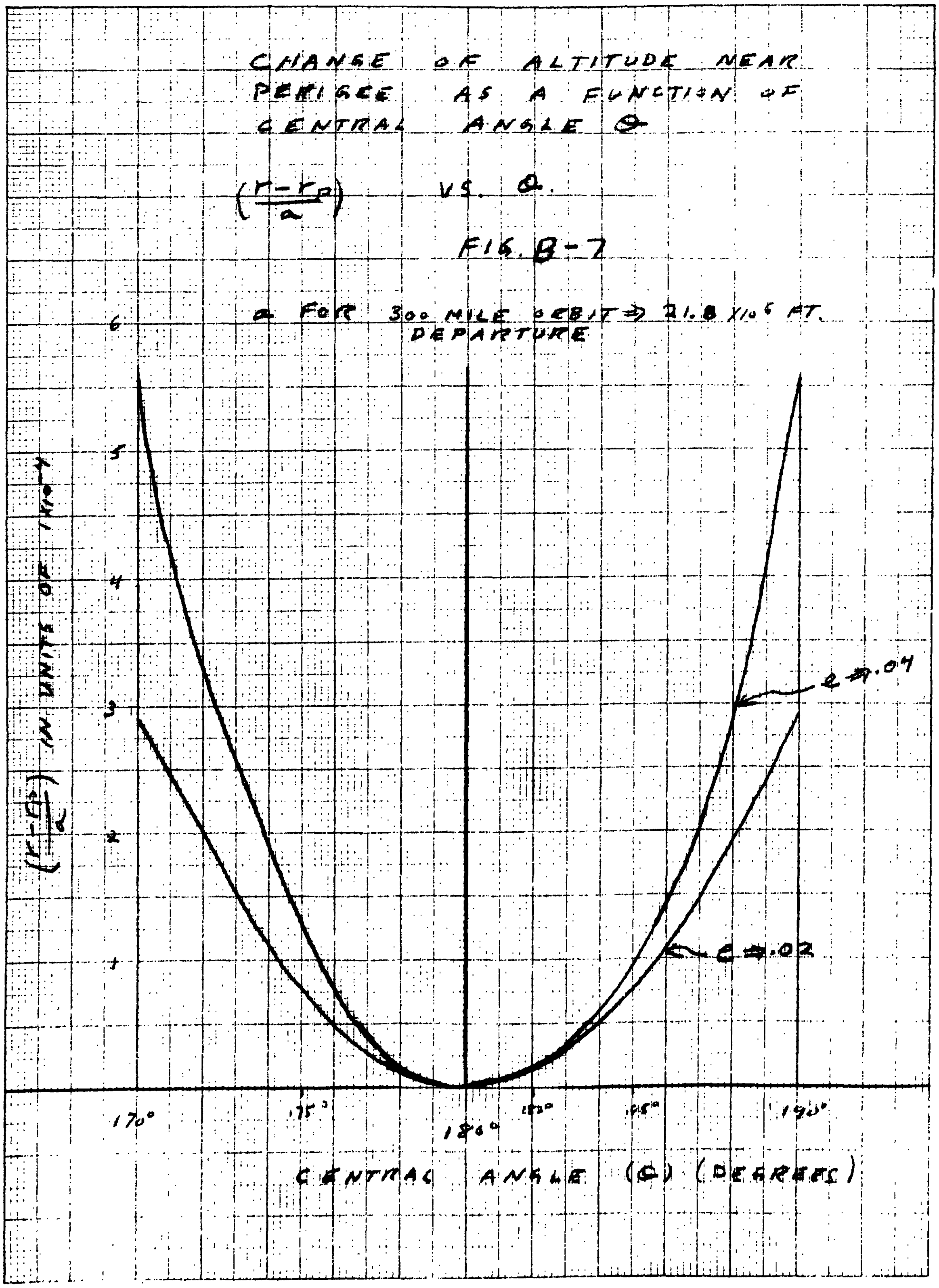

CHANGE OF ALTITUDE NEAR PERIGEE AS A FUNCTION OF CENTRAL ANGLE θ
(r−r_p / a) VS. θ.
FIG. B-7
a FOR 300 MILE ORBIT ⇒ 21.8 ×10^6 FT.
DEPARTURE
(r−r_p / a) IN UNITS OF 1×10^-4
6
5
4
3
2
1
e = .04
e = .02
170°
175°
180°
185°
190°
CENTRAL ANGLE (θ) (DEGREES)

APPENDIX C

ATMOSPHERIC TRAJECTORY

In order to determine what information a navigation system should supply to the pilot of the vehicle and what accuracy requirements would be placed on the system in order to supply it, the flight regime below 300,000 ft., where the atmosphere is considered to become appreciable, is examined in some detail. This examination is also necessary to gain some knowledge of where the descent from orbit should be initiated in order to land at some desired spot.

The flight regime below 300,000 ft. may conveniently be subdivided into two parts. The first is the re-entry phase in which the flight path of the vehicle is governed by the amount of heating and the magnitude of the drag decelerations to which it is desirable to subject the vehicle and its occupant. The second, or gliding, phase begins when the vehicle velocity has decreased sufficiently to allow maneuvering without experiencing undue heating or accelerations and the vehicle can be glided to a landing at the desired spot.

1. Trajectories

Mr. Dean R. Chapman has derived the following equation for vertical motion of a lifting vehicle on re-entry into the atmosphere[32]

$$\bar{u}\frac{d}{d\bar{u}}\left(\frac{dZ}{d\bar{u}} - \frac{Z}{\bar{u}}\right) - \frac{1-\bar{u}^2}{\bar{u}Z}\cos^4\phi + \sqrt{\beta r}\,\frac{L}{D}\cos^3\phi = 0.$$

[redacted]

Where:

$$Z = \frac{\overline{\rho}_o}{2\left(\frac{m}{C_D A}\right)} \sqrt{\frac{r}{\beta}}\ \overline{u}\, e^{-\beta y}$$

$$\overline{u} = \frac{u}{\sqrt{gr}}$$

A	reference area for drag and lift.
C_D	drag coefficient.
g	gravitational acceleration.
g_c	gravitational conversion constant.
D	drag force.
L	lift force, lbs.
m	mass of vehicle, slugs.
r	distance from planet center, ft.
u	circumferential velocity component normal to radius vector, ft/sec.
u_c	circular orbital velocity, V_{gr}, ft/sec.
y	altitude, ft.
β	atmospheric density decay parameter, ft^{-1}.
ϕ	flight path angle relative to local horizontal direction, positive for climbing flight, negative for descent.
ρ	density, slugs/ft.
$\overline{\rho}_o$	represents the intercept of the straight line which best fits a curve of log ρ vs. altitude and is not the same as the true sea level density ρ_o.

In deriving this equation, Mr. Chapman made the following assumptions[33]:

[redacted]

1. Atmosphere and earth are spherically symmetric.

2. Atmospheric density ρ_{∞} varies exponentially with altitude.

3. Peripheral velocity of the earth is negligable compared to the velocity of the entering vehicle.

4. In a given increment of time, the fractional change in the geocentric radius to the vehicle is small, compared to the fractional change in velocity.

5. For lifting vehicles, the flight path angle ϕ relative to the local horizontal direction is sufficiently small that the component of lift in the horizontal direction is small compared to the drag.

This equation was integrated by members of the Instrumentation Laboratory staff, using the IBM 650 computer, for a number of initial conditions. Since it was not the intent of this thesis to study the dynamics of re-entry, one of the vertical flight profiles thus obtained was selected as representative, and the results (height and time) used to obtain representative horizontal flight paths over the earth's surface. The profile chosen was for a lift to drag ratio of 1, initial angle ϕ of 0^{o}, and initial velocity with respect to the air mass of 25,445 ft. per second. To obtain horizontal flight paths, the following equations were derived:

$$V_{IA} = W_{IE} (R_E + h) \cos Lat$$

$$\frac{dV_{IV}}{dt} = - a \left[\frac{V_{IV}^2 - V_{IA}^2 + V_{AV}^2}{2 V_{IV} V_{AV}} \right] - W_{IE} \left[(\cos Lat) \frac{dh}{dt} + (\sin Lat)(R_E + h) \frac{dLat}{dt} \right] \left[\frac{V_{IA}^2 + V_{IV}^2 - V_{AV}^2}{2 V_{IA} V_{IV}} \right]$$

$$\frac{d\,Lat}{dt} = \frac{V_{IV}}{(R_E + h)} \sqrt{1 - \left[\frac{V_{IA}^2 + V_{IV}^2 - V_{AV}^2}{2\,V_{IV}\,V_{AV}}\right]^2}$$

$$\frac{d\,Lon}{dt} = W_{IE}\,(1 - \cos Lat) + \frac{V_{AV}}{(R_E + h)\cos Lat}\left[\frac{V_{IV}^2 - V_{AV}^2}{2\,V_{IA}\,V_{AV}}\right]$$

Where:

Lat	latitude.
Lon	longitude.
W_{IE}	earth's angular rate of rotation.
R_E	average earth's radius.
h	height above earth's surface.
a	deceleration of the vehicle due to drag.
V_{IA}	velocity of the atmosphere with respect to an earth centered inertial coordinate frame.
V_{IV}	velocity of the vehicle with respect to an earth centered inertial coordinate frame.
V_{AV}	velocity of the vehicle with respect to the air mass.

In these equations the following assumptions were made in addition to those made by Mr. Chapman:

1. Motion was over a rotating earth rather than a nonrotating earth as assumed by Mr. Chapman.

2. On the average, the atmosphere is nonrotating with respect to the earth.[34]

3. The results obtained from integration of Mr. Chapman's equations are valid for motion with respect to the atmosphere.

4. No control is exercised over the vehicle by the pilot.

These equations were integrated for three sets of conditions of initial latitude, initial longitude, and initial velocity with respect to inertial space using the IBM 650 computer (see program). The results are shown in Figures C-1 thru C-3.

The assumption that the pilot cannot exercise control over the vehicle during the re-entry phae is based on the following facts:

a. Any aerodynamic loads applied through the controls would increase the amount of heating and acceleration experienced by the vehicle.

b. No appreciable changes in direction of travel by application of controls can be made without experiencing high accelerations due to the very high velocity of the vehicle.

c. If continual lateral control in one direction a majority of the time were required to follow a more arbitrary path, then the pilot would be restricted in the amount of control remaining available to him for correction of errors.

When the velocity of the vehicle has been reduced by air drag to the point where the vehicle may be maneuvered, it may be considered to be in the glide phase of its flight. The exact point at which the re-entry phase is completed and gliding flight begins is not well defined and will depend on the structure of the vehicle, the desirable acceleration maximum, etc. For purposes of this thesis it was assumed to begin when the aircraft velocity, with respect to the air mass, had dropped to approximately one tenth of its original value or about 2,500 feet per second. For the representative vertical flight profile used, this occurred at a height of about 91,000 feet.

Once the velocity has decreased to this lower value, the

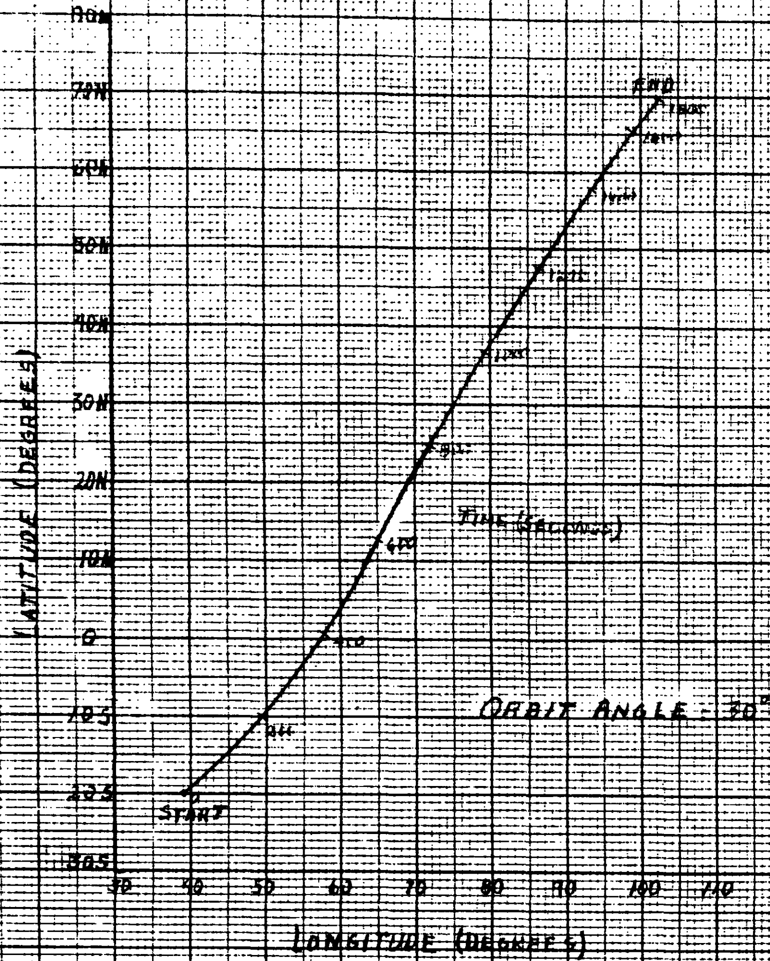


FIGURE C-11 VEHICLE GROUND TRACK

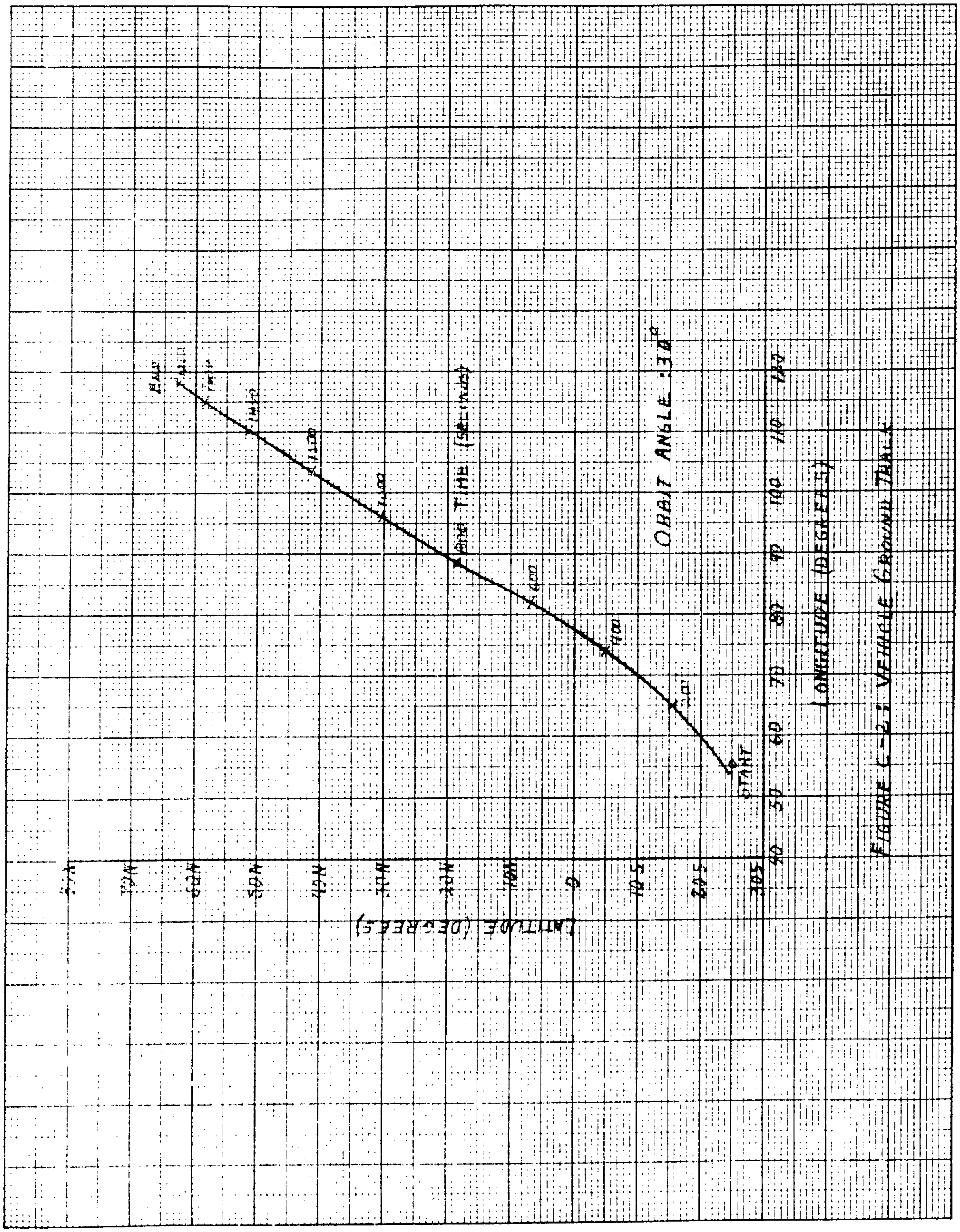


FIGURE C-2: VEHICLE GROUND TRACK

[redacted]

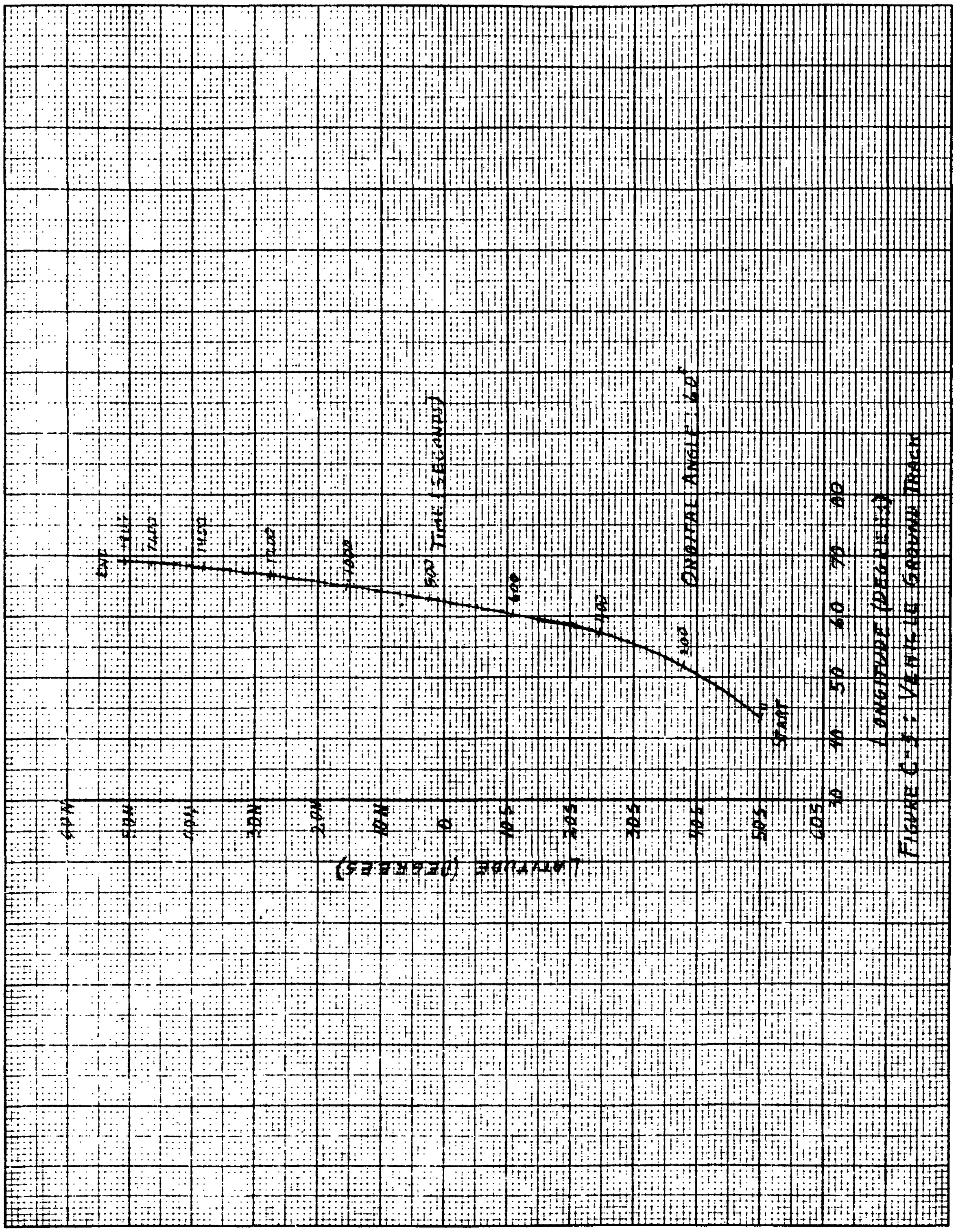


Figure C-5: Vehicle Ground Track

[redacted]

lateral accelerations experienced in following a great circle path are very moderate, never exceeding 0.1 gravity[35] and decreasing as the square of velocity.

Since the actual flight path to be followed during the gliding phase is at the option of the pilot, only the limits over which the pilot could control the vehicle, i.e. the dimensions of the area in which he could land, were sought. For these purposes it was assumed that the vehicle could fly at any lift to drag ratio between 1 and 4. The distance he can travel during glide is given approximately by the energy equation:

$$\Delta s = \frac{L}{D}\left[h + \frac{V_1^{\,2} - V_2^{\,2}}{2\,g}\right]$$

Where:

L	lift.
D	drag.
h	height.
Δs	distance traveled.
V_1	velocity at beginning of glide.
V_2	velocity at termination of glide.
g	gravity (assumed constant).

If a landing speed of 300 feet per second and an initial height of 90,000 feet are assumed with $V_1 = 2,500$ feet per second, values of Δs for L/D equal to 1 and 4 are 185,000 feet and 740,000 feet or 35 miles and 140 miles respectively. Thus the range of control available to the pilot over distance traveled (about the distance traveled at an average L/D of 2.5) is $\pm$ 52.5 miles. The horizontal track covered by the vehicle in traveling the distance Δs is at the option of the pilot, but the area formed by all points at which he can land is bounded by a shape similar to a Cardioid.[36]

The mose desirable ground track during the glide phase, for planning purposes at least, is one in which the final heading at the end of re-entry phase is continued over a great circle route (for practical purposes at the short glide distance available - a straight line) at a lift to drag ratio in the middle of the available range, in this case 2.5. Thus, the most desirable glide track over the ground is straight and is 87.5 miles in length.

Combining the re-entry phase ground track with the glide phase ground track gives the desired re-entry point (and initial condition of velocity) for a given landing point.

2. Derivations

The re-entry phase of the vehicle flight is considered to start at an altitude of 300,000 feet where the effect of the atmosphere becomes appreciable. It lasts until the vehicle velocity has been decreased sufficiently by atmospheric drag to allow the vehicle to enter a gliding flight (see text).

During the re-entry phase, there exists the possibility of the vehicle being subjected to severe heating and accelerations. In order to minimize these quantities, the vertical flight profile must be carefully planned. The problem becomes, then, one of finding what ground track will be flown when a given flight profile is flown.

For use in maing this computation, data from solution of Mr. Chapman's equations was used and is as follows:

t time (seconds).

h height above the earth's surface (assumed spherical) in feet.

V_{AV} velocity of the vehicle with respect to the air mass expressed as

$$U = \frac{V_{AV}}{\sqrt{\mu / r}}$$

Where:

μ	constant.
r	$R_E + h$
Re	radius of earth

a deceleration of the vehicle.

Definition of symbols:

V_{IV} velocity of the vehicle with respect to inertial space.

V_{IA} velocity of the air mass with respect to inertial space (assumed stationary with respect to the earth's surface).

V_{AV} velocity of the vehicle with respect to the air mass.

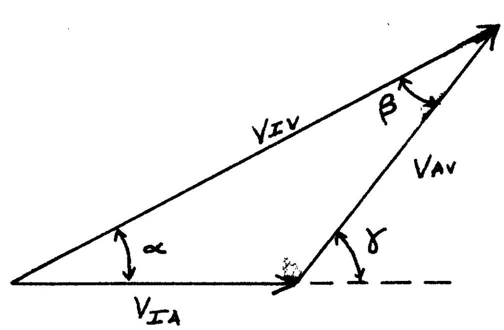
Lon longitude.

Lat latitude.

$$\Delta q = \int_0^t dq/dt$$

$$q = q_o + \Delta q$$

α, β, γ = angles as shown below



W_{IE} angular velocity of the earth with respect to inertial space.

Determination of initial values

Lon_o	derived from $lat_o + i$.
Lat_o	assumed (consistant with i)
i	orbital inclination (assumed)
H_o	initial heading angle: Determined as follows:

Let $Lo = - Lon \text{ (of line of nodes)} + Lon_o$

$La = Lat_o$

Transforming from position expressed in orbital quantities to an expression in earth coordinates and equating like terms gives:

$$\sin H_o = \frac{\cos i \cos L_o + \sin L_o \left[\dfrac{\cos i \sin L_o \cos Lat + \sin i \sin Lat}{\cos L_o \cos Lat}\right]}{\sqrt{1 + \left[\dfrac{\cos i \sin L_o \cos Lat + \sin i \sin Lat}{\cos L_o \cos Lat}\right]}}$$

This determines H_o in terms of L_o, i, and Lat.

$$V_{IAo} = W_{IE} (R_E + H_o) \cos Lat_o$$

V_{AVo} = given from other program

$$V_{IVo} = V_{IAo} \sin H_o + \sqrt{V_{AVo}^2 - (V_{IAo} \cos H_o)^2}$$

$$V_{IVo} = V_{IAo} \sin H_o + \sqrt{V_{AVo}^2 - V_{IAo}^2 + V_{IAo}^2 \sin^2 H_o}$$

Initial conditions used for curves C-1, C-2, and C-3 were:

1. Curve C-1

Lat_o	20.000^o
Lon_o	39.081^o
V_{IV_o}	26,485 feet/second

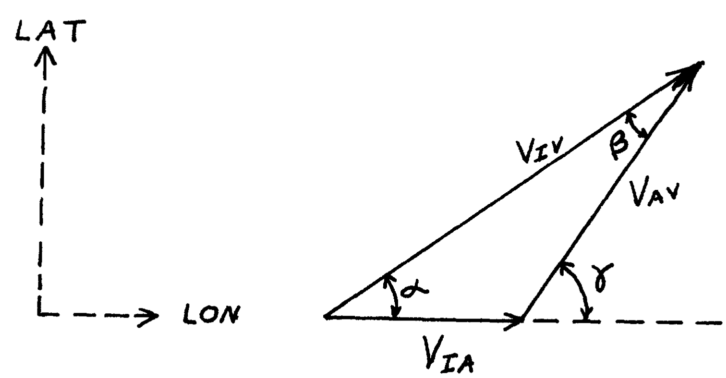
2. Curve C-2

Lat_o	25.000^o
Lon_o	53.869^o
V_{IV_o}	26,488 feet/second

3. Curve C-3

Lat_o	50.000^o
Lon_o	43.477^o
V_{IV_o}	25,890 feet/second

Derivation of equations to be solved: The triangle formed by the velocity vectors is:



From this diagram:

$$\frac{d\,(\Delta\ \mathrm{Lat})}{dt} = \frac{V_{AV}}{(R_E + h)} \sin\gamma$$

$$\frac{d\,(\Delta\ \mathrm{Lon})}{dt} = \frac{V_{IA}}{(R_E + h)\cos \mathrm{Lat}} + \frac{V_{AV}}{(R_E + h)\cos \mathrm{Lat}} \cos\gamma - W_{IE}\cos \mathrm{Lat}$$

$$\gamma = \alpha + \beta$$

$$V_{IA} = W_{IE}\,(R_E + h)\,\cos \mathrm{Lat}$$

$$\frac{d\,(\Delta\ \text{lon})}{dt} = W_{IE}\,(1 - \cos \text{Lat}) + \frac{V_{AV}}{(R_E + h)\cos \text{Lat}} \cos \gamma$$

$$\sin \gamma = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\cos \gamma = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\cos \alpha = \frac{V_{IA}{}^2 + V_{IV}{}^2 - V_{AV}{}^2}{2\, V_{IA}\, V_{IV}}$$

$$\sin \alpha = \sqrt{1 - \frac{V_{IA}{}^2 + V_{IV}{}^2 - V_{AV}{}^2}{2\, V_{IA}\, V_{IV}}} = [A]$$

$$\cos \beta = \frac{V_{IV}{}^2 - V_{IA}{}^2 + V_{AV}{}^2}{2\, V_{IV}\, V_{AV}}$$

$$\sin \beta = \frac{V_{IA}}{V_{AV}} \sin \alpha$$

$$\sin \gamma = A \left[\frac{V_{IV}{}^2 - V_{IA}{}^2 + V_{AV}{}^2}{2\, V_{IA}\, V_{AV}} + \frac{V_{IA}{}^2 + V_{IV}{}^2 - V_{AV}{}^2}{2\, V_{IA}\, V_{IV}} \left(\frac{V_{IA}}{V_{AV}} \right) \right]$$

$$\sin \gamma = \left[\sqrt{V_{IV}{}^2 - \left[\frac{V_{IA}{}^2 + V_{IV}{}^2 - V_{AV}{}^2}{2\, V_{IA}} \right]^2} \right] \frac{1}{V_{AV}}$$

$$\cos \gamma = \left[\frac{V_{IA}{}^2 + V_{IV}{}^2 - V_{AV}{}^2}{2\, V_{IA}\, V_{IV}} \right] \left[\frac{V_{IV}{}^2 - V_{IA}{}^2 + V_{AV}{}^2}{2\, V_{IV}\, V_{AV}} \right]$$

$$+ \frac{V_{IA}}{V_{AV}} \left[\left(\frac{V_{IA}{}^2 + V_{IV}{}^2 - V_{AV}{}^2}{2\, V_{IA}\, V_{IV}} \right)^2 - 1 \right]$$

$$= \ldots V_{IA} V_{IV}{}^2 V_{AV} \ (V_{IA}{}^2 + V_{IV}{}^2 - V_{AV} \ldots$$

[redacted]

$$= \frac{1}{4\, V_{IA}\, V_{IV}^{2}\, V_{AV}} \left[(V_{IA}^{2} + V_{IV}^{2} - V_{AV}^{2})(V_{IA}^{2} + V_{IV}^{2} - V_{AV}^{2} + V_{IV} - V_{IA} + V_{AV}^{2}) \right]$$

$$- \frac{V_{IA}}{V_{AV}}$$

Then:

$$\cos \gamma = \frac{(V_{IA}^{2} + V_{IV}^{2} - V_{AV}^{2})}{2\, V_{IA}\, V_{AV}} - \frac{V_{IA}}{V_{AV}}$$

$$= \frac{V_{IV}^{2} - V_{AV}^{2}}{2\, V_{IA}\, V_{AV}}$$

$$\frac{d\, V_{IV}}{dt} = \frac{d\, V_{AV}}{dt} \cos \beta + \frac{d\, V_{IA}}{dt} \cos \alpha$$

$$\frac{d\, V_{AV}}{dt} = - a \text{ from other program}$$

$$V_{IA} = W_{IE}\, (R_E + h) \cos \text{Lat}$$

$$\frac{d\, V_{IA}}{dt} = W_{IE} \left[- \sin \text{Lat}\, (R_E + h) \frac{d\Delta\, \text{Lat}}{dt} + \cos \text{Lat}\, \frac{dh}{dt} \right]$$

$$\frac{d\, V_{IV}}{dt} = - a \left[\frac{V_{IV}^{2} - V_{IA}^{2} + V_{AV}^{2}}{2\, V_{IV}\, V_{AV}} \right]$$

$$- W_{IE} \left[(\cos \text{Lat}) \frac{dh}{dt} + (\sin \text{Lat})(R_E + h) \frac{d\, \text{Lat}}{dt} \right]$$

$$\left[\frac{V_{IA}^{2} + V_{IV}^{2} - V_{AV}^{2}}{2\, V_{IA}\, V_{IV}} \right]$$

$$\frac{d\, \text{Lat}}{dt} = \frac{V_{IV}}{(R_E + h)} \sqrt{1 - \left[\frac{V_{IA}^{2} + V_{IV}^{2} - V_{AV}^{2}}{2\, V_{IA}\, V_{IV}} \right]^{2}}$$

[redacted]

[redacted]

$$\frac{d\,Lon}{dt} = W_{IE}\,(1 - \cos Lat) + \frac{V_{AV}}{(R_E + h)(\cos Lat)}\left[\frac{V_{IV}^{\,2} - V_{AV}^{\,2}}{2\,V_{IA}\,V_{AV}}\right]$$

M0010 READ V_{IV} , LAT , LON, DT 1

S0020

M0030 PUNCH HDG, SP2

E0040 TIME LAT LONG V_{IV}

M0050

S0060 SECS DEGS DEGS FPS

M0070 N = 0

M0080 R = 20902900 EARTH'S RADIUS

S0090 E

M0100 T = 0

M0110 W = .000071972 EARTH'S RATE (RADS / SEC)

S0120 IE

M0130 C = 4

M0140 READ A, H, V_{AV} , DH 2

S0150

M0160 $V_{IA} = W_{IE}\,(R_E + H)$ COS (LAT) 3

S0170

[redacted]

E0220

M0230 D LAT/DT = $(V_{IV}$ SQRT $(1 - M^2))$

S0240

M0250 / $(R_E + H)$

S0260

28397 FLAD SYSTEM 0000 04-08-59

E0270

M0280 D LON/DT = W_{IE} (1 - CL) + V_{AV} $((V^2_{IV}$ -

S0290

E0300

M0310 $V^2_{AV})$ / $2V_{IA}$ V_{AV})/$(R_E$ + H) CL

S0320

E0330

M0340 DV_{IV}/DT = - A $((V^2_{IV} - V^2_{IA} + V_{AV}{}^2)/2$

S0350

M0360 V_{IV} $V_{AV})$ - W_{IE} (CL DH + SIN (LAT)

S0370

M0380 $(R_E$ + H) DLAT/DT) M

S0390

M0400 IF C-4 NZ, GO TO 8

M0410 IF N-15 ZERO, GO TO 6

M0420 PUNCH T/54,(57.296 LAT) /52,

M0430 (57.296 LON) / 53, V_{IV}/ 55, SP2

S0440

```
M0450   N = N + 1
M0460   GO TO 7
M0470   PUNCH T/54, (57.296 LAT) /52,      6
M0480   (57.296 LON/53, V_IV/55, SKIP
S0490
M0500   PUNCH HDG, SP 2
E0510   TIME LAT LONG V_IV
M0520
S0530   SECS DEGS DEGS FPS
M0540   N = 0
28397   FLAD SYSTEM                        0000 04-08-59
M0550   C =0                               7
M0560   DIFEQ T, DT                        8
M0570   C = C + 1
M0575   IF C-4 ZERO, GO TO 2
M0580   GO TO 3
M0590   START AT 1
```

APPENDIX D

USE OF ELECTROMAGNETIC ENERGY AS A MEANS OF NAVIGATION

In examining the various possible means of navigation of a satellite supply vehicle consideration must be given to the possibility of using as a means of navigation, or as an assist to another means of navigation, information gained through the use of electromagnetic energy propogation.

D-1. Propagation

Propagation of electromagnetic energy through the atmosphere has been studied for many years and its properties are well known. Due to the relatively great height of the satellite supply vehicle, however, the possibility exists of using so-called "line of sight" frequencies for contacting the vehicle at great range, the propagational horizon having been greatly extended by the height of the vehicle (see Figure D-1). If the electromagnetic energy is to be utilized only for communication or establishing a data-link between the vehicle and the ground the great range presents only one problem - that of obtaining sufficient transmitted power[37].

For electromagnetic navigational systems, however, an atmospheric effect, pronounced at great ranges, which will have considerable effect on the accuracy of the system, is refraction of the energy by the atmosphere in the vertical plane.

Electromagnetic energy is refracted by the atmosphere at a standard value of radius of curvature of four-thirds of the earth's radius. Meteorological conditions other than standard will cause this value to vary from about 1.1 earth's radius to 1.6 earth's radius[38].

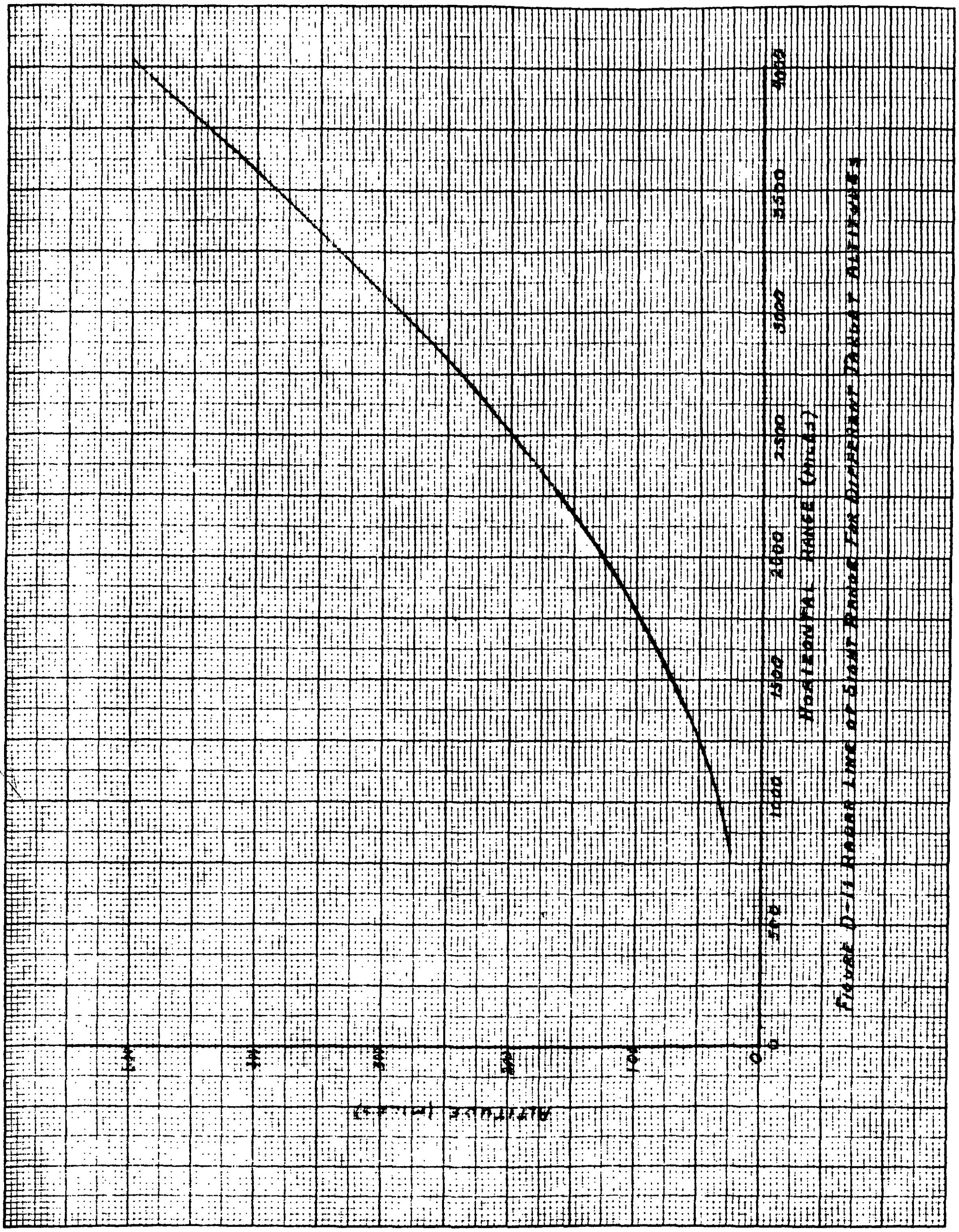


FIGURE D-11 RADAR LINE OF SIGHT RANGE FOR DIFFERENT TARGET ALTITUDES

Under some conditions, particularly trapping of a cool, moist layer of air under a warmer layer, complete beam trapping may occur giving greatly extended horizontal coverage but blind vertical coverage.

The effects of the variation in atmospheric bending are three-fold. First, the "horizon" of propagation will vary with the amount of refraction experienced; obviously the condition of least bending, where the horizon is at its shortest distance from the transmitting site would have to be provided for, to give reliability of coverage in any electromagnetic navigation system envisioned. Secondly, for any navigation system utilizing the measurement of time of travel of energy for establishing position, as range for conventional radar, there exists an uncertainty in the length of the propagation path, due to the variation in refraction, which may be considerable. Thirdly, for conventional directional radar systems height measurement becomes inaccurate and unreliable.[39]

If extremely high frequencies are to be considered, an additional factor bearing consideration is the absorption of electromagnetic energy by oxygen and water vapor, the only atmospheric gases having both permanent dipole moments and energy level spacings of the appropriate value to be of interest in the propagation of energy at wave lengths above 1 millimeter. The absorption effects of these gases become noticeable at wave lengths below 1.5 centimeter, and are very pronounced below 1 millimeter, the total effect being to render the atmosphere nearly opaque for wave lengths between 1 millimeter and the visible region.[40]

Ionospheric effects on propagation must be considered if electromagnetic navigation is to be used at heights in or above the ionosphere. Of first concern in this respect is the well known reflection of electromagnetic energy by the ionospheric layers. It has been shown[41] that the energy transmitted through a stratified ionosphere of four layers is not appreciably attenuated due to reflection and magnetoionic splitting at frequencies above 40 megacycles per second, the ratio of transmitted energy to incident energy for an incidence angle of 10^{o} at 40 megacycles per second being .99984. These results can be extended to other incidence angles since it has been shown[42] that oblique-incidence results at frequency (f) can be related to results at other incidence angles using the relation, f cosi, where (i) is the angle of incidence.

Introduction of noise into radar signals by the ionosphere and by meteoric trails is present and could be a factor for consideration. Insufficient knowledge is available on the subject to give quantitatively these effects.

Investigation of the propagation of electromagnetic energy through the ionized air surrounding a hypersonic aircraft[43] has shown that for speeds up to about 25,000 feet per second and altitudes up to about 250,000 feet vacuum wave lengths less than about one millimeter are necessary fo transmission through the nose region of a blunt nosed aircraft, and wave lengths less than about one meter are necessary for transmission through a high speed boundary layer. Little data is available for transmission through the wake region.

The conclusion indicated by the above data is that electromagnetic propagation to and from a satellite supply vehicle is possible and reliable, within the limits of ability to predict the propagational path, at frequencies corresponding to wave lengths between one meter and one-and-a-half centimeters. This is possible throughout the entire flight. Height information obtained by conventional radar at long ranges is, however, to be considered unreliable.

D-2 Systems

In studies of the flight path of the satellite supply vehicle it can be seen that the total distance flown, from the time it leaves the circular satellite orbit until the gliding phase of flight begins, encompasses more than three quarters of the earth's circumference.

To place the entirity of this distance within the range of the stations of a ground based electromagnetic navigation system would require at least eight powerful stations for a given descent path. To provide a system which would give such coverage for a number of different satellite orbits and vehicle landing points would require a very extensive system of powerful navigation stations and associated communications and computation facilities. An alternative to this type of system is one which is contained within the supply vehicle. Examples of self-contained systems utilizing electromagnetic propagation are those which navigate through radar map-matching techniques and Doppler velocity and positioning techniques. The former, although possibly useful over part of the flight, cannot be used over the entirity since a good portion of the flight must be assumed to be over water, which offers no identifiable radar echoes useful for navigation. The latter system is essentially an accurate dead-reckoning system. It requires a knowledge of the vertical direction or its equivalent in order to resolve the measured velocity components.

In addition to the systems which use electromagnetic propagation exclusively there exists the possibility of their utilization for occasional fixes or for furnishing only part of the navigational information (as radar height finders) in a system which basically uses some other means of navigation, such as stellar or inertial. The decision to use or not to use such a system must weight accuracy gained and relaxation of necessary specifications on other equipment and against additional weight in installing such an assist, additional complexity, etc.

APPENDIX E

CELESTIAL NAVIGATION

In order to determine one's position in three dimensional space, three independent coordinates are required. In classical celestial navigation, the surface of the earth forms one of these coordinates. Another convenient coordinate is the locus of points forming a fixed angle with two arbitrary points; in particular, the cones formed by the locus of points having a given altitude angle between a point at infinity, and a point located at the center of the earth. It is seen that two cones plus the surface which is the locus of points located a distance from the center of the earth will give two point locations in three dimensional space, and an ambiguity must be solved. This is true since the intersection of each cone with the sphere is a circle, and the intersections of the two circles will result in two points.

From Figure E-1 it is seen that if the point 0 is taken as the center of the earth, then the direction 0-AP is the direction of the vertical.

From Figure E-2 the intersection of each cone with the sphere is a circle, and the intersection of the two circles is the position on the earth specified by the angles ϕ_1 and ϕ_2 with respect to the points at infinity. The two infinite points may be chosen so that the undesired solution is remote compared to the desired one, and the ambiguity is resolved.

With a basic knowledge of celestial navigation[44] the minimum requirements for the instrumentation of the solution of the navigation problem may be determined.

There are:

1. Knowledge of the approximate position

2. A device to measure the altitudes of the stars.

3. An accurate knowledge of time.

4. Storage for information on stellar positions.

5. Computing devices and associated circuitry.

The chief advantage is gained in knowing the approximate position when the stars are obscured, or when the tracking mechanism loses the star for a period of time. In such cases, a form of dead reckoning may replace the stellar navigation until such time as the stars may again be tracked. A device to measure the altitude angle of the stars is of course necessary, since this is the essence of celestial navigation, and the accuracy to which position may be determined is based on the accuracy with which the altitude of the stars can be measured from some useable reference plane. The requirement for accurate knowledge of time is obvious.

In an automatic celestial navigation system, the human operator is eliminated, and a substitute must be made for his use of charts and tables in the solution of the problem. This is easily done by putting the required data in a computer type storage, and having automatic access at a predetermined time. The same reasoning tells us that we need automatic computing devices to solve the problem, given all of the necessary information.

Consideration of the factors involved here show that there are at least two methods by which we may proceed in the solution of the problem. We may assume our position, and by consulting our time standard and the tabulated star data, position the telescope in the sighting head so as to bring the desired stars into the field of view. Any errors which the sighting head detects may then be applied to the assumed position so as to give the correct position. We may, however, assume a position on a predetermined course, and proceed as before, except that we will now use the errors detected by the sighting head to control the vehicle control system so as to drive the vehicle back onto the precomputed course.[45]

Moreover, it is evident that if we are on a known track, such an automatic stellar sighting and computing device will give us an excellent indication of true heading and as such may be used in the orientation of the vehicle. In this case, the stars azimuth, as determined from the position along track, is computed. Then the true heading is the difference between the relative bearing and the azimuth.[46]

In this thesis it is anticipated that the use of the device will be as an astro compass for vehicle orientation during the orbital period, and as a vertical indicating device during the latter part of the descent.

The conventional complete star tracker[47] may be divided into three groups:

1. The tracking head.
2. The control and data section.
3. Computer and associated electronics.

The tracker is mounted against the top skin of the aircraft with a small dome protruding above the skin. The tracker contains a small photoelectric or other light sensing device, with a telescope which tracks the stars, and is controlled about two axes; relative bearing and altitude. The telescope is mounted on a stabilized platform. The telescope must be arranged so that it may be swung through angles corresponding to time, siderial hour angle, and the longitude of the craft. (See Fig. E-3).

An illustrative example of an existing star tracker will be discussed to illustrate the accuracy, etc. of such a device. The telescope and associated light sensitive device will be of the standard "chopper" type having a disc in front of the light sensitive material which, when the rotation of the disc is compared to the light falling on the sensing material will give a phased difference if there is an error in positioning. (See Fig. E-4).

It is presumed that the system will be capable of detecting 2nd magnitude stars, and should be made as small as possible due to anticipated lack of space. Assuming a 1 inch diameter lens, on the earth's surface, a second magnitude star gives the light of 2×10^{-10} lumens of collected light and this figure should be many times greater above the atmosphere. A field of view approximately 1 to $2\frac{1}{2}$ degrees in diameter will probably be required to assure acquisition of the star, and, in conditions of full daylight, the light from 1 degree of sky will be 10^6 times that from a second magnitude star. Therefore, background illumination must be minimized by appropriate chopping and filtering.

It is presumed that while in orbit, the position of the vehicle can be precomputed to the order of feet; therefore, the position of the stars as a function of time is known with excellent accuracy. These may be stored on perforated discs or on steel magnetic tape, and by using an electrically driven tuning fork as a time reference can be fed to the tracking system in the form of aximuth and altitude. When the telescope is thus positioned on a star, any error in sighting the star can be fed out to display to the pilot as error in heading (assuming the craft to be stabilized in pitch and roll).

The above discussion is easily extended such that if we use two telescopes and have our data precomputed properly for any desired track, any error that the sensing devices detect will be an indication of the position error, and by using "feed-back" techniques a continuous indication of position error can be presented to the pilot, and the proper corrective action may be taken. (See Fig. 5).

With equipment now in use, time may be resolved to .0006 seconds and the telescope can track a star to within 15 seconds of arc.[48] It is estimated that the above described system, including the tracking instruments and computers, would weight approximately 250 to 300 pounds and will occupy approximately 15 feet ^{3}

of space. We would be able to track a star of 4.0 corrected relative magnitude only 5° from a full moon and a first magnitude star[49] to within 15° of the sun.

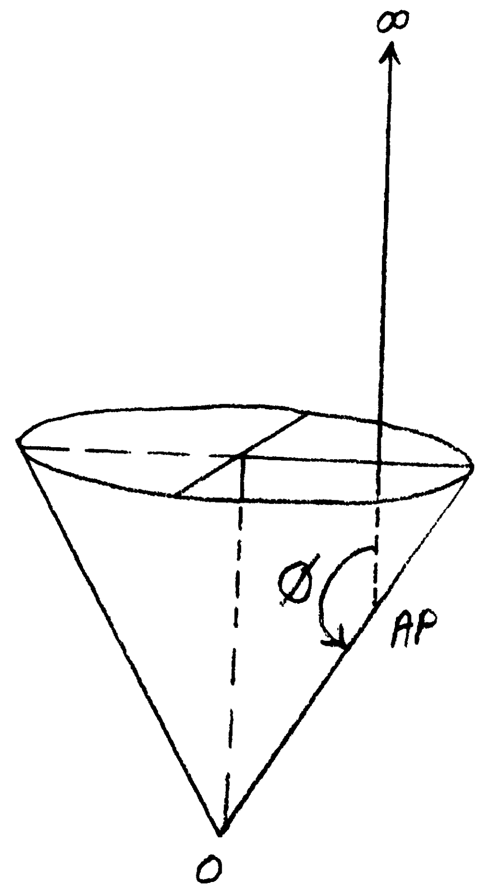


Figure E-1: Locus of points forming the angle Ø with a point at infinity and a point O at the earth's center.

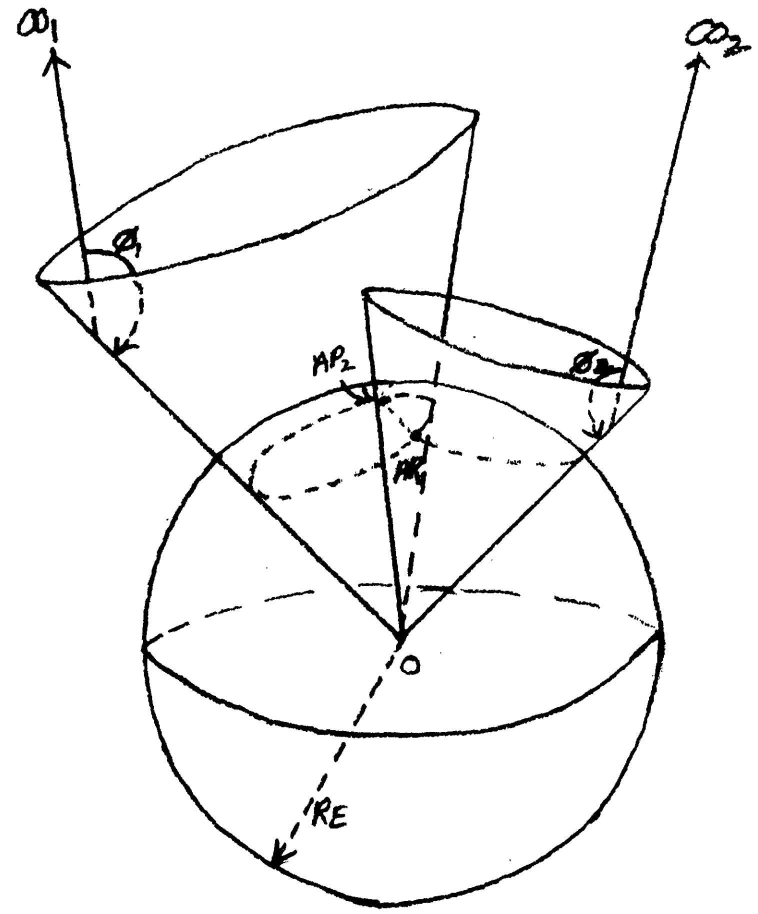


Figure E-2: Points AP_1 and AP_2 are solutions of the intersection of the loci forming the two cones, and the locus forming the sphere.

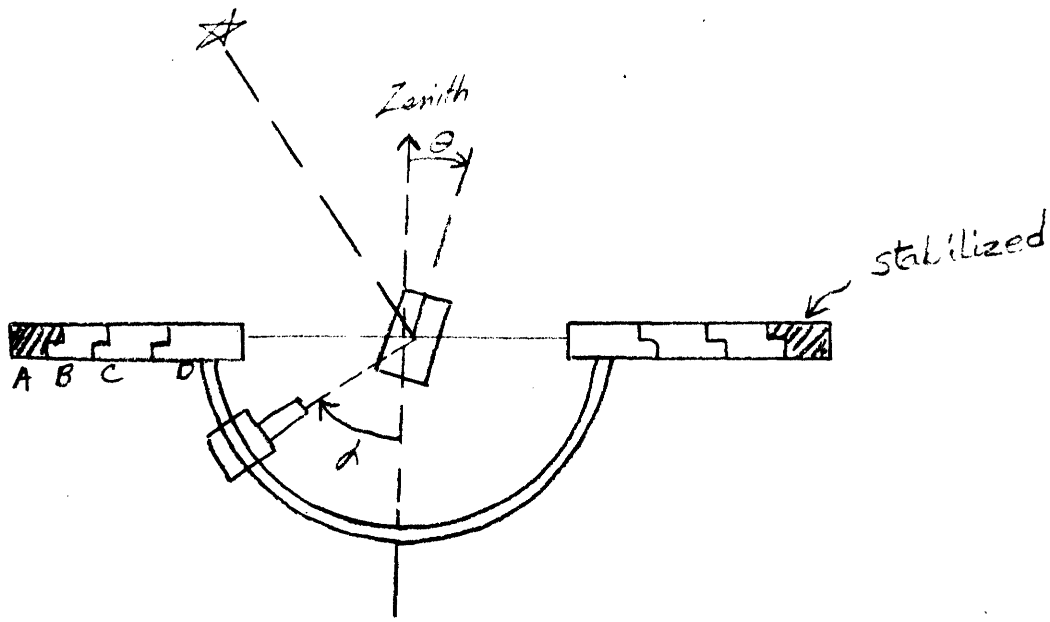
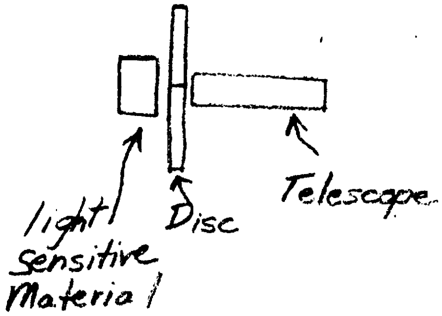
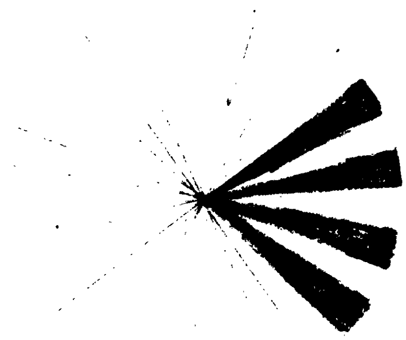


Figure E-3: Ring A is turned so as to indicate the course and position, the mirror axis is east and west. It is also mounted on ~~part of~~ the stable platform.
Ring B turns through an angle equal to SHA.
Ring C is positioned according to time. 50
Ring D is positioned according to longitude.

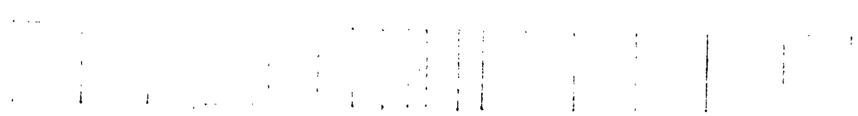
$\theta = \pi/4 - \text{lat}/2 \qquad \alpha = \pi/2 - \text{dec}$





Disc with alternate clear and dark segments

Signal when star is centered in tracker.



Signal with star not centered in tracker.

Figure E-4

[redacted]

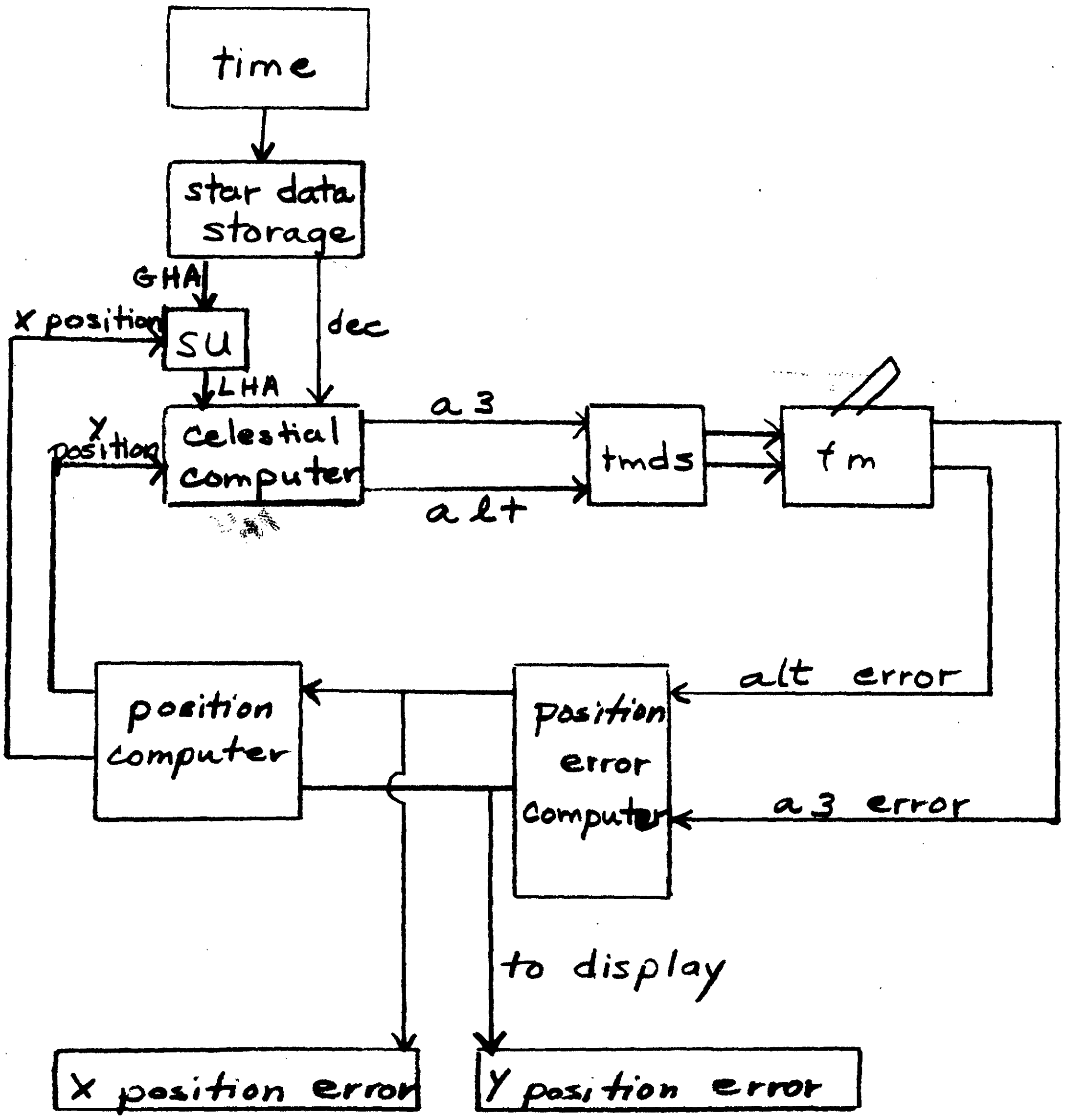


Figure E-5: Illustration of the use of tracking errors to give position errors.

APPENDIX F

NAVIGATION SYSTEM REQUIREMENTS

F-1. Mission

The navigation system must be capable of giving guidance information of sufficient accuracy and utility to the pilot, or automatic control system such that the supply vehicle may use this information to accomplish a satisfactory return to earth from an orbital condition to a desired landing area. This capability should be limited only by the inherent control capabilities of the supply vehicle and such other limitations placed on the vehicle and/or its occupant as maximum tolerable accelerations, heating, etc.

F-2. Landing Area

The landing area referred to in the mission requirements above requires further elucidation before any of the accuracy requirements of the system can be evaluated. It is assumed for the purposes of this thesis that the landing area is a well defined area on the earth's surface, much like a conventional landing field, with whatever modifications might be necessary to allow landings of the supply vehicle at the end of the return trajectory. By this assumption, very stringent navigation accuracy requirements are placed on the navigation system. The navigation system proposed by this thesis is designed to meet these stringent requirements. If the specifications on the desired landing area are relaxed for one reason or another, the requirements placed on the navigation system are correspondingly relaxed. Indeed, with a sufficiently large region in which landing is to take place, the navigation requirements degenerate to such an extent that they become almost control system requirements, such as a reference for altitude stabilization, etc. It would be an intersting and important area for further research to see just how far the landing area requirements would have to be relaxed in order to simplify and/or eliminate portions of the navigation system proposed by this thesis. For example, it might be possible to eliminate the need for altitude information during the flight if the resultant position errors at the end of the trajectory are compensated for by a comparable extension of the permissible landing area.

F-3. Flight Path Control

The portion of the return trajectory which takes place in the atmosphere was studied in this thesis on the basis of assumed initial conditions and with the aid of a high speed calculating machine (see Appendix C). The study was purely exploratory and was carried out in order to get ground track information on a typical supply vehicle return trajectory. As such, it was not deemed feasible to extend the study to a machine aided error analysis wherein the sensitivity of the atmospheric trajectory to small variations in the initial conditions would have been determined. In place of this, a very approximate linear error analysis was carried out to establish system requirements. In any future research done to establish how far the stringent requirements of this study could be relaxed, a necessary preliminary would be the machine aided error analysis of the assumed atmospheric trajectory.

A glance at the assumed initial conditions for the atmospheric trajectory (see Appendix C) and a comparison of these assumptions with the possible variations in their magnitudes (i.e. variations in the atmospheric density structures,[51] the inaccurate estimation of aerodynamic coefficients,* etc.), plus a look at some typical re-entry trajectory studies wherein the initial conditions were

*Note from Appendix C that for a portion of the trajectory the variation in velocity is in the fifth significant figure, while the aerodynamic coefficients can be considered accurate to only 2 or 3 significant figures.

varied[52] (such as re-entry angle,* velocity,** etc.) makes it apparent that the uncertainties involved in the prediction of the exact atmospheric trajectory that would be taken by the satellite supply vehicle under some control condition, such as $L/D = 1.0$ as assumed for this thesis, are too great*** to allow the supply vehicle to proceed through the atmospheric re-entry trajectory without the application of control corrections based on navigational information. In short, a flight path control system is necessary to accomplish the return mission. The initial estimate of the trajectory to be followed need then only be sufficiently accurate to keep the control corrections from forcing the vehicle into an unacceptable flight condition with respect to heating or acceleration, etc.

F-4. Initial Condition Errors

It may be seen from Appendix B that the 300 mile initial orbit places the most stringent conditions on most of the variables of interest; therefore, the 300 mile orbit will be taken for the evaluation of the remainder of the navigation system requirement considered in this thesis.

Velocity in orbit is known to ± 1.00 foot/second for time up to one-half an orbital period after the last ground tracking period, and to ± 1.15 feet/second for time up to two days after the last ground tracking period (see Appendix B-1). Initial flight path angle

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*An extrapolation of the curves of NACA TN 4276 gives an estimated sensitivity of ground range in going from 300,000 feet altitude to 90,000 feet, approximately 30 miles $1/4^{\circ}$ change in re-entry angle around the assumed re-entry angle of 0°. As the angle of re-entry increases, the sensitivity of range to re-entry angle increases enormously.

**An extrapolation of the curves of NACA TN 4276 gives an estimated sensitivity of ground range in going from 300,000 feet altitude to 90,000 feet altitude to initial velocity of approximately 30 miles/foot/second.

***It is to be noted that the ability of the supply vehicle to exert control and correct for positional errors at 90,000 feet in the glide from 90,000 feet to the landing point is limited (see Appendix C).

is known to $\pm$ .2 minutes of arc (see Appendix B-3). Initial position is known to either $\pm$ .5 miles or $\pm$ 5 miles depending upon the time between ground tracking and the firing of the first retro-thrust as above (see Appendix B-1).

F-5. Initial Retro-Thrust Requirements

While in orbit the supply vehicle must get from some source

a. the time of firing,

b. the magnitude of velocity change to be applied, and

c. the direction of the retro-thrust application.

F-5a. Time of Firing

With a clock aboard the vehicle with an accuracy of 1 part in 10^6, the time at the supply vehicle may be determined to within $\pm$ .1728 seconds for periods as long as two days. Travelling at 25,000 feet/second this would represent a range increment of $\pm$ 4,320 feet, or less. The same clock would, if operated over a one half orbital period, give an incremental error corresponding to 75 feet, or less. Such a clock is considered adequate for the determination of time at the supply vehicle. The time to fire the first retro-thrust is a ground computed quantity based upon the knowledge of the satellite orbit; therefore, this quantity may be transmitted to the supply vehicle during the last period of ground tracking.

F-5b. The magnitude of velocity change

The incremental velocity to be applied to the supply vehicle in order to have a perigee of 300,000 feet is also a precomputed quantity based upon ground tracking information, and as such it may also be transmitted to the supply vehicle during the last period of ground tracking prior to orbital departure of the supply vehicle. The signalled magnitude may have any number of significant figures, but it will be assumed that the supply vehicle can control this retro-thrust magnitude to only ± 1 foot/second accuracy.

F-5c. Retro-Thrust Direction

The direction in which to point the retro-thrust must be referred to at least one physical direction held somewhere within the supply vehicle. This direction might conveniently be the direction of the indicated velocity of the supply vehicle, but if any orbital corrections were envisioned using the retro-thrust aligned to some direction other than the direction of motion, one more physical reference direction is necessary. This direction might easily be the indicated vertical, but it would not necessarily have to be. The indicated directions would not necessarily coincide with the true directions, so that orbital perturbances from retro-thrust misalignment are to be considered probable.

$$\gamma d = \tau \frac{\Delta V}{V} = .028\tau \quad \text{for 300 mile orbit}$$

(see Appendix B-9)

$$\gamma d = .0007^\circ \text{ or } .42' \quad \text{for } \tau = \frac{1}{4}^\circ \text{ or } 15'$$

(see Appendix A-1 for symbols used in this appendix)

F-6. Initial Error Propagation

From B-12 and B-3 it is seen that

$$\frac{\partial \Delta V}{\partial \gamma_o} \text{ required} = (-22.2 \times 10^4)\, \gamma_o$$

which becomes (at a maximum) for the 300 mile orbit

$$(-22.2 \times 10^4)\left(\frac{.08}{57.3}\right) = -\ 310\ \frac{\text{ft/sec}}{\text{radian}}$$

From B-3, the maximum error in γ_o at the first retro-thrust is ± .2 minutes of arc or .003 degrees. Assuming for the moment that the control of the retro-thrust direction is to within $\frac{1}{4}^\circ$ of the desired direction, the resultant total error in γ_o after the first retro-thrust becomes..01 degrees. Thus, the first retro-thrust velocity may be in error by as much as $\pm$ .054 foot/second due solely to this cause. This is certainly a neglible figure and will be neglected.

The error in γ_0 propagates to the re-entry condition as an error of approximately the same magnitude in the same variable. Added to this is another misalignment error due to the second retro-thrust. Assume this second retro-thrust also to be controlled to within $\frac{1}{4}^{\circ}$ of the correct direction, and the resultant re-entry angle error might be as large as $\pm$ .017 degrees. A linear approximation to the errors caused by such re-entry direction errors follows.

Extrapolating the curves of NACA TN 4276 to find the figure of range error at 90,000 feet, due to re-entry angle, we arrive at approximately 120 miles per degree. The above figure would correspond to a range error at 90,000 feet of approximately 2 miles. The total range travelled is approximately 7,200 miles. The height error at the end of this travel, due to initial misalignment, is approximately 120 miles per degree of initial misalignment. Therefore, the error in height at what is supposed to be 90,000 feet could be as great as two miles, due to re-entry angle errors alone.

Any range or height errors of larger magnitude than this will be considered as unacceptable for the present navigation system, and the following system requirement is thereby established; the vertical direction and the direction of motion (or whatever other references might be used) must be known at both retro-thrust conditions to within $\frac{1}{4}$ degree for the purposes of controlling the retro-thrust.

The velocity error at the first retro-thrust will propagate as a time or position error at the second retro-thrust and will also give a velocity error at the second retro-thrust.

$$V_P = \sqrt{\frac{\mu}{a}}\sqrt{\frac{1+e}{1-e}}$$ which for small eccentricities becomes

[redacted]

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$$V_P = \sqrt{\frac{\mu}{a}}\,(1 + e)$$

$$EV_P = \frac{\partial V_P}{\partial a}\,\frac{\partial a}{\partial \Delta V}\cdot E\Delta V + \frac{\partial V_P}{\partial e}\cdot\frac{\partial e}{\partial \Delta V}\cdot E\Delta V$$

$$= (-1 - 1.77)\,E\Delta V = -2.77\,E\Delta V$$ for the 300 mile orbit.

Thus, the uncertainty in re-entry velocity could be as high as (2) (2.77) = $\pm$ 5.54 feet/second for the case where the first retro-thrust occurs soon after the last tracking interval, and 5.96 feet/second for the case where a delay of two days is experienced prior to firing the first retro-thrust. For simplicity, call the short delay case 1 and the long delay case 2.

The uncertainty of control in the second retro-thrust must be added to this, and assuming this control is also accurate to $\pm$ 1 foot/second, the re-entry velocity uncertainty becomes $\pm$ 6.5 or $\pm$ 7.0 feet/second for cases 1 or 2 respectively. Uncorrected, these velocity errors at re-entry propagate to range errors at 90,000 feet at the rate of approximately 30 miles/foot/second. This would infer a range error at 90,000 feet altitude of as much as 195 miles for case 1 or 210 miles for case 2 in the absence of a flight path control system to eliminate these errors.

In the event of a flight path control system being utilized which was unaware of these initial velocity errors, we can assume that these velocity errors remain practically constant over the time of flight of the programmed trajectory, approximately 1,800 seconds. These velocity errors then give rise to range errors of $\pm$2.2 miles and $\pm$ 2.4 miles respectively, far more nearly acceptable figures !

The height error at re-entry due to the velocity error at 300 miles becomes:

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[redacted]

$$Eh_P = \frac{\partial h_P}{\partial \Delta V} \, E\Delta V \text{ (see Appendix B 5)}$$

$$= - \; 3,342 \; E\Delta V$$

$$= \mp 18,500 \text{ ft. for } E\Delta V = 5.5 \text{ ft/sec.}$$

$$= \mp 19,900 \text{ ft for } E\Delta V = 5.96 \text{ ft/sec.}$$

The errors in height at re-entry generated by initial velocity errors lead to over all range errors which oppose the effect on range errors of the velocity errors at re-entry generated by the same initial velocity errors at orbital departure. Thus, if it were not for the height divergence problem, these two might be said to cancel each other out somewhat and their effects might be neglected in the resultant trajectory. The height divergence problem is discussed in F-9.

The range error at re-entry due to an initial velocity error becomes:

$$(ET)\ (25,000 \text{ ft/sec}) = 25,000 \, \frac{\partial T}{\partial a} \, \frac{\partial a}{\partial \Delta V} \; E\Delta V$$

$$= (- \; .311 \text{ sec/ft/sec})(25,000) \; E\Delta V$$

$$= 8.15 \text{ miles for case 1.}$$

$$= 8.75 \text{ miles for case 2.}$$

If we add to these the range errors due to uncertainty of initial conditions (and time determination for case 2) we get range errors at re-entry of 8.65 miles for case 1 and 10.17 miles for case 2.

Finally, if we add to these inherent range errors the navigational uncertainty of the direction of the vertical of up to $\frac{1^\circ}{4}$ during the re-entry period, (which amounts to a fifteen nautical mile range uncertainty) we note that in the absence of control corrections, the range errors at 90,000 feet might be as large as $\pm$ 25 miles due to these causes alone. This is considered unacceptable for the present navigation system, and it sets the following requirement; sometime prior to re-entry, the supply vehicle must be given a

[redacted]

navigational fix in a form which it is capable of utilizing during the re-entry period in order to cut down the range errors noted above. This navigational assist may take any form whatsoever, but based upon flexibility arguments it would seem better to have the assisting equipment mobile. A large specially instrumentad aircraft has been assumed as the assisting unit for this thesis. The aircraft tracks the supply vehicle for a short period of time prior to re-entry and then passes on all of the navigation information obtained which is of any use to the supply vehicle. It is not sufficient to tell the supply vehicle its present position prior to re-entry, but some means must also be found for orienting properly the flight path control instrumentation, i.e. essentially the navigation system during re-entry. This means that through information contained in electromagnetic signals from the assisting aircraft the supply vehicle must be able to physically orient some reference direction. This is a trick not easily accomplished, and it leads to the next porition of the system instrumentation.

F-8. Star Trackers

Various methods may be utilized to transfer electromagnetically a physical reference direction. Most of these require a penalty in weight not commensurate with the inherent accuracy achievable (such as active radar tracking by the supply vehicle and thereby establishing the line of sight, etc.).

One straightforward and extremely accurate method of transfer is to utilize star trackers to monitor a stable platform and then specify the angles which a reference direction on the stable platform should make with the directions to the stars (see Chapter 5). Accuracies of better than 15 seconds of arc seem possible by this method (see Appendix E). The stable platform may be utilized as an inertial reference with a high degree of accuracy for short periods of time, and the star trackers provide the means for correcting the errors generated with time in the stable platform.

[redacted]

During the re-entry phase, the stable platform will be utilized as the prime component of a precision vertical indicating system which is Schuler tuned to eliminate the dynamic efforts of linear acceleration.

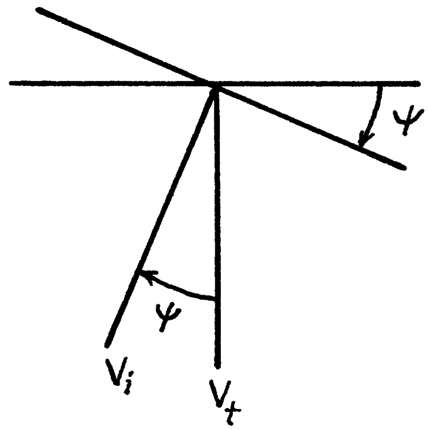
The problems associated with the calculation of height information in utilizing such an inertial system lead to the inclusion of a separate height measuring device to complete the flight path control system (along with a special purpose computer, of course).

F-9. Height Divergence

In the problem of using an inertial platform for navigational data the question of how to instrument the knowledge of altitude becomes important. The usual method is to calculate the motion of the vehicle with respect to inertial space including a term for the calculated value of gravity that corresponds to the calculated position. In any such closed loop process the possiblity of divergence exists. Russell[53] has shown that such a divergence does exist in the knowledge of the height coordinate, by a simple calculation involving a linear approximation to the gravity term around an inertial point. An extension of this approximation is required for this thesis. The period during which the inertial navigation system is to be utilized to command control corrections is the re-entry period, and an accurate knowledge of altitude is essential to that control. Consequently, it is only in the re-entry period that the divergence error becomes important.

A glance at the assumed flight trajectory of this thesis shows (see Appendix C), 1) a constant L/D of 1.0 is assumed, 2) the acceleration of the vehicle in the horizontal plane can be fairly well approximated as due to drag alone, since the local flight path angle is small for most of the trajectory; making this approximation leads to an approximation of this acceleration of (- .0164t) ft/second^2, 3) and the vertical lift contribution can therefore be approximated as (+ .0164t) ft/sec^2 .

Let ψ be the angle between the true and indicated verticals in the plane of the true vertical and the velocity vector measured (+) clockwise from the true vertical to the indicated vertical.



In the case of a vehicle flying in a condition where a net specific vertical force exists, ψ tends to be oscillatroy and bounded.[54] For the chosen trajectory a lift force acts essentially in the upward vertical direction, so that any value of ψ tends initially to decrease with time or converge toward zero. The resultant period of the oscillatory motion is so long, however, that the tendency to converge can be completely neglected over the short time of re-entry. By thus neglecting this convergent tendency, the worst misalignment possible may be examined. ψ may be due to:

1. an initial value ψ_o
2. drift times time $(D \cdot t)$
3. initial re-entry velocity uncertainty creating platform rate error

$$\left(\frac{EV_o}{R_o} \right)$$

4. uncertainty in knowledge of height coupling through system dynamics into ψ (neglect this source entirely for the small height errors required of this system).

Therefore, $$\psi = \psi_o + \frac{(EV_o)}{R_o} t + Dt$$

Let:

a_v be the reading of a truly vertical accelerometer $= .0164t$

a_{vi} be the reading of an accelerometer whose input axis is the indicated vertical

$$a_{vi} = a_v \cos \psi + a_H \sin \psi \approx a_v (1 - \psi)$$

$$a_{vi} = a_v + Ea_v \quad \text{(where } Ea_v \text{ is an error in the vertical accelerometer reading)}$$

$$Ea_v = - a_v \psi$$

$$Ea_v = - .0164t\ \psi_o - \frac{EV_o}{R_o} t^2 (.0164) - .0164\ Dt^2$$

Regardless of the coordinate system used for the computation in the general navigation problem, the calculation of the height with respect to the earth comes down to the solution of the equation $\ddot{h} = - g + a_v + \frac{V^2}{R}$, with known initial conditions.

The instrumentation of this equation would give;

$$\ddot{h}_i = - g_i + a_{v_i} + \frac{{V_i}^2}{R_i}$$

(i) as a subscript here refers to both indicated and calculated quantities.

Therefore;

$$E\ddot{h} = (-Eg) + (Ea_v) + (E\frac{V^2}{R})$$

Using a linear approximation to the variation of g with h,

$$E_g = - \frac{2g_o}{R_o} (Eh)$$

For convenience take, $g_o = 32.2$ ft/sec., $R_o = 21.2 \times 10^6$ ft.

$$\frac{2g_o}{R_o} = 3.035 \times 10^{-6} \qquad \sqrt{\frac{2g_o}{R_o}} = 1.742 \times 10^{-3}$$

and an approximation to,

$$E\frac{V^2}{R} = \frac{2V(EV_o)}{R_o}$$

where $V = V_o - .0164t$

Thus,

$$E\frac{V^2}{R} = \frac{2(EV_o)\,V_o}{R_o} - \frac{2(EV_o)(.0164t)}{R_o}$$ take V_o = 25,000 ft/sec for convenience.

Therefore,

$$E\ddot{h} - (1.742 \times 10^{-3})^2 \; Eh = -(.774 \times 10^{-9}(EV_o) + .0164D)t^2$$
$$-(1.548 \times 10^{-9}(EV_o) + .0164\psi_o)t$$
$$+ (2.36 \times 10^{-3} (EV_o)$$

Solving analytically and assuming $E\dot{h}_o = 0$ (which is not strictly true);

$$Eh = (.5Eh_o + .305 \times 10^3 \, EV_o - .177 \times 10^{10} D + .140 \times 10^7 \psi_o)e^{-1.742 \times 10^{-3} t}$$
$$+(.5Eh_o + .305 \times 10^3 \, EV_o - .177 \times 10^{10} D + .140 \times 10^7 \psi_o)\, e^{-1.742 \times 10^{-3} t}$$
$$+ (.255 \times 10^{-3} \, EV_o + .54 \times 10^4 D)t^2$$
$$+ (.510 \times 10^{-3} \, EV_o + .54 \times 10^4 \, \psi_o)t$$
$$+ (.355 \times 10^{10} D - .61 \times 10^3 \, EV_o)$$

Setting t = 1,800 seconds,

$$Eh = 11.5 \, Eh_o + 7.2 \times 10^3 \, EV_o - 2 \times 10^{10} D - 2.25 \times 10^7 \psi_o$$

Assume a very good fix just prior to re-entry;

Assumption	Contribution to Eh at t = 1,800 sec.
$Eh_o \leq 50$ ft	525 ft
$EV_o \leq 1$ ft/sec	7,200 ft
$D \leq 1.2$ minutes/hour	2,000 ft
$\psi_o \leq 1'$ or 2.9×10^{-4} rad.	6,500 ft.
Total	16,225 ft. Eh at 90,000 ft.

Recall, this ignores $E\dot{h}_o$ which could be appreciable !

F-10. Height Information

Since height information is one of the necessary coordinate values utilized in a flight path control system, it must be a known or accurately calculable quantity during the re-entry portion of navigation. Since the calculation process is shown to lead to unacceptable height errors, the alternative appears to be to include a height sensing device within the vehicle for this information. After considering the problems of mounting pressure measuring devices, etc. as discussed in Chapter 2 of this thesis, it was decided that a radar altimeter was the necessary choice for this altitude information. As long as altitude errors do not exceed 4 or 5 thousand feet, they will then have neglible effect upon the rest of the system's accuracy. Thus, a small and fairly long pulse radar altimeter may be utilized for this information.

F-11. Drift Rate

With the system envisioned in this thesis, a navigational fix may be supplied to the supply vehicle just prior to re-entry and thereby eliminate the accumulated drift effects on platform misalignment. The resulting re-entry trajectory occupies too short a time to allow very serious navigation errors due to drift rate. The criterion as to how much drift is acceptable then becomes the already selected lvel of $\frac{1^{\circ}}{4}$ control over the direction of the first retro-thrust combined with the amount of monitoring of the stable platform by star trackers desired. Taking the platform by itself and assuming it perfectly aligned during a portion of the last ground tracking interval, then a drift rate of .31 minutes of arc/hour would lead to an error of $\frac{1^{\circ}}{4}$ after a two day delay interval up to the time of firing the first retro-thrust.

The above figure is a fairly low drift rate, but it is feasible with present equipment.[55] A better solution to this problem would appear to be to work out a monitoring program whereby the star trackers corrected for platform drift. This is what is done in the system designed in this thesis. On this basis, the drift rate may be as high as 5 minutes per hour with a resulting error in position of only 2.5 miles at 90,000 ft. altitude due to this source.

F-12. Summary of Requirements

a. Gyro monitored inertial platform used as a precision vertical indicator during re-entry.

b. Precision accelerometers associated with the vertical indicator.

c. Gyro drift rate low, i.e. 5 minutes/hour or less, and the knowledge of reference directions to $\frac{1^\circ}{4}$ or less prior to the application of any retro-thrust.

d. Radar altimeter for height information.

e. Star tracking equipment to monitor the inertial platform and to provide a means of transferring information contained in electromagnetic signals into useful physical orientation of apparatus.

f. Special purpose computer for the navigation problem in the supply vehicle.

g. Precision tracking equipment and special purpose computers, plus an accurate navigation scheme for the assisting aircraft which eliminates many of the supply vehicle's navigational errors by providing a fix to the supply vehicle just prior to re-entry.

h. Precision tracking equipment located at the ground station and utilized in conjunction with high speed digital calculations. The same or similar equipment might be utilized to provide the ILS or GCA type approach information from 90,000 feet on down until landing.

The above equipment should provide an accurate and reliable method of navigation for the satellite supply vehicle from orbit to an earth landing at a desired landing field.

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