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SOME COMPLEXITY RESULTS ABOUT PACKET RADIO NETWORKS *

by

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ABSTRACT

It is shown that the decision problem regarding the membership of a point in the capacity region of a packet radio network is NP-hard. The capacity region is the set of all feasible origin-to-destination message rates where feasibility is defined as the existence of any set of rules for moving the data through the network so that the desired rates are satisfied.

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■ : marks the end of a proof.

$\in$: is an element of, belongs to.

$\notin$: is not an element of.

$\cup$: union of sets.

$\cap$: intersection of sets.

$\geq$: greater than or equal to.

$\subset$: is a subset of.

I. INTRODUCTION

A packet radio network (PRN) is a collection of geographically distributed and possibly mobile users which share a common radio channel for exchanging messages among each other. Among the distinctive features of a PRN are the burstiness of message traffic and the fact that not all users are necessarily within the line of sight of one another. PRNs present problems which are different in nature and more difficult than those encountered in wire or satellite networks, [1]-[4]. For a complete survey of the issues concerning PRNs, we refer to [1].

We are interested here mainly in the capacity region of a PRN which is defined as the set of all origin-to-destination (o-d) message rates that are achievable via any arbitrary protocol. (This definition will be made more precise in the next section; we should note however that the term capacity is not used in the information theoretical sense.) We show that the problem of determining whether a given point belongs to the capacity region of a PRN is NP-hard. This implies that there exists no polynomial time algorithm for determining the feasibility of a set of o-d rates unless there exist polynomial time algorithms for a large class of well-known combinatorial problems such as CLIQUE, 3-COLORABILITY etc., which have resisted such solutions despite many efforts.

Our analysis is based on time-division-multi-access (TDMA) schemes for which conditions for the feasibility of desired rates can be expressed as a linear program. The end result however is entirely general and does not depend on the use of TDMA schemes at all.

II. A PRN MODEL

We represent a PRN by a directed graph G=(N,A) such that for each user of the network there is one and only one node in N and there exists a link (a,b) in A iff node b is within the transmission range of node a.

Messages in a PRN are transmitted in the form of varible length packets. We assume that at any time at most one packet can be sent over any link. When a node sends a packet to one of its neighboring nodes, because of the broadcast property of the network, this packet reaches all the neighboring nodes whether or not each one of them is an intended receiver. For clarity, we say that a packet is transmitted over link (a,b) iff that packet is transmitted by node a and node b is an intended receiver.

A transmission over link (a,b) is said to be successful iff the packet in question is received correctly by node b. In our model the only cause of unsuccessful packet transmissions is interference between the transmissions of different users. Interference can be described as a binary relation on the set of links: $(c,d) \in A$ interferes with $(a,b) \in A$ iff $a \neq c$ and either $c = b$ or else $(c,b) \in A$. (Figure 1 a,b,c.)

We say that $(c,d) \in A$ conflicts with $(a,b) \in A$ iff either (c,d) interferes with (a,b) or $a = c$ and $b \neq d$. (Figure 1.d.) The latter case corresponds to the exclusion of simultaneous transmission of two or more different packets by the same node. We define C_{ab} to be the set of all links that conflict with link (a,b); thus, $C_{ab} = \{(c,d) \in A: \ c \neq a, \ (c,b) \in A\} \cup \{(a,d) \in A: d \neq b\} \cup \{(b,d) \in A\}$. The significance of C_{ab} is that a transmission over (a,b) is successful iff no link in C_{ab} attempts to transmit another packet simultaneously.

We assume that the average desired traffic rates are fixed for each o-d pair and denote the collection of all o-d rates by a column vector $\vec{r}$ whose $(x,y)^{th}$ row, r_{xy}, is the desired rate for o-d pair $(x,y) \in N \times N$, $x \neq y$.

If there exist network protocols which satisfy the desired rates for each o-d pair, then these o-d rates are called feasible. The capacity region $C(G)$ of a PRN G is the set of all feasible o-d rate vectors.

The problem of major interest here is to determine whether a given o-d rate vector $\vec{r}$ belongs to the capacity region $C(G)$ of a given PRN G. The $\vec{r}$-feasibility problem, as this problem is called, is difficult to formulate in a general setting due to the vague notion of existence of certain network protocols. For this reason, we initially consider a restricted problem by assuming that 1) $r_{xy} = 0$ unless $(x,y) \in A$ and 2) each (x,y)-packet (i.e. a packet with origin x and destination y) is sent directly over link (x,y) whenever $(x,y) \in A$.

Under restrictions 1 and 2, the desired flow rate f_{xy} across link (x,y) equals r_{xy} for each $(x,y) \in A$. We represent link flow rates by a column $|A|$-vector $\vec{f}$, and use the term $\vec{f}$-feasibility problem for the restricted version of the $\vec{r}$-feasibility problem.

The feasibility of $\vec{f}$ depends only on the timing and duration of individual transmissions; i.e. it involves no routing decisions. This is a scheduling problem where the set of rules determining the schedule is usually called a multi-access scheme. In the next section, we shall analyze the $\vec{f}$-feasibility problem under TDMA schemes.

As a final simplifying assumption, each link in the network will have the same capacity. Link traffic rates will be normalized with respect to this capacity so that $\vec{0} \leq \vec{f} \leq \vec{1}$ for any feasible $\vec{f}$.

III. TDMA SCHEMES

Consider a variable slot length TDMA scheme where all nodes are synchronized so that each transmission slot as perceived by different users starts and ends simultaneously. With each slot associate a column L-vector $\vec{t}$ where L is the number of links in the network and $t_i = 1$ if link i transmits in that slot; $t_i = 0$ otherwise, for $i = 1,\ldots,L$.

The transmission vectors that we consider here are conflict-free in the sense that whenever $t_i = 1$ for some link i, $t_j = 0$ for all $j \in C_i$.

The set of all links used by a transmission vector $\vec{t}$, i.e. the set $\{i \in A: t_i = 1\}$, is called the transmission set of $\vec{t}$. A transmission set is called maximal if it is not contained in any other transmission set. A maximal transmission vector is one whose transmission set is maximal. As the PRN in Figure 2 shows, the number of maximal transmission vectors need not be polynomially bounded in the number of links in the network. This fact has important consequences in terms of the complexity of the feasibility problems as will be explored later.

Let $\vec{t}_1, \vec{t}_2, \ldots, \vec{t}_K$ be an ordering of all transmission vectors and let T be the LxK matrix whose i^{th} column is $\vec{t}_i$ $(i = 1,\ldots,K)$. The particular ordering of the transmission vectors as columns of T is not important in this formulation, so T is treated as if it is unique and called the transmission matrix.

With no loss of generality, we can consider variable slot length TDMA schemes in which each transmission vector is used only once in a frame. We let $x_i \geq 0$ be the slot length of time in a frame for which transmission vector $\vec{t}_i$ is used. Also with no loss of generality, we let the frame length to be 1. Clearly, $\vec{f}$ is feasible under TDMA iff $1 \geq \min \vec{1}\cdot\vec{x}$ subject to $T\vec{x} = \vec{f}$ and $\vec{x} \geq \vec{0}$.

The fact that the TDMA-$\vec{f}$-feasibility problem can be formulated as a linear program does not directly guarantee its solution in time polynomially bounded in L because, as we have shown, the number of columns of T need not be bounded polynomially in L even when only maximal transmission vectors are counted.

Before giving the complexity results about the feasibility problems, we need to define them in a more precise way.

A TDMA scheme is a three-tuple $\langle G,T,\vec{x}\rangle$ where G is a PRN, T is the transmission matrix of G and $\vec{x}$ is a column K-vector such that $\vec{1}\cdot\vec{x} \leq 1$ and $\vec{x} \geq \vec{0}$.

FF (TDMA-$\vec{f}$-feasibility problem)

Instance: $\langle G,\vec{f}\rangle$ where G is a PRN and $\vec{f}$ is a column L-vector (one element for each link) with $\vec{0} \leq \vec{f} \leq \vec{1}$.

Question: Does there exist a TDMA scheme $\langle G,T,x\rangle$ such that $Tx = \vec{f}$?

RF ($\vec{r}$-feasibility problem)

Instance: $\langle G,\vec{r}\rangle$ where G is a PRN and $\vec{r}$ is a non-negative column vector with one element for each o-d pair in G.

Question: Is it true that $\vec{r} \in C(G)$ where C(G) is the capacity region of PRN G?

The main result is the following:

Theorem. FF is NP-complete. RF is NP-hard.

The proof is given in the appendix. NP-hardness of RF follows easily from the NP-completeness of FF and section 2 of the appendix containing this proof can be read independently.

IV. CONCLUSIONS AND SOME EXTENSIONS

For their proper and efficient operation, PRNs need fast and reliable algorithms that can adapt to changes in data traffic and network topology. In this respect,it is discouraging to see that some of the most fundamental problems about PRNs are intractable even in their simplest forms. In fact, it can further be shown that not even approximate solutions can be obtained in polynomial time for FF and RF, unless similar polynomial time algorithms exist for the CLIQUE problem, which is an extensively studied problem for which no such approximation algorithm is known, [5], [6].

Intractability of FF and RF prevails also in cases where slots are constrained to be greater than a fixed length in duration (i.e. $x_i=0$ or $x_i \geq c$ for some constant c) or fixed in size (i.e. $x_i=0$ or $x_i=c$ for some constant c). For proofs and other extensions of the above results, we refer to [5].

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APPENDIX

Notation.

We use a shorthand notation for the cartesian product of an arbitrary set V and an index set I which requires some explanation. For $x \in V$, $i \in I$, we denote $(x,i) \in V \times I$ by x_i and denote $\{(x,i) : x \in V\}$ by $V_i = \{x_i : x \in V\}$. We refer to V_i as the i^{th} copy of V.

1. NP-Completeness of FF

To prove that FF is NP-complete we first show that FF $\in$ NP and then give a polynomial reduction from a related NP-complete problem.

Lemma 1. FF $\in$ NP

Proof. If $\vec{f}$ is feasible, then the linear program of section III has a basic feasible solution, i.e., there exists L linearly independent transmission vectors $\vec{t}_{i_1}, \vec{t}_{i_2}, \ldots, \vec{t}_{i_L}$ such that $B\,\vec{x}_B = \vec{f}$ and $\vec{1}\cdot\vec{x}_B \leq 1$, where B is the LxL basis matrix whose j^{th} column is $\vec{t}_{i_j}$. Therefore, a concise certificate for a YES instance of FF is the basis matrix B. Note that B^{-1} can be computed in time polynomially bounded in L and $\vec{x}_B$ can be computed in time polynomially bounded in L and the size of the input $\langle G,\vec{f}\rangle$. ∎

Consider now the following problem which will be shown to be NP-complete and polynomially transformable to FF, thereby completing the proof that FF is NP-complete.

Definition. The Transmission Set Cardinality Problem (TSC)

Instance: $\langle G,D,k\rangle$ where G = (N,A) is a PRN, $D \subset A$ is a set of links, and k is a positive integer not greater than $|D|$.

Question: Does there exist a transmission set S of G such that $|S \cap D| = k$?

Lemma 2. TSC is NP-complete.

Proof. The reduction is from the CLIQUE problem: Given $\langle H,k\rangle$ where H = (V,E) is an undirected graph and k is integer with $0 < k \leq |V|$, does H contain a clique of size k, i.e. a complete subgraph with k nodes? CLIQUE is NP-complete [6] and the following algorithm transforms CLIQUE to FF.

Algorithm A1.

Input: <H,k> (an instance of CLIQUE),

Output: <G,D,k> (an instance of FF).

G = (N,A) is related to H = (V,E) as follows.

$N = V_1 \cup V_2$, $A = \{(a_1,b_2),(b_2,a_1) : a \in V,\ b \in V,\ (a,b) \notin E\}$, $D = \{(a_1,a_2) : a \in V\}$.

The algorithm is illustrated in Figure A1 by an example.

If Q is a clique in H with $|Q| = k$, then $S = \{(a_1,a_2) : a \in Q\}$ is a transmission set of G and $|S \cap D| = k$. Conversely, if S is a transmission set of G with $|S \cap D| = k$, then $Q = \{a \in V : (a_1,a_2) \in S \cap D\}$ is a clique of size k in H. Therefore, <H,k> is a YES instance of CLIQUE iff <G,D,k> is a YES instance of TSC. To complete the proof that TSC is NP-complete we note that a concise certificate for a YES instance of TSC is the set S. ∎

To show that TSC is reducible to FF we need a special construction which is introduced next.

Definition. The m^{th} power of G = (N,A) is $G^m = (N^m, A^m)$ where $N^m = \bigcup_{i=1}^{m} N_i$, $A^m = \bigcup_{i=1}^{m} \bigcup_{j=1}^{m} A_{ij}$,

$$A_{ii} = \{(a_i,b_i) : a_i \in N_i,\ b_i \in N_i,\ a_i \neq b_i\}, \quad i = 1,\dots,m,$$

$$A_{ij} = \{(a_i,b_j) : a_i \in N_i,\ b_j \in N_j,\ a = b \text{ or } (a,b) \in A\}, \quad i \neq j,$$

$i = 1.,\dots,m,\ j = 1,\dots,m.$

In Figure A2, G^2 is shown as an example for a small graph G. Note that (N_1,A_{11}) and (N_2,A_{22}) are complete directed graphs. If we regard G and G^2 as PRNs, then it can be seen that whenever (x,y) and (u,v) are non-conflicting links in G, (x_1,y_1) and (u_2,v_2) are non-conflicting in G^2.

Lemma 3. Let $G = (N,A)$ be a PRN and $G^m = (N^m,A^m)$ be the m^{th} power of G. For any two links $(x,y) \in A$ and $(u,v) \in A$, (x,y) does not conflict with (u,v) in G iff (x_i,y_i) does not conflict with (u_j,v_j) in G^m for all i and j such that $i \neq j$ and $1 \leq i,j \leq m$.

Proof. Suppose (x,y) does not conflict with (u,v). Let i and j be such that $i \neq j$ and $1 \leq i, j \leq m$. By the hypothesis, $x \neq v$ and $(x,v) \notin A$. Hence $(x_i,v_j) \notin A^m$ and it follows that (x_i,y_i) does not conflict with (u_j,v_j).

Suppose (x,y) conflicts with (u,v). Then either $x = v$ or $(x,v) \in A$. In each case $(x_i,v_j) \in A_{ij}$ for all i,j such that $i \neq j$ and $1 \leq i, j \leq m$; hence (x_i,y_i) conflicts with (u_j,v_j). ∎

Lemma 4. Let <G,D,k> be an instance of TSC and $D^k = \bigcup_{i=1}^{k} D_i$. The following statements are equivalent:

(i) There exists a transmission set S of G such that $|S \cap D| = k$.

(ii) There exists a transmission set S^k of G^k such that $|S^k \cap D^k| = k$.

Proof. (i) $\Longrightarrow$ (ii): Let $S \cap D = \{\ell_1, \ell_2, \ldots, \ell_k\}$ and let ℓ_{ij} be the copy of ℓ_i in A_{jj}, i.e., if $\ell_i = (a,b)$, then $\ell_{ij} = (a_j,b_j)$ for $1 \leq i \leq k$ and $1 \leq j \leq k$. By Lemma 3, $S^k = \{\ell_{ii} : i = 1,\ldots,k\}$ is a transmission set of G^k and $|S^k \cap D^k| = k$.

(ii) $\Longrightarrow$ (i): S^k must contain exactly k links with one link from each D_i $(i = 1,\ldots,k)$ because any two distinct links in D_i conflict with each other. Therefore $S = \{(a,b): (a_i,b_i) \in S^k \text{ for some } i \in \{1,\ldots,k\}\}$ has k elements and is a transmission set of G. Since $S \subset D$, $S \cap D$ also has k elements. ∎

Finally, we consider the algorithm that transforms TSC to FF.

Algorithm A2.

Input: <G,D,k> (an instance of TSC)

Output: <G',f'> (an instance of FF) where $G' = (N',A')$ and $G = (N,A)$

are related as follows:

$$N' = N^k \cup P \cup Q$$

$$N^k = \bigcup_{i=1}^{k} N_i$$

$$P = \{p_1, p_2, \ldots, p_k\}$$

$$Q = \{q_1, q_2, \ldots, q_k\}$$

$$A' = A^k \cup Y \cup Z$$

$$A^k = \bigcup_{i=1}^{k} \bigcup_{j=1}^{k} A_{ij}$$

$$A_{ii} = \{(a_i, b_i) \in N_i \times N_i : a \neq b\} \quad , i = 1,\ldots,k,$$

$$A_{ij} = \{(a_i, b_j) \in N_i \times N_j : a = b \text{ or } (a,b) \in A\} \quad , \quad i \neq j,$$

$i = 1,\ldots,k,\ j = 1,\ldots,k,$

$$Y = (P \times Q) \cup (Q \times P)$$

$$Z = \bigcup_{i=1}^{k} \bigcup_{\substack{j=1 \\ j \neq i}}^{k} Z_{ij}$$

$$Z_{ij} = \{p_i\} \times N_j \cup N_j \times \{p_i\} \quad , j \neq i,\ j = 1,\ldots,k,\ i = 1,\ldots,k$$

$$f'_{xy} = \begin{cases} \delta & ; (x,y) \in D^k \\ (|D|-1)\,\delta & ; (x,y) \in C \\ 0 & ; \text{otherwise,} \end{cases}$$

where $\delta = \dfrac{1}{1 + k(|D|-1)}$,

$$D^k = \bigcup_{i=1}^{k} D_i \ , \quad C = \{(p_i, q_i) \in P \times Q : i = 1,\ldots,k\}.$$

Note that the k^{th} power of G, (N^k, A^k), is part of G'. The complete structure of G' is illustrated in Figure A3 for the case of $|D| = 4$ and $k = 3$. The part of G' enclosed in the rectange is G^3. As seen in the figure, (P,Q,Y) is a complete bipartite graph, i.e. $(x,y) \in Y$ iff ($x \in P$ and $y \in Q$) or ($x \in Q$ and $y \in P$). So, transmission sets of G' can have at most one link in common with Y. Likewise, $(\{p_i\}, N_j, Z_{ij})$ is a complete bipartite graph for $i \neq j$ and if a transmission set S' contains a link of the form (p_i, x) for some $x \in N'$, then $S' \cap A_{ij} = \emptyset$. In fact, if $(p_i, x) \in S'$, then S' can have at most two elements, the other one being any link in A_{ii}. The reader should verify the above statements for himself before proceeding with the following lemma.

<u>Lemma 5</u>. Let $<G,D,k>$ be an input of algorithm A2 and $<G',\vec{f'}>$ be the corresponding output. Then, $<G,D,k>$ is a YES instance of TSC iff $<G',\vec{f'}>$ is a YES instance of FF.

<u>Proof</u>. Suppose $<G,D,k>$ is a YES instance of FF. By Lemma 4, there exists a transmission set S' of G' such that $S' \subset D^k$ and $|S'| = k$. Thus, if we use S' for δ time units, then we can satisfy the desired flow rates of k links in G, with one link from each set D_i $(i = 1,\ldots,k)$. There remains k-1 links in each D_i $(i = 1,\ldots,k)$ which are not included in S', so let $D_i \setminus S' = \{(a_{ij}, b_{ij}) : j = 1,\ldots k-1\}$ be a labelling of such links. Define transmission sets $S_{ij} = \{(a_{ij}, b_{ij}), (p_i, q_i)\}$ for $i = 1,\ldots,k$, $j = 1,\ldots,k-1$. Now, it can be verified easily that $\vec{f'}$ is feasible under a TDMA scheme which uses the transmission sets S' and S_{ij} $(i = 1,\ldots,k,\ j = 1,\ldots,k-1)$ each for δ fraction of the time.

Suppose that $\vec{f'}$ is feasible under TDMA. Observe that any TDMA scheme under which $\vec{f}'$ is feasible must spend at least $1-\delta$ fraction of the time in satisfying the traffic assignments on links in C. In the remaining δ fraction of the time at least k links in D^k must be allowed to transmit simultaneously. This implies that there exists a transmission set S' of G' with $|S' \cap D^k| = k$. By Lemma 4, there exists a transmission set S of G with $|S \cap D| = k$. This completes the proof of the lemma and the NP-completeness of FF.

2. NP-hardness of RF

Define the total traffic rate associated with node x of a PRN $G = (N,A)$ as $R_x = \sum_{y \in N} r_{xy} + \sum_{y \in N} r_{yx}$. R_x is the minimum fraction of time node x is busy with receiving or transmitting messages which either originate at x or terminate at x. Clearly, if $<G,\vec{r}>$ is a YES instance of RF, then we must have $R_x \leq 1$ for all $x \in N$. The following algorithm makes use of this fact in reducing FF to RF.

Algorithm A3.

Input: $<G,\vec{f}>$ (an instance of FF)

Output: $<G',\vec{r}'>$ (an instance of RF) where $G' = (N',A')$ and

$G = (N,A)$ are related as follows:

$$N' = N \cup N_1 \cup N_2$$

$$A' = A \cup B \cup C \;, \quad B = \{(a_1,a_2) : a \in N\}$$

$$C = \{(a,a_2),\ (a_2,a),\ (a,a_1),\ (a_1,a),\ (a_2,a_1) : a \in N\}$$

$$r'_{ab} = f_{ab} \qquad ; \quad (a,b) \in A$$

$$r'_{a_1 a_2} = 1 - \sum_b (f_{ab} + f_{ba}) \qquad ; \quad (a_1,a_2) \in B$$

$$r'_{xy} = 0 \qquad ; \quad (x,y) \in C \text{ or } (x,y) \notin A'$$

The algorithm is illustrated in Figure A4.

Lemma 6. Let $<G,\vec{f}>$ and $<G',\vec{r}'>$ be the input and the output of Algorithm A3, respectively. Then, $<G,\vec{f}>$ is a YES instance of FF iff $<G',\vec{r}'>$ is a YES instance of RF.

<u>Proof</u>. Suppose $<G,\vec{f}>$ is a YES instance of FF. Since $r'_{xy} = 0$ for all $(x,y)\notin A'$, each packet can be transmitted directly from its origin to its destination. Let this be our routing rule for $<G',\vec{r}'>$. The resulting link traffic rates satisfy $f'_{xy} = r'_{xy}$ for all $(x,y)\in A'$.

Let S' be a transmission set of G' containing no links of the form (a,y) or (x,a) for some node $a\in N\subset N'$. Then $S'\cup\{(a_1,a_2)\}$ is a transmission set of G'. In this way the transmission sets of G used by the TDMA scheme under which $\vec{f}$ is feasible can be augmented to yield a TDMA scheme for G' under which $\vec{f}'$ is feasible.

Suppose $<G',\vec{r}'>$ is a YES instance of RF. Since $R'_a + R'_{a_1} = 1$ and $R'_a + R'_{a_2} = 1$ for all $a\in N\subset N'$, sending each packet directly from its origin to its destination is the only routing rule under which $\vec{r}'$ can be feasible.

Furthermore there is no room for collisions or any other overhead traffic in the form of protocol information, reservation packets, etc.. Whatever protocol is used, it must enable transmission over links which constitute a transmission set of G'. As a result, $f'_{xy} = r'_{xy}$ for all $(x,y) \in A'$ and hence $\langle G,\vec{f}\rangle$ is a YES instance of FF.

This completes the proof that RF is NP-hard. Note that,for a YES instance of RF, a certificate would be a protocol under which the desired rates are feasible. However, since we have put no restrictions on the type of protocols, the verification of such a protocol for validity may not be possible in polynomial time. Nevertheless, it is not difficult to convince oneself that there exists a concise certificate for every YES instance of RF in the form of a stationary routing rule and a collision free multi-access scheme. However, a formal proof of this requires making the notion of an arbitrary protocol more precise, so that given an arbitrary protocol under which a set of o-d rates is feasible, we can show there exists a protocol under which the same set of rates is feasible and which can be verifiable in polynomial time. In a sense, such a sharpening of the result is possible only at the expense of some generality; for this reason we prefer to leave the result in this form.

References

1. R.E. Kahn, S. A. Gronemeyer, J. Burchfiel, and R. C. Kunzelman, "Advances in packet radio technology," Proc. IEEE, vol. 66, pp. 1468-1496, Nov. 1978.
2. P. D. Shapiro, "Conditions for optimal routing in a radio network," B.S. Thesis Report, M.I.T., May 1979.
3. M. Sidi and A. Segall, "Two interfering queues in packet radio networks," IEEE Trans. Comm., vol. Com-31, number 1, pp. 123-129.
4. S. Toueg and K. Steiglitz, "The complexity of optimal addressing in radio networks," IEEE Trans. Comm.,vol. Com-30, number 11, pp. 2455-2457, Nov. 1982.
5. E. Arikan, " Multi-access in packet radio networks," M.S. Thesis, LIDS-TH-1234, M.I.T., August 1982.
6. C.H. Papadimitriou and K. Steiglitz, Combinatorial Optimization, Englewood Cliffs, New Jersey 07632, Prentice Hall, Inc., 1982.

List of Figure Captions

Fig. 1. Situations in a PRN in which (c,d) conflicts with (a,b).

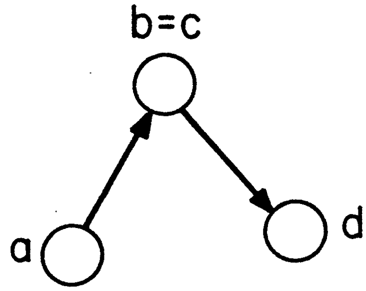
Fig. 2. A chain PRN with 3m+2 nodes containing at least 2×3^m maximal transmission vectors. (Use induction on m.)

Fig. A1.

Fig. A2.

Fig. A3.

Fig. A4.



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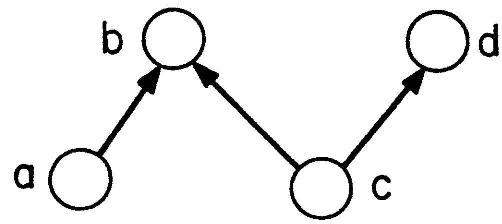

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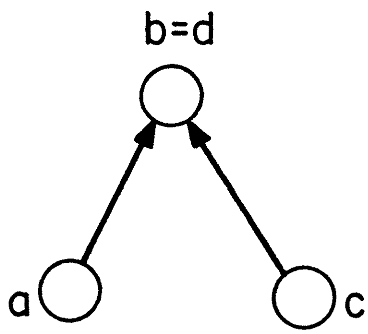

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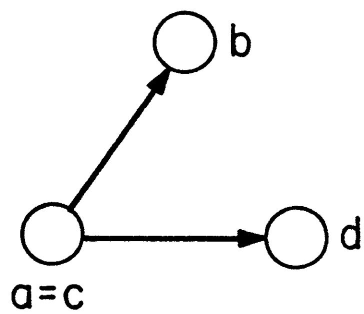

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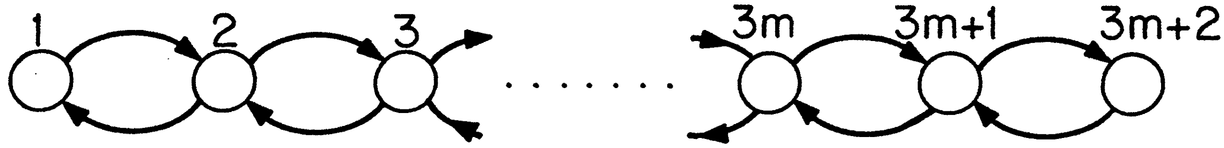
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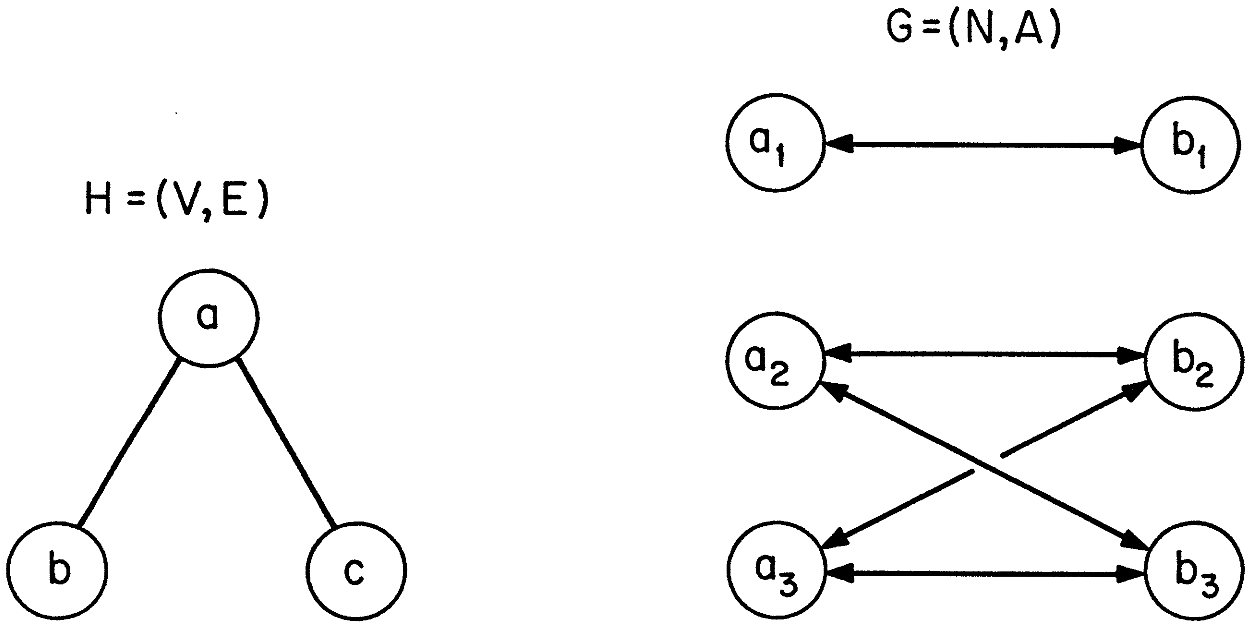
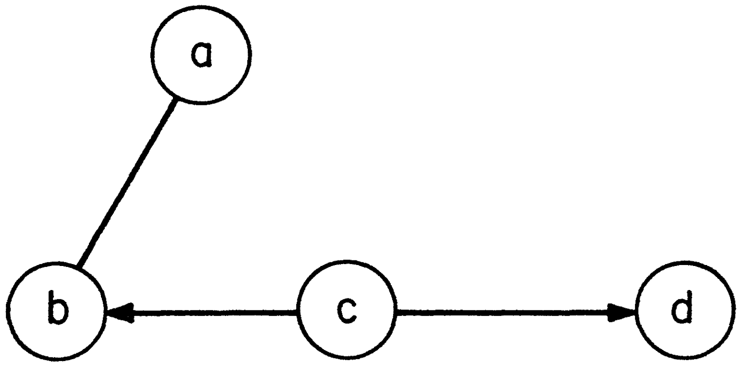
2
3
3m
3m+1
3m+2



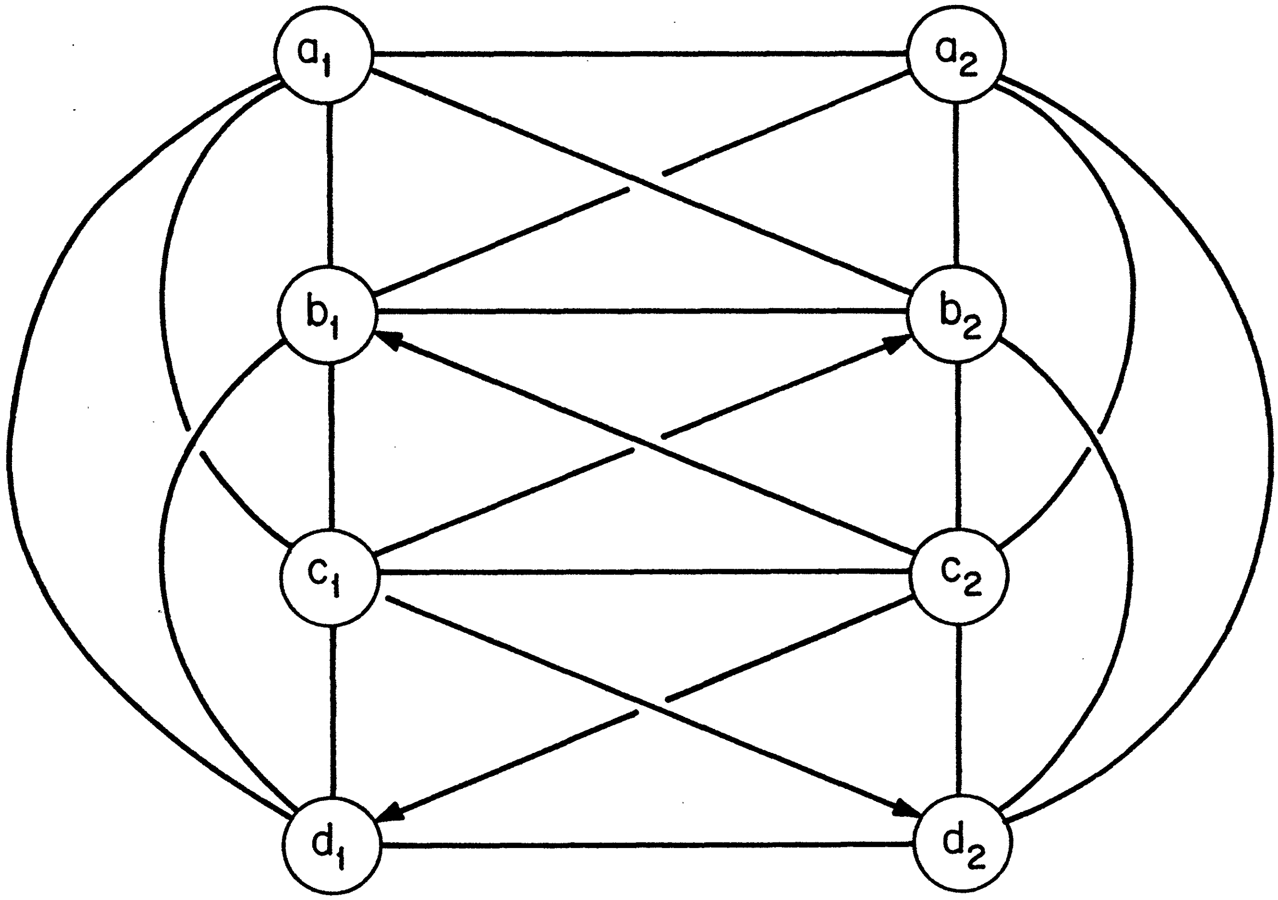
(a) (b)

$$B = \{(a_1, b_1), (a_2, b_2), (a_3, b_3)\}$$

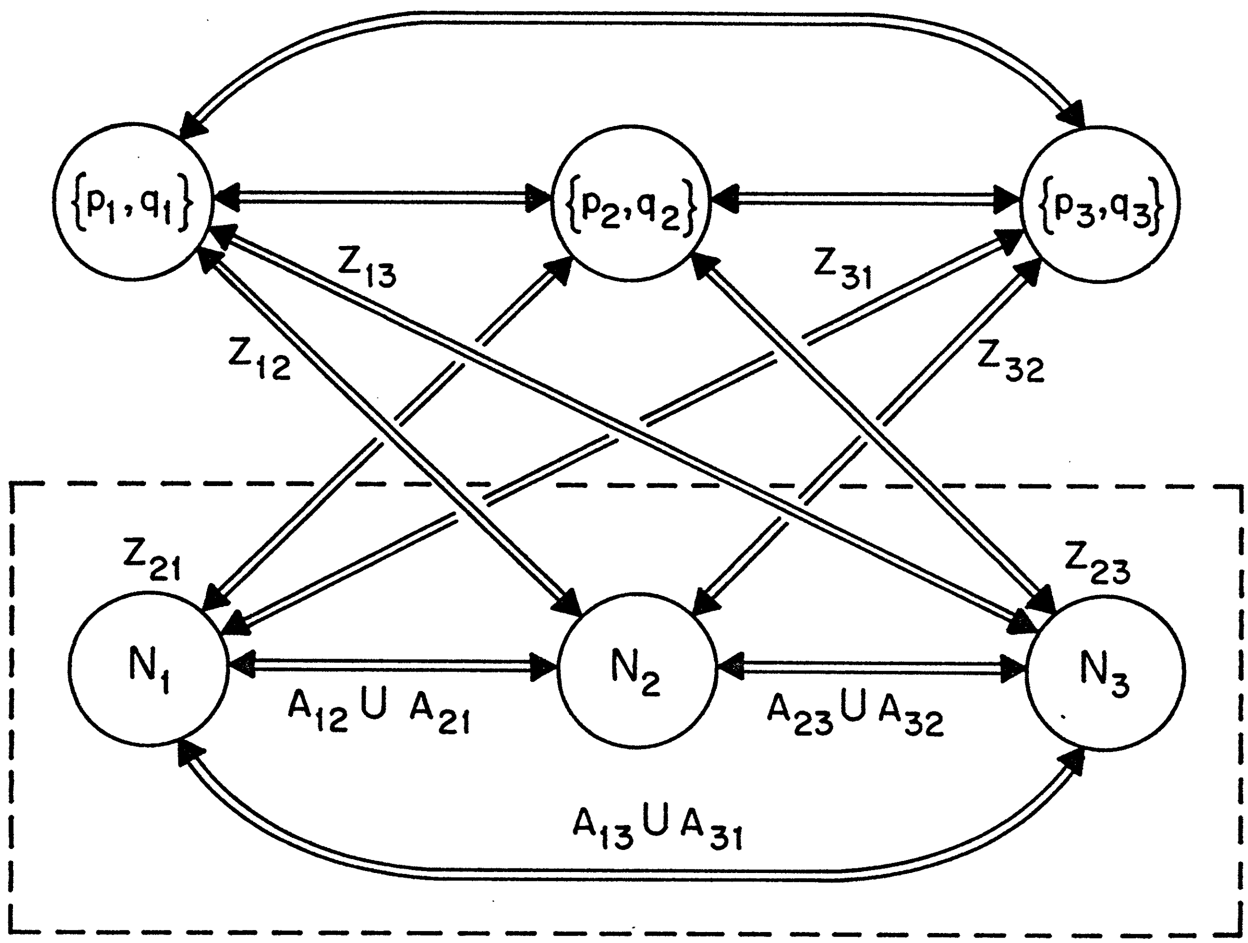
(A link without an arrow stands for two oppositely directed links in this figure.)



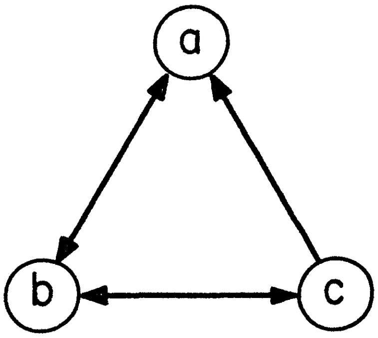
(a) $G = (N, A)$



(b) $G^2 = (N^2, A^2)$


{p1, q1}
{p2, q2}
{p3, q3}
Z13
Z31
Z12
Z32
Z21
Z23
N1
N2
N3
A12 U A21
A23 U A32
A13 U A31

G' =(N', A')



$f_{ab} = 0.2$

$f_{ba} = 0.3$

$f_{bc} = 0.1$

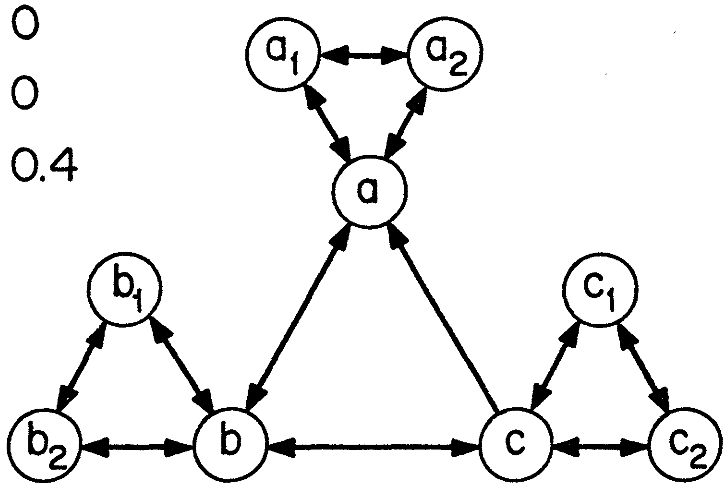
$f_{cb} = 0$

$f_{ca} = 0.5$

(a) $G = (N, A)$

$r'_{ab} = 0.2$	$r'_{ba} = 0.3$
$r'_{bc} = 0.1$	$r'_{cb} = 0$
$r'_{ca} = 0.5$	$r'_{a_1 a_2} = 0$
$r'_{b_1 b_2} = 0.4$	$r'_{c_1 c_2} = 0.4$

All other o-d rates are zero.



(b) $G' = (N', A')$