Coordinating the Multi-Retailer, Single Supplier Procurement Processes for a Seasonal Product with Supply Contracts

by

Craig K. Prisby
B.S.E., Industrial and Operations Engineering
University of Michigan, 1993

Submitted to the Engineering Systems Division
In Partial Fulfillment of the Requirements for the Degree of

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_________________________
Signature of Author

_________________________
Engineering Systems Division
May 23rd, 2003

_________________________
Certified by

_________________________
Executive Director, Master of Engineering in Logistics Program
Thesis Supervisor

_________________________
Accepted by

_________________________
Professor of Civil & Environmental Engineering
Professor of Engineering Systems
Co-Director, Center of Transportation and Logistics
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ABSTRACT

Supply contracts are used to maximize profits in a supply chain by coordinating order quantities between the suppliers and retailers. In traditional supply contracts, retailers use a newsvendor approach to maximize their profits, while the supplier's profits increase linearly as a function of the number of units supplied to retailers. Initially, retailers assume risk in the supply chain because they are facing an unknown demand, and the suppliers assume no risk.

This thesis looks at an example from the garment industry where retailers order to replenish stock after a small assortment buy is placed at the start of the finite selling season. The suppliers must place production orders for the entire selling season before the selling season begins. It is clear see that the retailers assume little risk in this model, while the supplier faces significant risk, especially if its forecasting methods are not accurate. The levels of risk that each assumes in this model are reversed when compared to the traditional supply contract model.

A method is developed that coordinates the retailer ordering with the supplier’s production schedule. It is shown that coordinating the supply chain’s ordering will lead to higher profits than the current, uncoordinated model.

Thesis Advisor: James M. Masters
Title: Executive Director, Master of Engineering in Logistics Program
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1. INTRODUCTION

1.1 Background

The fashion industry has been sourcing a greater percentage of goods overseas over the past century. American companies first attempted to fight this trend through tariffs (Harvard Business School, 1991). Unfortunately, there were many loopholes in this legislation and trade restrictions failed. Other trade bills were rejected by presidential veto.

A group of businesses in the industry established the “Crafted with Pride in the U.S.A.” Council to compete with foreign produced garments. This was initially a marketing campaign to get the public to support American workers by purchasing goods made in the U.S. This had some positive impact on the consumer, but the impact was not strong enough to offset the increased profit margins of sourcing overseas.

After failing in these efforts, the industry needed to find a competitive advantage that foreign suppliers could not provide. The Crafted with Pride in the U.S.A. Council looked at the geographical advantage of U.S. manufacturers and determined that local producers can supply finished garments to the American consumer with significantly shorter lead times. If the lead time can be short enough so that manufacturers can react to sales patterns, risk can be virtually eliminated for retailers selling American made goods.

This new strategy was called Quick Response (QR). To fully implement QR, every party in the supply chain must work together. Retailers will need to share Point of Sales (POS) and forecast data with manufacturers. Manufacturers will need to employ agile manufacturing techniques to respond to customer demand. Textile producers will
need to modify their equipment to accept various batch sizes while decreasing change-over costs.

QR was a success for non-seasonal products, but it failed for single-period products in the fashion industry. There are several reasons why QR failed for seasonal products. But one aspect of QR remained in the industry, the retailer ordering policies.

Retailer ordering policies under QR have been very beneficial for the retailers and extremely risky for manufacturers that haven’t implemented QR throughout the entire supply chain. Retailers place an assortment order (typically 35 - 60% of the season’s expected demand) at the beginning of the season or sales period. As they get a better feel for the demand of each product, the retailers will place replenishment orders for products that are selling.

These ordering policies benefit retailers by 1) transferring demand forecast risk to the manufacturers, and 2) reducing the resources that once were needed to accurately forecast demand. The problem with retailers reducing their forecasting activities is that the retailers know and understand the customers in their region or industry segment better than anyone else in the supply chain. They are the link in the supply chain that can best forecast demand for any particular product or garment. Any supply chain that does not integrate retailer forecasts will certainly incur more risk.

Over the years, manufacturers have lost power in the supply chain to the retailers. Retailers now require that all manufacturers allow them to place assortment orders, even if the manufacturers are not operating under QR.

Manufacturers that source their product overseas are trapped in between retailers replenishing items during the selling season and textile manufacturers that still supply
goods with long lead times. The retailers do not share POS information with them to warn the manufacturers of unexpected sales trends. Manufacturers have no methods that can accurately forecast demand for a seasonal item. The lead times for producing a garment are almost as long as the selling season. Life for the overseas manufacturer is becoming increasingly difficult.

This paper attempts to undo some of the damage caused by the results of not fully implementing QR. It establishes a means to coordinate manufacturing orders with the retailers’ forecasting data. It will show that if the manufacturer can use the retailers’ forecasts, total supply chain profits will increase. Additional risk will be placed on the retailers, but the manufacturer will share the increased profit with them through discounted wholesale prices.

1.2 Literature Review

Several forms of literature seem to capture some portions of the problem addressed in this paper. Many sources have documented that coordinating the supply chain leads to greater profits. This is the intent here as well.

Price discounts have been shown (Klastorin, 2002) to decrease inventory costs when they are used to get retailers to place orders coinciding with the beginning of the manufacturing cycle in an infinite period model. In this case, the retailers were given a discount to make an assortment buy and paid the original wholesale price for reorders that occurred during the selling season. While this model shows that inventory costs will decrease when coordination takes place, it does not capture stochastic demand, nor does it consider the effect of expected profits when the retailers order earlier.
Discounted early sales (Weng and Parlar, 1999) can be used to reduce variation in demand forecasts. The idea here is to offer products with discounted prices to consumers at the beginning of the period, and to use the sales data during this time to gauge how demand will occur throughout the period. Their work determines the optimal early sale price, which suggests that an optimal discount can be calculated for the model developed in this paper.

Rudi (2000) determines optimal strategies for initially sourcing garments using low-cost suppliers with long lead times and replenishing inventory during the selling season using local higher-cost suppliers with short lead times. In the example illustrated in this paper, the expected profits increased by approximately 6%.

Cachon (2002) and Li (2002) discussed several documented forms of supply contracts. In each case, the manufacturer (or supplier) attempts to induce the retailer to purchase the supply chain optimizing order quantities. These contracts are not effective for garment manufacturers selling to retailers using QR because the current assortment ordering policy is too profitable for the retailers, and the manufacturer cannot lower the wholesale price enough to get the retailers to agree to any form of contract documented.

There have also been numerous publications that deal with coordinating prices or clearance markdowns to demand. These types of models are counterproductive for companies that have built a high cost, high quality reputation. Discounting such products will deteriorate the brand name that these companies have built.

2. MODELS

The three models covered are: 1) Current Assortment Buy Model, 2) Retailers Order One-Time Early for the Whole Season, and 3) Early Assortment Ordering. All
data and assumptions to follow are notional, but are based on actual industry cases and experience.

2.1 Assumptions

2.1.1 Retailer Assumptions

Retailers are responsible for maximizing their own profits. Because the retailers have channel power in the supply chain, enticing them to order when the manufacturers place production orders will be difficult. The current practices have the retailers placing assortment orders near the start of the selling season. Replenishment orders are then placed as the retailers’ inventory depletes.

Two alternatives will be modeled to increase the supply chain profits. In the first model, retailers place orders for the whole season before production schedules are made. The second alternative looks at maintaining current assortment purchasing practices, except that these purchases are made early enough to coordinate with the manufacturing lead times. In addition to placing an early assortment order, retailers must also share their forecast information with the manufacturer. In both cases, discounted wholesale prices will be offered to the retailers to share the benefits of the new business practices.

Retailers buy \(Q_R\) goods from the manufacturer at the wholesale price \(w\) and sell goods to their customers for the retail price \(p\). At the end of the season, they sell products to their customers at a salvage value \(s_R\) (where \(s_R\) is a percentage of the retail price). If the retailer does not have a specific unit available (size, style, and color) that the customer wants, the retailer faces a lost sale cost of \(v\).
Profits $\Pi_R$ are a function of the individual retailer’s order quantity and are maximized at $Q_R^*$. Retailers face a demand probability function with the expected value of demand $E(D_R)$ and standard deviation $\sigma_R$.

The manufacturer sells to $n$ identical retailers with uncorrelated demand distributions. To simplify calculations in the model, the order quantities and demand distributions will be uniformly equal and independent for all $n$ retailers. From this, the manufacturer’s expected value of demand is

$$E(D_M) = \sum_{i=1}^{n} E(D_R^i) = n \cdot E(D_R),$$

where $D_R^i = \text{Demand For Retailer } i$.

### 2.1.2 Manufacturer Assumptions

The manufacturer is responsible for the design and assembly of the seasonal product. They contract the assembly work to a foreign contract manufacturer who places orders with the textile and materials suppliers. The lead times from manufacturer order to arrival on the retailers’ shelves are dependent on the product being manufactured. Some products take three months from order to finished product, while others take four months. Typical order lead times are shown in Figure 1.

The total selling season lasts for 6 months (180 days). In order for the retailers to have inventory available for the last month of sales, all production orders should be placed sometime before the selling season.

Manufacturers place an order quantity $Q_M$ at a per unit cost of $c$. Units are sold to retailers at the wholesale price $w$, and the manufacturer has the option to sell remaining units at the end of the season to outlet stores or discount retailers for a salvage value $s_M$. 

9
(where \(s_M\) is a percentage of the wholesale price). The manufacturer faces an expected demand \(E(D_M)\) with standard deviation of demand \(\sigma_M\).

**Figure 1.** Lead times (in days) from textile ordering to retailer shelf.

<table>
<thead>
<tr>
<th>Product:</th>
<th>“3 Month Production Lead Time”</th>
<th>“4 Month Production Lead Time”</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) Production Order to Finished Good</td>
<td>104.7 +/- 3.9</td>
<td>134.7 +/- 3.8</td>
</tr>
<tr>
<td>(b) Factory to DC</td>
<td>31.6 +/- 0.8</td>
<td>31.5 +/- 0.8</td>
</tr>
<tr>
<td>(c) DC to Retailer</td>
<td>24.0 +/- 0.0</td>
<td>24.0 +/- 0.0</td>
</tr>
<tr>
<td>(d) Retailer to Shelf</td>
<td>6.5 +/- 0.7</td>
<td>6.5 +/- 0.7</td>
</tr>
<tr>
<td><strong>Total Lead Time (days):</strong></td>
<td><strong>166.8 +/- 4.0</strong></td>
<td><strong>196.8 +/- 3.7</strong></td>
</tr>
</tbody>
</table>

Manufacturers attempt to maximize their profits by placing production order quantities that optimize the profit function \(\Pi_M(Q_M)\). The optimal order quantity will be denoted with an asterisk \(Q_M^*\).

A summary of these parameters and variables is shown in Table 1.

**Table 1.** Parameters and Variables Included used Models.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Manufacturer</th>
<th>Retailer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Purchase Cost</td>
<td>(c)</td>
<td>(w)</td>
</tr>
<tr>
<td>Selling Price</td>
<td>(w)</td>
<td>(p)</td>
</tr>
<tr>
<td>Unit Salvage Price</td>
<td>(s_M)</td>
<td>(s_R)</td>
</tr>
<tr>
<td>Lost Sale Cost</td>
<td>(v)</td>
<td></td>
</tr>
<tr>
<td>Demand</td>
<td>(D_M = \Sigma D_R)</td>
<td>(D_R)</td>
</tr>
<tr>
<td>Standard Deviation of Demand</td>
<td>(\sigma_M)</td>
<td>(\sigma_R)</td>
</tr>
<tr>
<td>Order Quantity</td>
<td>(Q_M)</td>
<td>(Q_R)</td>
</tr>
<tr>
<td>Profit</td>
<td>(\Pi_M)</td>
<td>(\Pi_R)</td>
</tr>
<tr>
<td>Assortment Percentage of Expected Demand</td>
<td>(m)</td>
<td></td>
</tr>
<tr>
<td>Number of Retailers</td>
<td>(n)</td>
<td></td>
</tr>
<tr>
<td>Increased standard deviation multiplier (Proposed)</td>
<td>(g)</td>
<td></td>
</tr>
</tbody>
</table>
2.2 Current Model

2.2.1 Retailers’ Model

Retailers place an assortment order $Q_R^o$ (any variable with $^o$ will pertain to current practice models throughout the rest of this paper) where $E(D_R) = \frac{1}{m} \cdot Q_R^o$ and $m$ is the assortment percentage. The profit function for the retailer is:

$$\Pi_R(Q_R^o) = p \cdot D_R + s_R \cdot (Q_R^o - D_R) - w^0 \cdot Q_R^o, \text{ when } D_R \leq Q_R^o$$

$$= p \cdot D_R - w^0 \cdot D_R, \text{ when } D_R > Q_R^o$$

(1).

Combining both instances from (1) and using probability distributions will result in the expected profits for each retailer function:

$$E[\Pi_R(Q_R^o)] = (p - s_R) \cdot \int_0^{Q_R^o} D_R \cdot f(D_R) \partial D_R + (s_R - w^o) \cdot \int_0^{Q_R^o} f(D_R) \partial D_R$$

$$+ (p - w^o) \cdot \int_{Q_R^o}^{\infty} D_R \cdot f(D_R) \partial D_R$$

(2).

Taking the derivative of (2) shows that the profits of the retailers are maximized when $Q_R^o = 0$. This is clear because when retailers can replenish their inventory at any time, beginning with no inventory will decrease their risk of not selling a product. However, $Q_R^o = 0$ is not very practical because the retailers need to have items on their shelves before they can sell anything to their customers. The second derivative proves that the profit function is indeed decreasing.

2.2.2 Manufacturer’s Model

The current manufacturing order practice is not known, but forecasted demand and production quantity information for a single season of a fashion manufacturer was provided and will be analyzed in Section 3. To compare the best profit maximizing
current model against alternatives, a simple newsvendor model (with salvage value revenue $s_M$) will show the current manufacturing ordering policy. This profit function is written as:

$$E[\Pi_M(Q^o_M)] = -c \cdot Q^o_M + w \cdot \int_{2u} Q^o_M \cdot f(D_M) \, dD_M$$

$$+ s_M \cdot \int_{2u} (Q^o_M - D) \cdot f(D_M) \, dD_M + w \cdot \int_{2u} D_M \cdot f(D_M) \, dD_M$$

(3).

Setting the first derivative equal to zero, the optimal current manufacturing $Q^*_{M}$ order quantity can be found when

$$\text{Prob}(D_M < Q^*_{M}) = \frac{w - c}{w - s_M}$$

(4).

2.3 Whole Season Early Model

In this model retailers will place their entire season of orders with the manufacturer before production orders are placed, eliminating the assortment order. The retailers will assume greater risk because their forecasts are not as certain when placing orders 5-6 months before the selling season. To model this effect, the retailers' coefficient of variation (defines as the standard deviation divided by the expected value of demand, and denoted c.v.) will increase by a percentage $g$ where

$$\frac{\sigma_R}{E(D_R)} = c.v. = g \cdot c.v. = \frac{g \cdot \sigma^o_R}{E(D_R)}.$$}

For sake of simplicity, we will assume that the expected values of demand, $E(D_R)$, will remain the same as the current model. Only the standard deviations increase when earlier orders are placed by the retailers.
2.3.1 Retailers’ Whole Season Early Model

The retailers now face a single period newsvendor model with salvage revenues and stock-out costs. The new profit function for each retailer is:

\[
\Pi_R(Q_R) = p \cdot D_R + s_R \cdot (Q_R - D_R) - w \cdot Q_R \quad \text{, when } D_R \leq Q_R
\]
\[
= p \cdot Q_R - v \cdot (D_R - Q_R) - w \cdot Q_R \quad \text{, when } D_R > Q_R
\]  
(5).

Rewriting it into a single expected profit function gives us:

\[
E[\Pi_R(Q_R)] = p \cdot \int_{Q_R}^{\infty} D_R \cdot f(D_R) \, dD_R + s_R \cdot \int_{0}^{Q_R} (Q_R - D_R) \cdot f(D_R) \, dD_R - w \cdot Q_R
\]
\[
+ p \cdot \int_{Q_R}^{\infty} Q_R \cdot f(D_R) \, dD_R - v \cdot \int_{Q_R}^{\infty} (D_R - Q_R) \cdot f(D_R) \, dD_R
\]  
(6).

It will be assumed that each retailer will order their profit optimizing order quantity \( Q_R^* \) which is found when

\[
1 - \text{Prob}(D_R < Q_R^*) = \frac{w - s_R}{p + v - s_R}
\]  
(7).

But since the goal is to optimize the supply chain profits, we must set \( w \) so that the retailers will order the supply chain profit maximizing order quantity \( Q^{SC} \). This will be addressed when the manufacturer’s profit function is modeled.

One thing to note about (7) is that \( w \) must be greater than \( s_R \). An optimal \( Q_{R}^* \) is not feasible when \( w < s_R \) because \( F(Q_{R}^*) > 1 \) (which can never be true).

2.3.2 Manufacturer’s Whole Season Early Model

With the retailers ordering before manufacturing orders are placed, the manufacturer’s profit function is now a linear function of the retailers’ ordering quantities.

\[
E[\Pi_M(Q_R)] = (w - c) \cdot n \cdot Q_R
\]  
(8)

To maximize the total supply chain profits function
\[ E[\Pi_{SC}(Q_R)] = E[\Pi_M(Q_R)] + n \cdot E[\Pi_R(Q_R)]. \]

we will take the first derivative with respect to \( Q_R \) and set it to zero. This gives us a supply chain profit maximizing order quantity when

\[ 1 - \text{Prob}(D_R < Q_{SC}^R) = \frac{s_R - c}{p + v - s_R} \]  \hspace{1cm} (9).

By setting \( Q_{SC}^R = Q^*_R \) and combining equations (7) and (9), we find that the wholesale price should be set at \( w = (2 \cdot s_R) - c \) when attempting to maximize the supply chain profits.

2.4 Early Assortment Order Model

The Early Assortment Order Model is essentially the same as the current model, except that the retailers’ assortment orders are coordinated with the production plan. Retailers will place assortment orders before production begins, and they will share demand forecast information with the manufacturer.

As in the case of the Whole Season Early Model, the retailers’ c.v. is increased by the factor \( g \), and the expected value for demand for a particular item will remain the same as the original model to help compare results.

The manufacturer will face a newsvendor problem similar to the current model, where the only difference is that the manufacturers will place orders based on the retailers’ demand forecast information instead of internally determined sales forecast data.

2.4.1 Retailers’ Early Assortment Order Model

The Expected profit of the retailer for this model is calculated in the same manner as in the current model. Three differences exist in that: 1) the retailers may be able to
order a different percentage $m'$ of the expected demand for their assortment order, and 2) a price reduction $w'$ (where $w = w^0 - w'$) may be given to the retailers for coordinating with the manufacturer.

The expected profit function is identical to equation (2) except that $w^0$ is replaced by $w$.

2.4.2 Manufacturer's Early Assortment Order Model

As in the case of the retailer, the manufacturer's model is the same as the current model (using a newsvendor equation) except that the expected value of demand and its distribution is modified to take the retailers' forecast data into account.

The expected demand distribution for the manufacturer is now expressed with:

\[
E(D_M) = n \cdot E(D_R)
\]
\[
\text{Var}(D_M) = n \cdot \text{Var}(D_R)
\]
\[
\sigma_M = \sqrt{n} \cdot \sigma_R
\]

One thing to note here is that pooling the retailers' forecasts significantly reduces the standard deviation of demand for the manufacturer because the retailers' standard deviations are not additive. It is this pooling effect that will increase supply chain profits.

3. FINDINGS

3.1 Information Gathering and Assumptions

Data for seasonal, fashion products were provided for a single selling season. In all, 85 different style/color combinations were considered from a single manufacturer.

The information was aggregated over all the sizes of each style.
The sales data received included 1) Manufacturer’s Expected Demand Values, 2) Retailer Assortment Purchase Quantities, 3) Pre-Season Production Order Quantities, and 4) End of Season Sales Quantities.

Cost and pricing information was provided for each of the 85 styles for: 1) Landed Manufacturing Costs $c$, 2) Current Wholesale Prices $w$, and 3) Retail Prices $p$. These costs and prices differ for each style.

The factors assumed in the model were: 1) Manufacturer Salvage Value ($s_m = c \cdot s_m^e$, where $s_m^e$ is assumed), 2) Retailer Salvage Value ($s_R = p \cdot s_R^e$, where $s_R^e$ is assumed), 3) Retailer Shortage Cost ($v = p \cdot v^e$, where $v^e$ is assumed), 4) Percentage of Expected Retail Sales that the retailer purchases in their assortment buy (described earlier as $m$), 5) the number of retailers $n$ purchasing each style, 6) each retailer’s c.v., 7) the manufacturer’s c.v., and 8) the percent increase of the retailer’s c.v. when ordering earlier (modeled as $g$).

The decision variables that can be manipulated to optimize profits are: 1) the manufacturer’s order quantity $Q_M$, 2) the retailer’s order quantity $Q_R$, 3) the wholesale price discount $w'$, and 4) the percentage of expected demand that the assortment order covers $m'$ for the two proposed models.

3.2 General Data Overview

A quick analysis of the data revealed that retailers were able to better predict demand overall. To calculate this, the percentage that the expected value differed from the actual season sales $\left( \frac{(sales - E(D))}{sales} \right)$ was averaged amongst all the style/color combinations. The standard deviations of these percentages were also calculated. We
should expect that the average of the differences should be zero, while more accurate forecasts will yield a lower standard deviation. These calculations were also performed for the combinations with the highest and lowest sales figures. Table 2 summarizes these findings.

Table 2. Analysis of Expected Demand Values (assuming \( m = 50\% \) for calculating retailer expected demand figures). The percentages in this table were calculated using:

\[
\% = \frac{(sales - E(D))}{sales}.
\]

<table>
<thead>
<tr>
<th></th>
<th>Retailer</th>
<th>Manufacturer</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>All Styles</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average</td>
<td>-69%</td>
<td>-106%</td>
</tr>
<tr>
<td>St. Dev.</td>
<td>93%</td>
<td>134%</td>
</tr>
<tr>
<td>Range</td>
<td>-358% / 91%</td>
<td>-830% / 91%</td>
</tr>
<tr>
<td><strong>Top 10 Sales</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average</td>
<td>10%</td>
<td>-11%</td>
</tr>
<tr>
<td>St. Dev.</td>
<td>61%</td>
<td>48%</td>
</tr>
<tr>
<td>Range</td>
<td>-85% / 82%</td>
<td>-80% / 51%</td>
</tr>
<tr>
<td><strong>Bottom 10 Sales</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average</td>
<td>-150%</td>
<td>-218%</td>
</tr>
<tr>
<td>St. Dev.</td>
<td>122%</td>
<td>245%</td>
</tr>
<tr>
<td>Range</td>
<td>-353% / 70%</td>
<td>-830% / 19%</td>
</tr>
</tbody>
</table>

Many inferences can be made from this information. It is easy to see that the retailers forecast demand significantly better for slow selling styles. Also notice how the forecast errors increase for products having the lowest sales volumes, compared with products selling in higher volumes. The last characteristic to note is that either the selling season analyzed was slower than expected or the sales forecasting is biased towards the high side.

3.3 Selection of Demand Distribution

A Gamma Distribution was chosen to represent the demand distribution that the retailers and manufacturer face. Several representations of the Gamma probability
distribution function are shown in Figure 2. The Gamma distribution seemed to fit best because the distribution cannot accept negative demand values and large c.v. values are acceptable.

Figure 2. Examples of the Gamma Probability Distribution Function.

The normal distribution was considered, but could not be used because a significant portion of the probability distribution function falls in a negative demand region when the coefficient of variation is large. A negative demand cannot exist. When a normal distribution with a high c.v. is used, the expected value of demand when
demand is less than the order quantity is less than the actual expected value when the integral is taken from zero to \( Q \) as shown in the following equation.

\[
\int_0^Q D \cdot f(D) \cdot \partial D < \int_0^Q D \cdot f(D) \cdot \partial D
\]

3.4 Model Results

Assumptions for the models are summarized in Table 3. Descriptions of each variable were defined in Section 3.1.

Table 3. Model assumptions used to compare the current model to the proposed models.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( s_M^c )</td>
<td>30%</td>
</tr>
<tr>
<td>( s_R^c )</td>
<td>40%</td>
</tr>
<tr>
<td>( v^c )</td>
<td>10%</td>
</tr>
<tr>
<td>( n )</td>
<td>100</td>
</tr>
<tr>
<td>( g )</td>
<td>100%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m )</td>
<td>50%</td>
</tr>
<tr>
<td>( m' )</td>
<td>50%</td>
</tr>
<tr>
<td>Retailer c.v.</td>
<td>0.75</td>
</tr>
<tr>
<td>Manufacturer c.v.</td>
<td>0.9</td>
</tr>
<tr>
<td>( w' )</td>
<td>$2.00</td>
</tr>
</tbody>
</table>

The first analysis performed looked at the expected profits in the current model. Table 4 contains the results of the expected profits. The left data column was calculated using the manufacturer’s profit maximizing ordering quantity \( Q_M^{*o} \). The right column shows the expected profits using this manufacturer’s current ordering policies. By simply adjusting their production order quantities based on their own forecasts, the manufacturer can increase profits by almost $570,000.

When looking at the actual end-of-season sales data, the manufacturer’s ordering policy would have made $5,872,892.83, and the optimal ordering policy would have
generated profits of $579,641.53. As discussed earlier, actual profits are worse for the optimal order quantity case because the data is likely to be biased data.

Table 4. Expected profits for the current model using an optimal manufacturing order quantity compared to current practices.

<table>
<thead>
<tr>
<th></th>
<th>Using $Q^*$</th>
<th>Actual Manuf. Orders</th>
</tr>
</thead>
<tbody>
<tr>
<td>Each Retailer</td>
<td>$384,061.65</td>
<td>-same-</td>
</tr>
<tr>
<td>All Retailers</td>
<td>$38,406,165.16</td>
<td>-same-</td>
</tr>
<tr>
<td>Manufacturer</td>
<td>$10,572,197.73</td>
<td>$10,004,846.14</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>$48,978,362.89</strong></td>
<td><strong>$48,411,011.30</strong></td>
</tr>
</tbody>
</table>

When looking at each model's expected profits in Table 5, one thing is evident: the current supply chain can make higher profits.

Table 5. Expected profits for each model.

<table>
<thead>
<tr>
<th></th>
<th>Current Using $Q^*$</th>
<th>Whole Season Early</th>
<th>Early Assortment</th>
</tr>
</thead>
<tbody>
<tr>
<td>All Retailers</td>
<td>$38,406,165.16</td>
<td>$7,698,142.51</td>
<td>$40,011,488.84</td>
</tr>
<tr>
<td>Manufacturer</td>
<td>$10,572,197.73</td>
<td>$35,116,667.68</td>
<td>$14,800,385.72</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>$48,978,362.89</strong></td>
<td><strong>$42,814,810.19</strong></td>
<td><strong>$54,811,874.57</strong></td>
</tr>
</tbody>
</table>

Notice that requiring retailers to order early for the whole season significantly decreases their profits, but if their standard deviation only increases by 10% (instead of the 100% increase modeled) then the expected retailer profits will be $20,847,253 (and supply chain expected profits will increase to $54,772,040).

Even if retailers have the ability to forecast demand more accurately, they would never agree to Whole Season Early model. The manufacturer would need to lower the
wholesale price below the manufacturing costs before retailers will agree to this contract under the pricing and costs information provided by the manufacturer. The advantage of placing an assortment order in the current model virtually assures that every unit ordered by a retailer is sold to their customers. This not only reduces their risk, but minimizes the number of units that need to be salvaged. A model was considered where the retailers could give back a certain portion of their order, but this lead to sub-optimal supply chain profits because the manufacturer’s salvage revenue per unit is lower than the retailers’.

The Early Assortment Order model gives us the best expected profits for the supply chain. This model combines the most significant aspects of the first two models to increase profits. It utilizes the assortment buy that provides large expected profits for the retailers, and it also coordinates the retailer forecasts and orders with the manufacturing schedule. Adjusting the assortment purchase percentage \( m' \), does little to effect the retailers’ expected profits and does not effect the manufacturer’s expected profits.

The main factor that increases profits in the Early Assortment Order model is the coordination between the retailers and their supplying manufacturer. When the retailers share forecast information with the manufacturer, the standard deviation of demand that the manufacturer uses to place order quantities is significantly reduced. In this model (where the retailers’ \( c.v. = 0.75 \) and \( n = 100 \)), the manufacturer’s \( c.v. \) decreased from 0.9 to 0.0825. So, pooling the retailers’ forecast data tightened the accuracy of the manufacturer’s expected demand, thus increasing their expected profits when producing the profit maximizing optimal quantity.
4. FUTURE CONSIDERATIONS

These models would be unnecessary if QR can be fully implemented where manufacturing and textile suppliers have short enough lead times so they can react to actual customer demand for a seasonal product. This stated, any manufacturer wishing to implement these models must realize that it was developed to reduce the adverse effects that strong retailers are placing on the manufacturers in the supply chain. The proposed models in this paper increase total supply chain profits because they promote a higher level of information sharing and risk sharing. If companies ever switch over to QR, supply chain partners will be more comfortable sharing information with each other (which is a must for full QR implementation).

Although this model mathematically proves the advantages of coordinating the retailer forecasts with the rest of the supply chain, there were many factors that could not be modeled. The first is that manufacturers typically do not produce the whole season of products at one time. If they place weekly or monthly orders with the factories, their orders will be spread out to avoid holding excessive inventory at the beginning of each selling season. The manufacturer may want to give the retailers the opportunity to adjust assortment orders and forecasts in accordance with the production scheduling. This will give the retailers some flexibility to change their entire season order quantity, and it also presents a situation where the retailers share updated forecast information with the manufacturer.

Another consideration that was not modeled, but probably practiced by most retailers, is maintaining a high service level. All the calculations here are based on profit maximizing order quantities. It is highly unlikely that retailers order to maximize their profits, as they compete against each other on service. In order to maintain a high service
level, retailers order more than $Q_r^*$. This will lead to sales volumes higher than that modeled for the manufacturer, thus increasing their profits more. If a higher service level needs to be modeled, $\nu$ can be adjusted to a greater value before $Q_r^*$ is recomputed.

This notion of service level can also be carried to the manufacturer. Some contingencies may need to be worked out to ensure that unexpected “hot” items can make it to the market. This may include actions such as manufacturing slightly more than what the retailers ordered or creating an expedited build process that will have minimal effects on operations. However in either case, the trade-off of suboptimal profitability vs. service level must be worked out.

One more simplification made in the models is the pricing strategies used by the retailers. Retailers frequently place products at discounted or promotional prices. These actions lead to lower “per unit” profits.

Another over-simplification of these models occurs with the assumption that the demand distributions are equally distributed for all retailers. In actuality, large retailers will have greater effects on this model than the smaller retailers. If the large retailers have worse forecasting tools, more variance can be added to the model. However, the opposite should also be true.

Coordination between retailers and manufacturers in the supply chain can lead to greater profits for all parties. The models developed in this paper further validate such a claim. For companies employing such processes, it is their responsibility to decide what to do with the additional profits: distribute it between themselves or pass the savings onto the consumers.
5. REFERENCES


6. APPENDIX

6.1 Derivative Calculations

The following calculation was used to obtain the derivatives of the integrals used in the profit functions.

\[
G(x) = \int_{a(x)}^{b(x)} F(x, y) \, dy
\]

\[
\frac{\partial}{\partial x} G(x) = \int_{a(x)}^{b(x)} \frac{\partial F(x, y)}{\partial x} \, dy + F(x, b(x)) \cdot \frac{\partial b(x)}{\partial x} - F(x, a(x)) \cdot \frac{\partial a(x)}{\partial x}
\]

6.2 Gamma Distribution Properties

The gamma distribution (Devore 1991) defines the expected value and variance as:

\[
E(x) = \alpha \cdot \beta
\]

\[
\text{Var}(x) = \sigma_x^2 = \alpha \cdot \beta^2
\]
where $\alpha > 0$ and $\beta > 0$ are the parameters of the gamma distribution function

$$f(x; \alpha, \beta) = \frac{1}{\beta^\alpha \cdot \Gamma(\alpha)} \cdot x^{\alpha-1} \cdot e^{-x/\beta} \quad \text{when } x \geq 0$$

and the gamma function $\Gamma(\alpha)$ is defined as

$$\Gamma(\alpha) = \int_0^\infty x^{\alpha-1} \cdot e^{-x} \cdot dx.$$  

Since $E(D)$ is known and the c.v. are is assumed, $\alpha$ and $\beta$ are defined by

$$\alpha = \left( \frac{1}{\text{c.v.}} \right)^2 = \left( \frac{E(D)}{\sigma} \right)^2$$

$$\beta = (\text{c.v.})^2 \cdot E(D) = \frac{\sigma^2}{E(D)}$$