A Game-Theoretic Analysis of Electronic Warfare Tactics with Applications to the World War II Era

by

David M. Blum

B.A. History
Columbia University, 2001

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ABSTRACT

This thesis considers electronic countermeasures as well thought-out signals sent by the "attacker" to a recipient, the "defender" in order to create uncertainty, and argues that tactics that incorporate the judicious use of bluffing further such uncertainty. I discuss two forms of bluffing, bluffing to create uncertainty as to the location of an attack (bluffing in space), and bluffing to create uncertainty as to the time of attack (bluffing in time). Two electronic warfare tactics used by the Allied air forces during World War II, representing an example of each, are modeled as dynamic zero-sum games with incomplete information. I show that in most instances, Perfect Bayesian Nash Equilibria dictate that the defender delay cuing his interceptors longer than he would so otherwise, and that in those situations where he should cue his interceptors, he must do so at random. Furthermore, except where the cost to bluff is prohibitive, the attacker always benefits from the use of tactics that incorporate bluffing, though bluffing in space is generally more effective than bluffing in time for a given set of detection probabilities.

Thesis Supervisor: James M. Snyder
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When I designed this project I did not realize the extent to which it would require technical expertise to manipulate and analyze large quantities of data. To that end, I must thank Matthew Drake, whose programming and analytic skills were instrumental in allowing me to vet the higher-dimensional solutions for each of the many equilibria generated by my models. Mr. Drake put in many hours assisting me with the technical analysis behind the "Sensitivity" and "Results" sections of this paper.

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1. INTRODUCTION

The use of radio emissions to aid in warfighting, known as electronic warfare (or EW), has a long and storied history dating back to the years preceding World War II, when British scientists invented radar. Since that time radar has become the most ubiquitous means of detecting an adversary beyond visual range on or above the Earth's surface, and in particular, has become the mainstay of most nations' air defenses. Historians and military analysts have discussed at length the tactics used in past wars to prevent or delay an adversary from using electronic means to engage friendly aircraft. They have drawn many lessons, one of which deserves particular mention – the transience of tactics. "Electronic warfare is a dynamic field of endeavor and no method or tactic will remain effective indefinitely," states Alfred Price in his treatise *The History of U.S. Electronic Warfare.* [1] Yet it is also evident that some of the most successful EW operations - the Allied strategic bombing campaign over Germany after 1944, Linebacker II in Vietnam, Israel's 1982 "Operation Peace for Galilee," and the opening night of Operation Desert Storm - featured tactics that incorporated judicious use of bluffing (defined in the subsequent section, *The Role of Subterfuge in Electronic Warfare*). In this paper, I aim to demonstrate why bluffing can be so useful in an EW campaign and why it has been incorporated into so many EW tactics.

[1] Alfred Price, *The History of U.S. Electronic Warfare,* Vol. 3, (2000) p. 555

A clear understanding of the character of EW as it relates to the give and take of information forms the basis of any effort to explain why a particular electronic warfare tactic is effective. I begin by characterizing electronic countermeasures as well thought-out signals sent by the "attacker" to a recipient, the "defender" in order to create uncertainty. Then without attempting to advocate specific tactics for actual use in war, I proceed to model two EW tactics that were used during the World War II era. I show formally that in such situations of uncertainty:

(1) the optimal use of and response to each tactic in many cases requires the defender to delay acting at least until after he has attempted detection, and it is never optimal for him to "cue" his response predictively based upon the presence of the countermeasure;

(2) the situations in which it is optimal for him to cue his response <u>randomly</u> based upon the presence of the countermeasure result from a low penalty for error in conjunction with a low reward for properly but belatedly intercepting the attack;

(3) the expected payoff to the attacker in equilibrium is, in all but a few instances, greater using jamming tactics that incorporate bluffing than it is using jamming tactics that do not.

Finally I compare the theoretical effectiveness of the two tactics and determine the circumstances in which one or the other is favorable. In concluding, I identify implications for other uses of bluffing, both in and beyond electronic warfare, in order to understand other situations that feature bluffing as a major element.

The Role of Subterfuge in Electronic Warfare

Electronic warfare as a whole does not concern itself with directly defeating the adversary. Rather, it is concerned with identifying or denying the adversary information that is necessary to prosecute war. Although EW has aspects that are offensive as well as defensive, in a conventional physical engagement both an 'attacker' and a 'defender' might engage in either one of these activities. Specifically, a belligerent may engage in an effort to increase its situational awareness as well as a corresponding effort to deny the other side situational awareness. During World War II, by detecting incoming aircraft hundreds of miles offshore, radar enabled countries to concentrate interceptors from distant bases in order to attrite attacks before they could reach their targets. Through the disruption of radar, countermeasures developed prior to and during the war served to deny the enemy information as to the time and location of an attack. The trouble with the unmoderated use of countermeasures was that such use did itself communicate information to the defender that could be used to cue interceptors or to otherwise allow the defender to take precautions. In other words, countermeasures served little purpose when implemented in a manner that allowed the defender to observe them and to positively identify them as countermeasures.

Edward Waltz writes the following of operational deception in his book *Information Warfare, Principles and Operations*

> Two categories of misconception are recognized: (1) *ambiguity* deception aims to create uncertainty about the truth, and (2) *misdirection* deception aims to create certainty about a falsehood. Deception uses methods of distortion, concealment, falsification of indicators, and development of misinformation to mislead the target to achieve surprise or stealth. Feint, ruse, and diversion activities are common military deceptive actions.[2]

To expand on Waltz's characterization, operational deception of either category involves sending a signal that conveys information to the adversary. This signal might look like the background, as in the case of camouflage, or it might look like the object the adversary seeks to detect except that it is intentionally delivered either at the wrong place or at the wrong time. This latter sort of signal is what Waltz refers to as a "feint," "ruse" or "diversion" and what I term a bluff. Whether given at the wrong place or the wrong time, a bluff will be mixed among, or substitute for, true signals.

[2] Edward Waltz, *Information Warfare, Principles and Operations,* (1998) p. 211

One of the EW tactics that I have selected for analysis consists of a signal intentionally given at the wrong time, and the other a signal given in the wrong place. Bluffing of either sort (wrong place or wrong time) can constitute "ambiguity deception" or "misdirection." In both cases it will have the effect of denying the adversary the information that is of interest because over time he becomes conditioned to the inherent uncertainty of the signal. In this sense the difference between "ambiguity deception" and "misdirection deception" is the extent to which the adversary misallocates his resources as a result. But whether a signal given at the wrong time is more or less effective at generating uncertainty than one given at the wrong place is not immediately obvious.

Defenders during World War II needed radar to provide the early warning necessary to intercept bombing raids while they were still relatively distant. By using countermeasures accompanied by bluffing tactics, the attacker not only denied the defender precise information as to the time and/or location of a true attack, but also presented him with a dilemma. Even if the defender knew for certain that the signal he was picking up was that of countermeasure, he could not be sure whether or not this countermeasure masked a bombing raid. If it did mask an actual raid, the defender logically would want to deploy his interceptors accordingly. But if it did not, misallocating interceptors might be disastrous should a raid later materialize. Alternatively, the defender could wait to deploy interceptors until either his radar was able to resolve an actual raid (if one were in fact underway) or until other sensors detected the raid closer in, possibly through visual or auditory means. Thus, just the fear of a bluff was, and is still, sufficient to cause a rational adversary to delay committing to an action based upon a single source of information.

Yet, as is evident from the above example, bluffing tactics might never be completely effective in causing a rational and knowledgeable adversary to ignore a signal and wait for independent corroboration. So long as a true signal is mixed in with a host of bluff signals, it may be in the adversary's interest to take action some of the time, albeit infrequently if either the cost for misallocation is high or if the bluffs are frequent relative to the true signals. In the World War II example, as bluffing was used to "provide a cover" for genuine use of countermeasures to mask attacks, the defender would sometimes decide that early allocation of interceptors made sense. Bluffing, moreover, does not come without a monetary cost. This cost will affect both the attacker's calculations as to how often to bluff, and by extension, the defender's reasoning as to how often he should act based upon the signal received.

The dilemma described above, which confronts both the attacker and the defender, may be thought of as a dynamic zero-sum game with incomplete information. The precise structure of the game will vary with the tactics employed by the attacker, the options available to the defender in allocating his defensive resources, and the technical characteristics of the countermeasures, the radars, and the weapons involved. Irrespective of the precise structure of the game, however, Perfect Bayesian Nash

Equilibria (PBNE) exist which will dictate how often the attacker should bluff and how often the defender should allocate his resources based upon the initial signal. Once the relevant PBNE has been obtained, it is a simple matter to determine the expected payoff of the tactic being analyzed. Doing so should give operations research analysts, political scientists, and national decision makers the ability to compare tactics and (in the case of other non-EW applications) strategies that involve bluffing.

2. METHOD

Past Approaches

To the extent that electronic warfare tactics are openly discussed in academic literature, formal analysis has largely taken a non-game theoretic approach. I found literature published by the Military Operations Research Society and the Department of Operations Research at the Naval Postgraduate School to be particularly helpful in understanding the military's approach to EW tactics. The analytic methods discussed in the research papers and theses make heavy use of integer programming techniques to compute optimal times and places for the employment of countermeasures or interdiction of radar nodes. The models used are specific to the tactics being analyzed, and I have every reason to believe that actual operational decisions regarding the use of various EW tactics are made using methods similar to those discussed. [3] Unfortunately these methods are limited in that they do not postulate a rational adversary that reacts dynamically to the tactics based upon whatever information is available.

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[3] Examples include Philip Whiteman, "A Target Selection Tool for Network Interdiction," *64th MORS Symposium Working Group 1*, Military Operations Research Society, (Sept. 11, 1996); Eitan Israeli, *System Interdiction and Defense,* Department of Operations Research, Naval Postgraduate School (1999); T. P. Krupenevich, *Network Representation for Combat Models,* Department of Operations Research, Naval Postgraduate School (1984).

In analyzing electronic warfare, academics have thus far utilized game theory only to a very limited degree, applying it to the general problem of network interdiction. Richard Wollmer, working at the time for RAND Corporation, pioneered the approach in a 1970 paper entitled "Algorithms for Targeting Strikes in a Lines-Of-Communication Network."[4] His method involved defining a space filled with nodes, in which any two nodes are connected by way of a finite number of pathways, along each of which commodities or signals may flow at a varying cost. By using matrices to keep track of the cost to a transmitor to transmit signals or commodities along each pathway, Wollmer could compute the total cost to transmit the signal or commodity from the initial node to the final node by any number of pathways. Meanwhile he supposed that an interdictor who interdicts a pathway along which signals or commodities following at any given moment receives a zero-sum payoff. This allowed him to identify optimal mixed strategies for the transmitor's transmission of signals and commodities along various pathways, together with the interdictor's simultaneous interdiction of pathways. These strategies constitute Nash equilibria.

Others have adopted Wollmer's framework as the basis for analyses of specific interdiction operations, involving for instance the Caribbean drug trade, but the framework is not easily applied to electronic warfare for two reasons.[5] First, Wollmer

[4] Richard Wollmer, "Algorithms for Targeting Strikes in a Lines -Of-Communication Network," *Operations Research*, Vol. 8, No. 3 (May-June, 1970), pp. 497-515
[5] See for instance A. Washburn and K. Wood, "Two Person Zero-Sum Games for Network Interdiction," *Operations Research*, Vol. 43, No. 2 (March – April, 1995), pp. 243-251

assumes that the interdictor is limited in his capacity to interdict, which is typically not the case in EW given the integrated nature of radar networks. To fit an analysis of EW tactics into Wollmer's framework, one would define the "transmittor" to be the attacker (who "transmits" bombing raids along various flight paths) and the "interdictor" to be the defender (who seeks to "interdict" the raids by detecting them). However prior to the development of anti-radiation missiles, the typical radar network was in constant operation in all possible locations. As such, the equilibrium strategies for the transmittor and interdictor are trivial. Where Suppression of Enemy Air Defense (SEAD) operations force the interdictor to limit his search efforts, Wollmer's method may be useful, provided, however, that both sides have complete information, which is its second limitation. This limitation is significant because, as I argue, electronic warfare tactics are designed to maximize the uncertainty they cause the adversary.

Lacking is an analytic method for EW tactics that combines the specificity of the military's integer programming models with the assumption that both sides are rational, as postulated by Wollmer et al. I attempt to fill this hole with my analysis. It is possible that with enough adjustments, the traditional framework for the game theoretic analysis of network interdiction could be made to fit the specific circumstances of EW tactics. The generic framework is indeed very powerful, particularly with respect to cost optimization. In a sense it is overkill. Rather than be bogged down manipulating cost matrices that would themselves require lots of computing power as well as detailed (and likely unavailable) data, I have elected to start fresh with models that are specially tailored to be as simple as possible while providing a reasonably realistic representation of specific EW tactics.

"Window" and "Mandrel Screen"

This analysis treats all bluffing behavior as rational and calculative, a result of the careful optimization of a number of intricate factors, including prediction of the defender's optimal response given detection probabilities. It is not necessarily the case, however, that during World War II all bluffing strategies were a result of such optimization. Though World War II saw the employment of thousands of operations research analysts to shape the war effort, with particular emphasis on the fields of search and detection, military commanders often experimented with unconventional tactics.[6] Occasionally this was even done at the suggestion of operations research analysts, who were trying to forecast the behavior of cutting edge technology, and thus welcomed the opportunity to test predictions. Nevertheless over the course of the Allied strategic bombing campaign, as in any situation where games are repeated, both sides came to recognize the effects of their tactics and began to make predictions regarding the other

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[6] See B.O. Koopman, *A Theoretical Basis for Methods of Search And Screening,* U.S. Office of Scientific Research and Development, National Defense Research Committee, vol. 2B, (1946); also, for more general information on World War II military operations research, see Morse and Kimball, *Methods of Operations Research,* Office of the Chief of Naval Operations, Operations Evaluation Group, (1946)

side's response. Thus, for situations like this one in which opposing sides have time to observe one another's strategies - either because they are played repeatedly or else because they are implemented gradually - it makes sense to treat bluffing as calculative, rational, and tending toward predicted equilibria.[7]

"Window" tactics are an example of such unconventional tactics evolving out of the experimentation. "Window" was the nickname for canisters of aluminum foil dropped out of aircraft, which were found to cause huge quantities of noise on radar scopes. (The foil later became known as chaff.) Initially such canisters were dropped haphazardly by hand out of hatches, but, as the tactic caught on, special ports were eventually installed on most bombers to aid in dispersing the foil. Officers in charge of planning were unsure of the exact effects of chaff on radar. They therefore directed crews to drop chaff from the bombers that were part of the raid on one day, on another day to drop it from aircraft flying parallel to the flight path of the raid, on another day to drop it from aircraft flying perpendicular to the raid, and so forth. [8] By varying the location from which the chaff was dispersed relative to the bombing raids, this experimentation somewhat coincidentally allowed the Allied air forces to reap the benefits of bluffing. German radar operators were never sure whether chaff in a given area masked a raid or was merely a noisy distraction from a raid elsewhere. One captured German radar operator exclaimed during interrogation, "Oh, your jamming was

[7] For an analysis of the tendency for systems to evolve toward equilibrium, see Peyton Young, *Individual Strategy and Social Structure: An Evolutionary Theory of Institutions,* (1998)

[8] Price, Vol. 1, pp. 168-168, 176

fantastic! It obliterated everything on our Wurzburg screens!" He went on to say that the Luftwaffe was so confused when the Americans were dropping chaff that they simply turned the radars off and resorted to firing indiscriminate flak. [9]

In contrast to "Window," the tactic known as "Mandrel Screen" was a carefully crafted attempt to use operations research methods to improve the odds of a raid avoiding detection by the Freya, Wurzburg, and Wasserman radars positioned along the coast. B-24 Liberators specially equipped with electronic jamming equipment flew in holding patterns off the coast of Continental Europe and jammed the early warning radars along the flight paths of raids – whether or not bombers were en route.[10] From the mouth of the Meuse northward along the coast all the way to Jutland, Germany maintained a series of complexes known as Himmelbett Stations.[11] Each station was composed of one Freya and three Wurzburg radars, although Wassermans later supplemented the Freyas. Ranges for these radars were up to 300 km (Wasserman), with the basic model Freya capable of detecting bombers out to a more reasonable 130 km.[12] The stations were spaced approximately 50 km apart (some were closer), and in places were arrayed inland as well, forming a formidable defense-in-depth with overlapping radar coverage.[13] As an attempt to desensitize the Luftwaffe to the jamming by blinding the corridors more than was

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[9] Ibid. pp. 169-170
[10] Ibid. p. 176
[11] James Crabtree, *On Air Defense,* (1994) p. 76
[12] H. Kummritz, "German Radar Development up to 1945," ed. Russell Burns, *Radar Development to 1945,* (1988) pp. 212-217
[13] Crabtree, p. 77

necessary, "Mandrel Screen" is an example of a tactic in which signals were intentionally sent at the wrong time.

It is not known the extent to which "Window" and "Mandrel Screen" tactics were optimized in order to keep attempted intercepts to a minimum. Analyzing records and conducting interviews with combatants, the U.S. War Department concluded that from the winter months of 1944 onward, the Luftwaffe steadily decreased its counter-air operations, making little attempt, for instance, to oppose the June 1944 Normandy invasion. [14] But attributing this data to any one factor is impossible. Was it due to "Window" and "Mandrel Screen," to a shortage of fuel and spare parts, to increased presence of Allied escort fighters, or to some other factor or combination of factors? The Allied strategic bombing campaign of Germany was so all-inclusive that we cannot be sure.

That said, it is known that Luftwaffe took particular action in response to Allied tactics that included adding Freya and Wurzburg radars to its ground controlled intercept scheme, known as the Kammhuber Line (traversing Germany, the line already included Himmelbett Stations spaced as closely as 30 km apart). And specifically in response to subterfuge, it fed the radar data to a series of divisional control centers to monitor and deconflict contacts and vector interceptors to bombing raids.[15] Given that attempted

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[14] United States Strategic Bombing Survey, *Summary Report (European War),* (Sept 30, 1945) pp. 7-8
[15] Crabtree, pp. 76-78

intercepts grew less frequent as the war progressed, while at the same time radars and control centers were added to the Kammhuber Line, it is fair to conclude that "Window" tactics contributed to the German decision to rely increasingly on flak, and that over time they were tailored for the task at hand. As an indicator of the effectiveness of "Mandrel Screen," Bomber Command requested assistance from the 8th Air Force in order to further perform the tactic at night.[16] In spite of German electronic counter-countermeasures, this effort is widely credited as having reduced the maximum detection range of the typical German radar by about 70%.[17]

Parameters Determining Effectiveness

The success or failure of "Window" and "Mandrel Screen" depended on many factors, some inherent in the tactics and others functions of the technical properties of the countermeasures. In this analysis I determine the theoretical value of the tactics themselves in terms of factors such as the technical effectiveness of each countermeasure and the beneficial or detrimental effects of interception and misallocation. To do so I identify seven parameters, and phrase the equilibria as well as the effectiveness of each tactic in terms of these parameters. Analogies to these parameters should be easy to

[16] Price, Vol. I, p. 177

[17] A. E Hoffman-Heyden, "German World War II Anti-jamming Techniques," ed. Russell Burns, *Radar Development to 1945,* (1988) p. 382

identify for other applications of bluffing. Of particular note, the relative locations of the defender's radars, as well as their susceptibility to the actual countermeasure, will determine the various probabilities of detection. I therefore consider the difference in overall effectiveness of the tactics, plotted against the appropriate probabilities of detection, as well as the value to the defender of any pause in the bombing campaign. Thus I determine the conditions in which one or the other tactic will be more effective.

This relationship is summarized by the generic function

$$U_{1W} - U_{1M} = f(p_{cd}, p_c \mid p_{ci}, , V_o, V', P, C)$$

where

U_{1W} = the attacker's expected payoff for employing "Window" tactics,

U_{1M} = the attacker's expected payoff for employing "Mandrel Screen"

p_{ci} = the minimally degraded probability of contact at a range r (a technical parameter) when "Window" tactics are employed

p_{cd} = the maximally degraded probability of contact at a range r(T) when "Window" tactics are employed

p_c = the probability of contact at range r(T) when "Mandrel Screen" is employed

V_o = the value to the defender of intercepting a raid early, while it is still distant

V' = the value to the defender of intercepting a raid late at a time T

L = the penalty to the defender for misallocating interceptors to a wrong location

C = the penalty to the attacker for allowing a pause in the bombing campaign

Probabilities of Contact

These parameters will depend in some manner upon the range of the raid to the various detectors, as well as upon the number of detectors in the defender's networked air defense system if one exists. They will likely change on a raid-by-raid basis, or in the case of continuous detection, will change over the course of a raid as the aircraft move. As such, I use these probabilities as the primary independent variables in my analysis of the relative effectiveness of the tactics. Though the other parameters are also independent, they will probably be the subject of much intelligence reporting in a given conflict, and are much more likely to remain fixed through the course of a bombing campaign. I have not included any explicit dependence on range in order to keep my models as simple as possible, and also to free the reader to make use of whatever model of search and detection is most appropriate for a given application.

The most common model for search and detection of aircraft used by World War II operations research analysts is known as the inverse cube law. It states that the instantaneous probability of detection of an object is proportional in the inverse cube of the range to that object, or

$$g(r) = \frac{kh}{r^3}$$

where k is the constant of proportionality and h is the difference in height of the detector and the object. It has been shown that for any given instantaneous probability of detection g(r), if multiple objects are present, the cumulative probability of detecting one object is

$$p_c = 1 - e^{-F_c}$$

$$F_c = \sum_{(\text{detectors})} \sum_{t=1}^{T} \ln[1 - g(r_t)]$$

where T is the time at which the cumulative probability is measured. Thus the cumulative detection probabilities may be found for any number of networked radars searching for the same track, using the inverse square law or any other model for the instantaneous detection probability of an object. [18] [19]

The parameters p_{ci}, p_{cd}, and p_c represent cumulative probabilities of detection under conditions where countermeasures are present. In the case of "Window," p_{cd} may be thought of as the resulting probability of detection at a given range when jamming is concentrated ahead of the bombers, along their flight path. Thus if the bombers were to fly directly over a radar, the jamming would maximally reduce the probability that this

[18] For a derivation of the inverse cube law as well as formulae for computing cumulative probabilities of detection in use during World War II, see Koopman, pp. 18-21

[19] Any given radar within the defender's radar network may be affected minimally or maximally by an instance of jamming depending on the geometry of the jamming. A single expression for the cumulative probability of detection after an elapsed time T is obtained, however, by summing over each individual detection attempt by each individual detector according to Koopman's equation.

radar detects them. The probability p_{ci} , on the other hand, should be thought of as the probability of detection at a given range when jamming is concentrated off the flight path of the bombers. This jamming maximally reduces the probability that some other radar off the axis of the flight path detects the bombers, but leaves the nearest radar relatively unaffected. It is assumed that the nearer radar has a greater instantaneous probability of detection than does the more distant off-axis radar, and therefore that

$p_{ci} \geq p_{cd}$ [20]

Meanwhile, the "Mandrel Screen" tactic does not involve the attacker varying the position of jamming with respect to the bombers and the radar network. Therefore the cumulative probability of contact associated with any given geometric arrangement of radars, bombing raids, and jamming aircraft may be represented by one expression, p_c, rather than by multiple expressions to account for all possible locations where an attacker may release countermeasures.

Effect of Interception

If it is the case that the attacker has calculated elsewhere the expected payoff for engaging in a bombing raid and has elected to go ahead with the strategy, that payoff may be taken as the baseline, and all subsequent payoffs calculated as deviations from it.

[20] The validity of this assumption depends on the ability (or lack thereof) of the various radars to reject interference from the "sidelobes". It is possible that more advanced radars may be unaffected by jamming that does not enter through the "mainlobe," in which case p_{ci} and p_{cd} may not have a simple correlation.

Parameters V_o and V' indicate deviations resulting from the interception of bombing raids carried out as part of the attacker's overall strategy. There are two rationales for why V_o, the value to the defender of intercepting that raid while it is still at a distance from its probable target, should be greater than V', the value of intercepting it when it is close. First, the target of the raid will generally not be known with certainty, in which case if the raid is intercepted at a distance, it is less likely to have already dropped its bombs than if interception is delayed. Second, the defender may be able to make use of extra time to send additional interceptors to the air battle.

There will be different methods of computing V_o and V' depending on the defender's uncertainty as to the target and also the defender's ability to make use of extra time to flow interceptors into a battle. As parameters V_o, V', C, and L are all defined without units, their magnitude is only significant relative to one another. I therefore normalize V_o to unity and compute equilibria and payoffs using parameters given relative to V_o. If uncertainty as to the target of the raid is the dominant factor, it may make sense to imagine a probability distribution of the target. V' can then be computed as

$$V' = 1 - \int_0^T \mathit{targprob}(t)\, v(t)\, dt$$

where v(t) is the airspeed of the raid and T is the time at which V' occurs. On the other hand, if the dominant factor is the likelihood the potential for additional interceptors to arrive after interception has occurred, then V' is smaller than V_o due to the defender's reduced ability to attrite the attacker. Use of Lanchester's equations for combat, which characterizes the population of two opposing forces over time as coupled differential equations, might prove to be a better way to compute V'. [21]

Effect of Misallocation

Parameter L should also be thought of as a deviation in the baseline expected payoff of a successful bombing raid, but in the direction opposite V_o and V'. If an electronic warfare tactic results in the defender's misallocating his interceptors, not only does he fail to intercept any raid in progress, but also interceptors might not be available to intercept a subsequent raid. Therefore L is of a larger magnitude when the frequency of raids is high or when the speed of the interceptors is low relative to the space they must patrol. On the other hand, L is smaller when raids are infrequent or the interceptors can be repositioned quickly. The exact magnitude of L may be difficult to compute as it must be normalized to V_o, but presumably this can be done using operations research techniques. Specifically an estimation of L must take into account the impact of a successful bombing raid on the progress of the war, as compared to the loss of a portion of the bomber fleet's anticipated contribution to the war if the raid had been intercepted.

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[21] Lanchester's theory for attrition in combat, in particular the "Lanchester Square Law," has received extensive discussion in the military operations research literature. For a simple overview see N.K. Jaiswal, *Military Operations Research: Quantitative Decision Making,* (1997) pp. 233-238. For a thoughtful critique of the theory, see J. W. R. Leppingwell, "The Laws of Combat? Lanchester Re-examined," *International Security,* Vol. 12, No. 1, (1987) pp. 89-139.

Effect of a Pause

A pause in the bombing, which may occur when the attacker employs a bluff while performing “Mandrel Screen,” takes the form of a cost to the attacker for using the tactic. As the attacker has computed a baseline expected payoff for the bombing campaign using a given engagement rate, the actual engagement rate will reflect the pause and thus yield a negative deviation. Additionally, whatever the attackers goals are for the bombing campaign, it is nearly certain that the defender may make use of time to repair the effects of bombing. If the campaign seeks to interdict supplies to front line troops, time permitting, the defender will repair his roads and bridges. If it seeks to derail the defenders wartime economy, a pause will permit workers to go about their work unhindered by air raid sirens. Any repairs that the defender is able to make represent negative deviations from the baseline expected payoff in their own right.

Even worse, resumption of supply distribution or of war production may have cascading effects on other aspects of the war effort. For instance if a pause in bombing permits supplies to move to front line troops engaged in battle, those troops may now be able to inflict casualties on the attacker’s ground forces that they would not otherwise be able to inflict. Or if the attacker uses the pause to upgrade his air defenses, when the campaign resumes the attacker is likely to suffer greater casualties than he would have otherwise. A pause therefore benefits the defender or hurts the attacker. Of course its exact value again must be normalized to V_o, and may be even more difficult to compute than was determining L, but again this calculation should be feasible using operations research techniques.

3. MODEL I

Definition: "Window" Tactics

There are two strategic players, player 1 (the attacker) and player 2 (the defender), along with Nature, a third player who makes choices randomly according to various probability distributions. In the notation that follows, index i is assumed to never equal j, while k is unrestricted and may equal i or j .

° In Period 1, Nature decides whether the attacker is of type 1 (t_1), type 2 (t_2), or type 3 (t_3), signifying different plausible flight paths along which to send a bombing raid to attack, according to a [1/3 , 1/3 , 1/3] distribution. Along each path the defender has a detector. [22]

° In period 2, the attacker observes Nature's move (t_i) and decides along which flight path to employ countermeasures, path 1 (m_1), path 2 (m_2), or path 3 (m_3). Choice ($m_i \mid t_i$) will correspond to jamming radars on-axis, and choices ($m_j \mid t_i$) will correspond to jamming radars off-axis. To consider mixed strategies, the attacker's strategy space is denoted as $\rho_k(t_i)$ where k represents the message sent by type t_i.

[22] Presumably there are other flight paths available to the attacker as well as other detectors available to the defender, but they do not appear in the model. For an explanation, see assumption *A1 Geometry*

° The defender does not observe Nature's move t_i but does observe the attacker's choice of m_i. Therefore m_i constitutes a signal to the defender, enabling him to form a belief (p_3) in accordance with Baye's rule as to the attacker's type. [23] Consistent with that belief, in period 3 the defender then chooses whether he wishes to reallocate interceptors to cover the blind corridor (a_1) or wait (w_1). If a_1 is chosen the game ends; otherwise it proceeds. Mixed strategies are denoted $[r_1 , 1\text{-}r_1]$.

° Time T is allowed to pass. In period 4 Nature randomly chooses whether the bombing raid has been detected (d) or not detected (nd) based upon the following probability distributions:

if player 1 type t_i played m_i; then prob(d) = p_{cd} , prob(nd) = 1- p_{cd}

if player 1 type t_i played m_j; then prob(d) = p_{ci} , prob(nd) = 1-p_{ci}

If d is chosen the game ends; otherwise it proceeds

° The defender observes Nature's choice, and if Nature chooses nd, he updates his belief (p_5) as to player 1's type again in accordance with Baye's rule. Consistent with that belief, in period 5 the defender decides again whether to reallocate the interceptors (a_2) to

[23] Baye's rule is: prob(A|B) = prob(B|A) * prob(A) / prob(B)

the jammed corridor or wait (w_2). In either case the game ends. Mixed strategies are denoted $[r_2 , 1-r_2]$

° Players 1 and 2 are given zero-sum payoffs $U_2 = -U_1$. Player 2 receives the following payoffs:

$$U_2(t_i , m_i , a_1) = V_o$$

$$U_2(t_i , m_i , w_1 , d) = V'$$

$$U_2(t_i , m_i , w_1 , nd , a_2) = V'$$

$$U_2(t_i , m_i , w_1 , nd , w_2) = 0$$

$$U_2(t_i , m_j , a_1) = L$$

$$U_2(t_i , m_j , w_1 , d) = V'$$

$$U_2(t_i , m_j , w_1 , nd , a_2) = L$$

$$U_2(t_i , m_j , w_1 , nd , w_2) = 0$$

Assumptions

A1 Geometry

Though this model considers only three possible flight paths and three detectors, it is consistent with a somewhat richer (but still symmetric) geometry. I assume that the defender has many detectors arranged in an arc, each along flight paths that extend radially outward from the raid's point of origin. Traditional radar has a detection range limited to its line-of-sight, so that at no time will all detectors be simultaneously capable of detecting the raid, though in the case of the German air defense network there was a certain degree of overlapping radar coverage. I limit the overlapping radar coverage to the detector along the raid's actual flight path as well as to each of the adjacent detectors on either side of the flight path. Therefore for any given raid, the attacker degrades the radar network only if he jams along a flight path that is either the actual flight path or one of the two adjacent flight paths. This is consistent with "Window" tactics, which in all cases sought to degrade the German radar network to provide a degree of cover to the bombers, though their jamming was not confined to the bombers' actual flight paths. Knowing this, after observing an instance of jamming (m_i), the defender may logically limit the plausible attacker types to three, the type that jams on-axis (t_i) and the two types that jam off-axis along adjacent flight paths (t_j).

A2 Symmetry

I further assume that the adjacent flight paths, and therefore the detectors, are spaced evenly. This permits detection probability p_{ci} to be used for both off-axis strategies. Though this might not represent reality in the strictest sense, maps of the Kammhuber Line show a general uniformity in the spacing of Himmelbett Stations. More importantly, however, this simplification allows one to treat the mixed strategy of attacker type t_i as $[\rho_i(t_i)\ ,\ 1\text{-}\rho_i(t_i)]$, permitting an analytic solution to the model.

A3 Defender's Strategy Space

The defender's options, upon receipt of a signal at the start of periods 3 and 5, are limited to reallocating interceptors to the jammed corridor and to waiting. I do not permit him to reallocate interceptors to other corridors. Were this an option to the defender, additional mixed strategy equilibria might result, but these equilibria would be strange, and from a purely military point of view, illogical. It is true that the underlying structure of the model resembles the classic "Matching Pennies" game, which would imply that occasionally reallocating forces to any plausible corridor is rational. [24] However the dilemma that usually confronts a general when faced with an adversary conducting

[24] "Matching Pennies" is a game in which one player chooses heads or tails without telling the other player, and the other player calls heads or tails in attempt to match it. Equilibrium strategies are [1/2 , 1/2] for both players.

electronic warfare is not, "Which corridor is the attack coming from?" but instead, "Do I have sufficient information to act now, or must I delay my response in the hope of acquiring better information?" The exception is in situations where there is jamming along multiple corridors or along *every* plausible corridor. In these cases the emphasis generally switches to the "Matching Pennies" dilemma because one assumes that extra time is unlikely to yield better information. That said, I am interested in the extent to which bluffing tactics cause a defender to delay his reaction when he otherwise would not, so I limit the defender's strategy space accordingly.

A4 Single Opportunity for Detection

Because detection probabilities p_{ci} and p_{cd} may be thought of as cumulative probabilities of detection, it is not completely accurate to say that in this model the defender has a single opportunity to detect the raid. Nonetheless, he is only given one opportunity to reallocate after attempting to detect the attacker, so the model is limited to a certain degree with regard to opportunities for detection. The model could be made richer by allowing for a repetition of periods 4 and 5 until either detection occurs or the defender decides to reallocate, the only limitation being the flight time of the raid to its target. However in doing so, uncertainty as to the raid's target must be explicitly treated as additional information chosen by Nature and observed only by the attacker. In this case the defender will need to formulate additional beliefs with extremely complex Bayesian updating processes, and solving the model will likely require the use of computer-based numerical methods. Furthermore, equilibria will likely exist at every opportunity given to the defender to reallocate his forces. Allowing the defender a single opportunity to reallocate his forces after attempting detection greatly simplifies the calculation of expected payoffs while preserving a diversity of equilibria and illustrating the characteristics of the dilemmas posed by bluffing.

Lesser Assumptions

There are two lesser assumptions inherent in this model that deserve mention. First, in awarding a payoff of V' to the defender after successful detection of the raid, I assume there is a perfect reallocation of interceptors. This might not be the case, but any failure to intercept the raid after detection has occurred is a result of an exogenous force that should be modeled separately. Bluffing is aimed at furthering the aims of electronic warfare, namely denying accurate information to the defender by supplanting it with false signals. Once the defender has obtained accurate information, the tactic is over, and hence I terminate the model. Second, I assume that an attack is always in progress – Nature is not given the opportunity to choose an attacker type that does not attack. This is necessary so as to isolate the non-repeatable effect of a surprise tactic being used in battle and producing lopsided results. The defender must be assumed to have foreknowledge of a tactic if the source of the tactic's effectiveness is to be identified.

Potential Equilibria and Expected Payoffs

The solution to Model I begins with a formulation of the defender's beliefs in period 3 and period 5 (p_3 and p_5). [25]

PROPOSITION 1:

$$p_3 = \rho_i(t_i)$$

PROOF:

The defender's belief in period 3 is defined as the probability that the observed signal m_i was sent by attacker type t_i, or equivalently, the probability that the attacker has jammed on-axis. This may be written as

$$p_3 = \text{prob}(t_i \mid m_i)$$

$$= \text{prob}(m_i \mid t_i)\text{prob}(t_i) \,/\, \text{prob}\,(m_i)$$

$$= \text{prob}(m_i \mid t_i)\text{prob}(t_i) \,/\, [\text{prob}(m_i\,,\, t_i) + \text{prob}(m_i\,,\, t_j) + \text{prob}(m_i\,,\, t_g)]$$

where g is neither i nor j, and k remains unrestricted

$$= \text{prob}(m_i \mid t_i)\text{prob}(t_i) \,/\, [\text{prob}(m_i \mid t_i)\text{prob}(t_i) +$$

$$+ \text{prob}(m_i \mid t_j)\text{prob}(t_j) + \text{prob}(m_i \mid t_g)\text{prob}(t_g)]$$

[25] For an explanation of the Bayesian updating of beliefs in games involving detection, see Pate-Cornell and Fischbeck, "A Two-Step Procedure for the Bayesian Updating of the Probability of Nuclear Attack on the United States," ed. Stanley Erickson Jr., *MAS Proceedings 1991, Military Modeling and Management,* Operations Research Society of America, (Nov. 12-14, 1991) pp. 36-52

Now, prob(m_i | t_k) = $\rho_i(t_k)$, which is defined as the probability that an attacker of any type t_k jams along a given flight path m_i. Meanwhile prob(t_k) is defined to be 1/3 for all types. Therefore,

$$p_3 = \rho_i(t_i) / \Sigma_k \, \rho_i(t_k)$$

By symmetry,

$$\rho_i(t_i) = \rho_j(t_j) \quad \text{, and } \rho_j(t_i) = \rho_g(t_i)$$

The probabilities that a particular attacker type sends any given message must sum to 1

$$\begin{aligned} 1 &= \rho_i(t_i) + \rho_j(t_i) + \rho_g(t_i) \\ &= \rho_i(t_i) + 2\rho_j(t_i) \\ &= \rho_j(t_j) + 2\rho_j(t_i) \end{aligned}$$

Therefore,

$$\rho_i(t_j) = [1 - \rho_i(t_i)] / 2$$

and so

$$\begin{aligned} p_3 &= \rho_i(t_i) / 1 \\ &= \rho_i(t_i) \end{aligned}$$

PROPOSITION 2:

$$p_5 = (1-p_{cd})\rho_i(t_i) / [(1-p_{cd})\rho_i(t_i) + (1-p_{ci})(\rho_i(t_j) + \rho_i(t_g))]$$

PROOF:

Just as in period 3, the defender's belief in period 5 is the probability that the observed signal m_i was sent by attacker type t_i. However the defender has also observed signal nd. Therefore the belief is the probability that the attacker has jammed on-axis given a failure to detect, or

$$p_5 = \text{prob}(t_i \mid m_i , \text{nd})$$

$$= \text{prob}(m_i , \text{nd} \mid t_i)\text{prob}(t_i) / \text{prob}(m_i , \text{nd})$$

$$= \text{prob}(m_i , \text{nd} \mid t_i)\text{prob}(t_i) / \Sigma_k \text{ prob}(m_i , \text{nd} \mid t_k)\text{prob}(t_k)$$

$$\text{prob}(m_i , \text{nd} \mid t_k) = \text{prob}(m_i , \text{nd} , t_k) / \text{prob}(t_k)$$

$$= \text{prob}(\text{nd} \mid m_i , t_k)\text{prob}(m_i , t_k) / \text{prob}(t_k)$$

$$= \text{prob}(\text{nd} \mid m_i , t_k)\text{prob}(m_i \mid t_k)\text{prob}(t_k) / \text{prob}(t_k)$$

Substituting in,

$$p_5 = \text{prob}(\text{nd} \mid m_i , t_i)\text{prob}(m_i \mid t_i) / \Sigma_k \text{ prob}(\text{nd} \mid m_i , t_k)\text{prob}(m_i \mid t_k)$$

$$= \text{prob}(\text{nd} \mid m_i , t_i)\text{prob}(m_i \mid t_i) / \text{prob}(\text{nd} \mid m_i , t_i)\text{prob}(m_i \mid t_i) +$$

$$+ 2 \text{ prob}(\text{nd} \mid m_i , t_j)\text{prob}(m_i \mid t_j)$$

The quantity $1\text{-}p_{cd}$ is defined to be the probability that no detection occurs when the attacker is jamming on axis, and $1\text{-}p_{ci}$ is the probability of no detection when the attacker is jamming off axis. Therefore

$$1\text{-}p_{cd} = \text{prob}(\text{nd} \mid m_i , t_i)$$

$$1\text{-}p_{ci} = \text{prob}(nd \mid m_i, t_j)$$

And given the previous definition of $\rho_k(t_i)$,

$$p_5 = (1\text{-}p_{cd})\rho_i(t_i) / [(1\text{-}p_{cd})\rho_i(t_i) + (1\text{-}p_{ci})(\rho_i(t_j) + \rho_i(t_g))]$$

PROPOSITION 3:

$$p_3 < p_5$$

PROOF:

Let $k = p_5 / p_3$. Then,

$$k = [(1\text{-}p_{cd})\rho_i(t_i) / [(1\text{-}p_{cd})\rho_i(t_i) + (1\text{-}p_{ci})(\rho_i(t_j) + \rho_i(t_g))]] / \rho_i(t_i)$$

$$= (1\text{-}p_{cd}) / [(1\text{-}p_{cd})\rho_i(t_i) + (1\text{-}p_{ci})(\rho_i(t_j) + \rho_i(t_g))]$$

$$= 1 / [\rho_i(t_i) + [(1\text{-}p_{ci}) / (1\text{-}p_{cd})](\rho_i(t_j) + \rho_i(t_g))]$$

$$0 < p_{cd} < p_{ci} < 1$$

$$1\text{-}p_{ci} < 1\text{-}p_{cd}$$

$$(1\text{-}p_{ci}) / (1\text{-}p_{cd}) < 1$$

$$\rho_i(t_i) + (\rho_i(t_j) + \rho_i(t_g) = 1$$

$$k > 1$$

$$p_3 < p_5$$

The implication is that as time passes and the defender still has not detected an attack, if he is aware that there is an attack somewhere, the probability grows that it is coming through the jammed corridor. With these observations about the defender's beliefs, it is now possible to deduce equilibria. In any equilibrium, symmetry requires that each attacker type employ identical strategies (recall $\rho_i(t_i) = \rho_j(t_j)$). Four types of equilibria are considered. In all cases, the zero-sum nature of the game forces the attacker to play a mixed strategy. If he were to play a pure strategy the defender would optimize his response and reallocate accordingly – the essence of bluffing is its unpredictable nature. In response to this mixed strategy, the defender might conceivably play: (1) a pure strategy of never reallocating (were he to always reallocate, the attacker would always bluff); (2) a period 3 mixed strategy and period 5 pure strategy of not reallocating; (3) a period 3 pure strategy of not reallocating and a period 5 mixed strategy; or (4) mixed strategies in both periods.

Where mixing exists for either player, it is only possible because the player's expected payoffs to pursue one or the other strategy are equal. The player is indifferent, enabling him to mix strategies without regard to the payoff. For the defender to mix strategies his belief for the period in question must correspond to this indifference. Thus I shall refer to the defender's expectation of equal payoffs regardless of the strategy he chooses as the indifference condition.

PROPOSITION 4:

For equilibria involving mixed strategies in period 3, the period 3 indifference condition yields belief

$$p_3{}^* = \frac{-L + p_{ci}V' + (1 - p_{ci})\, r_2 L}{V_o - L + (p_{ci} - p_{cd})V' - (1 - p_{cd})\, r_2 V' + (1 - p_{ci})\, r_2 L}$$

PROOF:

Setting the defender's expected payoffs for playing strategies a_1 and w_1 equal to one another,

$$p_3{}^*V_o + (1- p_3{}^*)L =$$

$$= p_3{}^* \, [p_{cd}V' + (1-p_{cd})r_2V'] + (1- p_3{}^*)[p_{ci}V' + (1-p_{ci})r_2L]$$

Solving for $p_3{}^*$ results in

$$p_3{}^* = \frac{-L + p_{ci}V' + (1 - p_{ci})\, r_2 L}{V_o - L + (p_{ci} - p_{cd})V' - (1 - p_{cd})\, r_2 V' + (1 - p_{ci})\, r_2 L}$$

PROPOSITION 5:

For equilibria involving mixed strategies in period 5, the period 5 indifference condition yields belief

$$p_5{}^* = \frac{-L}{V' - L}$$

PROOF:

Setting the defender's expected payoffs for playing strategies a_2 and w_2 equal to one another,

$$p_5{}^*V' + (1 - p_5{}^*)L = 0$$

Solving for $p_5{}^*$ results in

$$p_5{}^* = \frac{-L}{V' - L}$$

Where the defender employs a pure strategy during one or both periods, the two indifference conditions also must correspond to his actions in that they act as the boundaries past which the pure strategies are impossible. Serving as restrictions, these beliefs enable the calculation of PBNE.

PROPOSITION 6:

Potential solutions for each of the four types of equilibria considered exist according to the following respective systems of equations and restrictions.

(1)

$$\rho_i(t_i) \leq \frac{p_{ci}V' - L}{V_o - L + V'(p_{ci} - p_{cd})}$$

$$r_1 = 0$$

$$r_2 = 0$$

with the attacker's expected payoff

$$U_1(\rho_i(t_i), r_1, r_2) = V'(p_{ci}-p_{cd})\rho_i(t_i) - V'p_{ci}$$

Solutions of this type do not meet the definition of Nash Equilibrium (see the subsequent section, Equilibrium Solutions and Sensitivity to Parameters, for an explanation).

(2)

$$\rho_i(t_i) = \frac{L(1-p_{ci})}{L(1-p_{ci}) - V'(1-p_{cd})}$$

$$r_1 = 0$$

$$r_2 = \frac{-Lp_{cd}(1-p_{ci}) - V'p_{ci}(1-p_{cd})}{2L(1-p_{ci})(1-p_{cd})}$$

$$U_1(\rho_i(t_i), r_1, r_2) = -\rho_i(t_i)[p_{cd}V' + (1-p_{cd})V'r_2] - (1-\rho_i(t_i))[p_{ci}V' + (1-p_{cd})Lr_2]$$

$$= -\frac{V'[Lp_{cd}(1-p_{ci}) - V'p_{ci}(1-p_{cd})]}{L(1-p_{ci}) - V'(1-p_{cd})}$$

with the following restriction

$$-L \geq \frac{V'^2 p_{ci}(1-p_{cd})}{V_o(1-p_{ci}) - V'(1-p_{ci}p_{cd})}$$

(3)

$$\rho_i(t_i) = \frac{V'p_{ci} - L}{V_o - L + V'(p_{ci} - p_{cd})}$$

$$r_1 = \frac{V'(p_{ci} - p_{cd})}{V_o - L + V'(p_{ci} - p_{cd})}$$

$$r_2 = 0$$

$$U_1(\rho_i(t_i), r_1, r_2) = -\rho_i(t_i)[r_1 V_o + (1 - r_1) p_{cd} V'] -$$
$$- (1 - \rho_i(t_i))[r_1 L + (1 - r_1) p_{ci} V']$$

$$= -\frac{V'(V_o p_{ci} - L p_{cd})}{V_o - L + V'(p_{ci} - p_{cd})}$$

with the following restriction

$$-L \leq \frac{V'^2 p_{ci}(1 - p_{cd})}{V_o(1 - p_{ci}) - V'(1 - p_{ci} p_{cd})}$$

(4)

No solutions of this type exist.

PROOF:

See Appendix

Equilibrium Solutions and Sensitivity to Parameters

The restrictions associated with Equilibrium Types 2 and 3 imply that PBNE corresponding to individual sets parameters will be of one type or the other, though there might be multiple equilibria where the restrictions overlap. Strategies $\rho_i(t_i)$, r_1, r_2, and payoff U_1 were computed for each potential equilibrium type using a range of values for each of the four parameters, V', L , p_{ci} , and p_{cd} (V_o was set to 1) in order to identify trends.[26] The parameters were constrained to the following ranges:

$0 \leq V' \leq V_o = 1$

$0 \leq p_{cd} \leq p_{ci} \leq 1$

$0 \geq L$

The results were vetted using the equilibrium restrictions as well as the requirement that $\rho_i(t_i)$, r_1 , and r_2 be between 0 and 1. Sample sets of valid parameters, along with their resulting solutions and payoffs, are given in Table 1.

Parameters					Solution Type 1				Solution Type 2				Solution Type 3			
L	p_{cd}	p_{ci}	V'	V_o	max ρ_i	r_1	r_2	max U_1	$\rho_i(t_i)$	r_1	r_2	U_1	$\rho_i(t_i)$	r_1	r_2	U_1
-3.6	0.2	0.5	0.9	1	0.8335	0	0	-0.2291	0.7291	0	0.2528	-0.2826				
-2.0	0.4	0.5	0.7	1	0.7655	0	0	-0.2964	0.7042	0	0.5083	-0.3681				
-1.4	0.3	0.7	0.9	1	0.7287	0	0	-0.3565	0.4406	0	0.8286	-0.4269				
-0.6	0.2	0.5	0.6	1	0.5056	0	0	-0.209	0.3846	0	0.625	-0.1538				
-1.5	0.4	0.7	0.3	1	0.6602	0	0	-0.1506					0.6602	0.0347	0	-0.1506
-0.2	0.2	0.3	0.7	1	0.3228	0	0	-0.1874	0.2	0	0.875	-0.054				
-1	0.2	0.4	0.5	1	0.5714	0	0	-0.1429					0.5714	0.0476	0	-0.1429
-0.7	0.3	0.6	0.3	1	0.4916	0	0	-0.1358					0.4916	0.0503	0	-0.1358
-0.4	0.1	0.2	0.9	1	0.3893	0	0	-0.145	0.2832	0	0.3368	-0.0408				
-0.2	0.3	0.6	0.2	1	0.254	0	0	-0.1048					0.254	0.0476	0	-0.1048
-1.5	0	0.4	0.4	1	0.6241	0	0	-0.0602					0.6241	0.0602	0	-0.0602
-4	0.6	0.6	0.5	1	0.86	0	0	-0.3					0.86	0	0	-0.3

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Trends cited in this section were proven analytically where possible and are listed in the Appendix. Where this was not possible or practical, inductive methods were used. To find solutions satisfying one or more equilibria, Visual Basic scripts in MS Excel were written to generate 10,000 random datasets along with corresponding strategies and payoffs for each equilibrium, where each parameter was randomly selected over its valid range. Valid equilibria were extracted from these sets. To corroborate hypothesized trends (e.g. one value is always greater in one particular equilibrium than in another), 100,000 random datasets were generated in the same manner and the strategy and payoff outputs scanned for contradictory results. No trend is cited for which there were any contradictory findings.

-2	0.8	0.9	0.2	1	0.7219	0	0	-0.1656					0.7219	0.0066	0	-0.1656
-1	0.2	0.4	0.3	1	0.5437	0	0	-0.0874					0.5437	0.0291	0	-0.0874
-1.5	0.6	0.9	0.1	1	0.6285	0	0	-0.0711					0.6285	0.0119	0	-0.0711
-0.5	0.2	0.6	0.2	1	0.3924	0	0	-0.0886					0.3924	0.0506	0	-0.0886

Table 1: Sample of Valid Parameter Sets

with Solutions and Payoffs Across All Equilibria

Equilibrium Type 1

Solutions of this type involve the defender never reallocating his interceptors except after he detects a bombing raid using his degraded detectors. A Nash Equilibrium is defined as a set of strategies for which each is the optimal response to the other(s). These solutions do not meet Nash's definition for equilibrium because, where the defender never reallocates his interceptors absent a detection, the strategy $\rho_i(t_i) = 1$ minimizes the probability of detection and so is the optimal response. The attacker has no incentive to play any $\rho_i(t_i) < 1$. I therefore do not consider Type 1 solutions any further.

The Equilibrium Boundary

Neither Equilibrium Type 2 nor Type 3 is valid over the entire four-dimensional parameter space. Even given the proper domain of each parameter, there will be sets of parameters yielding values for $\rho_i(t_i)$, r_1, or r_2 that are less than 0 or greater than 1. Fortunately within the region of valid parameter space that is bounded by Type 2's restriction, Type 2 solutions are valid everywhere, and the same is true for Type 3. The sole exception is narrow region of parameter space where the restrictions overlap,

$$-L = \frac{V'^2 p_{ci}(1 - p_{cd})}{V_o(1 - p_{ci}) - V'(1 - p_{ci}p_{cd})}$$

which I refer to s the "boundary value." Where this condition holds, so long as the parameters themselves are valid, Type 3 was found to be valid everywhere while Type 2 was found to be valid for some parameter sets and not others. Furthermore, where Type 2 is valid given this condition, the solutions were found to yield a U_1 that is less than the corresponding Type 3 U_1, as well as a $\rho_i(t_i)$ less than the corresponding Type 3 $\rho_i(t_i)$. The implication is that for any given set of valid parameters, one and only one PBNE exists.

For

$$-L > \frac{V'^2 p_{ci}(1 - p_{cd})}{V_o(1 - p_{ci}) - V'(1 - p_{ci}p_{cd})}$$

Type 2 solutions are PBNE.

For

$$-L \leq \frac{V'^2 p_{ci}(1 - p_{cd})}{V_o(1 - p_{ci}) - V'(1 - p_{ci}p_{cd})}$$

Type 3 solutions are PBNE.

Generally speaking, Type 3 PBNE will exist where the cost of misallocation (-L) is low along with the value of a late interception (V'), while Type 2 PBNE will exist where this is not true. The dependency of the boundary value of L on the three parameters V', p_{ci}, and p_{cd} is illustrated using comparative statics in Figure 1. Because of the requirement that L always be negative (or that - L always be positive), regions of space where the - L boundary value dips below 0 imply that a valid value of –L would necessarily be greater than the boundary value. In these regions of space, the Type 2 restriction always holds, and therefore Type 2 solutions are PBNE.

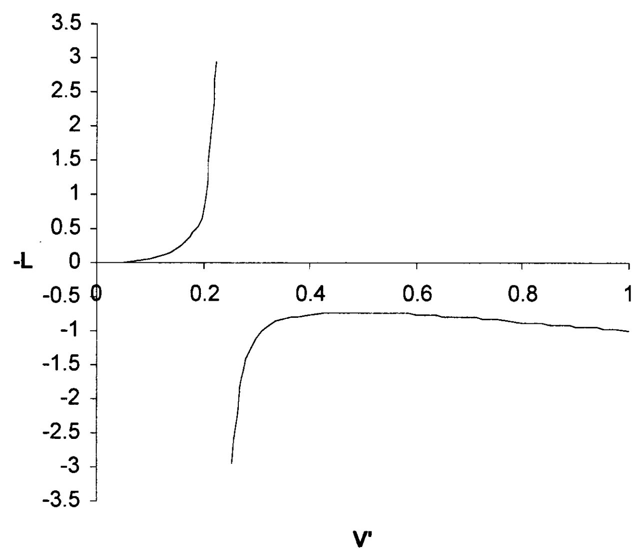

Figure 1: Dependency of the boundary value of –L on V'
p_{cd} = 0.2 p_{ci} = 0.8

The equation governing the boundary value is hyperbolic, with a vertical asymptote at

$$V' = V_o(1 - p_{ci})/(1 - p_{ci}\,p_{cd})$$

For all valid values of V_o, p_{cd}, and p_{ci}, this vertical asymptote clearly lies between 0 and 1. And because all values of –L to the right of the asymptote are negative, only those values of V' to the left of the asymptote will produce a positive (i.e. valid) –L boundary. Examination of the boundary value equation reveals that

$$-L(V' = 0) = 0 \quad \text{and} \quad \frac{\partial(-L)}{\partial V'}\Big|_{V'=0} = 0$$

Therefore as V' increases from 0, the -L boundary value grows hyperbolically, approaching its asymptote. For all V' between the asymptote and 1, the boundary value is negative. Type 3 equilbiria exist along and below the boundary value, but only where the boundary value is positive. Above the boundary value, and in all places to the right of the vertical asymptote, where the boundary value is negative, Type 2 equilibria exist.

The intuition associated with the sensitivity of –L on V' resembles a situation where the defender seeks to outsmart the attacker up until a point, doing the opposite of what the attacker expects him to do. Beyond that point, however, he reasons that the risk associated with trying to fool the defender is not worth the added reward. It is logical that Type 2 equilibria, which represent the defender's decision to delay reallocation until after having first attempted to detect the attacker, would exist where the cost of misallocation is higher than some boundary, while Type 3 equilibria exist below the boundary. The existence of a threshold (the vertical asymptote), above which Type 3 equilibria are entirely unsustainable, corresponds to some critical value of late reallocation (V') relative to early reallocation (V_o). When V' is above this critical value, the defender cannot ever justify the gamble associated with sending interceptors early, no matter how small the cost of misallocation. At the same time, however, under the logic of the Type 3 equilibrium, for greater values of V' the attacker bluffs less often because he knows the defender has a greater interest in attempting to detect him, and he wishes to minimize the probability of detection (see the section *Equilibrium Type 3*). The defender knows this, and beneath the critical value of V', he reasons that the bluffs are sufficiently infrequent, and the difference between V_o and V' sufficiently great, as to warrant reallocating early, even given increasing penalties for misallocation.

Varying p_{ci} and p_{cd} changes the slope of the other asymptote of the hyperbola, which for all valid probability values is negative. For small values of p_{cd} and large values of p_{ci}, the slope is steep. As p_{cd} is increased, or as p_{ci} is decreased, the slope becomes less steep. The position of the vertical asymptote along the V' axis adjusts accordingly, as the point (V'=0 , - L=0) must remain a local minimum. Larger values of p_{cd} shift the asymptote to the right, while larger values of p_{ci} shift the asymptote to the left. Under the logic of the Type 2 equilibrium, where p_{ci} is larger, the defender is in a better position to reason that, "Having failed to detect the attack, the jamming is more likely to be on-axis." (See *Proposition 3*.) Therefore the critical value above which the defender can never justify early allocation decreases, broadening the conditions under which the defender seeks a Type 2 equilibrium. Larger values of p_{cd} weaken the above line of reasoning, thus decreasing the inventive of late allocation and broadening the conditions under which the defender seeks a Type 3 equilibrium.

Equilibrium Type 2

Comparative statics are also used to illustrate the sensitivity of the Type 2 $\rho_i(t_i)$ to parameters L, V', p_{ci}, and p_{cd}. The results appear in Figures 2 through 4. These figures are intended to aid in developing intuitions about how the Type 2 $\rho_i(t_i)$ depends on the four parameters, and thus do not consider the boundary values of L except where explicitly noted. Also, valid Type 2 solutions will only exist where $L < 0$, $p_{cd} < 1$ and $p_{ci} < 1$. Failure to meet any of these three conditions yields an indefinite value for r_2. In the following sensitivity analysis I assume that these conditions are met.

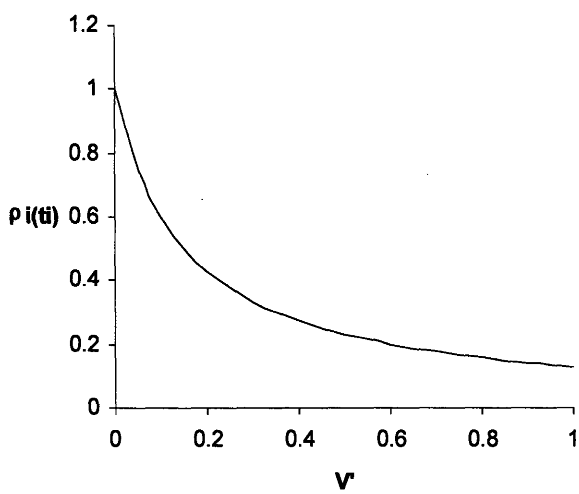

Figure 2: Dependency of $\rho_i(t_i)$ on V'

$L = -0.6$ $p_{cd} = 0.2$ $p_{ci} = 0.8$

$\rho_i(t_i) = 1$ where V' = 0, irrespective of the other parameters. $\rho_i(t_i)$ approaches 0 asymptotically as V' increases. Intuitively, increasing the value to the defender of late reallocation would tend to increase the defender's willingness to reallocate late. The attacker counters this willingness through a lower $\rho_i(t_i)$, which means more frequent bluffing.

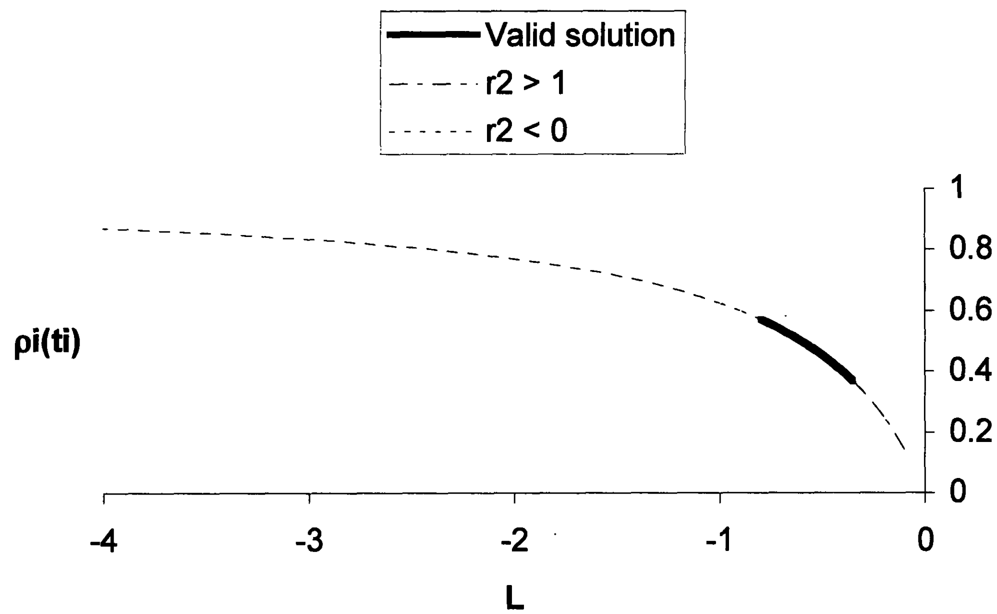

Figure 3: Dependency of $\rho_i(t_i)$ on L

V' = 0.3 $p_{cd} = 0.6$ $p_{cl} = 0.8$

Looked at in isolation, $\rho_i(t_i)$ begins at 0 and approaches 1 asymptotically as the penalty for misallocation increases. Intuitively, when the penalty for misallocation is large, the defender is more reluctant to reallocate his interceptors in the absence of detection, enabling the attacker to bluff less frequently. The degenerate case exists where

V' = 0, in which case the attacker never bluffs. This occurs because the defender has no incentive ever to reallocate late, so the attacker's optimal strategy is simply to minimize the probability of detection, which he does by playing $\rho_i(t_i) = 1$.

However valid solutions are constrained by r_2, which may have a minimum of 0 and a maximum of 1, corresponding to the boundary with the Type 3 Equilibrium.

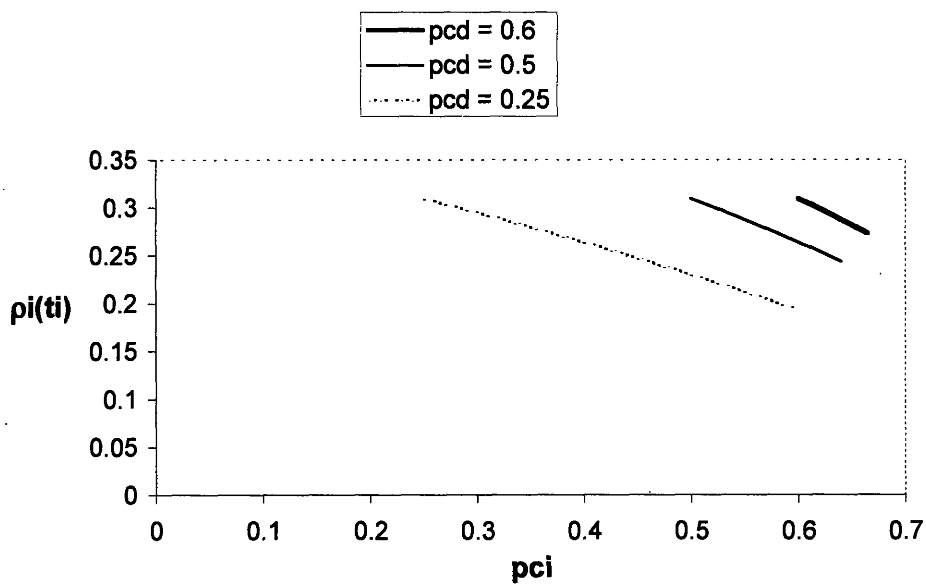

Figure 4: Dependency of $\rho_i(t_i)$ on p_{ci} for different values of p_{cd}

V' = 0.67 L = -0.6

$\rho_i(t_i)$ also varies hyperbolically on the probabilities of detection, p_{ci} and p_{cd}. Values of p_{ci} and p_{cd} producing valid solutions are bounded on one end by the restriction that $p_{cd} \leq p_{ci}$, and on the other end by $r_2 \leq 1$. The maximum value for $\rho_i(t_i)$ occurs where $p_{cd} = p_{ci}$, in which case $\rho_i(t_i) = L / (L-V')$, unless $|L| > 2V'$. Here $\rho_i(t_i)$ reaches its valid maximum at 1.

As p_{ci} increases towards 1, $\rho_i(t_i)$ tends toward 0, while as p_{cd} increases toward 1, $\rho_i(t_i)$ tends toward 1. In the figure below, increasing p_{cd} appears to shift $\rho_i(t_i)$ to the right, resulting in larger values of $\rho_i(t_i)$ for the same p_{ci}.

The intuition behind these dependencies is based on the fact that whenever the defender chooses not to reallocate his interceptors early, a failure to detect the attack indicates an increased likelihood that the attack is on axis. Therefore given a larger p_{ci}, the defender places increasing weight on the assumption that if jamming were off-axis, he would have detected the attack, and so has a greater incentive to reallocate late. The attacker counters this incentive by bluffing more often. Larger values of p_{cd} weaken the above line of reasoning, thus lessening the need for the attacker to bluff in order to counter the defender's incentive to reallocate late.

Equilibrium Type 3

Along and below the boundary value of –L, solutions of Type 3 are PBNE. I again test the sensitivity of the Type 3 $\rho_i(t_i)$ to the four parameters using comparative statics. The results appear in Figures 5 though 8.

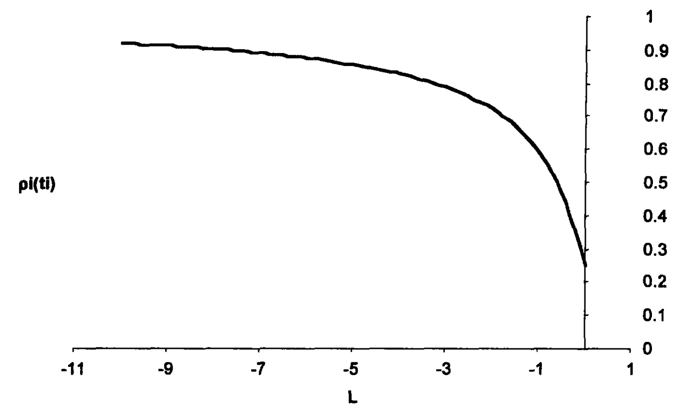

Figure 5: Dependency of $\rho_i(t_i)$ on L

$p_{cd} = 0.2$ $p_{ci} = 0.5$ $V' = 0.6$

$\rho_i(t_i)$ has a minimum value of $V'p_{ci} / [V_o+V'(p_{ci}-p_{cd})]$ for L equal to 0 and asymptotically approaches 1 as L approaches $-\infty$. Where either V' or p_{ci} is equal to 0, $\rho_i(t_i) = 0$. As the cost of misallocation increases the defender is less prone to reallocate absent detection, allowing the attacker to bluff less frequently.

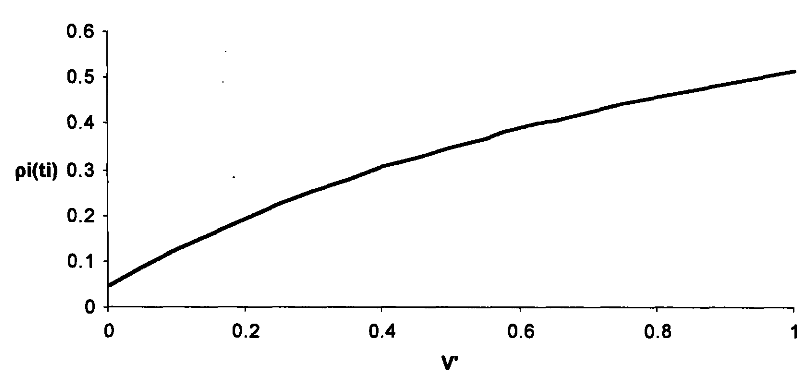

Figure 6: Dependency of $\rho_i(t_i)$ on V'

$p_{cd} = 0.05$ $p_{ci} = 0.95$ $L = -0.05$

$\rho_i(t_i)$ has a minimum value of $-L / (V_o-L)$ for V' equal to 0 and a maximum value of $(p_{ci}-L) / (V_o-L+ p_{ci}-p_{cd})$ for V' equal to 1. Under the logic of the Type 3 equilibrium, the defender never reallocates interceptors after attempting detection if the attempt fails. Therefore a high value for late reallocation serves to restrain the defender in reallocating prior to the attempt, which he may now rely on to a greater degree. This restraint permits the attacker to bluff less frequently, though he must bluff some of the time in order to sustain the equilibrium. If the maximum $\rho_i(t_i)$ were 1, the defender would simply cue his response to the jamming and always reallocate early. In the figure above, extreme values of parameters were used to emphasize the hyperbolic dependency of $\rho_i(t_i)$ on V'. Given less extreme values, the dependency is appears nearly linear.

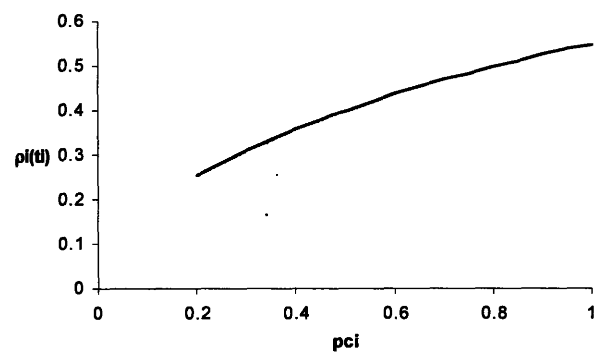

Figure 7: Dependency of $\rho_i(t_i)$ on p_{ci}

$p_{cd} = 0.2$ $L = 0.1$ $V' = 0.9$

$\rho_i(t_i)$ has a minimum value of $-L / (V_o-L)$ for p_{ci} equal to p_{cd} and a maximum value of $(V'-L) / [V_o-L+V'(1-p_{cd})]$ for p_{ci} equal to 1. As p_{ci} increases, the defender's detection attempt becomes more useful. He therefore has less of an incentive to reallocate, in turn permitting the attacker to bluff less often. Extreme values of parameters were again chosen to emphasize the hyperbolic dependency of $\rho_i(t_i)$ on p_{ci}.

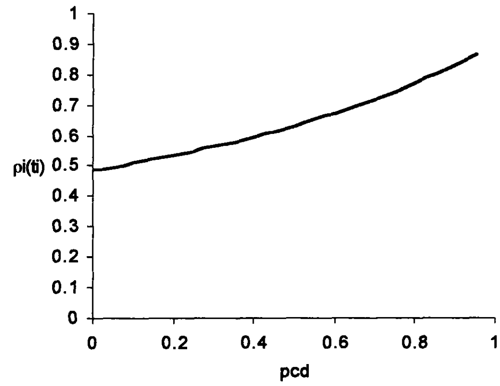

Figure 8: Dependency of $\rho_i(t_i)$ on p_{ci}

p_{ci} = 0.95 L = 0.1 V' = 0.9

$\rho_i(t_i)$ has a minimum value of $(V'p_{ci}–L) / (V_o\text{-}L+V'\ p_{ci})$ for p_{ci} equal to 0 and a maximum value of $(V'p_{ci}–L) / (V_o\text{-}L)$ for p_{cd} equal to p_{ci}. Similar to p_{ci}, as p_{cd} increases the defender has less of an incentive to reallocate early, permitting the attacker to bluff less often. Extreme values of parameters were again chosen to emphasize the hyperbolic dependency of $\rho_i(t_i)$ on p_{cd}.

No Bluffing

To compare the effectiveness of the PBNE to alternative jamming tactics that are straightforward and involve no bluffing, the defender's optimal response to $\rho_i(t_i) = 1$ is postulated. In this case the defender could achieve his maximum $U_2 = V_o$ by playing $r_1 = 1$, $r_2 = 0$. The defender cues his response to the jamming, reallocating the moment he detects countermeasures because he knows that they are a perfect indicator of the flight path that the raid will follow.

This is reasonable logic when the attacker never bluffs, but if the defender tries to use this strategy in the face of bluffing, he is likely to face disaster. The 1967 American raid codenamed "Operation Bolo" in Vietnam, and the Suppression of Enemy Air Defense (SEAD) campaign in support of the 1982 Israeli "Operation Peace for Galilee," both illustrate what can happen when a defender equates the location of jamming with the flight path of an impending raid (see Implications for Modern SEAD Tactics). Through the use of bluffing the attacker can keep a rational defender to a payoff as low as

$$U_2(\text{Type 2 PBNE}) = \frac{V'[Lp_{cd}(1 - p_{ci}) - V'p_{ci}(1 - p_{cd})]}{L(1 - p_{ci}) - V'(1 - p_{cd})}$$

$$U_2(\text{Type 3 PBNE}) = \frac{V'(V_o p_{ci} - Lp_{cd})}{V_o - L + V'(p_{ci} - p_{cd})}$$

For all valid values of parameters these payoffs are less than V_o, and by extension the attacker's payoff in equilibrium is greater than $-V_o$.

4. MODEL II

Definition: "Mandrel Screen"

There are two strategic players, player 1 (the attacker) and player 2 (the defender), along with Nature, a third player who makes choices randomly according to various probability distributions. Player 1 is said to be jamming at all times, resulting in a probability of detection p_c while a raid is in progress and 0 while no raid is in progress. An opportunity cost C is imposed on player 1 while no raid is in progress.

° In period 1, the attacker decides whether to attack (A) or not to attack (NA). If he chooses A, the raid will fly along a flight path coinciding with the jammed corridor. Mixed strategies are denoted [q , 1-q] . Because payoffs will again be given as deviations from the expected payoff of a bombing campaign decided upon exogenously, one must consider choice NA as a pause in the campaign, which includes a total of N attacks.

° The defender does not observe the attacker's move, therefore holds a prior belief q that the attacker chose A. Consistent with the prior, in period 2 the defender chooses whether he wishes to reallocate interceptors to cover the jammed corridor (a_1) or wait (w_1). If a_1 is chosen the game ends; otherwise it proceeds. Mixed strategies are denoted [r_1 , 1-r_1].

° Time T is allowed to pass. In period 3 Nature randomly chooses whether the bombing raid has been detected (d) or not detected (nd) based upon the following probability distributions:

if player 1 played A; then prob(d) = p_c , prob(nd) = 1- p_c

if player 1 played NA; then prob(d) = 0 , prob(nd) = 1

If d is chosen the game ends; otherwise it proceeds.

° The defender observes Nature's choice, and if Nature chooses nd, he updates his belief (p_4) as to player 1's period 1 choice in accordance with Baye's rule. Consistent with that belief, in period 4 the defender decides again whether to reallocate the interceptors (a_2) to the jammed corridor or wait (w_2). In either case the game ends. Mixed strategies are denoted [r_2 , 1-r_2]

° Players 1 and 2 are given zero-sum payoffs $U_2 = -U_1$. Player 2 receives the following payoffs:

$U_2(A , a_1) = V_o$

$U_2(A , w_1 , d) = V'$

$U_2(A , w_1 , nd , a_2) = V'$

$U_2(A , w_1 , nd , w_2) = 0$

$U_2(NA , a_1) = L\text{-}C$

$U_2(NA , w_1 , d) =$ [cannot happen]

$U_2(NA , w_1 , nd , a_2) = L\text{-}C$

$U_2(NA , w_1 , nd , w_2) = -C$

Assumptions

A1 Geometry and Symmetry

The "Window" assumptions as to geometry and symmetry are relaxed because when player 1 attacks, his flight path always coincides with the jammed corridor. Whether there are two detectors or ten, their arrangement does not matter as long as their cumulative probability of detecting the raid by time T is p_c.

A2 Single Opportunity for Detection

This assumption remains from Model I. See Part 3 Assumptions, *A4 Single Opportunity for Detection.*

A3 Zero Payoff

Previously, denoting payoffs as deviations from those expected for a successful bombing raid was logical because the bombing raid always took place. In Model II the bombing raid does not necessarily take place. Therefore one may argue that a zero payoff means two different things: in one sense it means that a raid bombed its target successfully and there are no deviations in payoff, and in another sense it means that no raid occurred and there are no deviations in payoff. This apparent contradiction may be unsettling. But in the context of a larger bombing campaign whose expected payoff, divided by the total number of expected attacks, is the baseline payoff, awarding a payoff of zero in the case of a delay (less the opportunity cost for pausing) is consistent. The length of the campaign must now be extended to include the attack that was called off, while the baseline payoff remains unchanged.

Lesser Assumptions

As was the case in Model I, in awarding payoff V' to the defender upon detecting the raid, I assume that reallocation of interceptors is perfect. The possibility for failure of command and control (C2) that causes the defender to miss the opportunity to intercept should be modeled separately. And again, as with Model I, the attacker's strategy spaced does not include multiple locations for attack. Including such strategies would make it impossible to isolate the effects of tactical ingenuity from the actual effectiveness of the tactic.

Potential Equilibria and Expected Payoffs

Similar to Model I, the solution to Model II begins with a formulation of the defender's posterior belief in period 4 (p_4).

PROPOSITION 7:

$$p_4 = (q - qp_c) / (1 - qp_c)$$

PROOF:

The defender's posterior belief in period 4 is defined as the probability that the attacker chose to attack given that the degraded detector network failed to detect the attack. This may be written as

$$p_4 = \text{prob}(A \mid nd)$$

$$= \text{prob}(nd \mid A)\text{prob}(A) / \text{prob}(nd)$$

$$1 - p_4 = \text{prob}(NA \mid nd)$$

$$= \text{prob}(nd \mid NA)\text{prob}(NA) / \text{prob}(nd)$$

Substituting in,

$$p_4 = \text{prob}(nd \mid A)\text{prob}(A)\,(1 - p_4) \,/\, [\text{prob}(nd \mid NA)\text{prob}(NA)]$$

The probability that no detection occurs when there is an attack present is defined to be 1-p_c, while the probability that no detection occurs when there is no attack present is defined to be 1. The probabilities of that attacker chooses to attack or not attack are q and 1-q respectively. Therefore

$$p_4 = (1\text{-}p_c)(1\text{-}\ p_4)q \,/\, (1\text{-}q)$$

Solving for $p_4$yields

$$p_4 = (q\text{-}qp_c) \,/\, (1\text{-}qp_c)$$

Again the zero-sum nature of the game forces the attacker to play a mixed strategy. Given the defender's strategy space, the same four types of equilibria are considered as in Model I: (1) a pure strategy of never reallocating (were he to always reallocate, the attacker would always bluff); (2) a period 2 mixed strategy and period 4 pure strategy; (3) a period 2 pure strategy of not reallocating and a period 4 mixed strategy; or (4) mixed strategies in both periods. Where mixing exists for either player, it is again only possible because the player's expected payoffs to pursue one or the other strategy are equal. For period 4 mixing this corresponds to the indifference condition.

PROPOSITION 8:

For equilibria involving mixed strategies in period 4, the period 4 indifference condition yields belief

$$p_4{}^* = -L / (V'-L)$$

PROOF:

Setting the defender's expected payoffs for playing strategies a_2 and w_2 equal to one another,

$$p_4{}^*V' + (1- p_4{}^*)(L-C) = (1- p_4{}^*)(-C)$$

Solving for $p_4{}^*$ results in

$$p_4{}^* = -L / (V'-L)$$

PROPOSITION 9:

For equilibria that require the defender to be indifferent as to his strategy in period 4, the attacker must play strategy

$$q^* = \frac{-L}{V'(1-p_c) - L}$$

PROOF:

Using the results of propositions 7 and 8, set $p_4 = p_4{}^*$ to obtain

$$(q^*-q^*p_c) / (1-q^*p_c) = -L / (V'-L)$$

Solving for q* yields

$$q^* = \frac{-L}{V'(1-p_c)-L}$$

PROPOSITION 10:

Potential solutions for each of the four types of equilibria considered exist according to the following systems of equations. There are no additional restrictions.

(1)

$$q \leq \frac{-L}{V_o - p_c V' - L}$$

$$r_1 = 0$$

$$r_2 = 0$$

with attacker's expected payoff

$$\begin{aligned} U_1(q, r_1, r_2) &= (-C - p_c V')q + C \\ &\leq \frac{L(C + p_c V')}{V_o - p_c V' - L} + C \end{aligned}$$

(2)

$$q = q^* = \frac{-L}{V'(1-p_c)-L}$$

$$r_1 = 0$$

$$r_2 = \frac{-C - p_c V'}{V'(1-p_c)-L}$$

$$U_1(q, r_1, r_2) = -q[V' r_2 + (1-r_2)V' p_c] - (1-q)(L r_2 - C)$$

$$= \frac{L(CL - p_c^2 V'^2 - CV')}{(1 - p_c)(V' - L)[V'(1 - p_c) - L]} - \frac{L(C + p_c V')}{V'(1 - p_c L)}$$

(3)

$$q = \frac{-L}{V_o - p_c V' - L}$$

$$r_1 = \frac{-C - p_c V'}{V_o - p_c V' - L}$$

$$r_2 = 0$$

$$U_1(q, r_1, r_2) = - q[V_o r_1 + p_c V'(1 - r_1)] - (1 - q)(L r_1 - C)$$

$$= C + \frac{L(C + p_c V')}{V_o - P_c V' - L}$$

(4)

No solutions of this type exist.

PROOF:

See Appendix

Equilibrium Solutions and Sensitivity to Parameters

As with Model I, the potential equilibria solutions for Model II were tested using various sets of parameters, and the resulting values for q, r_1, and r_2 vetted to determine which solutions could be PBNE and for which parameter values. Parameters were restricted to the following ranges:

$0 \le V' \le V_o = 1$

$0 \le p_c \le 1$

$0 \ge L$

$0 \ge C$

Sample sets of valid parameters, along with their resulting solutions and payoffs, are given in Table 2. Comparative statics are used to illustrate the sensitivity of q to parameters L, V', and p_c. C is also considered insofar as it affects payoff U_1. The results appear in Figures 9, 10, and 15. Figures 11 – 14 depict the boundary of the parameter space in which Type 2 solutions constitute PBNE.

Parameters					Equilibria Type 1 and 3					Equilibrium Type 2			
P	p_c	V'	V_o	C	q	Type 1 r_1	Type 3 r_1	r_2	U_1	q	r_1	r_2	U_1
-2	1	1	1	-1	1	0	0	0	-1	1	0	0	-1
-1	0.3	0.7	1	-2	0.559	0	1	0	-1	0.671	0	1.201	-0.799
-1	0.25	0.75	1	-1.5	0.552	0	0.724	0	-0.776	0.64	0	0.84	-0.66
-2	0.75	0.5	1	-1.5	0.762	0	0.429	0	-0.643	0.941	0	0.529	-0.441
-1	0.5	0.5	1	-1	0.571	0	0.429	0	-0.571	0.8	0	0.6	-0.4
-2	0.75	0.5	1	-0.5	0.762	0	0.048	0	-0.405	0.941	0	0.059	-0.382
-4	0.5	0.5	1	-2	0.842	0	0.368	0	-0.526	0.941	0	0.412	-0.353
-0.5	0.5	0.5	1	-0.5	0.4	0	0.2	0	-0.4	0.667	0	0.333	-0.333
-2	0.25	0.5	1	-1	0.696	0	0.304	0	-0.391	0.842	0	0.368	-0.263
-0.5	0.75	0.25	1	-0.3	0.381	0	0.086	0	-0.257	0.889	0	0.2	-0.2

Table 2: Sample of Valid Parameter Sets

with Solutions and Payoffs Across All Equilibria

Equilibrium Type 1

Model I Type 1 solutions did not constitute Nash Equilibria because they did not permit the attacker to optimize his strategy to the defender's response. There the defender chose never to reallocate unless he detected an attack, motivating the attacker to minimize the probability of detection by never bluffing. Model II differs from Model I in that there is only a single probability of detection, p_c. The attacker might conceivably "minimize" the probability of detection by choosing never to attack - that is, always bluffing. However such a strategy violates assumption *A3 Zero Payoff*. If the attacker is to employ "Mandrel Screen" to enhance a bombing campaign, the attacker is required to attack some of the time. Given an a priori decision to carry out a set number of attacks, the overall probability of detection when the attacker uses "Mandrel Screen" will remain constant regardless of q. Therefore I will proceed to analyze the sensitivity of Type 1 solutions, as a strategy q that is less than 1 is indeed consistent with the defender's response $r_1 = 0$, $r_2 = 0$.

Type 1 solutions for Model II exist throughout the valid portion of the parameter space. Furthermore the Type 1 solutions provide the maximum payoff to the attacker when he plays the maximum allowable value q (except when $C > p_cV'$), and this q value and the resulting U_1 are identical to the corresponding Type 3 q and U_1 values (see Appendix for proof). Barring $C > p_cV'$ (which is discussed at the end of this section), once again the defender never has an incentive to reallocate before attempting detection, and anywhere in the parameter space that a Type 2 solution does not exist will produce a PBNE of Type 1. However Type 1 equilibria payoffs never exceeds those of the corresponding Type 2 equilibria. Nor is the maximum q value that any given Type 1 solution produces ever larger than the corresponding Equilibrium Type 2 q value (see Table 2). In fact the opposite is true. For any given set of parameters that yields a valid Type 2 solution, the solution's q value will be less than the maximum allowable q value for the corresponding Type 1 solution (see Appendix). Therefore the Type 2 solutions, where they exist will be PBNE, and in those cases the Type 1 solutions will not be PBNE.

The maximum value of q (hereafter simply q, as the maximum value provides the attacker with the maximum payoff), depends on parameters L, V' and p_c as follows.

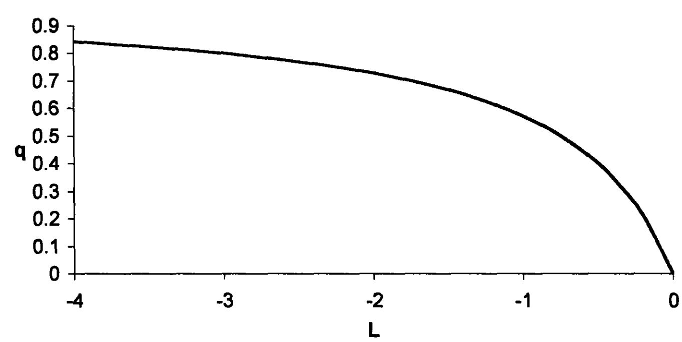

Figure 9: Dependency of q on L

$p_c = 0.5$ V' = 0.5 C = -1

q has a hyperbolic dependency on L, asymptotically approaching 1 in the limit as L approaches -∞. In the degenerate case where $p_cV' = 1$, then $q = 1$. As p_cV' approaches 1, the hyperbola is "pulled up" toward the asymptote. There is no simple intuition for the dependency of q on L under the logic of the Type 1 equilibrium, as misallocation can never occur. The dependency occurs because "q" really represents the maximum allowable q to sustain the equilibrium. For any given value of q, if the penalty for misallocation is not sufficiently high, the defender will not be motivated to wait until detection to reallocate. The less frequently the attacker bluffs, the higher the penalty for misallocation must be in order to restrain the defender from reallocating interceptors without first detecting the attack.

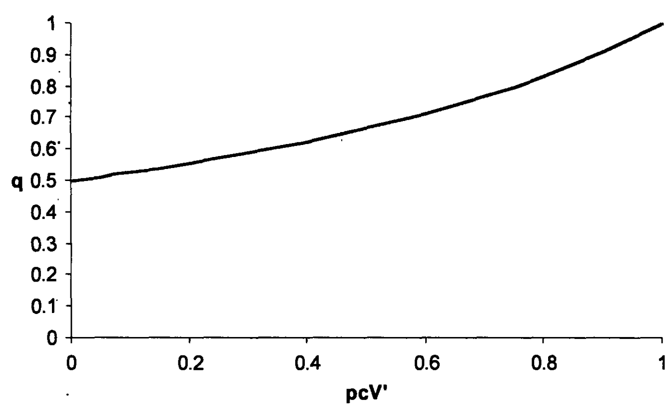

Figure 10: Dependency of q on p_cV'

L = -1 C = -1

All of the dependencies on p_c or V' occur in the form p_cV'. Therefore I consider p_cV' as a single quantity for purposes of analyzing the sensitivity of q. This dependency is also asymptotic, with the minimum $q = -L / (V_o - L)$ occurring where $p_cV' = 0$. In the degenerate case where L is 0, then $q = 0$. Again, all values of q less that the maximum allowable are valid, and p_cV' must be sufficiently high to sustain any given value of q. Within this equilibrium type, the defender only receives payoff V' when he detects the attacker, so p_cV' may be thought of as the expected value of detection. The lower the defender's expected value of detection, the more often the attacker must bluff in order to sustain the equilibrium by denying the defender motivation to reallocate in the absence of detection.

Earlier in this section I referred to $C < p_cV'$ as an exception to the validity of Equilibrium Type 1. For all values of C the attacker's payoff U_1 varies linearly with his strategy q from maximum $q = 0$ to maximum $q = 1$. Where q is 0, U_1 is the opportunity cost of not attacking, C. The slope of U_1 for positive values of q may either be positive, 0, or negative, depending on the value of $C + p_cV'$. Situations of interest correspond to $C + p_cV' \leq 0$ because where $C + p_cV' > 0$, the slope of U_1 is negative, a result of assumption *A3 Zero Payoff*. For such situations the opportunity cost of not bombing is so small that the attacker never has a motivation to bomb. This amounts to a logical flaw in the model. A simple analogy may clarify this point. Consider a person whose employer advances him his weekly income of $320. His job pays an hourly wage of $8 and offers flex hours, though the person is required to work a total of 40 hours per week. With $320 already in hand, on any given morning the person lacks an immediate incentive to go to work (his flex hours enable him to sleep in, and he already has the money to meet his needs for the week), but if he never goes to work he will default on his employer, and his future income in unsustainable. Similarly, when $C + p_c V' \leq 0$ the attacker lacks an incentive to launch any given individual bombing raid. However his baseline payoff assumption is unsustainable if he does not motivate himself to launch a total of N raids. That said, if this analysis is to have any value at all, we must accept this small logical flaw and move on.

Equilibrium Type 2

As stated above, Type 2 solutions are PBNE within the parameter space for which they are valid. This is not the entire parameter space, however. The boundary of parameter space governed by Equilibrium Type 2 is restricted by the value of r_2, which must be between 0 and 1. Sensitivity of r_2 to parameters p_c, V', L or C is illustrated in Figures 8-11. Where Type 2 solutions produce invalid values for r_2, Type 1 solutions constitute PBNE.

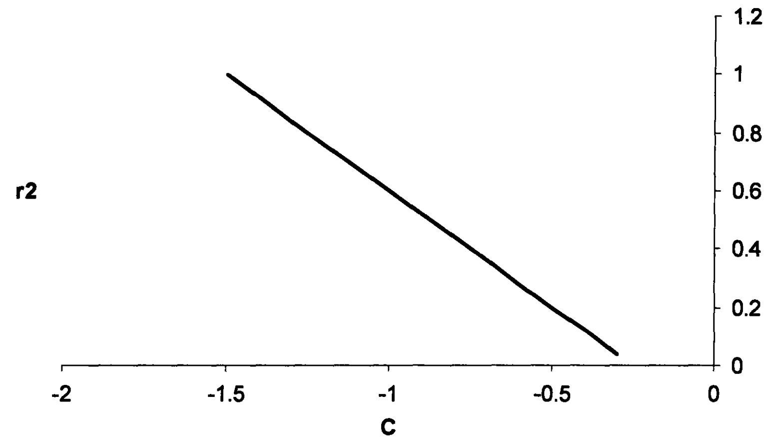

Figure 11: Dependency of r_2 on C showing valid values of r_2 only

P = -1 V' = 0.5 pc = 0.5

r_2 has a straightforward linear dependency on C, which results from the fact that the defender profits by the amount of C regardless of his strategy whenever the attacker bluffs. Therefore the larger the cost to the attacker for employing "Mandrel Screen", the more the defender can feel at liberty to gamble by reallocating when he fails to detect an attack. If the attack is actually occurring, the gamble results in an interception. If the defender guesses wrong and there is no attack, he suffers a loss of the amount L but it is made up for by the amount C.

The slope of r_2 is always between 0 and –1. r_2 crosses 0 at $C = -p_cV'$, and crosses 1 at $C = L-V'$. Therefore r_2 is invalid for all values of C not between these two points. The lower boundary ($-p_cV'$), which was discussed in the previous section, results from the logical flaw in Model II created as a result of *Assumption A3.* The upper boundary exists because, where the cost to bluff is too large, the defender cannot possibly reallocate his interceptors often enough after failing to detect an attack. That said, the reversion to Equilibrium Type 1 is somewhat puzzling. However, the Type 3 solutions produce the same payoffs as Type 1 solutions (see *Equilibrium Type 3* and appendix), and it is intuitive that where the defender seeks to capitalize on the high cost to bluff by reallocating, at some point he might begin to reallocate early. In any case, the dependency of r_2 on other parameters will be more complex than for C, but these two boundary points will always remain, affecting the Type 2 boundary with respect to the other parameters due to algebraic equivalence.

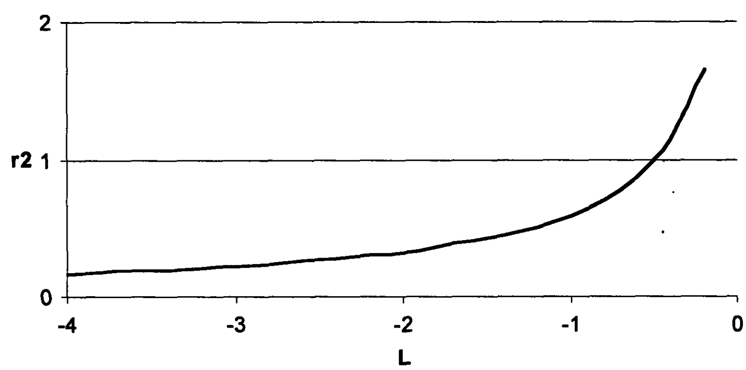

Figure 12: Dependency of r_2 on L showing invalid values of r_2 above 1

$p_c = 0.5$ $V' = 0.5$ $C = -1$

r_2 increases hyperbolically as L approaches 0. This is logical, as the defender's willingness to reallocate his interceptors absent detection should increase as the penalty for misallocation falls off. r_2 reaches its valid maximum of 1 where $L = V' + C$, which is equivalent to the cost parameter's upper boundary. In the limit as L approaches $-\infty$, r_2 approaches 0.

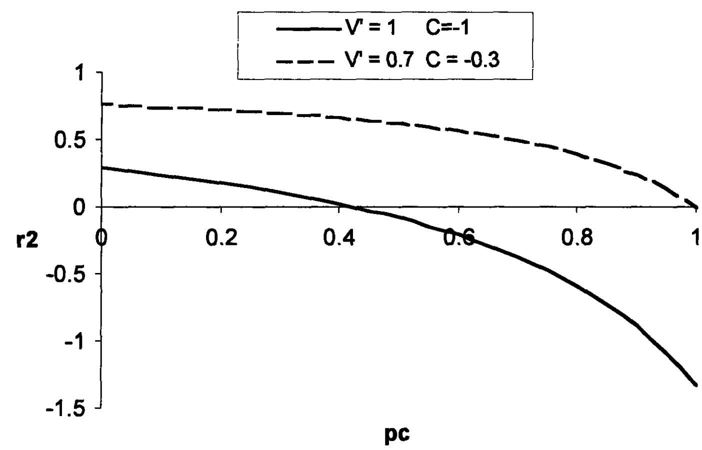

Figure 13: Dependency of r_2 on p_c

showing invalid values of r_2 below 0

L = -0.3

r_2 depends hyperbolically on p_c with a horizontal asymptote at $r_2 = 1$. Whether r_2 approaches 1 from the right, as is shown in Figure 10, or whether it approaches from the left depends on if C is greater or less than L-V' (in both the above cases C is greater). C = L-V' produces the degenerate case, $r_2 = 1$. Where C is less than L-V', there are no valid values of r_2, corresponding to the upper boundary discussed earlier. Considering then the dependency illustrated if Figure 13, where the probability of detecting an attack is high, an absence of detection indicates an increased likelihood that the attacker is bluffing. The defender should therefore be less inclined to reallocate late and risk the penalty for misallocation.

When p_c is 0 then $r_2 = C/(L - V')$, which may or may not be less than 1. r_2 crosses 0 (from either the top or the bottom, depending on L) at $p_c = -C/V'$. This r_2 intercept may be less than or greater than $p_c = 1$, and is equivalent to the cost parameter's lower boundary.

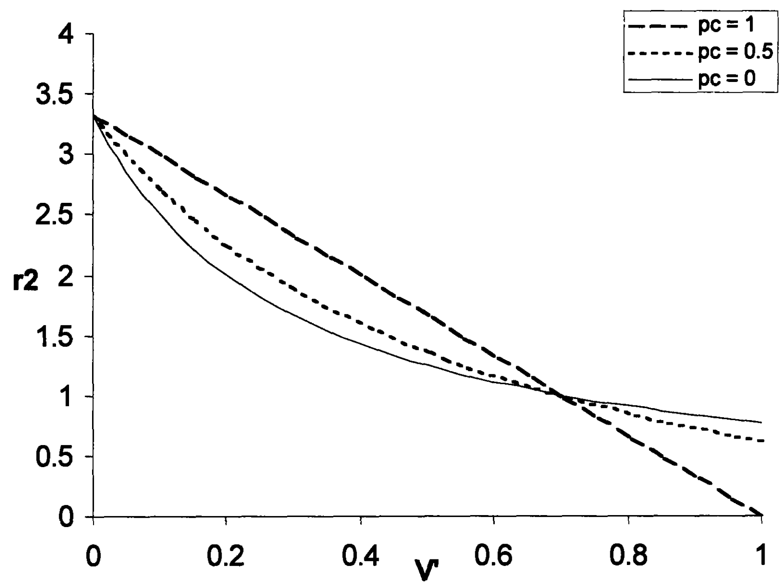

Figure 14: Dependency of r_2 on V'

showing invalid values of r_2 above 1

L = -0.3 C = -1

Because V' occurs in the numerator and the denominator, r_2 asymptotically approaches 0 as V' approaches 1. This occurs because a larger reward to the defender for intercepting the attacker motivates the attacker to bluff more often, and thus the defender responds by reallocating less frequently so not to be penalized for misallocation. When $p_c = 1$ the V' terms in the denominator cancel completely, and we get the degenerate case, a line. r_2 crosses 0 at V'= $-C/p_c$, which may be greater or less than V' = 1. r_2 crosses 1 at V'= L-C, which may be any number greater than L. Because of this condition it is possible that for certain combinations of L and C there will be no valid values of r_2, regardless of V' or p_c. These boundaries are again equivalent to those of cost parameter.

Having described the region in which Type 2 solutions are PBNE, I am now free to analyze the sensitivity of q to the parameters V', p_c, L, and C. The analysis is straightforward, as the Type 2 q is nearly identical to the Type 1 q except for a single term in the denominator. The Type 1 denominator is slightly larger than the Type 2 denominator (the former contains the term V_o where the latter contains V'), indicating that for Type 2 the attacker bluffs less often with respect to every parameter. This occurs because where otherwise the attacker would never profit from a misallocation on the part of the defender (who for Type 1 only reallocates after detecting an attack), now the defender receives additional payoff whenever the attacker makes a mistake. This frees him to bluff slightly less often and save on some of the cost of bluffing.

The behavior the Type 2 q with respect to parameters C, L, and p_c is identical to that of the Type 1 q for those same parameters, except that q is shifted up slightly. For V', the behavior will be different, and is shown in Figure 15.

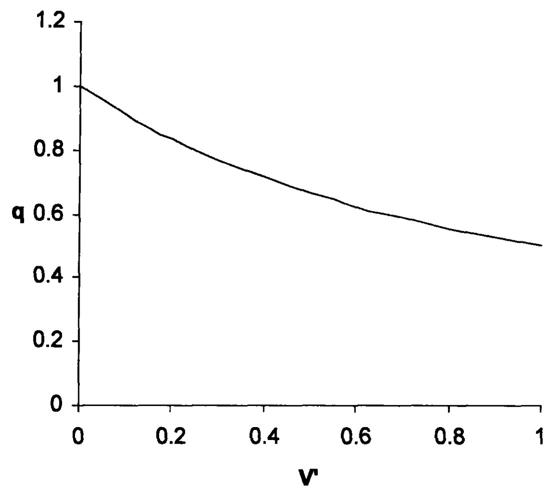

Figure 15: Dependency of q on V'

showing invalid values of r_2 above 1

L = -0.7 pc = 0.3

q now decreases asymptotically toward 0 as V' increases rather than increasing toward 1. Its maximum value is 1 at V' = 0, and its minimum value is L / (L + p_c −1) where V' = 0. As was implied in the discussion of the dependence of r_2 on V', q is decreasing in nature because a larger reward to the defender for intercepting the attacker motivates the attacker to bluff more often.

Equilibrium Type 3

Where solutions of Type 3 do exist, both the q value and the payoff are identical to that corresponding to Equilibrium Type 1 (for proof see Appendix). Therefore there are no conditions for which a Type 3 solution is a unique PBNE, although intuitively it should occur for $C < L-V'$ as discussed in the previous section. I do not discuss the sensitivity of Type 3 solutions, as it would not add any new insight.

No Bluffing

To compare the effectiveness of the PBNE to alternative jamming tactics that are straightforward and involve no bluffing, the defender's optimal response to $q = 1$ is postulated. As in Model I, the defender could achieve his maximum $U_2 = V_o$ by playing the strategies $r_1 = 1$, $r_2 = 0$. Again the defender reallocates the moment he detects jamming because he knows that the jamming indicates perfectly whether or not a raid is in progress. On the other hand, depending on which PBNE the parameters produce, through the use of bluffing the attacker can keep a rational and knowledgeable defender to a payoff as low as

$$U_2\,(\text{Type 1 PBNE}) = -\,C - \frac{L\,(C + p_c V')}{V_o - P_c V' - L}$$

or

$$U_2\,(\text{Type 2 PBNE}) = \frac{L\,(C + p_c V')}{V'(1 - p_c L)} - \frac{L\,(CL - p_c^{\,2}\, V'^2 - CV')}{(1 - p_c)\,(V' - L)\,[V'(1 - p_c) - L]}$$

In the limit as C approaches $-\infty$, both of these payoffs may be greater than V_o. Therefore it is not always the case that employing the "Mandrel Screen" is profitable. However for any reasonable value of C such that $0 \geq C \geq -1$, U_2 will be significantly less than V_o, so use of the tactic will be advantageous for the attacker.

5. RESULTS

Having characterized the PBNE for both Model I and Model II, representing "Window" and "Mandrel Screen" tactics, I may now compare the effectiveness of each tactic, denoted as U_1(Window) and U_1(MS). Given any valid values for parameters discussed in the Part 2 (Method), both tactics will prevent a rational adversary from cuing his interceptors predictively based upon the jamming. Depending on the exact values of these parameters, "Mandrel Screen" may result in an additional delay ad infinitum if the defender fails in his detection attempt, or the defender may find it in his interest to reallocate randomly following a failure to detect the raid. When "Window" is employed, the defender will occasionally find it in his interest to reallocate forces randomly prior to detection where both the cost of misallocation and the value of a belated interception are small, but will generally delay reallocation until after attempting to detect an attack.

Comparison with "Mandrel Screen" Equilibrium Type 1

Where C is less than L-V' or greater than $-V'p_c$, "Mandrel Screen" will have a PBNE for Type 1. Recall, however, that values of C greater than p_cV' create a logical flaw in Model II as a result of *Assumption A3*. Therefore I only consider situations where C is less than L-V'. In these situations, for all valid values of V', L, p_c, p_{cd}, and p_{ci}, I found that "Window" equilibria generate greater values of U_1 than does "Mandrel Screen" Equilibrium Type 1. This trend is likely due to the high cost to the attacker to employ "Mandrel Screen."

Comparison with "Mandrel Screen" Equilibrium Type 2

"Mandrel Screen" Type 2 solutions are PBNE where C is between the values of L-V' and –V'p_c. Within these values of C, when faced with "Mandrel Screen," the defender will delay responding to jamming until after having attempted detection, but assuming he fails to detect the raid, he will find it profitable to cue his interceptors to the jamming. Because C is unconstrained below zero and influences only the effectiveness of "Mandrel Screen," it is the primary determinant of relative effectiveness. "Window" will always be more effective for large values of C because "Mandrel Screen" is unaffordable. For most of this region of the parameter space, U_1(Window) will exceed U_1(MS), though certain values of p_c, p_{cd}, and p_{ci} can cause this trend to reverse. When p_{cd} and p_{ci} are near 1, low values of C result in superior equilibrium payoffs for "Mandrel Screen" as shown in Figure 12. Note that the "Window" Type 2 and Type 3 equilibria exist within distinct regions of the parameter space, so the Figure 16 is contrived in that only a single "Window" equilibrium should be compared to "Mandrel Screen" for any parameter set. However the figure serves to illustrate the point that for most parameter space in which "Mandrel Screen" Type 2 equilibria exist, U_1(Window) exceeds U_1(MS).

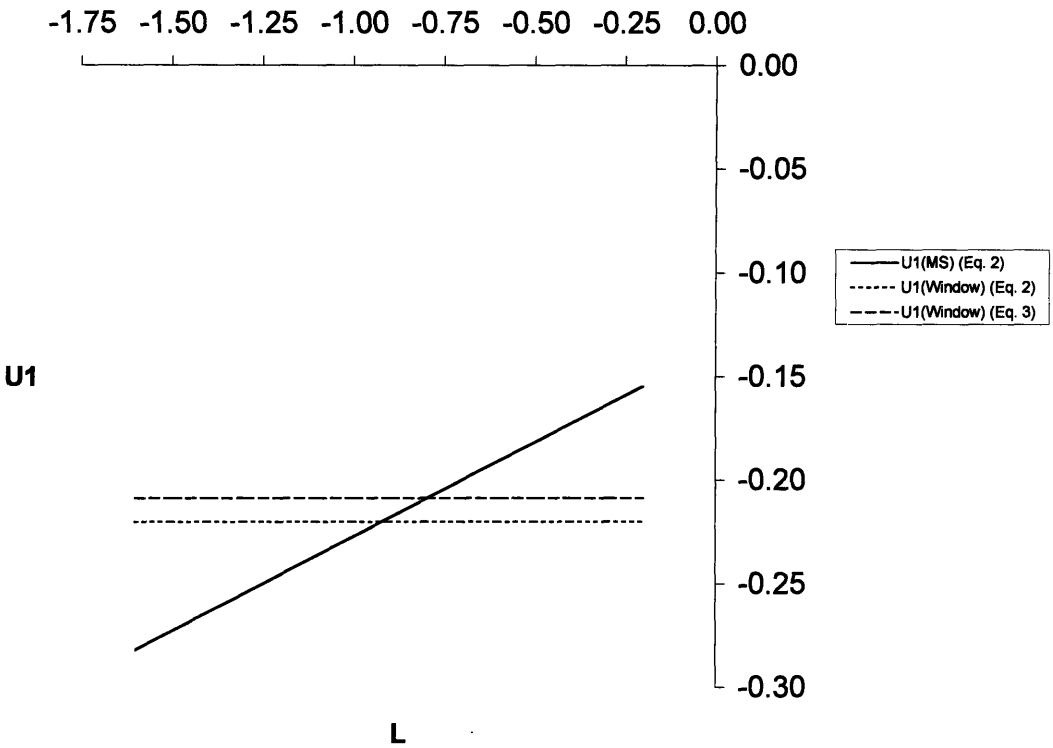

Figure 16: Expected payoffs for values of C within the bounds of "Mandrel Screen" Type 2 PBNE

$p_c = 0.5$ $p_{cd} = 0.6$ $p_{cl} = 0.9$ $V' = 0.3$ $L = -1.5$

Historically, "Window" and "Mandrel Screen" tactics were associated with different countermeasures, hence the reason p_c and p_{cd} are treated as independent parameters. To determine how the technical effectiveness of various countermeasures influences the relative effectiveness of the tactics, U_1(Window) – U_1(MS) is considered as a function of p_c and p_{cd}. Because Models I and II each produce two equilibria depending on the parameter set, the analysis of U_1(Window) – U_1(MS) is repeated for both "Window" Type 2 and Type 3 solutions. However, the analysis omits further discussion of the equilibrium boundaries. In the figures that follow, positive values along the vertical axis favor "Window," and negative values favor "Mandrel Screen".

For all values of C producing a Type 2 PBNE for "Mandrel Screen," a comparison with the two "Window" equilibria yield a planar relationship, owing to the linear dependence of $U_1(MS)$ on C. (see Figure 17).

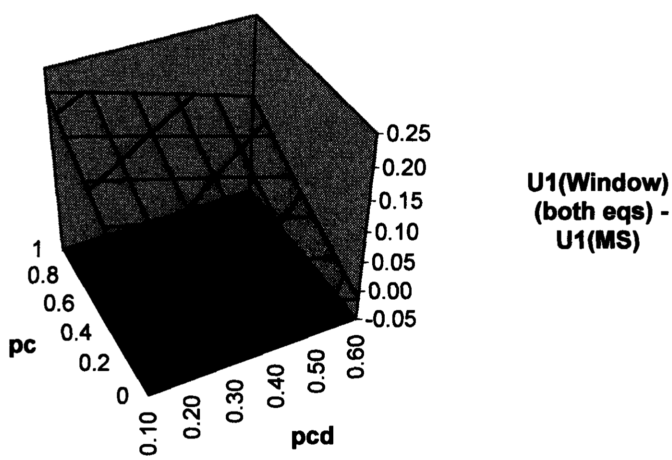

Figure 17: Dependency of the difference in U_1

for each tactic on p_c and p_{cd}

$P = -1.5$ $C = -0.5$ $p_{ci} = 0.6$ $V' = 0.3$

Where p_c and p_{cd} are comparable or nearly comparable, the results strongly favor "Window" over a broad range of C. A similar dependency exists when p_c is held constant and one looks at the difference as a function as p_{cd} and p_{ci} as shown in Figures 18 and 19. The catastrophe on the left side of the figures is the result of p_{cd} exceeding p_{ci}

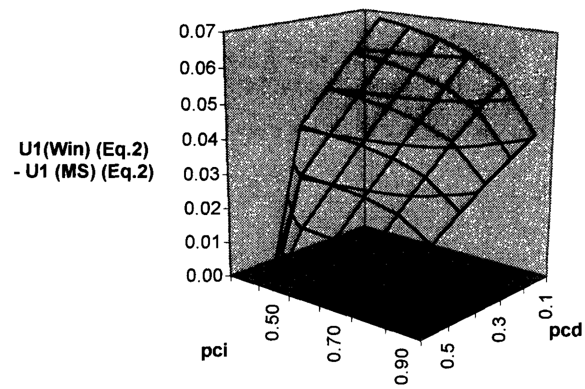

Figure 18: Dependency of the difference between

U_1(Window) (Eq. 2) and U_1(MS) on p_{cd} and p_{ci}

L = -1.5 C = -1 pc = 0.5 V' = 0.1

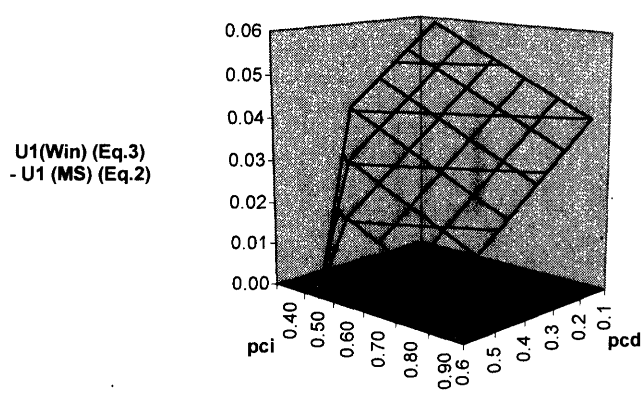

Figure 19: Dependency of the difference between

U_1(Window) (Eq. 3) and U_1(MS) on p_{cd} and p_{ci}

L = -1.5 C = -1 pc = 0.5 V' = 0.1

These results support the intuition that smaller probabilities of detection associated with either tactic increases the profitability of employing that tactic.

Lastly, it should be noted that neither V' nor L have noteworthy impact on the relative effectiveness of the two tactics within the range of C being considered, though within a narrow band of C, p_c, p_{ci}, and p_{cd} values, where L takes on a large magnitude U_1(MS) will exceed U_1(Window) for both equilibria types (shown in Figure 20). There is no simple intuition for this result, except to say that the defender seemingly has a greater tendency to misallocate his interceptors when he is uncertain as to the time of an attack rather than the place of an attack. Such a tendency would cause greater penalties for misallocation to increase the profitability of "Mandrel Screen" more than they do the profitability of "Window."

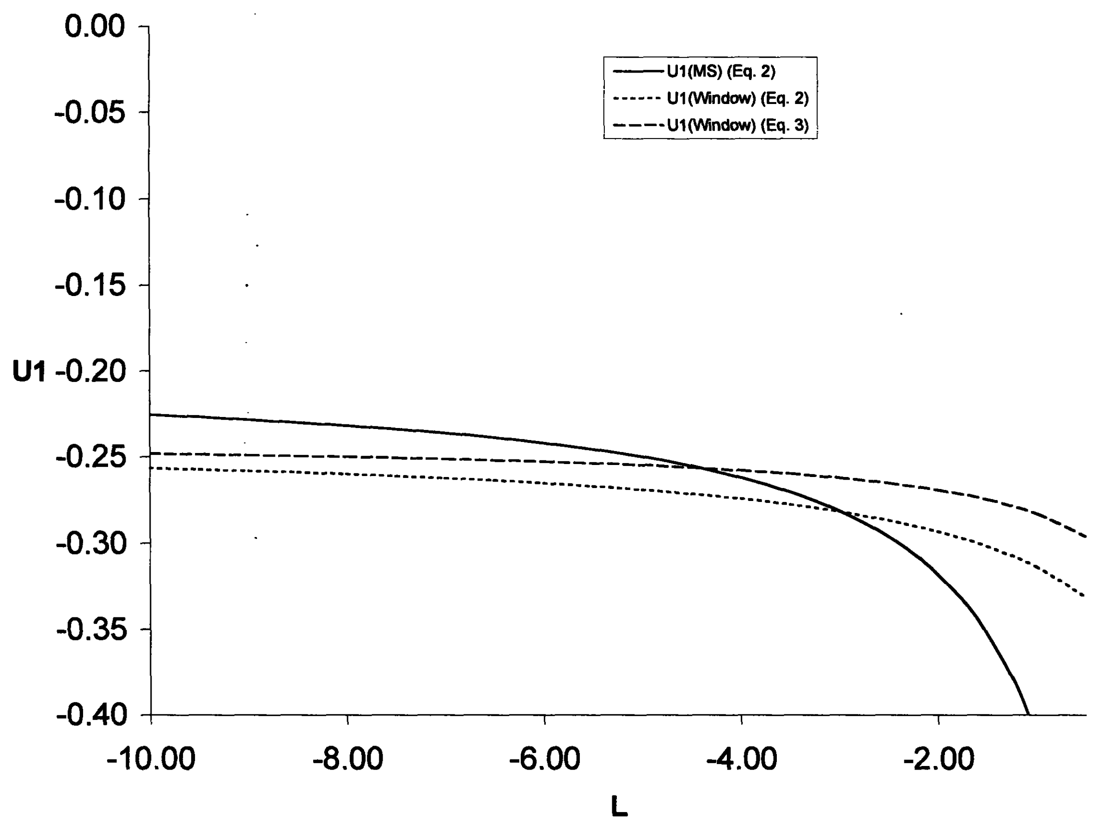

Figure 20: Expected payoffs as functions of L

$p_c = 0.5$, $p_{cd} = 0.6$, $p_{ci} = 0.9$, $V' = 0.4$, $C = -1.5$

6. DISCUSSION

Implications for Modern SEAD Tactics

Though the methods and, by association, the tactics of electronic warfare have evolved over the years, there is no reason that modern technology should make obsolete the general principle of bluffing. By assuming that the defender has prior knowledge of the tactic employed by the attacker, the models isolate the effect of tactical ingenuity and focus instead on the principles by which the tactics operate. This implies that similar measures to create uncertainty as to the location or time of an attack ought to have effects similar to those observed for “Window” and “Mandrel Screen” when incorporated into more modern tactics. Indeed, a number of modern EW operations have incorporated such measures and have met with great success. This is especially interesting in the case of two operations from the Vietnam War, “Bolo” and “Linebacker II”. The protracted nature of that conflict permits comparison with operations that did not feature tactical bluffing.

To apply the foregoing analysis, it is assumed that these modern EW operations may be modeled with values V_o, V’, P, C, p_c, p_{ci}, and p_{cd} that do not test the extreme limits of their respective valid ranges. It follows that the success rate of attacks will be higher for operations incorporating measures to create uncertainty as to either the place of an attack or the time of an attack. However, signals that create uncertainty about the place of attack are predicted to be more effective in this respect.

Elements of Bluffing in Vietnam Era Tactics

During "Rolling Thunder," the bombing campaign directed against North Vietnam from 1965 to 1968, the North Vietnamese Air Force acquired a reputation for relatively skillful use of their MiG fighter aircraft to intercept American air raids. Kenneth Werrel writes in his book *Archie, Flak, AAA, and SAM,* "This Air Force proved as elusive as the Vietcong, using guerrilla tactics of hit and run, and fighting only when circumstances were favorable." [27] In particular, MiGs were used to intercept F-105 "Wild Weasel" aircraft (the code name given to aircraft carrying anti-radiation missiles) performing SEAD missions. The F-105s were effective in forcing air defense radars to cut their emissions but were vulnerable in air-to-air combat. "Operation Bolo" was launched in January 1967 in order to ferret out the elusive MiGs, and represents the United States' greatest air-to-air combat success of the war. For a week, groups of US F-4 Phantom air-to-air fighters flew flight profiles identical to those of the "Wild Weasels" (same formations, altitudes, speeds, and routes), while jamming aircraft provided support similar to that given to the F-105s. Over the course of the week, nine MiG-21s were destroyed without loss as they attempted to intercept what they thought were F-105s,

[27] Kenneth Werrel, *Archie, Flak, AAA, and SAM,* (1988) pp. 101, 102

following which the NVAF stood down for a number of weeks to regroup.[28]

"Operation Bolo" amounted to bluffing in time, analogous to "Mandrel Screen". Each incursion, whether F-4 or F-105, constituted a signal, while only the F-105 raids were true signals – the F-4 raids were bluffs masquerading as true signals. The operation resulted in air-to-air victories because the NVAF did not see the bluff coming. Once the NVAF incorporated the bluffing tactics used in "Bolo" into its planning process, to the extent that it stood down, its rate of attempted interception dropped dramatically as predicted. It eventually rebounded somewhat, though presumably the ground-control intercept radar operators were more wary of their contacts.

The USAF also reintroduced the chaff corridor during the 1972 operation "Linebacker I". [29] In contrast to World War II, however, the corridors were laid strictly along the intended paths of the bombers, partly because new "short pulse" radars improved the defender's ability to detect aircraft not wholly within the corridors (equivalent to increasing p_{ci} in Model I). Although the corridors were somewhat successful in spoofing surface-to-air missiles (SAMs) shot at aircraft flying within them, MiG-21s and MiG-19s quickly located American aircraft. in spite of the corridors On May 10, the opening day of "Linebacker I," the MiGs laid an ambush that resulted in

[28] Carl Berger, *The United States Air Fore in Southeast Asia, 1961- 1973,* Office of Air Force History, (1984) pp. 82-83
[29] Werrel, p. 117

large dogfights and the downing of several Phantoms.[30] This action was typical of "Linebacker I," though the Americans learned to expect the MiGs. Later that year, however, during "Linebacker II," the USAF switched to laying multiple chaff corridors and eventually entire "chaff blankets" to cover the multiple approaches taken by B-52 Stratofortresses to Hanoi. [31]

The MiG threat never materialized during the course of "Linebacker II," though this was probably due to a combination of factors such as the nightly raids on their airfields, the bad weather, the presence of fighter escorts, as well as the chaff. [32] Nonetheless, the complete absence of MiGs during the opening days of "Linebacker II" is somewhat surprising, considering their heavy presence during "Linebacker I." American F-111 night-capable fighters had been in present during "Linebacker I," [33] and the USAF had fielded large numbers of fighter escorts, [34] so the revised tactics used to employ electronic countermeasures deserve much of the credit. The chaff tactics used in "Linebacker II" amount to bluffing in space, and were nearly identical to "Window" tactics used in World War II (the chaff blanket was a contemporary innovation). The marked contrast in the MiG presence over North Vietnam for the two bombing campaigns attests to the effectiveness of bluffing in space.

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[30] Price, Vol. 3, pp.187-189
[31] Werrel pp. 123-125
[32] Price, Vol. 3 208 - 212
[33] Berger, p. 70
[34] Werrel, pp. 118, 119

Bekaa Valley and the Birth of Modern SEAD

In 1982, in response to the use of Lebanon by terrorists as a base from which to stage raids, Israel launched "Operation Peace for Galilee," the invasion of Lebanon. At that time, Syria maintained a sophisticated Soviet designed integrated air defense system in Lebanon, which included a number of advanced SA-6 and SA-8 surface-to-air missile sites backed by MiG interceptors. Having lost a sizable portion of its air force to Syrian and Egyptian SAMs nine years earlier in the 1979 Yom Kippur War, Israel devised an operation featuring extensive use of deception to completely neutralize the SAMs. Samson decoy drones with radar signatures mimicking those of aircraft were used in the initial phase of the assault to entice the Syrian SAM radars to emit. The emissions were triangulated and used as aim points for bombs, artillery and missiles. [35] Also, jamming along corridors off the axis of actual flight paths induced Syrian radar operators into vectoring MiGs to intercept phantom attacks. Israeli fighters lying in wait ambushed the MiGs, resulting in as many as 90 Syrian aircraft lost for no Israeli losses. [36] Syria withdrew its remaining air defense forces.

Both sorts of bluffing contributed to the Israeli victory – the drones represent bluffing in time while the off-axis jamming represents bluffing in space. However, the item of note is not the lopsided Israeli victory in downing so many Syrian aircraft, but

[35] James Brungess, *Setting the Context*, (1994) pp. 19-21
[36] Charles Mayo, "Lebanon: An Air Defense Analysis," *Air Defense Artillery*, (Winter 1983) p. 24

Syria's decision to withdraw forces. Israel achieved its immediate military goals by employing sophisticated tactics that Syria did not anticipate. In the face of those tactics, the optimal Syrian response would have been to withhold the MiGs and to cease all SAM radar emissions, at least until after attempting to fix the location of Israeli air raids. In withdrawing their forces, Syria expressed some rudimentary understanding of what had happened and what it should expect if it continued to fly its aircraft and operate its air defenses in Lebanon. The predicted equilibrium was reached by trial and error.

Soviet Anti-Carrier Tactics

In 1976 the Soviet Union transferred half of its fleet of supersonic Tu-22 Backfire bombers to its Naval Aviation service. It began testing a variety of tactics involving the use of air launched cruise missiles to saturate the air defenses of U.S. carrier battle groups. The U.S. Navy witnessed the introduction of EW aircraft and fighter escorts into the mix of Backfires in a series of Soviet fleet exercises held in 1985. [37] According to analysts, the purpose of the EW aircraft was to jam a battle group's early warning radar along multiple corridors, while down one corridor the Backfires would fly, shooting their massive package of cruise missiles.[38] The tactic did have two notable limitations – it required that the Soviets know the battle group's location and that the battle group be within range of the land-based Backfires. Nonetheless, as late as 1990, the U.S. Navy still

[37] Richard Gross, "Soviet Forces Simulate US Aircraft Carrier Attack," *Jane's Defense Weekly*, 27 April 1985, p. 701
[38] Milan Vego, *Soviet Naval Tactics*, pp. 210-219

had not devised a means to effectively counter the tactic except to operate farther off the Soviet coast.[39]

Thankfully, the United States never went to war with the Soviet Union, and no country today other than Russia maintains the capability to carry out such a saturation tactic incorporating EW bluffing. This situation highlights the increased effectiveness of creating uncertainty as to the location of an attack rather than the time of an attack. The U.S. Navy's acute difficulty in developing a response resulted from the limited number of fighters - 24 F-14 Tomcats - aboard a carrier available to intercept the massive bomber fleet. A generous estimate is that half may have been airborne on Combat Air Patrol in the most heightened state of alert. This force of twelve aircraft would have been hard pressed to intercept a massed raid of 20 or more Backfire bombers even if its flight path were known. Split up to cover three or four axes simultaneously, the interceptors faced a near-impossible task. Had the U.S. experienced such a tactic in real war, the battle group would likely have delayed allocating its limited number of Tomcats until the location of the bombers was fixed, possibly resulting in disaster. Creating uncertainty as to the time of an attack might have stretched U.S. resources thin, but would not have similarly required an interceptor to be in two or three places at once.

[39] Donald Chipman, "The Transformation of Soviet Maritime Air Operations, Implications for US Maritime Strategy," *Airpower Journal* (Summer 1990) p. 5

Operation Desert Storm, Opening Night

The SEAD campaign that took place during the opening night of "Operation Desert Storm" appears to have been lifted straight from the history books on the Bekaa Valley operation. Most of the details are still classified, but Central Command and US Air Force planners appear to have made use of decoy drones as well as selective jamming of Iraqi radar systems in order to stimulate Iraqi radars to emit, thereby creating targets for anti-radiation missiles. [40] Air Force BGEN Buster Glosson, in charge of developing the air campaign, expressed concern over the possibility that operators of Iraq's formidable integrated air defense system might cue their weapons based upon American jamming. [41] The need to incorporate elements of bluffing into coalition SEAD tactics was therefore taken quite seriously, and will probably continue to be taken seriously in any future electronic warfare campaign.

Broader Implications

Elements of bluffing may be embedded within elaborate tactics, such as the Soviet anti-carrier tactic discussed above, but they can also take very simple forms. Whenever a rational actor must base an action upon a certain signal, bluffing may be used

[40] Michael Gordon and Bernard Trainor, *The General's War,* (1995) pp. 112, 118
[41] ibid. p. 117

to inject uncertainty into the signal. A planner can then identify parameters determining the probability that the signal is genuine or fake along with the values associated in both cases with acting or not acting, and thereafter perform a game theoretic analysis similar to the analysis in this paper to determine equilibrium strategies.

Beyond the world of electronic warfare, there are a variety of simple examples of bluffing where uncertainty created in both location and time of attack might provide good case material to test the predictions of the "Window" and "Mandrel Screen" models. Examples of the former include maneuvers in ground combat that involve feinting, "shoot and scoot" artillery displacement tactics, and "wolfpack" tactics in submarine warfare. [42] The latter will include dress rehearsals for attacks that are observed by the defender prior to an actual occurrence (such as the Egyptian 1973 attack across the Suez Canal), as well as methods of cryptography where a message is embedded within noise with the key acting as the filter.

Perhaps the most common use of bluffing in the modern era of warfare is the use of decoys on the battlefield. Decoy tanks and radio traffic were used during World War II to deceive the Germans as to the location of the 1944 cross-channel attack; the German Army incorrectly suspected Calais. More recently, Serbia made extensive use of decoys to mask the location of their military hardware during NATO's "Operation Allied Force". But the grimmest use of decoys to create uncertainty, and that most worthy of study, is

[42] In the latter, bluffing might occur when a single submarine attacks a convoy and forces the escorts decide how to respond given the potential threat of a nearby "wolfpack".

without a doubt the inclusion of decoy warheads on intercontinental ballistic missiles. With the weight of a decoy being a fraction of that of a genuine warhead, an attacker can place a large number of decoys onboard a missile for a relatively small penalty in performance. This is analogous to a very small C in Model II. Recall that in such situations, the defender's equilibrium strategy was to do nothing absent a confirmed detection, indicating that missile defense is likely to be futile. Of course the value of intercepting a genuine warhead is incredibly high. As all parameter values are normalized to this value, it is possible (in a model analogous to Model II) that this will offset the small C. Nonetheless, the analogy appears to be both timely and apt. Further analysis should help in determining if missile defense has a place in America's national military strategy.

In whatever context, electronic warfare or otherwise, judiciously used bluffing permits an actor to augment his other efforts to degrade his adversary's situational awareness by creating uncertainty. If an equilibrium is allowed to form (most likely by trial and error), the result will likely be a delay in the adversary's response and a higher success rate for the attack. However, the reality is that all to often, military planners are enthralled by the effects of tactical ingenuity and continue to implement newer and more elaborate tactics rather than wait for the defender to adjust and establish an equilibrium. I do not mean to imply that this is a bad thing. If the goal of a mission is to destroy enemy interceptors en masse, the tactics that are used to draw the interceptors out will probably work only once. But returning to Alfred Price's lessons of electronic warfare, it is cautioned that "there is no bottomless bucket of electronic warfare tactics," and thus repetition in protracted conflict is inevitable. [43] This should not be a cause for fear. So long as tactics contain within themselves elements that cause the adversary some inherent uncertainty, planners may continue to use them even after the adversary has adjusted, enjoying the benefits of equilibrium in addition to the initial effect of surprise.

[43] Price, Vol III, p. 554

APPENDIX

Proposition 6 Proof

Equilibrium Type 1

For this equilibrium to exist the defender must always decide to wait in period 3 and period 5 (i.e. , $r_1 = 0$, $r_2 = 0$). Therefore

$$p_3 \leq p_3^*$$

$$p_5 \leq p_5^*$$

Proposition 3 states that in all cases $p_3<p_5$, so the latter condition is redundant.

Proposition 2 states

$$p_3{}^* = \frac{-L + p_{ci}V' + (1 - p_{ci})\, r_2 L}{V_o - L + (p_{ci} - p_{cd})V' - (1 - p_{cd})\, r_2 V' + (1 - p_{ci})\, r_2 L}$$

Proposition 1 states $p_3 = \rho_i(t_i)$. Combining these conditions and setting $r_2 = 0$ yields

$$\rho_i(t_i) \leq \frac{p_{ci}V' - L}{V_o - L + V'(p_{ci} - p_{cd})}$$

Expected payoff U_1 will be defined as

$$\sum_{\text{strategy space}} (\text{strategy})\,(\text{payoff of strategy})$$

Therefore

$$U_1(\rho_i(t_i)\,,\, r_1\,,\, r_2) = -\rho_i(t_i)(p_{cd}V') - (1-\rho_i(t_i))(p_{ci}V')$$

$$= V'(p_{ci}-p_{cd})\rho_i(t_i) - V'p_{ci}$$

However solutions of this type are not Nash Equilibria, as explained in the section <u>Potential Equilibria and Expected Payoffs</u>.

Equilibrium Type 2

Where this equilibrium to exists, the defender always waits in period 3 but mixes in period 5 (i.e. $r_1 = 0$, r_2 is mixed). Therefore

$$p_3 \leq p_3{}^*$$

$$p_5 = p_5{}^*$$

Substituting terms into the latter condition yields

$$(1-p_{cd})\rho_i(t_i) \,/\, [(1-p_{cd})\rho_i(t_i) + (1-p_{ci})(\rho_i(t_j) + \rho_i(t_g))] = p_5{}^*$$

Isolating the denominator of the left half of the above equation,

$$(1-p_{cd})\rho_i(t_i) \,/\, p_5{}^* = [(1-p_{cd})\rho_i(t_i) + (1-p_{ci})(\rho_i(t_j) + \rho_i(t_g))]$$

$$= (1-p_{ci})(\rho_i(t_j) + \rho_i(t_g)) \,/\, (1- p_5{}^*)$$

which by symmetry

$$= (1\text{-}p_{ci})(1\text{-}\rho_i(t_i)) \,/\, (1\text{-}\ p_5{}^*)$$

Substituting in for $p_5{}^*$

$$\rho_i(t_i) = -\ (1\text{-}\rho_i(t_i))(L \,/\, V')[\ (1\text{-}p_{ci}) \,/\, (1\text{-}p_{cd})]$$

Solving for $\rho_i(t_i)$,

$$\rho_i(t_i) = \frac{L\,(1 - p_{ci})}{L(1 - p_{ci}) - V'\,(1 - p_{cd})}$$

The period 5 indifference condition requires that the defender's expected payoffs in that period be equal. Therefore

$$\rho_i(t_i)[p_{cd}V' + (1\text{-}p_{cd})V'r_2] = (1\text{-}\rho_i(t_i))[p_{ci}V' + (1\text{-}p_{cd})Lr_2]$$

Isolating $\rho_i(t_i)$ obtains

$$\rho_i(t_i) = \frac{V'p_{ci} + L\,(1\ -\ p_{ci})\,r_2}{V'p_{ci} + L\,(1\ -\ p_{ci})\,r_2 + V'p_{cd} + V'\,(1\ -\ p_{cd})\,r_2}$$

Substituting in for $\rho_i(t_i)$ and solving for r_2,

$$r_2 = \frac{-Lp_{cd}\,(1 - p_{ci}) - V'p_{ci}\,(1 - p_{cd})}{2L\,(1 - p_{ci})\,(1 - p_{cd})}$$

Using the definition of expected payoff

$$U_1(\rho_i(t_i)\ ,r_1\ ,\ r_2) = -\rho_i(t_i)[p_{cd}V' + (1\text{-}p_{cd})V'r_2] - (1\text{-}\rho_i(t_i))[p_{ci}V' + (1\text{-}p_{cd})Lr_2]$$

$$= -\frac{V'[Lp_{cd}(1-p_{ci}) - V'p_{ci}(1-p_{cd})]}{L(1-p_{ci}) - V'(1-p_{cd})}$$

Returning to $p_3 \leq p_3^*$ and substituting in terms yields the additional restriction

$$\rho_i(t_i) \leq \frac{-L + p_{ci}V' + (1-p_{ci})r_2L}{V_o - L + (p_{ci} - p_{cd})V' - (1-p_{cd})r_2V' + (1-p_{ci})r_2L}$$

Substituting in for r_2 and $\rho_i(t_i)$ and solving for L produces the restriction

$$-L \geq \frac{V'^2 p_{ci}(1-p_{cd})}{V_o(1-p_{ci}) - V'(1-p_{ci}p_{cd})}$$

Equilibrium Type 3

Where equilibrium to exists, the defender always mixes in period 3 but waits in period 5 (i.e. r_1 is mixed, $r_2 = 0$). Therefore

$p_3 = p_3^*$

$p_5 \leq p_5^*$

Just as with equilibrium Type 2, substituting into the former condition yields

$$\rho_i(t_i) = \frac{-L + p_{ci}V' + (1-p_{ci})r_2L}{V_o - L + (p_{ci} - p_{cd})V' - (1-p_{cd})r_2V' + (1-p_{ci})r_2L}$$

After setting $r_2 = 0$ this simplifies to

$$\rho_i(t_i) = \frac{V'p_{ci} - L}{V_o - L + V'(p_{ci} - p_{cd})}$$

Furthermore, as a necessary condition to his willingness to play mixed strategy $\rho_i(t_i)$, the attacker must receive an equal payoff regardless of the strategy he plays.

When an attacker of type i jams on-axis (that is, sends message i), he expects payoff

$$-V_0 r_1 - p_{cd} V'(1-r_1)$$

When an attacker of type i jams off-axis (that is, sends message j), he expects payoff

$$-L r_1 - p_{ci} V'(1-r_1)$$

Setting these two equations equal and solving for r_1 yields

$$r_1 = \frac{V'(p_{ci} - p_{cd})}{V_o - L + V'(p_{ci} - p_{cd})}$$

Using the definition of expected payoff

$$\begin{aligned} U_1(\rho_i(t_i), r_1, r_2) &= -\rho_i(t_i)[r_1 V_o + (1 - r_1) p_{cd} V'] - \\ &\quad - (1 - \rho_i(t_i))[r_1 L + (1 - r_1) p_{ci} V'] \\ &= -\frac{V'(V_o p_{ci} - L p_{cd})}{V_o - L + V'(p_{ci} - p_{cd})} \end{aligned}$$

Returning to $p_5 \leq p_5^*$ and substituting in yields the additional restriction

$$\frac{(V'p_{ci} - L)(1 - p_{cd})}{(V_o - V'p_{cd})(1 - p_{ci}) + (V'p_{ci} - L)(1 - p_{cd})} \leq \frac{-L}{(V' - L)}$$

Solving for L produces the restriction

$$-L \le \frac{V'^2 p_{ci}(1 - p_{cd})}{V_o(1 - p_{ci}) - V'(1 - p_{ci}p_{cd})}$$

Equilibrium Type 4

For this equilibrium to exist, the defender must always decide to mix in periods 3 and 5 (i.e. r_1 is mixed, r_2 is mixed). Therefore

$$p_3 = p_3^*$$

$$p_5 = p_5^*$$

By the same logic as Equilibrium Type 2, the former condition implies

$$\rho_i(t_i) = \frac{-L + p_{ci}V' + (1 - p_{ci})r_2L}{V_o - L + (p_{ci} - p_{cd})V' - (1 - p_{cd})r_2V' + (1 - p_{ci})r_2L}$$

while the latter condition implies

$$\rho_i(t_i) = \frac{L(1 - p_{ci})}{L(1 - p_{ci}) - V'(1 - p_{cd})}$$

Just as with Equilibrium Type 3, as a necessary condition to his willingness to play mixed strategy $\rho_i(t_i)$, the attacker must receive an equal payoff regardless the strategy he plays. Imposing this condition would yield a third expression for $\rho_i(t_i)$, this time in terms of both r_1 and r_2. However it is not necessary to formulate this third expression, as the first two are sufficient to show that r_2 is undefined in all cases, and therefore there are no solutions of this type.

Isolating r_2 in the first $\rho_i(t_i)$ expression produces

$$r_2 = \frac{[V_o - L + (p_{ci} - p_{cd})V']\rho_i(t_i) + L - p_{ci}V'}{(1 - p_{cd})\rho_i(t_i)V' + (1 - \rho_i(t_i))(1 - p_{ci})L}$$

The second expression for $\rho_i(t_i)$ may now be substituted into the above expression for r_2, and both the numerator and the denominator multiplied by

$$L(1 - p_{ci}) - V'(1 - p_{cd})$$

for the purpose of simplification.

The denominator of the resulting fraction is

$$LV'(1 - p_{ci})(1 - p_{cd}) - LV'(1 - p_{ci})(1 - p_{cd})$$

This is just 0. The expression for r_2 is therefore undefined, and therefore no solution of Equilibrium Type 4 can exist.

Proposition 10 Proof

Equilibrium Type 1

Where this equilibrium exists, the defender always decides to wait in periods 2 and 4 (i.e. $r_1 = 0$, $r_2 = 0$). This necessitates the posterior belief

$p_4 \leq p_4^*$, which requires

$q \leq q^*$

Thus,

$$q \leq \frac{-L}{V'(1 - p_c) - L}$$

Because the defender decides to wait in period 2 also, his prior belief must likewise be that it is profitable to wait in the first round. Momentarily assume, however, that his prior belief supported indifference. As a necessary condition to his willingness to play mixed strategy r_1, the defender would have to receive an equal payoff regardless of the strategy he plays.

When the defender reallocates in period 2, he expects payoff

$V_o q + (1-q)L$

When he waits in period 2, he expects payoff

$$[p_cV' + (1-p_c)r_2V']q + (1-q)[p_cV' + (1-p_c)r_2L]$$

Setting r_2 to 0 and equating these two expressions yields the strategy q necessary for period 2 indifference, q`

$$q` = \frac{-L}{V_o - p_cV' - L}$$

However, because the defender is not indifferent, $q \leq q`$, and therefore

$$q \leq \frac{-L}{V_o - p_cV' - L}$$

Because by definition $V' \leq V_o$, it must be true that $q` \leq q^*$, so the first restriction on q is redundant. Using the definition of expected payoff

$$\begin{aligned} U_1(q, r_1, r_2) &= qp_c(-V') + (1-q)C \\ &= (-C - p_cV')q + C \\ &\leq \frac{L(C + p_cV')}{V_o - p_cV' - L} + C \end{aligned}$$

Note that U_1 varies linearly with q and has a positive slope where $C < p_cV'$. Therefore for all values of $C \leq p_cV'$, which are the only values considered, the attacker maximizes U_1 by playing the maximum allowable q.

Equilibrium Type 2

Where this equilibrium exists, the defender always decides to wait in period 2 and to mix in period 4 (i.e. $r_1 = 0$, r_2 is mixed). Therefore

$p_4 = p_4^*$, and so

$$q = \frac{-L}{V'(1 - p_c) - L}$$

(Note that Equilibrium Type 2 q is equal to or greater than the maximum value of Equilibrium Type 1 q because the denominator of the former contains V' rather than V_o, and $V' \leq V_o$.)

As a necessary condition to his willingness to play mixed strategy q*, the attacker must receive an equal payoff regardless of the strategy he plays.

Knowing the defender will play $r_1 = 0$, when the attacker chooses to attack, he expects payoff

$$-V'r_2 - p_cV'(1-r_2)$$

When he chooses not to attack, he expects payoff

$$(C-L)r_2 + C(1-r_2)$$

Setting these two expressions equal yields

$$r_2 = \frac{-C - p_c V'}{V'(1 - p_c) - L}$$

Using the definition of expected payoff,

$$U_1(q, r_1, r_2) = - q[V'r_2 + (1 - r_2)V'p_c] - (1 - q)(Lr_2 - C)$$

$$= \frac{L(CL - p_c^2 V'^2 - CV')}{(1 - p_c)(V' - L)[V'(1 - p_c) - L]} - \frac{L(C + p_c V')}{V'(1 - p_c L)}$$

Equilibrium Type 3

For this equilibrium to exist, the defender must always decide to mix in period 2 and to wait in period 4 (i.e. r_1 is mixed, $r_2 = 0$). Therefore

$$p_4 \leq p_4^*$$

By the same logic as in Equilibrium Type 1, $q = q`$. Thus

$$q = \frac{-L}{V_o - p_c V' - L}$$

As $q` \leq q^*$, this fulfills the condition $p_4 \leq p_4^*$.

Again, however, as a necessary condition to his willingness to play mixed strategy $q`$, the attacker must receive an equal payoff regardless of the strategy he plays.

Knowing the defender will play $r_2 = 0$, when the attacker chooses to attack, he expects payoff

$-V_o r_1 - p_c V'(1-r_1)$

When he chooses not to attack, he expects payoff,

$(C-L)r_1 + C(1-r_1)$

Setting these two equations equal yields

$$r_1 = \frac{-C - p_c V'}{V_o - p_c V' - L}$$

The expected payoff is just

$$U_1(q, r_1, r_2) = -q[V_o r_1 + p_c V'(1 - r_1)] - (1 - q)(L r_1 - C)$$

$$= C + \frac{L(C + p_c V')}{V_o - P_c V' - L}$$

Note that Equilibrium Type 3 q and U_1 are equivalent to Equilibrium Type 1 maximum q and U_1 for $C \le p_c V'$.

Equilibrium Type 4

For this equilibrium to exist, the defender must decide to mix in both periods 2 and 4 (i.e. r_1 is mixed, r_2 is mixed). Therefore

$p_4 = p_4{}^*$

and also the defender's prior belief must support indifference.

The former condition necessitates that

$$q = \frac{-L}{V_o - p_c V' - L}$$

To impose the latter condition, the same logic is used as in Equilibrium Type 1 to obtain

$$V_o q + (1-q)L = [p_c V' + (1-p_c)r_2 V']q + (1-q)[p_c V' + (1-p_c)r_2 L]$$

Substituting in q, all terms containing r_2 cancel, leaving behind

$$V' = V_o$$

This restriction is not acceptable, as it does not allow the model sufficient utility. Indifference as to the time it takes to intercept a raid implies total certainty as to the target of a raid. Such certainty is not impossible, but it generally results in a "goal tending" strategy, in which the defender forgets about early warning entirely and dedicates all available resources to protecting a particular target. Modeling the uncertainty surrounding detection then becomes irrelevant.

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