DISTRIBUTED DECISIONMAKING WITH CONSTRAINED DECISION MAKERS - A CASE STUDY†

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ABSTRACT

A specific distributed decisionmaking problem is formulated that includes processing load constraints on team members. Solutions are possible that place team members in regions where random behavior is required and/or where individual errors are likely.

I. INTRODUCTION

A main goal in most distributed decisionmaking formulations, particularly team theoretic ones, is to obtain normative decision rules that represent the desired behavior of each decision agent or team member [1]. This paper considers a modified team theoretic problem that incorporates decision rules that are descriptive of actual human behavior, and furthermore takes into account the processing load incurred to execute these decision rules. The problem formulated is motivated by considerations in the design of human information processing organizations of the type where organization members perform routine processing tasks under the pressure of time. Examples of such organizations are found in air traffic control and command and control situations. In this context, the usual team problem can be taken as a model of organization structure, and decision rules as idealized behavior for organization members. When models for actual human behavior are substituted for the normative decision rules in the team structure, team behavior in general changes. Furthermore, the workload of team members may be such that desired team operation exceeds human processing limitations. Thus, given the basic team structure, a problem can be formulated to choose decision rules, to be realized by actual human behavior for best team performance, subject to their feasibility with respect to team member processing load.

The specific team structure considered is that of a two-member, tandem distributed detection network. Section II describes this structure and reviews the characteristics of theoretical team member behavior. A key feature of the decision rules is the presence of thresholds, which each member uses to make comparison tests. A model for the information processing required to execute such a test is then described, with processing time used as the measure of workload. The complete model for each member's actual behavior includes a second element, however, which accounts for behavior when processing time for threshold tests exceeds the time allowed. This element derives from human ability to trade accuracy for speed. Two different mechanisms for doing this are incorporated, one for each member. The overall actual behavior and processing load realized is parameterized by the thresholds used and other parameters that figure in the speed/accuracy tradeoff capability. The modified team theoretic problem is then to place these parameters for minimum team error, subject to processing time used being less than processing time available for each member. Section III discusses the characteristics of the problem solution. A particular consideration of interest is whether, and if so under what conditions, it remains desirable to retain the thresholds obtained in the original (unconstrained) team problem. Section IV investigates a special case of the problem, from which principles of general interest are apparent. Finally, Section V summarizes the paper.

†This research was supported by the Office of Naval Research under grants ONR/N00014-77-C-0532 (NR 041-519) and ONR/N00014-84-K-0519 (NR 649-003).

II. PROBLEM FORMULATION

A. Team Structure

Consider the two member, tandem, distributed detection network shown in Figure 1. Each team member re-

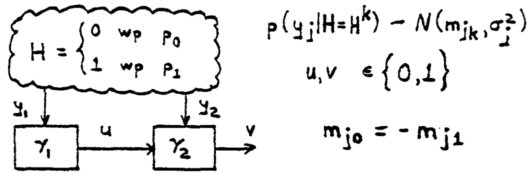

Figure 1 Team Structure

ceives a conditionally independent, gaussian observation on the presence or absence of a given phenomenon H. Based on his observation, the first member selects one of two symbols to send to the second member. The latter then incorporates his own measurement with the received symbol to make a detection decision for the network. The optimal decision rules γ_j^* for each team member that minimize the probability of error in detection are known [2]. They are threshold tests as given in (1).

$$\begin{array}{ll} \gamma_1^*: & \gamma_2^*: \\ & \text{if } u = i \quad (i = 0,1) \text{ and} \\ \text{if } y_1 \geq t_1^* \quad u = 1 & \text{if } y_2 \geq t_{2i}^* \quad v = 1 \\ \text{if } y_1 < t_1^* \quad u = 0 & \text{if } y_2 < t_{2i}^* \quad v = 0 \end{array} \quad (1)$$

Basically, the first member biases the second member's choice by selecting the latter's threshold.

B. Information Processing Models

Now consider that the threshold comparison tests in (1) are to be accomplished by humans. For example, the observation could be displayed visually as a horizontally displaced dot, with the threshold also displayed as a vertical line displaced according to its value. Viewing such a display and selecting a response takes time. Furthermore, threshold position with respect to the likely position of observations will have an effect on the time required to select a response. In particular, assume that a comparison with threshold t requires, on the average, $\bar{t}_p$ seconds to make, where

$$\bar{t}_p = \bar{t}_p(t) = a - b\cdot(t)^2 \quad a > 0,\ b \geq 0 \quad (2)$$

Given that observations are predominantly near zero, the model in (2) reflects the observed behavior that response time decreases as the uncertainty decreases in the response required. In eq.(2), as t becomes large in absolute value ($b \neq 0$), the likelihood that observations will fall only on one side of t is high.

<u>First Team Member</u>

The first team member performs his task using a single threshold. The processing time required to do this test is given by eq.(2); specifically, it is $\bar{t}_{p11}(t_1) = a_1 - b_1\cdot(t_1)^2$. In addition, it is assumed that the input/output behavior realized is such that a flawless comparison can be made. Denote by $\bar{k}_{11}$ the conditional distribution $p(u|y_1)$ realized using the threshold test. The model is then that of

$$\bar{k}_{11}: \quad \text{if } y_1 \geq t_1 \quad u = 1; \quad \text{else } u = 0 \quad (3)$$

Suppose now that the operation of the team is such that the member must complete comparison tests at the rate of one every τ_1 seconds. If it happens that t_1 is set such that $\bar{t}p_{11}(t_1) > \tau_1$, the member will be overloaded. Therefore, an alternative processing mode is provided: an option to "guess", i.e. to essentially ignore the observation y_1 and to arbitrarily respond $u = 1$ with some guessing bias g_1. Input/output behavior when guessing is modeled by the conditional distribution $\bar{k}_{12}$:

$$\bar{k}_{12} : \quad u = 0 \text{ wp } 1 - g_1 \quad \text{and } u = 1 \text{ wp } g_1 \tag{4}$$

To make this a viable option, assume that the time required to exercise this option, denoted by $\bar{t}_{p12}$, is less than $\bar{t}p_{11}(t_1)$ for some range of t_1 values.

Finally, because the team member has two options, there will be an additional amount of processing time required to switch between them. Switching overhead depends on switching frequency, as given by the expression

$$d_1 \cdot (1 - q_1) \cdot q_1 \tag{5}$$

where q_1 is the fraction of guessing and d_1 is a scale factor. If one option is used exclusively, (5) is zero. Thus, the first team member has an input/output behavior modeled by K_1 that requires a processing time of T_{p1}:

$$K_1 = (1 - q_1) \cdot \bar{k}_{11} + q_1 \cdot \bar{k}_{12} \tag{6}$$

$$T_{p1} = (1 - q_1) \cdot t_{p11}(t_1) + q_1 \cdot \bar{t}_{p12} + d_1 \cdot (1-q_1) \cdot q_1 \tag{7}$$

The model given in eq.(6) and eq.(7) is basically the so-called Fast-Guess model [3], which reflects one mechanism whereby humans can trade speed for accuracy.

Second Team Member

The second team member switches between two thresholds. Assuming an overhead for switching similar to (5), the average time required to accomplish this task depends on the threshold values, and the relative frequency of using them:

$$T_{p2} = \sum_{i=0}^{1} \left[p(u = i) \cdot (a_{2i} - b_{2i} \cdot (t_{2i})^2) \right] + d_2 \cdot p(u = 0) \cdot p(u = 1) \tag{8}$$

As with the first team member, it assumed that the second member is subject to a processing time limit; in this case it is assumed to be a like a deadline constraint τ_2. So long as $T_{p2} \leq \tau_2$, the team member can accomplish this processing without error. Errors will be made, if however, if $p(u)$, t_{20}, and t_{21} are such that $T_{p2} > \tau_2$. The likelihood of errors depends on the difference between the deadline imposed, denoted t_d, and the processing time required T_{p2}. Thus the input/output behavior of the second member, K_2, is as follows:

K_2: if $u = i$ $(i = 0,1)$ and (9)

$y_2 \geq t_{2i}$, then $v = 1$ wp $1-q_2$; $v = 0$ wp q_2

$y_2 < t_{2i}$, then $v = 0$ wp $1-q_2$; $v = 1$ wp q_2

where

$$q_2 = q_2(T_{p2}, t_d) = (1 + e^{f(T_{p2}, t_d)})^{-1} \tag{10}$$

and

$$f = \begin{cases} f_s \cdot (t_d - T_{p2}) + f_m , & T_{p2} \geq t_d \\ f_m , & T_{p2} < t_d \end{cases} \tag{11}$$

In words, the second member performs the threshold comparison test correctly a fraction $(1 - q_2)$ of the time, and makes an error on the fraction q_2 of the observations processed. For analytical convenience it is assumed that $f_m < \infty$, which effectively means that the minimum value of q_2 is non-zero. Eqs.(8)-(11) form the model of the second team member. It reflects a second mechanism of trading speed for accuracy exhibited by humans. In particular, the log-linear relationship between the "odds-ratio" $(1-q_2)/q_2$ and processing time is derived from Pew [4]. Note that values of f_s and f_m are not selected; they represent fixed human behavior.

C. Problem Statement

Five independent variables have been specified within the models of team members. They include the three comparison thresholds (t_1, t_{20}, t_{21}), the amount of guessing by the first member (q_1), and the processing time deadline for the second member (t_d). Substituting K_j for γ_j^* and adding the processing time constraints for each member, a constrained optimization problem can be formulated to minimize the detection error probability for the organization, subject to meeting the processing time limitations of each member. Denote by J_o the detection error probability. Then formally stated, the problem is as follows.

Problem A1 (Constrained Optimization Problem)

$$\min_{t_1, t_{20}, t_{21}, q_1, t_d} \quad J_o(q_1, t_1, t_{20}, t_{21}, t_d)$$

$$\text{s.t.} \qquad T_{p1} \leq \tau_1 \quad ; \quad t_d \leq \tau_2$$

III. SOLUTION CHARACTERISTICS

There are several issues of interest with respect to the solution of Problem A1. One is whether it is ever to any advantage to set the deadline t_d for the second member to be strictly less than τ_2. This is shown not to be the case, due to the monotonicity of q_2 in t_d. A second issue is whether a possible solution is to leave the thresholds at their unconstrained optimal values, i.e. $t_1^*, t_{20}^*, t_{21}^*$, and to tolerate any consequent input/output errors (q_2) or guessing (q_1). At the other extreme is the possible solution of adjusting thresholds such that q_1 and q_2 are minimized. The basic consideration is one of whether it is better to absorb guesses and input/output errors some of the time in order to use quality thresholds most of the time, or to use an "inferior" set of thresholds all of the time. In Problem A1, so long as the thresholds t_{20}, t_{21} affect processing time of the second member, it is better to adjust them. However, solutions to Problem A1 do not necessarily minimize q_2 and q_1.

Examination of Problem A1 is greatly facilitated by taking advantage of the fact that the joint distribution $p(u, H)$ completely characterizes the analytical link between team members [2]. Thus the minimization in Problem A1 can proceed in two stages. First t_{20}, t_{21} and t_d can be placed as a function of $p(u,H)$. Since there is a 1-1 relationship between (q_1, t_1) pairs and $p(u,H)$ distributions, a second minimization can be performed over these distributions to place q_1 and t_1, and thereby solve Problem A1. Denote by p_{ik} the quantity $p(u=i, H=H^k)$. Then it is convenient to represent the distribution $p(u,H)$ as a vector Π, where

$$\Pi = [p_{00}, p_{10}, p_{01}, p_{11}]' \tag{12}$$

Furthermore, possible Π values depend on t_1 and q_1 according to

$$\Pi = (1-q_1) \cdot \left[p_{00t}(t_1), p_0 - p_{00t}(t_1), p_1 - p_{11t}(t_1), p_{11}\right]' + q_1 \cdot \left[(1-g_1) \cdot p_0, g_1 \cdot p_0, (1-g_1) \cdot p_1, g_1 \cdot p_1\right]' \triangleq \Pi(t_1, q_1) \tag{13}$$

where

$$p_{00t}(t_1) = \Phi\left(\frac{t_1 - m_{10}}{\sigma_1}\right) \cdot p_0 \tag{14a}$$

$$p_{11t}(t_1) = \left[1 - \Phi\left(\frac{t_1 - m_{11}}{\sigma_1}\right) \right] \cdot p_1 \tag{14b}$$

and $\Phi(\cdot)$ is the unit normal cumulative distribution function. From eq.(13) it is evident that $\mathbf{T}$ is determined as a combination of two $\mathbf{T}$ vectors, one corresponding to exclusive use of the threshold and one corresponding to exclusive use of guessing.

Define

$$J(\mathbf{T}, t_{20}, t_{21}) = p_{00} \cdot \left[1 - \Phi\left(\frac{t_{20} - m_{20}}{\sigma_2}\right)\right] + p_{01} \cdot \left[\Phi\left(\frac{t_{20} - m_{21}}{\sigma_2}\right)\right] + p_{10} \cdot \left[1 - \Phi\left(\frac{t_{21} - m_{20}}{\sigma_2}\right)\right] + p_{11} \cdot \left[\Phi\left(\frac{t_{21} - m_{21}}{\sigma_2}\right)\right] \tag{15}$$

Eq:(15) represents the detection error probability of the team as a function of $\mathbf{T}$, t_{20}, and t_{21}, assuming $q_2 = 0$. Rewriting J_0 using J and showing the decomposition by stages, Problem A1 becomes

$$\min_{\substack{t_1, q_1 \\ \text{s.t.} \\ T_{p1} \le \tau_1 \\ \mathbf{T} = \mathbf{T}(t_1, q_1)}} \quad \min_{\substack{t_{20}, t_{21}, t_d \\ \text{s.t.} \\ t_d \le \tau_2}} \left[1 - 2 \cdot J(\mathbf{T}, t_{20}, t_{21})\right] \cdot q_2(T_{p2}, t_d) + J(\mathbf{T}, t_{20}, t_{21})$$

Finally, before proceeding to an analysis of solution characteristics, it is convenient to formulate a modified version of Problem A1. It is true that explicit dependence on thresholds t_{20} and t_{21} occurs in Problem A1 only in the function J and in the determination of processing time T_{p2}. Therefore it is possible to aggregate these thresholds into the single variable $\bar{T}_{p2}$ and to substitute a new function $\bar{J}$ for J, where

$$\bar{J}(\mathbf{T}, \bar{T}_{p2}) = \min_{\substack{t_{20}, t_{21} \\ \text{s.t.}}} J(\mathbf{T}, t_{20}, t_{21}) \quad T_{p2} = \bar{T}_{p2} \tag{16}$$

In other words, given a $\mathbf{T}$ and $\bar{T}_{p2}$ value, the relationship of t_{20} and t_{21} is defined (in fact they describe an ellipse). The minimization in eq.(16) generates threshold values $\bar{t}_{2i}$ that are the solutions to eq.(16) as a function of $\bar{T}_{p2}$. Using this aggregation, Problem A1 an be stated in terms of $q_1, t_1, \bar{T}_{p2}$ and t_d as:

<u>Problem A2</u>

$$\min_{\substack{q_1, t_1 \\ \text{s.t.} \; T_{p1} \le \tau_1 \\ \mathbf{T} = \mathbf{T}(t_1, q_1)}} \quad \min_{\substack{t_d, \bar{T}_{p2} \\ \text{s.t.} \\ t_d \le \tau_2}} \left[1 - 2 \cdot \bar{J}(\mathbf{T}, \bar{T}_{p2})\right] \cdot q_2(\bar{T}_{p2}, t_d) + \bar{J}(\mathbf{T}, \bar{T}_{p2})$$

<u>Assigning Deadline</u>

Consider now the inner minimization in Problem A2. For given $\bar{T}_{p2}$, necessary conditions for a solution value of t_d [5] are given by

$$\frac{\partial \bar{J}}{\partial t_d} \cdot \left[1 - 2 \cdot q_2\right] + \frac{\partial q_2}{\partial t_d} \cdot \left[1 - 2 \cdot \bar{J}\right] + \mu = 0 \tag{17a}$$

$$\mu \cdot (t_d - \tau_2) = 0 \tag{17b}$$

$$\mu \ge 0 \tag{17c}$$

The first term in eq.(17a) is zero since $\bar{J}$ does not depend on t_d. The first factor in the second term is negative, since q_2 is monotonically decreasing with respect to increasing t_d. Furthermore, $\bar{J}$ is bounded above by 0.5. The latter derives from the interpretation of J as the detection error probability of the team when $q_2 = 0$. A value of $J \ge 0.5$ implies that the thresholds are being used to give observations an opposite interpretation, one which results in worse than chance behavior. Assuming that the minimization in eq.(16) assures that at least chance performance will obtain, i.e. $\bar{J} < \min(p_0, p_1) \le 0.5$, then eq.(17) implies that $t_d = \tau_2$. That is, always place the deadline at the maximum allowable. This result is valid independent of $\bar{T}_{p2}$ and $\mathbf{T}$ values.

<u>Using Unconstrained Optimal Thresholds</u>

Continuing with examination of the inner minimization, consider the question of whether the unconstrained optimal thresholds can be a solution to Problem A2. Because of the reformulation in terms of $\mathbf{T}$ and the stagewise minimization structure, this question must be answered in a more general way. Whereas the minimization in eq.(16) resulted in the construction of two functions $\bar{t}_{2i}(\bar{T}_{p2}, \mathbf{T})$, performing the minimization of J without the constraint in eq.(16) results in two different functions that represent the unconstrained optimal values of t_{2i} for a given $\mathbf{T}$ value. Included in this set is the pair of thresholds that define of γ_2^*. Indeed, if the functions defined by the unconstrained minimization are denoted $t_{2i}^*(\mathbf{T})$, then

$$t_{2i}^* = t_{2i}^*(\mathbf{T}(t_1^*, 0)) \tag{18}$$

The investigation below proceeds in terms of $\mathbf{T}$ and determines whether $t_{2i}^*(\mathbf{T})$ represent a possible solution to the inner stage minimization. Denote by $T_{p2}^*(\mathbf{T})$ the processing time required by the second member when unconstrained optimal thresholds are used. Setting $t_d = \tau_2$ in Problem A2, the inner stage minimization becomes that of finding a value of $\bar{T}_{p2}$ that solves

$$\frac{\partial \bar{J}}{\partial \bar{T}_{p2}} \cdot \left[1 - 2 \cdot q_2\right] + \frac{\partial q_2}{\partial \bar{T}_{p2}} \cdot \left[1 - 2 \cdot \bar{J}\right] = 0 \tag{19}$$

The issue at hand is whether $T_{p2}^*(\mathbf{T})$ satisfies eq.(19). Because $\bar{T}_{p2}^*(\mathbf{T})$ represents a global minimum of $\bar{J}$, the first term in eq.(19) is zero. Now, if $T_{p2}^*(\mathbf{T}) \le \tau_2$ the second term is also zero, since q_2 does not depend on $\bar{T}_{p2}$ in this region. Thus unconstrained optimal thresholds are solutions when the processing time they require does not exceed the deadline. This is reasonable, since any adjustment of thresholds would have no effect on input/output errors; hence the thresholds can be left at their unconstrained optimal values.

However, for $T_{p2}^*(\mathbf{T}) > \tau_2$ a different result obtains. In this situation, q_2 is monotonically increasing with $\bar{T}_{p2}$. Furthermore, since $\bar{J} < 0.5$, as discussed earlier, it is true that the second term is non-zero and hence $T_{p2}^*(\mathbf{T})$ does <u>not</u> solve eq.(19). This result means that if the processing time required by use of the unconstrained optimal threshold values is greater than that allowed, it is always desirable to adjust t_{20} and t_{21} to reduce T_{p2} and thereby reduce the input/output error q_2.

<u>Minimizing Second Member Input/Output Errors</u>

The discussion above has concluded that, when it is an issue, it is more advantageous to reduce the second member's input/output errors than to retain the best thresholds for processing observations. The question arises as to whether input/output errors should be minimized as much as possible, at the expense of the threshold settings. In terms of Problem A2, this issue is one of whether $\bar{T}_{p2} = \tau_2$ is a solution to the inner minimization, given that $T_{p2}^*(\mathbf{T}) > \tau_2$, or whether $\bar{T}_{p2} > \tau_2$ is a solution instead. Its resolution depends on how drastically the trade of speed for accuracy is made by the team member, which is modeled by the parameter f_s.

To properly investigate this issue, it is necessary to add another constraint to Problem A2 in the inner stage that restricts values of $\bar{T}_{p2}$ according to the region of interest. The result is the problem

$$\min_{\bar{T}_{p2}} \quad \bar{J}(\mathbf{T},\bar{T}_{p2}) + \left[1 - 2\cdot\bar{J}(\mathbf{T},\bar{T}_{p2})\right]\cdot q_2(\bar{T}_{p2},\tau_2) \qquad (20)$$
$$\text{s.t. } \tau_2 \leq \bar{T}_{p2}$$

where it is assumed that $T^*_{p2}(\mathbf{T}) \geq \tau_2$. The necessary conditions for a solution value of $\bar{T}_{p2}$ are

$$\frac{\partial\bar{J}}{\partial\bar{T}_{p2}}\cdot\left[1 - 2\cdot q_2\right] + \frac{\partial q_2}{\partial\bar{T}_{p2}}\cdot\left[1 - 2\cdot\bar{J}\right] - \mu = 0 \qquad (21a)$$

$$\mu\cdot(\tau_2 - \bar{T}_{p2}) = 0 \qquad (21b)$$

$$\mu \geq 0 \qquad (21c)$$

and the issue is whether $\bar{T}_{p2} = \tau_2$ is a solution to (21). If so, $\mu > 0$. Furthermore, it must be true that the first two terms in (21a) are positive in sum. The second of the two is always positive, as discussed previously. However, the first is always negative for the region of $\bar{T}_{p2}$ of interest. This assumes that $q_2 < 0.5$, which is again the assumption that the second member's processing behavior is better than chance level. Furthermore, in the interval where $\tau_2 \leq \bar{T}_{p2} \leq T^*_{p2}(\mathbf{T})$, $\bar{J}$ monotonically decreases with increasing $\bar{T}_{p2}$. That is, as $\bar{T}_{p2}$ forces the thresholds t_{20} and t_{21} to move away from $t^*_{2i}(\mathbf{T})$, $\bar{J}$ increases.

Thus it is unclear whether $\bar{T}_{p2} = \tau_2$ satisfies (21a). A more specific test to resolve the ambiguity can be derived as follows. At $\bar{T}_{p2} = \tau_2$, q_2 is at its minimum: $q_2 = (1 + \exp(f_m))^{-1} = q_{2m}$. Furthermore

$$\frac{\partial q_2(\tau_2,\tau_2)}{\partial\bar{T}_{p2}} = f_s\cdot(e^{f_m})\cdot(q_{2m})^{-2} \qquad (22)$$

Substituting (22) into (21a) and rearranging gives

$$f_s > -(q_{2m})^2\cdot e^{-f_m}\cdot\left[\frac{1-q_{2m}}{1-\bar{J}(\mathbf{T},\tau_2)}\right]\frac{\partial\bar{J}(\mathbf{T},\tau_2)}{\partial\bar{T}_{p2}} \triangleq F_s \qquad (23)$$

which must be satisfied if $\bar{T}_{p2} = \tau_2$ is a solution. F_s is a non-negative quantity. The parameter f_s models the rate at which input/output errors increase as the processing time required increases beyond the deadline. If $f_s > F_s$, then the marginal increase in q_2 is great enough such that it is optimal to minimize input/output errors and to adjust thresholds accordingly. If $f_s < F_s$, then there exists a compromise between the two extremes – minimum q_2 at $\bar{T}_{p2}$ or minimum $\bar{J}$ at $\bar{T}_{p2} = T^*_{p2}$ – that gives better overall team performance.

Guessing by First Member

Discussion thus far has considered solution characteristics in terms of $\mathbf{T}$, and the conclusions reached pertain to the second member. Turning now to the outer minimization in Problem A1, the question arises as to if and under what circumstances the problem solution involves guessing by the first member. This issue can be resolved by considering, in geometric terms, how feasible (t_1,q_1) values map to $\mathbf{T}$ values.

For fixed a priori probabilities on H (i.e. p_0,p_1), it is possible to characterize all $\mathbf{T}$ values in the (p_{00},p_{11}) plane as t_1 and q_1 range over their values. A region is determined typically as shown in Figure 2. The upper boundary of the region is the locus where $q_1 = 0$. Points Y and Z correspond to where $t_1 \to -\infty$ and $+\infty$, respectively. The lower boundary is the locus of points determined when $q_1 = 1$ and the guessing bias ranges from 0 to 1. Point S corresponds to $g_1 = 0.5$. When viewed as part of the lower boundary, points Y and Z correspond to g_1=1 and 0, respectively. In terms of the underlying (t_1,q_1) values, any point in the interior or on the

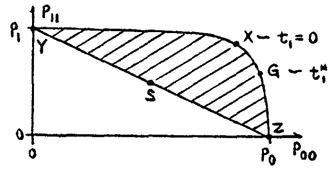

Figure 2 Typical Region of $\mathbf{T}$ Values in (p_{00},p_{11}) Plane

lower boundary represents a non-zero guessing frequency by the team member. Note that the unconstrained optimal value of t_1=t_1^* is therefore on the upper boundary as illustrated. Finally, the geometric representation in Figure 2 has many properties in common with the Receiver Operating Characteristic in signal detection theory [6]. Besides the association of the lower "diagonal" to guessing, it is also the case that better team performance results when the operating point in the (p_{00},p_{11}) plane moves nearer to (p_0,p_1), where perfect discrimination between hypotheses is made (by the first member).

Consider now the outer minimization of Problem A2. While Figure 2 represents possible $\mathbf{T}$ values, not all of them will be feasible due to the constraint on T_{p1}. Figure 3a shows typically how this constraint restricts

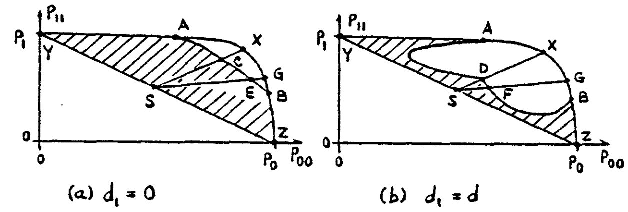

Figure 3 Constraint on T_{p1} in (p_{00},p_{11}) Plane; g_1=0.5

$\mathbf{T}$ values for $d_1 = 0$, i.e. when the first member has no switching overhead. A guessing bias of 0.5 has been assumed. The arc ACB represents the locus where $T_{p1} = \tau_1$, and the shaded area designates the region of feasible $\mathbf{T}$ values. A similar depiction is given in Figure 3b, except for the case where d_1 has increased from zero to a relatively significant value. Again, the arc ADB represents the locus where $T_{p1} = \tau_1$.

The solution to Problem A2 is found by searching over regions such as those in Figure 3. It can be shown, however, that a solution to A2 is such that either $q_1 = 0$ or $T_{p1} = \tau_1$. This means that the upper boundary of the feasible region contains the solution of Problem A2. In Figures 3a and 3b, therefore, the solution must be on the arcs YACBZ or YADBZ, respectively. In particular, it is possible that solutions will be obtained on the arcs ACB or ADB, i.e. it may be optimal to guess. This can be explained qualitatively as follows. All other things being equal (i.e. neglecting the second member), it is desired to operate in the (p_{00},p_{11}) plane as close as possible to the point where $q_1 = 0$ and $t_1 = t_1^*$. In Figure 3, neither region admits the unconstrained optimal solution as feasible. In Figure 3a, however, point E is closer than point B, where the former is such that $q_1 \neq 0$ and the latter is the nearest feasible point where $q_1 = 0$. In Figure 3b, point B is closer to the unconstrained optimal point. Thus the situation in (a) is likely to have a solution where $q_1 \neq 0$, while in (b) the solution will likely be at point B. Though shown for cases where $d_1 = 0$ or $d_1 \neq 0$, this behavior does not represent a special case, tied to the presence of switching overhead, nor is it dependent on having the bias in guessing at 0.5. Figure 4 shows the same constraints for a bias of $g_1 = 0.75$.

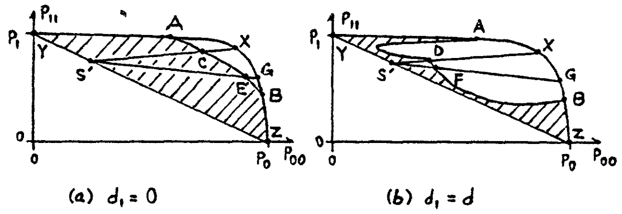

Figure 4 Constraint on T_{p1} in (p_{00}, p_{11}) Plane; g_1=0.75

V. SPECIAL CASE

To highlight particular mechanisms of how one member can affect the other and also team performance, consider the following special case. Suppose that the second member's processing time is independent of the threshold positions, but that it takes longer to use threshold t_{20} than t_{21}. Also, assume that the switching overhead for the second member is significant and that the deadline τ_2 affects the use of t_{20} but not that of t_{21}. That is, mathematically assume that

$$b_{2i} = 0; \quad a_{21} > \tau_2 > a_{20} \tag{24}$$

Also, assume that the first member is unconstrained. For this special case, Problem A1 can be summarized in terms of Figure 5. Since T_{p2} is independent of t_{2i}, its

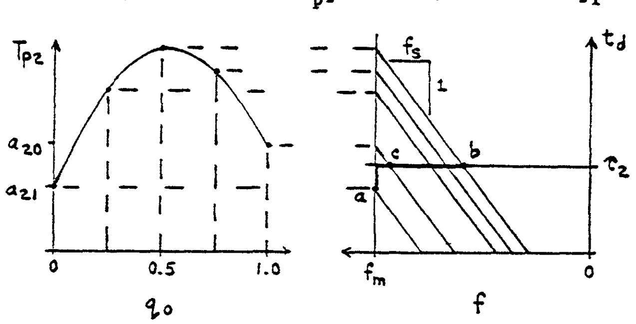

Figure 5 Illustration of Special Case Solution

variation is due entirely to variation in p(u), which is determined by the first team member through placement of t_1. The dependence of T_{p2} on p(u=0) (denoted = q_0) is shown in the left part of Figure 5. The relationship between T_{p2} and input/output errors q_2 (through f) is shown in the right part of the figure. Recall from eq.(11) that a given value of T_{p2} determines a locus of f values as a function of t_d. With $t_d = \tau_2$, a specific operating point on this locus is selected. As q_0 moves from 0 to 1, the resulting T_{p2} values trace out feasible operating points in the right part of the figure, moving from a to b and back to c. Each point on this locus has a minimum detection error probability obtained by solution of the first stage of the minimization. The overall solution thus becomes a matter of searching over t_1 (and thereby q_0) values. The interesting feature of the minimization in this special case is that the trade-off between speed and accuracy required for the second member is governed entirely by the first member. Furthermore, a reduction in T_{p2} depends mostly on reducing the switching frequency. If t_1^* is somewhere near 0, then $q_0 \approx 0.5$ and the optimization problem is essentially one that must weigh two alternatives: either degrade the first member's quality of processing by adjusting t_1 to reduce switching load of the second member and thereby reduce q_2; or accept the higher input/output error rate of the second member in favor of retaining a higher quality of processing by the first.

Once the solution is obtained, the thresholds will be set at the solution values and the team will presumably operate as modeled. By way of illustrating how processing load and performance can interrelate, suppose that after the team has been set into operation the constraint on the first member becomes binding, say due to external factors that reduce the value of τ_1. As per design, the team member can resort to guessing to meet the constraint. Figure 6 shows a trajectory in the

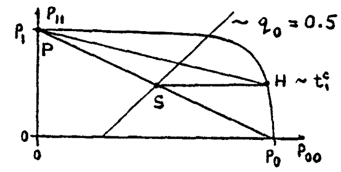

Figure 6 Illustration of Special Case Operation

(p_{00}, p_{11}) plane corresponding to increasing q_1 for two biases in guessing. Point H corresponds to the solution operating point (with $t_1 = t_1^c$). Points S and P correspond to completely random operation with guessing biases of 0.5 and 1.0, respectively. The locus of where q_0 = 0.5 has also been shown. As q_1 increases the operating point moves away from H to either S or P. Because the movement is toward the diagonal "guessing" line, team performance will generally be worse. However, a significant qualitative difference is apparent. Along the trajectory HS, T_{p2} is increasing and in fact comes to rest where switching frequency is at its maximum. Performance thus not only degrades because of changes in J but also because of an increase in q_2. Along the trajectory HP, however, T_{p2} first rises due to the increase in switching, but decreases as switching overhead goes to zero. In this case the contribution to performance degradation due to input/output errors is less. Within these cases are examples of increasing processing load and degrading performance, as well as decreasing processing load and degrading performance.

V. SUMMARY

The addition of processing time constraints to a team theoretic problem modifies team operation. In particular, partially random behavior by team members can be optimal, either by a member's choice, through the selection of an option to guess; or by design, through selection of thresholds such that processing time exceeds a deadline, which in turn makes processing errors more likely. Furthermore, the special case considered has demonstrated that a variety of relationships can exist between team performance and member workload. Because of this variability, a general guideline is suggested, whereby a first step toward understanding a particular structure might be to identify which of the possible relationships actually exists. The effects of switching, as seen in the special case, also suggest a principle of general interest. Given that changing tasks or procedures may require processing resources, and that the necessity to switch may be governed by another team member, the recognition of the potential for switching within a team structure may lead to a better understanding of team behavior.

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