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A DESCRIPTOR APPROACH TO ECONOMIC NETWORK EQUILIBRIUM OF A FOOD PROCESSING AND DISTRIBUTION SECTOR*

by

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ABSTRACT

ALINET is a network model designed for the analysis of energy use in the food processing and distribution sector and for the evaluation of the potential effectiveness of energy conserving technologies the basic structure of the model is that of a material flow network where nodes represent the specific state of the material at that stage and links represent the processes that the material is undergoing. The descriptor variable approach is used for determining the steady state flow of commodities through the system. Two iterative solution algorithms are considered for determining the steady state flow of commodities. The possibilities for using parallel algorithms for overcoming the numerical difficulties are also addressed. Finally, selected simulation results are presented to demonstrate the use of the methodology.

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ALINET is a network model designed for the analysis of energy use in the food processing and distribution sector and for the evaluation of the potential effectiveness of energy conserving technologies. The basic structure of the model is that of a material flow network where nodes represent the specific state of the material at that stage and links represent the processes that the material is undergoing. The descriptor variable approach is used for determining the steady state flow of commodities through the system. Two iterative solution algorithms are considered for determining the steady state flow of commodities. The possibilities for using parallel algorithms for overcoming the numerical difficulties are also addressed. Finally, selected simulation results are presented to demonstrate the use of the methodology.

KEYWORDS

Modeling, Descriptor Variables, Agriculture, Food-Processing.

1. INTRODUCTION

Economic systems continue to receive attention from control engineers: large scale economic models which provide forecasts and information about possible policy alternatives are now a common feature in policy analysis.

One such modeling effort is based on a material flow network representation of an engineering-economic system in which system variables describe the system operation while preserving its structure. The need to analyze energy use in the energy intensive food processing sector and to evaluate specific energy conserving technologies provided part of the motivation for the development of ALINET (Levis et al, 1980a; Ducot and Levis, 1980; Levis et al, 1980b) a network model of the food processing and distribution sector that is the starting point for the work presented in this paper.

The modeling and analysis methodology has been applied to a pilot project carried out with the cooperation of a Yugoslav enterprise. The network model is used to calculate the energy needed in the production of specific products and at each stage of processing, as well as to calculate the costs of production. In order to determine the steady-state flow of commodities through the system, a descriptor variable approach is used (Luenberger, 1977; Stengel, 1978a; 1978b). The descriptor variables, in this case, are the prices and the quantities of commodities. The use of descriptor variables to determine the network equilibrium is described in Section 3, while in Section 4 two iterative algorithms are presented that can be used to overcome the numerical difficulties associated with the equilibrium determination. In Section 5, it is shown that parallelism can be used within the equilibrium algorithms to increase processing speed.

2. THE FLOW MODEL

ALINET is a network model designed for analysis of energy use in the food processing and distribution sector and for the evaluation of the potential effectiveness of energy conserving technologies. The quantities that flow through the network are agricultural commodities. They start as agricultural products at the farm level. The processing of these commodities is represented by a sequence of transformation, each of which changes the state of the material. A state is represented by a node, while transformations or processes are represented by links or branches. The use of nodes and links results in the representation of the food processing and distribution sector by a directed graph.

The flow model of a particular Yugoslav agricultural enterprise that processes wheat is represented by a network with 53 nodes and 68 links. The processing of wheat was selected for the pilot project because it is a leading food crop in the Yugoslav economy. The material flow begins with wheat grain from the field (Node CV1 in Fig. 1). Grain is transported via trucks and tractors to the dryers and mills. The rest is modeled as going to sink nodes CV4 and CV7. Wheat processing also includes the transportation of different types of flour, the manufacture of "multi-mixture" flours, different types of breads; cakes, biscuits and pies, and final distribution of the plant products to their destinations. The complete material flow graph for wheat processing is presented in Figure 1.

The structure of this graph with the commodity flowing from left to right, can be described in terms of a sequence of levels, each containing a set of nodes. The levels correspond to physical stages in the processing of wheat.

Moving grain from production areas to dryers and millers, and flour from mills to bakers and other processors, involves elaborate distribution networks. The additional source nodes CV18 and CV19 represent corn flour and rye flour which are used as additives for "multi-mixture" flours. Nodes CV18 and CV19 depict the interconnection of the wheat sector with two others sectors, corn and rye production. The user can create sink nodes from which outgoing links are not considered. In the case of wheat, nodes CV4, CV7, and CV10 represent sink nodes, i.e., the points where material flows exit the system without going through a market. Final market points at the retail level are defined by nodes CV11, CV20, CV22, CV50, CV51, and CV53.

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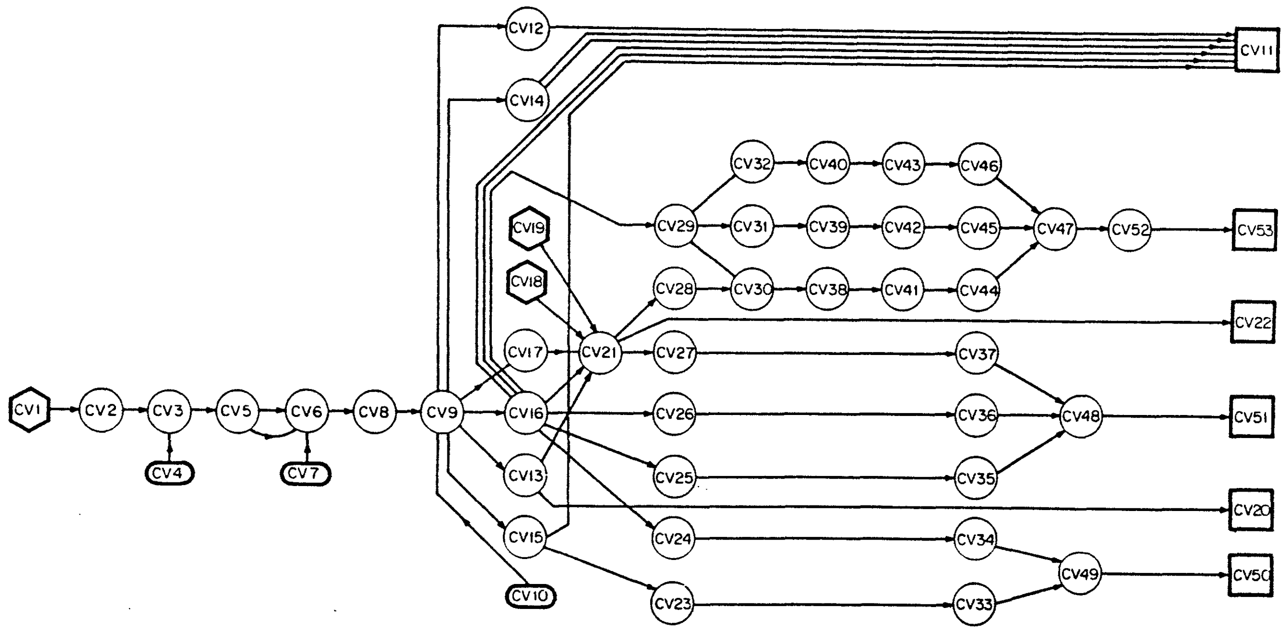

Figure 1. Structure of ALINET graph for Yugoslav Enterprise

Once the structure of the flow graph has been established, the supplies at the source nodes, the allocation coefficient for each link, and the attributes of the processes depicted by the links are determined. The allocation coefficient denotes the fraction of the supply at the node allocated to each outgoing link. With the supplies specified at the source nodes and the prices and allocation coefficients estimated for each link, the material flows in each link and at each node can be determined. Twelve such solutions were made, one for each month in the baseline year June 1983–May 1984. The simulation results were then compared to the data to verify that the model calculates material flows accurately. The largest difference between the simulation results and the data from the actual physical system were less than 2 percent.

The use of electricity, natural gas and diesel oil was also examined. In addition, the model was used to calculated the energy needed for the production of a specific product and to calculate the total energy use in each stage of processing. Table 1 shows the total direct energy use, by fuel type, and cost for the whole production. It includes energy used for transportation, drying and milling of grain, transportation of flour and "multi-mixtures", "multi-mixture" manufacture, baking, and final product distribution.

TABLE 1 Direct Energy Use from Farm to Consumer

	ENERGY	COST
Electricity	8433 MWH	19.228×10^6 din.
Natural Gas	869713 Nm	7.879×10^6 din.
Diesel Oil	54692 Lit	2.6492×10^6 din.

3. A DESCRIPTOR VARIABLE APPROACH TO EQUILIBRIUM DETERMINATION

Descriptor variable theory (Luenberger, 1977) is an approach to analyzing systems that preserves structure and allows considerable flexibility in modeling. The basis of this modeling methodology is to exploit the structure of the actual system being modeled. If the system representation is given in terms of physical or economic quantities that describe the system operation, the model will reflect the natural structure of the actual system. Initially, no attempt is made to select a minimum set of variables, as in the state space approach (Zadeh and Desoar, 1963).

The descriptor variable approach is particularly useful for modeling and analyzing systems with chain structure. In a system possessing a chain structure, each system variable can be assigned to exactly one of a sequence of sets, such that each mathematical relationship involves variables in one set or in sequentially adjacent sets. In this context, the descriptor variable approach is very powerful in determining the equilibrium of large-scale economic systems consisting of a supplier sector, producer sectors, and a consumer sector, with no flow loop.

In order to determine the equilibrium of the wheat production, processing, and distribution sector, a simplified model was constructed (see Figure 2). In this case, the chain-structure was not immediate, but was created by adding "dummy" sectors (Stengel, 1978b). Prices and quantities of commodities are considered as descriptor variables.

The decentralized control approach to sector modeling for equilibrium (Stengel, 1978) was chosen. In this approach supplier submodels respond to a demand for outputs by announcing the prices of which they will provide that flow of goods. Producers are given the flow rate of outputs they are to produce and the unit prices of their inputs, and they announce a demand for inputs and the prices of their outputs. Consumer sectors are price takers; they choose their consumption of goods.

Let q_j be a vector representing the flow of outputs in sector j and let p_j be the vector of corresponding prices. The model for suppliers in index set o is given by

$$p_o = f_o + S_o q_o \qquad (1)$$

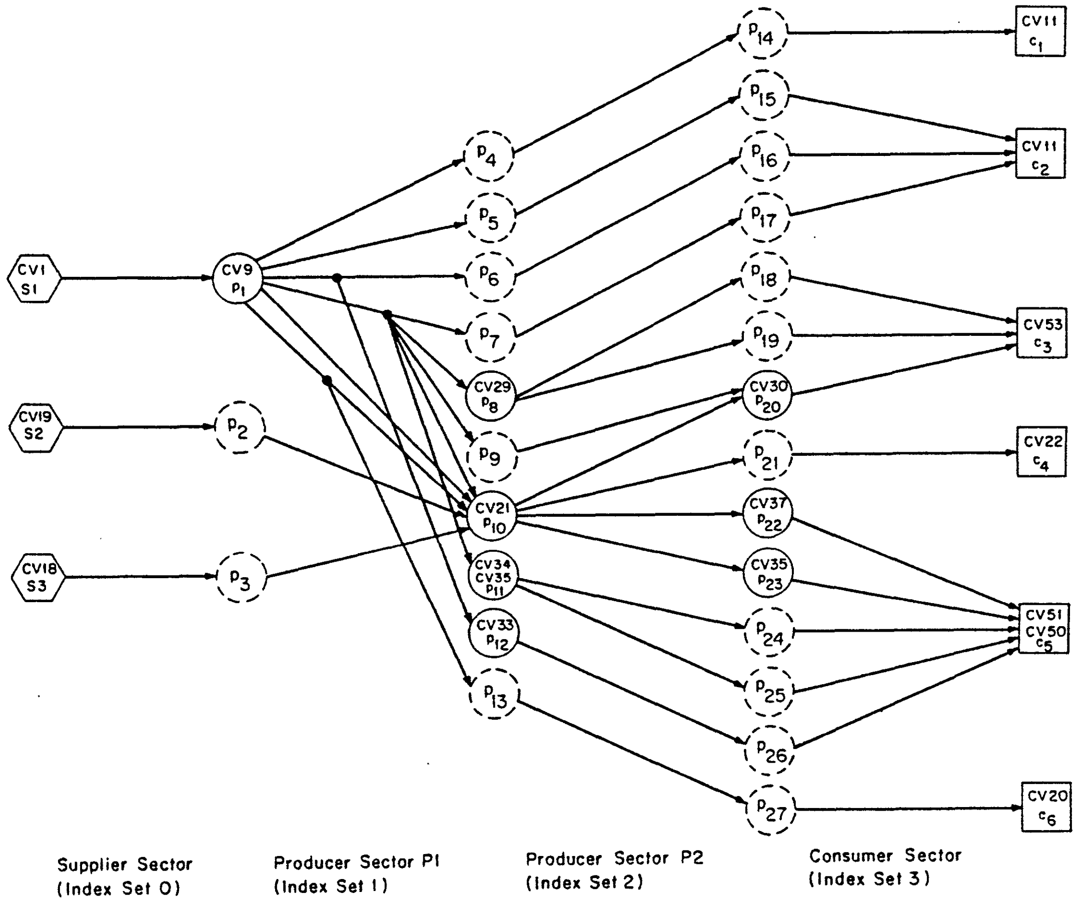

Figure 2. Network model consisting of a chain of sectors of a Yugoslav Enterprise

where

$p_0 \in R^3$, $f_0 \in R^3$, $q_0 \in R^3$ and $S_0 \in R^{3 \times 3}$

The producers in index set j, $1 \leq j \leq 3$ are given by

$$q_{j-1} = g_{j-1} + A_{j-1}\, p_{j-1} + B_j\, q_j \qquad (2)$$

$$p_j = f_j + C_{j-1}\, p_{j-1} + D_j\, q_j \qquad (3)$$

where

$q_1 \in R^{14}, p_1 \in R^{14}, q_2 \in R^{15}, p_2 \in R^{15}, q_3 \in R^{14}, p_3 \in R^{14}$

and A_{j-1}, B_j, C_j and D_j are matrices of corresponding dimensions.

The model for the consumer, index set 3, is characterized by

$$q_3 = g_3 + R_3\, p_3 \qquad (4)$$

The linear time-invariant descriptor system (1)-(4) can be rewritten in the following block matrix form

$$Ap = g \qquad (5)$$

where

$$A = \begin{bmatrix} I & -S_0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -A_0 & I & 0 & -B_1 & 0 & 0 & 0 & 0 \\ -C_0 & 0 & I & -D_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -A_1 & I & 0 & -B_2 & 0 & 0 \\ 0 & 0 & -C_1 & 0 & I & -D_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & -A_2 & I & 0 & -B_3 \\ 0 & 0 & 0 & 0 & -C_2 & 0 & I & -D_3 \\ 0 & 0 & 0 & 0 & 0 & 0 & -R_3 & I \end{bmatrix} \qquad (6)$$

$$p = [p_0,\ q_0,\ p_1,\ q_1,\ p_2,\ q_2,\ p_3,\ q_3]$$

$$g = [f_0,\ g_0,\ f_1,\ g_1,\ f_2,\ g_2,\ f_3,\ g_3]$$

The system Eq. (5) is solvable if and only if the matrix A is nonsingular. However, due to the high dimensionality of this matrix, it is useful to establish properties for the submatrices in Eq. (6) that insure solvability. In this case, sufficient conditions for solvability of the network are that S_k be positive definite $S_k = S_k^T > 0$ and R_k be negative definite $R_k = R_k^T < 0$ for all k, $0 \leq k \leq 3$, where each S_k is determined recursively from S_0 by,

$$S_{k+1} = C_k\, (I - S_k A_k)^{-1}\, S_k\, B_{k+1} + D_k \qquad (7)$$

and each R_k is determined recursively from R_3 by

$$R_{k-1} = B_k\,(I-R_k D_k)^{-1} R_k\, C_{k-1}\, A_{k-1} \qquad (8)$$

It has been shown (Stengel, 1978b) that the positive definiteness of the matrix S_k and negative definiteness of matrix R_k are guaranteed by the fact that $S_0 = S_0^T > 0$ and $R_3 = R_3^T < 0$. Notice that the positive definiteness of the matrix S_0 is a direct consequence of the fact that Eq. (1) corresponds to an inverse supply curve, and the negative definiteness of the matrix R_3 follows from the fact that Eq. (4) corresponds to a consumer demand function.

4. ITERATIVE ALGORITHMS

Many problems in practice that involve high dimensional systems of linear algebraic equations are solved by iterative methods. In what follows, we will consider two approaches proposed by Stengel (1978b) towards more efficient algorithms for solving equation (5).

Simultaneous Displacements

One common method for iterative solution of a system of linear algebraic equations is the so-called Simultaneous Displacements methods. Using this approach the iterative scheme for equation (5) can be presented in the form

$$\begin{bmatrix} p_0 \\ q_0 \\ p_1 \\ q_1 \\ p_2 \\ q_2 \\ p_3 \\ q_3 \end{bmatrix}_{i+1} = \begin{bmatrix} 0 & S_0 & 0 & 0 & 0 & 0 & 0 & 0 \\ A_0 & 0 & 0 & B_1 & 0 & 0 & 0 & 0 \\ C_0 & 0 & 0 & D_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & A_1 & 0 & 0 & B_2 & 0 & 0 \\ 0 & 0 & C_1 & 0 & 0 & D_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & A_2 & 0 & 0 & B_3 \\ 0 & 0 & 0 & 0 & C_2 & 0 & 0 & D_3 \\ 0 & 0 & 0 & 0 & 0 & 0 & R_3 & 0 \end{bmatrix} \begin{bmatrix} p_0 \\ q_0 \\ p_1 \\ q_1 \\ p_2 \\ q_2 \\ p_3 \\ q_3 \end{bmatrix}_{i} + \begin{bmatrix} f_0 \\ g_0 \\ f_1 \\ g_1 \\ f_2 \\ g_2 \\ f_3 \\ g_3 \end{bmatrix} \qquad (9)$$

where i, $i = 0,1,2,\ldots,$ is the index of iteration.

To start the iterative procedure, one should make an initial guess:

$$p_0^0,\ q_0^0,\ p_1^0,\ q_1^0,\ p_2^0,\ q_2^0,\ p_3^0 \text{ and } q_3^0$$

As mentioned earlier, because of its high dimensionality, equation (5) cannot be solved easily by a direct method; an alternative, but equivalent, approach is the following semi-iterative procedure.

Semi-Iterative Procedure

In this approach only some of the variables are re-estimated each time. By examining the equations (1)-(4), it can be easily concluded that p_{j-1} and q_j are used to compute q_{j-1} and p_j, $0 \le j \le 3$. Therefore, if q_0, p_1, q_2, p_3 are known, one can compute p_0, q_1, p_2 and q_3. This suggests the following two-step semi-iterative procedure.

Step 1:

$$\begin{bmatrix} q_0^{i+1} \\ p_1^{i+1} \\ q_2^{i+1} \\ p_3^{i+1} \end{bmatrix} = \begin{bmatrix} A_0 & B_1 & 0 & 0 \\ C_0 & D_1 & 0 & 0 \\ 0 & 0 & A_2 & B_3 \\ 0 & 0 & C_2 & D_3 \end{bmatrix} \begin{bmatrix} p_0^{i} \\ q_1^{i} \\ p_2^{i} \\ q_3^{i} \end{bmatrix} + \begin{bmatrix} g_0 \\ f_1 \\ g_2 \\ f_3 \end{bmatrix} \qquad (10)$$

Step 2:

$$\begin{bmatrix} p_0^{i+1} \\ q_1^{i+1} \\ p_2^{i+1} \\ q_3^{i+1} \end{bmatrix} = \begin{bmatrix} S_0 & 0 & 0 & 0 \\ 0 & A_1 & B_2 & 0 \\ 0 & C_1 & D_2 & 0 \\ 0 & 0 & 0 & R_3 \end{bmatrix} \begin{bmatrix} q_0^{i+1} \\ p_1^{i+1} \\ q_2^{i+1} \\ p_3^{i+1} \end{bmatrix} + \begin{bmatrix} f_0 \\ g_1 \\ f_2 \\ g_3 \end{bmatrix} \qquad (11)$$

It is clear that an iteration by simultaneous displacement Eq. (9) requires the same number of computations as the two-step semi-iterative procedure given by Eqs. (10) and (11). It should be pointed out that neither with simultaneous displacement nor with the semi-iterative procedure is convergence guaranteed. The general convergence will depend on the eigenvalues of the matrix A in Eq. (5). However, as it is shown by Stengel (1978b), the semi-iterative procedure will converge whenever the simultaneous displacements procedure converges. Furthermore, if there is convergence, then the semi-iterative procedure will converge twice as fast.

When the simultaneous displacements was implemented for the network model considered, the algorithm converged very slowly, resulting in excessive computation cost. In order to improve convergence, the semi-iterative procedure was used. As predicted by theory, the semi-iterative approach was far more effective: convergence was achieved twice as fast.

The equilibrium values for the quantities and prices for sets j, $j = 0,1,2,3$, are given in Table 2.

TABLE 2 Equilibrium Quantities and Prices

q_0 =	54990.0, 16.69, 151.0
p_0 =	15450.0, 21000.0 , 21000.0
q_1 =	8506, 1711, 8930, 2996, 16830, 282.4, 42350, 3279, 2941, 522.8, 15.89, 7.764, 16.69, 15100
p_1 =	8500, 28250, 24750, 18750, 26850, 20480, 20480, 20480, 20480, 25100, 5000, 5000, 5000, 21000, 21000
q_2 =	8506, 1711, 8930, 16830, 301.9, 356.6, 423.5, 41.72, 3406, 278, 202.5, 252.8, 3835, 4195, 7.703
p_2 =	8500, 28450, 24750, 25850, 168000, 185.6, 20480, 27000, 36700, 27000, 27000, 92500, 46250, 30000, 5000
q_3 =	8506, 1711, 8930, 16830, 301.9, 356.6, 961.7, 3463, 389.2, 269.5, 257.8, 3835, 4195, 7.664
p_3 =	8500, 28450, 24750, 26850, 168000, 188.6, 192000, 36700, 62000, 52800, 92500, 46250, 30000, 5000

5. PARALLEL PROCESSING

In order to improve further the speed and efficiency of the computation, a parallel algorithm for multiprocessors was introduced, i.e., we defined a collection of processes* that may operate simultaneously for solving the problem considered. In other words, we were interested in designing algorithms for which multiprocessors can be employed to speed the computation of the iterative algorithm for equilibrium. To achieve this goal, a synchronized parallel algorithm was used. In such an algorithm, each iteration is decomposed so that each step is done by more than one process, and the processes are synchronized at the end of each iteration.

For simplicity, let us assume that we are interested in constructing a parallel algorithm consisting of two processes, i.e., suppose that we want to compute p_0, q_0, p_1, q_1, p_2, q_2, p_3, and q_3 by two processes. We may construct a parallel algorithm by creating process π_1 consisting of one stage: calculation of p_0, q_0, p_1, and q_1, and process π_2 consisting of one stage: calculation of p_2, q_2, p_3, and q_3. Clearly, the activation of the second process π_2 is subject to the condition that process π_1 is completed. Thus, this is a synchronized parallel algorithm.

The most natural approach in this case is to decompose each vetor p^{i+1} into two segments p_1^{i+1} and p_2^{i+1} each of the same size, $p_1^{i+1} \varepsilon R^{23}$, $p_2^{i+1} \varepsilon R^{23}$, and update them by two parallel process as follows,

$$\begin{bmatrix} p_1^{i+1} \\ p_2^{i+1} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} p_1^{i} \\ p_2^{i} \end{bmatrix} + \begin{bmatrix} g_1 \\ g_2 \end{bmatrix} \tag{12}$$

where

$$p_1^{i+1} = A_{11}\, p_1^i + A_{12}\, p_2^i + g_1 \tag{13}$$

and

$$p_2^{i+1} = A_{21}\, p_1^i + A_{22}\, p_2^i + g_2 \tag{14}$$

That is, at an iterative step, each process updates half of the components, and starts the next iteration only after both processes have finished the updating.

Having in mind that the mathematical model of the material flow through the network was based on the descriptor variable approach (which preserves the natural structure of the system being modeled) the decomposition of the vector p^{i+1} into two segments can be done using the semi-iterative procedure. In that case

$$p_1^{i+1} = \begin{bmatrix} g_0^{i+1} \\ p_1^{i+1} \\ g_2^{i+1} \\ p_3^{i+1} \end{bmatrix}, \quad p_2^{i+1} = \begin{bmatrix} p_0^{i+1} \\ g_1^{i+1} \\ p_2^{i+1} \\ g_3^{i+1} \end{bmatrix} \tag{15}$$

Since in both cases the computation of p_1^{i+1} and p_2^{i+1} involves practically the same amount of work, one might tempted to conjecture that this is the best scheme using two processors.

*We view a process as the execution of a procedure in a multi-processor operating system.

6. CONCLUSIONS

Two realizations of ALINET were developed. The first one included the analysis of energy consumption in the wheat processing and distribution sector. The second one was used for determining the steady-state flow of commodities and corresponding prices through the system. Two iterative algorithms, the simultaneous displacements and a two-step semi-iterative procedure, were used to overcome numerical difficulties associated with determination of the equilibrium. The network modeling system was used as a tool for analyzing wheat production and processing in a Yugoslav enterprise.

It should be pointed out that the primary purpose of the network model was not to propose definite policies for the wheat production and distribution sector. Rather, by means of detailed and meticulous modeling of the network, the model is intended to provide a framework within which a variety of analyses may be carried out in cooperation with the Yugoslav enterprise.

7. REFERENCES

Levis, A. H., I. S. Levis and E. R. Ducot (1980), "Energy Conservation Analysis in the Food Processing Sector: A Simulation Approach," Int. Journal of Policy Analysis and Information Systems, Vol. IV, No.1.

Ducot, E. R. and A. H. Levis (1980), "User Perspective and Design Emphasis: Experience with a Network Model of the Food Processing Systems (ALINET)," Proc. ASAE Winter Simulation Conference, Orlando, Florida.

Levis, A. H., E. R. Ducot, F. Webster and I. S. Levis (1980), "ALINET: A Model of Assessing Energy Conservation Opportunities in Food Processing," Proc. ASAE National Energy Symposium, Kansas City, MO.

Luenberger, D. G. (1977), "Dynamic Equations in Descriptor Form," IEEE Trans. on Automatic Control, AC-22, No. 3.

Stengel, D. N. (1978a), "A Descriptor Approach to Economic Network Equilibrium," Proc. Symposium of Large Engineering Systems, University of Waterloo, May 15-16.

Stengel, D. N. (1978b) "Descriptor Analysis of Economic Systems," Ph.D. Thesis, Dept. of Engineering-Economic Systems, Stanford University, Stanford, CA.

Zadeh L. A. and C. A. Desoer (1963), Linear System Theory - The State Space Approach, McGraw-Hill Co., New York.