MARCH 1986 LIDS-P-1542

A TUTORIAL ON THE LQG/LTR METHOD

by

Michael Athans
Laboratory for Information and Decision Systems
Department of Electrical Engineering and Computer Sciences
Massachusetts Institute of Technology
Cambridge, Mass. 02139

ABSTRACT

In this paper we overview the so-called Linear-Quadratic-Gaussian method with Loop-Transfer-Recovery (LQG/LTR). Our objective is to provide a pragmatic exposition, with special emphasis on the step-by-step characteristics for designing multi-variable feedback control systems.

1. INTRODUCTION

Control engineers need systematic design procedures for designing high performance feedback control systems for both single-input single-output (SISO) and multi-input multi-output (MIMO) systems. It is also essential that readily available and reliable CAD software be used in the design process. No matter how powerful the design methodology is, a typical application requires several iterations. Hence, it is imperative that the design procedures are transparent, are conducive to educated trial and error design iterations, and that the number of design parameters be kept at an absolute minimum.

At the present time, the LQG/LTR design methodology has many of the required characteristics of an easy-to-use design method for SISO and MIMO feedback control. Moreover, commercially available CAD software, e.g. MATRIX-X, CTRL-C, LQGALPHA, Program CC etc, can readily handle the computations associated with the LQG/LTR method in both the time-domain and the frequency-domain. On the other hand, there are very few published papers that deal with either the method itself or with applications oriented studies that have exploited the LQG/LTR methodology. As a consequence, this powerful method is not readily accessible to the practicing engineer. It is the goal of this paper to present the key ideas behind the LQG/LTR method in as simple a way as possible. In this manner, we hope that the practicing engineer can appreciate both the advantages and the shortcomings of this design methodology. Together with the other papers in the 1986 ACC invited session on LQG/LTR, references [1] to [5], there is enough material to survey the method and typical ways in which it is applied. Thus, we hope that the practicing control engineer can find enough material so as to have a clear idea of how this method may address his/her design problems.

We stress that there are no new results presented in this paper. In point of fact, we only overview the simplest aspects of the LQG/LTR method. The interested reader is refered to the paper by Stein and Athans [6] for extension of the basic ideas to a much more general and formal design framework.

2. THE DESIGN PLANT MODEL AND MODEL ERRORS.

In this section we summarize the definition and equations of what we call the design plant model (DPM) which we shall use in conjuction with the LQG/LTR method. We remark that the design plant model not only includes a nominal model of the dynamics of the physical process that we are going to control, but also reflects the scaling of the variables and may include augmentation dynamics (such as integrators) that the designer has appended

* This research was supported by the NASA Ames and Langley Research Centers under grant NASA/NAG-2-297, by the Office of Naval Research under contract ONR/N00014-82-K-0582 (NR 606-003) and by the National Science Foundation under grant NSF/ECS - 8210960.

PROC. AMERICAN CONTROL CONFERENCE, Seattle, WA, June 1986.

to the plant model to meet special command-following and disturbance-rejection performance specifications.

We assume that the design plant model (DPM) is linear and time-invariant (LTI) with m controls, m outputs, and n state variables. In the time domain the DPM obeys the dynamics

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) \quad (1)$$

$$\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t). \quad (2)$$

The transfer function matrix (TFM) $\mathbf{G}(s)$ of the DPM is a square mxm matrix given by

$$\mathbf{G}(s) = \mathbf{C}\mathbf{\Phi}(s)\mathbf{B}, \quad (3)$$

where

$$\mathbf{\Phi}(s) \equiv (s\mathbf{I} - \mathbf{A})^{-1}. \quad (4)$$

In the sequel, we shall assume that $[\mathbf{A},\mathbf{B}]$ is stabilizable, i.e. all unstable modes of (1) are controllable, and that $[\mathbf{A},\mathbf{C}]$ is detectable, i.e. all unstable modes in (1) and (2) are observable.

Knowledge of the DPM is necessary, but by no means sufficient, for the design process. The control engineer must have some information about the size of the modeling errors. In general, high frequency model errors impose a limit upon the control system bandwidth and, thereby, limit the performance of the feedback system. In the version of the LQG/LTR method that we shall employ, we must reflect all model errors to the output of the DPM, using the so-called multiplicative model error representation; see Doyle and Stein [7], and Lehtomaki et al [8], [9].

Suppose that $\mathbf{G}_A(s)$ denotes the actual design plant dynamics. The multiplicative error matrix, $\mathbf{E}(s)$, reflected at the plant output is defined by:

$$\mathbf{G}_A(s) \equiv [\, \mathbf{I} + \mathbf{E}(s)\,]\mathbf{G}(s) \equiv \mathbf{L}(s)\mathbf{G}(s) \quad (5)$$

Obviously, the engineer does not have a detailed expression for $\mathbf{E}(s)$. Rather, we assume that it is possible to bound the size of the maximum possible error in the worst possible direction as a function of frequency ω, i.e. we assume that

$$\sigma_{max}\mathbf{E}(j\omega) < e_m(\omega) \quad (6)$$

and that the bound $e_m(\omega)$ is known as part of the modeling process.

2. FEEDBACK DESIGN

We shall imbed the design plant model $\mathbf{G}(s)$ in the standard negative identity MIMO feedback loop configuration shown in Fig. 1. This configuration shows explicitly the tracking error vector $\mathbf{e}(s)$. The impact of all disturbances acting on the physical process is accounted for as an equivalent additive disturbance vector $\mathbf{d}(s)$ acting on the DPM output. In Fig. 1 we can see that the only measurements are the output variables, which of course include the effects of all disturbances. To maximize clarity we do not show explicitly sensor noise and errors.

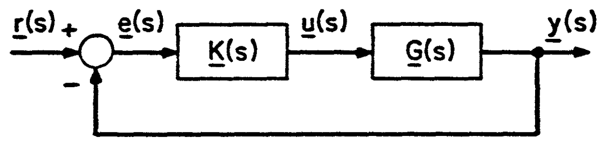

Figure 1. The MIMO feedback loop.

The bottom line, of course, is to specify the dynamic compensator $\mathbf{K}(s)$ in Fig. 1. The compensator $\mathbf{K}(s)$ is assumed to be LTI, and it has m inputs (the tracking error signals) and m outputs (the inputs to the DPM). Of course, we cannot design $\mathbf{K}(s)$ unless we are given reasonable specifications. These specifications relate to:

(a) Nominal stability.

(b) Stability-robustness to modeling errors.

(c) Good performance.

Performance specifications relate to good command following, good disturbance rejection, insensitivity to sensor noise and nominal modeling errors. All of these relate to the impact of uncertainty in the feedback loop in Fig. 1. Remember that the only reason for using feedback in the first place, rather than open-loop control, is because of the ever present uncertainty.

3. THE PHILOSOPHY OF THE LQG/LTR METHOD

The LQG/LTR design methodology seeks to define the MIMO compensator $\mathbf{K}(s)$ so that the stability-robustness and performance specifications are met to the extent possible. We remark that, although the LQG/LTR method is clearly applicable to SISO designs, it is an inherently multivariable design method for MIMO systems. By this we mean that the LQG/LTR method does not reduce the MIMO design problem into a sequence of SISO design problems; rather, it attacks the MIMO design problem directly and the same steps and philosophy are used independent of the number of state variables (n), and control and output variables (m).

The LQG/LTR method involves two basic steps. In the first step we generate a MIMO target feedback loop; let us suppose that this target design meets the posed performance specifications without violating the stability-robustness constraints. In the second step a special compensator **K**(s) is used in Fig. 1. The LQG/LTR compensator **K**(s) has some adjustable parameters that can be "tweaked" in a clever but straightforward way, so that the performance of the feedback system in Fig. 1 approximates the performance of the target feedback loop established in step one. The degree of approximation is governed by any nonminimum phase characteristics of the design plant model (DPM). If the DPM is minimum phase, then the degree of approximation, or the "recovery", of the target feedback loop can be arbitrarily good. If the DPM is nonminimum phase, then the quality of the "recovery" will depend on the location of the nonminimum phase zeros. We shall elaborate on this point in the sequel.

4. THE TARGET FEEDBACK LOOP (TFL).

In this section we shall discuss the definition of the TFL. As mentioned above, the TFL must meet the imposed stability-robustness and performance specifications.

The structure of the TFL is shown in Figure 2. It is simply defined by the parameters **C** and $\Phi(s)$ of the DPM, see eqs. (1)-(4), and by an mxn constant matrix **H** which we shall call the filter gain matrix.

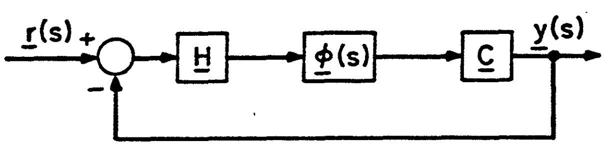

Figure 2. The target feedback loop structure.

If we break the loop at the output or, equivalently, at the error signal, we readily obtain the loop TFM associated with the TFL which we shall denote as $G_{KF}(s)$:

$$G_{KF}(s) = C\Phi(s)H \tag{7}$$

We remark that the assumption that **[A,C]** is detectable implies that there exist matrices **H** which will make the TFL of Fig. 2 stable. The loop TFM $G_{KF}(s)$ defines the sensitivity TFM $S_{KF}(s)$

$$S_{KF}(s) = [\, I + G_{KF}(s)\,]^{-1} \tag{8}$$

and the closed-loop (or complementary sensitivity) TFM $C_{KF}(s)$

$$C_{KF}(s) = [\, I + G_{KF}(s)\,]^{-1}\, G_{KF}(s). \tag{9}$$

For any given filter gain matrix **H**, regardless of how it is found, we can evaluate whether or not the resulting TFL meets the stability-robustness constraints and performance specifications. For stability-robustness to hold the following inequality must be true for all ω:

$$\sigma_{max}\, C_{KF}(j\omega) < 1/e_m(\omega) \tag{10}$$

where $e_m(\omega)$ is defined in eq. (6). Command-following and disturbance-rejection frequency domain properties of the TFL can be checked from the obvious relations

$$y(s) = C_{KF}(s)r(s)\text{ , when } d(s) = 0 \tag{11}$$

$$e(s) = S_{KF}(s)[\, r(s) - d(s)\,] \tag{12}$$

Note that actual transient responses can be carried out by simulating the TFL in Fig. 2, and injecting command vectors **r**(t) and/or disturbance vectors **d**(t) that correspond to those that are expected to appear in the actual system.

Let us suppose that, somehow, we have found a matrix **H**, and hence a loop TFM $G_{KF}(s)$ that we like (we shall return to methods that help us accomplish this in Section 6). It makes no difference whatsoever how we have arrived at the matrix **H**. Obviously we cannot implement the TFL; there are no controls! However, since the TFL we designed is robust to model errors and since we like its command- following and disturbance-rejection properties, we can wonder whether we can construct a compensator **K**(s) in Fig. 1 with the property that the feedback system of Fig. 1 approximates the behavior of the TFL in Fig. 2. This would happen if the following equality were true:

$$G(s)K(s) = G_{KF}(s) \tag{13}$$

Since both $G(s)$ and $G_{KF}(s)$ are n-th order systems and $K(s)$ is a dynamic compensator the equality (13) cannot be true. However, for the purposes of design it is not necessary that we have exact equality as in eq. (13). Indeed, if we had a way of finding $K(s)$ so that we had the approximate relation

$$G(j\omega)K(j\omega) \approx G_{KF}(j\omega) \tag{14}$$

over the band of frequencies relevant to our concerns for robustness and performance, then we would have been satisfied. This is precisely what the LQG/LTR method allows us to do, under certain assumptions.

5. THE LQG/LTR COMPENSATOR, K(s).

In this section we summarize the structure of the LQG/LTR compensator that will accomplish our design objectives.

The LQG/LTR compensator belongs to the class of the so-called model-based compensators (MBC). As illustrated in Fig. 3 an MBC contains a replica of the design plant model (DPM) together with two feedback loops. One feedback loop involves a gain matrix G, while the other feedback loop involves a gain matrix H. These two matrices are the "free" design parameters in the compensator.

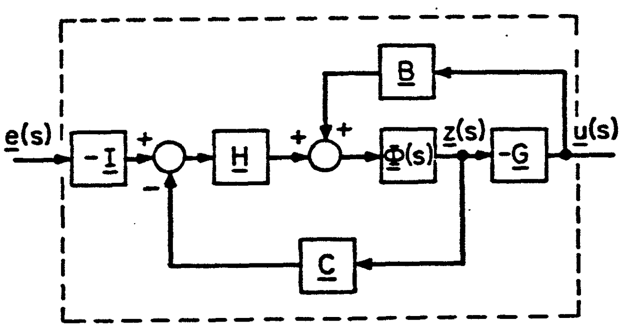

Figure 3. The structure of an MBC and an LQG/LTR compensator.

One can readily compute the transfer function matrix of the MBC in Fig. 3. This is defined by

$$u(s) = K(s)e(s) \tag{15}$$

with

$$K(s) = G(sI - A + BG + HC)^{-1}H \tag{16}$$

In the time domain, if we let $z(t) \in R^n$ denote the state vector of the compensator $K(s)$, then it is defined by:

$$\dot{z}(t) = (A - BG - HC)z(t) - He(t)$$
$$u(t) = -Gz(t) \tag{17}$$

The special properties of the MBC are due to the separation property - see, for example, Kwakernaak and Sivan [10] - which states that the 2n closed-loop poles of the feedback system in Fig. 1, when the compensator is given by eq. (16), are the eigenvalues of $(A - BG)$ and $(A - HC)$.

What we mean by a LQG/LTR compensator is an MBC in which the two matrices H and G in eq. (16) or (17) are computed in a special way. The filter gain matrix H is fixed to be that found in the TFL, as discussed in Section 4. The only other remaining design parameter in $K(s)$, the so-called control gain matrix G, is computed via the solution of the so-called cheap-control Linear-Quadratic Regulator (LQR) problem as outlined below.

We remark that the choice of the signs in eq. (17) was made so that when $r(t) = 0$, the compensator state vector $z(t)$ corresponds to the state estimate in a classical LQG regulator setup.

To compute G for the LQG/LTR compensator we solve the algebraic Riccati equation (ARE)

$$0 = -K_\rho A - A'K_\rho - C'C + (1/\rho)K_\rho BB'K_\rho \tag{18}$$

for $\rho \to 0$, and then compute the matrix G_ρ by

$$G_\rho = (1/\rho)B'K_\rho. \tag{19}$$

The main result, the so-called Loop Transfer Recovery (LTR), generally credited to Doyle and Stein [7], is as follows:

LTR Result: If the DPM $G(s) = C(sI - A)^{-1}B$, defined in eqs. (1) to (4), has only minimum phase transmission zeros then pointwise in s

$$\lim_{\rho \to 0} C(sI-A)^{-1}BG_\rho(sI-A+BG_\rho+HC)^{-1}H \to C(sI-A)^{-1}H \tag{20}$$

which implies that

$$\lim_{\rho \to 0} G(s)K_\rho(s) \to G_{KF}(s) \tag{21}$$

The LTR result accomplishes then what we set out to do. The loop TFM, $G(s)K(s)$, of the system we are going to build (Fig. 1), approximates the loop TFM, $G_{KF}(s)$, of the target feedback loop (Fig. 2), provided that the minimum phase condition on $G(s)$ holds. Thus, if we like the response of our target feedback loop, then we can use the LQG/LTR compensator to "recover" it.

In the frequency domain, the LQG/LTR method yields good agreement between the loop, sensitivity, and closed-loop TFM singular values of the actual and TFL systems for frequencies well beyond crossover. In general, at high frequencies the singular values of $G_{KF}(j\omega)$ roll-off at -20 db/dec, while those of $G(j\omega)K(j\omega)$ eventually roll-off at -40 db/dec. Thus, LQG/LTR loops offer some additional robustness to high frequency unmodeled dynamics as compared to the TFL. The command-following and disturbance-rejection performance, in the low frequency region, between the TFL and the LQG/LTR systems will be essentially the same.

The LQG/LTR compensator essentially generates the best stable inverse of the DPM $G(s)$ and substitutes the desired dynamics defined by $G_{KF}(s)$. The zeros of $K(s)$ correspond to the zeros of $G_{KF}(s)$. Some poles of the compensator $K(s)$ are used to cancel the transmission zeros of $G(s)$; this is why the LTR method does not "work" for nonminimum phase plants since this would involve cancellations of poles and zeros in the right half s-plane. Some of the poles of $K(s)$ go to infinity, as $\rho \to 0$, in such a way so that the nominal loop stability is preserved; this can be exploited since one can approximate the high frequency dynamics of $K(s)$ by feed-through terms (see Athans et al [11] for a concrete example).

6. HINTS FOR DESIGNING THE TARGET FEEDBACK LOOP USING KALMAN FILTER TECHNIQUES.

In this section we present some methods for designing the target feedback loop (TFL) in Fig. 2, and in particular the loop TFM of eq. (7), i.e.

$$G_{KF}(s) \equiv C\Phi(s)H \tag{22}$$

We have remarked that as far as the LTR step is concerned, the filter gain matrix H can be arbitrary as long as the TFL is stable. Thus, we want to have at our disposal some "easy" methods for selecting H so that we can shape in the frequency domain the singular values of the loop (7) or (22), sensitivity (8), and closed-loop (9) TFM's.

We can exploit the solution of a fictitious continuous time Kalman Filter to find H. We warn the reader that we are using Kalman Filter formulas and concepts as a means to an end, rather than in a precise optimal stochastic estimation and control context.

The Kalman Filter problem formulation that will generate the formulas that we shall use involves the stochastic state dynamics

$$\dot{x}(t) = Ax(t) + L\xi(t) \tag{23}$$

$$y(t) = Cx(t) + \theta(t) \tag{24}$$

where L is an nxm matrix, the process noise $\xi(t)$ is white, zero mean, with identity intensity matrix, and the measurement noise $\theta(t)$ is white, zero mean, and with intensity matrix equal to μI.

The solution to the Kalman Filter problem yields the formula for calculating the filter gain matrix H,

$$H = (1/\mu)\Sigma C' \tag{25}$$

where the symmetric, and at least positive semidefinite, matrix Σ is found from the solution of the algebraic Ricatti equation,

$$0 = A\Sigma + \Sigma A' + LL' - (1/\mu)\Sigma C'C\Sigma \tag{26}$$

From now on we shall view the nxm matrix L and the scalar parameter $\mu>0$ as the design parameters at our disposal that will specify the filter gain matrix H.

Note that if $[A,L]$ is stabilizable and if $[A,C]$ is detectable, then we are always guaranteed the noninal stability of the TFL for any choice of the design parameters μ and H.

The choice of the design parameters is facilitated by the following frequency domain equality, [6], [7], [10], [12], involving the singular values of the return difference TFM associated with the TFL:

$$\sigma_i[I + G_{KF}(j\omega)] = \sqrt{1 + (1/\mu)\sigma_i^2 [\, C\Phi(j\omega)L \,]} \tag{27}$$

Equation (27) facilitates the choice of μ and L because these design parameters appear in the right hand side of the equation, while the frequency domain characteristics of the TFL appear in the left hand side.

One advantage of using the Kalman filter methodology for calculating H is that the TFL has certain automatic performance and robustness guarantees which can be directly obtained from eq. (27). These are:

$$\sigma_{max}\, S_{KF}(j\omega) \leq 1 \quad \text{for all } \omega \tag{28}$$

and

$$\sigma_{max}\, C_{KF}(j\omega) \leq 2 \quad \text{for all } \omega \tag{29}$$

Equation (28) guarantees that the TFL will never amplify disturbances (reflected at the plant output) at any frequency. Equation (29) implies that the TFL will never go unstable to multiplicative model errors (again reflected at the plant output) as long as

$$\sigma_{max}E(j\omega) \leq 1/2 \quad \text{for all } \omega \tag{30}$$

We shall now illustrate a popular procedure which demonstrates how to arrive at a suitable DPM starting from a description of the plant dynamics, and how to select the design parameters μ and **L** to shape the singular values of the TFL in the frequency domain.

Suppose that our plant is described by

$$\dot{\mathbf{x}}_p(t) = \mathbf{A}_p\mathbf{x}_p(t) + \mathbf{B}_p\mathbf{u}_p(t)$$
$$\mathbf{y}(t) = \mathbf{C}_p\mathbf{x}_p(t) \tag{31}$$

with $\mathbf{x}_p(t) \in R^n$, $\mathbf{u}_p(t) \in R^m$, and $\mathbf{y}(t) \in R^m$. Also suppose that $\mathbf{A}_p^{-1}$ exists, so that the plant has no poles at the origin. We wish to design an LQG/LTR feedback system which has the property that it has zero steady-state error to arbitrary constant (step) commands and/or disturbances; this specification implies that we must have integrators in each and every channel without any feedback. Also, we would like to have all loop singular values to be identical at both low and high frequencies; this requirement often leads to designs in which all crossover frequencies are approximately the same, so that the MIMO system has about the same speed of response in all directions.

Since we are using the LTR method, all the desirable attributes of the design must be reflected in the TFL. To meet the zero steady-state error specifications we must first define correctly the design plant model (DPM) so that it contains the necessary "free" integrators. This can be accomplished by adding one integrator in each control channel of our plant (31). Mathematically, we define the vector $\mathbf{u}(t) \in R^m$ by:

$$\dot{\mathbf{u}}_p(t) = \mathbf{u}(t) \tag{32}$$

or, equivalently,

$$\mathbf{u}_p(s) = (1/s)\mathbf{u}(s) \tag{33}$$

The DPM is then defined by the augmented dynamics (those of the plant and of the added integrators) and it is now an (n+m)-dimensional system. In eqs. (1) and (2) we use the matrices

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}_p & \mathbf{B}_p \\ \mathbf{0} & \mathbf{0} \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} \mathbf{0} \\ \mathbf{I} \end{bmatrix}, \quad \mathbf{C} = [\,\mathbf{C}_p \;\; \mathbf{0}\,] \tag{34}$$

Now we shall choose the design matrix **L** to cause the TFL singular values to be identical at both low and high frequencies. First we decompose **L** as follows:

$$\mathbf{L} = \begin{bmatrix} \mathbf{L}_L \\ \mathbf{L}_H \end{bmatrix} \tag{35}$$

We shall use the nxm matrix $\mathbf{L}_L$ to influence the low frequency behavior of the singular values. We shall use the mxm matrix $\mathbf{L}_H$ to influence the high frequency behavior of the singular values.

Let us examine the matrix $\mathbf{C\Phi}(j\omega)\mathbf{L}$ in eq. (27). Straight-forward linear algebra calculations can be used to show that

$$\lim_{j\omega\to 0} \mathbf{C\Phi}(j\omega)\mathbf{L} = -(1/j\omega)\mathbf{C}_p\mathbf{A}_p^{-1}\mathbf{B}_p\mathbf{L}_L \tag{36}$$

and that

$$\lim_{j\omega\to\infty} \mathbf{C\Phi}(j\omega)\mathbf{L} = (1/j\omega)\,\mathbf{C}_p\mathbf{L}_H \tag{37}$$

From eq. (36) we can see that, in order to make the singular values of $\mathbf{C\Phi}(j\omega)\mathbf{L}$ identical at low frequencies, we can select the design matrix $\mathbf{L}_L$ as follows:

$$\mathbf{L}_L = -[\,\mathbf{C}_p\mathbf{A}_p^{-1}\mathbf{B}_p\,]^{-1} \tag{38}$$

so that,

$$\lim_{\omega\to 0} \sigma_i\, \mathbf{C\Phi}(j\omega)\mathbf{L} = (1/\omega) \tag{39}$$

From eq. (37) we see that, in order to make the singular values of $\mathbf{C\Phi}(j\omega)\mathbf{L}$ identical at high frequencies, we can select the design matrix $\mathbf{L}_H$ as follows:

$$\mathbf{L}_H = \mathbf{C}_p'(\,\mathbf{C}_p\mathbf{C}_p'\,)^{-1} \tag{40}$$

so that,

$$\lim_{\omega\to\infty} \sigma_i\, \mathbf{C\Phi}(j\omega)\mathbf{L} = (1/\omega) \tag{41}$$

From eqs. (39) and (41) and from eq. (27) we see that this specification for the design matrix **L** leads to the following approximations:

<u>At low frequencies:</u>

$$\sigma_i G_{KF}(j\omega) \approx 1/\omega\sqrt{\mu} \tag{42}$$

<u>At high frequencies:</u>

$$\sigma_i G_{KF}(j\omega) \approx 1/\omega\sqrt{\mu} \tag{43}$$

Thus all singular values of the TFL loop TFM roll-off at -20 db/dec at both low and high frequencies. The design parameter μ can be selected to adjust the crossover frequency consistent with the stability-robustness constraints.

In summary, the design of the TFL can be carried out using Kalman Filter methods. The frequency domain properties of Kalman Filters, as given by eq. (27), can be used in a proactive mode to help the designer select the design parameters, the scalar μ and the matrix **L**.

7. NONMINIMUM PHASE PLANTS AND THE LQG/LTR METHOD.

When the plant G(s) in eq. (3) has one or more nonminimum phase zeros, the LTR method cannot recover the TFL. Nonminimum phase zeros do present limitations in the achievable performance of a feedback loop independent of the design methodology used; see Freudenberg and Looze [13]. Such limitations constrain the achievable sensitivity and closed-loop transfer function matrices. It should be noted that the nonminimum phase characteristics of **G**(s) are not reflected in the loop TFM $\mathbf{G}_{KF}(s)$ of the TFL. This can be seen by examining eq. (7), $\mathbf{G}_{KF}(s)=\mathbf{C}\mathbf{\Phi}(s)\mathbf{H}$, which shows that only the **A** and **C** plant matrices are involved in the definition of the TFL. Indeed, it can be shown that $\mathbf{G}_{KF}(s)$ is always minimum phase when designed using the Kalman Filter method. The nonminimum phase plant characteristics become apparent when all three plant matrices, **A**,**B**, and **C**, are specified. Thus, there are no inherent limitations in the performance of the TFL. On the other hand, we do have performance limitations in the LQG/LTR loop.

We can certainly apply the LQG/LTR method to nonminimum phase plants as described. In fact we recommend to use the LQG/LTR method whether or not the plant has nonminimum phase characteristics. The resulting design will not "recover" the characteristics of the TFL; it will exhibit the limitations inherent with the presence of the nonminimum phase zeros. If the nonminimum phase zero frequencies are beyond the bandwidth of the TFL, then "recovery" will take place in low frequencies, and for all practical purposes the presence of far-away nonminimum phase zeros does not degrade the low frequency command-following and disturbance-rejection properties of the design. If there are low frequency nonminimum phase zeros, then the sensitivity of the LQG/LTR loop will not be small in that range of frequencies. In MIMO designs this implies that there are certain frequencies and directions in the space of commands and disturbances, controlled by the location and directions of the nonminimum phase transmission zeros of **G**(s), for which we cannot have as good a performance as we may like. However, there is very little that we can do about that anyway. The reader is refered to [6] for more insights on the relations between the LQG/LTR method and nonminimum phase plants.

8. CONCLUDING REMARKS

In this paper we presented an overview of the LQG/LTR method together with some hints about specific design tricks. We reiterate that the method is relatively simple and systematic. The burden is to design a target feedback loop (TFL) with realistic stability-robustness and performance properties; use of Kalman Filter methods together with plant augmentation help the designer in defining a "good" TFL. The "recovery" of the TFL properties, consistent with any nonminimum phase behavior, is completely straightforward.

In the past five years the author has supervised over a dozen master's theses at MIT that have applied the LQG/LTR method for MIMO feasibility studies involving aircraft, jet engines, submarines, and helicopters. In each case there were no surprises; the methodology "worked" as the theory suggested. Indeed in each case study, the major effort expended was to specify reasonable performance tradeoffs, stability-robustness constraints, and to establish reasonable scaling parameters for the control and output variables. The details of the LQG/LTR design, once the design problem was well formulated, accounted for less than 10% of the total design effort.

REFERENCES

[1]. W. H. Pfeil, M. Athans, and H. A. Spang III, "Multivariable Control of the GE T700 Engine Using the LQG/LTR Design Methodology", Proc. American Control Conference, Seattle, WA, June 1986.

[2]. R. Martin, L. Valavani, and M. Athans, "Multivariable Control of a Submarine Using the LQG/LTR Design Methodology", Proc. American Control Conference, Seattle, WA, June 1986.

[3]. J. Korn, G. C. Goodman, and D. L. Kleinman, "New Concepts for a Submarine Depthchanging System", Proc. American Control Conference, Seattle, WA, June 1986.

[4]. A. Rodriguez and M. Athans, "Multivariable Control for a Twin-Lift Helicopter System Using the LQG/LTR Design Methodology", Proc. American Control Conference, Seattle, WA, June 1986.

[5]. G. Stein, "A Critique of the LQG/LTR Design Method", Proc. American Control Conference, Seattle, WA, June 1986.

[6]. G. Stein and M. Athans, "The LQG/LTR Procedure for Multivariable Feedback Control Design", Report LIDS-P-1384, MIT Lab. for Information and Decision Systems, Cambridge, Mass., May 1984 (to appear in the IEEE Trans. on Automatic Control)

[7]. J. C. Doyle and G. Stein, "Multivariable Feedback Design: Concepts for a Classical/Modern Synthesis", IEEE Trans. on Automatic Control, Vol. AC-26, February 1981, pp. 4-16.

[8]. N. A. Lehtomaki, N. R. Sandell, Jr., and M. Athans, "Robustness Results in Linear-Quadratic-Gaussian Based Multivariable Control Designs", IEEE Trans. on Automatic Control, Vol. AC-26, February 1981, pp. 75-92.

[9]. N. A. Lehtomaki, "Practical Robustness Measures in Multivariable Control System Analysis", Ph.D. Thesis, LIDS-TH-1093, MIT, Dept. of Electrical Engineering and Computer Science, Cambridge, Mass., May 1981.

[10]. H. Kwakernaak and R. Sivan, Linear Optimal Control Systems, Wiley-Interscience, NY, 1972.

[11]. M. Athans, P. Kapassouris, E. Kappos, and H. A. Spang III, "Linear-Quadratic-Gaussian with Loop-Transfer-Recovery Methodology for the F-100 Engine", AIAA Journal of Guidance, Control, and Dynamics, Vol. 9, January 1986, pp. 45-52.

[12]. G. Stein, " LQG-Based Multivariable Design Frequency Domain Interpretation", AGARD Lecture Notes 117, NATO, 1981.

[13]. J. S. Freudenberg and D. P. Looze, "Right Half Plane Poles and Zeros and Design Tradeoffs in Feedback Systems", IEEE Trans. on Automatic Control, Vol. AC-30, June 1985, pp. 555-561.