The Economics of Personal Bankruptcy

by

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2
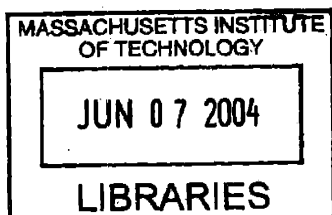
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Abstract

This thesis examines the effect of bankruptcy law on consumer borrowing and welfare. The thesis consists of four theoretical chapters and two empirical chapters.

Chapter 1 presents a simple model of consumer borrowing where the repayment of debt is governed by a bankruptcy law which allows a consumer to protect income below a given exemption level from creditors. Increasing bankruptcy exemption levels are found to increase borrowing and to increase consumer welfare so long as the consumer is borrowing less than the maximum amount possible. If consumers are borrowing the maximum amount possible, increasing exemption amounts increases credit constraints and decreases borrowing. Consumer welfare is maximized at the point where the marginal benefit the amount of insurance provided by the bankruptcy regime equals the marginal cost to reducing borrowing.

Chapter 2 expands the model described in chapter 1 to include consumers who differ as to either their demand for credit or their ability to repay loans. The optimal exemption level is found to occur where the marginal cost due to increasing credit constraints to consumers with a higher demand for credit or a lower ability to repay is balanced against the increased insurance benefit provided to other borrowers.

Chapter 3 considers the effect of bankruptcy law on credit markets with asymmetric information. I find that the possibility to receive a discharge of debt provided by bankruptcy law may cause consumers to distort their borrowing choices. Optimal exemption levels balance costs due to distortions in borrowing with benefits associated with increases in insurance.

Chapter 4 presents a model of the effect of bankruptcy law on incentives to work. I find that increasing exemption levels may either increase or decrease incentives to work or to take risk.

Chapter 5 examines the effect of exemption levels on household borrowing. I find that increasing personal property exemption levels are associated with higher levels of home mortgage debt and decreased probabilities that non-homeowners have greater than $50,000 in debt. Homestead exemptions are negatively associated with homeownership.

Chapter 6 finds that personal property exemption levels are positively related to bankruptcy filing rates.

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3

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Table of Contents

Chapter 1: A One-Type Model of Bankruptcy

It is often said that the purpose of the personal bankruptcy code is to provide debtors who encounter financial difficulty with a "fresh start." As Justice Sutherland famously wrote in a 1934 Supreme Court Opinion, the purpose of bankruptcy laws is to give "to the honest but unfortunate debtor ... a new opportunity in life and a clear field for future effort, unhampered by the pressure and discouragement of preexisting debt."

This paper develops a theoretical model of consumer borrowing in the presence of bankruptcy to analyze the potential effects of the bankruptcy fresh start on consumer borrowing decisions and on consumer welfare. It argues that the potential to receive a discharge of unpaid debts while protecting a minimum level of assets from seizure by creditors provides consumers with valuable insurance, but may create credit constraints which limit the consumers ability to borrow. The optimal asset exemption level balances the marginal insurance benefit to increasing asset exemption levels against the marginal loss in utility due to credit constraints.

Although economists have long recognized that the potential to receive a discharge of unpaid debts in bankruptcy might provide borrowers with a kind of insurance in the event that a borrower receives an adverse income realization which makes repayment of the debt more burdensome than expected, this paper is the first to consider a tractable model of consumer borrowing with bankruptcy which can be used to describe the effect of exemption levels on borrowing and consumer utility.

Rea (1984) argues that in order to provide debt relief only to those debtors who have had bad income realizations, bankruptcy law must be structured to encourage only needy debtors to seek bankruptcy protection.

Dye (1986) presents a simple model of the bankruptcy which suggests that, while increasing exemptions may have short term benefits to potential bankrupts, in the long run, more generous bankruptcy laws may harm borrowers.

Alder, Polak and Schwartz (2000) use a simple model of bankruptcy to suggest that risk-averse consumers benefit from the insurance-like benefits of the bankruptcy discharge, while potential incentive effects make the potential discharge of debts provided by the bankruptcy system inefficient if consumers are risk-neutral. However, their model does not allow them to

evaluate potential tradeoffs between the two effects that they identify, nor does it allow them to describe an optimal bankruptcy regime.

Other papers have focused on the structure of the bankruptcy code and the ability of consumers to behave strategically. White (1998) has argued that consumers who behave strategically can shelter most of their assets from creditors and inefficiently raise interest rates for strategic and non-strategic consumers alike. Wang and White (2000) suggest that given a consumer's ability to hide their assets, the efficiency of the system can be improved by requiring consumers to contribute some future income, which is difficult to disguise, towards payment of their debts.

The remainder of this paper proceeds as follows: Section 1.1 describes a simple two period model of borrowing where repayment is subject to a bankruptcy regime that allows consumers to receive a discharge of their unpaid debts by declaring bankruptcy and turning over to creditors any second period income in excess of a fixed income exemption. In Section 1.2, a consumer's optimal choices of borrowing and debt are determined. Because consumer preferences are not convex, consumers may borrow nothing at all, at the point where the marginal utility of consumption in the first period is equal to the marginal utility of consumption in the second period, conditional on the consumer avoiding bankruptcy or at the point where the consumer borrows as much as possible given the requirement that lenders breakeven. Section 1.3 considers the affect of the bankruptcy exemption level on borrowing. Higher exemption levels increase borrowing for consumers borrowing less than the maximum amount possible, but decreases borrowing among consumers who are borrowing the maximum amount. The optimal bankruptcy exemption level is considered in Section 1.4. It is argued that the optimal bankruptcy exemption level occurs where consumers borrow as much as possible and the exempt amount is equal to first period consumption. Section 1.5 compares borrowing with bankruptcy to models of borrowing where default is not possible. Finally, in Section 1.6, I conclude.

1.1 The Basic Model Described

The model consists of two periods. In the first period, a risk averse consumer receives an income y_1 and can borrow an amount B from a risk-neutral, price-taking lender in exchange for a

promise to repay a debt D in the second period subject to the bankruptcy system described below.

A consumer's second period income is a random variable, y_2, drawn from a continuous distribution with density function $f(y_2)$ and cumulative distribution function $F(y_2)$, where $y_2 \in$ [0,M] and $f(M)=0$.

The bankruptcy system exempts an amount of income A from attachment by creditors to satisfy the debt. Consequently, a consumer with an income less than A, consumes his entire income. A consumer with income greater than A must contribute any excess over A to pay his debts until they are paid in full. The consumers second period consumption is therefore:

$$(1.1) \ c_2 = \begin{cases} y_2, & y_2 < A \\ A, & A < y_2 < A + D \\ y_2 - D, & A + D < y_2 \end{cases}.$$

It should be noted that, in reality, bankruptcy law provides for different exemption levels for different types of assets. By modeling exemption levels as applying to income, I am not considering the possibility that consumers respond to exemption levels by altering their asset purchasing decisions.

Assuming the consumer's discount rate between periods is zero, the consumer's utility is:

$$(1.2) \ U(B,D) = u(y_1 + B) + \int_0^A u(y_2)f(y_2)dy_2 + u(A)[F(A+D) - F(A)] + \int_{A+D}^M u(y-D)f(y)dy.$$

I assume u'>0, u''<0 and u'''≥0. That is, marginal utility is positive, decreasing and quasi-convex. The assumption that marginal utility is quasi-convex implies that expected marginal utility is weakly decreasing with a mean-preserving spread. As a result, the amount of borrowing for a consumer at an interior solution will be weakly decreasing with a mean-preserving spread..

The lender receives nothing if the consumer's income is less than A, and y_2-A if income is between A and A+D. If income is greater than A+D, the lender receives D. Assuming the real interest rate is zero, the expected value of a promise to pay D subject to the bankruptcy system is therefore:

$$(1.3) \ EV(D,A) = \int_A^{A+D} [y_2 - A]f(y_2)dy_2 + D[1 - F(A+D)].$$

11

1.2 The Consumers' Choice of Borrowing and Debt

The consumer chooses B and D to maximize utility subject to the constraint that the lender must earn at least zero profits in expected value terms. The consumer's maximization problem is depicted below. The curve BE represents the lender's break even constraint. The curve $U^*(B,D)$ is the consumer's indifference curve at the optimum level of borrowing. B^* and D^* depict the optimal level of borrowing and debt respectively.

Figure 1.1:
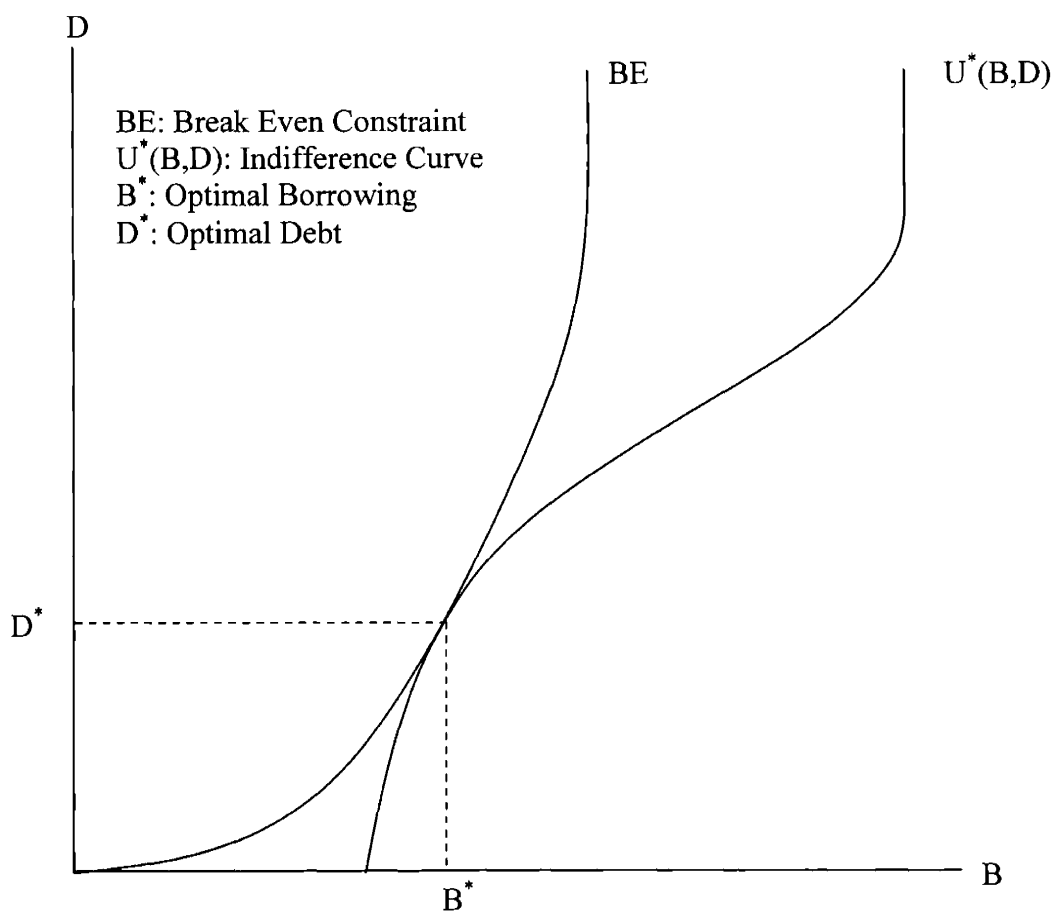
Proposition 1.1: The breakeven constraint, BE, i.e. the combinations of D and B which give the lender zero profits in expected value terms, is increasing and convex.

Proof and discussion:

The slope of the breakeven constraint is:

$$(1.4) \quad \frac{dD}{dB} = \begin{cases} \dfrac{1}{1 - F(A + D)}, & D \leq M - A \\ \infty, & D > M - A \end{cases}$$

Because F(A+D) is a cumulative distribution function, $\dfrac{1}{1 - F(A + D)}$ is always greater than or equal to zero, so the breakeven constraint is increasing.

The second derivative is:

$$(1.5) \quad \frac{d^2 D}{dB^2} = \frac{f(A + D)\dfrac{dD}{dB}}{[1 - F(A + D)]^2} \geq 0$$

The breakeven constraint is therefore convex.

Intuitively, the slope of the breakeven constraint is positive up to the point where D=M-A because up to that point, a promise to repay a larger amount increases the expected future value to be received by the lender. Because the promise to repay additional debt increases the likelihood that the consumer will default, the slope of the breakeven constraint is increasing over that range. For D>M-A, a promise to pay additional debt will not increase the amount received by the lenders because the borrower has already committed all of his potential earnings above A to repay the lender, and the slope will be infinite.

Proposition 1.2: The consumer's preferences are strictly increasing in B and strictly decreasing in D so long as A+D<M. If A+D≥M consumers are indifferent to greater levels of debt. The indifference curves are weakly upward sloping but are not uniformly concave. Preferences are therefore not convex.

Proof and discussion:

The derivatives of the consumer's utility function with respect to B and D are respectively:

$$(1.6) \quad \frac{dU(B,D)}{dB} = u'(y_1 + B) \text{ , and}$$

(1.7)

$$\frac{dU(B,D)}{dD} = \begin{cases} -\displaystyle\int_{A+D}^{M} u'(y_2 - D)f(y_2)dy_2 = -E[u'(y_2 - D) \mid y_2 \geq A+D][1 - F(A+D)] & D \leq M - A \\ 0, & D > M - A \end{cases}$$

,

where E[u'(y₂-D)|y₂≥A+D] is the expected marginal utility of second period consumption conditional on the consumer not going bankrupt.

Because u'>0, equation (1.6) is always positive and preferences are strictly increasing in B. Equation (1.7) is always negative for D<M-A for the same reason and equal to zero for D≥M-A, so the consumer's preferences are weakly decreasing in D.

The slope of each indifference curve is given by:

$$(1.8) \quad \frac{dD}{dB} = \begin{cases} \dfrac{u'(y_1 + B)}{\displaystyle\int_{A+D}^{M} u'(y_2 - D)f(y_2)dy_2} = \dfrac{u'(y_1 + B)}{E[u'(y_2 - D) \mid y \geq A+D][1 - F(A+D)]}, & D \leq M - A \\ \infty, & D > M - A \end{cases}$$

.

Because u'>0, equation (1.8) will be positive for all values of D<M-A and equal to zero for D≥M-A.

The second derivative is:

$$(1.9) \quad \frac{d^2D}{dB^2} = \begin{cases} \dfrac{1}{[\displaystyle\int_{A+D}^{M} u'(y_2 - D)f(y_2)dy_2]^2} \{u''(y_1 + B)\displaystyle\int_{A+D}^{M} u'(y_2 - D)f(y_2)dy_2 \\ \quad + [u'(A)f(A+D) + \displaystyle\int_{A+D}^{M} u''(y_2 - D)f(y_2)dy_2]\dfrac{dD}{dB}u'(y_1 + B)\}, & D \leq M - A \\ Undefined, & D > M - A \end{cases}$$

.

Equation (1.9) will not, in general, be uniformly positive or negative. The term

$$\frac{1}{[\displaystyle\int_{A+D}^{M} u'(y_2 - D)f(y_2)dy_2]^2}$$ is always positive, meaning the sign of the second derivative will be

the same as the sign of.

$$u''(y_1 + B)\int_{A+D}^{M} u'(y_2 - D)f(y_2)dy_2 + [u'(A)f(A+D) + \int_{A+D}^{M} u''(y_2 - D)f(y_2)dy_2]\frac{dD}{dB}u'(y_1 + B).$$

14

The term $u''(y_1 + B) \int\limits_{A+D}^{M} u'(y_2 - D) f(y_2) dy_2$ will always be negative. The term

$[u'(A) f(A + D) + \int\limits_{A+D}^{M} u''(y_2 - D) f(y_2) dy_2] \dfrac{dD}{dB} u'(y_1 + B)$ may be positive or negative depending

on whether u'(A)f(A+D), which is positive, is larger in absolute value terms than

$\int\limits_{A+D}^{M} u''(y_2 - D) f(y_2) dy_2$ which is negative.

The expression $\int\limits_{A+D}^{M} u''(y_2 - D) f(y_2) dy_2$ represents the increase in the marginal disutility

of debt due to the fact that the marginal utility of consumption is decreasing. The term
u'(A)f(A+D) represents the decrease in the marginal disutility of debt due resulting from the
increase in the probability that the consumer will go bankrupt and not have to pay his debt in

full. If u'(A)f(A+D) is greater than $\int\limits_{A+D}^{M} u''(y_2 - D) f(y_2) dy_2$, then the marginal disutility of debt

is decreasing and the term $[u'(A) f(A + D) + \int\limits_{A+D}^{M} u''(y_2 - D) f(y_2) dy_2] \dfrac{dD}{dB} u'(y_1 + B)$ will be

positive.

If $[u'(A) f(A + D) + \int\limits_{A+D}^{M} u''(y_2 - D) f(y_2) dy_2] \dfrac{dD}{dB} u'(y_1 + B)$ is positive and greater in

absolute value than $[u'(A) f(A + D) + \int\limits_{A+D}^{M} u''(y_2 - D) f(y_2) dy_2] \dfrac{dD}{dB} u'(y_1 + B)$, then the second

derivative will be positive and preferences will not be convex.

While it is not obvious from equation (1.9) that the second derivative must become
positive, it is clear that preferences will not be convex. Because the slope of the indifference
curves is positive and finite for levels of D<M-A, and equal to infinite for values of D>M-A,
preferences cannot be convex. Instead, as depicted in the figure, the indifference curve must
steepen at levels of D approaching M-A.

Intuitively, this occurs because as default becomes more likely a promise to pay
additional debt becomes less costly at the margin since it is increasingly unlikely that the

borrower will have to pay back the full amount D. Once D>M-A, the consumer will never have to pay off the entire amount D and is therefore indifferent between all levels of D≥M-A.

Proposition 1.3: Consumers will necessarily choose to borrow money if either or both:

    a) the marginal utility of consumption evaluated at the first period income is less than the expected marginal utility of consumption in the second period conditional on second period income being greater than the exempt amount A; and/or

    b) utility from borrowing as much as possible is greater than utility from borrowing nothing.

Proof and discussion:

First, if $u'(y_1)[1 - F(A)] > \int_A^M u'(y_2) f(y_2) dy_2$, then slope of the indifference curve will be

steeper than the slope of budget constraint at B=0, D=0. That is, the marginal benefit of borrowing will be greater than the marginal cost of incurring more debts and consumers will prefer borrowing at the margin. Rearranging, I find:

$$(1.10) \quad u'(y_1) > \frac{\int_A^M u'(y_2) f(y_2) dy_2}{[1 - F(A)]}.$$

The left hand side of equation (1.10) is, of course, the expected marginal utility of consumption in the second period conditional on second period income greater than A.

Second, even if the consumer does not prefer borrowing at the margin, because preferences are not convex it is possible that the consumer will prefer borrowing as much as possible to borrowing nothing at all. Specifically, the consumer will prefer to borrow B_{max}, the maximum amount that he can borrow, to borrowing nothing if:

$$(1.11) \quad u(y + B_{max}) + \int_0^A u(y_2) f(y_2) dy_2 + u(A)[1 - F(A)] \geq u(y_1) + Eu(y_2).$$

It should be noted that even if neither of these conditions hold it is still possible that the consumer will choose to borrow at an interior point which is preferred to the zero borrowing point, because the indifference curve is not uniformly concave or convex.

16

Proposition 1.4: If the consumer chooses to borrow, he will select a level of borrowing either:

  a) *where the indifference curve is tangent to the lender's breakeven constraint at a level of borrowing less than the maximum possible level of borrowing; or*

  b) *where the consumer has borrowed as much as possible given the lender's breakeven constraint.*

Proof and discussion:

The interior solution is depicted in the above figure by D* and B*. In that case, the indifference curve never crosses the breakeven constraint so the interior tangency point is preferred to any other feasible combination of borrowing and debt.

While the diagram depicts a case in which there is only one interior tangency point between an indifference curve and the break-even constraint, it is important to note that there may be more than one such point.

At any point where the indifference curves and the breakeven constraint are not tangent, the marginal rate of substitution between borrowing and debt would not be the same as the tradeoff required by the lender's breakeven constraint. In such a circumstance, the borrower would prefer to remain on the breakeven constraint and borrow either more (if the indifference curve is steeper than the breakeven constraint) or less (if the indifference curve is flatter than the breakeven constraint). Consequently, those points cannot be an equilibrium and the only possible equilibrium occurs at an interior point of tangency.

It is also possible that the equilibrium might occur with the consumer borrowing as much as possible. At any point where A+D≥M, i.e. where the consumer borrows the maximum amount, both the indifference curve and the breakeven constraint will have an infinite slope and will therefore be tangent. If the indifference curve at that point never crosses the breakeven constraint, then the consumer will prefer to borrow the maximum.

The consumer's maximization problem can be written symbolically as:

$$(1.12) \quad \max_{B,D} u(y_1 + B) + \int_0^A u(y_2)f(y_2)dy_2 + u(A)[(F(A+D) - F(A)] + \int_{A+D}^M u(y_2 - D)f(y_2)dy_2$$

subject to:

$$(1.13) \quad \int_A^{A+D}[y_2 - A]f(y_2)dy_2 + D[F(A+D) - F(A)] - B \geq 0.$$

The Lagrangian for the problem is:

$$L = u(y_1 + B) + \int_0^A u(y_2)f(y_2)dy_2 + u(A)[(F(A+D) - F(A)] + \int_{A+D}^M u(y_2 - D)f(y_2)dy_2$$

(1.14)

$$+ \lambda\{ \int_A^{A+D} [y_2 - A]f(y_2) + D[1 - F(A+D)] - B\}$$

Assuming the consumer chooses to borrow a positive amount, the first order conditions with respect to B and D are given in equations (1.15) and (1.16) respectively:

(1.15) $u'(y_1 + B) - \lambda = 0$.

(1.16) $-\int_{A+D}^M u'(y_2 - D)f(y_2)dy_2 + \lambda[1 - F(A+D)] = 0$.

If the optimum occurs at a level of debt less than M-A, i.e. if the consumer chooses to borrow less than the maximum, then the optimum occurs at an interior tangency point and both first order conditions must hold. Combining the two I find:

(1.17) $u'(y_1 + B) = \dfrac{\int_{A+D}^M u'(y_2 - D)f(y_2)dy_2}{1 - F(A+D)}$, or

(1.18) $u'(y_1 + B) = E[u'(y_2 - D) | y_2 \geq A + D)$,

where B is the amount borrowed at the interior tangency point. The consumer maximizes utility by balancing the marginal utility of consumption in the first period with the expected marginal utility of consumption in the second period conditional on the consumer paying his debts in full.

The second possibility is that the consumer will chose to borrow as much as possible. In that case, the derivative of the Lagrangian with respect to D, yields only 0=0. Because the marginal utility of borrowing is always positive, the consumer prefers to borrow as much as he can and the equilibrium level of B is determined by the lender's breakeven constraint, or:

(1.19) $B_{max}(A) = \int_A^M [y_2 - A]f(y_2)dy_2$.

It should be noted here that the maximum amount that can be borrowed, B_{max} is a function of the exempt amount A as well as the probability distribution of second period income. For notational simplicity, however, the A will usually be suppressed and the maximum amount borrowed will be written simply as B_{max}

18

Whether the consumer chooses an interior solution or to borrow as much as possible depends, of course, on which possible solution gives him higher utility.

1.3 The Effect of the Bankruptcy Exemption on Consumer Borrowing

Proposition 1.5: As the amount of income exempted from attachment by creditors, A, increases, the consumer's utility associated with borrowing at an interior tangency point increases. The level of borrowing and debt chosen by a consumer borrowing at an interior point of tangency will also increase.

Proof and discussion:

From the envelope theorem, the change in the consumer's utility at the interior tangency point is:

$$(1.20) \quad \frac{dU}{dA} = [F(A+D) - F(A)][u'(A) - \lambda],$$

where $\lambda = u'(y_1+B) = E[u'(y_2-D)|y_2>A+D]$. Because $E[u'(y_2-D)|y_2>A+D]>u'(A)$, equation (1.20) is positive and an increase in A increases the consumer's utility from borrowing at the tangency point.

Intuitively, increasing A shifts the consumer's obligation to repay the debt to states of nature where marginal utility is smaller. In so doing, increasing A increases the consumer's consumption in states of nature where marginal utility is high and decreases consumption in state of nature where marginal utility is lower. Put another way, increasing A increases the amount of insurance given to the consumer and therefore increases welfare.

Moreover, if there exist two or more interior points of tangency between the consumer's indifference curves and the break even constraint, equation (1.20) demonstrates that increasing A will increase utility at points associated with high levels of borrowing (and associated debt) at a higher rate than those with comparatively lower levels of borrowing. This implies that the consumer's choice may jump discontinuously from a tangency point with a low level of borrowing to a point with a higher level of borrowing in response to an increase in A, but will not jump discontinuously to a point of lower borrowing in response to an increase in A.

Each interior tangency solution is defined by two conditions: First, the slope of the indifference curve and the budget constraint must be equal:

$$(1.21) \quad u'(y_1 + B)[1 - F(A + D)] - \int_{A+D}^{M} u'(y_2 - D)f(y_2)dy_2 = 0.$$

Second, the combination of B and D must satisfy the budget constraint:

$$(1.22) \quad \int_{A}^{A+D} [y_2 - A]f(y_2)dy_2 + D[F(A + D) - F(A)] - B = 0.$$

The implicit function theorem can be used to determine how the tangency point changes with A. Specifically:

$$(1.23) \quad \begin{bmatrix} \partial B / \partial A \\ \partial D / \partial A \end{bmatrix} = -W^{-1}X, \text{ where}$$

$$W = \begin{bmatrix} u''(y_1 + B)[1 - F(A + D)] & -1 \\ f(A + D)[u'(A) - u'(y_1 + B)] + \int_{A+D}^{M} u''(y_2 - D)f(y_2)dy_2 & [1 - F(A + D)] \end{bmatrix}$$

and

$$X = \begin{bmatrix} f(A + D)[u'(A) - u'(y_1 + B)] \\ [F(A + D) - F(A)] \end{bmatrix}.$$

Solving for ∂B/∂A, I find:

(1.24)

$$\frac{\partial B}{\partial A} = -\frac{[(1 - F(A + D)]f(A + D)[u'(A) - u'(y_1 + B)] + [F(A + D) - F(A)]}{u''(y_1 + B)[1 - F(A + D)]^2 + f(A + D)[u'(A) - u'(y_1 + B)] + \int_{A+D}^{M} u''(y_2 - D)f(y_2)dy_2}.$$

Each of the probability terms in the numerator, [1-F(A+D)], f(A+D) and [F(A+D)-F(A)] are positive. Moreover, at the optimum, u'(A) > u'(y₁+B) so [u'(A)-u'(y₁+B)] is also positive. Therefore, the numerator as positive.

If the tangency point occurs where the consumer's utility function is concave, the second derivative of the indifference curve must be positive. Combining equation (1.9) and equation (1.5) tells us that if the second derivative of the indifference curve is greater than the second derivative of the breakeven constraint, as must be the case at an interior maximum, then

$$\frac{u''(y_1 + B) \int\limits_{A+D}^{M} u'(y_2 - D)f(y_2)dy_2 + [u'(A)f(A+D) + \int\limits_{A+D}^{M} u''(y_2 - D)f(y_2)dy_2]u'(y_1 + B)\frac{dD}{dB}}{[\int\limits_{A+D}^{M} u'(y_2 - D)f(y_2)dy_2]^2} \le \frac{f(A+D)\frac{dD}{dB}}{[1 - F(A+D)]^2}.$$

Substituting $\dfrac{dD}{dB} = \dfrac{1}{1 - F(A+D)}$ from equation (1.4) and $1 - F(A+D) = \dfrac{\int\limits_{A+D}^{M} u'(y_2 - D)f(y_2)dy_2}{u'(y_1 + B)}$

from equation (1.17) and multiplying each side by $-[\int\limits_{A+D}^{M} u'(y_2 - D)f(y_2)dy_2]^2$ and rearranging, I

find that

(1.25) $u'(A)f(A+D) + \int\limits_{A+D}^{M} u''(y_2 - D)f(y_2)dy_2 + u''(y_1 + B)[1 - F(A+D)]^2 - u'(y_1 + B)f(A+D) \le 0$.

Because the numerator is positive, the denominator is negative and the whole fraction is multiplied by -1, dB/dA is positive. In other words, so long as the optimum remains at an interior tangency point, increasing the exempt amount, A, increases the consumer's level of borrowing.

Proposition 1.6: Increasing the exempt amount A will decrease the maximum amount that a consumer can borrow and may increase or decrease the consumer's utility from borrowing the maximum amount possible.

Proof and discussion:

As discussed above, the amount borrowed by consumers borrowing all that they can, B_{max}, is determined by equation (1.19). Differentiating with respect to A, yields:

(1.26) $\dfrac{dB_{max}}{dA} = -[1 - F(A)]$.

Equation (1.26) is, of course, negative. Therefore, increasing A decreases the maximum amount that a consumer can borrow.

The effect of changing A on the consumer's utility from borrowing the maximum is:

(1.27) $\dfrac{dU_{max}}{dA} = [1 - F(A)]u'(A) - \dfrac{dB_{max}}{dA}u'(y_1 + B_{max)} = [1 - F(A)][u'(A) - u'(y_1 + B_{max})]$.

21

Equation (1.27) can be either positive or negative depending on whether A is less than or greater than B_{max}. If $A > B_{max}$, then $u'(A) < u'(y_1+B_{max})$ and equation (1.27) is negative, i.e. increasing A decreases the consumer's utility from borrowing the maximum amount. Similarly, If $A < y_1 + B_{max}$, then $u'(A) > u'(y_1+B_{max})$, equation (1.27) is positive and increasing A increases the consumer's utility from borrowing the maximum amount.

Because a consumer borrowing as much as possible will always go bankrupt, The consumer will either consume his entire income (if he earns less than the exempt amount) or will consume an amount A (if he earns more than the exempt amount. Increasing the exempt amount therefore benefits the consumer by decreasing the amount that the consumer has to pay on his debts in the event that he earns more than the exempt amount A. At the same time, increasing the exempt amount harms the consumer by decreasing the amount that he can borrow. As described above, if $A < y_1 + B_{max}$ then the marginal utility to decreasing the amount that must be paid in the second period is greater than the marginal disutility in the first period resulting from the reduction in the amount borrowed, and an increase in A would increase the consumer's utility. Conversely, if $A > y_1 + B_{max}$, the marginal disutility of decreasing the amount that can be borrowed outweighs the marginal utility of decreasing the amount that must be paid back in the second period, and the consumer's utility will decrease if A is raised.

Proposition 1.7: Assuming that at A=0, the consumer chooses to borrow at an interior point, there exists an A=A_I, at which the consumer is indifferent between borrowing at the interior point and borrowing the maximum amount possible. For values of A<A_I, the consumer will prefer to borrow at a tangency point. For values of A>A_I, the consumer will prefer to borrow the maximum amount possible.

Proof and discussion:

Assuming that at A=0, the consumer prefers to borrow at an interior point, then the consumer's utility from the interior solution must be greater than the consumer's utility from borrowing as much as possible. Put another way, the difference between the consumer's utility at the interior solution and the consumer's utility from borrowing the maximum amount possible is positive.

Combining equations (1.20) and (1.27), I find that the change in the difference in utility associated with the interior solution and the maximum borrowing point is:

(1.28)

$$\frac{d}{dA}(U_T - U_{max}) = [F(A+D) - F(A)][u'(A) - u'(y_1 + B_T)] - [1 - F(A)][u'(A) - u'(y_1 + B_{max})]$$

Because $[1-F(A)]$ is greater than $[F(A+D)-F(A)]$, and $[u'(A)-u'(y_1+B_{max})]$ is greater than $[u'(A)-u'(y_1+B_T)]$, the difference is decreasing.

Because the difference in the consumer's utility at the tangency point and at the maximum borrowing point is strictly decreasing, the consumer will be indifferent between the tangency point and borrowing as much as possible for at most one level of $A=A_I$. For all levels of $A<A_I$, the consumer will prefer to borrow at the interior tangency, while for all $A>A_I$, the consumer will prefer to borrow the maximum amount.

Moreover, for an amount A slightly less than M, the consumer will prefer to borrow at B_{max}. Consequently, there will be one and only one level of $A=A_I$ where the consumer is indifferent between borrowing as much as possible and borrowing at the interior tangency point.

1.4 The Optimal Bankruptcy Exemption Level

Proposition 1.8: The optimal level of income exempted from bankruptcy occurs at a level of exempt income where consumers borrow as much as possible and where the level of exempt income equals consumption in the first period, i.e. where $A = y_1 + B_{max}$.

Proof and discussion:

Call the consumer's indirect utility function $V(A)$. The social planner's maximization problem is then:

(1.29) $\max_A V(A)$.

Proposition 5 tells us that for levels of A at which the consumer chooses to borrow less than the maximum amount possible, increasing the amount of exempt income, A, increases the consumer's utility. Because increasing A over this range always leads to greater utility, the optimal level of A must occur at a level where consumers choose to borrow as much as possible.

As discussed previously, given that the consumer chooses to borrow as much as possible, the derivative of the indirect utility function is:

$$(1.30) \quad \frac{dV(A)}{dA} = [1 - F(A)][u'(A) - u'(y_1 + B_{max})].$$

Setting equation (1.35) equal to zero, I find that if borrowing is to occur the maximum must occur at:

(1.31) A=y_1+B$_{max}$(A).

Put another way, the optimal level of A occurs a point such that a consumer who pledges all of his income above the exempt amount to creditors can borrow enough so that first period consumption is equal to the exempt amount A.

This result is easily understood. Increasing A does not change the expected value of the amount promised to creditors. Increasing A shifts the obligation to pay in the second period from states of nature where the marginal value of consumption is relatively high to states of nature where the marginal value of consumption is lower. As a result, even though the level of debt that has to be incurred must increase in order to keep the budget constraint satisfied, raising the level of exempt income, A, increases second period utility. In this way, a higher level of exempt income can be thought of as providing the consumer with greater insurance, which, because the consumer is risk-averse, improves second period utility.

Increasing the level of exempt income, however, decreases the maximum amount that a consumer can borrow. Raising A may, therefore, decrease first period utility if the consumer cannot borrow as much as he otherwise would have.

If the consumer is borrowing at an interior tangency point, increasing A will not prevent him from borrowing the same amount that he would have previously. Consequently, there is no cost to raising the level of exempt income and raising A will increase the consumer's utility.

If the consumer is borrowing the maximum amount possible, then increasing the level of exempt income will decrease the amount borrowed and will decrease first period utility. At the optimum, this marginal cost in terms of decreased first period utility must be balanced against the marginal benefit of providing greater insurance.

The optimum occurs where first period consumption equals the level of exempt income in the second period. At this point, the marginal utility of increasing A, u'(A), equals the marginal disutility of decreasing consumption in the first period, u'(y_1+B).

The effect of the bankruptcy exemption level on the level of borrowing and on consumer utility is illustrated in Figure 1.2.

Figure 1.2
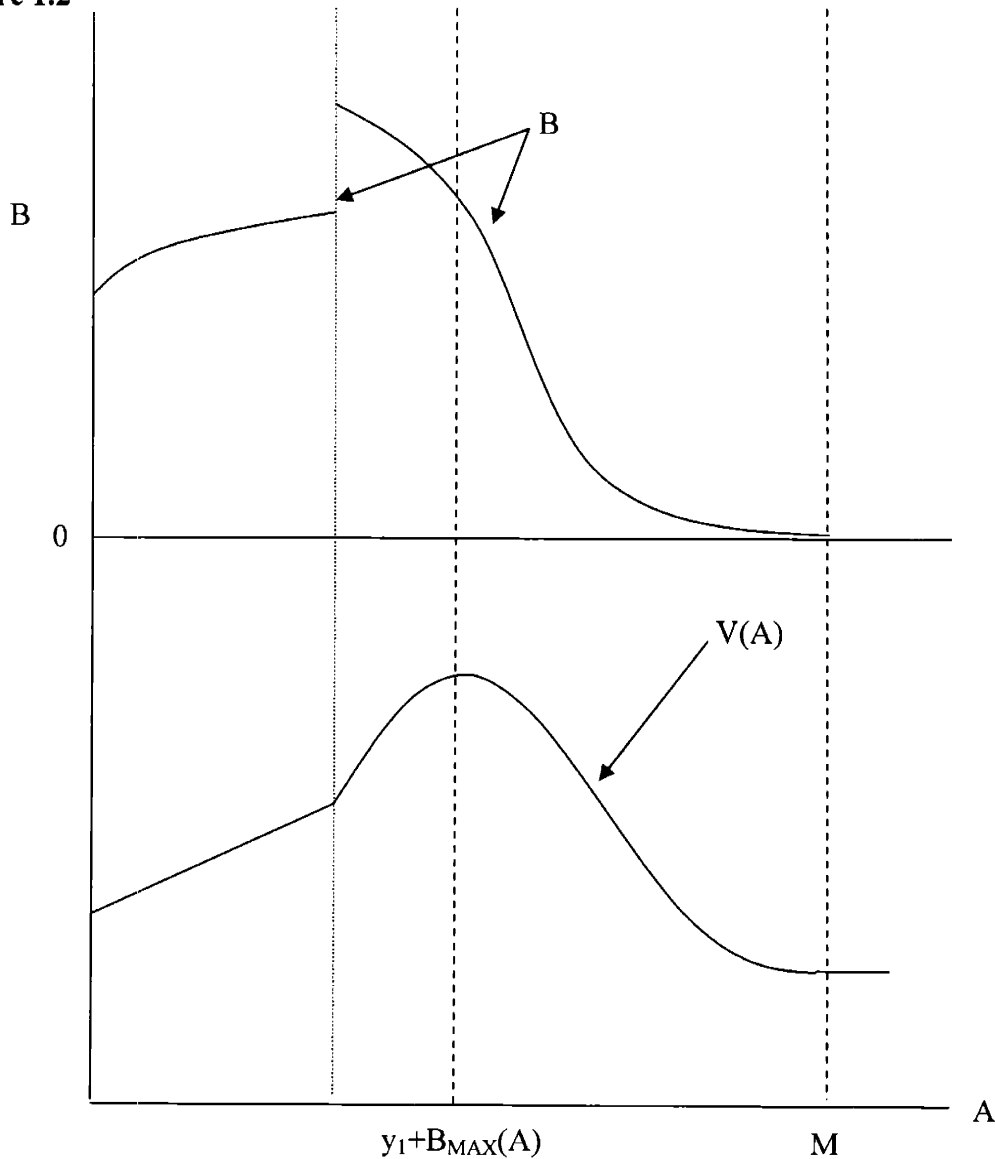
As described in propositions 1.5, 1.6 and 1.7 borrowing increases with the bankruptcy exemption level until such time as the consumer switches between borrowing at a point of internal tangency between his indifference curve and the break even constraint and borrowing as much as possible. Since the bankruptcy exemption level lowers the maximum amount that can be borrowed, the amount borrowed declines as the exemption level increases beyond the point where the consumer chooses to borrow as much as he can.

There is a kink in the V(A) curve at the point where the consumer switches between borrowing at the tangency point and the maximum amount possible because when the amount of debt jumps, the marginal value of increasing the exemption level increases discontinuously.

Finally, the maximum of the consumer's indirect utility function occurs at the point where first period consumption is equal to the exempt amount.

1.5 A Comparison of Results with Models Without Bankruptcy

Proposition 1.9: Consumers borrowing subject to a bankruptcy regime which exempts an amount of income A from attachment by creditors, who borrow at an interior point, borrow weakly more than is predicted by models in which bankruptcy is not possible and consumers never default.

Proof and discussion:

The simplest models of consumer borrowing with uncertain future income generally assume that consumers cannot default, either because $y_2 > B$ or because negative consumption is permitted to allow the consumer to pay back creditors in the event that $y_2 < B$. In either case, these models predict that that consumers will borrow to the point where:

(1.32) $u'(y_1 + B) = E[u'(y_2 - B)]$.

In other words, the consumer in these models borrows until marginal utility of consumption in the first period is equal to the expected marginal utility of consumption in the second period.

The model with bankruptcy differs in that it predicts that a consumer borrows until the marginal utility of first period consumption is equal to the expected marginal utility of second period consumption *conditional on the consumer not declaring bankruptcy.*

It should be noted that the two predictions are the same so long as a consumer defaults with zero probability.

If the consumer may default with a positive probability, the model presented in this paper suggests that the consumer will borrow more than is predicted by models in which the consumer cannot default. The reason for this difference is easily understood. Whereas in a model without the possibility of default, an increase in the amount borrowed decreases consumption in the second period regardless of the realization of y_2, in the model with bankruptcy, a marginal

increase in borrowing and the corresponding increase in debt decreases second period consumption only in states of nature where the consumer does not declare bankruptcy, i.e. where $y_2 > A+D$. In essence, the bankruptcy system shifts the obligation to repay the amount borrowed to states of nature in which $y_2 > A+D$. In so doing, the bankruptcy system reduces the marginal cost to borrowing and leads consumers to borrow more.

To understand why this is so mathematically, consider the following: The fact that $u'' < 0$ and $u''' \geq 0$, i.e. consumers marginal utility is decreasing and quasi-convex, combined with the fact that the bankruptcy system shifts the obligation to pay debts to states of nature where income is higher while keeping the expected value of the obligation the same implies that the expected marginal utility of consumption is weakly smaller when bankruptcy is possible than when it is not, i.e.

$$(1.33) \quad E[u'(y_2 - B)] \geq \int_0^A u'(y_2)f(y_2)dy_2 + u'(A)[F(A+D) - F(A)] + \int_{A+D}^M u'(y_2 - D)f(y_2)dy_2 .$$

Moreover, because the marginal utility of consumption is decreasing,

$$(1.34) \quad \int_0^A u'(y_2)f(y_2)dy_2 + u'(A)[F(A+D) - F(A)] + \int_{A+D}^M u'(y_2 - D)f(y_2)dy_2 \geq$$
$$E[u'(y_2 - D) | y_2 \geq A + D]$$

that is, the average marginal utility of consumption given that income is above $A+D$ is greater than the average marginal utility of consumption across all possible income realizations. By transitivity, then:

(1.35) $E[u'(y_2-B)] > E[u'(y_2-D)|y_2 \geq A+D]$.

Because a consumer who chooses to borrow at an interior tangency point chooses B and D such that

(1.36) $u'(y_1+B) = E[u'(y_2-D)|y_2 \geq A+D]$,

the consumer will borrow more than he would in the absence of the bankruptcy system when he would equate $u'(y_1+B)$ with $E[u'(y_2-B)]$.

1.6 Conclusion

The model presented in this paper provides a tractable framework for considering the effect of bankruptcy law on consumer borrowing. I find that the bankruptcy regime causes consumer preferences to be non-convex. As a result, consumers may borrow nothing, at an interior tangency between their indifference curves and the lender's breakeven constraint or the maximum amount possible given the requirement that lenders breakeven.

The potential for consumers borrowers to receive a discharge of their debts without paying them in full provides risk averse consumers with valuable insurance which makes borrowing less risky. Bankruptcy exemptions shift the obligation to repay debt to states of nature where income is higher and therefore less valuable to the borrower. Borrowing is, therefore, less costly in utility terms and consumers who are not credit constrained will increase borrowing as exemption levels increase. If a consumer is credit constrained, higher exemption levels will exacerbate the credit constraint and borrowing will decrease.

The optimal bankruptcy exemption level for a particular consumer is found to occur at the point where consumers are borrowing as much as possible and the exempt amount is equal to first period consumption. Higher exemption levels would prevent consumers from optimally smoothing consumption across periods, while lower exemption levels would provide too little insurance.

Optimal exemption levels when consumers differ as to first period income or the distribution of second period income are considered in Chapter 2.

Chapter 2: Credit Markets with Two Types of Borrowers and Bankruptcy

The potential to receive a discharge of unpaid debts provided by bankruptcy law will affect borrowers differently depending on their demand for credit and their ability to repay their creditors. Since bankruptcy exemption levels apply to all consumers, an optimal bankruptcy exemption policy must balance the interests of borrowers for whom the optimal exemption level may differ.

To analyze the effects of bankruptcy law on different types of borrower, this chapter extends the model developed in Chapter 1 to allow for comparison between the borrowing choices and utility of consumers who differ by first period income or who differ by the distribution of second period income, assuming that lenders can distinguish between the different types.

Section 2.1 describes the differences in borrowing choices for consumers who differ only according to their first period incomes. Consumers with greater first period income have a lower marginal utility of first period consumption and therefore have a lower demand for borrowing. As a result, borrowing is weakly decreasing with first period income. The optimal exemption level is found to be greater than the optimal exemption level for consumers with low first period income and higher than the optimal exemption level for consumers with high first period income. The optimal exemption level occurs at the point where utility costs to increased credit constraints for consumers with low first period income are balanced against utility gains to consumers with high first period income.

Section 2.2 presents a model where the second period income distribution for type L consumers is first order stochastically dominated by the second period income distribution for type H consumers. The marginal disutility of debt is weakly lower for type H consumers, who, consequently, borrow weakly more than type L consumers. The optimal bankruptcy exemption level balances increased credit constraints for type L consumers against increased insurance benefits for type H consumers. The optimal exemption level is therefore above the optimal level for type L consumers and below the optimal level for type H consumers.

In Section 2.3, I consider a model where consumers' second period income distributions have the same mean, but differ as to the level of risk. Unlike Sections 2.1 and 2.2, it is not clear,

in general, which type of consumer will borrow more. Again, the optimal exemption level will occur where the marginal cost of increasing credit constraints on one type of consumer is equal to increase in the insurance benefit to the other type of consumer. However, it is impossible to determine which type of consumer will be credit constrained.

Section 2.4 concludes.

2.1 Consumers With Differing First Period Income

I begin by considering a model with two types of consumer, type L and type H who differ only by their first period income.

Specifically, I assume that type L and type H consumers receive a first period income y_{1L} and y_{1H} respectively, where $y_{1L} < y_{1H}$. Each consumer's second period income is assumed to be a random variable drawn from the same distribution $f(y_2)$.

Both type H and type L consumers can choose to borrow an amount B from a risk neutral lender in exchange for a promise to repay an amount D in the second period subject to a bankruptcy regime which exempts an amount of income A from attachment by creditors. Since the probability distribution of second period income is the same for both types, they face the same budget constraint.

The optimal levels of borrowing and debt for both type L and type H consumers are depicted in Figure 2.1

Figure 2.1
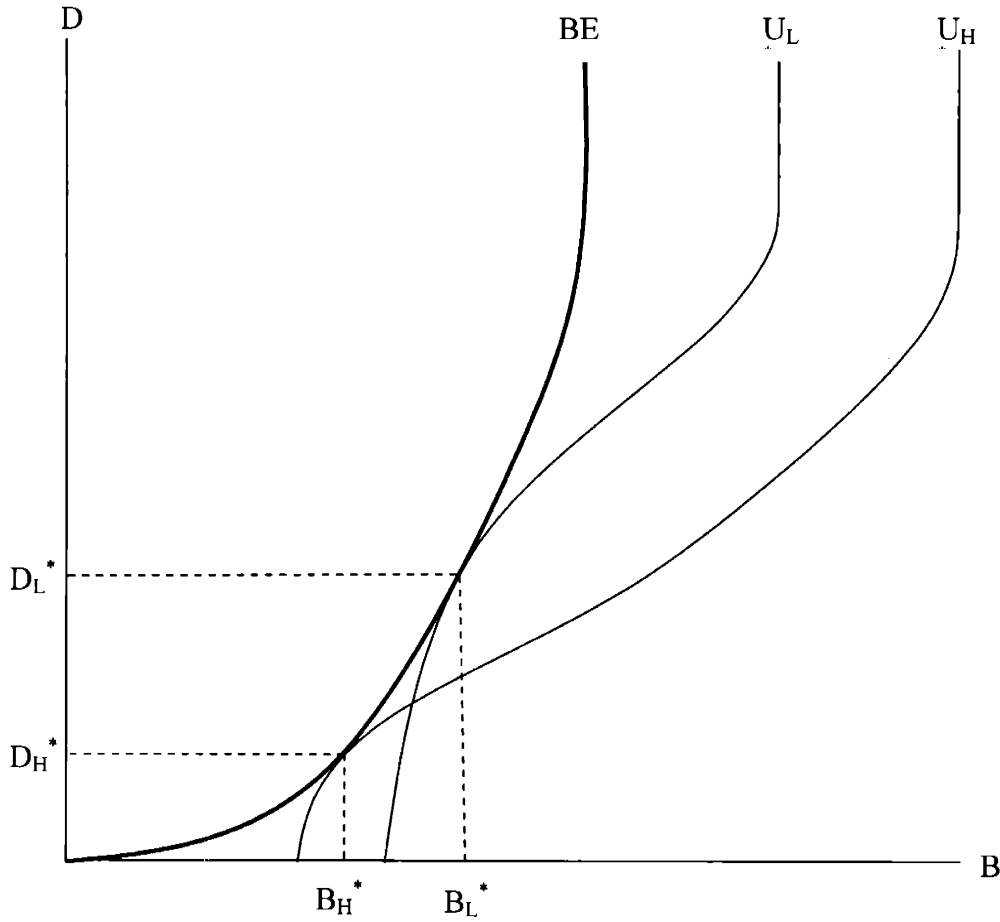
Proposition 2.1: At any combination of B and D where D<M-A, the indifference curves for type H consumers will be flatter than the indifference curves for type L consumers.

Proof and discussion:

From equation (1.8), we know that the slope of a consumer's indifference curve is given by:

$$(2.1) \quad \frac{dD}{dB} = \begin{cases} \dfrac{u'(y_{1_i} + B)}{\displaystyle\int_{A+D}^{M} u'(y_2 - D)f(y_2)dy_2}, & D \le M - A \\[4mm] \infty, & D > M - A \end{cases} \quad \text{, where i=L or H.}$$

Since $y_{1H} > y_{1L}$ and u''<0, the numerator of the expression for the slope, u'(y_{1i}+B), is smaller for type H consumers than for type L consumers. Because the denominator is the same for both types of consumer, the slope will be smaller for type H consumers than for type L consumers.

Intuitively, since a type L consumer has smaller first period income than a type H consumer, for any given level of borrowing, the first period marginal utility of consumption will be larger for type L consumers than for type H consumers. Because the marginal benefit to borrowing is greater for type L consumers, while the marginal cost to incurring debt is the same for both types of consumer, it will require a larger increase in debt to make a type L consumer indifferent than for a type H consumer.

Proposition 2.2: So long as both types borrow positive amounts, type L consumers will borrow more than type H consumers unless both types of consumer borrow the maximum amount possible, in which case they will borrow the same amount.

Proof and discussion:

Each type of consumer maximizes:

$$(2.2) \quad \max_{B,D} u(y_{1i} + B) + \int_0^A u(y_2)f(y_2)dy_2 + u(A)[(F(A+D) - F(A)] + \int_{A+D}^M u(y_2 - D)f(y_2)dy_2$$

subject to:

$$(2.3) \quad \int_A^{A+D}[y_2 - A]f(y_2)dy_2 + D[F(A+D) - F(A)] - B \geq 0.$$

Because the budget constraint defines D as a function of B, a type i consumer' maximization problem can be rewritten as:

(2.4)

$$\max_B u(y_{1i} + B) + \int_0^A u(y_2)f(y_2)dy_2 + u(A)[(F(A + D(B)) - F(A)] + \int_{A+D(B)}^M u(y_2 - D(B))f(y_2)dy_2$$

Taking the derivative with respect to B, I find:

$$(2.5) \quad \frac{dU}{dB} = \left\{ u'(y_{1i} + B) - \int_{A+D(B)}^M u'(y_2 - D(B))f(y_2)dy_2 \frac{dD}{dB}, \quad B \leq B_{max}. \right.$$

Substituting $\dfrac{1}{1-F(A+D)}$ for $\dfrac{dD}{dB}$, I find that if A+D<M, i.e. if both consumers borrow

less than the maximum amount possible:

(2.6)

$$\frac{dU_i}{dB} = u'(y_{1_i} + B) - \frac{\int\limits_{A+D(B)}^{M} u'(y_2 - D(B))f(y_2)dy_2}{1-F(A+D)} = u'(y_{1_i} + B) - E[u'(y_2 - D(B)) \,|\, y_2 \geq A + D_i].$$

The difference between $\dfrac{dU_H}{dB}$ and $\dfrac{dU_L}{dB}$ is therefore:

(2.7) $\dfrac{dU_H}{dB} - \dfrac{dU_L}{dB} = u'(y_{1H} + B) - u'(y_{1L} + B)$.

Because u'(y₁H+B)<u'(y₁L+B), this difference is negative. Since the marginal utility of

borrowing for type H consumers is less than the marginal utility of borrowing for type L

consumers, type L consumers will borrow more than type H consumers. Therefore, so long as

both consumers borrow less than the maximum amount possible, type L consumers will borrow

more than type H consumers.

Because marginal utility of borrowing is always greater for type L consumers than for

type H consumers, type H consumers will never borrow the maximum amount possible unless

type L consumers do so as well. It is, however, possible that the type L consumer will borrow the

maximum amount while the type H consumer borrows less than that. In this case, the type L

consumer will, of course, borrow more. Since both types face the same budget constraint, if each

type of consumer borrows the maximum amount possible, then they will borrow the same

amount.

Proposition 2.3: Where the bankruptcy exemption level is allowed to vary by type of consumer,

the optimal level of the bankruptcy exemption will be higher for type H consumers than for type

L consumers.

Proof and discussion:

Proposition 1.9 says that the optimal level of income exempted from bankruptcy for

consumers of a particular type occurs at a level of exempt income where consumers borrow as

much as possible in the first period and $A=y_1+B_{max}$. Since the budget constraint is the same for both type L and type H consumers, the maximum amount that a consumer can borrow is the same for both types of consumer. Because $y_{1L}<y_{1H}$, the optimal exemption level for type L consumers will be less than the optimal exemption level for type H consumers.

Proposition 2.4: If the bankruptcy exemption level is not allowed to differ by a consumer's type and if two types of consumer differ only as to first period income, the optimal level of the bankruptcy exemption will be higher than is optimal for type L consumers and lower than is optimal for type H consumers.

Proof and discussion:

The social planner's objective function is:

$$(2.8) \quad \max_A V(A) = w_L V_L(A) + w_H V_H(A),$$

where w_L and w_H are the weights placed on type H and type L consumer's utility respectively.

At the optimum, the derivative of the social planner's objective function with respect to A must equal zero, i.e.

$$(2.9) \quad \frac{dV(A)}{dA} = w_L \frac{dV_L(A)}{dA} + w_H \frac{dV_H(A)}{dA} = 0$$

or rearranging:

$$(2.10) \quad w_L \frac{dV_L(A)}{dA} = -w_H \frac{dV_H(A)}{dA}.$$

This implies that the optimal level of A will occur where $\dfrac{dV_L(A)}{dA}$ and $\dfrac{dV_H(A)}{dA}$ must be of opposite signs or must both equal zero.

As discussed following proposition 1.9, the derivative of the indirect utility function with respect to A is given by:

$$(2.11) \quad \frac{dV(A)}{dA} = [u'(A) - u'(y_1 + B)][F(A+D) - F(A)].$$

The derivative of the indirect utility function will, therefore, be positive if $A<y_1+B_{max}$ and negative if $A>y_1+B_{max}$.

36

If $A<y_{1L}+B_{max}$, then both $\dfrac{dV_L(A)}{dA}$ and $\dfrac{dV_H(A)}{dA}$ will be positive, and the level of exempt

income will be below the socially optimal level. If $A>y_{1H}+B_{max}$, then both $\dfrac{dV_L(A)}{dA}$ and

$\dfrac{dV_H(A)}{dA}$ will be negative, and A will be above the socially optimal level.

The optimal exempt amount must therefore occur where $y_{1L}+B_{max}<A<y_{1H}+B_{max}$, that is, between the optimal levels of exempt income for type L and type H consumers.

Intuitively, the optimal exemption level balances the loss in utility to the increasingly credit-constrained type L consumers against the increased insurance benefit to type H consumers. Because, at the social optimum, $A>y_{1L}+B_{max}$, increasing A further decreases the amount that the type L consumer can borrow below the optimal level for type L consumers. Moreover, because $A<y_{1H}+B_{max}$, increasing A provides greater insurance to type H consumers without decreasing the amount that can be borrowed below the optimal level for type H consumers. Consequently, the type H consumer's utility increases. The optimal exemption level is set by balancing the reduction in the type L consumers' utility against the increase in the type H consumers' utility.

The effect of the bankruptcy exemption level on borrowing for type H and type L consumers is illustrated in Figure 2.2.

Figure 2.2
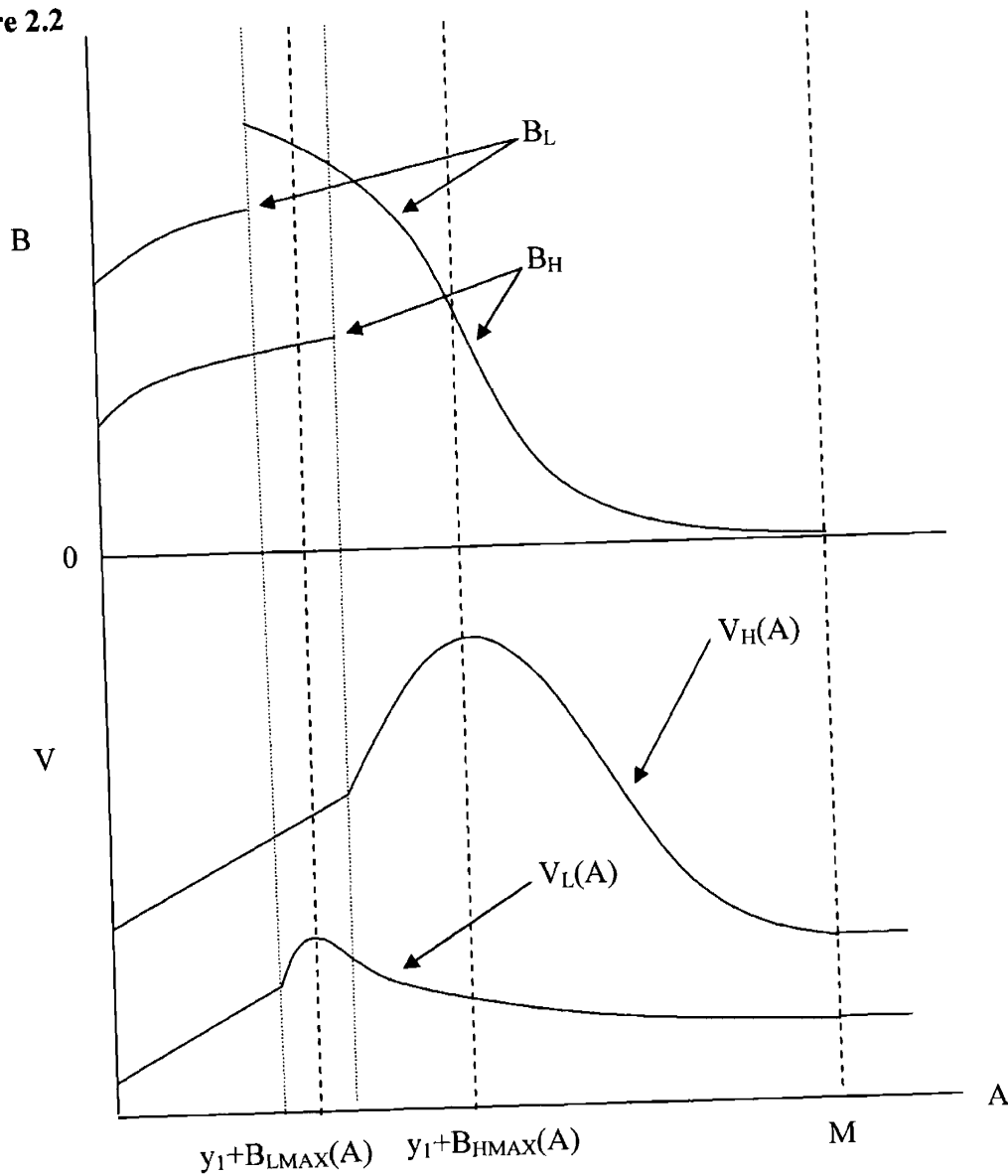
Borrowing for both types of consumer increases with the bankruptcy exemption level until the point where the consumer switches from borrowing at the internal tangency point to borrowing as much as possible. Because the marginal utility of borrowing is greater for type L consumers, they switch to borrowing as much as possible sooner than do type H consumers.

The utility for type L consumers peaks earlier than for type H consumers because they get more utility from borrowing but have the same ability to pay as do type H consumers. As a result, the point where worsening the credit constraint on type L consumers outweighs the benefit of higher exemption levels is lower for type L consumers than for type H consumers.

2.2 Consumers With Differing Second Period Income

I now consider a model in which there are two types of consumer with the same first period income but with differing cumulative distribution functions for second period income. Specifically, I assume there are two types of consumer, L and H, with cumulative distribution functions $F_L(y_2)$ and $F_H(y_2)$ respectively, where $F_L(y_2) \geq F_H(y_2)$ for all y_2, i.e. where F_H dominates F_L in the sense of first order stochastic dominance. The two types of consumer are assumed to have the same first period income and the same utility function and to be subject to the same bankruptcy regime.

The model is depicted graphically below. As was the case with the Figure presented in Chapter 1, the vertical axis represents the level of debt incurred. The horizontal axis represents the amount of borrowing. The curve U_H^* and U_L^* are the type H and type L consumer's indifference curves at the optimal level of borrowing for each type when a borrower's type is observable. The curves BE_H and BE_L represent the break even constraints for lenders lending to type H consumers and type L consumers respectively.

The point B_L^*, D_L^* represents the optimal level of borrowing and debt for type L consumers when types are observable. Similarly, B_H^* and D_H^* are the optimal levels of borrowing and debt for type H consumers when types are observable.

Figure 2.3
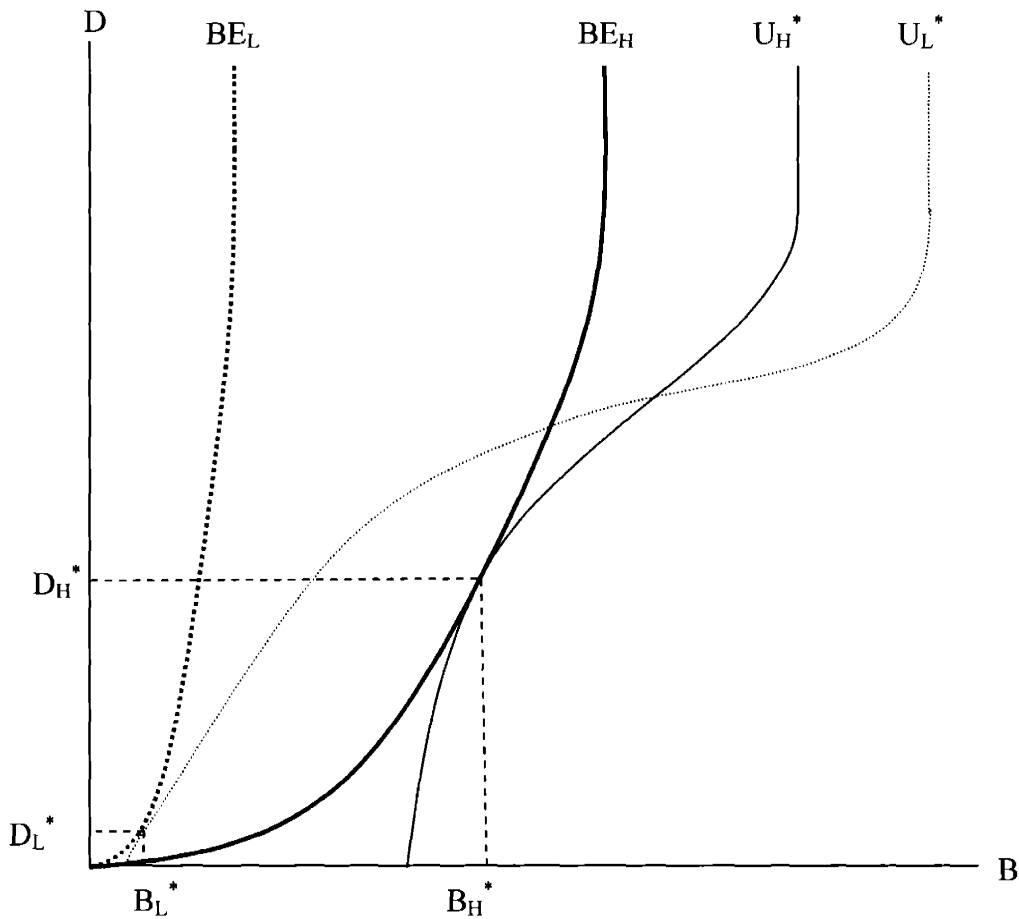
Proposition 2.5: The break even constraint for type L consumers lies to the left of the breakeven constraint for type H consumers.

Proof and discussion:

As was discussed earlier, the breakeven constraint for a particular consumer is given by:

$$(2.12) \quad \int_{A}^{A+D} [y_2 - A] f(y_2) dy_2 + D[1 - F(A+D)] = B$$

.

Because the type L consumer's distribution is first order stochastically dominated by the type H consumer's period two income distribution, the break even level of debt will necessarily be less for a type H consumer than a type L consumer and the breakeven constraint for lenders lending

to a type L consumer will lie to the left of the breakeven constraint for lenders lending to a type H consumer.

Proposition 2.6: In general, it is not possible to determine whether the slope of an indifference curve for type L consumers may be greater than, less than or equal to the slope of an indifference curves for type H consumers at any given point..

Proof and discussion:

As was demonstrated in the previous section, the slope of a consumer's indifference curve is given by:

$$(2.13) \frac{dD}{dB} = \begin{cases} \dfrac{u'(y_1 + B)}{\int_{A+D}^{M} u'(y_2 - D)f(y_2)dy_2} = \dfrac{u'(y_1 + B)}{E[u'(y_2 - D) | y \geq A + D][1 - F(A + D)]}, & D \leq M - A \\ \infty, & D > M - A \end{cases}$$

At any point, B and D will be the same for both type L and type H consumers. The only difference in the slopes of the two indifference curves comes from differences in $f(y_2)$.

At any point where D<M-A, the slope of the type L consumer's indifference curves might be flatter than the slope of a type H consumer's indifference curve because there will be a higher probability that the type L consumer receives lower income realizations. As a result, the average marginal utility for the type L consumer will be higher than that of a type H consumer, which may cause the slope for the type L consumer's indifference curve to be less than that for the type H consumer.

However, because the slope only depends on the integral of the product of the marginal utility function and probability density over the range of income realizations where the consumer does not go bankrupt, it is possible that even though type L consumer's indifference curve will not be steeper if the type L consumer's probability of bankruptcy is greater than that of a type H consumer. Indeed, since $F_L(A+D) \geq F_H(A+D)$, the probability that the type L consumer will go bankrupt is weakly greater than for the type H consumer. As a result, the probability that the type L consumer has to pay off his debts in full is lower than that for a type H consumer, potentially making the term $\int_{A+D}^{M} u'(y - D)f(y_2)dy_2$ smaller for type L consumers than for type H consumers.

41

If that is the case, then the slope of the type L consumer's indifference curve would be steeper than that of a type H consumer.

Finally, at some points, including all levels of D≥M-A, the slopes of the indifference curves for both types will be the same.

The figures depicted in this section are drawn to illustrate the situation in which the type L consumer's indifference curves are flatter than a type H consumer's indifference curves at low levels of A and D, where there is a lower probability that either consumer will go bankrupt. At higher levels of A and D, the type L consumer's indifference curves are depicted as steeper than a type H consumer's indifference curves.

Proposition 2.7: Type H consumers will borrow more than type L consumers, unless
$F_H(y_2)=F_L(y_2)$ *for all $y_2 \geq A^1$, in which case they will borrow the same amount.*

Proof and discussion:

If each consumer's type can be observed by lenders, then consumers will choose levels of borrowing and debt to maximize expected utility subject to the lender's break even constraint in the same way that they did in chapter 1.

As was discussed above, because the budget constraint defines D for each type of consumer as a function of B, a type i consumer' maximization problem can be rewritten as:

(2.14)

$$\max_B u(y_1 + B) + \int_0^A u(y_2)f_i(y_2)dy_2 + u(A)[(F_i(A + D_i(B)) - F_i(A)] + \int_{A+D_i(B)}^M u(y_2 - D_i(B))f_i(y_2)dy_2$$

Taking the derivative with respect to B, I find:

$$(2.15)\ \frac{dU}{dB} = u'(y_1 + B) - \int_{A+D(B)}^M u'(y_2 - D_i(B))f_i(y_2)dy_2 \frac{dD_i}{dB}.$$

Substituting $\dfrac{1}{1 - F_i(A + D_i)}$ for $\dfrac{dD_i}{dB}$, I find

---

[1] Even though the distribution of second period income for type H consumers stochastically dominates the distribution of second period income for type L consumers, it is still possible that $F_H(y_2)=F_L(y_2)$ for all y_2 if the distributions differed only as to the probabilities of incomes less than A.

(2.16)

$$\frac{dU_i}{dB} = u'(y_1 + B) - \frac{\int_{A+D_i(B)}^{M} u'(y_2 - D_i(B))f_i(y_2)dy_2}{1 - F_i(A + D_i)} = u'(y_1 + B) - E_i[u'(y_2 - D(B)) \mid y_2 \ge A + D_i].$$

Taking the difference between $\dfrac{dU_H}{dB}$ and $\dfrac{dU_L}{dB}$, I find

(2.17)

$$\frac{dU_H}{dB} - \frac{dU_L}{dB} = E_L[u'(y_2 - D_L(B)) \mid y_2 \ge A + D_L(B)] - E_H[u'(y_2 - D_H(B)) \mid y_2 \ge A + D_H(B)].$$

So long as $F_H(y_2) < F_L(y_2)$ for some $y_2 > A$, the expected marginal utility of second period consumption conditional on $y_2 \ge A+D$ will be greater for the type L consumer than for the type H consumer because: 1) the level of debt, D_i, will be less for type H consumers than for type L consumers and 2) the type L consumer has a higher probability of lower incomes than does the type H consumer. As a result this difference is positive.

Because this difference in the marginal utility of borrowing is positive, the type H consumer borrows more than the type L consumer.

Since the level of consumption if income is less than A does not depend on the amount borrowed, differences in probabilities of income below A do not affect a consumer's borrowing decision. Therefore, if the probability distribution of second period income is the same for both types of consumers for all levels of income above A, then the two types of consumers will borrow the same amount.

Proposition 2.8: When the exemption level is not permitted to differ by type of consumer, the optimal level of A will be weakly higher than the optimal amount for the type L consumer and weakly lower than the optimal amount for the type H consumer.

Proof and discussion:

Denoting the indirect utility functions for types L and H $V_L(A)$ and $V_H(A)$ respectively, the social planner's objective function is:

(2.18) $\max_A V(A) = w_L V_L(A) + w_H V_H(A)$

where w_L and w_H are the weights placed on type L and type H's welfare.

43

The consumers utility $V_L(A)$ and $V_H(A)$ are depicted in Figure 2.4

Figure 2.4

$y_1 + B_{LMAX}(A)$ $y_1 + B_{HMAX}(A)$ M
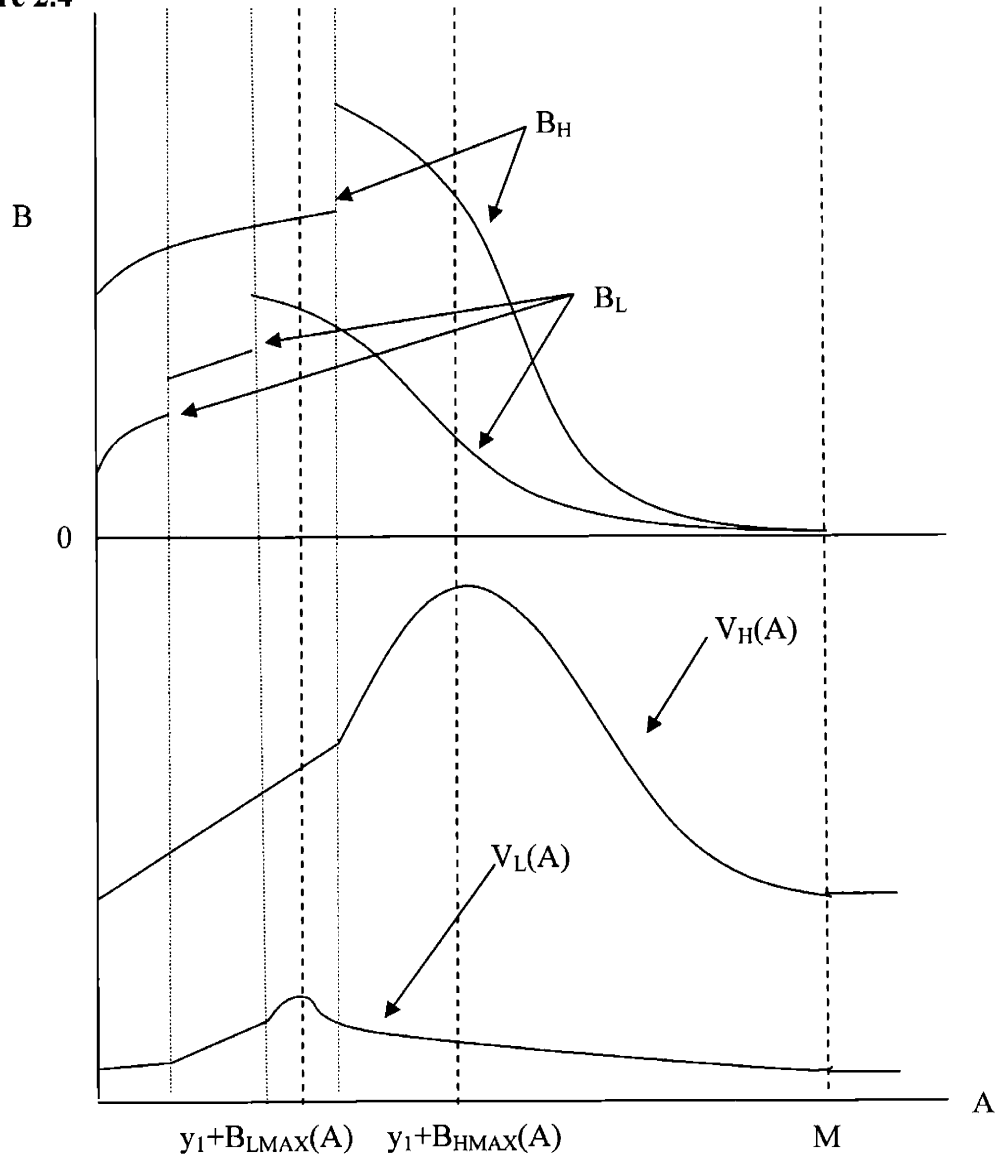
below:

At the optimum, the derivative of the social planners function with respect to A must equal zero, i.e.

$$(2.19) \quad \frac{dV(A)}{dA} = w_L \frac{dV_L(A)}{dA} + w_H \frac{dV_H(A)}{dA} = 0,$$

or, rearranging,

44

$$(2.20) \quad w_L \frac{dV_L(A)}{dA} = -w_H \frac{dV_H(A)}{dA}.$$

Equation (2.20) implies that the optimal level of A must occur where $\frac{dV_L(A)}{dA}$ and

$\frac{dV_H(A)}{dA}$ must be of opposite signs or both must be equal to zero.

Equation (2.20) will not hold at levels of A where both types choose to borrow at an interior point. From equations (1.20) and the discussion following proposition 1.5, I know that if A is such that both type L and type H consumers borrow less than the maximum the lender's breakeven constraint will allow them, then increasing the exempt amount A will increase utility

for both the high and low type consumers, i.e. $\frac{dV_L(A)}{dA} \geq 0$ and $\frac{dV_H(A)}{dA} \geq 0$. In other words, if

both consumers are borrowing at an interior solution, increasing the level of exempt income provides both types of consumers with greater insurance and therefore increases utility for both types. As a result, equation (2.19) cannot hold and the social optimum cannot occur at levels of A where neither type L or type H consumers borrow the maximum possible.

Similarly, the optimum cannot occur at levels of A at which type H consumers borrow as much as possible while type L consumers borrow at an interior point. For only type H consumer to borrow at the maximum, A must be less than $y_1 + B_{Hmax}$. Equation (1.27) and the discussion following proposition (1.6) implies that increasing A will increase utility for the type H consumer so long as $A < y_1 + B_{Hmax}$. Furthermore, because the type L consumer would be borrowing at an interior point, increasing A would increase the utility for the type L consumer as

well. Because $\frac{dV_L(A)}{dA} \geq 0$ and $\frac{dV_H(A)}{dA} \geq 0$, the social optimum cannot occur at levels of A

where only type H consumers borrow at the maximum.

Consequently, the optimum must occur at levels of A where type L consumers borrow the maximum amount.

If a type L consumer borrows as much as possible, then increasing A will decrease the amount that a type L consumer can borrow and therefore reduce first period utility. At the same time, it will increase consumption in the second period in any states of nature where income is greater than the level of exempt income. If $A < y_1 + B_{Lmax}$, then the decrease in first period utility

will be outweighed by the increased utility in the second period. Therefore, $\dfrac{dV_L(A)}{dA} > 0$.

Moreover, because the maximum amount that a type L consumer can borrow will be less than or equal to the maximum amount that a type H consumer can borrow, if $A < y_1 + B_{Lmax}$, then $A < y_1 + B_{Hmax}$ and utility will increase for type H consumers as well. Because $\dfrac{dV_L(A)}{dA} > 0$ and

$\dfrac{dV_H(A)}{dA} > 0$ for levels of $A < y_1 + B_{Lmax}$, the socially optimal level of A cannot occur at levels of $A < y_1 + B_{Lmax}$.

If $A > y_1 + B_{Lmax}$, then the decrease in first period utility due to the decreased amount that can be borrowed at the maximum will outweigh the increase in second period utility due to the higher level of exempt income. Increasing A will, therefore, decrease the utility for a type L consumer, i.e. $\dfrac{dV_L(A)}{dA} \leq 0$.

If type H consumers borrow at an interior point, then increasing A will raise give the type H consumer more insurance without decreasing the amount he can borrow and the utility for type H consumer will increase. Moreover, increasing A will increase the utility of a type H consumer borrowing at the maximum so long as A is less than $y_1 + B_{Hmax}$. Put another way, so long as $A < y_1 + B_{Hmax}$, $\dfrac{dV_H(A)}{dA} > 0$.

If $A = y_1 + B_{Lmax} = y_1 + B_{Hmax}$, as would be the case if the second period income distributions is the same for both types of consumer for incomes above $y_2 = A$, then $\dfrac{dV_L(A)}{dA} = 0$ and $\dfrac{dV_H(A)}{dA} = 0$. In that case $A = A = y_1 + B_{Lmax} = y_1 + B_{Hmax}$ would be the optimal level of A.

Otherwise, the optimum must occur where $\dfrac{dV_L(A)}{dA}$ and $\dfrac{dV_H(A)}{dA}$ are of opposite signs.

Because $\dfrac{dV_L(A)}{dA}$ and $\dfrac{dV_H(A)}{dA}$ are either of opposite signs or both zero only where type L consumers borrow as much as possible and $y_1 + B_{Lmax} \leq A \leq y_1 + B_{Hmax}$, the optimum level of A must occur in this range. Moreover because $A = y_1 + B_{Lmax}$ and $A = y_1 + B_{Hmax}$ are the optimums for type L and type H consumers respectively, the socially optimal level of A will be weakly higher

than the optimum for type L consumers and weakly lower than the optimum for type H consumers.

Substituting for $\dfrac{dV_L(A)}{dA}$ and $\dfrac{dV_H(A)}{dA}$ into equation (2.10), I find:

(2.21)
$$\frac{\partial V(A)}{\partial A} = w_L\{u'(A)[F_L(A+D_L)-F_L(A)]-\lambda_L[F_L(A+D_L)-F_L(A)]\}$$
$$+ w_H\{u'(A)[F_H(A+D_H)-F_H(A)]-\lambda_H[F_H(A+D_H)-F_H(A)]\} = 0$$

where $\lambda_L = u'(y_1 + B_{L\max})$ and $\lambda_H = u'(y_1 + B_H)$.

Intuitively, at the optimum, the marginal loss to type L consumers due to decreasing the amount that they can borrow must be balanced against the marginal increase in utility for type H consumers from shifting the obligation to repay the amount borrowed to states of nature where income and consumption are higher.

Again, the amount of borrowing for each type of consumer increases with the bankruptcy exemption level up until the point where the consumer switches from borrowing at an interior point to borrowing as much as possible. As discussed above, the type L consumer borrows less than the type H consumer because he has fewer resources in the future.

Since the type L consumer's second period income distribution is first order stochastically dominated by the type H consumer's second period income distribution, he has fewer expected future resources from which creditors can be repaid. As a result, the point at which the decreased utility resulting from further credit constraining type L consumers exceeds the increased insurance provided by higher exemptions is lower for type L than for type H consumers.

Though it is not considered here, the fact that bankruptcy law specifies particular assets which can be protected from creditors, most notably home equity, may lessen some of the tension between providing more insurance and increasing credit constraints. If, for example, consumers with lower expected future income have less home equity than consumers with greater expected future income, then the *de facto* bankruptcy exemption level may be lower for consumers with less expected future income and higher for consumers with higher future expected income. In this way, exemptions for home equity may be an efficient way of providing greater bankruptcy relief to consumers with higher expected future income who will not be credit constrained by the higher exemptions.

2.3 Differing Risks

I now consider a model where the distributions of second period income for the two types of consumer have the same mean but different levels of risk. Specifically, I assume that $E_H(y_2)=E_L(y_2)$, that is that the distributions have the same means, but that the probability of having an income realization that is a given amount either greater or less than a level of income y_c is greater for type L than for type H consumers, i.e. $F_H(y_2) \leq F_L(y_2)$ for $y_2 < y_c$ and $F_H(y_2) \geq F_L(y_2)$ if $y_2 > y_c$. In other words, y_c is the crossing point between the cumulative distribution functions for type H and type L consumers.

The breakeven constraints, representative indifference curves and optimal levels of borrowing for both type L and type H consumers are illustrated for two levels of the bankruptcy exemption A in Figures 2.5 and 2.6.

Figure 2.5
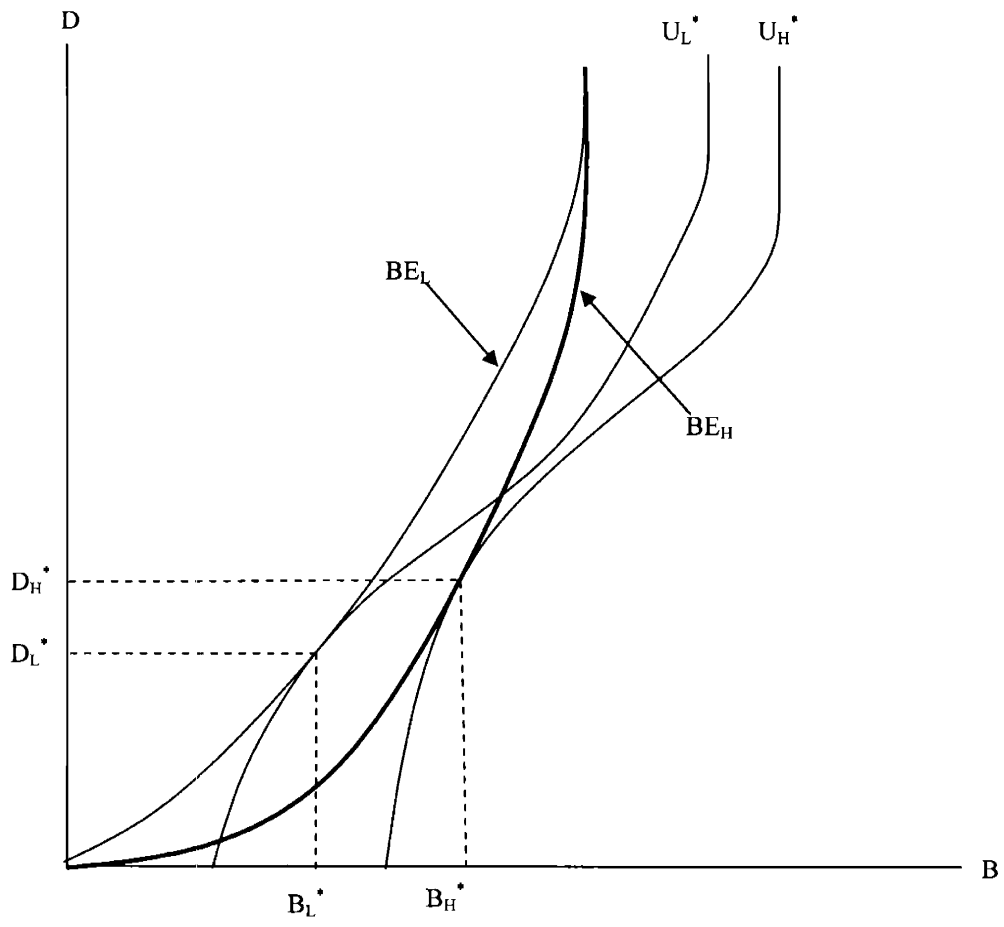
Figure 2.6
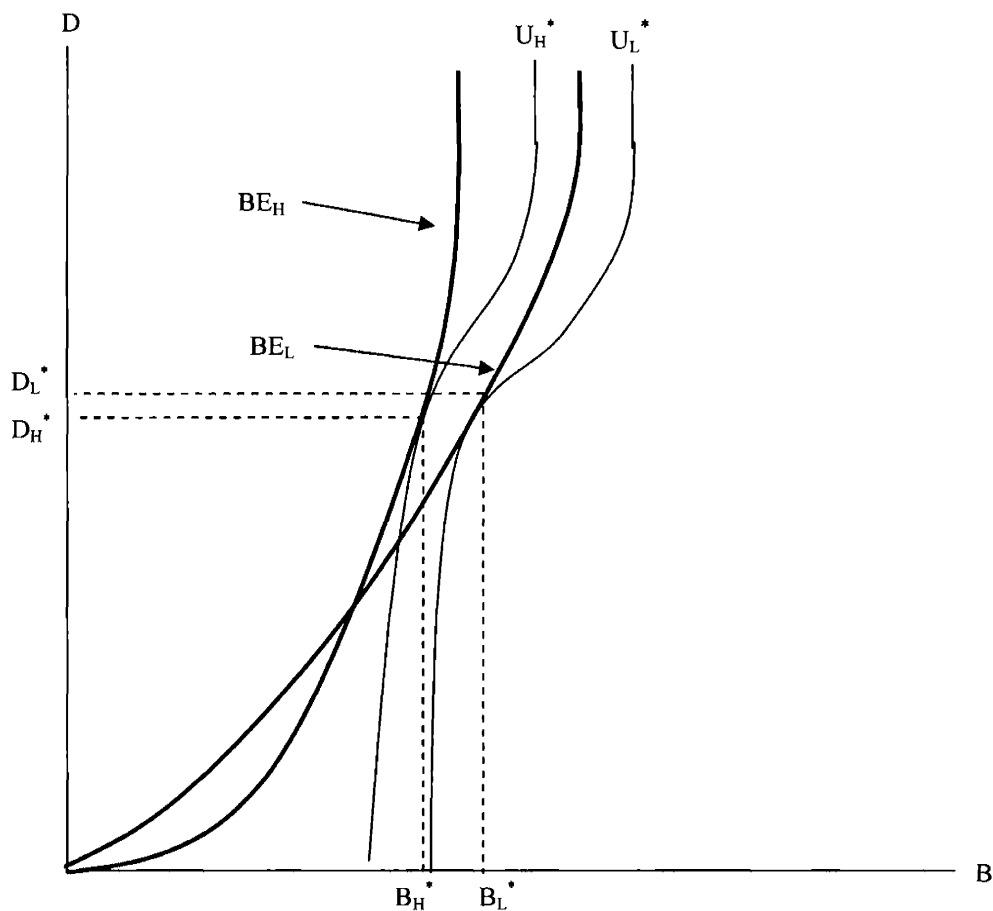
Figure 2.5 depicts the equilibrium for a level of A=0, while Figure 2.6 depicts the equilibrium for a level of A greater than zero but less than the y_c for second period income. Again, BE_L and BE_H represent the breakeven constraints for type L and type H consumers respectively, while U_L^* and U_H^* represent the indifference curves for type L and type H consumers at the optimum level of borrowing.

Proposition 2.9 The breakeven constraint for type L consumers lies:

　　a)　*weakly to the left of the breakeven constraint for type H consumers for low levels of*
　　　　A≥0.

　　b)　*Partly to the left and partly to the right of the breakeven constraint for type H*
　　　　consumers for some levels of A between 0 and M.

50

c) *Weakly to the right of the breakeven constraint for type H consumers for high levels of A≥y_c.*

Proof and Discussion:

The slope of each breakeven constraint is given by: $\dfrac{dD}{dB} = \dfrac{1}{1 - F_i(A + D)}$. Consequently,

the breakeven constraint for type H consumers will be flatter than the breakeven constraint for type L consumers for A+D<y_c and steeper than the breakeven constraint for type L consumers for A+D>y_c. As a result, so long as the exemption is less than the mean of the income, the slope of the breakeven constraint for type H consumers will be flatter for very low levels of debt and the type H consumers' breakeven constraint will be to the right of the type L consumers' breakeven constraint, at least for low levels of debt.

If A=0, then because the means of second period income are the same for each type of consumer, the maximum amount that can be borrowed is the same for each type. Consequently, the breakeven constraints for the type H and type L consumers in Figure 2.5 touch at both the origin and at the maximum amount borrowed. Since the slope of the type H consumers' breakeven constraint must be flatter than the type L consumers' breakeven constraint for low levels of D and steeper for high levels of D, the two cannot cross if they are to end up at the same point. For this reason, the breakeven constraint for type L borrowers must be to the left of the breakeven constraint for type H borrowers at all points other than the origin and the maximum amount borrowed.

It is possible that for some levels of positive A<y_c, that the breakeven constraint for the type L consumers will be strictly to the left of the breakeven constraint for type H consumers. The maximum amount that can be borrowed for each type is given by:

(2.22) $\displaystyle\int_A^M y_2 f_i(y_2)dy_2 = Ey_2 - \int_0^A y_2 f_i(y_2)dy_2$.

Because the expected value of y_2 is the same for each type, the maximum amount that can be

borrowed will be smaller for type L consumers than for type H consumers if $\displaystyle\int_0^A y_2 f_i(y_2)dy_2$ is

larger for type L consumers than for type H consumers. Because the probability of having low

51

incomes is less for type H consumers than type L consumers, it is possible that $\int_0^A y_2 f_i(y_2)dy_2$ is

greater for type L consumers (since $f_i(y_2)$ would be larger). In that case, the maximum amount

borrowed would be lower for type L consumers, and since the breakeven constraints cannot cross

more than once, the entire type L budget constraint, excluding the origin, would be to the left of

the type H budget constraint.

For higher levels of A, the maximum amount which can be borrowed by type L

consumers will be greater than the amount which can be borrowed by type H consumers. For

$A>y_c$, the slope of the breakeven constraints for type H consumers will always be steeper than

the breakeven constraints for type L consumers (since the probability of having income greater

than A will be greater for the consumer with the riskier second period income). In that case, the

breakeven constraint for type H consumers, excluding the origin, will lie entirely to the left of the

breakeven constraint for the type L consumers.

In the circumstance depicted in Figure 2.6, A is less than y_c, so initially the type H

breakeven constraint is flatter than the type L breakeven constraint for small levels of D and is

initially to the right of the type L constraint. However, for levels of A slightly less than y_c, the

breakeven constraints must cross since the amount which can be borrowed by type L consumers

will be greater for high levels of debt.

Intuitively, the riskier consumers can borrow more and at lower interest rates for some

levels of A because the distribution of income below A is relevant to the lender only in so far as

it effects the probability that income will be greater than A. For example, if $A=y_c$, then the

probability that income is greater than A is equal for both types of consumer. It is irrelevant to

the lender that type L consumers have a much greater probability of having very low income (as

opposed to having an income jut slightly below A) since the lender will not receive anything in

either case. However, the lender benefits from the fact that type L consumers are more likely to

have higher incomes relative to type H consumers since they will receive a larger repayment if,

for example, the consumer makes enough to pay in full than if he does not. Consequently, the

lender will offer lower interest rates to the riskier consumer.

Moreover, the riskier customers might borrow more in equilibrium, as for example, if

$A>y_c$ and both types of consumer borrow the maximum amount possible.

Proposition 2.10: The indifference curves for type H consumers may be flatter than, steeper than or the same slope as the indifference curves for type L consumers.

Proof and discussion:

As was shown above, the slope of a consumer's indifference curve is given by:

$$(2.23)\frac{dD}{dB} = \begin{cases} \dfrac{u'(y_1 + B)}{\displaystyle\int_{A+D}^{M} u'(y_2 - D)f(y_2)dy_2} = \dfrac{u'(y_1 + B)}{E[u'(y_2 - D)\,|\,y \geq A + D][1 - F(A + D)]}, & D \leq M - A \\[4ex] \infty, & D > M - A \end{cases}$$

Because B and D will be the same for both types of consumer at any point on the graph, the differences in slope of the indifference curves for each type come entirely from differences in $f(y_2)$. For A+D=0, the denominator of the fraction will be greater for type L consumers since u''' is assumed to be positive and the probability of y_2 being greater than zero is equal to one for both types. As a result, the type H consumers indifference curves will be steeper.

For values of A+D which are greater than zero it is not possible to determine the sign of the denominator. For levels of A+D which are less than y_c, the probability of having income greater than A+D will be greater for type H consumers than for type L consumers. This will tend to cause the denominator to be larger for type H than for type L consumers. The fact that u'' is negative means that $u'(y_2)$ is greater for lower levels of y_2 which may be more likely for type L consumers, tending to make the denominator larger for type L consumers. Similarly, for levels of A+D which are greater than the mean of second period income, the probability of having income greater than A+D is greater for type L consumers, but marginal utility is greater for smaller levels of y_2, which may be more prevalent for type H consumers. Because the two effects act in opposite directions, it is impossible, in general, to determine the relative slopes of the indifference curves for type L and type H consumers.

Proposition 2.11: When the bankruptcy exemption level is not allowed to differ by type of consumers, the optimal bankruptcy exemption level will be:

  a) weakly higher than the optimal bankruptcy exemption level for type L consumers and weakly lower than the optimal bankruptcy exemption level for type H consumers if the maximum amount that a type L consumer can borrow is less than the maximum amount a type H consumer can borrow,

53

b) weakly lower than the optimal bankruptcy exemption level for type L consumers and weakly higher than the optimal bankruptcy exemption level for type H consumers if the maximum amount that a type L consumer can borrow is more than the maximum amount a type H consumer can borrow.

Proof and discussion:

The proof of proposition 2.11 closely resembles the proof of proposition 2.8. Briefly, the social planner's objective function is:

$$(2.24) \quad \max_A V(A) = w_L V_L(A) + w_H V_H(A)$$

where w_L and w_H are the weights placed on type L and type H's welfare. Taking the derivative of the utility function with respect to A and rearranging, I find:

$$(2.25) \quad w_L \frac{dV_L(A)}{dA} = -w_H \frac{dV_H(A)}{dA},$$

which implies that either both derivatives must equal zero, or the derivatives of the social welfare functions must have equal and opposite signs.

As has been discussed previously in sections 2.1 and 2.2, if both consumers are borrowing less than the maximum amount possible, increasing A will increase the welfare to each type of consumer since it will shift the obligation to repay the debt to a state of nature where consumption is less valuable and, since they are borrowing less than the maximum amount, will not lead to credit constraints.

The optimal level of A must, therefore, occur at a point where at least one consumer is borrowing as much as possible. If type L consumers are borrowing as much as possible, then increasing A will increase utility if $A < y_1 + B_L$ and will decrease utility if $A > y_1 + B_1$. The optimum for type L consumers occurs where $A = y_1 + B_L$. Similarly, if a type H consumer is borrowing as much as possible then increasing A will increase utility if $A < y_1 + B_H$ and will decrease utility if $A > y_1 + B_H$. The optimum for type H consumers occurs where $A = y_1 + B_H$.

If the maximum amount that a type H consumer can borrow is more than the maximum amount that a type L consumer can borrow, then equation 2.25 can hold only if $y_1 + B_L < A < y_1 + B_H$, i.e. if it is greater than the optimal exemption levels for type L consumers and less than the optimal exemption level for type H consumers.

54

Alternatively, if the maximum amount that a type L consumer can borrow is more than the maximum amount that a type L consumer can borrow, then equation 2.25 can hold only if $y_1+B_H<A<y_1+B_L$, i.e. if it is greater than the optimal exemption levels for type H consumers and less than the optimal exemption level for type L consumers.

2.4 Conclusion

Where consumers differ either as to first period income or the probability distribution of second period income, the socially optimal bankruptcy exemption level will occur where the marginal cost of increasing credit constraints for one type of consumer is equal to the marginal insurance benefit provided to the other type of consumer.

Consumers with either a higher demand for credit or a lower ability to repay debt will be credit constrained at lower exemption levels than consumers with a lower demand for credit or a greater ability to repay debt. At the socially optimal exemption level, the marginal cost of exacerbating these credit constraints will be balanced by the marginal insurance benefit provided to consumers with a smaller demand for credit relative to their ability to repay the debts.

Because higher first period income reduces first period marginal utility (and therefore the marginal benefit to borrowing), consumers with low first period income have a greater demand for credit than consumers with higher first period income. The optimal bankruptcy exemption level will, therefore, balance the marginal cost of increasing credit constraints on consumers with low first period incomes against the insurance benefits provided to consumers with higher first period incomes.

Similarly, consumers who have lower expected second period income have a lower ability to repay their debts. Because consumers with lower expected second period income will be credit constrained at lower exemption levels, the optimal bankruptcy exemption level will balance the loss of utility due to worsening credit constraints on consumers with lower expected second period incomes against the marginal insurance benefits provided to consumers with higher expected second period income,

The tradeoff between credit constraints and insurance is somewhat more complicated when consumers differ as to the riskiness of their second period income because it is not clear, in general, which type of consumer will become credit constrained at the lowest exemption level. If

consumers with safer second period incomes are credit constrained at lower exemption levels than consumers with riskier second period incomes, then the socially optimal exemption level will be higher than the optimal exemption level for consumers with safer second period income and lower than the optimal exemption level for riskier consumers. Alternatively, if the riskier consumers are credit constrained at lower exempt amounts, then the socially optimal exemption level will be higher than for the optimal exemption level for riskier consumers and lower than the optimal exemption level for safer consumers.

In any case, the general lesson remains the same. Where consumers differ as to either their demand for credit or their ability to pay off their debts, the socially optimal bankruptcy exemption level will balance the marginal costs of worsening credit constraints for some groups against increased insurance benefits for other groups.

Chapter 3: Credit Markets with Bankruptcy and Asymmetric Information

The theoretical effect of bankruptcy law on consumer credit markets with heterogeneous consumers will depend, in part, on whether lenders can observe a borrowers' ability to repay a loan. The models considered in Chapter 2 assume that the borrowers' second period distribution of income, and thus his credit worthiness, are known to both borrowers and lenders. This chapter alters that assumption and considers the affect of bankruptcy on consumer borrowing, consumer utility and social welfare in a credit market where a consumer's second period distribution of income is not observable to lenders.

As described in greater detail in Section 3.1, this chapter considers possible equilibria in a model like that discussed in Section 2.2, where a consumers' second period income may be drawn from one of two distributions, one of which first order stochastically dominates the other. Unlike the analysis presented in Chapter 2, it is assumed here that lenders cannot observe a borrower's type.

Section 3.2 describes potential separating equilibria. For levels of the bankruptcy exemption where default is not a realistic possibility for either type of borrower, consumer borrowing is found to be the same as if the borrower's type can be observed. For higher levels of the bankruptcy exemption level, borrowers with the higher expected second period income will have to distort their borrowing in order to prevent less credit worthy borrowers from mimicking their borrowing choices in order to receive a lower interest rate. If the utility cost to these distortions worsens as the exemption level increases, it will offset some or all of the social benefit of providing greater insurance to borrowers. Social welfare may be maximized in a separating equilibrium at the point where the marginal benefit to increasing insurance for borrowers is balanced by the marginal cost due to distortions in borrowing by consumers with favorable second period income distributions. Alternatively, because the utility costs of these distortions will decrease as the exemption level increases for large levels of the bankruptcy exemption level, A, the socially optimal bankruptcy exemption level may occur where the marginal cost to increasing credit constraints for borrowers with lower expected future income is equal to the marginal benefit for borrowers with higher expected future income from greater insurance and a reduction in the distortions needed to signal their type to lenders.

Section 3.3 discusses potential pooling equilibria. It is argued that a pooling equilibrium can only occur at bankruptcy exemption levels which are sufficiently high that both types of consumer prefer the pooling equilibrium to any separating equilibrium. As a result, any small change in the bankruptcy exemption level which would cause a the equilibrium to shift from a pooling to a separating equilibrium would result in a welfare reduction for all consumers. It is, therefore, possible that the socially optimal bankruptcy exemption occurs at an exemption level where a small change would cause the type of equilibrium to change. Furthermore, since the amount of borrowing at a pooling equilibrium will change with the bankruptcy exemption, the increased insurance benefits for both types of consumers must be balanced against any losses in social welfare due to shifts in borrowing required to maintain the pooling equilibrium.

Section 3.4 considers potential semi-separating equilibria, where some consumers who are poor credit risks mimic the borrowing of consumers with dominant second period income distributions and some do not. It is argued that a semi-separating equilibrium can occur only when it is weakly preferred to all separating equilibria by both types of consumer. As was the case with separation and with pooling equilibria, changes in the bankruptcy exemption level may lead to costs due to worsening distortions in borrowing which must be balanced against the increased insurance provided by higher exemptions. The socially optimal exemption level may occur in a semi-separating equilibrium where the marginal costs associated with increased distortions in borrowing are equal to the marginal insurance benefit to increasing the exemption level.

Section 3.5 concludes.

3.1 The Model Described

To begin, I assume that there are two types of consumers: L and H. Type L and type H consumers differ only with respect to the distribution of second period income. Specifically, type L and type H consumers are assumed to have second period income distributed according to cumulative distribution functions $F_L(y_2)$ and $F_H(y_2)$ respectively, where $F_L(y_2) \geq F_H(y_2)$. Type L and type H consumers are assumed to have the same first period income and to maximize the same utility function.

The probability that a consumer is a type L consumer, $\pi(L)$, is common knowledge.

The borrower moves first by proposing to borrow an amount B. Lenders then offer to make the loan in exchange for a promise by the borrower to repay an amount D. The borrower will then choose to borrow from the lender which has offered the lowest D.

Repayment of debt is governed by a bankruptcy regime which exempts an amount A from attachment by creditors.

The game is depicted graphically below:

Figure 3.1
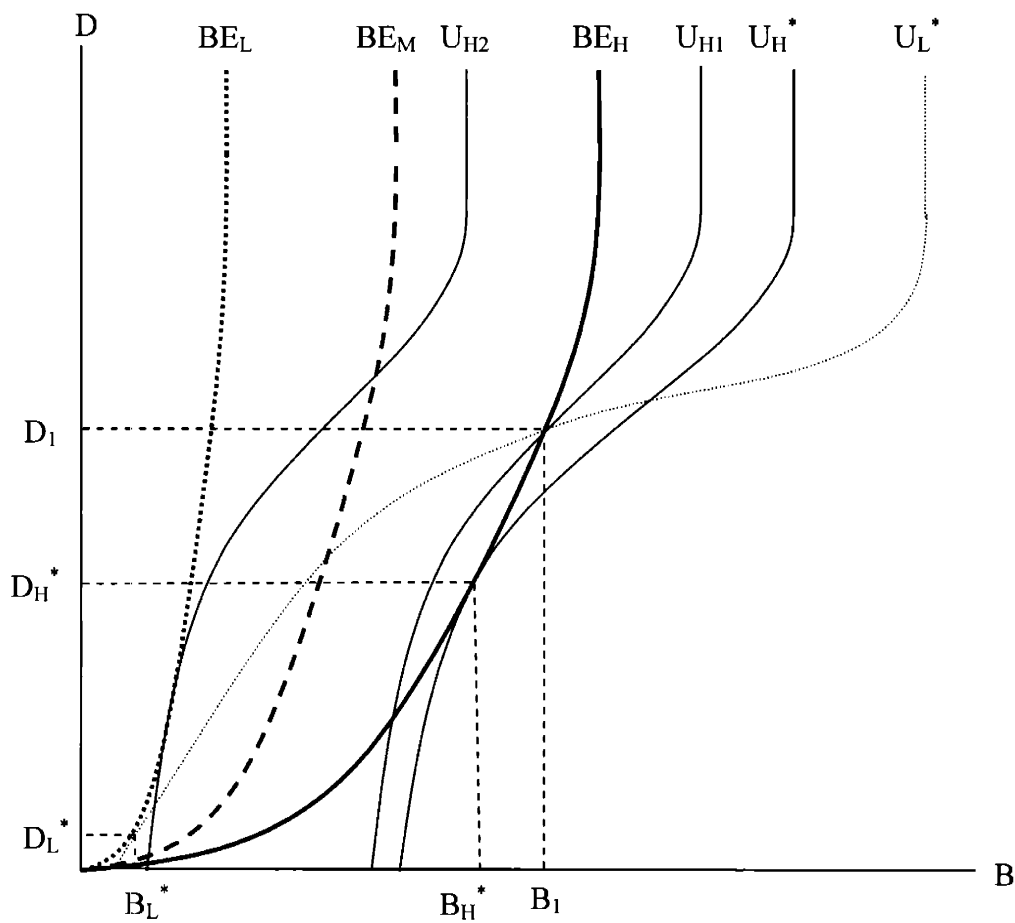
As in Figure 2.1, the curves BE_H and BE_L represent the breakeven constraints for type H and type L consumers respectively. The curve BE_M represents the combinations of borrowing and debt that will allow a lender lending to both type L and type H consumers in the same proportion as their prevalence in the population.

The curves U_H^* and U_L^* represent the indifference curves for type H and type L consumers at the level of borrowing and debt chosen by type H and type L consumers when the consumer's type can be observed. The curve U_{H1} represents a type H consumer's indifference curve passing through the point where U_L^* crosses the type H consumer's breakeven curve, BE_H. While the upper intersection of U_L^* and BE_H is depicted in Figure 3.1 as being above (B_H^*, D_H^*), that intersection may occur either above or below (B_H^*, D_H^*). The curve U_{H2} represents the combination of points that give a type H consumer utility equivalent to the highest level of utility that a type H consumer could achieve on the type L consumer's breakeven constraint, BE_L.

In order to determine the potential effect of bankruptcy exemption levels on the amounts borrowed by each type of consumer, I will describe the set of Bayesian-Nash equilibria that can be supported by weakly communication proof beliefs.

Three categories of Bayesian-Nash equilibria will be considered: separating, semi-separating and pooling. To make clear which results depend on the weakly communication proof restriction on lenders' beliefs, I begin by considering the set of Bayesian-Nash equilibria for each type of equilibria, i.e. the set of strategies and beliefs for which a player's actions are utility maximizing given his beliefs about the other player's type and for which those beliefs are consistent with the other players' types. Following the discussion of the potential Nash equilibria, the weakly communication proof restriction on beliefs is considered. Finally, given each equilibrium type, the effect of exemption levels on social welfare is considered.

3.2 Separating Equilibria

3.2.1 Bayesian-Nash Separating Equilibria

I first consider the set of separating equilibria, that is the set of equilibria where type H and type L consumers borrow different amounts. To begin, I consider the behavior of lenders in any Nash equilibrium.

Proposition 3.1: In any Nash equilibrium, lenders will earn zero profits.

Proof and discussion:

Lenders cannot earn positive profits in a Nash equilibrium because other lenders would have an incentive to undercut them, and cannot earn negative profits because lenders could do better by not lending.

More formally, if a lender's offer of D would result in positive profits given the proportion of type H and type L consumers that choose to accept that offer, then the lender not chosen by the borrower would prefer to have offered to make the loan in exchange for slightly less than D, in which case he would make positive profits. Because the other lender would have been better off by offering to make the loan in exchange for less debt, an offer which would result in positive profits cannot occur in an equilibrium.

If the D proposed would result in negative profits, then the lender would be better off charging a higher price or not making the loan at all. Such a level of D cannot, therefore, occur in an equilibrium.

The only possible equilibrium, then, occurs when lenders earn zero profits. In that case, each lender is maximizing his expected profits given the action of the other lender.

Proposition 3.2: In any separating equilibrium, the type L consumer chooses borrowing and debt levels that are the same as when lenders can observe his type.

Proof and discussion:

It follows from the requirement that lenders earn zero profits, that in any separating equilibrium, the combination of B_L, D_L must satisfy:

$$(3.1) \quad \int_{A}^{A+D_L} [y_2 - A] f_L(y_2) dy_2 + D_L[1 - F(A + D_L)] - B_L = 0.$$

That is, any lender lending B_L in exchange for a promise to pay D_L in a future period will earn zero profits.

B_L and D_L must also be the point on the lender's zero profit constraint that maximizes the type L consumer's utility. That is, it must solve:

$$(3.2) \quad \max_{B,D} u(y_1 + B_L) + \int_{0}^{A} u(y_2) f_L(y_2) dy_2 + u(A)[(F_L(A + D_L) - F_L(A)] + \int_{A+D_L}^{M} u(y_2 - D_L) f_L(y_2) dy_2$$

subject to:

$$(3.3) \quad \int_{A}^{A+D_L} [y_2 - A] f_L(y_2) dy_2 + D_L[1 - F(A + D_L)] - B_L = 0$$

By definition, the consumer prefers this point to any other point on the lender's type L breakeven constraint..

This is, of course, the same maximization problem as determines the consumer's level of borrowing and debt in the case where his type is observable and has the same solution.

As was discussed in Chapter 1, the type L consumer may prefer to borrow at either an interior tangency between the breakeven constraint and his indifference curve or he may prefer to borrow as much as possible. Figure 3.2 below depicts breakeven constraints, consumer indifference curves and potential equilibrium levels of borrowing in a separating equilibrium where type L consumers prefer to borrow at an interior tangency, while Figure 3.3 depicts the case where type L consumers prefer to borrow as much as possible.

Figure 3.2

Figure shows curves labeled BE_L, BE_M, U_{H2}, BE_H, U_{H1}, U_H^*, U_L^* on axes D (vertical) and B (horizontal), with dashed reference lines at D_1, D_H^*, D_L^* and B_L^*, B_H^*, B_1.

Possible Levels of Borrowing and Debt Points for Type H Consumers in Separating Equilibrium

Figure 3.3

D

BE_L BE_H

$D_L{}^*$

$U_L{}^*$

U_{H2}

$U_H{}^*$

B

Possible Levels of Borrowing and
Debt Points for Type H Consumers
in Separating Equilibrium
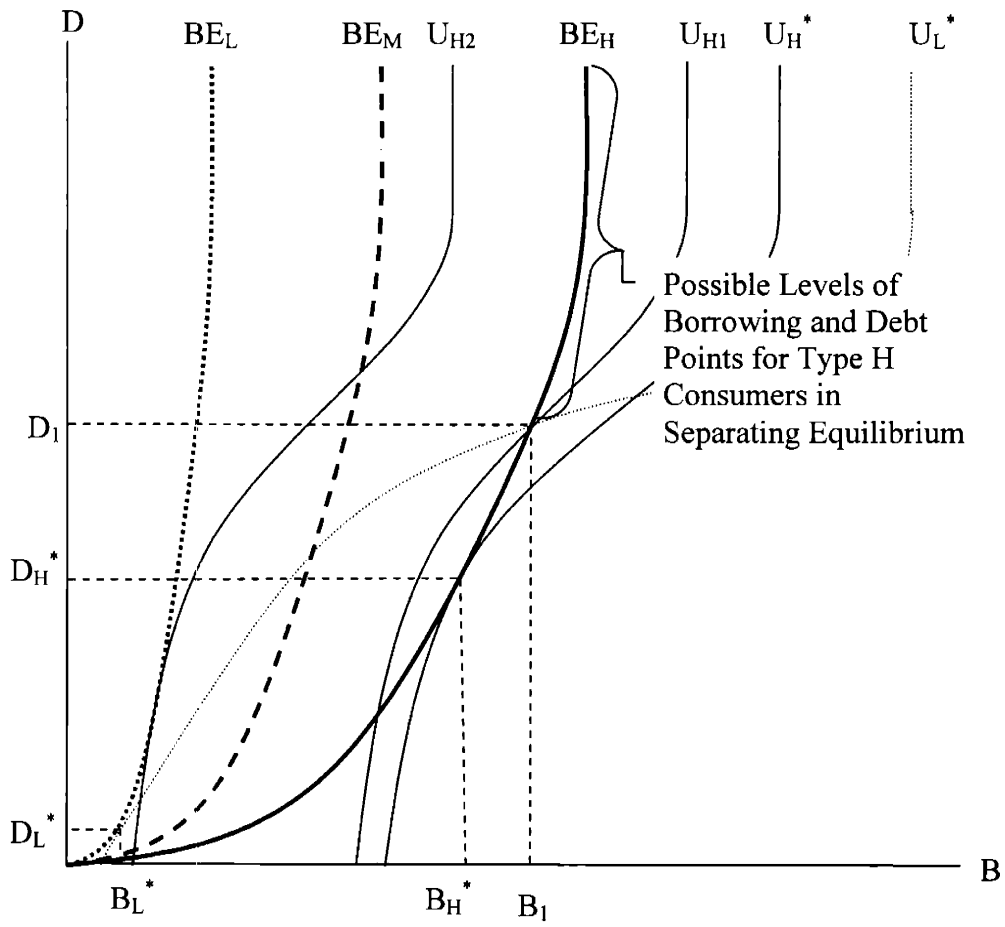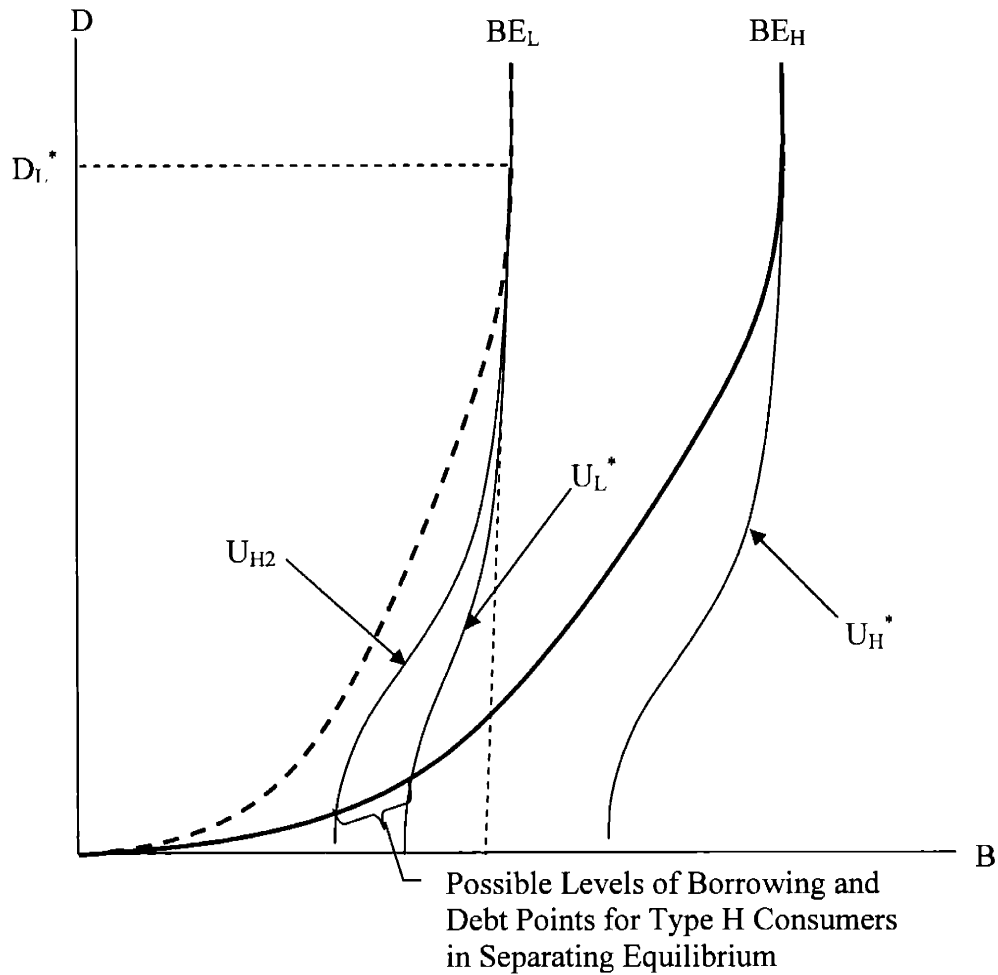
Proposition 3.3: In a separating equilibrium, any level of borrowing by type H consumers on the breakeven constraint BE_H that is weakly preferred by the type H consumer to anywhere on BE_L and which a type L consumer does not prefer to $B_L{}^*$, $D_L{}^*$ can occur in a Bayesian-Nash equilibrium.

Proof and discussion:

The fundamental difference between the model presented in Chapter 2, where a consumer's type is observable and this model, where a consumer's type is not observable, is that

the type L consumer may wish to pretend to be a type H consumer in order to receive a lower interest rate, i.e. promise to repay a lower amount D. Moreover, lenders are aware of the possibility that type L borrowers would like to emulate type H consumers and will base the interest rates they charge on their beliefs about the probability that a borrow is a high or low type.

For type H consumers to borrow money and to incur a debt that reflects their credit-worthiness (i.e. to pay a low interest rate), lenders must correctly believe that the amount of borrowed and debt incurred by type H consumers is not preferred by type L consumers to B_L^* and D_L^*. Consequently, a type H consumer seeking a low interest rate may choose to borrow an amount other than the amount he would choose to if his type were observable.

Put more formally, in a separating equilibrium, the type H consumer will chose the level of borrowing, B_H to maximize their utility subject to the following constraints:

First, lenders lending to type H consumers must earn zero profits., i.e. the choice of D_H and B_H must satisfy:

$$(3.4) \quad \int_A^{A+D_H} [y_2 - A] f_H(y_2) dy_2 + D_H[1 - F(A + D_H)] - B_H = 0.$$

Second, a type H consumer must prefer B_H, D_H to any other level of B and D such that D is consistent with the lender's beliefs about the type of consumer seeking to borrow B.

Finally, the type L consumers must prefer the point B_L, D_L to B_H, D_H.

These criteria do not, in general, produce a unique Bayesian perfect equilibrium. Any combination of B_H and D_H which

    (a) satisfy the lender's zero profit constraint;

    (b) are not preferred to by the type L consumer to B_L, D_L; and

    (c) are not preferred by the type H consumer to any point on BE_L

can be part of an equilibrium. As it is drawn in Figure 3.2, any level on BE_H that is above B_1 can be the equilibrium if supported by appropriate beliefs. For example, the belief that anyone wanting to borrow less than any $B' \in [B_1, B_2]$ was a type L consumer while anyone seeking to borrow more than B' was a type H consumer could support B' as an equilibrium level of B_H.

Finally, if as is depicted in Figure 3.3, the lower intersection of U_{H2} and BE_H is below and to the left of the lower intersection of U_L^* and BE_H then an equilibrium where type H

consumers choose to borrow at a point on BE_H between the lower intersections of U_{H2} and U_L^* with BE_H.

Levels of borrowing by the type H consumer less than B_1 cannot occur in a separating equilibrium unless the lower intersection of U_L^* and BE_H occurs above the lower intersection of U_{H2} and BE_H, because the type L consumer would prefer those levels of borrowing (and corresponding levels of debt on BE_H) to B_L^* and D_L^*.

Three points from the above discussion should be highlighted.

First, because expected second period income is higher for type H consumers than for type L consumers, incurring debt may be more costly for type L consumers. For this reason, debt may be able to be used to separate the two types when the types are not observable.

Second, because a willingness to take on higher levels of debt may serve as a signal to lenders that a borrower is a type H consumer, type H consumers may choose to borrower more or less than they would if their type were observable.

Finally, whether a separating equilibrium occurs and, if it does occur, the exact combination of B and D that occur in equilibrium depend on a combination of consumers' preferences, lenders' breakeven constraints and lenders' beliefs. As was discussed above, the consumer's preferences and the breakeven constraints define the set of possible separating equilibria, but exactly which equilibrium occurs is largely determined by lenders' beliefs.

3.2.2 Weakly Communication Proof Separating Equilibria

In order to reduce the number of possible equilibria, it is necessary to consider some restrictions on the set of possible beliefs. Specifically, I require that beliefs are "weakly communication proof."[2]

The weakly communication proof test requires that a lender's beliefs be reasonable in the sense that it requires the lender to believe a consumer who seeks to borrow an amount that is preferred to the equilibrium allocations only by type H (type L) consumers, to be a type H (type L) consumer. Put rather formally, the weakly communication proof restriction rules out any equilibria that are supported by beliefs which assign a probability less than one to a type seeking

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[2] This is identical to Cho and Krebs "intuitive criterion".

to borrow an amount B' if 1) B' is strictly preferred by that type to the equilibrium level of B and 2) B is strictly preferred to B' by all other types .

Proposition 3.4: A Bayesian equilibrium can be supported by beliefs which satisfy the weakly communication proof test if and only if there is no point which satisfies the breakeven constraint for a lender lending to a type H consumer which is preferred to the equilibrium choices for type H consumers only by type H consumers.

Proof and discussion:

Proposition 3.4 is a straight forward application of the definition of weakly communication proof beliefs.

If there is no point that satisfies the breakeven constraint for a lender lending to a type H consumer which is preferred to the equilibrium allocation level of borrowing and debt for type H consumers only by type H consumers, then the weakly communication proof test will not restrict lenders' beliefs at any point where it might affect a lender's choice of strategy. Because the restriction on beliefs will not change lenders' strategies, it will not affect the payoffs to borrowers or their choices. Consequently, the equilibrium can be supported by beliefs which satisfy the weakly communication proof test.

If there is a point that satisfies the breakeven constraint for a lender lending to a type H consumer which is preferred to the equilibrium choices for type H consumers only by type H consumers, then the weakly communication proof test requires that lenders believe that any consumer willing to borrow at that point be a type H consumer. Because the choices of borrowing and debt are continuous, there must be a point which is preferred to the equilibrium point by type H consumers and which would give lenders positive profits. Since they could earn positive (as opposed to zero) profits, lenders would maximize expected profits by being willing to lend at that point. Moreover, type H consumers could improve their utility by proposing to borrow at that point.

Because neither lenders nor type H borrowers would be maximizing their utility given beliefs which satisfy the weakly communication proof test, the equilibrium cannot be supported by weakly communication proof beliefs.

Proposition 3.5: Given the weakly communication proof restriction on beliefs, if the incentive compatibility constraint does not bind, the model's unique equilibrium occurs with type L consumers and type H consumers borrowing the same amount and incurring the same levels of debt as they would if their types were observable.

Proof and discussion:

In any equilibrium, type L consumers will borrow at a point that gives them at least as much utility as if they borrow B_L^* and promise to repay D_L^*. The weakly communication proof test, therefore, requires that lenders believe that any consumer who borrows at a point under and to the right of U_L^* and above and to the left of U_{H2} will be a type H consumer.

If the incentive compatibility constraint does not bind, the point B_H^*, D_H^* will be above and to the left of U_L^*. Consequently, the weakly communication proof test requires that lenders must believe that any borrower who would borrow at B_H^*, D_H^* would be a type H consumer. The lender would therefore be expect to earn zero profits by lending at that point and, since the lender can never earn positive profits in equilibrium, would be willing to do so.

Moreover, given the lenders beliefs, a type H consumer maximizes his utility by choosing to borrow B_H^* and incur debt D_H^*.

Finally, if a lender believes that anyone borrowing at a point below and to the right of U_L^* to be a type L consumer, then the type L consumer would maximize utility by borrowing B_L^* in exchange for a promise to repay D_L^*.

Since lenders and both types of consumer would be maximizing utility, this is a Nash equilibrium. In addition, because there is no point which satisfies the lender's breakeven constraint which is preferred to the equilibrium by only type H consumers, the equilibrium can be supported by weakly communication proof beliefs.

All other equilibria are eliminated because there exists a point arbitrarily close to B_H^*, D_H^* that is preferred to the proposed equilibrium point only by type H consumers and which would give lenders positive profits if they could lend only to type H consumers. Because the weakly communication proof test requires that lenders believe that they would lend only to type H consumers at that point, they could improve their profits by changing their strategies and lending to borrowers who seek to borrow at that point. Since the lenders could not be

70

maximizing profits at such a proposed equilibrium given weakly communication proof beliefs, it cannot be an equilibrium.

If, as depicted in Figure 3.4, the exemption level, A, is low enough that default is not a realistic possibility, the incentive compatibility constraint will not bind. Since neither type of consumer will ever go bankrupt, the budget constraint for both type H and type L consumers will be a line extending from the origin as is depicted in Figure 3.4. Because the budget constraint is the same for both types of consumer, there is no reason for one type of consumer to mimic the borrowing choices of the other.

Figure 3.4
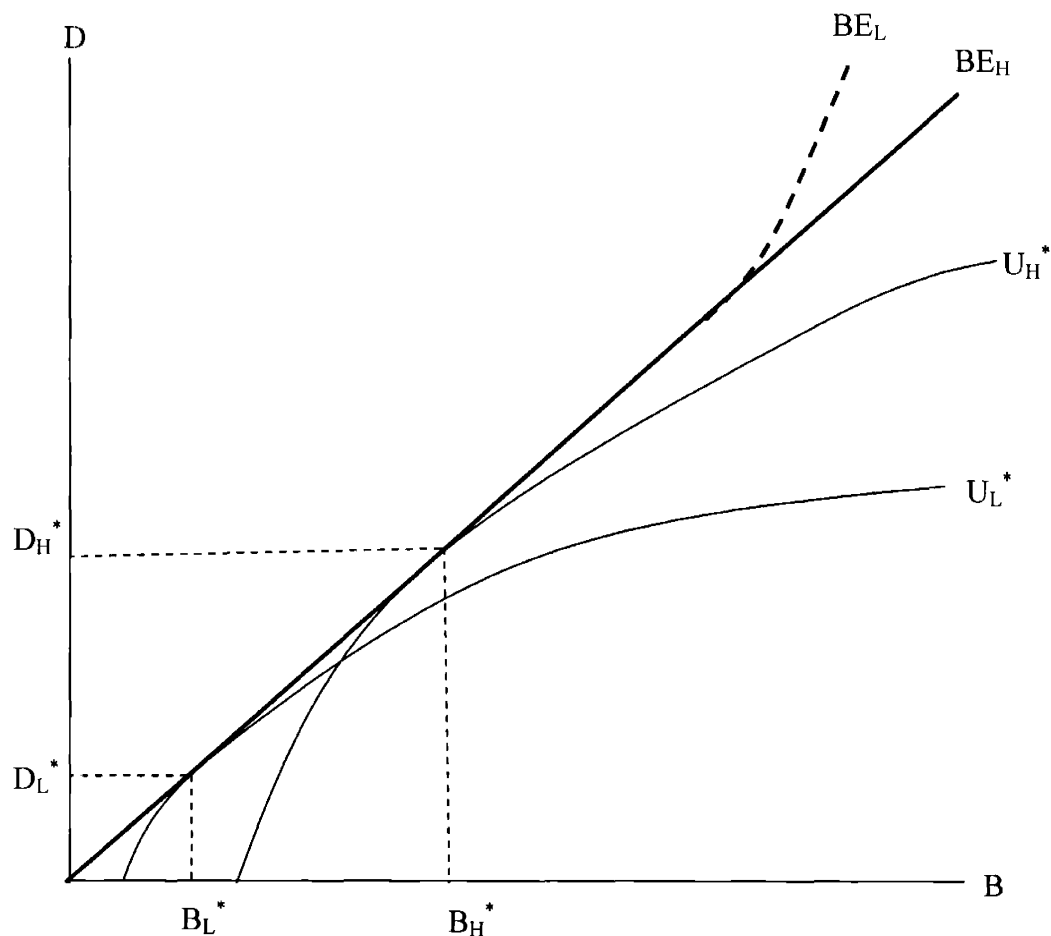
Following proposition 3.5, since the incentive compatibility constraint does not bind, both type L and type H consumers will choose to borrow the same amounts as they would if their types were observable.

As A increases, each type will continue to borrow the same amount as if their types were observable so long as type L consumers prefer B_L^* and D_L^* to B_H^* and D_H^*. The marginal change in utility to a type L consumer of borrowing the optimal amount given his own breakeven constraint, BE_L, is given by:

$$(3.5) \quad \frac{dU_L}{dA} = [u'(A) - u'(y_1 + B_L^*)][F_L(A + D_L^*) - F_L(A)] .$$

The marginal change in utility to a type L consumer of mimicking a type H consumer and borrowing B_H^* and D_H^* is given by:

$$(3.6) \quad \frac{dU_L}{dA} = u'(A)[F_L(A + D_H^*) - F_L(A)] + u'(y_1 + B_H^*)\frac{dB_H^*}{dA} - \int_A^{A+D_H^*} u'(y_2 - D_H^*)f_L dy_2 \frac{dD_H^*}{dA} .$$

While equation 3.5 is neither uniformly greater than nor uniformly less than equation 3.6., each equation is continuous. Since, type L consumers will prefer to mimic type H consumers if A is high enough (for example, if A is slightly less than M), the continuity of equations 3.5 and 3.6 implies that as A increase from levels where each type of consumer borrows the same amount as they would if their type were observable, exemption levels will eventually reach a point where type L consumers are indifferent between borrowing B_L^* and mimicking type H consumers and borrowing B_H^*. For smaller levels of A, the incentive compatibility constraint will not bind and levels of borrowing for each type will be the same as if their types were observable. Moreover, because the derivative of the difference in utility for type L consumers borrowing B_L^* and borrowing B_H^* is not monotonic, there may also be higher levels of A such that a type L consumer would prefer not to mimic a type H consumer borrowing B_H^*.

For other values of A, a type H consumer must either distort his borrowing choices to prevent a type L consumer from imitating his borrowing choices or to borrow the same amount as a type L consumer.

Figure 3.5 depicts a unique separating equilibrium of the game where type H consumers choose to borrow more in order to prevent type L consumers from mimicking their borrowing choices.

Figure 3.5
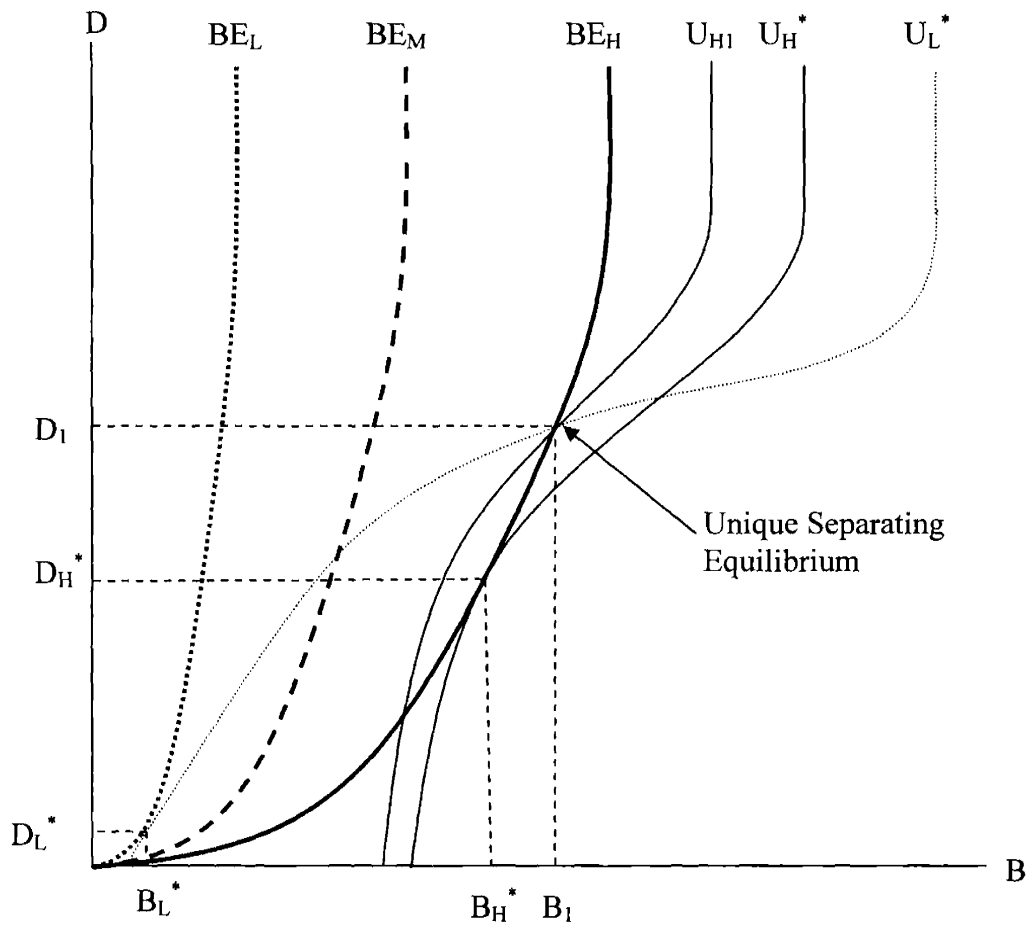
Proposition 3.6 If, as is depicted in Figure 3.5,

    a) *the incentive compatibility constraint binds;*

    b) *there is no point which satisfies the breakeven constraint for lending to type H consumers and the incentive compatibility constraint and is preferred to the upper most intersection of BE_H and U_L^* only by type H consumers; and*

    c) *type H consumers prefer the upper intersection of U_L^* and BE_H to any point on BE_L*

then, a Bayesian perfect equilibrium

    1) *with type L consumers borrowing the same amount as they would if lenders could observe their type;*

    2) *type H consumers borrowing the minimum amount possible such that the incentive compatibility constraint is satisfied; and*

    3) *lenders earn zero profits*

can be supported by weakly communication proof beliefs.

Proof and discussion:

As is the case with many signaling models, the weakly communication test eliminates all separating equilibria other than the best separating equilibrium, i.e. the separating equilibrium which gives type H consumers the highest utility. All other separating equilibria are eliminated because they are supported by beliefs which are unreasonable in the sense that they require a lender to believe that a consumer seeking to borrow an amount that would make a type L consumer worse off might never-the-less be a type L consumer.

If lenders believe that any consumer seeking to borrow an amount greater than or equal to B_1 to be a type H consumer and anyone seeking to borrow less than B_1 to be a type L consumer, then a lender lending an B_1 to a consumer in exchange for a promise to repay D_1 in the second period, would expect to earn zero profits by lending at any point on BE_H above B_1 and would therefore be willing to do so.

Given the lenders' beliefs, a utility maximizing type H consumer would choose to borrow B_1 and incur debt D_1.

Moreover, given the lender's beliefs a type L consumer would be maximizing utility by borrowing B_L^* and incurring debt D_L^*.

Thus, the borrower's behavior would be in accordance with the lender's beliefs and the strategies by consumers and lenders constitute a Bayesian Nash equilibrium.

Furthermore, because there is no point to the left of BE_H that is preferred to the type H consumer's equilibrium allocation only by type H consumers, the equilibrium can be supported by weakly communication proof beliefs.

Intuitively, because the type H consumer has greater expected second period income, the marginal disutility of debt may be lower to the type H consumer than for the type L consumer. For this reason, increased borrowing by the type H consumer may prevent the type L consumer from mimicking his borrowing. Because the type H consumer may be able to pay a lower interest rate by borrowing more, he will be willing to do so.

Proposition 3.7: If, as is depicted in Figures 3.4 or 3.5, the type H consumer's equilibrium choices of borrowing and debt, described in propositions 3.5 or 3.6 are preferred by the type H consumer to any point on BE_M, then the equilibrium will be unique.

Proof and discussion:

All Bayesian perfect equilibria, other than that described in proposition 3.5 or 3.6, are eliminated because they are supported by beliefs that violate the weakly communication test. Applying proposition 3.4, because the type H consumer's choices of borrowing and debt described in propositions 3.5 or 3.6 would be preferred to levels of borrowing and debt chosen by type H consumers in any other equilibrium only by type H consumers, those other equilibria cannot be supported by weakly communication proof beliefs.

Thus, in the circumstances described in proposition 3.6, the model's unique equilibrium will be the "best" separating or semi-separating equilibrium in the sense that of all the possible separating equilibria, the weakly communication proof equilibrium will result in the highest possible utility for type H consumers.

Because type H consumers will declare bankruptcy less often than type L consumers for a given amount of debt, it is also possible debt will be more costly to incur for type H consumer than for type L consumers. In that case, the type H consumer may borrow less than the type H consumer in a separating equilibrium. Such an equilibrium is depicted in Figure 3.6 below.

Figure 3.6
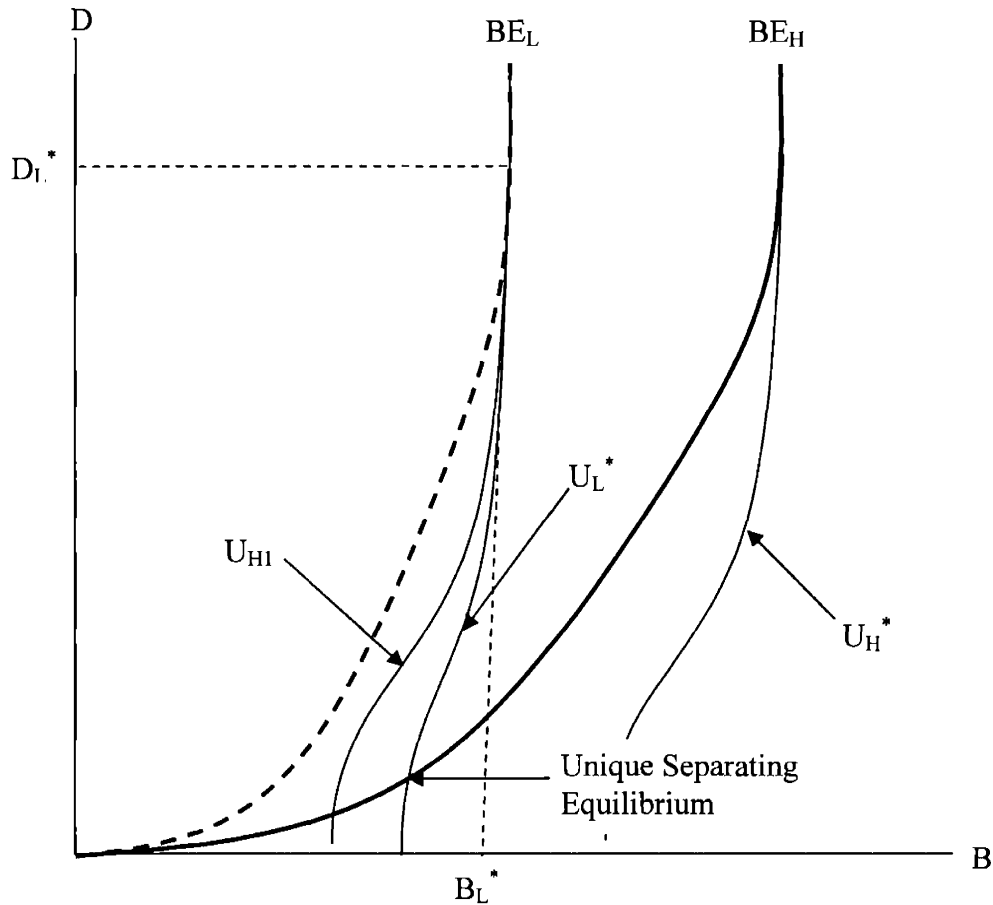
Proposition 3.8 If, as is depicted in Figure 3.6,

    a) the incentive compatibility constraint binds;

    b) there is no point which satisfies the breakeven constraint for lending to type H consumers and the incentive compatibility constraint and is preferred to the lower most intersection of BE_H and U_L^* only by type H consumers; and

    c) type H consumers prefer the lower intersection of U_L^* and BE_H to any point on BE_L

then, a Bayesian perfect equilibrium

    1) with type L consumers borrowing the same amount as they would if lenders could observe their type;

2) type H consumers borrowing the maximum amount possible such that the incentive compatibility constraint is satisfied; and

3) lenders earn zero profits

can be supported by weakly communication proof beliefs.

Proof and discussion:

Again, only the separating equilibrium which gives the type H consumer the highest possible utility given the incentive compatibility constraint can satisfy the weakly communication proof test. All other potential equilibria are eliminated because the weakly communication proof test requires lenders to believe a consumer seeking to borrow an amount slightly less than the lower (or only) intersection between U_L^* and BE_H to be a type H consumer. Given the lenders beliefs, the type H consumer could do better by borrowing an amount arbitrarily close the intersection between the U_L^* and BE_H. Consequently, the only possible separating equilibrium is the best separating equilibrium.

In contrast to the equilibrium depicted in Figure 3.5, where the type H consumer increases borrowing in order to satisfy the incentive compatibility constraint, here the type H must decrease borrowing to satisfy the incentive compatibility constraint. Because the type H consumer has a greater likelihood of having income above the bankruptcy exemption level, the marginal benefit to lowering debt below the maximum amount may be greater for type H consumers than for type L consumers. The type H consumer may, therefore, be able to satisfy the incentive compatibility constraint by borrowing less the type L consumer (and less than he would if his type were observable).

The bankruptcy exemption level will determine whether the type H consumer must distort his borrowing in order to satisfy the incentive compatibility constraint and if so whether he will borrow more or less than he would if his type were observable.

As discussed above, the incentive compatibility constraint will eventually bind as A rises from a point where type L consumers would not prefer to imitate a type H consumer borrowing B_H^*.

At exemption levels such that the incentive compatibility constraint just begins to bind, the type H consumers will prefer the upper intersection of U_L^* and BE_H to any point on BE_M and to any point on BE_L. Increases in the exempt amount which tighten the incentive compatibility

constraint will cause type H consumers to borrow more than they would if their type were observable in an effort to signal their type to lenders. Because the marginal utility of consumption is decreasing, the cost of incurring debt will be greater for consumers with lower expected future income. As a result, incurring more debt allows type H consumers to prevent type L consumers from mimicking their borrowing choices, even though the type H consumers are more likely to pay off that debt.

For higher exemption levels, debt will be more costly to incur for type H consumers since the probability that they will have income above the exempt amount is greater. At that point, it will be cheaper for type H consumers to satisfy the incentive compatibility constraint by lowering their borrowing than be raising it.

At exemption levels where the incentive compatibility constraint does not bind, the type H consumer will prefer the separating equilibrium to any point on BE_M or BE_L. Increases in the exempt amount will not monotonically decrease the difference in utility between the potential separating equilibrium point and points on BE_M and BE_L. However, at some point rising A will cause the type H consumer to prefer a point on BE_M to the intersection of U_L^* and BE_H, at which point the separating equilibrium need not be unique since, as is discussed in section 3.3, a pooling equilibrium is also possible.

Finally, as is described in section 3.4, if A is such that the type H consumer prefers some point on U_L^* between BE_M and BE_H to the intersection of U_L^* and BE_H, then unique equilibrium will be a semi-separating equilibrium.

3.2.3 The Optimal Bankruptcy Exemption Level Given a Separating Equilibrium

Having described the set of weakly communication proof Bayesian Nash separating equilibria, I now consider the effect of bankruptcy exemption levels on social welfare given that a separating equilibrium exists.

Proposition 3.9: Given weakly communication proof beliefs, the optimal bankruptcy exemption cannot occur at a point where both type L and type H consumers consume the same amount as they would if their types were observable.

Proof and discussion:

In any separating or semi-separating equilibrium, the utility for type L consumers is given by:

(3.7)

$$\max_{B_L, D_L} u(y_1 + B_L) + \int_0^A u(y_2) f_L(y_2) dy_2 + u(A)[(F_L(A + D_L) - F_L(A)] + \int_{A+D_L}^M u(y_2 - D_L) f_L(y_2) dy_2$$

subject to:

$$(3.8) \quad \int_A^{A+D_L} [y_2 - A] f_L(y_2) dy_2 + D_L[F_L(A + D_L) - F_L(A)] - B_L \geq 0$$

and the type H consumer's utility is given by:

(3.9)

$$\max_{B_H, D_H} u(y_1 + B_H) + \int_0^A u(y_2) f_H(y_2) dy_2 + u(A)[(F_H(A + D_H) - F_H(A)] + \int_{A+D_H}^M u(y_2 - D_H) f_H(y_2) dy_2$$

subject to:

$$(3.10) \quad \int_A^{A+D_H} [y_2 - A] f_H(y_2) dy_2 + D_H[F_H(A + D_H) - F_H(A)] - B_H \geq 0$$

and

$$u(y_1 + B_L) - u(y_1 + B_H) + u(A)[F_L(A + D_L) - F_L(A + D_H)] +$$

$$(3.11) \quad \int_{A+D_L}^M u(y_2 - D_L) f_L(y_2) dy_2 + \int_{A+D_H}^M u(y_L - D_H) f_L(y_2) dy_2 \geq 0$$

As was the case in Chapter 2, the socially optimal choice of A can be found by

$$(3.12) \quad \max_A V(A) = w_L V_L(A) + w_H V_H(A).$$

From the envelope theorem, I find:

$$\frac{\partial V(A)}{\partial A} = w_L \{u'(A)[F_L(A + D_L) - F_L(A)] - \lambda_L[F_L(A + D_L) - F_L(A)]\}$$

$$(3.13) \quad + w_H \{u'(A)[F_H(A + D_H) - F_H(A)] - \lambda_H[F_H(A + D_H) - F_H(A)] +$$

$$\mu\{u'(A)[F_L(A + D_L) - F_L(A + D_H)]\}\}$$

where λ_i is the Lagrange multiplier on the breakeven constraint for consumer of type i and μ is the Lagrange multiplier on the incentive compatibility constraint in the type H consumer's maximization problem.

79

If the incentive compatibility constraint does not bind, $\mu=0$ and equation (3.11) is identical to equation (2.12), the social welfare function in the case that the consumers' types are observable. Moreover, since neither type of consumer will borrow as much as possible for levels of A where the incentive compatibility constraint does not bind, equation (3.11) will be positive for all levels A where both types of consumer borrow the same amount as when their types were observable.

Put another way, if the incentive compatibility constraint does not bind, a marginal increase in the level of exempt income will not cause the type H consumer to distort his borrowing choices. The only effect of increasing the exemption level would be to provide greater insurance to both types of consumer. Consequently, there marginal value of increasing the exemption level will always be positive if both types of consumer borrow the same amount as they would if their types were observable.

Proposition 3.10: Given weakly communication proof beliefs, the optimal exempt amount may occur in a separating equilibrium either:

    a) *at the point where the marginal benefits of to providing more insurance to borrowers are equal to the marginal social loss due to type H consumers altering their borrowing behavior to signal their type; or*

    b) *at the point where the marginal cost due to credit constraining type L consumers is balanced against the marginal insurance benefit for type H consumers and the marginal social benefit due to relaxing the incentive compatibility constraint.*

Proof and discussion:

Taking the derivative of the type H and type L consumer's maximization problems with respect to B_H and B_L respectively, I find that

(3.14) $u'(y_1 + B_L) - \lambda_L = 0$,

and

(3.15) $u'(y_1 + B_H) - \lambda_H - \mu u'(y_1 + B_H) = 0$.

Setting equation (3.13) equal to zero, substituting for u'(y₁+B_L) for λ_L and u'(y₁+B_H)[1-μ] for λ_H and rearranging, I find that:

(3.16)

$$w_L\{[u'(A)-u'(y_1+B_L)][F_L(A+D_L)-F_L(A)]\} + w_H\{u'(A)-u'(y_1+B_H)][F_H(A+D_H)-F_H(A)]\} =$$
$$w_H\mu[u'(A)[F_L(A+D_H)-F_L(A+D_L)]-u'(y_1+B_H)[F_H(A+D_H)-F_H(A)]]$$

The left hand side of equation (3.16) is the same as equation (2.12) and represents the gain in welfare due to providing increased insurance. As mentioned in chapter 2, the left hand side will be positive if neither type of consumer is borrowing as much as possible, because increasing the exempt amount shifts the obligation to repay debts to states of nature where income is higher and therefore less valuable. The right hand side of equation (3.16) represents the marginal social cost of changes in borrowing by type H consumers to signal their type.

For separating equilibria where the type H consumer incurs more debt than the type L consumer, the left hand side will be positive, since neither type will be borrowing as much as possible. In that case, the derivative might be equal to zero if the right hand side of equation 3.16 is also positive, i.e. if increasing A increases the social cost to distortions in the type H consumers borrowing to signal his type.

Whereas in the complete information case, there was no cost to utility associated with raising the level of exempt assets unless consumers were credit-constrained, here raising the exemption level may cause distortions in the credit market even if neither type of consumer is credit constrained. If the marginal social cost due to signaling by type H types is equal to the marginal social benefit of providing increased insurance, then the derivative of the social welfare function will be equal to zero. In that case, socially optimal level of A may occur in a separating equilibrium where type H consumers borrow more than they would if their type were observable. In such a circumstance, the exempt amount would occur at a lower level than is optimal when a consumer's type is observable.

In the case depicted in Figure 3.6, where the type H consumer borrows less and incurs less debt than the type L consumer, the right hand side of equation 3.16 will be negative. Put another way, increasing the exempt amount will slacken the incentive compatibility constraint.

For equation 3.16 to hold, the left hand side must also be negative. Since the left hand side represents the tradeoff between insurance and credit constraints, for it to be negative, the type L consumer must be credit constrained and must be consuming less than A in the first period. In that case, increasing A will worsen the credit constraint for a type L consumer and will

81

decrease the type L consumer's utility. If the marginal cost to the type L consumer's utility is exactly balanced by the combination of increased insurance for the type H consumer and the positive effect of the exempt amount on the incentive compatibility constraint, then the derivative of the social welfare function will be equal to zero. The socially optimal exempt amount may, therefore, occur at a level higher than the social optimum if a consumer's type is observable if the marginal reduction in type L consumers' utility due to increasing credit constraints is balanced by increases in type H consumers' utility resulting from increased insurance and a reduction in the severity of the incentive compatibility constraint.

3.3 Pooling Equilibria

3.3.1 Nash Pooling Equilibria

The potential Bayesian Nash pooling equilibria, i.e. the set of potential equilibria in which type L and type H consumers borrow the same amount, is depicted in Figure 3.7 below.

Figure 3.7
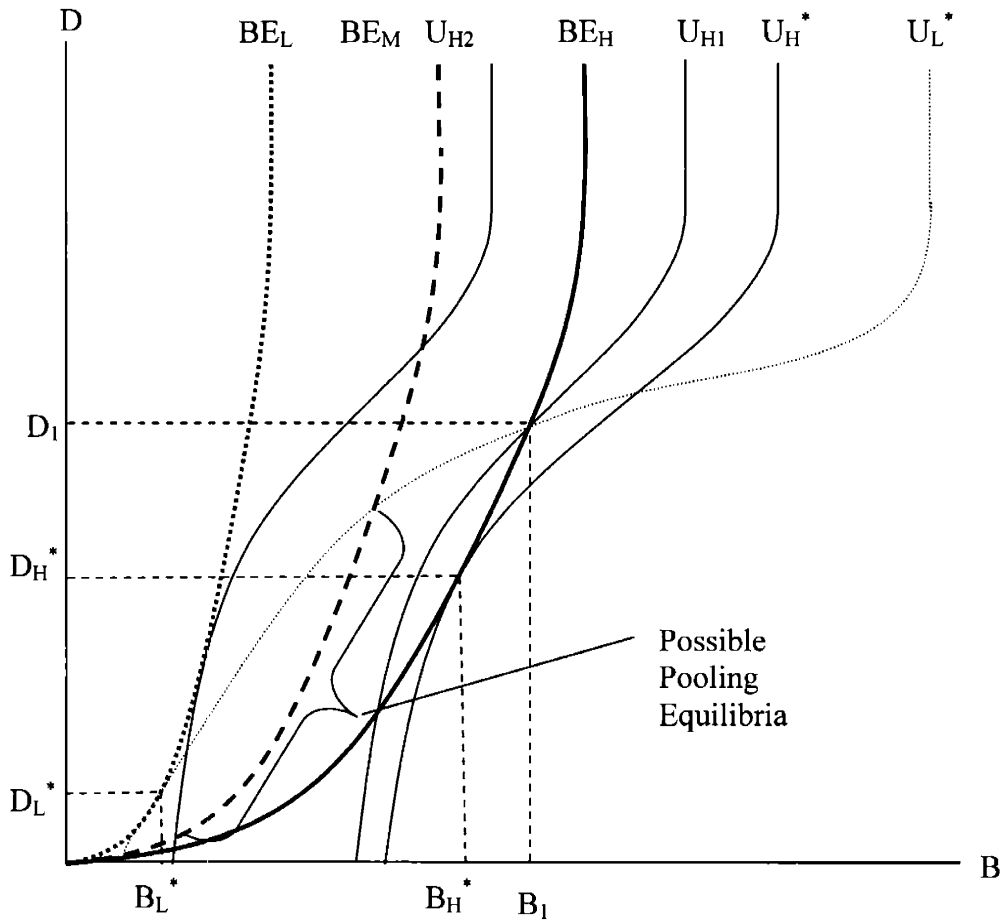
Proposition 3.11: Any point on the breakeven constraint for lenders lending to both type L and type H consumers that is preferred by both type L and type H consumers to any point on the breakeven constraint for lenders lending to only type L consumers can form a pooling equilibrium. Graphically, the equilibrium can occur at any point on BE_M, between the lower of the two intersections between U_{H2} and BE_M and the upper intersection between U_L^* and BE_M. If U_L^* does not intersect BE_M, then a pooling equilibrium can occur at any point on BE_M above the lower intersection of U_{H2} and BE_M.

Proof and discussion:

A pooling equilibrium can occur whenever the lenders' beliefs are such that both the type H and type L consumers prefer borrowing at a point on BE_M than at any other point at which lenders would expect to at least break even.

As always, any combination of borrowing and debt in a pooling equilibrium must give lenders lending to both types zero profits. That is, the combination of B and D must satisfy:

$$
\begin{aligned}
&\pi(L)\{ \int_A^{A+D_L} [y_2 - A] f_L(y_2) dy_2 + D_P[1 - F(A + D_P)]\} + \\
&(1 - \pi(L))\{ \int_A^{A+D_H} [y_2 - A] f_H(y_2) dy_2 + D_P[1 - F(A + D_P)]\} - Bp = 0
\end{aligned}
$$

(3.17)

Any level of borrowing and debt satisfying the lender's breakeven constraint such that neither type prefers a point on the BE_L to that point could be an equilibrium if supported by appropriate beliefs. Graphically, neither type L or type H consumers will prefer a point on BE_L between the lower of the two intersections between U_{H2} and BE_M and the intersection of U_L^* and BE_M. Any point in that range could be an equilibrium supported by the belief that any consumer seeking to borrow a different amount was a type L consumer.

3.3.2 Weakly Communication Proof Separating Equilibria

To narrow the number of potential pooling equilibria, I again require lender's beliefs to satisfy the weakly communication proof test. A possible weakly communication proof pooling equilibrium is depicted in Figure 3.8 below:

Figure 3.8
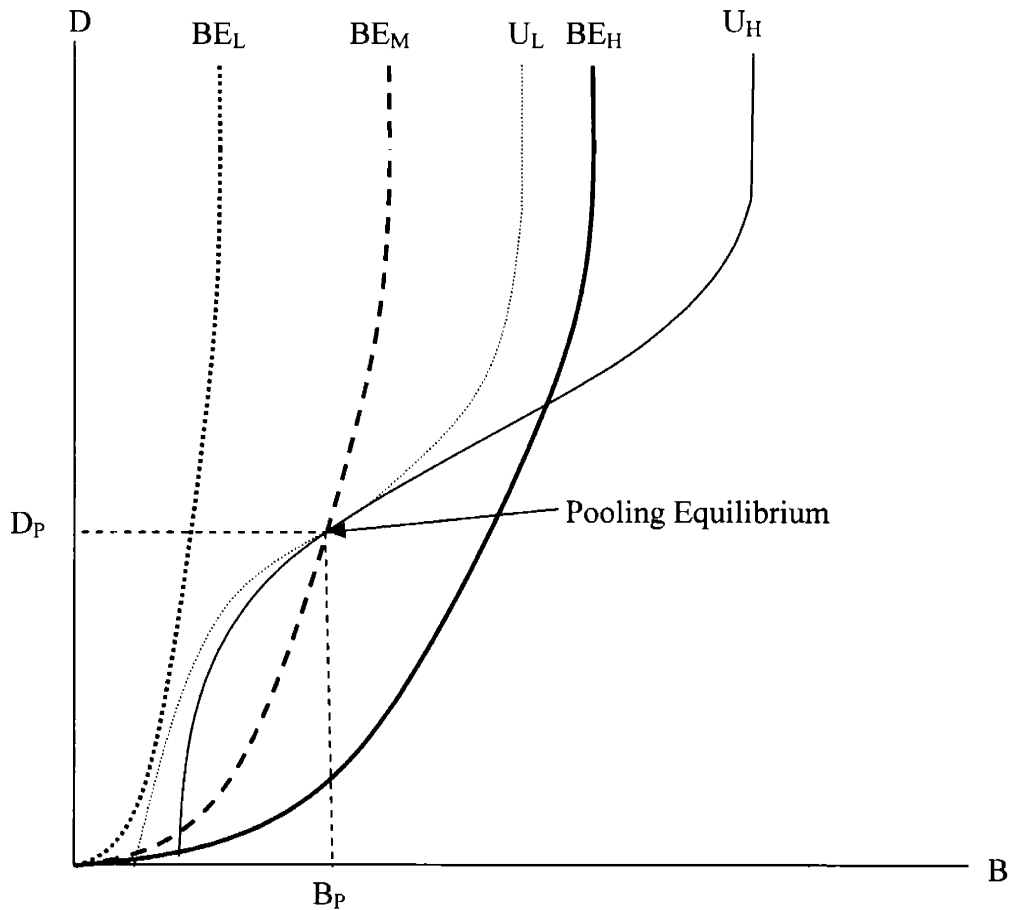
Proposition 3.12: A Bayesian perfect pooling equilibrium, such as that depicted in Figure 3.8, can be supported by beliefs which satisfy the weakly communication proof test if and only if:

a) *there exists a point of tangency between indifference curves for type H and type L consumers on BE_M;*

b) *the point of tangency between the indifference curves is preferred by type L and type H consumers to any point on BE_L; and*

c) *there is no point satisfying the breakeven constraint for type H consumers that is preferred to the point of tangency only by type H consumers.*

The only possible pooling equilibria occur with both type H and type L consumers borrowing at a point on BE_M where the indifference curves for type H and type L consumers are tangent to each other.

Proof and discussion:

In circumstances where a type H consumer prefers a point on BE_M to any point on U_L^* between BE_M and BE_H, a pooling equilibrium which satisfies the weakly communication proof test may exist at a point where the indifference curves for type H and type L consumers are tangent.

Assuming that there exists a point of tangency between type H and type L consumer's indifference curves on BE_M that is preferred to any point on BE_L by both type H and type L consumers, the point of tangency could be supported by lenders' beliefs that the probability that a consumer borrowing at that point is a type L consumer with probability $\pi(L)$, the unconditional probability of a consumer being a type L consumer, and that any consumer seeking to borrow an amount other than the amount of borrowing at that tangency point on BE_M is a type L consumer. Given these beliefs lenders would expect to earn zero profits by lending at the point on BE_M where the consumers' indifference curves are tangent. Lenders would expect to earn negative profits by lending at any point below and to the right of BE_L. Consequently, in equilibrium lenders with these beliefs would be willing to lend at any point on BE_L and on BE_M at the point of tangency between type L and type H consumer's indifference curves.

Given the lenders' strategy, utility maximizing type L or type H consumers would choose to borrow at the point on BE_M where the indifference curves for type H and type L consumers are tangent. Since consumers of both types borrow at that point with certainty, the consumers' behavior is consistent with lenders' beliefs. Finally, because if there is no point which is preferred to the equilibrium point only by type H consumers, the lenders' beliefs satisfy the weakly communication proof test. Consequently, a Bayesian Perfect Nash equilibrium with type L and type H consumers borrowing at a point on BE_M where the indifference curves between for type L and type H consumers are tangent to each other can be supported by beliefs which satisfy the weakly communication proof test.

If the consumer's indifference curves were not tangent to each other at the point on BE_M where they were borrowing, there would be a point which is preferred to the equilibrium borrowing point only by type H consumers. Therefore, the equilibrium could not be supported by weakly communication proof beliefs.

As mentioned above, for low levels of A such that type H consumers prefer the intersection of U_L^* and BE_H to any point on BE_M, a pooling equilibrium is not possible. For higher levels of A such that type H consumers prefer a point on BE_M to distorting their borrowing choices to borrow at a point to the right of and below BE_M (in either a separating or semi-separating equilibrium), a pooling equilibrium is possible.

3.3.3 Optimal Bankruptcy Exemptions in a Pooling Equilibrium

I now consider the optimal bankruptcy exemption level given a weakly communication proof pooling equilibrium.

Proposition 3.13: Given weakly communication proof beliefs, the socially optimal exempt amount may occur in a pooling equilibrium either

> a) *where a change in the exempt amount would cause lenders beliefs to change in such a way to create a separating or semi-separating equilibrium instead of a pooling equilibrium, or*

> b) *where the marginal benefit to providing greater insurance is equal to the marginal cost of changing the level of debt.*

Proof and discussion:

Given weakly communication proof beliefs a pooling equilibrium may occur only when both type H and type L consumers prefer the pooling equilibrium point to the levels of borrowing and debt that they would receive in a separating equilibrium. A change in A which would cause lenders' beliefs to change such that the pooling equilibrium would be replaced by a separating equilibrium would reduce the utility of both types of consumer. Consequently, the optimal level of A can occur at a level such that a pooling equilibrium can exist but for which a small change would cause the type of equilibrium to change from a pooling equilibrium to a separating or semi-separating equilibrium.

If changing the level of A does not change the type of equilibrium, determining the effect of A on the equilibrium is somewhat more complicated.

As described in proposition 3.12, given weakly communication proof beliefs, a pooling equilibrium can occur only where the combination of borrowing and debt lies on BE_M and the indifference curves for the two types of consumer are tangent to each other.

Since B and D must be such that they satisfy BE_M and the indifference curves for type H and type L consumers are tangent, they must satisfy:

$$(3.18) \quad \begin{aligned} &\pi(L)\{ \int_A^{A+D}[y_2 - A]f_L(y_2)dy_2 + D[1 - F(A+D)]\} + \\ &(1-\pi(L))\{ \int_A^{A+D}[y_2 - A]f_H(y_2)dy_2 + D[1 - F(A+D)]\} - B = 0 \end{aligned},$$

and

$$(3.19) \quad \frac{u'(y_1 + B)}{\int_{A+D}^M u'(y_2 - D)f_L(y_2)dy_2} = \frac{u'(y_1 + B)}{\int_{A+D}^M u'(y_2 - D)f_H(y_2)dy_2}$$

The utility for type L and type H consumers in a pooling equilibrium is given by:

$$(3.20) \quad u(y_1 + B) + \int_{A+D}^M u(y_2)f_i(y_2)dy_2 + u(A)[F_i(A+D) - F_i(A)] + \int_{A+D}^M u(y_2 - D)f_i(y_2)dy_2,$$

where i=L or H.

Taking the derivative with respect to A, I find that the change in utility for each consumer due to a change in A is equal to:

$$(3.21) \quad \frac{dU_i}{dA} = u'(A)[F_i(A+D) - F_i(A)] + u'(y_1 + B)\frac{dB}{dA} - \int_{A+D}^M u'(y_2 - D)f_i(y_2)dy_2\frac{dD}{dA}.$$

From equation (3.20) the derivative or B with respect to A is:

$$(3.22) \quad \begin{aligned} \frac{dB}{dA} &= \pi\{-[F_H(A+D) - F_H(A)] + [1 - F_H(A+D)]\frac{dD}{dA}\} \\ &+ (1-\pi)\{-[F_L(A+D) - F_L(A)] + [1 - F_L(A+D)]\frac{dD}{dA}\} \end{aligned}.$$

Taking the derivative of the social welfare function (equation 3.21) and substituting from equation (3.21) I find:

$$(3.23) \quad \begin{aligned} \frac{dV}{dA} &= w_L\{u'(A)[F_L(A+D) - F_L(A)] + u'(y_1 + B)\frac{dB}{dA} - \int_{A+D}^M u'(y_2 - D)f_L(y_2)dy_2\frac{dD}{dA}\} + \\ &w_H\{u'(A)[F_H(A+D) - F_H(A)] + u'(y_1 + B)\frac{dB}{dA} - \int_{A+D}^M u'(y_2 - D)f_H(y_2)dy_2\frac{dD}{dA}\} \end{aligned}$$

88

Assuming that the weights in the social welfare function are equal to the population fraction for each type, and substituting for $\frac{dB}{dA}$, equation (3.23) becomes:

$$\frac{dV}{dA} = [u'(A) - u'(y_1 + B)][F_m(A + D) - F_m(A)]$$

(3.24)
$$+ \{u'(y_1 + B)[1 - F_m(A + D)] - \int_{A+D}^{M} u'(y_2 - D)f_m(y_2)dy_2\}\frac{dD}{dA},$$

where F_m and f_m indicate the population weighted average of the cumulative distribution function and the density function respectively.

Because u'(A) is greater than u'(y_1+B) and F_m(A+D) is greater than F_m(A), $[u'(A) - u'(y_1 + B)][F_m(A + D) - F_m(A)]$ is always positive. As has been the case throughout this paper, this term represents the gain in utility due to the increase in insurance provided by a higher exemption level. If the equilibrium level of debt did not change as A changed, this would be the change in utility due to the change in A. As was the case in Chapter 2, the marginal utility to increasing A would positive so long as A<y_1+B and the optimal level of A would be A= y_1+B.

However, since the weakly communication proof test requires that the type L and type H consumers' indifference curves be tangent at the equilibrium, the equilibrium level of debt will have to change when A changes. As is discussed below, that change in the level of debt can have either positive or negative effects on the consumer's utility depending on whether the amount of debt increases or decreases and whether the budget constraint is steeper or flatter than the indifference curves.

The sign of the expression which is multiplied by $\frac{dD}{dA}$,

$u'(y_1 + B)[1 - F_m(A + D)] - \int_{A+D}^{M} u'(y_2 - D)f_m(y_2)dy_2$, will depend on whether the indifference curves are flatter or steeper than the budget constraint, BE_M, at the equilibrium point. The expression will be negative if BE_M is steeper than the indifference curves and positive if it is flatter. Intuitively, if the breakeven constraint is steeper than the indifference curves, then both types of consumer would be made better off by a move down the breakeven constraint, so the sign of the expression must be negative. Similarly, if the breakeven constraint is flatter than the

indifference curves, both consumers could be made better off by a move up the indifference curve so the sign of the expression is negative.

The derivative of D with respect to A can be calculated from equation (3.19):

(3.25)

$$\frac{dD}{dA} = \frac{u'(A)[f_L(A+D) - f_H(A+D)]}{\int_{A+D}^{M} u''(y_2 - D)f_L(y_2)dy_2 + u'(A)f_L(A+D) - \int_{A+D}^{M} u''(y_2 - D)f_H(y_2)dy_2 - u'(A)f_H(A+D)}.$$

Because not much can be said, in general, about the relative sizes of the positive and negative terms in equation (3.23), it is not clear whether $\frac{dD}{dA}$ is positive or negative.

If $\frac{dD}{dA}$ is the same sign as $u'(y_1 + B)[1 - F_m(A+D)] - \int_{A+D}^{M} u'(y_2 - D)f_m(y_2)dy_2$, then the net effect of changes in the level of debt due to an increase in A would be positive, while if they are of opposite signs it would be negative. Put another way, if increasing A pushes the level of borrowing towards a level which is preferred by both types, then this effect is positive. If, on the other hand, increasing A pushes the level of equilibrium debt in a direction which is not preferred by debtors, then the effect is negative.

If the net marginal effect of changes in the level of equilibrium debt due to changes in A is negative and is equal in size to the marginal benefit to consumers as a result of providing increased levels of insurance then the total derivative with respect to A will be zero and it is possible that the optimal level of A can occur at this point.

To summarize, increasing the bankruptcy exemption level will both provide increased insurance and will cause a shift in borrowing. If the shift in borrowing is advantageous for borrowers, then the increase in the exemption level will increase utility. If the shift in borrowing is disadvantageous for borrowers, then the cost associated with the shift in borrowing must be balanced against the increased insurance benefits.

3.4 Semi-separating Equilibria

Semi-separating equilibria, where type L consumers mix between borrowing the same amount as the type H consumers and borrowing another amount that is chosen only by type L consumers, are also possible model equilibria. As was the case with separating and pooling equilibria, I first discuss all possible Bayesian Nash semi-separating equilibria and then eliminate those equilibria which cannot be supported by weakly communication proof beliefs.

3.4.1 Bayesian Nash Semi-Separating Equilibria

Two possible types of semi-separating equilibria are possible. First, type L consumers may mix between borrowing at a point on BE_L and pooling with type H borrowers at a point below and to the right of BE_M. In Figure 3.9, below, these possible equilibrium pooling points are labeled "a) Possible Pooling Points in a Semi-Separating Equilibrium." Second, it is possible that type H consumers will mix between borrowing at a point on BE_H and pooling with type L borrowers at a point above and to the left of BE_M. These pooling points are labeled in Figure 3.9 as "b) Possible Pooling Points in a Semi-Separating Equilibrium."

Figure 3.9
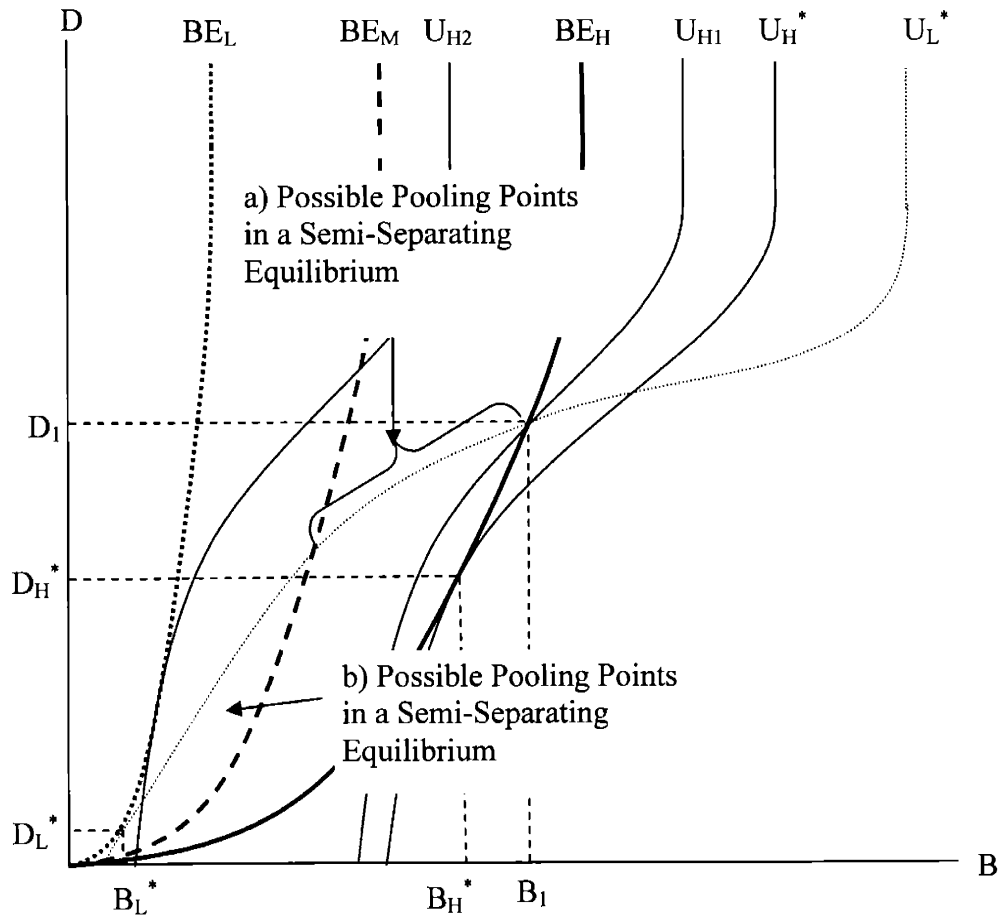
Proposition 3.14: The following are possible combinations of borrowing and debt chosen by type L and type H consumers in a semi-separating equilibria:

a) All or some type H consumers borrowing at points on U_L^* between BE_M and BE_H with the remainder (if any) borrowing at a point on BE_H such that they are indifferent to that point an the point on U_L^* and type L consumers mixing between the point on U_L^* chosen by some or all type H consumers and B_L^*, D_L^* such that lenders earn zero profits.

b) All type L consumers borrowing at a point in the area bounded by BE_M, U_L^* and U_{H2}, and type H consumers mixing between borrowing at the same point as type L consumers and borrowing at a point on BE_H such that they are indifferent between

92

the point chosen by the type L consumers and the point on BE_H and lenders earn zero profits.

Proof and Discussion:

As was the case for type H consumers in a separating equilibrium, both type H and type L consumers may alter their choices of borrowing and debt in response to a lenders beliefs. As described below, if lenders' beliefs are such that consumers are indifferent between borrowing at different points, the result may be a semi-separating equilibrium.

Because Type L consumers are indifferent between borrowing at any point on U_L^*, a semi-separating equilibrium may occur with type L consumer mixing between borrowing at B_L^*, D_L^* and borrowing at any point on U_L^*.

Depending on the lenders' beliefs, a type H consumer may be maximizing his expected utility by either always or with some positive probability borrowing at a point above U_{H2}. For example, if the lenders believe that any borrower seeking to borrow at an other level was a type L borrower and could be a mixture of type L and type H borrowers at a specific point below and to the right of U_{H2}, then the type H borrower would prefer to borrow at that point. If, given the lenders beliefs, the consumer is indifferent between borrowing on BE_H and borrowing at a point under BE_H, the type H consumer may mix between borrowing on BE_H and at a point underneath BE_H.

Included in the set of points that a utility maximizing consumer might choose given some lenders beliefs is, of course, the portion of U_L^* between BE_M and BE_H.

Thus, given appropriate lenders' beliefs, type H and type L consumers could both choose to borrow at any point on U_L^* between BE_M and BE_H. So long as the appropriate portion of type L and type H consumers choose to consume at that point, lenders would break even.

In that case, all the requirements for an equilibrium would be met.

Type L consumers prefer all points in the area labeled "b) Possible pooling points in semi-separating equilibria" to any point on BE_L. Thus, if given the opportunity to borrow at a point in that area, type L consumers would choose to borrow at a point in that area.

Furthermore, as discussed above, given appropriate beliefs, a type H consumer may be indifferent between borrowing at any point in area B and a point on BE_H.

93

If the right proportion of type H consumers choose to borrow at a particular point in the area bounded by BE_M, U_L^* and U_{H2}, then lenders can earn zero profits.

Consequently, a semi-separating equilibrium could be supported by appropriate beliefs with all type L consumers borrowing at a point in the area bounded by BE_M, U_L^* and U_{H2}, and type H consumers mixing between borrowing at the same point as type L consumers and borrowing at a point on BE_H such that they are indifferent between the point chosen by the type L consumers and the point on BE_H.

Again, it is important to note that whether a semi-separating equilibrium exists and, if it does, the exact combinations of B and D that occur depend on a combination of consumers' preferences, breakeven constraints and lenders beliefs.

3.4.2 Weakly Communication Proof Semi-Separating Equilibria

In order to reduce the number of possible semi-separating equilibria, I again require that lenders' beliefs satisfy the weakly communication proof test. Like the separating equilibrium case, I find that only the semi-separating equilibrium which gives type H consumers the highest utility consistent with the incentive compatibility constraint can be supported by weakly communication proof beliefs.

Figures 3.10 and 3.11 illustrate potential weakly communication proof semi-separating equilibria. Figure 3.10 illustrates a potential semi-separating equilibrium where type H consumers incur more debt than type L consumers borrowing on BE_L, while Figure 3.11 illustrates a semi-separating equilibrium where type H consumers incur less debt than type L consumers borrowing B_L^*.

Figure 3.10
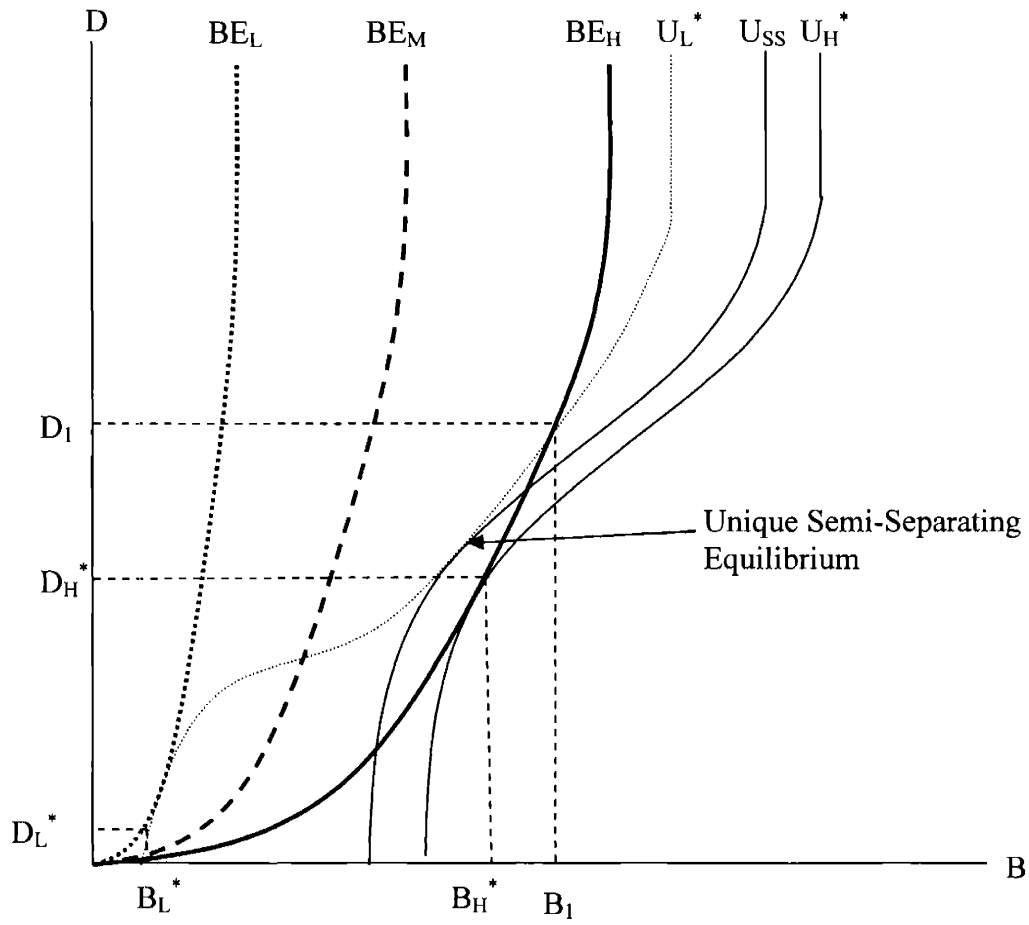
Figure 3.11
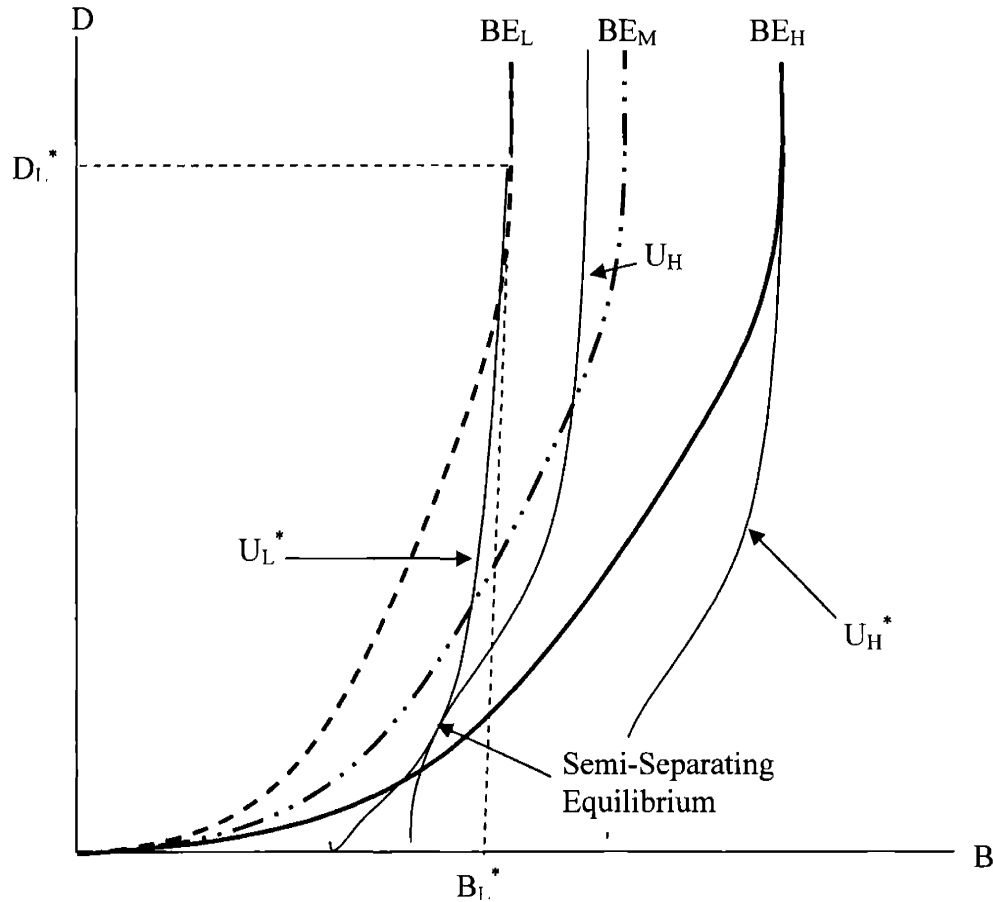
Proposition 3.15: If, as depicted in Figure 3.10 or 3.11:

   a)   the incentive compatibility constraint binds;

   b)   there exists a point which satisfies both the breakeven constraint for lending to type
        H consumers and the incentive compatibility constraint and is preferred to the upper
        most intersection of BE_H and U_L^* only by type H consumers; and

   c)   there is a point on U_L^* which satisfies the breakeven constraint for a type H
        consumer, that a type H consumer prefers to any point on BE_L,

then, a Bayesian perfect equilibrium with

1) type H consumers borrowing at the point (or points) on U_L^* between BE_H and BE_M which gives the type H consumer the most utility;

2) type L consumers mixing between borrowing the same amount as they would if lenders could observe their type and at the same point on U_L^* as type H consumers; and

3) with lenders earning zero profits

can be supported by beliefs which satisfy the weakly communication proof restrictions on beliefs.

Proof and discussion:

Again the weakly communication proof test eliminates all semi-separating equilibria where type L consumers mix between borrowing on BE_L and pooling with type H consumers at a point on U_L^* other than the semi-separating equilibrium which gives the highest utility to type H consumers. All other equilibria violate the weakly communication proof test because the type H consumer, and only the type H consumer, would prefer another point under BE_H to the potential equilibrium point. Following proposition 3.4, these equilibria cannot be supported by weakly communication proof beliefs.

The semi-separating equilibrium which gives type H consumers the highest possible utility given the incentive compatibility constraint can be supported by weakly communication proof beliefs. For example, suppose that lenders believe that the probability that a consumer borrowing an amount between the level of borrowing at the upper most intersection of U_L^* and BE_M and B_1 is such that a lender would breakeven by making the loan in exchange for a promise to repay an amount D such that the point lies on U_L^* and that a consumer borrowing more the B_1 would certainly be a type H consumer. In that case, he would expect to earn zero profits by entering into a loan contract at any point on U_L^* between BE_M and BE_H or at any point on BE_H above B_1. Since he would expect to break even, the lender would be willing to make a loan at any such point.

Given the lenders beliefs, a utility maximizing type H consumer would, naturally, choose the point on U_L^* between BE_H and BE_M that gives him the highest utility.

Type L consumers are indifferent between borrowing at any point along U_L^*. For this reason, utility maximizing type L consumers would be willing to mix between borrowing at B_L^* and D_L^* and the point on U_L^* preferred by type H consumers.

So long as the fraction of type L consumers choosing the point preferred by type H consumers is such that lenders earn zero profits, the borrowers behavior will be consistent with the lender's beliefs and, since all actors are maximizing their utility subject to the behavior of the others, the strategies constitute a Bayesian Perfect Nash equilibrium.

Finally, because there is no point to the left of BE_H that is preferred to the type H consumer's equilibrium choices of borrowing and debt only by type H consumers, the equilibrium can be supported by weakly communication proof beliefs.

Proposition 3.16: If, as is depicted in Figure 3.10, the type H consumer's equilibrium choices of borrowing and debt, described in propositions 3.15 are preferred by the type H consumer to any point on BE_M, then the equilibrium will be unique.

Proof and discussion:

All Bayesian perfect equilibria, other than that described in proposition 3.15, are eliminated because they are supported by beliefs that violate the weakly communication test. Applying proposition 3.4, because the type H consumer's choices of borrowing and debt described in propositions 3.15 would be preferred to levels of borrowing and debt chosen by type H consumers in any other equilibrium only by type H consumers, those other equilibria cannot be supported by weakly communication proof beliefs.

The model's unique equilibrium is the best separating or semi-separating equilibrium in the sense that of all possible equilibria, the weakly communication proof equilibrium will result in the highest possible utility for type H consumers.

Semi-separating equilibria cannot occur at low levels of A where the incentive compatibility constraint does not bind. For higher levels of A such that type H consumers prefer borrowing at a point on U_L^* between BE_M and BE_H to any point on BE_M or to the intersection of U_L^* and BE_H, the semi-separating equilibrium will be the unique weakly communication proof equilibrium.

If A is such that the consumer prefers a point on BE_M to the separating equilibrium, that equilibrium will not be unique since a pooling equilibrium is also possible.

Proposition 3.17: Semi-separating equilibria where type H consumers mix between borrowing at a point on BE_H and borrowing at a point to the left of BE_M cannot be supported by beliefs which satisfy the weakly communication proof test, except in the circumstance that both the type L and type H consumers are indifferent between the two points that type H consumers are mixing between and there is no point under BE_H that is preferred to the type H consumers' choices of borrowing and debt only by type H consumers.

Proof and discussion:

In a semi-separating equilibrium where type L consumers choose to borrow only at a point under BE_M, type L consumers must weakly prefer that point to the point on BE_H at which type H consumers are borrowing with positive probability. If the type L consumers strictly prefer to borrow at the point under BE_M, then there exists a point under BE_H which is strictly preferred to the type H consumers' equilibrium choices of borrowing and debt only by type H consumers. Applying proposition 3.4, these equilibria cannot be supported by beliefs which satisfy the weakly communication proof test.

If the type L consumers are indifferent between borrowing at the point under BE_M and the point on BE_H that type H consumers borrow at, and there are no other points that are preferred to the type H consumer's equilibrium choices of borrowing and debt only by type H consumers, then the equilibrium can be supported by beliefs which satisfy the weakly communication proof test.

Because the indifference curves are not convex, it is possible that a type H and type L consumer's indifference curves would cross more than once. For that reason, it is impossible to rule out the possibility that both a type H and a type L consumer will be indifferent between borrowing at a point on BE_H and a point below BE_M. However, there is no reason to expect this condition to be satisfied in most cases. It is therefore possible to rule out separating equilibria where a type H consumer mixes between borrowing at a point on BE_H and at a point under BE_M in all but the special case where both types of consumer happen to be indifferent.

3.4.3 Optimal Bankruptcy Exemption Levels Given a Semi-Separating Equilibrium

Having described the model's potential weakly communication proof semi-separating equilibria, I now consider the effect of the bankruptcy exemption level on social welfare at those exemptions.

Because type L consumers in a weakly communication proof semi-separating equilibrium borrow the same amount as they would if their type were observable, the effect of the bankruptcy exemption level on a type L consumer's utility is the same in a semi-separating equilibrium as if their type were observable. The effect of changes in exemption levels on a type L consumer's utility is therefore given by:

$$(3.26) \quad \frac{dU_L}{dA} = [u'(A) - u'(y_1 + B_L{}^*)][F_L(A + D_L{}^*) - F_L(A)].$$

Again, equation 3.26 represents the marginal gain due to greater insurance or the marginal loss due to increasing credit constraints when A is raised. So long as $A < y_1 + B_L$, equation 3.26 will be positive. If $A > y_1 + B_L$, the type L consumer will be credit constrained and increasing A will decrease utility for type L consumers.

In a semi-separating equilibrium, type H consumers solve the same maximization problem as consumers in a separating equilibrium described in section 3.1. Specifically, the equilibrium choice of B_H and D_H solves:

(3.27)

$$\max_{B_H, D_H} u(y_1 + B_H) + \int_0^A u(y_2) f_H(y_2) dy_2 + u(A)[(F_H(A + D_H) - F_H(A)] + \int_{A+D_H}^M u(y_2 - D_H) f_H(y_2) dy_2$$

subject to:

$$(3.28) \quad \int_A^{A+D_H} [y_2 - A] f_H(y_2) dy_2 + D_H[F_H(A + D_H) - F_H(A)] - B_H \geq 0$$

and

$$(3.29) \quad \begin{aligned} &u(c_L) - u(c_H) + u(A)[F_L(A + D_L) - F_L(A + D_H)] + \\ &\int_{A+D_L}^M u(y_2 - D_L) f_L(y_2) dy_2 + \int_{A+D_H}^M u(y_L - D_H) f_L(y_2) dy_2 \geq 0 \end{aligned}$$

The derivative of the Lagrangian for the maximization problem with respect to B_H and D_H are respectively:

(3.30) $u'(y_1 + B_H) - \lambda - \mu u'(y_1 + B_H) = 0$

and

(3.31) $- \int_{A+D_H}^{M} u'(y_2 - D_H)f_H(y_2)dy_2 + \lambda[1 - F_H(A + D_H)] + \mu \int_{A+D_H}^{M} u'(y_2 - D_H)f_L(y_2)dy_2 = 0$

where λ is the Lagrange multiplier on the breakeven constraint and μ is the multiplier on the incentive compatibility constraint. Since the breakeven constraints do not bind in a semi-separating equilibrium, λ is zero, and equation 3.28 reduces to:

(3.32) $\mu=1$.

Given that $\mu=1$ and $\lambda=0$, the effect of a small change in the exemption level on a type H consumer's utility is given by:

(3.33) $\dfrac{dV_H}{dA} = u'(A)[F_H(A + D_H) - F_H(A)] + u'(A)[F_L(A + D_L) - F_L(A + D_H)]$.

The first term, $u'(A)[F_H(A+D_H)-F_H(A)]$ represents the increase in utility resulting from higher consumption in the event that the consumer declares bankruptcy and makes some payments to his creditors. The second term, $[u'(A)[F_L(A + D_L) - F_L(A + D_H)]$, represents the change in utility resulting from shifts in U_L^*. If $D_H>D_L^*$ this term will always be negative. Consequently, increasing A results in a shift in the tangency between U_L^* and U_H which reduces the type H consumer's utility for levels of A where the type H consumer incurs more debt than the type L consumer borrowing on BE_L. If, as depicted in Figure 3.11, $D_H<D_L^*$, the second term will be positive, indicating that the shift in borrowing resulting from the increase in A is beneficial to type H consumers.

The social welfare function is given by:

(3.34) $\max_A V(A) = w_L V_L(A) + w_H V_H(A)$.

Taking the derivative of equation 3.32 and substituting for $\dfrac{dV_L}{dA}$ and $\dfrac{dV_H}{dA}$, I find:

(3.35) $\dfrac{dV(A)}{dA} = w_L \{u'(A)[F_L(A + D_L) - F_L(A)] - u'(y_1 + B_L)[F_L(A + D_L) - F_L(A)]\}$

$+ w_H \{u'(A)[F_H(A + D_H) - F_H(A)] + [u'(A)[F_L(A + D_L) - F_L(A + D_H)]\}$

101

The change in utility for type L consumers will be positive if the type L consumer will not be credit constrained. Moreover, $u'(A)[F_H(A+D_H) - F_H(A)]$ will always be positive, reflecting the increase in utility from increasing consumption in the event that second period income is between A and A+D$_H$. Finally, as discussed above $[u'(A)[F_L(A+D_L) - F_L(A+D_H)]$ will be negative if D$_H$>D$_L^*$. The optimal exemption level can occur in a semi-separating equilibrium where D$_H$>D$_L^*$ if $[u'(A)[F_L(A+D_L) - F_L(A+D_H)]$ is large enough to exactly offset the other gains in utility from increasing A. In that case, the derivative would be equal to zero and that level of A might be the socially optimal level.

Alternatively, the socially optimal exempt amount may occur at a separating equilibrium at a point where D$_H$<D$_L^*$ if the social loss due to increasing credit constraints on type L borrowers is offset by gains to type H borrowers. In that case, the term

$$[u'(A) - u'(y_1 + B_L^*)][F_L(A+D_L^*) - F_L(A)]$$ would be negative since type L consumers are credit constrained. At the same time, $u'(A)[F_H(A+D_H) - F_H(A)]$ would be positive since increasing the bankruptcy exemption level provides greater insurance to type H borrowers. Finally, as discussed above, the term $[u'(A)[F_L(A+D_L) - F_L(A+D_H)]$ is positive if D$_H$<D$_L^*$ indicating that higher exemption levels relax the incentive compatibility constraint. If the marginal insurance benefit plus the benefit due to relaxing the incentive compatibility constraint offset the loss to type L consumers due to increasing credit constraints, equation 3.35 will be equal to zero and that exemption level might be a social optimum.

To summarize, the socially optimal exemption level may occur in a semi-separating equilibrium either 1) where the increased insurance benefit to both type L and to type H consumers is balanced by increases in marginal incentive costs to type H consumers or 2) where the cost to increasing credit-constraints to type L consumers is balanced by the marginal insurance benefit provided to type H consumers and the benefit associated with relaxing the incentive compatibility constraint.

3.5 Conclusion

Asymmetric information in credit markets can cause distortions in borrowing as consumers attempt to either signal their credit worthiness to lenders or to conceal it from them. For levels of the bankruptcy exemption which are low enough that default is not a realistic option for any borrowers, the presence of asymmetric information will not cause any distortions in the borrowing choices of either type of consumer. In this circumstance, small increases in the bankruptcy exemption level will provide greater insurance to both types of consumer, but will not cause any distortions in borrowing. Consequently, social welfare will increase.

For higher exemption levels, consumers with higher expected second period incomes will have to distort their borrowing in order to maintain a separating equilibrium. In that case, increases in the bankruptcy exemption level may worsen the distortions needed to maintain a separating equilibrium. The costs to social welfare of these distortions in borrowing must be balanced against the insurance benefit provided by higher exemptions. The socially optimal exemption level may occur at the point where the marginal cost due to increased distortions is equal to the marginal insurance benefit. Alternatively, the socially optimal bankruptcy exemption level may occur where the cost due to increasing credit constraints on type L consumers is balanced against the benefit of an increase in insurance provided to type H consumers plus the benefit resulting from a relaxing of the incentive compatibility constraint.

A pooling equilibrium can occur at exemption levels which are high enough such that both types of consumer prefer a pooling equilibrium to the best separating equilibrium. For this reason, a small change in the bankruptcy exemption levels which causes the type of equilibrium to shift will reduce welfare for both types of consumer. Furthermore, since the amount of borrowing at a pooling equilibrium will change with the bankruptcy exemption, losses in social welfare may result from shifts in borrowing away from the optimal level of borrowing in a pooling equilibrium. Consequently, given a pooling equilibrium, the optimal bankruptcy exemption level may occur where either a small change in the exemption level would change the type of equilibrium or where the insurance benefits provided by higher exemption levels are balanced against costs due to changes in borrowing away from the ideal level of borrowing given a pooling equilibrium.

Finally, semi-separating equilibria can occur if consumers with high expected second period income prefer borrowing at a point inside their budget constraint to any incentive compatible point on their budget constraint. Again, changes in exemption levels will cause distortions in the equilibrium amount borrowed which must be balanced against either the increased insurance provided by higher exemptions or the increased credit constraints resulting from higher exemption levels.

In each type of potential equilibria, changes in the exemption level have the potential to create distortions in borrowing choices. The costs associated with these distortions must be balanced against the insurance benefits provided by higher exemptions in setting an optimal exemption policy.

Chapter 4: Personal Bankruptcy and Incentives to Work

Like all systems of insurance, the possibility to avoid repayment of debts provided by personal bankruptcy will affect incentives for borrowers to work hard and incentives for borrowers to take risk. For example, because the bankruptcy discharge increases a borrower's consumption in states of nature where his income is low, the possibility to receive a discharge of unpaid debts may decrease the consumer's incentive to exert effort in order to avoid low income realizations.

Alder, Polak and Schwartz (2000) present a simple model of bankruptcy with exogenous borrowing where a borrower can have either low or high income realizations. When consumers are risk averse and the probability of having a low income realization is not affected by the consumer's work effort, the optimal bankruptcy exemption level provides as much insurance as possible while allowing creditors to earn zero expected profits. If consumers are risk neutral, and the probability of having a low income realization depends on a borrowers work effort, then the optimal bankruptcy exemption level is equal to zero. Alder, Pollack and Schwartz argue that just as the tension between providing incentives to work and reducing the variation in consumption reduces the optimal level of insurance, so too incentive effects should lower the optimal bankruptcy exemption level.

This chapter presents the results of a more complicated model of bankruptcy with exogenous borrowing and a continuous second period income realizations which challenge the idea that moral hazard will necessarily reduce incentives to work, increase incentives to take risk or lower the optimal bankruptcy exemption level. I argue that there are two fundamental differences between determining optimal bankruptcy exemption levels and determining the optimal level of insurance. First, even very high exemption levels do not provide complete insurance. Because relief granted by a bankruptcy court is limited to granting a discharge of an insolvent borrower's debts, higher exemption levels will not help a borrower who has less than the exempt amount in assets. Higher exemption levels increase consumption only for debtors who file for bankruptcy and who make some payments to their creditors. If those debtors are more likely for a job with a higher expected income or less risk, then increasing exemption levels can provide borrowers with a greater incentive to work and to reduce risk.

Second, whereas the only option for improving incentives in a typical insurance model is to allow for more variation in consumption, i.e. to provide less insurance, with bankruptcy exemption levels, incentives can be improved either by reducing the insurance benefit of the system by lowering exemptions or by increasing exemptions and exacerbating a consumer's credit constraint. Whether the optimal exemption level is higher or lower in the presence of incentive problems depends on whether it is cheaper, in utility terms, to improve incentives by reducing the amount of insurance provided by the system or by reducing the amount that a consumer can borrow.

To examine the effect of bankruptcy exemption levels on consumer borrowing, job choice and utility where borrowers can determine how hard to work and how much risk to take, I modify the model presented in Chapter 1 to allow the probability distribution of second period income to depend on a consumer's choice. I assume that a consumer can choose between two jobs in the second period, which will have different probability distributions of income and which will require different levels of effort. Section 4.1 describes the effect of bankruptcy exemption levels on consumers' borrowing and job choices and consumers' utility in a model where consumers choose between two second period jobs, one of which has a second period income distribution which first-order stochastically dominates the other. In general, increasing the bankruptcy exemption level will neither monotonically increase nor decrease the attractiveness of either job relative to the other. Moreover, the optimal exemption level may be higher than, lower than or the same as would be the case if the borrower could commit in advance to his second period job choice.

In section 4.2, I consider a model where the consumer must choose between two jobs which differ in terms of risk, i.e., where the probability distribution of second period income from one job is a mean preserving spread of the probability distribution of income for the other. Again, increasing the bankruptcy exemption level will neither monotonically increase nor decrease the incentive for the consumer to choose either job. Indeed, at high exemption levels, consumers may choose jobs which are safer than they would be if exemption levels were lower. Again, the optimal exemption level may be higher than, lower than or the same as it would if there were no incentive problem.

Section 4.3 concludes.

4.1 First Order Stochastic Dominance

4.1.1 Equilibrium

I begin by assuming that the consumer can choose between two jobs in the second period, labeled job J and job K. Job J is assumed to be first-order stochastically dominated by job K, i.e. $F_J(y_2) \geq F_K(y_2)$ for all y_2 and $F_J(y_2) > F_K(y_2)$ for some y_2. The consumer will choose whichever job maximizes utility in the second period. That is, the consumer's job choice will be the solution to the problem:

$$(4.1) \quad \max_i = U_{2i} = \int_0^A u(y_2) f_i(y_2) dy_2 + u(A)[F_i(A+D) - F_i(A)] + \int_{A+D}^M u(y_2 - D) f_i(y_2) dy_2 - e_i$$

Given that, in the second period, the consumer will choose the job which maximizes his second period utility, he will choose a level of borrowing that maximizes:

(4.2)

$$\max_{B,D} u(y_1 + B) + \int_0^A u(y_2) f_i(y_2) dy_2 + u(A)[F_i(A+D) - F_i(A)] + \int_{A+D}^M u(y_2 - D) f_i(y_2) dy_2 - e_i \text{ subject}$$

to:

$$(4.3) \quad \int_A^{A+D} [y_2 - A] f_i(y_2) dy_2 + D[1 - F_i(A+D)] - B \geq 0$$

and

$$(4.4) \quad
\begin{array}{l}
\int_0^A u(y_2) f_i(y_2) dy_2 + u(A)[F_i(A+D) - F_i(A)] + \int_{A+D}^M u(y_2 - D) f_i(y_2) dy_2 - e_i \geq \\
\int_0^A u(y_2) f_{-i}(y_2) dy_2 + u(A)[F_{-i}(A+D) - F_{-i}(A)] + \int_{A+D}^M u(y_2 - D) f_{-i}(y_2) dy_2 - e_{-i}
\end{array}$$

where equation (4.3) is the lender's breakeven constraint and equation (4.4) is the incentive compatibility constraint.

Figure 4.1 depicts graphically the utility maximization problem for consumers facing as choice between jobs J and K, where job J is first order stochastically dominated by job K but requires less effort than job J.

107
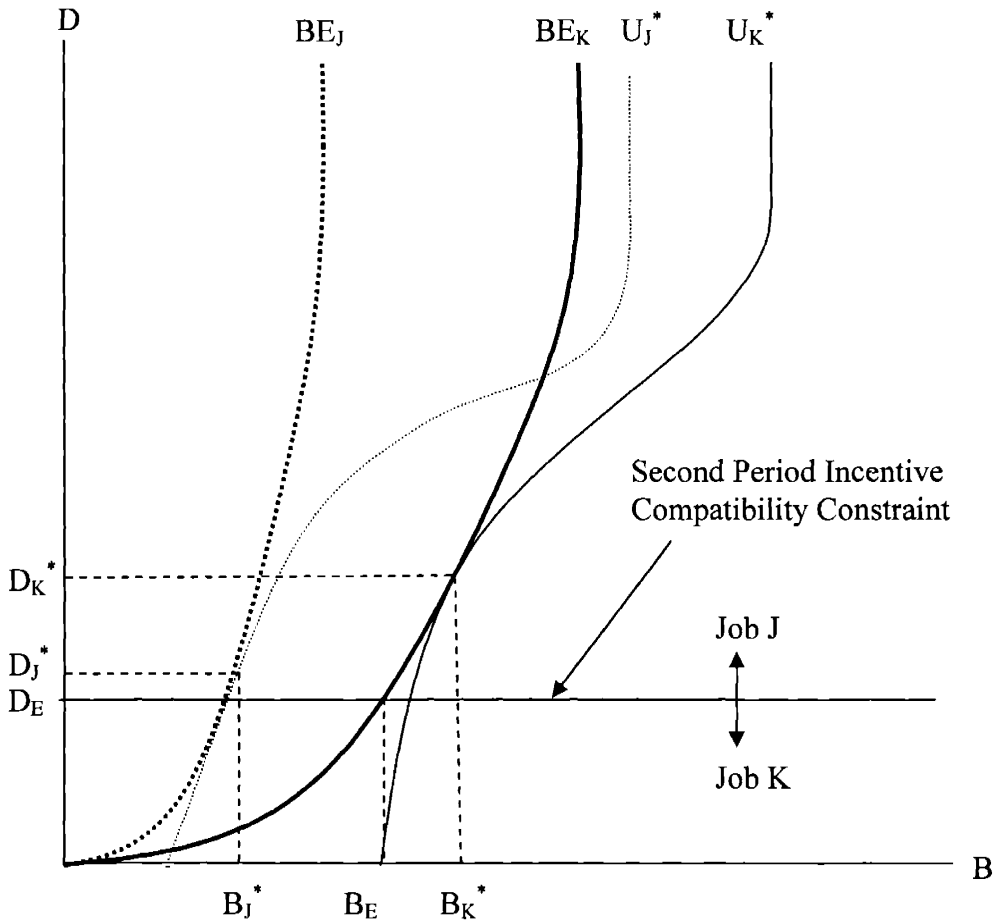
Figure 4.1

Most of the curves in Figure 4.2 are identical to those depicted in section 2.2, where consumers differed as to differences in future income. The curves BE_J and BE_K are the breakeven constraints for lenders lending to consumers choosing jobs J and K respectively, while the curves U_J^* and U_K^* are the indifference curves for consumers with type J and type K jobs respectively at the optimal borrowing choice for those jobs if there were no incentive problems.

The difference between the model depicted in Figure 4.2 and those discussed in Section 2.2, where consumers differed as to distributions of second period income, but where there was no incentive problem, is the second period incentive compatibility constraint. Given the bankruptcy exemption level, A, and the second period income distributions associated with each job, equation 4.1 describes a consumer's job choice as a function of the level of debt held by the consumer. In Figure 4.1, the incentive compatibility constraint is depicted as a level of debt, below which the consumer will choose job K and above which he will choose job J. While this is

one possible scenario, there is no reason why the consumer's job choice cannot change back and forth as debt rises.

In the consumer's maximization problem depicted in Figure 4.1, the consumer cannot borrow the optimal amount for consumer's with job K, B_K^*, because the amount of debt required to satisfy the breakeven constraint would violate the incentive compatibility constraint. He must therefore choose to borrow either at B_E, the maximum amount which satisfies both the incentive compatibility and breakeven constraints or he must borrow an amount B_J^* and incur a debt D_J^* which would cause him to choose job J in the second period.

Proposition 4.1: When the consumer's distribution of second period income depends on his own effort, the consumer will borrow:

a) *An amount such that the marginal utility of first period consumption equals the expected marginal utility of consumption in the second period conditional on the consumer paying his debts in full or the maximum amount possible, if the incentive compatibility constraint does not bind on the margin.*

b) *An amount larger than described in (1) if larger levels of debt increase the relative attractiveness of job K relative to job J in the second period.*

c) *A smaller amount than that described in (1) if larger levels of debt decrease the attractiveness of job J relative to job K in the second period.*

d) *The maximum amount possible given the breakeven constraint for the job which the consumer will choose if his debt, D≥M-A.*

Proof and Discussion:

The level of borrowing and debt chosen by a utility maximizing consumer can be found by setting the derivatives of the Lagrangian for the maximization problem described in equations 4.2, 4.3 and 4.4 to zero. Taking the derivative, I find that the consumer will choose B and D to satisfy:

(4.5) $u'(y_1 + B) - \lambda = 0$

and

$$- \int_{A+D}^{M} u'(y_2 - D) f_i(y_2) dy_2 + \lambda [1 - F_i(A+D)] +$$

$$(4.6)$$

$$\delta [\int_{A+D}^{M} u'(y_2 - D) f_{-i}(y_2) dy_2 - \int_{A+D}^{M} u'(y_2 - D) f_i(y_2) dy_2] = 0$$

where λ and δ are the Lagrange multipliers associated with equations 4.3 and 4.4 respectively.

If the incentive compatibility constraint (equation 4.4) does not bind, then the consumer will choose to borrow either at an interior tangency or will borrow as much as possible given the budget constraint. If the consumer borrows at an interior tangency, he will choose B and D such that:

$$(4.7) \quad u'(y_1 + B) = \frac{\int_{A+D}^{M} u'(y_2 - D) f_i(y_2) dy_2}{[1 - F_i(A+D)]},$$

that is such that the marginal utility of consumption in the first period equals expected consumption in the second period given that the consumer does not go bankrupt. This is, of course, the same condition that characterizes the level of borrowing when the consumer's second period distribution of income does not depend on which job he chooses.

If the incentive compatibility constraint does bind, then, rearranging equation (4.6), I find:

$$u'(y_1 + B)[1 - F_i(A+D)] - \int_{A+D}^{M} u'(y_2 - D) f_i(y_2) dy_2 =$$

$$(4.8)$$

$$\delta [\int_{A+D}^{M} u'(y_2 - D) f_i(y_2) dy_2 - \int_{A+D}^{M} u'(y_2 - D) f_{-i}(y_2) dy_2]$$

Because, the right hand side of the equation is not zero, a consumer borrowing at an interior tangency will distort his borrowing choice such that the incentive compatibility constraint is satisfied.

The direction of the distortion will depend on the relative sizes of

$\int_{A+D}^{M} u'(y_2 - D) f_i(y_2) dy_2$ and $\int_{A+D}^{M} u'(y_2 - D) f_{-i}(y_2) dy_2$. The integrals represent the effect of a

small change in the amount of debt on the relative attractiveness of jobs i and –i.

Where, as here, job J has a second period income distribution which is first order stochastically dominated by job K, increasing the amount of debt has two effects on the relative

110

attractiveness of jobs J and K. First, because having more debt decreases consumption only in states of nature when the consumer doesn't go bankrupt and because the consumer will go bankrupt less often if he chooses job K, a larger level debt may decrease the return to choosing job K in utility terms. Put another way, since increasing debt decreases consumption in states of nature which are more likely to occur with job K than with job J, increasing debt may decrease the consumer's incentive to choose the job with the higher average income in the second period. Second, because u''<0, the changes to utility will be larger for income levels which are just high enough to avoid bankruptcy and which, depending on the level of A and D, might be more common for consumers with job J. This will tend to cause an increase in debt to make job K relatively more attractive to consumers even if it reduces income in states of nature which are, on average, more likely if the consumer chooses job K.

If the first effect dominates and the incentive compatibility constraint binds, the consumer will borrow less than he otherwise would have in order to give himself motivation to work hard in the second period. The right hand side of equation 4.8 would be positive and borrowing would be lower than in equation 4.6.

Returning to Figure 4.1, the incentive compatibility constraint requires that debt be less than D_E if the consumer was to choose job K in the second period. Because the optimal level of borrowing for consumers with job K in the absence of the incentive constraint is not feasible, the consumer must borrow less than he would have in the absence of the potential incentive effect associated with holding particular levels of debt.

If, however, the second effect dominates, i.e. if marginal utility is sufficiently greater for states of nature which are more likely under job J than for states of nature that are more likely for job K, then the right hand side of equation (4.8) would be negative and the amount of borrowing would be higher than if the incentive compatibility constraint did not bind.

Alternatively, the consumer may again borrow as much as possible given the incentive compatibility and break even constraints. In Figure 4.1, since a consumer with a high level of debt would choose job J, the consumer might borrow the maximum amount possible given the breakeven constraint BE_J.

111

4.1.2 The Effect of the Bankruptcy Exemption Level on Job Choices

Because the bankruptcy exemption level alters the amount of consumption that debtors receive at different income levels, it has the potential to affect consumer's decisions about how much to borrow and which job to take.

Proposition 4.2: As the bankruptcy exemption level increases, the difference in utility associated with each job will not, in general, increase or decrease monotonically. Consequently, the marginal effect of raising the bankruptcy exemption on work effort is ambiguous.

Proof and discussion:

Applying the envelope theorem to the maximization problem described in section 4.1.1, the derivative of the consumer's utility from borrowing choices that will implement a certain job are given by equation 4.10.

$$(4.9) \quad \frac{dV(A)}{dA} = [u'(A) - u'(y_1 + B)][F_i(A + D) - F_i(A)]$$
$$+ \delta u'(A)\{[F_i(A + D) - F_i(A)] - [F_{-i}(A + D) - F_{-i}(A)]\}$$

The change in the difference in utility between job K and job J is therefore given by:

$$\frac{dV_K}{dA} - \frac{dV_J}{dA} = [u'(A) - u'(y_1 + B_K)][F_K(A + D_K) - F_K(A)]$$
$$(4.10) \quad + u'(A)\delta_K\{[F_K(A + D_K) - F_K(A)] - [F_J(A + D_K) - F_J(A)]\}.$$
$$- [u'(A) - u'(y_1 + B_J)][F_J(A + D_J) - F_J(A)]$$
$$- u'(A)\delta_J\{[F_J(A + D_J) - F_J(A)] - [F_K(A + D_J) - F_K(A)]\}$$

Equation (4.9) will not, in general, be uniformly positive or negative. First, while the insurance benefit associated with raising the exemption level, $[u'(A) - u'(y_1 + B_i)][F_i(A + D_i) - F_i(A)]$, will increase for both job J and job K so long as A<y₁+Bᵢ, it is not clear for which job the benefit will increase the most. Indeed, it is likely that for some values of A, the increase in the insurance benefit is larger job J, while the increase is larger for job K at other values of A.

Second, the change in utility associated with incentive effects,

$\delta_i u'(A)\{[F_i(A + D_i) - F_i(A)] - [F_{-i}(A + D_i) - F_{-i}(A)]\}$. will be different for the maximum utility associated with choosing job J and the maximum utility associated with choosing job K.

To understand how a marginal change in the exemption level effects the incentive cost to the maximum utility associated with each job, consider the example depicted in Figure 4.1. In that case, a change in A which moves the second period incentive compatibility constraint upwards decreases the incentive cost to utility associated with job K, but, because the constraint is not binding for job J, the change does not affect the incentive cost associated with job J.

If, contrary to the way it is drawn in Figure 4.1, the incentive compatibility constraint were above D_H^*, then a change in A which moved the constraint upwards would decrease the incentive cost associated with job J, but would not effect the incentive cost to utility associated with job K.

Moreover, even the direction of the effect of changes in the bankruptcy exemption level on the incentive compatibility constraint is unclear.

Figure 4.2
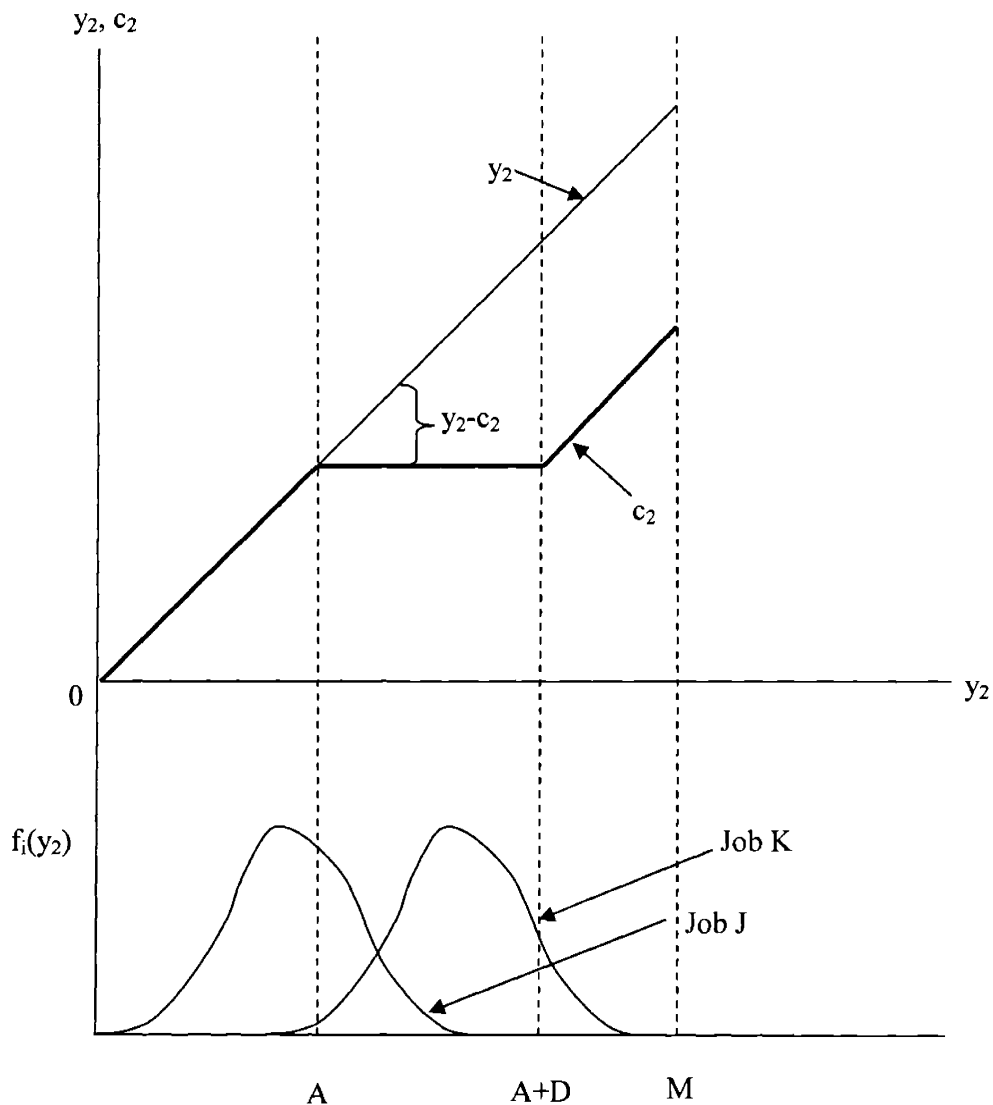
For instance, given the probability distributions of income associated with jobs J and K depicted in Figure 4.2, a small increase in A when A is as is depicted in the figure or larger, would increase consumption in states of nature which are more likely for consumers choosing job K than those choosing job J. As a result, an increase in A at that level would increase the attractiveness of job K relative to job J.

At the same time, if A is small, the flat portion of consumption that would be directly effected by a small change in A would be further to the left. As a result, a small increase in A when A is small would increase consumption in states of nature that are more likely for

consumers choosing job J than for consumers choosing job K and therefore should increase the attractiveness of job J relative to job J.

Unlike the model presented by Alder, Polak and Schwartz, which has only two possible outcomes, here there are three ranges of possible outcomes: income below the exempt amount, income above the exempt amount but not high enough to pay creditors in full, and income high enough for all debts to be paid in full. With only two possible outcomes, raising exemption levels raises consumption in the state of nature with the lowest income. Since the low income realization is more likely when the consumer works less hard, increasing exemption levels decreases the incentive to work hard.

Raising exemption levels in a model with a continuum of possible income realizations only increases consumption in the middle range of outcomes, where borrowers earn enough to make some payments to creditors. Since this middle range might be more likely for consumers with a job with a higher expected income, it is possible that increasing exemption levels will increase incentives to work hard.

4.1.3 The effect of the Bankruptcy Exemption Level on Consumer Borrowing

The effect of an increase in the bankruptcy exemption on the consumer's borrowing decision will depend on whether the incentive compatibility constraint binds, how the incentive compatibility constraint changes with the exemption level and whether an increase in the exemption level would cause the consumer to switch jobs.

Proposition 4.3: If the incentive compatibility constraint does not bind, increasing the exemption level will increase borrowing if the consumer is not credit constrained and will decrease borrowing if the consumer is credit constrained.

Proof and discussion:

If the incentive compatibility constraint does not bind, then the consumer's borrowing decision will be the same as in Chapter 1 and the effect of changes in the bankruptcy exemption level will be the same as for when the consumer cannot effect the distribution of second period income by choosing between two jobs. Specifically, so long as the consumer does not borrow as

much as possible, increasing the exemption level will shift the obligation to repay debt to states of nature where the marginal utility of consumption is lower. As a result, borrowing will be cheaper in utility terms and consumers will borrow more. If, on the other hand, the consumer is borrowing the maximum amount possible, then increasing the exemption level will reduce the amount borrowed because it decreases the expected value of future income which can be committed to repaying a loan.

Proposition 4.4: *If the incentive compatibility constraint does bind, but consumers will not switch jobs as a result of a small change in exemption level, increasing the exemption level*

    a) *will decrease borrowing if increasing the exemption level shifts the incentive compatibility constraint downwards; and*

    b) *may either increase or decrease borrowing if increasing the exemption level shifts the incentive compatibility constraint upwards*

Proof and discussion:

    If the incentive compatibility constraint does bind, the amount borrowed will be determined by the intersection of the incentive compatibility constraint and the breakeven constraint for whichever job the consumer will choose in the second period. Since the incentive compatibility constraint will hold with equality, it defines the consumer's choice of debt as a function of the exempt amount A, D(A). Substituting D(A) into the breakeven constraint for job i, I find that if the incentive compatibility constraint binds, the consumer's level of borrowing is determined by:

$$(4.11) \quad B = \int_{A}^{A+D(A)} [y_2 - A] f_i(y_2) dy_2 + D(A)[1 - F_i(A + D(A))].$$

Taking the derivative of equation 4.12 with respect to A, I find that if the incentive compatibility constraint binds, but the consumer does not switch jobs when the exemption is raised,

$$(4.12) \quad \frac{dB}{dA} = -[F_i(A + D(A)) - F_i(A)] + [1 - F_i(A + D(A))]\frac{dD(A)}{dA}.$$

    The first term, $-[F_i(A + D(A)) - F_i(A)]$, represents the decrease in the amount that can be borrowed if the amount of debt remains constant as the exempt amount increases. This first

116

term is always negative because increasing the exempt amount decreases the amount that creditors are paid in bankruptcy.

The second term, $[1 - F_i(A + D(A))]\dfrac{dD(A)}{dA}$, represents the change in the amount borrowed when the amount of debt changes. Because increasing A can either shift up or down the incentive compatibility constraint, the sign of $\dfrac{dD(A)}{dA}$ can be either positive or negative. Consequently, the change in borrowing due to changes in the amount of debt incurred can be either positive or negative.

If the incentive compatibility constraint shifts downward, then both the first and second terms in equation 4.11 will be negative and borrowing will fall when the exemption level rises.

If, on the other hand, the incentive compatibility constraint shifts upward, then the first term will be negative and the second term will be positive. The direction of the net effect will, of course, depend on which term is larger, i.e. whether the reduction in the amount received by creditors due to the increase in the exemption level is larger or smaller than the increase in the amount received by creditors due to the increased amount of debt resulting from the shift upwards in the incentive compatibility constraint.

Proposition 4.5: If a consumer switches jobs as a result of an increase in the exemption level, the consumers may either increase or decrease borrowing when the exemption level increases.

Proof and discussion:

Consider again the consumer's maximization problem depicted in Figure 4.1. In that case, the consumer will borrow either $B_J{}^*$, incur debt $D_J{}^*$ and choose job J in the second period, or will borrow B_E, incur debt D_E and choose job K in the second period depending on which combination gives him the higher utility.

If the consumer is indifferent between each combination, then increasing A may cause the consumer to switch between the two possible combinations. Whether this increases borrowing will depend on, among other things, the direction of the change and the effect of the change on preferences, the budget constraint and on the incentive compatibility constraint. For example, suppose that, in Figure 4.1, an increase in the exempt amount causes a consumer to

switch from borrowing at B_J^* to B_E and causes the incentive compatibility constraint to shift upwards. In that case, the increase will have caused the consumer to borrow more. If instead, it caused a shift from B_J^* to B_E, but that the incentive compatibility constraint shifted down by a sufficiently large amount, the amount of borrowing may decrease.

4.1.4 The Optimal Bankruptcy Exemption Level

Proposition 4.6: The optimal bankruptcy exemption level when a consumer's second period distribution of income depends on his work effort will be:

    a) equal to first period consumption if the incentive compatibility constraint does not bind,

    b) greater than first period consumption if:

        1) the incentive compatibility constraint binds and,

        2) the loss in utility due to increasing credit constraints required to provide the consumer with the incentive to choose job K is less than the loss in utility due to a decrease in insurance required to provide the consumer with the incentive to choose job K by lowering the exempt amount.

    c) smaller than first period consumption if:

        1) the incentive compatibility constraint binds and,

        2) the loss in utility due to increasing credit constraints required to provide the consumer with the incentive to choose job K is greater than the loss in utility due to a decrease in insurance required to provide the consumer with the incentive to choose job K by lowering the exempt amount.

Proof and discussion:

      Applying the envelope theorem to the maximization problem described in equations 4.2, 4.3 and 4.4, I find that:

$$(4.13) \quad \frac{dV(A)}{dA} = u'(A)[F_i(A+D) - F_i(A)] - \lambda[F_i(A+D) - F_i(A)]$$
$$+ \delta\{u'(A)[F_i(A+D) - F_i(A)] - u'(A)[F_{-i}(A+D) - F_{-i}(A)]\}$$

Substituting u'(y₁+B) for λ and rearranging, I find that at the optimum:

$$(4.14) \quad \frac{dV(A)}{dA} = [u'(A) - u'(y_1 + B)][F_i(A+D) - F_i(A)]$$
$$+ \delta u'(A)\{[F_i(A+D) - F_i(A)] - [F_{-i}(A+D) - F_{-i}(A)]\}$$

The first term in equation 4.13, $[u'(A) - u'(y_1 + B)][F_i(A+D) - F_i(A)]$ represents the increase in utility associated with providing greater insurance. As discussed in Chapter 1, so long as A>y₁+B, this benefit will be positive. If, however, A<y₁+B, then consumer will be credit constrained and any insurance benefits associated with raising A will be more than offset by losses in utility due to the fact that the consumer is credit constrained.

If the incentive compatibility constraint does not bind, then δ=0 and equation 4.13 reduces to the same condition as describes the optimum level of A in the case where the consumer's second period income is independent of his choice of jobs. In that case, the optimum exemption level occurs at the point where A=y₁+B_max.

If the incentive compatibility constraint does bind, then there are two possibilities. First, the exempt amount might be set lower than the point where A=y₁+B_max at the point where the marginal insurance benefit to increasing A is equal to the marginal benefit to loosening the incentive compatibility constraint. In this way, the amount of insurance provided by the bankruptcy system would be reduced in order to provide incentives to work in the second period.

A second possibility is that the exempt amount might be set higher than the point where A=y₁+B_max at the point where the marginal loss due to worsening incentive compatibility constraints is equal to the marginal benefit to loosening the incentive compatibility constraint. Because increasing the exempt amount increases the payoff to having an income greater than the exempt amount, which is more likely to occur for consumers with job K than with job J increasing the exempt amount above the point where A=y₁+B_max will increase the incentive that the consumer has to choose job K. In the extreme, setting A=M would result in a situation where the consumer could not borrow at all (since he would never pay anything back), but where there would be no distortions in his job choice since he would consume his entire second period income.

It should be noted that because the term

$\delta u'(A)\{[F_i(A+D)-F_i(A)]-[F_{-i}(A+D)-F_{-i}(A)]\}$ is not either uniformly increasing or

decreasing with A, that there may be more than one point at which the derivative of the indirect

utility function is equal to zero. Consequently, the fact that equation 4.13 is equal to zero is a

necessary rather than sufficient condition for the optimal level of A.

The optimal exempt amount will occur at the point where equation 4.13 is equal to zero

which results in the highest level of utility for the consumer. If that point occurs where

$A<y_1+B_{max}$, then it can be inferred that reducing the amount of insurance provided relative to the

optimum is cheaper in utility terms than increasing the exempt amount and worsening consumers

credit constraints. If the utility cost to worsening the credit constraints for consumers is cheaper

in utility terms than reducing the amount of insurance provided by the bankruptcy system, then

the optimal exempt amount will occur at a point where $A>y_1+B_{max}$.

The effect of moral hazard on the optimal bankruptcy exemption differs from the effect

of moral hazard on the amount of insurance provided in a typical insurance model in that either

raising or lowering the bankruptcy exemption level may provide incentives to exert additional

effort required for a job with higher earnings. In a typical insurance model, complete insurance,

which provides a constant level of consumption, is optimal in the absence of any potential moral

hazard. In that case, the only way to provide an incentive to exert additional effort is to provide

some differential in consumption depending on how much is earned, i.e. to reduce the amount of

insurance provided.

With bankruptcy, however, raising the exemption level has two effects. First, raising the

exemption increases consumption for those who declare bankruptcy, but who make some

payment to their creditors. Second, raising the exemption level lowers consumption for

consumers who do not declare bankruptcy since they will face higher debts. Since raising the

exemption levels raises consumption for some income realizations and lowers exemption levels

for other income realizations, either raising or lowering exemption levels may provide additional

incentives to work.

4.2. Consumer Incentives to Take Risk

Section 4.1. considered the potential effects of bankruptcy exemption levels on the incentive for a consumer to chose a job which requires greater effort, but which also pays more. This section considers the effect of bankruptcy exemption levels on the incentive for a consumer to take risk. Specifically, I consider the same model as was discussed in Section 4.1. with the modification that the consumer can choose between two jobs, P and Q, where the probability distribution of income for job P is a mean preserving spread of the probability distribution of income for job Q, i.e. job P is riskier than job Q. Finally, it is assumed that job P requires an effort with utility cost e_P and job Q requires effort with utility cost e_Q. Although many of the results from Section 4.1 carry over to the distributions considered in this section, the interpretation of the relative distributions differ,

4.2.1 Model Equilibrium

The consumer's maximization problem is the same as that in section 4.1.1 with the exception that income distributions for the two jobs are different.

The consumers maximization problem for two different levels of A is depicted in Figures 4.3 and 4.4 below:

Figure 4.3
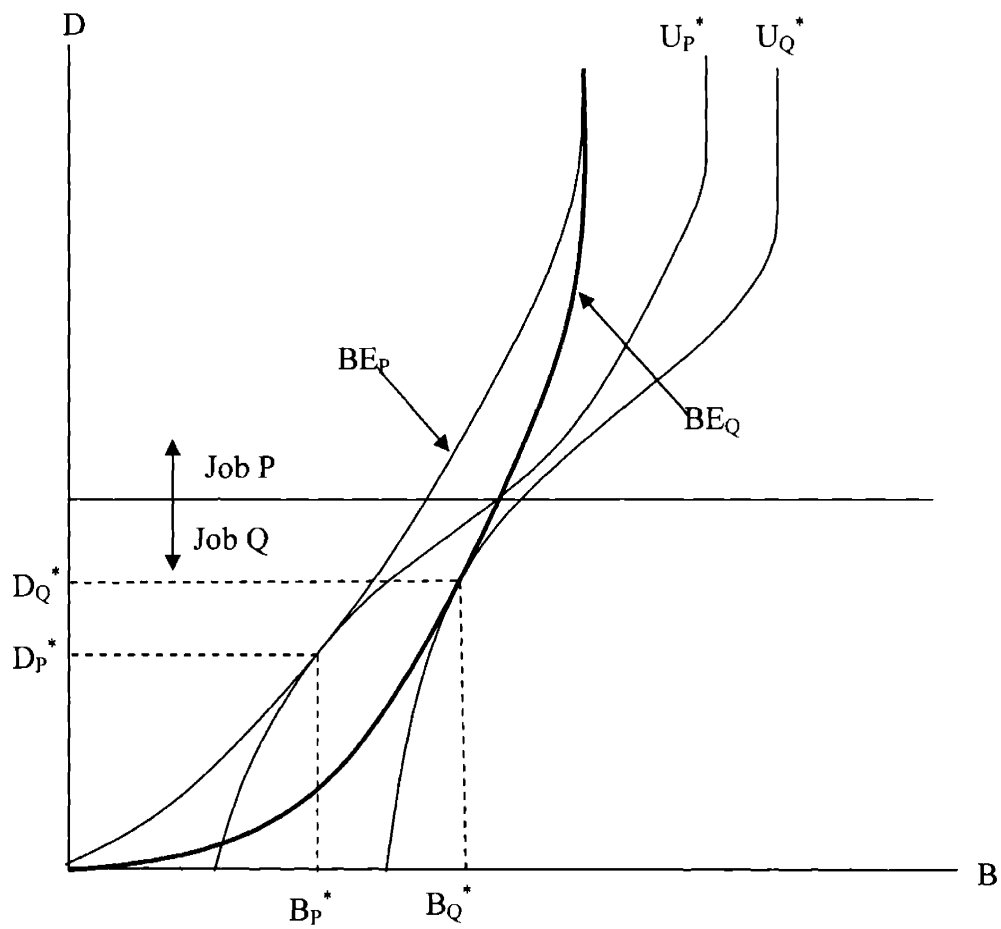
Figure 4.4
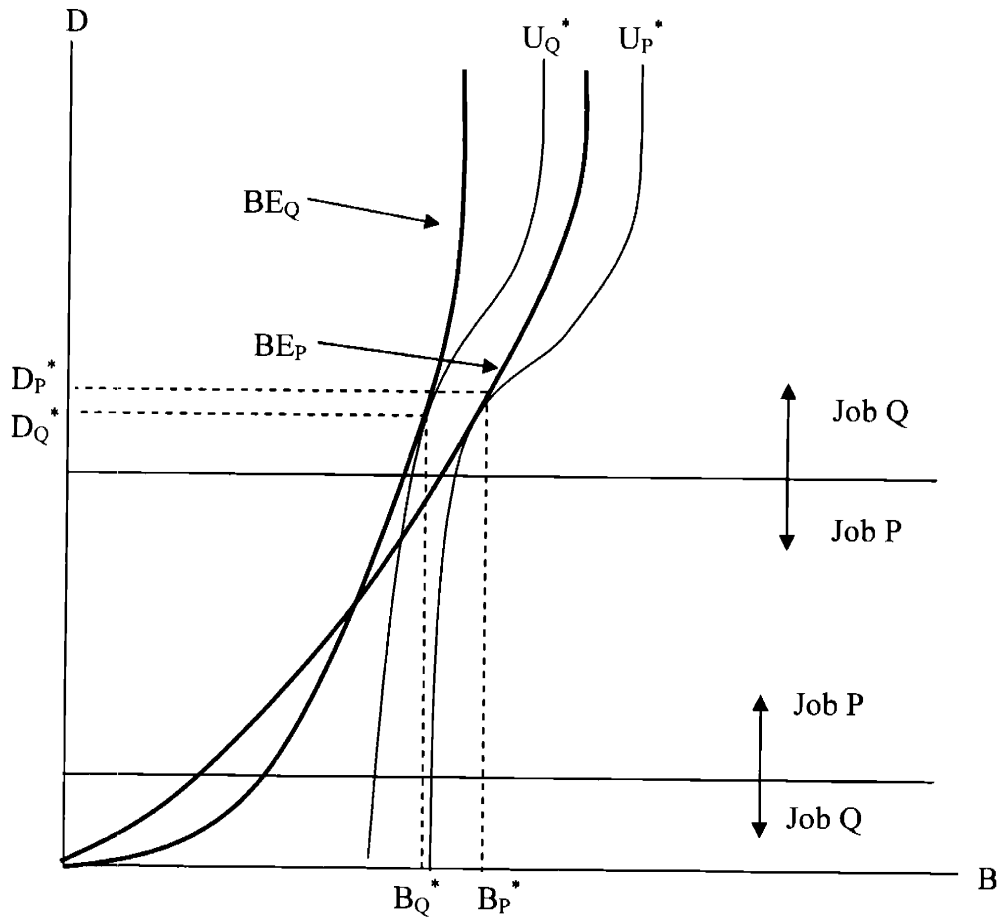
Figures 4.3 and 4.4 are essentially the same as the figures depicting the optimal level of borrowing for consumers in section 2.3, where consumers' second period income distributions differed according to the level of risk, but with the addition of the incentive compatibility constraints. Figure 4.3 illustrates the breakeven constraints and indifference curves facing the consumer if $A=0$, while figure 4.4 depicts the breakeven constraints and indifference curves facing a consumer if $0<A<y_c$, where y_c is the crossing point between the low and high risk cumulative distribution functions.

The greater the difference in risk between the two jobs, the flatter the budget constraint for the safer job will be relative to the riskier job below y_c and the steeper it will be relative to the riskier job above y_c. In the extreme, if the safe job gives income y_c with certainty, the budget constraint of the safe job will have a slope equal to one for borrowing and debt less than y_c-A and an infinite slope for levels of debt greater than y_c-A.

123

A consumer's consumption as a function of his second period income for different levels of debt is depicted in Figure 4.5 if A=0.

Figure 4.5
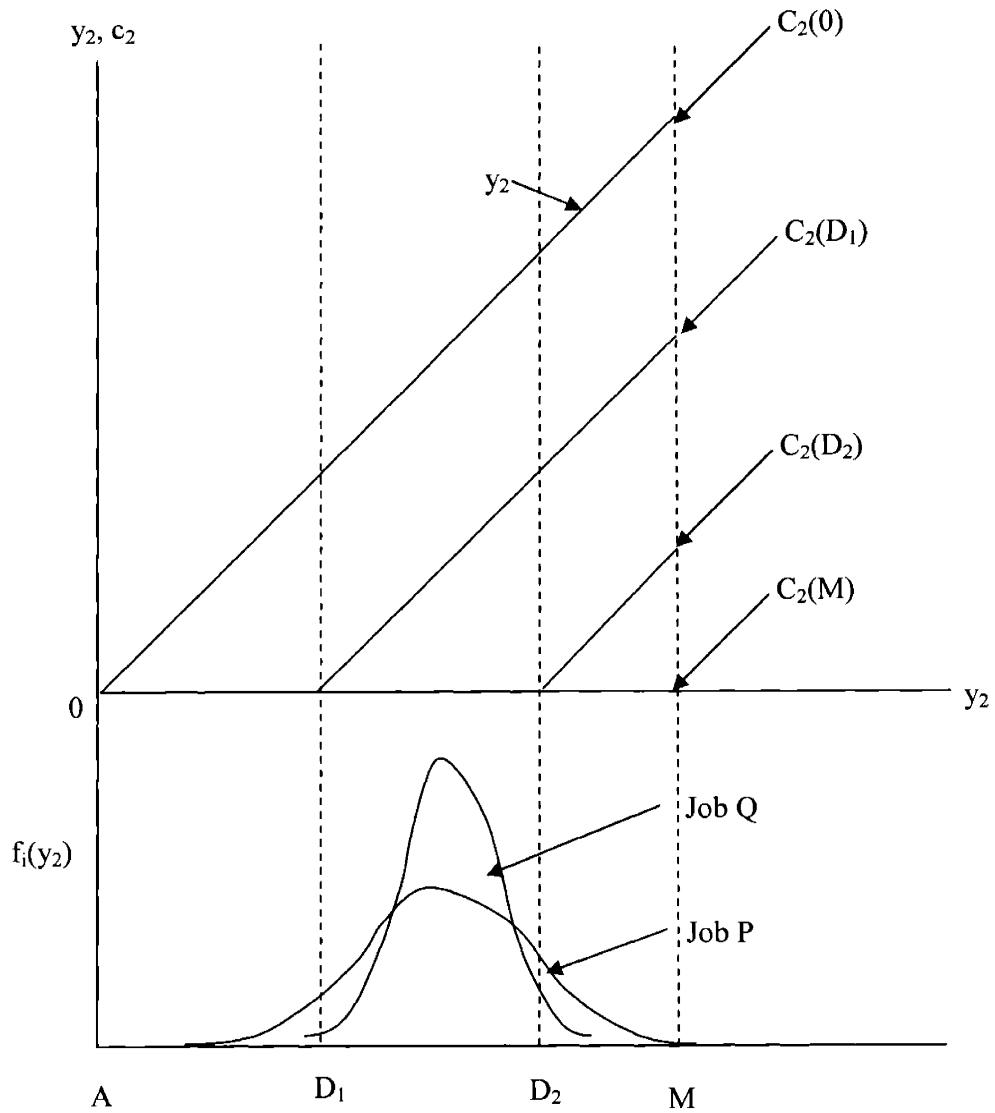
A consumer's consumption as a function of his second period income for different levels of debt is depicted in Figure 4.6 if $0<A<y_c$.

Figure 4.6
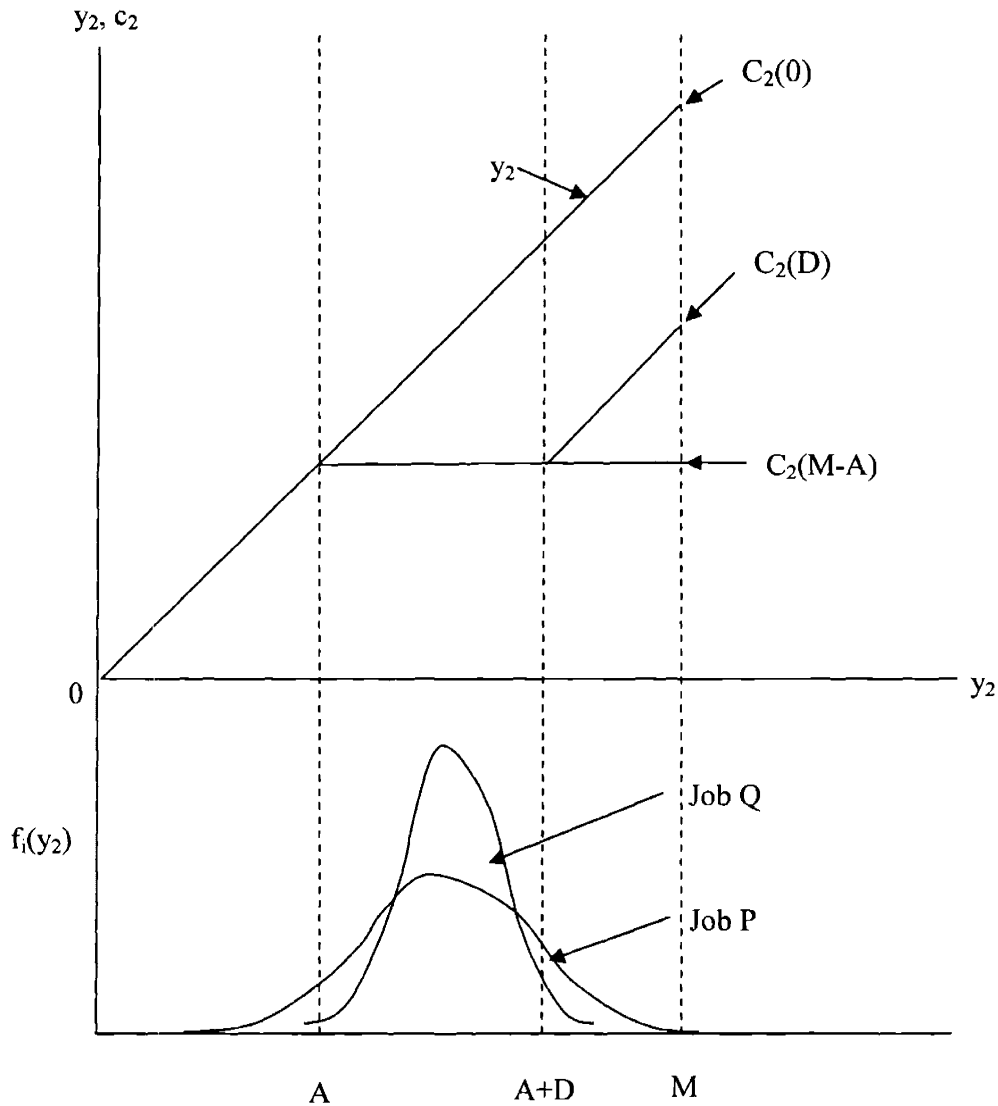
In both Figures 4.5 and 4.6, the lines $C_2(\cdot)$ represent second period consumption as a function of second period income for a given level of debt.

In either case, if $D=0$, the bankruptcy system is irrelevant and the consumer will choose whichever job gives him the greatest expected utility. Because job P is riskier than job Q, if the difference between e_Q and e_P is small enough, i.e. if the safer job doesn't require too much additional effort, the consumer will prefer the safer job. Consequently, in both Figures 4.3 and 4.4, consumers are depicted as choosing job Q for low levels of debt.

If A=0 and D>y_c, then the consumer's consumption will be flat (equal to zero) for levels of income below D, but positive and increasing for levels of income greater than D. As a result, the consumer has an incentive to chose job P, the riskier job, since he is indifferent between the two distributions for low levels of income, but prefers the riskier distribution because it gives him a greater chance of having higher consumption. In Figure 4.3, consumers are depicted as choosing job P for high levels of debt.

If A>0, then the consumer's consumption varies with his income for realizations of second period income less than A and above A+D, but is flat between A and A+D. As a result, the consumer experiences the downside of the risky job for income realizations less than A and less than y_c (a probability of having a very low income) and the upside of the risky job for income realizations greater than A+D and greater than y_c (a higher probability of having a very high income realization). If the flat portion of consumption occurs (mostly) at levels of income less than y_c, then the consumer will not experience much of the cost of the riskier distribution because his consumption is flat, but will experience the benefit to the riskier distribution because his consumption will be higher in states of nature which are more likely with the riskier job. For this reason, in Figure 4.4, the consumer is depicted as preferring the riskier job for some low levels of debt.

If, as is the case in Figure 4.5 with debt=D, the probability of having income less than A is approximately the same as the probability of having income greater than A+D, then the consumer's preference for certainty should cause him to prefer job Q over job P for moderate levels of debt. For high levels of debt, including D=M-A, the consumer will experience even less of the benefit of having a job with a risky second period income and will therefore continue to prefer the less risky job.

Unlike the case where A=0, when A>0, high levels of debt cause the consumer to be less willing to take risk, because consumption below the exempt amount depends on income, a consumer with risky income is more likely to experience very low consumption relative to a consumer with less risky income. At the same time, a consumer with risky income is less likely to experience the benefit of having a risky income if he has a large amount of debt, since much of that benefit will accrue to creditors.

Like the equilibrium discussed above in section 4.1, where jobs differed as to their mean earnings, with jobs of differing risks consumers will borrow at the point where the marginal

utility of first period consumption equals the expected marginal utility of second period consumption conditional on income being high enough to avoid bankruptcy if the incentive compatibility constraint does not bind. If the incentive compatibility constraint binds, they will borrow either more or less than the point where the marginal utilities are equal in order to satisfy the incentive compatibility constraint. For example, in either Figures 4.3 or 4.4, the consumer could either borrow at B_Q^*, D_Q^* and choose job Q in the second period, or could borrow at the intersection of BE_P and the incentive compatibility constraint and choose job P in the second period, depending on which choice gives the consumer greater expected utility.

The consumer's maximization problem and the first order conditions are the same as section 4.1, except that the distributions of second period income will differ. Specifically, if the consumer borrows at an interior tangency, he will satisfy equation (4.8):

$$(4.8) \quad \begin{aligned} u'(y_1 + B)[1 - F_i(A + D)] - \int_{A+D}^{M} u'(y_2 - D)f_i(y_2)dy_2 = \\ \delta[\int_{A+D}^{M} u'(y_2 - D)f_i(y_2)dy_2 - \int_{A+D}^{M} u'(y_2 - D)f_{-i}(y_2)dy_2] \end{aligned}$$

Again, if the incentive compatibility constraint does not bind, the consumer will borrow at the point where the marginal utility of first period consumption equals the expected marginal utility of second period consumption given that the consumer's income is high enough that he doesn't declare bankruptcy.

If the incentive compatibility constraint binds, the consumer will distort his borrowing in order to satisfy the incentive compatibility constraint. The direction of the distortion will again depend on the relative sizes of $\int_{A+D}^{M} u'(y_2 - D)f_i(y_2)dy_2$ and $\int_{A+D}^{M} u'(y_2 - D)f_{-i}(y_2)dy_2$, i.e. whether increased debt increases or decreases the attractiveness of the job the consumer wants to choose in the second period relative to the other job. For example, in Figure 4.4 reducing the amount of debt from D_P^* will increase the attractiveness of the riskier job. If the consumer wishes to choose the riskier job in the second period, he must therefore borrow less than he would if he could commit to choosing job P in the first period.

127

4.2.2 The Effect of the Bankruptcy Exemption Level on Job Choice

As was the case in Section 4.1, the effect of the bankruptcy exemption level on the relative attractiveness of different jobs is ambiguous. As a reminder, applying the envelope theorem to the consumer's maximization problem and taking the difference, we find that:

$$\frac{dV_Q}{dA} - \frac{dV_P}{dA} = [u'(A) - u'(y_1 + B_Q)][F_Q(A + D_Q) - F_Q(A)]$$
$$(4.15) \quad + u'(A)\delta_Q\{[F_Q(A + D_Q) - F_Q(A)] - [F_P(A + D_Q) - F_P(A)]\},$$
$$- [u'(A) - u'(y_1 + B_P)][F_P(A + D_P) - F_P(A)]$$
$$- u'(A)\delta_P\{[F_P(A + D_P) - F_P(A)] - [F_Q(A + D_P) - F_Q(A)]\}$$

which can be broken down into the difference in insurance benefits:

$$(4.16) \quad [u'(A) - u'(y_1 + B_Q)][F_Q(A + D_Q) - F_Q(A)] - [u'(A) - u'(y_1 + B_P)][F_P(A + D_P) - F_P(A)],$$

and the differential effect on incentives to choose each job:

$$(4.17) \quad \begin{aligned} &u'(A)\delta_Q\{[F_Q(A + D_Q) - F_Q(A)] - [F_P(A + D_Q) - F_P(A)]\} \\ &- u'(A)\delta_P\{[F_P(A + D_P) - F_P(A)] - [F_Q(A + D_P) - F_Q(A)]\} \end{aligned}.$$

Again, it is not possible to determine, in general, whether raising the exempt amount increases the relative attractiveness of job P or job Q.

For example, consider the equilibrium illustrated in Figure 4.3. In that case, a consumer choosing job P in the second period would have to borrow more and incur more debt than a consumer choosing job Q in the second period. As a result, equation 4.16 would be negative since $B_P > B_Q$ and $F_P(A+D_P) > F_Q(A+D_Q)$. Increasing A would also increase the relative attractiveness of the riskier job. Since the incentive compatibility constraint would not bind for a consumer choosing job Q, $\delta_Q = 0$. Furthermore, since $A=0$, $F(A)=0$ and $F_P(A+D_P)-F_Q(A+D_P)>0$, so equation 4.17 is also negative. The net effect is, therefore, that increasing A increases the relative attractiveness of the riskier job.

The result might be different for equilibria such as that depicted in Figure 4.4. In that case, it is not clear whether the insurance benefit to increasing A is greater for a consumer choosing job P or for job Q, because while $B_P > B_Q$, it is not clear whether $F_P(A+D_P)-F_P(A)$ is greater or less than $F_Q(A+D_Q)-F_Q(A)$. Similarly, it is not clear whether increasing A increases or decreases the relative attractiveness of job P or job Q, because it is not clear whether

$[F_P(A+D_P)-F_P(A)]$ is greater or less than $[F_Q(A+D_P)-F_Q(A)]$, that is which distribution has the highest probability of having income high enough to make some payment to creditors, but not enough to avoid bankruptcy given the risky consumer's choice of debt. That having been said, if the region where $A \leq y_2 \leq A+D$ is close to the crossing point between the two cumulative distributions, then it might be expected that $[F_Q(A+D_P)-F_Q(A)] > [F_P(A+D_P)-F_P(A)]$ and equation (4.17) would be positive. If the incentive effect acts in the same direction as, or is larger than, the effect of the relative increases in the insurance benefit, then increasing A would increase the relative attractiveness of the safe job.

4.2.3 The Effect of Bankruptcy Exemptions on Consumer Borrowing

It is not possible to determine in general the marginal effect of the bankruptcy exemption level on borrowing. As was the case in section 4.1.3, a small increase in the exemption level has a variety of potential effects on a consumer's choice of borrowing. First, increasing the exemption level will tend to make borrowing cheaper in utility terms since it shifts the responsibility to repay the debt to states of nature where the marginal utility of consumption is lower. As a result, if the incentive compatibility constraint doesn't bind and if the consumer isn't borrowing as much as possible, the borrowing will increase if the exemption is raised.

If, however, the incentive constraint binds, then the direction of the effect of increasing the bankruptcy exemption level on borrowing will depend on how the increase affects the incentive compatibility constraint and whether the consumer will switch jobs as a result of the increase. For example, if increasing the exemption level allows consumers to incur more debt and keep the same job, i.e. if the incentive compatibility constraint shifts up, then increasing the exemption level will increase borrowing. If the incentive compatibility constraint shifts down, then it's possible that borrowing decreases.

Finally, if increasing the exemption levels causes the consumers to shift jobs, the direction of the effect on consumer borrowing will depend on whether the consumer shifts from the risky to the less risky job, the location of the incentive compatibility constraint, and the level of A, among other things.

4.2.4 The Optimal Exemption Level

Again, the factors which must be considered in setting the optimal bankruptcy exemption are essentially the same if the distributions have the same mean but have different levels of risk, as if one distribution first order stochastically dominates the other. As a reminder, proposition 4.6 states that if the incentive compatibility constraint does not bind, the optimal bankruptcy exemption occurs at the point where first period consumption is equal to the bankruptcy exemption level. If the incentive compatibility constraint does bind, then the optimal bankruptcy exemption level will either be higher or lower than first period consumption in order to provide the consumer with the incentive to choose the desired job. Specifically, the exemption level will be lower than first period consumption if reducing the insurance benefit to provide the consumer with incentives to choose the desired job is cheaper in utility terms than increasing credit constraints by increasing the exemption level to provide incentives to choose the right job.

For example, if the consumer should choose a job with no uncertainty in second period income, then the insurance benefit to increasing exemption levels is zero since consumer will not be exposed to any risk. In that case, the optimal exemption level is clearly zero since there is no insurance benefit but there may be a cost due to worsening incentive effects.

It should be noted that it is possible that the consumer may take either too much or too little risk relative to the optimum if the exempt amount is set such that A is equal to first period consumption.

Figure 4.7
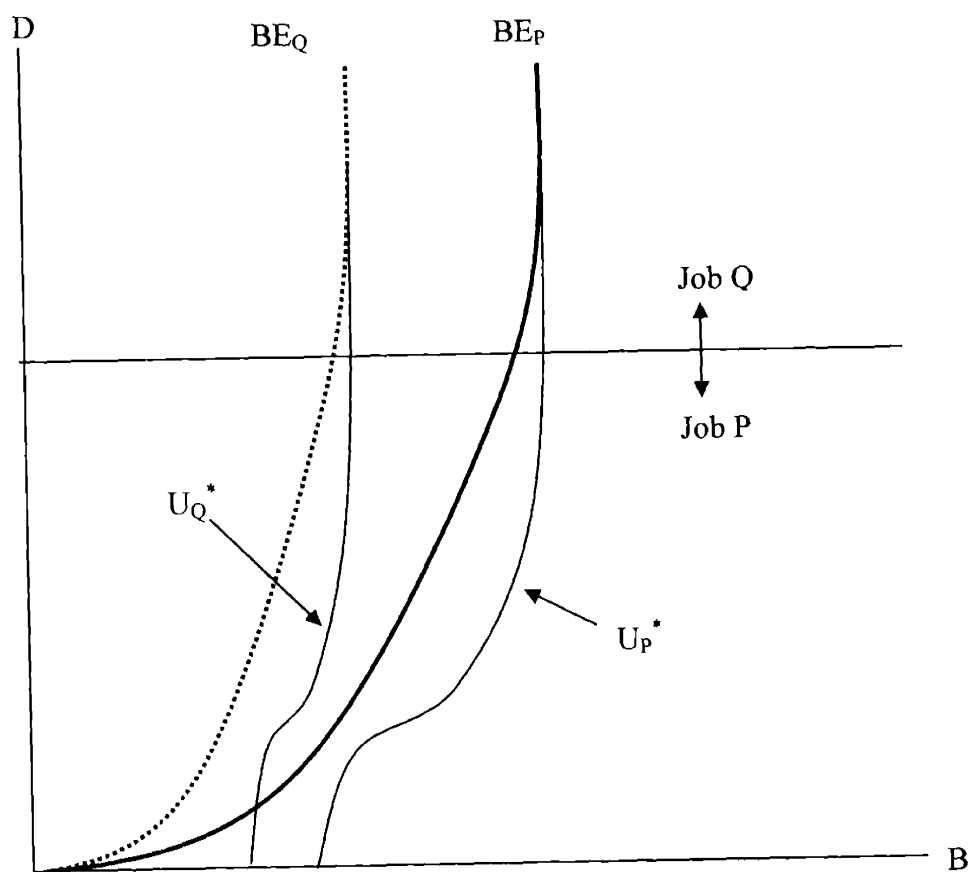
For example, consider the situation depicted in Figure 4.7. In that case, the consumer can either borrow as much as possible and choose the less risky job Q or he can borrow less than the maximum amount and choose the riskier job P. Suppose that the exemption level in Figure 4.7 is such that it would equal the first period consumption if a consumer borrowed the maximum amount possible given that he had the risky job P, i.e. the exemption level is the same as the optimal exemption level for a consumer who had no choice but to take job P. In that case, raising the exemption level, which would shift up the incentive compatibility constraint since it increases consumption at high levels of income which are more likely for riskier consumers. This easing of the incentive compatibility constraint will increase utility for a consumer choosing job P and will lower utility for a consumer choosing job Q. If the effort required for job Q is sufficiently large relative to the effort required for job P, then the consumer will choose job P and this easing of the incentive constraint increases welfare, even if it increases credit constraints

on the consumer beyond what would be optimal in the absence of the incentive compatibility constraints. Put another way, increasing the exemption level beyond the level which optimally balances the insurance benefit with credit constraints may be optimal if it eliminates the consumers incentive to inefficiently chose a less risky job.

Alternatively, the optimum could occur at a lower level of A such that the consumer chooses job P and the increase in the insurance benefit is balanced against an increase in the incentive cost. In this case, the insurance benefit would be reduced in order to improve the consumer's incentives to take the riskier job.

Finally, if the consumer would get higher utility from choosing the less risky job, then the optimal exemption level for the situation depicted in Figure 4.7 would be the same as the optimal exemption level for a consumer who had no choice but to have job Q in the second period.

Whether the exemption level is lower, higher or the same as first period consumption depends on which potential solution gives the consumer the highest utility. For instance, if the utility cost to increasing credit constraints to loosen the incentive compatibility constraint is less than the cost to lowering the amount of insurance provided to loosen the constraint and less than the cost to taking the less risky job, then the optimal exemption level would be higher than first period consumption. Similarly, if lowering the level of insurance were the least expensive, the optimal exemption level would be lower than first period consumption. If simply taking the less risky job is the cheapest, then the exemption level will be the optimum for a consumer choosing job Q.

4.3 Conclusion

The models presented in this chapter challenge the assumption that more generous bankruptcy exemption levels will decrease incentives to work hard and will increase incentives to take unnecessary risk. In contrast to most insurance models, where greater insurance increases consumption in states of nature where income realizations are low, more generous bankruptcy exemption levels cannot increase consumption in states of nature where income is less than the exempt amount. Instead, greater exemption levels increase consumption only for borrowers with incomes which are greater than the exempt amount but which are not high enough for creditors to be paid in full. Since this income range may be more likely for consumers with jobs with

higher expected income or less risk, it is possible that raising exemption levels will encourage work and discourage risk taking.

Moreover, because incentives can be improved either by reducing exemption levels, and thereby reducing the level of insurance provided by bankruptcy, or by increasing exemption levels and reducing the amount that consumers can borrow, it is possible that incentive costs will cause the optimal exemption level to be either higher or lower than would be the case if incentive costs were ignored.

Chapter 5: The Effect of Bankruptcy Asset Exemption Levels on Household Borrowing

The repayment of consumer debt in the United States is governed by a combination of state and federal laws which describe the circumstances in which creditors can compel repayment of debt and the circumstances in which debtors can receive a discharge of their debt. Chief among these laws is the Federal Bankruptcy Code which allows debtors to receive a discharge of their unpaid debts in certain circumstances. Despite the important role played by the Bankruptcy Code in regulating the collection of debt, there have been relatively few papers exploring effects of the Bankruptcy Code on consumer borrowing.

Previous chapters have argued that more lenient bankruptcy laws benefit borrowers who are not credit constrained (who are predicted to borrow more when exemption levels increase) and may harm borrowers who are credit constrained (and will be forced to borrow less if exemption levels increase). This chapter attempts to identify those groups that are potentially benefited or harmed by increases in the amount of property that bankrupt debtors are allowed to protect from creditors. Specifically, this chapter uses changes in bankruptcy exemption levels from 1984-2000 and data on household borrowing from the Survey on Program Participation to estimate the effect of bankruptcy asset exemption levels on household borrowing decisions and to identify those groups of households which are potentially benefited or harmed by more lenient bankruptcy laws.

The only previous empirical study of the effect of bankruptcy exemption levels on total household borrowing, Gropp, Scholz and White (1997), argues that higher exemption levels cause households with low levels of assets to be denied credit. They argue that bankruptcy redistributes credit from households with below average levels of assets to households with above average asset holdings. The results described in this chapter present a dramatically different picture of the effect of bankruptcy exemption levels on household borrowing. While homestead exemptions are found to be negatively correlated with the probability of having any debt, it appears that much of this effect is due to a reduction in the probability that a household will own a home or have a mortgage. Because homestead exemptions should not affect the repayment of mortgage debt, this result is inconsistent with a theory that higher exemption levels increase credit constraints on borrowers. The results presented below indicate that personal

property exemptions are positively correlated with homeownership, mortgage debt and total household debt and negatively correlated only with the probability that non-homeowners have large levels of debt. At least as far as personal property exemption levels are concerned, the data suggests that far more households increase borrowing as a result of higher exemption levels than are credit constrained by them.

I begin by presenting a brief discussion of bankruptcy law in Section 5.1. Section 5.2 discusses the existing empirical literature concerning the effect of bankruptcy law on consumer credit markets. Section 5.3 briefly describes a theoretical model of the effect of asset exemption levels on consumer borrowing decisions. Households which are not credit-constrained are predicted to increase borrowing in response to higher exemption levels. Households which are credit constrained are predicted to borrow less when exemption levels rise. The data and empirical strategy are described in section 5.4. Results are presented in section 5.5. I find evidence that higher personal property exemption levels are associated with higher levels of mortgage debt incurred by homeowners and lower probabilities that non-homeowners borrow more than $50,000 in 2000 dollars. Higher homestead exemption levels are associated with lower probabilities of homeownership and with lower probabilities that a non-homeowner has positive debt. In section 5.6, I discuss the results presented in the context of the predictions of the theoretical model, and in section 5.7, I conclude.

5.1 The Role of Asset Exemptions in State Debt Collection Law and Bankruptcy

The collection of money from debtors is governed by a combination of state and federal laws including, among others, state laws governing the seizure of property by creditors to satisfy unpaid debts and the Federal Bankruptcy Code, which provides for the discharge of unpaid debts under certain circumstances.

By way of example, suppose that a consumer debtor fails to pay an unsecured debt owed to a lender. If self help remedies such as calling the debtor repeatedly demanding payment fail, the lender can compel payment by suing and receiving a judgment ordering the debtor to pay the amount owed. If the debtor still fails to pay, the lender can go back to court and seek an order allowing them to seize assets or garnish wages in order to collect the judgment.

State laws governing the seizure of assets and the garnishment of wages provide some protection against creditors who seek to enforce judgments by seizing property or garnishing wages, without the need to declare bankruptcy. A typical state law allows debtors to exempt at least some property from seizure by creditors and provides limits on the ability of creditors to garnish wages. These state laws do not extinguish the consumer's debts, but merely restrict the ability of creditors to collect those debts. Nevertheless, if the consumer has no non-exempt assets and no wages, the creditor will be unable to collect on the debt regardless of whether the individual declares bankruptcy.

In practice, unsecured creditors rarely resort to legal action to compel the repayment of debt, relying instead on extra-legal means to cajole creditors into paying.

If a debtor fails to pay an amount owed to a secured creditor (including a mortgage creditor), the creditor's position is somewhat different. Under Article 9 of the Uniform Commercial Code (UCC), which governs security interests in personal property and, as the name suggests, is nearly identical in every state, a secured creditor may seize the collateral securing the loan without a court order so long as the seizure could be accomplished without a breach of the peace. Moreover, in most cases[3], the collateral can be seized and sold even if it would have been exempt from attachment by an unsecured creditor.

If the sale of the collateral does not generate enough money to satisfy the secured debt, the creditor can pursue the debtor for the amount unpaid of the debt in the same way as an unsecured creditor would. For example, suppose a consumer owes $10,000 that is secured by an automobile worth only $6,000. If he does not pay, the creditor can seize the automobile and sell it, using the $6,000 proceeds to satisfy part of the debt. The remaining $4,000 is then treated as unsecured debt and the creditor can proceed against the debtor as any other unsecured creditor would.

In the case of default on a mortgage, the mortgage creditor must seek a court order to foreclose on the property but can do so even if the property would have been exempt from attachment by a non-mortgage creditor. In all but five states, if the proceeds from the sale of the property in foreclosure are not enough to satisfy the debt, the mortgage creditor can pursue the debtor for the deficiency in the same way as any unsecured creditor.

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[3] Under the UCC, non-purchase money security interests in consumer goods as well as blanket security interests in an individual's property are not generally enforceable.

Bankruptcy provides debtors with additional protection from creditors and gives debtors the possibility of having at least part (and often all) of their unpaid debts discharged. When a debtor files for bankruptcy, an automatic stay is issued halting all legal proceedings against the debtor, including all actions by secured creditors to repossess or foreclose on the debtor's property.

In general, debtors wishing to have their debts discharged or restructured in bankruptcy may file under either Chapter 7 or Chapter 13 of the bankruptcy code. Debtors filing under Chapter 7 must turn over all property held as collateral by secured lenders to those lenders and must turn over all non-exempt property to the trustee to be sold with the proceeds (if any) being distributed *pro rata* to all unsecured creditors. Any unpaid debts are then discharged.

The property exemptions used in Chapter 7 can come from either state or federal law. The 1978 bankruptcy reform established uniform federal bankruptcy exemptions levels for certain categories of property, including home equity, automobiles, jewelry and other personal property, but allowed states to opt out of the federal system and to establish their own bankruptcy exemptions. By 1983, every state had done so, typically importing exemption levels from state debt collection law, although 16 states allowed debtors to choose between the federal and state exemptions.

Debtors filing under Chapter 13 of the bankruptcy code may keep all of their property, including property held as collateral by secured creditors, but must propose a payment plan under which each creditor will receive at least as much as he would have under Chapter 7 and which contributes all of a debtor's "disposable income" for three years to paying his creditors. After the payment plan is completed, all unpaid debts are discharged.

5.2 Existing Literature

There have been relatively few empirical studies of the relationship between bankruptcy law and consumer borrowing. Gropp, Scholz and White (1997) present the first, and to my knowledge only, study of the relationship between bankruptcy asset exemption levels and total household borrowing. Using data from the 1983 Survey of Consumer Finances, they estimate a double selection model of household borrowing which adjusts for truncation of the desired debt

level at zero and for the possibility that household debt is less than their desired level of debt due to credit constraints.

To allow for the possibility that bankruptcy exemption levels affect low-asset and high-asset households differently, Gropp, Scholz and White interact asset quartile dummies with the state's bankruptcy exemption level[4] and find that, among households which are not denied credit or discouraged from borrowing, bankruptcy asset exemption levels are negatively related to borrowing for low-asset households and positively related to borrowing for high-asset households. They also find that households in states with unlimited asset exemptions are more likely to be turned down for credit and that low-asset households pay higher interest rates on auto loans in states with higher bankruptcy exemption levels.

They argue that more lenient bankruptcy laws redistribute credit from households with low asset levels to those with higher levels of assets and suggest that bankruptcy laws harm poor households.

Lin and White (2001) examine the relationship between mortgage loan approval rates and bankruptcy asset exemption levels. Using data collected pursuant to the Home Mortgage Disclosure Act, they estimate a linear probability model of the relationship between the probability of a loan application being rejected and a state's homestead and personal property exemption levels. They find that homestead exemption levels are positively and significantly related to the probability that a loan application is rejected in models both with and without state fixed effects.

5.3 The Effect of Exemption Levels on Household Borrowing and Asset Choices

5.3.1 Exemption Levels and Household Borrowing

To provide a framework for thinking about the effect of bankruptcy on consumer borrowing and social welfare, I consider a simple two period model where the consumer's income in the second period is a random variable. Consumers borrow money in the first period,

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[4] Gropp, Scholz and White do not distinguish between homestead exemption levels and personal property exemptions in their analysis. Because homestead exemptions are typically much larger than personal property exemptions, most of the variation in value between states in results from differences in homestead exemptions.

which they must repay in the second period subject to a bankruptcy system which exempts an amount A from attachment by creditors.

Second period consumption under such a bankruptcy system is depicted in Figure 5.1 below:

Figure 5.1
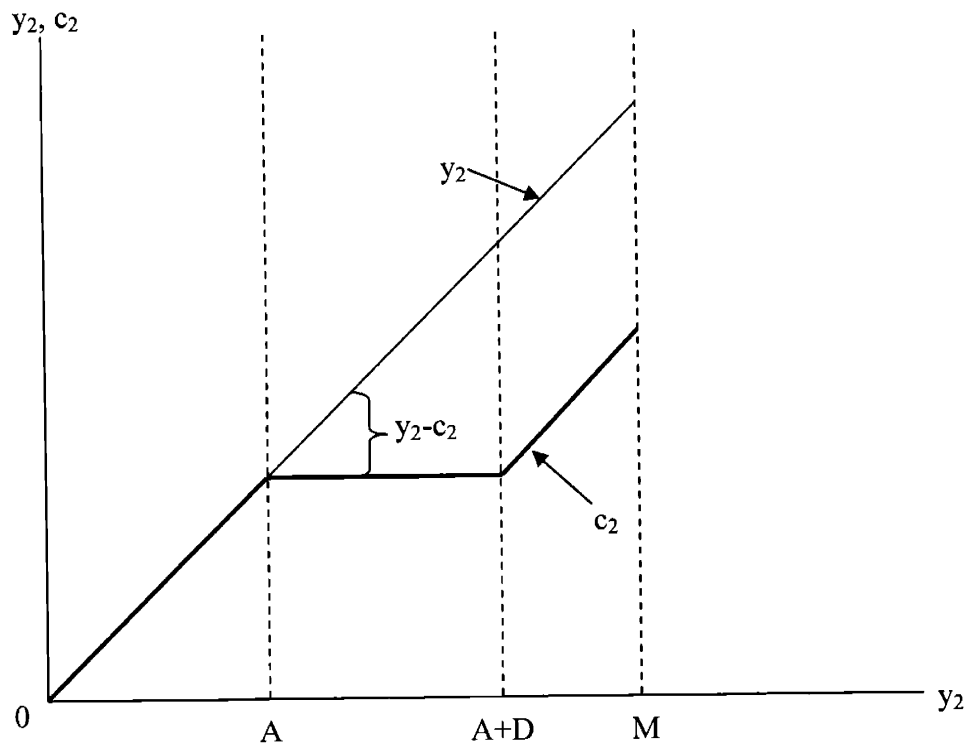
Since all income below the exempt amount A is exempt from attachment by creditors, if a consumer earns less than A, he consumes his entire income. If he earns more than the exempt amount, but not enough to pay off his debts, he consumes A and his creditors receive y_2-A. If he earns more than A+D, he pays his debts in full and consumes y_2-D.

If the exempt amount A increases, the obligation to repay the debt is shifted to states of nature where the marginal utility of consumption is lower. Consequently, borrowing is cheaper in utility terms (even though nominal interest rates will rise to compensate lenders for the increase risk of default) and consumers will wish to borrow more. So, long as the consumer can compensate the lenders for the increased risk of default by paying higher interest rates, lenders will be willing to lend them more and borrowing will increase. In that case, the borrower will be better off since the utility cost of borrowing will be lower.

If, however, consumers cannot compensate lenders for the increased default risk with higher interest payments (either because of usury laws which limit the maximum interest rate that can be charged, or because even an infinite interest rate is not enough to compensate lenders for the increased risk), then they will be credit constrained if the exempt amount is increased. In such a circumstance, consumers would borrow less as the level of exempt assets increases. If the consumer is credit constrained, increasing the exemption level will decrease the borrower's utility if the loss in utility due to the credit constraint is greater than any gains in second period utility resulting from the increased exemption amount.

Put another way, borrowers who are not credit constrained will borrow more when the bankruptcy exemption level is increased because increasing the exemption level reduces the cost of borrowing in utility terms. At the same time, borrowers who are credit constrained will borrow less when the exemption level is increased since the increase in the exemption level will exacerbate the credit constraint.

The effect of an increase in the bankruptcy exemption level on the level of borrowing is therefore likely to be nonlinear. Consumers with higher demand for borrowing or with lower ability to repay debts should be more likely to be credit constrained by increases in the level of bankruptcy exemption amounts and would therefore borrow less if exemption amounts increased. At the same time, consumers with lower demand for borrowing or a greater ability to repay their debts should be less likely to be credit constrained and should borrow more when exemption amounts increase.

Social welfare will be maximized at the point where the marginal loss in welfare due to increasing credit constraints on borrowers with high demand for credit or with a low ability to repay is balanced against the increase in welfare resulting from the reduction in the utility cost of borrowing to non-credit constrained borrowers when the exempt amount is increased. One of the goals of this paper is to determine which groups of consumers are likely benefited or harmed by changes in bankruptcy exemption levels.

5.3.2 Exemption Levels and Household Asset Choices

Asset exemption levels in general, and homestead exemption levels in particular, have the potential to affect the level and type of assets held by debtors. Given that higher homestead exemptions allow debtors to protect more home equity from creditors in the event that a homeowner defaults on a loan, it might be expected that higher homestead exemptions would increase demand for homeownership.

The value of the homestead exemption level should not affect the probability that a mortgage creditor is repaid. If the value of the home is greater than the amount owed on the mortgage, the creditor will be paid in full if he forecloses on the property. If, however, the value of the home is less than the amount owed on the mortgage, there will be no home equity and the value of the homestead exemption is irrelevant. Mortgage lenders should be indifferent as to the homestead exemption level. Increasing homestead exemption levels should therefore increase the probability that a debtor owns a home and increase the average value of homes owned by debtors.

Since most homes are purchased with mortgage loans, which the homeowner may have difficulty repaying if the value of the home falls, it might be expected that higher personal property exemption levels would similarly increase the demand for homeownership and might make individuals more willing to borrow more to purchase a more expensive home. At the same time, higher personal exemptions might make mortgage lenders less willing to lend money since it reduces their ability to pursue other assets in the event that the borrower defaults and the home value is not enough to repay the debt. Because higher personal property exemptions decrease the risk of incurring mortgage debt, but also reduce the assets available to creditors in the event of a bankruptcy, the net effect of personal property exemption levels on homeownership is unclear.

142

5.4 Data and Empirical Strategy

5.4.1 Data

To examine the effect of bankruptcy exemption levels on consumer borrowing, I combined the data on state and bankruptcy prevention levels with data on household assets, liabilities, income and demographic characteristics from the Survey on Income and Program Participation (SIPP) for households surveyed from 1984 through 2000. Debt and asset levels are obtained from the asset and liability topic modules, while income and demographic characteristics come from the core survey. Because the SIPP does not uniquely identify five small states, it was not possible to determine which state bankruptcy exemption applied for households from those states. After excluding those five states from my sample, my data set includes a total of 278,891 observations.

Homeowners account for 64.3% of the observations. Fifty-seven percent of non-homeowners and 81.2% of homeowners had positive levels of debt.

Summary statistics for total debt, mortgage debt, non-mortgage debt unsecured debt and home value are listed in Table 1.

143

Table 1: Summary Statistics

Variable	Total Sample	Homeowners	Non-Homeowners
Number of Observations	279,271	179,447	99,824
Total Debt			
Unconditional			
Mean	35,891	51,879	7,150
Standard Deviation	77,669	90,916	27,067
Conditional on Positive Debt			
Number of Observations	203,230	145,781	57,449
Mean	49,320	63,861	12,423
Standard Deviation	87,335	97,002	34,749
Mortgage Debt			
Unconditional			
Mean	22,982	35,766	-
Standard Deviation	42,846	48,987	-
Conditional on Positive Debt			
Number of Observations	110,642	110,642	0
Mean	58,008	58,008	-
Standard Deviation	51,009	51,009	-
Non-Mortgage Debt			
Unconditional			
Mean	13,230	16,672	7,150
Standard Deviation	59,549	71,454	27,067
Conditional on Positive Debt			
Number of Observations	186,102	128,655	57,447
Mean	19,635	22,855	12,423
Standard Deviation	71,672	82,812	34,749
Home Value			
Mean	65,316	101,664	-
Standard Deviation	82,344	82,815	-

The state level home price index from the Federal Office of Housing Enterprise Oversight is used in some regressions to account for changes in state housing markets which may be correlated with debt levels.

As described above, bankruptcy exemption levels can be defined by either state or federal law for different categories of debt. State and federal laws define separate exemption levels for various categories of property such as home equity, automobiles, clothing, cash, jewelry and tools of the trade, among others.

I divided exemption levels into homestead exemptions (available for home equity in an owner occupied house) and personal property exemptions, which were calculated as the sum of automobile, cash and general personal property exemption levels. I did not include exemption levels for jewelry, tools of the trade or personal property exemptions for specific categories of

144

goods such as "wearing apparel" which were frequently unlimited. My exemption levels appear to be the same as those used by Fay, Hurst and White (2002). They differ from Gropp, Scholz and White (1997) in that they don't include jewelry exemption levels of tools of the trade exemptions. Differing definitions of personal property exemptions do not significantly affect the results.

Where both federal and state exemption levels are available, the exemption level that I used depends on whether federal or state law allows homeowners or non-homeowners to exempt more property. If federal law allows homeowners to protect more property than state law, then the federal homestead exemption will be used and vice versa. Similarly, if federal personal property exemptions are larger, then they are used and if state personal property exemptions are higher, then the state exemptions are used. It is therefore possible that in some states the maximum homestead exemption is given by state law while the maximum personal property exemption comes from federal law or vice versa.

It is not possible to include variables for asset exemption levels available in bankruptcy as well as those available under state debt collection law since asset exemption levels rarely change in states which allow the use of federal exemptions. Consequently, there is not sufficient variation to estimate the effect of bankruptcy and non-bankruptcy exemption levels separately.

The homestead and personal exemption levels for each state are summarized in the appendix following this chapter.

Some states allowed unlimited homestead exemptions. Following Lin and White (2001), the homestead exemption level is coded as $500,000 in those cases . The specific value does not significantly effect the results since only Iowa (from a limited exemption to an unlimited exemption) and Minnesota (from an unlimited exemption level to a limited exemption) change homestead exemption levels either to or from an unlimited homestead exemption. Re-coding the unlimited exemption level or excluding these states from the analysis has very little effect on the final results.

5.4.2 Empirical Strategy

I use linear and probit regression to analyze the effect of the bankruptcy exemption levels on levels of total debt, mortgage debt, non-mortgage debt and unsecured debt as well as on home value.

I begin by regressing the log of total debt, mortgage debt, non-mortgage debt and home value on the log of homestead and personal property exemption levels along with dummies for deciles of household income, marital status, race, family size, the age of the household head as well as state and year fixed effects.

Because the relationship between asset exemption levels and borrowing is expected to be non-linear, I also run a series of probit regressions to examine the effect of the bankruptcy exemption level on the cumulative distribution of each type of debt and of home prices. A negative effect of the bankruptcy exemption level on the probability that a household had borrowed more than a particular amount would indicate that on average consumers seeking to borrow more than that amount were credit constrained and may be harmed by higher exemption levels. A positive effect of exemption levels on the probability that a household has at least a certain amount of debt indicates that consumers seeking to borrow more than that amount are not credit constrained and are made better off by higher exemption levels. For example, since theory predicts that consumers with higher demand for credit will be more likely to be credit constrained, bankruptcy exemption levels might negatively effect the probability that households have large amounts of debt. Consumers with more moderate demand for credit will be less likely to be credit constrained and are more likely to be able to increase their borrowing in response to increases in exemption levels. Exemption levels may therefore be positively related to the probability that consumers have moderate levels of debt or greater.

The use of probit regressions to account for the potential non-linearity of the effect of bankruptcy exemptions on consumer borrowing is preferable to the methodology employed by Gropp, Scholz and White, which allowed the effect of bankruptcy exemption levels to differ depending on a household's asset levels. Since debt is often incurred to purchase assets, a household's asset and debt levels are likely to be jointly determined. It is, therefore, problematic to use an interaction with assets as an exogenous explanatory variable. Using probit regressions

146

to identify groups of borrowers who are benefited or harmed by changes in exemption levels avoids this potential problem because assets are not used as an explanatory variable.

To account for the possibility that exogenous changes in the housing market may be correlated with the bankruptcy exemption levels, I include the log of the housing price index as an explanatory variable in all but the home value regressions.[5]

Standard errors for the least squares and probit regressions are clustered on the state level.

Because of the importance of homestead exemptions, I repeat each analysis for the sample as a whole and for homeowners and non-homeowners (where appropriate).

5.5 Results

5.5.1 The Effect of Bankruptcy Exemption Levels on Total Household Debt

I begin by considering the effect of bankruptcy exemption levels on the cumulative distribution function of total debt for all households, homeowners, and non-homeowners. The coefficient estimates, robust standard error estimates and t-statistics for the two bankruptcy exemption variables in the least squares and probit regressions are presented in Table 2. Along with the log of the asset exemption levels, the regressions for which results are presented in Table 2 include demographic controls, state and year fixed effects and the log of the home price index.

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[5] I have also run all of the regressions presented here without home prices as an explanatory variable and the results are very similar.

Table 2: Effect of Bankruptcy on Total Debt						
	Entire Sample		Homeowners		Nonhomeowners	
	Log Homestead Exemption	Log Personal Property Exemption	Log Homestead Exemption	Log Personal Property Exemption	Log Homestead Exemption	Log Personal Property Exemption
Mean Log Debt	0.0393	0.0948	0.0838	0.0322	0.0805	0.0983
	(0.0451)	(0.0388)	(0.0713)	(0.0546)	(0.0520)	(0.0734)
	0.87	2.44	1.17	0.59	1.55	1.34
Prob Debt>0	-0.0502	-0.0256	-0.0006	-0.0522	-0.0244	-0.0093
	(0.0287)	(0.0315)	(0.0527)	(0.0425)	(0.0144)	(0.0145)
	-1.75	-0.08	-0.01	-1.23	-1.70	-0.64
Prob Debt>5000	-0.0700	0.0242	-0.0568	-0.0017	-0.0458	0.0191
	(0.0199)	(0.0332)	(0.0356)	(0.0430)	(0.0344)	(0.0453)
	-3.51	0.73	-1.59	-0.04	-1.33	0.42
Prob Debt>10000	-0.0695	0.0559	-0.0508	0.0296	-0.0030	0.0450
	(0.0291)	(0.0308)	(0.0391)	(0.0352)	(0.0377)	(0.0240)
	-2.39	1.81	-1.30	0.84	-0.08	1.88
Prob Debt>25000	-0.1077	0.0774	-0.0400	0.0379	-0.0778	-0.0838
	(0.0442)	(0.0442)	(0.0397)	(0.0436)	(0.0771)	(0.0751)
	-2.44	1.75	-1.01	0.87	-1.01	-1.12
Prob Debt>50000	-0.1011	0.0965	-0.0377	0.0655	0.0936	-0.2399
	(0.0523)	(0.0354)	(0.0414)	(0.0205)	(0.0758)	(0.0638)
	-1.93	2.73	-0.91	3.19	1.23	-3.76
Prob Debt>100000	0.0907	0.0152	0.1405	-0.0052	-0.0363	-0.2272
	(0.0640)	(0.0549)	(0.0867)	(0.0764)	(0.1404)	(0.1919)
	1.42	0.28	1.62	-0.07	-0.26	-1.18

In the least squares regression of the log of total debt for the entire sample, the coefficient on the log of personal property exemption levels is positive and significant, suggesting that the mean level of debt conditional on debt being positive increases when the level of borrowing increases. Doubling the personal property exemption levels is estimated to lead to an increase of approximately 9.5% in average levels of debt among those with positive levels of debt. The coefficient on the log of the personal property exemption level is higher when the sample is restricted to non-homeowners and lower for the sample of homeowners, but is not significant in either case.

With regard to the probit regressions, the homestead exemption level has a negative and statistically significant effect (at least a 10% level of significance and in many cases at a 5% level of significance) on the probability that a household of any kind has a positive level of debt and that the household has greater than $5,000, $10,000, $25,000 and $50,000 in debt. The estimates in Table 2 suggests that doubling homestead exemption levels would lead to a 1.5 to

1.6% reduction in the number of households with positive levels of debt and a reduction of 1% to 3% in the number of households with greater than the specified levels of debt.

The coefficient estimates on the homestead exemption level on the probability of having positive debt is also negative and significant (although only at a 10% level of significance) in the non-homeowner sample as is the coefficient on the homestead exemption in the probability of having greater than $5,000 in debt in the homeowners sample (again only at a 10% level of significance). Taken together, these results suggest that the higher homestead exemption levels reduce the probability of having at least some low or moderate levels of debt, which may suggest that higher homestead exemption levels lead to credit constraints for people who are borrowing low to moderate amounts. The fact that the homestead exemption appears to negatively affect the probability that a non-homeowner will have a positive level of debt is puzzling, since higher homestead exemption levels can be applied only to owner-occupied homes.

The coefficient for personal property exemption levels in the least squares regression for the total sample is both positive and significant. The estimated elasticity of debt with respect to personal property exemptions is 0.095 in the regression where home prices are not included and 0.089 if average home prices are included as an explanatory variable. In either case, it appears that higher personal property exemptions are associated with significant increases in average debt levels.

Personal property exemptions are also positively and significantly correlated with the probability that a household has greater than $10,000, $25,000 and $50,000 in total debt, with the coefficient estimates varying between 1.3% and 2%.

Personal property exemption levels are significantly negatively related to the probability that non-homeowners have debt in excess of $50,000. While the coefficient estimate is only 0.0035, the effect is relatively large considering that only 1.75% of non-homeowners in the sample have debt over $50,000.

5.5.2 The Effect of Bankruptcy Exemption Levels on Homeownership

To explore the possible effect of bankruptcy exemption levels on home ownership, I estimated a least squares regression of log home values and probit regressions for the probability of having a home worth at least a certain value on the same explanatory variables as were used in

the equations presented above. The results for the bankruptcy exemption variables are presented in Table 3 below.

Table 3: Effect of Bankruptcy on Homeownership				
	Entire Sample		Homeowners	
	Log Homestead Exemption	Log Personal Property Exemption	Log Homestead Exemption	Log Personal Property Exemption
Mean Log Home Value			0.0308	0.0490
			(0.0551)	(0.0364)
			0.56	1.34
Prob Homeownership	-0.02444	0.02126		
	(0.0088)	(0.0115)		
	-2.77	1.85		
Prob Home Value>50000	-0.08244	0.02992	-0.0394	0.0368
	(0.0404)	(0.0342)	(0.0405)	(0.0193)
	-2.05	0.88	-0.97	1.91
Prob Home Value>100000	-0.00013	0.00511	0.0215	0.0069
	(0.0086)	(0.0101)	(0.0247)	(0.0178)
	-0.01	0.51	0.87	0.39
Prob Home Value>200000	-0.0001	0.0001	-0.0002	0.0002
	(0.0001)	(0.0001)	(0.0002)	(0.0003)
	-1.36	0.71	-1.36	0.75

Neither homestead nor personal property exemption levels have a statistically significant effect on the mean home price. The coefficient on the log of the homestead exemption level is negative and significant in the probit regressions for owning a home, owning a home worth at least \$25,000 or at least \$50,000. Conditional on homeownership, the value of the homestead exemption level is not significant in any regressions.

The state home price index is not included as an additional control in the regressions presented in Table 3 since home value is the left hand side variable. The significant negative relationship between the probability of homeownership and state homestead exemption levels remains negative and significant even if average home prices are included.

The negative coefficients for the homestead exemption levels in the probit regressions is rather surprising since, as was discussed above, higher exemption levels should make owning a home more valuable. Since most homeowners have mortgage loans, the fact that homeownership is negatively related to homestead exemptions may explain the negative relationship between the probability of having debt and homestead exemption levels presented in Table 2. Because the

negative relationship between homeownership and homestead exemptions does not correspond with the theoretical predictions, it is difficult to conclude that the negative relationship between homestead exemptions and the probability of having positive debt as due to credit constraints.

The coefficient estimates for effect of the value of the personal property exemptions are positive and significant in the probit regressions for the probability of homeownership. Conditional on owning a home, the personal property exemptions are positive and significant at a 10% level in the regressions for the probability of owning a home worth at least $25,000 and at least $50,000.

5.5.3 The Effect of Bankruptcy Exemption Levels on Mortgage Debt

The effect of the bankruptcy exemption levels on levels of mortgage debt is, of course, closely related to the effect of bankruptcy exemption levels on the probability of home ownership and on home value.

To examine the effect of the bankruptcy exemption levels on levels of mortgage debt, I begin by estimating probit regressions for the probability of having a mortgage worth at least a given value (in 2000 dollars) on the log of homestead and personal property exemption levels, demographic controls and state and year fixed effects. The results for the bankruptcy exemption variables are presented in Table 4 below.

Table 4: Effect of Bankruptcy on Mortgage Debt				
	Entire Sample		Homeowners	
	Log Homestead Exemption	Log Personal Property Exemption	Log Homestead Exemption	Log Personal Property Exemption
Mean Log Debt			0.0335	0.0699
			(0.0307)	(0.0201)
			1.09	3.47
Prob Mortgage Debt>0	-0.0293	0.0111	-0.0202	-0.0055
	(0.0097)	(0.0113)	(0.0133)	(0.0107)
	-3.03	0.98	-1.51	-0.52
Prob Mortgage Debt>5000	-0.0320	0.0246	-0.0220	-1.8800
	(0.0099)	(0.0105)	(0.0117)	(0.0102)
	-3.25	2.36	-1.88	1.56
Prob Mortgage Debt>10000	-0.0247	0.0213	-0.0092	0.0127
	(0.0083)	(0.0073)	(0.0106)	(0.0068)
	-3.00	2.94	-0.87	1.86
Prob Mortgage Debt>50000	-0.0068	0.0059	-0.0054	0.0058
	(0.0030)	(0.0023)	(0.0032)	(0.0025)
	-2.32	2.60	-1.66	2.32
Prob Mortgage Debt>100000	0.0000	-0.0001	0.0001	-0.0003
	(0.0001)	(0.0001)	(0.0002)	(0.0002)
	0.43	-1.18	0.71	-1.49

The coefficient on the value of the personal property exemption levels is positive and significant in the least squares regression of the log of mortgage debt, suggesting that conditional on having positive mortgage debt, higher personal property exemption levels are correlated with higher levels of mortgage debt.

The coefficients on the homestead exemption levels is negative and significant for the probit regressions for the probability of having positive levels mortgage debt or having mortgage debt of at least $5000, $10,000, and $50,000, but is not significant in any of the regressions conditional on homeownership, suggesting that the negative effect in the unconditional regression is due to the reduced probably of homeownership.

The coefficients on the personal property exemption levels are positive and significant in several of the probit regressions for the entire sample and are significant at a 10% level in two of the probit regressions where the sample is restricted to homeowners only. This is consistent with the hypothesis that higher personal exemption levels make mortgage debt less risky in the event that the value of the home is less than the value of the mortgage.

5.5.4 The Effect of Bankruptcy Exemption Levels on Non-mortgage and Unsecured Debt:

Estimates of the relationship between bankruptcy exemption levels and levels of non-mortgage debt are presented in Table 5. Though it probably obvious, it should be noted that for non-homeowners, non-mortgage debt and total debt are the same.

Table 5: Effect of Bankruptcy on Non-Mortgage Debt						
	Entire Sample		Homeowners		Nonhomeowners	
	Log Homestead Exemption	Log Personal Property Exemption	Log Homestead Exemption	Log Personal Property Exemption	Log Homestead Exemption	Log Personal Property Exemption
Mean Log Debt	0.0291	0.0501	-0.0002	0.0528	0.0805	0.0983
	(0.0476)	(0.0388)	(0.0605)	(0.0492)	(0.0520)	(0.0734)
	0.61	1.29	0.00	1.07	1.55	1.34
Prob Debt>0	-0.0229	-0.0094	-0.0174	-0.0099	-0.0244	-0.0093
	(0.0091)	(0.0120)	(0.0106)	(0.0124)	(0.0144)	(0.0145)
	-2.52	-0.78	-1.64	-0.80	-1.70	-0.64
Prob Debt>1000	-0.0029	-0.0111	-0.0068	-0.0084	0.0045	-0.0117
	(0.0092)	(0.0115)	(0.0104)	(0.0139)	(0.0108)	(0.0123)
	-0.31	-0.97	-0.65	-0.60	0.42	-0.95
Prob Debt>5000	-0.0164	0.0010	-0.0212	0.0022	-0.0097	0.0041
	(0.0083)	(0.0141)	(0.0131)	(0.0196)	(0.0073)	(0.0096)
	-1.98	0.07	-1.62	0.11	-1.33	0.42
Prob Debt>10000	-0.0075	0.0002	-0.0130	-0.0016	-0.0003	0.0047
	(0.0078)	(0.0044)	(0.0122)	(0.0079)	(0.0039)	(0.0025)
	-0.96	0.05	-1.06	-0.20	-0.08	1.88
Prob Debt>50000	0.0012	-0.0007	0.0015	0.0010	0.0013	-0.0034
	(0.0024)	(0.0011)	(0.0045)	(0.0018)	(0.0011)	(0.0009)
	0.50	-0.60	0.32	0.56	1.23	-3.76
Prob Debt>100000	0.0009	-0.0005	0.0019	-0.0001	-0.0002	-0.0010
	(0.0012)	(0.0010)	(0.0023)	(0.0015)	(0.0006)	(0.0008)
	0.73	-0.46	0.82	-0.09	-0.26	-1.18

Neither of the coefficients on the bankruptcy exemption variables is significant in the least squares regression of the log of non-mortgage debt.

The coefficient on the log of the homestead exemption level is negative and significant in the probability of having positive non-mortgage debt and having non-mortgage debt greater than $5,000 for the sample as a whole. The coefficients for homestead exemptions is also negative and significant for the probability of homeowners having non-mortgage debt greater than $5,000

(at a 10% level) and for non-homeowners having positive levels of non-mortgage debt and having non-mortgage debt greater than $5,000 (again, at only a 10% level).

The coefficients on the personal property exemption has a small positive and significant (at a 10% level) effect on the probability that non-homeowners have at least $10,000 in non-mortgage debt. At the same time, personal property exemption levels have a significant negative effect on the probability that non-homeowners have at least $50,000 in non-mortgage debt, again suggesting that higher personal property exemption levels may increase credit constraints on some non-homeowners borrowing relatively large amounts of money.

I have also run the same series least squares and probit regressions for unsecured debt and found no significant effects.

5.6 Discussion and Summary of Results

The theoretical predictions and the results of the least squares and probit regressions are summarized in Table 6. The label "No Prediction" indicates that the theory suggests that some households will be credit constrained, in which case they will borrow less as exemptions rise, while others will not be credit constrained and will borrow more. In those categories, a negative observed relationship suggests that, on average, consumers may be credit constrained and a positive observed relationship suggests that consumers are not credit constrained. For example, personal property exemption levels have a positive effect on the amount of total debt for the sample as a whole, they are associated with lower probabilities that non-homeowners have large amounts of debt, suggesting that while many households are benefited by higher exemption levels, households seeking to borrow large amounts of money may be credit constrained by higher personal property exemptions.

Table 6: Summary of Predicted and Observed Results				
	Homestead Exemptions		Personal Property Exemptions	
	Predicted Relationship	Observed Relationship	Predicted Relationship	Observed Relationship
Total Sample				
Total Debt	No Prediction	Negative	No Prediction	Positive
Homeownership	Positive	Negative	No Prediction	Positive for Probability of at least $50,000
Mortgage Debt	Positive	Negative	No Prediction	Positive
Nonmortgage Debt	No Prediction	Negative	No Prediction	Negative for Probability of at least $50,000
Homeowners				
Total Debt	No Prediction	No Significant Effect	No Prediction	Positive for Probability of at least $50,000
Mortgage Debt	Positive	No Significant Effect	No Prediction	Positive
Nonmortgage Debt	No Prediction	No Significant Effect	No Prediction	No Significant Effect
Non-Homeowners				
Nonmortgage Debt	No Effect	Negative for Probability of Any Debt	No Prediction	Negative for Probability of at least $50,000

Overall, the results suggest four trends. First, higher levels of homestead exemption levels seem to lower the probability that a household owns a home. Second, higher homestead exemption levels lower the probability that a household will have positive non-mortgage debt. Third, higher personal property exemption levels are associated with higher levels of mortgage debt and therefore higher levels of overall debt – especially for homeowners. Fourth, higher personal property exemption levels lower the probability that households, especially non-homeowners, have moderately large or larger levels of non-mortgage debt.

5.6.1 The Effect of Homestead Exemption Levels on Borrowing

As indicated in Table 6, the correlation between higher homestead exemption levels and lower probabilities of home ownership was not expected, although it does correspond to Lin and White's finding that higher homestead exemption levels are correlated with an increase in the rejection rate of mortgage applications. Lin and White argue that higher foreclosure costs inside bankruptcy can explain the negative relationship homestead exemption levels and home mortgage acceptance rates. Briefly, they suggest that consumers with positive home equity who cannot afford to keep their homes will declare bankruptcy more frequently if they can keep more of their home equity. If foreclosure is more costly in bankruptcy, and banks are not compensated for those increased costs, then higher home equity exemption levels might increase expected foreclosure costs to mortgage lenders. Mortgage lenders may, in turn, be less willing to approve loans when homestead exemptions are higher.

Lin and White's explanation is problematic, however, because foreclosure is likely less expensive in bankruptcy than outside of bankruptcy since foreclosure can be handled by the bankruptcy trustee and creditors do not have to incur the expense of complying with state foreclosure laws. Moreover, there do not appear to be significant numbers of debtors with positive home equity who declare bankruptcy.

It is also possible that those who have a high probability of declaring bankruptcy will bid up the prices of homes when the bankruptcy exemption level is higher. In that case, some potential home buyers who have a low probability of declaring bankruptcy might find purchasing a home to be a less attractive option. Although the inclusion of the home price index should account for changes in average home prices, it is possible that bankruptcy laws affect only prices of homes in a particular range of values. In that case, the inclusion of average home prices might not adequately account for the effect of bankruptcy on home prices.

Whatever the explanation, the negative effect of homestead exemption levels on total household borrowing seems to result, in large part, from the negative relationship between homestead exemption levels and the probability of homeownership (and therefore the probability of having a mortgage). For this reason, Gropp, Scholz and White's interpretation of the negative effect of homestead exemptions on the probability that a household has positive debt as

indicative of credit constraints is problematic. Since repayment of mortgage debt should be unaffected by homestead exemption levels, any credit constraints resulting from increased homestead exemption levels would not be observed as a change in the probability of homeownership. The negative relationship between homestead exemption levels and debt must therefore be explained in some other way.

While it is only significant at a 10% level of significance, it is also surprising that higher homestead exemption levels should lower the probability that non-homeowners should have any debt. This negative relationship between homestead exemption levels and the probability that a homeowner has positive debt might be explained in two ways.

First, the exemption levels apply at the time a debtor declares bankruptcy and not at the time the borrowing decision is made. Because any debtor might buy a home (indeed, might intentionally buy a home to shelter his assets) between the time he incurs a debt and the time he considers declaring bankruptcy, in principle, there is no reason why the non-homeowners should not care about the maximum exemption for homeowners. While it is undoubtedly true that current homeowners are more likely to be homeowners at the time of a bankruptcy, the size of the effect should depend on the joint probability of bankruptcy and homeownership. It might be the case that current non-homeowners are more likely to go bankrupt than homeowners and that the joint probability of bankruptcy and homeownership might be just as large for homeowners as non-homeowners.

Second, it might be the case that lenders have tightened credit requirements across the board in response to the increase in bankruptcy exemption levels. In that case, non-homeowners, who may be less credit worthy than homeowners, might be denied credit more frequently than homeowners. Lenders might raise interest rates across the board in order to compensate for the perceived increase in the risk of default. If non-homeowners receive no benefit to the increased exemption level, but must pay higher interest rates, they may be less willing to borrow money.

In any case, it is difficult to evaluate the effect of increases in homestead exemptions on social welfare since the estimated effects of homestead exemption levels on borrowing are not consistent with the predictions of the theoretical model of bankruptcy exemption levels which inform my expectations as to the relationship between borrowing and household utility.

5.6.2 The Effect of Personal Property Exemption Levels on Borrowing

The third and fourth results are not surprising. The model presented above suggests that higher exemption levels should lower the utility cost of borrowing and should increase borrowing among consumers who are not credit constrained. Higher personal property exemption levels should make borrowing money to purchase a home safer because households can protect their other assets from creditors in the event that they have negative home equity. It is, therefore, to be expected that higher personal property exemptions would be associated with higher levels of mortgage debt and therefore higher levels of overall debt for homeowners. The theoretical model suggests that since these consumers are borrowing more when exemption levels are increased, they have been made better off by increases in bankruptcy exemptions.

It is also to be expected that higher levels of exemptions should lead to credit constraints for those borrowing large amounts of money – especially if they do not have a home which they can use as collateral for the loan. It appears from the results of the probit regressions presented in Table 5 that non-homeowners are less likely to borrow amounts in excess of $50,000 when personal property exemption levels are increased, suggesting that debtors seeking to borrow large amounts of non-mortgage debt may be credit constrained by higher personal property exemptions. As discussed above, these consumers may be made worse off by increases in personal property exemption levels.

Taken together, these results suggest that increases in personal property exemption levels benefit homeowners and non-homeowners with relatively small levels of non-mortgage debt and may credit constrain borrowers seeking to borrow large amounts of non-mortgage debt. It should be noted that rising personal property exemption levels from 1984-2000 have likely increased borrowing for many more households than have been credit constrained by higher exemption levels. Doubling personal property exemption levels is estimated to increase average debt among those with positive levels of debt by 8-9% and to increase average mortgage debt among those with mortgages by 7-8%. The probability of households having greater than $10,000 in total debt is estimated to increase by 2% and the probability of having greater than $50,000 by 1%. In contrast, doubling personal property exemptions would decrease the probability of non-homeowners having greater than $50,000 in debt by only 0.3%. While it is not possible to determine the net effect of increases in personal property exemption levels on social welfare,

these results suggest that the number of households benefited by increases in exemption levels is larger than the group harmed by those increases.

These results paint a very different picture of the groups benefited and those harmed by higher exemption levels than do Gropp, Scholz and White's results. Whereas Gropp, Scholz and White argue that higher exemption levels harm households with low levels of assets and benefit households with greater asset holdings, I find that higher personal property exemptions harm only those households seeking to borrow large amounts of non-mortgage debt. This suggests that it is not poor households who are denied credit as a result of more generous personal property exemptions, but rather households with relatively high demand for credit.

At least as far as personal property exemption levels are concerned, the evidence suggests that increasing exemption levels benefits many households by reducing the riskiness associated with incurring mortgage debt when purchasing or refinancing a home, while harming a relatively small number of households who wish to incur large levels of non-mortgage debt.

5.7 Conclusion

This chapter examines the effect of homestead and personal property exemption levels on consumer borrowing and welfare from 1984-2000. The least squares and probit regressions described in this chapter indicate that homestead exemption levels are negatively related to household borrowing. Because much of this negative relationship appears to result from a decrease in the probability of homeownership when exemptions are increased, it is not consistent with the theory that higher exemption levels increase credit constraints on households. Further research is required to explain the negative relationship between homestead exemptions and homeownership.

The observed effects of personal property exemption levels on consumer borrowing are more consistent with theoretical predictions. The positive relationship between personal property exemption levels and household mortgage debt and total household debt suggests that higher personal property exemption levels benefit a majority of households. As personal property exemption levels rise, borrowing becomes less risky and households that are not credit constrained borrow more. The negative relationship between personal property exemption levels and the probability that a non-homeowner has greater than $50,000 in debt suggests that

159

households seeking to borrow large amounts of money may be credit constrained by increases in personal property exemption levels. While it is not possible to determine the net effect of bankruptcy exemptions on social welfare, the data indicates that there are far more households which benefit from higher personal property exemptions than are harmed by them.

Appendix: State Bankruptcy Exemptions

State	Homestead Exemption	Personal Property Exemptions
Alabama		
1984-present	5000	3000
Alaska		
1984-1989	24000	6000
1990-2003	60000	6000
Arizona		
1984-1989	50000	5650
1990-present	100000	5650
Arkansas		
1984-1994	Unlimited	9100
1995-1998	Unlimited	18700
1999-2001	Unlimited	20125
2002-present	Unlimited	21725
California		
1984-1990	30000	1200
1991-1995	50000	1200
1996-present	50000	1900
Colorado		
1984-1990	20000	2500
1991-2000	30000	2500
2001-present	45000	6000
Connecticut		
1984-1994	7500	9100
1994	76000	9100
1995-1998	76000	18700
1999-2001	76000	20125
2002-present	76000	21725
Delaware		
1984-present	0	75
District of Columbia		
1984-1994	7500	9100
1995-1998	15000	18700
1999-2001	16150	20125
2002-present	Unlimited	21725
Florida		
1984-present	Unlimited	2000
Georgia		
1984-2001	5400	9900
2001-present	10400	14100
Hawaii		
1984-1994	20000	9100
1995-1998	15000	18700
1999-2001	16150	20125
2002-present	17450	21725

Appendix: State Bankruptcy Exemptions (cont.)

State	Homestead Exemption	Personal Property Exemptions
Idaho		
1984-1989	30000	5500
1990-present	Unlimited	5500
Illinois		
1984-1989	7500	1200
1990-present	7500	3200
Indiana		
1984-present	10000	4000
Iowa		
1984-1988	500	2200
1989-present	Unlimited	5000
Kansas		
1984-present	Unlimited	20000
Kentucky		
1984-present	6000	6500
Louisiana		
1984-2000	15000	0
2001-present	25000	0
Maine		
1984-1989	7500	1200
1990-1991	26200	1200
1992-1994	27500	2500
1995-present	28300	2500
Maryland		
1984-1992	3000	3000
1993-present	5500	5500
Massachusetts		
1984-1985	60000	9100
1986-1994	100000	9100
1995-1998	100000	9100
1999-2000	100000	18700
2001	300000	20125
2002-present	300000	21725
Michigan		
1984-1994	7500	9100
1995-1998	15000	18700
1999-2001	16150	20125
2002-present	17450	21725

Appendix: State Bankruptcy Exemptions (cont.)

State	Homestead Exemption	Personal Property Exemptions
Minnesota		
1984-1992	Unlimited	9100
1993	Unlimited	9750
1994	200000	9750
1995-1998	200000	18700
1999-2001	200000	20125
2002-present	200000	21725
Mississippi		
1984-1991	30000	1000
1992-present	75000	10000
Missouri		
1984-present	8400	2900
1984-1995	40000	5700
1996-1999	60000	5700
2000-2001	60000	7000
2002-present	100000	7000
Nebraska		
1984-1987	6500	2500
1988-1997	10000	2500
1998-present	12500	2500
Nevada		
1984-1990	90000	4500
1991-1995	95000	4500
1996-1997	125000	4500
1998-present	125000	7500
New Hampshire		
1984-1992	5000	3500
1993-1994	30000	3500
1995-1997	30000	8000
1998	30000	18700
1999-2001	30000	20125
2002-present	50000	21725
New Jersey		
1984-1994	7500	9100
1995-1998	15000	18700
1999-2001	16150	20125
2002-present	17450	21725
New Mexico		
1984-1993	20000	9100
1994	20000	9100
1995-1998	30000	18700
1999-2001	30000	20125
2002-present	30000	21725

Appendix: State Bankruptcy Exemptions (cont.)

State	Homestead Exemption	Personal Property Exemptions
New York		
1984-1992	10000	1000
1993-present	10000	2400
North Carolina		
1984-present	10000	8500
North Dakota		
1984-present	80000	9700
Ohio		
1984-present	5000	3000
Oklahoma		
1984-present	Unlimited	3000
Oregon		
1984-1993	15000	3950
1994-present	25000	6900
Pennsylvania		
1984-1994	7500	9100
1995-1998	15000	18700
1999-2001	16150	20125
2002-present	17450	21725
Rhode Island		
1984-1994	7500	9100
1995-1998	15000	18700
1999-2001	16150	20125
2002-present	17450	21725
South Carolina		
1984-1994	7500	9100
1995-1998	15000	18700
1999-2001	16150	20125
2002-present	17450	21725
South Dakota		
1984-1998	Unlimited	2000
1999-present	Unlimited	4000
Tennessee		
1984-present	5000	4000
Texas		
1984-present	Unlimited	30000
Utah		
1984-1999	10000	3500
2000-present	20000	3500
Vermont		
1984-1994	30000	10000
1995-1996	30000	18700
1997-1998	75000	18700
1999-2001	75000	20125
2002-present	75000	21725

Appendix: State Bankruptcy Exemptions (cont.)

State	Homestead Exemption	Personal Property Exemptions
Virginia		
1984-present	5000	7000
Washington		
1984-1992	20000	9100
1993-1994	30000	9100
1995-1998	30000	18700
1999-2001	30000	20125
2002-present	30000	21725
West Virginia		
1984-present	15000	27200
Wisconsin		
1984-1986	25000	9100
1987-1994	40000	9100
1995-1998	40000	18700
1999-2001	40000	20125
2002-present	40000	21725
Wyoming		
1984-1996	10000	4000
1997-present	10000	4400

Chapter 6: The Effect of Asset Exemptions on Bankruptcy Filing Rates

The number of personal bankruptcy filings in the United States has risen dramatically from under 300,000 in 1984 to over 1.5 million in 2002. Losses to creditors from debts discharged in bankruptcy proceedings have been variously estimated at between $38 billion and $47 billion per year in the late 1990s. More than 1% of American households now file for bankruptcy each year.

Several bankruptcy reform proposals have been made in response to the large and growing numbers of bankruptcy filings. In 1997, the National Bankruptcy Reform Commission proposed reforming bankruptcy law to provide for a minimum value of assets which debtors would be allowed to protect from seizure by creditors. Another proposal, the Bankruptcy Abuse Prevention Act, a version of which has been proposed in each of the last four Congresses, would require certain debtors to make future payments to creditors for a period of years before receiving a discharge of their unpaid debts.

There is no consensus in the academic literature as to the likely effect of these proposed changes in the generosity of bankruptcy law on bankruptcy filing rates. Because the level of assets that can be protected by creditors differs across states, numerous papers have used variations in asset exemption levels to determine how changes in the generosity of the bankruptcy system may affect the number of debtors filing for bankruptcy. Specifically, several papers use state-level data to examine the effect of state asset exemption levels on bankruptcy filing rates. White (1988) finds a positive correlation between bankruptcy filing rates and bankruptcy exemption levels in cross-sectional data from the early 1980s. Shiers and Williamson (1987) and Buckley and Brinig (1998) find that exemption levels have a negative effect on filings. Ellis (1998) finds no relationship between the filing rate and asset exemption levels.

Pomykala (1997) examines the 1994 bankruptcy reform which doubled federal bankruptcy exemptions and found that filing rates rose more quickly in states which allowed debtors to choose federal exemptions than in those which did not.

The most recent attempt to quantify the effect of asset exemption levels on bankruptcy filing rates, Fay, Hurst and White (2002), uses micro data from the Panel Survey on Income

Dynamics to estimate the relationship between a debtor's potential financial gain from declaring bankruptcy and the probability that he declares bankruptcy. Finding a positive relationship between financial gain and bankruptcy, they argue that since increasing bankruptcy exemption levels would increase the "financial gain" to declaring bankruptcy, higher exemption levels should increase the number of bankruptcy filings.

This chapter uses changes in bankruptcy asset exemption levels from 1984-2002 to examine the effect of asset exemption levels on state bankruptcy filing rates. The results of that analysis are then used to predict the likely effect of proposed changes in the bankruptcy law. Section 6.1 describes the state and federal laws governing debt repayment and the way in which the Bankruptcy Abuse Prevention Act or the proposal by the National Bankruptcy Review Commission would change those laws. Section 6.2 sets out a simple model of the bankruptcy filing rate which suggests that, contrary to the assumption of many commentators, the effect of asset exemption levels on the bankruptcy filing rate need not be positive. In section 6.3, I estimate the effect of homestead and personal property exemption levels on bankruptcy filing rates using a simple fixed effects model. I find a statistically significant relationship between personal property exemption levels and bankruptcy. In section 6.4, I use the results obtained in section 6.3 to predict the effect of the National Bankruptcy Review Commission's proposed reforms on bankruptcy filing rates and discuss the potential effect of the Bankruptcy Abuse Prevention Act on filing rates.

6.1 Bankruptcy and Debt Collection Law

As described in Chapter 5 above, the collection of money from debtors is governed by a combination of state and federal laws including, among others, state laws governing the seizure of property by creditors to satisfy unpaid debts and the Federal Bankruptcy Code, which provides for the discharge of unpaid debts under certain circumstances.

Lenders can compel payment of unpaid debts by suing and receiving a judgment ordering a debtor to pay the amount owed, which can be enforced by a court order seizing assets or garnish wages. State laws governing the seizure of assets and the garnishment of wages provide some protection against creditors without the need to declare bankruptcy. A typical state law allows debtors to exempt at least some property from seizure by creditors and provides limits on

the ability of creditors to garnish wages. The consumer's debts are not extinguished. Only the creditors ability to collect on those debts are restricted. Nevertheless, if the consumer has no non-exempt assets and no wages, the creditor will be unable to collect on the debt regardless of whether the individual declares bankruptcy.

Bankruptcy provides debtors with additional protection from creditors and gives debtors the possibility of having at least part (and often all) of their unpaid debts discharged. Debtors wishing to have their debts discharged or restructured in bankruptcy may file under either Chapter 7 or Chapter 13 of the Bankruptcy Code. Debtors filing under Chapter 7 must turn over all property held as collateral by secured lenders to those lenders and must turn over all non-exempt property to the trustee to be sold with the proceeds (if any) being distributed *pro rata* to all unsecured creditors. Any unpaid debts are then discharged.

The property exemptions used in Chapter 7 can come from either state or federal law. The bankruptcy code defines federal bankruptcy exemptions levels for certain categories of property, including home equity, automobiles, jewelry and other personal property, but allows states to opt out of the federal system and to establish their own bankruptcy exemptions. Every state has done so, typically importing exemption levels from state debt collection law, although 16 states allowed debtors to choose between the federal and state exemptions.

Debtors filing under Chapter 13 of the bankruptcy code may keep all of their property, including property held as collateral by secured creditors, but must propose a payment plan under which each creditor will receive at least as much as he would have under Chapter 7 and which contributes all of a debtor's "disposable income" for three years to paying his creditors. After the payment plan is completed, all unpaid debts are discharged.

Among other things, the National Bankruptcy Review Commission proposed setting minimum and maximum asset exemption levels that would be available to debtors in bankruptcy. Specifically, the commission recommended that homeowners be permitted to claim exemptions of at least $20,000 in home equity and $20,000 in personal property. Homestead exemptions would be capped at $100,000. Non-homeowners would be permitted to claim exemptions of at least $35,000.

The heart of the Bankruptcy Abuse Protection Act is a requirement that prohibits certain debtors from filing under Chapter 7 of the Bankruptcy Code. Specifically, debtors who earn more than the median income in their county would be required to file under chapter 13 and

make minimum payments to creditors for three years. Debtors affected by the bill would be required to make greater payments to creditors than are required under existing law.

6.2 Theoretical Effects of Asset Exemption Levels on Bankruptcy Filing Rates

The theoretical effect of exemption levels on bankruptcy filing rates is not clear cut. To understand how the level of a bankruptcy exemption can affect a debtor's decision to declare bankruptcy differently depending on whether it is a state law exemption available both inside and outside of bankruptcy or a federal exemption available only in bankruptcy, consider the following simple model.

Debtors are assumed to have debt level D and to live in a state with a state asset exemption level A_S which is available outside of bankruptcy and asset exemption level A_B which is available to debtors who declare bankruptcy.

The utility cost of carrying a given amount of debt to a debtor who does not declare bankruptcy is given by the function $C(D,A_S)$, where that $\frac{dC}{dD} > 0$ and $\frac{dC}{dA_S} \leq 0$. Intuitively, carrying more debt is costly to the consumer, while the availability of a higher state exemption level reduces the cost of carrying a high debt.

If a consumer declares bankruptcy, he is assumed to incur a cost $W_i-B(A_B)$, where W_i is a measure of debtor i's non-monetary costs to utility associated with going bankrupt and $B(D,A_B)$ is a function representing the benefit gained from debt relief provided by the bankruptcy process. I assume that B>0 and $\frac{dB}{dA_B} > 0$, i.e. the benefit associated with debt relief is always positive and increasing with the exemption level. I further assume that W_i is distributed according to a cumulative distribution function F(W). Because F(W) is a cumulative distribution function $0 \leq F(W) \leq 1$ and F'>0.

A cost minimizing consumer will declare bankruptcy whenever the costs of doing so are less than the costs of carrying the debt without declaring bankruptcy, i.e. whenever:

(6.1) $W_i-B(D,A_B)<C(D,A_S)$.

Or, rearranging, whenever

(6.2) $W_i<C(D,A_S)+B(D,A_B)$.

Because W_i is distributed according the cumulative distribution function W_i, the bankruptcy rate, R, is given by:

(6.3) $R = F(W_i) = F(C(D, A_S) + B(D, A_B))$

It should be noted that the bankruptcy rate in this simple model depends on both the exemption level available outside of bankruptcy and the exemption level available in bankruptcy. Moreover, because $\dfrac{dC}{dA_S} \leq 0$ and F'>0, for given bankruptcy exemption A_B, there will be a weakly higher bankruptcy filing rate, among debtors with the same amount of debt, if the exemption available outside of bankruptcy is less than the exemption available in bankruptcy than if the two exemptions are the same. Put another way, when the exemption level available outside bankruptcy is less generous than that available in bankruptcy, more consumers will prefer to declare bankruptcy than if the two exemption levels where the same.

Holding the level of debt constant, the marginal effect of increasing the state exemption level available outside of bankruptcy is given by:

(6.4) $\dfrac{dR}{dA_S} = F'(C(D, A_S) + B(D, A_B)) \dfrac{dC}{dA_S}$,

and the marginal effect of increasing the exemption available in bankruptcy is given by:

(6.5) $\dfrac{dR}{dA_B} = F'(C(D, A_S) + B(D, A_B)) \dfrac{dB}{dA_B}$.

If the two exemption levels are the same, the effect of increasing the common exemption is therefore:

(6.6) $\dfrac{dR}{dA} = F'(C(D, A) + B(A))[\dfrac{dC}{dA} + \dfrac{dB}{dA}]$.

Two things are worth noting. First, the marginal effect of increasing only A_B on the bankruptcy filing rate is always positive Moreover, so long as $\dfrac{dC}{dA_S} < 0$, the marginal effect of increasing A_B will be greater if A_S is not changed as well. Consequently, I would expect the marginal effect of increasing the bankruptcy exemption on the bankruptcy rate to be positive and to be higher in states where the highest available exemption level comes from Federal law than when it comes from state law.

171

Second, it is not clear *a priori* that increasing a state exemption level that is available both to debtors outside of bankruptcy and in bankruptcy will increase the bankruptcy rate. This will be true if and only if $-\frac{dC}{dA} < \frac{dB}{dA}$, that is if the marginal benefit to increasing the exemption is greater inside bankruptcy than outside.

Because creditors have other means of collecting debts outside of bankruptcy such as garnishing wages, but can only collect from the non-exempt assets of a debtor who has filed for bankruptcy, the marginal benefit of increasing the exemption may be expected to be greater for debtors inside than outside of bankruptcy. However, that need not necessarily be the case.

When the potential effects of exemption levels on debt are included in the analysis, the total effect of exemption levels on bankruptcy filing rates is even more complicated. When the potential effects of changes in debt are included, the marginal effect of increasing the state exemption level available outside of bankruptcy is given by:

$$(6.7)\quad \frac{dR}{dA_S} = F'(C(D,A_S) + B(D,A_B))[\frac{dC}{dA_S} + (\frac{dC}{dD} + \frac{dB}{dD})\frac{dD}{dA_S}]$$

and the marginal effect of increasing only the exemption available in bankruptcy is given by:

$$(6.8)\quad \frac{dR}{dA_B} = F'(C(D,A_S) + B(D,A_B))[\frac{dB}{dA_B} + (\frac{dC}{dD} + \frac{dB}{dD})\frac{dD}{dA_B}].$$

Finally, the marginal effect of increasing both exemption levels is given by:

$$(6.9)\quad \frac{dR}{dA} = F'(C(D,A) + B(A))[(\frac{dC}{dA} + \frac{dB}{dA}) + (\frac{dC}{dD} + \frac{dB}{dD})\frac{dD}{dA}].$$

The total effect of an increase in the level of the bankruptcy exemption outside of bankruptcy is no longer clearly positive. Even though increasing the bankruptcy exemption level increases the benefit to declaring bankruptcy, it may lower the filing rate if the effect of changes in the debt on the filing rate are negative and sufficiently large. For example, if the effect of the exemption level on the debt, $\frac{dD}{dA_B}$, is positive and the marginal cost of debt is greater outside of bankruptcy than the marginal benefit of debt relief in bankruptcy, i.e. if $\left|\frac{dC}{dD}\right| > \left|\frac{dB}{dD}\right|$, then the filing rate may fall if the exemption level is increased. Similarly, if the effect of the exemption

level on debt levels is negative and if $\left|\dfrac{dC}{dD}\right| < \left|\dfrac{dB}{dD}\right|$, then the effect of raising the exemption level on the filing rate may also be negative.

Finally, equation (6.9) indicates that if both exemption levels are raised at the same time, the net effect depends on a combination of whether the difference between the marginal benefit of increasing the exemption levels is greater inside or outside of bankruptcy and the benefits (or costs) inside and outside of bankruptcy to changes in the level of debt held. The net effect of a change in the exemption level on the filing rate is therefore ambiguous. Moreover, it is no longer clear that increasing exempt amounts inside bankruptcy will have a greater effect on the filing rate than increasing exempt amounts both inside and outside of bankruptcy.

6.3 Effect of Exemption Levels on Bankruptcy Filing Rates

6.3.1 Data

I use state level bankruptcy filing rate data from the Administrative Office of the Courts combined with state level unemployment rates gathered from the Bureau of Labor Statistics and state exemption levels that I obtained for each state from 1984-2003 from state statute books to examine the effect of exemption levels on Bankruptcy filing rates.

As described above, bankruptcy exemption levels can be defined by either state or federal law for different categories of debt. State and federal laws define separate exemption levels for various categories of property such as home equity, automobiles, clothing, cash, jewelry and tools of the trade, among others.

I divided exemption levels into homestead exemptions (available for home equity in an owner occupied house) and personal property exemptions, which were calculated as the sum of automobile, cash and general personal property exemption levels. I do not include exemption levels for jewelry, tools of the trade or personal property exemptions for specific categories of goods such as "clothing" which were frequently unlimited.

Where both federal and state exemption levels are available, the exemption level that I used depends on whether federal or state law allows homeowners or non-homeowners to exempt more property. If federal law allows homeowners to protect more property than state law, then

the federal homestead exemption is used and vice versa. Similarly, if federal personal property exemptions are larger, then they are used and if state personal property exemptions are higher, then the state exemptions are used. It is therefore possible that in some states the maximum homestead exemption is given by state law while the maximum personal property exemption comes from federal law or vice versa.

It is not possible to include variables for asset exemption levels available in bankruptcy as well as those available under state debt collection law since asset exemption levels rarely change in states which allow the use of federal exemptions. Consequently, there is not sufficient variation to estimate the effect of bankruptcy and non-bankruptcy exemption levels separately.

Some states allowed unlimited homestead exemptions. Following Lin and White (2001), the homestead exemption level is coded as $500,000 in those cases . The specific value does not significantly affect the results since only Iowa, the District of Columbia (from a limited exemption level to an unlimited exemption) and Minnesota (from an unlimited exemption level to a limited exemption) change homestead exemption levels either to or from an unlimited homestead exemption. Re-coding the unlimited exemption level or excluding these states from the analysis has very little effect on the final results.

6.3.2 Empirical Strategy

To estimate the effect of the bankruptcy exemption level on the bankruptcy filing rate, I regress the log of a state's bankruptcy filing rate on the log of the value of homestead and of personal property exemptions for each state, state level unemployment rates and an index of state housing prices, as well as state and year fixed effects. Because state and year fixed effects are included, the effect of the bankruptcy exemption levels on filing rates is identified off the changes in exemption levels.

In order to test whether federal exemption levels (which are available only to debtors who file for bankruptcy) have a larger effect on the filing rate than state exemption levels (which are available to debtors whether or not they file for bankruptcy), I allow the effect of bankruptcy exemptions to differ depending on whether the exemption comes from federal or state law.

6.3.3 Results

Since 1984 there the number of bankruptcy filings by individuals has increased from less than 300,000 in 1984 to over 1.5 million in 2002. Over that time, the value of asset exemptions available in bankruptcy changed in 35 states and in the District of Columbia.

In 1994 Federal exemptions, which are available to debtors in 16 states and the District of Columbia, were doubled as part of a relatively minor revision to the bankruptcy code. Since then, Federal exemption levels have been increased automatically to adjust for inflation in 1998 and 2001. Figure 2 depicts the rise in the number of bankruptcy filings for two groups of states – those which allow debtors to choose between federal and state exemption levels (labeled "yes" in Figure 1) and those which do not (labeled "no" in Figure 1).

Figure 6.1:
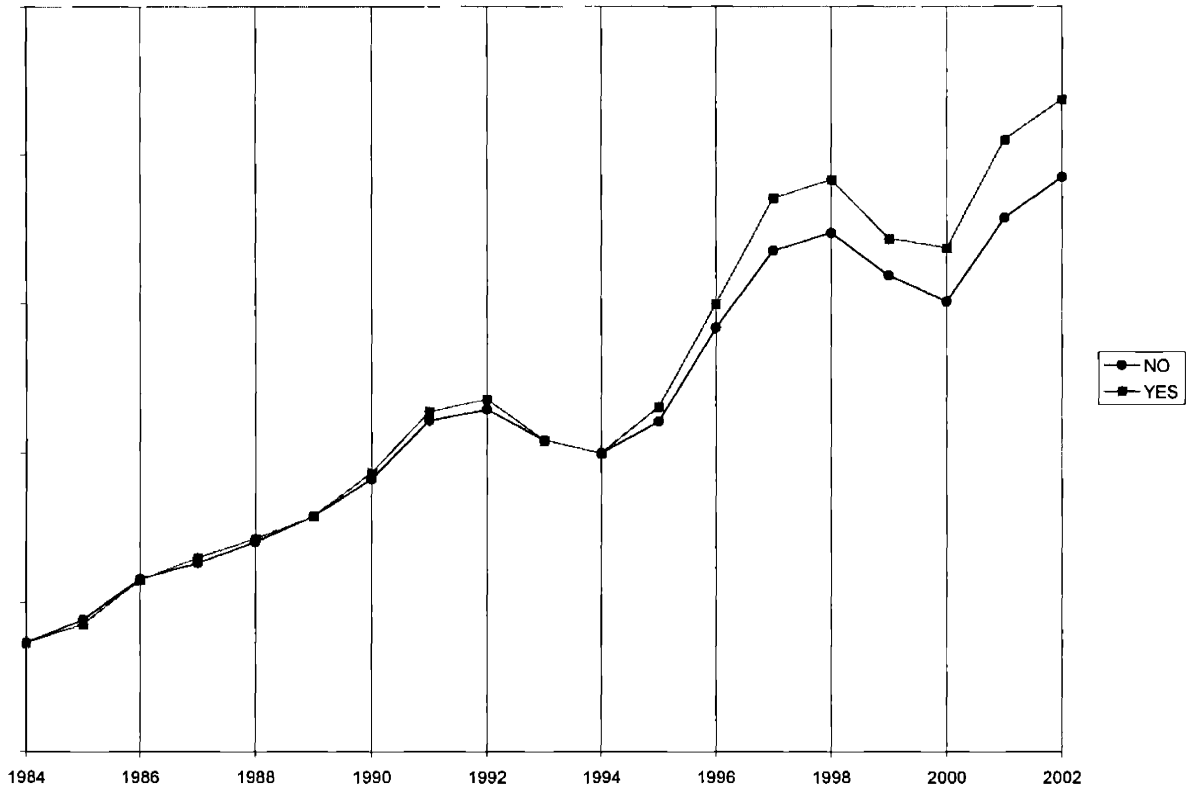
The clear increase in the number of bankruptcies in the states that allow debtors to choose the federal exemption level relative to those states which do not, following the 1994 increase in the federal exemption levels suggests that the 1994 reform led to an increase in the number of filings.

The results for the regressions of the log of the filing rates are presented in Table 1 below. Model (1) presents the results for the model where the effect of the bankruptcy exemption levels are pooled and bankruptcy exemption levels, state unemployment rate and state and year fixed effects are included as explanatory variable. Model (2) presents the results when federal exemption levels are allowed to have different effects than state exemption levels. Models (3) and (4) are the same as models (1) and (2) with the addition of state home prices as an additional explanatory variable. Standard errors are clustered on the state level.

Table 1: Effect of Asset Exemption Levels on Bankruptcy Filing Rates

Variable	Model (1)	Model (2)	Model (3)	Model (4)
Log of Homestead Exemption				
Coefficient	0.0074	0.0132	0.0026	0.0093
Standard Error	0.0287	0.0298	0.0259	0.0278
t-stat	0.26	0.44	0.10	0.33
Additional Effect if Federal Exemption				
Coefficient		0.0106		0.0128
Standard Error		0.0240		0.0203
t-stat		0.44		0.63
Log of Personal Exemption				
Coefficient	0.1945	0.1640	0.1969	0.1677
Standard Error	0.0845	0.0814	0.0905	0.0875
t-stat	2.30	2.02	2.18	1.92
Additional Effect if Federal Exemption				
Coefficient		0.0244		0.0224
Standard Error		0.0246		0.0243
t-stat		0.99		0.92
Unemployment Rate				
Coefficient	0.1197	0.1216	0.1214	0.1220
Standard Error	0.0228	0.0246	0.0230	0.0247
t-stat	5.24	4.95	5.27	4.93
Log Home Prices				
Coefficient			-0.4834	-0.4831
Standard Error			0.1997	0.2072
t-stat			-2.42	-2.33

In none of the four models does the homestead exemption level have a significant effect on the bankruptcy filing rate. The personal exemption level has a positive and significant effect on the bankruptcy filing rate in each model. When the effect is allowed to differ if exemption levels are federal rather than state exemptions, the point estimates suggest that the effect of federal bankruptcy exemption levels on bankruptcy filing rates is slightly higher than the effect of state bankruptcy exemptions, but the difference is not statistically significant.

Finally, in each model the unemployment rate is positively and significantly related to the bankruptcy filing rate. Home prices are negatively and significantly related to bankruptcy filing rates.

The estimated effects of the changes in personal property exemption levels are relatively large. For example, in Model (3) a doubling of the bankruptcy exemption level is estimated to

lead to increase in the bankruptcy filing rate that it is approximately the same that is caused by an increase in the unemployment rate of 1.6 percentage points or a 40% fall in home prices.

In the context of the model presented above, the small difference between the effect of federal and state exemptions is consistent with the hypothesis that the marginal benefit of an increase in a state property exemption outside of bankruptcy is relatively small compared to the marginal benefit of an increase in the exemption level available in bankruptcy.

The fact that the filing rates seems to be related to personal property exemption levels and not homestead exemption levels may indicate that insolvent debtors have little home equity and are therefore unaffected by increases in homestead exemptions. At the same time, insolvent debtors may have personal property in excess of the amount exempted from attachment by creditors. Increasing personal property exemption levels would therefore increase the benefit of declaring bankruptcy and would increase the number of bankruptcy filings.

6.4 The Effect of Proposed Reforms on Filing Rates

The results discussed in section 6.3 can be used to assess the potential effects of the National Bankruptcy Review Commission's recommendation and the Bankruptcy Abuse Prevention Act on the number of debtors filing for bankruptcy. As was mentioned above, the National Bankruptcy Review Commission recommended that homestead exemption levels be at least $20,000 and not more than $100,000 while personal property exemption levels be at least $20,000 for homeowners and $35,000 for non-homeowners. In the context of the model presented above, the commission's proposal would require that homestead exemptions be between $20,000 and $100,000 and that personal property exemptions be at least $35,000.

Because homestead exemptions do not seem to be related to bankruptcy filing rates, the restrictions on the level of homestead exemption levels is unlikely to significantly affect the number of debtors filing for bankruptcy. For that reason, I estimate the number of additional bankruptcy filings which would likely result from an increase in personal property exemptions to $35,000. Table 2 lists the number of additional bankruptcies in each state which would result from an increase in the personal property exemption level to $35,000. The number of additional bankruptcies is calculated using the results of Model (4) presented in table 2. Because the proposed law would affect exemptions available in bankruptcy only, the number of additional

bankruptcies is calculated using Model (4) assuming that the available exemptions come from Federal law, i.e. the elasticity of bankruptcy filings with respect to personal property exemption levels is assumed to be equal to 16.77% + 2.24%, or 19.01%.

As Table 2 depicts, increasing the personal property exemption levels to $35,000 would increase the expected number of bankruptcy filings by over 200,000 each year. The increase in filings in each state is calculated by multiplying .1901 by the difference in the log personal property exemption levels between the state's 2002 exemption levels and $35,000, the amount proposed by the commission. Louisiana is excluded because it's exemption level is zero, making a prediction using changes in log exemption levels impossible.

Table 2: Estimated Effect of National Bankruptcy Review Commision Proposal

State	Personal Property Exemptions	Proposed Change	Filings	Increase
Alabama	$3,000	$32,000	41,478	19,371
Alaska	$6,000	$29,000	1,359	456
Arizona	$5,650	$29,350	28,960	10,040
Arkansas	$21,725	$13,275	23,264	2,109
California	$1,900	$33,100	144,766	80,179
Colorado	$6,000	$29,000	20,769	6,963
Connecticut	$21,725	$13,275	11,570	1,049
Delaware	$75	$34,925	3,140	3,668
District of Columbia	$21,725	$13,275	2,451	222
Florida	$2,000	$33,000	90,374	49,173
Georgia	$14,100	$20,900	74,354	12,851
Hawaii	$21,725	$13,275	4,432	402
Idaho	$5,500	$29,500	11,454	4,030
Illinois	$3,200	$31,800	8,653	3,935
Indiana	$4,000	$31,000	81,091	33,437
Iowa	$5,000	$30,000	52,859	19,553
Kansas	$20,000	$15,000	14,731	1,567
Kentucky	$6,500	$28,500	26,823	8,584
Louisianna	$0	$35,000	26,461	-
Maine	$2,500	$32,500	4,321	2,168
Maryland	$5,500	$29,500	34,700	12,207
Massachusetts	$21,725	$13,275	17,019	1,543
Michigan	$21,725	$13,275	54,805	4,968
Minnesota	$21,725	$13,275	17,758	1,610
Mississippi	$10,000	$25,000	22,088	5,260
Missouri	$7,000	$28,000	51,655	15,804
Nebraska	$2,500	$32,500	1,958	982
Nevada	$7,500	$27,500	8,581	2,513
New Hampshire	$21,725	$13,275	7,569	686
New Jersey	$21,725	$13,275	3,822	346
New Mexico	$21,725	$13,275	40,310	3,654
New York	$2,400	$32,600	19,274	9,819
North Carolina	$8,500	$26,500	3,942	1,061
North Dakota	$9,700	$25,300	35,806	8,735
Ohio	$3,000	$32,000	69,705	32,554
Oklahoma	$3,000	$32,000	77,287	36,095
Oregon	$6,900	$28,100	23,890	7,375
Pennsylvania	$21,725	$13,275	23,043	2,089
Rhode Island	$21,725	$13,275	53,538	4,854
South Carolina	$21,725	$13,275	4,842	439
South Dakota	$4,000	$31,000	15,575	6,422
Tennessee	$4,000	$31,000	2,540	1,047
Texas	$30,000	$5,000	62,473	1,831
Utah	$3,500	$31,500	77,056	33,729
Vermont	$21,725	$13,275	41,861	3,795
Virginia	$7,000	$28,000	21,527	6,586
Washington	$21,725	$13,275	1,735	157
West Virginia	$27,200	$7,800	24,439	1,171
Wisconsin	$21,725	$13,275	38,545	3,494
Wyoming	$4,400	$30,600	10,109	3,985
Total Predicted Change				**216,981**

Because the increase in personal property exemption levels is substantially greater than the personal property exemption levels in most states, the estimated increase in the number of bankruptcy filings may be smaller than the number in Table 2. For example, if the number of potentially bankrupt debtors with assets in excess of $15,000 or $20,000 is small, then increasing exemption levels might increase the benefit of declaring bankruptcy for only a fraction of debtors and may have a smaller effect than Table 2 suggests.

The effect of the Bankruptcy Abuse Prevention Act on the number of bankruptcy filings is more difficult to predict. Because the Act would not alter asset exemption levels, but rather would require some debtors to make payments to creditors out of future income, the results described in section 6.3 cannot be used to estimate the number of debtors filing for bankruptcy. While numerical estimates of the effect of the Bankruptcy Abuse Prevention Act on the number of bankruptcy filings are impossible to calculate, the results presented in section 6.3 suggest that provisions of the Act which would require debtors to make larger payments to creditors would reduce the numbers of bankruptcy filings. In so far as requiring debtors to make payments out of future income is analogous to requiring debtors to give up larger amounts of personal property to creditors, higher future payments by debtors should lead to a smaller number of bankruptcy filings.

6.5 Conclusion

Debtors seem to respond to financial incentives contained in bankruptcy law when deciding whether to file for bankruptcy. Estimates of the effect of bankruptcy asset exemption levels on bankruptcy filing rates suggest that debtors file bankruptcy more frequently when personal property exemption levels are increased.

The estimated relationship between personal property exemption levels suggests increases in the value of personal property exemption levels such as those proposed by the National Bankruptcy Review Commission would increase the number of bankruptcies dramatically if the commission's proposal were implemented. On the other hand, if proposals such as the Bankruptcy Abuse Prevention Act which increase the value of payments required of debtors in order to receive a discharge of their unpaid debts, were to become law, the number of

debtors filing for bankruptcy would likely decrease, although it is impossible to estimate the size of the potential effect.

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