MINERAL DEPLETION, WITH SPECIAL REFERENCE TO PETROLEUM

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ABSTRACT

Two implications of received theory are (1) mineral net prices rise at the riskless interest rate, and (2) in-ground value is equal to the current net price. Both propositions are false. A correct theory has been joined to mistaken premises.

Mineral resources are inexhaustible. The economic problem is not the intertemporal allocation of a stock but coping with the cost of a flow of reserve accretions. Mineral scarcity and price are the uncertain fluctuating result of a tug-of-war between diminishing returns versus increasing knowledge. Hence minerals are risky assets.

Development cost, finding cost, and user cost (the penalty for development/production today instead of tomorrow) are all substitutes. Hence change in any one is a proxy for change in any other. Development cost is observable, and has been stable in many countries for prolonged periods. User cost was also stable in the USA. There is no sign of any pattern of gradual depletion and rising cost.

A simple model of an individual reservoir explains observed relations of value and price. The rate of interest has both a positive and negative effect upon the rate of reservoir depletion. The net effect of a change is therefore weak.

Expropriation of low-cost oil fields, had they been operated independently to maximize value, would have led to drastic increases in depletion rates. The fact of decrease proves collusive restriction of output to maintain prices.

All life is an experiment. Every year if not every day we have to wager our salvation upon some prophecy based upon imperfect knowledge. Oliver Wendell Holmes Jr.

THE PROBLEM STATED

"The price of oil should increase through time, growing at the rate of interest." [Starrett [Arrow rev.] 1987] "No viable alternative paradigm exists." [Miller & Upton 1985a, p.24] Figure 1 [IEW 1988] shows the consensus of rising prices. (See also [EMF 6 1982], [Hogan & Leiby 1985]) The vision of a tide lapping forever upward is in strong contrast with the historical record of Figure 2.

[Figure 1]

[Figure 2]

It is widely believed that the 1973 oil price was untenably low, because of long-run resource scarcity. The 1978 price, several times as high as in 1973, would soon have become untenably low. [Gately 1984, p. 1113] [Samuelson 1986, p.896-97]. Some suggest that even the 1981 price was artificially low, held down by Saudi Arabia.

The value of the mineral holdings (chiefly petroleum) of the United States Government was recently estimated at $860 billion, on the assumption of 1981 oil prices rising at 3 percent real per year, the revenue flow discounted at a riskless 2 percent. [Boskin et al 1985] The 1987 price should on that reckoning have been $51, nearly three times actual. The current private value is roughly 7 percent of their estimate.[1] A lower "social discount rate" to allow for taxation (page 8 below) and for alleged portfolio effects could hardly account for much of the 93 percent difference. For 1982, the total present discounted value of future net cash flows out of domestic oil and gas reserves held by all publicly quoted corporations was estimated under SEC rules at only $212 billion. [Andersen 1982 p.137]

The [Boskin et al] estimate includes the value of undiscovered oil. Taking only proved reserves: Mexico in 1981 had 57 billion barrels [OGJ:WWO 1981]. Their 1981 net price per barrel was above $30. With 3 percent increase and 2 percent discount, the asset was worth nearly $2,000 billion. It was--or seemed--ultra-conservative to borrow only $60 billion against it. The results were horrendous.

It is time to look at the underlying theory, which can be stated very simply. The present value of any mineral asset is equal to its net price (net of operating expenses) in any future year, discounted at the appropriate risk-adjusted rate:

$$(1) \quad V(0) = P(t)\, e^{-it}$$

where V(0) is today's value, P(t) is the net price in any year t, and i is the discount rate. Since the mineral stock is fixed, P will increase at some rate g:

$$(2) \quad P(t) = P(0)\, e^{gt}$$

Substituting into (1):

$$(3) \quad V(0) = P(0)\, e^{-(i-g)\,t}$$

If i exceeded g, the owners would forthwith sell off the mineral, in order to avoid holding an asset which earns less than the rate i. Contrariwise, if g exceeded i, owners would hold the below-ground asset, since it would grow in value more than if sold and the proceeds reinvested.

Therefore, in equilibrium g=i, and the Hotelling Valuation Principle (HVP):

$$(4) \quad V(0) = P(0)$$

Equations [2] and [4] give us two hypotheses for testing: minerals prices will rise over time, and the value of a barrel of oil in the ground will approximate the current net price.

THE RISING-PRICE PARADIGM

Suppose that over the long run mineral prices can be described by a set of random walks, some rising some falling with no significant average change. That would disprove the paradigm. The facts are much stronger. For 35 years it has been apparent that over the long term, most minerals prices have actually declined.[2] [Paley 1952] [Potter & Christy 1962]

[Barnett & Morse 1963] [Manthy 1978] [World Bank 1984]. In oil, before 1970 there was no sign of any upward price trend. The "renewable" resource timber has generally risen in price.

[Table I]

According to [Smith 1978], one can not reject the hypothesis of minerals prices not actually falling. Non-rejection is not acceptance. But the null hypothesis of stable prices, if true, would refute the paradigm of prices generally rising. Smith's data ended with 1973. There is a widespread impression of "a marked upward drift in the prices of oil and most other industrial materials which began in the early 1970s and accelerated further at the close of the decade." [Miller & Upton 1985a, p. 2[3]] In fact, during 1970-86, the real prices of nonfuel minerals declined nearly 40 percent. [IMF 1987]

[Slade 1982] estimated a set of quadratic function which showed mineral prices bottoming out by 1970, then rising. Table I applies the Slade estimating equation and deflator for each of the nonfuel minerals in her study. Actual prices in 1985 were invariably lower than predicted, on average one-third lower.

The price of uranium ore, which next to oil probably made the greatest impression in the 1970s, soared and then fell until by 1984 it reached (and has stayed at) an all-time low for the 37 years during which any price existed. [Neff 1984, p.16[4]]

THE "HOTELLING VALUATION PRINCIPLE" (HVP)

According to HVP, simplified in Equation [4] above, the present value of a unit of the asset is equal to the current net price. For oil, the net price of U.S.crude oil was for many years quite stable around nearly two-thirds of the gross wellhead price. [Adelman 1988] Hence, if the principle is correct, value should also have been about two thirds of gross wellhead price.

During 1946-1973, the average U.S. in-ground value of a barrel of oil, compiled by the J. S. Herold company, whose estimates are widely used and quoted in the financial press, stayed

[Figure 3]

in the range of 40-50 percent of the net price. Over the next 13 years, values ranged between 45 and 55 percent of the net price. [Adelman, DeSilva, & Koehn 1987]. Value never approached net price even during the highly unrepresentative period December 1979-August 1981 used by Miller & Upton, who could not replicate their results for any later period. ([Miller & Upton 1985b] See also [Gulley 1987])

For any given year, subtraction of development cost from value is an estimate of the pure resource value of the undeveloped resource, an approximation to user cost. As Figure 3 shows, it was approximately flat in nominal terms through 1972, hence declining in real terms. The later erratic bumps were obviously effect not cause of the price increases.

Another test of the HVP is in reservoir engineering practice. For many years, the Society of Petroleum Engineers' standardized nomenclature has included the "deferment factor," which is present value as a decimal fraction of future expected undiscounted net revenues. [PPH 1962 page 38-1] [PEH 1987, page 41-36] [Garb 1985]

If prices usually rose at or near the discount rate, a deferment factor around unity would be an important rule of thumb. Despite "the opinion of many operators [that] the long term inflationary trend may put a premium on future income..." [PEH 41-5], the recommended practice, in 1962 and 1987 alike, has been to assume the current price over the life of the project. One should discount the net cash flow first by a "safe rate," 5 percent [PPH1962] or 10 percent [PEH1987], then by a factor to allow for risk, to arrive at "fair market value," which is then stated as a percentage of the undiscounted value.

One classic yardstick for estimating the value of oil in the ground has remained reasonably constant through the years.... Oil reserves in the ground had a market value of approximately one third of their [current] posted wellhead price. [PEH 1987 p.41-5]. [See also OGJ 1980, Beninger & Arndt 1987].

One third of the wellhead price, i.e. one half of the net price, is close to the observed mean of the Herold valuations. It is altogether incompatible with the HVP. (We offer an explanation below; see "Valuation of Developed Reserves").

These value factors are averages of reservoirs with widely varying deferment factors. (The higher the reserves:production (R:P) ratio, the greater the deferment.) But both the 1962 and the 1987 handbook cite two studies, made in the late 1950s, of property transactions in widely varying locations. They are consolidated in Table II, and shown in Figure 4.

[Figure 4]

The first column is the average reserve:production ratio (R/P) in the subset. The next two columns show the fair market value as a percent of the aggregate undiscounted net cash flow at the constant price (We apply this concept below, pp. 19ff). The two sets of results are very close to each other, and are merged by regression in column 4.

If the HVP were correct, the degree of deferment should make no difference to current value. In fact there is a strong inverse relation: for every 1 percent increase in deferment, cur rent value declines by 0.44 percent. An R/P ratio of 11.5, which is close to the average for the United States in the mid-1950s, has a combined discount factor of just over 50 percent of net cash flow, which agrees approximately with the Handbook and the Herold valuations (The reader should disregard the last column until page 27, "The Valuation of Developed Reserve").

[Table II]

There are no later published studies of the relation of deferment to value. However, a recent paper [Horn 1986] gives a nomogram for relating the reserve:production ratio to market value. We take the middle of his range of discount rates. As the reserve:production ratio increases from unity (reserve depleted in a year) toward 17, the value declines by about two-thirds. This is not far from the 1950s, i. e. $(V1/V2)=(R1/R2)^x, 0.33=17^x$, and $x=-0.39$, as compared with the -0.44 exponent in Table II.

In a recent valuation of Naval Petroleum Reserve 1 (Elk Hills) [Horn, for Shearson Lehman Brothers 1987, Addendum H], post-tax cash flow is estimated at $6.8 billion; sales value at $3.4 billion, or 50 percent. Since the reserves were 9.9 times initial output, the ratio calculated from Table I is fairly close:56 percent. Incidentally, the present value of Federal tax payments was calculated at $0.8 billion; hence value to the public sector is 61.7 percent of net

cash flow. Thus the public discount rate might be loosely reckoned at 0.81 times the private rate. This is far above any riskless rate.

Martin B. Zimmerman has tabulated coal properties sold during 1980-82, as reported in the press. The sample is small but at least suggestive. Table III shows that the ratio of value to price is only a small fraction of what it is in oil. This follows logically from the low ratio of coal production to reserves, hence the high deferment factor.

[Table III]

[Figure 5]

GOOD THEORY, BAD PREMISES

The rising-price paradigm and the HVP are false. But the Hotelling theory [Hotelling 1931] is not only logically consistent, but provides a basic insight into mineral economics. The proposition, that a negotiable asset will only be held if the price net of cost rises at the risk-adjusted rate of return, not only seems "obviously" true, but has been verified often. The most familiar example is the price of a farm crop held from harvest to harvest, rising at the rate of interest in order to offset the cost of storage.

When a theory is logically correct, yet predicts wrongly, it must have been joined to at least one false premise. Here we have at least two. One, the neglect of investment, is discussed below, (see "Discount Rate and Depletion Rate"). The more basic error is the assumption of "an exhaustible natural resource...We have a fixed stock of oil to divide between two [or more] periods." [Stiglitz, 1976] and see [Dasgupta & Heal p.5] In terms of Figure 5, the fixed stock is the special case of only one harvest. In fact, there is no fixed stock to divide. There <u>are</u> additional harvests, but with no regular annual pattern.

Resources are inexhaustible.

More precisely, the total mineral in the ground is an irrelevant non-binding constraint. It would not matter if a resource were infinite -- never mind that "the great globe itself" is finite. If finding-development costs come to exceed the expected net price, investment dries

up, and the mineral is no longer sought. The amount still left in the ground is unknown, probably unknowable, but surely unimportant: a geological curiosity not an economic asset. Cost is all that matters, finite-ness not at all. The occasional estimates of worldwide "ultimate" or "potential" reserves are either an ordinal ranking of areas [Weeks 1969]; or more usually an implicit untestable prediction of future costs and prices. (Estimates for geologically defined smaller areas or "plays" are something altogether different (see Kaufman in AHKZ 1983)).

We cannot save the principle of a fixed stock by equating it to "the economic portion" of the mineral in the ground. That is circular reasoning. The "economic portion" is an estimate derived from future costs and prices. We might as well say that in the beginning there was some limited amount of buggy whips to be produced, and when the industry reached it, their earthly task was done. A logistic production curve for a mineral should be no more or less impressive than the many logistic curves for manufactured products. [Burns 1934]

The great contribution of Hotelling and other pioneers was to show how an industry with very large inventories (proved reserves) adjusts to expected price changes, which increase or decrease the value of current reserves, hence the "user cost" of consuming them today. But to assume that user cost is set merely by "future alternative use" is a one-bladed scissors. There is also a supply function, to which we now turn, after a tribute to a predecessor.

A notorious forgotten contribution

When W. Stanley Jevons sounded the alarm on coal supply [Jevons 1865, 1965, pages xxix-xxx], he was clear and emphatic: "Our mines are literally inexhaustible. We cannot get to the bottom of them." The problem was rising cost at the margin.

Jevons' failure is as instructive as his success. In trying to estimate <u>how much</u> marginal development cost might increase in Britain, he had to calculate how much coal existed at various depths and other cost-determining conditions. He went increasingly then ludicrously wrong as he extended the limited knowledge of 1865 for decades ahead. The same mistake has been made many times since, but with less excuse, even as to coal. [Steenblik 1986]

Jevons then multiplied this error by grossly overestimating the effect of rising coal costs on the national economy. It came almost to a coal theory of value.[5] **The Coal Question** is poorly regarded today. But the errors do not taint the insight.

A THEORY OF MINERAL COST[6]

The distinctive trait of mineral production is a very large shelf inventory, "proved reserves," constantly depleted at one end and replenished at the other. We need no assumption of a fixed stock, but only of constantly diminishing returns to investment, to get rising prices and support for Equations [1] - [4] above. As will be seen below, there is plenty of empirical support for diminishing returns. Yet somehow we have not gone forever from good to bad to worse. We need to look at the cost of creating inventory.

Development creates reserves

Exploration, discussed shortly, locates new reservoirs. Development investment, mostly drilling, creates proved reserves and producing capacity. Reserve-additions are not "discoveries." The distinction between knowledge and its application is basic. The study of copper and iron ore by [Trocki 1986] shows how a discovery initiates a long sequence of reserve additions. It is equally striking in oil.

The first Persian Gulf discovery was in 1908. Reserves in 1944 were reckoned by a special expert mission at 16 billion barrels proved, 5 probable. [OGJ 1944] By 1975, those same fields, excluding later discoveries, had already produced 43 billion barrels, with 74 billion "remaining." [IPE 1976] The numbers are larger today, but we cannot update because of the shrinking data base.

The 1944 geological team knew their business, and estimated correctly, from data known then. But Persian Gulf development investment, supplemented by some exploration on the borders and in the interstices, expanded the reserves.

In 1945, the United States (ex-Alaska) was considerably more drilled-up than is any other area today: 1.3 million holes dug, cumulative production 30 billion barrels, "remaining

recoverable reserves" 20 billion. But over the next 40 years, cumulative production (ex-Alaska) was 100 billion barrels, and 20 billion remained at the end of that time. It was not because of large new discoveries, which peaked before 1930, and declined rapidly thereafter. Of the 1985 reserves in large fields, about 80 percent was in fields discovered before 1945. [Adelman 1988]

The continuing increments to reserves in old fields resulted partly from increased recovery factors, partly from redetermination of oil-in-place. The expansion varied from negligible to massive. Kern River field in California was discovered in 1899. After 43 years, it had 54 million "remaining reserves." In the next 43 years, it produced 781 million, and had 877 million "remaining" at end-1986[7].

Persian Gulf costs declined before 1969 and were static thereafter. [Adelman 1972] [Adelman 1986a] In the US, there was a marked decrease in unit investment requirements per barrel added in 1918-30, plausibly the result of a stream of large discoveries. Cost remained stable for the next forty years, to the early 1970s. During the frenzied boom from 1973 to 1985, real cost approximately doubled. We have not been able to partition this increase between the depletion effect--if any--and the general waste and inefficiency of the unprecedented drilling boom which sagged after 1981 and collapsed in 1986. [Adelman 1988]

To conclude merely that "there's more oil out there than you think" is to trivialize the problem. The new reserves in old fields were neither a gift of nature, nor the geologists' supposed "conservatism" (i. e. righteous or pious fraud). There was continuing heavy investment.[8]

Models which postulate a phase of "exploration," and a temporary price drop, followed by the final irreversible price rise ([Dasgupta and Heal 1979], and many others) ignore the many sources of a large and uncertain flow of reserve-additions even without discoveries. One cannot expect a model to capture all of reality; but these ignore what is most essential.

The array of reservoirs: diminishing returns

It has long been a commonplace that "the better quality deposit will be mined first until it is exhausted, and the lower quality deposit will be mined subsequently. This is precisely what considerations of efficiency dictate." [Dasgupta & Heal 1979, page 173] This is an

important half truth, and half falsehood, because it ignores rising marginal cost. Within any reservoir, the more intensive the development, the higher the required investment per unit of output. It follows that the greater is cumulative depletion, the higher the investment coefficient also. ([AHKZ, chapter 1, and **passim.**] For an elegant development, see Bradley in [Gordon, Jacoby & Zimmerman eds. 1986].) We examine this in detail below (see "Discounting and Forecasting").

As one develops the cheapest deposit, its marginal development cost must rise to the point where it equals initial marginal cost in the second-cheapest. Henceforth both deposits will be developed at rising cost, until one reaches equality with initial marginal cost in the third-cheapest, and so on. At any given moment, a number of deposits are being expanded. In equilibrium, the level of marginal costs is equal everywhere. Movement toward equilibrium, which may take a long time, is the process of moving along the varying slopes toward equality in the levels.

Instead of whole deposits, one should think of tranches of each deposit. Under competitive conditions, the market serves as a sensing-selective mechanism, scanning all deposits to take the cheapest increment into production at any given moment. [Lee and Aronofsky 1958] There is of course a lag. Newer lower-cost fields expand more rapidly than, or at the expense of, older higher-cost fields. Therefore when high cost areas actually expand and low-cost areas contract, it is a sure sign of a block to competitive adjustment. Never has this anomaly been more glaring than since 1973. [Adelman 1986a]

But there is an offset to rising marginal costs: increasing knowledge of the particular reservoir and its neighbors, and also improved technology. The uncertain expectation of which effect will dominate, over what period, determines the in-ground value of reserves.

Exploration is research

It does not add reserves but knowledge: the probability of a mineral occurrence in a given place, and some indications of its size and development cost. Finding-plus-development

cost of the new pools must be less than continued more intensive development in the old ones, or the investment is misdirected.

Output from exploration effort is, at best, very uncertain. As just seen, the effects may extend over decades. As in time, so in space: new knowledge in one area may be a joint product with new knowledge in another. Therefore unit exploration cost is usually though not always unknowable, for the same reasons as unit research cost. The "finding costs" reported in the financial press are an illogical mix, often absurd on their face, though they may be ordinally correct. [Adelman 1988]

Diminishing returns to exploration

Within a given basin, and in any given state of technology, the size of new-found fields diminishes over time. The larger deposits would be found earlier even by chance, because they are larger. The better informed is the search, the more clear is the progress downward. [Kaufman in AHKZ 1983] [Smith & Paddock 1984]

Diminishing field size holds only for a given basin or "play" containing a population of reservoirs which is sampled by exploration. To extrapolate into a larger area "breaks the model's legs in several different ways." [Kaufman in AHKZ 1983, p.294] The whole world is not one great petroleum play, with diminishing size throughout.

But even within a given basin, and a fortiori for a larger area, a constantly decreasing average size of new fields does not necessarily imply an increasing exploration cost per unit over time, because over time the ceteris paribus assumption cannot hold. Let the reader imagine a few known reservoirs sprinkled over an area. Their discovery and development has an externality: knowledge of the geology of the intervening areas. The practice of "dry hole money" confirms this.[9] The later discoveries in those areas will probably be smaller, but so will be the resources expended to find and develop them. The net result may go either way.

Moreover, the relation between smaller fields and development cost is weaker still. Smith and Paddock [1984] have shown that in a number of large producing countries, the size

of onshore fields cannot be related to cost. We pointed out earlier that it did not hold in the USA after 1930.

The marginal equalities of mineral investment

Additional oil or gas reserves can be obtained in a known exploited field, by (1) more development drilling in known exploited reservoirs, (2) drilling in known but hitherto unexploited ones, (3) finding and developing new reservoirs which are believed to exist in the proximity of old ones. The next step is (4) to search for new fields in old basins. The most venturesome is (5) to search for new basins. Overarching all these steps is (6) the search for new better i.e. cheaper methods in any of the five classes. All these forms of investment are alternatives. In any given undeveloped pool, exploration and development are complementary; outside of it, they are competitive. If one does none of the above, but simply uses up some of the existing inventory of reserves, then incurring the loss of its value, or "user cost," should not exceed the cost of a barrel gained by the best alternative investment.

Development cost change as proxy for inground value and exploration cost

Cost at any given margin is constantly driven toward equality with cost at every other margin. Therefore change in any one type of cost is a proxy for change in every other. If newer fields are becoming smaller, deeper, more faulted, etc., then more development investment is needed to book an incremental reserve barrel.[10] Thus changes in development cost indicate at least the direction of changes in finding cost, which is usually unobservable.

At any given moment, the avoided cost of additional development measures the value, hence the user cost, of a newly-found barrel. For example, a crude simulation with 1962-64 data estimated the value of newly discovered Persian Gulf oil, or "maximum economic finding cost" (MEFC). Strong assumptions were made to put the maximum likely strain upon existing fields: zero new-field discoveries, and also higher consumption growth, helped by disappearance of most coal worldwide, and zero nuclear construction. The resulting massive depletion from existing Persian Gulf fields greatly increased development costs relatively, but not much absolutely: about 5.3 cents per barrel by 1980. [Adelman 1966, p. 59]

Adjusted to 1976 factor price levels [IPAA], this estimate was 13.8 cents. It is comparable with a uniquely well-informed estimate, made that year, of both the value and the cost of a newly-found barrel. When Saudi Arabia "bought" the assets of the Arabian American Oil Co, Aramco stayed as operator. The agreed-upon fee payable for a barrel of newly-discovered oil was six cents. We would expect it to be lower than our estimate because interim depletion was much lower than we assumed. (The six cents of user cost, plus operating and development cost of perhaps 30 cents, compare with a then-current price of $12.50.)

If future development is expected to cost more than current, the value of oil in a developed reserve is higher. Some part of the stock will become more profitable to hold than to extract, production will be lower, and prices higher today. Thus the markets which value below-ground inventories (reserves) exert an influence over, just as they are influenced by, the market in current production. Rising values are a forecast of increasing scarcity, and an inducement to react to buffer the change. Mankind is not in danger of driving blindly off the cliff of inadaquate oil supply.

Mineral user cost is, as Marshall said of the rent of land, "not a thing in itself, but the leading species of a large genus." The value of any asset, in any industry, is equal to the <u>lesser</u> of (a) the present discounted value of its future surplus over operating costs, or (b) its replacement cost. The ratio (a/b) has become well known as "Tobin's Q." In a mining industry, the higher the in-ground value, the greater the development incentive. Conversely, the higher the development cost, the greater the value of a barrel already developed. Value and replacement cost are forever gravitating toward one another, and pushed apart by exogenous forces.

The valuation and sale of existing assets is a routine business activity, supported by uncertain changeable forecasts, corrected and updated when possible. There is nothing unique about minerals. Only the illusion of a fixed mineral stock has given such great but (in my view) mistaken importance to the question whether there exist perfect forward or futures markets in minerals to reflect all possible states of the world. Of course not, any more than in non-minerals. It should disappoint nobody but Dr. Pangloss.

What we have instead is a system which tends to stability because the opposing forces of depletion and of progress play upon a very large number of old and new deposits. The various methods of increasing supply, all competing for capital funds, make a mineral industry stable. Steady supply does not depend on a few lucky throws of the dice, but rather on a continuing game played incessantly for small stakes and occasionally for large.

DISCOUNTING AND FORECASTING

There have been some attempts to relate mineral price changes to the discount rate. They seem without exception to assume riskless or very low rates.[11] This is reasonable, if--but only if--net minerals prices must inevitably rise smoothly. But the price and cost of a mineral are the uncertain fluctuating resultant of diminishing returns versus increasing knowledge. The question is how much of the variance about the expected price can be diversified away by folding into the portfolio of the markets in assets and incomes, or the "portfolio" of the whole national income.

Much mineral price risk is correlated with the economy, hence un-diversifiable. The fluctuations of raw-material prices in the 1970s ([IMF 1986]) look like an exaggerated version of fluctuations in economic growth.

Long-term, the downside risk is from greater knowledge. Much of the progress in science and technology is autonomous and irregular, hence diversifiable. But much is not. The more the price rises, the greater the rewards of acquiring knowledge. To an important extent, therefore, the downside risk is also not diversifiable.

Thus there is no good reason to question the popular or business perception that investment in minerals is no less risky than investment in other branches of economic activity, perhaps even a shade more risky, but not strikingly different. In fact, corporations specialized in oil and gas exploration and development have a cost of equity capital close to that of corporations at large, roughly 10 percent real this last decade. [Table I in Adelman 1986b] Reserve purchases seem to imply 10 to 12 percent. [Diggle & David 1987][Lohrenz 1989] This cannot be

reconciled with the rising-price paradigm, and the vision of oil as a low-risk appreciating asset. But it is compatible with the theory sketched here.

In considering future prices, we should be guided by [Koopmans 1974]:

Do these past [declining price] trends extend into the future? I submit that an answer to that question is not within the capability of the economist. To the extent that an answer is possible, the main contribution should come from scientists and engineers, and especially from those among them who spend a good deal of time and effort looking ahead as to what future technology and resource availability may be. (p.11)

Koopmans is right, but the economist can observe a capital market, which is also a market in ideas. A few brief examples will show the kind of knowledge, guesses, and hunches of scientists and engineers which feed the perceptions of investors and shape the values of assets.

"Unconventional" hydrocarbons

Sixty years ago, shale oil appeared like a good bet for near-term development, but it is no closer today. But oil under the ocean was an "unconventional" hoped-for source as late as 1945, and today it supplies a large and growing fraction.

Heavy oil has been known for centuries. Even 15 years ago, there was no indication of early development. But by the end of 1986 it was 38 percent of total Canadian conventional crude reserves, and increasing rapidly. One large reservoir contains about 25 billion barrels, of which 5 billion could be recovered with current technology. Development has thus far created only 450 million barrels "net proved oil reserves," highly profitable at less than 1987 prices. [Randol & Macready 1987] But the Venezuelan deposits, many times greater and much cheaper, [Parra 1981] [PIW 1986], are largely blocked from use by Venezuelan participation in the cartel, and by the Venezuelan success, since nationalization, in development of even cheaper conventional oil, mostly in old fields.

The amount of natural gas dissolved in water trapped in underground formations in South Louisiana is many times as great as all known natural gas found and produced so far; but so far it supplies not an mcf of reserves [FERC 1978], and there is no indication when it ever will.

These are the kinds of scientific-engineering and economic data which a capital market reduces to a common currency: their effect on values estimated today. Those estimates fluctuate above and below the true values. Markets are not precision instruments. But they do not systematically and permanently undervalue in-ground assets, as compared with other assets.

DISCOUNT RATE AND DEPLETION RATE

A good statement of the consensus is in [ICF 1979]:

> *If extractors become less fearful that their fields will be expropriated, it is no longer rational to pump as vigorously, much as if there were an unanticipated reduction in the interest rate.*

This is very revealing. How can a lower interest rate reduce investment hence production? The implicit major premise is: zero investment; one has only to "pump" more or less "vigorously."[12] Only a minority of economists have considered the investment process. (See [Gordon 1966], pp.332ff.]; [Adelman 1972, pp.38-40]; [Watkins 1978, pp.321 - 29])

The optimal depletion rate

To incorporate investment, we set up a very simple model of development of an oil or gas reservoir.

Let P = constant expected price, net of current operating cost

V = net present value in ground

Q = initial output, barrels per year

a = exponential depletion/decline rate, percent per year

k = the investment factor, an empirical constant reflecting more or less favorable reservoir conditions

K = projected capital expenditures = kaQ

R = reserves = cumulative output expected

i = discount rate, percent per year

We simplify by assuming exponential decline, as is conventional among reservoir engineers. Using the more rigorous hyperbolic decline function would considerably increase reserves, and

also complexity. In terms of present value an infinite time period does not (usually) make much difference. Then,

$$(5) \quad R = \int_0^{\infty} Qe^{-at}dt = Q/a, \text{ or } Q = Ra$$

The derivative da/dQ = 1/R = a/Q. Then the net present value, say VR, is,

$$VR = PQ \int_0^{\infty} e^{-(i+a)t}dt - kaQ$$

$$(6) \quad VR = (PQ/(a+i)) - kaQ$$

In words: net present value is future output at a constant price, subject to the compound discount of production decline and of futurity, <u>less</u> up-front capital expenditures, kaQ.[13] As mentioned earlier, costs increase with intensity of exploitation. Moreover, too high a depletion rate would damage the reservoir and lose recovery. Within this limit, which varies greatly among reservoirs, the operator must choose the optimum depletion rate.

To make all projects comparable, we divide through equation [6] by PR, price multiplied by reserves. PR is the deposit's limiting value, if it could all be brought up instantaneously with zero investment.

$$VR/PR = (PQ/PR)/(a+i) - kaQ/PR$$

Substituting from [5] and cancelling, we have:

$$(7) \quad VR/PR = a/(a+i) - ka^2/P$$

We have already (Figure 4 above) met the ratio VR/PR as the deferment factor used by reservoir engineers. The quadratic term captures rising costs. However, the true curve is often much flatter, up to the point or neighborhood of some maximum tolerable rate, after which it rises abruptly. Over the range where cost is a linear or even horizontal function of

development intensity in a given pool, the optimal depletion rate might be considerably higher than as suggested in Figure 6. However, for a whole area including a number of reservoirs, the quadratic may be an acceptable envelope curve.

Given depletion intensity a, average cost of the incremental tranche is marginal cost of the deposit. This is calculated ex post by setting VR to zero, and taking P' as breakeven price or cost:

(8) $P' = ka\,(a+i)$

But ex ante, a is not exogenous, it is the decision variable. To maximize present value, we differentiate with respect to a, obtaining the optimal depletion rate a* :

(9) $d(VR/PR) / da = i / (a+i)^2 - 2ka / P = 0$

(10) $a^* = \sqrt{Pi/2ka} - i$

Equation (10) is a form of the cubic:

(10a) $a^3 + 2a^2 + ai^2 - Pi/2k = 0$

Gordon M. Kaufman has demonstrated that one root of (10a) is real, two are imaginary. This is logically necessary for applications of the equation. For calculation, especially on a personal computer, it is easier to solve by numerical approximation of Equation (10).[14] The discount rate has both a positive and negative effect on the depletion rate, which is maximized when:[15]

(11) $P/k = 4a\,(a+i)$

Where P/k is less than 4a(a+i), an increase in the discount rate lowers the optimal depletion; where it is greater, a higher discount rate raises optimal depletion. User cost is implicit in Equations [5] - [10], since at optimum depletion there is no gain in moving production from one to another time period.

Now let us assume a price increasing at g percent per year; then we substitute (i-g) for i throughout. Assume now that i=g, as in Equation [3]. Then optimal investment and depletion is zero. This is consistent, since the basis for Equations [5] - [10] is a tradeoff between the incremental value and cost of quicker recovery. If the price is rising at the rate of interest, the quicker recovery is not worth anything.

On the horizontal axis of Figure 6 is a range of depletion rates, which in practice rarely exceed 20 percent per year. On the vertical axis both gross present value and expenditures are stated as a percent of PR. In the hypothetical "high cost" reservoir, the investment factor k is five times that in the "low cost" pool. Here 14 percent depletion, even if technically feasible, would cost more than the total undiscounted value of the reserve.

There are three sets of present values, corresponding to the three hypothetical discount rates: 5, 10, and 20 percent per year. For either reservoir, present value is optimized where the slope of the curve showing present value of receipts equals the slope of the investment requirements curve.

[Figure 6]

Within the range of commercial discount rates, 5 to 20 percent, the chaged discount rate has very little effect upon optimal depletion. At a given discount rate, optimal depletion dwindles as cost rises. But, as discount rates increase, high-cost pools become marginal then completely non-viable. Far from speeding up production, very high discount rates force a shutdown.

At their respective optima, total expenditures (normalized as percent of PR) are approximately the same in the high-cost and in the low-cost deposit. This is the expected equality at the margin.

Reserves as options

The owner of an undeveloped deposit has an option on a developed reserve, whenever he is willing to pay the development cost. In our example, if the discount rate is as high as 25 percent, the high cost reservoir is an option not worth exercising now. But a higher price-cost ra-

tio (P/k) would rotate the investment curve clockwise. Therefore the probability of a higher price, or a lower cost, or a lower discount rate would give the reservoir some positive value, not to develop now, but to hold.[16] [Paddock Siegel & Smith 1983, 1989] show how an undeveloped deposit whose contents were worth $10.61 per expected barrel in 1980, and would cost $11.79 to develop (i.e., a negative conventional net present value), had on some reasonable assumptions an option value of $.052. This is a more refined estimate of user cost. The better the deposit, the less important the option value. Where deposits are not marginal, "we simply develop the property immediately." The value of waiting for an uncertain gain converges to zero as the value of immediate development increases.[17]

The valuation of developed reserves[18] was discussed above (Hotelling Valuation Principle). Since the investment has already been made, the cost is zero, and the value of the reserve, in relation to the net price, is:

(12) $V/P = a / (a+i-g)$

(13) $g = i + a(1 - (P/V))$

HVP is a special case of [12], for if g=i, then V=P. Alternatively, suppose that g=0, and that a is approximately equal to i, as was true for many years after World War II. Then V=P/2. This explains why the in-ground value has long fluctuated around a mean of one-half of the net price (one-third of gross wellhead price).[19]

The reader should now look at Table II, column 5, and at Figure 4. Assuming i=8 percent for a developed reserve and a=Q/R (Equation 5), this gives the predicted ratio of present value to expected undiscounted receipts. It is very far of course from the constant 100 percent of HVP, but there is systematic understatement, the error perhaps increasing with the degree of deferment. A different discount rate might reduce the discrepancy. Moreover, the depletion is really over a finite period, hence the decline rate is overstated at low values.[20] I suggest a more important source of error: many properties were not fully developed when sold. (This has been

the case in recent California sales: [Miller & Vasquez 1988] If so, it would pay to increase the depletion rate, thereby raising the present value of the receipts. But this would require additional investment. In short there is an unknown but positive error in the assumption of a fully developed lease and known factor a. The error is an indicator of the option value of increasing the reserve.

Two corollaries in taxation and nationalization

(1) Mineral or oil production is commonly taxed on a royalty basis, the sovereign taking either a given percentage of the output, or of total market value. In many countries, the percentage is very high, with major effects on depletion. For example, if the sovereign took 80 percent of output, or of market value, that would in effect reduce P/k by 80 percent, exactly the same as multiplying k five-fold, and rotating the cost line from "low" to "high." At 20 percent discount, it would be barely viable. (Even in the United States, where rates are much lower, the effect is not neglible [Lohrenz et al 1981]. Consider now even a mild increase in risk, and in the discount rate. Even the low-cost project is not worth investing in.

Contrariwise: a reduction of the royalty tax rate increases the optimal depletion rate. A reduction in risk may also increase it. Such reductions go far to explain why, despite a drop of over 60 percent in real oil prices since 1981, non-cartel production outside the US has actually increased. A better solution would be to tax profits. [Bradley & Watkins 1987][Smith 1987]

(2) The price explosions of the 1970s have been explained as a competitive response to a drastic fall in discount rates. Private corporations rightly expected confiscation. Their high implicit discount rates allegedly made earlier production more attractive relative to later. Hence production was higher, and the price lower, than the long-run competitive level. But the expropriating nations had no such fear, used properly low rates, and therefore depleted more slowly. Lower production and higher prices were not a cartel result; they were actually closer to the long-run competitive norm. [Mead 1979] [Samuelson 1986]

Anyone familiar with Persian Gulf depletion rates must be astonished at the suggestion of too <u>rapid</u> depletion, since rates there were the lowest in the world, about one-sixth of the in-

dustry's rule-of-thumb normal. [Adelman 1972] Moreover, governments always demanded higher output from resident companies, who tried to stave off overproduction that would undermine prices. Iraq expelled the companies in 1972 because they did not produce enough. I believe that OPEC governments face higher risks than private companies and use higher implicit interest rates. [Adelman 1986b] But as just seen, discount rate changes do not seem that important. But expropriation abolished the excise tax in low-cost areas. The investor now received all of the net price, not a minor fraction. In terms of Figure 6, the effect was exactly the same as dividing the investment factor k by a factor of 4. It rotated the investment line clockwise, from "high cost" to "low cost." Under competitive conditions, the removal of the excise tax would greatly <u>increase</u> the optimum depletion rate. Equation [8] shows how user cost is directly proportional to investment cost, because both depend on the worsening tradeoff between holding and developing. Expropriation lowered user cost.

By competitive standards, therefore, the 1970 price was not too low but too high, a distortion supported by the excise tax. This assumes a constant price, however. Although there is good reason for that assumption, we relax it now to explore the consequences.

THE GAIN OR LOSS TO POSTPONEMENT

We now assume that the price will increase at g percent per annum. In Equations [5] to [8] above, we substitute (i-g) for (i). To restate net present value as a function of time, we drop the a* designation, and treat the depletion rate as predetermined.[21] The net present value of the project initiated in any year t is:

$$(14) \quad VR(t) = [(PQ\,e^{gt}/(a+i)) - kaQ]\,e^{-it}$$

$$= (PQ/(a+i))\,e^{(g-i)t} - kaQ\,e^{-it}$$

Differentiating with respect to t:

$$(15) \quad d(V(t))/dt = [(PQ/(a+i))\,(g-i)\,e^{(g-i)t} + kaQi\,e^{-it}$$

To find the best time to commence development (see also [Paddock Siegel & Smith]), we set the derivative to zero, transpose one term, divide both sides by -Q, then by $(i-g)e^{it}$ obtaining:

$$(P/(a+i))\ (g-i)\ e^{(g-i)\,t} = -\ kai\ e^{-it}$$

$$e^{gt} = (kai\,/\,(i-g))\ \ ((a+i)\,/\,P)$$

Taking logarithms, and dividing both sides by g:

$$(16) \qquad t = (\ln k + \ln a + \ln i + \ln(a+i) - \ln(i-g) - \ln p)\ /\ g$$

Thus the optimal time to start depletion, which may or may not be the present (time zero) depends on current price, the expected rate of price increase, investment requirements perunit, the discount rate, and the predetermined optimal depletion rate.[22] A negative t means that the investment should have been made that many years ago. Its postponement to the present has been a loss.

At one extreme, suppose the project is costless, i.e. the investment factor k=0. Then:

$$(17) \qquad VR\,(t) = PQ/(a+i)e^{(g-i)\,t}$$

$$(18) \qquad d\,(VR\,(t))/dt = (PQ/(a+i))\ (g-i)\ e^{(g-i)\,t}$$

Here postponement depends strictly on the expected rate of increase in price versus the discount rate.

Case	Derivative	Investment Action
g>i	Positive	Postpone investment
g=i	Zero	None: indifference
i>g	Negative	Speed up investment; project is overdue

As with constant prices, the Hotelling Valuation Principle (HVP) is the special case of g=i: the present value of producing this year is the same as in any other year.

Suppose now that the project would barely repay the cost of capital, i. e. $PQ/(a+i) = kaQ$, hence $VR(0) = 0$. Therefore:

$$(19) \quad VR(t) = (PQ/(a+i))\, e^{-it}[e^{gt}-1]$$

In this case, VR(t) is always positive if g exceeds zero, negative if g is negative, and stays at zero if g is zero. The derivative is:

$$(20) \quad d(VR(t)/dt = (PQ/(a+i))\, e^{-it}[(g-i)\ e^{gt}-i]$$

Here waiting raises the price to be received, but does not affect the needed investment.[23] Whatever the cost of capital, the investor is better off waiting for a higher price. If the current return is negative, all the more reason. Thus a "dog" of a project is always worth postponing, and has only option value. A good project should usually not be postponed.

THE SUPPLY CURVE AND EXPECTED PRICES

So far, we have considered the single deposit or field in isolation, with the price exogenous. Now, if we knew the values of k, a, i, and g for all reservoirs, then we could calculate both the optimal decline rate under a static price, and the optimal postponement period under any expected price increase. We could model the response of the system to exogenous shocks or assumed rates of reserve deterioration and rising development costs. Suppose that k-factors were expected to rise as reserves added were always higher-cost than those used up. Depletion rates would fall, and lower production would force up prices. There would be conflicting effects: higher prices would promote development, but some development would be postponed for the expected still-higher prices. This iteration would go on until the system was again in balance, at a higher price level.

Existing data is far from allowing any such modeling design, which is perhaps why past modeling efforts have not been successful. However, we can get some answers to limited questions, by making use of Equations [10] and [16].

For Saudi Arabia in 1970, we make a test of the pressure on resources. The private company's net price was the approximate arm's-length price ($1.25) less operating expenses of 5 cents and less an excise tax (in form an income tax) of 88 cents. The actual depletion rate was 1 percent; the calculated private optimal was nearly three times as high. (It was probably much higher, since as explained above, the quadratic exaggerates the cost increase.) Since actual was below optimum, it would have paid to expand, i.e.user cost was negative. This indicates a substantial repression of output to avoid spoiling the market.

[Table IV]

The government operation is assumed in line (2) to use the same discount rate. Its higher net price calls for an optimal depletion rate seven times the actual, and nearly three times the private. In line (3), we reduce the discount rate by a factor of nearly 10, to be barely in excess of the expected rate of price increase, in order to make t=0. The optimal starting time is now the present, instead of being 45 years overdue. This makes little difference for the optimal depletion rate. In short, given 1970 prices and costs, and assuming competitive hence purely autonomous decision-making, net present value would have been vastly increased with more output, started sooner.

In lines (4) and (5), we assume much higher prices, and the result is greatly to speed up depletion, though not to accelerate the optimal starting date. We then arbitrarily increase the capital coefficient by a factor of 10, and it much reduces the optimal depletion rate, which is however 9 times the actual rate in that year or later ones.

We then assume that depletion cannot exceed 10 percent without a drastic increase in costs, because of reservoir damage and loss of reserves. Here the effect is a sharp increase in negative user cost in respect of starting the investment.

All the numerical factors are rough approximations at best, but even gross changes would hardly alter any conclusions. Table III shows how far-fetched is the notion that 1970 prices were "untenably low," and had to rise to induce investment in oil production. Prices were untenably high for a competitive market, hence competition had to be suppressed.

The higher prices and even lower depletion rates since 1970 have increased the long-term surplus. Despite the tiny discovery effort at the Persian Gulf, many new-found fields were not delineated, let alone developed. Saudi Arabia has proved 53 commercial fields [Aramco 1986], of which only 15 operate [OGJ:WWO 1986]; nobody knows the contents of the other 38. In Kuwait, for many years there was not even a desultory oil discovery effort, but in 1980 there was an attempt to find gas for local power generation. "They found only [sic] oil" [World Oil 1983], a 30-odd billion barrel field with development costs so low that it will be started up to replace some current production. This was the result of a small inadvertent effort in a tiny area; one can only guess what a serious effort would turn up over a much larger area.

In short, the hypothesis of an owner holding back on development because of a lower discount rate, or because of the paradigm of prices rising at the riskless rate, made no sense even before 1970, and still less for the years since.

CONCLUSIONS

1. The rising-price paradigm is false because it joins an important insight, that mineral deposits are assets, to the mistaken factual premise of "a fixed stock... to divide between two [or more] periods." [Stiglitz, 1976] There is no fixed stock. Its allocation over time to do justice as between us and our posterity is like calculating the number of angels who can dance on the point of a needle: an intricate difficult non-problem.

2. The Hotelling Valuation Principle (HVP) is false because it ignores the investment process needed to provide the flow of new mineral inventory (proved reserves).

3. In oil market models, the assumption of a fixed stock may be an acceptable proxy for rising costs. Otherwise, the assumption is wrong, and the models are irrelevant.

4. The shelf inventory, proved reserves, is continuously depleted and replenished. Greater mineral scarcity means higher replacement cost. Expenditure per incremental unit of capacity or of in-ground reserves is a current or coincident indicator of changes in scarcity. In-ground value is a leading indicator.

5. Mineral scarcity is the uncertain fluctuating resultant of two opposing forces: decreasing returns versus increasing knowledge. Between their endless jar the replacement cost may rise and fall, and with it the competitive price and in-ground value. Therefore mineral deposits are risky assets.

6. Both decreasing returns and increasing knowledge guide investment at many margins. Break-throughs or discoveries operate slowly, through development investment. Hence the private discount rates on mineral assets reflect the normal range of risk. If the social discount rate is lower than the private (an issue we do not reach), it is not because private markets cannot take sufficient account of mineral depletion.

7. User cost enters the finding investment decision and it is part of the development investment decision on the optimum depletion rate. It is the least of (a) the avoided cost of more intensive development or (b) the cost of additional discovery or (c) the discounted value of future use. In equilibrium, (a)=(b)=(c).

8. There is no need to postulate a rising price to prevent arbitrage, i.e. to induce the owner to keep the mineral in the ground instead of selling it off forthwith. Depletion has a built-in brake, rising cost at the margin. By the same token, there is an inescapable holding cost. The case is especially clear for fluid minerals like oil and gas.

9. In determining optimal depletion, the role of the interest rate is two-faced, equivocal. Macbeth's porter said of drink: "Lechery, Sir, it provokes and unprovokes: it provokes the desire, but it takes away the performance." A higher discount rate makes quick extraction more desirable but less accessible. In any given deposit, the net effect of a changed discount rate can go either way; for the whole population of deposits, it probably is not important.

10. Under the assumption of a fixed mineral stock, the time-pattern of depletion and prices under competition is similar to, and may even be identical with, the pattern under monopoly. [Stiglitz 1976, Pindyck 1978] But the assumption is wrong, hence there is no family resemblance. Higher oil prices made investment surge in high-cost areas where producers were

price takers, while price-makers in low cost areas cut back investment drastically. In sharp contrast, higher uranium prices made investment surge everywhere because the industry was not monopolized. [Neff 1984]

11. We can apply the Hotelling theory, as distinct from the paradigm. Before 1970, market prices were declining, contract prices were not above spot prices, development investment requirements per unit were stable in the United States and declining elsewhere, capital values of oil reserves in the U.S. (the only ones observable) were stable to declining, and companies and governments were trying to stave off excess supply. The expropriation of oil reserves during the 1970s would have led to much faster depletion, hence lower prices, had each nation-owner operated in independent pursuit of maximum present value. The conclusion is that the world price of oil was above the competitive level in 1970, not to mention later years.

It is a familiar argument, not always self-serving [Kaldor 1983], that the price of some products must be kept above the market clearing level to avoid insufficient investment, declining output, and eventually a higher price. The underlying assumption is that private markets systematically undervalue some assets, and lead owners to sell them off too cheaply. But private markets do not undervalue in-ground assets, as compared with other assets,. They may often be beautifully wrong as to both. [24]

As for the attempts to explain the post-1970 price levels by user cost and Hotelling rent: as Winston Churchill might have said, rarely have so many made so much of so little.

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1 If we multiply their estimate by $18/51 to allow for actual 1987 price, and discount by 10 percent instead of 2 percent, the private present value is about 7 percent of their estimate, i.e. $61 not $860 billion.

2 There is a well-known caution best given by example. Only 7 percent of the 1968 price of ingot aluminum represented ore cost; for copper, it was 99 percent. [Harris 1985] This is generally and I think properly neglected for long term movements.

3 [Boskin et al 1985, p. 926] make the highly misleading statement that "the average annual rise in real oil prices received by U.S. producers was 3.5 over the period 1950-82." The price declined consistently for the twenty years 1950-1972, rose for the next nine years, then declined by 16 percent in their terminal year 1982.

4 Dr. Neff has kindly updated his published price series to the end of 1987.

5 Ernst Berndt and Anthony Scott have drawn my attention to an 1866 House of Commons speech by John Stuart Mill, who was so impressed by Jevons' vision of the high coal price throttling back British economic growth toward zero that he insisted the national debt be paid off before coal production dwindled and growth ceased.

6 Ernst Berndt and Anthony Scott have drawn my attention to an 1866 House of Commons speech by John Stuart Mill, who was so impressed by Jevons' vision of the high coal price throttling back British economic growth toward zero that he insisted the national debt be paid off before coal production dwindled and growth ceased.

7 The process continues. Prudhoe Bay started with 9.6 barrels, a number still repeated. By early 1987, 5 billion had been produced. [OGJ 1987Mar] Hence there should be 4.6 billion "remaining." It is now estimated at 8.2 billion. [Salomon 1987 April]

[8] In uranium, in the U.S., there was "... an inexorable progression toward lower grade, deeper and more costly sources of supply .... [But ... there has been a radical change in geologic horizons outside the U.S., with many rich new deposits being added to world reserve lists, and indications of many more....Undoubtedly there is much more uranium in much richer deposits than was imagined even ten years ago; I suspect that we have only found the tip of the iceberg." [Neff 1985, p. 6]

[9] Operator A promises to share the cost of drilling a wildcat well on Operator's B's lease, if the well turns out to be dry, since the drilling record will supply information valuable to A.

[10] Suppose all pools to be at the same depth, circular, and homogeneous. then the total number of wells drilled into any pool is proportional to its area. The number of dry holes is proportional to the circumference which they outline. The ratio of circumference to area is $2/r$, where r = radius, so the smaller the area the higher the dry-hole ratio. Heterogeneous reservoirs are in effect aggregations of smaller circles, with higher dry-hole ratios. Non-circular areas have a higher ratio of circumference to area. Over time, an increasing ratio of dry development wells to all development wells would indicate diminishing returns. It is not observable in the USA.

[11] [Heal & Barrow 1981: U.K. Treasury bills] [V.K. Smith, 1981: prime commercial loans, high grade municipals, one-year corporate bonds, 30 year corporate bonds, and stock exchange call loans.] [Devarajan & Fisher 1981: no reference to appropriate discount rates] [Boskin et al 1985: 2 percent] [Marshalla and Nesbitt 1986: a range of 2 to 6 percent]

[12] I find it hard to understand this fixation of even professional economists on current variable costs, as if they were marginal costs; and, consequently, the neglect of investment.

[13] Calculation of factor k for any given a. Total capital expenditures $K = kaQ$.

(1) If K is in dollars per barrel of production per day, $365K/Q$:

then: $(365K/Q) = 365kaQ/Q = 365ka$

$k = (365K/Q)/365a$

Example: let $a=0.10$, and $(365\ K/Q) = \$10{,}000$ per daily barrel,

then $k = \$10{,}000/365(.1) = \$274$

(2) If K is in dollars per barrel of reserves booked, K/R:

$K/R = K/Q/a = Ka/Q = ka^2$

$k = (K/R)/a^2$

Example: let $a=.10$, and $K/R = \$2.74$ per barrel of newly booked reserves, then $k=\$2.74/.01 = \$274$.

[14] In numerical work, the known depletion rate may not agree with the calculated optimal rate, which balances Equation [10]. The discrepancy shows the degree but not the source of error: functional form other than the quadratic, poor data, including averaging over many reservoirs, etc. Most important, it may show a monopoly restraint on investment and production.

[15] Note that equation [9] may be written as, $a(a+i)^2 = Pi/2k$. Differentiating both sides with respect to i, and setting to zero, we obtain Equation (11).

[16] A simple heuristic: assume 10 percent discount rate, current price $1, expected to increase at 5 percent per year. If cost is 50 cents, net profit is 50 cents this year, 55 cents next year. The present value of this year's and next year's profit is the same, and the seller is indifferent as between selling and holding. If cost is below 50 cents, the increase in the profit is less than 10 percent, and he should sell not hold. If cost is above 50 percent, he should hold not sell. The higher is cost, the greater the penalty for not holding, i.e., the higher is user cost. This is consistent with--or helps explain why--the better deposit or tranche is exploited earlier.

[17] The greater the expected price variance, the greater the option value, because the better chance of a very high price is worth something, while a lower price cannot make the reservoir be worth less than zero.

[18] Some economists have used the concept of a "backstop technology," i.e. a substitute available in unlimited amounts at a constant price. Its present value serves as a limit to the price. [Nordhaus 1974][Das Gupta & Heal 1979] This is useful to work out a worst case scenario: e.g., zero accretions to reserves. Then by the Year X, the price will be up to $Y, at which point the economy will switch over to the backstop, whose present value sets the limit to price today. The present value is usually quite small.

[19]For calculations of g, the implied price forecast, see [Adelman 1986, 1988]

[20] For example, suppose R/Q = 9.09, and Q/R = .11, but that T is 25 years instead of infinity. Then for R/Q to equal 9.09, a must equal .102, not .11. The lower the value of a, the less the error, hence it cannot explain the larger errors at lower values.

[21] There is nothing wrong with making optimum depletion a joint function of time and of the other variables. But aside from realism, our aim is to focus on postponement holding all else constant.

[22] At first glance, Equation [16] looks odd, as though postponement time is inversely proportional to g, the growth rate of price. But as g increases and approaches i, (i-g) becomes very small, its negative logarithm becomes very large, and the formula makes it positive. Thus the higher is g, the higher is t, which however vanishes when g goes to zero.

[23] This assumes constant factor prices. An empirical rule among engineers and bankers is that factor prices will change about equally with oil prices. [Horn 1987] The unstated assumption is a rightward shift in the factors' demand curve. Then higher demand for investment goods presses against inelastic factor supply, as was spectacularly true in the 1970s.

[24][Black 1986] calls a market efficient when the observed value is no more than twice, and no less than half of the true value.

Figure 1

International Price of Crude Oil
Actual and Six Successive IEW Polls

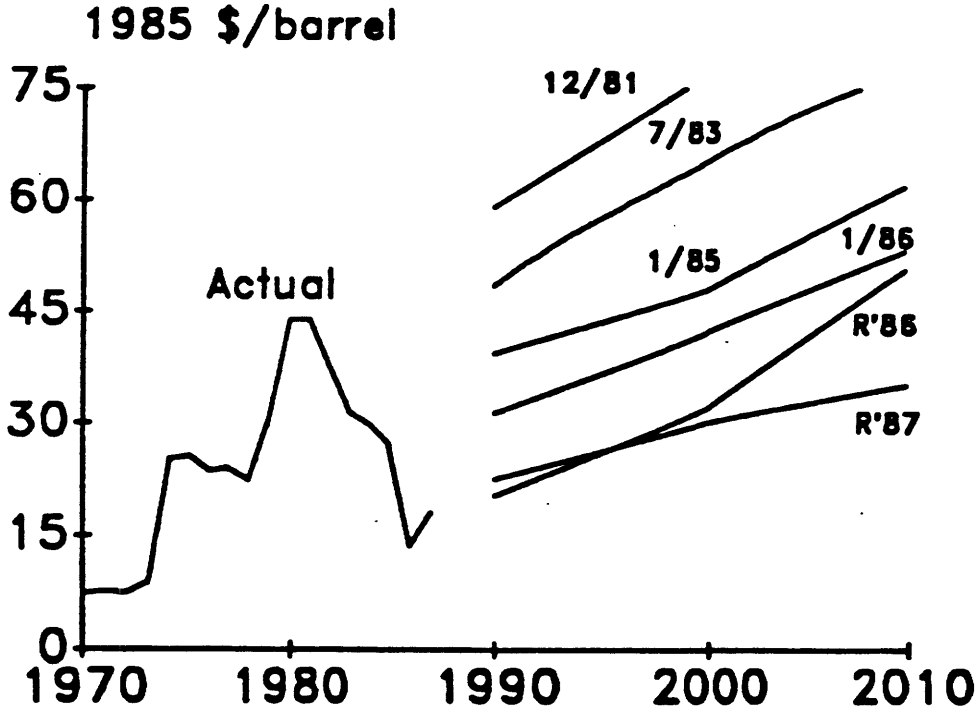


Figure 2

Crude Oil Prices
USA (1912-87), Mideast Light, Spot (1947-87)

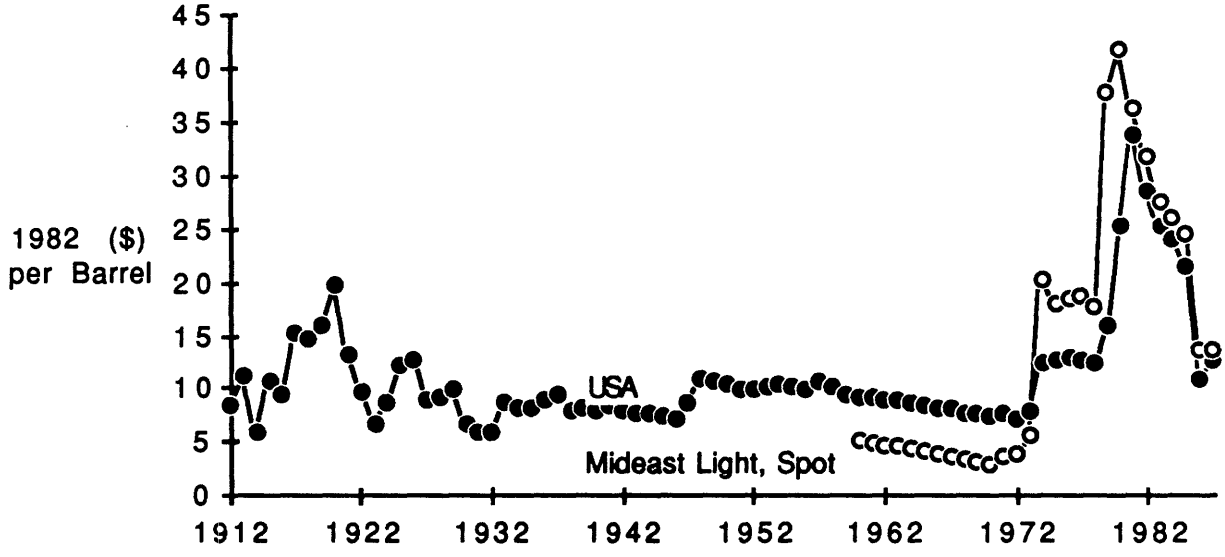


Figure 3

User Cost (Value Less Development Cost)
USA Crude Oil 1955-1984

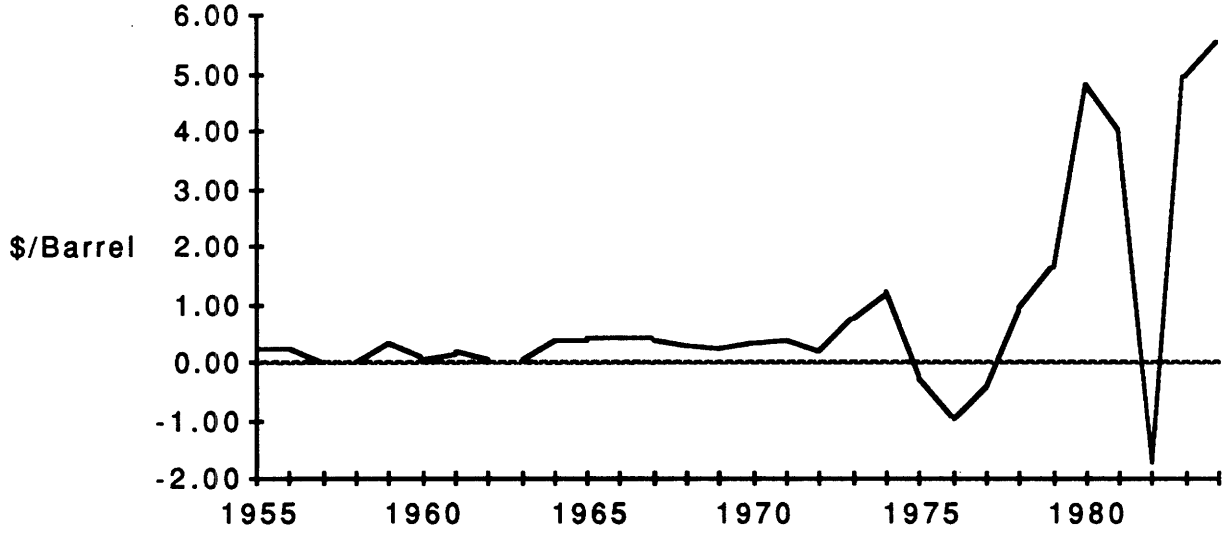


Figure 4

Actual and Theoretical Reserve Values
Related to R:P Ratio
(Value as Percent of Future Net Cash Flow)

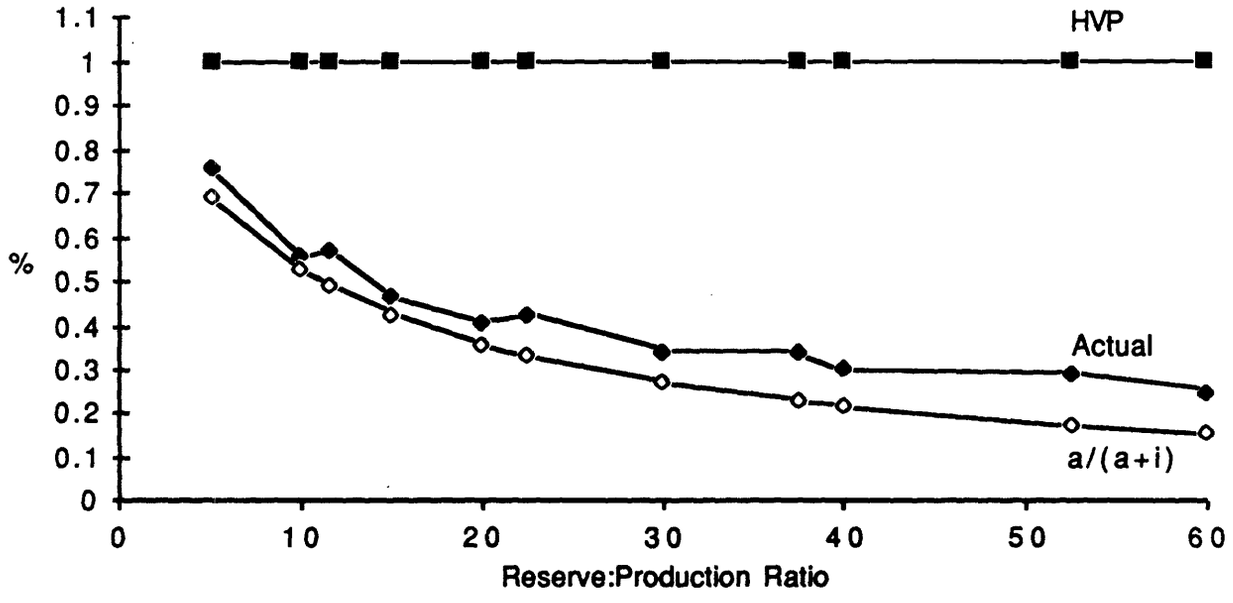


Figure 5

Farm Prices versus Minerals Prices

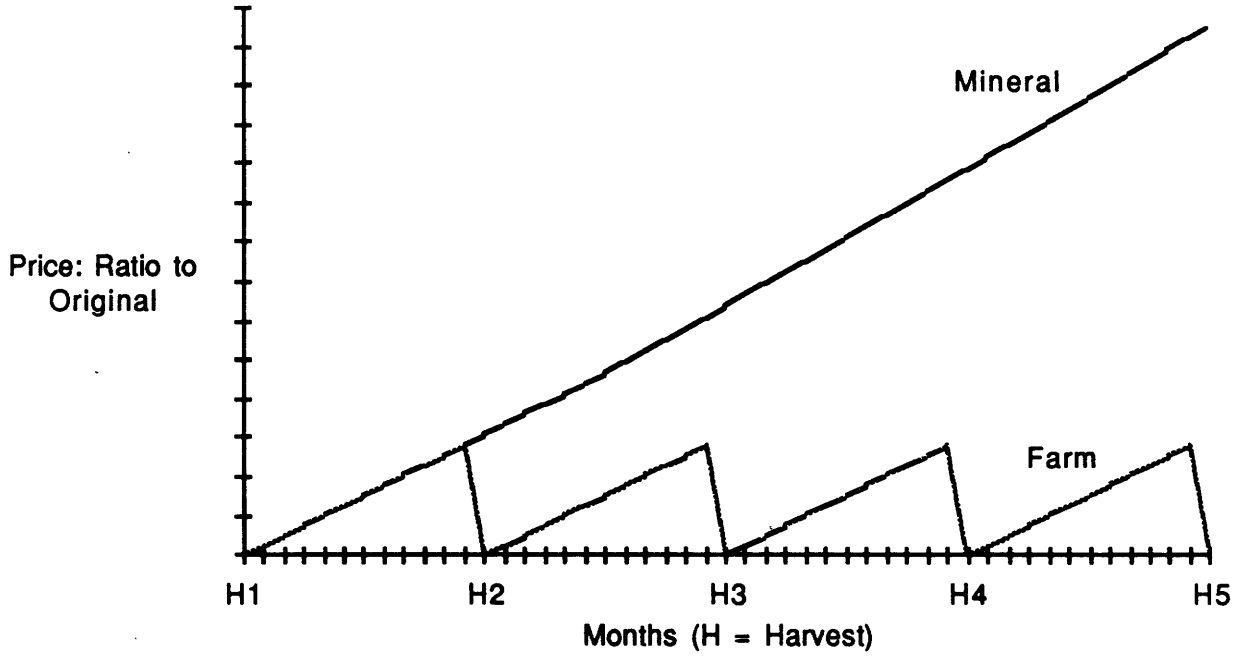
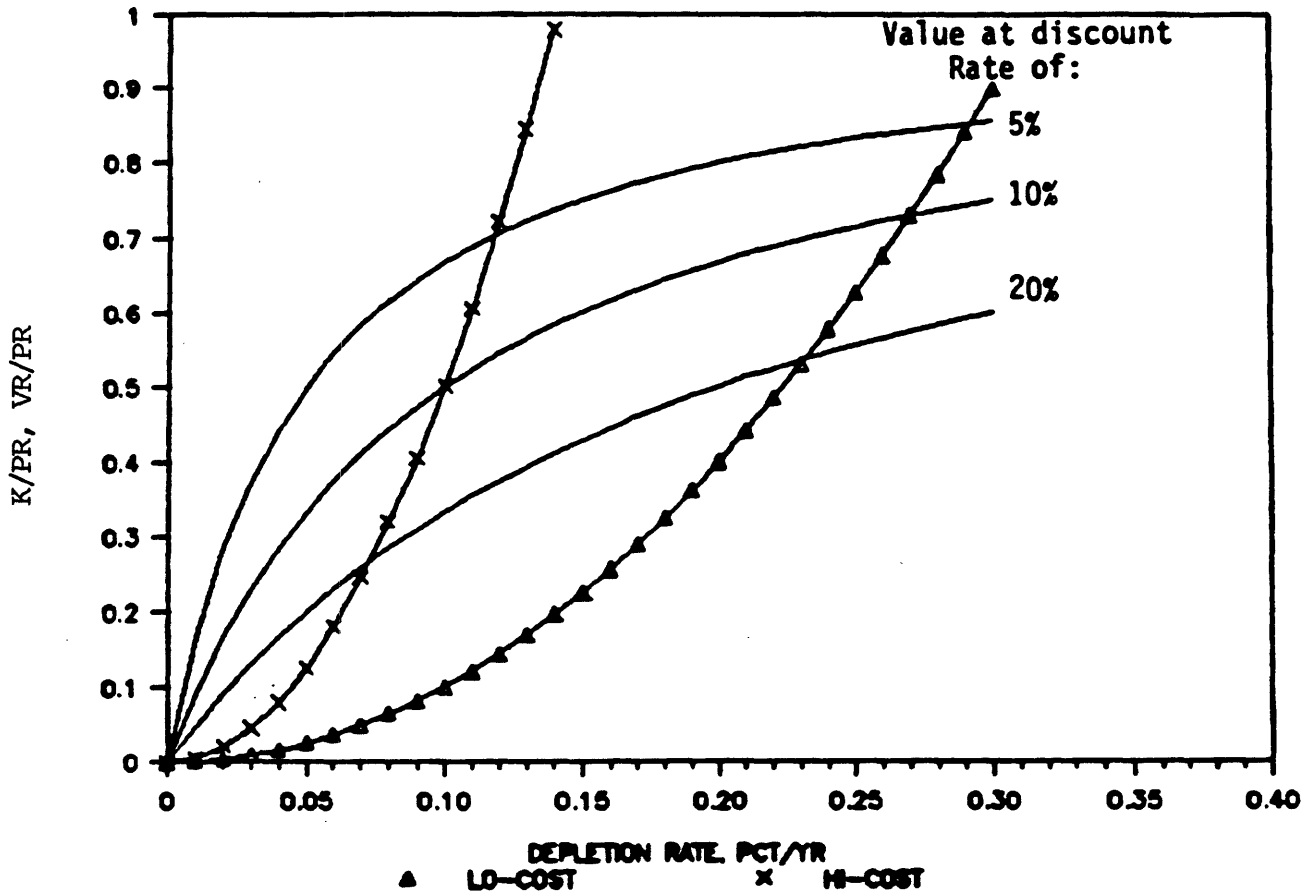


Figure 6

Value as Function of
Depletion and Discount Rates



K = Total Investment
PR = Undiscounted Value
VR = Discounted Value

Table I

CALCULATED AND ACTUAL PRICE CHANGES:
NONFUEL MINERALS 1970-1985

1985 PRICE AS PERCENT OF 1970

Mineral	Calculated	Actual	A/C	Average Annual Error
Aluminum	1.25	1.01	0.81	-0.014
Copper	1.20	0.41	0.34	-0.069
Iron*	1.24	0.99	0.80	-0.014
Lead	1.06	0.44	0.41	-0.058
Nickel	1.42	0.61	0.43	-0.055
Silver	1.54	1.24	0.81	-0.014
Tin	1.21	1.22	1.01	0.001
Mean	1.25	0.86	0.69	-0.029

SOURCE: Calculated, Margaret Slade, "Trends in Natural-Resource Commodity Prices," Journal of Environmental Economics and Management, vol. 9, 122-137 (1982). Actual, Statistical Abstract of the U.S. (Washington, 1987), pp. 678, 690, 692, 693. Deflated by Producer Price Index, All Commodities.
*Calculated: finished iron; actual, iron ore 1970-84

Table II

EFFECT OF DEFERMENT ON MARKET VALUE

Reserves/ Production (R/Q=1/a)	Value as Percent of Undiscounted Net Cash Flow at Present Prices			
	ARPS	FAGIN	Regression Estimate	Theoretical a/(a+i)
(1)	(2)	(3)	(4)	(5)
5	--	0.695	0.756	0.714
10	--	0.577	0.556	0.556
11.5	0.540	--	0.572	0.521
15	--	0.497	0.464	0.455
20	--	0.428	0.408	0.385
22.5	0.458	--	0.425	0.357
30	--	0.343	0.341	0.294
37.5	0.345	--	0.338	0.250
40	--	0.308	0.300	0.238
52.5	0.281	--	0.291	0.192
60	--	0.227	0.251	0.172

Source: J.J. Arps, "Valuation of Oil & Gas Reserves," ch. 38 in Frick, ed., PETROLEUM PRODUCTION HANDBOOK (1962), p.38-9. (Included also in PETROLEUM ENGINEERING HANDBOOK (1987).

Regression Estimate:

$\ln(ValPct) = .436-(.44\ln(R/Q)) + .09dum$, Arps=1, Fagin=0

R^2=.96, t-stats 4.9, 15.6, 2.1

"Theoretical": a=Q/R, i=.08

Table III

RELATION OF IN-GROUND VALUE TO NET PRICE
BITUMINOUS COAL 1980-82

A. Relation of Net to Gross Receipts, 1982

1	Value of Shipments	28261	
2	Value Added	18455	
3	Payrolls	6736	
4	Net receipts (inc. taxes, royalties)	11719	
5	Net/gross, unadjusted		0.415
6	Less taxes, royalties		0.100
7	Net/gross, adjusted		0.315

B. Relation of Values to Net Prices

	Year	Average Gross Price ($/short ton)	Average In-Ground Value	Relation: Value to Adjusted Net Price
8	1980	24.51	0.82	0.106
9	1981	26.30	0.56	0.68
10	1982	27.12	0.21	0.025

Sources: Lines 1-3, Statistical Abstract 1987,p. 673
Line 4 = (2) - (3)
Line 5 = (4)/(1)
Line 6, rough allowance
Lines 8-10: prices, Statistical Abstract 1987, p. 675
In-ground sales values, unpublished tabulation by Martin B. Zimmerman
Relation, value divided by 31.5 percent of sales

Table IV

COMPARISON OF OPTIMAL DEPLETION RATES AND OPTIMAL POSTPONEMENT: PRIVATE AND PUBLIC OPERATION, SAUDI ARABIA 1970

Line	Operator	(1) Net Price ($) P	(2) Assumed Discount Rate i	(3) Capital Coeffic-ient k	(4) Optimal Depletion Rate a*	(5) Actual Depletion Rate a	(6) Expected Price Rise g	(7) Optimal Postpone-ment (years)
1.	Aramco	0.32	0.2	21.5	0.023	0.0098	0.02	-35
2.	S.A. Govt.	1.2	0.2	21.5	0.074	0.0098	0.02	-46
3.	"	1.2	0.023	21.5	0.072	0.0098	0.02	0
4.	"	10	0.023	21.5	0.160	0.0098	0.02	-33
5.	"	20	0.023	21.5	0.205	0.0098	0.02	-44
6.	"	20	0.023	215.0	0.088	0.0098	0.02	-8
		----------Depletion cannot exceed 10 percent----------						
7.	"	10	0.023	21.5	0.100	0.0098	0.02	-76
8.	"	20	0.023	21.5	0.100	0.0098	0.02	-111

NOTE: Negative number in colum (7) indicates it would have been optimal to commence that many years ago.

SOURCES: Cols. 1,2,3,5: Adelman (1972), pp. 209 (assuming 5 cents operating costs), 314. Capital coefficient $k=K/aQ$, where K=capital expenditures per annual barrel of added capacity, a=depletion rate, Q=annual capacity added. Cols. 4,7: see text for derivation of formulas.

NOTE: The postponement factors assume complete freedom to optimize. Hence the depletion rate used to calculate the postponement in column 7 is the optimal not the actual depletion rate. Use of the actual rate would accelerate the start of production, i.e. greatly increase the negative values of column (7).