ORGANIZING PRINCIPLES UNDERLYING THE FORMATION OF ARM TRAJECTORIES

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Submitted in Partial Fulfillment
of the Requirements for the Degree of

DOCTOR OF PHILOSOPHY
at the
MASSACHUSETTS INSTITUTE OF TECHNOLOGY

May 1983

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Submitted to Harvard-M.I.T Division of Health Sciences and Technology
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ABSTRACT

The control of multi-joint movements involve complex kinematic and dynamic issues which do not arise in the case of single-joint movements. This thesis explores these issues, in the context of human planar two-joint arm movements.

Kinematic features of planar arm movements indicate that the spatial trajectory of the hand, is explicitly specified, and that there exist some relation between speed and form of movement. Treating the hand as a point mass, and minimizing the rate of change of the hand acceleration (jerk), hand trajectories were mathematically modeled. Predicted trajectories, for curved and target-to-target movements, showed a very good qualitative and quantitative fit to measured human movements. Minimum-jerk trajectories have also been shown to successfully match single-joint forearm movements. These combined findings may be indicative of a single organizing principle underlying trajectory formation for various types of arm movements.

Movement of multiple segment limbs requires generation of appropriate joint torques which include terms arising from dynamic interactions among the moving segments. The significance of the individual interaction torques, arising from inertial, centripetal, and Coriolis forces for different movement paths and movement speeds has been assessed. Trajectory formation strategies which simplify the solution of the problems of dynamics are presented.

Thesis Supervisor: Dr. Emilio Bizzi, Eugene McDermott Professor in the Brain Sciences & Human Behavior.

Acknowledgements

I feel very lucky to have been given the opportunity of bringing my wish for doing research in motor control into realization. I am particularly grateful to Professor Emilio Bizzi for affording me this unique opportunity of studying and working in such an inspiring and challenging atmosphere. I thank him for his guidance, contribution to my understanding of motor physiology, and his will to advise and to give of his time whenever needed. I greatly appreciate his encouragement, support and generosity throughout this period. They were very valuable in making this research work an interesting, enjoyable and pleasant experience.

Professor John Hollerbach has made a significant contribution to this work in numerous ways. I am especially grateful to him for opening before me the fields of robotics and computational approaches to motor control. I thank him for contributing from his ideas and knowledge to my understanding. His close collaboration with me on the study of the dynamics of movement, his guidance, and the time and effort he has contributed, were very important for the materialization of this study.

Many thanks to Professor Neville Hogan, whose work has inspired the portion of this thesis which deals with the kinematics of arm movements. His enthusiasm and encouragement and his generous will to guide me and to discuss with me various aspects of this work, have been of great value. I thank him for giving from his insight and knowledge and for the contribution that his many suggestions and his criticism and comments have made to this research.

I thank the other two members of my committee Professor Richard Cohen and Professor Mark Hallett, for their suggestions and for their comments, criticism and careful reading of this thesis. I thank Dr. Hallett for the exposure I had in his laboratory to the study and treatment of movement disorders. I thank Professor Cohen for his encouragement and the help he has given me on many occasions throughout my studies in MIT.

I thank Professor Morasso, Professor Bizzi and particularly Dr. William Abend, for providing the experimental results on the basis of which the optimization model was suggested. I am grateful to Dr. Abend for the many discussions we had and for his help. Many fellow students and members of the laboratory have contributed to my learning and to the progress of this work. I thank Chris Atkeson, Bill Chapple, Jonathan Delatizky, Shraga Hocherman, Claude Prablanc, Barbara sakitt and Tom Zeffiro. I am grateful to Terry Heyward and to Gayle Morrison for their help in many different ways. Great many thanks to Kathy Duffin for being such a good friend and editing this thesis.

The excellent facilities of the Artificial Intelligence Laboratory and the help of its members have been essential for the execution of this work.

I greatly appreciate the support and the opportunity that the Harvard MIT division of Health Sciences and Technology has given me throughout my enrollment as a graduate student. I would especially like to thank Professor Ernest Cravalho for his help and patience as my Medical Engineering Medical Physics advisor.

My family and especially my father and mother have always been supportive and understanding. Their love and care have sustained me throughout the years and their encouragement have made me capable of studying and dealing with periods of frustration. This is largely due to you that I have been able to reach this stage.

Finally, I thank Shalom my husband and Shira my daughter who lived with this work on a daily basis. Thank you Shira for being such a source of joy and thank you Shalom for giving me love, understanding and help and putting up with me throughout all of this. I thank you for being the way you are and for making me capable of starting, continuing and completing this work.

This research was supported in part by the National Institute of Health Research Grant AM27610 and EY02621. Personal support was provided in part by the Young Men's Philanthropic League and by the Whitaker Health Sciences Fund, Whitaker College.

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Chapter 1

Introduction.

The capability of humans to walk or to move their arms seems a natural and integral part of existence. Nevertheless, the understanding of the processes which underlie the planning and control of even the simplest movement has thus far eluded us, to a great extent.

In his review of the history of motor control Granit [1981] has commented on the shift in emphasis that has occurred in motor control studies since the early twenties. Traditionally, investigators in the field were interested mainly in anatomical and physiological questions. Currently, however, the purposive and organizational elements in motor control are increasingly being sought. Thus, at present, more so than in the past, investigators are trying to understand motor phenomena in terms of the variables which are being controlled and the mechanisms which are used to control these variables. Moreover, in contrast to previously prevailing ideas in the field, motor control scientists have started to look beyond servo control for other control mechanisms since servo control cannot by itself account for the speed and flexibility of most movements. Recently, therefore, more emphasis has been placed upon preplanning and open-loop control. Evidence for the important role played by preplanning in motor control has been gathered in many physiological and behavioral studies [Keele 1981]. In addition, with the help of computational approaches, the importance of distinguishing between the hardware of the system and the programs installed by the system has been emphasized.

In contrast to the sensory systems (visual, auditory), the motor control system is not concerned merely with the processing of input information, but rather with the generation of actions which transport body segments, or enable the organism to interact with the environment. Moreover, since movements can be generated in the absence of somatosensory input, insight into the strategies which are employed in planning movements cannot be gained merely through the manipulation of this input. Therefore, in this search for an understanding of the purposive and organizational aspects of motor control it has been doubted whether understanding of the system can be gained without quantitative analysis of movements [Kelso & Wallace 1978]. For these reasons many scientists in the field, including Muybridge, Holmes and the Russian school of motor control (e.g. Bernstein, Gurfinkel) have directed their efforts towards quantitative analysis of motor output.

For the same reasons, in the course of this work, we studied, quantitatively described and analyzed a certain class of human movements: planar two-joint arm movements. One of our ultimate goals in movement studies in general, and in this work in particular, is to investigate the underlying organization of the motor control system. Therefore, in order to explain the concepts and ideas on which this work is based, and the logic behind our methodology, views on, and various approaches to the study of motor organization will be introduced first. This will be followed by a presentation of the objectives of this thesis.

1.1. Organization of the motor control system.

1.1.1. Levels of motor representation.

When dealing with a highly complex system, such as the motor system one is confronted with a multiplicity of observable variables and therefore, with a difficulty of interpreting an observation once it is made. Each movement can be represented, for example, as a set of E.M.G signals, a set of joint rotations, or joint torques etc. [Saltzman 1979]. The identification and use of such levels of representation of movements can simplify the characterization and analysis of movements. The use of various levels for movement representation, however, is not only convenient but is also based on our current views on the organization of the motor control system. A common belief held by many motor control scientists is, that the motor system is hierarchically structured [Keele 1981, Schmidt 1975]. Moreover, it is believed that a motor plan evolves through various stages of processing and translation from a more general and abstract schema higher in the hierarchy, to a more detailed presentation at lower levels [Keele 1981]. According to this view, once a decision to generate a movement is made, an action plan (i.e. a generalized motor program), is chosen and processed by the central nervous system at different levels, until it is finally translated into a set of neural signals to a specific set of muscles acting on a specific limb [Saltzman 1979]. A simple fact, the human capability for generating very similar strings of handwriting by means of different mechanical systems (e.g. the hand versus the arm), provides evidence for the this view. In addition the CNS receives information from the periphery and this information enables the system to generate accurate and well oriented movements. Organization or order, should not be expected to be found only with respect to the overall structure of the entire system. One should expect to find at each level certain strategies and principles that have evolved to deal with the processing and translation of motor programs at that specific level. These principles can be revealed through the identification of certain regularities in various representations of the system output. As part of the attempt to reveal the underlying organization of the system, a few of these underlying principles have already been identified. These include, for example, the size principle which, with few exceptions, dictates the order of recruitment of motor units [Henneman 1979]. Another principle which has been found in behavioral analysis of human movement is the speed-accuracy trade-off [Fitts 1954, Fitts & Peterson 1964].

1.1.2. Approaches to the study of motor organization.

At some levels of motor representation, however, the identification of such rules or principles is very difficult. Therefore one should first make an attempt to identify what the specific problems are that the system needs to solve at these levels. If these problems are identified, the next step should involve the formation of certain hypotheses on the nature of the strategies which the system uses to solve them. This can serve as a basis for predictions of the system's behavior under various conditions. Comparisons of the predicted with the real behavior of the system can be used to test the validity of these hypotheses.

Another approach which can help us in the investigation of motor organization is the classification of biological movements according to the specific tasks involved in their generation. Since the repertoire of human movements is very rich, it is conceivable to assume that even if there exist only a few general planning strategies, in generating different types of movements, these plans are implemented by means of different control mechanisms . Thus, for each class of movements a certain set of controlled variables can be assumed. Therefore, one should expect to find, for example, that the CNS controls different variables for the insertion of a peg in a hole than the ones used to control free reaching movements. Movements which involve contact with a surface are defined in robotics as compliant movements. Such movements include manipulations of tools or the exertion of forces on external objects. The generation of compliant movements probably involves, either combined position and force control (with position control along certain coordinate axes and force control along the remaining axes) [Mason 1982], or stiffness control [Hogan 1980]. Contact movements complicate the control problems considerably, and therefore, have not been studied as extensively as free movements until now [Hollerbach 1982].

In addition, even though the same biological hardware is at the disposal of the system at all times, it is reasonable to assume that different neural subsystems are activated, or are of more or less significance depending on the nature of the generated movement. Thus, for example, the CNS may rely much more heavily on alpha-gamma coactivation in the generation of fine, slow, finger movements than in the generation of fast, large, forearm movements. Moreover, it has been indicated that different cortical and subcortical centers play different roles in controlling different kinds of movement (e.g. spinal generators in gait generation [Grillner 1975], pyramidal tract in fine distal movements, cerebellum in programming movement rhythm [Brooks 1979, Holmes 1939].)

Although there are reasons to assume that different classes of movements are generated through the use of different control mechanisms, motor control and planning can be more efficient if the system uses a single global principle for the generation of different kinds of movement. This would require changes only in the details when such a principle is applied under different conditions. If such motor organization is hypothesized, the specification of particular control variables or of a specific hardware (neural and skeletomuscular) can be done only at the final stage of the implementation of such a principle.

As a result, when one investigates motor organization, an attempt must be made to classify movements according to the control issues involved in their generation. On the other hand, one must find similarities in various behavioral characteristics between movements which belong to different classes. To sum, because studies which intend to shed light on the organizational and purposive aspects of movement are apt to face tremendous complexities, one should restrict such studies to the investigation of only a single narrow class of movements. Since movement classification, especially on the basis of the controlled variables, is not always obvious one has to propose testable models or to make certain hypotheses. However, after the models for a specific class of movements have been validated experimentally, one should proceed to suggest more general theories or to look for unifying principles.

1.2. Thesis scope and objectives.

Adult humans are capable of generating a very rich variety of movements. Some of these movements are learned from a very young age while others demand different degrees of practice until they reach a high level of performance. Humans are capable of performing skillful movements which have not been practiced previously, and of making the necessary adjustments in their movements whenever the external conditions change. They can readily and effortlessly adjust to changes in locations of objects they intend to reach, the existence of obstacles along movement paths or the need to change movement speed. Humans also adjust easily to changes in the masses of lifted objects and in the weight of their own limbs. Even in cases where the sensory input is artificially distorted [Hein & Held 1962, Held 1965] humans can still perform appropriately.

The question is, what are the strategies for movement generation which result in such goal-directed and purposeful movements but also allow for their flexibility and richness?

This thesis attempts to address a few aspects of this general question through the study of trajectory formation in human planar two-joint reaching movements. Reaching movements describe the movement of the arm when a human subject reaches for an object in his environment. The term trajectory refers to the configuration of the arm in space and to the speed of movement as the hand moves from its initial to its final position, and trajectory formation refers to the process of the planning and control of these movements. Through the study of trajectory formation in human planar arm movements we hope to shed some light on the organization of motor planning. In addition to this general goal, this thesis has two more specific objectives. The first objective is to identify some of the problems that the CNS must face in the generation of multi-joint arm movements. In this work only planar two-joint arm movements have been studied, but the control problems for these movements are in essence the same as for less constrained arm movements. The second objective is to investigate what strategies the system uses to solve these problems.

To achieve these objectives, this study concentrates on two aspects of trajectory formation : kinematics and dynamics. Kinematics deals with such variables as positions, velocities and accelerations, and dynamics deals with forces and torques. Adopting the concept of movement representations, a reaching movement can be kinematically represented by the path or geometrical curve in space along which the hand moves between its initial and final positions. Since the hand path alone is not sufficient to describe the movement, the kinematic representation should also include the time history of hand positions. Since arm movements involve rotations of arm segments, about their respective joints, joint angles and their change with time, are also included in a kinematic representation of movement. For joint rotations to take place, appropriate joint torques must be generated. The time history of joint torques gives a dynamic representation of movements.

The kinematic and dynamic aspects of arm trajectory formation have been investigated in this thesis side by side but separately. The purpose of addressing the kinematic and dynamic aspects of trajectory formation separately is to simplify the analysis of movements. Moreover, theoretical considerations [Bernstein 1967] support the view that movements are indeed planned in two stages, first at the kinematic level and then at the dynamics level. According to this view once a decision to move has been made by the CNS, a kinematic representation, a trajectory, is planned for this limb. This kinematic representation is then translated into a dynamic representation, or directly into a particular pattern of muscle activation to generate the appropriate joint torques, which are required to achieve the desired trajectory. The variability of specific muscular responses, with circumstances, in such a way as to produce the same movement, offers an evidence for this hierarchical order, at least at the level of muscular activity [Saltzman 1979].

Even if one does not adopt this view, one can explore the possible content of motor plans through the study of various aspects of trajectory formation at these two levels. Therefore, several questions were addressed in the course of this thesis, some of which deal with kinematic aspects and others which deal with dynamic aspects of trajectory formation in multi-joint arm movements. Among these questions the ones which correspond to the kinematic aspects of trajectory formation are:

1. Once the CNS has knowledge of the location of the object and a decision to move the arm has been made, what are the principles which govern the selection or dictate the execution of specific trajectories among the vast number of possible ones?

2. What are the specific features or invariant properties of these movements and what can these characteristics teach us about these principles?

The questions which address the dynamic aspects of arm trajectory formation are:

1. What is the nature of the mechanical problems involved in moving the arm once a certain movement has been selected?

2. What are the solutions that the system is using to solve these mechanical problems ?

Since most human arm movements in the course of everyday life usually involve two or more joints, there is a great significance in understanding the issues involved in their control. In the past, investigation centered on single joint movements. However, this simpler situation does not allow us to assess the scope of the complexities of the problems which the CNS must confront in the control of multi-joint movements. Since the appreciation of these complexities is one of the main concerns of this work, I have chosen to progress beyond the study of single joint movements to two-joint movements involving rotations about the shoulder and elbow joints. Experiments were conducted in which subjects reached for visual targets in a horizontal plane. The experimental setup in these studies is shown in Figure 1-1.

The apparatus used in these studies is made of two mechanical links. The subject grasping the handle of the manipulandum was asked to direct it below lit targets. The data on the manipulandum joint angles were obtained by means of calibration and analysis of signals from potentiometers located at the apparatus joints. Through the application of simple trigonometric relations, this data were then used, to compute the values of the subject's shoulder and elbow angles. The shoulder was constrained from movement throughout these experiments and the subject's wrist was braced. According to the classification of compliant versus free movements these are compliant movements but since they involved the exertion of only a negligible force on the environment, they can be regarded as free movements. Since these movements were confined to a single horizontal plane, gravity exerts no forces that affect them, and therefore, they represent a subclass of free, three dimensional movements. Nevertheless, most of the other control problems for free multi-joint arm movements in a three dimensional space already exist in the control of planar two-joint motions.

1.3. Kinematics and dynamics of arm trajectory formation.

The physiology of one-degree-of freedom movements has been studied extensively. These movements have been characterized according to their speed, accuracy, duration and the relation among these parameters. The trajectories of these movements and the forces involved in their generation have also been quantitatively

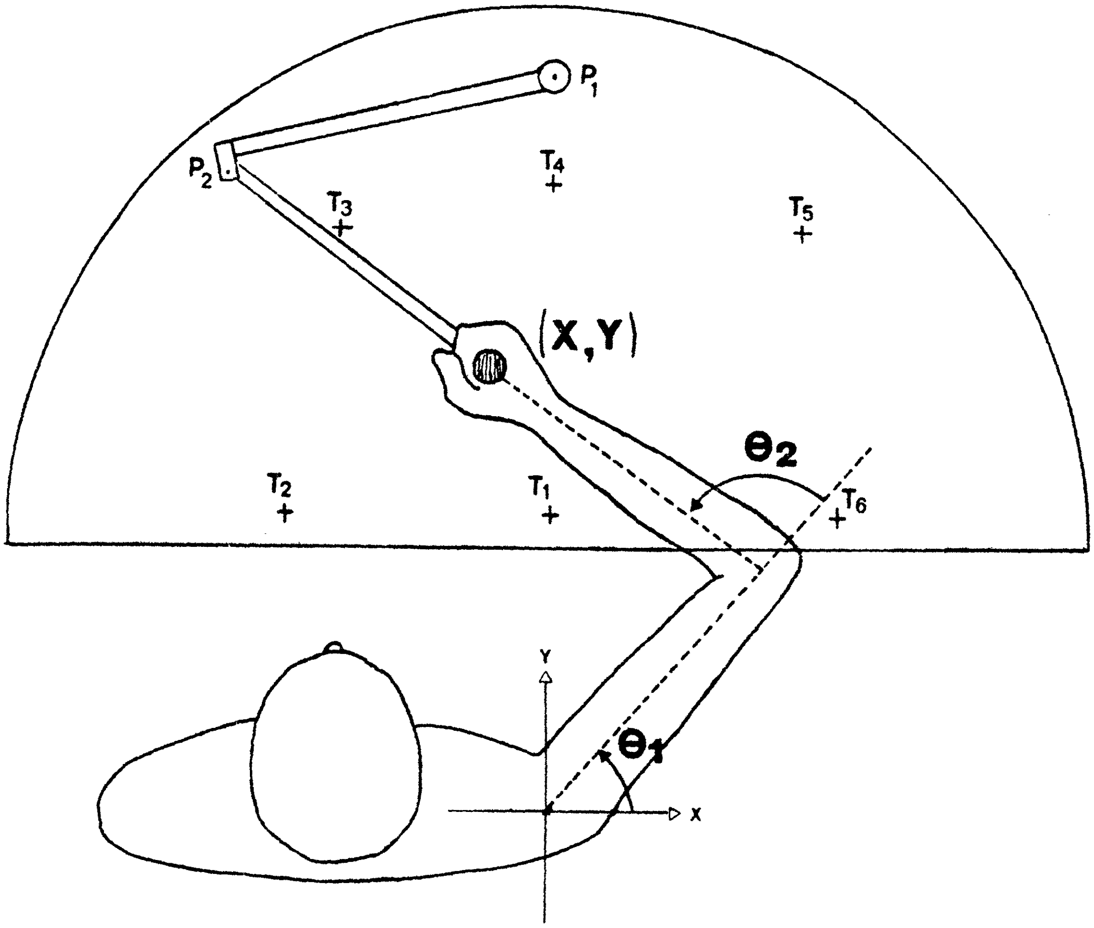


Figure 1. Experimental apparatus for measuring arm trajectories in a horizontal plane.

described. By contrast, relatively few studies have dealt with the kinematic and the dynamic aspects of multi-joint arm movements as they evolve in time. In particular, only very few investigators have analyzed kinematic features of hand trajectories and the relations which exist between these trajectories in space and joint rotations. The reasons for this paucity of experimental data, stem from the lack of adequate recording techniques, providing a full and accurate kinematic descriptions of multi-joint free arm movements in space. Similarly, because of the lack of such techniques, and because of the complicated nature of the dynamics of multi-joint movements, the joint torques and muscle tensions responsible for the generation of these movements have been evaluated only in relatively few studies. Recently, however, a few studies of human or monkey multi-joint arm movements have been conducted with the objective of identifying stereotyped patterns which characterize certain elements of a particular class of movement [Morasso 1981, Abend et al. 1982, Soechting & Lacquaniti 1981, Georgopoulos et al. 1981]. The supposition of these studies was, that if common kinematic features or stereotyped patterns of muscle activations do exist, their identification can enable one to gain insight into the strategies and rules used by the central nervous system to control skilled movements and can teach one about the organization of the motor control system.

1.3.1. Multi versus single joint movements.

Single-joint movements, which involve rotations of a single segment about a single joint axis (e.g. forearm movements), are both kinematically and dynamically simpler than multi-joint movements. Kinematically, they are simpler since they consist of variations in a single degree of freedom. Hence, for example, in forearm movements the hand is constrained to move only along a circular arc. Dynamically, they are simpler than multi-joint movements since they require the generation of a single joint torque which results from the forces exerted by muscles inserting on the forearm. If the forearm rotates about the elbow with an angular acceleration, $\ddot{\theta}$, the joint torque, N, is:

$$N = I\ddot{\theta} \tag{1}$$

where I is a moment of inertia of the forearm. In the case of one-degree-of-freedom movement the moment of inertia is a scalar. In contrast, multi-joint movements, or even planar movements performed in a horizontal plane with two degrees of freedom are already kinematically more complicated than single joint movements. Thus, even for the relatively simple configuration studied by us, there is already a very wide range of possible movements for the arm, and therefore, for the hand. Dynamically, muscles acting on both the forearm and the upper arm must be activated, in order to generate appropriate torques at both the shoulder and elbow joints. In addition, muscles which span more than one joint, introduce joint interactional effects. Moreover, each joint torque no longer depends only on the angular accelerations at the same joint but also on velocities and accelerations at both joints. This dependency stems from the mechanical interactions between the moving segments of the arm. Furthermore, the inertia is no longer a constant scalar but changes continuously as the configuration of the arm changes. Thus, the dimension of the kinematic and dynamic problems increases dramatically. In the next two sections the problems of the dynamics and kinematics of trajectory formation for multi-joint limbs will be discussed in more detail and the specific issues addressed in this study will be presented.

1.4. Kinematics of trajectory formation: issues and approaches.

In dealing with the kinematics of arm movements we need first to explore, whether the CNS explicitly specifies the trajectory of the arm. In the above section we have mentioned the view that that indeed is the case. The opposite view has also been expressed by some authors [e.g. Kelso & Holt 1980] who suggested that the CNS only plans explicitly the final position for the limb, and that the actual trajectory is therefore merely a byproduct of properties of the musculoskeletal apparatus. This conclusion was drawn based on the results from experiments in which fingers of blindfolded subjects were anesthetized by means of blood deprivation. The subjects, were capable, nevertheless, to generate finger movements to learned locations, even in trials in which a torque motor momentarily retarded the movement during its course [Kelso & Holt 1980]. Studies on monkeys, however, have indicated that simple single-joint pointing movements are controlled by preprogrammed signals which specify a trajectory as well a as final position [Bizzi et al. 1982]. Studies on various free two-joint reaching movements performed between different pairs of targets, have revealed that although different movements involve different contraction patterns for the muscles involved, the trajectories of the hand have certain invariant kinematic features [Soechting & Lacquaniti 1981]. Another question which is still open, is whether the CNS plans trajectories computationally or analytically [Morasso & Ivaldi 1982], or whether trajectories are specified by neural and muscular properties. In such a control scheme the system does not perform computational and symbolic operations at higher levels. Instead, the emerging trajectories are dictated by factors such as patterns of activation of muscle synergies, spring-like properties of muscles, or other low level mechanical effects.

When a subject reaches for a target in the environment the hand moves along a certain path from its initial to its final position. The movement of the hand is achieved by rotations of the various arm segments which lead to the hand. If we accept the view that arm trajectories are explicitly planned, the second question we must ask, therefore, is whether trajectories are planned in terms of hand coordinates in the external space, or whether they are planned in terms of joint coordinates. In the second mode there is no explicit planning of the trajectory of the hand in the extracorporal space while in the second mode trajectories are explicitly planned in hand spatial coordinates.

Again, there exists a controversy among motor control researchers with regard to the answer to this question. Some investigators have argued that the system maintains constant ratios between angular velocities at various joints [Greene 1972], or relates motion in a few joints which belong to a certain "functional synergy" [Saltzman 1979]. Soechting and Lacquaniti [1981], for example, have claimed that the ratio of elbow angular velocity to shoulder angular velocity during forward projections of the hand toward a target is maintained constant especially in the terminal phases of the movement. Several authors, including the above, have suggested that the purpose of introducing such constraints is to reduce the complexity of the problem by reducing the number of degrees of freedom of the system. Soechting and Lacquaniti [1981] have interpreted their results as indicating that trajectories are planned in joint variables, rather than in hand Cartesian coordinates.

Against this view some motor theorists [Bernstein 1967, Lashley 1951] have suggested that simplicity of motor control is achieved through the planning of hand trajectories in the extracorporal space, and therefore joint rotations are only

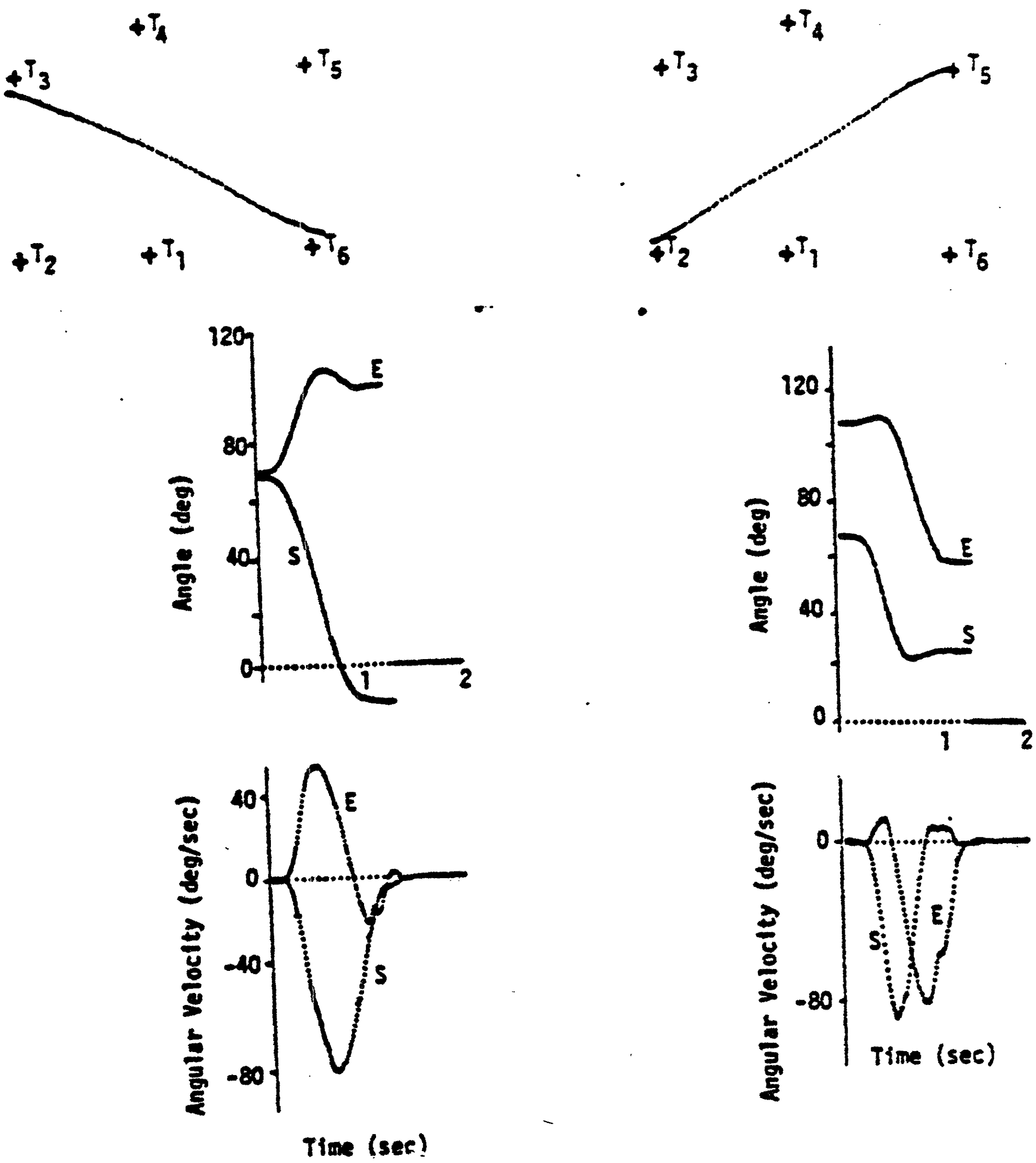


Figure 2. Point-to-point movements in different parts of the work space.

tailored to produce these desired hand movements.

Recently, this view has been supported by the studies of [Morasso 1981, Abend et al. 1982, Georgopoulos 1981]. In Morasso's study, subjects were instructed to generate arm movements between pairs of targets located in a horizontal plane. The movements involved rotations about the shoulder and elbow joints. Figure 1-2 displays typical results for two target-to-target movements similar to the results presented by Morasso.

The resulting movements exhibited a few invariant characteristics. The first invariance was a hand path invariance. When the subject was asked to move his hand to a new target, the hand moved along a roughly straight path in any part of the movement space even though no instructions were given to the subjects with regard to their hand trajectories. The second invariance was exhibited in the hand tangential velocity profile. This profile was always bell shaped and had a single peak independently of the part of the movement space in which the movement was generated. This speed profile was similar to the one obtained for simple one-degree-of freedom forearm movements and for single-joint movements of other body parts [Hogan 1982a]. In contrast to the invariant characteristics of hand trajectories the time-history of joint angles and joint angular velocities at both shoulder and elbow were highly dependent on the movement space region. These results have led Morasso to the conclusion that hand trajectories rather than joint rotations are explicitly planned. Furthermore, these findings have indicated that the CNS explicitly selects certain stereotypical hand trajectories for point-to-point movements from the vast number of possible trajectories.

As an extension of this study, Abend et al. [1982] have conducted a series of experiments in which subjects were instructed to generate curved movements. The results from these experiments have again indicated explicit planning of hand trajectories, but in addition other unexpected kinematic characteristics of curved movements were found. Typical curved movements made in different parts of the work space are shown in Figure 1-3.

Although, for curved movements the hand velocity profiles were expected to be single-peaked as for straight movements, they usually exhibited two peaks and a speed valley in between, and occasionally even three or more peaks. In addition, although the hand paths appeared smooth, the hand trajectories showed a few midcourse curvature peaks which corresponded temporally to the hand tangential velocity valleys. Thus, the majority of hand paths appeared as made of two or more segments of gently curved paths meeting at a more highly curved region. Temporal coupling between velocity and curvature has also been observed in handwriting and drawing [Viviani & Terzuolo 1980] and in infant reaching movements [von Hofsten

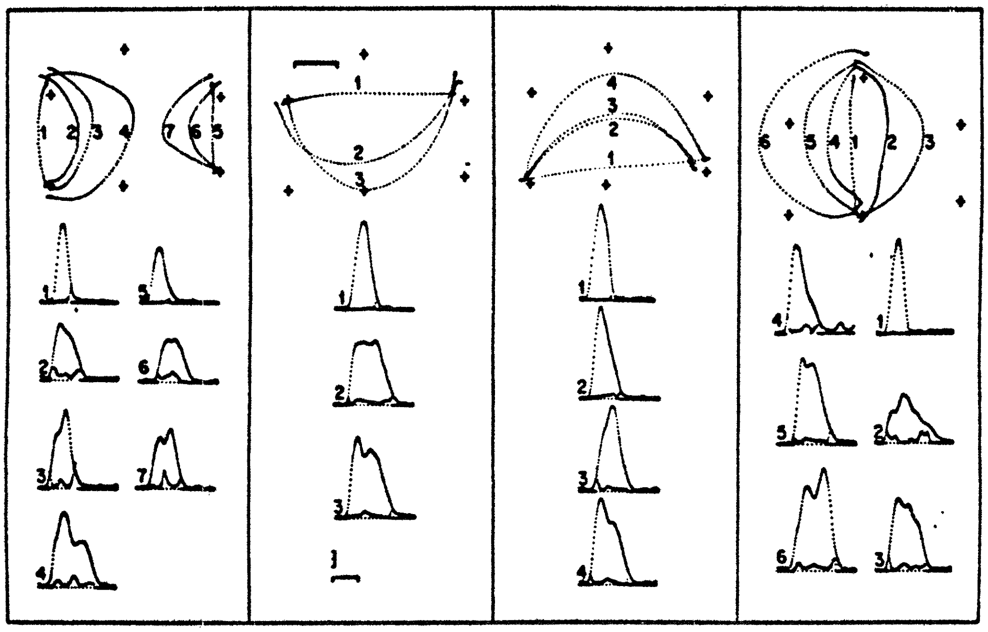


Figure 3. Curved movements in different parts of the work space by the same subject.

1979].

On the basis of these results, several questions can be raised. First, in point-to-point movements, why is there a tendency to move the hand along straight paths and why does the hand speed have this particular tangential velocity profile? Secondly, why do curved movements exhibit multi-peaked curvature and speed profiles, and why is there a coupling between hand curvature and speed? Thirdly, how or why does the CNS select these particular hand trajectories from the vast number of possible trajectories?

With regard to the second question, a possible cause for the curvature and speed features of curved movements may involve mechanical factors. For this to be considered as a possible cause, such factors should impose certain constraints only on curved and not on point-to-point or single joint movements. As possible mechanical factors one may consider the interaction forces between moving segments or effects due to the activation of two joint muscles [Hogan 1980].

Abend et al. [1982], Morasso and Ivaldi [1982], and Viviani and Terzuolo [1982] have suggested that since the central controller (for unknown reasons) prefers to generate straight hand paths between targets, curved movements are also planned as a sequence of roughly straight segments which are then spliced together to give the characteristic curved trajectory. This may offer an explanation for the speed and curvature features of curved movements, but does not explain why there is a tendency for the hand to move along straight lines, and offers only a partial answer to the question of why only particular trajectories are selected.

It is possible that the hand trajectories are planned with the objective of optimizing some kinematic or dynamic variable [Abend et al. 1982]. Such an optimization may result in the selection of curved and straight trajectories with the above kinematic features, and therefore may offer answers to all the above questions.

1.4.1. Specific issues addressed at the kinematic level.

In this part of the work I have looked for a possible explanation for the above kinematic features of hand trajectories. The goal of this work was to suggest a model which, based on a single principle, can account for the global aspects as well as the details of the kinematic presentation of planar arm movements for point-to-point and curved movements.

Evolutionary theory suggests that a fundamental question, which needs to be asked, is what are the variables which the nervous system optimize in controlling movement. Hence, the use of optimization techniques to model natural behavior is appealing because of the analogy it bears to the optimization presumed to occur as a result of natural selection.

Single joint movements have been successfully modeled [Hogan 1982a] based on the assumption that these movements are performed with the objective of maximizing the smoothness of the movement from one equilibrium position to another. Maximizing the smoothness of a motion implies minimizing the jerk. Jerk is the third derivative of position and is the rate of change of acceleration. In this work we explored the possibility that a model which is based on jerk minimization can successfully describe two-joint planar arm movements and can account for the majority of the kinematic features of straight and curved trajectories. The reasons for attempting to use a similar kinematic optimization criterion for planar movements are based on our desire to look for a unifying principle for trajectory formation of different kinds of movements [see section 1.1.2].

The movements investigated in the course of this work display invariant characteristic only of trajectories represented in hand Cartesian coordinates and not in joint variables. It is possible to treat the hand as a point mass, moving in the plain. The location of this mass with respect to a Cartesian planar coordinate system is specified by a position vector. The jerk of the disembodied hand is the third derivative of this position vector. The model, which is presented here, is based on the minimization of this jerk squared.

Using dynamic optimization techniques, the model yielded mathematical expressions for hand trajectories. Different kinds of planar movements have been modeled including straight movements, obstacle avoidance movements, and curved movements. Comparisons of these model-based trajectories with experimentally derived records of human planar arm movements enabled us to assess the success of the model in matching measured movements. In addition to jerk we tested also other objective criteria. These were hand snap squared (snap is the fourth derivative of position) and hand acceleration squared.

1.5. Dynamics of trajectory formation: issues and approaches.

As mentioned above, the musculoskeletal system is a skeletal assemblage whose members are acted upon by muscular actuators controlled by neural commands. The individual bony segments can be considered as rigid bodies connected at joints and forming mechanical chains with a certain number of degrees of freedom, depending on the number of joints and the degrees of freedom at each joint. For example, the primate and human arm has seven degrees of freedom not including the intrinsic degrees of freedom of the hand (three degrees of freedom at the humerus/scapula joint, two degrees of freedom at the elbow joint and two degrees of freedom at the wrist).

The dynamics of such an assemblage can be described by a set of nonlinear second order differential equations, the equations of motion, which express the dependency of joint torques on the kinematic variables of the movement. The derivations of these equations of motion are based on the principles of Lagrangian mechanics. For a multi-joint mechanical chain the system of equations of motion can be written as:

$$\underset{\sim}{N} = J(\underset{\sim}{\theta})\ddot{\underset{\sim}{\theta}} + \underset{\sim}{C}(\underset{\sim}{\theta}, \dot{\underset{\sim}{\theta}}) + \underset{\sim}{B}(\dot{\underset{\sim}{\theta}}) + \underset{\sim}{G}(\underset{\sim}{\theta}) \qquad (2)$$

where $\underset{\sim}{N}$ is the joint torque vector, $\underset{\sim}{G}$ is the gravitational torque vector, $\underset{\sim}{B}$ is the frictional torque vector, $\underset{\sim}{C}$ is the inertial velocity dependent torque vector, and J is the moment of inertia matrix. $\underset{\sim}{\theta}$, $\dot{\underset{\sim}{\theta}}$ and $\ddot{\underset{\sim}{\theta}}$ vectors are the angles, angular velocities and angular accelerations.

Assuming that all the angles, velocities and accelerations at all joints for all degrees-of-freedom are known, these equations can be solved and the magnitude and direction of joint torques can be computed. This problem of computation of joint torques for a given set of kinematic variables describing the movement of the chain is called the "Inverse Dynamics" problem [Chao & Rim 1973]. The opposite problem is the "Direct Dynamics" problem which allows us to predict the movement of the mechanical chain from a known set of input forces and torques.

For a chain of skeletal segments such as the upper limb with its seven degrees of freedom, the derivation of these equations and the calculations of the torques are quite involved. In addition, during movement the configuration of the chain, its moment of inertia and the angular accelerations and velocities keep changing and, therefore, the equations must be continuously updated and solved repeatedly if one wishes to compute joint torques correctly. All these complexities result in a very heavy computational load. In the case of man-made manipulators, it was believed that the complexities described above make an on-line, real time guiding of artificial manipulators infeasible [Waters 1979]. Therefore, joint torques were computed using various approximations or simplifications of the dynamic equations, and the degree of success of the manipulator in following a desired trajectory has depended very heavily on feedback corrections. Another suggestion for overcoming the problems associated with real-time computations is the use of look-up tables stored in memory [Raibert 1978, Horn & Raibert 1978]. By using such an approach one avoids the computational load but instead one needs to rely on a very large memory storage. Recently, however, several mathematical formalisms [Luh et al. 1980, Hollerbach 1980, Walker & Orin 1981, Armstrong 1979] have been suggested which render on-line real-time dynamic computations feasible.

The equations of motion are the same whether movements of an artificial manipulator or of a human arm are being considered, and therefore the biocontroller and the controller of an artificial manipulator must solve similar problems if joint torques must be derived. For biological movements the question is how does the CNS arrive at the appropriate joint torques required to carry the arm along a desired trajectory. Since various lines of evidence [Bossom 1974, Rack 1981] lead to the rejection of the idea that feedback control is essential in the generation of movement, we must find out what biological strategies have evolved to allow open-loop preprogramming of movements. Various suggestions for such strategies have been raised. One theory assumes that the CNS performs on-line accurate computations of joint torques [Saltzman 1979, Hollerbach 1982]. Such a strategy is highly advantageous since it eliminates the need for immense memory storage and adapts easily to a priori known changes in loads or in inertial properties of body parameters. Nevertheless, it is not yet known whether the CNS actually solves the equations of motion each time a movement must be executed. Other authors have suggested that the CNS uses sparse look-up tables in both motor learning and in open-loop motor programming [Raibert 1978]. Another model has suggested that through the control of the end-effector impedance the CNS may bypass the need to solve the inverse dynamics problem [Hogan 1980].

It is not clear yet whether any of the above proposed strategies is indeed used by the CNS in the solution of dynamic problems. Nevertheless, theoretical modeling and consideration of the complexities of the computations involved in the solution of similar problems for artificial manipulators can help us in suggesting testable models for the biological system. Moreover, the fields of artificial intelligence and mechanics can also provide us with useful tools for the simulation and analysis of movements. Since motor research suffers from the lack of knowledge of the exact mechanical output of the system, such tools are very useful in allowing us to compute joint torques and muscle forces.

One of the goals of this work is to assess the complexities of the dynamic problems in the context of biological movement. For this purpose the work done in robotics has served us in suggesting testable ideas; therefore approaches and solutions used in the control of artificial manipulators will be referred to many times in the course of this thesis. It can be argued that such an approach leads us on the wrong route because it can be seen as another example of modeling the work of the brain on the analogy to the latest technological invention (e.g. memory-holography). Nevertheless, this argument does not hold in this case since the biocontroller and the controller of an artificial manipulator do need to solve similar problems in the control of similar effectors. Furthermore, although this work was initially intended for the purpose of studying biological movements, some of the main conclusions of the thesis are of interest and significance for robotics.

As mentioned above one of the factors which contribute to the complexities of dynamic problems for multi-joint movements is the existence of interaction forces between the moving segments of the arm. The contribution of these forces to the net joint torques have been evaluated and their significance has been assessed. The reasons for assessing their significance will be discussed in the next section.

1.5.1. Interaction forces.

Forces which the system has to deal with include reaction forces. These forces are introduced at various joints as a passive dynamic consequence of velocities and accelerations at remote joints. These forces arise from mechanical coupling between the segments, whenever a distal segment is moving with respect to a proximal rotating link. Obviously interaction forces are also introduced in biological multi-joint movements. Nevertheless, it is not clear a priori that interaction forces are important in the course of normal movements. In the control of artificial manipulators, because of the desire to develop faster and more efficient algorithms, one tries to avoid complex dynamic computations. Since it was noticed [Waters 1979] that the introduction of interaction forces increases dramatically the computational load it was suggested that velocity torques, the torques which are proportional to the product of joint velocities of two joints, can be omitted from the computations. The justification for this omission was based on the argument [Paul 1981] that these terms are much less significant than inertial torques, torques proportional to joint acceleration, or gravity torques. At high speeds, however, the velocity torques contribute large forces and ignoring them in schemes which include computations only of inertial torque terms and not of frictional or elastic forces, introduces large errors into the computations [Luh et al. 1980].

If the interaction forces are important during normal movements, then they will affect the trajectory of the hand. Therefore, in one movement the reactive forces may contribute to the desired trajectory while in another design they may oppose the trajectory unless the appropriate torques for the movement, which include contribution for interaction forces, are generated. Bernstein [1967] has expressed the view that the highest degree of coordinative skill "corresponds to a degree of coordination at which the organism is unafraid of reactive phenomena in a system with many degrees of freedom but is able to structure its movements so as to utilize entirely the reactive phenomena which arise". Based on his studies of running in very young children, he has argued that such skills develop with motor learning and practice. Moreover, it was suggested [Fowler & Turvey 1978] that a subject naive to a specific skill may immobilize some of the joints which are used when the task is performed skillfully, or may put restrictions on the range of motion of other joints. Such immobilization of some of the joints, does not eliminate all the reactive forces, at the immobilized joints, and at other joints, but diminishes their magnitude. Thus, the increase in the number of controllable degrees of freedom may be equivalent to increasing the degree of expertise since by this the subject increases the potential for reactive forces.

The question is does the CNS "structures its movements" so as to utilize this reactive phenomena, or does the system care more about the generation of desired trajectories [Morasso 1981, Abend et al. 1982] and therefore need to do an adequate job in computing or compensating for these forces regardless of their magnitude or the direction in which they affect the movement?

Another issue which must be addressed at the dynamic level is, how does the system adapt to requirements to change the speed of movement or the weight of carried loads? If it uses a computational mode, does it solve from scratch dynamics problems each time a movement is generated or does it use strategies which simplify the adaptation to changes in speed?

1.5.2. Studies of dynamics: specific objectives and methods.

The first specific objective of this part of the thesis was to calculate the net joint torques and to assess the significance of interaction forces during human two-joint arm movements at various speeds. The second objective was to consider plausible mechanisms for dynamics computation or compensation for interaction forces by the human motor system.

For these purposes, this work combined computer simulations of the dynamics of multi-segment linkages and analysis of experimentally obtained records of target-to-target planar arm movements. Subjects were required to move their hands between different pairs of targets at various movement speeds. The relative significance of various interaction torques was assessed.

1.6. Organization of the thesis.

The main objectives and concerns of this work have been presented in this chapter. The nature of the problem of trajectory formation and the main two levels at which this problem has been approached have been discussed. Chapter 2 contains a discussion of some of the relevant physiological background and a review of experimental work and theoretical modeling on the control of single and multi joint movements. Various approaches to trajectory planning and dynamics for artificial manipulators are also discussed. Chapter 3 presents the optimization model for the kinematics of hand trajectories, the predictions derived from this model and descriptions of the experiments conducted to test the model. The main conclusions from this part of the work are discussed separately in chapter 3. In chapter 4 the problems of dynamics and the various kinds of interaction forces are discussed. The methods used for the computation of joint torques and interaction forces and the results from these computations are described. Various mechanisms for compensation for the interaction forces are discussed. Finally chapter 5 contains an overall discussion of the thesis and presents the main conclusions drawn from this work. This chapter also presents suggestions for the extension of this study.

Chapter 2

Movement generation: related physiological and theoretical studies.

Before proceeding to the discussion of various theories of motor control, it is useful to review some of the underlying physiology of events associated with the generation of movement. This discussion will be followed by a description of various types of simple movements and of the basic features of complex movements. finally, various models proposed for the control of simple and complex movements will be presented. In the discussion of complex movements various pertinent approaches and methods, used in robotics for the control of artificial manipulators, will be described.

2.1. Physiological background.

2.1.1. Basic structure of the musculoskeletal system.

Human and primate limbs are made of a certain number of segments that can basically be described as rigid bodies since their deformations during movement are typically very small compared to the gross movements of these segments. The segments are connected by joints which can be classified according to the number of degrees of freedom allowed at them. Thus, for example, the shoulder is a ball and socket joint and has three degrees of freedom. The elbow can be described as two hinge joints with flexion/extension movements at the humerus-ulna joint and pronation/supination movements at the radius-ulna joint. The human wrist is a complex structure but it behaves basically as a universal joint and all its movements can be described as combinations of flexions or extensions and radial or ulnar deviations.

Each joint is spanned by many muscles. Most muscles are attached to two or more segments usually by way of tendons but occasionally directly to bones. The majority of muscles span only one joint but some muscles span two (e.g. long head of biceps) or even three joints. A movement at a hinge joint requires only two muscles, a flexor and an extensor. It can be shown, however, [Alexander 1981] that for a structure like the elbow, with its two degrees of freedom, three muscles, are sufficient to provide all possible combined movements at the two joints, and n+1 muscles are sufficient to produce all possible movements of a system with n degrees of freedom. Nevertheless, most limb segments are worked by far more muscles than are actually needed. The reasons for this redundancy or the rules for the selection of certain muscles for certain tasks are not entirely clear.

Within each muscle, bundles of fibers are arranged in various ways. Some muscles are parallel fiber muscles while others are either unipannate or bipannate so that their muscle fibers run at certain angles to the direction at which the muscle as a whole pulls.

2.1.2. Muscle innervation.

The smallest element of a skeletal muscle which is capable of independent activity is the motor unit. A motor unit consists of a number of contractile fibers which are innervated by a single efferent α motoneuron. The number of motor units within muscles range from a relatively small (e.g. flexor digitorum longus) to a very large number (e.g. medial-gastrocnemius) [Burke 1981]. The descending motor command appears as an action potential that travels down the axon branches which innervate the individual muscle fibers of the motor unit. Until recently it was believed that the fibers belonging to a single motor unit are arranged as a localized collection of fibers which in turn are scattered through a more extensive territory. Recently, however, glycogen depletion experiments have shown [Burke & Tsairis 1973] that muscle fibers belonging to the same unit are scattered evenly through an extensive territory. Each fiber is innervated by a single α motoneuron and there is no polyneural innervation of an adult muscle fiber. In addition, all muscle fibers belonging to the same motor unit are usually activated by each motoneuron spike [Burke 1981].

The group of motor units innervating a particular muscle forms a motor pool in the ventral horn. Motoneurons are recruited in an orderly manner which correlates with the cell size. Thus the larger motoneurons which have faster motor units and produce more force are recruited after the smaller motoneurons. This is called the size principal by Henneman [1979]. Muscle output is graded by this recruitment order and by the modulation of the firing rates of single motoneurons. This fixed recruitment order which is kept even in ballistic movements [Desmedt 1980] makes the control of movement simpler for the CNS.

Most muscles contain three muscle fiber types [Burke 1981]. Muscle fibers which belong to the same unit are essentially identical in histochemical appearance and therefore probably also in their mechanical properties. The characteristics of the three muscle fiber types balance functional demand against metabolic cost. Many characteristics of the motoneuron (e.g. duration of spike after-hyperpolarization and speed of axonal conduction) also match the function of the muscle fiber type which it innervates [Burke 1981]. The three types of motor units are: S, FF and FR.

Type S fibers are resistant to mechanical fatigue and are more efficient when the fiber length remains constant. They produce less force per unit cross sectional area than fast twitch muscle fibers. Type S motor units have the lowest functional threshold and are particularly specialized for postural maintenance.

The second type, FF fibers, generate large forces and are susceptible to fatigue. They have the highest functional threshold. These units are involved in fast movements and are ideal for bursts of activity. Type FF units have a relatively low cost of maintenance.

Between the two extremes is the FR type which is characterized by fast contraction and relaxation and high fatigue resistance. This type has a high metabolic cost of maintenance and is better adapted to phasic movements.

The population of motor units within muscles is matched to the functional demands placed on the muscle. Several motoneurons are recruited in most movements and each twitch tension is superimposed mechanically upon the force generated by other units. Because motor unit fibers are scattered through extensive overlapping territories, the mechanical interactions between the fibers help to smooth the summation of their active tension during synchronous low frequency firing of motoneurons [Partridge & Benton 1981].

The innervation of intrafusal muscle fibers will be discussed in the in the section below.

2.1.3. Proprioceptive sensors.

In addition to the outflow of neural signals from the brain to the periphery through α and γ motoneurons, proprioceptive fibers conduct information from muscles, tendons, ligaments, joints and skin. Although these messages are essential for kinesthetic sense and awareness of the position of limbs and their movements, most of these messages are spread to various areas of the brain which do not function at a conscious level. The proprioceptive information is detected by muscle spindles, tendon organs, joint receptors and other mechanoreceptors in the skin.

Joint receptors are encapsulated receptors within articulated joints. Their discharge facilitates the activation of flexors and inhibits extensor muscles. In the past, several authors [e.g. Marteniuk & Roy 1972] have claimed that joint receptors are active throughout the entire range of joint positions. Recently, however, new data have indicated a lack of activity from joint receptors when the joint is held at its midrange position compared to when it approaches its maximum range [Clark & Burgess 1975]. This makes these receptors unsuitable for signalling the whole range of joint positions, but may indicate a role for them as limit detectors. Since joint receptors may effect the timing of the stepping cycle [Lundberg et al. 1978], they may play an important role in locomotion.

Most skeletal muscles contain a large number of muscle spindles. A muscle spindle [Matthews 1972, 1981] consists of a bundle of specialized muscle fibers, the intrafusal fibers, which exert a powerful force on the receptor terminals that are situated on them. Two types of afferent axons leave the spindles, large fast conducting group Ia (primary) axons, and smaller slower conducting secondary type axons. The primary and secondary fibers show a static response to length. Primary fibers show also a marked velocity sensitivity.

Since the spindle lies parallel to the extrafusal muscle fibers, contraction of the latter unloads the spindle unless the spindle activity is biased by the contraction of the intrafusal fibers. There are three types of intrafusal fibers [Boyd 1980]: nuclear bag fibers, bag1 and bag2, and smaller nuclear chain fibers. Most of the efferent innervation of intrafusal muscles consists of static and dynamic γ motoneurons which are a little slower than alpha motoneurons. Bag1 is innervated by dynamic fibers. Bag2 and nuclear chain fibers are innervated by static γ. All three intrafusal fiber types have primary sensory spirals around them. All these terminations are branching from the same primary axon. Secondary sensory endings have spiral endings around each of the nuclear chain fibers and less extensive spray-like terminations on the nuclear bag2 fibers and all these terminations are connected to the same secondary axon.

The spindle's secondary and primary response is to muscle stretch and to contraction of intrafusal muscles. When the intrafusal muscle fiber is being activated, since its sensory zone displays less contractile activity than its poles, it is stretched. This stretch causes the receptor response to rise. When the extrafusal muscle lengthens, since the intrafusal muscle lies in parallel to the extrafusal fibers, it also lengthens thus again causing a discharge. Conversely extrafusal shortening causes decreased afferent discharge. In effect the spindle discharge corresponds to the difference between extrafusal and intrafusal lengthening. Static γ input increases the sensitivity of spindle afferents to length but decreases their sensitivity to velocity. Dynamic γ input increases the sensitivity of primary fibers to velocity but decreases their sensitivity to length.

The Golgi tendon organ lies in series with the muscle and therefore it discharges during the contraction of the muscle [Houk & Henneman 1967]. Tendon organs are selectively sensitive to the force of muscle contraction independently of the length of the muscle. They are sensitive to force variations throughout its physiological range. Tendon organ Ib axons activate interneurons and these inhibit motoneurons innervating the muscle from which the Ib axons arise.

2.1.4. Spinal and supraspinal stretch reflexes.

The spinal stretch reflex involves closed loops within the spinal cord which consist of afferent fibers and efferent motoneurons. Most of the stretch reflex activity arises from muscle spindles. A stretch of a muscle extends the sensory endings and this leads to an increase in the afferent input to the alpha motoneurons. This increased input predominantly causes motoneurons to be recruited although it may also increase their firing rates and this in turn leads to the contraction of the muscle. This reflex contraction of the muscle is further helped by the inhibition of the antagonist muscle. This principle of excitation of one group of muscles while its antagonist group is being inhibited was called reciprocal innervation by Sherrington [1906].

Information from the peripheral sensory organs seem to involve reflex loops which extend beyond the level of the spinal stretch reflex [Conrad & Meyer-Lohmann 1980, Lee & Tatton 1975, Mardsen et al. 1976]. In response to perturbations to limbs, short and intermediate bursts of muscle excitation can be demonstrated upon the rectified EMG. The short response (M1) is consistent with the monosynaptic reflex and is small and brief compared to M2 M3 responses. The latter responses have been described as the long-latency component. The latencies of these responses to biceps stretch, for example, are 50-60 msec as opposed to about 110 msec latency of the voluntary response and 18 msec to the stretch-reflex response [Hammond 1960]. The gain of these reflexes can be modified, although not to a great extent, by previous instructions, [Gottlieb & Agarwal 1980]. The pathways of long-latency stretch reflexes presumably involve longer supraspinal loops. This view was reinforced by Phillips et al. [1971] who described cortical neurons in the 3a area of the cortex which are responsive to activation of Ia fibers. Studies have shown that corticospinal neurons project back to the same muscles from which afferent information has been received [Conrad & Meyer-Lohmann 1980]. This indicates that the pyramidal tract neuron is similar to the motoneuron since this cell is also a summing point for central programs and peripheral feedback [Miles & Evarts 1979].

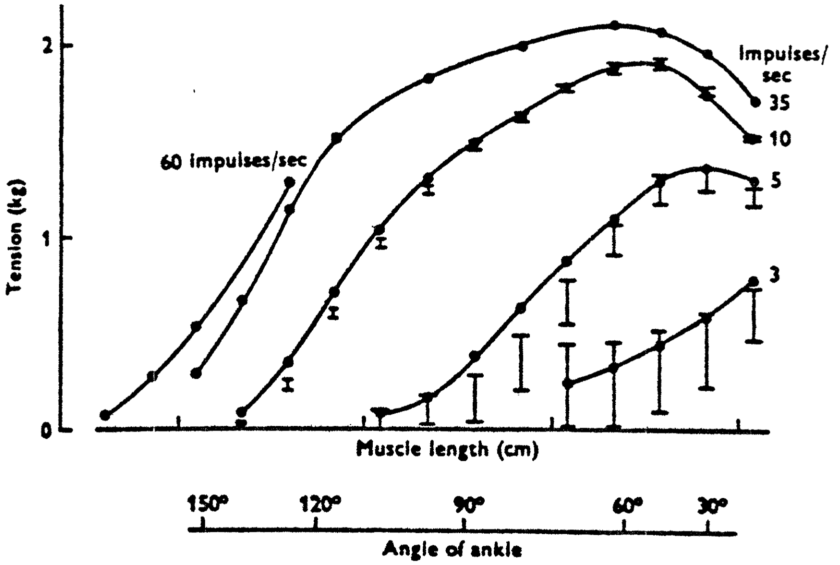


Figure 1. Length-tension curves for soleus muscle. From [Rack & Westbury 1969].

2.1.5. The intrinsic properties of a skeletal muscle.

In this section only very basic properties of skeletal muscles which are necessary for the discussion of current theories of motor control will be described.

When a length of a muscle increases, the muscle tension increases roughly linearly with the length [Rack & Westbury 1969], except at maximum lengths where there is a decrease in muscle tension. This can be seen in Figure 2-1. Muscles normally operate on the ascending portion of the length-tension curve.

The static stiffness of muscle, which is defined as the increment in muscle tension divided by the increment in muscle length, is appreciable at intermediate lengths and decreases almost to zero at maximum lengths. On the other hand, the passive stiffness of a muscle is negligible at intermediate lengths but increases markedly in both ends of the range of the physiological length of muscle fibers. This information was gathered through studies of isolated muscles [Rack & Westbury 1969]. Length-tension curves of the motor servo, consisting of the muscle and the set of autogenic pathways have been studied in decerebrated cats [Nichols 1973].

The effect of recruitment of motor units on the isolated muscle length-tension curves was studied by comparing the length-tension curves when different number of motor units are electrically stimulating at physiological rates [Houk et al. 1970]. This effect amounts to a scaling of the entire length-tension curve in proportion to the level of recruitment as can be expected from an addition of contractile elements in parallel. The effect of recruitment of additional motor units is equivalent to an increase in muscle stiffness [Houk & Rymer 1981]. An increase in firing rate can increase, decrease, or leave unchanged the muscular static stiffness depending on the particular firing rate and the initial length of the muscle about which the change of the slope of the length-tension curve is assessed (see Figure 2-1). However, as can be seen from Figure 2-1, increasing the rate of firing of motor units causes characteristic shifts of the length-tension curves to the left, or a decrease in the muscle rest length [Rack & Westbury 1969, Joyce et al. 1969].

Therefore, basically areflexic muscles behave like springs with changeable stiffness and rest length.

In addition to the dependency of muscle tension on muscle length, muscle tension depends also on the velocity of muscle shortening or lengthening. By stimulating different subdivisions of ventral roots in rotations it was possible to obtain smooth contractions of muscles [Joyce & Rack 1969]. Typical results from this study conducted on isolated soleus muscle are shown in Figure 2-2.

During shortening the tension always fell below the isometric force. At low rates of stimulation the tension fell abruptly at the beginning of the shortening movement but later there was only a gradual fall in force. During lengthening at high rates of stimulation the force always rose above the isometric tension. At lower rates of stimulation the tension rose at the beginning of the extension but decreased later to the same level or below the isometric tension. As the velocity of shortening or lengthening increases, the tension in the muscle decreases. The initial change of muscle tension with muscle length is called short range stiffness relating to the stiffness of stretched but unbroken crossbridge bonds [Joyce, Rack and Westbury 1969]. This stiffness is rather high. If stretch continues beyond the limit of strength of the bond yield, a decrease in stiffness occurs as can be seen from Figure 2-3.

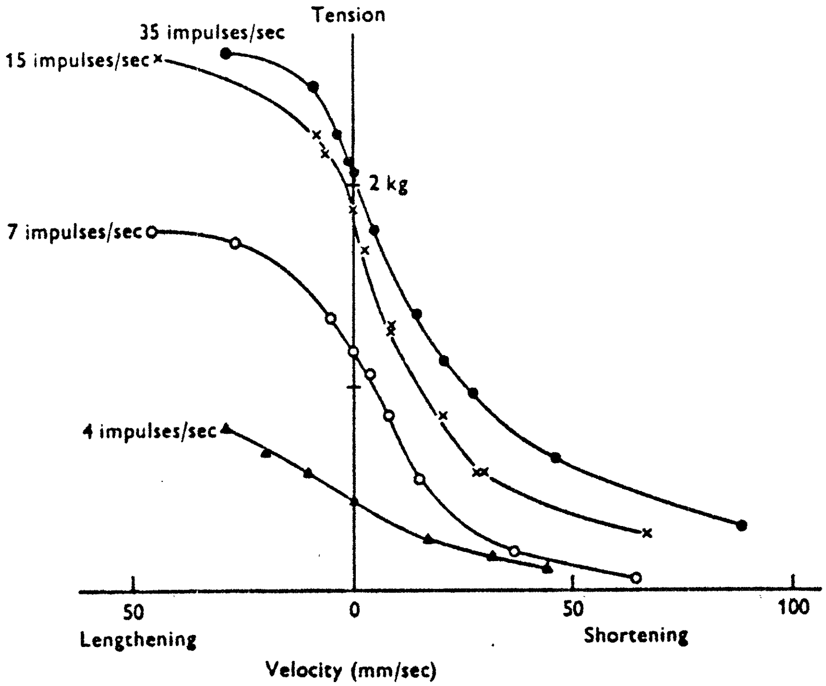


Figure 2. Velocity-tension relations in cat soleus. From [Joyce and Rack 1969].

The behavior of a muscle in response to lengthening or shortening of the muscle is referred to as viscoelastic resistance although a muscle does not follow the behavior of an ideal viscoelastic element.

2.1.6. CNS sequence of neural events preceding voluntary movements.

No attempt will be made in this section to describe the vast amount of information available on the involvement of various parts of the brain in the control of voluntary movement. For a review of the various central neuronal systems involved in the generation of reaching movements see Humphrey [1979]. Only a brief description of the sequence of CNS events preceding the generation of voluntary movements as was suggested by Allen and Tsukahara [1974] will be presented. EEG records show a "readiness potential" about 800 msec before the onset of muscle action [Kornhuber 1974]. This is followed by a slow sharply negative wave 0.5 to 0.1 second before movement. Thus, preparation for movement takes about 700 msec before any firing from the motor cortex. During this interval signals are sent from the associative cortex to the motor cortex directly and through two subcortical loops

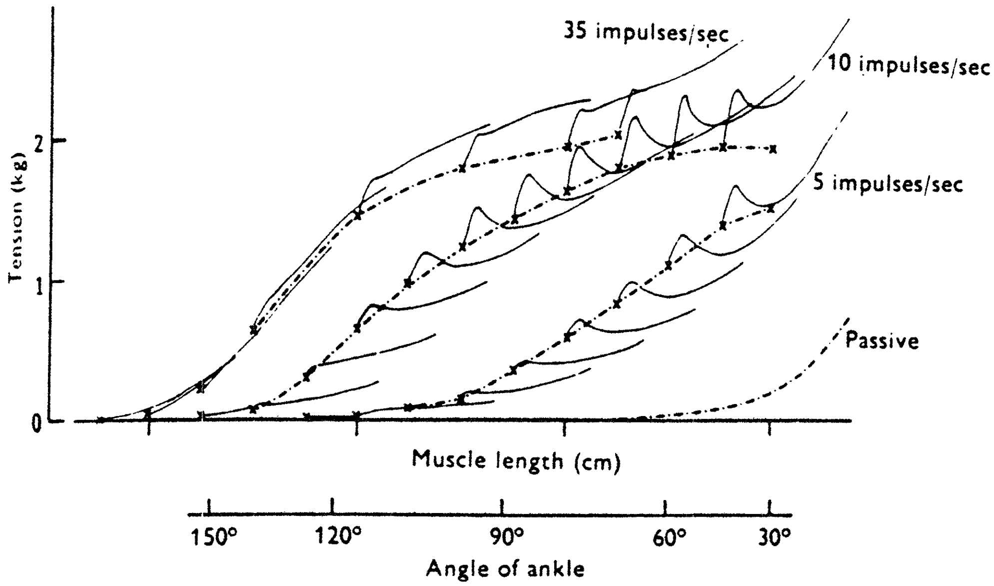


Figure 3. Force-length trajectories for constant velocity stretchings of the muscle beginning in different operating points (solid lines) superimposed on length-tension curves (broken lines). From [Joyce et al. 1969]

via the basal ganglia and lateral cerebellum and subsequently via the ventrolateral nucleus of the thalamus back to the motor cortex. Next, the output is sent via the corticospinal tract to spinal motor pools. Additional output is sent via branches of the pyramidal tract fibers which after a synaptic relay in the pons give input to the intermediate zone of the cerebellum. After an interval of processing in the intermediate cerebellum, neural signals are returned to the cortex via the thalamus, but are also sent via the rubrospinal tract to the spinal motor neurons. Other input to motoneurons is sent through the ventromedial brainstem pathway by reticulospinal and vestibulospinal tracts. The reticulospinal and vestibulospinal tracts are mainly involved with posture, gross limb movements and integration of the two. The reticulospinal tract gives the capability for independent use of the extremities and the corticospinal tract contributes to the fine control of distal extremity muscles

[Kuypers 1973]. Proprioceptive information is mediated by posterior columns and the anterior and lateral cerebellar tracts, to eventually reach either the sensorimotor cortex or the cerebellum.

2.2. Behavioral analysis of movement.

2.2.1. Classes of simple movements.

In this section we will describe the main features of simple movements. Simple movements are movements which involve rotations about one joint in a single plane [Holmes 1939]. Steton [1935] has described three classes of simple movements:

(i) Slow tension: slow moving fixation movements.

(ii) Ballistic movements:

1. Loose ballistic movements with no contraction of opposite muscles.
2. Stiff ballistic movements with continuous tension between opposing muscles.

This division of simple movements (e.g. flexion of the upper arm or the elbow) into three classes was supported by the experiments of Brooks et al. [1973] conducted on monkeys. Monkeys were trained to perform simple movements to target zones with no mechanical stops and without seeing their hands or the apparatus. Brooks et al. [1973] have described two types of movements: continuous and discontinuous. The discontinuous movements which lasted about 1 second, had a few velocity peaks and exhibited 3 Hz oscillations in the acceleration trace. In these movements the handle was placed within the target zone after some tremor. These movements correspond to the slow tension movements. The second kind of movements which Brooks [1979] called continuous movements lasted about 0.5 seconds and had a single velocity peak and fine oscillations of 6 Hz in the acceleration trace. These movements correspond to the stiff ballistic type. Initially the monkeys performed discontinuous movements but after they learned the task, continuous movements could be made without visual or tactile information. In cases where the targets were placed unexpectedly in some other part of the trajectory, the movements ran along their usual course while slow discontinuous movements consistently overshot the target when target cues were omitted.

In monkeys performing ballistic (lasting $\approx$ 0.2 seconds) wrist flexion/extension movements against mechanical stops, when the mechanical stops were moved unexpectedly, the animal exerted muscular force for the previous duration of time [Brooks 1979]. Mean and peak accelerations and velocities and the durations of movement were unchanged. For energetic ballistic movements since these movements are very brief they preclude adjustments by visual or even proprioceptive information. Brooks [1979] has concluded that the rhythm of the muscular effort or some correlate of it seems to have been preprogrammed.

Normally the muscular activity during ballistic movements has three phases [Hallett and Marsden 1979, Desmedt & Godaux 1978]. At the first phase there is a burst of agonist activity. This is followed by a silent period in which there is a burst of antagonist activity and the arm decelerated and finally there is a second burst of agonist activity. Lestienne [1979] has described human flexion/extension movements of the forearm in the horizontal plane. The subjects were asked to perform three types of movements: low speed movements (LS), normal speed movements (NS), and high speed movements (HS). The NS and HS movements were characterized by having a single acceleration phase and a single deceleration phase. The cessation of activity of the agonist muscles occurred approximately when the first peak of acceleration was reached. The onset of antagonists began before the peak velocity. For HS movements, a substantial overlapping of agonists and antagonists appeared during the acceleration component but the amplitude was greater in these two bursts than in the NS movements. For LS movements there was a sustained level of EMG activity in the agonist which ceased near the peak velocity and there was a pause of antagonist activity lasting the entire movement. When the inertial load was increased, LS movements showed a burst in antagonist activity.

2.2.2. Complex movements.

By the term complex movements we will refer in this chapter to multi-joint movements of a single limb as well as to simultaneous and sequential movements which involve one or more body segments [Keele 1981]. It is much harder to classify complex movements than simple movements. Some characteristics of two-joint movements have been described in chapter 1. For detailed reviews of the behavioral analysis of movements see Keele [1981] and Poulton [1981]. Keele describes various studies dealing with sequential movements. He has reviewed evidence showing that complicated sequences of movements or utterance of words are broken into syllables before their execution. Moreover, the same sequence of actions can be performed at fast or slow rates and can be executed by different arms or even by different body parts (e.g. handwriting). A considerable interval exists between the signal to move and the movement itself even for a well practiced movement. The duration of this interval, which is a measure of the reaction time, depends on various factors such as the number of actions in the sequence their timing and the total time duration of the sequence. Once a sequence of movements begins however, it appears to be timed according to a preset clock.

2.2.3. Speed-accuracy trade-off

Fitts [1954] has shown that large movements take longer than small movements if they need to be performed with the same accuracy. On the other hand movements of the same amplitude take longer if more accuracy is required. This speed-accuracy trade-off has been expressed mathematically as [Fitts & Peterson 1964]:

$$MT = a + b \log_2 \frac{2D}{W} \tag{3}$$

where MT is the movement time, D is the distance moved, W is the width of the target and a and b are empirically determined constants. Because this equation applies to many settings it is called Fitts law. It was shown to be correct for many different types of human movements including single-joint wrist rotations as well as multi-joint movements [Keele 1981] and was shown to describe monkey movements as well [Brooks et al. 1973]. Various explanations have been suggested to account for Fitts law including the assumption that this law results from the application of multiple corrections based on visual feedback during the last part of the movement [Keele 1968]. This explanation has been rejected by Keele [1981] on the grounds that these corrections are not seen in the movement trace and because movements to narrower targets are slower during the entire movement. Another explanation of the trade-off between accuracy and speed [Schmidt et al. 1979] will be described below.

2.3. Open-loop versus closed-loop motor control.

This section starts with the discussion of various views on the function of reflexes in motor control. It is followed by a description of the limitations of reflexes and of evidence in support of theories of open-loop planning and control of movements. This then leads to a presentation of the idea of motor programming and to a discussion of some models for control of simple and complex movements.

2.3.1. The role of reflexes in motor control.

Although proprioceptive information and reflexes have been studied extensively their role is still unclear. Not very long ago reflexes and specifically the stretch reflex were though to play a dominant role in motor control. The early ideas were based on the reflex-chaining hypothesis which suggested that some central event initiates a movement and the feedback from the first muscular contraction triggers the next event in the chain and so on. This hypothesis has suffered from a number of lines of evidence most severely from the repeated finding that certain movements can be performed nearly normally in deafferented limbs.

The follow-up servo hypothesis by Merton [1953] suggested that most movements are initiated by commands sent via γ motoneurons which cause shortening of the polar zone and this results in an increased spindle discharge. This increased discharge causes activation of motoneurons and therefore extrafusal shortening. This unloads the spindles and in turn causes the muscle to come into balance at a shorter length. Experimental evidence has shown, however, that gamma motoneuron excitation does not precede alpha motoneuron excitation. In voluntary movements the onset of EMG activity preceded the acceleration of spindle discharge by an interval of at least of some tens of milliseconds [Hagbarth et al. 1975]. This interval is consistent with simultaneous activation of skeletomotor and fusimotor neuron pools by descending motor commands. Since there is substantial evidence that there is no fusimotor activity in non-contracting muscles, it seems likely that descending motor commands recruit gamma and alpha motor neurons in parallel, which has come to be known as gamma alpha coactivation. This does not mean, however, that the activity of gamma motoneurons is rigidly linked to that of alpha motoneurons. The balance between the skeletomotor and fusimotor drives to contracting muscles can be changed by reinforcement maneuvers or anticipation, which increase the responsiveness of stretch reflexes. Stimulations of skin and muscle mechanoreceptors can also change this balance. Therefore it can be expected that different muscles would have a different alpha-gamma balance and changes can occur even for a single muscle if its role is changed from a prime mover to posture fixer [Burke 1980].

It has been suggested that afferent feedback from muscle spindles is used, in a servo-like fashion, in the moment to moment regulation of movement. Muscle spindles are thought to control muscle length and limb position by comparing the actual length with an instructed or desired length and using the difference as an error signal to generate a corrective movement. This is supposedly achieved by means of activation of the intrafusal fibers and a feedback signal is conditional on the existence of an error or a difference between extra and intrafusal fiber lengths. Thus fusimotor together with extrafusal contraction would be required in order to maintain muscle spindle discharge in spite of muscle shortening [Burke 1980]. Such a system may help to simplify the task of the higher centers and supply load compensations and corrections for undesired or unexpected disturbances. Studies have shown, however, that the fusimotor system is ineffective in maintaining spindle discharge for fast movements, and does not result in significant sensitivity of spindles to disturbances of movement trajectories (see below in the section on the limitations of stretch reflexes). Since it appears that the stretch reflex does not play a major role in load compensation or corrections for unexpected disturbances it was speculated [Burke 1980] that the fusimotor activity plays an important role mainly in controlling slow or small movements. Moreover since the performance of unfamiliar tasks initially involves such movements, these are the very circumstances when feedback from muscle spindles must be essential [Burke 1980].

Other authors have suggested that stretch reflexes may help in the local management of muscles. It has been argued that proprioceptive reflexes may be more important in producing corrections for small instabilities arising from the neuromuscular system of the organism itself than for reactions to disturbances. Goodwin et al. [1978] have observed that muscle tremor with frequencies between 3 and 8 Hz has increased after deafferentation and therefore suggested that stretch reflexes may help to reduce such tremor.

Nichols and Houk [1976] have claimed that stretch reflexes contribute to the improved linearity of muscle properties by regulating muscle stiffness. When a load is added to a reflexic muscle there are three components of the static response to the muscle stretch. One component is due to the muscle's mechanical properties. The second response is mediated through the spindle pathway and causes an increase in muscle force and therefore an increase in muscle stiffness. The third is a force feedback response mediated by the tendon organ pathway and causes an inhibition of the motor output which tends to decrease force and therefore muscle stiffness. Houk and Rymer [1981] have argued that since length and force cannot be well regulated at the same time length and force feedback are combined to regulate the stiffness of the motor servo. When a muscle is stretched at a constant rate, force builds up but beyond a certain point muscles yield and the stiffness decreases abruptly [Nichols & Houk 1976]. Joyce Rack and Westbury [1969] have proposed that this yield is due to a detachment of cross bridges when the amplitude of stretch exceeds their elastic limit. The static characteristics of the motor servo resemble the length-tension curves of the areflexic muscle. Nevertheless, comparing the reflex responses with mechanical responses in decerebrated cats revealed that the initial region of high stiffness continues smoothly beyond the point where yield would have occurred [Nichols & Houk 1976]. Hoffer and Andressen [1981] confirmed these results in premammillary decerebrated cats [1981]. Also based on experiments with decerebrated cats it was argued that the overall dependency of reflex stiffness on initial force and length is modest compared to the muscle mechanical stiffness [Hoffer & Andressen 1981, Houk & Rymer 1981]. The third way in which reflexes can regulate stiffness is by decreasing the dependency of the stiffness of an electrically stimulated muscle on the direction of an imposed velocity change. Comparisons of the response of spindles to stretch versus release reveals asymmetry with a greater increase in motor output during stretch [Hasan & Houk 1975]. This asymmetry is opposite to the asymmetry of the mechanical component. Therefore, it appears that the asymmetry in the reflex action serves as a compensatory mechanism which contributes to stiffness regulation [Crago, Houk & Hasan 1976]. In summing up, it seems that the force developed in a reflexic muscle in response to stretch or release is more closely proportional to the amplitude of change in muscle length than the force of the mechanical component alone. All the above evidence indicates that the stretch reflex helps to maintain the spring like properties of the muscle.

2.3.2. Evidence against feedback control of movement.

The main difficulties with motor control theories which depend primarily on feedback arise from the limitations of stretch reflexes [Rack 1981]. Other evidence which contradicts such models is provided from studies on deafferented animals or from humans who, as a result of certain pathological processes, lack proprioceptive information.

In general, a servo control system operates effectively when it has a high loop gain so that a small error would give rise to a large correction. Nevertheless, measurements made on muscles of decerebrated cats [Matthews 1972], human fingers [Vallbo 1974] or neck muscles of conscious monkeys [Bizzi et al. 1978] show that the gains of the stretch reflex are rather small. Based on a biomechanical model of the head and neck, and on analysis of head movements in intact and deafferented monkeys, Bizzi et al. [1978] compared the ability for load compensation prior and after deafferentation and have evaluated the contribution of the stretch reflexes to be $\approx$30% of the load compensation. According to their computations intrinsic properties of muscles and limb contribute $\approx$60% of the load compensation. In addition, it seems that coactivated fusimotors can maintain spindle discharge under special conditions only. Thus, for example, Prochazka et al. [1979] have suggested that if the rate of muscle shortening in voluntary contraction exceeds 20% of its rest length per second the fusimotor is ineffective in maintaining this discharge. Furthermore the transport of the afferent information from the sensory organs to the contracting muscle involves considerable delays. These reflex delays have two portions. One portion is due to the time it takes to conduct the neural impulses from the peripheral sensory organ until it reaches the activated muscle. The other delay is due to the difference in times between the time the signals reach the muscle fibers till the resulting force is generated. If the neural delay is measured according to the time an EMG signal is first detected after a limb is perturbed (e.g. extension of the achilles tendon), this delay lasts 30-40 msec for the gastrocnemius muscle. This is approximately the time for a spinal stretch reflex. There are other later waves of EMG activity the latest after 120 msec [Nashner 1976]. In the biceps the spinal reflex delay is 18 msec with a later burst after 50 msec [Hammond 1960]. The delay between neural activation and peak tension of muscle twitch is about 20 msec. It was suggested that reflexes are used in a comparator system [Bernstein 1967] as a basis for corrections and modifications of an ongoing movement. Given these delays, feedback correction signals will be out of phase for the movement for which they are intended. In general, a feedback system with a high gain and long delays can lead to undesirable oscillations. A high stiffness (high gain) of the stretch reflex might have been advantageous in order for the system to be immune to external disturbances and changes in loads. Nevertheless given the long delays in the biological feedback systems this would have resulted in low resonant frequencies which might have been close to the working range of the system. Therefore, the low gain of the stretch reflex helps to reassure the stability of the system [Rack 1981].

Although some reservations on deafferentation procedures still exist, the evidence in support of the idea that animals can perform movement without sensory feedback is rather convincing [Bossom 1974, Taub & Berman 1968]. Taub and Berman [1968] have shown, for example, that monkeys can still succeed in climbing or performing fine movements such as picking raisins between thumb and finger. Hallet et al. [1975] have examined the EMG pattern of ballistic elbow movements generated by a human subject who has been deafferented due to peripheral neuropathy. Comparisons of the EMG pattern to that of normal subjects have shown that the pattern of muscular activity is similar. In the studies of Kelso and Holt [1980], functional deafferentation was obtained using a cuff pressure technique. In spite of this deafferentation subjects could still make finger movements to preset targets.

2.3.3. Central control of movements.

The evidence presented above indicates that movements (at least simple movements) can be made by humans in the absence of information from the periphery. Although this does not mean that ordinarily feedback plays no role, this suggests that the role played by reflexes and proprioceptive information in the control of movement must be more subtle than was once imagined. Therefore, movements must be centrally planned and controlled independently of afferent signals. It was suggested that a central motor control has two essential components: preprogramming and a monitoring of the efferent signal. The latter is often referred to as feedforward control [Houk & Rymer 1981, Miles & Evarts 1979].

Motor programs in the narrower sense mean that there exists a certain representation of a skill that can lead to a movement in the absence of feedback. In a more general sense, however, motor programs may include specifications of the ways to use feedback at different times [Keele 1981]. According to Welford [1974] central programming of movement is made at 3 stages: at the first stage input (e.g. visual) is analyzed and may be stored briefly. The second stage is concerned with relating input to response and the third stage is concerned with the execution of the selected response.

Feedforward control detects potential disturbances or their effects on regulated variables in contrast to feedback control which detects the effects themselves [Houk & Rymer 1981]. The controller extrapolates from out-of-date feedback information and knowledge on the behavior of the system at the time that the correction will reach the system. Feedforward control or corollary discharge has been indicated in various human and animal movements where an illusion is produced that a commanded movement has been carried out, even though the peripheral source of proprioceptive input may have been disabled in some way [McCloskey 1981]. Supraspinal signals command muscle movements and notify other centers that movement has been commanded although not yet executed. Internal feedback or corollary discharge provides an estimate of position and velocity of the limb before the movement occurred or sensory information got back. It computes the consequence of the command and directly issues corrective commands which depend on extrapolation. Therefore, in a feedforward strategy the model of the system being controlled and the representation of the external world must be quite accurate.

In the next section various models for the generation of simple movements will be presented.

2.4. Theoretical models for the control of simple movements.

2.4.1. What variables does the CNS control?

Much debate has accumulated around the question what muscle and movement variables the nervous system controls. Studies with behaving monkeys on correlations between discharge of precentral neurons and various movement parameters did not lead to a conclusive answer although the neural firing was best correlated to joint torque [Evarts 1968].

In recent years two contradictory models for the control of simple movements have emerged. One model (or theory) is the force-timing theory [Schmidt et al. 1978, Schmidt 1980] and the second model is final position control [Feldman 1966a, 1966b, Bizzi et al. 1976, Bizzi et al. 1978, Polit & Bizzi 1979]. The force-timing (impulse-timing) theory assumes that the nervous system uses a strategy in which it calculates distance from starting position to target, computes the accelerative and decclerative forces that must act on the limb and then programs the timing and sequence of muscles force for the entire duration of movement. This theory views muscles as pure force generators [Schmidt et al. 1978, Schmidt 1980]. The final position model proposes that the final location of the limb and not distance is being planned. Muscles are not viewed as pure force generators, instead this model suggests that the system takes advantage of muscle intrinsic spring-like properties in order to achieve this final position. Among researchers who hold this view, some authors have argued that muscle zero length is being controlled [Feldman 1966a, 1966b, Houk & Rymer 1981] while others have suggested that muscle stiffness is the controlled variable [Cooke 1980, Grillner 1972]. Other authors have argued that although the stiffness of a muscle may not be controlled the system controls the joint impedance [Hogan 1982b, 1983, Nichols 1982, Feldman 1980a, 1980b]. These various theories are based mainly on experimental evidence derived from the study of simple movements and have mainly addressed the control problems of one-degree-of freedom movements. Therefore, even though some of these theories have been extended to deal with the control of complex movements, we will first discuss them in the context of simple movements.

2.4.2. Force-timing theory

This theory is based on the idea of motor schema where a schema is a learned rule for selecting the parameters of motor programs [Schmidt, 1975]. According to the force-timing hypothesis motor schemas contain sequencing, phasing and magnitudes of muscle forces. Schmidt et al. [1978] have suggested that variability in the selection of force-time profiles should affect the trajectories and the end point achieved. Increasing the movement amplitude requires an increase in the force and therefore an increase in force variability and in the variability in movement endpoint. Likewise, faster movements increase the force variability but decrease the time variability since the time of action of the impulse is shorter. These considerations have led to a new relation between movement accuracy (defined as effective target width) and movement speed, in which the variation in movement end point (W_e expressed in standard deviations units) is linearly related to the movement amplitude (D) and inversely related to movement time (MT):

$$W_e = a + b\frac{D}{MT} \tag{4}$$

where a and b are empirically determined constants.

The impulse-timing theory of movement has been challenged by another view which assumes that only the final position of the limb is being controlled [Feldman 1966a 1966b, Asatryan & Feldman 1965] and timed forces are not represented in the program. According to the impulse-timing model an unexpected change in the initial position of the limb would cause the limb to miss the final target. A similar result is expected if there is a sudden loading of the limb and then a release. The results from studies on monkey head [Bizzi et al. 1976] and arm movements [Bizzi & Polit 1979] contradict this prediction.

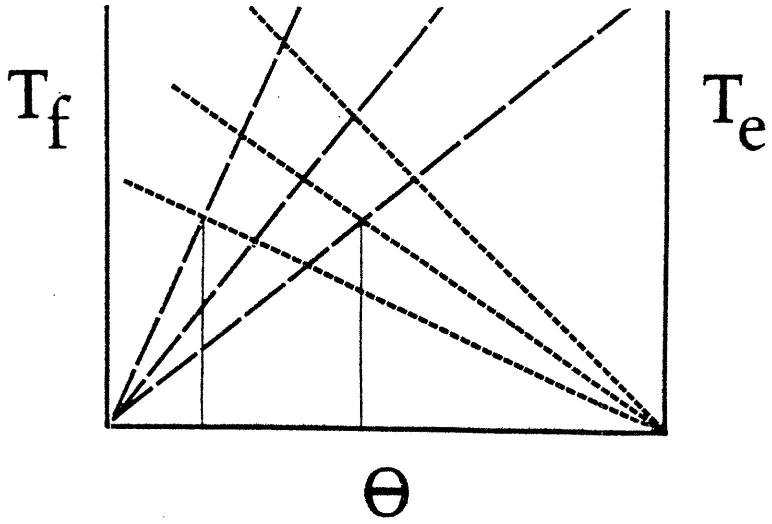


Figure 4. Schematic representation of flexor (T_f) and extensor (T_c) length-tension curves, θ represents the joint angle. From [Bizzi et al. 1982]

Schmidt [1980] has claimed, however, that multi-directional movements may require timed forces. In addition, there exists some evidence for the programming of timed forces in ballistic movements. This evidence will be discussed at the end of this chapter.

2.4.3. The final position model.

This idea maintains that posture can be viewed as an equilibrium point dependent on the firing rate of alpha motoneurons innervating agonists and antagonists, the length-tension properties of the muscles and the magnitude of external loads. Since for a given level of excitation the intersection of length-tension curves of the agonist and antagonist muscle groups specifies a unique angle at which the net torque on the limb is zero, a new equilibrium point can be achieved through the selection of new length-tension curves for one or both muscle groups. The concept can be schematically represented by Figure 2-4, in which idealized length-tension curves of agonist/antagonist muscles are represented as lines with different slopes.

As a result, movement can be achieved as a smooth transition from one equilibrium point to another without explicit planning of the entire trajectory. Faster movements can be produced according to this concept, by selecting an equilibrium point shift larger than necessary and following this shift by a return to the required static level [Ghez & Vicario 1978a, 1978b].

This idea got support from the experiments of Bizzi et al. [1976, 1978] on monkey head movements. Monkeys were trained to make horizontal eye-head movements directed at visual targets. In the intact animal the application of an unexpected load was followed by an increase in EMG activity in the neck muscles presumably as a result of spindle activity. In spite of this activity the head reached its intended final position after the load was removed. In another set of experiments the inertial load was unexpectedly increased, but although the head trajectories changed, the intended head position was reached even in the absence of sensory (visual or vestibular) clues [Bizzi et al. 1978]. Additional support of equilibrium point control is provided by experiments by Feldman [1966a, 1966b] in which human elbow rotations were studied and by experiments on elbow rotations in intact monkeys [Polit & Bizzi 1979]. All these experiments have shown that instructed final positions were achieved in spite of disturbances imposed during the movements, even when these disturbances were large enough to cause a barrage of neural impulses induced by stretch reflexes. Even in experiments in which the initial position was perturbed, the final postural state was achieved [Bizzi & Polit 1979]. In monkeys without proprioceptive information (eliminated by cutting dorsal roots c1 to t3), head movements towards visual targets achieved the same final position even when the inertia of the head was changed. Similar results were obtained in monkeys with a deafferented arm. In these experiments, however, there was an increased amount of co-contraction as a mean to achieve stiffening of the joint. Unimpaired achievement of final positions, in spite of (functional) deafferentation, was also demonstrated in experiments by Kelso and Holt [1980] on finger movements.

Recently, Bizzi et al. [1982] have presented evidence against the idea that the CNS controls simple movements by specifying only final position. Instead their findings indicate that the CNS actively controls the trajectory in addition to the final position. Monkeys performing simple pointing tasks were studied. As the monkeys began their arm movements toward the target the arm was quickly forced by the action of a brief torque pulse into the intended final position. It did not remain there as could have been expected from the final position theory, instead it returned toward its initial position before reversing direction again and continuing toward the final target. In another set of experiments on deafferented animals the arm was driven and maintained at the location of the next target before this target light was illuminated. After the appearance of this light the arm was released but instead of remaining at this target it moved toward the initial position from which it was initially displaced and then it returned to the final target. When the servo was maintained for a period corresponding to the normal duration of movement no significant movement of the arm was seen after its release. In both sets of experiments while the arm moved back and forth the agonist muscle developed an EMG pattern comparable to the one seen in undisturbed movements. Thus, in the presence of flexor activity, extension of the forearm was generated (or vice versa). This cannot be explained if muscles are regarded as pure force generators. The above findings suggest the existence of a gradually changing control signal to muscles during forearm pointing movements and not a step-like shift to the equilibrium point. These results have been interpreted as indicating that the neural input to muscles is specifying intermediate equilibrium positions which constitute a reference trajectory.

The effect of the recruitment of motor units amounts to a change in the slope of the length-tension curve and hence in muscle stiffness [Houk & Rymer 1981]. Increasing the discharge rate of motor units can be approximately described as decreasing the rest length of the muscle [Grillner, 1972]. Therefore, in order to achieve a new equilibrium point between the torques exerted by the two antagonist groups the nervous system can use either or both mechanisms in the selection of new length-tension curves for the muscles.

2.4.3.1. Rest length versus stiffness control.

Feldman [1966a, 1966b] has suggested that neural control signals act mainly by altering the muscle rest length, the length at which there is no force exerted by the muscle. The model of Feldman assumes that the neural control is equivalent to a shift in the length-tension curves along the length axis by merely shifting the zero length. Another possible model [Sakitt 1980, Cooke 1980] assumes that a shift in the length-tension curve is achieved by changing the slope of the curve which is equivalent to a change in muscle stiffness. Cooke [1980] has modeled simple elbow rotations by assuming a pulse like shift only in agonist stiffness. He has shown that the peak velocity during movement is linearly related to the amplitude from start to target and that peak velocity is reached about half way through the movement. Observed human forearm movements have matched these predictions of the model.

Against this view Houk and Rymer [1981] have argued that there is a very little change in stiffness during movement and therefore conclude that motor commands actually control the rest length of the muscle. They base their argument on the studies of Nichols and Houk [1976] and Hoffer and Andressen [1981] which show that the pattern of the motor servo is such that it tends to reduce variations in stiffness. Moreover based on studies on human subjects reacting to perturbation on an instruction basis they argue that higher centers change the set point of reflexes rather than the slope of the length tension curves. Thus, they maintain that although muscle stiffness does increase when the musculature is brought from the relaxed to the excited state, there appears to be a little additional capability for controlling stiffness after a movement has started.

2.4.3.2. Impedance control.

The final position model suggests that the CNS specifies the new equilibrium point through a shift in the length-tension curves for the agonist/antagonist muscle groups. Nevertheless, the intersection of many pairs of length-tension curves can give the same final position. One way in which the system can select among the vast number of possible length-tension curves is through the control or specification of a desired joint stiffness. The joint stiffness can be controlled independently of the net joint torque through a mechanism of co-contraction of antagonist muscle groups [Hogan 1982b, 1983]. This is because the torques generated by agonist and antagonist groups subtract whereas their stiffnesses add. Since the resistance to disturbances depends on the joint stiffness, and since the first part of resistance to movement is a property of muscle tissue (and not of reflex activity), the amount of resistance can be preset by altering the force of contraction in the participating muscles [Feldman 1980a, 1980b, Hogan 1982b, Hoffer & Andressen 1981]. Therefore, we can co-contract the agonist and antagonist groups to stiffen our joints when we carry a fluid-filled cup, or when we try to draw a very straight line on paper. Sakitt [1980] has shown that during posture of the forearm the integrated EMG ratio of biceps to triceps correlates well with elbow angle and that there is much less variability in this parameter compared to the variability in the electromyographic signals of these two muscles separately. Asatryan and Feldman [1965] also found that co-contraction increased the slopes of torque angle curves in the vicinity of the zero net torque. Hogan [1982b, 1983] has discussed the significance of the controlling joint impedance (stiffness and viscosity) for postural stability. Biomechanical modeling led to a prediction of simultaneous activation of antagonist muscles in the maintainance of upright posture of the forearm. Experimental evidence has shown that the increase in the load carried by the hand has led to greater co-contraction. Other mechanisms for impedance control such as differential activation of synergists [Smith et al. 1981] may also change the impedance of the limb.

2.4.4. An overview on the control of simple movements.

At this stage it is possible to first confront the two possible models of feedback control and preprogramming for the control of simple movements. Secondly, for each class of simple movements, we can ask which one of the above control theories can better account for the behavioral characteristics of this specific class. Given the limitations of the stretch reflexes they seem incapable of dealing with large disturbances. On the other hand, preprogramming which does not foresee certain disturbances, will not be capable of dealing with it either. The stretch reflex may, nevertheless, be tuned up in cases where disturbances are foreseen, as has been described in the Russian literature [Asatryan & Feldman 1965, Gurfinkel & Paltsev 1965] or, the gain of the stretch reflexes can be modified by descending paths from higher centers or by information from the peripheral sensory organs. Marked adaptive modifications in reflex responses to ankle rotations as well as changes in the coupling between muscles have been described by Nashner [1976].

Slow movements (slow tension movements, see the classification in section 2.2.1) seem to be made primarily under the action of feedback control. This suggests that feedback control is important in learning a new task. On the other hand, fast movements (loose ballistic in the above classification) are too fast to depend on proprioceptive information or visual feedback and therefore must be preprogrammed. Terzuolo and coworkers [1974] looked for evidence against the idea of complete preprogramming of ballistic movements. Deafferented animals continued to make ballistic movements but the biceps activity was not interrupted by pauses. Instead, the biceps activity ended abruptly or subsided gradually while the antagonist activity changed randomly. Ghez and Martin [1982] have shown recently that only the initial agonist burst is preprogrammed. The antagonist and second agonist bursts are mediated through reflex activity and their amplitudes depend on the initial velocity of movement.

Whether there is preprogramming of timed muscle forces or whether the system relies mainly on muscle and limb properties is another issue. For fast movements the burst in agonist and antagonist muscle is predetermined by the speed of movement selected before the movement starts. It seems that the burst of antagonist activity has a constant duration [Freund & Budingen 1978] whatever the amplitude or force of movement. Experiments have shown [Lestienne 1979] that when the inertial load is increased, what is increased after adaptation, is the global level of agonist and antagonist excitation and not the timing of their activity. This indicates that the speed controlling system operates by selecting the same timing for the cessation of agonist activity and the beginning of antagonist activity, independently of the expected change in mechanical conditions. Additional evidence that ballistic movements may involve timed forces comes from Freund and Budingen [1978] and Ghez and Vicario [1978a, 1978b]. Freund and Budingen have shown that for isotonic movements, movement time was the same for different distances while for isometric movements the time to peak force was the same regardless of its value. Ghez and Vicario found, similarly, that for limb movements in the cat the time to peak force was essentially constant.

Continuous movements (stiff ballistic, see section 2.2.1) also seem to be preprogrammed [Brooks 1979]. They involve activation of the agonist which propels the limb and is balanced by viscoelastic forces rising in the antagonist which brake the movement [Lestienne 1979]. Keele [1981] has suggested that humans invoke two different movement strategies for simple movements. Relatively slow and smooth movements (continuous: type 2, see in section 2.2.1) may involve either a change in stiffness or in the rest length of muscles according to the final position model. Ballistic movements may consist of three phases: A propulsive phase, a braking phase, and a fixation phase. The fixation phase may be planned in a way similar to the slower movements while the two initial phases or at least the first phase may involve the programming of timed forces.

2.5. Theoretical models for the control of complex movement.

2.5.1. Generalized motor programs

Much less work has been done on complex movements than on single-joint movements. In addition, very few analytical and testable models for the control of these movements have been suggested.

The stimulus-response chaining theory has suggested that each movement is stimulated by changes in the environment made by the preceding movement. Incorporation of the knowledge of results from visual feedback, however, is too slow to be effective since the processing of the information leading to a new movement requires on the order of 150-200 msec. This has accentuated the importance of kinesthetic information but kinesthetic reaction time also falls in the range of 110-160 msec, the fastest being 70 msec [Evarts & Vaughn, 1978]. In addition, studies have shown that animals can generate even complex movements in the absence of kinesthetic feedback.

Initially Keele [1968] has defined motor programs as prestructured set of muscle commands which determine the completion of the act, uninfluenced by peripheral feedback. Because the storage of all muscle commands for all possible movements would have required immense memory, recently the ideas of abstract generalized motor programs [Keele 1981] or action plans [Turvey 1977] have been suggested. An action is defined as a sequence of movements needed to perform a task such as lifting, opening the door etc. An action plan represents the parameters or rules for translating an intention into an action [Newell 1978]. Although an action is a more general term than a movement, the concept of generalized motor programs [Schmidt 1975, Keele 1981] which relates to the planning of a sequence of movements is almost analogous to the idea of action plans; therefore these two will be used interchangeably here. The idea of generalized motor programs suggests that an abstract representation of movements (or actions) exists at higher levels of the program and is broken into more specific components at descending levels of the hierarchy. Thus at lower levels, different specifiers are applied to the same abstract structure from above. These specifiers include movement effectors, activated muscle groups, or various parameters such as time, work-space regions, locations of objects, etc. [Keele 1981].

We have very little understanding of the rules that are involved in an action plan and the manner in which plans for action (or motor programs) develop. Gel'fand and Tsetlin [1971] and Bernstein [1967] have suggested that the development of such rules is achieved through a continuous process of searching for an optimal motor solution to a motor act. Therefore, any problem of a skilled action can be described, in part, as a problem of optimizing a function of several variables; skill acquisition thus amounts to finding the set of relevant constraints. These constraints set boundary conditions on the involved effectors or their segments and therefore on muscle activation patterns. Gel'fand and Tsetlin [1971] have suggested that humans can successfully apply a certain method of search through the space of constraints, although they are ignorant of the organization of the problem, and it is only in the course of this search that the optimal pattern is disclosed. It was suggested that the organization of the action may be described as a function that is preserved invariantly over changes in the specific values of the participating elements [Fowler & Turvey 1978]. Thus, for example, to minimize the displacement of the point of intersection of the line of aiming with the target, experienced marksmen constrain the joints of the weapon arm in such a way that the horizontal displacement of the individual kinematic links are reciprocally related [Aryutunyan, Gurfinkel & Mirskii 1968].

2.5.2. Proposed models for the control of complex movements.

Most, if not all theories of motor control of complex movements agree with the idea of hierarchical levels of motor presentations. Nevertheless, very few theories have suggested specific levels of motor representations or specified the problems which are solved at each level. In addition, these models cannot be as easily classified as the control models for simple movements, on the basis of controlled variables. Recently, however, two more specific classes of models for the control of muti-joint movements have been suggested. One such class consists of models which we will call computational models. Computational models, such as the ones proposed by Hollerbach [1982], Saltzman [1979] and Benati et al. [1980] assume that the generation of desired trajectories is achieved through processes which are analogous to computational solutions of dynamic and kinematic problems. The premise of these models is that at the final stage of movement planning, hand positions are first converted to joint angles, joint torques are then computed, and finally these torques are distributed among various muscles. Such models, therefore, view muscles basically as force generators although they do not exclude the possibility that the system takes into account the spring-like behavior of muscles. The second class of models is comprised mainly of a single alternative model which we will call the impedance control model [Hogan 1980, 1982b]. This model proposes that trajectories are generated by setting the end-effector impedance. The main premise of this model is that impedance is controlled through the use of factors intrinsic to the biomechanical structure of the arm, such as the spring-like properties of muscles and the configuration of the arm muscles, in order to set up a potential energy valley for the end effector.

2.5.3. Computational models.

Computational models for motor control of multi-joint complex movements have been proposed by several authors [Saltzman 1979, Hollerbach 1982, Benati et al. 1980]. These authors have based most of their suggestions on methods used to control artificial manipulators and have combined these with various ideas on biological motor control. Saltzman has discussed two types of representations of action plans: a working representation which relates to the current of the task-environment and the musculoskeletal system and a more abstract representation. He has suggested that action plans are represented at seven major levels: the conceptual, environmental-space motion, effector, body-space motion, joint motion, joint torque, and muscle level.

At the conceptual level, the action is defined in terms of symbolically coded actions such as reaching, lifting, etc. At the environmental-space motion level, the locations of objects are established with respect to an environmental fixed Cartesian coordinate system and their spatiotemporal motions are outlined. At the effector level, the acting member of the body is selected with its relevant sets of muscles, and at the body-space level the spatiotemporal movement of the end-effector (e.g. hand) and the object, with respect to a body fixed coordinate system (polar or Cartesian), is selected. At the joint level, this motion is transformed into a representation in terms of joint variables. Joint torques are then computed and finally muscle forces and muscle innervations are specified. Hollerbach [1982] has also discussed a hierarchical order for motor planning and has suggested three levels: an object level, a joint level, and an actuator level. At the object level, the task command is converted into a planned trajectory for the end-effector represented in Cartesian coordinates. At the joint level, this plan is translated into the coordinated movements of the joints and at the muscle level the specific activations of various muscles are specified. Nevertheless, Hollerbach proposed that at each level, four planning and control modules can be developed. This idea is based on the modular structure of planning and control which has emerged in robotics. The structure consists of the following modules: a trajectory planning module, a compliance module, a dynamics module and a feedback control module. This structure allows for more flexibility than the one proposed by Saltzman.

Only a few of the aspects of movement generation addressed by computational models will be discussed here.

Object level trajectories planning:

Most of the authors who proposed computational models for biological trajectory have argued that the CNS uses object level Cartesian planning. Morasso and Ivaldi [1982] for example, basing their argument on kinematic analysis of planar movements and of handwriting, have suggested that the system plans end-effector trajectories through computational splicing of basic curved elements. Saltzman[1979] and Raibert [1978], on the other hand, have suggested that the preferred Cartesian trajectories result from a compromise between optimizations of various penalty functions. In the control of industrial manipulators, the approach of Cartesian path control is widely used [Paul 1981]. Usually straight trajectories are chosen for the end-effectors. The reasons for this choice result from the wish to minimize distance and the inertial forces on objects. Moreover, straight trajectories are advantageous for obstacle avoidance planning [Hollerbach 1982].

Joint level planning:

Most computational models [Hollerbach 1982, Saltzman 1979] propose that the next stage after object level planning involves computations of joint angles from hand positions. This is called the "inverse kinematics" problem. The problem of transforming joint angles into hand positions is called the "direct kinematics" problem. Part of the inverse kinematics problem involves transformations of hand velocities into joint angular velocities. This can be achieved through an inversion of the Jacobian if the Jacobian is nonsingular and the system is not redundant. Singularities of the Jacobian can occur at the boundaries of the arm reaching space. A system is kinematically redundant if the number of joint degrees of freedom exceeds the number of specified end-effector kinematic variables.

In robotics, the numerical methods used for the inversion of the Jacobian are too time consuming for object level planning. Nevertheless, time-efficient analytical computations exist under certain cases. One such condition is when the hand is connected to a wrist with three degrees of freedom [Pieper 1968]. The human arm has a similar structure, and therefore, it may allow object level of trajectory planning [Hollerbach 1982].

If the system is redundant, one can specify additional variables besides those for the end-effector. Since the human and primate arm is a redundant system (it has a total of 7 degrees of freedom compared to 6 kinematic parameters specifying the location and orientation of the hand in a three dimensional space) the specification, for example, of elbow elevation above the plane, can allow to overcome the redundancy problem. Other ways to overcome the problem involve either freezing certain degrees of freedom, or imposing certain relations among various joint velocities. Therefore, the motor control system may use anatomical or functional joint fixations, or may subdivide the group of all the participating joints into smaller subgroups and impose certain velocity ratios among joints in the same particular subgroup [Green 1972]. It has been proposed that this mechanism is used at early motor developmental stages or when a skill is being learned and that these constraints are removed later [Fowler & Turvey 1978].

Joint torque level:

Most of the discussion at this level will concentrate on methods used for the control of robots. Traditional manipulators have been controlled with analog servos closed separately around each joint. This control mode, however, could not deal at high speeds with the varying inertias and the cross coupling between joints. In recent years there has been a considerable interest in efficient formulation of manipulator dynamics due to the desire to have real time feedforward control (the term "feedforward" as it is used here refers to precomputations of joint torques). Feedforward open-loop control was thought to be advantageous since more accurate solutions of dynamic problems reduce the need for feedback corrections. Feedback cannot be relied upon to correct for large errors in fast movement since the elevation of feedback loop gains causes instability.

Initially, the work on manipulator dynamics [Uicker 1965, Kahn 1969] was based on Lagrange formalism and on the use of 4x4 rotation translation matrices which transform a vector from a link-fixed coordinate system to a fixed(inertial) coordinate system. The Lagrangian formalism involves the computation of generalized forces from second order differential equations (see equation 2 chapter 1) derived through various differentiations of the system Lagrangian (a scalar). Initially such computations lasted too long [Luh et al. 1980] which led to the conclusion that real-time on-line dynamic computations are infeasible.

Two types of schemes have been proposed to render the Lagrangian dynamics formalism feasible: simplifications of the dynamics or tabularizing of the solutions. Since the computations needed for the calculation of Coriolis and centripetal forces comprise 46 % of the total number of computations versus 6 % for gravity forces and 48 % for inertial forces [Waters 1979], the most common method to simplifying the dynamics was to omit these second order terms from the computations. This was based on the notion that if the arm is moved slowly, ignoring these terms will introduce only small errors [Paul 1981]. At high speeds, however, the errors which are introduced by such an omission become large and cannot be corrected easily through feedback, because of the excessive requirements on the gains of the feedback loops.

Albus [1975], Raibert [1978], and Raibert and Horn [1978] introduced the idea of look-up tables as a way to deal with computational complexity. If the trajectory is divided into very short time intervals, joint angles and joint angular velocities can be considered constant and therefore during a specific time interval the various joint torque terms can also be considered constant. These precomputed stored dynamic terms corresponding to a certain limb state can be retrieved and the joint torques can be easily derived [Raibert 1978]. This approach nevertheless requires immense tables and introduces problems of memory storage and retrieval. Raibert next proposed a discrete state space in which each dimension of space (position and velocity of each joint) is divided into a large number of smaller intervals. Thus, retrieving is reduced to retrieving within one space interval. In another approach, motion coefficients are indexed according to configuration alone (joint angles) [Raibert and Horn, 1978]. This demands a little more computation but reduces storage problems. Nevertheless, tabular solutions are not as accurate and as general as analytic solutions since they require quantization of continuous variables and do not adapt readily to mechanical and load changes. In addition, the table cannot be easily generalized to unlearned movements.

In recent years, however, fast algorithms and reformulations of the dynamic equations which decrease the number of operations in these computations have been suggested. Some algorithms are based on a Lagrange formalism [Hollerbach 1980] and others on the Newton Euler formalism [Orin et al. 1979, Luh et al. 1980, Armstrong 1979]. In the Newton Euler formalism the laws governing the motions of rigid bodies are obtained from Newtonian mechanics. In this formalism the forces and torques acting on each segment are derived separately. The initial algorithms [Stepanenko and Vukobratovic 1976, Orin et al. 1979], based on the Newton-Euler approach, were too time consuming. Recently, however, very fast recursive algorithms based on Newton- Euler formalism have been developed [Luh et al. 1980, Armstrong 1979]. The Newton-Euler formalism and these recursive computations will be described in chapter 3. Hollerbach [1980] has applied a recursive method to Lagrangian formalism but his formalism still required five times the number of additions and multiplications than Euler-Newton approach. In addition, he replaced the use of 4x4 matrices by 3x3 rotation matrices. This has reduced the number of additions and multiplications to 2.5 times Newton-Euler formalism. Silver [1982] has shown that a Lagrange formalism which uses vector representation of angular velocities instead of matrix representation, results in the same formalism as the Newton-Euler formalism.

The computational models of motor control suggest that the CNS has designed means by which it can solve the inverse dynamics problem each time a movement is about to be executed. Since analytical computations of inverse dynamics are plausible in robotics, these models argue that similar mechanisms may be also plausible in biological motor control.

Muscle level:

The computational models suggest that the next level after the joint torque level involves distribution of joint torques among all the muscles which participate in the generation of a movement. Since each muscle contributes a certain torque depending on its tension and its moment arm about the joint, the sum of all these torques should be equal to the joint torque, and hypothetically one can calculate muscle forces. In practice, because of the redundancy in muscles, there is an indeterminacy in the system and an infinite number of solutions can satisfy these dynamic constraints. It was proposed that the CNS overcomes this indeterminacy by using one of two mechanisms or both [Saltzman 1979, Hardt 1978]. One suggestion is that the CNS groups functionally similar muscles into larger synergies and requires a certain force level from the functional synergy as a whole and not from individual muscles. A synergy was defined by Sherrington as an anatomically based reflex linkage of a group of muscles. Gel'fand et al. [1971] and Gurfinkel et al. [1971] have used this term to refer to a functional group whose structure can change with motor acts and with time. It was speculated that such a mechanism is used by the CNS in order to reduce the number of independently controlled parameters [Gel'fand et al. 1971]. The second proposed mechanism is based on the selection of muscle activation patterns based on an optimization of a certain objective function. This idea was often suggested in gait research [Hardt 1978 , Pedotti et al. 1978, Hatze 1980] and less often in arm movement studies [Ayoub et al. 1974 Crowningshild et al. 1981]. Various such criteria have been suggested, including the minimization of input chemical energy or maximization of endurance. Nevertheless, it was shown that the emerging pattern of muscle activation depends to a greater extent on the geometry of the musculoskeletal system [Hardt 1978, Petriarco et al. 1982] than on the choice of an optimization criteria.

Finally, according to this computational model once muscle forces have been specified, the neural activation of muscles to generate these forces must be determined.

2.5.4. The impedance control model.

As an extension of the impedance control idea for simple joint movement, a Cartesian impedance control model for multi-joint movements has been recently suggested [Hogan 1980, 1982b, 1983 (in preparation)].

Since biological feedback cannot provide an effective mechanism for postural stability and corrections of unexpected disturbances, it was suggested that this can be efficiently achieved through the control of the impedance of the end-effector. This mechanism suggests a unifying control mechanism allowing for both interactive behavior with the environment (e.g. the use of tools) and accurate Cartesian trajectory generation in free arm movements. Hogan [1983, in preparation] has further argued that to achieve such adaptable behavior, the planning of kinematic variables (positions, velocities etc.) and the derivation and control of torques is not sufficient. It is also necessary to control the mechanical impedance at the point of mechanical interactions with the environment. Through the control of both impedance and torques for a given desired trajectory, the controller specifies the virtual equilibrium trajectory at which the arm stabilizes after the disturbance has ceased. Here we will discuss only those aspects of the impedance control model which apply to preprogramming of free multi-joint movements.

When a spring is stretched, potential energy is stored in it giving rise to a force that tends to restore a mass connected to the spring to its rest position. For a system of springs attached to a mass in two dimensions, a combined force field results from the summation of the force fields of all the muscles. Therefore, this combined force field can be characterized by iso-potential lines connecting all the points in the plane in which the same level of force acts on the mass. The collection of all these iso-potential lines forms a valley whose bottom corresponds to the equilibrium position. The steepness of the valley walls is determined by the stiffness of the springs. As was described above, a muscle behaves like a spring with adjustable stiffness; therefore, through the control of the stiffnesses for the antagonist muscle groups the CNS can set a potential valley for the hand.

For a multi-link arm the potential valleys which arise at the various joints are not independent, due to the existence of two joint muscles [Hogan 1980]. For the hand we can associate a net potential valley from the combined potential valleys of all the joints leading to the arm. The stiffness of the hand in this case is a tensor quantity which specifies the change in the restoring force as a function of the hand displacement from its equilibrium point. The CNS can adjust the shape and direction of the potential valley and the location of the equilibrium point. Therefore, for example, the valley walls can be made steep in one direction and shallow in another so that the bottom of the potential valley is along any desired direction toward the equilibrium point. Therefore, if the CNS shifts the equilibrium point from the initial position of the hand to the target, the hand would follow the bottom of this valley. In addition to stiffness, the neural input to muscles also specifies the joint viscous properties, and hence, the viscosity at each joint. In a way similar to that of the stiffness tensor, this defines a viscosity tensor for the hand. Thus, the CNS can specify not only the resistance of the hand to displacement, but also its resistance to the rate of change of displacement. Adjustments of the hand viscosity tensor can permit the system to have more control over the velocity pattern of the hand. Additional freedom in shaping hand trajectories can be achieved through allowing the potential field to change in time by defining a time-history of equilibrium points as an extension of the reference trajectory idea. In summary, the model suggests that the CNS can directly achieve desired hand trajectories by tuning the impedance of the hand through the use of intrinsic viscoelastic properties of muscles. This model suggests, therefore, that the system does not need to solve the inverse dynamics problem or to distribute joint torques among muscles as was suggested by the computational model.

Chapter 3

An optimization model for hand trajectories.

3.1. Introduction.

The issues relating to kinematic aspects of trajectory formation, which are investigated in this work, were presented in chapter 1. In this chapter we present a model which offers a mathematical description of the experimentally observed motor behavior of a few different kinds of planar two-joint arm movements [Morasso 1981, Abend et al. 1982]. These are: unconstrained point-to-point arm movements and unconstrained curved and obstacle avoidance movements. This work derives from an optimization model proposed by Hogan [1982a] for the description of single-joint forearm movements. In extending this model to two-joint movement we attempt to show that a single principle can describe a broad spectrum of motor behavior. The experimentally observed characteristics of single and planar two-joint arm-movements, were described at more length in chapter 1. Here we just briefly list these findings:

1. Single-joint movements:
   - (i) The trajectories exhibit an invariant unimodal velocity curve.
   - (ii) This invariance shows up for movements of many different body parts.
2. Planar two-joint unconstrained point-to-point movements:
   - (i) These movements show work-space invariances in hand Cartesian path and tangential velocity.

(ii) Joint angles and angular velocity tracings show no work-space invariances.

(iii) The hand path is roughly straight.

(iv) The hand tangential velocity curve is identical to the velocity curve for single-joint movements.

3. Planar curved two-joint movements:

(i) The movements show multimodal hand tangential velocity curves.

(ii) The movements appear as if made of two or more segments of gently curved paths meeting at a more highly curved region.

(iii) Curvature peaks and velocity valleys appear simultaneously.

To suggest a model that can account for all the above experimental observations, optimization techniques have been used. These techniques are based on an optimization of an objective function. The objective function is expressed mathematically as a time integral of a performance index, which generally depends on the system inputs and outputs. The optimization of the objective function describes the goal or objective of the movement. A set of differential equations are formulated, which describe the response of the system to a certain set of inputs. The methods of calculus of variations and optimal control theory make it possible to find the inputs, which subject to the constraints imposed by the system differential equations and to the end-point boundary conditions, yield an optimal objective function. In some problems, some of the system inputs or state variables may also be subject to certain constraints. This defines a subspace of admissible inputs which include only those inputs which do not violate these additional constraints.

Variational principles have found a broad spectrum of applications in physics and in engineering, specifically in control theory [Bryson & Ho 1975]. A few authors have also emphasized the appeal of using optimization techniques for the description of complex biological systems [Rosen 1967]. Similarly, in motor control research, a few studies have recently appeared, in which these methods were used to model motor behavior. Such studies have included the modeling of head [Zangenmeister et al. 1981] and eye movements [Clark & Stark 1975], the modeling of a whole body movements (e.g. high jumping [Hatze 1980]), or as in gait research, the use of the optimization of various objective functions, in order to distribute computed joint torques among redundant lower extremity muscles [Hardt 1978].

A critical part of suggesting a model based on optimization, for the description of some behavior of a physiological system, is the selection of an appropriate objective function. This choice should be guided by the nature of the behavior to be modeled. Additional considerations should be given to the objectives of such modeling. A common objective in attempting to model motor behavior using optimization techniques, is to find the set of inputs which controls the system at a much higher hierarchical level that at which the desired observed behavior is described. Hence, for example, in attempting to model the kinematic features of arm movements, one can introduce all the complexities of the neuromuscular system, hoping to find the muscle activities or the neural signals responsible for the generation of such trajectories. For a complex system such as the motor system, this requires developing a very complicated set of mathematical equations based on physiological data which, at the current stage of our knowledge, we do not have. An alternative approach may require building a model based on an array of theoretical assumptions that, again, currently, cannot easily be justified.

Given the above considerations, our choice of an objective function was guided by two different factors. Firstly, we limited the objective of this modeling to obtaining a faithful description of arm trajectories only, and not to finding, for example, the joint torques or neural inputs which may be responsible for these kinematic features. Secondly, we were guided by previous observations on the kinematics of arm trajectories which we wanted to account for.

When using an objective function which depends on dynamic control variables, such as torques, it becomes difficult to find analytic forms of optimal solutions because the system equations are generally of a high order and nonlinear. Moreover, the dependence of the dynamic equations on joint angles causes a strong dependence of the resulting trajectories on the region of the work space in which the movements take place. Because of the invariability of the trajectories displayed by human behavior, with regard to space, it is unlikely that an optimization criterion which is based on dynamic variables will succeed in modeling the observed behavior. This has led us to propose an objective function which depends only on kinematic variables relating to movement trajectories, and to define as inputs to the system, the set of time-varying hand positions. Hence, the optimization procedure yields the one set of such inputs which brings the criterion function to a minimum.

Observations on single joint elbow rotations have indicated that these movements can be successfully modeled, assuming that the underlying objective is the generation of the smoothest movement which carries the hand from one equilibrium position to another [Hogan 1982a]. Simulations based on this model succeeded in matching both free and perturbed monkey arm movements. This was achieved by assuming that single joint movements are such that jerk (which is defined as the rate of change of acceleration) is being minimized. Minimization of jerk is implied by the objective of maximizing smoothness. The minimum-jerk trajectory for a single-joint movement is a fifth order polynomial in time.

For planar arm movements, as the arm trajectory is executed, the tip of the hand traces a curve in the plane. The hand can be treated as a point mass or a disembodied figure and one can define a position vector for this point mass with respect to a laboratory-fixed planar coordinate system, and by differentiating this position vector three times, one obtains the Cartesian jerk for the disembodied hand. This vector will be termed the "hand jerk" from now on. We have chosen a cost function (objective function) which is the time integral of the square of the hand jerk for the movement duration. The reasons for minimizing time integral of the square of the hand jerk and not of the the jerk magnitude will be explained in the following section. Minimizing this objective function, we tested if the trajectories which our model predicts, match unconstrained point-to-point and curved planar arm movements similar to the movements which our subjects tended to generate.

To model point-to-point movements, the minimum-jerk trajectories had to satisfy only initial and final hand position, velocity and acceleration boundary conditions with no other path or velocity constraints. In modeling curved movements and movements with obstacle avoidance, we imposed interior point equality constraints, assuming that the hand is required, in its movement between the end points, to pass through a third specified via point. The time at which the hand had to pass through this point was not specified. Rather, the optimization technique in finding the minimum-jerk trajectories forces the hand to move through this point in a particular time. The model was successful in yielding trajectories which had all the kinematic characteristics described above. The model led also to predictions of additional kinematic features, beyond those that were described in the beginning of this section. To test these predictions we designed experiments in which subjects were instructed to generate two-joint planar arm movements between two end-points through a third via point. These movements showed the same invariances as previously described [Morasso 1981, Abend et al. 1982]. They also validated our predictions. To check if only jerk minimization can offer a good model for movement description, we also tested the match between real movements and trajectories derived from the minimization of higher-order and lower-order derivatives of hand position, i.e snap and acceleration.

3.2. Mathematical Formulation.

3.2.1. Straight movements: minimum-jerk.

Forearm movements have been successfully modeled, with the assumption that they are generated with an underlying objective of bringing the limb from its initial to its final position, with the smoothest motion [Hogan 1982a]. It was assumed that maximizing smoothness implies minimization of jerk. Smoothness characterizes also humans' (and primates') hand motions in planar two-joint arm movements. This suggested that jerk minimization may serve as an appropriate model for the description of these movements as well. Nevertheless, in this case one has to make a choice between an objective function which depends on the hand jerk, expressed in Cartesian coordinates, and an objective function which depends on the shoulder and elbow angular jerks. Seeing that two-joint arm movements exhibit invariances in Cartesian hand paths and tangential velocities, we chose an objective function which depends on a third derivative of the position vector of the hand treated as a point mass. This vector specifies the location in space of the disembodied hand with respect to a fixed, Cartesian, coordinate system. In doing so, therefore, we disregarded the rest of the arm and the problem turned simply into an optimization of kinematic variables.

One can expect to find that the physical system which generates the movement (i.e. the neuro-musculoskeletal system) imposes certain constraints on some of the kinematic or dynamic variables. Hence, such constraints may set upper limits on the hand speed or acceleration, due, for example, to limitations on the maximum torques that the system can generate and on how rapidly they can be changed. Constraints on the duration of movements may be explicitly specified or result from the force limits. Other constraints on the movement paths, for example, may result from the requirements of a specific movement task (e.g. the hand must pass through a certain point in the plane). None of the movements studied in this work was extremely fast nor did any of the movements require the generation of very high torques or joint stiffnesses. Therefore, the subjects did not operate anywhere near the limits of the neuromuscular system. In addition, in experiments with point-to-point movements, no instructions were given to the subjects, and therefore the task did not impose position constraints on the movements.

For unconstrained point-to-point movements the objective of the optimization can be stated as follows:

> "Generate the smoothest hand motion between the initial and final hand positions in a given duration."

Jerk is defined as the rate of change of acceleration, (i.e the third time derivative of the position vector). Therefore, if the hand position coordinates in the horizontal plane are x and y, the hand jerk is:

$$\underset{\sim}{jerk} = \frac{d\underset{\approx}{a}}{dt} = \frac{d^3x}{dt^3}\hat{i} + \frac{d^3y}{dt^3}\hat{j} \tag{3.1}$$

We suggested that in moving from an initial to a final position in a given time t_f, the square of the jerk is minimized. Therefore, the cost function (objective function) to be minimized is:

$$C = \frac{1}{2}\int_0^{t_f} (\frac{d\underline{a}}{dt})^2\, dt = \frac{1}{2}\int_0^{t_f} ((\frac{d^3x}{dt^3})^2 + (\frac{d^3y}{dt^3})^2)\, dt \qquad (3.2)$$

In general, no minimum will exist for problems in which the performance index and the system constraints are linear in the state and control variables unless inequality constraints on the state and/or control variables are specified. If the inequality constraints are linear and are placed only on the control variables, a minimal solution if it exists, will always require the controls to be at one point or another of the boundary of the feasible control region ("bang-bang" control) [Bryson & Ho 1975]. In the case of the problem treated here, the system constraints are linear in the state variables (positions velocities and accelerations) and in the control variable (the jerk vector). Had the performance index been defined as the jerk magnitude (a linear function in the control variable) and had we placed linear inequality constraints on the jerk, the optimization would have resulted in a hand jerk which behaves like a "bang-bang" control. Since the analysis of measured movements shows that the hand jerk does not behave like a "bang-bang" control, the time integral of the jerk vector squared and not of the jerk magnitude was chosen to be minimized.

Time-varying hand positions are to be found, which bring the criterion function in equation 3.2 to a minimum. Since we are dealing here with movements which are nowhere near the boundaries of the system performance, the techniques of classical variation calculus can be applied to this problem as shown in Appendix B. Dynamic optimization problems, in which the system dynamics impose certain constraints, can be solved using various techniques [Bryson & Ho 1975]. Here, the method of Pontryagin is described in Appendix A and was applied to our problem to derive the necessary conditions for the existence of a minimum. Hence both methods were applied to the problem of unconstrained point-to-point movements as a means of checking the mathematical results. In addition, the Pontryagin's method for dynamic optimization was applied to curved movements.

The objective criterion determines the form of the movement trajectory. Nevertheless, to get hand trajectories with the right details of movement, certain boundary conditions must be satisfied, at onset and termination of movement.

These boundary conditions relate to hand positions, velocities and accelerations, at these fixed times. Problems of this kind are classified as optimization problems with some state variables specified at the onset and at a fixed terminal time.

Both methods gave a minimum jerk trajectory which is a fifth order polynomial in time for the x(t) and y(t), hand position coordinates:

$$\begin{aligned} x(t) &= a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5 \\ y(t) &= b_0 + b_1 t + b_2 t^2 + b_3 t^3 + b_4 t^4 + b_5 t^5 \end{aligned} \tag{3.3}$$

In order to specify the trajectories in their entirety, a sufficient set of boundary conditions are the position coordinates, and the velocity and acceleration components along the two axes, at the start of the movement ($t = 0$) and at the end ($t = t_f$). Since the movements started and finished at rest, the following boundary conditions were applied:

$$\begin{array}{ll} x(0) = x_0 & y(0) = y_0 \\ x(t_f) = x_f & y(t_f) = y_f \\ \dot{x}(0) = 0 & \dot{x}(t_f) = 0 \\ \dot{y}(0) = 0 & \dot{y}(t_f) = 0 \\ \ddot{x}(0) = 0 & \ddot{x}(t_f) = 0 \\ \ddot{y}(0) = 0 & \ddot{y}(t_f) = 0 \end{array} \tag{3.4}$$

These were substituted in the appropriate expressions, and the values of the coefficients $a_1 \ldots a_5$ and $b_1 \ldots b_5$ were calculated. This procedure yielded the following expressions for the hand trajectory:

$$\begin{aligned} x(t) &= x_0 + (x_0 - x_f)(15\tau^4 - 6\tau^5 - 10\tau^3) \\ y(t) &= y_0 + (y_0 - y_f)(15\tau^4 - 6\tau^5 - 10\tau^3) \end{aligned} \tag{3.5}$$

where $\tau = t/t_f$.

A typical predicted hand path and hand tangential velocity curve for a point-to-point unconstrained movement are seen in Figure 3-1. Hand tangential velocity was computed according to the expression:

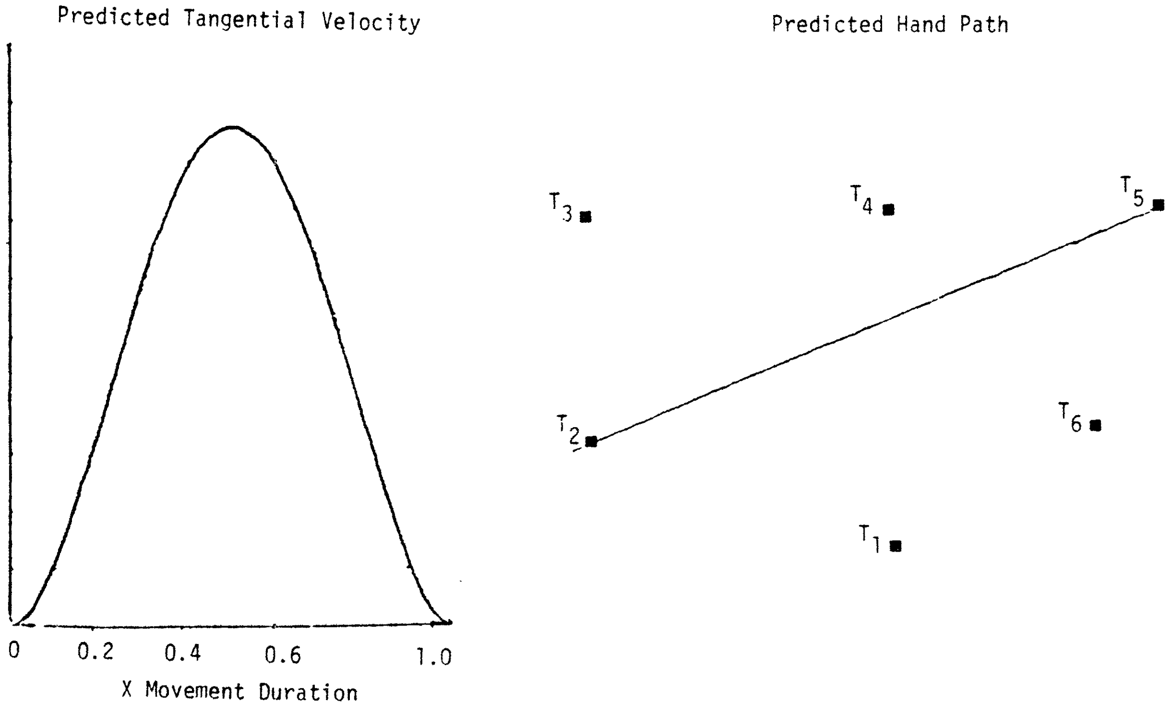


Figure 1. Predicted hand path and hand tangential velocity profile for point-to-point unconstrained movement.

$$T = \sqrt{(\dot{x})^2 + (\dot{y})^2} \tag{3.6}$$

where $\dot{x}$ and $\dot{y}$ are the velocity components of the hand along x and y coordinates respectively.

Since the mathematical expressions in (3.5) depend only on initial and final position coordinates of the hand, the same results, straight hand paths and a single-peak hand tangential velocity profile will be obtained for movements in different parts of the work-space independently of the arm shoulder and elbow angles and angular velocities. There is a good qualitative fit between this predicted trajectory and the observed behavior [Morasso 1981]. In addition, it is important to emphasize that the results are independent of the physical system generating the motion and did not require the modeling of the neuromuscular system. The expressions for $x(t)$ and $y(t)$ in (3.5) show that the shape of the predicted trajectory does not change with amplitude or duration of movement. Changes in movement amplitude or duration merely serve to change the scale of the position and time axes respectively. It is of interest to note also, that if one rotates the planar coordinate system so that one of the axes is directed along the hand path, and transforms the above expressions (3.5) to this new coordinate system, one would obtain a single polynomial in time, that except for units, is identical to the polynomial obtained for single-joint rotations.

3.2.2. Curved movements: minimum-jerk

Extending the model to curved movements or to movements with obstacle avoidance, the objective can be stated as follows:

> "Generate the smoothest motion which will carry the hand from the starting position to the target position at a given time. The hand is required to move to the final target through a specified point in space (a via point) at an unspecified time."

The requirement that the hand should move through a specified via point in the plane, defines equality constraints; the hand position is prescribed at some intermediate time between the onset and the termination of the movement. Hence, if the location of the via point with respect to a Cartesian coordinate system is given, by the coordinates $(x_1,\ y_1)$, the equality constraints on the hand position coordinates, x(t) and y(t), at time t_1, are:

$$\begin{aligned} x(t_1) &= x_1 \\ y(t_1) &= y_1 \end{aligned} \tag{3.7}$$

Problems of this kind are defined as optimal control problems with interior point equality constraints on the state variables [Bryson & Ho 1975]. The method of Pontryagin is applied in Appendix C to derive the solution of the problem defined here. This technique allows us to find the optimal trajectory for the entire movement between the initial and final position. Again the details of the movement depend on boundary conditions at time $t = 0$ and $t = t_f$ and on the interior point constraints. To model curved movements, the via points were assumed to be located at the maximum-curvature points of the measured trajectories. Hand curvature is defined as:

$$C = \frac{\dot{x}\ddot{y} - \dot{y}\ddot{x}}{((\dot{x})^2 + (\dot{y})^2)^{\frac{3}{2}}} \tag{3.8}$$

where $\dot{x}$ and $\dot{y}$ are the hand velocity components along the x and y axes, respectively, and $\ddot{x}$ and $\ddot{y}$ are the corresponding hand acceleration components.

Given the initial and the final boundary conditions, and the interior point position constraints, the optimization procedure yields the optimal trajectory, including the value of the time t_1, at which the optimal hand movement passes through the specified via point. The only additional requirement of this technique is the continuity of the state variables at the interior point, i.e. at time t_1 [Bryson & Ho 1975]. For our problem the state variables are the velocity and acceleration components. Therefore, at time t_1 these requirements are as follows:

$$\begin{aligned} \dot{x}^+(t_1) &= \dot{x}^-(t_1) \\ \dot{y}^+(t_1) &= \dot{y}^-(t_1) \\ \ddot{x}^+(t_1) &= \ddot{x}^-(t_1) \\ \ddot{y}^+(t_1) &= \ddot{y}^-(t_1) \end{aligned} \tag{3.9}$$

where $x^-(t)$, $y^-(t)$ are the hand position coordinates for $t \leq t_1$ and $x^+(t)$, $y^+(t)$ are the hand position coordinates for $t \geq t_1$. For arm movements these are also reasonable requirements, because if the velocities and accelerations are not continuous at some intermediate time, this will give infinite accelerations and jerks at this point.

Applying the optimization technique one obtains an expression for the position component x(t) at all times $t \leq t_1$:

$$\begin{aligned} x^{-}(\tau) = \frac{t_f^5}{720}\Big(\pi_1\Big(\tau_1^4(15\tau^4 - 30\tau^3) + \tau_1^3(80\tau^3 - 30\tau^4) - 60\tau^3\tau_1^2 + 30\tau^4\tau_1 - 6\tau^5\Big) \\ + c_1\Big(15\tau^4 - 10\tau^3 - 6\tau^5\Big)\Big) + x_0 \end{aligned} \tag{3.10}$$

and for times $t \geq t_1$ the expression is:

$$\begin{aligned} x^{+}(\tau) &= \frac{t_f^5}{720}\Big(\pi_1\Big(\tau_1^4(15\tau^4 - 30\tau^3 + 30\tau - 15) + \tau_1^3(-30\tau^4 + 80\tau^3 - 60\tau^2 + 10)\Big) \\ &\quad + c_1\Big(-6\tau^5 + 15\tau^4 - 10\tau^3 + 1\Big)\Big) + x_f \\ &= x^{-}(\tau) + \pi_1 \frac{{t_f}^5(\tau - \tau 1)^5}{120} \end{aligned} \tag{3.11}$$

These equations depend on the dimensionless variables $\tau_1 = t_1/t_f$ and $\tau = t/t_f$, and c_1 and π_1 are constants. The mathematical expressions for these constants are given in Appendix C.

The same expressions are obtained for y^{+} and for y^{-} with other sets of constants, π_2 and c_2 replacing π_1 and c_1 in equations 3.10 and 3.11. These constants, π_1 and c_1, π_2 and c_2 depend on the position coordinates at the boundaries and at the interior point and on the time t_1 which the technique allows us to find (see Appendix C).

3.2.2.1. Model predictions.

A typical predicted minimum-jerk curved movement is shown in Figure 3-2. This figure shows the hand path (Figure 3-2A) and the hand tangential velocity and curvature profiles (Figure 3-2B). Hand speed was computed according to the expression in (3.6) and hand curvature was computed according to (3.8). In addition, the two velocity profiles along the x axis, V_x, and the y axis, V_y, are also shown in Figure 3-2B.

As this example demonstrates, the predicted trajectories display the majority of observed features of real curved hand trajectories. If the real curved hand movements can be adequately described by the proposed model, one should expect

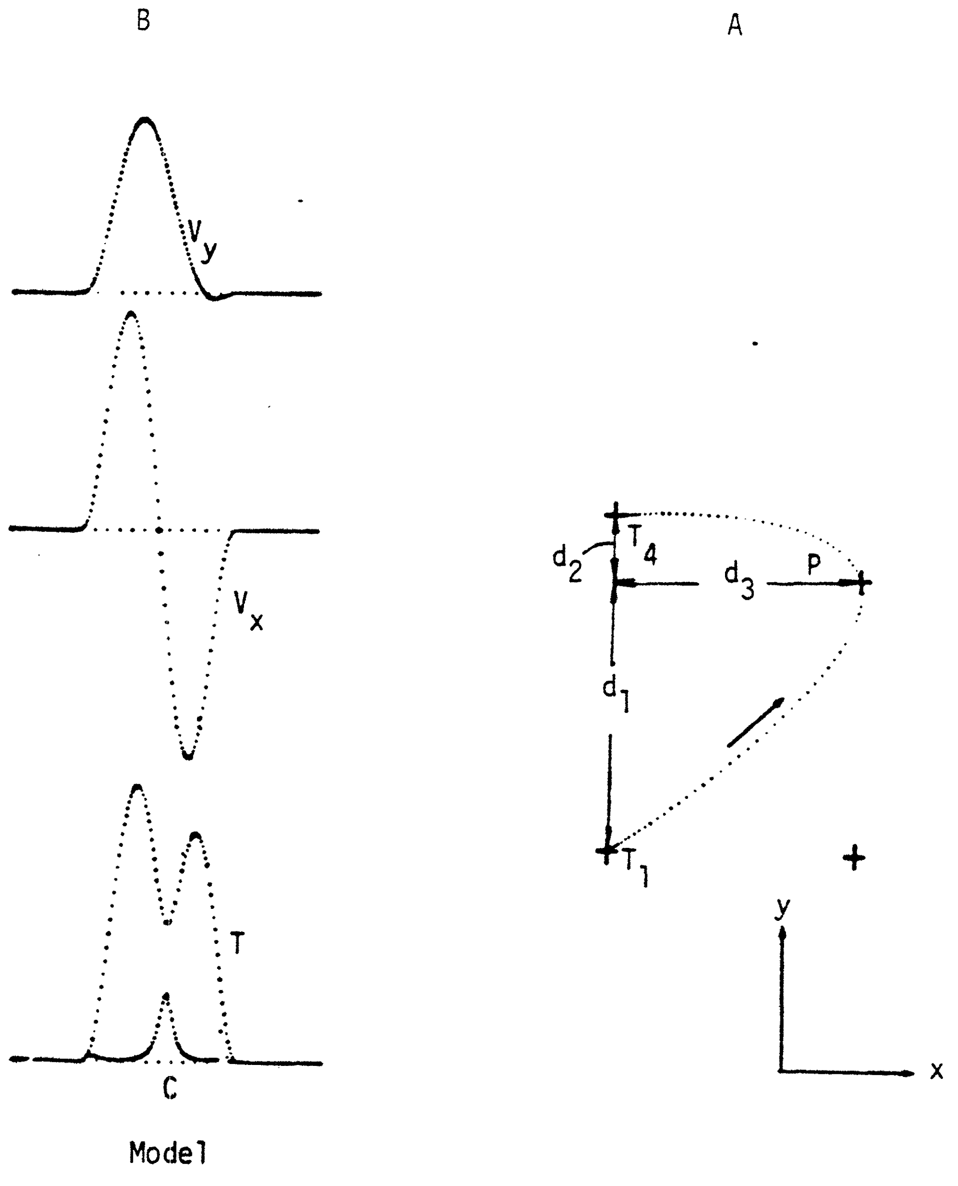


Figure 2. A representative example of a predicted curved minimum-jerk trajectory. (A) The hand path, P, is illustrated by plotting the predicted hand location every 10 msec. The meanings of d_1, d_2 and d_3 are explained in the text. (B) Hand tangential velocity T, hand curvature C, and hand velocity components along the x axis, V_x, and along the y axis, V_y.

these real trajectories to exhibit certain kinematic features, which were not included in the initial observation base for which this model has hoped to account, but are displayed by the predicted minimum-jerk trajectories. These predicted features are as follows:

1. Height of tangential velocity peaks: If the via point is displaced in any direction, towards either the initial, or final target, the velocity peak for the longer segment of the motion (in terms of distance) should be higher. To make this point clearer we can use a simple illustration. As shown in Figure 2, one can pass a line between the movement end-points. Another line, perpendicular to the first line, which passes through the via point, divides the first line into two segments, d_1 and d_2. We define d_3 as the segment which lies along the second line and connects the first line with the via point. If d_1 is longer than d_2, the velocity peak on the first portion of the movement, between the initial target and the via point will be higher and vice versa. Whereas if the d_1 and d_2 are of equal lengths, the height of the two velocity peaks will be equal.

2. Depth of velocity valley, and height of the curvature peak: If two movements, have the same values of d_1 and d_2, but d_3 is longer for one of these two movements, for this movement the hand curvature peak will be higher and the tangential velocity valley will be deeper than for the other movement.

3. Isochrony principle: The times it take the hand to move along the two portions of the movement, from the start to the via point, and from the via point to the end of the movement, should be equal, except for cases in which the via point is very close to either one of the movement end-points. Therefore, if one plots $t_1/(t_1 + t_2)$ values versus $d_1/(d_1 + d_2)$ values, for many different movements, one should expect to find that $t_1/(t_1 + t_2)$ is equal to 0.5 for most of the values of $d_1/(d_1 + d_2)$ between 0 and 1 except for $d_1/(d_1 + d_2)$ values quite close to 0 or 1 The model also predicts that for the same, or close values of $d_1/(d_1 + d_2)$, the $t_1/(t_1 + t_2)$ values will be the same, independently of d_3 values. This behavior of the minimum-jerk trajectories is the cause for the differences in heights of the two velocity peaks on both sides of the velocity valley because, if both parts of the movement have the same duration but

in one part the hand must travel a longer distance, then the velocity in this segment must be higher.

4. Time scaling: The shapes of the predicted curved trajectories, similarly to straight trajectories, scale with time as one changes the movement duration. Therefore, the path of the hand should look the same for fast and slow movements, while the velocity amplitudes should scale numerically by the ratio between the fast and slow movement durations.

3.2.3. Analysis for other optimization criteria.

To test whether other similar kinematic objective functions can also yield, as good, or even better models, minimizations of other criteria have been performed. These criteria are acceleration squared and snap squared (snap being the time derivative of jerk). These minimizations were used, again, to derive mathematical expressions, for both unconstrained point-to-point movements and motions through via points. The minimization techniques are similar to the ones described above for jerk minimization. Nevertheless, with acceleration minimization we can specify boundary conditions only with regard to position and velocity, requiring that the hand velocity will be zero at the onset and termination of movement. When snap is being minimized we need to add boundary conditions relating to the jerk value at the start and end of the movement. It is hard to establish what the real jerk values are at the end and at the beginning of the movement (see in the results section); we therefore chose the simplest boundary condition, zero jerk at both boundaries.

3.2.3.1. Unconstrained point-to-point movements.

The expressions for $x(t)$ and $y(t)$, for these movements, which result from the minimization of acceleration squared, are:

$$\begin{aligned} x(t) &= (x_f - x_0)(3\tau^2 - 2\tau^3) + x_0 \\ y(t) &= (y_f - y_0)(3\tau^2 - 2\tau^3) + y_0 \end{aligned} \tag{3.12}$$

Where $\tau = t/t_f$. The expressions for $x(t)$, $y(t)$ resulting from snap minimization are:

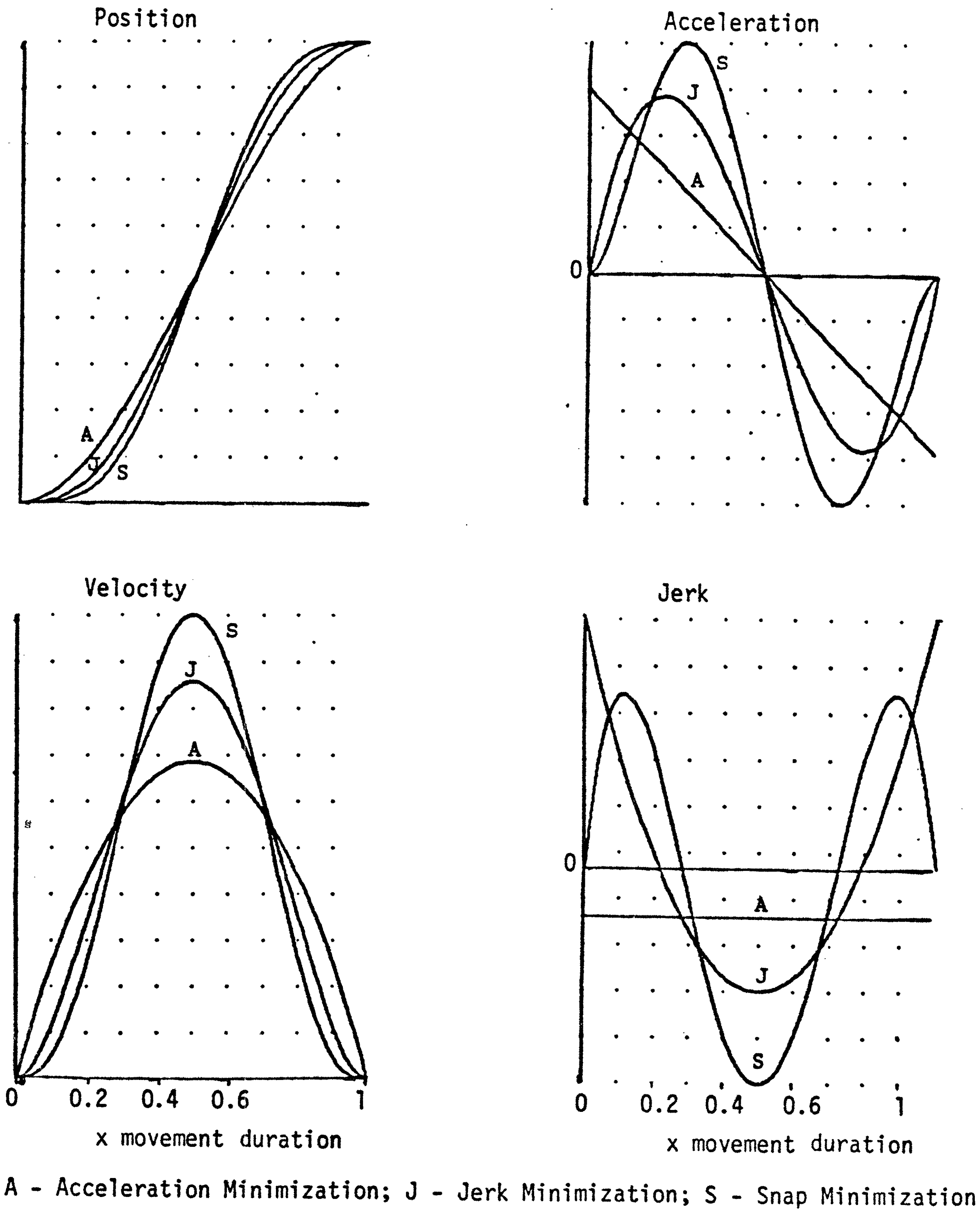


Figure 3. Position, velocity, acceleration and jerk graphed against relative time for trajectories resulting from the minimization of acceleration squared (A), jerk squared (J) and snap squared (S).

$$\begin{aligned} x(t) &= (x_f - x_0)(-20\tau^7 + 70\tau^6 - 84\tau^5 + 35\tau^4) + x_0 \\ y(t) &= (y_f - y_0)(-20\tau^7 + 70\tau^6 - 84\tau^5 + 35\tau^4) + y_0 \end{aligned} \tag{3.13}$$

Similar expressions are derived for $y(t)$.

Position, velocity, acceleration and jerk graphed against relative time for trajectories resulting from the minimization of the three criteria, are shown in Figure 3-3. The curves are denoted as follows: (A) for acceleration minimization, (J) for jerk minimization and (S) for snap minimization. The position curves for all three criteria and the velocity and acceleration curves resulting from snap and jerk minimization are quite similar. Notice, also the symmetry with relation to time of the velocity and acceleration curves for all criteria. Nevertheless, the velocity curve for acceleration minimization is parabolic, unlike the other two. Note, that with acceleration minimization the acceleration does not vanish at the boundaries. The differences between minimum jerk and minimum snap results, are in the acceleration and velocity amplitudes, and in the times at which acceleration and deceleration reach their maxima. In addition, for jerk minimization there is a non-zero jerk at the boundaries, while for snap minimization we selected zero jerk values at the movement end-points.

3.2.3.2. Other criteria: curved movements.

We next applied methods similar to those used in the the jerk minimization procedure, to solve the problem of acceleration minimization for movements which are constrained to pass through a via point. The following expressions for the trajectories are obtained:

For $t \leq t_1$:

$$x^{-}(\tau) = \frac{t_f^3}{12}\Big(\pi_1(2\tau_1^3 - 6\tau_1\tau^2 + 3{\tau_1}^2\tau^2) + c_1(2\tau^3 - 3\tau^2)\Big) \tag{3.14}$$

and for $t \geq t_1$:

$$\begin{aligned} x^{+}(\tau) &= \frac{t_f^3}{12}\Big(3\pi_1{\tau_1}^2(\tau-1)^2 + c_1(2\tau^3 - 3\tau^2 + 1)\Big) \\ &= x^{-}(\tau) + \frac{\pi_1 {t_f}^3}{6}(\tau-\tau_1)^3 \end{aligned} \tag{3.15}$$

where c_1 π_1 are constants which again depend on the boundary conditions,the interior point constraints and the time t_1. The expressions for these constants appear in Appendix C. Similar expressions are obtained for y^{-} and y^{+}.

Similarly, curved movement trajectories have been derived assuming that snap is being minimized. These long expressions will not be given here, but we again have a solution with:

$$x^{+}(\tau) = x^{-}(\tau) + \frac{\pi_1 {t_f}^7(\tau-\tau_1)^7}{5040} \tag{3.16}$$

and a similar relation between $y^{+}(\tau)$ and $y^{-}(\tau)$ where π_1 and π_2 depend again on the positions at the beginning of the movement, at the via point and at the end of the movement.

3.3. Methods

Human arm movements were recorded and hand trajectories were compared to mathematically predicted hand trajectories. The experimental data was gathered in part by us and from experiments conducted by Abend et al. [1982] at this laboratory. The latter included curved and obstacle avoidance movements. In addition, data was recorded from subjects who were instructed to move from an initial to a final target, via an intermediate target set by the experimenter. Overall nine subjects participated in these experiments.

The apparatus used in these experiments was shown in Figure 1-1 (chapter 1). The subject was seated in front of the apparatus holding the handle of the two joint mechanical manipulandum. In most experiments movements were constrained only to elbow and shoulder rotations, but in some experiments wrist movements were also allowed. Most movements took place at a horizontal plane at the subject's shoulder level but in a couple of experiments subjects were instructed to move their arm at a plane passing through their waist. Targets consisted of red light-emitting diodes and the experimental paradigm required the subject to move the handle of the manipulandum under targets upon their illumination. The targets were mounted on a horizontal plexiglass panel just above the apparatus. Visual information on the arm location was eliminated from the subject by darkening the room in which recordings took place.

The joint angles of the mechanical apparatus were monitored by means of precision potentiometers. The potentiometers' voltage signals were digitized at a rate of 100 samples per second and stored on magnetic tapes. These signals were filtered with an upper cut-off frequency of 5.2 Hz. A calibration procedure, before each experiment, enabled the conversion of these voltage signals to apparatus joint angles. Offline data analysis, using trigonometric relationships, allowed the computing of the subject's hand positions and joint angles every 10 msec. The Lagrange polynomial differentiation method was used to obtain subject joint angular velocities and hand velocities along the two orthogonal axes. Hand tangential velocity which is equivalent to the hand speed was computed according to the expression in equation (3.6). Hand curvature was computed according to equation (3.8). The handle of the apparatus was mechanically driven at a series of known speeds. Handle movements as small as 1.0 mm could be detected and the error in the computed hand speed was less than 4%. The error in hand curvature was less than 6%.

We begin by briefly describing the experiments of Abend et al. [1982]. In the first paradigm the subjects were instructed to move their hand from an initial target to another target upon its illumination. No instructions were given regarding the type of path between the two targets. In the experiments of Abend et al. no instructions were given, with regard to the movement speed. In similar experiments, which are described in the next chapter, subjects were asked to move their arms at various speeds specified by movement durations ranging from about 0.5 to 1.0 secs. These movements were also used here. In the second paradigm subjects were instructed to move their hand to a target by means of a curved path in the absence of a guide or an obstacle. The instructions did not constrain the speed of movement but the 20-40 cm distance was usually covered in 800-1500 msec. The curved trajectories ranged from very shallow to highly curved ones as seen in Figure 1-3 in chapter 1. In another group of experiments, subjects were instructed to mimic curved guides which consisted of constant curvature arcs. The fourth paradigm required obstacle avoidance. Subjects were instructed to move their hand from an initial to a final target, while avoiding an illuminated bar of light. The maximum length of obstacles, used in these experiments, was 25 cm. The obstacle was placed between the targets with its long axis perpendicular to the line connecting the two targets.

In order to compare experimental results to model predictions, values of a few kinematic variables had to be derived from the experimental data for both straight and curved trajectories. For straight trajectories, these values consisted of the start and goal target locations, initial and final velocities along the two orthogonal axes, initial and final accelerations, and movement durations. The coordinates of the initial position of the hand were obtained by placing a cursor on the tangential velocity curve and visually locating the start of the movement. The position components at this time were computed using the data analysis program. The final position coordinates were obtained similarly, and the movement durations were derived by subtracting the initial from the final time. Occasionally, however, a small correctional movement towards the goal target was detected. In these instances the correction movement was disregarded and we used the end of the trajectory just before this small correction, as the final position. For curved and obstacle avoidance movements, the position components along the two axes at the via point had to be specified. This location was determined by finding the maximal curvature point. The actual time at which the hand passed through this via point, was not used in our model, as mentioned above, but it served us as an additional measure in evaluating the success of the model in matching the real behavior.

To model curved movements the virtual via point locations were derived from the real trajectories, but in these experiments no actual constraints were imposed on the subject's hand positions at intermediate points between the initial and final targets. We were interested, therefore, in finding out what effect introducing real position constraints, would have on the movements. This was in order to determine whether under such conditions, the same kinematic features as for unconstrained curved movements could be observed. In addition, we were interested in checking the sensitivity of the resulting trajectories to shifts in via point locations, in relation to the initial and final targets. For these purposes, experiments were conducted in which subjects were instructed to move from an initial to a final target through a real third target. No instructions were given with regard to the path that the hand should take, but the subjects were instructed to generate a single continuous movement from the initial to the final target on the route of the intermediate target. This intermediate target was lit throughout the movement.

Since we were interested in testing the model for both fast and slow movements, the subjects were instructed to move at a normal rate first, and then at higher speeds. In this set of experiments with a third target 10 different movements were studied. The movements consisted of three different groups: two groups of four movements each, and a third group of three movements. In each of the above two groups, the initial and final targets were the same (one pair for each group) but the via point was differently located in each of the four movements. The third group consisted of three totally different movements, each with different locations for all three targets.

In all these experiments, the locations of the via points were assumed to be the locations of the intermediate targets. All the other parameters were derived in the same way as described above. The numerical values for target locations and movement durations were then substituted in the mathematical expressions for hand point to point movements (see Appendix C). For movements through via points these values were used in the derivation of mathematical expressions describing the x and y components of the trajectories (see Appendix C).

The numerical values for hand positions along both coordinates, using the mathematical expressions resulting from the minimization of the three criteria, were computed for every 10 msec analogously to the real movements. These values were then differentiated, using the same method as for the real movements, to get hand velocity components along the two axes, and differentiated again to derive hand accelerations. Tangential velocity and curvature were also computed. The resulting simulated trajectories were then compared to experimental results for all the paradigms discussed above.

3.4. Results.

In this section we will first compare minimum-jerk predicted trajectories to real trajectories for unconstrained point-to-point, unconstrained curved and obstacle avoidance movements. This comparison will allow us to test whether our proposed model can account for the majority of the previously observed ([Morasso 1981, Abend et al. [1982]) kinematic features of these trajectories. Next we will examine if the movements recorded by us (through real third intermediate targets) have kinematic features similar to those observed for unconstrained curved trajectories and whether these movements fulfill the various predictions of the model. We will also discuss a few of the kinematic characteristics for which our model has failed to account. Finally the predicted trajectories from acceleration and snap minimization will be compared to the real and to minimum-jerk trajectories.

Typical results for three unconstrained point-to-point movements are shown in Figure 3-4 and Figure 3-5 together with the jerk minimization simulated trajectories for these movements. In these figures we compare predicted and experimental hand paths, hand tangential velocity curves and acceleration curves.

As can be seen, the simulated results match real movement kinematic variables quite well. The simulated trajectories agree with real movements with regard to the steepness of the rising and falling parts of the tangential velocity and acceleration curves, and with regard to the time at which maximum acceleration is reached. Note that the experimental results display asymmetry in the tangential velocity curve since the peak velocity is slightly displaced to the left, while the optimal solution predicts symmetry. Nevertheless, this deviation is quite small. The discrepancies between the measured and predicted acceleration profiles are mainly due to the deviation of the real hand paths from strictly straight lines between the initial and final targets (compare the movements between targets 1 and 4 with the movements between targets 3 and 6 and between targets 2 and 5.)

We tested also the fit between the temporal behavior of real and simulated

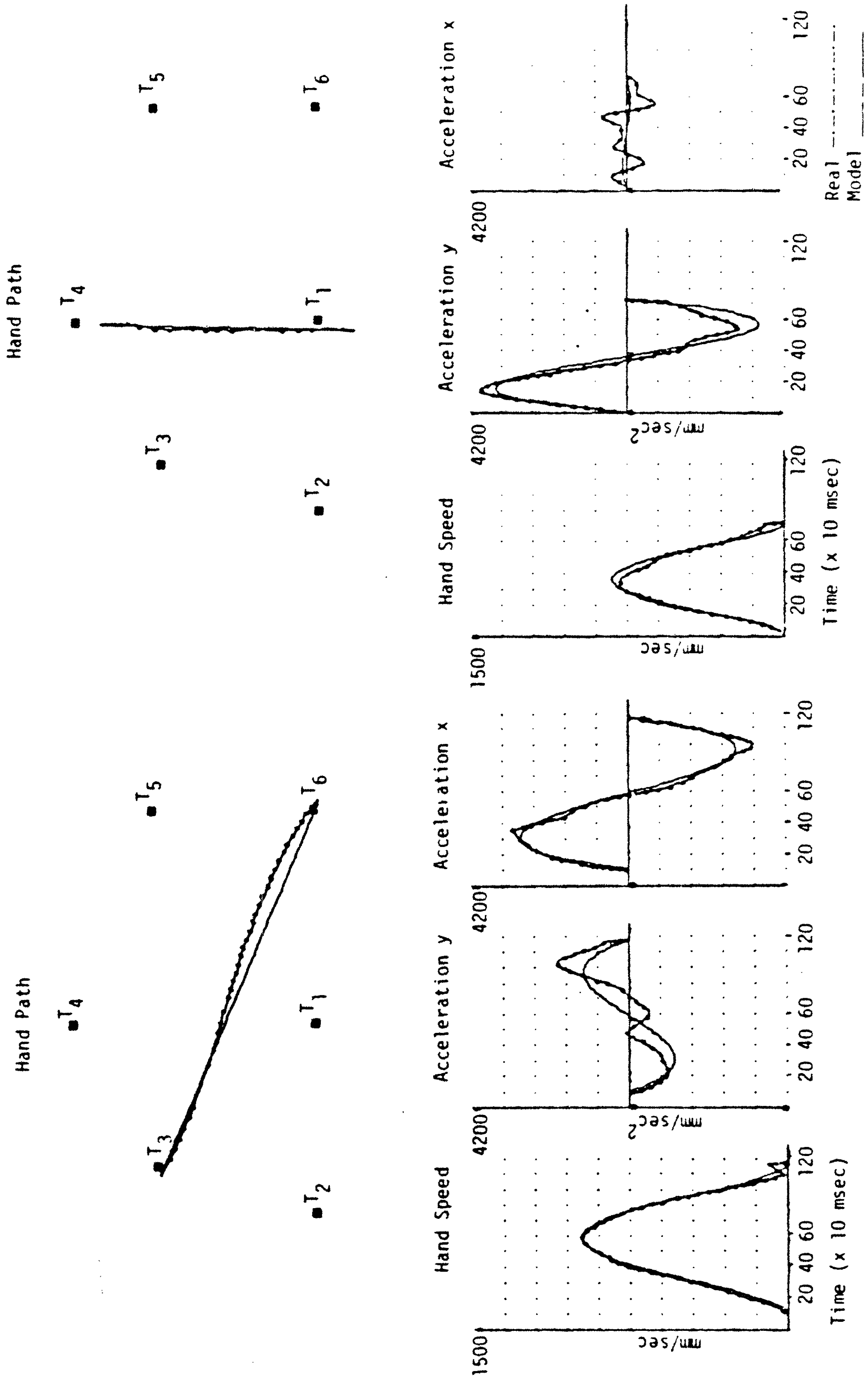


Figure 4. Overlapped predicted (solid lines) and measured (dotted lines) hand paths, speeds and acceleration components for two unconstrained point-to-point movements.

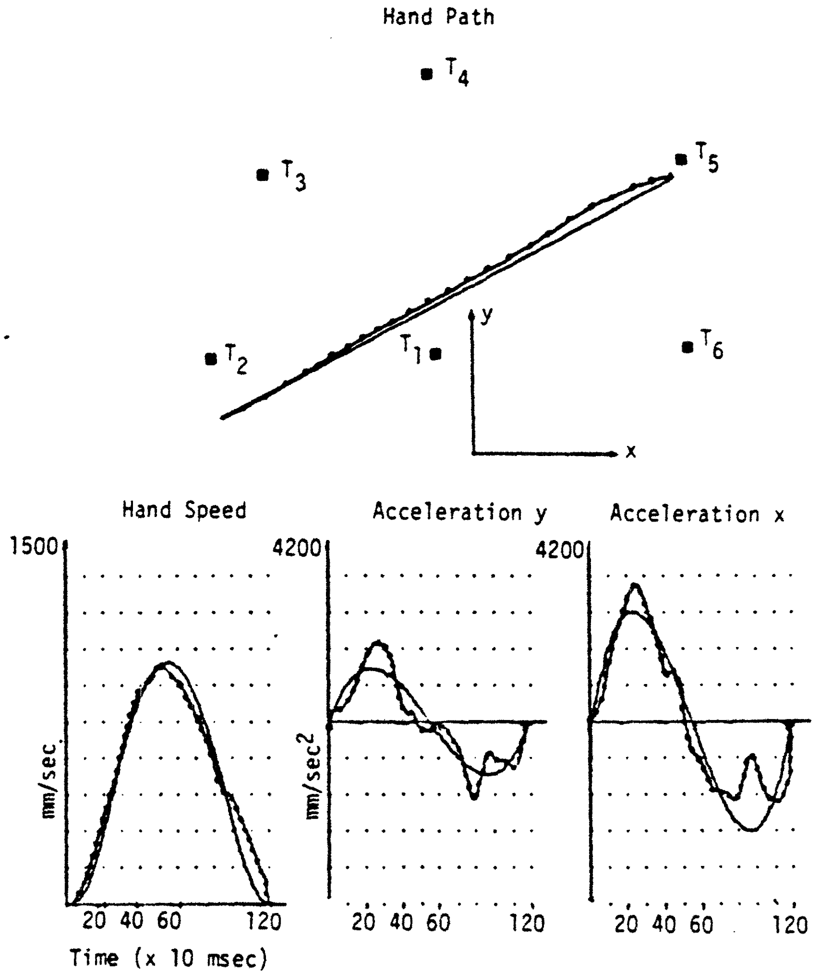


Figure 5. Overlapped measured (dotted lines) and predicted (solid lines) hand paths and hand speed and acceleration profiles for a movement between targets 2 and 5.

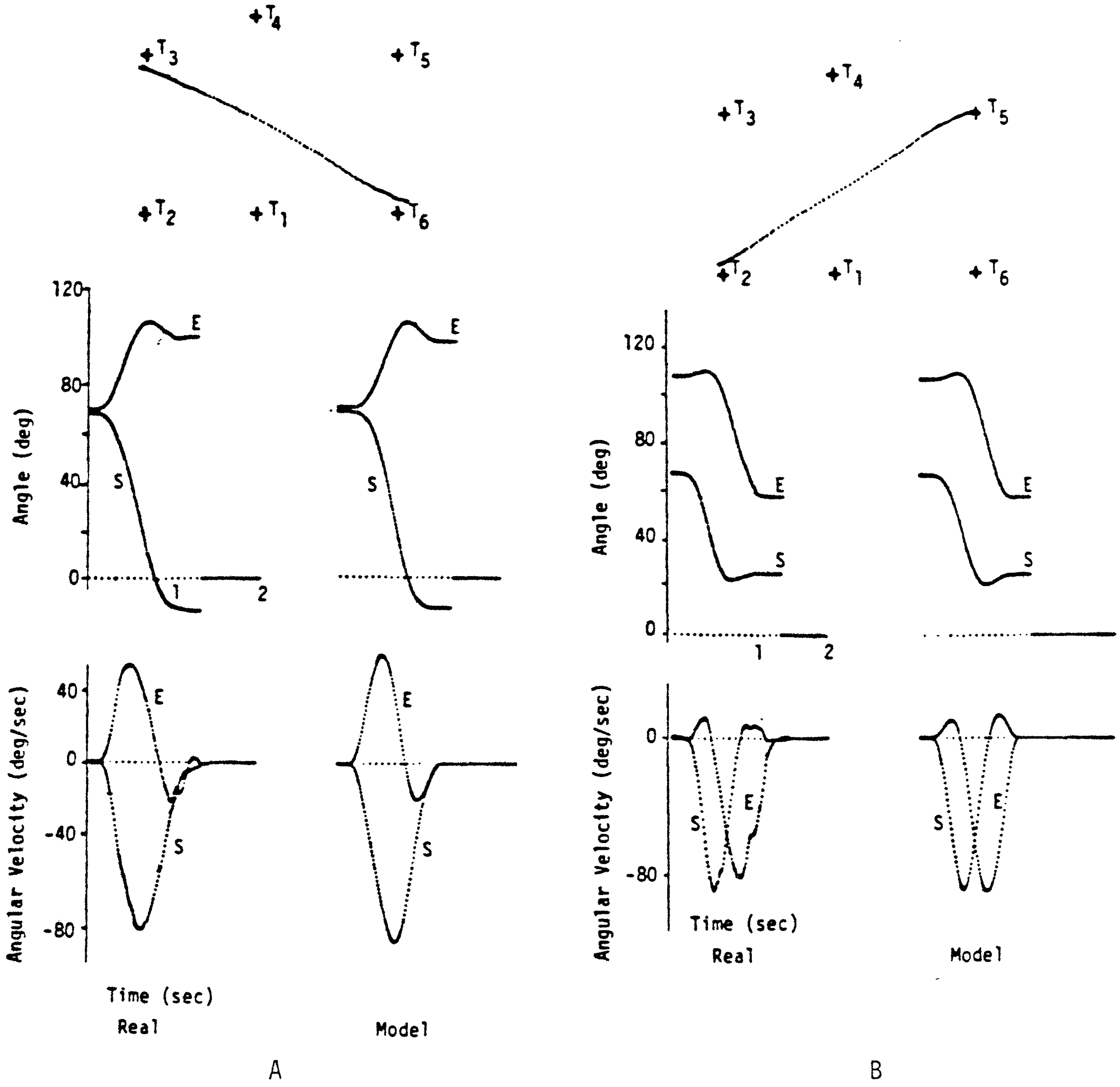


Figure 6. Measured and predicted shoulder (S) and elbow (E) joint angle and angular velocity profiles. (A) A movement between targets 3 and 6. (B) A movement between targets 2 and 5.

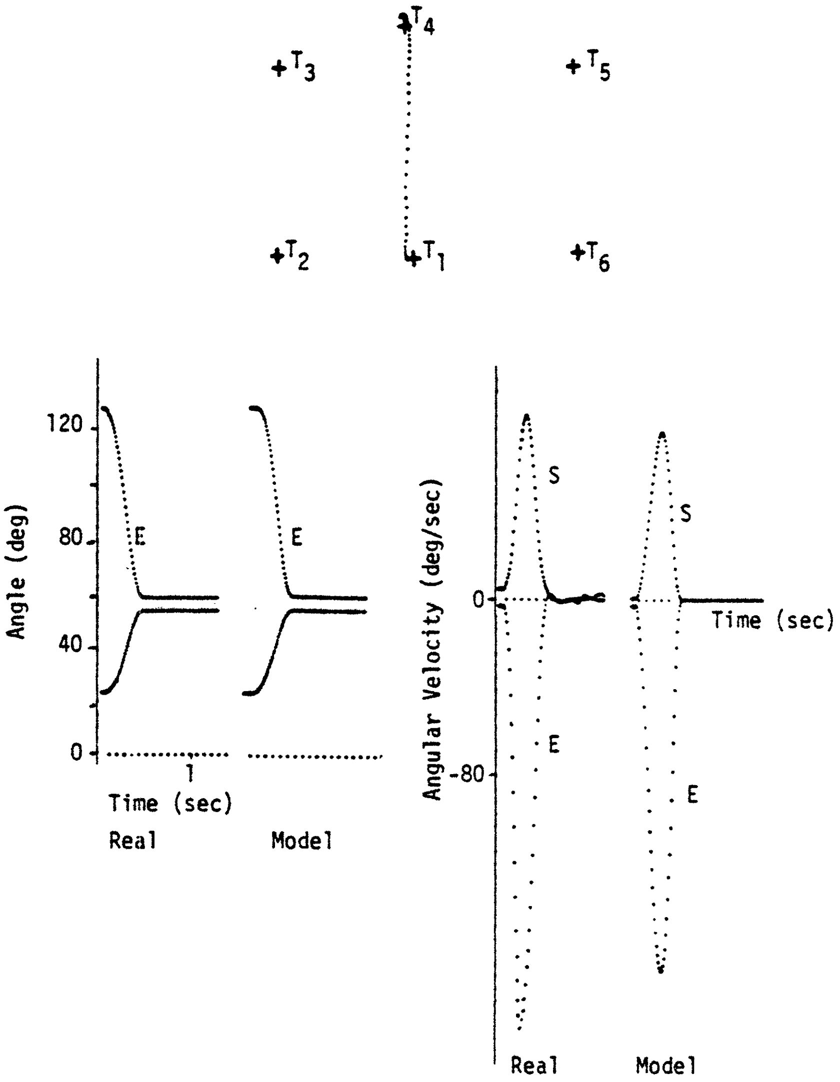


Figure 7. Comparisons between measured and predicted shoulder (S) and elbow (E) angles and angular velocity profiles for a movement between targets 1 and 4.

joint angles and angular velocities. To obtain joint angles and angular velocities, the inverse kinematics problem was solved. Comparisons between three real and simulated shoulder and elbow angles and angular velocities are shown in Figure 3-6. and Figure 3-7. Since for planar two-joint movements, hand trajectories uniquely define angular parameters, this comparison serves only to ascertain the success of the model. As can be seen in Figure 3-6 and Figure 3-7, there are only minute discrepancies between simulated and real results. Moreover, the amplitudes of real and simulated angular velocities agree surprisingly well.

Typical results from jerk minimization for four obstacle avoidance movements are shown in Figures 3-8 to 3-11. Representative results for three different unconstrained curved movements are shown in Figures 3-12, 3-13 and 3-14B. Results for a guide-mimicking movement are shown in Figure 3-14A. In all these figures, the predicted minimum-jerk hand paths, curvature profiles, tangential velocity profiles and the velocity components along the x and y axes, are displayed in the right column (Real). The corresponding experimental recorded hand paths, velocities and curvature profiles are displayed in all these figures in the left column (Model).

The simulated trajectories capture the following phenomena discussed by Abend et al. [1982]:

1. The hand curvature is not constant. Instead, it increases to reach a maximum, and then decreases again as the movement approaches the final position. The tangential velocity curves exhibit, in most cases, two peaks and the minima in the tangential velocity curves occur at the same time as maximum curvature points. In most of our results there is a very good fit between the time of the maximum curvature point in the simulated trajectories and the corresponding time for the real trajectories.

2. In the simulated trajectories the path from the initial target towards the via point is roughly straight. As the hand approaches the via point the path curves, and in the second segment, it is again roughly straight until it reaches the final position.

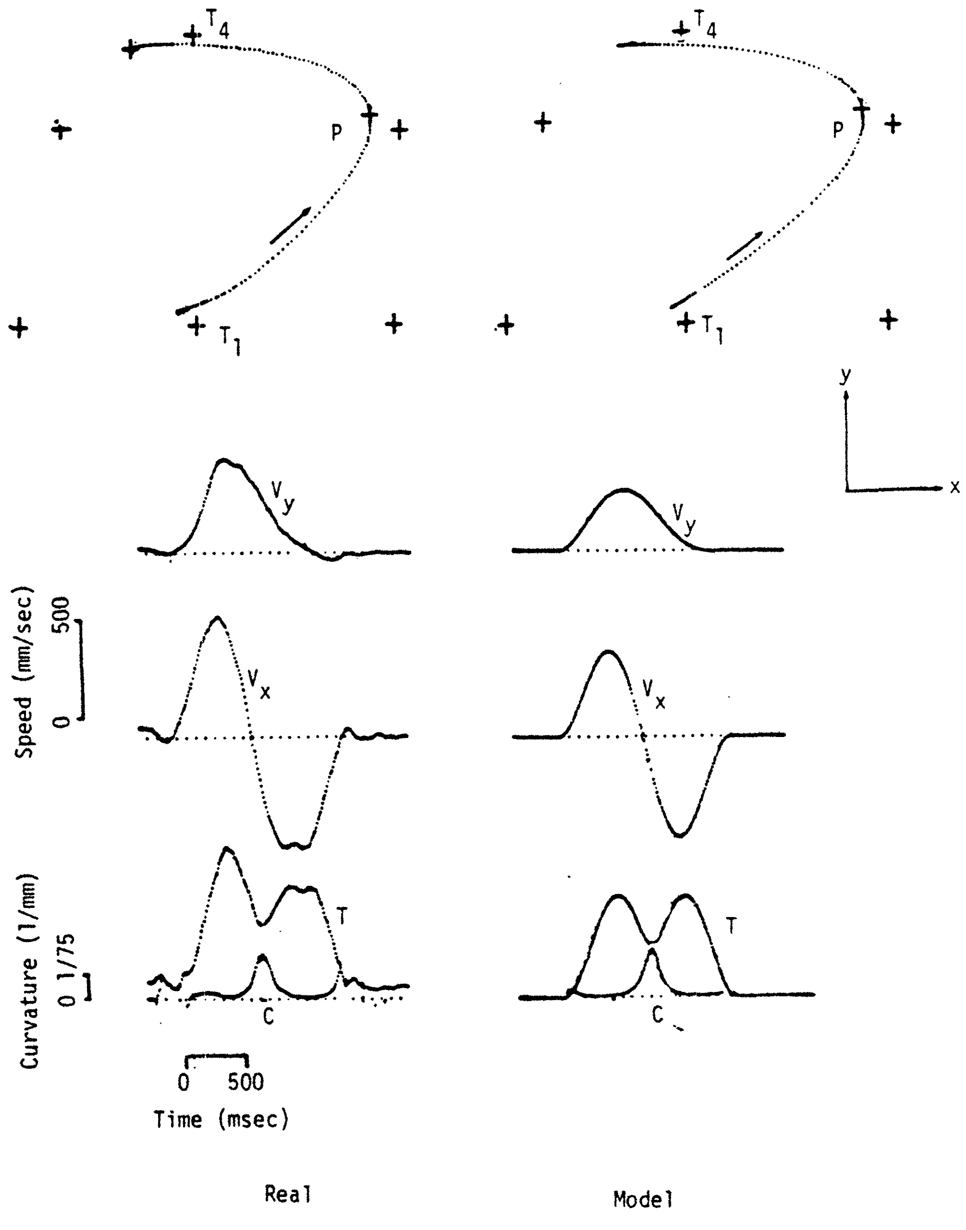


Figure 8. An obstacle avoidance movement. Measured (real, left column) and predicted (right column) hand paths (P) and plots of hand speed (T) curvature (C) and velocity components, V_x and V_y, versus time.

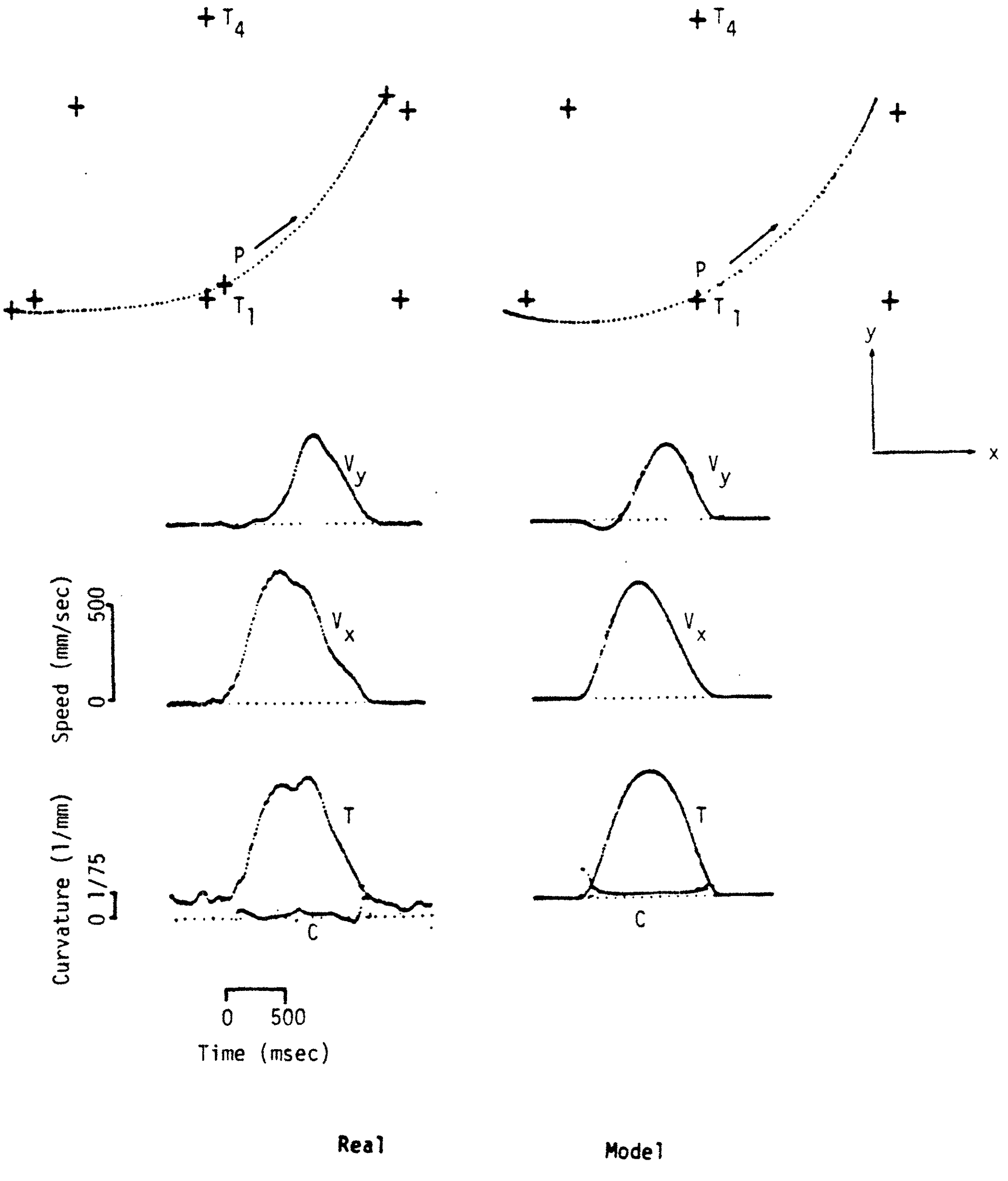


Figure 9. An obstacle avoidance movement. See Figure 3-8 for legend description.

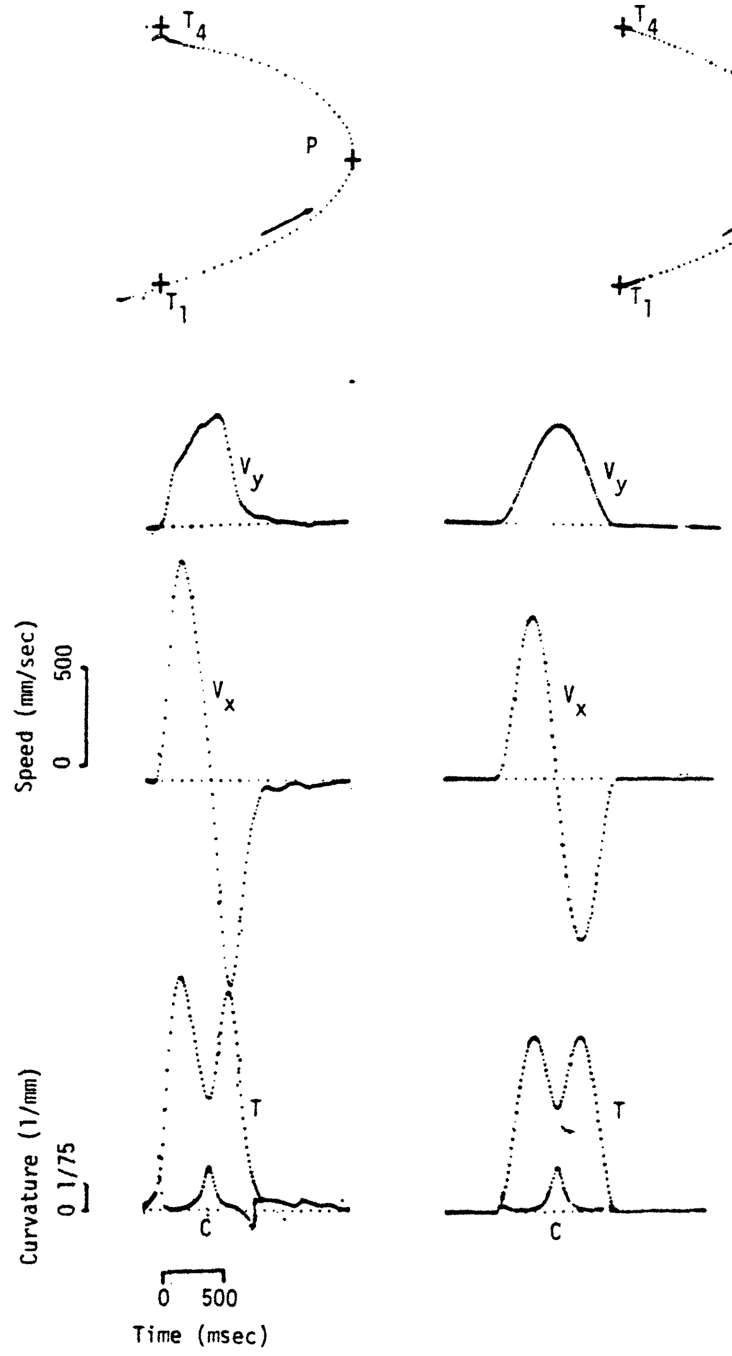
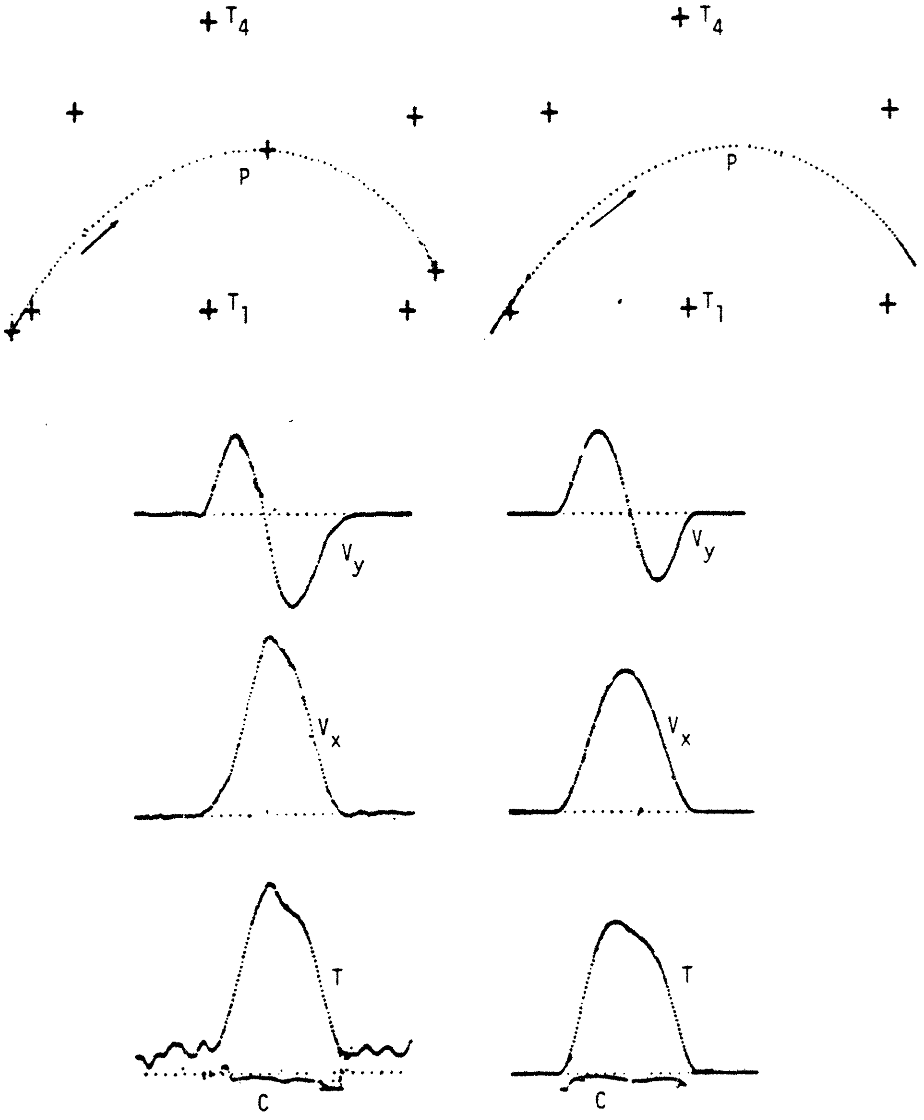
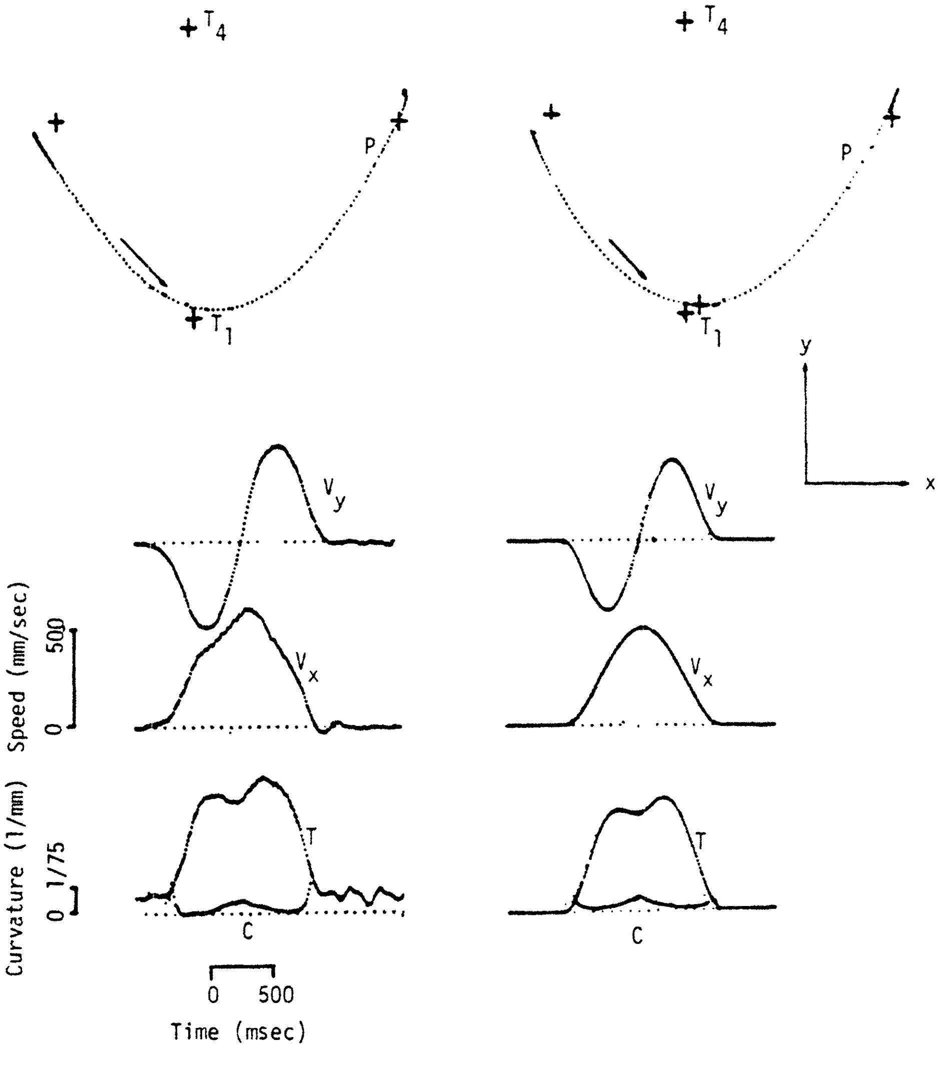


Figure 10. An obstacle avoidance movement. Legend description as in Figure 3-8.



Real Model

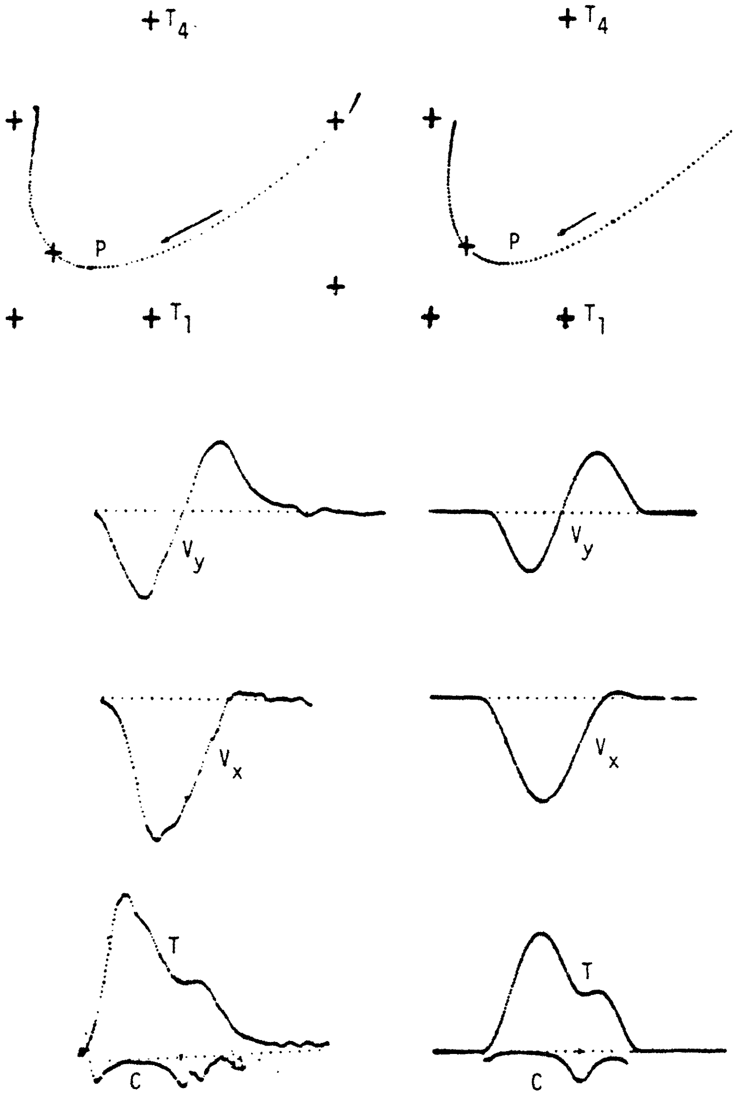
Figure 11. An obstacle avoidance movement. See previous figures for legend description.



Real

Model

Figure 12. An unconstrained curved movement. Legend description as in previous figures.



Real

Model

Figure 13. An unconstrained curved movement.

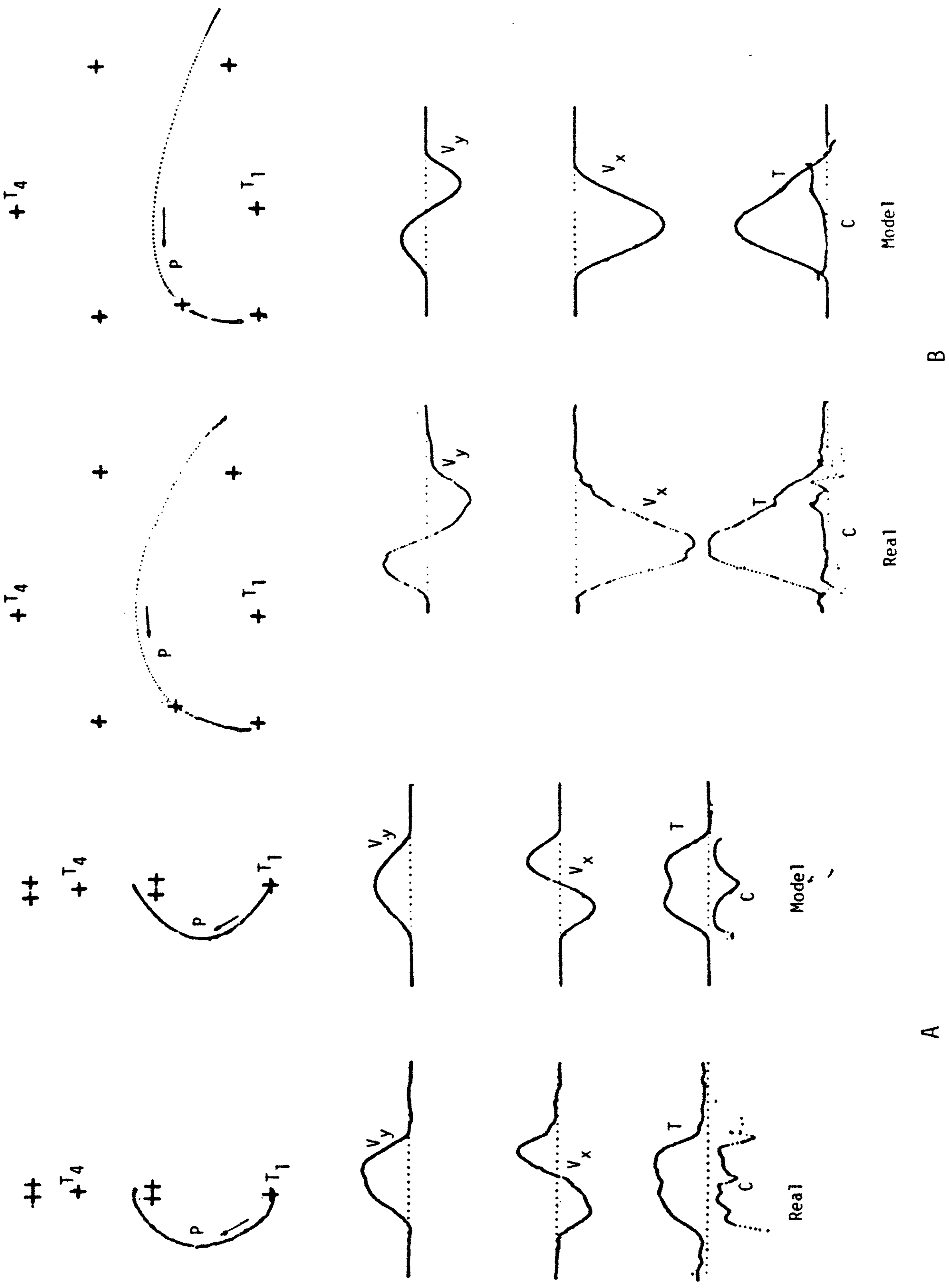


Figure 14. (A) A guide-mimicking movement. (B) An unconstrained curved movement.

3. For more highly curved movements the tangential velocity dip is more pronounced.

4. For some curvature peaks, usually those which appear at the beginning or towards the end of the movement, there is no corresponding velocity dip.

The above behavior is displayed in the experimental records, regardless of the part of the work space in which the hand is moving. Note also the similarities in the kinematic characteristics of the obstacle avoidance movements, the guide-mimicking movement, and movements from experiments in which the subjects were merely instructed to generate curved paths.

3.4.1. Results from via points experiments.

If we next examine the movements generated by subjects in experiments with real intermediate targets (Figures 3-15 to 3-24), it is apparent that these movements are quite similar to obstacle avoidance and unconstrained curved movements. Representative results from trials in which the hand had to reverse its direction of movement along the x axis, are shown in Figures 3-15 to 3-18. In all these figures, as in previous figures, measured hand paths and profiles of the hand speed, velocity components and curvature are shown in the left column and corresponding plots for the minimum jerk trajectories are shown in the right column.

In this group of movements, the intermediate target was first placed at equal distances from the initial and final targets, closer to (Figure 3-16) or further away from (Figure 3-15) the line connecting these two targets, and then was symmetrically offset from the center in both directions along a line passing parallel to the y axis (Figure 3-17 and Figure 3-18). Similar kinds of shifts in the locations of the intermediate target were repeated in trials which required a reversal in hand movement direction along the y axis (Figures 3-19 to 3-22). In addition, two movements which required a reversal in movement direction along both the x and y axes are shown in Figure 3-23 and in Figure 3-24. In all these figures, simulated hand paths, tangential velocity profiles, velocity component profiles and curvature profiles are compared to corresponding curves from the real movements.

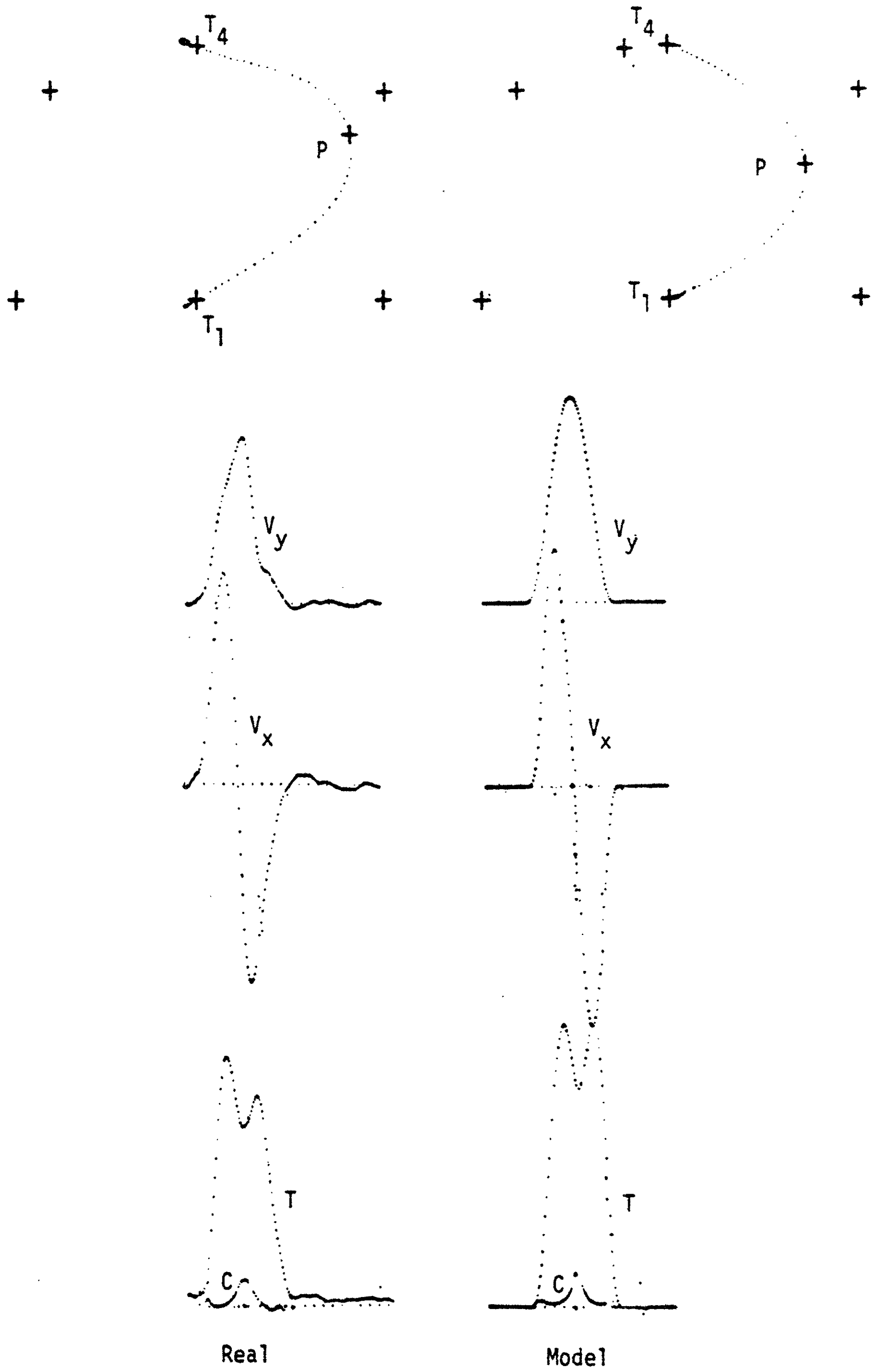


Figure 15. A movement from a "via-point" experiment. The movement reversed its direction along the x axis. The intermediate target was located at equal distances from the initial and final targets.

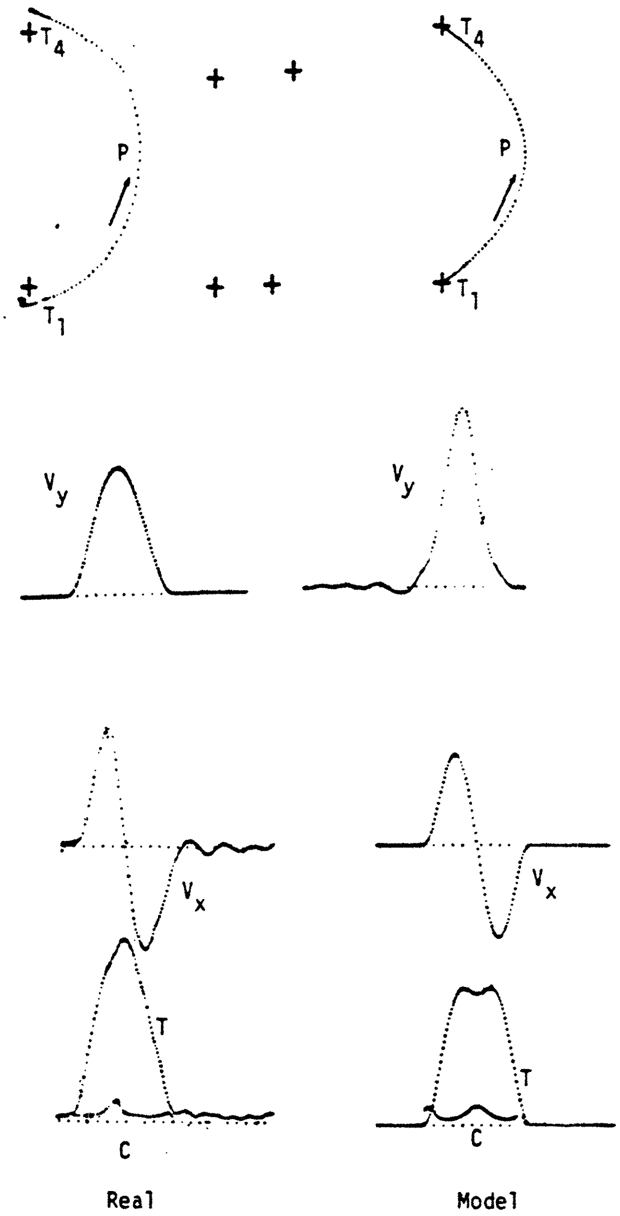


Figure 16. A reversal of movement direction along x-axis. The intermediate target was at equal distances from the movement end-targets but closer to line connecting them.

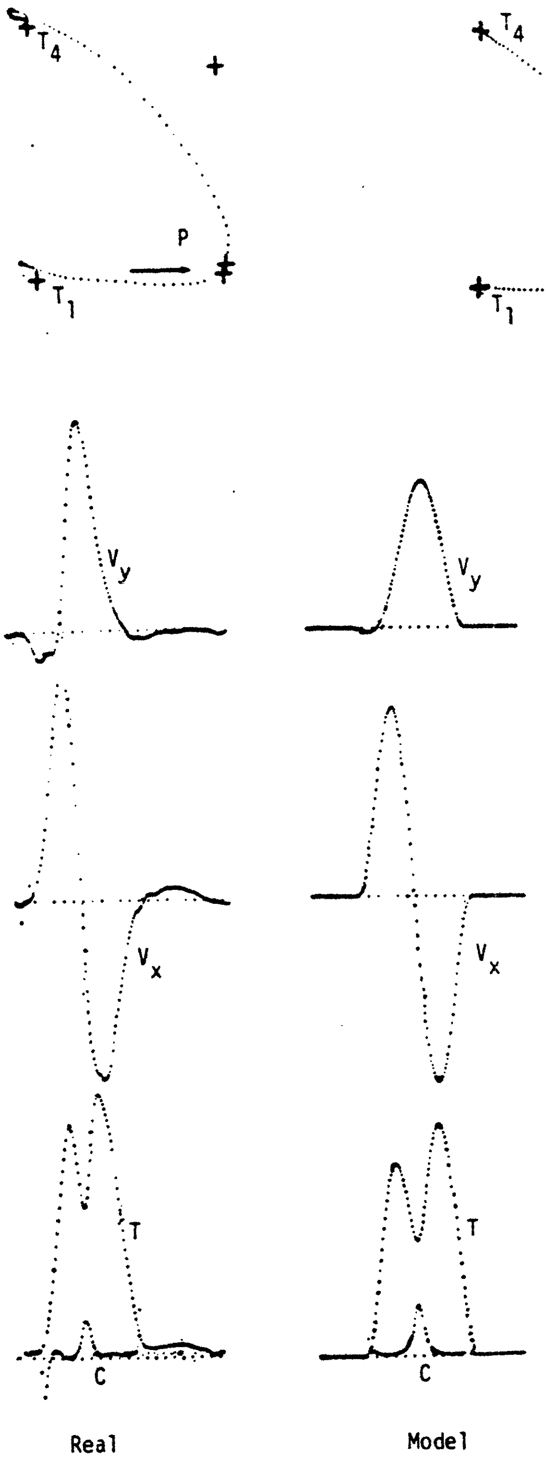


Figure 17. A reversal of movement direction along x-axis. The intermediate target was closer to the initial target.

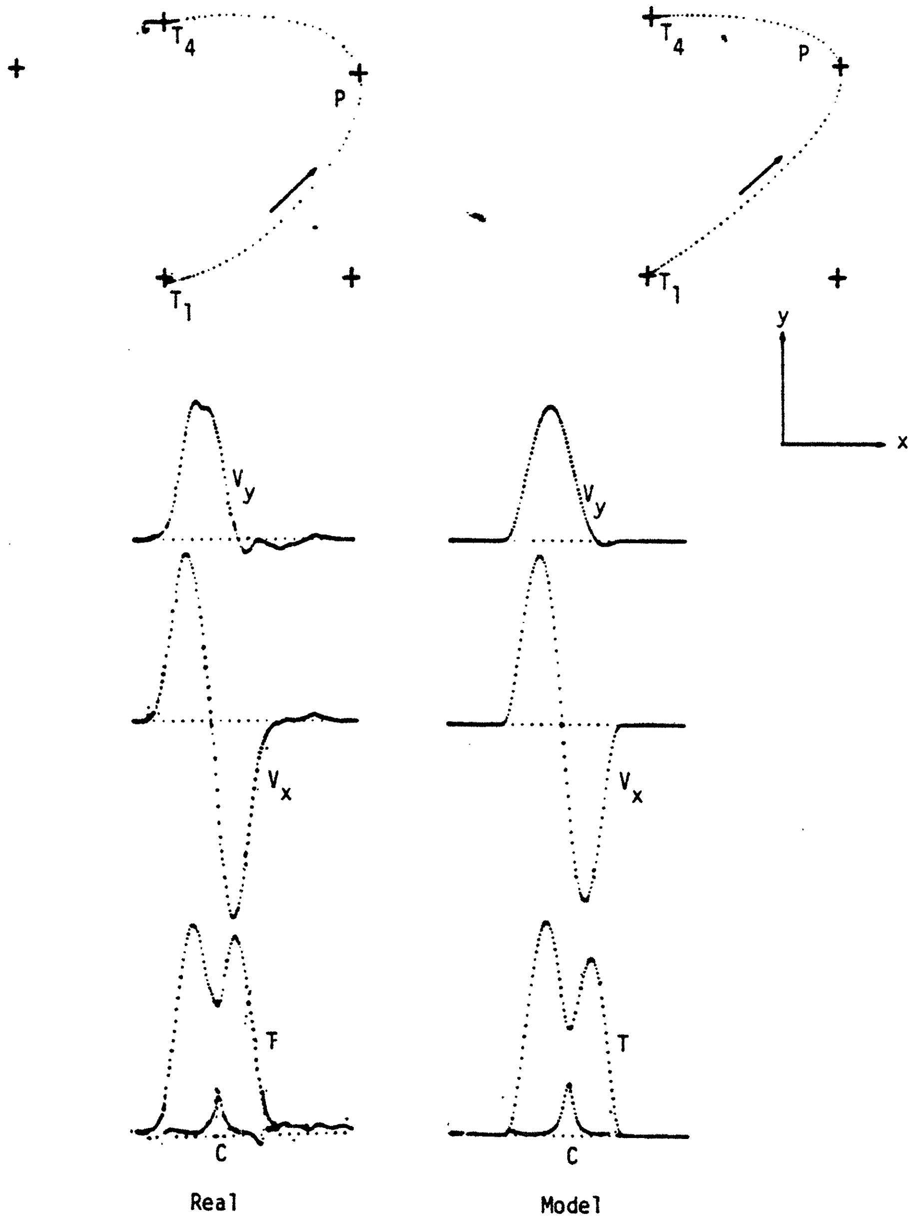


Figure 18. A reversal of movement direction along x-axis. The intermediate target was closer to the final target.

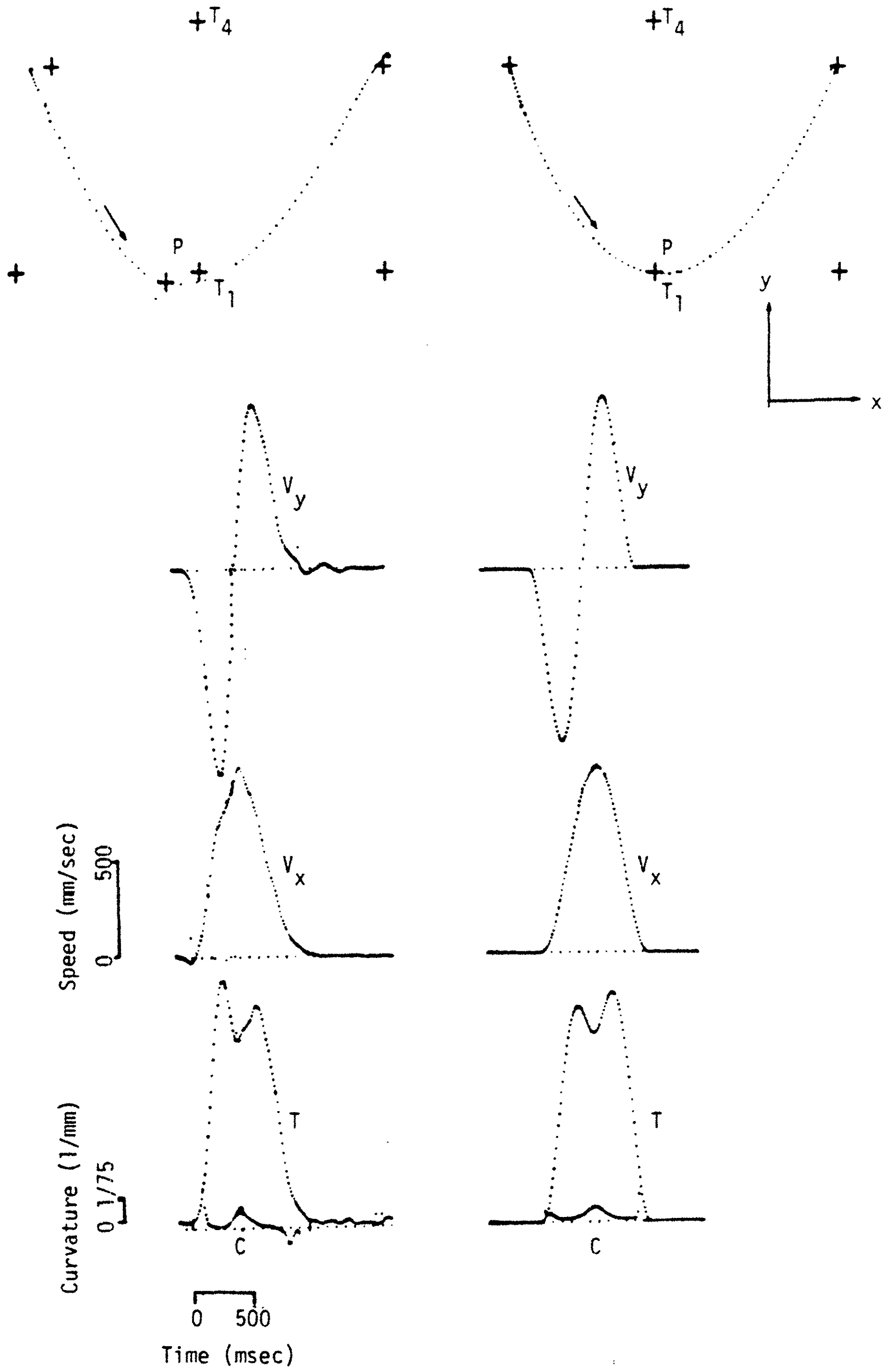


Figure 19. A movement from a "via-point" experiment. The movement reversed its direction along the y axis. The intermediate target was located at equal distances from the initial and final targets.

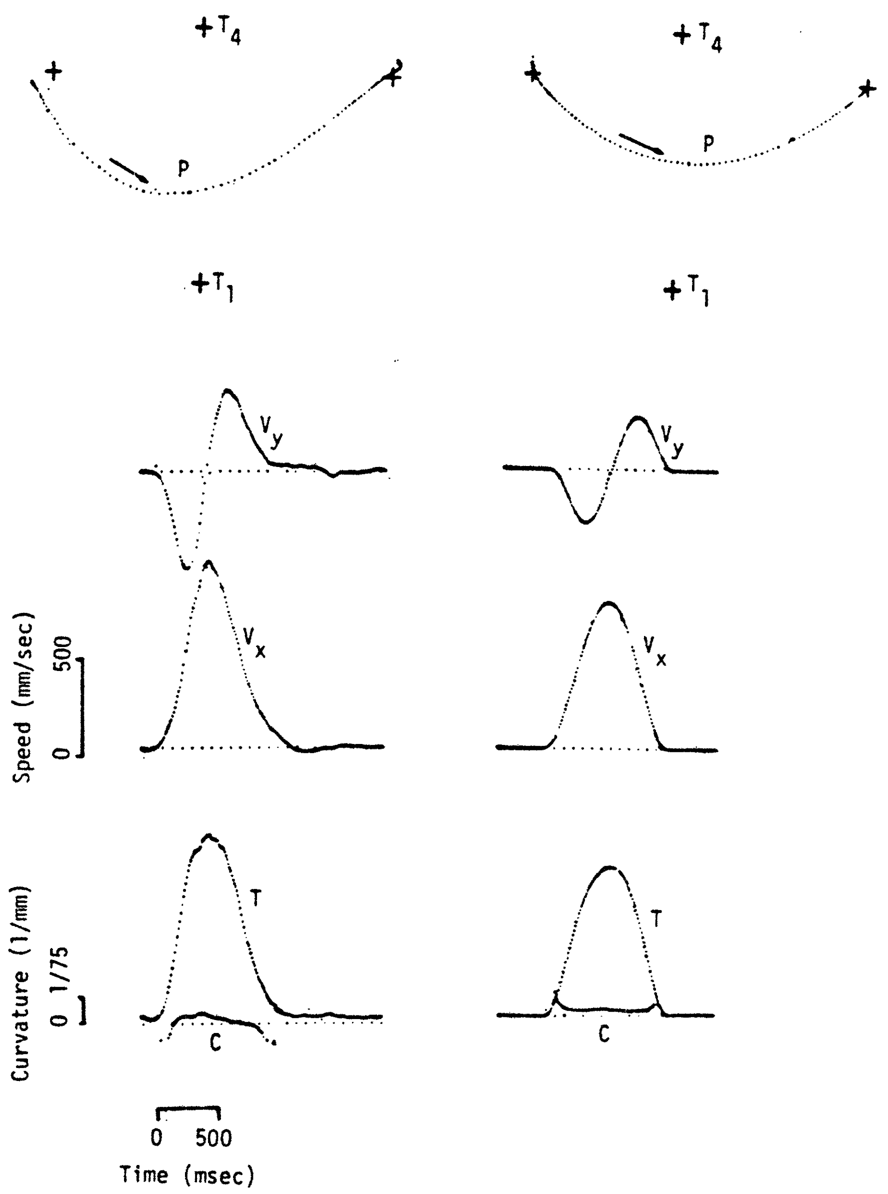


Figure 20. A reversal of movement direction along y-axis. The intermediate target was at equal distances from the movement end-targets but closer to line connecting them.

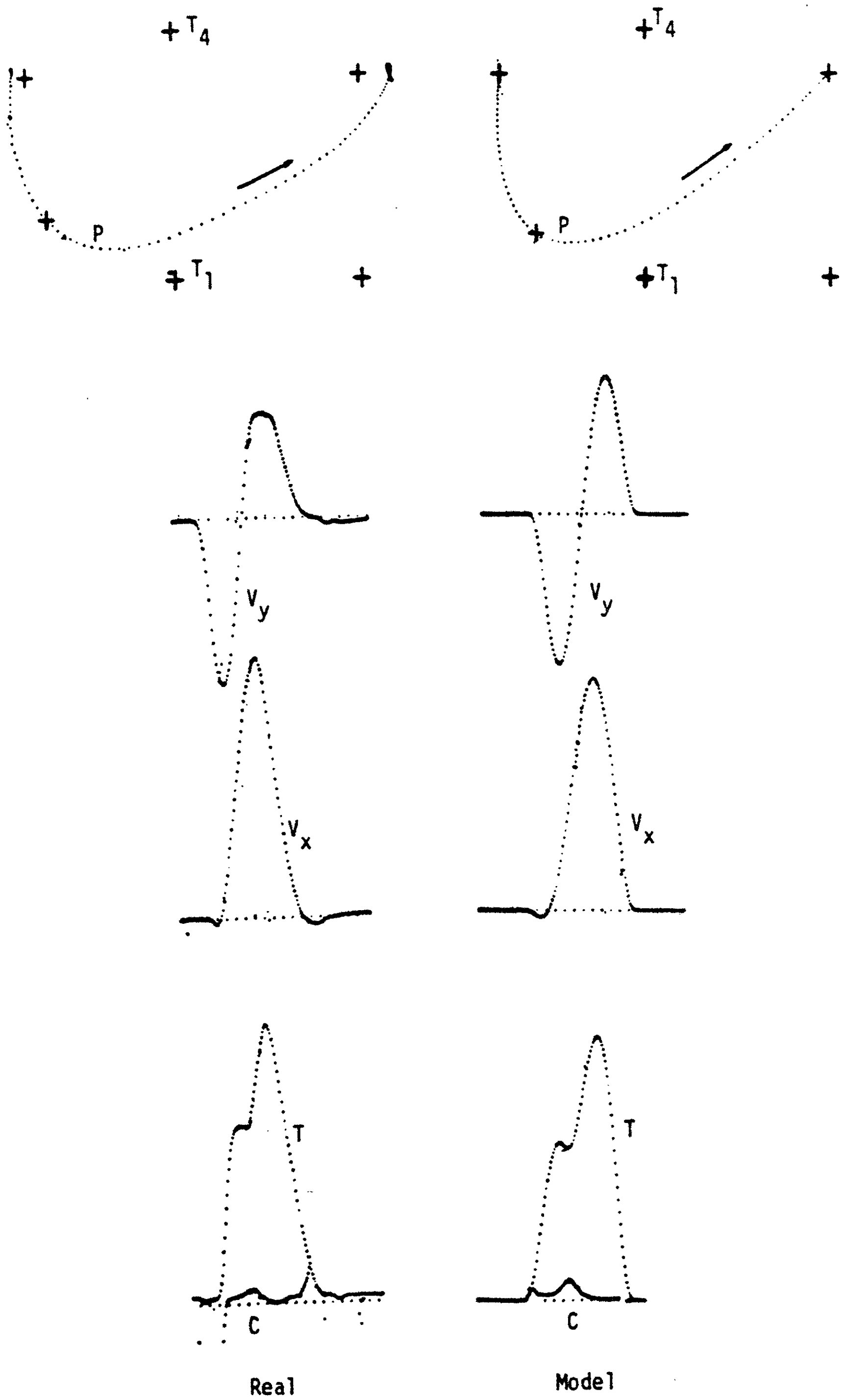


Figure 21. A reversal of movement direction along y-axis. The intermediate target was closer to the initial target.

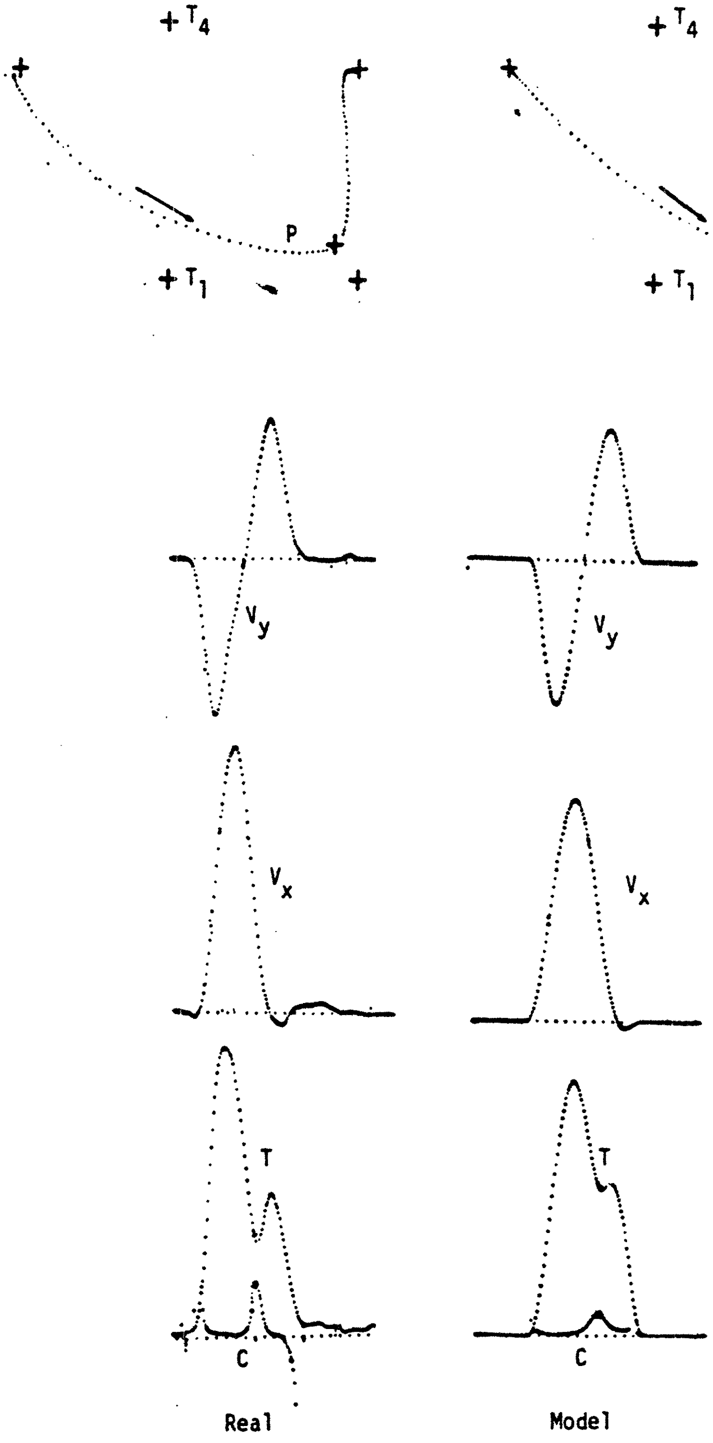
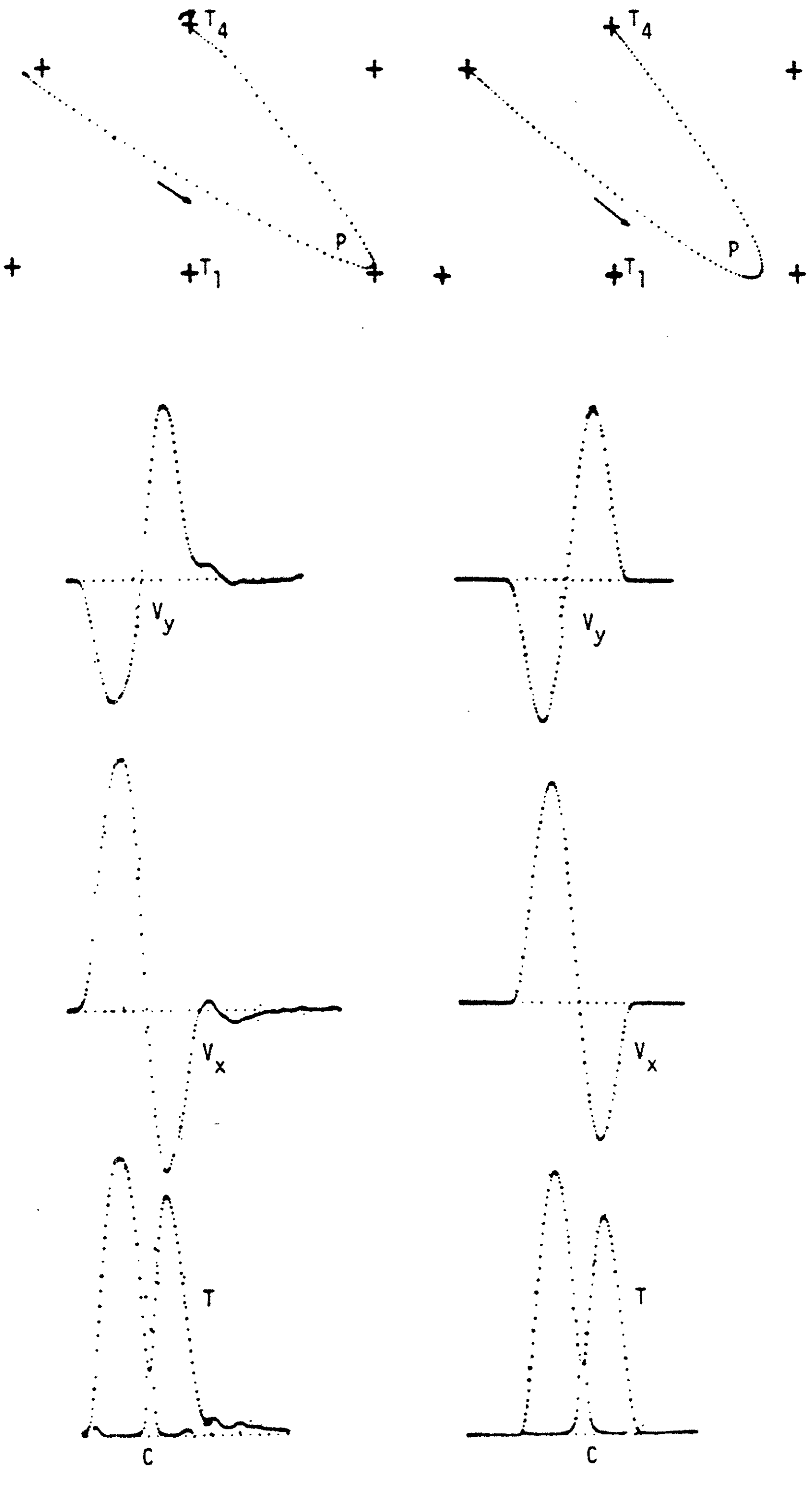


Figure 22. A reversal of movement direction along y-axis. The intermediate target was closer to the final target.



Real Model

Figure 23. A reversal in movement direction along both the x and y axes.

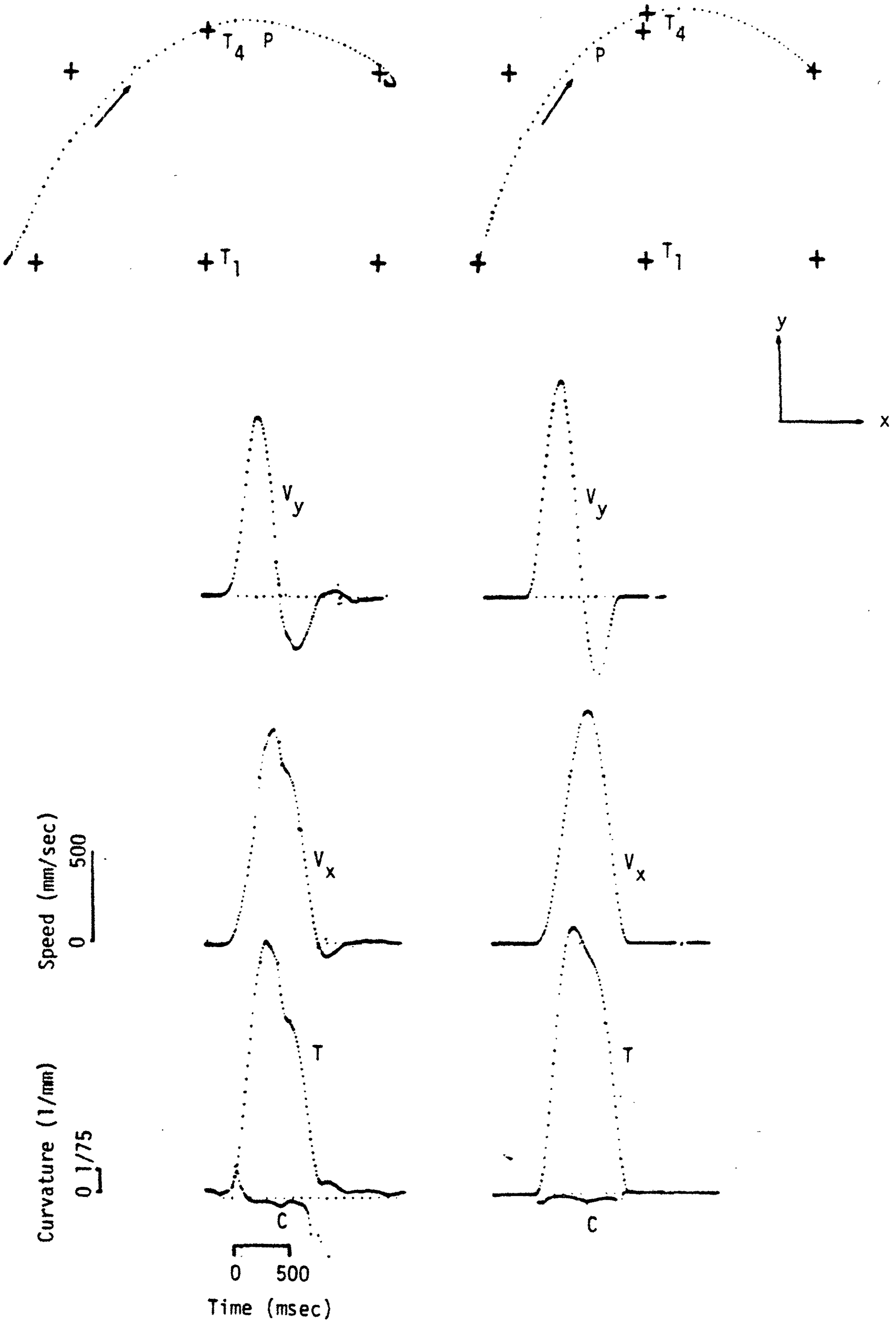


Figure 24. Another example of a reversal in movement direction along both the x and y axes.

In general, the simulated trajectories match the real movements quite well. With regard to the location of the via point, the predicted trajectories show that the point of reversal in x or y direction, or both, is usually also the point of maximum curvature and minimum tangential velocity (e.g. Figures 3-15 and 3-19). Occasionally, however, the point of change in direction of movement along one of the axes does not coincide with the maximum curvature point (e.g. Figures 3-18, 3-21). In these cases the same phenomenon is displayed in the real trajectories.

The model predicts the same hand trajectories regardless of the specific joints involved in the generation of the movement or whether they are generated in a horizontal plane at the level of the subject shoulder or at other levels. This was found to be the case in the work of Abend et al. [1982] for planar three-joint arm movements in which the arm configurations were redundant. In our own experiments, for movements in which the hand brace was removed and rotations at the wrist joint were allowed, and in experiments in which the movements were generated at a horizontal plane passing at the level of the subject 's waist, the trajectories were not different from the trajectories described in the above figures.

3.4.2. Verifying model predictions.

As predicted, the height of the two peaks of the tangential velocity on both sides of the velocity dip depend on the location of the via point. Thus, whenever the obstacle is offset from the center of the line passing through the via point, parallel to the line connecting the initial and final targets, the velocity peak on this side will be lower. As can be seen, for example, in Figures 3-17, 3-18, 3-21 and 3-22, the observed behavior matches this prediction. Similarly, unconstrained curved and obstacle-avoidance movements display this predicted feature (e.g. Figure 3-13). As predicted also, the trajectories will have higher curvature peaks and deeper velocity valleys depending on d_3, the distance of the via point along the perpendicular to the line connecting the initial and final targets, which passes through the via point. This was found to be the case (compare Figure 3-19 with Figure 3-20). Unconstrained curved and obstacle avoidance movements also match these predictions. Moreover, the above predictions were matched for movements in all parts of the work space.

As predicted by the "isochrony principle" the duration of the first part of the movement, till the via point, should be roughly equal to the duration of the second part, except for extreme cases where the via point is much closer to the start or to the goal target.

To test this prediction, points corresponding to measured $t_1/(t_1 + t_2)$ values for various experimentally set $d_1/(d_1 + d_2)$ parameters are superimposed on the predicted $t_1/(t_1+t_2)$ versus $d_1/(d_1+d_2)$ curve, as shown in Figure 3-25A. Similarly, points corresponding to measured $t_1/(t_1+t_2)$ versus d_3 for various movements, with similar values of $d_1/(d_1 + d_2)$, are superimposed on the corresponding predicted curve, as shown in Figure 3-25B. These two figures are derived from our "via point" experiments while similar plots, for which the data were taken from Abend et al.' [1982] records, are shown in Figures 3-26A and 3-26B.

In both sets of figures the real points fall very close to the predicted curves. In addition, the predicted plots in Figure 3-25A and in Figure 3-26, for example, have a very similar profile. Nevertheless, there is less scatter of the experimentally derived points around the predicted curve in Figure 3-26A compared to Figure 3-25B. The reasons for this difference will become clearer when we discuss movement variability in the next section.

Next we tested whether the real trajectories scale with time as predicted by the model. We compared movements with the same three targets generated in different speeds. The time scaling prediction was substantiated by the experimental data. The scaling of real movements in time is demonstrated in Figure 3-27, where the movement duration is 1.1 sec versus Figure 3-28, with a movement duration of 0.85 sec, and in Figure 3-29 (a movement duration of 0.9 sec) versus Figure 3-30 (a movement duration of 0.65 sec). In all these figures, we again display predicted and measured trajectories. As can be seen, the fact that the subjects move faster or slower does not affect the model performance.

3.4.3. Movement characteristics not accounted for by the model.

Thus, the model succeeds in accounting for the majority of the kinematic features of the real trajectories. However, a few features displayed in the real trajectories are not captured in the simulated trajectories. These include, for

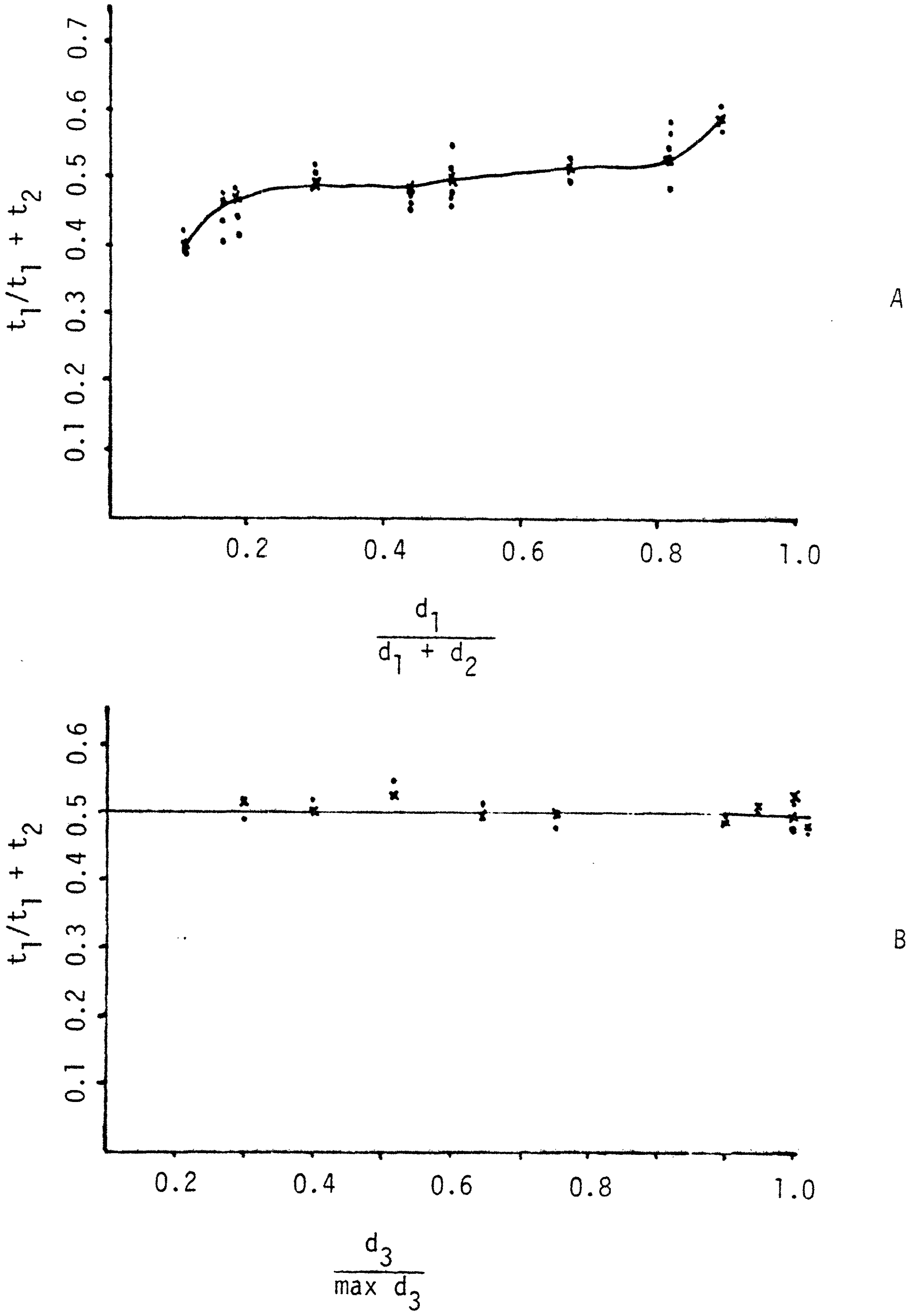


Figure 25. Via-point experiments. (A) A predicted $t_1/(t_1 + t_2)$ versus $d_1/(d_1 + d_2)$ curve was plotted through predicted points (x). The corresponding measured $t_1/(t_1 + t_2)$ are drawn by dots. (B) Predicted (solid line plotted x-es) $t_1/(t_1 + t_2)$ versus d_3 plot and measured superimposed values (dots).

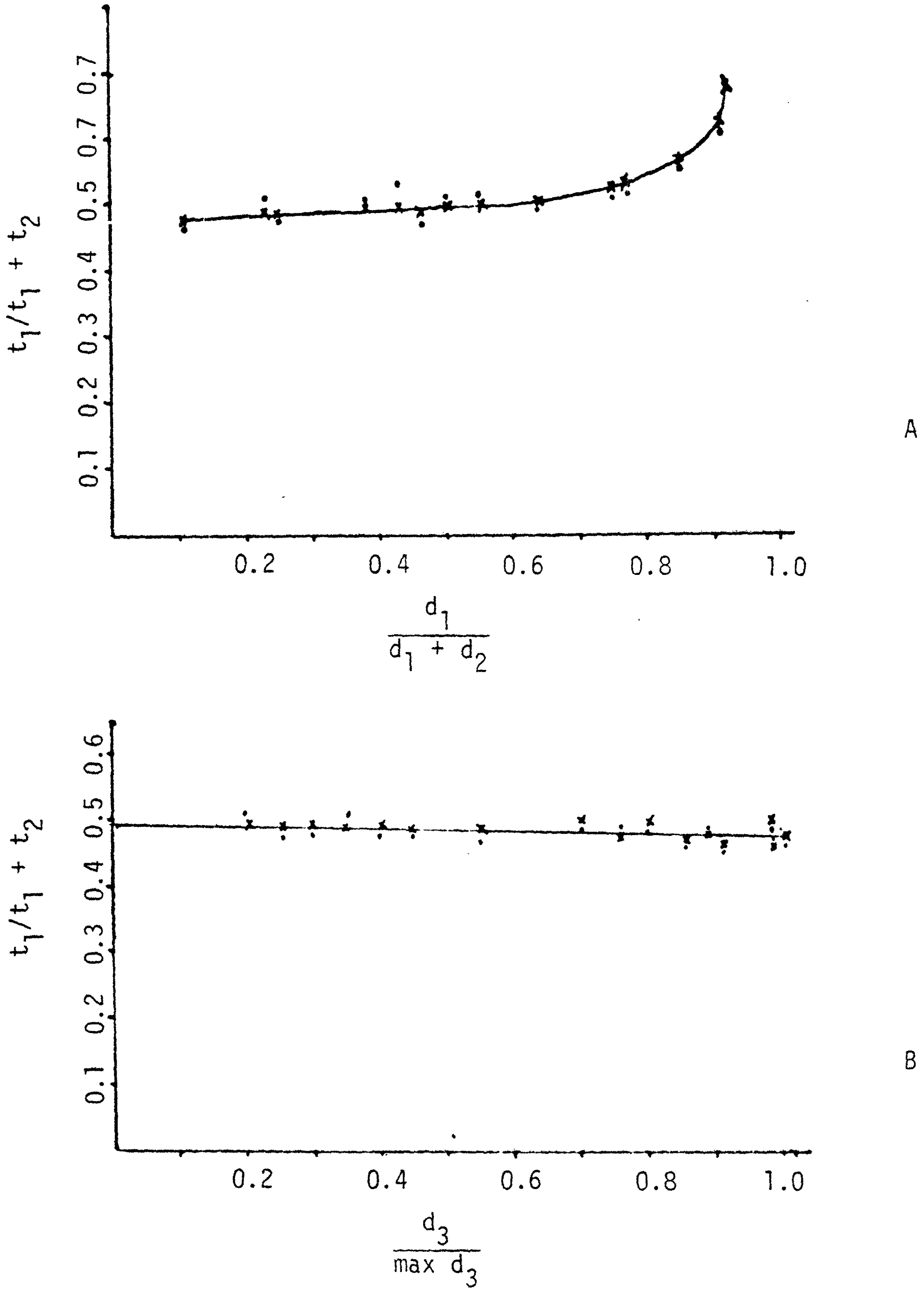


Figure 26. Results from unconstrained curved movements. See previous figure legend for notations.

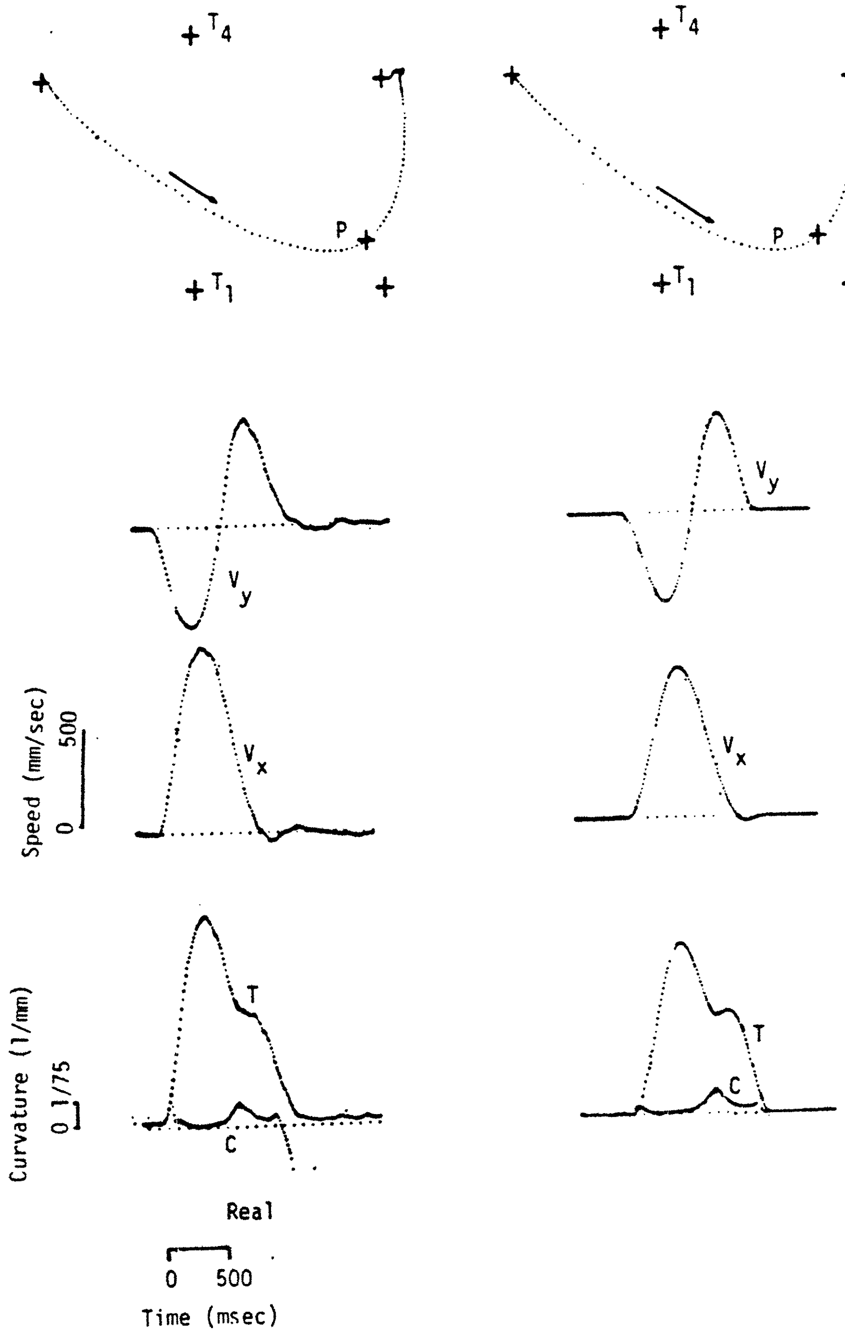
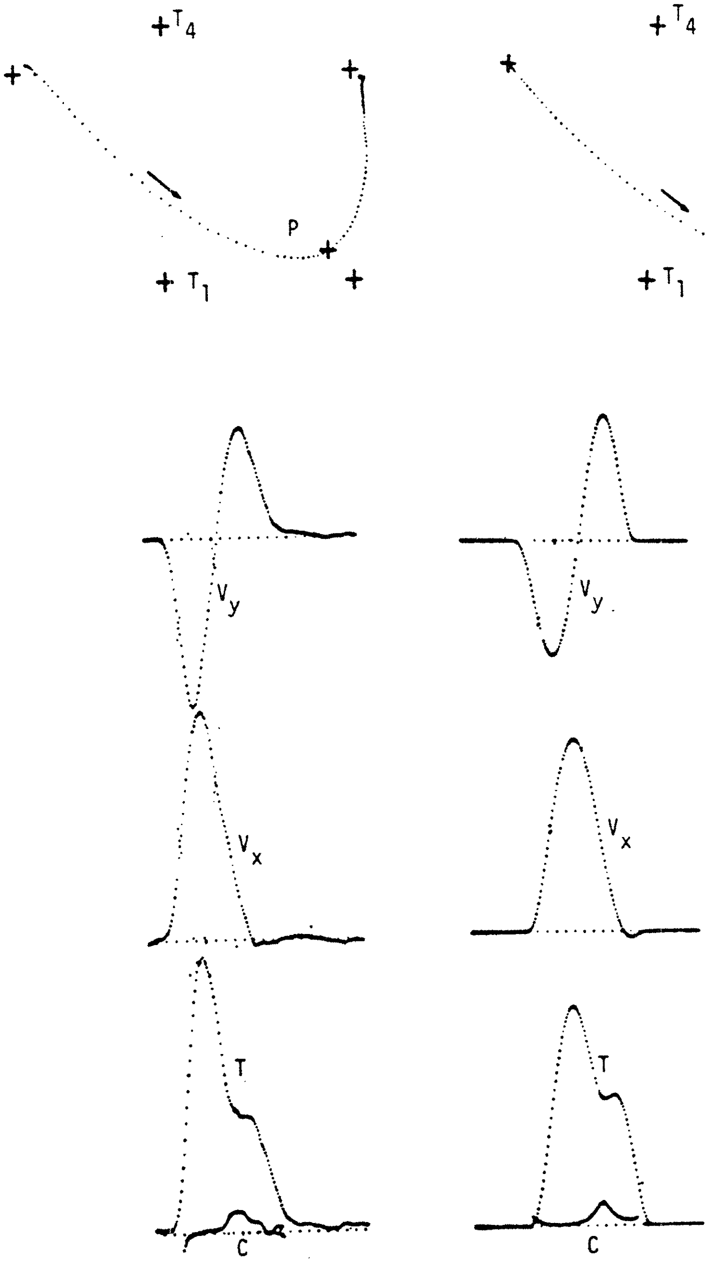


Figure 27. A 1.1 sec movement via an intermediate target. See Figure 3-8. for legend description.



Real

Model

Figure 28. A 0.85 sec movement via an intermediate target. The locations of this target and the initial and final targets are the same as in the movement displayed in Figure 3-27.

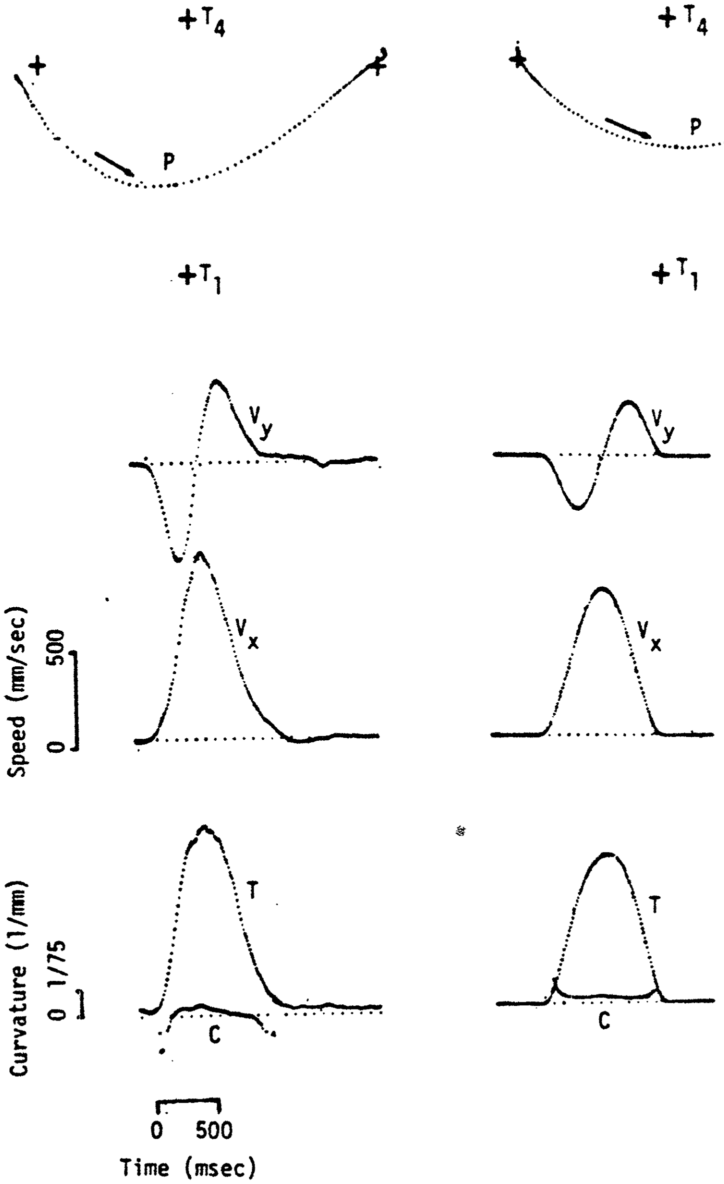
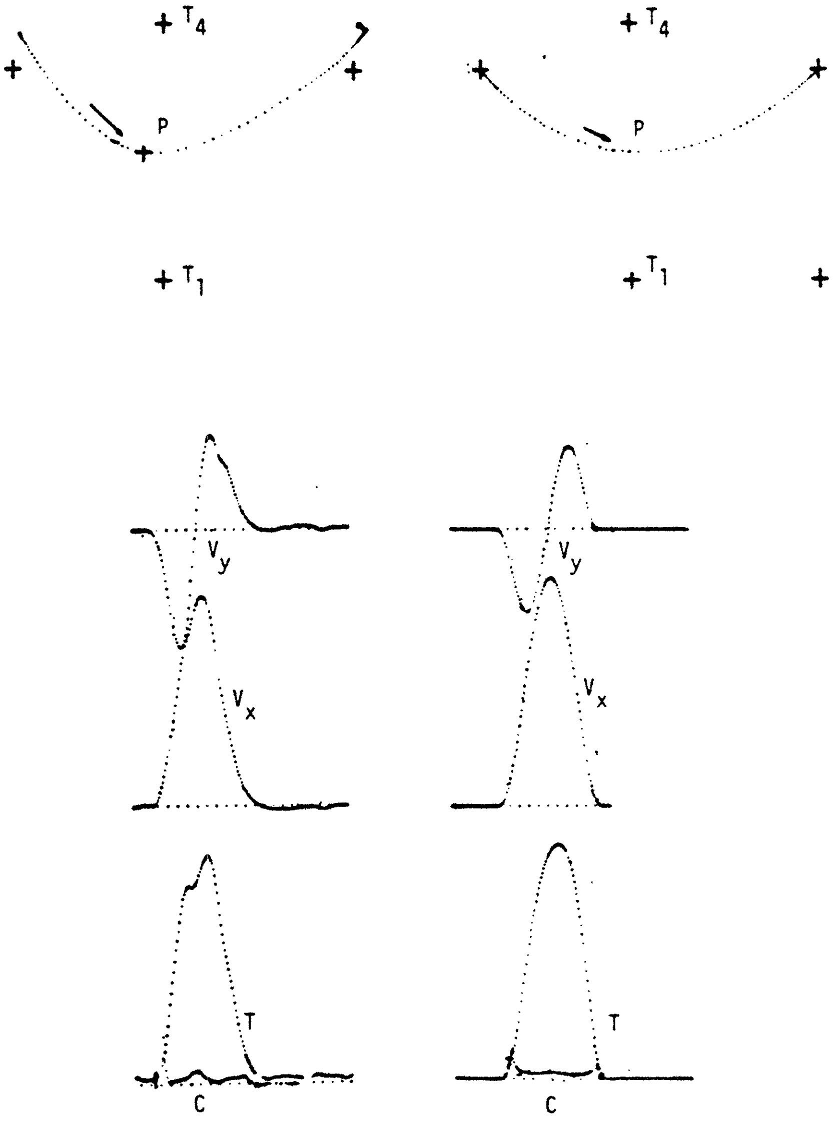


Figure 29. A 0.9 sec movement via an intermediate target. See Figure 3-8 for legend description.



Real

Model

Figure 30. A 0.65 sec movement via an intermediate target. The locations of the three targets are the same as in the movement displayed in Figure 3-29.

example, the tendency of the first velocity peak to be higher than the second peak in movements where the start target was located along the longitudinal axis directed away from the body with the shoulder as the origin (Figure 3-31).

Another feature is the existence of irregularities in the hand path (the little "hooks") and hand speed as the hand approaches the goal target (e.g. Figures 3-27, 3-30). It seems that, in general, the simulated trajectories fit the real movements better, in the first half of the movement, before the via point, than in the second half.

The majority of subjects generated movements such as those shown, for example, in Figure 3-27. In these movements, when the task required reversal of direction along one of the axes, the velocity along this axis, for obvious reasons, became zero at this point of reversal. Nevertheless, the curve for the other velocity component had a single peak with no velocity dips. This matches a similar behavior of the predicted velocity curves (e.g. Figures 3-15,3-27). Nevertheless, in a few movements, at the time that the hand reversed its direction of movement along one axis, there was a dip in the other velocity component curve. Such a movement is shown in Figure 3-32. This does not agree with the predictions of the model. In the majority of these cases, however, the real path does not differ very much from the path predicted by the model. For one of the subjects, however, whose movements consistently exhibited this type of velocity curve, the hand paths appeared as if made of two segments spliced together.

3.4.4. Movement variability.

To demonstrate the variability in movements between different trials under similar task requirements, the results from one subject, on succeeding runs, with the same target locations are shown in Figure 3-33. We see that the resulting trajectories vary very little between trials. The differences which do exist among movements generated on succeeding runs, such as those shown in Figure 3-33 may result from variations in the perceived locations of the three targets, or from variations in motor performance.

Most of the subjects did not accurately reach the final target and did not move their hands exactly through the via points. On the other hand, in the mathematical

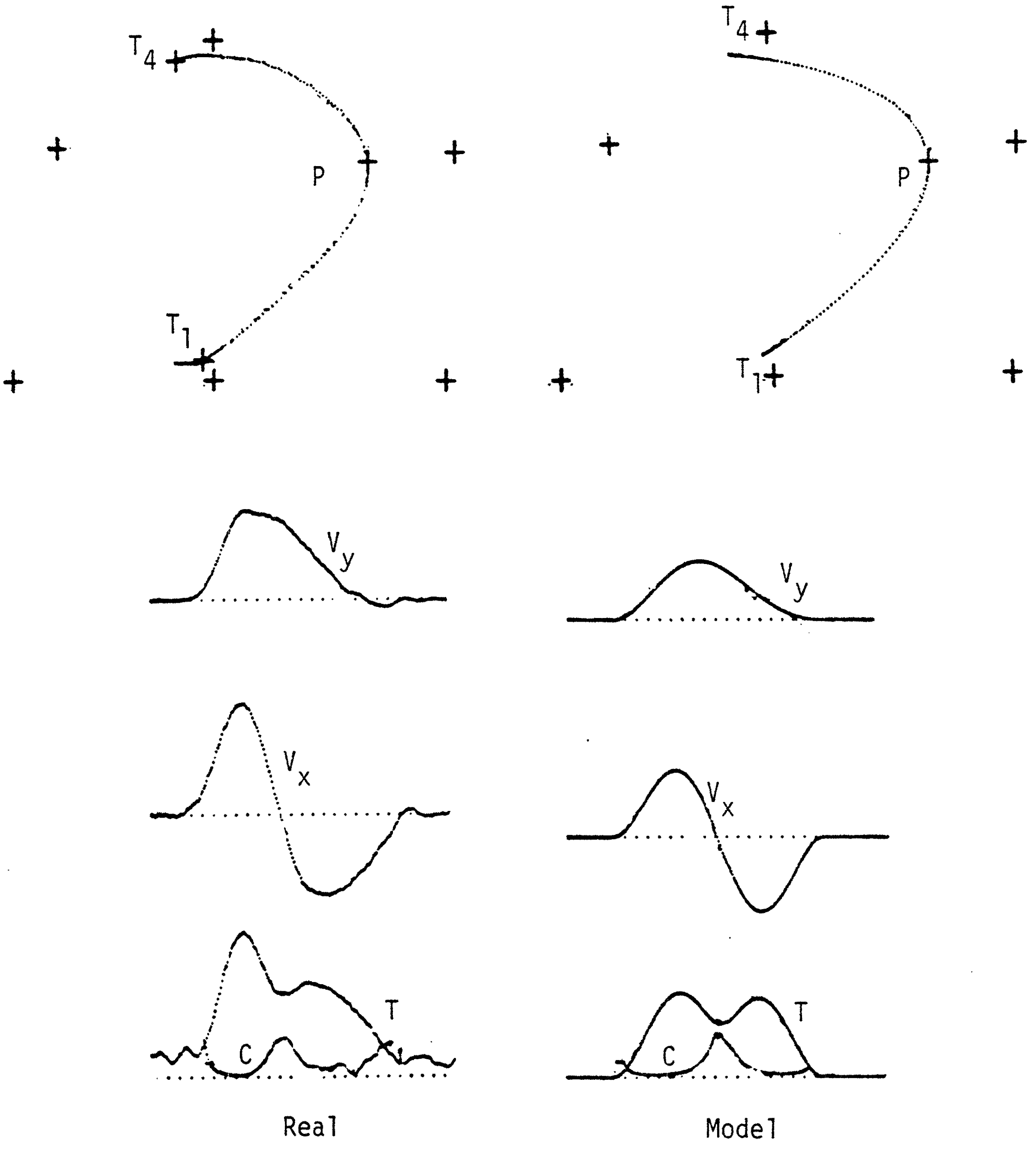


Figure 31. An example of a movement in which the first hand tangential velocity peak is higher than the corresponding peak in the predicted trajectory.

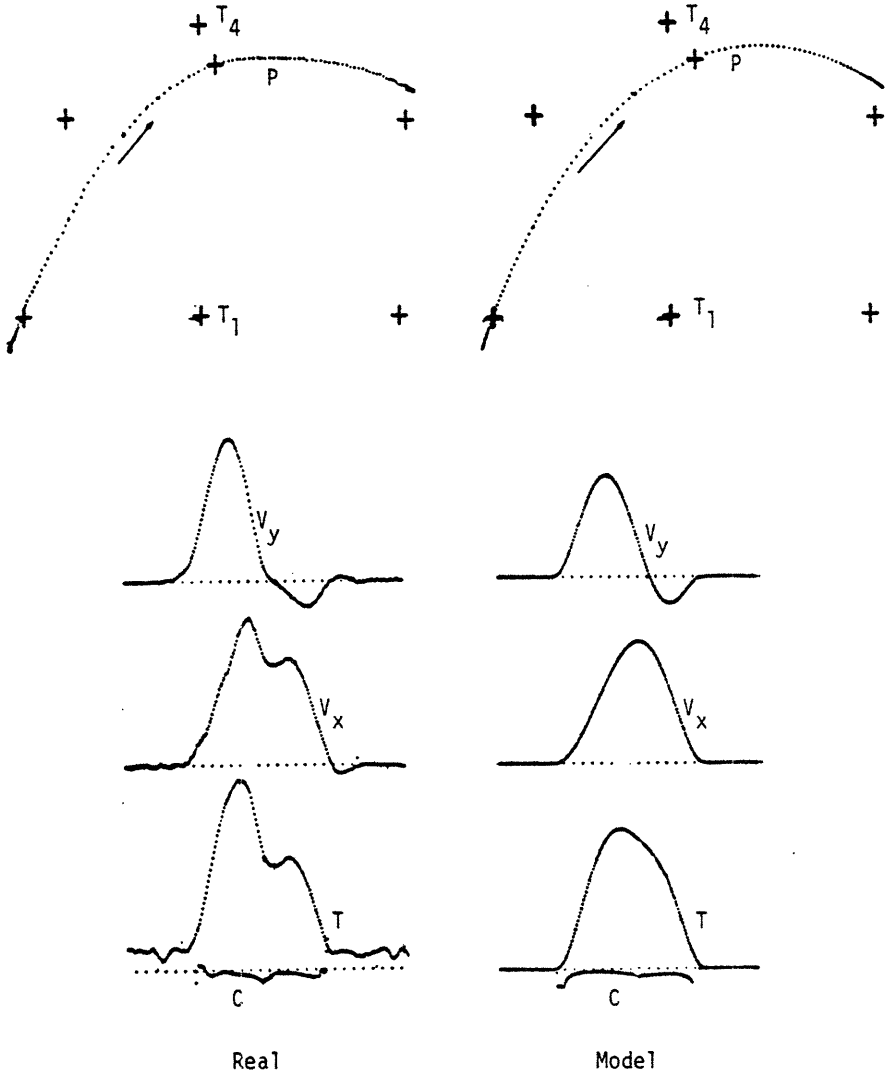


Figure 32. A reversal in movement direction along the y axis. The measured movement (Real) has a dip in the curve of the velocity component along x axis, V_x.

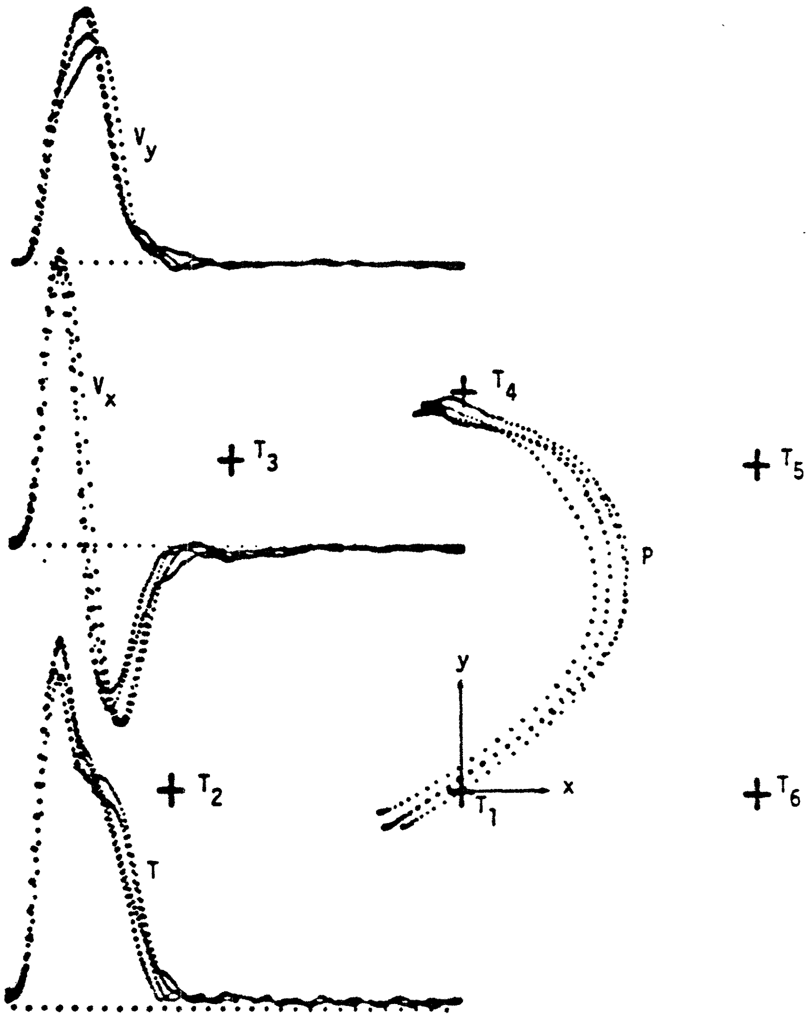


Figure 33. Four measured movements generated by the same subject, in succeeding trials, with the same set of initial, intermediate and final targets.

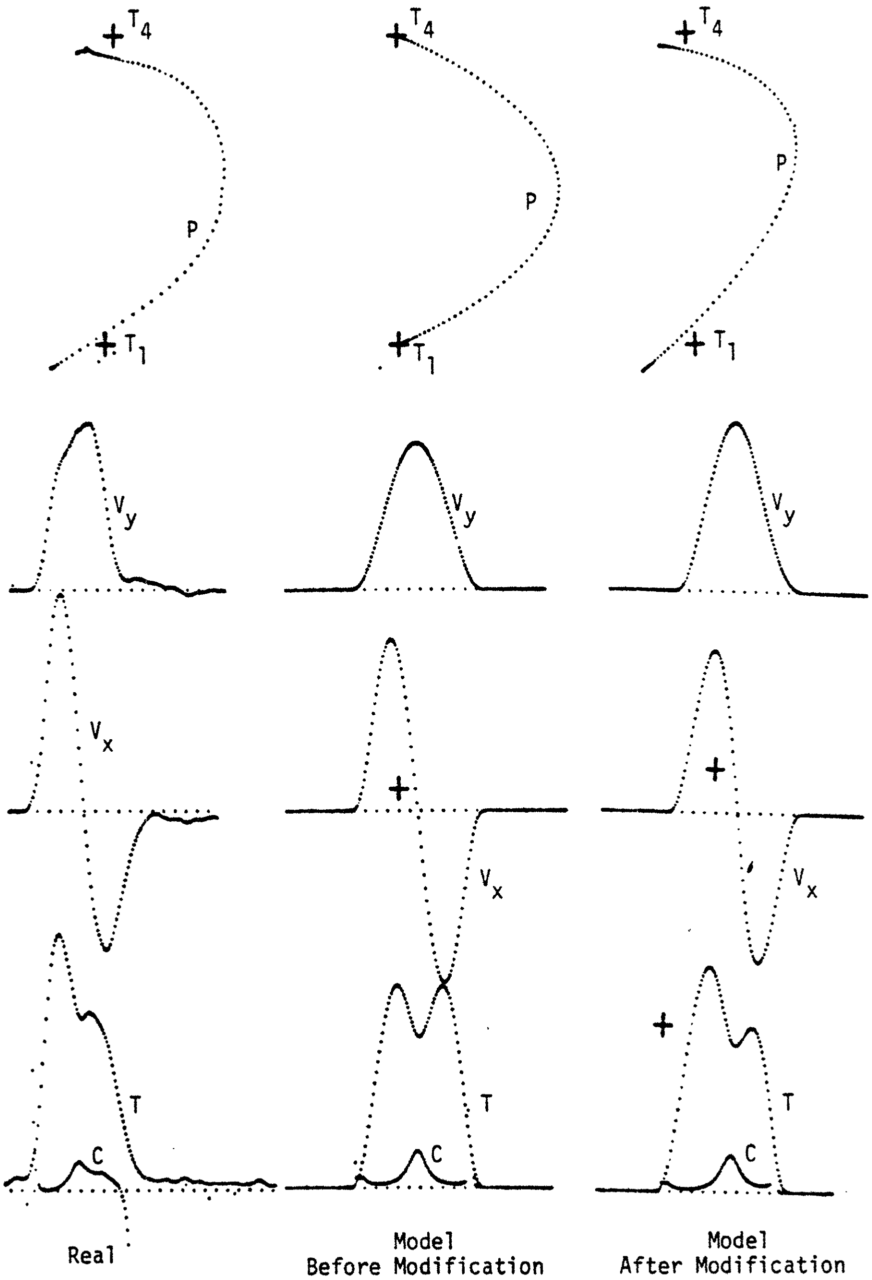


Figure 34. (A) a measured trajectory . Predicted hand trajectories before, (B), and after (C), the position boundary conditions were changed.

simulations of all hand constrained curved movements (via point experiments) the locations of the LEDs and not the actual, intermediate (maximal curvature) and final positions of the hand were used. Therefore, it is reasonable to assume that some of the discrepancies between the real and simulated trajectories are due to positioning inaccuracies on the part of the subjects, and hence, in our application of inappropriate position boundary conditions. To test this assumption we compared the match between the real and simulated trajectory before and after we introduced a change in these boundary conditions. This involved repeating the simulation with a new set of position parameters derived from the real data instead of the original position parameters (i.e. the locations of the LEDs). An example of the resulting trajectories before and after such a change is demonstrated in Figure 3-34.

3.4.5. Testing the success of all 3 objective functions.

Next, we have compared simulated trajectories resulting from acceleration, snap, and jerk minimization to examine which of these models succeeds best in fitting real movements. Figures 3-35 to 3-38 show the predicted hand speed and the profiles for the hand acceleration and jerk components along the y axis, for four different point-to-point movements, resulting from the minimization of acceleration, jerk, and snap. The profiles of the corresponding kinematic variables for the real movements are shown in the left columns of these figures.

If one examines the acceleration minimization results for unconstrained point-to-point movements (e.g. Figure 3-35 or Figure 3-36) it can be easily seen that although this minimization results in straight trajectories, other features do not agree with the corresponding features of the real movements. Thus the minimization of this criterion yields a parabolic tangential velocity curve unlike the real curve, the amplitude of the tangential velocity is lower than the real amplitude and the movements start and end with a non-zero acceleration which is unrealistic.

Next, if we compare the success of snap and jerk minimization models in fitting kinematic variables of real trajectories, it is harder to establish which model offers a better fit. As can be seen,for example, from Figure 3-38, the amplitude of the tangential velocity curve obtained from snap minimization is closer to the real amplitude than the minimum-jerk velocity amplitude for this movement and the

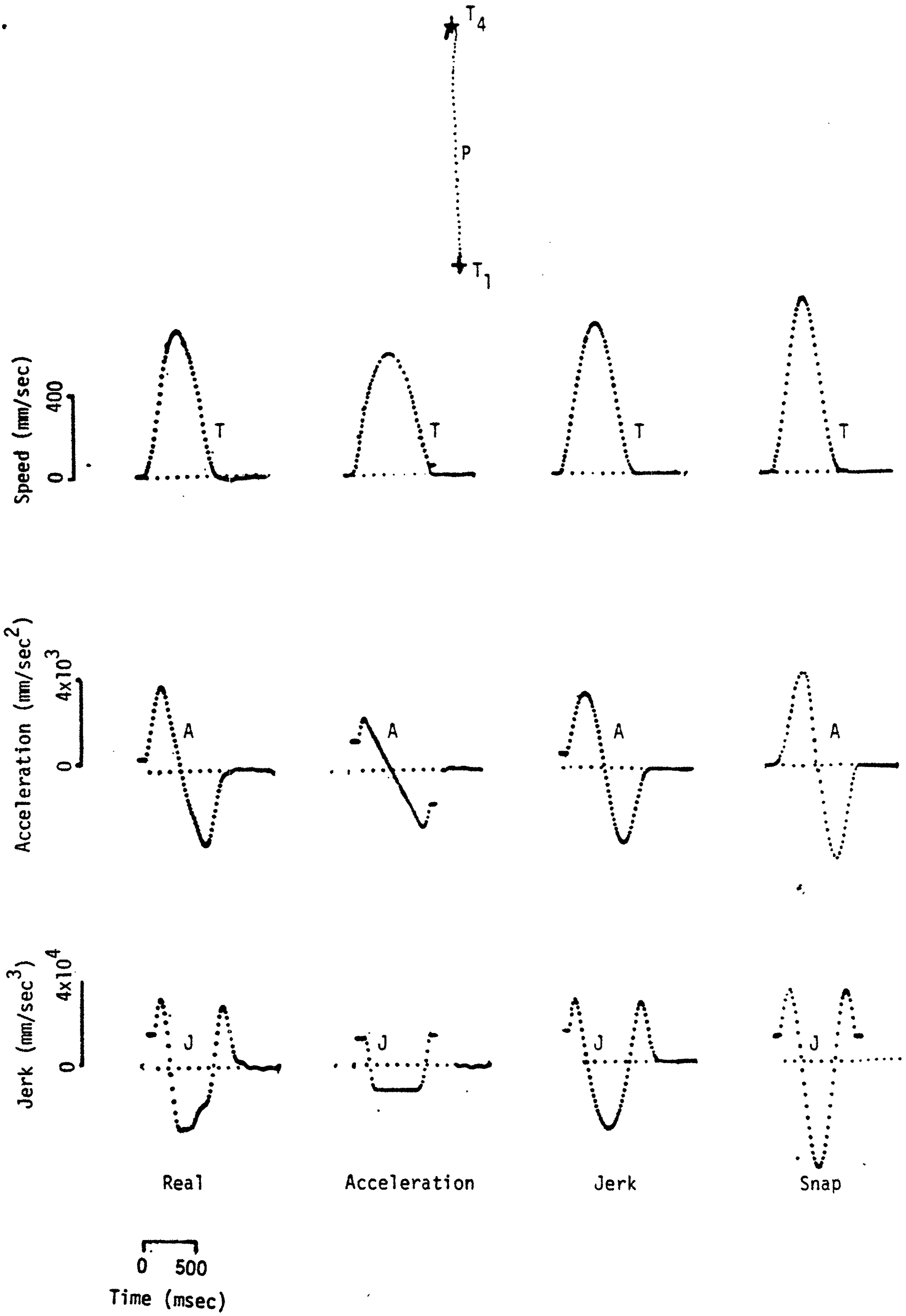


Figure 35. The hand speed (T), and hand acceleration (A) and jerk (J) components along the y axis for a measured (Real) movement, between targets 1 and 4, are shown in the left column. The corresponding results for minimum-acceleration, minimum-jerk and minimum-snap trajectories are shown in the other three columns.

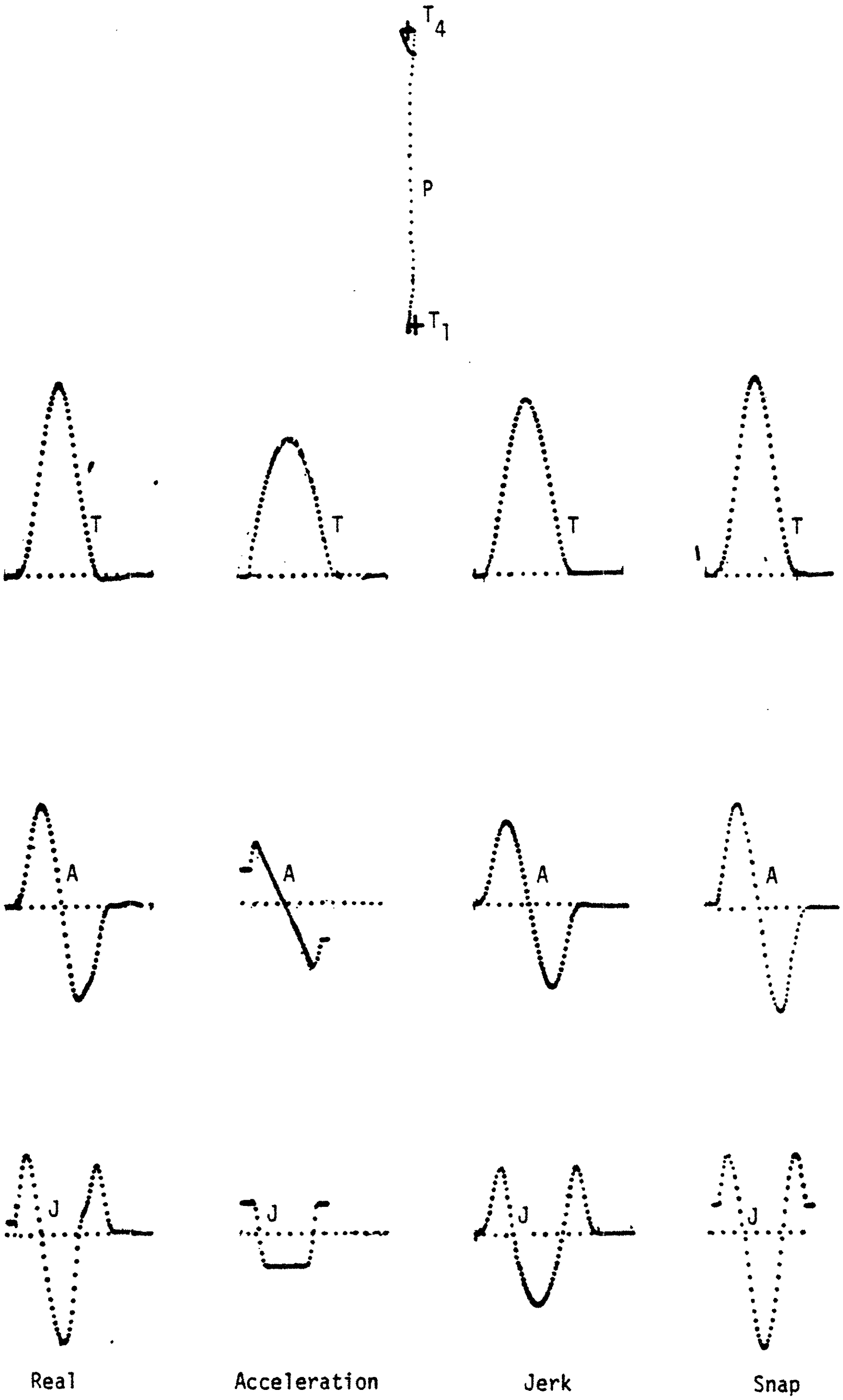


Figure 36. Real, minimum-acceleration, minimum-jerk and minimum-snap kinematic variable profiles for a 1 to 4 movement. See Figure 3-35 for legend description.

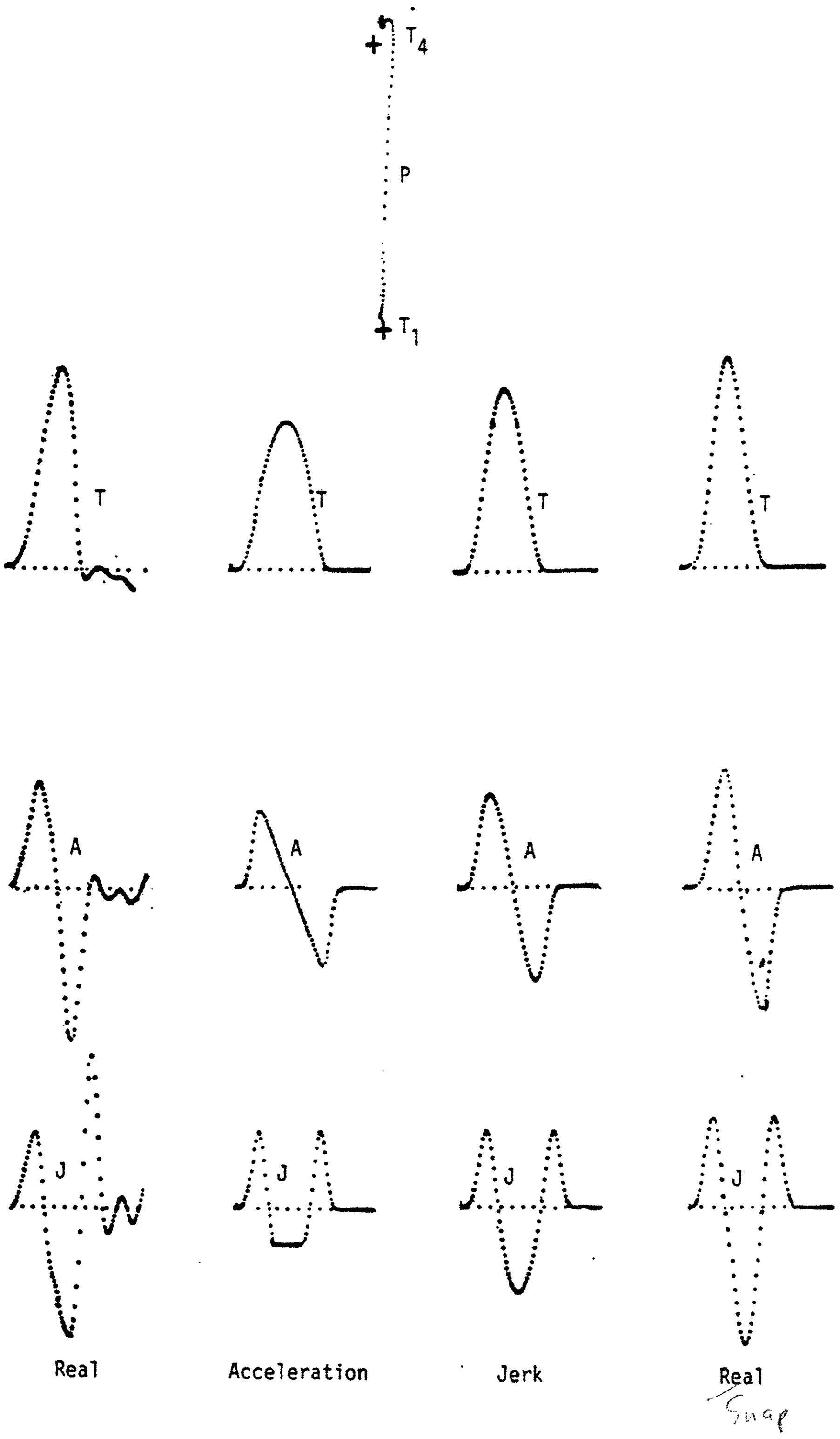


Figure 37. Real, minimum-acceleration, minimum-jerk and minimum-snap kinematic variable profiles for a 1 to 4 movement. See Figure 3-35 for legend description.

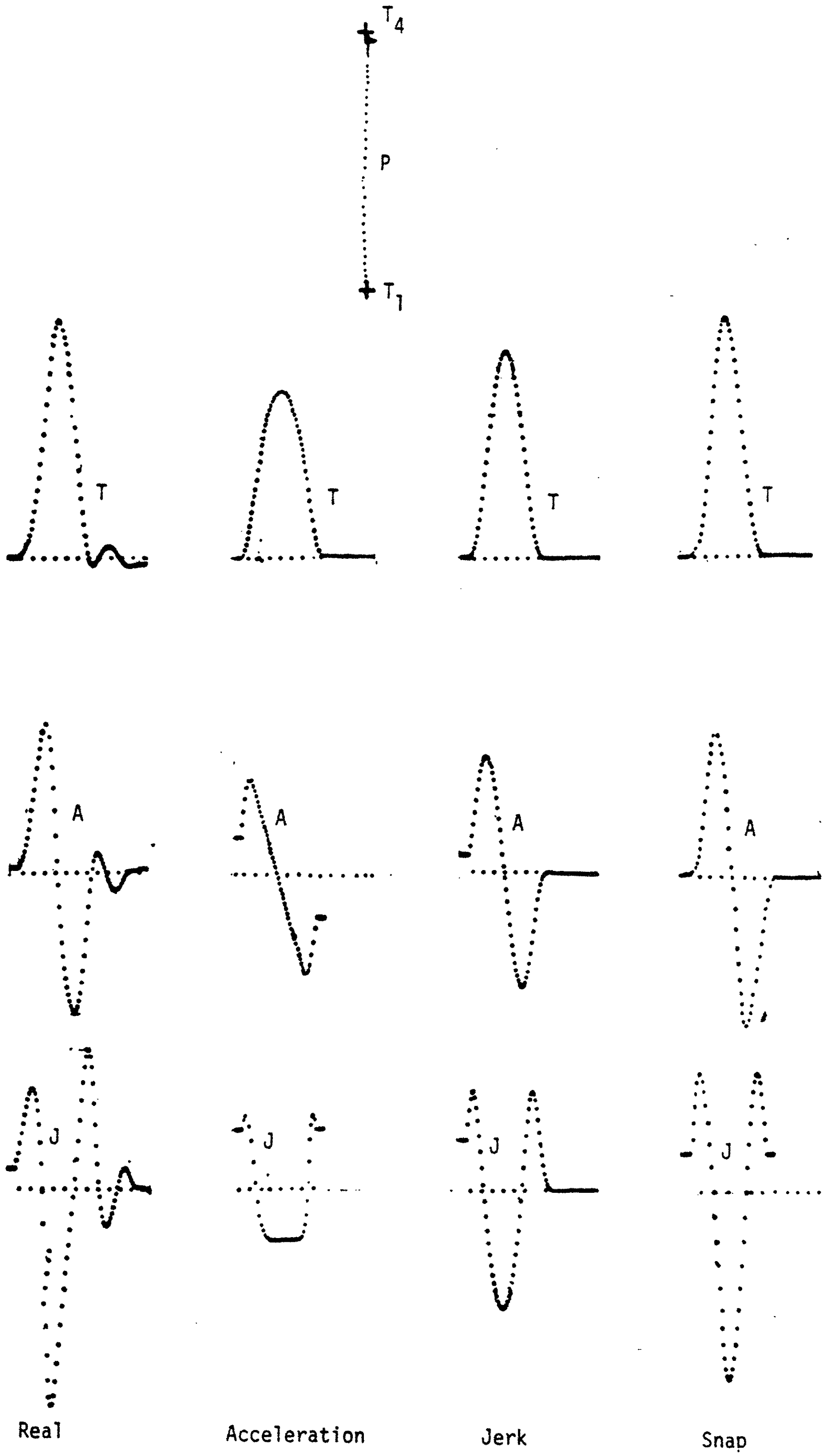


Figure 38. Real, minimum-acceleration, minimum-jerk and minimum-snap kinematic variable profiles for a 1 to 4 movement. See previous figures' legend description.

snap-minimization model offers a better fit. Similarly, snap-minimization succeeds better in predicting the tangential velocity curve for the movement shown in Figure 3-36. On the other hand, the tangential velocity amplitude predicted by this model is too high for the movement shown in Figure 3-35

As can be seen from Figures 3-35 to 3-38 the jerk minimization model predicts non-zero jerk values at the end points. For simulations based on snap minimization, we picked zero jerk at both boundaries. As can be seen from Figures 3-35 and 3-36, real movements start and end with non-zero jerk. We could have changed the boundary conditions for snap minimization according to the real movements. Nevertheless, in deriving the values of high derivatives of position (jerk, snap) at the boundaries, there is a technical difficulty since higher order differentiations result in noisy signals. Thus, if one computes numerically, at the beginning and at the end of the movement the higher derivatives of the various polynomials used in this work, these values differ from the values one would obtain from a simple substitution of $t = 0$ and $t = t_f$ in the corresponding mathematical expressions. This happens because of baseline effects and the specific method of differentiation used in this study. Judging, however, from the steepness of the acceleration curves at the beginning and at the end of the movement, and the time at which maximum acceleration is reached, the minimum-jerk trajectories succeed better than minimum-snap trajectories in predicting these features (see Figure 3-3 with better resolution and compare the predicted curves from jerk and snap minimization to the real curves in Figure 3-37).

In Figures 3-39 to 3-42 curved movements resulting from acceleration, jerk and snap minimization and the corresponding real movements are compared. As can be seen, acceleration minimization fails to predict realistic tangential velocity and velocity components. The amplitude is again too low, the shape is wrongly parabolic etc. Comparing minimum-snap trajectories with minimum-jerk trajectories, the maximum tangential velocity amplitude predicted by the snap-minimization model is too high in Figure 3-42, but fits better than the speed amplitude predicted by jerk-minimization in Figure 3-40. Nevertheless, judging from the steepness of the velocity curves, the predicted minimum-snap hand speed curves, are less steep

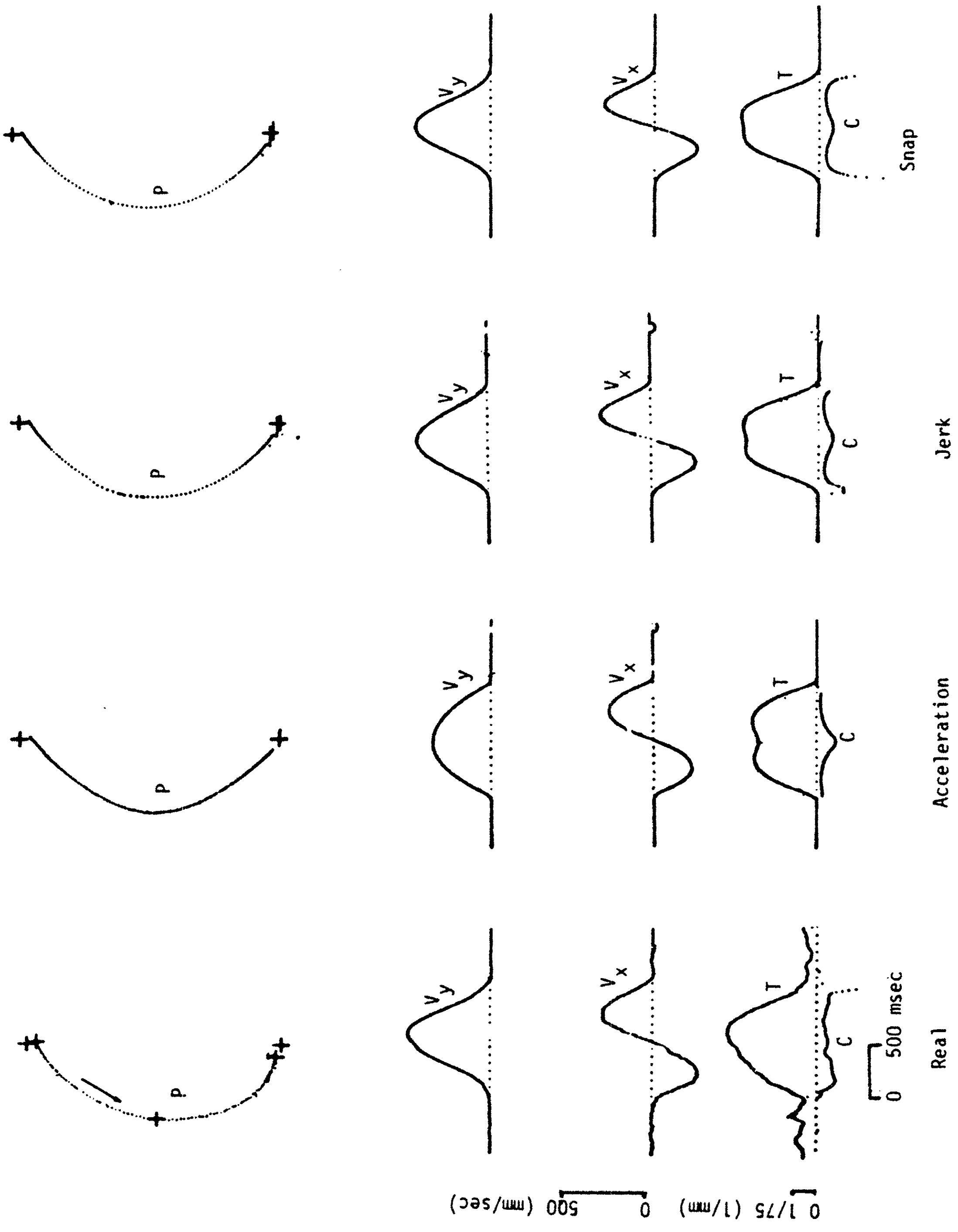


Figure 39. A curved movement. Hand speed (T), curvature, (C), and velocity components, (V_x) and (V_y) for measured, minimum-acceleration, minimum-jerk, minimum acceleration, and minimum-snap trajectories.

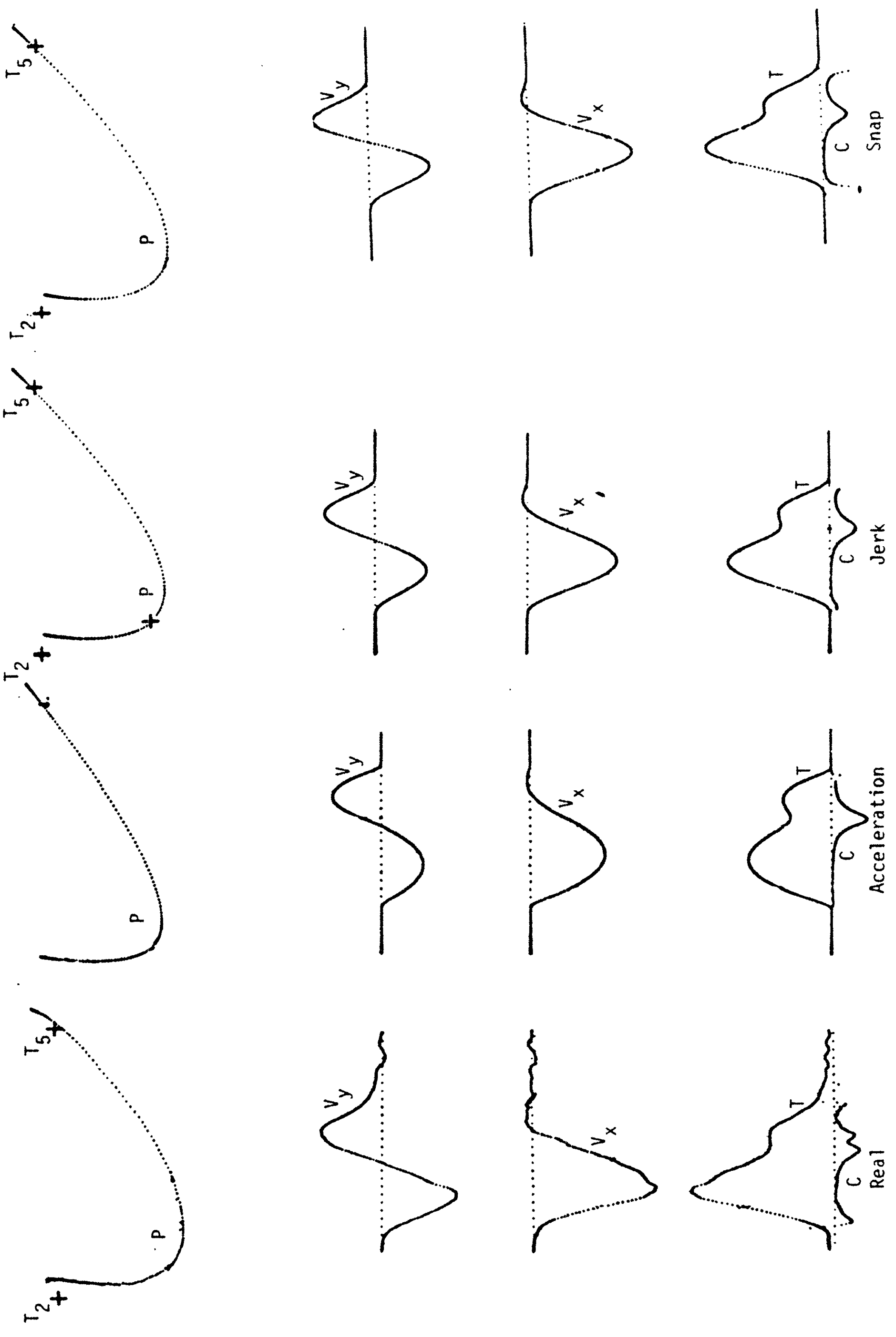


Figure 40. Real, minimum-acceleration, minimum-jerk, and minimum-snap curved trajectories. See Figure 3-39 for legend description.

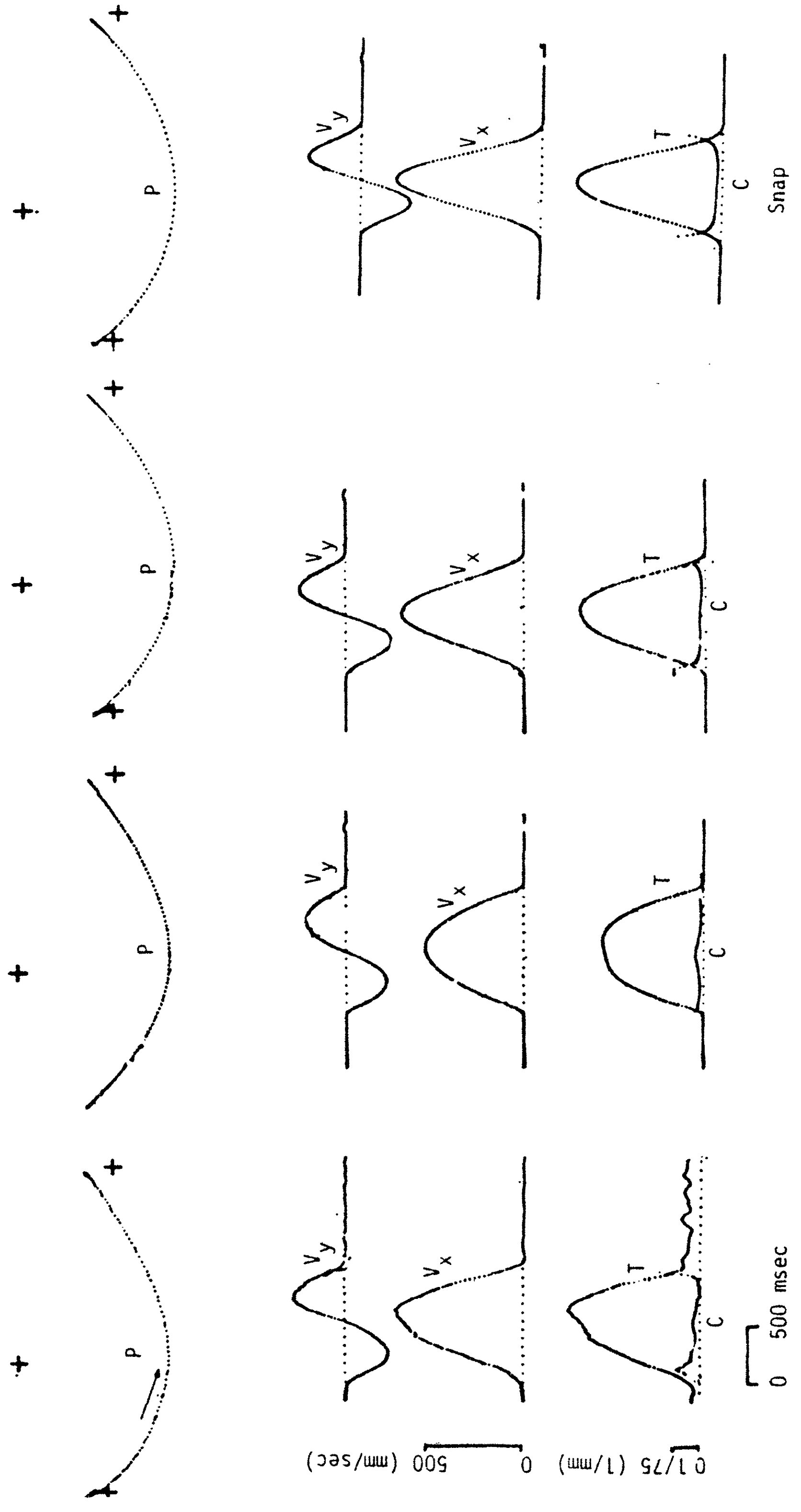


Figure 41. Real, minimum-acceleration, minimum-jerk, and minimum-snap, curved trajectories.

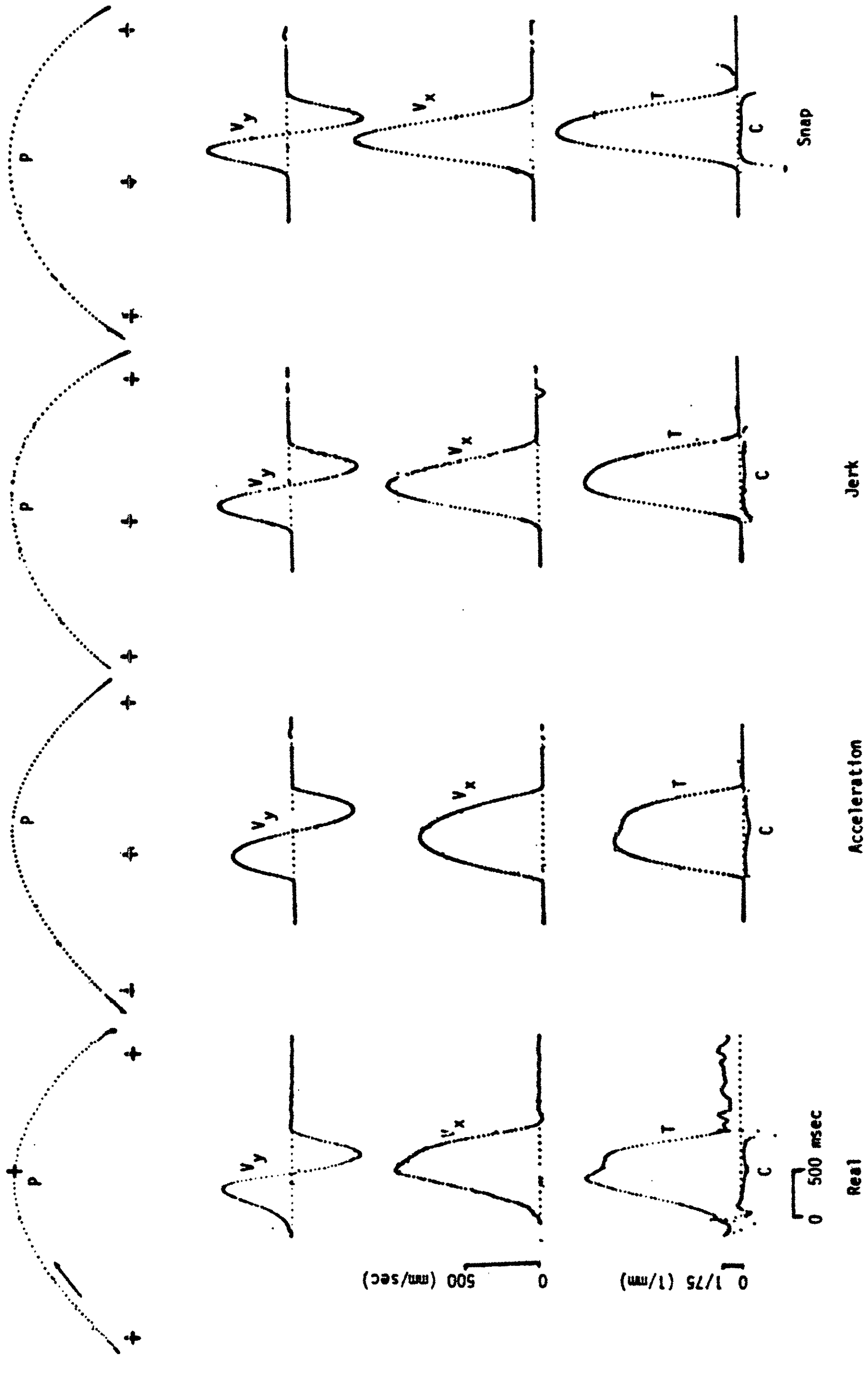


Figure 42. Real, minimum-acceleration, minimum-jerk, and minimum-snap curved trajectories.

than the experimental curves at the beginning and end of the movements (Figures 3-41,3-42). Snap minimization also fails in predicting correctly the right time for the maximum curvature point (Figure 3-40.). In addition, it also fails in matching the shape of the velocity component curves, especially for the axis along which the hand reverses its direction (Figures 3-41 and 3-42). The results shown here are representative of the results obtained with the majority of curved movements. On the whole, therefore, snap minimization succeeds better in matching the experimental results for unconstrained point-to-point movements but jerk-minimization succeeds better in matching curved movements.

3.5. Discussion.

The model presented in this paper yielded a mathematical description that matches observed human planar two-joint arm movements. The optimization technique permitted, through the choice of an objective function, the capture of the global aspects of the movement. In return, it produced movements that displayed path velocity and curvature profiles, very similar to the ones displayed by the real movements. Thus, our choice of objective function was guided by the view that planar arm movements are a work-space invariant, but the success of our optimization model also offers a strong evidence in support of this view. For unconstrained point-to-point movements, the model generated straight paths and hand tangential velocity curves comparable in amplitude and shape to the real hand trajectories. For unconstrained curved and obstacle avoidance movements, again, the predicted trajectories, qualitatively and quantitatively, matched real movements.

In addition the jerk-minimization model enabled us to make a few predictions and, therefore, is more powerful than simple curve fitting, in which an analytical function is fitted separately to each movement record. In doing so, the model enabled us to account for a few kinematic characteristics of movement which have been described in the motor control literature. Among these are the scaling of hand trajectories with time [Schmidt 1980], the "isochrony principle" [Viviani & Terzuolo 1982], and the temporal coupling between hand curvature and speed [Viviani & Terzuolo 1980, Abend et al. 1982]. With regard to the isochrony principle, it has long been noticed, that the peak velocity increases as an approximately linear function of the distance to be traveled, so as to keep the duration of the movement roughly constant [Viviani & Terzuolo 1982]. The evidence for time invariance and its implication for the content of motor programs has been discussed by Schmidt [1980]. Movement scaling with time has also been observed for many different movements including two-joint arm movements, reaching movements in three dimensional space, handwriting, wrist movements etc. [Schmidt 1980]. Temporal coupling between velocity and curvature has also been observed in handwriting and drawing [Viviani & Terzuolo 1980] and in in infant reaching movements [von Hofsten 1979]

Although the model has accounted for the majority of kinematic features displayed in hand trajectories, it did not succeed in capturing some of the more fine-grained details. This included a slight asymmetry in the tangential velocity profile, the small corrections towards the final target and for some movements, the especially high first velocity peak. We have observed, nevertheless, that the asymmetry of the tangential velocity curve is typical for point-to-point movements generated after some practice in performing the experimental task, while the speed profile is quite symmetrical for naive subjects. Therefore, this asymmetry may be mainly due to the subject's effort, to reach the final target more accurately after the movement has become well-practiced. The small hooks at the end of the path also may result from the subjects intentions to correct the errors in reaching the goal target and therefore they may represent fine modifications in the same basic trajectory plan. It is also possible that these small hooks appear due to the elastic behavior of the musculoskeletal apparatus [Delatizky 1982].

The variability in trajectories generated by different subjects or the same subject on succeeding trials, may be due to slight changes in the perceived locations of points in space to which the hand moves, or through which it passes. Another possible source of movement variability is motor variability, possibly resulting from changes in the commanded patterns of muscle activations on succeeding trials for the same subject, or from slight variations in such commands among various subjects. Thus, the fact that the same skill is seldom performed twice in exactly the same way, does not mean that different programs or rules are used each time, but rather that there is more variability in the product than in the program since the final products depend also on the translation of the motor plan into muscular activity and on the interaction of the arm with the environment [Keele 1981].

Comparing the trajectories, which resulted from the three different optimization criteria, jerk minimization was found to offer the best fit to the experimental behavior although some trajectories were more successfully modeled using snap minimization. Nevertheless, given the fact that snap-minimization, for some movements, yielded better results, it would be of interest to further pursue the issue of whether there are specific subclasses of trajectories (fast?, slow?) for which minimal snap and not minimal jerk movements offer a better description. It is also possible that the minimization of some combination of the two criteria, for some classes of movements, succeeds even better in predicting the observed behavior.

Through the definition of the objective function, we were able to derive explicit analytic expressions for the description of many different hand trajectories. The derivation of these expressions depended on the specification of a limited number of parameters relating, for unconstrained point-to-point movements, only to the locations, velocities and accelerations at the two end-points of the movement and to the movement duration. The extension of the model from straight to curved movements required only the addition of two more parameters, the position coordinates of a single via point. In return, the optimization yielded the time at which the hand passes through the via point and a detailed time-history of the hand positions. Therefore, the optimization method brings a dramatic reduction in the dimensionality of the problem of describing movement kinematics. In addition, it did not require including in the model any representation of the arm dynamics, nor using any arbitrary form for the neural input signals as assumed in some other proposed models which attempted to account for movement kinematics [Zangenmeister et al. 1981]. Moreover, this work provides a testable model of the planning and organization of voluntary movements since it is based on a single principle for both single-joint and planar two-joint arm movements.

In chapter 1, three questions relating to kinematic aspects of hand trajectory formation, were raised. Among these questions, the two questions which related to more limited aspects of this topic were: first, why is there a tendency to generate straight paths in point-to-point movements? and second, why is there a temporal coupling between hand speed and curvature? The more general question is, how does the CNS select hand trajectories among the vast number of possible trajectories? In the context of this chapter we will treat only the first two questions. As a result of the success of the optimization model, one may speculate that optimization principles underlie biological trajectory planning and may offer some answer to the third question. However, we leave most of the discussion of this hypothesis to chapter 5, where we will discuss the overall implications of this work.

Only a few models for the mechanisms underlying trajectory formation have been proposed. In a model proposed by Hollerbach [1981], handwriting production is viewed as a constrained modulation of an underlying oscillatory process. Coupled oscillations in horizontal and vertical directions produce letter forms. This theory applies generally to a number of acceleration oscillation patterns such as sinusoidal, rectangular or other oscillations. This model which assumes a piecewise constant modulation of the phase angle of the basic oscillation pattern can generate trajectories with multi-modal curvature and speed profiles. Since our model generates basic polynomial acceleration patterns in the vertical and horizontal directions, similarly to Hollerbach's idea, these can be viewed as the basic patterns underlying the generation of hand trajectories in two-joint arm movements.

Recently the phenomenon of the temporal coupling of hand path and speed has been addressed in a couple of studies [Morasso & Ivaldi 1982, Viviani & Terzuolo 1982]. These two groups of investigators have suggested that this phenomenon is due to a central mechanism which plans trajectories as sequences of movement units or segments. These separate strokes are then superimposed and overlapped in time. Given the tendency to generate roughly straight hand paths in point-to-point movements, Abend et al. [1982] and Viviani and Terzuolo [1982] have suggested that a curved movement may also be planned as a sequence of such segments may also be centrally planned also curved movements. Viviani and Terzuolo [1982] state the claim, in support of the segmentation idea, that movements obey the "Isogony

Principle". This principle states that the ratio between tangential velocity and curvature is piecewise constant. Our analysis of arm movements did not support this finding. Morasso and Ivaldi [1982] also argue that curved movements are generated from separate strokes and they base their hypothesis on the presence of of few peaks in the hand tangential velocity in curved versus straight movements. They do not assume that all the segments are roughly straight. Instead, they suggest, that each stroke is characterized by various geometric parameters, such as length, total angular change etc. Different movements may be comprised from different strokes, with different geometric parameters, and all these strokes are stored in memory. Thus, according to them, even the simplest curved movement is composed of at least two strokes if there is a visible speed valley in the recorded velocity trace. Their model implies also the need for a huge dictionary in human memory, where all possible strokes are stored and from which they can be retrieved.

Both papers [Viviani & Terzuolo 1982, Morasso & Ivaldi 1982], however, do not make it clear whether there exist any rules or principles according to which the kinematic variables, even for point-to-point movements, are chosen, or why there is a tendency to generate straight paths in unconstrained movements. The work by Morasso and Ivaldi, which suggests a computational model for trajectory planning, does not clarify what kind of an algorithm is used to overlap succeeding strokes in time nor does it suggest any explanation for the isochrony principle.

This idea of planning sequences of movement segments, is analogous to ideas which were suggested in order to account for differences in reaction time in utterance of lists of words of various lengths or in generating lists of typed signs. The computational model for human trajectory planning, proposed by Morasso & Ivaldi, also derives from robotics. In robotics, more complicated movements or movements through via points, for artificial limbs, are planned as series of simple polynomials which are spliced together [Brady 1983]. Besides initial and final position constraints on the end-effector, in Cartesian trajectory planning, via point constraints are used to prevent the arm from colliding with an object in the work space. It is always possible to plan a trajectory which can satisfy an arbitrary number of constraints, using a sufficiently high degree polynomial. Nevertheless, the computations of all the parameters of a polynomial trajectory must be repeated from scratch, each time a via or an end point changes. This problem is usually solved by using splines consisting of low degree polynomials smoothly joined together at knot points.

In contrast, although the model presented here uses via points, it applies the optimization procedure globally to the entire trajectory. Going back to the idea of segmentation in biological trajectory planning, if indeed all curved movements are planned by joining a few, less curved segments together, one should expect these trajectories to exhibit a certain kinematic feature. This feature is as follows: if the hand, for example, moves from the initial point to the via point, in the positive direction of both the x and y axes, and reverses its direction of movement along, for example, the x axis, one should expect to find a dip in the y-axis velocity curve corresponding temporally to the valley in the tangential velocity curve. This is so because, according to the segmentation explanation, the hand moves toward the via point almost as if it were the end point of a straight segment. Occasionally, such behavior was observed in a few movement records. It is important to emphasize, however, that the majority of curved movements did not display this expected feature. These movements, too, nevertheless, exhibited a valley in the tangential velocity of the hand. This suggests, therefore, that the slowing down of the hand, more often than not, is not due to segmentation.

The fact that the minimum-jerk predicted trajectories showed a good fit to the experimentally recorded movement, may suggest that free reaching planar arm movements, whether straight or curved, are minimum-jerk movements. Hence, in trying to suggest a physiological interpretation for this qualitative and quantitative good fit, it is tempting to suggest that the CNS uses a trajectory planning strategy analogous to the computational algorithm presented here. Given this speculation, one may hypothesize, therefore, that single-joint and curved and straight planar movements, are planned according to some globally applied optimization objectives similar to the ones presented here. On the other hand, since the planning of long and complicated sequences of movement, may require the selection of an indefinite large number of via points, one might speculate that for the planning of complex movements, the CNS uses a segmentation strategy. Conversely, however, the optimal nature of arm movements may not result from a central computational algorithm of trajectory planning. Instead it may represent the sum total of the structure and rules of operation that have evolved at the various levels of the motor system, which are involved in movement generation. Hence, this behavior may be due to hard-wired rules and not central computational trajectory planning.

The rational for jerk minimization in biological trajectory planning, unlike that for energy or time minimization, does not lend itself to self evident, causal explanations. One might speculate that this strategy has evolved to eliminate the need for abrupt changes in muscle forces, and hence, to minimize the "wear and tear" on the system. Given the fact, however, that the movements under consideration are at moderate speeds, and do not subject the system to a large stress, this does not seem to be a likely explanation. Instead, one might speculate that hand trajectory plans, which require abrupt and large changes in acceleration, would have imposed unwanted, abrupt changes in the forces transmitted to objects carried by the hand. Presently, however, we cannot offer a better explanation for the rationale behind the tendency to generate minimum-jerk hand trajectories.

This work enhances the previously made arguments [Morasso 1981, Abend 1982] that arm trajectories display kinematic invariances only in hand Cartesian motions. The model presented here accounted for these features and predicted other invariant characteristics and movement regularities. Moreover the minimum-jerk model presents an objective function for the description of a diverse class of movements which may indicate an organization principle underlying movement generation.

Chapter 4

Dynamics problems for planar two-joint arm movements

4.1. Introduction

The main objectives of the work presented in this chapter are to evaluate quantitatively the joint torques involved in the generation of planar arm movements and to assess the significance of dynamic interactions between limb segments, during these movements. Another objective is the consideration of plausible mechanisms for dynamic computation or compensation for these forces by the human motor system.

Dynamics converts a trajectory, described in terms of joint angles, rates, and accelerations to the joint torques which are required to generate this trajectory. As discussed in chapter 1, there are two types of problems of dynamics . The solution of the integral dynamics problem yields a trajectory given a time sequence of torque inputs to the joints, while the inverse dynamics yields the required torques, given a trajectory description. Given a trajectory plan, it is, therefore, the inverse dynamics problem which must be solved in order to arrive at the joint torques. The dynamic equations, which yield mathematical expressions for joint torques, can be derived from the application of kinematics, the Newton-Euler or Lagrange equations, and d'Alembert's Principle (Luh et al. 1980).

Although many works dealing with the evaluation of lower extremity joint torques have been published, there exist almost no equivalent studies on arm movements. It is important to evaluate the magnitude of the upper extremity joint torques during movement as an end in itself. Moreover, such information can be very valuable in an investigation of the mechanical function of various arm muscles. Therefore, in this work, general-purpose methods for the solutions of problems of dynamics were developed and were applied to human planar arm movements.

It is important to emphasize, however, that in this work, the torque expressions on which the dynamic computations were based include no explicit terms for viscosity and elasticity due to tendons, connective tissue, and muscles. The elbow and shoulder joint torques calculated in this work, represent only inertial effects. Therefore the contributions from all the above sources must sum up to generate these net torques. If one wishes to determine the actual forces contributed by the muscles, then one should have a suitable model of the viscoelastic effects in order to distribute the joint torques between muscles and viscoelastic forces.

The computational methods used in this study are based on up-to-date dynamic formalisms used in robotics [Luh et al. 1980, Armstrong 1979] and on an anthropometric model of the human body for the derivation of segmental inertial parameters [Hatze 1979]. Nevertheless, in some portions of the computations used here, some of these formalisms were adapted to fit our own purposes. Moreover, a few computational approaches suggested by various authors were combined to achieve a more efficient technique. Since the development of efficient techniques for the dynamic analysis of biological movement comprised an important part of this work, these techniques will be discussed in detail in the methods section. In addition, the various approaches used in this work will be compared to other available techniques.

4.1.1. Arm trajectories, dynamics and interaction forces

As discussed in chapter 1, the presence of interaction torques during multi-joint movements may introduce significantly complicating factor in the dynamics of these movements. On the other hand, if the interaction forces are small compared to other torques, such as inertial non-interaction torques or the gravity torque, their effect on the musculoskeletal apparatus may be negligible. As was mentioned in previous chapters, in the field of robotics it is usually assumed that the inertial torques, which are the torques proportional to joint acceleration, are usually much more significant than the velocity torques, which are the torques proportional to the product of joint velocities of two joints. This argument is used to justify dynamics simplification by eliminating the velocity torques. Given the fact that the solutions of problems of dynamics are highly dependent on the system that generates the movement, the relative significance of various torque terms cannot be assessed independently of this physical system or the movement trajectory. Therefore, in this work, we were interested in assessing the relative significance of all interaction torques in normal human arm movement. Moreover, in order to examine whether the linearization of dynamics can be considered a plausible biological mechanism for dynamics simplification, we were interested to assess the validity of the above preconception in robotics, with regard to biological movement.

If the interaction torques are significant during normal movement, then they will influence the trajectory of the arm. In order to control the path that the hand takes during movement, such as a straight line in point-to-point movements, it would then be necessary to generate the appropriate torques for the movement, which include contributions for the interaction terms. Possible sources for the generation of appropriate torques include open-loop solutions of dynamics, feedback corrections through loops including proprioceptive sensors, and intrinsic muscle or joint stiffness.

One cannot appreciate the character and the complexity of interaction forces without a close inspection of the equations of motion. Hence, in order to clarify the origin of various interaction forces and to demonstrate how they vary with path, a simplified kinematic and dynamic analysis of planar two-joint arm movement will be presented.

4.1.1.1. A simplified model of the upper extremity

A simplified model of the kinematic linkage of the human arm is presented in Figure 4-1. This actually is a very good approximation to the biomechanics of a real human arm. However, to derive more accurate values of joint torque in the actual computations, the computer simulation discussed in the methods section was used. During arm movement the hand describes some plane curve, and the relation between the joint angles θ_1, θ_2 and the position x, y of the hand is given by :

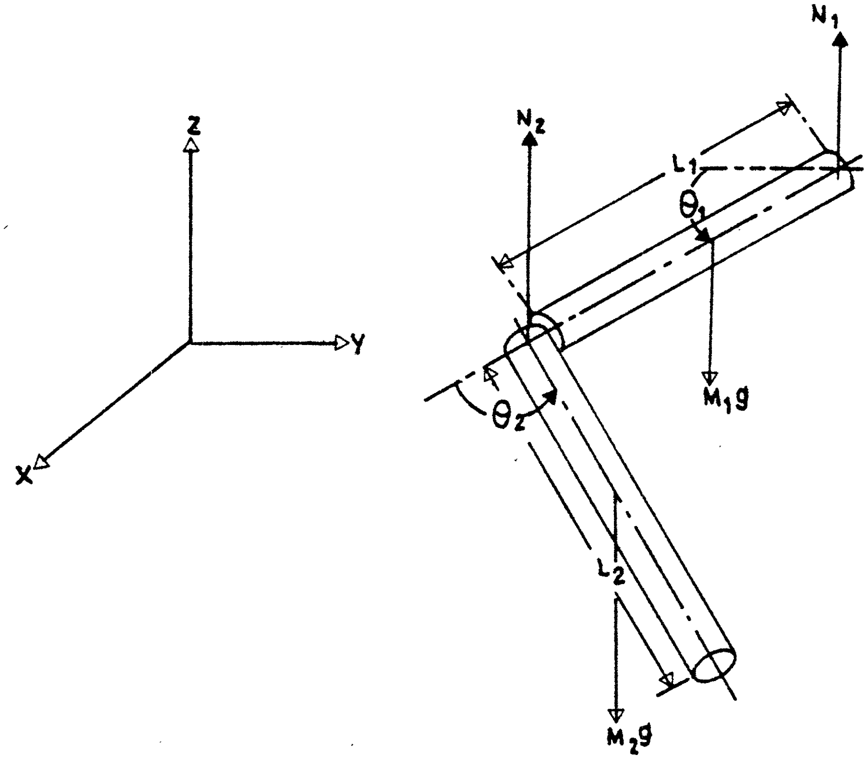


Figure 1. A simplified model of the human arm involving two links with rotary joints.

$$\theta_2 = \cos^{-1}\left(\frac{x^2+y^2-l_1^2-l_2^2}{2l_1l_2}\right) \tag{4.1}$$

$$\theta_1 = \tan^{-1}\left(\frac{y}{x}\right) - \tan^{-1}\left(\frac{l_2\sin\theta_2}{l_1+l_2\cos\theta_2}\right) \tag{4.2}$$

In our simplified model of the arm, based on a straightforward application of kinematics, the Newton-Euler equations, and d'Alembert's Principle, (the general formalism is presented in Appendix E), the resulting expressions for the elbow and shoulder torques, n_2 and n_1, are:

$$\begin{aligned} n_2 =& \ddot{\theta}_1\left(I_2 + \frac{m_2 l_1 l_2}{2}\cos\theta_2 + \frac{m_2 l_2^2}{4}\right) + \ddot{\theta}_2\left(I_2 + \frac{m_2 l_2^2}{4}\right) \\ & + \frac{m_2 l_1 l_2}{2}\dot{\theta}_1^2 \sin\theta_2 \end{aligned} \tag{4.3}$$

$$\begin{aligned} n_1 =& \ddot{\theta}_1\left(I_1 + I_2 + m_2 l_1 l_2 \cos\theta_2 + \frac{m_1 l_1^2 + m_2 l_2^2}{4} + m_2 l_1^2\right) \\ & + \ddot{\theta}_2\left(I_2 + \frac{m_2 l_2^2}{4} + \frac{m_2 l_1 l_2}{2}\cos\theta_2\right) \\ & - \frac{m_2 l_1 l_2}{2}\dot{\theta}_2^2 \sin\theta_2 - m_2 l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \sin\theta_2 \end{aligned} \tag{4.4}$$

where m_1, m_2 are the masses and I_1, I_2 are the rotary inertias of links 1 and 2 respectively. Since the above equations represent only the joint torque components along the joint axes about which the two segments rotate, these torques do not include any contributions due to gravity. In movements which occur in the horizontal plane, since the elbow is a single degree of freedom hinge joint and the axis of rotation is parallel to the gravity vector, gravity exerts no torque perpendicularly to the horizontal plane. Moreover, although the shoulder is a three degree of freedom joint, since both the forearm and the upper arm movements occur in the horizontal plane, gravity again exerts no torque along the shoulder axis that can affect the movement.

The inertial torques, which are proportional to the angular accelerations, include both the normal inertial terms, which represent a single joint movement and an interaction term due to movement of the other link. For the elbow torque expression (4.3), for example, the inertial torque proportional to $\ddot{\theta}_2$ is the normal inertial term which would arise in a single joint elbow movement, while the inertial torque proportional to $\ddot{\theta}_1$ is an interaction due to the shoulder movement.

There are two types of velocity torques, both of which represent interactions. If the torque terms are proportional to a multiplication product of two different joint velocities, these torque terms are called Coriolis torques. In the shoulder torque expression (4.4) the Coriolis torque is proportional to $\dot{\theta}_1\dot{\theta}_2$, but there is no Coriolis torque acting on the elbow joint. When the velocities in the product pair represent the same joint, the terms proportional to this product are called centripetal torques. In the elbow torque expression (4.3), for example, there is a centripetal torque due to the shoulder movement which is proportional to $\dot{\theta}_1^2$. It should be emphasized that although the distribution of the velocity interaction forces between the different terms (i.e. Coriolis and centripetal) at the two joints, may depend on a specific definition of joint angles, the presence of these forces results from true mechanical effects, and is not just an artifact resulting from this definition.

Although all the planar movements studied in the course of this thesis are reaching movements, from a dynamic standpoint, these movements can be broadly classified as whipping and reaching actions [Hollerbach & Flash 1982]. A whipping action involves rotations about the elbow and shoulder joints in the same direction, while a reaching action involves rotations about the elbow and shoulder joints in opposite directions. Some motions are comprised of a whipping action during one portion of the movement, and a reaching action in the other portion, but for most of the arm trajectories measured in the course of this research, this categorization holds. This classification can help to illustrate how for some trajectories the various interaction forces can oppose the movements of the two segments, while for other trajectories they can aid these movements, depending on the interplay between the rotations about the two joints.

During whipping movements the joint velocities of the shoulder and elbow joints have the same sign at the same point in the trajectory, and the joint accelerations have the same sign as well. In terms of the torque at the shoulder joint (4.4), the elbow inertial torque provides an additional acceleration to the upper arm in the direction in which it moves. The same is true for the elbow joint torque. The effect of the velocity torques, which work together because they have the same sign, depends on the shoulder acceleration. For a negative acceleration, the velocity torques aid the upper arm movement, for a positive acceleration the velocity torques oppose it.

During reaching movements the joint velocities and joint accelerations of the shoulder and elbow have opposite signs. In the shoulder torque expression (4.4) the inertial torque from the elbow opposes the movement of the upper arm, while the velocity torques have opposite signs, and therefore, may partially cancel each other out. For one particular movement the expression for the shoulder joint has a particularly simple form because the velocity terms precisely cancel each other out. This is a reaching movement in which the hand generates a straight line path which intersects the shoulder joint. Since the upper arm and the forearm/hand combination in humans are nearly of equal length, the equation of a straight line through the origin is $y = mx$ where m is the slope. Substituting into (4.1) and (4.2) one finds that $\theta_1 = \tan^{-1} m - \theta_2/2$ and after differentiation $\dot{\theta}_1 = -\dot{\theta}_2/2$. From this latter relation and (4.4) it is clear that the Coriolis and centripetal torques at the shoulder precisely cancel each other out. The inertial torques take on a simpler form as well, since $\ddot{\theta}_1 = -\ddot{\theta}_2/2$, which means that the shoulder torque can be computed merely from the time dependence of one of the angle accelerations:

$$n_1 = \ddot{\theta}_1\left(I_1 - I_2 + \frac{m_1 l_1^2 - m_2 l_2^2}{4} + m_2 l_1^2\right) \qquad (4.5).$$

Since in this particular case the shoulder torque has no dependence on the elbow angle θ_2, the upper arm is not affected by the movement about the elbow joint and, therefore, this movement behaves as if it were a single joint movement. The elbow joint torque, however, does not simplify as much.

From the above discussion it is apparent that the effects of various interaction forces are highly dependent on the shoulder and elbow angles and on the directions of the rotations of the two arm segments. Therefore, the tendency to generate arm trajectories which display invariant characteristics in the Cartesian space of the hand and not in the joint space of the arm, increases, rather than diminishes, the complexities of dynamics for two joint arm movements. Moreover, the increased complexity of a two joint movement over a single joint movement is apparent. If the interaction torques are a significant factor during arm movement, a complex burden will be imposed on the motor system in order to control the movement.

Following the methods section, the results from dynamic computations for experimentally recorded arm movements will be presented. These results indicate that the interaction torques are significant for two joint arm movements over a range of movement speeds, and of movement paths, performed in different parts of the work space. In addition, it was found that the relative significance of the various interaction torques does not vary with the speed of movement. A fundamental time scaling property of the dynamics has been identified as the reason for this invariance, and is suggestive of a simplifying strategy for dynamics compensation which may combine kinematic and dynamic aspects of trajectory formation. It is concluded that there must exist control strategies, which either compensate for the presence of interaction torques during the movement itself, or preprogram precise joint torques, including interaction forces, prior to the movement execution phase.

4.2. Review of methods for dynamic computations

As was mentioned above, to perform dynamic computations, one has to apply principles of kinematics and of dynamics. The data required for the solutions of the dynamic problems consist of the kinematic parameters of movements and of the inertial parameters of the moving segments. As part of this work, general-purpose computer simulation programs for dynamic computations were developed. To justify our choice of particular computational algorithms, and of particular methods for data gathering, the advantages of these algorithms and techniques, compared with other available approaches, will be briefly described.

4.2.1. Application of kinematics and kinematic measurements.

It is assumed that each body segment behaves, approximately, like a rigid body [Alexander 1982]. To compute dynamic forces for a kinematic chain, such as the human arm, it is useful to define a few link-fixed coordinate systems. Since in most arm or lower extremity movements, one limb segment is rotating relative to the preceding segment in the chain, such a definition helps to simplify the derivation of kinematic variables for all links. The orientation of one coordinate system with respect to another coordinate system can be described by Euler angles. In this work, we defined 3 link-fixed coordinate systems, for each link, based on the convention of Orin et al. [1979] (see Appendix D). The movement of each link, relatively to the preceding link, can be either a translation movement or a rotation. Hence, if the translations and rotations of all links in a kinematic chain, relatively to their preceding links, are known, one can obtain the positions, orientations, linear and angular velocities and accelerations of all links. The equations which enable to derive these kinematic variables are described in Appendix E.

Various techniques have been developed to measure the positions in space and orientations of human body segments during movement [Hardt 1978]. A highly advanced system, currently available, the Selspot System, enables one to measure three dimensional movements of limb segments, treating each link as a separate rigid body regardless of the linkage which leads to this segment. Nevertheless, for the purpose of measuring two-joint planar arm movements, the simple apparatus which was already described in chapter 3 is accurate enough.

4.2.2. Body segmental parameters.

Body segmental parameters refer to the masses, lengths, the locations of the center of mass and the inertia tensors of various body segments. A few methods have been developed for the derivation of inertial parameters for various body parts. Some of these methods require measurements performed on human cadavers [Braune & Fischer 1889] or on human subjects [Drillis & Contini 1964], while other methods are based either on mathematical modeling of body segments [Hannavan 1964, Hatze 1979] or on optimization techniques [Vaughan et al. 1982]. The measurements of masses, centers of gravity and moments of inertia, performed on a few human cadavers, were reduced to a series of coefficients [Braune & Fischer 1889] or regression equations [Dempster 1955] which, when applied to living subjects, permitted the derivation of their corresponding inertial parameters. Since there are postmortum changes in the masses and shapes of body segments, the application of data derived from cadavers, to living subjects, results in large errors. Direct measurements on living subjects [Drillis & Contini 1964, Hatze 1975] enable us to derive these parameters only for distal body segments (e.g forearm). Hanavan [1964] has proposed a computational technique in which each limb segment is modeled as a simple truncated cone and a uniform density is assumed for all body segments, hence his model is far from being realistic. Hatze [1979] has introduced an improved computational model which is claimed to have a worst-case relative error of 5%. While it is unlikely that his technique yields precise parameters, given the flaws of all the other techniques, it is the best available. Another technique, recently proposed by Vaughan et al. [1982], requires the minimization of the differences between calculated and measured forces on the most distal segment of a limb, and is therefore more appropriate for the calculation of the lower extremity inertial parameters.

4.2.3. Algorithms for dynamic computations.

Since the Newton-Euler formalism was used in this work, we will not discuss the Lagrange formalism any further. In the Newton-Euler formalism, the description of motion is broken into the linear motion of the center of mass, and a rotation about the center of mass. The total force acting at the center of mass is related to the acceleration of the center of mass by Newton's second law. The total torque about the center of mass is related to the angular velocity and angular acceleration by the Euler equation. The basic dynamics equations of the Newton-Euler formalism are presented in Appendix E. The computational algorithm used here is based on recursive computations [Orin et al. 1979]. The computations are made by progressing from the base of the chain to its last member, to compute the velocities and accelerations of all the members, and moving in the opposite direction to compute joint forces and torques. Recursive dynamics computations were first proposed by Orin et al. [1979] but their method has the disadvantage that the forces are referred to the base coordinates. Luh et al. [1980] have used a similar algorithm for the inverse dynamics problem but in their formalism, all input joint torques are represented in their own coordinate system. Angular and linear velocities and accelerations are all calculated in link coordinates. This dramatically reduces the number of coordinate transformations and allows the inertia tensor to be fixed in each link coordinate system. In this work, therefore, to solve the inverse dynamics problem, we have developed general-purpose computation programs based on the recursive formalism of Luh et al. The basic equations of this formalism are presented in Appendix E.

Armstrong has applied the recursive Newton-Euler formalism to the integral dynamics problem but his formalism refers to coordinate systems located at the links proximal joints, while Luh's forces and kinematic variables are referred to the center of mass. The integral dynamics computations used in this study are based on Armstrong 's formalism. Nevertheless, to make these computations compatible with the inverse dynamics computations, Armstrong's formalism was modified, referring it to the coordinate system which is located at the link center of mass. This modified formalism is described in Appendix F.

4.3. Methods used in this study

In order to compute joint torques and to assess the significance of the interaction forces, measurements of human arm movement in a horizontal plane involving the shoulder and elbow joints, were made. Using the general-purpose simulation program, the resultant kinematic data on the time sequence of joint angles, joint velocities, and joint accelerations were converted into a time sequence of joint torques by solving the inverse dynamics problem. In addition, special subroutines were written to compute the magnitude of the various interaction terms. Upper arm and forearm segmental parameters were derived using a computational model by Hatze [1979].

In order to investigate the effect that the omission of various interaction terms has on the resulting trajectories, simulated trajectories were obtained by solving the integral dynamics problem. By subtracting either the velocity terms, or all the interaction terms, from the net torques which were derived from the solution of the inverse dynamics problem, command torques were obtained. Using these torques as inputs to the integral dynamics computation module, we were able to predict what the trajectories would have looked like had there not been provision for the interaction terms.

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4.3.1. Measurement apparatus

Arm movements were measured with the same apparatus that was used in the experiments described in chapter 3. Similarly to the experimental paradigm used in those measurements, a seated subject gripped the manipuladum handle and moved it in a horizontal plane between specified targets (Figure 1-1 chapter 1). Shoulder movement of subjects was restrained by means of straps to the back of the chair, and the wrist was immobilized by bracing. To obtain hand positions and velocities and elbow and shoulder angles and angular velocities, the same data

gb2ps.zip

analysis program as the one described in chapter 3 was used. This program yields these values at a sampling rate of 100 per second.

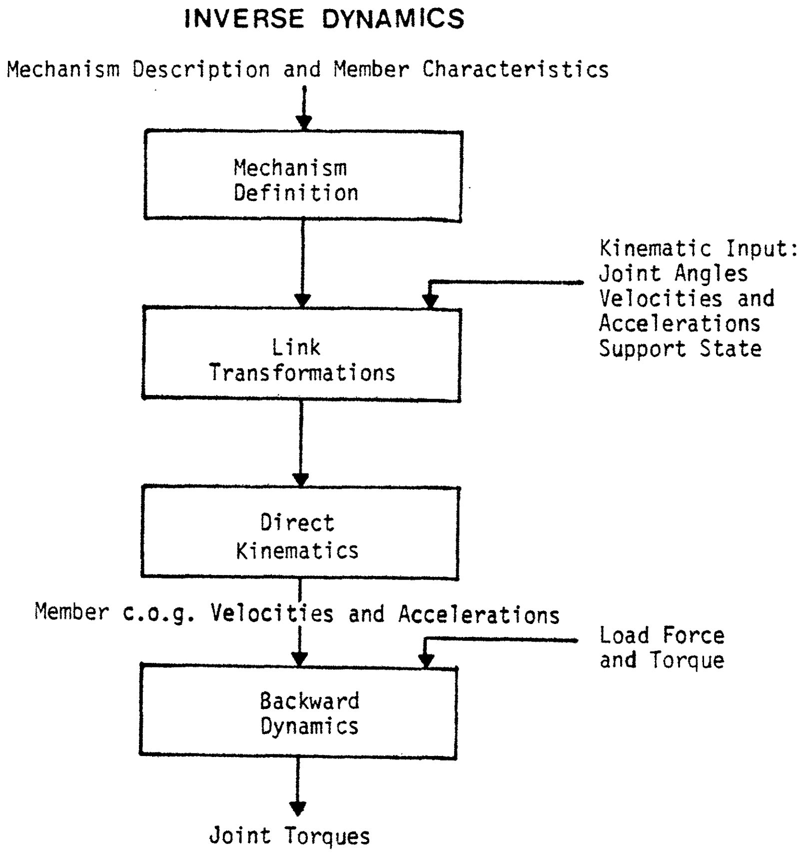
Six LED targets, which are numbered consecutively for reference purposes, from 1 to 6 in the clockwise direction starting from directly in front of the shoulder (Figure 1-1 chapter 1), were mounted on a plexiglass cover just above the apparatus. Subjects were asked to move their arms between pairs of targets at various speeds. The desired movement duration was indicated to the subject by illuminating the target towards which the next movement had to be generated, for a time period, equivalent to this desired duration. The movement durations ranged from about 0.4 to 1.0 second. No explicit instructions were given to the subject with regard to the type of path between targets. Two subjects were tested.

4.3.2. Determination of Segmental Parameters

According to the method developed in [Hatze 1979], the human body is divided into 17 segments, and the shoulders are modeled as separate entities. The method subdivides segments into small mass elements of various geometrical shapes, thus, allowing for the shape and density fluctuations of segments to be modeled in detail. In general, no assumptions are made on segmental symmetry. Furthermore, the model differentiates between male and female subjects and accounts for the specific body characteristics of each subject. The computations of body parameters are based on a battery of anthropometric measurements taken directly from the subjects. These include the varying lengths and circumferences of the arm segments, widths of segments, etc.

4.3.3. The Simulator

The simulator, diagrammed in Figure 4-2 has two complementary parts. The first part allows computation of the inverse dynamics, based on the formalism in appendix E. The second part allows computation of the integral dynamics, based on a formalism adapted from Armstrong [1979] (see Appendix F). The computer simulation programs were implemented on a LISP machine in the Artificial Intelligence Laboratory. These programs were written as general-purpose programs so as to allow the user to apply them to any system of mechanical chains.



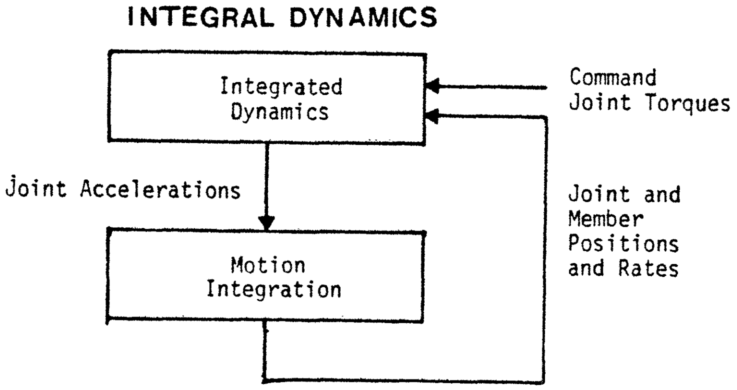


Figure 2. Block diagram of the simulator for open loop kinematic chains.

Secondly, these simulation programs were applied to analysis of human planar arm movements.

The first module of the simulator sets up the mechanism definition. This includes (1) geometric properties such as the number of chains, the number of links in each chain, internal link coordinate systems, relative movement axes between the links, and internal link lengths, and (2) the inertial properties of mass, location of the center of gravity, and principal inertias for each link. Three coordinate systems associated with each link are defined following the convention suggested by Orin et al. [1979]. These systems include a proximal system, located at the proximal joint of the link, a distal coordinate system, located at the distal joint, and a principal coordinate system aligned with the principal axes of the link, with its origin located at the link center mass. These coordinate systems are described in Appendix D.

The next two modules of the simulator involve a kinematic analysis of the motion. From the link coordinate definitions and a knowledge of joint angles, the transformation matrices between link coordinate systems, using Euler angles, and between a given link proximal coordinate system and the reference base can be computed. These 3x3 transformation matrices are defined in Appendix D. The angular and linear velocities and accelerations can then be computed recursively from the base to the most distal link. The movement of the base must be specified as the initial conditions for this computation.

The backwards dynamics module computes the joint torques and forces recursively from the most distal member towards the base (see Appendix E). The initial conditions required for this computation are any forces or torques exerted by the environment on the chain last link. For free arm movement these external forces and torques are zero.

The integral dynamics modules are shown in the lower part of the block diagram. The integral dynamics solution proceeds by formulating a linear relation between the link linear and angular accelerations and also a linear relation between the link joint forces at the proximal joint and the linear accelerations [Armstrong 1979, Walker et al. 1981]. The coefficients of the linear relations are computed recursively, distally to proximally. After specifying the initial positions and velocities of all links, the joint accelerations are computed from torques applied at the joints (see Appendix F). Integrating by means of a truncated (first order) Taylor expansion, the velocities and positions of links are obtained at the next time.

Elbow and shoulder forces and torques were computed for arm configurations sampled at a sampling rate of 100 per second. These torques and forces were first computed, with reference to the respective proximal coordinate systems of the upper arm and forearm and through the available transformation matrices were referred finally to the laboratory-fixed inertial coordinate system.

The computed values of the shoulder and elbow joint torques which are presented in the next section correspond only to the z components of these torques. As was discussed above, these are the only torque components which affect the movement. Estimates of the gravity torque will be considered in the context of the magnitudes of the interaction torques for comparison purposes.

To compute interaction forces, special subroutines were developed as part of the computer simulation programs. Since the gravity torques were excluded from the net joint torques, by considering the two joint velocities to be zero, the inertial torques for both joints were computed. One of the joint accelerations was then considered to be zero, and the inertial interaction torque at this joint due to the other joint acceleration was computed (e.g. the torque term at the elbow in equation 4.3, which is proportional to the shoulder acceleration, $\ddot{\theta}_1$). The repetition of this procedure, this time by considering the other joint acceleration to be zero, yielded the other inertial interaction torque. Next the two joint accelerations were considered to be zero, and the net velocity torques at each joint were calculated (at the elbow this torque is equivalent to the centripetal torque). The Coriolis torque at the shoulder (the term in equation 4.4 which is proportional to $\dot{\theta}_1\dot{\theta}_2$) was then computed. Finally, the subtraction of this term from the net shoulder velocity torque yielded the shoulder centripetal torque.

4.4. Results

4.4.1. Interaction torques during reaching and whipping movements

All measured movements exhibited straight hand paths. Representative

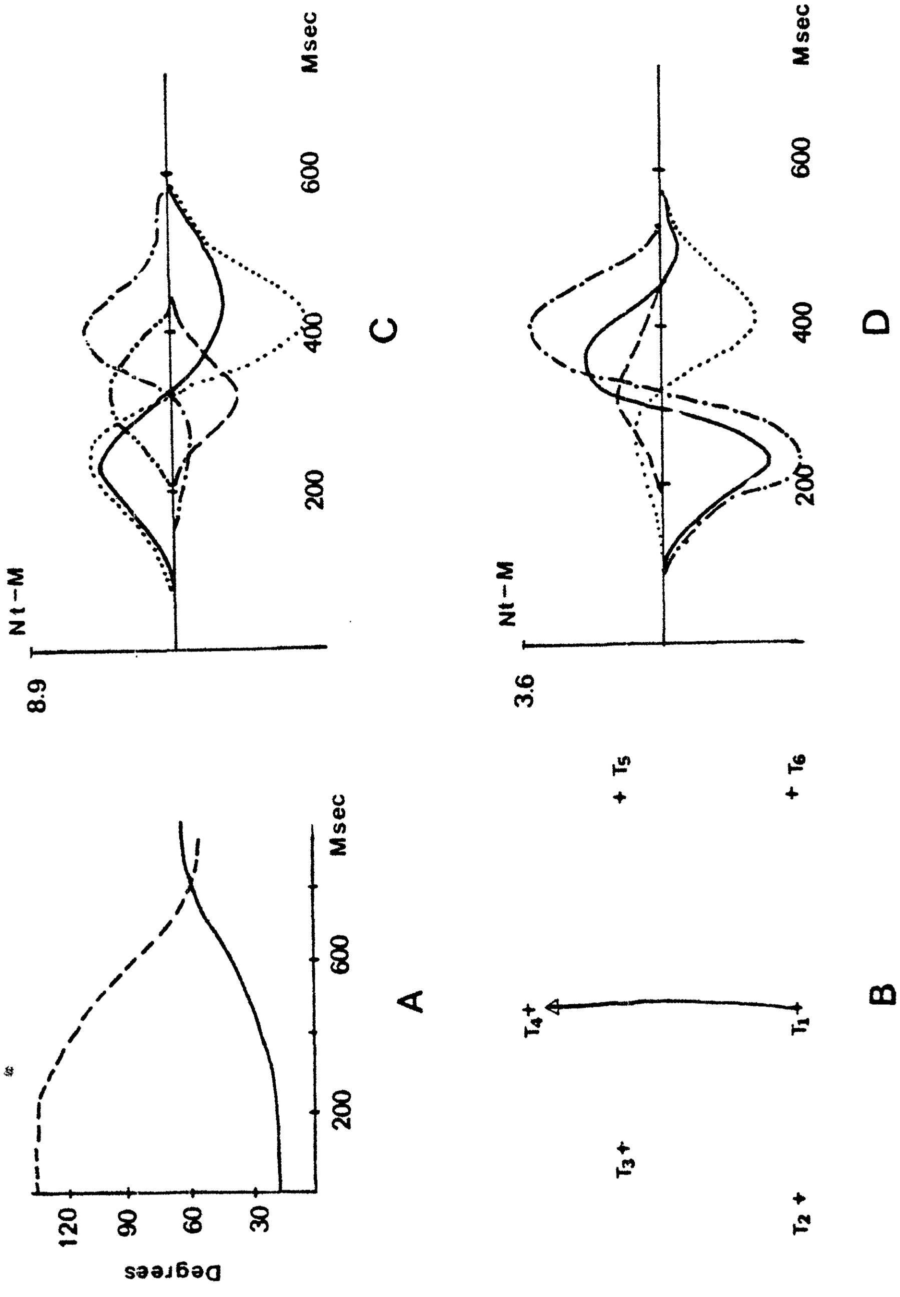


Figure 3. (A) Joint angle plots for the elbow (dashed lines) and shoulder (solid lines) joints for a 0.5 second movement from target 1 to target 4. (B) A plot of the hand path between the targets. (C-D) Components of joint interaction torques at the shoulder (C) and at the elbow (D). Solid line: the net torque at the shoulder in (C) and at the elbow in (D). Dotted line: the shoulder inertial torque. Alternating dots and dashes: the elbow inertial torque. Dashes: the centripetal torque due to the elbow in (C) and due to the shoulder in (D). Two dots with a dash: the Coriolis torque at the shoulder in (C).

movements of reaching actions will be shown first. Figure 4-3 shows a representative 0.5 movement from target 1 to target 4. The plots of shoulder and elbow angles in Figure 4-3A show that the two angles change in opposite directions; therefore, this movement can be classified as a reaching action. The shoulder angle, however, changes less than the elbow movement, and the shoulder angular velocity, especially in the first half of the movement, is approximately half the elbow angular velocity.

The profiles for the net shoulder torque (solid line) and for the shoulder interaction torques due to the forearm movement are shown in Figure 4-3C. The torque profiles for the elbow are shown in Figure 4-3D. As can be seen in Figure 4-3C, the Coriolis (two dots with a dash) and centripetal (dashes) torques at the shoulder cancel each other out, as was predicted for straight line paths through the shoulder. The elbow inertial torque at the shoulder (alternating dots and dashes), on the other hand, represents a substantial fraction of the net shoulder torque and its peak amplitude is even slightly higher than the shoulder maximum net torque. At the elbow the net elbow torque (alternating dots and dashes) is generated predominantly by the elbow inertial torque during the first half of the movement, but in the second half of the movement, the shoulder inertial torque term at the elbow (dotted line), contributes substantially to the net torque. The contribution of the centripetal torque at the elbow is relatively significant, comprising about 30% of the maximum net elbow torque.

A representative 0.5 second movement from target 3 to target 6 is shown in Figure 4-4. The plot of joint angles in Figure 4-4A indicates that this movement involves predominantly shoulder rotation. This movement can also be classified as a reaching action. In Figure 4-4D the elbow torque plot shows the most complexity in the interaction terms. The inertial and centripetal torques at the elbow, due to the shoulder, represent a significant fraction of the net elbow torque. This can be explained based on the high rate of change of the shoulder angle. For this movement, the maximum centripetal torque is as large and even slightly larger than the maximum net elbow torque. In contrast, for this specific movement, the shoulder torque plot in Figure 4-4C indicates a small contribution, due to interaction terms from the elbow, which can again be explained on the basis of the small

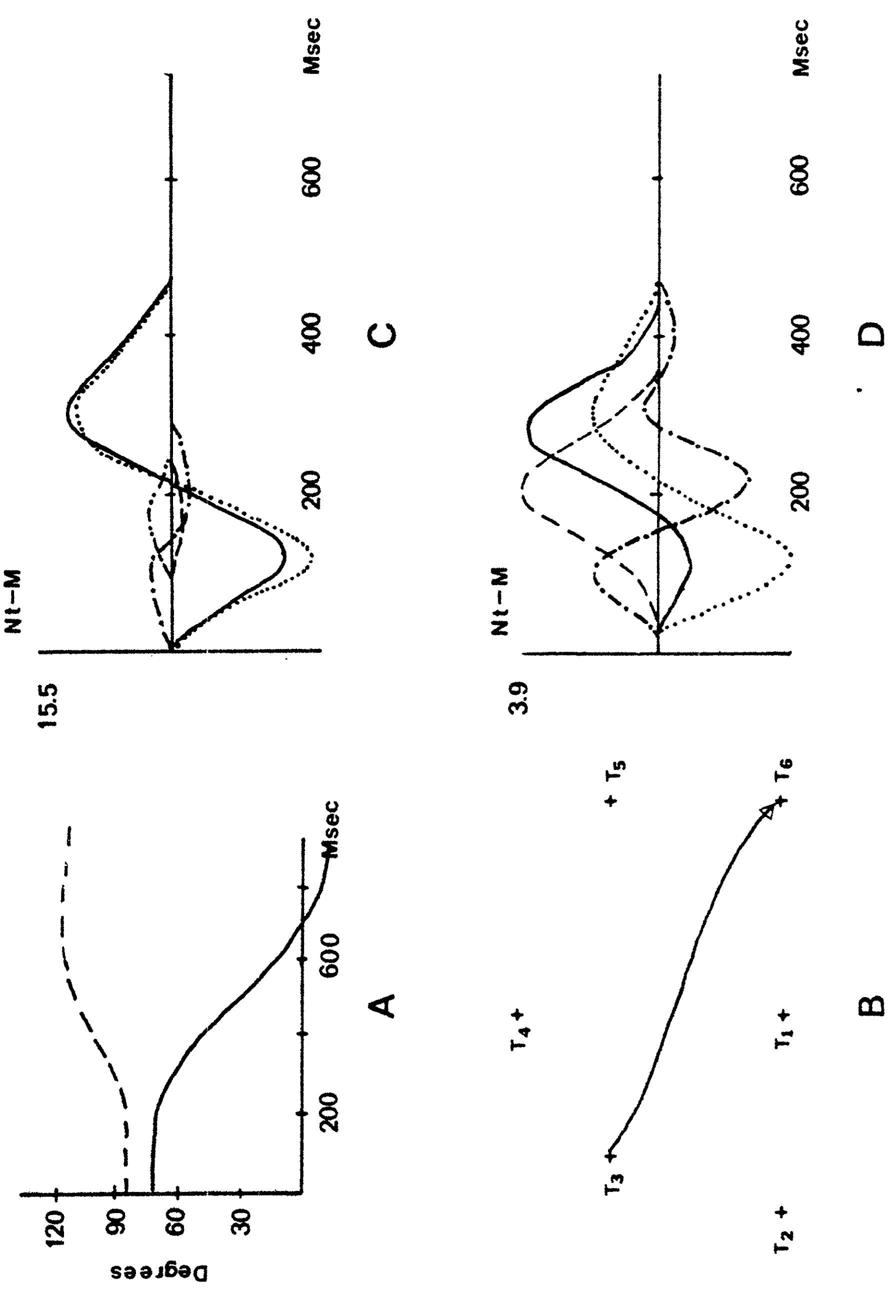


Figure 4. Plots of joint angles, hand path, and interaction torques for a 0.5 second movement from target 3 to target 6. See the previous figure legend for the description.

and relatively slow change in elbow angle. The centripetal torque is almost zero, whereas the elbow inertial torque and the Coriolis torque individually reach 20% of the maximum net shoulder torque. For this particular movement, therefore, the torque profile of the shoulder is close to that for a single joint movement, whereas the elbow torque profile is quite complicated.

The torque profiles for a representative 0.5 second movement from target 2 to target 5 are shown in Figure 4-5. The joint angle plot Figure 4-5A. indicates that the shoulder and elbow joints rotate in the same direction, which means that this movement is a whipping action. Moreover, in this movement, the maximum rate of change of elbow angle is roughly twice the rate of the shoulder angle. As was predicted for such movements in the introductory section, the velocity terms for the shoulder reinforce each other. Moreover, because of the above ratio between the maximum velocities of the elbow and shoulder, the maximum amplitude of the centripetal and Coriolis torques at the shoulder is roughly equal. The inertial torque profiles, however, do not clearly indicate a reinforcing effect because of a delay in the onset of elbow movement relative to shoulder movement. The movement path is oblique as in the 3 to 6 movement, but with a reversed direction with regard to the positive y axis. In spite of the similarity in hand paths for these two movements, the torque profiles between these two movements at the shoulder and elbow, are very different.

4.4.2. Dependence of interaction torques on speed

To examine whether, and to what extent, the significance of the various interaction torques depends on speed, a 1.0 second movement between targets 3 and 6 is presented in Figure 4-6. The shapes of the torque profiles in Figure 4-6 are similar to those in Figure 4-4, which represents a movement along the same path but at twice the speed. As can be seen from comparison of the torque profiles in the two figures, the velocity terms seem to have the same significance relative to the inertial terms regardless of the speed of movement. Moreover, the relative contribution of all the torque terms does not change with speed. This was proved to be true for all the movements measured here, over the entire range of measured speeds. This finding, which was first observed by examining the torque profile results, was

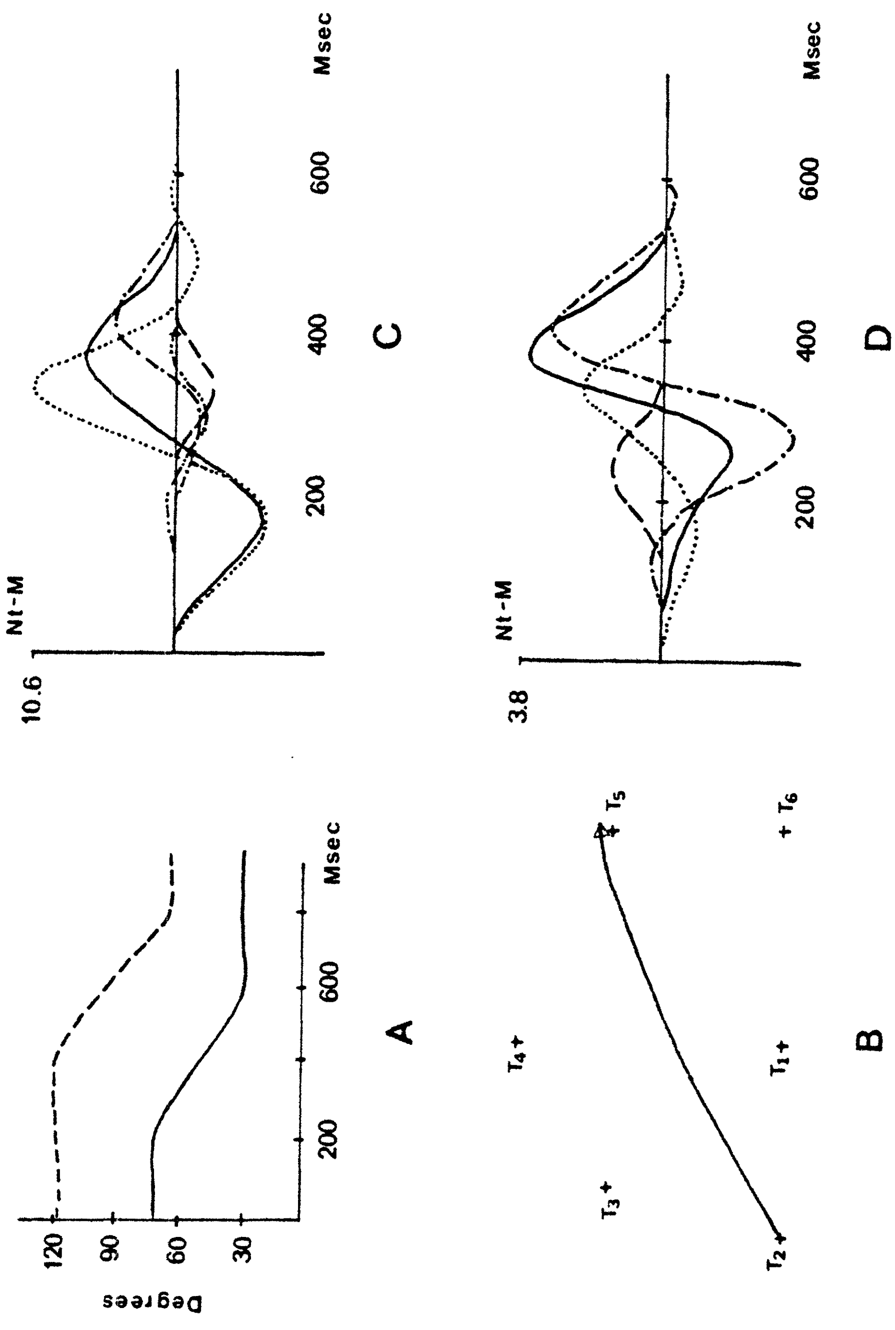


Figure 5. Plots of joint angles, hand path, and interaction torques for a 0.5 second movement from target 2 to target 5.

rather surprising given the preconception that the velocity torque terms become significant only at relatively high speeds. The interpretation and implication of this observation are discussed in a following section.

At the point of greatest angular velocity, the angular acceleration passes through zero to deceleration. This means that the joint torques at the approximate movement midpoint are dominated by the centripetal and Coriolis interaction terms, because the velocity terms are greatest when the inertial terms are zero. At the elbow joint this effect is even more dramatic than for the shoulder because of the large centripetal torque component due to the shoulder movement (see Figure 4-5). Therefore, the velocity terms have a dominant effect at the time that the other interaction terms nearly vanish.

4.4.3. The gravity contribution

The gravity contribution averages about 8 newton-meters at the shoulder, which is greater than the net shoulder torque for the slow movement, but less than the net shoulder torque for the fast movements. The gravity torque at the shoulder is larger in arm movements which occur in the horizontal plane at the level of the shoulder, than for any other movement. Nevertheless, it is clear that even for movements in this horizontal plane, the interaction forces are significant relative to gravity and for the faster movements, significantly dominate gravity.

4.4.4. Simulated trajectories without interaction terms

The results in this part of the work corroborated our previous findings that subjects tend to generate straight hand paths in movements between two targets. To examine what a hand trajectory would have looked like had there been no provision or compensation for joint interactions, movements were simulated with modified torques which did not contain these torque terms. The modified torques were generated from the net torques of recorded movements by subtracting from the net torques first, all velocity torques, and second to all interaction terms. The simulated trajectories which result from the modified torques are shown in Figure 4-7 for the fast movements between targets 3 to 6 (Figure 4-7A) and targets 2 to 5 (Figure 4-7B). If we examine Figure 4-4C again, the removal of the velocity torques

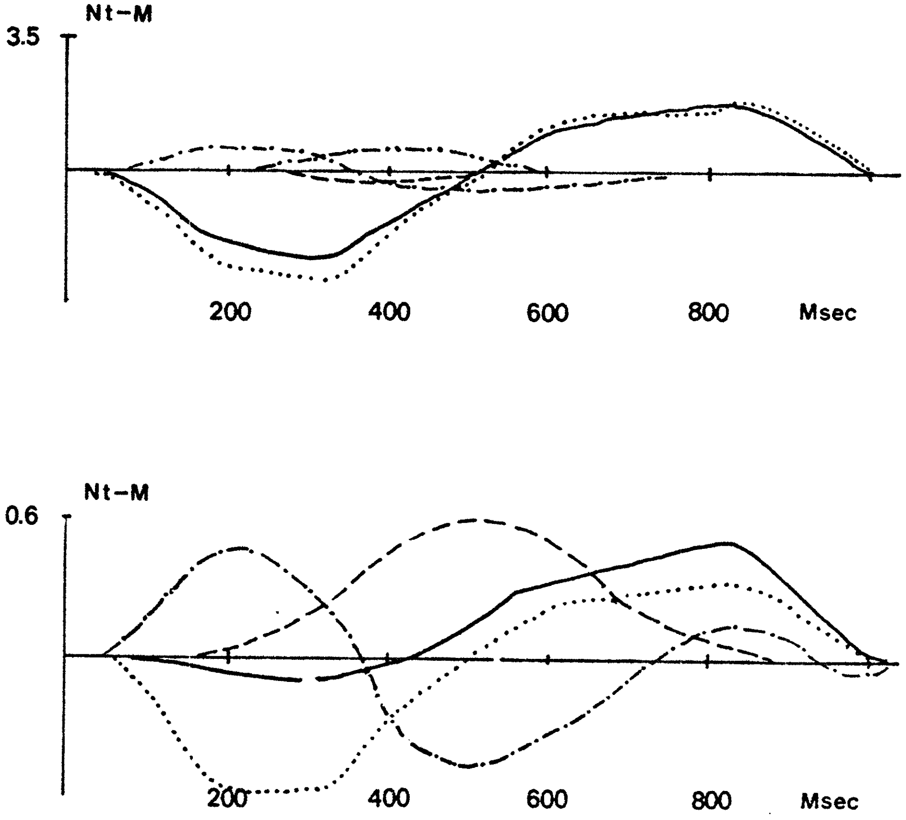


Figure 6. Components of joint interaction torques for a 1.0 second movement from target 3 to target 6.

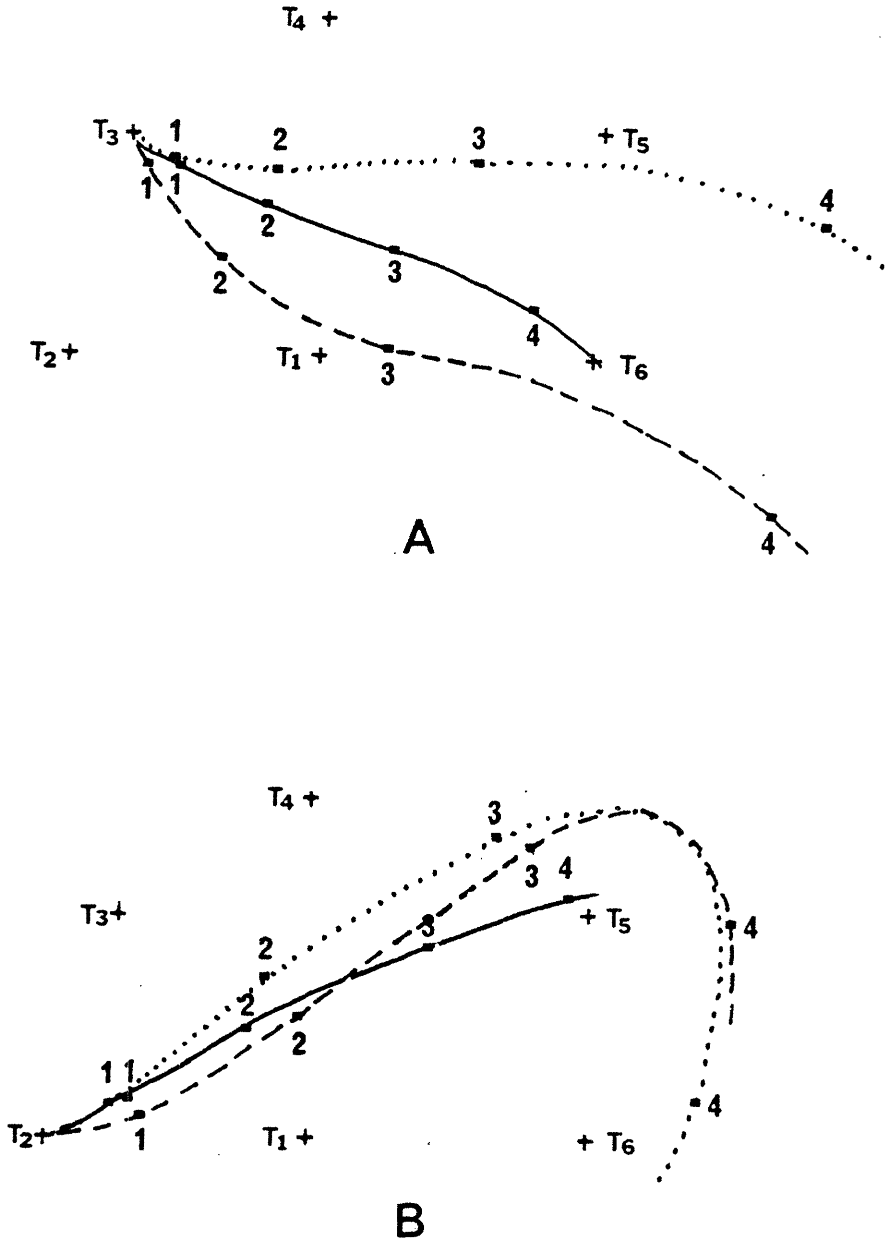


Figure 7. Synthetic trajectories generated from a measured trajectory of the hand (solid line) from target 3 to target 6 (A) and from target 2 to target 5 (B). By removing contributions to the net joint torques due to all velocity terms (dotted line) and due to all interaction terms (dashed line), trajectories were obtained to indicate how the movement might deviate from a straight line. Corresponding points in the trajectories spaced equally in time are indicated by numbered points.

from the elbow net torque will result in a more negative net torque during most of the movement, in contrast to the positive values of the net elbow torque when the centripetal term is not removed. Since, in the 3 to 6 movement, the movement involves predominantly elbow rotation, the elbow will extend and the hand will flail away from the body. On the other hand, if all the torque terms are removed, the net elbow torque in the beginning of the movement will still be positive, and then will be negative. Hence, in the beginning of the movement, the elbow will flex more strongly compared to the real 3 to 6 movement but in the second half of the movement will reverse direction. These effects of the removal of the various torque terms can indeed be seen in Fig 4-7A.

The removal of velocity torques for 2 to 5 movement should not have such a dramatic effect on the hand trajectory because, as can be seen from Fig 4-5C and Figure 4-5D, these torques contribute less significantly to the net torques both at the shoulder and at the elbow. Since the relative significance of the inertial interaction torques at the two joints is also less than for movement 3 to 6, the hand trajectory will not deviate as much from the real trajectory as for a movement between targets 3 to 6. The results in Figure 4-7B substantiate these predictions. Nevertheless, both examples show that there are substantial deviations from the target and from linearity. Even more pronounced than the spatial deviation, is the temporal deviation, since in the vicinity of the end point the movements are not at rest. When the torque profiles are generated, without provision for interaction terms, there is no guarantee that the movement will stop at the desired end point.

4.5. Discussion

The work presented in the previous chapter has supported the view that a human arm movement, in a task which involves pointing or reaching, results in a straight line path of the hand [Morasso 1981, Abend et al. 1982]. This observation has been substantiated by the movements which served as a basis for dynamic computations in this study. The results show that the interaction torques are significant relative to gravity for normal movement speeds. In addition, the Coriolis and centripetal torques have the same significance relative to the inertial torques at all movement speeds. The velocity interaction torques in fact completely dominate the dynamics at the movement midpoint because the inertial torques go through zero as the movement switches from acceleration to deceleration and the arm is moving the fastest. Simulated trajectories obtained by eliminating various interaction terms from the computed torque profiles of measured trajectories show substantial deviations from the measured straight line paths. These deviations of the simulated trajectories appeared in the simulated trajectory immediately at the onset of the movement and the simulated trajectory failed to stop at the vicinity of the end-point.

Given the significance of the interaction torques in normal human movements, it is apparent that they introduce substantial complications to the control of multi-joint versus single-joint movements. Therefore, it is questionable whether the control strategies which have evolved for the generation of single joint movements are the same strategies used to control multi-joint movements. This issue is discussed at greater length in the discussion chapter. Not only are the interaction torques significant, their presence adding, therefore, to the movement complexities, but in addition, for movements along straight line paths in different regions of the work space, the torque profiles are quite different. Therefore, although the invariant hand path is a simple straight line, there is no general rule for the generation of appropriate joint torques for point-to-point movements in different parts of the work place. Because the contribution of interaction forces is non-negligible, and in the majority of the movements is even substantial, and because the motor control system seems to generate movements which exhibit hand path invariances for all regions of the work space, one cannot argue that it does not do an accurate enough job in accounting for the dynamic interaction between limbs. It can be concluded, therefore, that the human motor system must have devised some means for precomputing or for compensating for the dynamic interactions.

4.5.1. Mechanisms for provision or compensation for dynamic interactions.

In this section we discuss the plausibility of open-loop and feedback control mechanisms in providing or compensating for dynamic interactions. One can suggest that biological motor feedback is important in correcting for simplified preprogrammed solutions of the inverse dynamics problem. Hence, one can propose a biological control model in which a simplified solution of dynamics, which omits interaction torques, is corrected through feedback loops involving proprioceptor sensors. An alternative strategy is an open-loop preprogramming of movement dynamics. A third alternative, such as final position control was shown to be not feasible since simulated movements based on such a control model do not display hand paths and tangential velocity curves similar to those generated by humans [Delatizky 1982]. One factor which was found to cause major deviations of the simulated trajectories from straight paths is the presence of interaction forces.

In the field of robotics, the most common method for controlling manipulator dynamics is to generate nominal torques based on a linearization of the dynamics and to correct errors due to interaction terms and other effects by feedback. The feedback is usually structured as an independent joint controller [Golla et al. 1981], in which, for purposes of design the manipulator is considered as composed of independent joints which do not interact. It was discussed, however, in chapter 2 that at high speeds, because velocity torques become significant, the requirements for high gains of the feedback control loops lead to instability. Furthermore, from the results presented here, it can be seen that for straight end-effector trajectories similar to those generated by humans, since the significance of velocity torques relative to inertial torques does not depend on speed, linearization of dynamics may introduce the same relative errors at slow speeds as at high speeds. As will be shown in the next section, this conclusion can be generalized for an even wider class of end-effector trajectories.

Moving next to a discussion of the feasibility of feedback corrections for the omission of interaction torques, it seems that for biological arm movements this is not a viable option. One may argue that proprioceptive sensors are sensitive enough so as to detect errors in muscle length (through α γ coactivation) or joint angles (by monitoring signals from joint receptors and from muscle spindles) or that reflexes have high enough gains (see evidence against it in chapter 2). Nevertheless, given the substantial delays which are involved in reflex loops, for faster movements by the time that the correction signal would have resulted in a modified command for muscle activation, the system would have evolved to a new state for which the corrective signal is inappropriate. Supraspinal reflex loops have delays of about 70 msec (until the detection of M2 EMG response). Hence, for fast movements in the range of 500-600 milliseconds, similar to the movements which were measured here, the supraspinal loop delay is too long to correct errors resulting from dynamics linearization. Given the relatively short delay which is involved in the monosynaptic stretch reflex (e.g. 20 msec for the biceps) one can suggest that the monosynaptic stretch reflex can serve in the role of a feedback controller. Nevertheless, given the large errors in the trajectories which result from dynamics linearization, the monosynaptic stretch response does not have a high enough gain (see [Bizzi & Polit 1979]) which is required for the correction of such large errors.

A plausible mechanism is an open loop solution of dynamics. The two alternative models for open-loop control of multi-joint complex movements have been discussed in chapter 2. One model has suggested that the CNS solves the inverse dynamics problem while the other model has suggested that through the control of the intrinsic mechanical impedance of the end-effector, the need to solve the inverse dynamic problem is bypassed [Hogan 1980]. This control scheme has the advantage that errors during a trajectory can be compensated for without delays. The question of whether such a scheme can effectively correct for simplification of dynamics, or whether such a hypothetical scheme will require unrealistically high impedance values is discussed in more detail in the next chapter.

The other control model, the computational model, suggests that substantially correct preprograms for solutions of the inverse dynamics problem must exist, which provide appropriate net joint torques including interaction terms.

While this control mechanism is quite plausible, since dynamic requirements for different trajectories are quite varied, the exact computation of the necessary joint torques might be too costly. This issue, and the capability of the two control models for correcting errors in movement trajectories resulting from unexpected disturbances, are further discussed in the discussion chapter. Our results have shown that the dynamics representation varies extensively from one trajectory to another. Therefore, it is pertinent to ask whether there exist any strategies of movement which can cause a simplification of dynamics so as to render new solutions of dynamic problems unnecessary or trivial, when the kinematic representation of a trajectory changes. As a result of our findings, such a strategy emerged which involves a simple way of scaling movement speed.

4.5.2. The scaling of movement speed

It was noted in the results section that the torque profiles remained substantially the same at different movement speeds. This result will now be explained. A simple scaling of movement speed should result in a simple scaling of the joint torques if gravity terms, which are only configuration and not rate dependent, are ignored [Hollerbach & Flash 1982]. More precisely, suppose there is a planar trajectory described by functions of time of the joint angles $(\theta_1(t), \theta_2(t))$. A similar trajectory but at a different speed can be obtained by scaling the time by a factor r to yield $(\theta_1\,(t), \theta_2\,(t)) = (\theta_1(rt), \theta_2(rt))$. If $r > 1$ the movement is speeded up, if $r < 1$ the movement is slowed down. The relation between the angular velocities and accelerations of the old and new trajectories for corresponding times is simply $(\dot{\theta}_1\,, \dot{\theta}_2\,) = r(\dot{\theta}_1, \dot{\theta}_2)$ and $(\ddot{\theta}_1\,, \ddot{\theta}_2\,) = r^2(\ddot{\theta}_1, \ddot{\theta}_2)$. By substituting these relations into equations (4.3) and (4.4) for the shoulder and elbow net joint torques, one finds that the new torques n_1 and n_2 are related to the old by $(n_1\,, n_2\,) = r^2(n_1, n_2)$.

To examine if the calculated joint torques, for a given human movement, do indeed have this time scaling property, the fast and slow movements from targets 3 to 6 illustrated in Figures 4-4 and 4-6 were compared. The factor r^2 was determined by taking the ratio of the maximum net torque amplitudes at the shoulder for the fast and slow movements. The plots of the net torque profiles for the elbow and for the shoulder for the fast and slow movements, were superimposed by compressing the time axis of slow movement by a factor r and multiplying the torques by a factor r^2. The solid lines in Figure 4-8 represent the net torques of the fast movement, and the dashed lines the net torques of the slow movement scaled in the manner just described. As can be seen the profiles overlap substantially. Between these two trials, therefore, the subject seems to have followed a strategy of changing movement speed by scaling the time dependent torque profile. Comparisons between other movements made at different speeds substantially agree with this result.

This finding suggests that the CNS uses strategies for trajectory formation

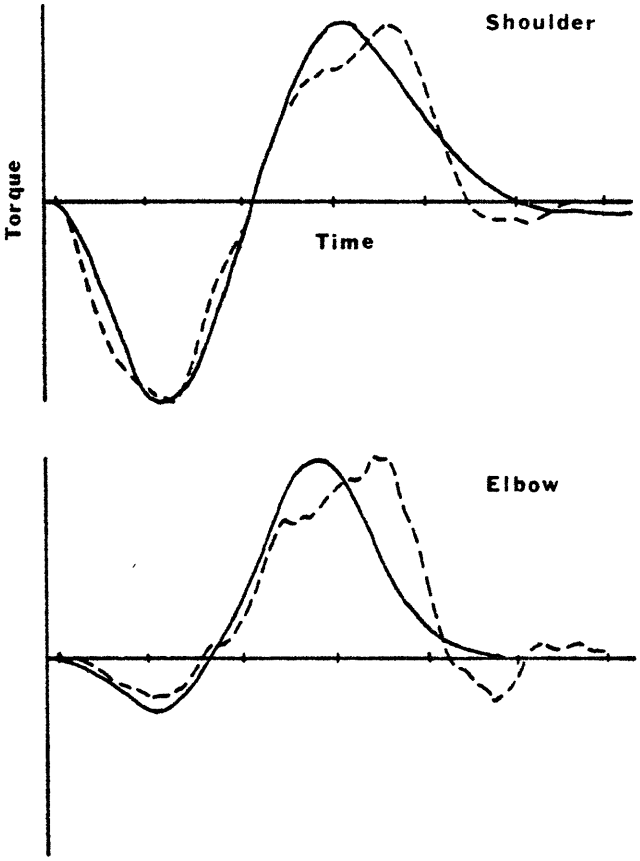


Figure 8. Overlapped net torque profiles for the shoulder and elbow joints from fast and slow movements between targets 3 to 6. The solid line represents the 0.5 second movement, the dashed line the 1.0 second movement. The torque values of the slow movement profiles were scaled by a factor r^2 determined from the ratio of the maximum net shoulder torques for the fast and slow movements, and the slow movement time axis was compressed by the same factor r.

which simplify dynamic computations or dynamic preprogramming. By scaling the time dependent portions of the joint torque profiles by a factor r^2, in order to speed up a movement by a factor r and then factoring in gravity separately, it is possible to change movement speed without deviating from the desired path. If one knows one particular way of making a movement between two points, it is, therefore, possible to make movements between the same two points at different speeds without a substantial dynamics recomputation. Such a strategy for the simplification of dynamic computations has a major significance and is pertinent also for the field of robotics. The scaling property of dynamics allows the movement speed to be changed without recomputation of dynamics for any trajectory in three dimensional space if the same path is generated in the fast and slow movements by simple scaling of the kinematic variables. This applies to any configuration of the arm because in the equations of motion, for any number of degrees of freedom, inertial terms are proportional to joint accelerations and velocity terms are proportional to the product of two joint velocities. Thus, in any trajectory plan where the same path has to be followed, be it a straight or a curved path, simple scaling can simplify dynamic computations when the speed has to change. This applies in cases where trajectory plans are done in terms of joint space. If trajectories are planned in Cartesian space and the inverse dynamics problem is non-redundant, the time scaling property immediately follows. Even in redundant systems, if the same joints are involved in the generation of the fast and the slow movements and the new joint angles are related to the old joint angles by the simple relation described above, the scaling of the rate dependent torque terms with time immediately follows.

Moreover, the time scaling of dynamics, for constant time scaling, results in the same relative significance of all interaction torques for fast and slow movements. Hence, this relative significance also remains the same, at various speeds, for the inertial interaction torques and not only for the velocity torques. Therefore, the argument that the latter are in general insignificant at slow movements for all kinds of trajectory plans, is false. Hence, if one simplifies dynamics by omitting the velocity torques in order to be consistent, one should also exclude from dynamic computations the inertial interaction torques.

The time scaling strategy suggested here combines kinematic aspects and dynamic aspects of trajectory formation. It was shown in the last chapter, that human hand trajectories, as was predicted by the jerk minimization model, indeed scale simply with time. If the time factor is scaled by r the results in the previous chapter have shown that $x(t) = x(rt)$ and $y(t) = y(rt)$. If we substitute it back in (4.1) and (4.2), the time scaling of joint angles promptly emerges. Hence, it seems that the system uses an organization principle which simplifies both kinematic and dynamic aspects of trajectory formation. This because it offers spatial invariances and time invariances for the movements of the hand, as well as a speed invariance of dynamics. Such a control strategy can also be of value in robotics.

As was discussed above, the time scaling property can simplify the problems of dynamics for the biocontroller if rate dependent torque terms are computed separately, and are added to gravity torques. It does not follow from the scaling observation, based on the calculated values of joint torques for human movements, that the human motor system actually carries out this scaling computation, since an alternative is that the motor system uses the same trajectory plan for different movement speeds and recomputes the dynamics each time. Moreover, even if the motor system uses this simplifying strategy, it still must be capable of generating the appropriate torque profiles for one particular movement speed. Nevertheless, the scaling observation does increase the attractiveness of a few suggested control schemes. This topic is further discussed in chapter 5.

Chapter 5

Discussion, conclusions and suggestions for further studies.

In this chapter, we discuss the interpretation of our findings and the answers they provide, to the questions concerning motor planning of complex movements, raised in the introductory chapter. We relate these findings to the few proposed models, for the generation of complex movements, presented in the literature review chapter. Broader implications of this study for the understanding of motor organization are also presented. Next, we make a few suggestions for the applications of the model and the methods which were developed here, in motor research studies. The implications of this study for robotics are also discussed. Finally, a few possible directions for the extension of this work are suggested.

5.1. Principles underlying the generation of complex movements.

5.1.1. Principles' generality: kinematics versus dynamics

In the introductory chapter, the question was raised whether, in planning an arm movement, the human motor system specifies trajectories in the Cartesian space of the hand or in the joint space of the arm. The observations of the kinematic features of measured movements and the success of our model in accounting for these features offer strong evidence for the view that arm trajectories are specified in hand cartesian space. This evidence, therefore, leads to the rejection of the views that arm trajectories are specified either in the joint space of the arm or at the muscle level. The view that arm trajectories are specified at the joint space is based on the argument that in movements which involve rotations about a few joints, the CNS maintains constant ratios among the various joint angular velocities

[Soechting & Lacquiniti 1981]. Our own findings did not support this argument. If one inspects our own results for point-to-point movements (e.g. Fig. 5 chapter 3), it is apparent that even during a single movement, the ratio between the shoulder and elbow joint velocities is not maintained as constant (except for straight paths along lines which pass through the shoulder). Obviously, therefore, there is no evidence to suggest that one can identify such a numerical constant for different movements in different parts of the work space.

The kinematic description of hand trajectories in point-to-point and curved movements is a work space invariant. Hence, it seems that the same general principle dictates the specification of trajectories for different movements. In contrast, the dynamic representation is much more complex since there is no readily applied rule for adapting or easily adjusting the torques required for the generation of a movement, performed in some region of the work space, to a movement performed in a different region. Hence, because different movements involve joint angular rotations and angular velocity and acceleration profiles which are highly dependent on the work-space region, the required non-interaction torque terms also show such dependency. Furthermore, during the movement of a multi-joint limb, the generation of appropriate joint torques for a trajectory is complicated by the presence of joint interactions due to inertial, centripetal, and Coriolis torques. Our results have shown that these joint interactions are significant and are highly dependent on the trajectory. Not only are the joint interactions significant, but since the system seems to specify the hand trajectory quite rigidly, the interaction forces are crucial for the generation of the desired hand trajectories as was shown by the trajectories which resulted from the elimination of interaction forces. We can, therefore, raise the question as to why does the system provide generality and simplicity in the case of kinematic, but not dynamic representations of the hand trajectories.

A possible explanation for this gap between the simplicity of the kinematic versus dynamic representations of complex movements is that since the goal of arm movements is the placing of the hand the system cares more about generating simple and predictable hand motions, than about the amount of complexity involved in generating appropriate joint torques. The alternative control scheme, which involves joint space planning, and which we have informally tested, would have meant no explicit control of the hand trajectory between positions in space due to the complicated relation between joint angles and Cartesian hand positions. The complexity of the dynamic representation of multi-joint movements also results from the large number of degrees of freedom of the arm. However, it is advantageous to have a large number of degrees of freedom, since it allows flexibility and versatility of movement.

Although our results have indicated that dynamics linearization is not a plausible strategy for the simplification of the problems of movement dynamics, we introduced the possibility that there exist strategies for trajectory formation which simplify these problems. Such a strategy of trajectory planning might be indicated by our results. It involves, as was suggested, a simple scaling of the trajectories with time which in turn results in a simple scaling of the rate-dependent torque profiles. Comparisons of human arm movements between the same targets but at different speeds show that subjects evidently adapt a strategy compatible with this hypothesis, to alter movement speed. One can conceive of a trajectory planning strategy which is based on joint space, and in which joint angles scale with time. Hence, in such a strategy, hand trajectories are invariant to changes in movement duration and dynamics scales with time, although these trajectories are not space invariant. In contrast to such a strategy, a strategy for trajectory planning which plans Cartesian hand trajectories that scale with time, achieves both goals. Firstly, it achieves simplicity and space invariance of hand trajectories, and secondly it achieves speed invariance and through this, a simplification of dynamics. The success of the optimization model may suggest that speed and space invariances both emerge from the objective of optimizing the smoothness of the hand's motion or optimizing a combination of kinematic variables represented in the Cartesian space of the hand.

5.2. Rules for selection of trajectories

A general question, which was presented in chapter 1, is how the CNS selects trajectories for the hand from the vast number of possible trajectories. Before proceeding with the discussion of this question, it is important to emphasize that in this and in previous studies, it was shown that both hand paths and hand velocities display stereotypical features. These findings indicate that the biocontroller specifies both hand paths and the time sequence of hand movements along a path. Hence, any model which suggests an answer to the question of how trajectories are selected, should account for both paths, and velocity and acceleration profiles of human movements.

In chapter 1, we also raised the question of why there should be a temporal coupling between speed and curvature in curved movements. Two possible explanations for this specific feature of curved movements were discussed in chapter 1. One explanation suggested that mechanical effects, due to the action of two-joint muscles or the presence of interaction forces, may account for this phenomenon. Based on our results, we are now in the position of rejecting such an explanation. Our dynamic computations have shown a very complex and variable picture of joint profiles in different parts of the work space. We found that in some movements the interaction forces are relatively small, while in others they are relatively high. Therefore, since the speed-curvature coupling is seen for movements generated in different parts of the work space, it is not reasonable to suggest that interaction forces with variable magnitudes and diverse mechanical effects (e.g. in whipping versus reaching actions), would cause a similar phenomenon regardless of the work space region. According to a similar rationale one should reject the explanation that two-joint muscles cause this phenomenon. In contrast, the idea that curved movements are planned as sequences of straight segments, may offer an acceptable explanation on logical grounds, for this phenomenon. This idea, however, does not answer the more general question of how hand trajectories are selected, since it does not provide an answer to the question of why point-to-point and curved movements display the particular time and space behavior exhibited by the observed movements. The optimization model presented here did account for the straight hand paths and hand velocities in point-to-point movements and for the paths and velocity curves in curved movements. It is tempting, therefore, to suggest that the objective of optimizing some kinematic variable such as jerk, dictates the selection of very specific trajectories; the optimal ones.

Such a constraint to generate the smoothest possible hand movement or the movement which requires a minimum change of the rate of change of hand acceleration with time, dictates the slowing down of the hand at maximum curvature points. In contrast, the maintainance of a bell shaped velocity curve even for curved trajectories, results in a higher value of jerk then the value corresponding to movements with a valley in the tangential velocity curve. Such speculation about the principle which underlies the selection of hand trajectories offers also a suitable explanation for Abend et al.'s [1982] findings in the experimental trials in which subjects were instructed to follow curved guides with their hands. These investigators have presented data which show that the subjects did not really mimic the curved guides accurately, but instead, generated less curved trajectories with a few more highly curved regions. These findings can be explained on the basis of the idea that the hand is constrained to form minimum-jerk trajectories.

Furthermore, we can speculate that this selection rule dictates the choice of desired hand trajectories not only in planar two-joint movements in the horizontal plane, but in movements occurring in other horizontal planes. Our results may indicate that that is the case since in experiments in which movements were generated in a horizontal plane passing at the level of the subject's waist, the trajectories had the same kinematic features as movements generated in a horizontal plane at the level of the subject's shoulder. In this discussion we do not mean to suggest, however, that the system does not have other objectives in the generation of curved as well as straight movements. Therefore, the tendency to move along straight lines between points may indicate, for example, the objective of minimizing both distance and vibrations transferred to a grasped object, or even the higher derivatives of acceleration.

5.2.1. Segmentation versus jerk-minimization

It is important to emphasize that the jerk-minimization model does not contradict the idea that trajectories are planned as sequences of several segments. Such a principle is implicitly embedded in the jerk-minimization model, because, if the system indeed constrains the hand to move in space through a few via points, the use and selection of such points result in an implicit segmentation. The difference between these two hypothetical schemes of trajectory planning lies, however, in the basic assumption of the optimization model. This model proposes that the planning of the second segment is coupled with the planning of the first segment, since the trajectories of both segments depend on the location of the initial, final and via point. This coupling of the two segments results in the isochrony principle, which in turn, causes the differences between the heights of the two velocity peaks for the two segments, on the two sides of the via point. In contrast, not only does the segmentation model not provide any explanation for the isochrony principle, but it implies that the two segments are independently planned. This is so because except for the requirement that there should be a partial overlapping of the two segments and a smooth transition between them, the segmentation model does not specify the duration of movement for each segment or the geometrical parameters which characterize the strokes and these can be chosen independently for each one of the two segments [Morasso and Ivaldi 1982].

Assuming that reaction time can be a measure of the complexity of movement planning, it has been shown that the reaction time to initiate curved movements is longer than to initiate straight movements [Mathison & Abend 1982]. On the other hand, when a movement made of a sequence of straight line paths was generated, the reaction time did not increase linearly with the number of segments but reached a plateau after a few segments. While these studies do support the hypothesis that curved trajectory planning is more complicated than the planning of straight trajectories, this evidence by itself can not help to establish which of the the two models mentioned above (i.e. segmentation versus optimization) is more realistic since both computational schemes are more involved and complicated for curved movements than for straight ones. We do tend to believe, however, that not every inflection point in the hand path and not every velocity valley, necessarily imply the beginning of a new stroke. This idea is supported by our own results (i.e. the lack of a dip in the velocity component curve along the non-reversed axis) as was discussed in chapter 3. These results indicate that the slowing down of the hand, more often than not, is not due to segmentation. Furthermore, the finding that measured hand trajectories do exhibit the isochrony principle, which is predicted by the optimization model, offers a strong support for the claim that a more basic principle than mere segmentation underlies the specification of hand trajectories. Additional evidence can be found in Wing's [1978] studies of handwriting. He argues that the positive temporal correlation between the up and down strokes in the letter (n), for example, may indicate that an up-down stroke is planned as a single unit rather than two separate strokes. This ties up nicely with the idea of the coupling between segments, which was discussed above.

5.2.2. Optimal control and motor learning

The evidence for the optimal nature of hand movements has great relevance to the topic of motor learning. Since with training and practice, movements tend to become more smoothly and skillfully performed, this may indicate an underlying implicit progress towards optimal motor performance. The optimal nature of movements has been referred to repeatedly in the motor control literature especially when discussing skill acquisition (see chapter 2 section 2.5). In studying two joint monkey arm movement Georgopoulos et.al. [1981] have shown that as a result of practice the variability in hand trajectories is reduced exponentially with time. These investigators suggest that two features may contribute to this reduced variability: an improved motor coordination in these compound movements and a better estimate of the best, appropriately aimed trajectory. Thus motor coordination can be viewed as a reduction in the degree of the independence of the angular rotations at the two joints so that with practice, improvement in the temporal correspondence between these two joint rotations results in an aimed hand trajectory so that the movement is performed as a whole. In addition, since there is still a vast number of possibilities for selecting hand trajectories, the fact that the number of executed trajectories reduces with practice and only a few emerge, may indicate an increase in the information available to the biocontroller about, which trajectories are optimal. Thus, practice and learning can be viewed as an adaptive process of search towards achieving better motor habits, which leads, eventually, to the final product of optimal movements.

5.3. Implications for the study of motor organization

In chapter 1, we discussed the topic of motor organization and the approaches which are, or should be used, to advance our understanding of the principles which underlie such an organization. We discussed one view, which suggests that motor programs are hierarchically planned, so that at higher levels there exists an ideal representation of movement which does not take into account the mechanical nature of the actual effectors but is abstract and general. This representation is then implemented by more specific commands to specific effectors as the hierarchy is descended, and finally firing patterns for motoneurons intended to activate specific muscles, are specified. In this section we will discuss our results in relation to this view of motor organization.

5.3.1. Single versus multi-joint movements

In chapter 1, we suggested that one should expect to find that different strategies, are used by the CNS, to control different classes of movements. On the other hand, it was suggested that at some level of organization the same principles may underlie motor programs for the generation of different kinds of movement. It was also emphasized that the generation of single joint and multi joint movements involve different types of kinematic and dynamic problems, and therefore, one can classify them as belonging to different classes of movements. The presence of interaction forces in multi-segment movements is understood on theoretical grounds. Our work has also shown, that interaction forces are significant, and therefore, one can conclude that strategies developed for single joint movements do not necessarily generalize to multi-joint movements. This conclusion is reinforced by Delatizky's [1982] work, in which the final position model was extended from single-joint to two-joint planar movements. The simulated movements predicted by this extended model, did not match realistic two-joint movements. We do not intend to argue against attempts to generalize various control models proposed for single-joint movements (e.g. impedance control [Hogan 1980]), to multi-joint movements. Instead, we argue that in any attempts of this kind, such models should take into account interaction forces, or should suggest ways of dealing with them. In contrast to the level of movement dynamics, at the kinematic level of movement representation, we have successfully accounted for the kinematic features of multi-joint movements, by generalizing an optimization principle which was applied to the modeling of single joint movements. This success in using the same principle for the modeling of both single and two-joint movements may indicate that although the generation of multi-joint movements is also kinematically more complicated than the generation of single-joint movements, a single unifying principle is used in the trajectory planning of both. Furthermore, computational models, which were proposed for the control of multi-joint movements, have suggested that the planning of the kinematic representations of movement is done at a higher hierarchical level, than the solution of dynamic problems. If indeed, there exists such a hierarchical order, we can speculate, based on our work, that at the higher level of the kinematics of movement, more general principles underlie the planning of trajectories, while at the lower level of dynamics, different, less general strategies, are used to implement these plans for multi-jointed as compared to single-jointed effectors.

Thus, a choice of a kinematic objective function for the description of hand trajectories is along the theories which view the motor system as being divided between higher levels which plan ideal trajectories for the end-effector, and lower level processes which translate them into torques and forces. In contrast, as was mentioned in chapter 3, including variables which relate to the musculoskeletal apparatus itself in the objective function that needs to be optimized, will result in threatening the dichotomy between higher and lower level processes [Raibert 1978].

5.3.2. Specification of movement parameters

The idea of generalized motor programs suggests that not only the specific commands to specific effectors or muscles, but even the temporal and spatial parameters of motor acts are applied to the same general and abstract internal representation of movement each time a movement is about to be generated. Movement durations and spatial position cues were suggested as such specifiers [Keele 1981]. In the sections below, our results will be discussed in view of these ideas.

5.3.2.1. The specification of movement duration

The idea that movement duration is an independent variable has been discussed by Schmidt [1980] and Keele [1981]. Keele has argued that timing appears to be an integral part of the motor program and that time representation is relative since different rates can be applied to the overall representation. This, he argues, supports the concept of a hierarchical program.

The optimization model yielded mathematical expressions for the predicted trajectories in which the movement duration appears as a multiplicative factor of polynomials which depend on a dimensionless variable (the ratio between the time that has elapsed from the beginning of the movement and the total movement duration). Hence, the movement duration appears as an independent variable. Given this characteristic of the optimal trajectories and our findings that human hand trajectories can be described with a great success, by such trajectories, we may speculate that the relative time representation of movement does indicate optimization principles which underlie internal motor representations. In connection to this implication of the optimization model, it is interesting to mention Viviani and Terzuolo findings of both time scaling and speed curvature coupling properties in handwriting and to cite their words that "if the central representation is specified by spatial aspects of movement such that there exists a relation between speed and form, a homotetic (speed invariance) behavior would necessarily follow" [Viviani & Terzuolo 1980]. It is of interest, also, to mention, in relation to the optimization model, Schmidt's [1980] conclusion that there might be an underlying representation of movement which manifests itself as a set of invariant properties and that phasing and relative timing might be inherent in the sequence.

The evidence in support of a hierarchic time conception of motor programs, is further reinforced by the finding that time scaling can simplify dynamic computations. Since for two joint planar movements (hand braced) the joint variables are uniquely defined by a specific choice of planar trajectory , the scaling of the time-dependent joint torque terms readily emerges. In movements where the hand brace was removed, the time scaling at least of hand trajectories, was conserved. It still remains to be seen, however, whether in arm movements, in which the system is kinematically redundant (more joint than hand degrees of freedom) a time invariance of hand trajectories leads to a simple scaling of joint angular velocities, and hence, to simple scaling of rate dependent joint torque terms.

5.3.2.2. Space invariance: position cues

The question as to whether trajectory plans are coded on of basis of spatial position cues, or on the basis of movement amplitude has been the topic of an ongoing controversy in motor literature for a long time. Russel [1976] has reviewed evidence which shows that spatial locations in space between and toward which, movements are generated, and not movement amplitudes, are encoded in memory. It has also been suggested that spatial locations of targets are used as position cues in speech production, in which case movements do not result from stored patterns of neural activity but are spontaneously generated on the basis of target locations [Russel 1976].

This idea has been proposed as an attractive alternative to the motor tape theory, which suggests that the system actually stores the detailed patterns of neural commands, or their sensory consequences, for all learned movements. Instead, the idea that the coded spatial location information is the information required to generate novel movements, and to reproduce learned movements, introduces the potential for a degree of plasticity, since the subject generates afresh the neural commands required to achieve the desired spatial locations.

The fact that man is able to direct his limbs towards visual targets indicates his ability to readily transform visual and auditory information into a schema used by the CNS to plan movements. Therefore, if position cues are the required information on the basis of which limb movements can be planned, this may suggest that similar reference spatial coordinates are used for the coding of both visual information and motor actions. If position cues are, indeed, used as a basis for internal motor representation, exteroceptive information processed by the visual system which specifies, for example, locations of objects in space, can be readily used in reaching movements. In other skills, for example, such as drawing or handwriting, position cues could be learned, stored and retrieved when needed. Lashley [1951] states of space reference systems: "Their influences pervade the motor system so that every gross movement of limbs or body is made with reference to the space system. The perception from the distance receptors, vision, hearing, and touch are also constantly modified and referred to the same space coordinates."

This work has presented an optimization model which uses exteroceptive (Cartesian) locations and yields a time history for the positions of the hand between the two end-points. For the generation of curved trajectories, this algorithm also uses position information: the spatial location of a via point. Since, mathematically, this is a simple enough algorithm, it is tempting to suggest that hard-wired or computational optimization algorithms, similar to the ones presented here, enable the translation of task objectives, coded by exteroceptive position cues, into trajectory plans.

5.4. Relevance to motor control theories for complex movements

A few control models which were proposed for multi-joint arm movements will now be discussed.

In chapter 4, it was argued that feedback control cannot be a plausible mechanism for the control and corrections of interaction forces for multi-joint arm movements, because delays from the proprioceptors render feedback correction infeasible. It was argued that computation or compensation for the dynamic interactions must already have occurred in the motor preprogram. Various methods of constructing motor programs have been discussed in chapter 2, in particular, computational models which are based on analytic solutions of problems of kinematics and dynamics or on tabularizations of portions of the dynamic equations, and the impedance control model. The theory of simple final position control [e.g. Sakitt 1979], where there is no explicit trajectory control, cannot be acceptable since it was shown that even for single-joint movements this theory cannot account for the results from experiments, conducted on monkeys, in which unexpected disturbances are introduced during movement [Bizzi et al. 1982]. In addition, this model was shown to be not suitable for predicting the kinematic characteristics of two-joint planar arm movements [Delatizky 1982]. Another concept, the motor tape concept, which suggests that the commands for all possible movements have been learned and stored and are played back whenever required, might be rejected on many reasonable grounds including the argument that there is no point in having common kinematic features if all possible movements are stored anyhow. Hence, the most plausible models are real-time computations of the inverse kinematics and dynamics, and the impedance control model. We will discuss these two models in relation to the main conclusions drawn from this work.

5.4.1. Schemes for the solution of dynamics problems

Any model which is put forth for the control of multi-joint movements, must be shown to be capable of compensating or providing for dynamic interactions. The real-time computation alternative, in principle, imposes no limitations on trajectory formation ability, but requires fast computation of the inverse dynamics and kinematics. Efficient recursive methods for computation of the inverse dynamics have been shown to be feasible in robotics [Luh et al. 1980, Hollerbach 1980]. Such methods for preprogramming of biological movements appear even more attractive, given the time scaling property of dynamics for trajectories of the kind that humans generate. Nevertheless, even in such schemes, in the face of unpredictable errors arising from inaccuracies of kinematic and dynamic models of the robot, limitations of computational precision and mechanical effects such as static friction or vibrations, feedback control is used to ensure that a planned sequence of motion will be executed correctly. Whether the brain is capable of performing computations by this or by analogue means for the solution of dynamics problems is an open question. Tabular solutions, trade off memory for computation but are not as general and do not allow for the same flexibility or adapt readily to mechanical changes. Nevertheless, tabularization strategies can also benefit from the time scaling property of dynamics since the time domain in tabularizations of the inverse dynamics could be compressed.

There is, however, one problem which computational models (for complex biological movements), as they are currently stated, cannot deal with. In such a scheme of motor control, any unexpected or unforeseen disturbance would require not just modifications of the inverse dynamics problem, but a repetition from scratch of the entire computation. Following an unexpected disturbance, the arm deviates from the initial trajectory and the amount of its deviation depends on the disturbance and the arm impedance. In addition, according to computational models, precomputed joint torques appropriate for the original trajectory plan, would continue to be generated regardless of the disturbance, until a recomputation of dynamics is completed. Only after about 200 msec (normal reaction time in movement generation) some corrections can be introduced and since the values of computed torques depend on the initial position, it is not clear how, even after recomputations, the controller can compute those torques, which are necessary to achieve the appropriate corrections.

Experiments conducted by Bizzi and collaborators have shown that when unexpected disturbances were introduced during two-joint movements, and the trajectory was deflected for 200 msec, and then released, the hand immediately moved back towards the initial trajectory without any wild trajectory in between. These findings cannot be reconciled with the behavior, expected from a motor system which uses computational strategies. Hence, one should modify or introduce other factors into these models in order to render them viable for biological motor control.

The impedance control model [Hogan 1980] suggests that a potential field is set by the CNS so that the net force on the hand, (which is defined as the gradient of the potential field), causes the hand to move along the desired path. Thus, hand trajectories result implicitly from a certain selection of intrinsic properties (stiffness, zero-lengths) of spring-like muscles. If a scheme is proposed in which linearization of dynamics is to be corrected by setting the impedance of the joints, the question is whether in movements in which interaction forces are large, that would not require impedances which are far above the normally low impedance in normal human movement. Based on his work, Delatizky [1982] has concluded that the spatial properties of the inertia tensor are more important than the generalized mechanical impedance. His results have also shown that in unloaded movements at reasonable speeds, it is highly unlikely that viscoelastic forces dominate the nonlinear interactional inertial forces.

On the other hand, an important advantage of an impedance control scheme is the ability it has to correct for unexpected disturbances. For free movements, and we will concentrate here only on free movements, when a disturbing force is imposed on the hand, a restoring force, which is directed towards the valley of the potential field would tend to restore the hand immediately towards the equilibrium point. Hence, although it is questionable whether intrinsic impedance can correct errors resulting from linearization of dynamics, this model does offer means to deal with unexpected disturbances. In contrast, the computational models provide for interaction forces but are unsuitable to deal with unexpected disturbances.

A control scheme was recently proposed [Hogan 1983, in preparation], in which given a desired time history of hand positions, joint torques, including interaction torques, are accurately computed. In addition, this model proposes that the CNS also controls the end-effector impedance. Through the control of this impedance, the CNS selects a virtual trajectory. This trajectory is defined as the time history of equilibrium points, at which no forces act on the hand. If a disturbance causes the arm trajectory to be deflected, after the release of the arm, the hand should return to this virtual trajectory. In reality, the actual trajectory which the hand will return to will be determined by inertial effects, the success in implementing a desired joint impedance, etc. This is a hybrid model combining the computational and the impedance control models, where joint torques are computed for a given time history of hand positions, but by controlling the impedance of the end-effector, corrections for disturbances are readily obtained.

In chapter 2 we discussed the possibility that by using the spring-like properties of muscles, the equilibrium point between the antagonist muscle groups is set not only for the final position of the trajectory but for intermediate points along the trajectory. Moreover, by shifting the potential valley along the trajectory one generates such a virtual path. Such a shift of the potential valley can be achieved by changing in time either the rest length or the stiffnesses of the antagonist muscle groups or both. Such a strategy of shifting the potential valley in time along a cartesian hand trajectory, would eliminate the need to solve the inverse kinematics and inverse dynamics problems. Although this is an attractive idea, since it eliminates the need for complicated computations, it is an open question whether or not this is a viable strategy for biological trajectory formation.

5.4.2. Relevance of the kinematic model to motor theories

If the CNS specifies hand trajectories in external space, the computational models, the impedance control model and the hybrid model, all require end-effectors trajectory plans, available through computations or through some hard-wired selection rules. As was discussed in previous sections the objectives of obtaining optimal trajectories may specify trajectories for the dynamic level of trajectory formation. We speculated above that optimizations are used as computational algorithms at the higher kinematic level as the first step for transforming a certain input (e.g. visual information) into motor output. An alternative idea may suggest that the optimal behavior of hand trajectories is an outcome of the operation of the entire mechanism as a whole: the neural and musculoskeletal systems. Thus, this optimal behavior may be a description at the level of emerging movements of intrinsic properties of the system. Such an idea, may represent at the level of the structure, rather than at the level of higher cognitive processes, the same basic principle: that evolutionary adaptations have led to an optimal behavior of biological systems. Thus, it is not clear which of the above models the optimization model can best fit into. Nevertheless, all the models discussed above, the computational models, the impedance control model or the hybrid model, all require end-effector trajectory plans, available either through computations or through some hard-wired selection rules.

5.5. Applications of this study

5.5.1. Applications to research in motor control

The methods developed in this thesis give us powerful tools for the description and analysis of human movements. As was shown in chapter 3, the jerk-minimization model is quite successful in predicting real human hand trajectories. Therefore, one can use the jerk-minimization model to evaluate the motor performance of intact humans and of patients with various motor disorders. One can also use the predictions of the jerk-minimization model to evaluate the performance of movement simulations predicted by control models, based on various theories for the control of multi-joint movements. Based on the model, performance measures can be developed, which can enable us to objectively assess the performance both of human subjects and of various hypothetical algorithms. Such performance measures can use either the integrated jerk values for the entire movement, or integrated values of such parameters as curvature, path length, hand speed etc. derived from the predicted trajectories. Recently, for example, such measures, derived from predicted minimum-jerk trajectories, were used to evaluate the match between movement simulations, resulting from a final position control model for two-joint arm movements, and measured two-joint human arm movements [Delatizky 1982].

In addition, the computer simulations developed here for the solutions of the inverse dynamics and the integral dynamics programs, can be used in combination with the jerk-minimization model, either for the computations of joint torques or for the prediction of arm trajectories for a given set of command torques. Such simulations can find a wide range of applications in motor control research.

5.5.2. Implication for Robotics

Although the main objective of this work was to study biological trajectory formation, a few of the conclusions from this study are pertinent to robotics. In addition, this work may offer a plausible algorithm for Cartesian trajectory planning in robotics.

In robotics a separate module is assigned to deal with trajectory planning. This module selects paths between locations in space along which the manipulator end-effector should move so that it can grasp objects in the environment and avoid collisions. In robotics, the two most common approaches in trajectory planning, are implicit and explicit planning [Brady 1982]. In the first approach, the programmer implicitly describes the trajectory to be followed by the arm, by specifying a set of constraints on its positions, velocities and accelerations at a number of points along a trajectory. The trajectory planner chooses one of a class of parameterized trajectories that satisfies the constraints. In the second approach the path is described explicitly by an analytic function, usually in Cartesian coordinates, and the goal is to plan a trajectory whose path closely approximates the desired path. For reasons discussed above, in robotics, trajectory plans usually constrain the end-effector to follow straight lines. First order polynomials are at present commonly used for straight paths [Taylor 1979]. When trajectories through via points are planned, second order spline polynomials are used to concatenate such simple segments. This method is used in order to overcome problems of using higher order polynomials required to satisfy position, velocity and acceleration constraints at the end-points and at via points. Constraint satisfaction approaches to trajectory planning, have the advantage that they are based on the use of position velocity and acceleration constraints only at a few points. On the other hand, if one uses high order polynomials the disadvantage is that the trajectories are only weakly constrained. A disadvantage of using lower order splines is that one can not guarantee that jerk and vibrations will not be introduced at via points. The jerk minimization algorithm presented here provides a rigorous way of trajectory planning for motions between end-points and for movements in which via points are introduced. While constraint satisfaction methods require that the number of parameters of a time dependent function be equal to the number of constraints, optimization methods provide a more general approach to constraints satisfaction even when the number of constraints and the number of parameters are different. Therefore, the jerk minimization algorithm depends on a small number of constraints (basically only position values at the end points and at via points if the movement is assumed to have zero acceleration and velocity at the end points) but guarantees a well-behaved movement with low values of jerk.

In addition, since this algorithm predicts roughly linear trajectories between points, this overcomes the problem of using constraint satisfaction methods, in which the curve followed by the end-effector is not explicitly planned and it is non-trivial to reassure that the end point will stay within the workspace. For a large number of via points one could still join together a few segments, each planned using the jerk-minimization algorithm.

One other important advantage of such a proposed algorithm for trajectory planning in robotics, is the fact that it results in trajectories which scale simply with time. The time scaling property of dynamics, which was identified in this study suggests that if the controller wishes to generate the same path, it can compress the total movement duration by a factor r. After the rate dependent components of the torques are scaled by r^2, the gravity contribution is added in separately and the faster movements are readily obtained. This observation can be generalized to any manipulator configuration with any number of degrees of freedom.

The time scaling property of dynamics identified here, leads to our rejection of the view that inertial forces completely dominate dynamics at slow speeds, for any manipulator, or trajectory plan, hence; one can eliminate velocity forces at slow speeds. For a large class of trajectories, in which the trajectory scales with time, all the torque terms in the equations of motion have the same relative significance. Our results have also shown that even at slow speeds, for some movements, Coriolis and centripetal forces are significant and the elimination of these forces results in large errors. It was shown, also, that even at slow speeds, these forces are significant relative to gravity. Furthermore, as was shown for human movements, at the movement midpoint, when the velocity is at its maximum, the acceleration is zero, and at this point, dynamics is completely dominated by velocity terms. Recently, the time scaling of dynamics was extended to cases where the manipulator moves along the same path with different speeds, but where r, the constant durations ratio, is now a time dependent monotonic function [Hollerbach 1982]. Hollerbach has proposed that the time-scaling property of dynamics can be exploited to determine whether a planned trajectory at higher speeds is dynamically realizable, given actuator torque limits, or how to modify it to bring it within dynamic and actuating constraints.

5.6. Suggestions for future extensions

In this section a few possible directions for the extension of this work are suggested. In this discussion we will progress from more specific suggestions to more general ones.

5.6.1. Studies of intact motor behavior

In the work, presented here, curved movements which show two velocity peaks were modeled. The majority of unconstrained curved movements and obstacle avoidance movements measured by Abend et al. [1982] and in this study had indeed only two velocity peaks. Occasionally, however, subjects generated movements with more curvature peaks and more corresponding velocity valleys. It would, therefore, be of interest to examine if the jerk-minimization model, using more via points, can account as successfully for the kinematic features of these curved movements as it accounted for the movements examined in this work. It was suggested above that the jerk minimization model might be successful in describing, mathematically, movements which do not require the use of a large number of via points, and that more kinematically complex movements might be described by concatenating, for example, a few minimum-jerk segments. It will be of interest to investigate, if one gradually increases the number of via points, what the limiting number is above which the model will no longer be successful in accounting for the observed behavior. The same phenomena of multimodal tangential velocity curves, multimodal curvature curves and temporal coupling between curvature and speed was observed [Viviani and Terzuolo 1980] in handwriting. We already mentioned a model proposed by Hollerbach [1981], in which the basic forms of cursive handwriting are generated by two orthogonal waveforms in two-dimensional space. Although, Hollerbach used a basic oscillatory pattern, he concluded that alternative basic patterns can also succeed in producing appropriate slants, heights, and widths of handwritten letters. Since the jerk-minimization model also generates end-effector trajectories along two orthogonal axes, and given the observed speed-curvature coupling in hand-writing, it is of interest to investigate whether the jerk-minimization model can succeed in accounting for other observed kinematic characteristics of handwritten strings.

An issue which should be pursued further is jerk versus snap minimization. As our results have shown, jerk-minimization gives better results in some movements, but in other movements snap minimization offers a better fit. Since, in this work the success of minimum-jerk versus minimum snap trajectories in matching real movements, was only qualitatively assessed, more quantitative methods for assessing their relative success should be developed. In addition one should investigate, what the characterization is, of the movements for which, each of these two objective functions gives a better mathematical description.

It is of interest, also, to identify the nature of the conditions under which the jerk-minimization model fails in predicting the observed behavior. In this work only mildly fast movements were investigated. On the other hand, the model may not provide a good fit for very fast movements, which require the generation of large joint torques, and modifications of the model, using, for example, constrained minimization may be required to render it successful. Such studies will enable to assess the generality of the model and to identify the boundaries of the subspace of admissible trajectories.

Other possible directions for the extension of this work, involve measurements and analysis of different kinds of movement than the ones studied here. In this work, the jerk-minimization model was tested only for planar two-joint arm movements. Other kinds of free arm movements for which this model can be tested include: planar (horizontal) three-joint movements, planar (vertical) movements and three dimensional movements. Similarly, movements which involve carrying loads should be recorded, analyzed and their trajectories compared to trajectories predicted by the jerk-minimization model.

Although the jerk minimization model accounted for hand trajectories for movements in which the wrist was not braced, it is of interest to collect more kinematic data from planar three-joint arm movements . Such a configuration is redundant since the number of joint variables is larger than the number of hand kinematic variables. Hence, one can examine the time history of the hand orientation and the time histories of shoulder and joint angles during such movements, to find out whether there are any principles which underlie their generation.

In arm movements in the horizontal plane, gravity torques do not affect the movement. To test whether movements which gravity torques affect can still be described by the jerk-minimization model, one could measure two-joint and three-joint movements in various vertical planes. Similarly, one could measure and analyze three dimensional free arm movements. As was done, for two-joint planar movements, one should obtain a detailed kinematic description of three-dimensional movements and should identify the regularities and invariances seen in such movements. It cannot be argued a priori that planar vertical and three-dimensional arm movements display the same Cartesian hand trajectories as planar arm movements, and therefore, it cannot be argued a priori that three-dimensional movements are not planned in joint space. Moreover, although the jerk-minimization model can be generalized mathematically to predict the minimal-jerk hand trajectories for movements in vertical planes and the three dimensional space curve, and perhaps also the hand orientation curve, for three dimensional movements, it cannot be clear a priori whether minimization of jerk is an appropriate objective function. If the model, in its original form, cannot predict the kinematic features of such movements, perhaps it will be possible to modify the objective function and to successfully predict the real behavior.

With regard to the study of dynamic aspects of trajectory formation, some of our conclusions, such as the relative significance of interaction forces or the time scaling property of dynamics, can be generalized to any arm configuration, involving more degrees of freedom. Nevertheless, three dimensional movements should be measured to examine whether, in these movements, humans generate end-effectors space curves and time histories of end-effector orientations which result in time scaling of joint rotations even in a redundant configuration. Moreover, it would be of interest to perform dynamic computations in movements in which loads of various inertias and masses are carried.

One question which remains open is the origin of the good fit between the predicted minimum-jerk and real trajectories. We have speculated that this good fit may either indicate the use of optimization algorithms in hand trajectory planning, or may result from hard-wired rules for the activation of muscles. Since the jerk-minimization model is based on the jerk of the hand and not of the joints or on a third derivative of joint torques, it is unlikely that the smoothness of muscle contractions is the source of this good match. One should investigate, however, whether the existence of muscle synergies, or the time history of end-effector impedance can cause such a behavior. For this purpose and other more general purposes of validating the various control models, further experimental and simulation studies are needed in order to be able to gather more information on the viscoelastic properties of the arm. Further experiments in which disturbances are applied to the arm during the movement, may offer insight into these issues. Studies of this kind will hopefully enable us to assess whether the impedance control model can offer plausible mechanisms for compensation or provision for interaction forces in multi-joint arm movements.

Although most of this work does not touch directly upon questions relating to muscle activity, it can be extended to investigate how the different dynamic representations, and the relative significance of interaction forces for different (whipping or reaching) movements are expressed at the level of the actuators. Of interest is to assess how muscle activity expressed in muscle EMG amplitude varies with speed of movements, in light of the scaling property of dynamics, which was identified here. It is important to emphasize that other control schemes, besides the computational one, can benefit from and utilize the scaling property of dynamics. In the model in which the CNS specifies intermediate equilibrium positions for the hand, even though this control scheme bypasses the need to solve computationally the inverse dynamics problem, the reference trajectory, which describes the time history of the equilibrium points for the hand, can be specified so that it scales with time. If the changes in rest length or stiffness of the various muscle groups also have the time scaling property, the values of these intrinsic parameters of muscles could be easily adjusted, so that fast and slow movements would not require entirely different sets of such parameters. It is of interest to mention that it was shown, in single joint ballistic forearm movements, that the EMG amplitude of the antagonist muscle group is scaled by r^2 when the movement duration is compressed by a factor r [Lestienne 1979]. Whether muscle activity in two-joint movements display the same behavior, still needs to be assessed.

One question which requires investigation, and is not derived directly from this work, but which relates to problems of dynamics in biological movements, is the question of identification of inertial parameters of body segments or tools. In order to provide appropriate joint torques, the nervous system must have a knowledge of the organism's body parameters (inertias of limb segments, masses etc.) and of carried tools. It must also be capable of dealing with forces exerted by the environment. Little is known about the processes that are used in the identification of body parameters although proprioceptive information may play a major role in this identification. Similarly, forces exerted by the environment might be estimated on the basis of afferent signals from various proprioceptive sensors and reflexes might provide load compensation. A better way to deal with forces exerted by the environment, however, might involve the control of the end-effector impedance.

Another issue, which is of relevance to the computational models, is the transformation from Cartesian space of the hand to joint space of the arm. Although efficient computations for the solution of this problem were suggested in robotics [Hollerbach 1982] this issue should be investigated for biological movements.

How the inverse kinematics problem is solved for redundant configuration is another open question although it was proposed that optimization methods may be involved in these solutions [Benati et al. 1983].

5.6.2. Pathological motor behavior

In this work, only movements of adult intact human subjects were measured and investigated. Given the methods developed here for the kinematic and dynamic analysis of movements, one can extend this study to investigate abnormal arm movements, in various motor disorders. As was discussed above, the jerk-minimization model offers a performance criteria for human arm movements since it can predict quite accurately using position information (i.e. the locations of targets), what a normal hand trajectory should look like. In certain disease processes affecting humans, such as basal ganglia disorders (Parkinson disease, Huntington Chorea) and in cerebellar disorders, patients generate abnormal looking movements. In order to gain some insight, at least at the level of kinematics and dynamics, into the specific characteristics in which these movements differ from normal movements, one could gather kinematic data from patients performing planar two-joint arm movements and assess their performance, using performance criteria derived from the trajectories predicted for intact humans under the same set of conditions. Currently, experiments in which movements which are generated by patients suffering from cerebellar disorders are conducted at the department of Psychology at the Massachusetts Institute of Technology and at the Brigham hospital by Abend. Attention is directed, in these studies, to the definition of the lesion region, since different roles for different cerebellar regions have been proposed on the basis of both clinical and animal studies. Such descriptive kinematic analysis of multi-joint arm movement, in cerebellar patients, is clearly required, since the only kinematic descriptions available for such movements date back to Holmes [1939]. Moreover, since many clinical findings in cerebellar patients are strongly tied to the issues of kinematic and dynamic aspects of trajectory formation, it is of interest to investigate the movements generated by such patients dealing with similar issues to the ones dealt with in this work. Movements in cerebellar patients have been described in the literature as discontinuous and jerky and the amplitude and onset of jerk depends on the severity of the syndrome. Even in simple movements, "within even short intervals their speed varies being sometimes slower, sometimes faster, than in the normal limb, the natural rhythm and regularity of motion is lost" [Holmes 1939], and "delay in starting movements and irregularity in its acceleration are probably fundamental disturbances" [Holmes 1939]. In compound movements (movements involving more then one joint), "there is a lack in synchrony in separate components of movement" thus, "the affected limb seems to decompose the movement in time into its constituent parts" [Holmes 1939]. Moreover "delay in the initiation of one component, relative to another, and excessive range of one element of the movement particularly at the proximal joint are the chief causes of decomposition, but another factor is the defective fixation of one or the other of the moving joints" [Holmes 1939]. Reading these descriptions one might speculate that in cerebellar disorders, the motor programs are defective either at the kinematic level of trajectory planning, or at the level of dynamics, particularly in the ability to provide or compensate for interaction forces. With regard to the capability of such patients for generating, or perhaps planning hand trajectories, it is interesting to mention that it was argued by Eccles [1973] that the essential feature in cerebellar disorders is the failure of smooth control and that patients cannot carry out smooth, integrated movements, but decompose the movements into actions at one joint after the other to bring about the desired position of the limb. Even more severe deviations are seen when cerebellar patients are required to change the direction of movement such as when drawing a square. It is of interest to mention, also, in connection with the study of curved versus straight movements, that in monkeys with laboratory produced cerebellar lesions, the hand trajectories are more prominently curved than in normal primates which tend to generate roughly straight paths [Gilman et al. 1976].

5.6.3. General suggestions

Theoretical studies should be conducted combined with experimental work, in order to investigate whether any of the currently proposed models for the control of complex movements, is biologically plausible. In addition, more theoretical studies should be conducted which may perhaps offer other suggestions for strategies of control and planning of complex movements. Any such theories should propose mechanisms for dealing with the various kinematic and dynamic problems which were investigated and identified in this study. In particular, any such theory should suggest how the proposed scheme deals with the problems of dynamics (e.g. interaction forces, corrections for unexpected disturbances) and should be capable of yielding hand trajectories which exhibit the same kinematic features as those displayed by human (and primate) hand trajectories, including the Cartesian path and speed invariances, the time scaling property, the isochrony principle and the speed-curvature coupling of curved movements.

Another general suggestion for the extension of this work involves a further investigation of what objectives are optimized in human movements. Such pursuits could hopefully enable us to suggest analytical models (not verbal, vague discussions) which relate to motor organization and may help to identify unifying principles which underlie motor behavior. Such studies, will also, hopefully, enable us to describe, analytically, the contents of motor programs at various hierarchical levels of motor presentation. The idea of an internal representation of movement has been referred to repeatedly in the motor control literature, but very few real hypotheses have been made which suggest what aspects of movement generation, are actually represented. With the exception of very few studies, even less work was done in which attempts were made, to suggest what kind of selection and translation processes are performed by the CNS in converting an intention to move, into a detailed patterns of neural signals. Optimal control theory, might offer such a possibility of suggesting such testable models [Pew & Baron 1978]. Moreover, although one should not overlook the possibility that various classes of movements may require different control strategies for their generation, as is the case in the work done here, attempts should be made to look for unifying strategies and principles. Therefore, although one should not conclude from the success of a mathematical model in describing a real behavior, that the biological system indeed uses an analogous strategy, the combination of specific mathematical simulations and predictions based on optimal control theory, with experiments, can permit the testing of such theories, and therefore, has many merits.

Along similar lines, man-developed approaches to the control of artificial manipulators should be combined with physiological experiments and behavioral analysis of movement. Such a combination can offer a framework in which ideas from robotics can be tested against experimentally observed motor behavior, and the analysis of such behavior can serve as a basis for suggesting strategies and algorithms for the control of mechanical arms.

Such approaches to the study of motor control are needed in order to create, hopefully, from the overwhelming number of bits and pieces (i.e. anatomical and physiological findings) of this tremendously complicated puzzle, a picture, which is complete in at least some of its parts.

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Appendix A

Dynamic optimization

The optimization method

We want to minimize the following objective function:

$$C = \frac{1}{2}\int_0^{t_f} (\frac{d\underset{\sim}{a}}{dt})^2 \, dt = \frac{1}{2}\int_0^{t_f} ((\frac{d^3x}{dt^3})^2 + (\frac{d^3y}{dt^3})^2) \, dt \qquad (A1)$$

Generally, optimization problems similar to the problem solved here involve a system which can be described by a set of nonlinear differential equations

$$\dot{\underset{\sim}{s}} = f[s(t), u(t), t] \qquad (A2)$$

where $\underset{\sim}{s}(t)$ is an n vector function of state variables and $\underset{\sim}{u}(t)$ is an m vector control function. The problem is to find the control $\underset{\sim}{u}(t)$ which in carrying the system from an initial state $\underset{\sim}{s}(0)$ to a final state $\underset{\sim}{s}(t_f)$, the cost function C(t) is optimized. C(t) is defined as:

$$C(t) = \int_0^{t_f} L[\underset{\sim}{s}(t), \underset{\sim}{u}(t), t] \, dt \qquad (A3)$$

where $L[\underset{\sim}{s}(t), \underset{\sim}{u}(t), t]$ is the performance index.

If there are equality or inequality constraints on some of the state or input variables, the problem can be solved by means of Pontryagin equations. One defines a n component co-state (Lagrange multipliers) vector $\underset{\sim}{\lambda}(t)$ and a scalar hamiltonian:

$$H[\dot{\underset{\sim}{s}}(t), \underset{\sim}{u}(t), t] = L[\underset{\sim}{s}(t), \underset{\sim}{u}(t), t] + \underset{\sim}{\lambda}^T(t) f[\underset{\sim}{s}(t), \underset{\sim}{u}(t), t] \tag{A4}$$

To solve the problem the following differential equations must be solved:

$$\dot{\underset{\sim}{s}}(t) = \underset{\sim}{f}(\underset{\sim}{s}(t), \underset{\sim}{u}(t), t) \tag{A5}$$

$$\dot{\underset{\sim}{\lambda}}(t) = -\frac{\partial H}{\partial s}(\underset{\sim}{s}(t), \underset{\sim}{u}(t), \underset{\sim}{\lambda}(t), t) \tag{A6}$$

$$\frac{\partial H}{\partial u} = 0 \tag{A7}$$

Equations A5, A6 and A7 are Pontryagin equations.

Jerk minimization: point-to-point movements.

For our problem we define a state vector $\underset{\sim}{s}^T(t) = [x, y, u, v, z, w]$ and a control vector $\underset{\sim}{u}^T(t) = [\delta, \gamma]$. The components of these vectors are defined by the system equations :

$$\begin{aligned}
\dot{x} &= u \\
\dot{y} &= v \\
\dot{u} &= \ddot{x} = z \\
\dot{v} &= \ddot{y} = w \\
\dot{z} &= \dddot{x} = jerk_x = \delta \\
\dot{w} &= \dddot{y} = jerk_y = \gamma
\end{aligned} \tag{A8}$$

and the hamiltonian is:

$$\begin{aligned}
H =& \lambda_x u + \lambda_y v + \lambda_u z + \lambda_v w + \lambda_z \delta + \lambda_w \gamma \\
&+ \frac{1}{2}(\gamma^2 + \delta^2)
\end{aligned} \tag{A9}$$

Thus Pontryagin equations become

$$
\begin{aligned}
-\frac{d\lambda_x}{dt} &= 0\\
-\frac{d\lambda_y}{dt} &= 0\\
-\frac{d\lambda_u}{dt} &= \lambda_x\\
-\frac{d\lambda_v}{dt} &= \lambda_y\\
-\frac{d\lambda_z}{dt} &= \lambda_u\\
-\frac{d\lambda_w}{dt} &= \lambda_v
\end{aligned}
\tag{A10}
$$

and the conditions on the control variables are:

$$
\begin{aligned}
\frac{\partial H}{\partial \delta} &= \delta + \lambda_z = 0\\
\frac{\partial H}{\partial \gamma} &= \gamma + \lambda_w = 0
\end{aligned}
\tag{A11}
$$

Applying the following boundary conditions:

$$
\begin{aligned}
x(0) &= x_0 & y(0) &= y_0\\
x(t_f) &= x_f & y(t_f) &= y_f\\
u(0) &= 0 & u(t_f) &= 0\\
v(0) &= 0 & v(t_f) &= 0\\
z(0) &= 0 & z(t_f) &= 0\\
w(0) &= 0 & w(t_f) &= 0
\end{aligned}
\tag{A12}
$$

we solve Pontryagin equations and get the following fifth order polynomials for the x(t) and y(t), position coordinates:

$$
\begin{aligned}
x(t) &= a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5\\
y(t) &= b_0 + b_1 t + b_2 t^2 + b_3 t^3 + b_4 t^4 + b_5 t^5
\end{aligned}
\tag{A13}
$$

Appendix B

Unconstrained optimization

Minimum-jerk point-to-point movements.

If the function x(t) is sufficiently differentiable in the interval, then the unconstrained cost function

$$C(x(t)) = \int_0^{t_f} L[t, x, \dot{x}, \ddot{x}, \dots \frac{d^n x}{d^n t}]\, dt \tag{B1}$$

assumes an extremum when x(t) is the solution of Euler-Poisson equation:

$\frac{\partial L}{\partial x}$

$$\frac{\partial L}{\partial t} - \frac{d}{dt}(\frac{\partial L}{\partial \dot{x}}) \dots\dots + (-1)^n \frac{d^n}{dt^n} \frac{\partial L}{\partial (^n x)} = 0 \tag{B2}$$

Since in our case

$$L = \frac{1}{2}((\dddot{x})^2 + (\dddot{y})^2) \tag{B3}$$

we get the equation

$$\frac{d^3}{dt^3}(\frac{\partial \dddot{x}^2}{\partial \dddot{x}}) + \frac{d^3}{dt^3}(\frac{\partial \dddot{y}^2}{\partial \dddot{y}}) = 0 \tag{B4}$$

We can uncouple the terms depending on the two position components to get:

$$\frac{d^6 x}{dt^6} = 0 \qquad \frac{d^6 y}{dt^6} = 0 \tag{B5}$$

The resulting solution to differential equations of this kind is given by a fifth order polynomials

$$\begin{aligned} x(t) &= a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5 \\ y(t) &= b_0 + b_1 t + b_2 t^2 + b_3 t^3 + b_4 t^4 + b_5 t^5 \end{aligned} \tag{B6}$$

Appendix C

Via point constrained optimization

Problems with interior point equality constraints.

For curved movements we assumed that the hand is constrained to pass through the via point at time t_1 and the coordinates of this point are x_1 and y_1. Problems of this kind are defined as *optimal control problems with interior point equality constraints on the state variables* [Bryson & Ho, 1975].

For such problems one has a set of constraints at some

$$\underset{\sim}{N}(\underset{\sim}{s}(t_1), t_1) = 0 \tag{C1}$$

where N is a q-component vector function. These interior point constraints can be augmented to the cost function by a multipliers vector $\underset{\sim}{\pi}$ so that the new cost function is:

$$C = \underset{\sim}{\pi}^T \underset{\sim}{N} + \int_0^{t_f} (H - \underset{\sim}{\lambda}^T \dot{\underset{\sim}{s}}) \tag{C2}$$

The solution is obtained by allowing discontinuities in the co-state variables (Lagrange coefficients) $\underset{\sim}{\lambda}(t)$'s and in the hamiltonian $H[t, \underset{\sim}{\lambda}(t), \underset{\sim}{s}(t)]$. One can define Lagrange coefficients $\underset{\sim}{\lambda}^+(t)$ and hamiltonian $H^+(t)$ for $t \geq t_1$ and vector $\underset{\sim}{\lambda}^-(t)$ and hamiltonian H^- for $t \leq t_1$.

At time t_1 these variables satisfy the equations:

$$\underset{\sim}{\lambda}^{-}(t_1) = \underset{\sim}{\lambda}^{+}(t_1) + \underset{\sim}{\pi}^{T} \frac{\partial \underset{\sim}{N}}{\partial \underset{\sim}{s}(t_1)} \tag{C3}$$

$$H^{-}(t_1) = H^{+}(t_1) - \underset{\sim}{\pi}^{T} \frac{\partial \underset{\sim}{N}}{\partial t_1} \tag{C4}$$

The q components of $\underset{\sim}{\pi}$ are determined by the constraint equations (C1) while time t_1 is fully determined by equation (C4).

Minimum-jerk trajectories with via point constraints.

For our specific problem the only constraints are on hand position along both axes at time t_1:

$$\begin{aligned} x(t_1) &= x_1 \\ y(t_1) &= y_1 \end{aligned} \tag{C5}$$

The hamiltonian H^{-} for all times such that $t \leq t_1$ is:

$$\begin{aligned} H^{-} = &\lambda_x^{-} u^{-} + \lambda_y^{-} v^{-} + \lambda_u^{-} z^{-} + \lambda_v^{-} w^{-} + \lambda_z^{-} \delta^{-} + \lambda_w^{-} \gamma^{-} \\ &+ \frac{1}{2}((\gamma^{-})^2 + (\delta^{-})^2) \end{aligned} \tag{C6}$$

and the hamiltonian H^{+} for times $t \geq t_1$ is:

$$\begin{aligned} H^{+} = &\lambda_x^{+} u^{+} + \lambda_y^{+} v^{+} + \lambda_u^{+} z^{+} + \lambda_v^{+} w^{+} + \lambda_z^{+} \delta^{+} + \lambda_w^{+} \gamma^{+} \\ &+ \frac{1}{2}((\gamma^{+})^2 + (\delta^{+})^2) \end{aligned} \tag{C7}$$

Since the only constraint equations relate to position the only discontinuities are in λ_x and λ_y and, therefore, according to equation (C3), we get :

$$\begin{aligned} \lambda_x^{-} &= \lambda_x^{+} + \pi_1 \\ \lambda_y^{-} &= \lambda_y^{+} + \pi_2 \end{aligned} \tag{C8}$$

while all the other Lagrange coefficients are continuous at $t = t_1$:

$$\begin{aligned} \lambda_u^{+}(t_1) &= \lambda_u^{-}(t_1) \\ \lambda_v^{+}(t_1) &= \lambda_v^{-}(t_1) \\ \lambda_z^{+}(t_1) &= \lambda_z^{-}(t_1) \\ \lambda_w^{+}(t_1) &= \lambda_w^{-}(t_1) \end{aligned} \tag{C9}$$

Since time t_1 is not explicitly specified, the hamiltonian must be continuous at t_1 as emerges from equation (C4) :

$$H^+(t_1) = H^-(t_1) \qquad (C10)$$

Next we can write Pontryagin equations as well as the requirement that the derivatives of the hamiltonian with respect to the controls will vanish . These equations are derived separately for $t \geq t_1$ and $t \leq t_1$. In addition we require continuity of velocities and accelerations at t_1, so that:

$$\begin{aligned} u^+(t_1) &= u^-(t_1) \\ v^+(t_1) &= v^-(t_1) \\ z^+(t_1) &= z^-(t_1) \\ w^+(t_1) &= w^-(t_1) \end{aligned} \qquad (C11)$$

These come as a result of the requirements of the optimization technique for continuity of the state variables at the interior points [Bryson & Ho, 1975] .

Solving Pontryagin equations and applying the boundary conditions at $t = t_0$ and at $t = t_f$ one obtains an expression for the x(t) at all times $t \leq t_1$:

$$\begin{aligned} x^-(\tau) = \frac{t_f^5}{720}\Big(&\pi_1\Big(\tau_1^4(15\tau^4 - 30\tau^3) + \tau_1^3(80\tau^3 - 30\tau^4) - 60\tau^3\tau_1^2 + 30\tau^4\tau_1 - 6\tau^5\Big) \\ &+c_1\Big(15\tau^4 - 10\tau^3 - 6\tau^5\Big)\Big) + x_0 \end{aligned} \qquad (C12)$$

and for times $t \geq t_1$ the expression is:

$$\begin{aligned} x^+(\tau) &= \frac{t_f^5}{720}\Big(\pi_1\Big(\tau_1^4(15\tau^4 - 30\tau^3 + 30\tau - 15) + \tau_1^3(-30\tau^4 + 80\tau^3 - 60\tau^2 + 10)\Big) \\ &\quad +c_1\Big(-6\tau^5 + 15\tau^4 - 10\tau^3 + 1\Big)\Big) + x_f \\ &= x^-(\tau) + \pi_1 \frac{t_f{}^5(\tau - \tau 1)^5}{120} \end{aligned} \qquad (C13)$$

where c_1 and π_1 are constants, $\tau = t/t_f$ and $\tau_1 = t_1/t_f$.

The same expressions are obtained for y^+ and for y^- with π_2 and c_2 replacing π_1 and c_1 . Substituting $x^+(t_1) = x^-(t_1) = x_1$ in equations (C12) and (C13) and solving for π_1 and c_1, we obtain the following expressions :

$$c_1 = \frac{1}{t_f^5 \tau_1^2 (1-\tau_1)^5} \Big((x_f - x_0)(300{\tau_1}^5 - 1200{\tau_1}^4 + 1600{\tau_1}^3) + {\tau_1}^2(-720x_f + 120x_1 + 600x_0) + (x_0 - x_1)(300\tau_1 - 200) \Big) \qquad (C14)$$

$$\pi_1 = \frac{1}{{t_f}^5 {\tau_1}^5 (1-\tau_1)^5} \Big((x_f - x_0)(120{\tau_1}^5 - 300{\tau_1}^4 + 200{\tau_1}^3) - 20(x_1 - x_0) \Big) \qquad (C15)$$

and for π_2 and c_2 similarly, with y_0 replacing x_0 and y_f replacing x_f etc .

Next ,we substitute the expressions for π_1 π_2 c_1 and c_2 in equation (C10) which reduces to

$$\pi_1 u(t_1) + \pi_2 v(t_1) = 0 \qquad (C16)$$

and we get a polynomial equation in $\tau_1 = t_1/t_f$. We find the real roots of this polynomial and accept only those roots that lie between 0 and 1. The polynomials we obtained had only one acceptable root. We then substitute this value for τ_1 in the expressions for π_1 π_2 c_1 and c_2 and we can finally get the expressions for x(t) and y(t) for the entire movement.

Acceleration and snap minimization with via-point constraints.

Applying a similar method to solve the problem of acceleration minimization for movements which are constrained to pass through a via points, the following expressions for the trajectories are obtained:

For $t \leq t_1$:

$$x^{-}(\tau) = \frac{t_f^3}{12}\Big(\pi_1(2\tau_1^3 - 6\tau_1\tau^2 + 3{\tau_1}^2\tau^2) + c_1(2\tau^3 - 3\tau^2)\Big) \qquad (C17)$$

and for $t \geq t_1$:

$$\begin{aligned} x^{+}(\tau) &= \frac{t_f^3}{12}\Big(3\pi_1{\tau_1}^2(\tau - 1)^2 + c_1(2\tau^3 - 3\tau^2 + 1)\Big) \\ &= x^{-}(\tau) + \pi_1\frac{{t_f}^3}{6}(\tau - \tau_1)^3 \end{aligned} \qquad (C18)$$

where c_1 π_1 are constants. Similar expressions are obtained for y^{-} and y^{+}. For c_1 and π_1 the expressions are:

$$c_1 = 12\frac{(x_0 - x_f)}{t_f^3} + \pi_1(2\tau_1^3 - 3\tau_1^2) \qquad (C19)$$

$$\pi_1 = 3\frac{\Big((x_1 - x_0) - (x_0 - x_f)(2\tau_1^3 - 3\tau_1^2)\Big)}{t_f^3\tau_1^3(1 - \tau_1)^3} \qquad (C20)$$

Similarily curved movement trajectories have been derived assuming snap is being minimized. These long expressions will not be given here, but we have again a solution with:

$$x^{+}(\tau) = x^{-}(\tau) + \frac{\pi_1 t\epsilon^7(\tau - \tau_1)^7}{5040} \qquad (C21)$$

and a similar relation between $y^{+}(\tau)$ and $y^{-}(\tau)$ where π_1 and π_2 depend again on the positions at the beginning of the movement, at the via point and at the end of the movement.

Appendix D

Coordinate systems: definition and transformations.

D.1. Definition of coordinate systems

To calculate joint forces and torques in an open kinematic chain linkage system, it is helpful to define a few coordinate systems for the mechanism. The definition of coordinate systems used in this work followes the convention suggested in Orin et al. [1979].

In Orin's notation, the links of a manipulator are numbered consecutively from 1 to n starting from the base to the tip. By convention the reference frame is numbered link 0. The joints are the points of articulation between links, and are numbered so that joint $i, i-1$ connects links i and $i-1$. The first subscript of a symbol associates it with the member of the linkage to which it is attached or referred, the second subscript, if used refers to a second member which is joined to the first.

Three orthogonal coordinate system are fixed in each link (Figure D-1), as follows:

Principal coordinates

The right-handed system of orthogonal unit vectors ($\hat{q}_i^1$, $\hat{q}_i^2$, $\hat{q}_i^3$) has its origin at the center of gravity of link i and is oriented so that all the products of inertia are zero relative to this system. This is a set of principal-axis unit vectors rigidly attached to link i. The origin of this set is denoted by O_i.

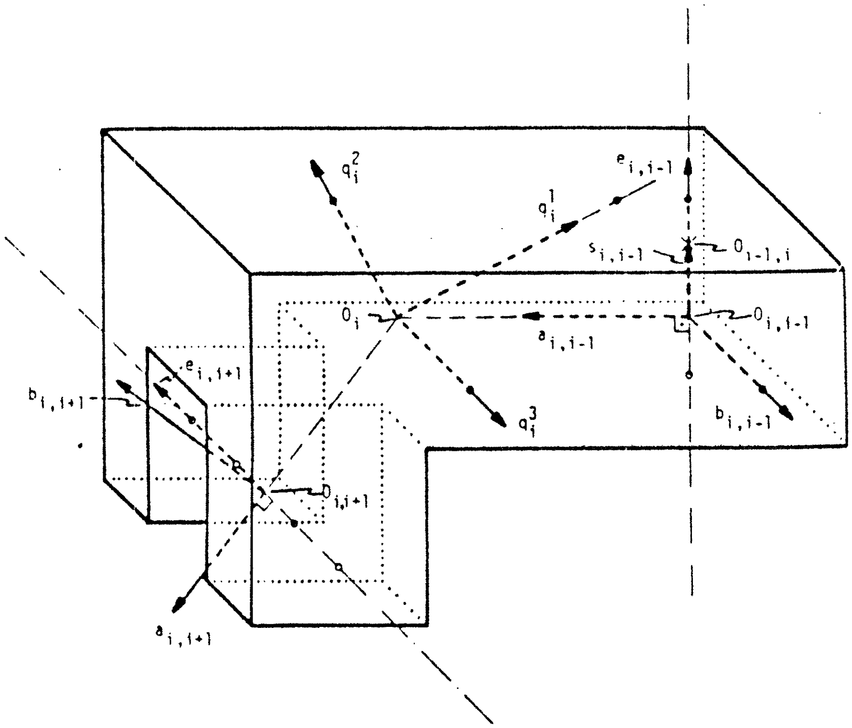

Figure 1. Orin et al. coordinate axes definition for connected links and relationships between neighboring coordinate axes.

Proximal joint-axis systems

Joint i,j connects links i and j. At each joint i,j two coordinate systems are defined, a proximal system at joint i, attached to link i, and a distal coordinate system, at joint i, for link j. The three unit vectors of the proximal system are defined as:

$\hat{e}_{i,i-1}$ is attached to link i and is directed along the axis of joint $i, i-1$,

$\hat{a}_{i,i-1}$ lies along the unique perpendicular to $\hat{e}_{i,i-1}$ which passes through the center of gravity of link i, and is oriented so that its positive sense is towards link n.

$\hat{b}_{i,i-1}$ completes the right-handed coordinate system.

Distal joint-axes system

The distal coordinate system is defined by the following unit vectors:

$\hat{e}_{i-1,i}$ is attached to link $i - 1$ at its distal end and is oriented in the same direction as $\hat{e}_{i,i-1}$.

$\hat{a}_{i-1,i}$ lies along the unique perpendicular to the joint axis, which passes through link $i - 1$ center of gravity, and is again directed towards the last member.

$\hat{b}_{i,i-1}$ completes the right-handed system.

The relative position of two adjacent links is completely described by the following two parameters:

s_i is the distance between $O_{i,i-1}$ and $O_{i-1,i}$ measured along $\hat{e}_{i,i-1}$.

$\alpha_{i,i-1}$ is the angle between the $\hat{a}_{i-1,i}$ and $\hat{a}_{i,i-1}$ axes measured in a right-handed sense about $\hat{e}_{i,i-1}$.

If the joint is rotational the joint variable will be $\alpha_{i,i-1}$, if translational the joint variable will be $s_{i,i-1}$. The symbol q_i will designate the variable for joint i whether it is $s_{i,i-1}$ or $\alpha_{i,i-1}$. Multiple rotational degrees of freedom can be expressed by joints with zero mass and zero length links.

Inertial coordinates

In addition to these link-fixed coordinate systems, a fourth inertial frame is defined. This system can be described by the unit vectors $(\hat{i}, \hat{j}, \hat{k})$.

D.2. Coordinate transformations

Transformations of vectors, from one coordinate system to another, may be easily accomplished through the use of 3x3 orthogonal transformation matrices. More specifically:

$$\begin{bmatrix} \hat{e}_{i,i-1} \\ \hat{a}_{i,i-1} \\ \hat{b}_{i,i-1} \end{bmatrix} = P_{i,i-1} \begin{bmatrix} \hat{q}_i^1 \\ \hat{q}_i^2 \\ \hat{q}_i^3 \end{bmatrix} \qquad (D1)$$

$$\begin{bmatrix} \hat{e}_{i,i+1} \\ \hat{a}_{i,i+1} \\ \hat{b}_{i,i+1} \end{bmatrix} = P_{i,i+1} \begin{bmatrix} \hat{q}_i^1 \\ \hat{q}_i^2 \\ \hat{q}_i^3 \end{bmatrix} \tag{D2}$$

$$\begin{bmatrix} \hat{e}_{i,i-1} \\ \hat{a}_{i,i-1} \\ \hat{b}_{i,i-1} \end{bmatrix} = P_{i,i-1} P_{i,i+1}^T \begin{bmatrix} \hat{e}_{i,i+1} \\ \hat{a}_{i,i+1} \\ \hat{b}_{i,i+1} \end{bmatrix} \tag{D3}$$

where the matrices $P_{i,i-1}$ and $P_{1,i+1}$ termed the Proximal-joint and the Distal-joint coordinate transformation matrices, respectively. These matrices are constant and are characteristics for each link i in the chain.

In order to define the orientation of the two coordinate systems at each joint with respect to each other (Distal of link i-1 and proximal of link i), another 3x3 orthogonal transformation matrix may be defined which varies with the relative angle at the joint:

$$\begin{bmatrix} \hat{e}_{i-1,i} \\ \hat{a}_{i-1,i} \\ \hat{b}_{i-1,i} \end{bmatrix} = T_{i-1,i} \begin{bmatrix} \hat{e}_{i,i-1} \\ \hat{a}_{i,i-1} \\ \hat{b}_{i,i-1} \end{bmatrix} \tag{D4}$$

where

$$T_{i-1,i} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha_{i-1,1} & \sin\alpha_{i-1,i} \\ 0 & -\sin\alpha_{i-1,i} & \cos\alpha_{i-1,i} \end{bmatrix} \tag{D5}$$

Other vectors, necessary for the definition of the mechanism, are described in appendix F. We define, also, a coordinate transformation between the inertial coordinate system and the proximal coordinate system, as followes:

$$\begin{bmatrix} \hat{i} \\ \hat{j} \\ \hat{k} \end{bmatrix} = A_i \begin{bmatrix} \hat{q}_i^1 \\ \hat{q}_i^2 \\ \hat{q}_i^3 \end{bmatrix} \tag{D6}$$

to find the elements of A_i one can progress from the base towards link i, so that:

$$A_i = A_{i-1} P^T_{i-1,1} P_{i,i-1} \tag{D8}$$

Appendix E

Recursive Newton- Euler inverse dynamics

E.1. Introduction

The inverse problem for manipulator dynamics, computes the joint torques which are required to produce given joint positions, velocities, and accelerations.

Recently a number of investigators have proposed an efficient recursive Newton-Euler formulation of manipulator dynamics [Luh, Walker, and Paul 1980] [Orin et al. 1979].

This formalism is based on the following scheme:

(i) Recursion of the velocities and accelerations working from the base of the manipulator to the end link.

(ii) Backward recursion of the forces and torques working from the end link to the base of the manipulator.

(iii) The use of 3x3 rotation matrices.

What followes is a brief revision of Newton-Euler equations [Luh, Walker, and Paul 1980], taken from [Hollerbach, 1980].

E.2. Recursive Newton-Euler Dynamics

In the following definitions and equations, capital letters represent 3x3 matrices, lower case letters represent 3x1 vectors. When a vector is presented without a left superscript, it is referred to the base coordinate orientation ($v = {}^{0}v$).

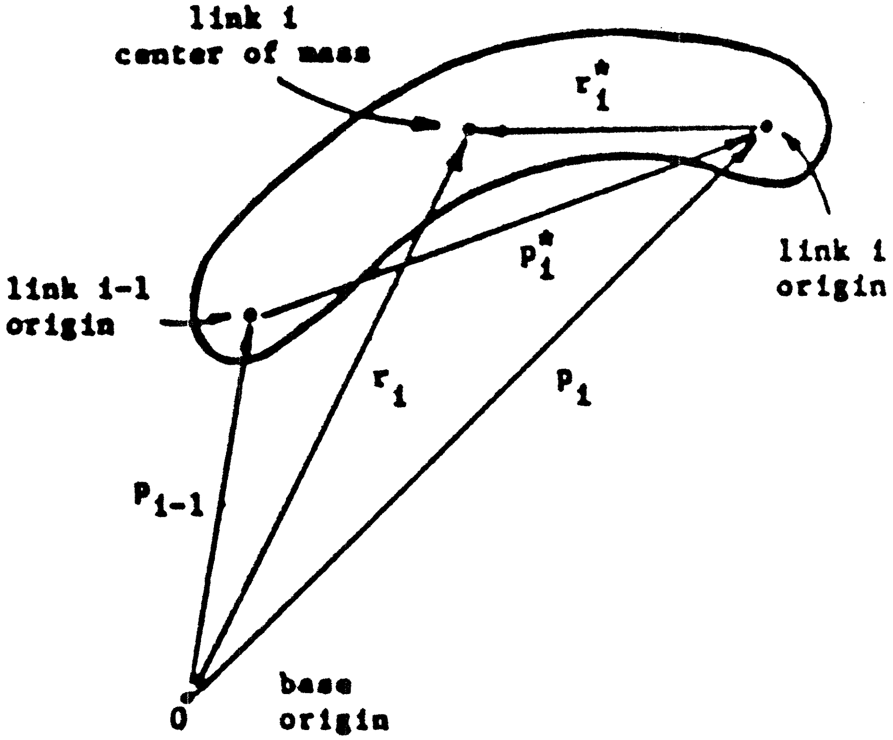

Figure 1. Vector definitions between the base origin and the link origins and center of masses.

For each link the following vectors are defined (see Figure E-1):

$\mathbf{p}_i$ is a vector from the base coordinate origin to the joint i coordinate origin,

$\mathbf{p}_i^*$ is a vector from coordinate origin $i-1$ to coordinate origin i,

$\mathbf{r}_i$ is a vector from the base coordinate origin to the link i center of mass,

$\mathbf{r}_i^*$ is a vector from coordinate origin i to the link i center of mass.

The forward recursion propagates angular velocities, angular accelerations, linear accelerations, total link forces, and total link torques from the base to the end link. For a rotational joint, the recursive equations are:

$$
\begin{aligned}
\underset{\sim}{\omega}_i &= \underset{\sim}{\omega}_{i-1} + \mathbf{z}_{i-1}\dot{q}_i \\
\dot{\underset{\sim}{\omega}}_i &= \dot{\underset{\sim}{\omega}}_{i-1} + \mathbf{z}_{i-1}\ddot{q}_i + \underset{\sim}{\omega}_{i-1} \times \mathbf{z}_{i-1}\dot{q}_i \\
\ddot{\mathbf{p}}_i &= \dot{\underset{\sim}{\omega}}_i \times \mathbf{p}_i^* + \underset{\sim}{\omega}_i \times (\underset{\sim}{\omega}_i \times \mathbf{p}_i^*) + \ddot{\mathbf{p}}_{i-1} \\
\ddot{\mathbf{r}}_i &= \underset{\sim}{\omega}_i \times (\underset{\sim}{\omega}_i \times \mathbf{r}_i^*) + \dot{\underset{\sim}{\omega}}_i \times \mathbf{r}_i^* + \ddot{\mathbf{p}}_i \\
\mathbf{f}_i &= m_i \ddot{\mathbf{r}}_i \\
\mathbf{n}_i &= I_i \dot{\underset{\sim}{\omega}}_i + \underset{\sim}{\omega}_i \times (I_i \underset{\sim}{\omega}_i)
\end{aligned}
$$

where the previously undefined terms are :

m_i is the mass of link i,

$\mathbf{z}_i$ is the unit vector along the joint axis,

q_i is the joint angle,

$\underset{\sim}{\omega}_i$ is the angular velocity of link i,

$\dot{\underset{\sim}{\omega}}_i$ is the angular acceleration of link i,

$\mathbf{f}_i$ is the total external force on link i,

$\mathbf{n}_i$ is the total external torque on link i, and

I_i is the inertia tensor of link i about its center of mass.

Te forward recursion propagates the forces and moments exerted on link i by link $i - 1$ from the end link of the manipulator to the base.

$$
\begin{aligned}
\mathbf{f}_{i-1,i} &= \mathbf{f}_i + \mathbf{f}_{i,i+1} \\
\mathbf{n}_{i-1,i} &= \mathbf{n}_{i,i+1} + \mathbf{n}_i + (\mathbf{p}_i^* + \mathbf{r}_i^*) \times \mathbf{f}_i + \mathbf{p}_i^* \times \mathbf{f}_{i,i+1} \\
\tau_i &= \mathbf{z}_{i-1} \cdot \mathbf{n}_{i-1,i}
\end{aligned}
$$

where

$\mathbf{f}_{i-1,i}$ is the force exerted on link i by link $i - 1$,

$\mathbf{n}_{i-1,i}$ is the moment exerted on link i by link $i - 1$, and

τ_i is the input torque at joint i.

In this formulation the implicit reference coordinate frame is the base coordinates. In the next section a more efficient formalism using a link-fixed reference frame is examined.

E.3. Recursive Newton-Euler Dynamics Referred to Link Coordinates

Orin et al. [1979] initially proposed that the forces and moments in the Newton-Euler formulation be referred to the link's internal coordinate system. Armstrong [1979] and Luh, Walker, and Paul [1980] extended this idea by calculating the angular and linear velocities and accelerations in link coordinates as well. The advantage of referring both the dynamics and kinematics to the link cordinates is to eliminate the need for a great deal of coördinate transformation and to allow the inertia tensor to be fixed in each link coordinate system.

Using Luh, Walker, and Paul's formulation once more, instead of rewriting all the Newton-Euler equations showing how they are referred to link-fixed coordinate systems, we present 3 examples of this reformulation. A more complete presentation, is described in [Luh, Walker, and Paul 1980].

$$
\begin{aligned}
{}^{i}\underset{\sim}{\omega}_i &= A_i^T\left({}^{i-1}\underset{\sim}{\omega}_{i-1} + {}^{i-1}\mathbf{z}_{i-1}\dot{q}_i\right) \\
{}^{i}\mathbf{n}_i &= {}^{i}I_i\,{}^{i}\dot{\underset{\sim}{\omega}}_i + {}^{i}\underset{\sim}{\omega}_i \times \left({}^{i}I_i\,{}^{i}\underset{\sim}{\omega}_i\right) \\
{}^{i}\mathbf{f}_{i-1,i} &= {}^{i}\mathbf{f}_i + A_{i+1}\left({}^{i+1}\mathbf{f}_{i,i+1}\right)
\end{aligned}
$$

Where A_i is a 3x3 rotation matrix relating the orientations of coordinate system $i-1$ and i. That is to say, if ${}^{i}\mathbf{v}$ is a vector expressed in terms of the orientation of coordinate system i axes, then ${}^{i-1}\mathbf{v} = A_i\,{}^{i}\mathbf{v}$. $A_i^T = (A_i)^{-1}$, and ${}^{i}\mathbf{v}$ denotes a vector v represented in the orientation of coordinate system j measured from the base origin. The number of additions and multiplications, required in this formalism varies linearly with the number of joints.

Appendix F

Recursive Newton- Euler integral dynamics

F.1 Introduction

The problem is to find link and joint accelerations for a given set of joint torques. This formalism is based on a formalism suggested by Armstrong [1979]. His formalism was modified referring the computations to coordinate systems located at the links center of mass and not to systems located at the proximal joints, as was done in Armstrong's original formulation. The solution proceeds by formulating two linear relations: between the link's linear acceleration and the angular acceleration at the proximal joint, and between the intersegmental force at the proximal joint and the link's linear acceleration. The coefficients of these relations are computed recrusively, distally to proximally. To compute joint accelerations the initial velocities and positions of all links must be known. When the accelerations of all the links are computed they can be integrated to yield the velocities and positions of all links.

F.2 The formalism.

We start by writing again the equations of motion. In this equations $R_{i,i+1}$ has the same meaning as A_{i+1} in Appendix E, and $\mathbf{a}_i = \ddot{\mathbf{p}}_i$.

$$\begin{aligned} \mathbf{a}_{i+1} &= \dot{\underset{\sim}{\omega}}_i \times \overset{*}{\mathbf{p}}_i + \underset{\sim}{\omega}_i \times (\underset{\sim}{\omega}_i \times \overset{*}{\mathbf{p}}_i) + \mathbf{a}_{i-1} && (F1) \\ \ddot{\mathbf{r}}_i &= \underset{\sim}{\omega}_i \times (\underset{\sim}{\omega}_i \times \overset{*}{\mathbf{r}}_i) + \dot{\underset{\sim}{\omega}}_i \times \overset{*}{\mathbf{r}}_i + \mathbf{a}_i && (F2) \\ \mathbf{f}_i &= m_i \ddot{\mathbf{r}}_i && (F3) \\ \mathbf{n}_i &= J_i \dot{\underset{\sim}{\omega}}_i + \underset{\sim}{\omega}_i \times (J_i \underset{\sim}{\omega}_i) && (F4) \end{aligned}$$

$$\begin{aligned} \mathbf{f}_{i-1,i} &= \mathbf{f}_i + R_{i,i+1}\mathbf{f}_{i,i+1} && (F5) \\ \mathbf{n}_{i-1,i} &= R_{i,i+1}\Big[\mathbf{n}_{i,i+1} + R^T_{i,i+1}\overset{*}{\mathbf{p}}_i \times \mathbf{f}_{i,i+1}\Big] \\ &\quad + (\overset{*}{\mathbf{p}}_i + \overset{*}{\mathbf{r}}_i) \times \mathbf{f}_i + \mathbf{n}_i && (F6) \end{aligned}$$

For an open kinematic chain where no forces or torques are exerted on the last link, link N, we have the following boundary conditions:

$$\mathbf{f}_{N+1,N} = \mathbf{n}_{N+1,N} = 0$$

$$\mathbf{f}_{-1,0} = \mathbf{n}_{-1,0} = 0$$

In the following equations $\overset{*}{p}_i = \overset{*}{\mathbf{p}}_i$, $\overset{*}{r}_i = \overset{*}{\mathbf{r}}_i$, $a_i = \mathbf{a}_i$, $n_{i,j} = \mathbf{n}_{i,j}$ and $f_{i,j} = \mathbf{f}_{i,j}$. Substituting (F2), (F3), (F4) and (F5) into (F6), we get the following expression:

$$\begin{aligned} n_{i-1,i} = {} & R_{i,i+1}\Big[n_{i,i+1} + R_{i+1,i}\overset{*}{p}_i \times f_{i,i+1}\Big] \\ & + (\overset{*}{p}_i + \overset{*}{r}_i) \times m_i\Big[\dot{\omega}_i \times \overset{*}{r}_i + \omega_i \times \omega_i \times \overset{*}{r}_i + a_i\Big] \\ & + J_i\dot{\omega}_i + \omega_i \times (J_i\omega_i) \end{aligned} \qquad (F7)$$

For any two vectors we can define $\mathbf{v} \times \mathbf{u} = \underline{u}\mathbf{v}$, where if $\mathbf{u} = [a, b, c]$, then:

$$\underline{\mathbf{u}} = \begin{bmatrix} 0 & -c & b \\ c & 0 & -a \\ -b & a & 0 \end{bmatrix}$$

Using this definition we derive the following expression for $\dot{\omega}$:

$$\begin{aligned}\dot{\omega}_i = \Big[J_i - m_i\underline{(p_i + \overset{*}{r}_i)}\Big]^{-1}\{&n_{i-1,i} - R_{i,i+1}(n_{i,i+1}\\ &+ R_{i+1,i}\overset{*}{p}_i \times f_{i,i+1})\\ &- (\overset{*}{p}_i + \overset{*}{r}_i) \times m_i(\omega_i \times \omega_i \times \overset{*}{r}_i)\\ &- (\overset{*}{p}_i + \overset{*}{r}_i) \times m_i a_i - \omega_i \times J_i\omega_i\}\end{aligned} \qquad (F8)$$

Since $\dot{\omega}$ and $f_{i,i-1}$ depend on a_i it is possible to find linear relations between these two vectors and a_i. Since both $\dot{\omega}$ and $f_{i,i-1}$ depend on $f_{i,i+1}$, the idea is to use recursive computation for the coefficients of these linear relations. These two linear relations, are:

$$\dot{\omega}_i = B_i^{\omega a} a_i + b_i^{\omega g} \qquad (F9)$$

$$f_{i-1,i} = B_i^{fa} a_i + b_i^{fg} \qquad (F10)$$

where $B_i^{\omega a}$ and B_i^{fa} are matrices and $b_i^{\omega g}$ and b_i^{fg} are vectors.

In the following development, expressions for only part of the coefficients are presented. For the last link, if one substitutes the boundary conditions on the last link, link N, into (F8) and (F5) one obtains:

$$B_N^{\omega a} = -T_N m_N\underline{(\overset{*}{p}_N + \overset{*}{r}_N)}$$

$$b_N^{\omega g} = T_N\Big[n_{N-1,N} - (\overset{*}{p}_N + \overset{*}{r}_N) \times m_N(\omega_N \times (\omega_N \times \overset{*}{r}_N)) - \omega_N \times (J_N\omega_N)\Big]$$

where

$$T_N = \Big[J_N - m_N\underline{(\overset{*}{p}_N + \overset{*}{r}_N)}\Big]^{-1}$$

Similarly, one can derive the expressions for the other last (for the last link) coefficients. We now substitute the expression for $\dot{\omega}_{i+1}$ from (F9) into (F1) for $i+1$ to find an expression for a_{i+1}:

$$a_{i+1} = C_{i+1}\Big[R_{i+1,i}a_i + \omega_{i+1} \times \omega_{i+1} \times \overset{*}{p}_{i+1} + b_{i+1}^{\omega g} \times \overset{*}{p}_{i+1}\Big] \qquad (F11)$$

where

$$C_{i+1} = (I + \underline{\overset{*}{p}_{i+1}} B^{\omega a}_{i+1})^{-1}$$

and I is the identity matrix. Since

$$f_{i,i+1} = B^{fa}_{i} a_{i+1} + b^{fg}_{i+1} \tag{F12}$$

substituting (F11) into (F12), and then substituting the resulting equation into (F8) we derive the following coefficient of a_i:

$$B^{\omega a}_{i} = t_i \Big[-\underline{s}_i m_i - R_{i,i+1} \underline{d}_i B^{fa}_{i+1} C_{i+1} R_{i+1,i} \Big]$$

where:

$$d_i = R_{i+1,i} \overset{*}{p}_i$$

$$s_i = \overset{*}{p}_i + \overset{*}{r}_i$$

Similarly from (F5), (F10) and (F12) we find:

$$\begin{aligned} b^{fg}_{i} = {} & R_{i,i+1} b^{fg}_{i+1} + m_i b^{\omega g}_{i} \times \overset{*}{r}_i \\ & + m_i \omega_i \times \omega_i \times \overset{*}{r}_i \\ & R_{i,i+1} B^{fa}_{i+1} C_{i+1} (\omega_{i+1} \times \omega_{i+1} \times \overset{*}{p}_{i+1} \\ & + b^{\omega g}_{i+1} \times \overset{*}{p}_{i+1}) \end{aligned}$$

From these expressions, it is clear how, in moving from the last link towards the base all the coefficients can be computed.

Starting now from the base and proceedings outwards, one can compute all the linear and angular accelerations for all links. Given the boundary conditions on the base we have:

$$a_o = -(B^{fa}_{o})^{-1} b^{fg}_{o}$$

$$\dot{\omega}_0 = B^{\omega a}_{o} a_o + b^{\omega g}_{o}$$

Therefore, using (F1) from this point on it is a simple matter to solve for the accelerations of all following links. After all these accelerations are computed they can be integrated once to give velocities. The positions of all links can be computed using the vectors p_i^*, r_i^* and the orientations of the links.