THE DESIGN OF AN INTEGRATED HAND AND WRIST MECHANISM

by

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ABSTRACT

A three degree of freedom, cable driven, integrated hand and wrist mechanism has been
developed which is suitable for retrofitting on the MIT Whole Arm Manipulator. The MIT
WAM is a high speed, low inertia, force controllable arm. Designing a wrist which can
both be carried by the WAM and meet its performance levels placed severe design
constraints on weight, volume, robustness and backdriveability. We met these goals with a
prototype wrist based on an \( n+1 \) cable actuation scheme. This thesis explores issues
relevant to the design including the identification of kinematic singularities and the
optimization of \( n+1 \) cabling arrangements. The work was done in conjunction with

Thesis Supervisor: Dr. J. Kenneth Salisbury, Jr.
Principal Research Scientist
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When my former roommates, Bob Borchers and Jim Hyde graduated, they joked that I would write on my acknowledgements page, "No no NO! I did it all myself! I don't thank anyone!" Well... I'd love to. It would be great to see their faces, but I won't.

First I am indebted to my thesis advisor, Ken Salisbury, who preceded me not only at Stanford, but also at Menlo-Atherton High School. The combination of a relaxed attitude about life with an innovative and demanding attitude about robot design was a good one for me. Many of the ideas in this thesis on cable mechanisms and kinematics came originally from discussions with Ken, and share an intellectual tradition with his colleagues and advisor from his own graduate days. It is gratifying to become a new link in that chain.

Cathy Anderson built half of the hardware in this project. Without her contribution, I'd be presenting a piece of plastic with lots of springs, bearings and pulleys instead of a robot hand and wrist. My roommates Brian Eberman and Dave Brock showed me that there really are people who can remember everything they ever learned, in addition to helping me with proofs, code debugging, and random intellectual challenges. Nick Swarup shared in the construction and programming of the control station.

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1 Introduction

This thesis presents the design of an integrated hand and wrist package to be mounted on the existing four degree of freedom MIT Whole Arm Manipulator (WAM), a high performance arm capable of both fine force control and high acceleration. The hand/wrist, like the WAM, is not designed towards one particular task. Rather, the goal of the project is to develop a system capable of working in completely unstructured environments. We would like the system to be able to maneuver among unknown objects in unknown locations, locating and acquiring objects as delicately as possible using tactile and force information.

There are many reasons for wanting to build such a system. One is that very few robots built as of today are able to work well in unstructured environments. The mechanical design of robots has lagged far behind the theoretical aspects such as sensing, kinematics, and controls. In addition, a number of urgent tasks exist today which require such high performance robots.

The most pressing of these tasks is the cleanup of contaminated nuclear storage bins maintained by the US Department of Energy. These bins are currently leaking toxic from the groundwater supply, and are far too highly contaminated for human cleanup. The work requires robots working either autonomously or under teleoperated control. The bins,
however, contain material unsuited to most robotic capabilities, from contaminated lab coats to milling machines. Furthermore, the contaminated material is piled haphazardly, and worst of all, no records of the contents exist! The current generation of robots is not designed to work in such uncontrolled surroundings.

Other uses for high performance robots might come either from space or the deep ocean. Archaeologists associated with the Woods Hole Oceanographic Institute have already used an arm based on concepts developed by this lab for the "Jason Project" to retrieve delicate objects from the sea floor. NASA researchers have also expressed great interest in systems which might be able to retrieve dropped and drifting tools or even rescue drifting astronauts. Such systems would have to be fast, precise, and force controllable in a wide variety of task configurations.

1.1 The WAM

The MIT Whole Arm Manipulator was developed by the WAM group at the MIT Artificial Intelligence Laboratory [Salisbury, Townsend, Eberman, DiPietro]. It was designed to contact and manipulate objects not only with a gripper, such as most robot arms, but with every surface on the arm. The WAM was designed with slender links and a clean exterior to maximize this ability. In order to be effective in whole arm manipulation, it was also necessary to design the arm to be extremely force controllable. Townsend showed mathematically what many robotics researchers are coming to realize, that good force control is never achieved by retrofitting a force sensor on a poor quality arm [Townsend]. In order to achieve accurate and sensitive force control, the arm needs an efficient transmission (or no transmission, as in the case of direct drive arms). It needs to have zero backlash, be light weight, and have the motor inertia accurately matched to the load inertia. The WAM group learned, in the process of designing and building the system, that the
same attributes which optimize an arm for force control also optimize it for speed. The result is an extremely high performance system.

Figure 1.1 The MIT Whole Arm Manipulator

The WAM has demonstrated the ability to explore its environment without force sensors. It is able to accomplish this using only feedback from motor position sensors and monitoring of drive currents because it has an extremely efficient, backdriveable, cable transmission.

Simple demonstrations of WAM capabilities have involved locating the surfaces of lightweight cardboard boxes by exploring them with the arm surfaces, rearranging boxes by pushing on them, and gripping smaller object with the inner surfaces of the arm. A more dramatic demonstration of the force control capabilities involves controlling the WAM with
a force reflecting master. Contacts with hard objects in the environment are both stable and easily detectable.

Figure 1.2 A Wam Inner Link Grasp

1.2 A Hand/Wrist for the WAM

At the beginning of the hand/wrist project, I designed and built a fast acting, one degree of freedom pneumatic gripper for the WAM. This gripper is currently under continuous use in successful attempts to catch a thrown object with a robot arm. To my knowledge, this is the first time any robot has succeeded in such a task. Two stationary
cameras mounted in the corners of the room triangulate to locate the ball. The arm accelerates to match the parabolic trajectory and closes the gripper to complete the catch.

The catching project is one of many applications which requires more dextrous grasping than is possible with a simple gripper. Having successfully demonstrated the WAM's ability to catch, the researchers working on that project are eager to move on to non-spherical (and probably tumbling) objects. Without a wrist, the necessary reorientation is impossible. A hand/wrist will, of course, add mass to the arm, and will slow it down considerably. This will probably require a whole new catching strategy, even for spherical objects. The current technique of trajectory matching requires very high accelerations and is a somewhat counterintuitive way to catch a ball. A person would probably put on a baseball glove and simply move the glove into the trajectory of the ball, cushioning the force slightly by partially matching speeds. With a hand and wrist, this type of catching strategy will become possible.

![Figure 1.3 The Pneumatic Gripper](image)

The gripper and forearm package weigh 2 lbs. The gripper has a closing time of 200 msec, and has been successfully used to catch a flying ball.
Other needs for a hand include any task involving small objects. The concept of “whole arm” grasping is well suited only to large objects. (Consider, for example, how we would carry a load of firewood.) Manipulation on a smaller scale naturally requires a smaller effector. It is important, however, to remember the attributes which distinguish the WAM from all other robots and to design a hand and wrist package which not only avoids sacrificing those special attributes, but also possess similar and consistent qualities.

Most robot arms are vehicles suitable only for conveying an end effector between tasks. The WAM arm is distinguished by its ability to interact with the environment at every point instead of just at the end effector. A hand which is consistent with this philosophy should be not just a machine for picking and placing objects, but a tool for actively sensing, exploring and manipulating the environment. It should have the ability to grasp objects and interact with objects on every surface, just as the WAM does, a concept we call “whole hand manipulation.”

1.2.1 Design Criteria

Some of the primary design criteria we considered in the design are mechanical bandwidth, backdriveability, torque output, weight, range of motion, speed, robustness, and exterior shape. I will discuss the thinking behind the relationship of these properties in the hand/wrist and the same properties in the WAM.

Mechanical bandwidth refers to the frequency with which the actuators can drive the joints of the system. Bandwidth is often a loosely used term. It varies with the position of the robot, and changes drastically when the robot comes in contact with an object. The bandwidth of the hand and wrist is primarily affected by the inertia of the links and the stiffness of the transmission. It is fairly clear that the bandwidth of the wrist should be higher than the bandwidth of the arm. Because the hand is smaller and has lower inertia than the arm, it is generally easy to keep the bandwidth of the hand higher. In addition, the
smaller hand and wrist is naturally suited to fine, high speed control, while the arm is better suited to gross positioning. See [Anderson] for more detail.

Backdriveability is a function of transmission efficiency and the gear ratio between the actuators and the joints. An efficient transmission and a low gear ratio will give a backdriveable arm with precise force control. In keeping with the WAM concept, we would like the hand and wrist to be as backdriveable as possible. However, if something has to be sacrificed, complete backdriveability of the hand/wrist is probably less important than bandwidth, torque output, weight, or range of motion. The arm is already adept at sensing contact, with or without a hand. The precision of the force control will not be reduced by the backdriveability of the wrist. Similarly, the ability of the system to avoid being damaged by moving aside in response to a force will also not be reduced. An extremely backdriveable wrist will provide additional detailed force information from contacts, and a completely nonbackdriveable system would be prone to damage by high speed contact with objects, but the qualities of the system as a whole will not suffer tremendously with a transmission which is less than 100% efficient. As it turns out, the package we built did turn out to be quite backdriveable, but this property was lower on our priority list during the design process than the others.

The proper torque output for a wrist is a topic of considerable disagreement among robot designers. We designed for the same relative strength as the arm. That means the wrist should be able to support the same maximum force as the arm at some designated location, such as the fingertip.

Torque output and weight are related by the size of the motors chosen, and must be balanced against each other. The total payload capacity of the WAM is only fourteen pounds (with no hand!), so every additional ounce added to the hand/wrist package subtracts from its ability to pick up objects. More importantly, the hand and wrist add
weight at the tip of the arm, where the effect on the inertia is the greatest. If the new hardware is made too heavy, the outstanding acceleration of the WAM will be sacrificed. See [Anderson] for a detailed analysis of the tradeoffs.

The range of motion of the wrist axes should be as large as possible. Not only does a large joint range greatly facilitate tasks, but a backdriveable mechanism is largely protected from damage by collisions as long the contact does not occur while the mechanism is at a joint limit. In general, the greater the range of motion the better, and it is worth considerable design effort to increase it.

The closing speed of the hand should also be fast. The current pneumatic gripper has a closing time of about 200 milliseconds. The members of the catching project are interested in having the new system close at least as quickly, so that old catching strategies as well as new ones will be possible. However the slower speed of the arm may demand a new catching strategy anyway, so I regard the 200 msec time as a goal to aim for, but one which could be relaxed if necessary.

The robustness of the system is an extremely important consideration. It may well be the most important and the hardest to achieve. The WAM arm itself is extremely robust and suffers repeated abuse without damage. Because the WAM is completely backdriveable, its power-off mode is to fall to the floor. At demonstration time, crashes are almost nonexistent, but during the testing of new code, crashes are frequent. Any hand mounted on the end of the arm will undoubtedly take the brunt of the abuse. If it is not able to withstand this type of collision, it will be worthless.

Finally, the wrist should have a completely clean exterior, like the WAM, so it can manipulate objects with any surface. Its actuators, reducers, and other hardware must fit inside the WAM forearm to avoid interfering with the WAM's useful manipulation abilities. This is a difficult but vital constraint.
1.3 Overview of the Thesis

I have discussed the background of the project and the design issues and constraints. In the remainder of the thesis, I will choose appropriate kinematics for the system (chapters 2 and 3), examine various cable actuation techniques (chapter 4), optimize the routing of the cables (chapter 5), examine implementation options (chapters 6 and 7), present results and discuss the future of the project (chapter 8).

In chapter 2, I show a technique for finding singularities of robots which can be used to evaluate kinematic arrangements. Singularities are among the more difficult problems for path planning algorithms to deal with. They also restrict the effective workspace of the arm and make structural damage to the arm possible in the event of collisions. Their location is vitally important in the selection of the kinematics. I use screw theory to generate a set of rules describing singular combinations of robot axes.

In chapter 3, I use this technique to evaluate several possible kinematic options for the wrist. The arrangement I choose minimizes the effect of singularities on the robot. I also present various criteria for the shape of a well designed hand, and choose a link configuration which can grasp both large and small objects equally well.

In chapter 4, I give an overview of cable mechanisms with detailed explanations of indeterminate systems, $n+1$ systems, and pretensioned systems. I choose a cable mechanism because of its transmission efficiency and backdriveability, and to minimize inertia by keeping the actuators as far from the end of the arm as possible. I choose an $n+1$ system to minimize weight and part count by keeping the number of cables and pulleys low.

In chapter 5, I describe a search of the possible tendon arrangements for the hand and wrist. I find the optimum cable routing for minimum peak cable tensions by
minimizing the condition number of the cabling matrix, and simulate a typical loading scenario for a hand/wrist cabled in this manner.

In chapter 6, I examine two possible implementations of the chosen wrist kinematics and cabling, and present prototypes of each. The first design has ideal kinematics, a very low part count, and infinite expansion possibilities. Unfortunately, it also suffers from wear and static friction problems. The second design is far more complicated and has slightly less desirable kinematics, but the lower static friction makes it a better choice overall.

In chapter 7, I address a number of implementation questions for the hand. The grasping behavior is governed by the choice of pulley diameters and centering spring stiffnesses. Dislocatable knuckles will hopefully save the hand from damage in the worst collisions.

In chapter 8, I present the benchtop prototype, discuss the lessons learned from building and controlling that model and speculate on the future directions of the project.
2 Finding Robot Singularities by Inspection

The first step in robot design is to choose a set of joint kinematics. This is often done haphazardly and much time and energy can be wasted on a fundamentally poor design. The most disturbing problem to discover after investing a large effort in detailed mechanical design would be an unwanted kinematic singularity in the interior of the workspace. Trajectory planners must avoid singular and near singular regions, and certain tasks may even prove to be impossible because of the need to avoid workspace singularities. In trying to evaluate several options for kinematic arrangements for a handwrist design, I discovered that no good method exists for identifying these singularities. The search for such a method became so interesting to me that it became a topic in itself. This chapter presents a set of rules for identifying singularities and the mathematical justification for the rules. I will use this method in chapter 3 to solve my original problem, the selection of a kinematic arrangement.

2.1 Introduction to Singularities

To arbitrarily position and orient an end effector in space, a robot manipulator needs a minimum of six degrees of freedom. A singularity occurs when one Cartesian motion is temporarily unavailable. More precisely, when the manipulator is in a singular configuration, some Cartesian velocity of the end effector will be physically impossible. This is demonstrated in figure 2.1 below, known as a "gimbal lock."
When pushed in one direction, the telescope is free to rotate, but when pushed in the other direction, the telescope is "locked."

This condition can also be observed mathematically, by writing the Jacobian matrix for the manipulator. The Jacobian maps joint velocities to Cartesian velocities

\[ J \dot{u} = \nu \]

and is easily generated. To convert Cartesian velocities to joint velocities, the Jacobian matrix must be inverted. If the determinant is zero, the matrix \( J \) is singular and inversion is not possible. In this case, a nonzero solution to the equation

\[ J \dot{u} = 0 \]

exists - or more concretely - some set of joint velocities will combine to produce no net motion of the end effector.

Any arbitrary six degree of freedom revolute manipulator will have a number of singular configurations within its workspace. The trajectory planner is generally given the task of avoiding these configurations, which is computationally expensive. Adding a seventh axis to the robot arm adds one redundancy - i.e. the arm will have one "self motion" in which the joints of the arm move, but the position and orientation of the end effector remain unchanged. This self motion can be used to avoid singularities, avoid
workspace obstacles, minimize joint loading on a particular joint which is nearing maximum torque, or minimize joint velocity on a joint which is approaching maximum velocity. It is important to remember that the additional degree of freedom can be used for only one criterion at once. Its use can certainly change over time during a trajectory, and it can be used to fulfill a weighted sum of criteria, but with one redundant axis, it can be used to fulfill only one extra constraint.

[Hollerbach] showed that all robot manipulators will have singularities in the interior of the workspace, regardless of the number of axes. He also showed that, given a wise selection of seven axes, all Cartesian positions and orientations in the interior of the workspace can be reached without entering a singular joint configuration. It is therefore unnecessary to design manipulators with more than seven axes for the purpose of avoiding singularities.

The Jacobian matrix for a seven degree of freedom manipulator is 7x6 and is always redundant. That is - there is always a null solution in which joint velocities will combine to produce no net motion at the end effector. If two independent null solutions exist, then the rank of $J$ must be 5, and the manipulator is at a singularity. This is the basis of the method I will outline here. I will describe all the possible geometric arrangements of joint axes which produce a null solution. (The number of such arrangements is surprisingly small.) The presence of one of these "self-motions" in a six degree of freedom manipulator or the presence of two simultaneous self-motions in a seven DOP manipulator indicates a singularity.

2.2 Rules for Singular Configurations of Axes

The following is the list of ways in which groups of revolute axes can be dependent (have a null solution). Derivations and discussion of the rules are presented in section 2.3.
1 axis
   1• - none

2 axes
   2• - two axes are colinear

3 axes
   3a• - three axes intersect in a single point and lie in the same plane
   3b• - three parallel axes lie in the same plane

4 axes
   4a• - four axes lie in the same plane
   4b• - The plane containing two parallel axes also contains the intersection of two axes and the parallel axes are parallel to the plane containing the intersecting axes.
   4c• - four axes intersect in a single point
   4d• - two sets of two intersecting axes exist where the line connecting the points of intersection lies in both planes
   4e• - four axes are parallel
   4f• - The planes containing two sets of two parallel axes are parallel.
   4g• - three different lines can be found which all intersect four skew axes

5 axes
   5a• - two lines can be found which intersect the same five axes
   5b• - three lines lie in a plane and two lines intersect the plane at a common point, or two lines lie in a plane and three lines intersect the plane at a common point.
   5c• - three lines lie in a plane and two lines are parallel to the plane, or two lines lie in a plane and three lines are parallel to the plane.
   5d• - one line can be found which intersects each of five lines all parallel to the same plane

6 axes
   6a• - a line can be found which intersects six axes
   6b• - six axes are all parallel to one plane
2. Finding Robot Singularities by Inspection

7 axes -
7+ - any seven axes

2.3 Discussion

2.3.1 How Many Lines Are There?

Any discussion of the techniques used to generate and prove the validity of the rules presented here requires a common language for describing the relative location and orientation of axes. To that end, I will present a brief discussion of twist coordinates.

The axis about which a revolute robot joint rotates is completely specified by a line in space (figure 2.2). A robot at a given instant is completely specified by the locations of the axes of its joints. As long as the axes are connected in series, the motion of the robot at that instant does not depend on how the axes are connected or in what order they are connected. Therefore, an angular velocity and a unique description of the line for each axis should completely determine the instantaneous rigid body translation and rotation for the end effector.

Figure 2.2 Two Revolute Joints

If, for a given set of axes, we can choose a non-zero set of joint angular velocities which produces zero endpoint motion of the robot, then we say that the axes are dependent. To most easily determine dependency, we need a vector algebra for lines which permits scaling and component by component addition and subtraction.
Any line can be uniquely specified relative to some base coordinate frame by four independent coordinates. For example, one could use an altitude and an azimuth to specify direction of a line, and two more coordinates to specify the location of the nearest point to the origin. One can also see this "by noting that of the six degrees of freedom possible in a three-dimensional space, the line has one translational and one rotational degree of freedom along and about its axis which is unspecified" [Shimano].

We say, then, that lines lie in four-space, or that there exist \( \infty^4 \) possible lines. Cartesian motions, by contrast, require six coordinates (three linear velocities and three angular velocities, for example). It makes sense, then, that at least six independent joints are required to arbitrarily position and orient a grasped object and that any set of seven joints has in general one redundant motion.

If at most six axes can be independent, and if robot axes are to be described by vectors, then each of these vectors must have exactly six components. Revolute axes, however, can be described by lines, and lines have only four independent coordinates!

It certainly seems as if there do not exist enough lines to do the job - that at most four lines could be independent of one another. If this were true, robots comprised of revolute joints only could never have all six Cartesian motions available. We know with certainty, however, that six revolute axes can indeed be independent, because six-jointed robots exist which can move an object in all directions.

To solve this paradox, we need to look a little deeper at what it means to add vector representations of lines. We said earlier that if a set of axes and angular velocities about those axes yielded zero motion, then the axes were dependent. Take an arbitrary set of \( n \) dependent axes. For the axes to be dependent, it must be possible to duplicate the motion produced by angular velocity about at least one of the axes by some combination of angular
velocities about the remaining axes. For the vectors describing the axes to be dependent, at least one axis vector must be a linear combination of the others.

When we speak of a linear combination of axes (or lines), then, we mean the resultant axis (or line), about which we could rotate the system to produce exactly the same motion. This matches both our physical understanding of what it means to add motions about robot axes, and our mathematical understanding of what it means to add vectors.

The core of the problem is that linear combinations of lines are not necessarily lines themselves. The motion produced by simultaneous rotation about multiple axes in series cannot necessarily be described as a pure rotation about some axis in space. In other words, the set of lines is not closed under addition. Combinations of joint velocities can produce not only pure angular rotation but they can also produce pure linear motion or a combination of linear motion and rotation.

The effect of simultaneous rotation about multiple lines in series can always be described as a "screw" - an axis along and about which translation and an associated rotation occur, with a given pitch and a given magnitude. It takes six coordinates to represent all possible screw motions (four to specify the screw axis, one for pitch, and one for magnitude). Lines can be viewed as screws with zero pitch and unit magnitude, yielding four independent coordinates. Six lines, then, (written as six-component vectors) can be independent of each other, not four, because linear dependence only has meaning if we can add vectors, and we cannot add lines written as four component vectors. Therefore (demonstrating what we all knew all along) it takes six axes (or lines) to span Cartesian space and to position and orient a robot arbitrarily.
2.3.2 Screws, Wrenches, Twists, and Herr Plücker

In the land of screws live twists (screw displacements), and wrenches (screw forces). An arbitrary twist can be represented by the three angular velocities and the three linear velocities at the origin when the coordinate frame is moved according to the twist. (If the twist has zero pitch, this corresponds to rotating the coordinate frame about the axis of the twist.) The vector can be written

\[ \{\tau_1, \tau_2, \tau_3, \tau_4, \tau_5, \tau_6\} \text{ or } \{\omega, v\} \]

where \( \omega \) and \( v \) are three component vectors. The six coordinates can also be thought of as the three directions and three "moments" of the twist about the origin. In this case, the vector is often written in Plücker coordinates as

\[ (l, m, n, p, q, r) \].

Figure 2.3 Illustration of Plücker Coordinates

In figure 2.3,
\[ p = \tau_y = y_1 \, dx - z_1 \, dy, \]
\[ q = \tau_z = z_1 \, dx - x_1 \, dz, \]
\[ r = \tau_x = x_1 \, dy - y_1 \, dx. \]

To represent a line, components \( l, m, \) and \( n \) of the twist give the \( x, y, \) and \( z \) components of the direction of the line. Components \( p, q, \) and \( r \) give the \( x, -y, \) and \( z \)-moments a unit force along the line would apply at the origin. Examples are shown in figure 2.4.

Figure 2.4 Examples of Plücker Coordinates

The pitch of the twist is the ratio of linear velocity to angular velocity - the linear distance moved per unit rotation about the axis, and is given by

\[ p = \frac{l p + m q + n r}{l^2 + m^2 + n^2} = \frac{\omega \cdot v}{\omega \cdot \omega}. \]
In the specialized case where the pitch is zero, the twist is a line. The equation for pitch can be applied to verify that all the twists shown in figure 2.4 are lines. I will use the terms "line" and "zero pitch twist" interchangeably. We will be primarily interested only in twists with zero pitch, because they can be used to represent revolute axes.

**Dependence of Lines**

If lines $L_1, L_2, \ldots$ are twists with zero pitch and

$$a L_1 + b L_2 + \ldots = 0,$$

the lines are linearly dependent [Ohwodepie]. This is exactly equivalent to saying that rotation about the lines $L_1, L_2, \text{etc.}$ with angular velocities $a, b, \text{etc.}$ produces no net motion of the robot end effector. This can be expressed in matrix form as

$$[L]([\omega]) = 0 \quad \text{or} \quad \text{Det}([LL^T]) = 0$$

where $L$ is the matrix whose columns are the vectors $L_n$ and $\omega$ is a vector of angular velocities.

**Intersection of Lines**

The mutual moment of two lines is $\alpha \sin(\theta)$ where $\alpha$ is the perpendicular distance between the lines and $\theta$ is the angle between them [Hunt]. This is given by

$$l_1 p_2 + m_1 q_2 + n_1 r_2 + l_2 p_1 + m_2 q_1 + n_2 r_1$$

or

$$\omega_1 \cdot v_2 + \omega_2 \cdot v_1.$$

If two lines have zero mutual moment, they either intersect or are parallel (intersect at infinity). This will later prove to be an exceedingly useful tool in examining combinations of lines.
2.3.3 Fundamental Line Combinations

In searching for robot singularities, we look for combinations of revolute joint axes which can produce zero motion with non-zero joint velocities. For a six degree of freedom robot, one such self motion indicates a singularity. For a seven degree of freedom robot, one self motion is always present and the presence of an additional one indicates a singularity.

To find a combination of lines which produces zero motion, we look for sets of lines dependent on a given group of "generator" lines. I refer to these sets of lines as "families." A line family is generated mathematically by finding all the zero pitch linear combinations of a set of generator lines. The rules for linear dependence will be the condition for an additional revolute axis to be included in the family of dependent lines.

I have numbered the line families (and the rules for dependence) by the number of robot axes which would imply dependence, if they were all to lie in the family. The smallest number of axes which can be dependent is two. Therefore, the first rule for dependence is rule 2, found below.

**One Line**

Linear combinations of one line are simply scalar multiples of the line. Therefore two axes can only be dependent if they lie along the same line (rule 2).

**Two Lines**

Two lines $L_1$ and $L_2$ can be either parallel, intersecting, or skew. To examine this situation, we define a coordinate system by placing the origin at the intersection of $L_1$ and the common normal between $L_1$ and $L_2$. We let the $z$-axis lie along $L_1$ and the $x$-axis lie along the common normal. $L_2$ then lies at a distance $a$ along the $x$-axis and an angle $\alpha$ in the $y$-$z$ plane.
With this, we can write the twist coordinates:

\[ L_1 = (0, 0, 1, 0, 0, 0) \]
\[ L_2 = (0, -\sin(a), \cos(a), 0, -a \cos(a), -a \sin(a)) \].

Consider the linear combination

\[ L_3 = k_1 L_1 + k_2 L_2 \, . \]

Then the pitch of \( L_3 \)

\[ p_3 = \frac{k_1 k_2 a \sin(a)}{k_1^2 + 2k_1 k_2 \cos(a) + k_2^2} \, . \]

We are interested only in the family of lines (zero pitch) which result, so we set the pitch equal to zero and get

\[ k_1 k_2 a \sin(a) = 0 \, . \]

The result is a line if
\[ k_2 = 0, k_2 = 0, a = 0, \text{ or } \alpha = 0. \]

Case 1: \( \alpha = 0 \) (Figure 2.6a)

If \( \alpha = 0, L_1 \) and \( L_2 \) intersect and they generate the family

\[ L_3 = \{ (0, -k_2 \sin (\alpha), k_1 + k_2 \cos (\alpha), 0, 0, 0) \}. \]

All the resultant lines lie in the \( y-z \) plane (because the twist has no \( x \)-component) and intersect the origin (because they have no moment about the origin). The family of dependent lines is called a planar pencil. If two robot axes form this pencil, and if a third axis is also in the pencil, then the three axes must be dependent (rule 3a).

Case 2: \( \alpha = 0 \) (Figure 2.6b)

If \( \alpha = 0 \), then \( L_1 \) and \( L_2 \) are parallel, and

\[ L_3 = \{ (0, 0, k_1 + k_2, 0, -k_2 a, 0) \}. \]

All the resultant lines are now parallel to the \( z \)-axis (because they have no \( x \) or \( y \)-components) and they lie in the \( x-z \) plane (because they have moments only about the \( y \)-axis). I call this family of dependent lines a "parallel pencil". If a third axis also lies in the parallel pencil, the three axes are dependent (rule 3b).

Case 3: \( k_1 \) or \( k_2 = 0 \) (Figure 2.6c)

If neither \( a \) nor \( c \) are zero, the two lines are skew and the resulting twist has nonzero pitch unless \( k_1 \) or \( k_2 \) is zero. If \( k_2 \) is zero, then

\[ L_3 = L_1 \]

and if \( k_1 \) is zero, then

\[ L_3 = L_2. \]

Therefore, the family of dependent lines contains only the original lines themselves. It is impossible for a third axis to lie in this family without itself being either \( L_1 \) or \( L_2 \), (which would be dependent by rule 2) so this configuration does not lead to a rule.
Three Lines

In moving to combinations of three lines, we can treat the families identified in the last section one at a time. It is not necessary to identify the particular lines which generated the family. Any two independent lines which generate the same family will do as well. In other words, we can replace rows of the matrix $L$ by linear combinations of the rows as long as we preserve independence.

Planar Pencil

We begin with the planar pencil. Consider three cases for the orientation of a third line. A third line can either lie in the plane of the pencil [case 1], be parallel to the plane of the pencil [case 2], or intersect the plane of the pencil [case 3]. We can describe a planar pencil in the x-z plane by two intersecting lines - the x axis and the z axis.

$L_1 = (0,0,1,0,0,0)$ \hspace{1cm} $L_2 = (1,0,0,0,0,0)$

Case 1: in the plane (figure 2.7)

A line $L_3$ in the x-z plane can be characterized by an angle $a$ and a distance $a$ from the origin.

$L_3 = (-\cos(a), 0, \sin(a), 0, -a, 0)$. 
Note, however, that we have the freedom to choose the coordinate frame as we wish. We could just as easily choose the x-axis parallel to \( L_3 \). We can then make \( a = 0 \) without loss of generality and write this line as

\[ L_3 = \{-1, 0, 0, 0, -a, 0\}. \]

Taking linear combinations of the three lines gives

\[
L_n = k_1 L_1 + k_2 L_2 + k_3 L_3
= (k_2 - k_3 \cos(a), 0, k_1 + k_3 \sin(a), 0, -(a k_3), 0).
\]

The pitch of \( L_n \) is zero regardless of the choice of \( k_2, k_3, \) and \( k_3 \), so all resultant vectors are lines (zero pitch twists). The shape of the family represented by all possibilities for \( L_3 \) can be seen by observing that the x-direction, the z-direction and the moment about y can all be chosen independently, so the family includes every line in the x-z plane (figure 2.7c). Therefore any three independent lines which all lie in a plane span all lines in the plane.

This leads to rule 4a - four lines are dependent if they all lie in a single plane.

![Figure 2.7 Combination of a Planar Pencil with a Line in the Plane](image)

Case 2: parallel to the plane (figure 2.8)

A line parallel to the x-z plane can be characterized by the same \( a \) and \( a \), and by a distance \( b \) from the plane.
$L_3 = \{ -\cos(a), 0, \sin(a), b \sin(a), -a, b \cos(a) \}$.

If we again choose the coordinate frame to make $a = 0$, this line can be written without any loss of generality

$L_3 = \{ -1, 0, 0, 0, -a, b \}$.

Taking a linear combination gives

$L_n = k_1 L_1 + k_2 L_2 + k_3 L_3$

$= \{ k_2 \cdot k_3, 0, k_1, 0, -(a \cdot k_3), b \cdot k_3 \}$.

The pitch of $L_n$ is

$p_n = \frac{b \cdot k_3 \cdot k_1}{k_1^2 + (k_2 \cdot k_3)^2}$.

Solving for the lines $L_n$ with zero pitch, we get

$b = 0 \text{ or } k_3 = 0 \text{ or } k_1 = 0$.

If $b = 0$, then $L_3$ lies in the plane, which was discussed in case 1. If $k_3$ or $k_1 = 0$, the family is generated either by two intersecting lines ($L_1$ and $L_2$) or by two parallel lines ($L_2$ and $L_3$) only, and is either a planar pencil or a parallel pencil. The combination of these families is the union of the planar and parallel pencils shown in figure 2.8c. Again - this shape is spanned by any three independent lines in the family, so a fourth line, also in the family, would be dependent (rule 4b).

Figure 2.8 Combination of a Planar Pencil with a Line Parallel to the Plane
Case 3: intersecting the plane (Figure 2.9)

A line which intersects the x-z plane intersects the pencil at one of its lines. We choose the coordinate frame so that the z-axis intersects $L_1$. $L_3$ can now be characterized by two angles $a$ and $\beta$, and a distance $d$ from the intersection.

$$L_3 = (-\cos(\beta) \sin(a), -\cos(\beta) \cos(a), \sin(\beta), a \cos(\beta) \cos(a), -a \cos(\beta) \sin(a), 0).$$

A linear combination gives

$$L_n = (k_2 - k_3 \cos(\beta) \sin(a), -k_3 \cos(a) \cos(\beta), k_1 + k_3 \sin(\beta), a k_3 \cos(a) \cos(\beta), -a k_3 \cos(\beta) \sin(a), 0).$$

The pitch

$$p_n = \frac{a k_2 k_3 \cos(a) \cos(\beta)}{k_2^2 \cos^2(a) + k_3^2 \cos^2(\beta) + (k_2 \cdot k_3 \cos(\beta)) \sin(a)^2 + (k_1 + k_3 \sin(\beta))^2}.$$

We find the resulting zero pitch lines by setting

$$a k_2 k_3 \cos(a) \cos(\beta) = 0.$$

If

$$\cos(a) = 0 \text{ or } \cos(\beta) = 0,$$

then $L_n$ is in the plane and the discussion of case 1 applies. If $a = 0$, then $L_1$, $L_2$, and $L_3$ intersect at a single point, and

$$L_n = (k_2 - k_3 \cos(\beta) \sin(a), -k_3 \cos(a) \cos(\beta), k_1 + k_3 \sin(\beta), 0, 0, 0).$$

Assuming that $L_2$ is not in the plane, then $\cos(a)$ and $\cos(\beta)$ are nonzero, so the $x$, $y$, and $z$-directions of $L_n$ can all be chosen independently and the three moments of $L_n$ are zero. The shape represented by all possibilities for $L_n$ is called a spherical bundle and is shown in Figure 2.9b. A fourth line, also in the spherical bundle, would be dependent (rule 4c).

If $k_2$ or $k_3 = 0$, the family is generated by two intersecting lines only ($L_1$ and $L_3$, or $L_1$ and $L_2$), and is the familiar planar pencil. The combinations form intersecting pencils and are shown in Figure 2.9c. Notice here especially, that any three independent lines in the family will generate the entire family. For this reason, a fourth line can lie in the family.
(and be dependent) without necessarily satisfying one of the three-line rules. Any four lines which lie in this family must always be dependent. (rule 4d)

Figure 2.9 Combination of a Planar Pencil with a Line Intersecting the Plane
A line intersecting a planar pencil forms a spherical bundle if it meets the point of intersection and intersecting pencils if it does not.

Parallel Pencil

Next we examine the case of the parallel pencil we identified before. Again, a third line can lie in the plane of the pencil [case 1], be parallel to the plane [case 2], or intersect the plane [case 3].

The parallel pencil (in the $x$-$z$ plane) can be described by the $z$-axis and a line parallel to the $z$-axis

$L_1 = \{0,0,1,0,0,0\} \quad L_2 = \{0,0,1,0,1,0\}$

Case 1: in the plane (figure 2.10)

For a line $L_3$ in the $x$-$z$ plane, we have either three parallel lines in the same plane, which would already be dependent (by rule 3a), or we have two parallel lines and a third line which intersects both. This case was already covered in the previous section by a planar pencil and a third line anywhere in the plane (with proper selection of the coordinate frame) and led to a complete plane.
Figure 2.10 Combination of a Parallel Pencil with a Line in the Plane
A parallel pencil and a line in the plane of the pencil span all lines in the plane.

Case 2: parallel to the plane (figure 2.11)

If \( L_3 \) is parallel to the x-z plane, we can specify it by a distance \( a \) from the plane, and an angle \( a \) from the z-axis. We pick the coordinate frame so that the origin lies on the common normal between \( L_2 \) and \( L_3 \). We can then write

\[
L_3 = (-\cos(a), 0, \sin(a), a \sin(a), 0, a \cos(a)).
\]

Taking linear combinations gives

\[
L_n = (-k_3 \cos(a), 0, k_1 + k_2 + k_3 \sin(a), a k_3 \sin(a),-k_2, a k_3 \cos(a)).
\]

The pitch of \( L_n \) is

\[
P_n = \frac{a (k_1 + k_2) k_3 \cos(a)}{(k_3 \cos(a))^2 + (k_1 + k_2 + k_3 \sin(a))^2},
\]

and we find the resulting zero pitch lines by setting

\[
a (k_1 + k_2) k_3 \cos(a) = 0.
\]

If \( a = 0 \) then \( L_3 \) lies in the x-z plane and the discussion of case 1 applies again. If \( k_3 = 0 \), the family would be generated by just two parallel lines, \( k_2 \) and \( k_2 \), and would produce one of the two line families already discussed. This leaves us with two ways to make the pitch of the resulting twist zero. Either \( \cos(a) = 0 \) or \( k_2 = -k_1 \).

Case 2a: \( \cos(a) = 0 \) (figure 2.11b)
If \( \cos(\alpha) = 3 \), then \( \alpha = \pi/2 \), which gives
\[
L_n = \{0, 0, 0, k_1 + k_2 + k_3, a, k_3, -k_2, 0\}.
\]
This twist has magnitude only in the z-direction and moments about x and y which can both be chosen independently. The result is any line in space parallel to the z-axis. Any three parallel lines (not all in the same plane) span this "parallel space." A fourth parallel axis would clearly be dependent. (rule 4e)

Case 2ii: \( k_2 = -k_1 \) (figure 2.11c)

The last way to produce a result with zero pitch is to set
\[
k_2 = -k_1.
\]
This case is VERY interesting, because
\[
k_1 L_1 + k_2 L_2 = \{0, 0, 0, 0, k_1, 0\}.
\]
This is a line removed to infinite distance. We can see this because it has moment but zero magnitude. Remember this concept. It will return later. When we set
\[
k_3 = -k_1,
\]
\[
L_n = \{-k_3 \cos(\alpha), 0, k_3 \sin(\alpha), 0, k_3 a k_3 \cos(\alpha)\}
\]
This twist is similar to \( L_2 \) (scaled by \( k_3 \)) except that it now has an arbitrary moment about the y-axis, so the result is a whole plane of lines parallel to \( L_3 \). The plane of lines parallel to \( L_1 \) and \( L_2 \) is, of course, included when \( k_3 = 0 \). This shape is shown below and generates rule 4f - If the planes containing two sets of two parallel axes are themselves parallel, the axes are dependent.
Figure 2.11 Combination of a Parallel Pencil with a Line Parallel to the Plane.

If $L_2$ is parallel to $L_1$ and $L_3$, the three lines generate parallel planes. If $L_2$ is not parallel to $L_1$ and $L_3$, the three lines generate two parallel pencils in parallel planes.

Case 3: intersecting the plane (Figure 2.12)

If $L_3$ intersects the x-z plane, we pick the coordinate frame such that the origin lies on $L_3$ and get

$$L_3 = (-\cos(\beta) \sin(a), -\cos(\beta) \cos(a), \sin(\beta), 0, 0, 0).$$

Taking linear combinations gives

$$L_2 = (-k_2 \cos(\beta) \sin(a), -k_2 \cos(a) \cos(\beta), k_2 + k_3 \sin(\beta), 0, -k_2, 0).$$

The pitch of $L_2$ is

$$\frac{k_2 k_3 \cos(a) \cos(\beta)}{k_1^2 + 2 k_1 k_2 + k_2^2 + k_3^2 + 2 k_1 k_3 \sin(\beta) + 2 k_2 k_3 \sin(\beta)}.$$

If $\cos(a) = 0$ or $\cos(\beta) = 0$, then $L_3$ lies in the x-z plane, producing case 1. If $k_2 = 0$, the family is generated by two intersecting lines. If $k_3 = 0$, the family is generated by two parallel lines. The resulting shape is the union of a parallel pencil and a planar pencil. This was shown earlier as 4b.
2 Finding Robot Singularities by Inspection

Figure 2.12 Combination of a Parallel Pencil with a Line intersecting the Plane

Skew Lines

Finally, we look at the case in which \( L_1 \) and \( L_2 \) are skew. Here, there are five cases. \( L_2 \) can intersect \( L_1 \) [case 1], be parallel to one [case 2], intersect one and be parallel to the other [case 3], intersect both [case 4], or be skew to both [case 5].

Case 1: \( L_2 \) intersects one of the two skew lines (figure 2.13a).

If \( L_3 \) intersects \( L_1 \), they form a planar pencil. \( L_2 \) will either be parallel to the plane containing this pencil, or intersect it. Both cases have already been studied.

Case 2: \( L_3 \) is parallel to one of the two skew lines (figure 2.13b).

If \( L_3 \) is parallel to \( L_1 \), they form a parallel pencil. Just as before, \( L_2 \) is either parallel to the plane of the pencil, or it intersects it. Again, all possible orientations of \( L_2 \) were studied earlier.

Case 3: \( L_3 \) intersects one line and is parallel to one line (figure 2.13c).

If \( L_3 \) intersects \( L_1 \) and is parallel to \( L_2 \), then \( L_2 \) is required to be parallel to the planar pencil formed by \( L_1 \) and \( L_3 \). Again, this case has been studied.

Case 4: \( L_3 \) intersects both of the two skew lines (figure 2.13d).

If \( L_3 \) intersects both \( L_1 \) and \( L_2 \), \( L_3 \) and \( L_1 \) form a planar pencil with \( L_2 \) intersecting it. Again, this case has already been studied.
Case 5: $L_3$ is skew to both lines (figure 2.14).

The only case which yields a new configuration is the case of three mutually skew axes. Linear combinations of three skew lines generate a family called a regulus. The regulus can be pictured in the following way. Through any three skew lines (the generators of the regulus) pass an infinite number of lines which intersect them all (the directrices of the regulus). Any line in the regulus intersects every directrix, and any directrix intersects every line in the regulus. Any line which intersects three of the directrices intersects them all, and is therefore in the regulus. Any three directrices also define a regulus, called the
conjugate reguli. If four lines lie in the same regulus, they must be dependent. Therefore, if three independent lines can be found which all intersect four mutually skew lines, the four lines must be dependent (rule 4g) [Veblen]

![Figure 2.14 Case 5 - Combination of Three Skew Lines - a Regulus](image)

With that, we have finished with all possible combinations of three lines. The seven three line families are summarized in figure 2.15.

**Four and Five Lines**

We now move to examining families generated by more than three independent lines. At this point, the approach I have been using becomes more and more difficult, and it becomes easier to tackle the problem in a different way. I will approach the last two sets of line combinations from the opposite direction, with the concept of reciprocal screws.

Two screws are said to be reciprocal to each other if their mutual moment is zero [Hess]. Two screw systems are reciprocal to each other if every screw in one system is reciprocal to every screw in the other. Every one-system is reciprocal to one and only one five-system. Every two-system is reciprocal to one and only one four-system. Every three-system is reciprocal to one and only one three-system.
Figure 2.15 The Seven Three Line Families
To prove this, we look at the formula for mutual moment.

\[ \{-w_1\} \cdot \{-v_2\} + \{-w_2\} \cdot \{-v_1\}, \]

where \(-w_n\) and \(-v_n\) represent three element vectors. Consider a five-system

\[ [L] = \begin{bmatrix}
    -v_1 & -v_1 & -w_1 \\
    -v_2 & -v_2 & -w_2 \\
    -v_3 & -v_3 & -w_3 \\
    -v_4 & -v_4 & -w_4 \\
    -v_5 & -v_5 & -w_5 
\end{bmatrix} \]

For a line \(L_n = \{-w_n, -v_n\}\) to be reciprocal to \(L\), the following must be true.

\[ \{-w_1\} \cdot \{-v_n\} + \{-v_1\} \cdot \{-w_n\} = 0 \]
\[ \{-w_2\} \cdot \{-v_n\} + \{-v_2\} \cdot \{-w_n\} = 0 \]
\[ \text{etc...} \]

In matrix form,

\[ [L] \begin{bmatrix} v_n \\ w_n \end{bmatrix} = 0 \]

The matrix \(L\) is 5 x 6, so if all five vectors are independent, the equation

\[ [L]x = 0 \]

has one solution. We can also say that the nullspace of \(L\) has rank one. Thus, there is one vector \(-w_n, -v_n\) which is reciprocal to every screw in \(L\). Therefore, if we take the solution to the equation

\[ [L]x = 0 \]

and exchange the first three components with the last three, we obtain the vector representation of the twist which is reciprocal to the 5-system \(L\).
Similarly - if \( L \) is composed of four independent vectors, it has rank four, and the
nullspace of \( L \) has rank two. Two independent screws will be reciprocal to every screw in
the system \( L \). If \( L \) is composed of three independent vectors, \( L \) has rank three and its
nullspace has rank three.

If \( L \) represents a five-system generated by \( \text{lines} \), and its reciprocal twist is a \text{line}
\( L_6 \), then \( L_6 \) intersects (or is parallel to) every line in \( L \). If the reciprocal twist is not a line (it
has nonzero pitch), we cannot make a comparable statement about its twist axis. Similarly,
for \( L \) of any rank generated by \( \text{lines} \), if all the reciprocal twists are lines, then they
intersect or are parallel to every line in \( L \). This means, of course, that they also intersect any
line which is dependent on the five lines which generate \( L \) and that no dependent line exists
which does not intersect the reciprocal line.

It follows simply, then, that if a line can be found which intersects six revolute
axes, the six axes are dependent, (rule 6a) and that if two independent lines can be found
which both intersect each of five revolute axes, the five axes are dependent. (rule 5a)

Unfortunately, the opposite statement is not guaranteed to be true. If \( n \) axes are
dependent, the \( 6 - n \) reciprocal twists may or may not have zero pitch. If they have nonzero
pitch, we can say nothing about the axes of the reciprocal screws.

We could leave the rules for combinations of four and five axes as stated in this
section and be correct, but the use of the rules will be simpler if we break them up by
finding reciprocal shapes for every one-, two-, and three-line family already described.

Beginning with the three-line families, a quick inspection will show the fascinating
fact that every three-line family is self-reciprocal.

There are three two-line families - the planar pencil, the parallel pencil, and two
skew lines. For each of these three families, we should be able to find four independent
lines which intersect every line in the family.
For the planar pencil - any three lines in the plane of the pencil will intersect every line. (Four lines in the plane would also intersect them all but would not be independent.) A fourth line, not in the plane, but intersecting it at the center of the pencil, is independent. These four lines generate a complete plane and a spherical bundle whose center lies in the plane. If five lines lie in this shape (the reciprocal of the planar pencil) they must be dependent.

![Planar Pencil and Spherical Bundle and Plane](image)

Figure 2.16 Reciprocal Farally to a Planar Pencil

For the parallel pencil - again, any three lines in the plane of the pencil intersect every line of the pencil. A fourth line, not in the plane, can only intersect every line of the pencil if it is parallel to all of them. (It intersects them at infinity.) Combinations of these lines produce a complete plane and parallel space. If five lines lie in this shape, they must be dependent.

Finally, we take two skew lines. Combinations generate no dependent lines, so we can say nothing more enlightening than that if two skew lines intersect five axes, the axes are dependent.
We can, however, gain some insight by allowing the two lines to be placed at infinity. Remember that a line at infinity can be written as a screw with zero magnitude, but nonzero moment. For example,

\[(0,0,9,1,0,0)\]

is a line at infinity in the \(yz\)-plane. This can be seen either by noting that it has moment about the \(x\)-axis only, or by noting that the \(y\)- and \(z\)-axes intersect it, but the \(x\)-axis does not (by taking mutual moments). This line can be thought of as a ring at infinite distance from the origin in the \(yz\)-plane. Any line which is parallel to the \(yz\) plane will have no magnitude in the \(x\)-direction and will therefore intersect this ring. Remember, also, that lines at finite distances can add to produce a line at infinity. For example,

\[
\begin{align*}
(0,0,1,0,0,0) & \text{ (the } x\text{-axis)} \\
- (0,0,1,-1,0,0) & \text{ (a line parallel to the } x\text{-axis)} \\
= (0,0,1,0,0,0) & \text{ (our example line at infinity)}
\end{align*}
\]

Combining these two lines produces a line at infinity. Combining rotations about the two axes (or rotating about the line at infinity) produces a pure translation along the \(x\)-axis.
In looking for families reciprocal to two-line families, first let one of the two lines be at infinity. For five lines to intersect both (the condition for dependence), all five must be parallel to the same plane and all must intersect a common line (rule 5d).

Figure 2.18 Reciprocal Family to Two Skew Lines, One of which Is at Infinity

If both lines which generate the two-line family are at infinity, they intersect at a point at infinity, and all lines which intersect both must have the same direction in space. No more than three lines, however, can point in the same direction and be independent (since three parallel lines span parallel space) so this configuration produces no new shape.

Figure 2.19 Reciprocal Family to Two Skew Lines, Both of which Are at Infinity
In examining families reciprocal to one line, there are only two possibilities. Either the line lies at infinity, or it does not. If the line does not lie at infinity, then five independent lines must intersect it, giving rule 6a - any six axes which intersect a single line are dependent. If the line does lie at infinity, then any line parallel to the plane in which it lies intersects it. Five independent lines span the reciprocal family, giving rule 6b - any six axes parallel to a single plane are dependent.

![Diagram](image)

Figure 2.30 Reciprocal Family to One Line which Lies at Infinity

**Six Lines**

Six independent lines span the space of all twists. Therefore, any seven axes must be dependent (rule 7).

2.4 Conclusions

The rules presented are sufficient to identify all singularities occurring in combinations of up to four axes and most singularities occurring in combinations of five or six axes. The additional singularities of five and six axes generally do not occur in manipulators with multiple axis intersections, but additional work in this area is needed to clarify the specific conditions which do permit them to occur.
With this set of tools, the robot designer should be able to evaluate many potential designs and currently existing systems. The information on the location of singularities is useful both in choosing robot kinematics and in writing path planning algorithms.

2.4.1 Example

The list is, for most robots, quite simple for the robot designer to search mentally, as I show in the following example. The joint configuration shown in figure 2.21b is one which I had to consider in the process of designing the hand and wrist. It would have been realized by adding a two-axis roll-pitch wrist to the existing arm (figure 2.21a).

To search the list of singular conditions, one mentally counts the robot about its joint axes in an attempt to meet the condition. This section describes the singularities of the arm with this wrist design. The singularities are shown in figure 2.22.

2a. Two axes can be made colinear in singularities I and II (figures 2.22a and 2.22b). Both are obvious singularities, which would probably be identified by even casual thought about the problem. I don't consider singularity II to be interesting because it occurs at the edge of the workspace. ALL points at the edge of any robot workspace are singularities.

3a. It is clear that three coplanar axes are parallel when singularity II occurs, so this configuration is singular for both reasons.

3b. Other than in the singularities already identified, three coplanar axes intersect only in singularity III (figure 2.22c). This is possible only if certain requirements on the link lengths are satisfied.

4a. Four axes will lie in the same plane in singularity I (always), in singularity II (sometimes), and in the following two cases. With axis 3 at zero, when axis 5 is rotated so that axis 6 lies in a vertical plane, we have singularity IV (figure 2.22d). When axis 3 is rotated 90 degrees, axis 5 can be rotated until axis 6 lies in the same plane as axis 2.
Examine these singularities closely. They would probably not ordinarily be identified without careful analysis.

6 - A line can be found intersecting six axes whenever the robot is at the edge of its workspace.

Figure 2.21 The Joint Axes of the WAM
Figure 2.22 Singularities of the WAM with a Roll-Pitch Wrist
3 Choosing the Kinematics

3.1 Choosing the Wrist Kinematics

The primary design choice I had to make in the design of the wrist was the choice of manipulator kinematics. The arm for which I designed the wrist, the WAM, has four degrees of freedom. I considered four possible kinematic arrangements for the wrist, two options with two degrees of freedom and two options with three degrees of freedom. The four arrangements can be called "pitch-roll", "pitch-yaw", "roll-pitch-roll", and "roll-pitch-yaw" (figure 3.1). I concluded that a three degree of freedom wrist has enough advantages to be well worth the additional design effort, and that the roll-pitch-yaw wrist gave by far the most useful kinematics for responsive, flexible, force-controlled manipulation. This section presents the reasoning behind that choice.

3.1.1 Six DOF Options: Pitch-Roll and Pitch-Yaw

The singularities of the WAM with a pitch-roll wrist were presented in the previous chapter (figure 2.22). In hindsight - we can group several of them for comparison of the possible designs. Singularity III was a subset of singularity IV, so they can be grouped together. Singularity II occurred at the edge of the workspace, and we now know that every point at the edge of any manipulator workspace produces a singular configuration, so for the purpose of comparing manipulator designs, we can consider only singularities in the interior of the workspace. And singularities IV and V have the same axis of self-motion, so
we can group them together. This leaves us with two types of singularities for the arm with a roll-pitch wrist. These are shown in figure 3.2.

![Four Options for Wrist Kinematics](image)

**Figure 3.1 Four Options for Wrist Kinematics**

![Two Singularities of the Arm with a Pitch-Roll Wrist](image)

**Figure 3.2 Two Singularities of the Arm with a Pitch-Roll Wrist**

If we instead choose a pitch-yaw wrist, we get three singularities in the interior of the workspace. They are shown in figure 3.3. Of the three, singularity III occurs at or
beyond what would probably be a joint stop of the wrist, so its importance is clearly diminished. Singularity II can be rendered impossible by proper choice of link lengths, so it, too, becomes less important. This leaves, effectively, just the shoulder singularity for this choice of axes. I conclude that from a kinematic point of view, the pitch-yaw design is preferable to the roll-pitch design.

Figure 3.3 Three Singularities of the Arm with a Pitch-Yaw Wrist
3.1.2 Seven DOF Options: Roll-Pitch-Roll and Roll-Pitch-Yaw

In order to evaluate seven degree of freedom manipulators, we will need some additional terminology. Seven DOF manipulators will always have at least one self motion. The "axis of self-motion" is the line about which we can rotate some part of the manipulator without changing the end effector position and orientation. If this axis changes or disappears as the robot moves through the self-motion, we will have to refer instead to an "instantaneous axis of self-motion."

Possible advantages to be gained from the self-motion are listed both in the previous chapter and in [Hollerbach]. Most manipulators will have one typical self motion which is present almost all of the time - around the "normal axis of self-motion." When the manipulator is in a "degenerate" configuration, the self motion will be different (and typically less useful) than the normal one. When the manipulator is in a singular configuration, at least two different self-motions will simultaneously be possible, and the rank of the Jacobian matrix will be less than six.

When a manipulator nears a singular configuration, finite motion commands for the end effector in Cartesian coordinates will demand large joint velocities. Similarly, when a redundant manipulator nears a degenerate configuration, the use of the self motion produces large joint velocities. Therefore, if the manipulator is to benefit from the self motion, degenerate configurations are also to be avoided by the trajectory planner.

Now we can turn to the analysis of the seven DOF configurations produced by adding a roll-pitch-roll or a roll-pitch-yaw wrist to the existing arm. Both configurations lead to a manipulator with two spherical joints (two sets of three intersecting axes). This makes the identification of the normal axis of self-motion simple. The axis lies along the line between the shoulder joint and the wrist joint (figure 3.4) [Hollerbach]. Reorientation of the manipulator about this line typically involves motion in six joints. In some
configurations fewer than six joints are involved in the self-motion, but the axis of self-motion is the same. Clearly, these cases are not degenerate.

Figure 3.4 The Normal Self-Motion of the WAM Arm with a Roll-Pitch-Roll Wrist
The normal self-motion of this arm involves a rotation of the arm about a line intersecting the shoulder and wrist joints.

Figure 3.5 A Degenerate Configuration
In this configuration, a different self-motion is available. The configuration is not singular because only one self-motion still exists.
Since the axis of self-motion always intersects all of the joints which are included in the self-motion, a self-motion of the manipulator pictured about a different axis can only be present in two cases. Either the degeneracy involves only axes of the shoulder or the wrist joint, but not both, or the degeneracy includes motion in the elbow joint. This greatly simplifies a search of the rules presented in the previous chapter.

Both roll-pitch-roll and roll-pitch-yaw designs have two possible degeneracies. A singularity occurs when both degeneracies occur simultaneously as in figure 3.6a. As shown in [Hollerbach], the singularity can be avoided through the use of the self motion (figure 3.6b). Neither design has any singularities within the workspace which cannot be avoided. The only significant difference between the designs is the location of the wrist degeneracy. In the roll-pitch-roll design, the degeneracy occurs in the middle of the joint range of the wrist. In the roll-pitch-yaw design, the degeneracy occurs somewhere near (or maybe beyond) a joint limit. This degeneracy would be much easier to avoid, and this gives the roll-pitch-yaw wrist design the best kinematics for a general purpose manipulator.

Figure 3.6 Avoiding the Singularity by Using the Self-Motion
3.1.3 Use of the Redundancy - Thoughts on Good 7DOF Design

I have chosen only the best kinematics for a three degree of freedom wrist to add to an already existing piece of hardware. I would argue, however that the full seven DOF system which will result is also near optimal. I believe that it is essential for a manipulator to have a continuous workspace, with no holes and no internal singularities which cannot be avoided. The minimum number of axes necessary to achieve this is seven. A seven degree of freedom manipulator must have at least two links of nonzero length for all seven axes to be independent. For kinematic simplicity, for minimum inertia, and for ease of hardware design, a manipulator should have as few links of nonzero length as possible.

If seven axes are to be used to avoid singularities, the manipulator should be designed to take full advantage of the single extra degree of freedom. To avoid workspace obstacles or change joint torque loadings, the self motion would be most useful if it changed the primary plane in which the manipulator works. To change joint velocities by superimposing a self motion on some set of joint velocities, the self motion would be most useful if it involved rotation in as many joints as possible.

If the manipulator is to have two nonzero length links and the best possible self motion, then the axis of self motion should intersect both ends of the manipulator and as many joints as possible. This thinking would lead to a two-link design with a spherical joint at either end, just as in the case of the WAM with the chosen wrist, and as in the case of the human arm.

3.1.4 Singularities and Peculiarities of the Human Arm

The one kinematic advantage of the human arm over the WAM is that the shoulder joint of the human arm is nearly a true spherical joint, instead of being made up of rotary joints. There is no shoulder degeneracy in our arm, and therefore no singularity other than at the edge of our workspace. Following the reasoning of the previous chapter, we should be able to understand why the outstretched human arm is at a singularity.
Human arms have seven degrees of freedom - three in the shoulder, one in the elbow, one in the forearm, and two in the wrist. Our normal axis of self motion is the line intersecting both the wrist and the shoulder. When the arm is at maximum extension (its only singularity) two self motions must be available. The normal self motion is still present here, and an additional one occurs (instantaneously) in rotation of the shoulder, elbow, and wrist such that the hand is pulled directly towards the shoulder. In the outstretched position, the ratio of linear motion of the hand to angular motion in the joints is zero, so instantaneously, this joint motion produces no net motion of the hand.

3.2 Choosing the Hand Kinematics

Any hand to be mounted on the WAM arm must have low enough mass to avoid seriously degrading the performance of the arm. At the same time, it should have enough versatility to acquire a wide variety of objects of different shapes, masses, and inertias. In order to be consistent with WAM capabilities, the hand will have to be able to work in completely unstructured environments, acquiring unknown objects at unknown locations either by accurate control of forces, feedback from haptic sensors, or mechanical behavior built into the system.

3.2.1 What Kind of Hand Should it Be?

I divide the world of robot hands into three broad categories - simple grippers such as the well known parallel jaw gripper, dextrous hands such as the Salisbury hand or the Utah/MIT hand, and conformal graspers such as the one built by [Hirose].

Simple grippers have one degree of freedom. They are typically not force controllable, but are simple, potentially lightweight, and good at picking up certain types of objects. Parallel jaw grippers, for example, are well suited to picking up objects with parallel sides. Industrial grippers and grippers for space applications are often designed along with a mating surface or handle to be mounted on all objects in the environment.
Simple grippers, however, are poorly suited to use in unstructured environments, where the objects of interest are unknown.

Dextrous robot hands begin to approximate the functionality of human hands. They have many degrees of freedom, at least some degree of force control, and are well suited to acquiring and responding to a wide variety of sensory information. A dextrous hand would be an ideal choice for use in unstructured environments if it were not for the mass of the actuators required. Nine degrees of freedom is about the minimum needed to accurately call a hand "dextrous", and the weight of the actuators required to drive this many joints with any kind of reasonable force prevents a dextrous hand from being a possibility for mounting on the WAM.

Conformal graspers, on the other hand, seem to be an excellent compromise. Other researchers at the MIT AI lab have developed robot end effectors with behaviors built into the design of the mechanism [Greiner]. (This gripper is presented in detail in section 4.1.) This type of manipulator has more joints than it has independently controllable degrees of freedom, and the design of the mechanism governs the behavior of the remaining joints. This strategy allows robust grasping strategies to be used without paying the cost of extra actuators mass.

3.2.2 One Finger, Two Fingers or Three Fingers

A conformal grasper to be used as a WAM end effector could be made in any number of shapes. A one-fingered design, for example, would consist simply of more links mounted after a wrist. This option was seriously considered for a time, as several members of our group felt that it would be more "WAM-like". In the end, the voices of those who felt that opposable thumbs really are a good invention prevailed. An opposable section allows a much more secure grasp and is the only practical way to achieve the high speed grasp necessary for catching flying objects.
In choosing the kinematics of a multifingered hand, I alternated between preferring two-fingered designs and three-fingered designs. A three-fingered design, in which the middle finger passes between the outer two, makes it simple to grasp small objects. It allows more flexibility for grasping non-cylindrical objects, and it allows the hand to apply torques to the object with the fingers. It is significantly more difficult, however, to actuate three fingers and to make them robust enough to survive impacts.

In the end, I decided that two broad, flat fingers would give me sufficient grasp stability and that I could choose the link lengths to still allow good grasping of small objects. I felt that the simplicity and robustness of the two-fingered design was worth some constraints on graspable objects.

3.2.3 Choosing the Link Lengths

Many studies have been done to determine the optimal link lengths for robot manipulators. Most optimize some extremely narrow criterion for a single task, or simplify the problem too far by neglecting either the thickness of the links or realistic joint limits. Nevertheless, I found it useful to keep the conclusions of those studies (and their limitations) in mind while choosing a design.

Design for Maximum Diameter Grasping

[Charnas] showed how to choose optimum link lengths for a stick figure manipulator to maximize the diameter of the largest possible cylinder graspable with form closure (a stable grasp even in the absence of friction). The shortest possible shape with $n$ straight sides which completely surrounds a circle is an $n$-sided regular polygon circumscribing the circle. For form closure, a gripper must have contacts at least halfway around the circle, so by cutting the polygon in half at points of intersection with the circle, one generates the shape of the shortest $n$-link manipulator which can grasp the circle. All the links of this manipulator have the same length except for the two distal links, which have lengths equal
to half of the length of the other links. The manipulator must have at least three links to grasp a circle of nonzero size, so this rule holds for any number of links $n$ greater than two.

[Chanmas] further argued that a five-link manipulator is sufficient for robotic grasping because additional joints (in a manipulator of constant length) add little to the graspable diameter. A six-link manipulator of the same length can only grasp a circle a few percent larger than the five-link manipulator can grasp.

To design a manipulator with tapered links instead of stick figures along these principles, one need only let the inner surfaces of the links (instead of the center distances of the links) have the proper ratios when in the grasp configuration (figure 3.7). This will change the center distances slightly.

Figure 3.7 Optimum Link Ratios for Maximum Diameter Grasping

This design is well optimized for grasping large cylinders, but it requires very large ranges of motion in the finger joints in order to enclose small objects securely (figure 3.8). This is not acceptable for any practical hand design.
Design for Minimum Diameter Grasping

I believe that the largest graspable diameter is only one criterion involved in the choice of link lengths. A more important consideration is that the hand be able to grasp a continuous range of sizes securely without extraordinary joint ranges.

If we modify the optimum for large cylinders slightly by adding one short link to make a 360 degree joint, we get the configuration shown in figure 3.9. This departure from the optimum sacrifices a little in maximum diameter but gives a completely continuous range of secure enclosing grasps from zero diameter up to the maximum. The asymmetric design is unusual, but there is no justification for adding yet another joint. The asymmetrical hand already fulfills all the design criteria nicely.

It is the eventual goal of the WAM group to build a two armed system to study multi-arm interactions. In a two armed system, the sacrifice in maximum diameter becomes less important, because two handed grasps will cover all the intermediate sizes between one handed grasps and two armed grasps, so no sacrifice is actually made in the size of the largest object graspable with both hands.
3.2.4 Choosing the Joint Range of Motion

Large joint ranges such as those realizable in a stick figure hand model are difficult to achieve with a real mechanism. They require hollow links and exposed parts. This problem was specifically pointed out by [Greiner] as an area of future work. Fortunately, the situation in a hand is the opposite of that mentioned previously for a wrist. With a good choice of link lengths, large joint ranges are not only unnecessary, but they can even be counterproductive. The hand shown in figure 3.9 does not need large joint ranges either for the grasping of small objects or for the grasping of large objects. Furthermore, the joint stops contribute to proper grasping behavior by preventing one link from curling completely closed before the others have curled around an object.
4 Cable Tools

When designed well, cable mechanisms have outstanding mechanical efficiency. They can be almost completely backdriveable and free of backlash. More importantly, however, cables allow us to get all the motors as far away from the hand as possible. Mounting the motors at the base of the forearm is an excellent way to keep the inertia of the package low enough for a hand and wrist to be feasible. Notice that this design principle is obeyed in nature as well. Except for the thumb, all the actuators which flex and extend human fingers are located back in the forearm, attached to tendons which pass through the wrist.

4.1 Indeterminate Systems

This section discusses a type of manipulator which has been built by Hirose in Japan and by Helen Greiner at MIT. In addition to exhibiting desirable innate mechanical grasping behavior, these manipulators illustrate a fundamental principle of cable actuation. Understanding this principle is essential to the discussion of more complex cable actuation schemes which follow.

4.1.1 Multijointed Carling Mechanisms

Consider the mechanism shown in figure 4.1. An idler pulley sits on bearings on the axis of a robot joint. A cable wraps 360° around the pulley, and terminates on the link. A force $F$ on the cable applies a torque

$$\tau = rF$$
to the joint. It does this even in the absence of friction between the cable and the pulley. This simple example illustrates something which is easily forgotten in complicated mechanisms, so I will prove it here.

![Figure 4.1 A One Cable - One Idler Pulley Illustration](image)

The easiest way to understand this mechanism and more complex versions is the principle of virtual work.

*For a system of initially stationary rigid bodies, the necessary and sufficient condition of equilibrium is that the virtual work of the active forces be zero for all possible virtual displacements consistent with the constraints [Stames].*

In this example, the active forces are the force $F$ on the cable and the corresponding torque $\tau$ on the joint sufficient to hold it in equilibrium. The only permissible virtual displacement is a rotation of the link about an angle $d\theta$, which is constrained by its relationship with the length of the cable

$$dx = \tau \, d\theta$$

If the joint were to move, the virtual work which would be done by the force on the cable through a displacement $dx$ must be equal to the virtual work done by the torque on the joint through a displacement $d\theta$.

$$dW = F \, dx = \tau \, d\theta$$

Substituting in the constraint, we get,
\[ F \tau \, d\theta = \tau \, d\theta \]
\[ \tau = r \, F \]

In plain English, if rotation of the joint displaces the cable, then a force on the cable must apply torque around the joint.

Consider now the two link system shown in Figure 4.2. By the same argument, a cable which passes around both idler pulleys and terminates on the distal link applies a torque

\[ \tau_l = r_l \, F \]

to each joint. Remember that the pulleys are free to spin as necessary and need not move with the link. A second cable, passing around another set of idler pulleys in the opposite direction, would apply torque to the joints in the same manner in the opposite direction. In this particular case, both cables could actually wrap around a single pulley at each joint, because the motion in the two cables is always exactly equal and opposite. This can be extended to an arbitrary number of links and idler pulleys.

Figure 4.2 A Two Cable - Two Pulley Mechanism

Hirose has built an innovative gripper using this principle to achieve curling, conformal grasping with uniform pressures around objects of unknown shapes. The
Hirose gripper consists of a large number of links, each of which has a pulley of the same diameter. A force on the cable therefore applies the same torque on every joint, producing the "soft" curling effect he describes (figure 4.3).

The MIT version shown in figure 4.4 is based on a similar principle, but uses decreasing pulley diameters to control the gripping behavior. Specifically, the diameter ratios are chosen in proportion to the link lengths to produce linearly decreasing torques along the gripper instead of uniform torques. In figure 4.3,

\[
\frac{r_3}{r_1} = \frac{l_3}{l_1 + l_2 + l_3} \quad \text{and} \quad \frac{r_2}{r_2} = \frac{l_2 + l_3}{l_1 + l_2 + l_3}.
\]

The two grippers are similar in that they both curl when the cables are pulled, and they both conform mechanically to grasped objects but the specifics of the interaction with objects differ. The decreasing pulley diameters in the Greiner PALM insist that the gripper will always curl in such a way as to improve the grasp. Wherever the contact occurs along the length of the gripper, only the joints more distal than the contact curl inward. If the contact occurs at the tip, no curling takes place at all (figure 4.5).

Both of these grippers are indeterminate systems. They have more joints than actively controllable degrees of freedom. The positions of the joints are constrained only by the relationship imposed on them by the cable, but can otherwise move in response to a contact force. In a determinate system, by contrast, the angle at each joint would be linked by some function. (See section 4.3.2 for an example.) In both of these indeterminate grippers, choosing the force on the cable determines the torque at every joint, but sets only one constraint on the joint positions.
Figure 4.3 The Hirose Gripper

Figure 4.4 The Greiner PALM

Figure 4.5 Positive Curling in Response to a Force
4.2 N+1 Systems

The problem with cables is that they can only pull, not push. Robot joints, however, need to move in both directions. It’s clear that two cables are sufficient to move one joint in both directions, and that one cable is not sufficient (unless opposed by a spring). The minimum number of cables required to fully determine bidirectional motion in \( n \) joints is \( n+1 \). This will be proved in section 4.2.4. With four degrees of freedom in the hand and the wrist, only five cables are required to pass through the wrist. Since each added cable requires several additional idler pulleys, keeping the number of cables to a minimum is absolutely vital to meet size and weight constraints.

\( N+1 \) systems are occasionally criticized because they are coupled. In other words, to move a particular joint, you must pull on combinations of cables. In reality, \( n+1 \) systems are no more or less coupled than any other system. A revolute robot moving in Cartesian space, for instance, must also move a combination of joints to produce the required endpoint motion.

In such a system, there are \( n+1 \) independent combinations of cables which can be pulled. \( N \) of these combinations actuate joints. The remaining combination controls the internal tension of the system. This “active pretensioning” is usually used to keep the tensions at the minimum necessary level. This minimizes the friction effects which increases the dynamic force range of the system. The active pretensioning could conceivably also be used to "stiffen" the system by adding extra static friction. The disadvantage of active pretensioning is that the system becomes completely slack when the power is turned off. This means that attention must be paid to the fact that cables can snarl, tangle, knot, jam, jump off of pulleys, and do any of the other annoying things that cables like to do while slack.
4.2.1 Simple $N+I$ Systems

Each cable of an $n+I$ system which passes over an idler pulley puts torque on the joint in the same manner as the cables in the Greiner PALM (by virtual work). The simplest $n+I$ system, the determinate case of one pulley and two cables (figure 4.6), was discussed in the previous section. Cable 1 puts torque about the joint in the negative direction. Cable 2 puts torque on the joint in the positive direction.

![Figure 4.6 A One Axis $N+I$ Mechanism](image)

The indeterminate case of two cables and two pulleys was also shown in the last section (figure 4.2). The determinate case of two pulleys and three cables is slightly more complicated (figure 4.7). For simplicity, assume that all pulley radii are equal. By the principle of virtual work, we can write the torques on each joint as,

$$\tau_1 = F_1 r + F_2 r - F_3 r$$

$$\tau_2 = F_1 r - F_2 r$$

It can be seen either from the equations or by inspection that joint 2 can be moved by moving cables 1 and 2 in opposition while holding cable 3 fixed. Joint 1 can be moved by moving cables 1 and 2 together as a group, in opposition to cable 3.
4.2.2 The Salisbury Hand

The Salisbury Hand was one of the first \( n+1 \) mechanisms built which used four cables to actuate three axes. The cabling is shown schematically in figure 4.8. As in the previous section, it can be verified either by inspection or by writing the equations, that there exists some unique and independent combination of cable tensions which can apply torque around each joint axis.

Note also, that unlike the simpler cases, there is more than one way to cable this set of pulleys. As the number of degrees of freedom increases, the number of possible ways to cable them increases dramatically. For this finger, there are 13 ways each cable could be routed (two which terminate after the first pulley, four which terminate after the second pulley, and eight which terminate after the third pulley). For four cables, each of which has 13 possible arrangements, there exist 715 ways to cable the finger. Most of these cable routings are singular. They cannot move each joint in both directions independently, but there do exist several non-singular routings.
4.2.3 The Jacobian Matrix for N+1 Systems

The cable routing of an N+1 system can be succinctly described by a matrix, usually called the R-matrix which maps joint angular velocity to cable velocity

\[ [R] \omega = \nu. \]

The vector \( \omega \) of joint velocities has \( n \) components, and the vector \( \nu \) of cable velocities has \( n+1 \) components, so the matrix \( R \) must be \( n+1 \) by \( n \). To construct this matrix for any mechanism, observe that each element of \( \omega \) multiplies a column of \( R \) to produce a vector of cable velocities. By mentally rotating each joint of the mechanism and noting the rate at which cables move, one can write down the columns of the matrix by inspection. The mapping for the Salisbury hand, for example, (assuming unity radii) is
\[
\begin{bmatrix}
1 & 1 & 0 \\
-1 & 1 & 1 \\
1 & -1 & 0 \\
\end{bmatrix}
\begin{bmatrix}
u_1 \\
v_2 \\
v_3 \\
\end{bmatrix}
= 
\begin{bmatrix}
u_f \\
v_2 \\
v_3 \\
\end{bmatrix}
\]

Each row of \( R \) represents the path of one cable. Each column of \( R \) represents the cabling at one joint.

The same matrix also embodies all the information about the relationship between forces in the cables and torques at the joints. To see this, we write the equation for conservation of energy for a motion given by

\[ \nu = [R] \omega. \]

Power input must be equal to power output, so

\[ f^T \nu = \tau^T \omega \quad \text{(conservation of energy)} \]

\[ f^T [R] \omega = \tau^T \omega \quad \text{(substitute } \nu = [R] \omega) \]

\[ \tau = [R]^T f \]

where \( \nu \) is a vector of cable velocities, \( f \) is a vector of cable forces, \( \omega \) is a vector of joint angular velocities and \( \tau \) is a vector of joint torques.

A particular Jacobian matrix must satisfy two conditions to permit \( n+1 \) cables to independently control \( n \) joints. First, for each joint to be independently controllable, the columns of \( R \) must be independent. \( R \) must have rank \( n \) in order to span the full space of joint velocities \( \mathbb{R}^n \). The test for this condition is

\[ \text{Det } [R^T R] = 0. \]

Second, for the mechanism to be physically controllable with flexible cables which can apply force in one direction only, the elements of the null space of \( R^T \) must all have the same sign. Because \( R \) has rank \( n \), but is non-square \((n+1) \times n\), there exists one non-zero solution to the equation

\[ [R]^T f = 0. \]
The solution to this equation, $f_0$, can be added to any other vector of cable forces without influencing the joint torques produced. If the elements of $f_0$ all have the same sign, then any vector $f$ can be biased by a multiple of this "pretension" vector up to the point where all forces are positive. (You try pushing on a cable sometime!)

### 4.2.4 $N+1$ Cables are Necessary and Sufficient to Control $N$ Joints

From the discussion of the previous section, each cable can be represented by an $n$-element vector. Because it represents a cable, this vector is "unidirectional." In other words, it can be multiplied only by non-negative constants. In order to show that $n+1$ cables are required to move $n$ joints bidirectionally, we will need to prove that $n+1$ of these unidirectional vectors are necessary and sufficient to span $n$-dimensional space. This is also the reason that a rigid body with $n$ degrees of freedom must be constrained by $n+1$ unisense contacts. Each contact applies a unidirectional wrench (usually just a pure force) to the body. A planar object must be constrained by four such contacts (Reuleaux), while an object in space must be constrained by seven contacts [Lakshminarayana].

![A Planar Object](image1.png)

![A 3-Dimensional Object](image2.png)

Figure 4.9 Objects Constrained by $N+1$ Forces
Lemma: For \( m = n + 1 \), there exists a unidirectional basis \( A \in \mathbb{R}^m \) which spans \( \mathbb{R}^m \).

Proof: If \( A \) spans \( \mathbb{R}^m \) unidirectionally, then for all \( b \in \mathbb{R}^m \) there exists an \( x \in \mathbb{R}^m \) such that

\[
[A]x = b \quad \text{and} \quad x_i \geq 0.
\]

Let

\[
[A] = \begin{bmatrix}
1 & -1 \\
1 & -1 \\
\vdots & \vdots
\end{bmatrix}
\]

Then

\[b_i = x_i - x_{n+1} \quad i \in (1, n).
\]

The solutions \( x_i \) are therefore given by

\[x_i = b_i + x_{n+1}
\]

where \( x_{n+1} \) is arbitrary. For some \( b \) let \( x_{n+1} = \min(b_i) \). This insures that all \( x_i \) are greater than or equal to zero, and therefore \( A \) is a unidirectional basis for \( \mathbb{R}^m \).

Lemma: For \( m \leq n \), there exists no unidirectional basis \( A \in \mathbb{R}^m \) which spans \( \mathbb{R}^m \).

Proof: It is sufficient to prove this for \( m = n \). Assume \( A \) is square and is unidirectional basis for \( \mathbb{R}^m \). Again, for all \( b \in \mathbb{R}^m \) there exists an \( x \in \mathbb{R}^m \) such that

\[
[A]x = b \quad \text{and} \quad x_i \geq 0.
\]

Apply Gram-Schmidt Orthogonalization to \( A \). If \( A \) spans \( \mathbb{R}^m \), there exist matrices \( Q \) and \( R \) such that

\[
[A] = [Q] [R]
\]

where \( Q \) is orthonormal and \( R \) is upper triangular. We can then write

\[
[Q] [R] x = b.
\]

Because \( Q \) is orthonormal,
\[ (Q^T)(Q) = I, \]

so

\[ [R] x = [Q]^T b. \]

The matrix \( Q^T \) also spans \( \mathbb{R}^n \), and \( b \) is an arbitrary vector. Therefore \( Q^T b \) is also an arbitrary vector, and we can write simply

\[ [R] x = b. \]

The matrix \( R \) is upper triangular, so choose

\[ b_n = -r_{nn} \]
to make \( x_n < 0 \), which contradicts the assumption that \( A \) is a unidirectional basis for \( \mathbb{R}^n \).

4.3 Pretensioned Systems

Pretensioned systems use one motor per joint with two pretensioned cables. In other words, \( 2n \) cables are required to control \( n \) joints. This type of arrangement has also been called "push-me-pull-you". As one side is reeled in, the other side is paid out. This type of system uses more cables than an \( n+1 \) system (2 \( n \) instead of \( n+1 \)), but only \( n \) motors. In addition, the mechanism is not coupled. One motor controls one joint, which is easy to understand and visualize. In situations where the number of cables is not critical, a pretensioned system system is usually a better choice than an \( n+1 \) system.

The pretension is required in order to keep the force output continuous even when changing directions. Without pretension, both cables would be momentarily slack while changing. This would make the dynamic behavior nonlinear (and unpredictable), and control algorithms less robust. Although it is not difficult to design a good tensioning system, almost all pretensioned cable designs do a miserable job of pretensioning.

4.3.1 The WAM Transmission

The WAM arm employs a creative solution to the pretensioning problem. Each motor pinion is manufactured as two separate pieces, which can rotate relative to each other
and can be tightened in place with a large locknut. Each of the two cables terminates on its own piece of the pinion. The two parts of the pinion are then rotated in opposite directions to tension the cables and locked in place.

4.3.2 Pretensioned Systems with Coupled Joints

Both of the grippers designed by Hirose and Greiner are indeterminate cable systems, with more joints than actuators. It is also possible to build a determinate cable driven system with more joints than actuators, where the motor position fully determines many joint positions (figure 4.10). A finger based on this principle was built at Irvine by McCarthy. It is possible to select any coupling of the joints by choosing the diameters appropriately. I don't feel that this system, as pictured, makes a very good gripper. The ability of the indeterminate grippers to conform to the shape of any object is a powerful tool. Restricting the joint motions to a coupled trajectory serves no useful purpose.

I am intrigued by one possibility involving a design of this nature. I believe that this kind of determinate system, with compliant tendons, could be made to have the same desirable passive curling characteristics as the Greiner and Hirose grippers by the proper choice of tendon stiffnesses and pulley ratios. If the compliance of the tendons were nonlinear (as human tendon is), one could then cocontract the tendons to stiffen the finger in any position. The ability to change the stiffness of an indeterminate conformal grasper would be a great advantage in force controlled interaction with objects. For an example of its usefulness, consider the last two links of your finger. They are coupled - that is, one input controls both joints and they move with a predetermined curling trajectory. Yet they can also deviate passively from that trajectory when they conform to an object, and they can be stiffened anywhere along the trajectory.

The gripper shown in figure 4.10 has a joint coupling which is completely determined by the pulley ratios.
\[ \frac{\delta_1}{\delta_2} = \frac{r_3}{(r_1 - r_2)} \]

The cables are directly connected to the first joint, so pulling a cable actuates both joints.

Figure 4.10 A Determinate Finger with One Actuator and Two Joints

4.4 Choosing a Cabling Arrangement for the Hand/Wrist

Most of the cable designs described in this chapter find their way into the hand/wrist somewhere. It makes the most sense to build the two opposing fingers as Greiner-style grippers. This gives good grasping performance without paying a cost in excess actuator weight. This type of system is also well suited to the high speed requirements for catching. The intrinsic grasping behavior of the fingers effectively has an infinite bandwidth, since it responds directly to applied forces without a need for sensing and actuation. Prototypes of these fingers mounted in opposition made a beautifully "grabby" hand, which seemed to want to hang on to just about everything.

The space constraints in the wrist make it difficult to pass cables through the wrist. Hence, the number of cables which pass through to the hand should be kept low and all cables should be utilized as effectively as possible, leading naturally to an \( n+1 \) cabling of...
the hand and wrist. It is awkward to include the roll axis of the wrist in an $n+1$ arrangement because of the irritating need for tensioned cables to lie on the outside of cylinders and not the inside. Fortunately, it is not necessary to include the roll axis in the $n+1$ scheme. It can be kept completely separate as a pretensioned system, and its cables do not need to pass through the wrist.
5 Selection of the Cable Routing

The kinematic analysis of chapters 2 and 3 motivated the choice of a three degree of freedom roll-pitch-yaw wrist. In chapter 4, I studied cable mechanisms and chose to actuate four of the axes in an \( n+1 \) configuration. To my knowledge, no four axis \( n+1 \) system has ever been built, and the mappings from motor motions to joint motions are somewhat less than intuitive. It is difficult simply to invent a cable routing which allows independent motion of all the joints. Given the extremely large number of possible cable routings, I felt that further analysis of this topic was needed.

5.1 Optimizing the \( N+I \) Routing

The \( R \)-matrix describing the cabling transforms the five cable forces to four joint torques. Since all the forces have the same range of tensions, we can imagine the five forces as representing a five dimensional sphere. The \( R \)-matrix transforms this sphere to a four-dimensional ellipse. The more the transformation distorts the sphere, the less balanced the cable forces will have to be when applying a full range of joint torques and the less productively the force output of the five motors of the \( n+I \) mechanism will be used. The measure of this “squashing” of the sphere is the condition number of \( R^T R \) - the square root of the ratio of the maximum eigenvalue to the minimum eigenvalue.

In order to test possible cable arrangements for their condition number, we need to write the routing each of the possible cables as a four-element vector

\[
\{v_1, v_2, v_3, v_4\}
\]
where \( \tau_1 \) and \( \tau_4 \) represent the torque a given cable puts on each of the four joints (wrist yaw, wrist pitch, finger 1, and finger 2) and can take on the values +1, 0, or -1 (assuming unity radii for the time being). Groups of five of these vectors can then be assembled into an R-matrix. The routing of each individual cable is subject to a number of constraints. It is impossible for any cable to actuate a finger without passing through the wrist, which means that any cable which has a nonzero value in \( \tau_3 \) or \( \tau_4 \) must also have nonzero values in both \( \tau_1 \) and \( \tau_2 \). Also, it is inconvenient to make a cable actuate both of the fingers, so cables may not have nonzero values in both \( \tau_3 \) and \( \tau_4 \).

![Diagram of cable routing](image)

**Figure 5.1 An Example Cable for this Four Axis System**

Within these constraints, there are four possibilities for cables to be routed around the two wrist joints,

\[
\{1, I, \tau_3, \tau_4\}, \{1, -I, \tau_3, \tau_4\}, \{-1, I, \tau_3, \tau_4\}, \{-1, -I, \tau_3, \tau_4\},
\]

and five possibilities for routing around the two fingers,

\[
\{\tau_1, \tau_2, 1.0\}, \{\tau_1, \tau_2, -1.0\}, \{\tau_1, \tau_2, 0.1\}, \{\tau_1, \tau_2, 0.0\}, \{\tau_1, \tau_2, 0.0\}.
\]
This makes 20 possible cables. Many $R$-matrices could be composed from these. Assembling 5 of the 20 cables into a matrix without repeating any gives

$$\frac{20!}{5! (20 - 5)!} = 15,504.$$  

From the last section, we saw that for a matrix to be usable in an $n+1$ configuration, it must have rank $n$ (determinant of $R^TR$ is nonzero) and all the elements of the null vector must have the same sign. The matrices which satisfy both of these constraints can be ranked by their condition number.

I wrote a program to evaluate each of the 15,504 matrices, checking for independence and for unique null vector, and then ranking the valid matrices by their condition number. Practically it turns out that many matrices have the same condition number. This occurs because many of the matrices actually represent identical cabling schemes. It does not matter which cable we choose to call #1, #2, etc. Nor does it matter which direction we choose to call positive at each axis. Each cable is represented by a row of the matrix, so changing the numbering of the cables corresponds to changing the ordering of the rows of $R$, and changing the definition of positive and negative at a joint corresponds to changing the sign of a column of $R$. Happily, neither of these operations affects the condition number of the matrix. It’s always nice when mathematics accurately matches reality.

The best matrix, ranked by condition number, is

$$R = \begin{bmatrix} 1 & -1 & 0 & 1 \\ -1 & -1 & 0 & -1 \\ 1 & 1 & 0 & -1 \\ -1 & 1 & -1 & 0 \\ -1 & 1 & 1 & 0 \end{bmatrix}$$
This matrix has a condition number of 1.52. For comparison, the identity matrix has a condition number of 1.0, the nearest other valid matrices all have condition numbers over 2.5.

The null space of \( R \) is the vector which applies no torque around any of the joints. For this \( R \) the null vector is any multiple of
\[
\{0.65, 0.33, 0.43, 0.43\}.
\]
We will add a multiple of this vector to any calculated vector of cable tensions when necessary to keep all cables under positive tension. The cabling which is represented by the chosen \( R \)-matrix can be seen in figure 6.2.

### 5.2 Simulation of a Hand and Wrist Cabled with the Chosen Matrix

In order to assess how well the \( R \)-matrix keeps the cable forces balanced when transforming torques from a typical loading situation, and to get an idea of the kinds of force levels which could reasonably be expected, I wrote a simulation for the mechanism.

I have assumed the following loading situation for the simulation. The hand holds a one pound object at the fingertips (the worst place at which to apply forces) with an additional pound of grip force. In addition, a two pound force acts at the fingertips and is applied in varying directions in the vertical plane from 0 to 360 degrees in order to get an idea of the way the force changes are reflected in the cable tensions. Tensions are calculated for each direction in which the rotating force is applied. Two different cases are run. The first models the hand held in an upright position. The second models the hand turned over, so that gravity acts in the opposite direction.

At each angle, these forces produce torques at the four joints. From section 4.2.3 we can calculate the cable forces from
\[
f = (R^T)^{-1} \tau.
\]
The simulation then calculates the minimum multiple of the null vector necessary to add to $f$ to make all the components of $f$ greater than or equal to zero. The results are shown in figures 5.2 and 5.3. Notice that one of the five cable tensions is always zero. This occurs because the tensions are always kept at the minimum level, which means that the lowest tension is always zero. The discontinuous derivatives in figure 5.3 are caused when the zero tension cable changes from cable 5 to cable 4.

![Graph showing cable tensions for the Hand/Wrist - Case 1](image)

Figure 5.2 Simulated Cable Tensions For the Hand/Wrist - Case 1
Condition number = 1.52
Figure 5.3 Simulated Cable Tensions for the Hand/Wrist - Case 2
Condition number = 1.52. Notice the change in the plot as different cables are biased to zero tension.
6 Design Choices for the Wrist

This chapter gives a quick overview of two different implementation possibilities for a pitch-yaw wrist (allowing the roll axis to be a separate mechanism). Both have similar kinematics, and both use the same $n+1$ cabling arrangement which includes the actuation of the fingers. Except for that, the two approaches differ radically.

6.1 The Strange Wrist

I started the design process with some rigid design criteria. The three axes of the wrist should all intersect. The joint ranges should be large. The part count should be kept low, the exterior clean, and the mechanism as backdrivable as possible. In trying to meet all of these criteria, I came up with a fairly radical idea for a wrist mechanism which does certain things very well.

The cables actuate this wrist in much the same way as a comparable $n+1$ system using pulleys would. In chapter 4, we saw that $n+1$ systems could be understood using the principle of virtual work. If a joint motion requires cables to move, then the cables must put torque on the joint. "The strange wrist" uses cables which slide through channels drilled in the wrist. Just as in pulley designs, independent combinations of the five cables can be actuated to move each joint independently.
Figure 6.1 Prototype of the Strange Wrist
Cables in this mechanism slide through channels drilled in the wrist. This is an extremely simple mechanism.

If the fingers are fixed relative to the wrist, and the wrist is moved through its range of motion, the cables do not slide in their channels. If the fingers are flexed or extended, however, the cables must slide over the teflon surface of the wrist. This design is largely patterned on the actuation scheme and kinematics of the human wrist. In the human wrist, just as in this design, the tendons which actuate the fingers are coupled into the wrist motion, and tendons must slide over other surfaces in constrained channels. In the human wrist, the constraints are ligaments, and the tendons slide in tendon sheaths lubricated with synovial fluid.

Like the human wrist, the actuation of the "strange wrist" is fundamentally nonlinear. In other words, the effective radius of the pulley over which the cable passes, changes with the angular position of the wrist. Initially, this nonlinearity might appear troublesome in any attempt to write simple control code, but the techniques of nonlinear controls are more than adequate to control such a system.
The great advantage of the "serange wrist" is that it is extremely simple. It has almost no parts - only three bearings in the wrist. The tendons are completely constrained and cannot jam, cross, or jump pulleys. The wrist is very robust and could easily withstand the rigors of use at the end of the WAM. In addition, there is no practical limit to the number of tendons which could be routed through the wrist in this manner. Cabling for a 15 degree of freedom hand and wrist could easily be accommodated.

The drawback to the design is the requirement that tensioned cables slide over teflon surfaces. The Spectra fiber we have chosen for tendon material has an extremely low coefficient of friction, and is very resistant to this type of wear [Anderson], but the large angle changes which would be required (up to 90 degrees) would cause at least a 5 - 10% power loss to static friction. This was deemed to be unacceptably high for precision force control, so other approaches to the design of the wrist had to be explored.

I believe that any future wrist designed to accommodate a dextrous hand and an arm with a clean exterior design like the WAM, will have to be designed along the lines of the serange wrist. The ability to guide many hand tendons through the wrist simply and cleanly, and to actuate the wrist with the same tendons is too great an advantage to be ignored. I hope that more research on lubricants and on "super-fibers" such as Spectra and Vectran will make this type of design practical.

6.2 The Stacked Pulley Design

The usual approach to designing \( n+1 \) systems is to employ stacks of pulleys, in which each cable rides on its own pulley. This type of mechanism has been built many times before, although I know of no four axis systems. The stacked pulley designs can be made to have very low friction, but they require many parts and many bearings, which can also make them more delicate systems.
The other significant drawback to a stacked pulley design was that it was necessary to give up on the three intersecting wrist axes. This makes the kinematics slightly less desirable because we move away from the simple self motion around the line connecting the shoulder and the wrist joints. An argument could be made, however, that these kinematics are in some sense better because the self motion now involves motion in all seven joints, instead of only six.

There is no doubt that this wrist can exert higher forces with more precise control than would have been possible with the strange wrist.

Figure 6.2 Close-up of a Stacked Pulley n+1 Wrist
Each of the 10 pulleys in this wrist can spin independently. High part count is traded for low friction operation.

The stacked pulleys gave rise to some tricky design problems. A cable which takes a full wrap on a pulley and continues, as the cables of all n+1 designs do, will travel laterally a distance of one cable diameter for every revolution of the pulley. The total
potential travel of the cable therefore places a minimum width requirement on the pulley to avoid rubbing against the flanges. The space constraints for the exterior of the wrist are also extremely tight, as it should not have parts which project beyond the diameter of the forearm of the WAM and interfere with its manipulation abilities.

This yielded a set of thin, large diameter pulleys, which might easily be subjected to large "overturning" moments if the pulleys were not supported laterally as well as radially. The space constraints precluded placing thrust bearings between the pulleys, so I built thrust bushings into the structure of the pulleys by dropping four suction balls into holes in the sides of the pulleys. The diameter of the balls is .005 inches larger than the depth of the holes, so they project out beyond the surface of the pulley and support any overturning moment.
7 Design Choices for the Hand

7.1 Selection of the Pulley Focus

The Greiner PALM shown in chapter 4 uses pulleys of decreasing diameter to control the contact behavior of the links. The pulleys are chosen in such ratios that two straight lines, tangent to each pulley, would intersect at the tip of the finger. The pulley ratios can, of course, be chosen so that this point of intersection, called the "pulley focus", occurs anywhere along or beyond the structure of the finger. Forces on the links of the gripper which occur proximal to the focus cause positive curling (the right kind for acquiring objects), forces which occur distal to the focus cause negative curling, and forces which act at the focus cause no curling. Choosing the focus at the tip of the finger causes the finger to always curl only at the joints which are more distal than the force.

I experimented briefly with other locations than the fingertip for the pulley focus. I found that a pulley focus location slightly beyond the tip of the finger allowed a cylinder of slightly larger radius to be grasped stably, but that this caused the fingers to slip on smaller objects and curl in on themselves. I speculate that a focus location slightly proximal to the tip might allow good fingertip (non-curving) grasping, which would be useful for flat objects like sheets of paper or cloth, at the expense of failing to grasp some objects which strike the fingers near the tip. For now, I have chosen to place the focus at the tip of the finger, but more work could be done in this area to determine the optimal location.
7.2 Dislocatable Knuckles

One of the primary areas of concern in the design of the hand is to make sure that the hardware I build is not destroyed the first time an errant trajectory attempts to drive the hand through a wall. The natural backdriveability of the wrist should save the hand from damage in most cases; the hand will simply be pushed to one side during a crash. It is always possible, however, that the hand may someday be crashed when the wrist is at a joint limit, providing an opportunity for major structural damage. It seemed impossible to make the structure of the hand strong enough to withstand all foreseeable collisions and still stay within reasonable weight constraints, so I took a different approach to the problem.

![Figure 7.1 A Dislocatable Knuckle](image)

The dislocatable knuckles should pop out in the event of a severe crash, saving the mechanism from damage.

I designed the hand with a deliberate "weak link" which should give way before the structure is damaged. The shafts easily pop in and out of the joint when the power is off,
but are held tightly in place when the hand is actively tensioned. The system has not yet been tested in the rigorous environment of the WAM, so it remains to be seen if this strategy will work as designed.

7.3 Spring Centering

Chapter 4 showed that the finger actuation is indeterminate - that is, that there are more joints than controllable degrees of freedom and that the joint positions are related only by the single constraint provided by the cable position. The finger would be a more effective gripper if we could insure that it would stay straight when not actuated and curl in a predictable way when it was actuated. Fortunately, this is easy to achieve with a small linear recentering spring at each joint.

[Chammas] showed that the joint trajectory for a finger which gave the largest grasp preimage - an area where an object was certain to be grasped - was one in which the joints moved simultaneously with a velocity proportional to the total angular distance the joints would move on the way to the grasp configuration. In this hand, each finger joint has approximately the same range of motion, so we can produce this type of trajectory (a simultaneous curling of all the joints) by choosing springs with a stiffness proportional to the diameter of the pulley at each joint.

7.4 Comparison with Human Hands

The human hand has more than \( n+1 \) tendons but less than \( 2n \) for \( n \) independently controllable degrees of freedom in the hand and wrist. (Each of the four fingers has three degrees of freedom because the last two joints of each finger are coupled.) Each additional tendon above \( n+1 \) should give an additional degree of freedom which can be used to cocontract a joint and control its stiffness. \( 2n \) tendons should allow us to cocontract every joint independently, giving us control over motion and stiffness at every joint. Because we have less than \( 2n \) tendons, we can’t do this. The point of this discussion is primarily that
our brain naturally adapts to the intrinsic hardware coupling in our hands, allowing us to control individual joints by contracting combinations of muscles in the forearm without even realizing that we are using a coupled system!
8 Results and Future Work

8.1 The Benchtop Prototype

We have constructed prototypes of the motor package [Anderson] and the hand/wrist, and mounted them in a benchtop version to test the performance of the design. The prototype met most of our expectations for it and exceeded a few, such as speed and backdriveability, so the experience was a positive one. We also learned some lessons in the process, which will lead to some minor design changes in the final version.

We also constructed a control station, centered around an IBM PC with motor control cards from Motion Engineering. Analog motor torque commands are sent from the PC to Copley Controls model 303 PWM amplifiers, which control current to the motors. Each motor position is measured by an encoder, and the encoder signals are read directly by the motor control boards. The control station is configured to simultaneously control 12 motors, and each channel can be separately enabled or disabled. In addition, a portable global "kill" switch is provided which can be used to quickly shut off power to all the motors when necessary. The motor controllers can be run either in a simple "position" mode, in which positions are commanded and a dedicated servo loop is closed in hardware, or in "current" mode, in which currents are commanded open loop and the programmer must close the loop in software.

We have implemented an impedance controller running in current mode. We sense motor positions, differentiate them to get motor velocities, and infer motor torque from the
commanded current. With these tools, we are able to command stiffness, damping, and a
nominal trajectory at each joint. We have shown that we can successfully command a very
high stiffness in the wrist and a very low stiffness in the hand. In addition, we can add
arbitrary torques (open loop) to those generated by the impedance controller. This allows
us to grasp objects with precise force control.

Figure 8.1 The Benchtop Prototype Hand/Wrist
Figure 8.2 The Hand/Wrist

Figure 8.3 The Hand Conforms to a Variety of Shapes
8.2 Lessons Learned from the Prototype

8.2.1 Software and Hardware Safeguards

Minor disasters when testing new control code are an inevitable part of any robot design process. We found that, like human muscle, the motors in the hand/wrist are indeed strong enough to do damage to the mechanism they are intended to control. Fortunately, mechanical safeguards we had designed earlier and software safeguards we implemented later were effective in preventing this kind of self-damage.

8.2.2 Wrist Pulley Thrust Bushings

The thrust bushing and radial bearing arrangement I chose for the pulley stacks did prove to be an effective way to squeeze many pulleys into a tight space. In badly designed n+1 systems, the cables tend to slide on the pulleys instead of spinning the pulleys at light loads, revealing the friction in the system. In the hand wrist, the pulleys spin beautifully at all loads, indicating that the thrust bushings add a negligible amount of friction to the system.

8.2.3 Finger Self-Centering

The radial bushings I chose for the joints of the fingers were unfortunately not effective. At high tendon tensions, such as when applying a large torque with the wrist, the internal tension in the fingers is high (because the same cables actuate both wrist and fingers), and the friction in each finger joint rises slightly. The difference in friction is not large enough to significantly affect the backdriveability of the finger, but it is enough to prevent the light springs at each joint from giving the desired self-centering effect described in the last chapter. Because the joints of the fingers are not independently controllable, the finger can "stick" in a curled position until driven against the opposite joint stops. This will make tasks such as catching difficult and needs to be remedied in later versions.
8.2.4 Pulley Jumping

The tendons occasionally jump their pulleys and pull down into the gaps between pulleys. This happens rarely - most often when the power is turned on incautiously, but is troublesome when it does happen. In the current benchtop prototype, most of the mechanism is exposed and fixing the problem is simple. When the hand is mounted on the WAM, this type of failure will be very difficult to fix quickly and is therefore unacceptable. Pulley jumping never happens when the hand/wrist is kept pretensioned, because the cables are pulled tightly against the pulleys and cannot escape over the flanges. Some have therefore suggested preventing pulley jumping by leaving the power on continuously, or by using a backup "tensioning" battery. Higher pulley flanges would also help, but space is extremely limited. The most practical ways to attack the problem would involve either some kind of "keeper" which surrounds the pulley and keeps the cable confined to the space over the pulley, or a spring tensioner in the forearm.

8.3 Future Work

Designing a hand/wrist for the WAM is just one step in an ongoing process of building high performance hardware and exploring the limits of the tasks it makes possible. This work proceeds in stages, with a hardware development stage followed by a task exploration stage, which shows the limits of the hardware and brings up needs for more and better hardware, repeating the cycle. A three degree of freedom force controllable wrist will make possible a whole new range of tasks in which the WAM can explore and manipulate its environment.

8.3.1 Hardware Development

In the next few months, we will be completing the design of the roll axis mechanism and the surrounding forearm, exploring ways to solve the problems we found in the prototype, and mounting the completed package on the WAM.
Other hardware aspects which need to be explored are sensors for the hand and a sensor bus for the entire system. Because the joint positions of the fingers of the hand/wrist are indeterminate, it would be extremely valuable to be able to sense the joint angles. Other work in this lab has focused on techniques for identifying grasped objects from joint positions [Siegel], and this hand would be an excellent system on which to implement them.

A set of contact sensors on the surface of the hand would also provide valuable information to use in the generalized grasping strategies developed in our group by [Chammas] and Brock. These sensors need not be complex tactile sensors. An array of 10 to 20 one bit sensors provides rich information to guide a grasping strategy. A variety of film transducers are available which make good one bit contact sensors.

Finally, other members of our group are developing a sensor bus which will be used to access sensors along the entire length of the arm, wrist, and hand. This is vital because it is impractical to run two or more wires for each sensor along the length of the arm.

8.3.2 Software and Communications

The prototype hand/wrist is controlled at this time by a PC with Motion Engineering motor control cards, while the WAM is controlled with 68030 processors linked on the Condor system developed at MIT. Ideally an integrated computer system should be built to accommodate the whole package - arm, hand and wrist. The current users, however, are unwilling to accept the down time this would require, so a temporary solution will be patched together.

This will involve a communication link between the WAM processors and the PC, probably through the serial port. The PC will continue to run the servo control code for the hand and wrist, but will receive a trajectory through the communication link at several
hundred Hertz. The programmer will then be able to control a full seven degree of freedom
trajectory at one location, ‘leaving the servo details of the hand and wrist’ to the smaller
system.

Once this is completed, new control software will have to be written to take
advantage of the new position and orientation capabilities of the arm, and then the fun work
of designing whole arm and hand tasks, grasping techniques, and catching strategies can
begin.

8.3.3 And on into the Future ...

The research done on new cable fibers by [Anderson] may lead to new possibilities
for a stronger and faster WAM, which is better able to carry the weight of a hand and wrist.
We already have experiments planned to test spectra and vectran fibers in place of the steel
cables in the arm.

Ultimately, we envision the future of the WAM project as a two armed system,
capable of passing small objects from one hand to the other, grasping basketball sized
objects between two hands, and holding and carrying large objects with both arms.
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