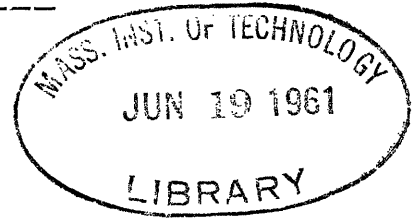


AN ELECTRONIC DATA PROCESSING DEVICE

FOR D.C. PROBES IN PLASMAS



by

Royce W. Fletcher

SUBMITTED IN PARTIAL FULFILLMENT OF THE
REQUIREMENTS FOR THE DEGREE OF
BACHELOR OF SCIENCE

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

June, 1961

Signature of Author
Department of Electrical Engineering

Certified by:
Thesis Supervisor

Accepted by:
Chairman Departmental Senior Thesis Committee

ABSTRACT

An electronic data processing device is discussed which rapidly determines the form of the function describing the distribution of electron random velocities in a plasma discharge.

The theory of probes is developed, in so far as it pertains to the operation of the device, and the results of such operation are discussed.

The important features of the operation of the data processing device are:

- 1) The form of the distribution is available without recourse to graphical differentiation.
- 2) Measurements are made over a relatively short period of time, thereby reducing the likelihood of an error's being introduced by slow changes, with time, of the probe characteristics or the discharge characteristics.
- 3) If the form of the distribution corresponds to a known distribution, e. g. Maxwellian, Druyvesteyn, etc., its important parameters may be found immediately from the oscilloscope display of the output of the device.

Quantitative estimates of the form of the distribution and deviations from expected distributions may be determined visually without further data reduction.

ACKNOWLEDGMENT

The author would like to thank Mr. R. S. Cooper, the thesis supervisor, for his help in evaluating the theory presented in this paper and for his assistance in making experimental measurements and in interpreting the results of the measurements.

Thanks are also due to my wife, Sue, for her efforts in typing the final manuscript and for her constant moral support.

TABLE OF CONTENTS

<u>SECTION</u>	<u>CONTENTS</u>	<u>PAGE</u>
1.00	INTRODUCTION	2
2.00	THE PROBE	4
2.10	The Operation of the Probe	4
2.20	Generalized Probe Theory	8
3.00	A MEANS FOR OBTAINING THE SECOND DERIVATIVE OF THE PROBE CURRENT WITH RESPECT TO THE VOLTAGE, V.	9
4.00	THE DATA PROCESSING DEVICE	12
4.10	The Linear Sweep Voltage Generator	13
4.20	The D. C. Biasing Source	14
4.30	The Sinusoidal Source	14
4.40	The Preamplifier	15
4.50	The Band-Pass Filter Network	16
4.60	The Display	18
5.00	RESULTS	18
6.00	CONCLUSIONS	20
7.00	FIGURES	22 - 33
7.10	Figures - 1 through 12	22 - 33
7.20	Table of Component Values	34
	BIBLIOGRAPHY	37
	APPENDIX	38

1.00 INTRODUCTION

A plasma is an ionized gas in which the principal negative charge carriers are electrons and the principal positive charge carriers are positive ions. In addition, the positive and negative carriers exist in approximately equal numbers.

A plasma may exist in the positive column of a gas discharge if the gas is relatively free from impurities, and if certain other rather general requirements are met. For a better description of these conditions, the reader is referred to L. B. Loeb, Basic Processes of Gaseous Electronics.¹

Throughout this paper, it will be assumed that plasma conditions do exist.

With gas pressures in the discharge tube in the millimeters of mercury range, the particles are accelerated along the positive column with frequent collisions producing a great deal of ionization and an overall randomness of motion. It has been suggested² that the random motions of the particles obey Maxwell-Boltzmann statistics so that the velocities of the particles may be described by a Maxwellian distribution function. The device discussed in this paper allows the determination of the deviation from the Maxwellian

¹ Superscripts refer to references listed in the Bibliography.

distribution and the determination of the form of the distribution function for the electrons.

The Maxwellian distribution of electron random speeds implies an equilibrium condition such as might exist in a completely stable discharge. In a stable discharge, the electric field is almost constant within the positive column; but oscillations within the plasma, striations in the field within the positive column, and similar non-equilibrium conditions might be expected to cause a deviation from the Maxwellian distribution of velocities. The device discussed in this paper provides a means for rapidly determining the form of the distribution function and, therefore, the deviation from a Maxwellian situation.

Information as to the distribution of electron random velocities is obtained through the use of small auxiliary electrodes which are inserted into the discharge. The theory of operation of the probe technique was first developed by I. Langmuir and H. M. Mott-Smith², but they assumed that the distribution function of the electrons was Maxwellian. Section 2.00 of this paper describes the operation of the probe in the same terms used by Langmuir and Mott-Smith, but develops the theory on the more general basis proposed by M. J. Druyvesteyn³.

The remainder of the paper discusses the implementation of a means for reducing the data which is obtained from the Druyvesteyn result, trying, wherever possible, to indicate conditions which may cause inaccuracies in the experimental observations.

2.00 THE PROBE

2.10 The Operation of the Probe

Figure 1 shows the physical arrangement of the probe in the discharge tube and defines the voltages to be used hereafter. Voltages V_k , V_b , and V_p are measured with respect to the anode and are the voltage of the cathode, the plasma space potential at the probe, and the voltage of the probe, respectively. In addition, another voltage will be defined as the probe voltage minus the plasma potential. This voltage will be designated by V . In all that follows, it will be assumed that the potential difference between the space at the probe and the anode is a constant. In the case of a time varying non-uniformity in the electric field in the positive column of the discharge, this may not be a valid assumption, in which case, the difference in potential as a function of time must be taken into account. Finally, the probe current, i_p is defined so that an increase in the electron current flowing to the probe from the plasma produces an increase in the probe current.

In order that the measurements made with the probe be accurate, it is desirable that the probe and its current disturb the conditions within the plasma as little as possible. This places certain restrictions on the probe and on the discharge conditions. First, conditions must be favorable to the formation

of a positive ion sheath on the probe. This sheath will be discussed in more detail later. Secondly, the discharge tube must be operated at very low pressures so that the mean free paths of the particles are large compared to the dimensions of the sheath, but small compared with the dimensions of the probe, or at high pressures where there is an abundance of particles. In the case of low pressures it may be assumed that the particles which contribute to the probe current come from a large distance from the probe and, therefore, do not disturb the local distribution of particles. In the case of high pressures, the situation of concern in this investigation, the assumption is that the probe current, while taking large numbers of particles from the plasma, does not appreciably disturb the distribution because of the larger number of particles which are available.

Having established the quantities to be used in the discussion of the probe theory and the conditions under which they apply, it seems in order to outline, qualitatively, the operation of the probe in terms of what to expect for the form of the probe i_p vs. V characteristic.

Figure 2 shows a typical probe characteristic with the voltage axis divided into three regions, A, B, and C. Region A, with the probe strongly negative with respect to the plasma

space potential, is a region in which all the positive ions which reach the vicinity of the probe contribute to the probe current, i_p . In this region, the positive ion current is space charge limited and a "sheath" of positive ions forms around the probe. ² This is a very useful phenomena in that it effectively isolates the field due to the probe voltage from the plasma field outside the sheath. A potential barrier is formed which keeps even the higher energy electrons from reaching the probe and from contributing to the probe current. The probe current is very slightly negative, due to the ion current, and is approximately a constant until the point Q is reached.

As the voltage, V , is increased, the higher energy electrons begin to penetrate the sheath and reach the probe. These electrons contribute to the probe current and the probe current begins to increase. At point Q, the ion current and the electron current are equal and no net probe current flows. The value of V_p at this point has been termed the wall potential since it is presumably the potential at which an insulating body would sit when inserted into the plasma. The wall potential is negative with respect to the plasma potential because the mobilities of the electrons are greater than those of the ions and this causes the electrons to diffuse toward the walls more rapidly than do the ions. The result is that, when

equilibrium is established, the wall may appear several volts negative with respect to the body of the plasma.

A further increase in the voltage, V , allows a larger and larger number of electrons to penetrate the sheath and contribute to the probe current, until point P is reached. At P, all the electrons which reach the sheath are able to contribute to the current. At this point, the probe voltage, V_p , is at the plasma potential and the probe should have no effect on the plasma.

As the probe characteristic enters region C, all the positive ions are repelled from the probe and the formation of an electron sheath occurs. For V greater than zero, the probe characteristic yields no useful information in terms of the investigation described in this paper.

2.20 Generalized Probe Theory

The Appendix includes a complete derivation of the results of the Druyvesteyn³ probe theory. Figure 10 shows the geometry which leads to the Druyvesteyn result. A small element of area at the origin represents an element of the surface of the sheath. The probe current due to electrons in all space is calculated. The electrons are assumed to have a distribution of speeds, $f(c)$, such that the probability of finding an electron with a speed between c and $c + dc$ in an element of volume, $d\mathcal{E}$, is $f(c) dc$. The probe surface is taken as parallel to the sheath surface and the current as a function of V is found. Upon taking the second derivative of the probe electron current with respect to the voltage, V , one finds that

$$i_{-}^{(2)}(V) = \frac{R}{V} f\left(\sqrt{\frac{2eV}{m}}\right). \quad (1)$$

The ion current is almost a constant so that the electron current will be assumed to be the total probe current when derivatives with respect to the voltage are being considered.

3.00 A MEANS FOR OBTAINING THE SECOND DERIVATIVE OF THE PROBE CURRENT WITH RESPECT TO THE VOLTAGE, V.

In order to use the result of the Druyvesteyn probe theory it is necessary to differentiate the probe current twice with respect to the voltage, V. A method of doing this was suggested in a private communication by Mr. R. S. Cooper at the Massachusetts Institute of Technology. It involves a result given in Reference 1 and outlined below.

If a small sinusoidal signal is applied to the probe, superimposed on the d. c. probe voltage from the biasing supply, and the resulting probe current is expanded in a Taylor Series, one finds that

$$\begin{aligned}
 i(V_p) = & \left\{ i(V_{p0}) + \frac{E^2}{4} i^{(2)}(V_{p0}) + \frac{E^4}{64} i^{(4)}(V_{p0}) + \dots \right\} \\
 & + \left\{ E i^{(1)}(V_{p0}) + \frac{E^3}{8} i^{(3)}(V_{p0}) + \dots \right\} \sin pt \quad (2) \\
 & - \left\{ \frac{E^2}{4} i^{(2)}(V_{p0}) + \frac{E^4}{48} i^{(4)}(V_{p0}) + \dots \right\} \cos 2pt \\
 & - \left\{ \frac{E^3}{24} i^{(3)}(V_{p0}) + \dots \right\} \sin 3pt + \dots,
 \end{aligned}$$

where $E \sin pt$ is the sinusoidal voltage, $i(V_p)$ is the probe current as a function of the probe voltage, and $i^{(k)}(V_{p0})$ is the k^{th} derivative of the probe current with respect to the probe voltage, evaluated at the average value of the probe voltage, V_{p0} .

One will note that in equation (2) the coefficient of the second harmonic of the sinusoidal function is composed of constants proportional to the second derivative and higher even derivatives of the probe current taken with respect to the probe voltage. Since the average probe voltage differs only by the addition of a constant from the average voltage V , one may, by the chain rule for differentiation, substitute V for V_{po} . Furthermore, if the value of E is kept small, it is possible to write the coefficient of the second harmonic term as $-\frac{E^2}{4} i^{(2)}(V)$.

If one applies the voltages as indicated in the above discussion, and filters the probe current to obtain only the signal which is proportional to the second harmonic of the sinusoidal voltage, then a function of the voltage, $h(V)$, will be obtained, where

$$h(V) = -G \frac{E^2}{4} i^{(2)}(V), \quad (3)$$

with G equal to the gain of the filter.

Substituting equation (1) into equation (3), one finds that

$$h(V) = -G \frac{E^2}{4} \frac{R}{V} f\left(\sqrt{\frac{2eV}{m}}\right) = \frac{B}{V} f\left(\sqrt{\frac{2eV}{m}}\right). \quad (4)$$

Equation (4) provides a means of evaluating the functional form of $f(c)$. The procedure for doing this is the following: -

- a) Make the substitution $\eta = \sqrt{\frac{2eV}{m}}$, and
 b) Plot $f(\eta) = \frac{m}{B} \frac{\eta^2}{2e} h\left(\frac{m\eta^2}{2e}\right)$ vs. η .

Of particular interest is the form of $h(V)$ for a Maxwellian distribution of velocities. In that case,

$$f(\eta) = S \eta^2 \exp\left(-\frac{m\eta^2}{2kT}\right), \quad (5)$$

and a proper substitution of variables will show that $h(V)$ is within a multiplicative constant of being exponential with a decay constant equal to the average electron energy, kT .

The data processing device to be discussed yields $h(V)$ with the additional feature of providing a slowly varying voltage in place of the d. c. voltage so that the probe voltages may be swept slowly from a very negative value of V_p to $V_p = V_b$. Sweeping the probe characteristic in such a manner will introduce a modulation of the sinusoidal signal so that the sweeping period should be kept large compared to the period of the sinusoidal source. An exact analysis of the error introduced by modulation of the sinusoidal signal is difficult, but it has been the observation of the author that, for a ratio of sweep period to sinusoid period of about 20, the

error is negligible.

4.00 THE DATA PROCESSING DEVICE

Figure 3 shows a block diagram of the data processing system. It consists of: -

- a) A. D. C. Biasing Source which may be adjusted to give a suitable operating point for the excitation system;
- b) A source of the sinusoidal voltage mentioned in Section 3.00;
- c) A linearly time varying sweep voltage generator to facilitate sweeping the probe characteristic over a suitable range;
- d) A resistor across which is measured a voltage proportional to the probe current;
- e) An amplifier which amplifies the voltage of d),
- f) A Band-Pass filter which passes only the component of the signal from the amplifier which is at twice the sinusoidal source frequency; and
- g) An oscilloscope which provides additional voltage gain and a display of the output of the filter (proportional to $h(V)$) vs. the linear sweep voltage received from the linear sweep voltage generator.

4.10 The Linear Sweep Voltage Generator

It was pointed out earlier that the period of the sweep generator should be made large compared to the period of the sinusoidal voltage source in order that excessive modulation of the sinusoidal signal might be avoided. On the other hand, the frequency of the sinusoidal source must be kept relatively low so as to prevent the excitation of oscillations in the plasma and to assure the response of the sheath and the plasma to the sinusoidally varying voltage. A sweep period of the order of a few seconds and a sinusoidal source with a frequency of about 300 cycles per second will satisfy the above conditions.

Figures 4 and 5 show the schematic diagram of the Sweep Generator. An external square wave generator provides the input to 6 frequency dividing flip-flops which, in turn, give an output square wave with a period which may be made as long as 15 seconds. The flip-flops operate into a transistorized relay driver which actuates a sensitive relay. This sensitive relay drives a heavier relay which alternately switches plus and minus 400 volts across an R-C integrating circuit.

The integrating circuit utilizes an R-C network with diode clipping to provide an output voltage which is a

linear function of time, to within 1%, and which sweeps the voltage from 0 to 45 volts. The d.c. output impedance of the sweep generator is about 5000 ohms, and its impedance at the sinusoidal frequency is negligible.

The zero reference of the sweep voltage is at the system ground, assuring a stable reference voltage. The sweep limit, at 45 volts, is determined by the 45 volt battery, providing a stable upper level.

4.20 The D. C. Biasing Source

The d.c. biasing source consists simply of a bank of batteries with part of the bank made up of a voltage dividing network to allow vernier manual adjustment of the total voltage over about a 90 volt range. The potentiometer used as a voltage divider is a helipot, assuring a stable output. The entire d.c. biasing source is bridged by an 8 mfd. capacitor to decrease noise pickup.

4.30 The Sinusoidal Source

The sinusoidal source is a Hewlett-Packard 200 D low frequency generator shunted by a 500 ohm resistor to

provide impedance matching and to assure a low output impedance.

The requirements of the sinusoidal generator are that it have a continuously variable frequency and that it have low harmonic content.

4.40 The Preamplifier

Figure 6a shows the schematic diagram of the preamplifier. Its power supply is in conjunction with that of the band-pass filter and is shown schematically in figure 7.

The input to the preamplifier contains two filters. Capacitor C_{25} acts as a shunt to ground for high frequency noise which may appear at the input due to oscillations within the discharge. L_4 and C_{24} form a shunt-resonant filter, resonant at 300 cps, to increase the ratio of the 600 cps. (second harmonic) signal to the 300 cps. (fundamental) signal.

The preamplifier, itself, is a high gain pentode amplifier having a gain of about 1000. All supply voltages are well regulated and filtered to insure stability and low noise pick-up.

4.50 The Band-Pass Filter Network

Figure 6b shows the schematic diagram of the narrow pass-band filter resonant at the second harmonic frequency of the sinusoidal source.

In order to achieve the narrow band width required to eliminate the fundamental frequency of the sinusoidal generator, and to achieve this narrow band width at frequencies of the order of a few hundred cycles per second, a Twin - T R-C filter ⁴ with a negative feedback arrangement was employed.

Figure 11 shows an s-plane plot of the poles and zeroes of the voltage transfer function of the twin - T filter. It will be noted that a zero appears at the resonant frequency. When this filter is inserted into a high gain, negative feedback loop, as in Figure 6, the zero at the resonant frequency of the twin - T filter becomes a pole at the same frequency in the overall filter system voltage transfer function. R_{20} adjusts the open loop gain, providing an adjustment of the magnitude of the Q achieved. The pole can be effectively moved at right angles to the imaginary axis of the s-plane plot by changing R_{20} and, with the gain above a certain value, oscillation can be made to occur. A maximum value of the Q is obtained just short of the point at which oscillation begins to occur. With the Q

at something less than its maximum value, R_{24} should be adjusted so that a relative maximum of the Q is obtained. With the adjustments made as indicated, it is possible to achieve Q 's of over 100. Unfortunately, at a very high Q , the response of the filtering system is slow, so that an experimental optimum setting must be found.

Cathode follower V_4 provides a low impedance signal source to match the twin - T rejection filter. Cathode follower V_5 acts as a high impedance load for the twin - T filter and effectively isolates the high gain amplifier from the rejection network. The high gain feedback amplifier is a pentode operating in its linear region with a gain capability of about 4000. Bias for the first cathode follower is obtained from the 150 volt point, Y , of the power supply. Bias for both pentode amplifiers is due to the drop across the cathode resistors. Bias for the second cathode follower is obtained by d. c. coupling through the twin-T filter to the first cathode follower.

As with the pre-amplifier, the power supply voltages are well regulated and filtered to insure maximum stability and minimum noise pick-up. The narrow band width of the filtering network does much to eliminate unwanted noise but precautions must be taken to insure that the 600 cycle

component of any noise source is negligible compared with the signals to be measured.

4.60 The Display

A high persistence oscilloscope capable of providing a displacement of several centimeters at 0.1 volt per centimeter vertically and 2 volts per centimeter horizontally has been used to display the output of the filtering network. The horizontal terminals are connected to the system ground and to the sweep generator output. The vertical terminals are connected to the system ground and the output of the filtering system. The oscilloscope must have good d. c. response in its horizontal amplifier circuit.

5.00 RESULTS

The envelope of the signal derived from the output of the filtering network is the function which was previously called $h(V)$. One such envelope is pictured in Figure 8. The un-normalized velocity distribution function which is obtained from this curve is shown in Figure 12. It will be noted from Figure 12, that the distribution of velocities covers a much narrower range than it would if the distribution were Maxwellian.

Figure 9 shows a function $g(V)$ which would be the expected envelope in a Maxwellian situation, with an average electron energy of 5 eV. The observed result, $h(V)$, is placed arbitrarily on the same voltage axis. As can be seen, the two differ quite markedly.

One can estimate, by such a comparison, that there will be fewer electrons with very high velocities, and fewer electrons with very low velocities than there would be if the distribution were Maxwellian. The reduced data of Figure 12 tends to bear this out.

It should be noted here that the Taylor Series expansion of equation (2) is not valid at a discontinuity such as there is at $V=0$ for a Maxwellian distribution. For values of V less than the zero to peak amplitude of the applied sinusoidal voltage, the results of the experiment become ambiguous. It is the conclusion of the author that, in the range of voltages $V < \frac{E}{2}$, an averaging may occur, tending to produce a change in the function $g(V)$, as indicated by the dashed line of Figure 9. This effect may be reduced by decreasing the sinusoidal source voltage, and the effect will be less noticeable if the average electron velocities are high.

When the discharge tube was operated in a region

conductive to a Maxwellian distribution of electron random velocities the function $h(V)$ approached the function $g(V)$. It was also the case, however, that under such conditions of operation, the average electron energy was quite large, tending to make the time constant of the exponential function $g(V)$ quite large, and in such a case, the averaging process described would have less effect.

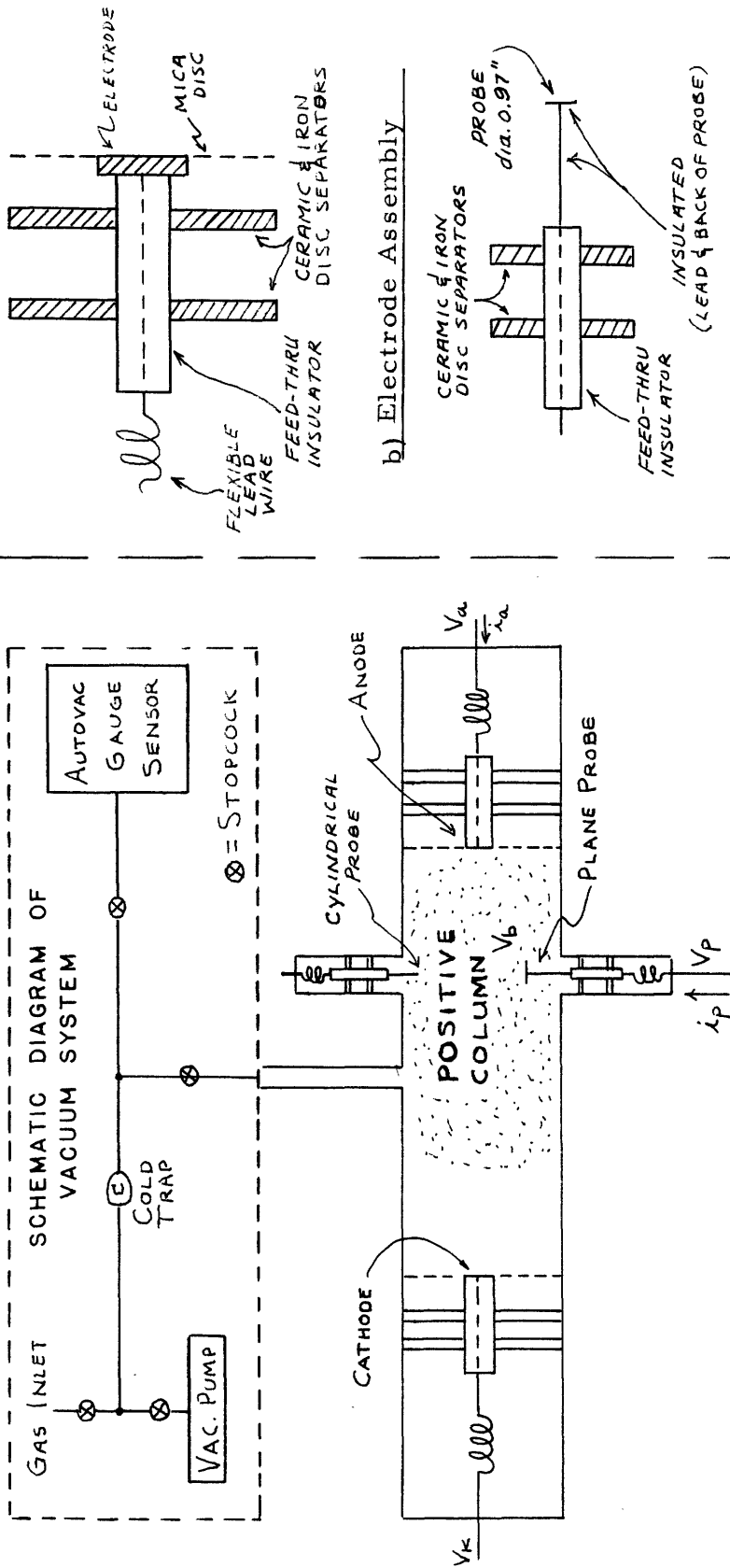
It is of interest that more information is available from the data processing system than has been described. Equation (2) shows that the coefficient of the signal at the frequency of the sinusoidal source is proportional to the first derivative of the probe characteristic with respect to the voltage, V . From this, one can see that tuning the sinusoidal source to the resonant frequency of the band-pass filter will cause the first derivative of the probe characteristic to be displayed in place of the second derivative.

It is also possible to display the probe characteristic by connecting the vertical terminals of the oscilloscope directly across the resistor, R_6 , as shown in Figure 3.

6.00 CONCLUSIONS

It is the conclusion of the author that a system such as the one described, and incorporating the suggested improvements,

will give results which lend themselves easily to interpretation and quantitative analysis. Although the results of the application of the electronic data processing device need a good deal more experimental verification, it is felt that the major effects which influence the operation of the device have been described in sufficient detail as to allow the device to be used to obtain the indicated quantitative information concerning the function describing the distribution of electron random velocities in a plasma.



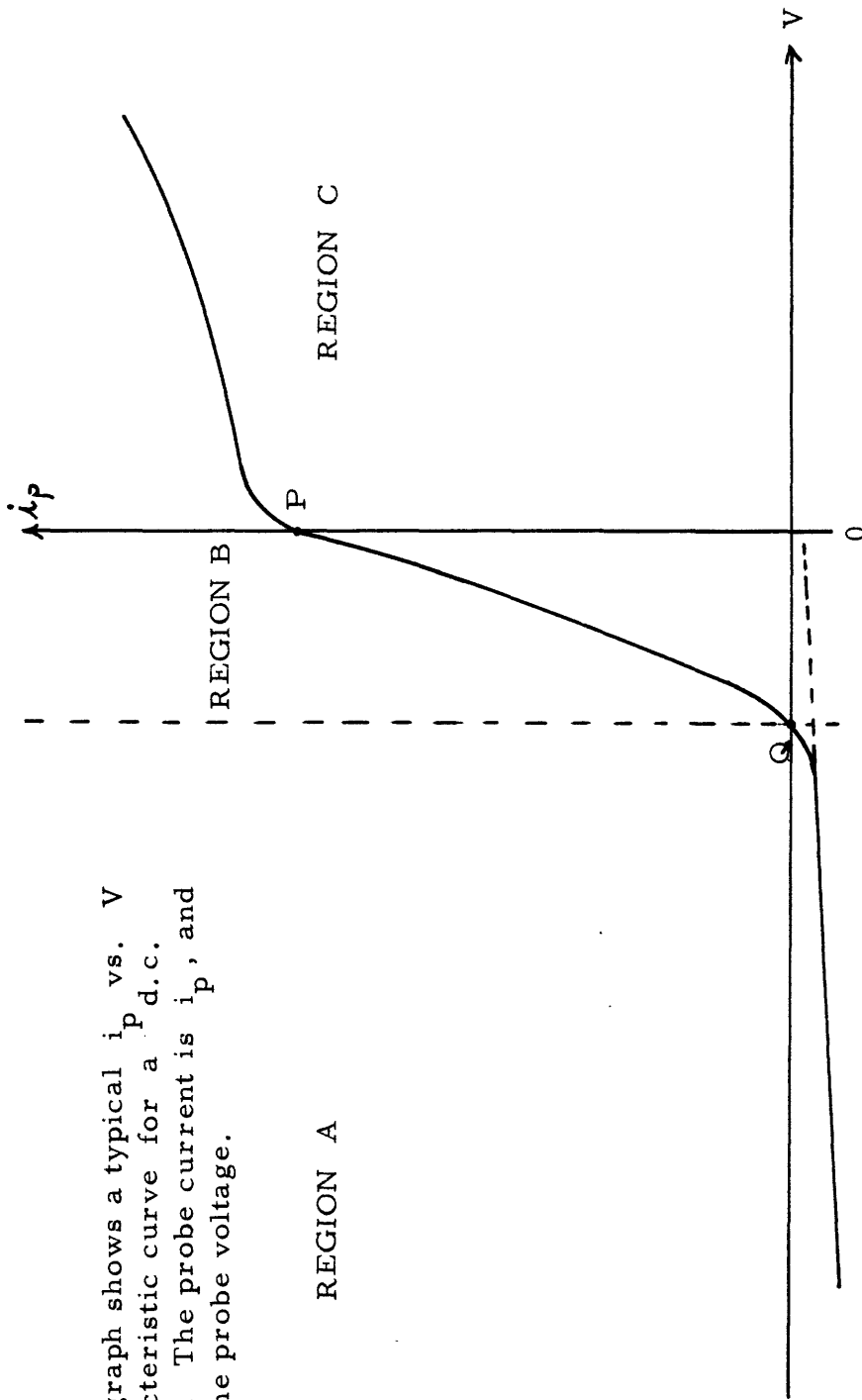
a) The Discharge Tube

b) Electrode Assembly

c) Probe Assembly

Fig. 1 A Schematic Diagram of the vacuum system and drawings of a) the discharge tube, b) the electrode assembly, & c) the probe assembly.

This graph shows a typical i_p vs. V characteristic curve for a d.c. probe. The probe current is i_p , and V is the probe voltage.



REGION A

REGION B

REGION C

Fig. 2 A typical probe characteristic

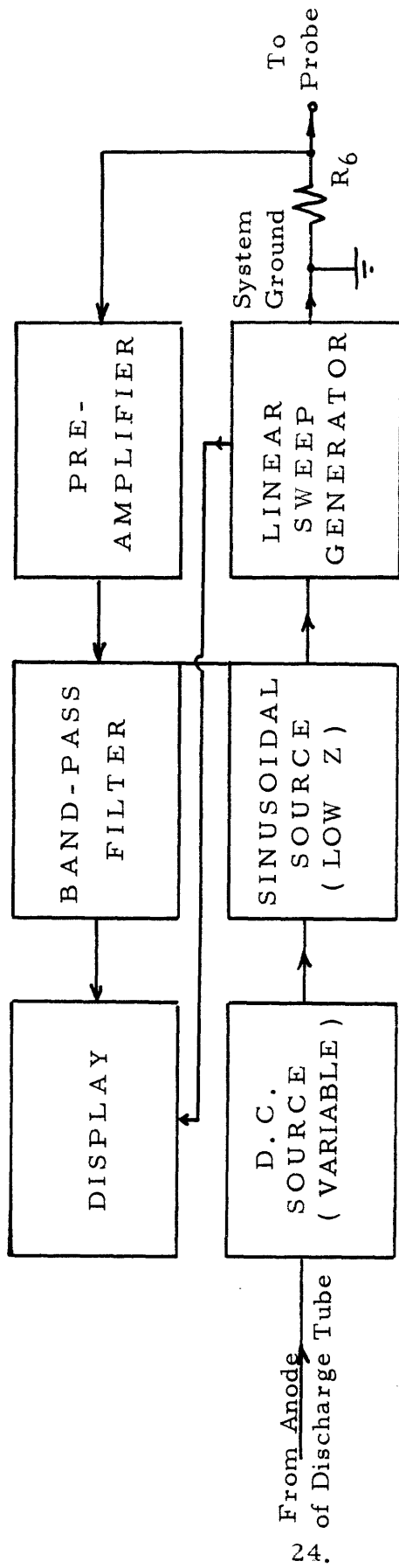


Fig. 3 A block diagram of the excitation and measuring system

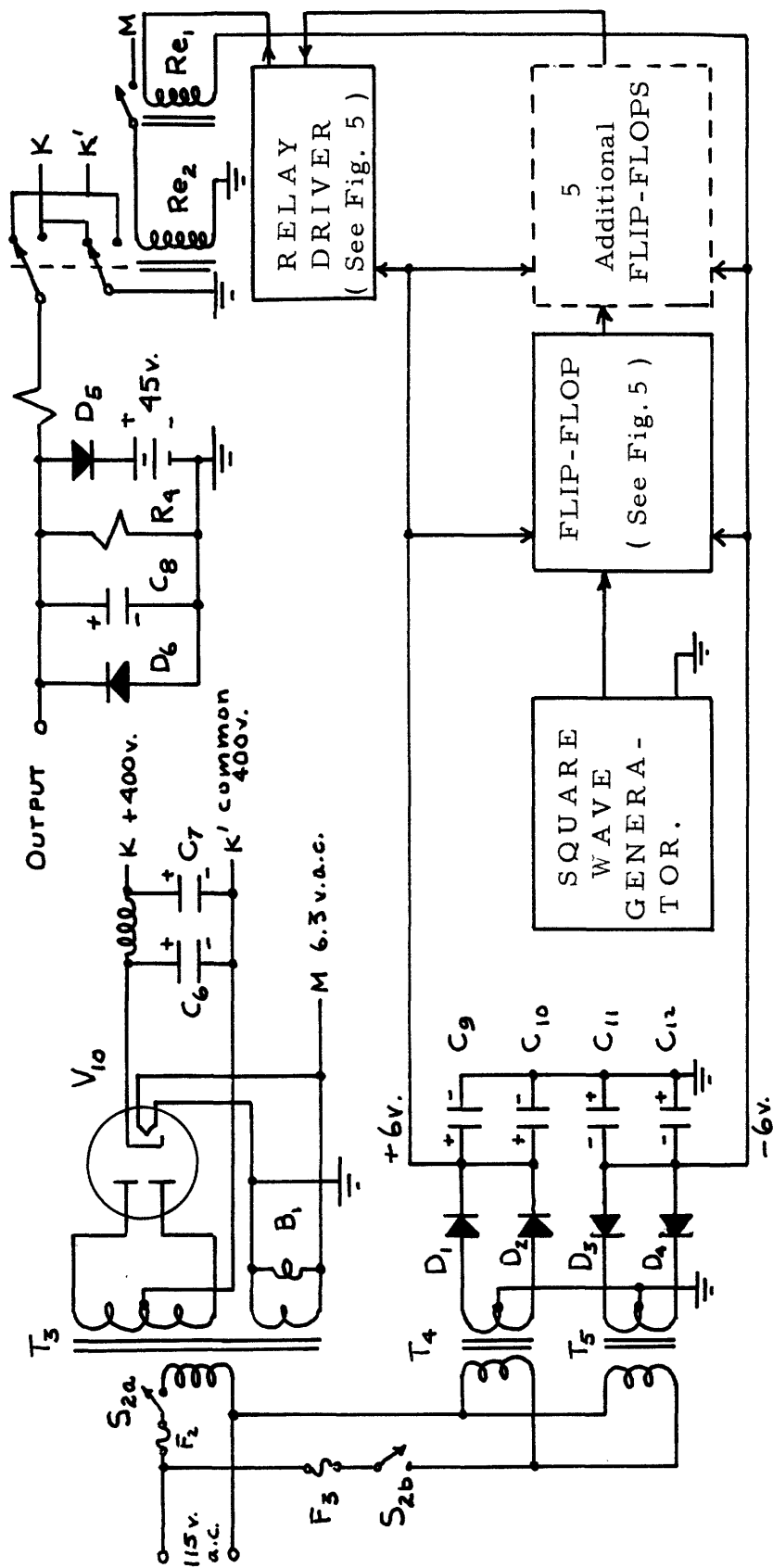


Fig. 4 The linear sweep voltage generator. The flip-flop and sensitive relay driver schematics are shown in figure 5 on the next page.

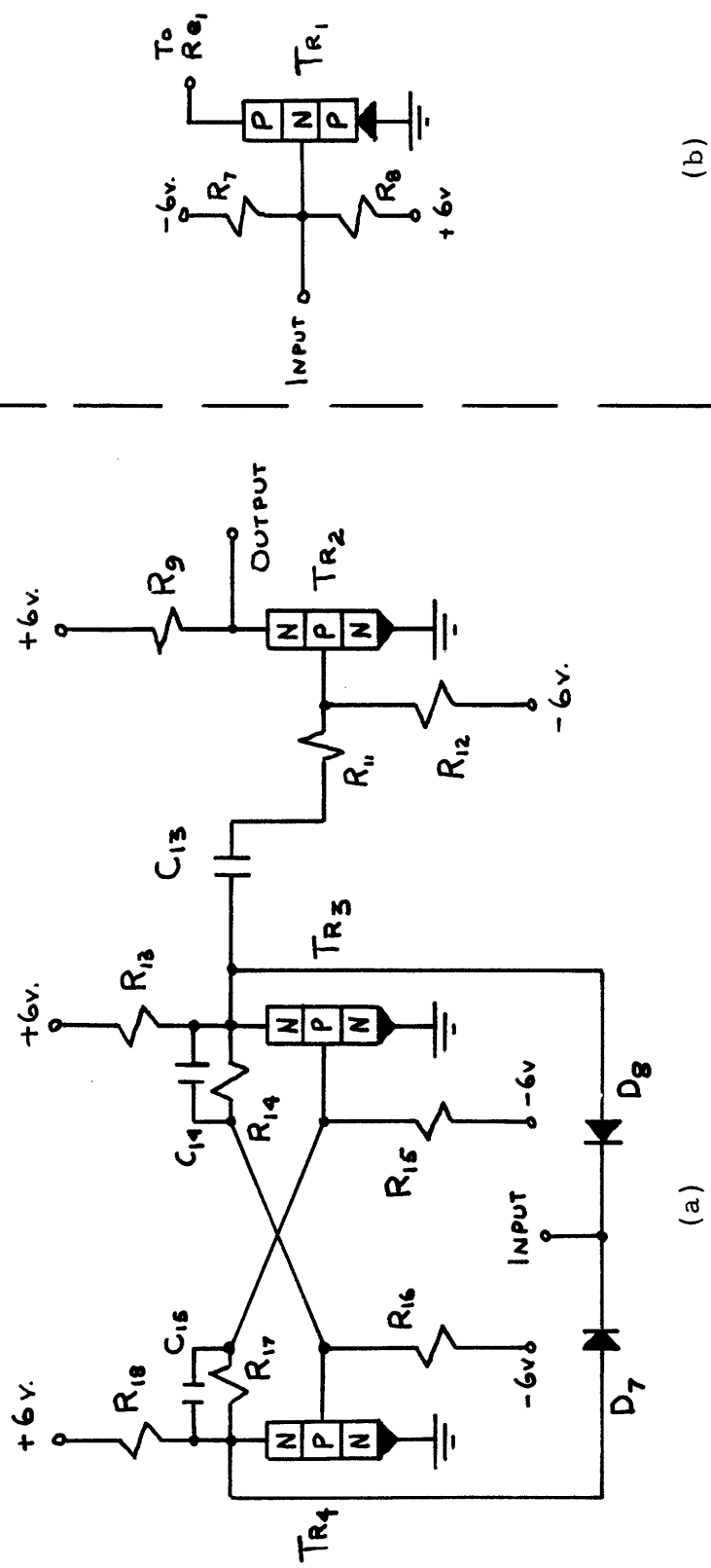


Fig. 5 Schematic diagrams of (a) the flip-flop, and (b) the sensitive relay-driver.

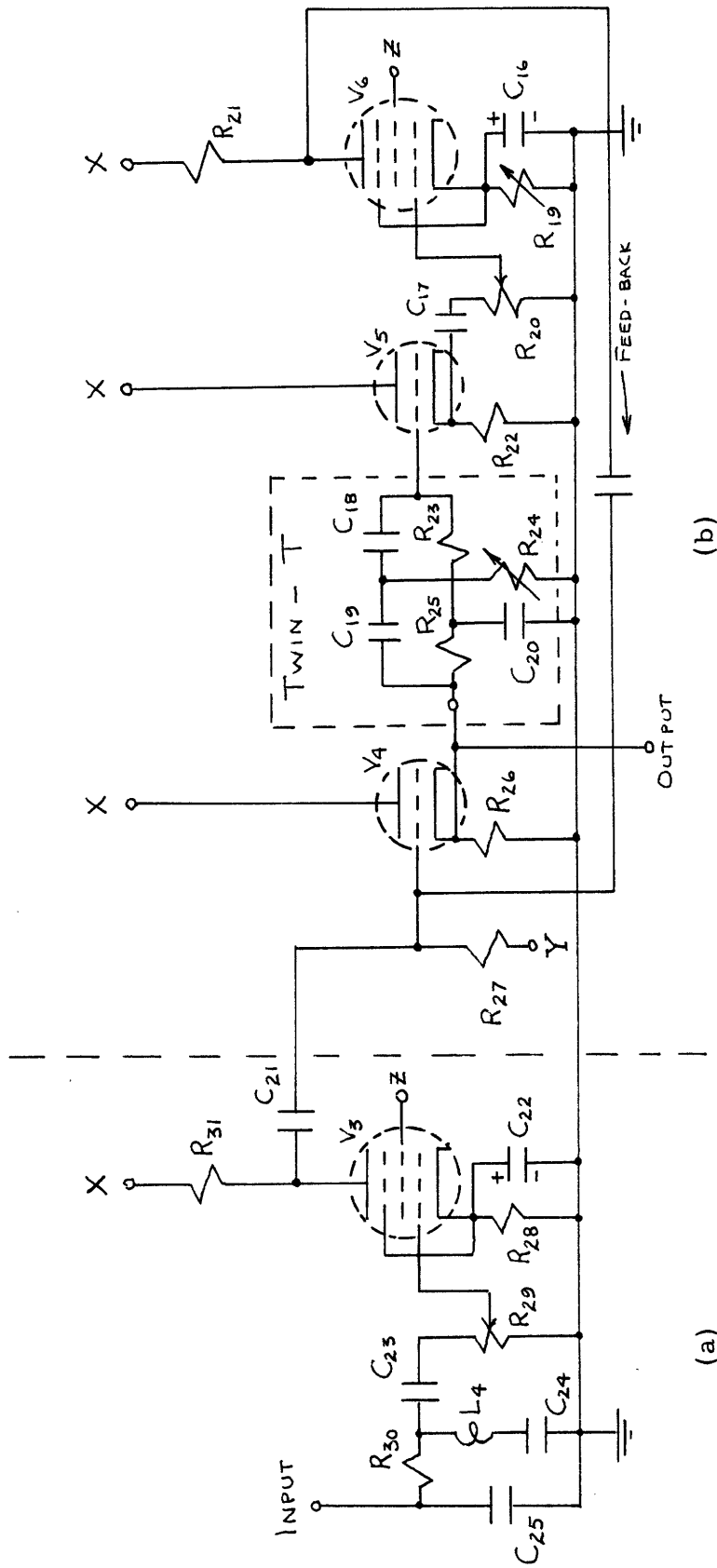


Fig. 6 Schematic diagrams of (a) the preamplifier, and (b) the band-pass filter network. The twin-T filter is shown in (b). The power supply for both (a) and (b) is shown in figure 7 on the next page.

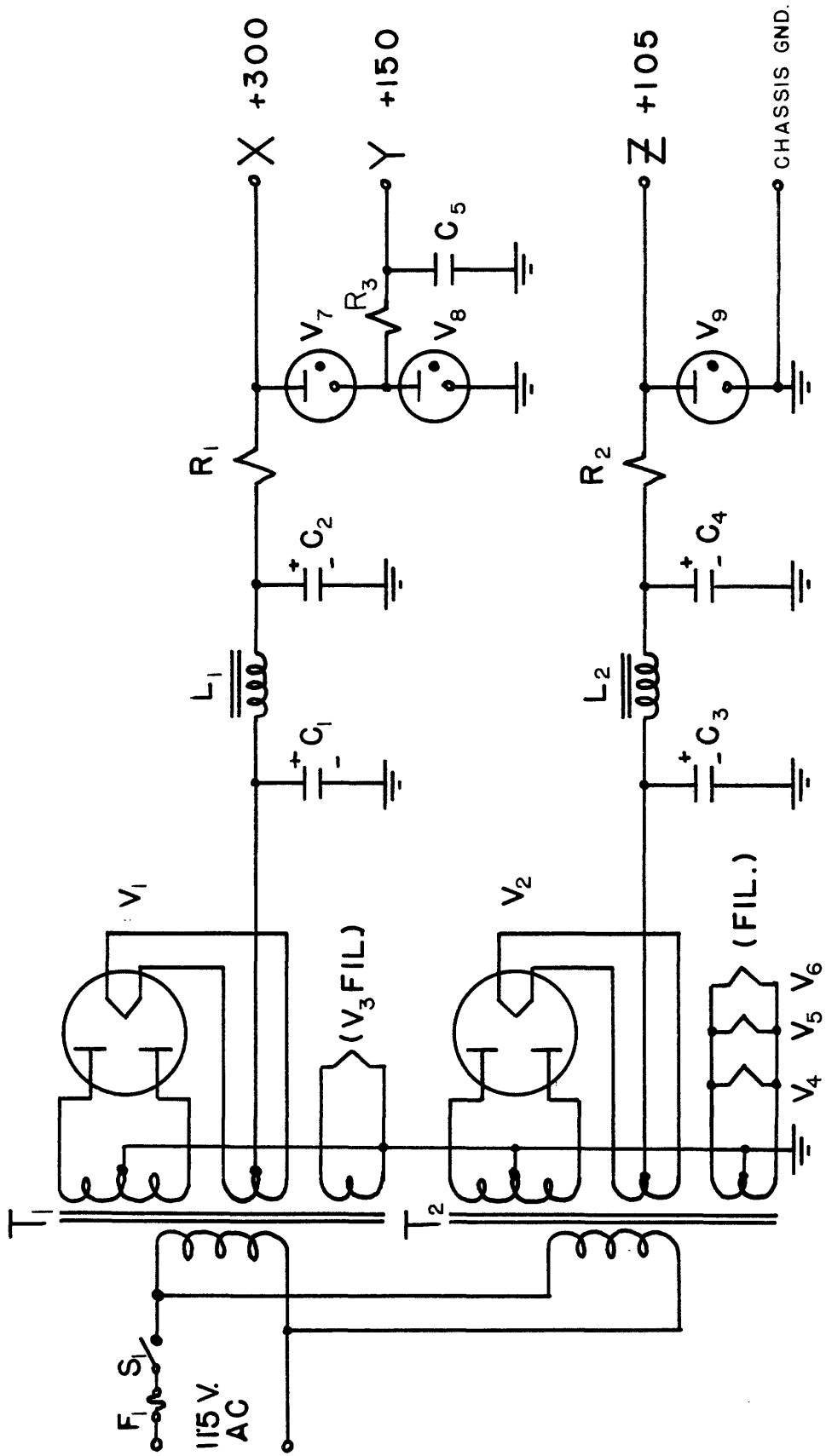
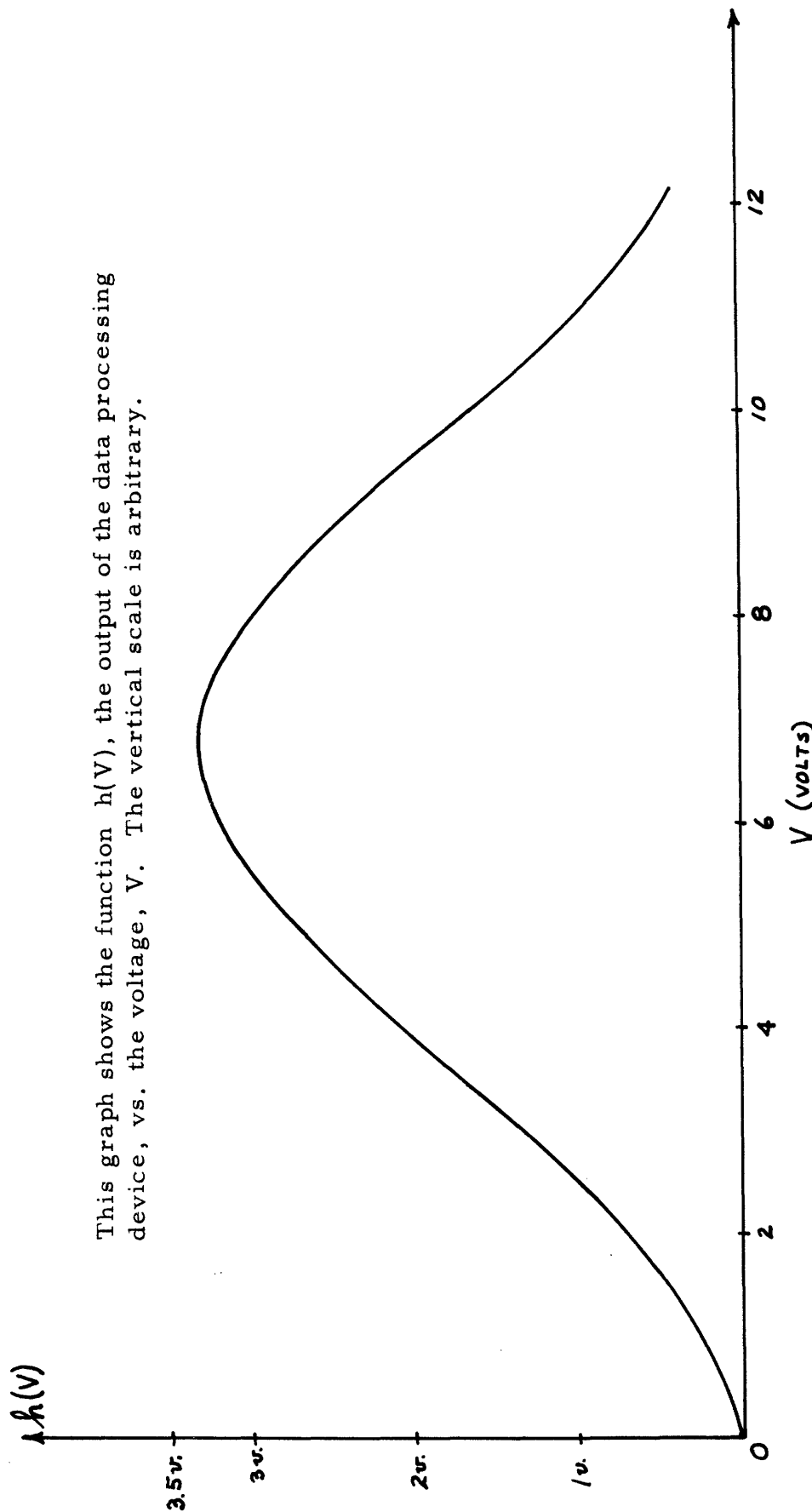


FIGURE 7 POWER SUPPLY (FOR AMPLIFIER & FILTER)



This graph shows the function $h(V)$, the output of the data processing device, vs. the voltage, V . The vertical scale is arbitrary.

Fig. 8 A typical output of the data processing device. The function, $h(V)$, represents the envelop of the actual output function.

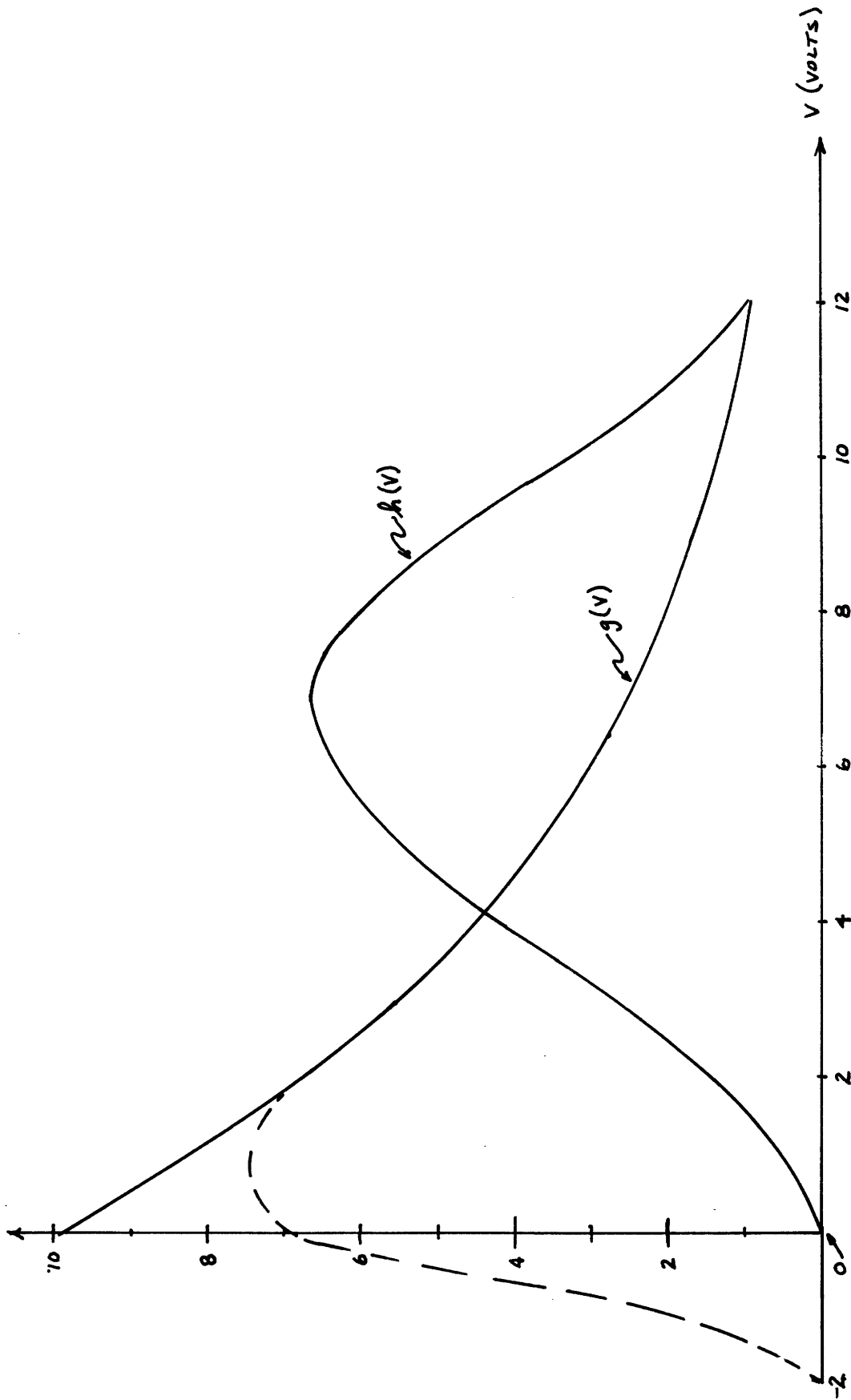


Fig. 9 A plot of the functions $h(V)$ and $g(V)$ as described on p. 19 of the text.

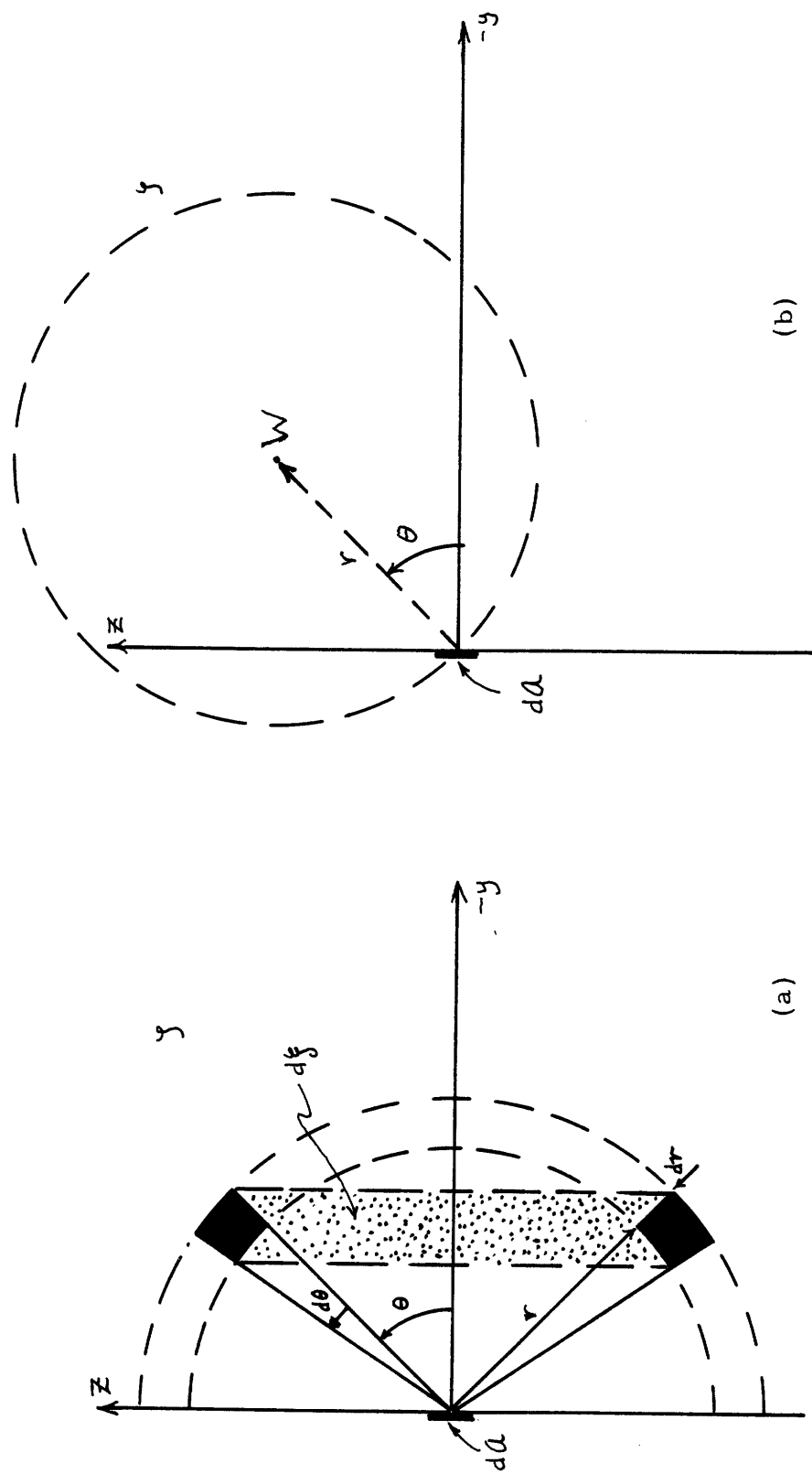


Fig. 10 A drawing of the geometries involved in the Druyvesteyn probe theory.

31.

This graph is an s-plane plot of the transfer function of the twin-T filter discussed on p. 16 of the text. The resonant frequency is approximately 600 cps.

o - zero
x - pole

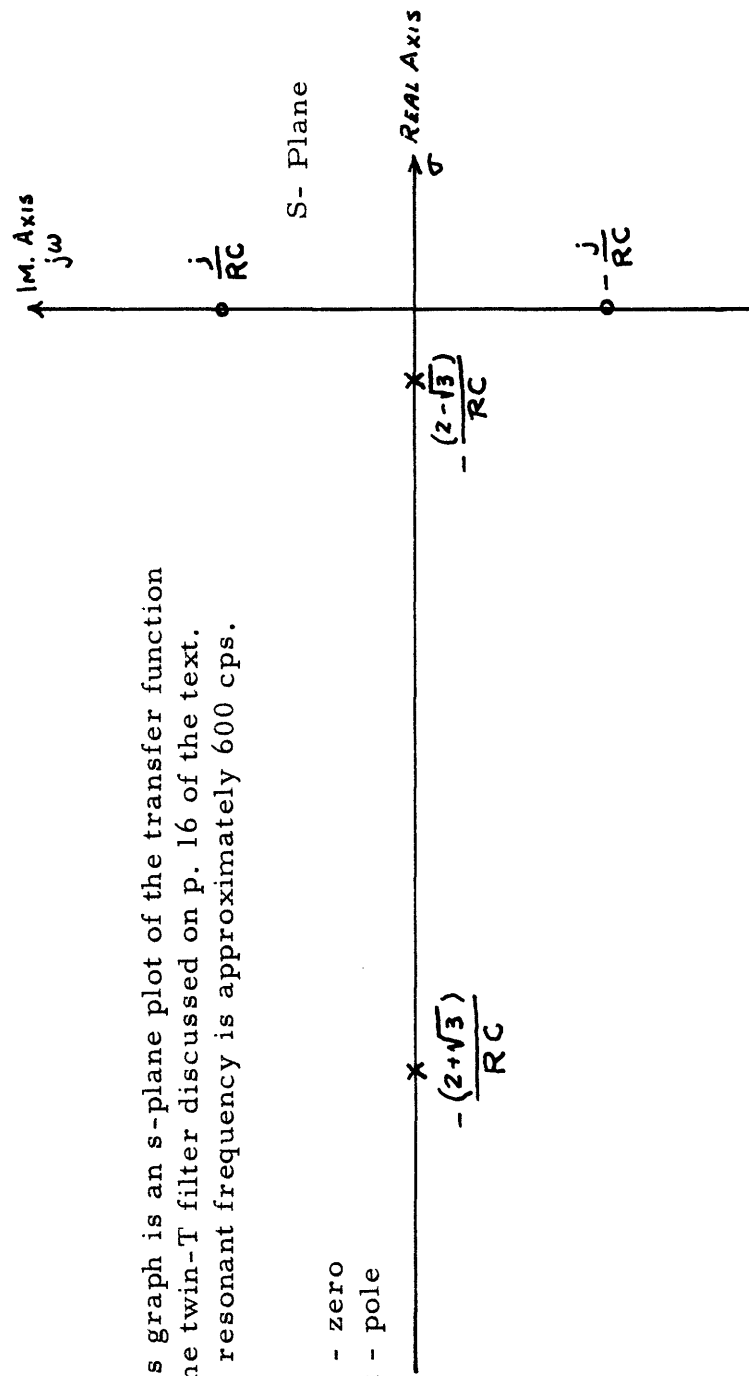


Fig. 11. An s-plane plot of the poles and zeroes of the twin-T filter voltage transfer function.

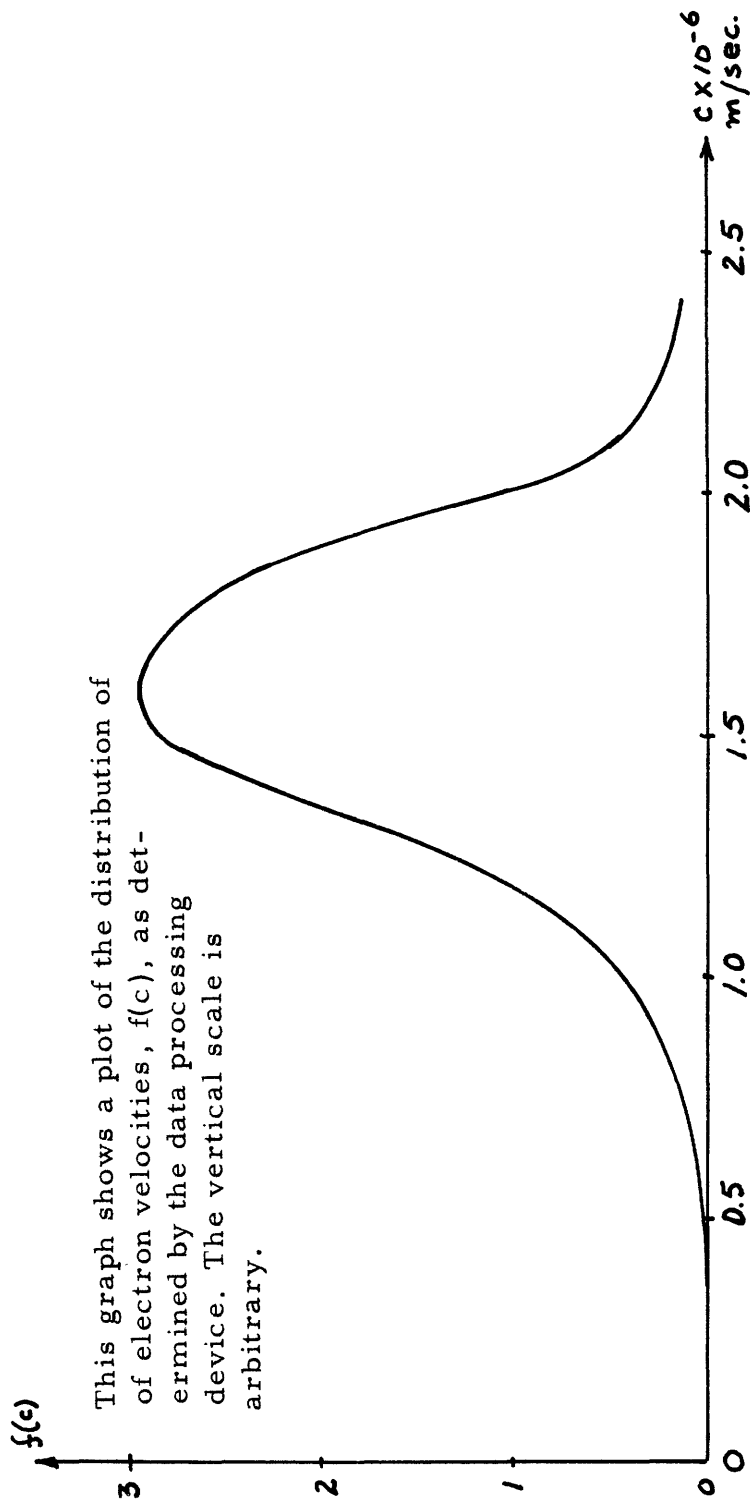


Fig. 12. A plot of the distribution of electron random velocities as found from the curve of figure 8, p. 29.

7.20 Table of Component Values

The following is a list of the component values of components labelled on the schematic diagrams.

<u>Resistors</u>	<u>Values and Descriptions</u>
1,	2500 ohm, 10 watt wire wound.
2	10k " , " " "
3	15k " , 1/2 watt composition
4	13k " , " " "
5	7500 " , 10 watt wire wound
6, 30	10k " , 1/2 watt composition
7, 11	5.1k " , " " "
8	7.2k " , " " "
9	10.5k " , " " "
12	3.9k " , " " "
13, 18,	1k " , " " "
14, 17	20k " , " " "
15, 16	39k " , " " "
21,	500k " , " " "
22, 26, 31	100k " , " " "
23, 25	30k " , " " "
27	1 megohm, " " "
28	2k " , " " "

Values and Descriptions

Resistors, Cont'd.

19	10k ohm, 1/2 watt potentiometer
20,29	1 meg, 1/2 watt potentiometer
24	30k ohm, 1/2 watt potentiometer

Capacitors

1,2,3,4	40 mfd. @ 450v. Electrolytic
5,17,21	1 mfd. @ 200v. paper
6,7,16,22	8 mfd. @ 450 v. Electrolytic
8	2000 mfd. @ 50 v. Electrolytic
9,10,11,12	1000 mfd. @ 25 v. Electrolytic
13	500 mmfd.@ 200 v. ceramic
14,15	220 mmfd.@ 200 v. ceramic
18, 19	0.01 mfd. @ 200v. ceramic
20	0.02 mfd. @ 400 v. ceramic
23,25	0.05 mfd. @ 200 v. paper
24	11.5 mfd. @ 200 v. paper

Miscellaneous

V _{1,2}	5U4 Rectifier
V _{3,6}	6AU6 Pentode
V _{4,5}	6AV6 Triode
V _{7,8}	VR 150 Voltage Regulator
V ₉	VR 105 " "
V ₁₀	6X5 Rectifier

Values and Descriptions

Miscellaneous, Cont'd.

Tr ₁	I. B. M. Type 015 P-N-P Transistor
Tr _{2,3,4}	2N585 N-P-N Transistor
L _{1,2,3}	7 hy. @ 50 ma. Choke
L ₄	24.5 mhy. Inductor
D _{1,2,3,4}	2 amp. Silicon Rectifier
D _{5,6,7,8}	1N34 Diode
F _{1,2,3}	1/4 amp. 3 AG. Fuse
S ₁	S. p. s. t. Toggle Switch
S _{2a,b}	D. p. d. t. Toggle Switch
B ₁	6.3 v. a. c. pilot lamp
T _{1,3}	400 v. @ 50 ma. Power Trans. (Stancor P6010)
T ₂	350 v. @ 40 ma. Power Transformer, (Thordarsen T 13R11).
T _{4,5}	12.6 v. c. t. @ 2amp. Trans. (Triad F 44X)
Re ₁	6 v. @ 10 ma. Relay (s. p. s. t. contact)
Re ₂	6.3 v. a. c. Relay (d. p. d. t. contacts)

BIBLIOGRAPHY

- ¹ Loeb, Leonard B. , Basic Processes of Gaseous Electronics; Univ. of Calif. Press, 1955; Ch. IV
- ² Langmuir, I. & Mott-Smith, H. M. , G. E. Review; (1924) Vol. 27, P. 449 and remainder of series.
- ³ Druyvesteyn, M. J. , Zeits, f. Phys. 64, 790, 1930.
- ⁴ Bachmann, A. E. , Proc. IEE; pt. B, Vol. 106, Pp. 107 - 174: (March, 1959).

APPENDIX

Consider an element of area, da , corresponding to an element of the sheath surface, located at the origin of the coordinate system of Figure 10a. Consider also an element of volume in the space, $d\mathcal{V}$, toroidal in shape, and oriented so that the plane of the toroid is perpendicular to the $x - z$ plane. The volume is given by

$$d\mathcal{V} = 2\pi r \sin \theta \, r \, d\theta \, dr. \quad (A1)$$

If the probability that a particle in \mathcal{V} has a speed between c and $c + dc$ is $f(c)dc$, and if the number of particles per unit volume is given by n , then one finds the number of particles in $d\mathcal{V}$ with velocity between c and $c + dc$ to be

$$n \, f(c) \, d\mathcal{V} \, dc. \quad (A2)$$

Reference to Figure 10b will show that the probability that a particle, located at point W a distance r from the origin, will be aimed so as to strike the elemental area, da , is

$$\frac{da \cos \theta}{4\pi r^2}. \quad (A3)$$

Combining equations (A2) and (A3) gives the number of particles, located in $d\mathcal{V}$, having a velocity between c and $c + dc$, which will, in time t , strike the elemental area, da ,

as

$$\frac{1}{2} dA n f(c) dc \sin \theta \cos \theta dr. \quad (A4)$$

If these particles are taken to be electrons, then the particles must have a y component of velocity at the origin such that $c_y \geq \sqrt{\frac{2eV}{m}}$, where e is the charge on the electron and m is the mass of the electron, in order to overcome the potential barrier presented by the sheath.

For a given value of c , only particles within a certain range of the angle, θ , will have the necessary y-directed velocity to penetrate the sheath. The maximum orientation angle, θ_a , is given by

$$\theta_a = \cos^{-1} \left(\sqrt{\frac{2eV}{m}} \right). \quad (A5)$$

The number of particles with a velocity between c and $c + dc$ which will reach the probe in time t and contribute to the current is then given by

$$\int_{r=0}^{ct} \int_{\theta=0}^{\theta_a} \frac{1}{2} dA n f(c) dc \sin \theta \cos \theta d\theta dr. \quad (A6)$$

Since the minimum speed which a particle must have to reach the probe is $\sqrt{\frac{2eV}{m}}$, one may write the total number of particles, from all ψ , which reach the probe as

$$N = \int_{c=\sqrt{\frac{2eV}{m}}}^{\infty} \int_{r=0}^{ct} \int_{\theta=0}^{\theta_a} \frac{1}{2} dA n f(c) \sin \theta \cos \theta d\theta dr dc. \quad (A7)$$

The probe current due to the electrons is given

by

$$i_- = \int_a \frac{Ne}{t}, \text{ where } a = \text{sheath area. (A8)}$$

Substituting equation (A7) into equation (A8) gives

the probe electron current as

$$i_- = \frac{ane}{2t} \int_{c=\sqrt{\frac{2eV}{m}}}^{\infty} \int_{r=0}^{ct} \int_{\theta=0}^{\theta_a} f(c) \sin\theta \cos\theta d\theta dr dc. \quad (\text{A9})$$

Carrying out the integration over r and θ gives

$$i_- = \frac{ane}{2} \int_{c=\sqrt{\frac{2eV}{m}}}^{\infty} c f(c) \left[1 - \frac{2eV}{m c^2} \right] dc. \quad (\text{A10})$$

Differentiation of i_- twice with respect to the voltage V gives the final result that

$$i_-^{(2)}(V) = \frac{\pi e^2 a}{4 m V} f\left(\sqrt{\frac{2eV}{m}}\right) = \frac{R}{V} f\left(\sqrt{\frac{2eV}{m}}\right). \quad (\text{A11})$$