

Fusing Strategies for the Dual-Voltage Fault

by

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B.Tech., Indian Institute of Technology, New Delhi (2003)

Submitted to the Department of Electrical Engineering and Computer Science
in partial fulfillment of the requirements for the degree of

Master of Science

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

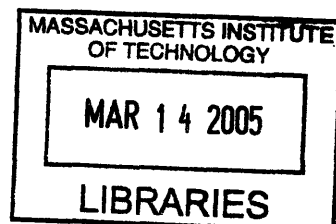
[February 2005]
January 2005

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Abstract

This thesis focuses on the 42V - 14V fault in a dual voltage system and discusses the possibility of effective fusing. A simple model for the system had been created from technical documentation. Based on the model and the goal of achieving a stable state through fusing, constraints are set on the load fuse currents and circuit parameters. Further, modifications to the circuit have been theorized and evaluated for their ability to bring the circuit to the stable state. The thesis concludes with an effective fusing strategy based on actively detecting the fault and taking corrective actions.

◁
Thesis Supervisor: Dr. Thomas Keim
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Acknowledgements

Many people have been a part of my graduate education, as friends, teachers, and colleagues. Dr. Thomas Keim, first and foremost, has been all of these. The best advisor and teacher I could have wished for, he is actively involved in the work of all his students, and clearly always has their best interest in mind. Thank you for guiding me. Time after time, his clear and deep rooted understanding of the electrical engineering fundamentals challenged me to achieve perfection even before I presented any results to him. On the personal side, he did not hesitate to entertain conversations ranging anywhere from pirate puzzles to cultural differences between India and the US. I was also fortunate to have been a member of the MIT-Automobile Industry Consortium of whom Dr. Keim is the Director. Working for the consortium helped me learn the practical way of looking at research. The very fact that the circuit I am working should be capable of surviving the real world environment inside an automobile, motivated me to strive towards that extreme of precision.

I would also like to thank Prof. David J. Perreault for his short but very insightful guidance to my project. His way of attacking assumptions has taught me to not just look for general trends but to bear in mind the importance of exceptions. I would also like to thank all the people associated with the Consortium including Gary Desgrosseliers, Karin Strasswimmer, Kiyomi Boyd and Vivian Mizuno for all the help they they did to make my stay at LEES comfortable and enjoyable.

In my office in the Laboratory for Electromagnetic and Electronic Systems (LEES), I was surrounded by knowledgeable and friendly people who helped me daily. My office mate, Vasanth Sarathy, has been a great source of practical information, as well as being happy to be the first to hear my outrage or glee at the day's current events. He has been a constant source of help inside and outside the office. I have also been able to make extensive use of his knowledge about the US and the way things work around here. My other neighbors, including Shiv Reddy, Tushar Parlikar and Timothy Neugebauer were each a great help in their own way. Tushar, thank you for all the coffee hours and the accompanying discussion about the American culture and the ongoing elections. Tim, thank you for your humor. Thanks also to Laura Zager, Francis O' Sullivan, Yihui Qui, Pdraig Murphy, Natalija Jovanovic, Ivan Celanovic, Leandro Lorilla, Rob Jensen and Riccardo Signorelli for all the lunch time discussions on almost any and every topic one can think of.

In addition to the people in LEES, I have been lucky enough to have the support of many good friends. Abhinandan Agrawal would surely be the first one to mention. Being from a similar background, I was able to associate with him a lot since I started here in MIT. Thanks to Vasanth Sarathy, Shubham Mukherjee and Laura Zager for all the drinks we had at almost every place that we could go to.

Acknowledgements

Finally, I would like to thank my parents, for their absolute confidence in me. Their belief in me helps me believe in myself.

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List Of Symbols

- I_{Fk} : Fuse current rating for the fuse in the k'th circuit
- I_{Lk} : Load current for the k'th circuit
- G_k : Load Conductance for the k'th circuit
- F_1 : Generic symbol for 42V fuse
- F_2 : Generic symbol for 14V fuse
- R_1 : Resistance of the 42V load
- R_2 : Resistance of the 14V load
- G_1 : Conductance of the 42V load
- G_2 : Conductance of the 14V load
- G_L : Combined parallel conductance of the shorted 42V and 14V load
- G_{1max} : Maximum possible conductance of the 42V load
- G_{2max} : Maximum possible conductance of the 14V load
- G_{Lmax} : Maximum value of G_L
- V_1 : Thevenin equivalent source of value 42V for the 42V bus
- V_2 : Thevenin equivalent source of value 14V for the 14V bus
- r_1 : Thevenin equivalent resistance for the 42V bus
- r_2 : Thevenin equivalent resistance for the 14V bus
- I_{F1} : Fusing current of Fuse F_1
- I_{F2} : Fusing current of Fuse F_2
- g_1 : $1/r_1$
- g_2 : $1/r_2$
- G_1 : $1/R_1$

$G_2 : 1/R_2$

i_1 : Current in fuse F_1 at the time of fault

i_2 : Current in fuse F_2 at the time of fault

i_{12} : Current in fuse F_1 after fuse F_2 blows (if fuse F_2 blows)

i_{21} : Current in fuse F_2 after fuse F_1 blows (if fuse F_1 blows)

I_{F1}^o : Maximum load current in the 42V load (R_1)

I_{F2}^o : Maximum load current in the 14V load (R_2)

$G_{La}, G_{Lb}, G_{Lc}, G_{Ld}, G_{Le}$: Symbols to locate specific points on the G_L space

i'_2 : Current in fuse F_2 at the time of fault with the Zener modification

V_Z : Zener breakdown voltage

ΔV_Z : Range of the Zener breakdown voltages for the Zener modification to work

g_Z : Zener conductance in the breakdown region

α : Symbol to define relationship between V_Z and V_1

V_a, V_b, V_p, V_q : Symbols to define certain voltages in the operation of the Zener modification and diode modification circuits

G_{Pcond} : PTC device (and any resistance in series with it) conductance

L_1 : Generic 42V load

L_2 : Generic 14V load

G_{42}, G_1 : Another symbol for the conductance of the 42V load

G_{14}, G_2 : Another symbol for the conductance of the 14V load

I_{42}, I_1 : Current rating of the 42V constant current load

I_{14}, I_2 : Current rating of the 14V constant current load

P_{42}, P_1 : Power rating of the 42V constant power load

P_{14}, P_2 : Power rating of the 14V constant power load

I_{MAX} : Maximum current in the 14V circuit

G_{1max} : Maximum value of G_1

G_{2max} : Maximum value of G_2

- I_{1max} : Maximum value of I_1
- I_{2max} : Maximum value of I_2
- P_{1max} : Maximum value of P_1
- P_{2max} : Maximum value of P_2
- V_x : Generic voltage bus (can be 42V or 14V)
- g_x : Equivalent conductance of the generic voltage bus (can be g_{42} or g_{14})
- G_x : Conductance of the load attached to the generic voltage bus (can be G_{42} or G_{14})
- I_x : Current rating of the load attached to the generic voltage bus (can be I_{42} or I_{14})
- P_x : Power rating of the load attached to the generic voltage bus (can be P_{42} or P_{14})
- v_{g_x} : Voltage drop across g_x
- V_{1max} : Maximum value of the 42V voltage bus
- V_{2max} : Maximum value of the 14V voltage bus
- V_{1nom} : Nominal value of the 42V voltage bus
- V_{2nom} : Nominal value of the 14V voltage bus
- V_{1min} : Minimum value of the 42V voltage bus
- V_{2min} : Minimum value of the 14V voltage bus
- j_{res} : Constant resistance load ratio parameter for bus voltage at its minimum value
- j_{cur} : Constant current load ratio parameter for bus voltage at its minimum value
- j_{pow} : Constant power load ratio parameter for bus voltage at its minimum value
- k_{res} : Constant resistance load ratio parameter for bus voltage at its nominal value
- k_{cur} : Constant current load ratio parameter for bus voltage at its nominal value
- k_{pow} : Constant power load ratio parameter for bus voltage at its nominal value
- l_{res} : Constant resistance load ratio parameter for bus voltage at its maximum value
- l_{cur} : Constant current load ratio parameter for bus voltage at its maximum value
- l_{pow} : Constant power load ratio parameter for bus voltage at its maximum value
- I_{L1} : Current in load L_1

I_{L2} : Current in load L_2

I_L : $I_{L1} + I_{L2}$

V : Common voltage at the shorted loads

$f(V)$: Function to relate I_L to V

$F1$: Ratio between maximum load current and fuse rating for the 42V fuse

$F2$: Ratio between maximum load current and fuse rating for the 14V fuse

Introduction To The Circuit And The Associated Problem

1.1 Introduction

Dual-voltage (e.g. 42V/14V) automobiles present the electrical system designer with challenges not found in today's cars. One such challenge is the design of a fault protection system to restore the electrical system to a safe state in the event of short circuits and/or faulted components. In conventional cars, simple fuses are the principal elements of the fault protection system. But in dual-voltage automobiles, a new class of fault (the bus-to-bus or 42V – 14V fault) can occur. It can be shown (and is shown here), that fuses alone cannot be used to protect against such an event. This thesis will show some methods which can be considered to achieve a dual voltage system which always returns to a safe condition after a bus-to-bus fault.

1.1.1 Current Fusing Circuits

Figure 1.1 presents a schematic representation of a typical 14V electrical system, showing fuses and loads represented by resistors. From [1], the largest fuse rating in the 1997 Ford Taurus was 40A. We will presume that this is typical for automobiles in general. Thus,

$$\max I_{Fk} = 40A \quad k = 1, 2, \dots, n \quad (1.1)$$

Here I_{Fk} designated the fuse current rating for the fuse in the k'th circuit. The fusing current for a circuit is by design more than the normal loading current in the circuit. Thus, if the load is modelled as a resistor then the fusing current sets a limit on how small the resistance of the load can be, or in other words, how large the conductance of the load can be. Putting the same in equation form, we get,

$$VG_k = I_{Lk} < I_{Fk} \quad (1.2)$$

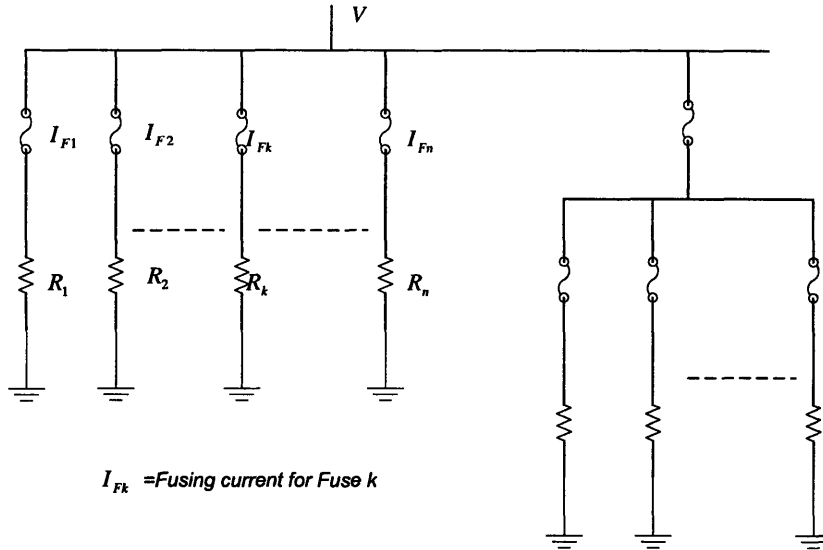


Figure 1.1: Current Fusing Circuits

where,

V =bus voltage,

$G_k=1/R_k$ =Load Conductance for circuit k ,

I_{Lk} =Load current for circuit k

If we substitute the maximum value of I_{Fk} then,

$$G_k < \max(I_{Fk})/V = G_{2max} \quad (1.3)$$

where, G_{2max} = maximum value for any 14V load conductance (the subscript 2 has been used to differentiate from G_{1max} = maximum value for any 42V load conductance). We will shortly introduce a second bus and will commonly use the subscript 2 to indicate the 14V system and the subscript 1 to represent the 42V system.

$$G_{2max} = 40A/14V \approx 3mhos \quad (1.4)$$

Thus, we have an upper bound on the value of the conductance that any load in any 14V circuit can have. Since, similar data for the 42V circuit does not exist at present, we assume that

$$G_{1max} = Kmhos. \quad (1.5)$$

If the 42V circuit is designed to cater to loads whose power demand is about 3 times that of 14V loads, then we can estimate K:

$$P_{42V} = 3P_{14V} \tag{1.6}$$

$$42^2 K = 3 * 14^2 * 3 \tag{1.7}$$

Hence,

$$K = 1 \tag{1.8}$$

We can also choose other values for K depending on the conditions.

1.1.2 Fault Considered

In a dual-voltage car, the second distribution system can be expected to look schematically just like Figure 1.1. Our concern in this paper is for faults between any branch of the 14V system and any branch of the 42V system. Figure 1.2 shows the fault considered for analysis. The explanations for the various current values and resistances are as follows:

R_1 = Resistance of the 42V load

R_2 = Resistance of the 14V load

I_{F1} = Fusing current of Fuse F_1

I_{F2} = Fusing current of Fuse F_2

Although Figure 1.2 is very simple, it can be used to represent any possible 42V – 14V

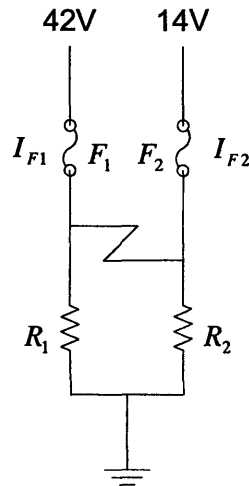


Figure 1.2: Fault Circuit Considered

fault. Faults between different circuits will result in different values of the parameters I_{F1} , I_{F2} , R_1 and R_2 . Note that even for a fixed pair of circuits, it is possible for R_1 and R_2 to assume different values at different times. In particular, it is quite common for electrical loads to be turned off in which case the corresponding load resistance is essentially infinite.

1.1.3 Safe Clearance Of The Fault

In the event of the fault, we define the safe clearance of the fault as any sequence of fuse blowing which will eventually result in a safe operating condition. With 2 fuses in place, we can have a total of 5 choices for fuse blowing events, i.e.

1. Neither of the fuses blowing
2. Fuse F_1 blowing only
3. Fuse F_2 blowing only
4. Fuse F_1 blowing, then Fuse F_2 blowing
5. Fuse F_2 blowing, then Fuse F_1 blowing

The start and the end results of all these combinations of fuse blowing are shown in Figure 1.3. Condition (1) is clearly an unacceptable outcome. The voltage applied to loads R_1

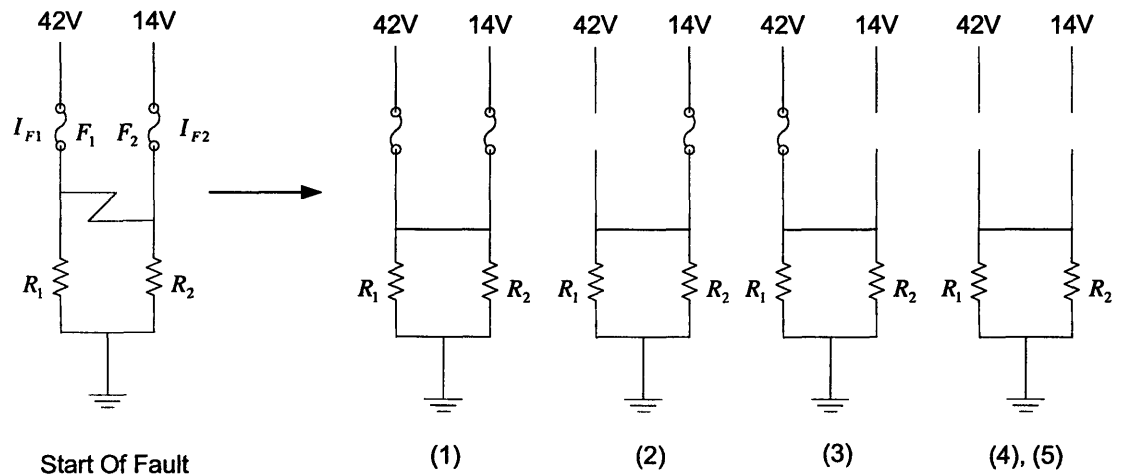


Figure 1.3: Clearance Conditions

and R_2 will be between the voltage of the buses; the exact value will be strongly influenced

by the magnitude of the effective series resistances of the two sources. (The simple diagram of Figure 1.3 does not explicitly indicate the source resistances.) In general, in state 1, there is a large current flowing in the fault and a high (possibly harmful) voltage on load R_2 . Condition (3) has the same detrimental features (current in the fault and high voltage on R_2) as Condition (1).

Some have argued that condition (2) is a safe outcome. The voltage across R_1 is lower than its intended value. Load R_1 may not function properly or at all with such a small voltage, but the low voltage is not likely to damage load R_1 . But current still flows in the fault. If we could be confident that the real world fault would behave as its model (a zero impedance short circuit), this line of reasoning might be worth considering. But the real world fault may have a small non-zero resistance. If the fault were to have a resistance which would support just a couple of volts at a few amps, the resulting heating, occurring in a very small volume, could cause excessive temperature. So, for the purposes of this study, we will consider only conditions (4) and (5) to be safe processes. Our methodology will enable us to design the fuse currents I_{F1} and I_{F2} and other circuit parameters such that the fault is safely cleared by one of these fusing events.

1.2 Equivalent Circuit At Fault

In this section, we describe an equivalent circuit for the system considered. Figure 1.4 shows the circuit. The equivalent circuit has the following features:

- Voltage bus : We have introduced an equivalent Thevenin circuit for the buses. Thus, we have replaced the 42V and the 14V bus by a voltage source of voltage 42V and 14V and a series resistance r_1 and r_2 respectively.
- Loads : The loads have been modelled as resistors of value R_1 and R_2 .
- Fault : The fault has been shown as a short circuit of zero resistance between the positive ends of the 42V and 14V loads.

Check Table 1.1 for clearer definitions. Since, the circuit topology makes it simpler to work with conductances rather than resistances, we will replace resistances with conductances defined accordingly. See Table 1.2. The value G_L is of importance. It represents the conductance of the *parallel* combination of R_1 and R_2 . And with the help of Eq. (1.5) and Eq. (1.4), we get a maximum G_L of

$$G_{Lmax} = 3 + K \tag{1.9}$$

Parameter	Definition
V_1	Thevenin equivalent source of value 42V for the 42V bus
V_2	Thevenin equivalent source of value 14V for the 14V bus
r_1	Thevenin equivalent resistance for the 42V bus
r_2	Thevenin equivalent resistance for the 14V bus
R_1	42V Load Resistance
R_2	14V Load Resistance
I_{F1}	Fuse current rating for the 42V load
I_{F2}	Fuse current rating for the 14V load

Table 1.1: Parameter Definition For The Equivalent Circuit

Conductance	Relation to resistance
g_1	$1/r_1$
g_2	$1/r_2$
G_1	$1/R_1$
G_2	$1/R_2$
G_L	$1/R_1 + 1/R_2$

Table 1.2: Conductance Definitions For The Equivalent Circuit

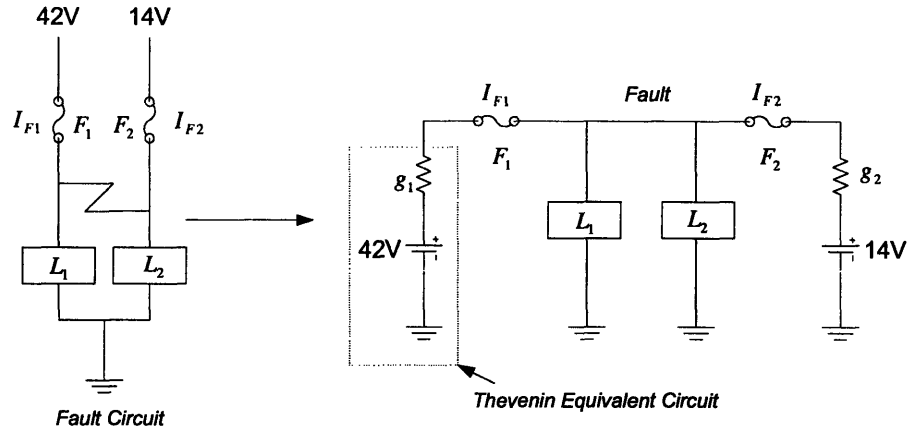


Figure 1.4: Equivalent Circuit At Fault

Current	Definition
i_1	Current in fuse F_1 at the time of fault
i_2	Current in fuse F_2 at the time of fault
i_{12}	Current in fuse F_1 after fuse F_2 blows (if fuse F_2 blows)
i_{21}	Current in fuse F_2 after fuse F_1 blows (if fuse F_1 blows)
I_{F1}^o	Maximum load current in the 42V load (R_1)
I_{F2}^o	Maximum load current in the 14V load (R_2)

Table 1.3: Current Definitions For The Faulted Circuit

Table 1.3 provides word definitions for the various currents in the circuit. Please refer to Figure 1.5 for the schematic definitions and sign conventions.

1.2.1 Current Expressions

Solving for the currents gives the following expressions:

$$I_{F1}^o = V_1 / (1/g_1 + 1/G_{1max}) \tag{1.10}$$

$$I_{F2}^o = V_2 / (1/g_2 + 1/G_{2max}) \tag{1.11}$$

$$i_1 = \frac{g_1[V_1(G_L + g_2) - V_2g_2]}{g_1 + g_2 + G_L} \tag{1.12}$$

$$i_2 = \frac{-g_2[V_2(G_L + g_1) - V_1g_1]}{g_1 + g_2 + G_L} \tag{1.13}$$

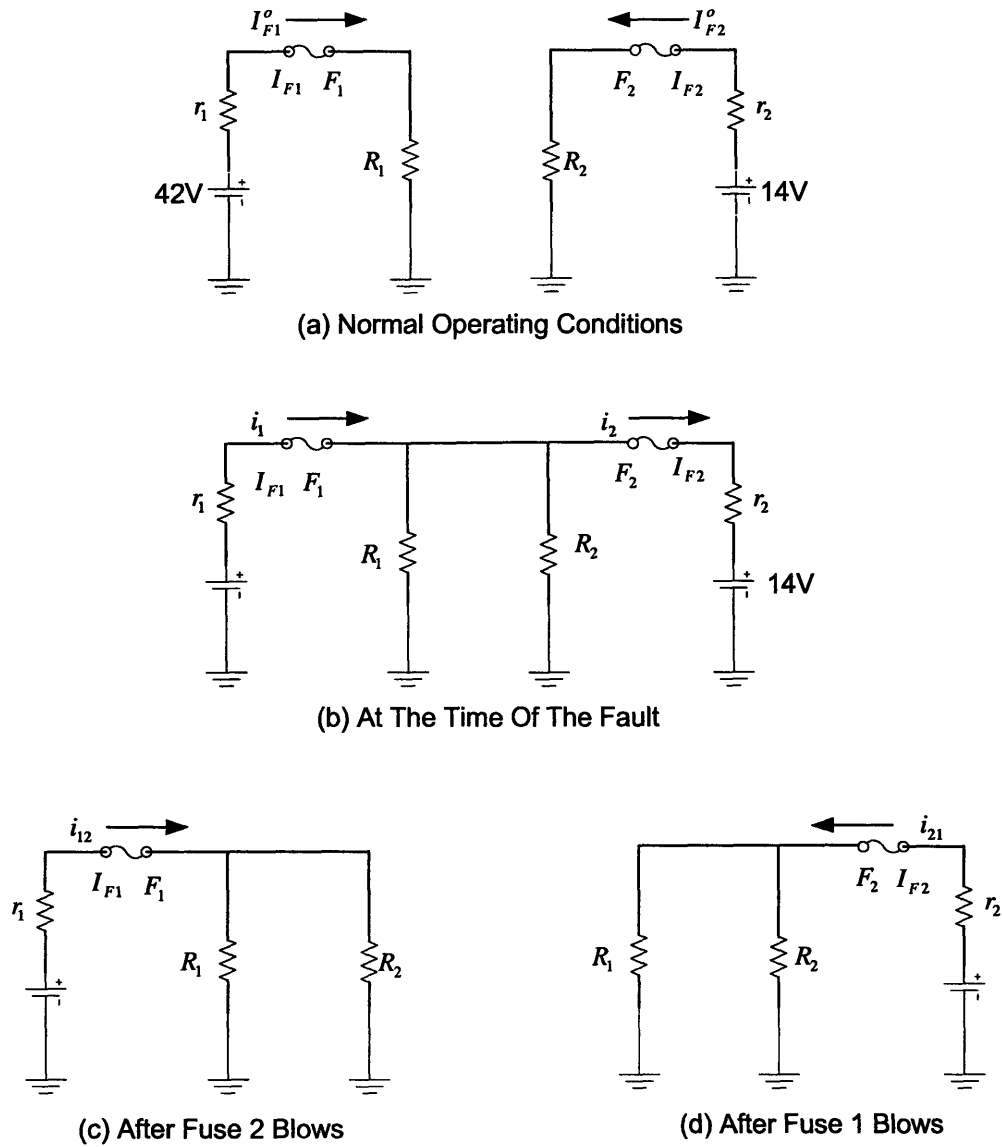


Figure 1.5: Current Definitions

$$i_{21} = \frac{V_2 G_L g_2}{g_2 + G_L} \quad (1.14)$$

$$i_{12} = \frac{V_1 G_L g_1}{g_1 + G_L} \quad (1.15)$$

1.3 Fusing Strategies And Conditions

In this section, we will summarize the fusing strategies and the conditions needed for them to be realized.

1.3.1 Fusing Strategy 1 : Fuse F_1 Blowing Followed By Fuse F_2

The conditions for fuse F_1 and Fuse F_2 blowing in succession are as follows:

1. $i_1 > I_{F1}$: This condition says that the fault current in fuse F_1 is more than its fusing current so that Fuse F_1 blows.
2. $i_2 < I_{F2}$: This condition says that the fault current in fuse F_2 is less than its fusing current so that F_2 does not blow.
3. $i_{21} > I_{F2}$: This condition says that the fault current in fuse F_2 after Fuse F_1 has blown is more than its fusing current so that F_2 does blow.
4. $I_{F1} > I_{F1}^o$: This condition says that the fusing current of Fuse F_1 is more than its maximum loading current, so that Fuse F_1 does not blow in the normal operating conditions.
5. $I_{F2} > I_{F2}^o$: This condition says that the fusing current of Fuse F_2 is more than its maximum loading current, so that Fuse F_2 does not blow in the normal operating conditions.

Note that condition (2) is not formally necessary for the F_1 then F_2 scenario to unfold. But if $i_2 > I_{F2}$, the amount of inequality will need to be small. If $i_2 > I_{F2}$, the only way to blow F_1 followed by F_2 is if Fuse F_1 is more overloaded with respect to its rating, so that it blows first.

1.3.1.1 Choice Of I_{F1} And I_{F2}

The first and fourth of conditions in section 1.3.1 can be satisfied by choosing I_{F1} such that

$$i_1 > I_{F1} > I_{F1}^o \quad (1.16)$$

For the conditions (2), (3) and (5) stated above to be satisfied, we can choose the fusing current I_{F2} such that

$$i_{21} > I_{F2} > i_2, I_{F2}^o \quad (1.17)$$

Here we have used the convention that $x > y, z$ is equivalent to $x > y, x > z$. Thus, the three conditions

$$i_1 > I_{F1}^o \quad (1.18)$$

$$i_{21} > i_2, I_{F2}^o \quad (1.19)$$

can ensure us the possibility to select I_{F1} and I_{F2} to implement Fusing strategy 1, with respect to the two circuits under consideration.

1.3.2 Fusing Strategy 2 : Fuse F_2 Blowing Followed By Fuse F_1

The conditions for fuse F_2 and Fuse F_1 blowing in succession are as follows

1. $i_1 < I_{F1}$: This condition says that the fault current in fuse F_1 is less than its fusing current so that Fuse F_1 does not blow.
2. $i_2 > I_{F2}$: This condition says that the fault current in fuse F_2 is more than its fusing current so that F_2 blows.
3. $i_{12} > I_{F1}$: This condition says that the fault current in fuse F_1 after Fuse F_2 has blown off is more than its fusing current so that F_1 does blow.
4. $I_{F1} > I_{F1}^o$: This condition says that the fusing current of Fuse F_1 is more than its maximum loading current, so that Fuse F_1 does not blow in the normal operating conditions.
5. $I_{F2} > I_{F2}^o$: This condition says that the fusing current of Fuse F_2 is more than its maximum loading current, so that Fuse F_2 does not blow in the normal operating conditions.

Here it is Condition (1) which is sufficient but not necessary for the F_2 then F_1 scenario.

1.3.2.1 Choice Of I_{F1} And I_{F2}

The first and the fourth conditions stated in section 1.3.2 can be satisfied by choosing I_{F2} such that

$$i_2 > I_{F2} > I_{F2}^o \quad (1.20)$$

For the conditions (2), (3) and (5) stated above to be satisfied, we can choose the fusing current I_{F1} such that

$$i_{12} > I_{F1} > i_1, I_{F1}^o \quad (1.21)$$

Thus, the three conditions

$$i_2 > I_{F2}^o \quad (1.22)$$

$$i_{12} > i_1, I_{F1}^o \quad (1.23)$$

can ensure us the possibility to select I_{F1} and I_{F2} to implement Fusing strategy 2, with respect to the two circuits under consideration.

1.3.3 Basic Plot Used

As was shown in the section 1.2.1, the currents i_1, i_2, i_{12}, i_{21} are functions of G_L rather than G_1 or G_2 , we will plot these currents versus G_L to get a clearer perspective of the safe operating conditions. To be noted is the fact that we need not consider the plots beyond G_{Lmax} (as defined in Eq (1.9)) because any conductance cannot go beyond this value. Figure 1.6 shows the basic working area that we will be using to define the safe clearance conditions. This plot shows G_L plotted on the x -axis from 0 to G_{Lmax} . The y -axis will have the plots of the fault currents and the maximum load currents separately for Fuse $F1$ and Fuse $F2$.

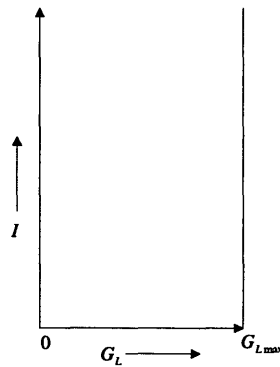


Figure 1.6: Basic Plot Used

1.4 Fusing Strategy 1: Fuse F_1 Blowing Followed By Fuse F_2

1.4.1 Ensuring Fuse F_1 Blows For All G_L

Plotted below in Figure 1.7 are the currents i_1 and $I_{F_1}^o$ for different values of g_1 and g_2 . Observe first Figure 1.7 (a). In the portion $G_L > G_{La}$, we find that $i_1 > I_{F_1}^o$ and thus condition in Eq. (1.18) is satisfied. Thus in circuits where the smallest possible G_L is large enough (which can occur if some loads are permanently energized) we can pick a value of I_{F_1} so that Eq. (1.18) is satisfied. In Figure 1.7 (b) and (c), which represent progressively larger values of g_1 and g_2 relative to Figure 1.7 (a), the range of allowable G_L grows. For large enough g_1 and g_2 , it is possible that

$$i_1|_{G_L=0} > I_{F_1}^o \quad (1.24)$$

In that case, it is possible to pick a fuse rating I_{F_1} which satisfies Eq. (1.18) for all possible G_L . The condition necessary for this to be true is

$$\frac{g_2 g_1 (V_1 - V_2)}{g_1 + g_2} > \frac{G_{1max} g_1 V_1}{G_{1max} + g_1} \quad (1.25)$$

which, for $V_1/V_2 = 3$, translates into

$$g_2 > \frac{3g_1 G_{1max}}{2g_1 - G_{1max}} \quad (1.26)$$

which turns out to be a relatively unrestrictive condition since g_2 is an order of magnitude higher than G_{1max} .

1.4.2 Ensuring Fuse F_2 Blows After Fuse F_1 For All G_L

Plotted below in Figure 1.8 are the currents i_2 , i_{21} and $I_{F_2}^o$ for some values of g_1 and g_2 . Since i_2 is a monotonically decreasing function of G_L and i_{21} is a monotonically increasing function of G_L , for the first of inequalities (Eq. (1.18)) to be true for even a range of G_L , we need

$$i_{21}(G_L = G_{Lmax}) > i_2(G_L = G_{Lmax}) \quad (1.27)$$

thus,

$$\frac{V_2 G_{Lmax} g_2}{g_2 + G_{Lmax}} > \frac{-g_2 [V_2 (G_{Lmax} + g_1) - V_1 g_1]}{g_1 + g_2 + G_{Lmax}} \quad (1.28)$$

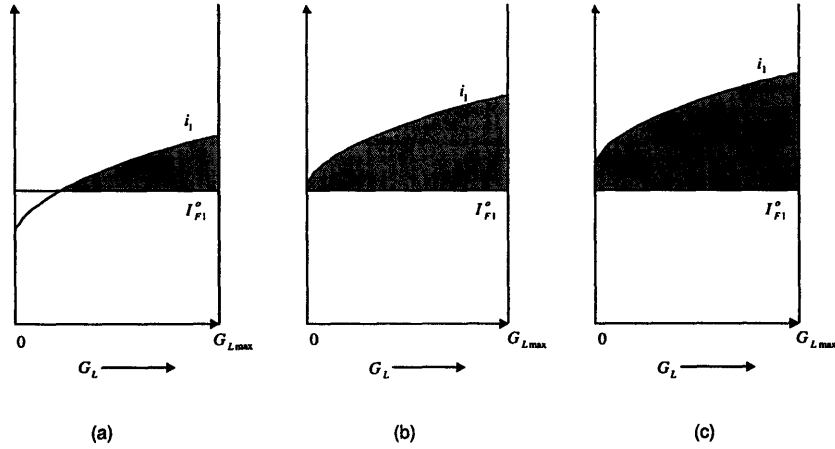


Figure 1.7: Plots Of Currents Through Fuse F_1 For Different Values Of g_1 And g_2

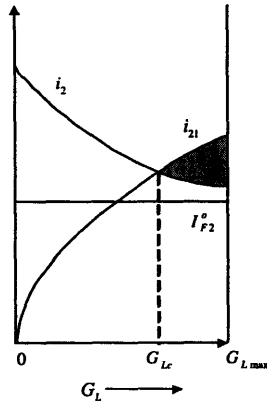


Figure 1.8: Plots Of Currents Through Fuse F_2

This, translates into (using $V_1/V_2 = 3$)

$$g_2 < \frac{G_{Lmax}(2G_{Lmax} - g_1)}{2(g_1 - G_{Lmax})} \tag{1.29}$$

g_1 is expected to be an order of magnitude higher than G_1 (since it is the series equivalent resistance and we do not want a high voltage drop across it). Thus $(2G_{Lmax} - g_1)$ in the numerator of the R.H.S. of Eq. (1.29) is a very small number, probably negative. Thus, this condition states that g_2 must be less than a small number which may be negative. Thus, for all practical cases, this condition is not obeyed. We may conclude that we cannot ensure blowing of Fuse F_2 after Fuse F_1 for many loading conditions.

1.4.3 Summary Of Fusing Strategy 1

As has been proved, we can ensure blowing of Fuse F_1 for many conditions but we cannot ensure blowing of Fuse F_2 after Fuse F_1 has blown. Thus, we cannot strictly implement this fusing strategy.

1.5 Fusing Strategy 2: Fuse F_2 Blowing Followed By Fuse F_1

1.5.1 Ensuring Fuse F_2 Blows For All G_L

Plotted below in Figure 1.9 are the currents i_2 and $I_{F_2}^o$ for different values of g_1 and g_2 . i_2 , being a function of g_1 and g_2 shifts as shown. Observe Figure 1.9 (a). In the portion $G_L < G_{Ld}$, we find that $i_2 > I_{F_2}^o$ and thus the condition in Eq. (1.22) is satisfied. Thus, in this shaded region we can choose I_{F_2} such that $I_{F_2}^o < I_{F_2} < i_2$. In (b) and (c), this condition is fulfilled for all values of G_L . The ability to draw a horizontal line in the shaded region from $G_L = 0$ to G_{Lmax} implies exactly this. And it is also clear that Figure (b) represents the boundary between (a) and (c). Thus, (b) can be used to define conditions on g_1 and g_2 so that the condition in Eq. (1.22) is satisfied for all G_L values. The condition shown in Figure (b) translates into current i_2 passing through point A (see Figure 1.9 (b)) or $(G_{Lmax}, \frac{V_2}{\frac{1}{g_2} + \frac{1}{G_{2max}}})$. Thus, substituting the values for the currents,

$$\frac{V_2}{\frac{1}{g_2} + \frac{1}{G_{2max}}} < -\frac{g_2[V_2(G_{Lmax} + g_1) - V_1g_1]}{g_2 + g_1 + G_{Lmax}} \quad (1.30)$$

This translates into (using $V_1/V_2 = 3$)

$$g_2 > \frac{-g_1G_{2max} + 2G_{2max}G_{Lmax}}{2g_1 - G_{Lmax} - G_{2max}} \quad (1.31)$$

$$g_2 > \frac{-3g_1 + 6(3 + K)}{2g_1 - 6 - K} \quad (1.32)$$

The condition in Eq. (1.32) is very easily met. The right hand side is always less than zero for reasonable values of g_1 , so the condition is met for any positive g_2 . This unusual result is a consequence of V_1/V_2 being exactly 3. But in a range of practical cases, including cases where V_1/V_2 deviates from 3, the condition is still easily met for most plausible values of g_1 , G_{1max} and G_{2max} .

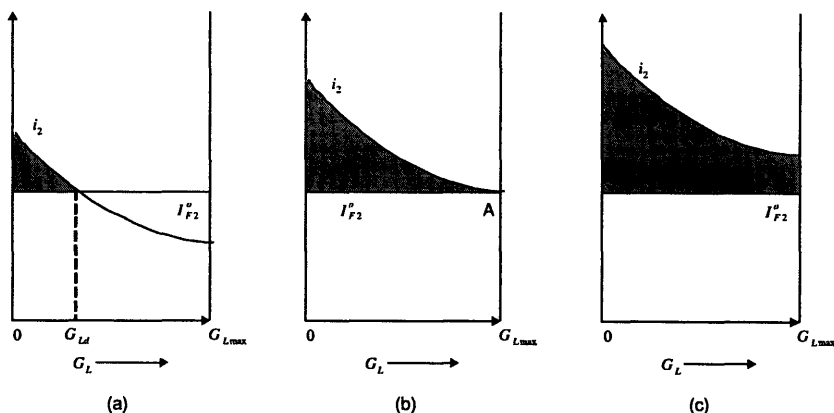


Figure 1.9: Plots Of Currents Through Fuse F_2 For Different Values Of g_1 And g_2

1.5.2 Ensuring Fuse F_1 Blows After Fuse F_2 For All G_L

Plotted below in Figure 1.10 are the currents i_1, i_{12} and $I_{F_1}^0$ for some values of g_1 and g_2 . In the portion $G_L > G_{Le}$, we find that $i_{12} > I_{F_1}^0, i_1$ and thus condition in Eq. (1.23) is satisfied. The solution for G_{Le} is given by

$$G_{Le} = 2g_1 \tag{1.33}$$

We want the point G_{Le} to be less than G_{Lmax} so that we can have some loads for which the Fuse F_1 blows after Fuse F_2 blows. However, g_1 is expected to be an order of magnitude higher than G_1 and G_{Lmax} . So $G_{Le} < G_{Lmax}$ is not possible. Thus, for all practical cases, Fuse F_1 cannot be made to blow after Fuse F_2 has blown.

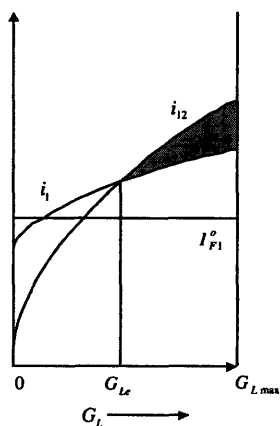


Figure 1.10: Plots Of Currents Through Fuse F_1

1.5.3 Summary Of Fusing Strategy 2

As has been proved, we can ensure blowing of Fuse F_2 but not blowing of Fuse F_1 after Fuse F_2 has blown. Thus, we cannot strictly implement this fusing strategy.

1.6 Conclusions

We have shown that the fusing strategies as envisaged theoretically cannot be implemented in practice. This result has been known for a long time by the automotive industry. With this chapter we have provided a mathematical footing and have shown that, without special measures, we cannot guarantee that both the fuses will blow in practical circumstances. In the next chapters, we will develop a strategy to get over this restriction and ensure both fuses blowing under all circuit operating conditions.

Passive Circuit Modifications To The Dual Voltage System

2.1 Introduction

As we saw before, the circuit in its working state cannot guarantee us the blowing of both the fuses. Hence, we look for modifications to the circuit, which do not affect the normal functionality of the circuit, but will become active in the event of a $42V - 14V$ fault.

2.2 Bottleneck Conditions

As we saw before, the conditions $i_{21} > i_2$ and $i_{12} > i_1$ are not satisfied for any circuit topology. In this section, we will introduce another condition that is not satisfied in many cases. Consider Eq.(1.19) as rewritten here

$$i_{21} > I_{F2}^o \quad (2.1)$$

or,

$$\frac{V_2 G_L g_2}{g_2 + G_L} > \frac{V_2 g_2 G_{2max}}{g_2 + G_{2max}} \quad (2.2)$$

Consider a $14V$ load which can in its operation range from 0 to G_{2max} . Now consider a $42V - 14V$ fault occurring when this load is switched off i.e. $G_2 = 0$. Thus, $G_L = G_1$ for the definition of i_{21} and $G_2 = G_{2max}$ for the definition of I_{F2}^o . Hence, this condition actually puts the condition (1.19) to the toughest test by minimizing i_{21} . Substituting as discussed,

$$\frac{V_2 G_1 g_2}{g_2 + G_1} > \frac{V_2 g_2 G_{2max}}{g_2 + G_{2max}} \quad (2.3)$$

This leads to,

$$G_1 > G_{2max} \quad (2.4)$$

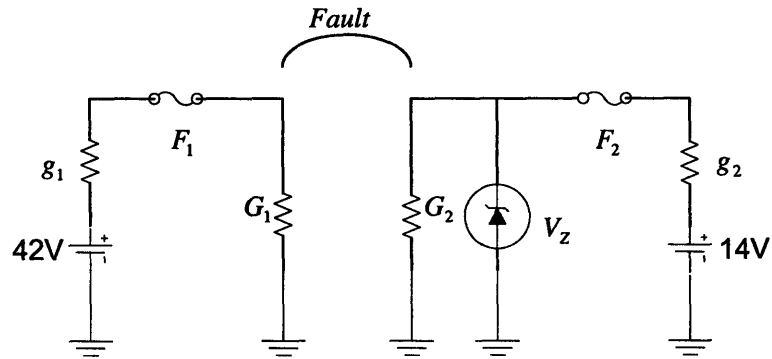


Figure 2.1: Circuit With The Zener In Place

It will be very difficult to assure this condition in generality. In particular, we will want to allow load G_1 to be switched off at the time of the fault (i.e. $G_1 = 0$). A similar condition in the 42V circuit may be established to yield the condition $G_2 > G_{1max}$. Thus, other strict conditions await us even if we resolve the $i_{21} > i_2$ or $i_{12} > i_1$ conditions.

2.3 Modification 1: Addition Of Zener

We will introduce a Zener diode to the circuit in the position shown in Figure 2.1. In the next section, we describe how we want the circuit to work.

2.3.1 Desired Sequence Of Events With The Zener Modification

We want the following operation sequence to be followed with the Zener in place:

- In the normal operation of the 14V circuit, the Zener is back biased but not to the Zener breakdown voltage. Thus, the Zener acts as an open circuit and therefore doesn't affect the circuit operation.
- In the event of a 42V – 14V fault, the Zener goes into reverse breakdown and provides a shorting path to the ground.
- Fuse F1 blows disconnecting the 42V circuit from the loads
- Zener goes into off state once again after Fuse F1 blows

- Fuse F2 blows

2.3.2 Effectiveness Of The Zener Modification

We will prove the efficacy of the Zener circuit with respect to the condition $i_{21} > i_2$. Consider Figure 2.2. This figure compares the circuit with and without the Zener after the fault occurs. Observe voltage V_a and V_b . With the Zener in place, voltage V_b tends to stay around the Zener breakdown voltage. (Lets call the Zener breakdown voltage V_Z). Now concentrate on currents i_2 and i'_2 .

$$i_2 = (V_a - V_2)g_2 \tag{2.5}$$

$$i'_2 = (V_b - V_2)g_2 \approx (V_Z - V_2)g_2 \tag{2.6}$$

Figure 2.3 shows the comparison of the circuits with and without the Zener for the case when Fuse F1 has blown and compares them for the current i_{21} . We find that since the Zener is off, the two circuits are the same and thus $i'_{21} = i_{21}$.

It is noteworthy that the we have to this point only postulated that the circuit will work this way. We have yet to establish the conditions in which the circuit actually works in this manner. Note that, i'_2 (or i_2 in the Zener circuit) is a function of the Zener breakdown voltage (V_Z)and is thus manipulatable by choice of V_Z . i_{21} on the other hand stays unaffected by the inclusion of the Zener. Thus, with a proper choice of the reverse breakdown voltage (V_Z), we can bring i_2 down so that it may be able to satisfy $i_{21} > i_2$.

2.3.3 Analyzing The Zener Circuit

Figure 2.4 shows the actual Zener model that we will use. We take the Zener as an open circuit when it is not conducting and as a voltage source with a series resistance when it is conducting. The voltage source being equal to the Zener breakdown voltage. We will also relate the Zener breakdown voltage to V_1 as

$$V_Z = \alpha V_1 \tag{2.7}$$

Envisaging a strategy involving Fuse F1 blowing and then Fuse F2 blowing, we need the following conditions to be true for the operating sequence that we have in mind to occur.

1. $V_Z > V_p$: With reference to Figure 2.5, this condition states that the Zener will not turn on (or break down) under normal operating conditions of the 14V circuit.

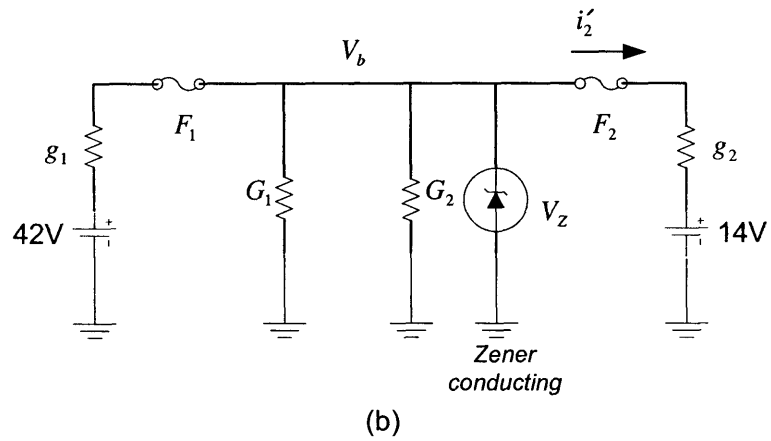
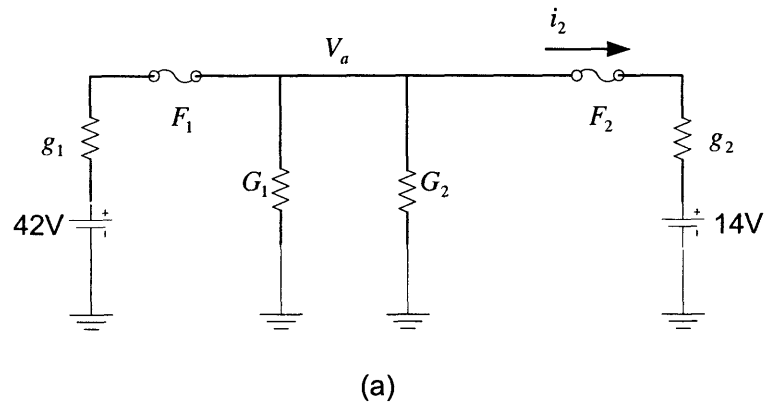
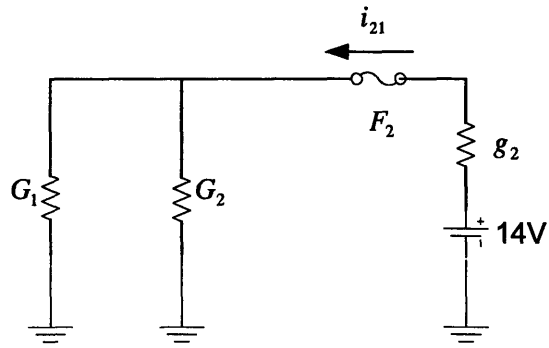
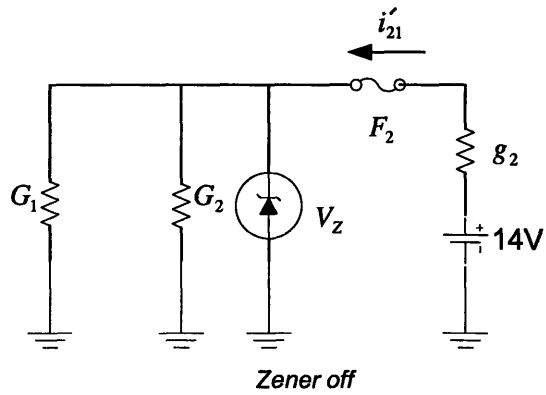


Figure 2.2: Comparison Of The Circuits With The Without The Zener At The Time Of The Fault



(a) Circuit without the Zener



(b) Circuit with the Zener

Figure 2.3: Comparison Of The Circuits With The Without The Zener After Fuse F1 Blows

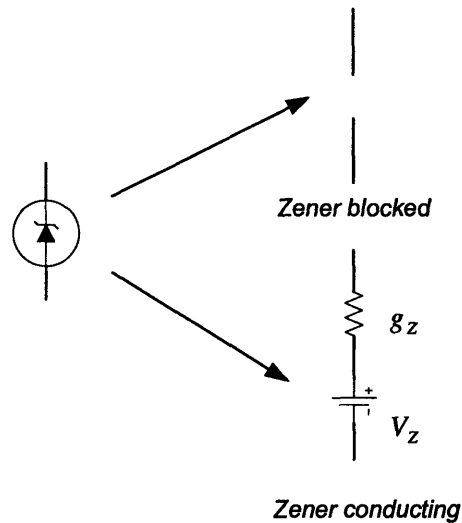


Figure 2.4: Zener Model

2. $V_Z < V_q$: With reference to Figure 2.6, this condition states that the Zener will turn on in the event of a fault. For the Zener to go into conduction, it is sufficient for the condition $V_Z < V_q$ to be met when the circuit is analyzed in the absence of the Zener. To indicate that, the Zener is shown in dotted lines.
3. $i_1 > I_{F1}$: With reference to Figure 2.7, this condition ensures that Fuse F1 will blow in the case of a fault.
4. $i_2 < I_{F2}$: With reference to Figure 2.7, this condition ensures that Fuse F2 will not blow in the case of a fault.
5. $i_{21} > I_{F2}$: With reference to Figure 2.8, this condition ensures that Fuse F2 will blow after Fuse F1 has blown.
6. $I_{F1} > I_{F1}^o$: This condition ensures that Fuse F1 will not blow under normal operating conditions of the 42V circuit.
7. $I_{F2} > I_{F2}^o$: This condition ensures that Fuse F2 will not blow under normal operating conditions of the 14V circuit.

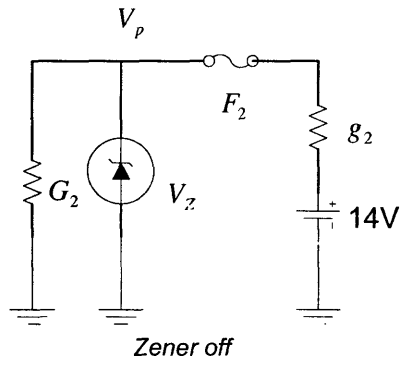


Figure 2.5: Figure For Condition 1

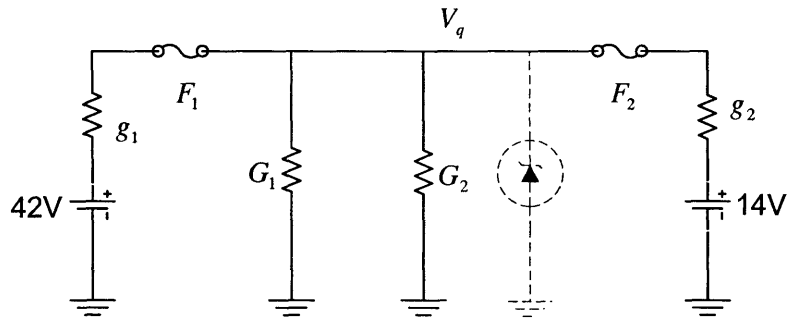


Figure 2.6: Figure For Condition 2

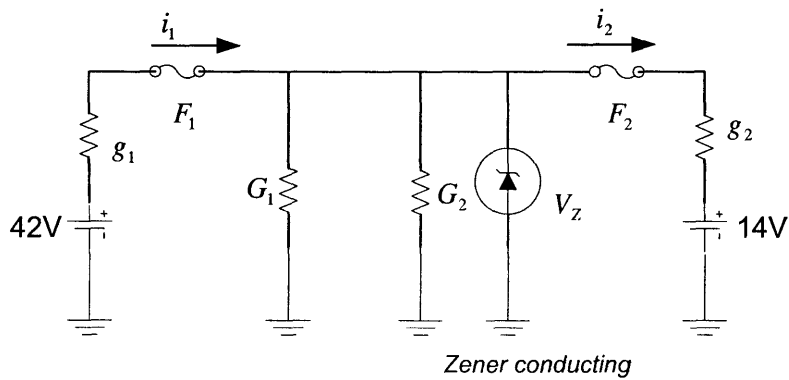


Figure 2.7: Figure For Condition 3 And 4

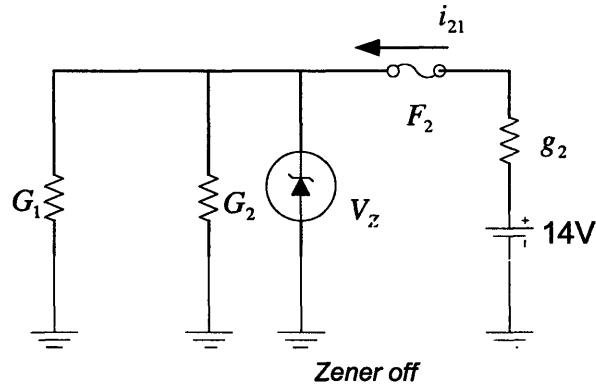


Figure 2.8: Figure For Condition 5

2.3.3.1 Condition 1

$$V_Z > V_p \tag{2.8}$$

$$\alpha V_1 > \frac{V_Z g_2}{G_2 + g_2} \tag{2.9}$$

or, if $V_1 = 3V_Z$,

$$\alpha > \frac{g_2}{3(G_2 + g_2)} \tag{2.10}$$

In the bottleneck case, when the fault occurs when there is no load on the 14V circuit, $G_2 = 0$. Thus,

$$\alpha > \frac{1}{3} \tag{2.11}$$

2.3.3.2 Condition 2

$$V_Z < V_q \tag{2.12}$$

$$\alpha V_1 > \frac{V_1 g_1 + V_Z g_2}{g_1 + g_2 + G_L} \tag{2.13}$$

or, if $V_1 = 3V_Z$,

$$\alpha > \frac{3g_1 + g_2}{3(g_1 + g_2 + G_L)} \tag{2.14}$$

2.3.3.3 Condition 3 And 6

Observe that if we can ensure $i_1 > I_{F1}^o$, then we can choose I_{F1} such that $i_1 > I_{F1} > I_{F1}^o$ and satisfy both of the conditions 3 and 6. Thus,

$$\frac{g_1[(V_1(g_2 + g_Z + G_L) - V_Z g_Z - V_2 g_2)]}{g_1 + g_2 + g_Z + G_L} < \frac{V_1 g_1 G_1}{g_1 + G_1} \quad (2.15)$$

Using, $V_1 = 3V_2$,

$$g_Z[3(g_1 - \alpha(g_1 + G_1))] > G_1 g_2 - 2g_1 g_2 - 3G_2 g_1 \quad (2.16)$$

Let us postulate that the coefficient of g_Z in Eq. (2.16) is a positive number. Then,

$$g_Z > \frac{G_1 g_2 - 2g_1 g_2 - 3G_2 g_1}{3(g_1 - \alpha(g_1 + G_1))} \quad (2.17)$$

This represents a condition on the on-state incremental conductance of the Zener, a condition which presumably can always be satisfied with a big enough device. The condition for the right hand side of Eq. (2.16) to be positive is

$$\alpha < \frac{g_1}{g_1 + G_1} \quad (2.18)$$

2.3.3.4 Condition 4, 5 And 7

Observe that if we can ensure $i_{21} > i_2, I_{F2}^o$, then we can choose I_{F2} such that $i_{21} > I_{F2} > i_2, I_{F2}^o$ and satisfy all of the conditions 4,5 and 7. Firstly, if we need i_2 to flow in the direction shown, then we need $V_b > V_2$ (Figure 2.2)

$$\frac{V_1 g_1 + V_Z g_Z + V_2 g_2}{g_1 + g_2 + g_Z + G_L} > V_2 \quad (2.19)$$

Using $V_1 = 3V_2$,

$$g_Z[3\alpha - 1] > G_L - 2g_1 \quad (2.20)$$

Using Eq. (2.11)

$$\alpha > \frac{1}{3} \quad (2.21)$$

so we can divide both sides of Eq. 2.20 by $(3\alpha - 1)$ without changing the direction of the inequality and,

$$g_Z > \frac{G_L - 2g_1}{3\alpha - 1} \quad (2.22)$$

This is a trivial condition which says that $g_Z >$ a negative number (check the numerator and remember $g_1 > G_L$). Now, we consider the regular conditions,

$$i_{21} > i_2 \quad (2.23)$$

$$\frac{V_2 g_2 G_L}{g_2 + G_L} > \frac{-g_2 [V_2 (g_1 + g_Z + G_L) - V_1 g_1 - V_Z g_Z]}{g_1 + g_2 + g_Z + G_L} \quad (2.24)$$

Using $V_1 = 3V_2$,

$$g_Z [G_L + (g_2 + G_L)(1 - 3\alpha)] > 2g_1 g_2 - 2G_L^2 + G_L g_1 - 2g_2 G_L \quad (2.25)$$

Now, the right hand side of Eq. 2.25 is a positive number for $g_1 > 2G_L$ (take $2g_2$ common from the first and fourth terms and G_L common from the second and third terms), and so should be the L.H.S. Thus,

$$G_L + (g_2 + G_L)(1 - 3\alpha) > 0 \quad (2.26)$$

or,

$$\alpha < \frac{g_2 + 2G_L}{3(g_2 + G_L)} \quad (2.27)$$

and,

$$g_Z > \frac{2g_1 g_2 - 2G_L^2 + G_L g_1 - 2g_2 G_L}{G_L + (g_2 + G_L)(1 - 3\alpha)} \quad (2.28)$$

Now, we remain with the condition,

$$i_{21} > I_{F2}^o \quad (2.29)$$

This results in the earlier bottleneck condition that we found out, namely Eq. (2.4) rewritten here

$$G_1 > G_{2max} \quad (2.30)$$

2.3.3.5 Collecting The Results And Analysis

Collecting all the constraints on α together, we have,

$$\frac{g_2}{3(G_2 + g_2)}, \frac{1}{3} < \alpha < \frac{g_1}{g_1 + G_1}, \frac{3g_1 + g_2}{3(g_1 + g_2 + G_L)}, \frac{2G_L + g_2}{3(g_2 + G_L)} \quad (2.31)$$

Collecting all the g_Z conditions we have,

$$g_Z > \frac{G_1 g_2 - 2g_1 g_2 - 3G_2 g_1}{3(g_1 - \alpha(g_1 + G_1))}, \frac{2g_1 g_2 - 2G_L^2 + G_L g_1 - 2g_2 G_L}{G_L + (g_2 + G_L)(1 - 3\alpha)} \quad (2.32)$$

We also have the bottleneck condition

$$G_1 > G_{2max} \quad (2.33)$$

Eq. (2.31),(2.32)and (2.33) summarize all the conditions needed for the strategy to work. Let us reduce the complexity. Consider first two expressions in the $\alpha < ()$ conditions. Let us postulate

$$\frac{3g_1 + g_2}{3(g_1 + g_2 + G_L)} < \frac{g_1}{g_1 + G_1} \quad (2.34)$$

This can be shown to be true if

$$g_2G_1 < 2g_1g_2 + g_1G_2 \quad (2.35)$$

which is satisfied always if $2g_1 > G_1$. Consider now two of the other expressions. Let us postulate

$$\frac{3g_1 + g_2}{3(g_1 + g_2 + G_L)} > \frac{2G_L + g_2}{3(g_2 + G_L)} \quad (2.36)$$

This holds if

$$g_1(2g_2 + G_L) > 2G_L(g_2 + G_L) \quad (2.37)$$

which again is true, as can be seen by comparison of each of the product constituents. Amongst the $() < \alpha$ conditions, it is also easy to see that

$$\frac{1}{3} > \frac{g_2}{3(G_2 + g_2)} \quad (2.38)$$

Thus, we are left with the final conditions on α as

$$\frac{1}{3} < \alpha < \frac{2G_L + g_2}{3(g_2 + G_L)} \quad (2.39)$$

Now let us consider the possible range within which α (and thus V_Z) can be chosen.

$$\Delta V_Z = \Delta \alpha V_1 \quad (2.40)$$

Equivalently,

$$\Delta V_Z = V_1 \left(\frac{2G_L + g_2}{3(g_2 + G_L)} - \frac{1}{3} \right) \quad (2.41)$$

Thus,

$$\Delta V_Z = \frac{14G_L}{g_2 + G_L} \quad (2.42)$$

or

$$14 < V_Z < 14 \left(1 + \frac{G_L}{g_2 + G_L} \right) \quad (2.43)$$

This is the allowable range of variability for V_Z , assuming the values of V_1 and V_2 remain fixed at exactly 42 and 14 volts. Since $g_2 \gg G_L$, this seems to be a very restrictive range.

2.3.3.6 Conclusions Concerning The Zener Circuit

After defining ΔV_Z as in Eq. (2.42), we will need to make sure that the Zener voltage can be maintained in that range, considering different operating temperature and device to device variability. Further, the circuit needs to work even if the bus voltages (in our model the bus voltages represent the 42V and 14V batteries) vary. Therefore the condition in Eq. (2.8) translates to

$$V_{Zmin} > V_{2max} \tag{2.44}$$

We have from [2] that the automobile will be expected to function with V_2 anywhere in the range of 9 to 16 volts, the protection circuit should also work in the face of variations of V_1 and V_2 . To rigorously assess these possibilities, it would be necessary to redo much of the foregoing, using different values of V_1/V_2 and different values for V_2 . But such extensive effort is probably unproductive.

The requirement in Eq. (2.44) and the required range of operation for V_2 tells us immediately that the lowest V_Z will be above 16 volts. Yet that same Zener diode is required to reliably turn on following a fault even when V_2 is as low as 9 volts and V_1 is at the same time way below nominal, perhaps as low as 30 volts. The requirement in Eq. (2.42) shows that for nominal V_1 and V_2 , for the circuit to operate as intended, the allowable range of V_Z is quite limited, perhaps only 10% of the nominal V_2 . If this analysis were repeated with $V_1 = 30V$ and $V_2 = 9V$, we would expect that the condition corresponding to Eq. (2.42) would restrict V_Z to within a small range (say 10%) above 9 volts.

Since it is impossible for V_Z to be both greater than 16 volts and also within 10% of 9 volts, the Zener circuit considered here works only if V_1 and V_2 are tightly constrained to their nominal values. When real world variability of operating conditions is considered, this circuit cannot always operate as intended.

2.4 Modification 2: Addition Of A Diode

We seek a further circuit modification which gives more flexibility in the selection of the Zener voltage V_Z to make the circuit work in all conditions. The condition $i_{21} > i_2$ is the hardest condition to fulfill. Observe that the most restrictive condition on α was given by Eq. (2.27) which was a result of the $i_{21} > i_2$ condition. Here we introduce a modification to the circuit that will get us rid of this condition totally.

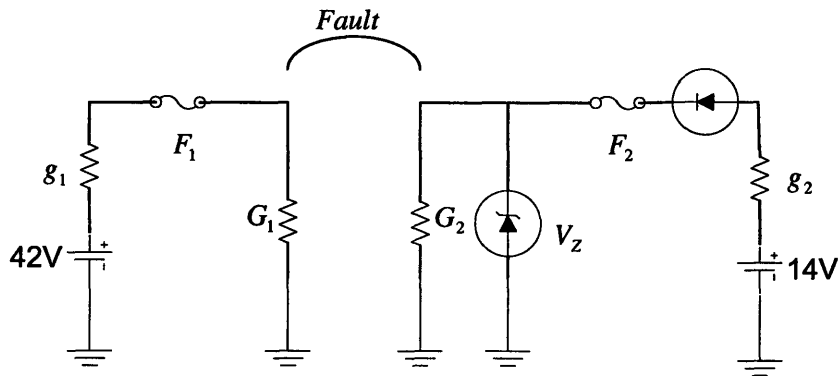


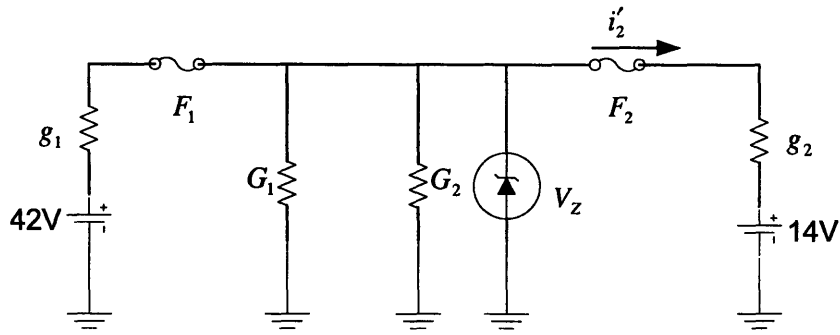
Figure 2.9: Circuit Modification With The Diode

The modification is in the form of a diode placed at the 14V circuit as shown in Figure 2.9.

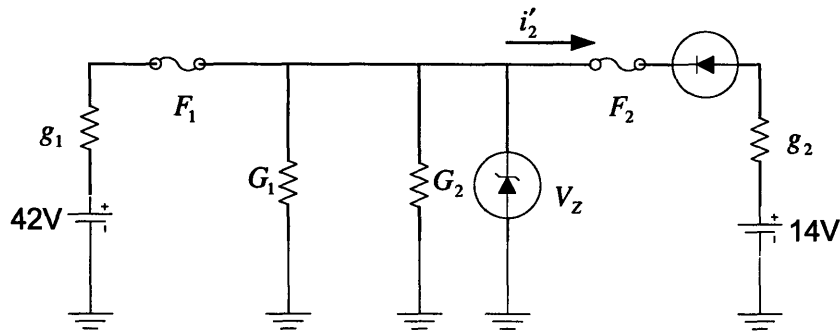
2.4.1 Desired Sequence Of Events With The Diode Modification

We want the following operation sequence to be followed with the Zener and the diode in place:

1. In the normal operation of the 14V circuit, the Zener is back biased but not to the Zener breakdown voltage. The diode is forward biased. Thus, the Zener acts as an open circuit and the diode acts as a short and therefore neither component affects normal circuit operation.
2. In the event of a 42V – 14V fault, the Zener goes into reverse breakdown and provides a shorting path to the ground. The diode however, become back biased and opens the circuit therefore isolating the 14V supply from the circuit.
3. Fuse F1 blows disconnecting the 42V circuit from the loads
4. The Zener goes into off state once again after Fuse F1 blows. The diode becomes conducting again.
5. Fuse F2 blows



(a) Circuit without the diode



(b) Circuit with the diode

Figure 2.10: Comparison Of The Circuits With And Without The Zener Plus Diode Modification At The Time Of The Fault

2.4.2 Effectiveness Of The Diode Modification

We will prove the efficacy of this modified circuit with respect to the condition $i_{21} > i_2$. Consider Figure 2.10. This figure compares the circuit with and without the modification before Fuse F1 blows. Observe that, since now the diode opens the circuit, there is no current i_2 flowing. Thus the condition $i_{21} > i_2$ is always satisfied. At this time, we will refrain from actually analyzing the circuit to quantify its superiority with respect to the Zener-only modified circuit. Instead, we will introduce another modification to the circuit which will get us through the condition in Eq. (2.4) as stated here again.

$$G_1 > G_{2max} \tag{2.45}$$

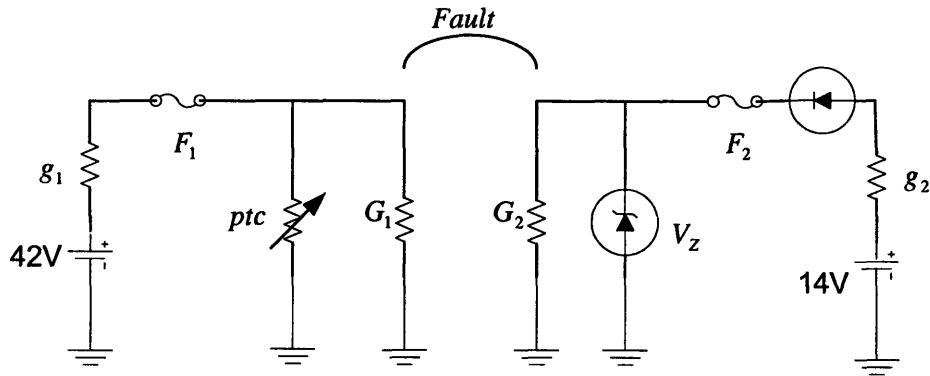


Figure 2.11: Circuit With The PTC Added

2.5 Modification 3: Addition Of A PTC (Positive Temperature Coefficient) Device

With this modification, we hope to get rid of the condition in Eq. (2.45). We add the PTC device to the circuit as shown in Figure 2.11. Please note that what we are calling a PTC device is an abstract device having the same circuit element properties as a real PTC device, for example the Polyswitch[®] family of devices manufactured by Tyco Electronics. When a small current is passed through these devices, they present a small resistance. When the current is raised above a threshold, the resistance quickly rises. As the resistance rises, it approaches the voltage of the source and the current in the device falls. The device remains in its new, high resistance state for as long as the potential is maintained across it. When the potential is removed (or reduced to a low enough value), the transition reverses and the device becomes a low resistance.

We postulate an abstract device because we have learned that Tyco's products, while exhibiting all the properties described above, will not provide adequate service life in the present application. We present this work because it illustrates the potential of a device with a strong positive temperature coefficient of resistivity. While we have not found a source for suitable devices, the existence of the Polyswitch[®] products offer hope that similar devices can be found. This same analysis also explains the performance of a circuit containing a synthesized circuit element (made e.g. with transistors) having the prescribed element properties.

2.5.1 Desired Sequence Of Events With The PTC Modification

We want the circuit to operate in the following sequence of events:

1. In the normal operating conditions, the Zener stays off and does not affect the functioning of the 14V circuit. The diode is forward biased and therefore does not affect the 14V circuit. The PTC device, due to the high voltage applied to it (42V), trips and becomes an open circuit. Thus, it does not affect the 42V circuit.
2. In the event of a fault, the Zener breaks down and conducts a large current so that Fuse $F1$ blows. The diode gets reverse biased and cuts off the 14V circuit. The PTC device stays tripped and thus is an open circuit still.
3. Fuse $F1$ blows.
4. The Zener goes back into off state due to the lessening of the voltage on it. The diode becomes forward biased and conducts again. The PTC device now working on a lower voltage goes into the conduction stage and thus provides a shorting path to the current from the 14V bus.
5. Fuse $F2$ blows.

2.5.2 Effectiveness Of The The PTC Modification

Keep in mind that all the conditions have been satisfied by the circuit without the PTC device except the one stated in Eq. (2.45). Observe that the PTC device has been added in parallel to the load G_1 . It becomes active only when Fuse $F1$ has blown and at that time it is in the conduction state. Let us call the conductance of the PTC device (and any resistance added in series to it) as G_{Pcond} . Thus, it *adds* to G_1 (since they are in parallel). Thus, the condition in Eq. (2.45) changes to

$$G_1 + G_{Pcond} > G_{2max} \quad (2.46)$$

Eq. 2.46 states that we should choose G_1 and G_{Pcond} such that their sum is more than G_2 . However, in practice, we cannot *choose* G_1 . Thus, we say that if we can satisfy

$$G_{Pcond} > G_{2max} \quad (2.47)$$

then we can be sure that the condition in Eq. (2.46) will always be satisfied. It is notable to see that the condition in Eq. (2.45) has been reduced to Eq. (2.47), effectively making it in our control. That is, we can design a PTC device such that it satisfies Eq. (2.47).

2.6 Conclusions

As discussed earlier, the PTC device that we have theorized is closest to a practical element known as the Polymeric Positive Temperature Coefficient or a PPTC device. We have analyzed the feasibility of the PPTC device in this circuit in Appendix A. Thus, in this chapter, we have tried to incorporate simple passive circuit elements to the original circuit and tried to implement the aforesaid fusing strategies. However, we have found that such implementations are either not robust enough to work on all operating conditions (Zener modification) or cannot be implemented with practical circuit elements (PTC device modification). Hence, we conclude that simple passive element modification will not be able to bring the circuit to a safe operating condition. In the next chapter, we will work on a strategy which works with active circuit elements and latching circuits to achieve the same end result.

Active Circuit Modifications For The Dual Voltage System

3.1 Introduction

We have seen in chapter 1 that the dual voltage fault by itself cannot clear the fuses. Circuit modifications by passive components gives us an effective scheme of fault clearance. However, its implementation with practical circuit elements is not feasible. Apart from this, we have only considered constant resistance loads which are switchable. In general, we can have any type of loads in an automobile. Thus, the discussions in the previous chapters was incomplete in that sense. In this chapter, we will incorporate different types of loads and introduce a new protection circuit based on fault sensing and active correction. The solution proposed here is simple and robust.

3.1.1 Fault Considered

As said earlier, we will be considering generic loads on the automobile. Thus, for sake of completeness, we will introduce the fault considered with general loads designated by L_1 and L_2 . The explanations for the various circuit parameters are as follows.

L_1 = Generic 42V load

L_2 = Generic 14V load

I_{F1} = Fusing current of fuse F_1

I_{F2} = Fusing current of fuse F_2

Figure 3.1 can be used to represent any possible 42V – 14V fault. Faults between different circuits will result in different values of the parameters I_{F1} and I_{F2} . Also L_1 and L_2 can take different configurations as purely resistive, constant current, or even constant power loads. Note that even for a fixed pair of circuits, it is possible for L_1 and L_2 to assume different values at different times. In particular, it is quite common for electrical loads to be turned off in which case the corresponding load is essentially absent.

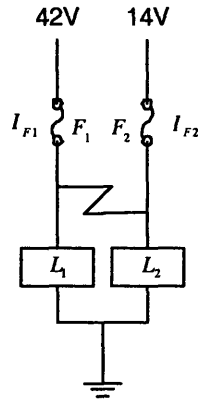


Figure 3.1: General Bus-To-Bus Fault

Parameter	Definition
V_1	Thevenin equivalent source of nominal value 42V for the 42V bus
V_2	Thevenin equivalent source of nominal value 14V for the 14V bus
g_1	Thevenin equivalent conductance for the 42V bus
g_2	Thevenin equivalent conductance for the 14V bus
L_1	42V Load
L_2	14V Load
I_{F1}	Fuse current rating for the 42V load
I_{F2}	Fuse current rating for the 14V load

Table 3.1: Parameter Definition For The Equivalent Circuit

3.2 Equivalent Circuit At Fault

We will introduce the equivalent circuit at fault with the generic loads in place instead of the constant resistance loads. Figure 3.2 shows the circuit. Table 3.1 defines the parameters of the circuit of Figure 3.2. We will also use the definitions of Table 3.2 for the various currents in the circuit. Please refer to Figure 3.3 for a diagrammatic definition of the currents.

3.3 Fault Clearance

In this section, we introduce the concept of an ideal fault detector system and try to develop its properties with respect to the actions it takes after the fault occurs. Assume that we

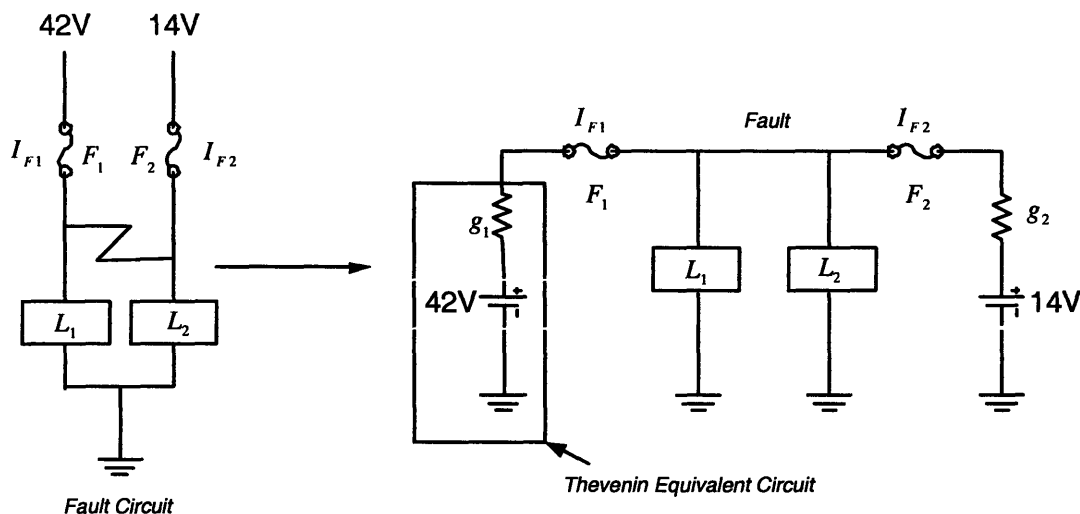


Figure 3.2: Equivalent Circuit At Fault

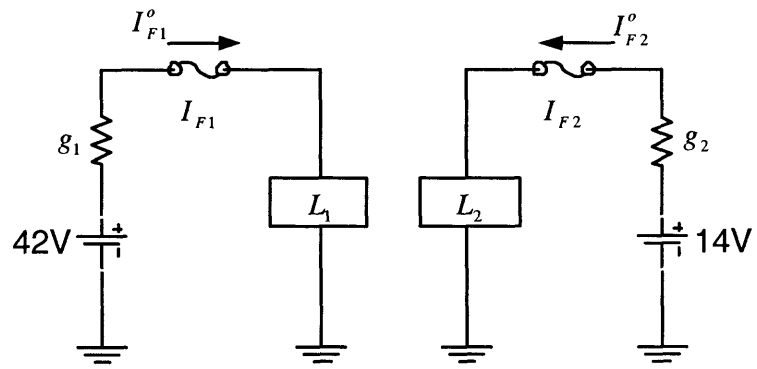
Current	Definition
i_1	Current in fuse F_1 at the time of fault
i_2	Current in fuse F_2 at the time of fault
I_{F1}^o	Maximum load current in the 42V load (L_1)
I_{F2}^o	Maximum load current in the 14V load (L_2)

Table 3.2: Current Definitions For The Faulted Circuit

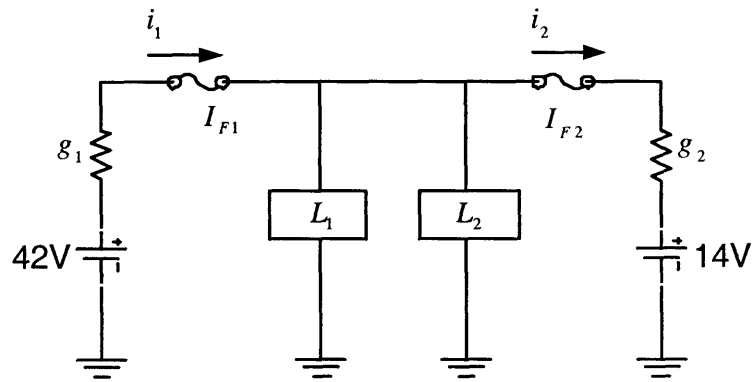
have a detector system which tells us exactly when the 42V – 14V fault occurs and assume also that it sits on the 14V circuit load. We want the following sequence of events to occur:

- The system detects the 42V – 14V fault.
- Once, it has detected the fault, the system shorts the 14V load directly to ground. Lets call this for the time being the "crowbar" action. Both the 42V and the 14V loads are now shorted directly to ground
- Since the 42V and the 14V fuses are designed to blow in the event of such a fault (i.e the loads shorting directly to ground), they blow out.
- Both the 42V and the 14V sources are therefore cleared from the fault.

Fig. 3.4 shows the above. It is also obvious that the detection system will be an active circuit which uses some voltages and currents in the circuit and makes a decision based on



(a) Normal operating conditions



(b) At the time of fault

Figure 3.3: Current Definitions

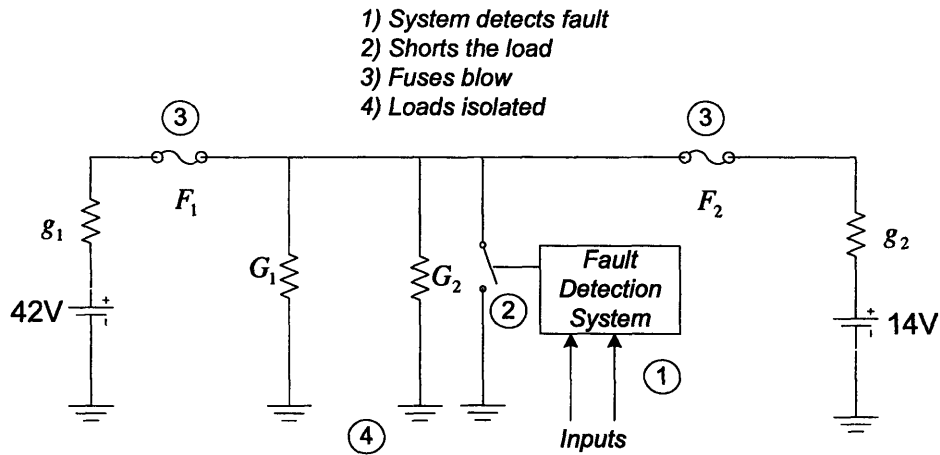


Figure 3.4: Operation Of The Fault Detector System

that. It is also clear that this detection system can be put on either of the 42V or the 14V circuits with essentially the same outcome.

3.4 Issues With The Fault Detection System

The ideal detection system that we just introduced needs to follow certain properties. Firstly, it preferably should not take inputs from both the 42V and the 14V circuits. That is because it does not know beforehand the other circuit to which the one associated with the protection device has faulted. If all the inputs to detection system come from the circuit to which it is linked, a simple system results. If information from the circuit to which it has faulted is required, it will be necessary to receive information from all the candidate circuits. Further, in the event of a fault, there will need to be means to determine which of the inputs from the candidate circuits contains actionable information. We would also want to inputs to this circuit to be simple and discrete. That will help to simplify the design of the system and reduce its cost.

3.5 Fault Indicators

For a 42V-14V fault, we find the following indicators that can help us detect the fault. Let's start with the fault indicator sitting on the 42V bus.

- Fuse F1 blowing: If the fuse blows in response to a 42V-14V fault, we can use this event to trigger our crowbar, which will also blow the 14V circuit fuse F_2 , since its load is also effectively shorted.
- Fuse F2 blowing: If the fuse blows in response to a 42V-14V fault, we can use this event to trigger our crowbar, which will also blow the 42V circuit fuse F_1 , since its load is also effectively shorted.
- Load voltage falling and Fuse Current rising in the 42V circuit: We can use this condition to trigger our crowbar. (Note that this indicator will not work if there is a constant power load attached to the bus under consideration.)
- Load voltage rising and Fuse current rising in the 14V circuit. (This indicator will also not work in the case of a constant power load attached to the circuit.)

Note that we did not separate voltage falling and current rising as independent indicators of the 42V-14V fault. This is so because in the accepted normal operation of the 42V circuit, the bus voltage (and thus the load voltage) may fall [3] and thus can trigger a false alarm. In a similar vein, we did not separate the voltage rise and current rise as two different indicators for the 14V circuit [2]. Also note that even if any one of the actions from above takes place in the event of a fault, we can use it effectively to activate our crowbar. That means that in one fault event Fuse F_1 might blow and trigger the crowbar sitting on the 42V load, while in another event Fuse F_2 might blow and trigger a crowbar sitting on the 14V load resulting in essentially the same action. The simplest and the most discrete signal might be blowing of the fuses F_1 or F_2 . In the next section, we will discuss its reliability as a fault indicator.

3.6 Fuse Blowing As A Fault Indicator

In the last section we talked about the crowbar action. It would be a simple act to provide a crowbar at each fuse. If we are able additionally to provide a reliable indication that a given circuit is involved is a bus-to-bus short circuit, we could trigger the corresponding crowbar and return the system to a safe state. In this section, we will show that the blowing

of a fuse can serve as the reliable indication we need.

To provide a reliable basis for action, it would be sufficient to know that in the event of any 42V-14V fault, either fuse F_1 or fuse F_2 will blow. To this point, we have not demonstrated that this is the case. In this section, we will develop the conditions under which we can rely on a bus-to-bus fault blowing either fuse F_1 or fuse F_2 . We will achieve this by placing restrictions on the range of values allowable for the equivalent source conductances g_1 and g_2 . In particular, we can achieve our guaranteed fuse-blowing objectives by making these conductances large enough so that the post-fault current will blow a fuse.

In practice, the conditions we will derive do not require the system designer to do things much differently than he would in the absence of the possibility of bus-to-bus fault. We will derive our conditions by requiring that under reasonable circumstances, only a small fraction of the source voltage appears across the source conductance, the larger fraction being applied to the load. In most cases, the system will achieve this objective at least approximately, even if it were designed without consideration of 42V-14V faults. All we will do is apply a quantitative restriction.

In a few cases, the system designer may need to change his design to meet our restriction. But if that happens, the change will probably be restricted to using a slightly larger wire cross section than would otherwise have been chosen. The cost and mass impact on the wire harness and the vehicle should be very modest.

For generality's sake, we will consider not only switchable resistance loads, but also switchable constant current and constant power loads. The reader will see that inclusion of these multiple load types substantially complicates the development which we will present. But even with this additional effort, we do not claim full generality. A circuit could, for example, serve a combination of these load types. It seems plausible that a system that is safe for both a purely resistive load and a purely constant-power load would be safe for a load which was a combination of these two types. But we do not provide a proof of this statement.

Even though this analysis lacks full generality, it nevertheless provides a substantial insight into a powerful method to make a system safe against 42V-14V faults. We show by example the methods necessary to expand this work to include other load types. If a system were to be designed using another load type, the designer could easily perform the indicated analysis, following by analogy the steps described here. Or he could restrict himself to load cases analyzed here.

Note that the proposed system of detection and protection will be designed for only the case of *hard* (i.e. low impedance) 42V-14V shorts. These will comprise the vast majority of

all 42V-14V faults. It is formally possible to have a high impedance short, which does not blow a fuse but still poses a hazard. The system being investigated cannot detect such a short. But this is probably an acceptable risk. The mechanisms which would produce such a high impedance short exist on today's auto fleet (where the risk is a high impedance short to ground). Today's fusing does not respond to high impedance shorts to ground, and the risk is evidently acceptable.

3.6.1 Circuit Model Considered

Fig. 3.3 (b) shows the general circuit that we will analyze. The character of the load present on any circuit at the time of a fault can vary widely. Quite commonly, the load will appear to the source to be a fixed resistance. In other cases, internal controls can cause a load to assume a constant power characteristics, in which the current drawn by the load rises as voltage falls. Constant current loads are a third possibility. Or the load on a given circuit could be some combination of these types.

We will consider all the cases where the load on any circuit has one of these characteristics (constant resistance, constant current and constant power). Note that there would be 9 different possible load combinations which will give rise to different circuits for analysis. These are shown in the following list. The symbol G_{42} stands for a 42V constant resistance load. Similarly I_{14} stands for a 14V constant current load. A load designated by the symbol P is a constant power load.

1. G_{42} shorting with G_{14}
2. G_{42} shorting with I_{14}
3. G_{42} shorting with P_{14}
4. I_{42} shorting with G_{14}
5. I_{42} shorting with I_{14}
6. I_{42} shorting with P_{14}
7. P_{42} shorting with G_{14}
8. P_{42} shorting with I_{14}
9. P_{42} shorting with P_{14}

3.6.2 Circuit Analysis

By considering the three kinds of loads, we have added complexity to the problem. Further, in the case of a constant power load, the current is a solution to a quadratic equation (see Appendix B). Although the circuits are simple enough that an analytical solution may have been possible, we chose a general numerical approach to the problem, using simulations in MATLAB to verify the circuit properties for all the operating conditions.

3.6.3 Electrical System Design Assumption

We will require that the electrical system feeding a given load be designed to assure that no more than a certain fraction of the open-circuit voltage will appear across the equivalent series resistance of the source, when the source is at its nominal value (42V or 14V). We will quantize this fraction with the symbol k . Thus, $k = 6$ means that $\frac{1}{6^k}$ or less of the voltage is dropped across the equivalent series resistance. If $k = 6$, than at least 83% (5/6) of the power delivered by the source is being consumed by the load. We will apply the same value of k to both the 42V and the 14V circuits. Thus the voltage drop across g_1 is no more than V_1/k , and the voltage drop across g_2 is no more than V_2/k .

Although we choose to use one value of k for both voltages, we will choose to use different values of k , depending on the type of load serves. Superficially, it would seem simpler to choose a single value of k , independent of the load type. But, when considered in more detail this choice leads to substantial complications. For example, a choice of $k = 6$ seems eminently reasonable for a resistive load. But for a constant power load, this same choice results in nearly half the source voltage being lost to a drop across the equivalent series resistance when the 14V source voltage is at the lower limit of its allowable range. This condition is at the minimum unattractive, and for many cases, may result in an unallowable temperature rise in the conductor serving the load.

So we choose to assign a different k to each load type, giving rise to the self explanatory symbols k_{res} , k_{cur} and k_{pow} . For a simpler discussion in the next sections, we will further define the ratio variables given in Table 3.3. To choose the various ratio variables, we will start with a choice of k_{res} , and then compute the remaining variables with the constraint that for a given equivalent series resistance, the maximum current, I_{MAX} , considering the full range of allowable source voltage, will be the same, independent of the type of load.

Ratio Variable	Definition
j_{res}	Constant resistance load ratio parameter for bus voltage at its minimum value
j_{cur}	Constant current load ratio parameter for bus voltage at its minimum value
j_{pow}	Constant power load ratio parameter for bus voltage at its minimum value
k_{res}	Constant resistance load ratio parameter for bus voltage at its nominal value
k_{cur}	Constant current load ratio parameter for bus voltage at its nominal value
k_{pow}	Constant power load ratio parameter for bus voltage at its nominal value
l_{res}	Constant resistance load ratio parameter for bus voltage at its maximum value
l_{cur}	Constant current load ratio parameter for bus voltage at its maximum value
l_{pow}	Constant power load ratio parameter for bus voltage at its maximum value

Table 3.3: Currents in the Different Loads

3.7 Generalized Approach To Circuit Analysis

The above assumptions are very useful in the sense that they help us to find the upper limit of the conductance of a constant resistance load, current in a constant current load and power in a constant power load. We show below how this is done. We will use the following nomenclature for the voltage source values (with "x" denoting that the voltage source can either be 14V or 42V):

1. $V_{x_{min}}$: Voltage V_x operating at minimum value.
2. $V_{x_{nom}}$: Voltage V_x operating at nominal value.
3. $V_{x_{max}}$: Voltage V_x operating at maximum value.

3.7.1 Constant Resistance Load

Relation Between Ratio Variables: For a constant resistance load served by a resistively modelled series impedance, the ratio of the voltage dropped on the series resistance will be a constant irrespective of the voltage impressed. Thus,

$$j_{res} = k_{res} = l_{res} \tag{3.1}$$

Limit On Load Values: Fig. 3.5(a) shows a voltage source driving a constant resistance load. The drop across g_x (lets denote it by v_{g_x}) in this case turns out to be

$$v_{g_x} = \frac{V_x G_x}{g_x + G_x} \quad (3.2)$$

Now since,

$$v_{g_x} < \frac{V_x}{k_{res}} \quad (3.3)$$

We get,

$$\frac{V_x G_x}{g_x + G_x} < \frac{V_x}{k_{res}} \quad (3.4)$$

which simplifies to

$$G_x < \frac{g_x}{k_{res} - 1} \quad (3.5)$$

or

$$G_{x_{max}} = \frac{g_x}{k_{res} - 1} \quad (3.6)$$

Thus, our assumption of proper wire harness design allows us to write an expression for the upper limit of how large the load can be, given the magnitude of the equivalent series source resistance. As source voltage varies, the current drawn by a constant resistance load is maximum when the voltage is maximum. Its value is

$$I_{MAX} = \frac{V_{x_{max}} g_x}{l_{res}} = \frac{V_{x_{max}} g_x}{k_{res}} \quad (3.7)$$

We will limit the current in the constant current and the constant power loads by this value and calculate the j , k and l values for them.

Let us calculate the ratio variables for the 14V circuit. If we choose a value of $k_{res} = 6$, then we have

$$j_{res} = k_{res} = l_{res} = 6 \quad (3.8)$$

With $I_{MAX} = 40A$, we get (from Eq. 3.7),

$$g_x = g_1 = 15 \text{ mhos} \quad (3.9)$$

3.7.2 Constant Current Load

Relation Between Ratio Variables: For a constant current load, the current remains constant as the input voltage varies from $V_{x_{min}}$ through $V_{x_{nom}}$ to $V_{x_{max}}$. Thus, we have

$$I_x = \frac{V_{x_{min}}g_x}{j_{cur}} = \frac{V_{x_{nom}}g_x}{k_{cur}} = \frac{V_{x_{max}}g_x}{l_{cur}} \quad (3.10)$$

Therefore,

$$j_{cur} = k_{cur} \frac{V_{x_{min}}}{V_{x_{nom}}}, \quad l_{cur} = k_{cur} \frac{V_{x_{max}}}{V_{x_{nom}}} \quad (3.11)$$

Limit On Load Values: This case is straightforward since the maximum current that can flow in a constant resistance load is also the limit on a constant current load. Thus,

$$I_x < I_{MAX} = \frac{V_{x_{max}}g_x}{k_{res}} \quad (3.12)$$

This condition can be used to define k_{cur} which is the drop at nominal voltage conditions. For a constant current load, the current remains the same irrespective of the voltage. Thus, if the load is drawing I_{max} at $V_{x_{max}}$, it will draw it at $V_{x_{nom}}$ too. Thus,

$$I_{MAX} = \frac{V_{x_{max}}g_x}{k_{res}} = \frac{V_{x_{nom}}g_x}{k_{cur}} \quad (3.13)$$

Thus, j_{cur} and l_{cur} can also be found.

With our earlier choice of $k_{res} = 6$, we get from Eq. 3.13 for $V_{x_{min}} = 9V$, $V_{x_{nom}} = 14V$ and $V_{x_{max}} = 16V$

$$k_{cur} = k_{res} \frac{V_{x_{nom}}}{V_{x_{max}}} = 5.25 \quad (3.14)$$

and from Eq. 3.11

$$j_{cur} = 3.375, \quad l_{cur} = 6 \quad (3.15)$$

3.7.3 Constant Power Load

Relation Between Ratio Variables: In this case, it's the power that remains constant as the voltage varies from $V_{x_{min}}$ through $V_{x_{nom}}$ to $V_{x_{max}}$. For details of the analysis of constant power loads attached to a constant voltage bus, refer to Appendix B. We have repeated Eq B.3 here for convenience

$$v_{g_x} = V_{x_{nom}} - \frac{V_x + \sqrt{V_x^2 - 4\frac{P_x}{g_x}}}{2} \quad (3.16)$$

Lets write this equation for the voltage at its nominal value.

$$v_{g_x} = V_{x_{nom}} - \frac{V_{x_{nom}} + \sqrt{V_{x_{nom}}^2 - 4 \frac{P_x}{g_x}}}{2} < \frac{V_{x_{nom}}}{k_{pow}} \quad (3.17)$$

where we have included the criteria of dropping less than $\frac{1}{k_{pow}}$ of the total voltage across the series resistance. This translates into

$$P_x < g_x V_{x_{nom}}^2 \left(\frac{k_{pow} - 1}{k_{pow}^2} \right) \quad (3.18)$$

For the voltage at its minimum value, we will have a similar expression

$$P_x < g_x V_{x_{min}}^2 \left(\frac{j_{pow} - 1}{j_{pow}^2} \right) \quad (3.19)$$

Since the power in both expressions is the same, we can equate them and also include the expression for the variables at $V_x = V_{x_{max}}$

$$g_x V_{x_{min}}^2 \left(\frac{j_{pow} - 1}{j_{pow}^2} \right) = g_x V_{x_{nom}}^2 \left(\frac{k_{pow} - 1}{k_{pow}^2} \right) = g_x V_{x_{max}}^2 \left(\frac{l_{pow} - 1}{l_{pow}^2} \right) \quad (3.20)$$

These equations define the relationship between j_{pow} , k_{pow} and l_{pow} .

Limit On Load Values: In the case of a constant power load, the maximum current is drawn when the voltage applied to the load is at its minimum value. Thus, I_{max} is achieved at $V_{x_{min}}$. Thus,

$$I_{MAX} = \frac{V_{x_{max}} g_x}{l_{res}} = \frac{V_{x_{min}} g_x}{j_{pow}} \quad (3.21)$$

Thus,

$$j_{pow} = l_{res} \frac{V_{x_{min}}}{V_{x_{max}}} \quad (3.22)$$

Thus, we can compute j_{pow} and l_{pow} too. The maximum power is given by Eq. 3.18 and is thus,

$$P_{x_{max}} = g_x V_{x_{nom}}^2 \left(\frac{k_{pow} - 1}{k_{pow}^2} \right) \quad (3.23)$$

The values for the ratio variables for the constant power loads will therefore be (from Eq. 3.22),

$$j_{pow} = 3.37 \quad (3.24)$$

And from Eq. 3.20 we have,

$$k_{pow} = 10.5 \quad (3.25)$$

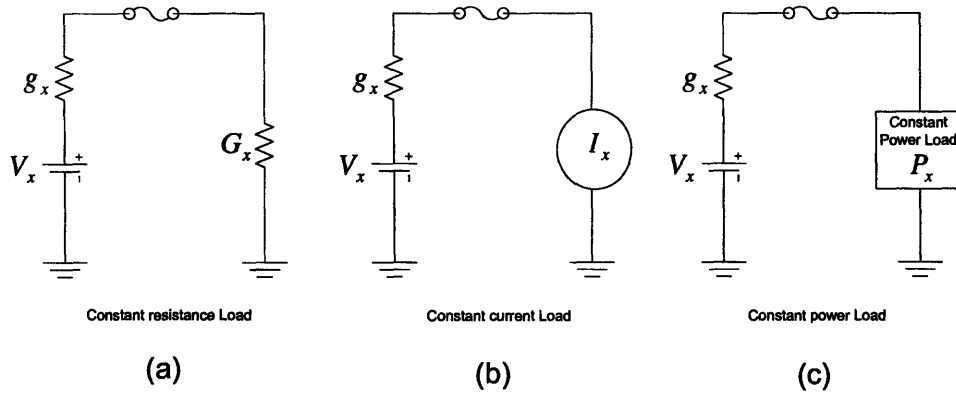


Figure 3.5: Load Value Limits

and,

$$l_{pow} = 14.081 \tag{3.26}$$

3.7.4 Load Value Limits And Design Criteria And The Value Of k_{res}

The aim of the previous discussion was to develop a strategy for choosing loads value limits (given by $G_{x_{max}}$, $I_{x_{max}}$ and $P_{x_{max}}$ in Eq. 3.6, Eq. 3.13 and Eq. 3.23) and design criteria to ensure these maximum values. The design criteria are in the form of the k values since an electrical system designer would want to work with nominal values. To summarize the discussions in the previous sections, we will state the final results as thus, starting with a chosen value of k_{res} . By, Eq. 3.13

$$k_{cur} = k_{res} \frac{V_{x_{nom}}}{V_{x_{max}}} \tag{3.27}$$

By eliminating j_{pow} between Eq. 3.21 and Eq. 3.20, we get,

$$k_{pow} = \frac{2}{1 - \sqrt{1 - \left(\frac{V_{x_{min}}}{V_{x_{nom}}}\right)^2 \left[1 - \left(\frac{k_{res} \left(\frac{V_{x_{min}}}{V_{x_{max}}}\right) - 2}{k_{res} \left(\frac{V_{x_{min}}}{V_{x_{max}}}\right)}\right)^2\right]}} \tag{3.28}$$

	Constant Resistance Load	Constant Current Load	Constant Power Load
<i>j</i>	$j_{res} = 6$	$j_{cur} = 3.38$	$j_{pow} = 3.38$
<i>k</i>	$k_{res} = 6$	$k_{cur} = 5.25$	$k_{pow} = 10.5$
<i>l</i>	$l_{res} = 6$	$l_{cur} = 6$	$l_{pow} = 14.08$
	$G_{2max} = 3 \text{ mhos}$	$I_{2max} = 40 \text{ A}$	$P_{2max} = 253 \text{ W}$

Table 3.4: Values For Ratio Variables And Maximum Loads For The 14V Circuit For $g_2 = 15 \text{ mhos}$

Thus, we have expressed all the k 's in terms of k_{res} . Now, we need to take care of the maximum load values. We have,

$$G_{xmax} = \frac{g_x}{k_{res} - 1} \quad (3.29)$$

$$I_{xmax} = \frac{V_{xnom} g_x}{k_{cur}} \quad (3.30)$$

$$P_{xmax} = g_x V_{xnom}^2 \left(\frac{k_{pow} - 1}{k_{pow}^2} \right) \quad (3.31)$$

Thus, started with choosing a value for k_{res} , we can decide the values of k_{cur} and k_{pow} and also come up with the maximum values for the load ratings of constant current and constant power loads.

So there are 3 k 's that we need to consider, namely k_{res} , k_{cur} and k_{pow} . A similar set of k 's exist for 42V and the 14V systems. So, our 14V electrical system designer would keep track of 3 values so that the maximum current in any 14V circuit is limited to a certain value. Similarly, the 42V electrical system designer will also keep track of the rest of the 3 k 's. For illustration, we have summarized the k values for the 14V circuit in Table 3.4. We have also included the maximum load ratings based on Eq. 3.29, Eq. 3.30 and Eq. 3.31. Thus, a 14V wire harness designer will keep the numbers $k_{res} = 6$, $k_{cur} = 5.25$ and $k_{pow} = 10.5$ in mind. For a choice of $k = 6$ for the 42V circuit we get the following table shown in Table 3.5. For reference, the voltages V_{1min} , V_{1nom} and V_{1max} are 30V, 42V and 48V. The assumption taken in calculating the values of the maximum loads is that the value of g_1 which gives the maximum current in the 42V circuit is the same as the value of g_2 which gives the maximum current in the 14V circuit. Thus, for the maximum current condition $g_1 = g_2 = 15 \text{ mhos}$. This choice is completely arbitrary and is at the sole discretion of the designer. If he wants to serve loads with current rating higher than 120A (or for that matter power rating higher than 2683W), he will need to use harnesses with higher conductance than 15 mhos.

	Constant Resistance Load	Constant Current Load	Constant Power Load
<i>j</i>	$j_{res} = 6$	$j_{cur} = 3.75$	$j_{pow} = 3.75$
<i>k</i>	$k_{res} = 6$	$k_{cur} = 5.25$	$k_{pow} = 8.90$
<i>l</i>	$l_{res} = 6$	$l_{cur} = 6$	$l_{pow} = 12$
	$G_{1max} = 3 \text{ mhos}$	$I_{1max} = 120 \text{ A}$	$P_{1max} = 2683 \text{ W}$

Table 3.5: Values For Ratio Variables And Maximum Loads For The 42V Circuit For $g_1 = 15 \text{ mhos}$

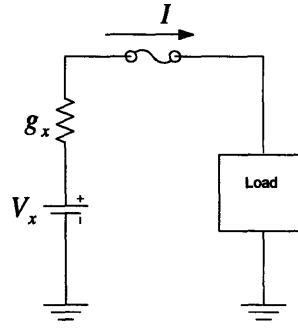


Figure 3.6: Circuit For The Discussion Of The Fuse Current Ratings

3.7.5 Fuse Ratings

The fuses are designed such that they should not blow in the event of supply voltage swings. This means that they allow a suitable range of supply voltage variation for which they would not blow. The fuse ratings are therefore closely linked to the maximum current that can flow in the circuit. Therefore, if we designate the upper limits of the range as V_{1max} and V_{2max} respectively for the 42V and the 14V circuits, we can determine the lower bound on the fuse rating current (defined in Table 3.2) as

$$I_{F1}^o = \frac{V_{1max}g_1}{k_{res}} \quad (3.32)$$

$$I_{F2}^o = \frac{V_{2max}g_2}{k_{res}} \quad (3.33)$$

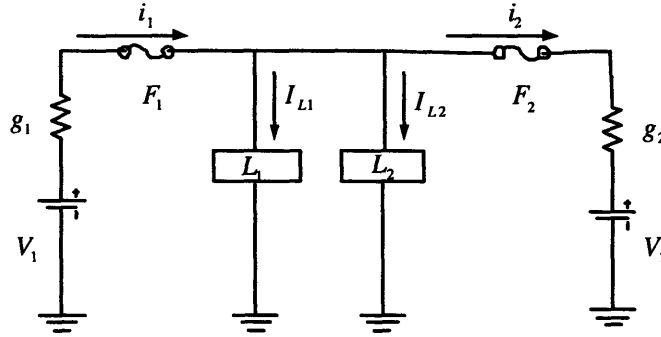


Figure 3.7: Circuit For The General Equation

Load	Current in the Load
Constant Resistance	VG_x
Constant Current	I_x
Constant Power	$\frac{P_x}{V}$

Table 3.6: Currents in the Different Loads

3.7.6 General Circuit Equation

Consider the generalized circuit model shown in Fig. 3.7. We want to analyze this circuit for the case when the loads 1 and 2 can be any of constant resistance, constant current and constant power. Consider Table 3.6. It shows the currents flowing in the loads given the voltage V applied on them. Thus, for example, if we want to analyze a 42V constant resistance load shorting with a 14V constant current load then we can find the *total load current* as

$$I_L = I_{L1} + I_{L2} = VG_1 + I_2 \quad (3.34)$$

Thus, for any load combination we will get the total load current I_L as a function of V . Lets call this function $f(V)$ for the time being.

$$I_L = f(V) \quad (3.35)$$

This I_L is also equal to $i_1 - i_2$. Thus

$$i_1 - i_2 = I_L = f(V) \quad (3.36)$$

or,

$$(V_1 - V)g_1 + (V_2 - V)g_2 = I_L = f(V) \quad (3.37)$$

3.7.7 Choice Of Bus Voltages V_1 And V_2

Another thing to be noted is that, in the event of a fault, the voltages may not be at their designed values of 42V and 14V. They may be undergoing a dc swing as explained earlier. We need to find the values of V_1 and V_2 which minimizes the fault currents i_1 and i_2 . Let us call this as the *worst case bus voltage* condition. This can be done by analyzing the derivatives $\frac{\delta i_1}{\delta V_1}$, $\frac{\delta i_1}{\delta V_2}$, $\frac{\delta i_2}{\delta V_1}$ and $\frac{\delta i_2}{\delta V_2}$. For example, to evaluate $\frac{\delta i_1}{\delta V_1}$, we will begin with Eq. (3.37) and take its derivative with respect to V_1 .

$$\left(1 - \frac{\delta V}{\delta V_1}\right)g_1 + \left(0 - \frac{\delta V}{\delta V_1}\right)g_2 = \frac{\delta f(V)}{\delta V} \frac{\delta V}{\delta V_1} \quad (3.38)$$

Thus,

$$\frac{\delta V}{\delta V_1} = \frac{g_1}{g_1 + g_2 + f'(V)} \quad (3.39)$$

where we have replaced $\frac{\delta f(V)}{\delta V}$ with $f'(V)$. Now, since

$$i_1 = (V_1 - V)g_1 \quad (3.40)$$

we get,

$$\frac{\delta i_1}{\delta V_1} = \left(1 - \frac{\delta V}{\delta V_1}\right)g_1 \quad (3.41)$$

Substituting $\frac{\delta V}{\delta V_1}$ from Eq. (3.39), we get,

$$\frac{\delta i_1}{\delta V_1} = \frac{g_1(g_2 + f'(V))}{g_1 + g_2 + f'(V)} \quad (3.42)$$

In a similar fashion, we can find out all the derivatives $\frac{\delta i_1}{\delta V_2}$, $\frac{\delta i_2}{\delta V_1}$ and $\frac{\delta i_2}{\delta V_2}$. Thus, all the derivatives are given by

$$\frac{\delta i_1}{\delta V_1} = \frac{g_1(g_2 + f'(V))}{g_1 + g_2 + f'(V)} \quad (3.43)$$

$$\frac{\delta i_1}{\delta V_2} = \frac{-g_1 g_2}{g_1 + g_2 + f'(V)} \quad (3.44)$$

$$\frac{\delta i_2}{\delta V_1} = \frac{-g_2(g_1 + f'(V))}{g_1 + g_2 + f'(V)} \quad (3.45)$$

$$\frac{\delta i_2}{\delta V_2} = \frac{g_1 g_2}{g_1 + g_2 + f'(V)} \quad (3.46)$$

Case	Load Combination	$f(V)$	$f'(V)$
1	$G_{42} - G_{14}$	$VG_1 + VG_2$	$G_1 + G_2$
2	$G_{42} - I_{14}$	$VG_1 + I_2$	G_1
3	$I_{42} - G_{14}$	$I_1 + VG_2$	G_2
4	$I_{42} - I_{14}$	$I_1 + I_2$	0
5	$G_{42} - P_{14}$	$VG_1 + P_2/V$	$G_1 - P/V^2$
6	$I_{42} - P_{14}$	$I_1 + P_2/V$	$-P_2/V^2$
7	$P_{42} - G_{14}$	$P_1/V + VG_2$	$-P_1/V^2 + G_2$
8	$P_{42} - I_{14}$	$P_1/V + I_2$	$-P_1/V^2$
9	$P_{42} - P_{14}$	$P_1/V + P_2/V$	$-P_1/V^2 - P_2/V^2$

Table 3.7: $f'(V)$ Values For The Different Load Combinations Under Consideration

Given,

$$g_2 + f'(V) > 0 \quad \text{and} \quad g_1 + f'(V) > 0 \quad (3.47)$$

or,

$$f'(V) > -g_1 \quad \text{and} \quad f'(V) > -g_2 \quad (3.48)$$

we can say that all the derivatives will either be positive ($\frac{\delta i_1}{\delta V_1}$ and $\frac{\delta i_2}{\delta V_1}$) or negative ($\frac{\delta i_1}{\delta V_2}$ and $\frac{\delta i_2}{\delta V_2}$). Thus, given condition in Eq. (3.48), we can say that i_1 and i_2 minimize when V_1 minimizes and V_2 maximizes. This is therefore the *worst case bus voltage* condition that we were looking for. In other words, if V_1 is at its minimum and V_2 is at its maximum (given Eq. (3.48)), we will have both the fault currents at their minimum values. Let us see the load combinations for which this condition is obeyed. See Table 3.7. The cases for $G_{42} - G_{14}$, $G_{42} - I_{14}$, $I_{42} - G_{14}$ and $I_{42} - I_{14}$ are pretty straight forward. For them, $f'(V) \geq 0$ and thus the condition in Eq. (3.48) is satisfied. Let us analyze the case for $I_{42} - P_{14}$. Condition in Eq. (3.48) would mean

$$f'(V) = -P_2/V^2 > -g_1 \quad (3.49)$$

Thus,

$$P_2/V^2 < g_1 \quad (3.50)$$

Let us try to maximize the left hand side of this equation. The maximum power that bus voltage V_2 can provide is given by (See Appendix B)

$$P_{2max} = \frac{V_2^2 g_2}{4} \quad (3.51)$$

Substituting,

$$\frac{V_2^2 g_2}{4V^2} < g_1 \quad (3.52)$$

Let us replace V_2^2 with V^2 since $V > V_2$,

$$\frac{V^2 g_2}{4V^2} < g_1 \quad (3.53)$$

Thus, we get

$$\frac{g_2}{4} < g_1 \quad (3.54)$$

which is plausible but not always true. The other condition

$$f'(V) = -P_2/V^2 > -g_2 \quad (3.55)$$

will lead to

$$\frac{g_2}{4} < g_2 \quad (3.56)$$

which is always satisfied. Thus, for the case $I_{42} - P_{14}$, we can be sure that the derivatives $\frac{\delta i_1}{\delta V_1}$ and $\frac{\delta i_2}{\delta V_1}$, will be positive and negative respectively. Hence, for this case, we can put V_1 at its extremum value (i.e. minimum), but we cannot do the same with V_2 . The case for $G_{42} - P_{14}$ also behaves in the same way since $f'(V)$ for that can be equal to $f'(V)$ for $I_{42} - P_{14}$ since,

$$G_1 - P/V^2 = -P/V^2 \quad \text{for} \quad G_1 = 0 \quad (3.57)$$

We will do a similar analysis for the case $P_{42} - I_{14}$.

$$P_1/V^2 < g_1 \quad (3.58)$$

Let us try to maximize the left hand side of this equation. The maximum power that bus voltage V_1 can provide is given by (See Appendix B)

$$P_{1max} = \frac{V_1^2 g_1}{4} \quad (3.59)$$

Substituting,

$$\frac{V_1^2 g_1}{4V^2} < g_1 \quad (3.60)$$

Let us replace V with V_2 to maximize the left hand side.

$$\frac{V_1^2 g_1}{4V_2^2} < g_1 \quad (3.61)$$

which leads to

$$V_2 > V_1/2 \quad (3.62)$$

Obviously, barring the case when $V_2 = 16V$ and $V_1 = 30V$, we are always sure that $V_2 < V_1/2$. Thus, we see that the condition in Eq. (3.48) is never satisfied for the case of the load being $P_{42} - I_{14}$. Similar statements can be made for the load cases being

$P_{42} - P_{14}$ and $P_{42} - G_{14}$. Thus, for those load combinations the *worst case bus voltages* are not the extremum values (i.e. minimum for V_1 and maximum for V_2).

Thus, for the first four load cases, we have the *worst case bus voltage* condition for both V_1 and V_2 , for the next two cases, we have the *worst case bus voltage* in an explicit form only for V_1 , while for the last three cases, we do not have the condition in any explicit form. We could, if we wanted, have three different types of simulation for these three types. However, for the sake of simplicity, we will treat the last five cases with one method of solution and simulate them for varying values of both V_1 and V_2 . This means that for the first four cases, the bus voltages can be fixed to their worst case values and the rest of the circuit can be simulated for the remaining independent parameters. For the last five load cases, the bus voltages will also be independent parameters which can vary over their entire range (i.e. $30V < V_1 < 48V$ and $9V < V_2 < 16V$).

3.8 Simulation Settings

Now that we have relations for the fault currents and the fuse ratings, we can simulate the circuit for a range of the independent circuit parameters and check whether one of the two fuses blow or not. We choose the following values for the parameters.

- $V_{1max} = 48V$
- $V_{2max} = 16V$
- $V_{1nom} = 42V$
- $V_{2nom} = 14V$
- $V_{1min} = 30V$
- $V_{2min} = 9V$
- $F1 = 1.5$
- $F2 = 1.5$
- $k_{res} = 6$ for both the 14V and the 42V circuits

The values $F1$ and $F2$ refer to the safety factor attached to the fuses. In circuit terms, it means that the fuse would not blow until its rated current is exceed by $F1$ times its value.

Mathematically, for fuse F_1 to blow,

$$i_1 > F1 * I_{F1}^o \quad (3.63)$$

3.8.1 Independent Circuit Parameters

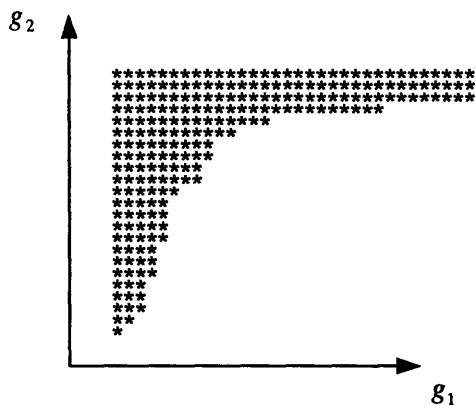
In this section, we will try to ascertain the independent circuit parameters for the simulation settings. Lets take the example of a 42V constant resistance load shorting with a 14V constant power load. The following can be separated as independent variables.

- The two conductances g_1 and g_2 can be varied independently of each other. Eq. 3.9 gives the value of $g_x = 15$ mhos for the 14V circuit. Thus, g_2 can vary from 0-15 mhos. Since, we do not know the value of the maximum current in the 42V circuit to carry out a similar calculation for g_1 , we will choose its range from 0-15 mhos too. Thus, g_1 and g_2 vary from 0-15 mhos.
- The conductance value of the 42V load (G_1) can be treated as independent provided it obeys the relation given in Eq. (3.5). Thus, it can be varied from 0 to $\frac{g_1}{5}$
- The power value of the 14V constant power load (P_2) can be treated as independent provided it obeys the relation given in Eq. (3.31). The computed value for k_{pow} is 10.5. And therefore, P_2 can be varied from 0 to $0.0861V_2^2g_2$
- As discussed earlier, this load case will treat V_1 and V_2 as independent parameters varying over their full ranges.

3.8.2 Goal Searching

In the output of this simulation, we will be searching for any combination of the independent parameters which can result in a condition in which neither of the fuses F_1 or F_2 blow. Thus, all the other combinations will lead to one of the fuses blowing (if both of the fuses *can* blow, then the one with the smaller time to blow will blow first). Thus, in the simulation we will search for points (in the independent parameter space) at which both of the following conditions are obeyed.

$$i_1 < I_{F1}^o \quad \text{and} \quad i_2 < I_{F2}^o \quad (3.64)$$



* denote where both the fuses wouldn't blow

Figure 3.8: Simulation Output Plot Used

3.8.3 Output Of The Simulation

To illustrate the output of the simulation, we will plot the values of g_1 and g_2 for which none of the fuses blow. Observe that there will be two or more additional independent parameters (in the case considered above there are four V_1 , V_2 , G_1 and P_2) but we do not care about their actual values, only whether they lead to a case in which neither of the fuses blow. Any useful result cannot include a situation where these other independent parameters lie within their allowable range, our solution must apply for any allowable value. On the other hand, we can design or at least choose g_1 and g_2 such that they may obey the relation we want (in this case, at least one of the fuses blowing, or the complement of the condition that we are looking at). Since our point search in the g_1 and g_2 plane is for discrete points, we will get an output of the kind shown in Fig. 3.8. The "*" signs on the graph indicate the values of g_1 and g_2 at which there exist some G_1 and P_2 for which neither of the fuses will blow for at least one combination of V_1 and V_2 . Thus, they represent values which we need to avoid. Fig. 3.9 and Fig. 3.10 shows the result plotted for g_1 on the x-axis and g_2 on the y-axis with values ranging from 0-15 for each conductance.

3.8.4 Simulation Code Example

The simulation code is present in Appendix C.

3.8.5 Simulation Results

We performed calculations as described above for all possible combinations of load types. For all values of $V_{1min} > 30V$, we do not get any values for g_1 and g_2 for which neither of the fuse blows. The graphs in Fig. 3.9 and Fig. 3.10 show the results of the simulation for $V_{1min} = 30V$.

3.9 Further Simulations

In this simulation, we used the values of the constant variables as given in Section 3.8. The choice of $V_1 = 30V$ has the relevance that this represents the lowermost value of the 42V bus voltage. With the values chosen for simulation right now, we could not achieve fuse blowing at $V_1 = 30V$. We can rerun the simulations for different values of the other parameters. In particular, we can start with a high value of k_{res} and with $F1 = F2 = 1$ and keep on increasing F1, F2 until we first fail to achieve fusing. Then we can choose a different value of k_{res} and repeat the same.

3.10 Conclusions

After the simulations run on these combinations of the fuse safety factors and k_{res} , we get the graph in Fig. 3.11, which shows choices of fuse safety factors and k_{res} which meet our goal. Each square corresponds to a case for which fusing takes place at a minimum of $V_1 = 30V$ (and consequently not at $V_1 = 29V$). Similarly, the triangles correspond to 29V. This result clearly shows that we can arrive at a strategy for choosing the appropriate fuse safety factors and the ratio k_{res} to ensure that for all possible combinations of the voltages and currents in the circuit, we will achieve fusing of at least one of the fuses. We can therefore activate a crowbar and achieve active response to a 42V-14V fault.

The beauty of this strategy is that it can do no harm. If following a fuse clearing, we actuate the corresponding crowbar, one of two results are obtained:

- If there is no 42V-14V fault, nothing happens.
- If there is a 42V-14V short, the action of the crowbar brings the circuit to a safe state.

Note that we do not claim to have done an exhaustive search over all possible load states for all circuits. Combination loads (e.g. constant resistance and constant power elements on the same circuit) can certainly exist. It is possible that combination loads will present protection challenges less demanding than the "pure" or "extreme" loads considered here, but we offer no proof. And other load characteristics can exist. One example is a dc motor with a high inertia.

It is possible that in evaluating the design for any given automobile, other calculation methods may prove more efficient. If the vast majority of possible faults are constant-resistance-to-constant-resistance, they can be quickly dispatched, and the remaining possibilities may be explicitly evaluated pairwise in an exhaustive search. Some possible circuit pairs may be removed from consideration (if, for example, they share no common loads or cable sections).

We do expect that the basic elements of the analysis presented here will continue to be useful. And we believe that the general design method we illustrated will be nearly universally effective at reaching satisfactory final designs. In particular, we refer to selecting higher values for k_{res} . In practice, this is equivalent to selecting heavier wires in the wire harness. Heavier wires deliver more fault current, clearing fuses in more cases. In practice, the need for heavier wires may well be limited to one or a few problem circuits. The selection of one number (k_{res}) to apply to every circuit in the car is a computational convenience which probably should not be used in the practical case.

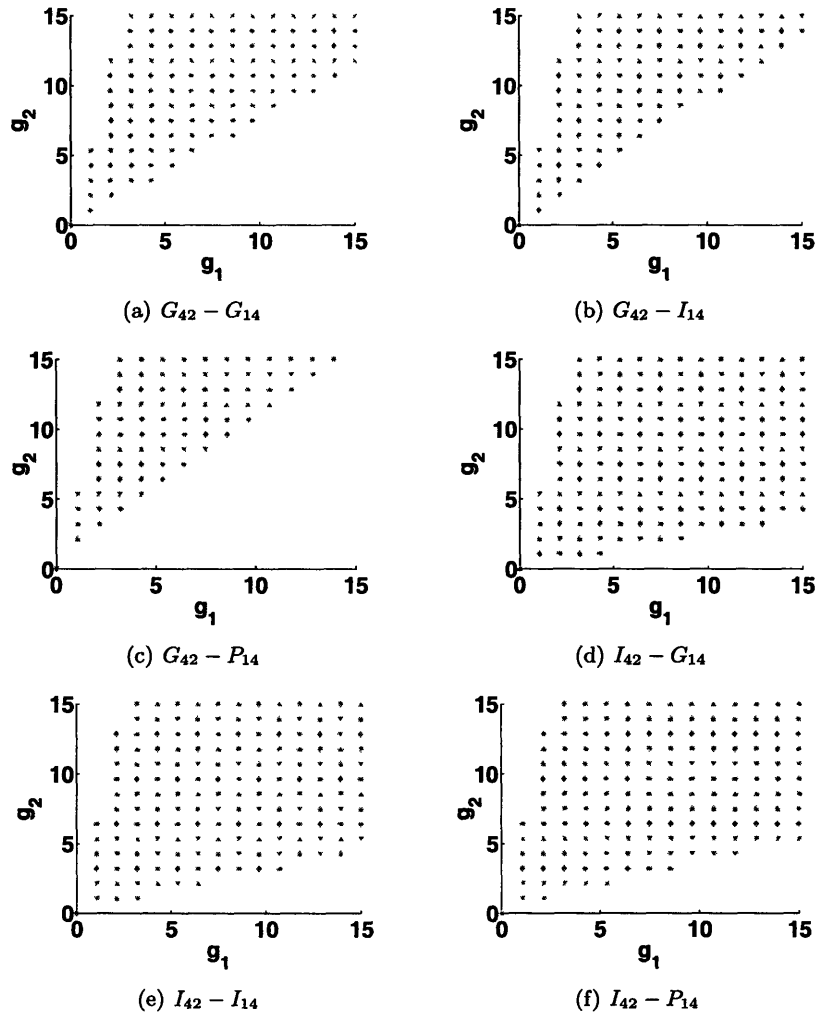


Figure 3.9: Simulation Results For $V_{1max} = 48V$, $V_{2max} = 16V$, $V_1 = 30V$, $V_2 = 16V$, $F1 = 1.5$, $F2 = 1.5$

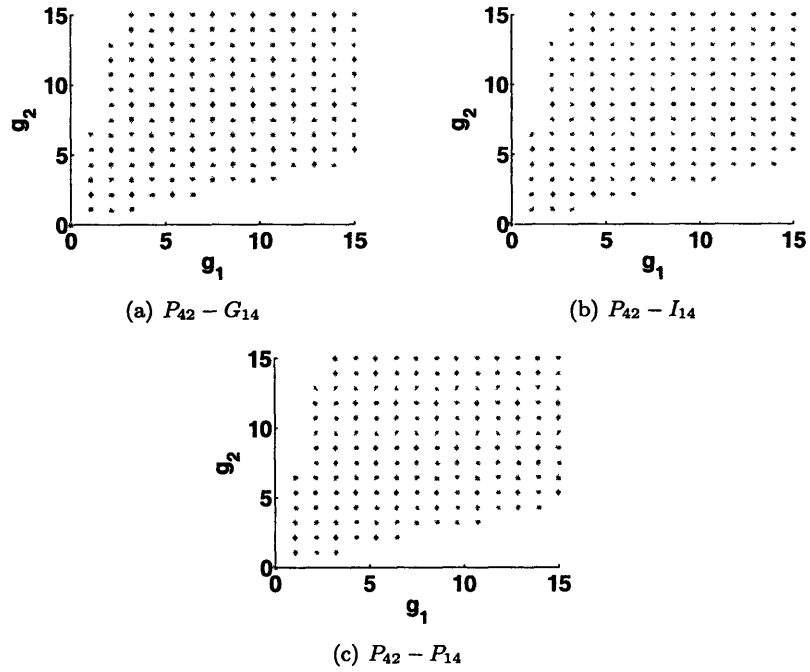


Figure 3.10: Simulation Results For $V_{1max} = 48V$, $V_{2max} = 16V$, $V_1 = 30V$, $V_2 = 16V$, $F1 = 1.5$, $F2 = 1.5$ (cont.)

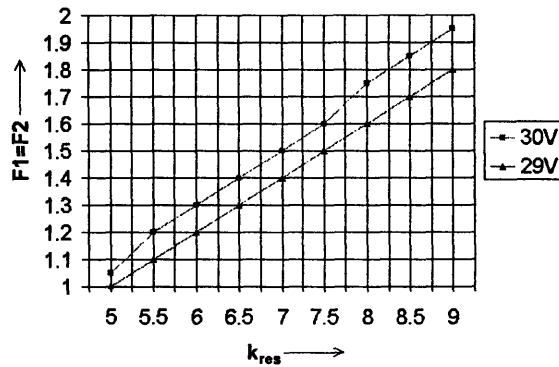


Figure 3.11: Lower Limits On The Choice For Fuse Safety Factors And k_{res}

4.1 Introduction

Dual-Voltage (e.g. 42V/14V) automobiles present the electrical system designer with challenges not found in today's cars. One such challenge is the design of a fault protection system to restore the electrical system to a safe state in the event of short circuits and/or faulted components. In conventional cars, simple fuses are the principal elements of the fault protection system. But in dual voltage automobiles, a new class of fault (the bus-to-bus or 42V – 14V fault) can occur. It has been shown that fuses alone cannot be used to protect against such an event. This thesis has shown some methods which can be implemented relatively simply to achieve a dual voltage system which always returns to a safe condition after a bus-to-bus fault.

4.2 Evaluation Of Thesis Objectives And Contributions

In chapter 1, we have shown that the fusing topologies currently in place cannot be relied upon to clear the bus-to-bus fault. Thus, left by itself, the bus-to-bus fault will not be able to clear both the fuses and bring the system to a stable state. This result has been known to the automobile industry for quite a while now. But, with the discussion in chapter 1, we have provided a mathematical footing to it. Chapter 1 also set the stage for further work in the form of modifications to the circuit.

In chapter 2, we worked upon passive circuit modifications to the dual voltage system. We came up with modifications in the form of Zener diodes, simple diode and a Positive Temperature Coefficient device. We have however, found that such modifications are either not robust enough to work on all operating conditions (Zener modification) or cannot be implemented with practical circuit elements (PTC modification). Hence, we concluded that simple passive element modification will not be able to bring the circuit to a safe operating condition. Thus, we were all set to work upon the next logical step which was incorporation

of active circuit elements and latching circuits to achieve the same result.

Chapter 3 worked on active circuit modifications to the dual voltage circuit. We simulated the circuit for all possible operating conditions in the form of the three main types of load that the electrical system of a car may have, and tested the latching circuit for its successful operation. Note that we do not claim to have done an exhaustive search over all possible load states for all circuits. Combination loads (e.g. constant resistance and constant power elements on the same circuit) can certainly exist. It is possible that combination loads will present protection challenges less demanding than the pure or extreme loads considered here, but we offer no proof. And other load characteristics can exist. However, we do expect that the basic elements of the analysis presented here will continue to be useful. And we believe that the general design method we illustrated will be nearly universally effective at reaching satisfactory final designs. These operating conditions included the types of load that an automobile can have, the possible switched state of these loads, the dc swing on the voltages of the voltage bus and certain transient voltage disturbances in the system. As a result of these simulations, we found out that within certain constraints, which are relatively practical, we can achieve fusing for both the fuses and bring the circuit to a safe state. We concluded the chapter by stating these constraints on the circuit which translate into design rules for the electrical system designer.

4.3 Future Work

The circuit modifications introduced in this thesis have been proven to work in all the operating conditions that the electrical system in a car may be subject to. Comprehensive simulations support the above claim. However, the final test of the proposed modification would be in a real life environment where we build the system and test it on an actual dual-voltage automobile. We can also test it on a current 14V automobile augmented with a 42V wire harness. Recommended future work would include tests like this and verification of the suitability of the proposed modifications.

Appendix-Analyzing The Circuit With The PPTC Device

We had stated in Chapter 2 that a PTC device can be theorized to have certain properties and implemented into the circuit so that the fusing strategy works. Here, we will try to evaluate a practical part similar to the idealized device, namely a PPTC (Polymeric Positive temperature Coefficient) device, for its suitability in this application. Before we go into analyzing the circuit with the PPTC, we will state the design conditions for a PPTC.

A.1 Design Criteria For A PPTC

A PPTC has to follow all of these conditions to work properly

- The maximum voltage across a PPTC should not exceed V_{max} .
- The maximum current should not exceed I_{max} .
- The minimum current needed to trip a PPTC is called I_{trip} .
- In order to reset a PPTC from its tripped state, the voltage in the circuit should decrease such that the power dissipation in the PPTC goes below a design parameter P_{trip} .
- The ON state resistance of the PPTC may vary between R_{min} and R_{max} . We will relax this condition and assume that the ON state resistance is a constant value R .

A.2 PPTC Model

From the above given conditions, we will be concentrating on the trip current I_{trip} , the trip power P_{trip} and the ON state resistance R . To understand these conditions, we will model the PPTC as follows. Figure A.1 shows the resistance to temperature behavior of the PPTC. Thus, the resistance stays at a constant value so long as the temperature is below a specific value T_{trip} . This resistance is R . As the voltage applied on the PPTC rises,

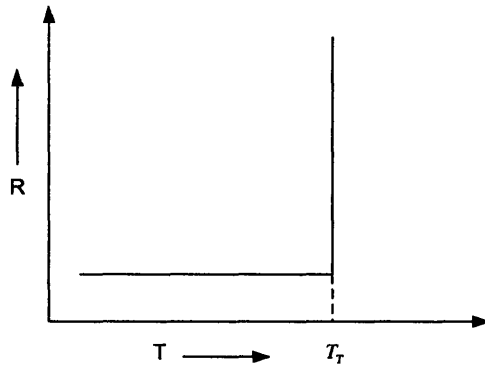


Figure A.1: Resistance To Temperature Behavior Of The PPTC

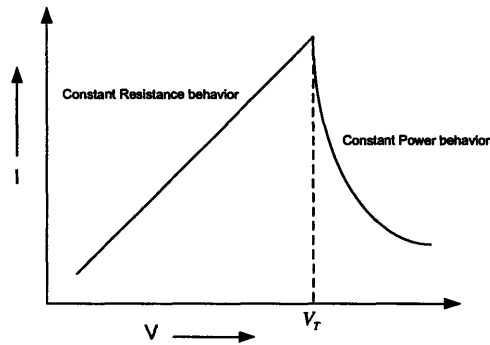


Figure A.2: Voltage Current Behavior Of The PPTC

the power dissipation rises till the temperature of the PPTC tends to go above T_{trip} . At this point the PPTC trips and the resistance rises to a value such that the temperature is maintained constant at T_{trip} . Thus, the PPTC now becomes a constant power device. Note that there is a voltage V_{trip} , which defines the trip point if the PPTC is connected to a voltage source. With this information, we can describe the voltage current characteristics of the PPTC as shown in Figure A.2. To bring the PPTC to its conducting state from the tripped state as given in its specification sheet, we need to reduce the power dissipated in it below a certain threshold. Lets call this power threshold as P_{trip} . This threshold is again defined by I_{trip} and V_{trip} .

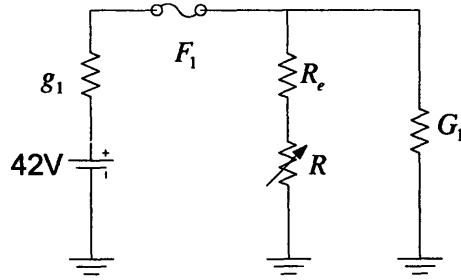


Figure A.3: Tripping Circuit For PPTC

A.3 Evaluating Conditions For The PPTC Design

In normal operation, our proposed circuit wants the PPTC device to stay in the high resistance state. The relevant circuit is shown in Figure A.3. Thus, we need the PPTC to "trip" to the high resistance state as soon as the circuit it turned on. The power dissipated in the PPTC is given by

$$P = \frac{V^2}{R} \quad \text{for } V \leq V_{trip} \quad (\text{A.1})$$

where,

V=applied voltage on the PPTC,

R=conducting state resistance of the PPTC

The PPTC temperature of the PPTC is given by

$$T = T_0 + \frac{P}{U} = T_0 + \frac{V^2}{UR} \quad (\text{A.2})$$

where,

U=temperature coefficient to power dissipation of the PPTC

At $P = P_{trip}$, $T = T_{trip}$. Thus,

$$T_{trip} = T_0 + \frac{P_{trip}}{U} \quad (\text{A.3})$$

Also, since power is defined in terms of the voltage, this point also specifies the trip voltage V_{trip} . Therefore,

$$T_{trip} = T_0 + \frac{V_{trip}^2}{UR} \quad (\text{A.4})$$

We need to find the expression for V_{trip} . The specification sheets define the tripping in terms of the current going above a given value I_{trip} . Thus, $I_{trip}V_{trip} = P_{trip}$. Therefore, from the specification sheets, we can find out V_{trip} .

We need the PPTC in the high resistance state in the normal functioning of the circuit. This, means that the effective voltage applied to it should exceed V_{trip} . To evaluate this

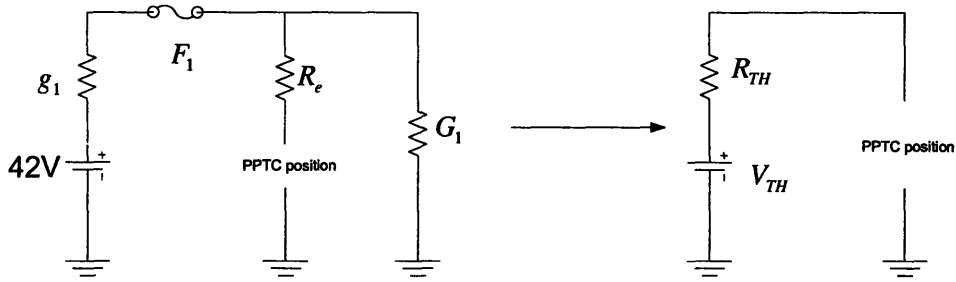


Figure A.4: Equivalent Circuit To Evaluate PPTC Tripping

condition, we will derive a Thevenin equivalent for the circuit in the normal operation of the PPTC. Figure A.4 shows the circuit and its Thevenin equivalent. The expressions for the circuit parameters are:

$$V_{TH} = \frac{V_1 g_1}{g_1 + G_1} \quad (\text{A.5})$$

$$R_{TH} = \frac{R_e(g_1 + G_1) + 1}{g_1 + G_1} \quad (\text{A.6})$$

Thus, to trip the device, we need

$$\frac{V_{TH} R}{R + R_{TH}} > V_{trip} \quad (\text{A.7})$$

Thus,

$$\frac{V_1 g_1 R}{(g_1 + G_1)(R + R_e) + 1} > V_{trip} \quad (\text{A.8})$$

Which translates to

$$R_e < \frac{V_1 R g_1 - V_{trip}}{V_{trip}(g_1 + G_1)} - R \quad (\text{A.9})$$

The other condition is the one pertaining to keeping the Zener turned off in the normal operation of the 14V circuit. The relevant circuit is shown in Figure A.5. Observe that for ascertaining the condition for the Zener to turn on, we need to check out the circuit without the Zener and make sure that the voltage goes above the Zener breakdown voltage. The Zener thus has been shown in dotted lines (to indicate its absence in the calculation) We will again relax this condition by simply specifying $V_Z > V_2$. This translates into,

$$\alpha > \frac{1}{3} \quad (\text{A.10})$$

where α is the same as defined in Section 2.3.3. The second condition in Section 2.5.1 says that the Zener should conduct in the event of a fault. The relevant circuit is shown in Figure A.6. Thus, we need $V_b > V_Z$ or,

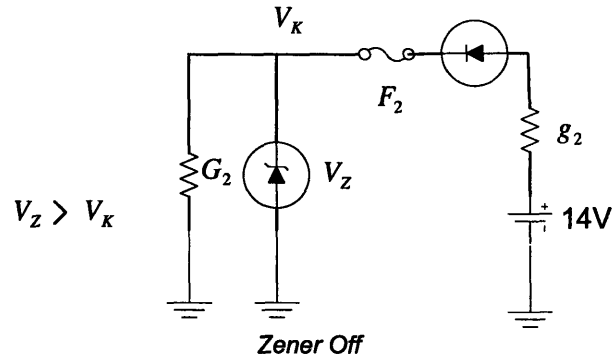


Figure A.5: Normal Operation Of The 14V Circuit

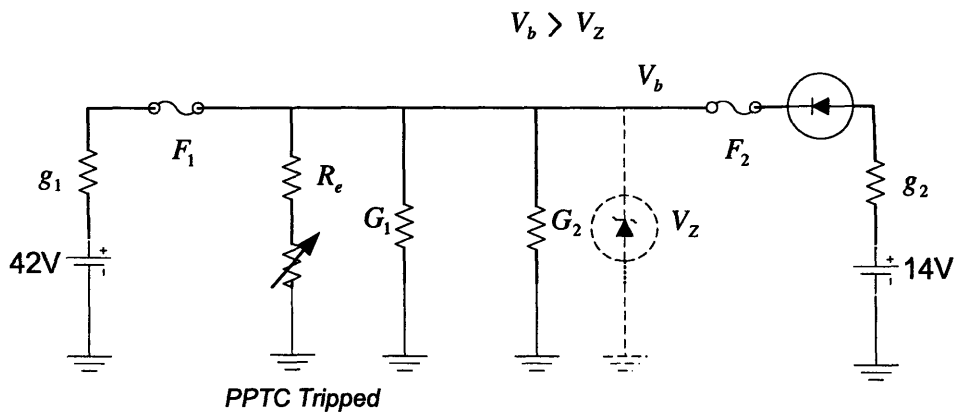


Figure A.6: Circuit Without The Zener To Ascertain The Condition For Its Blowing

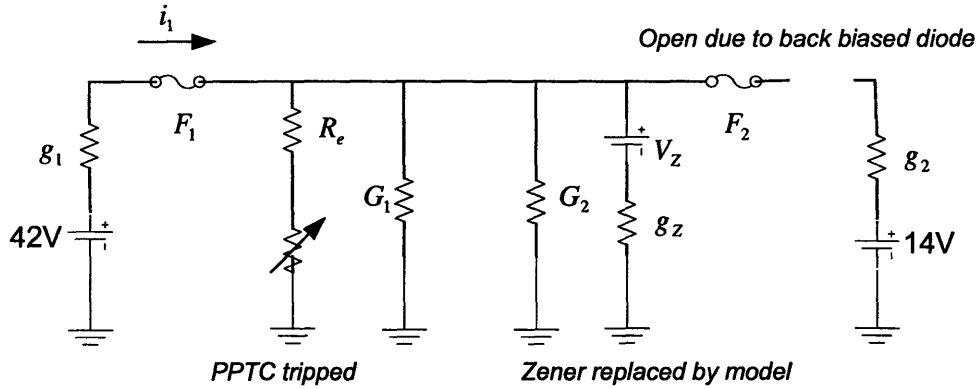


Figure A.7: Circuit In The Event Of The Fault

$$\frac{V_1 g_1}{g_1 + G_L} > \alpha V_1 \quad (\text{A.11})$$

Simplifying,

$$\alpha < \frac{g_1}{g_1 + G_L} \quad (\text{A.12})$$

Fuse F_1 should blow according to the third condition. The relevant circuit is shown in Figure A.7. We therefore need,

$$i_1 > I_{F1}^o \quad (\text{A.13})$$

$$\frac{g_1 [V_1 (g_z + G_L) - V_Z g_z]}{g_1 + g_z + G_L} > \frac{V_1 g_1 G_1}{g_1 + G_1} \quad (\text{A.14})$$

Simplifying,

$$g_z [(1 - \alpha)(g_1 + G_1) - G_1] > g_1 (G_1 - G_L) \quad (\text{A.15})$$

With the same reasoning that was used in Section 2.3.3.3, we can say the the multiplicand to g_z will be a positive number. Or,

$$\alpha < \frac{g_1}{g_1 + G_1} \quad (\text{A.16})$$

Working on the fourth condition, we need to keep the Zener off after Fuse F_1 blows. We find that the condition developed in Eq. (A.10) will ensure us that the Zener will never conduct when connected to the 14V circuit. However, the other condition is that of the PPTC reverting to the conducting state. Before we do that, let us try to elaborate on the PPTC design rules we gave in Section A.1. For a circuit given in Figure A.8, the PPTC will go back to conducting state when the following condition is met.

$$\frac{V^2}{4R_s} < P_{trip} \quad (\text{A.17})$$

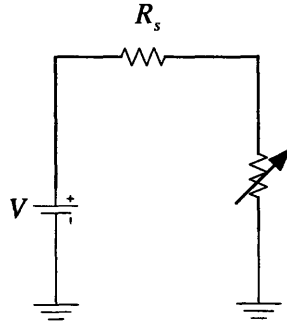


Figure A.8: Explanation For The PPTC Turn Off Condition

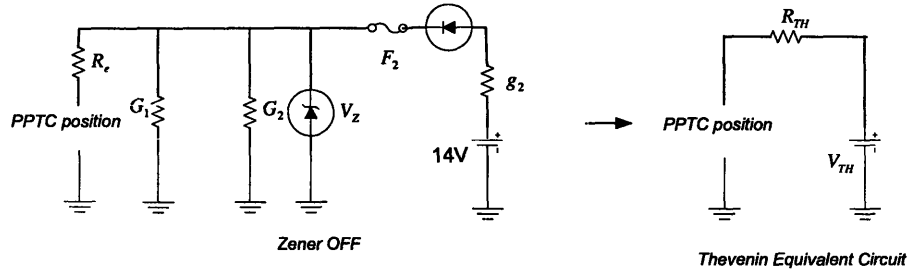


Figure A.9: Circuit Transformation For The Implementation Of The PPTC Turn Off Condition

To implement the same condition in our circuit, we need to do the following transformation to the Thevenin equivalent circuit as shown in Figure A.9 Now, we implement the turn off condition as follows,

$$\frac{V_{TH}^2}{4R_{TH}} < P_{trip} \quad (A.18)$$

This simplifies into,

$$R_e > \frac{V_2^2 g_2^2 - 4P_{trip}(g_2 + G_L)}{4P_{trip}(g_2 + G_L)^2} \quad (A.19)$$

Finally, after this we need Fuse F_2 to blow. If we notice carefully, this condition of $i_{21} > I_{F_2}^o$ was the first bottleneck condition we found in Section 2.2 and resolved in Section 2.5.2 giving rise to the condition

$$G_{Pcond} > G_{2max} \quad (A.20)$$

A.4 Concluding The PPTC Modification Circuit

Collecting all the R_e conditions (Eq. (A.9) and (A.19))

$$\frac{V_2^2 g_2^2 - 4P_{trip}(g_2 + G_L)}{4P_{trip}(g_2 + G_L)^2} < R_e < \frac{V_1 R g_1 - V_{trip}}{V_{trip}(g_1 + G_1)} - R \quad (\text{A.21})$$

Collecting all the α conditions i.e. Eq. (A.10),(A.12) and (A.16)

$$\frac{1}{3} < \alpha < \frac{g_1}{g_1 + G_1}, \frac{g_1}{g_1 + G_L} \quad (\text{A.22})$$

The g_Z condition Eq. (A.15)

$$g_Z > \frac{g_1(G_1 - G_L)}{[(1 - \alpha)(g_1 + G_1) - G_1]} \quad (\text{A.23})$$

And the condition

$$G_{Pcond} > G_{2max} \quad (\text{A.24})$$

which is closely related to the R_e conditions. These summarize the conditions needed for the proper operation of the fusing strategy. We use this condition (Eq. (A.24)) to pick up a value of G_{Pcond} . Since, $G_{2max} = 3$ (as found in Eq.(1.4)), we *choose* a value for $G_{Pcond} = 4$ and use the conditions in Eq. (A.21) to find out the bounds on the design parameters of the PPTC. Since,

$$G_{Pcond} = \frac{1}{R_e + R} = 4 \quad (\text{A.25})$$

For $R > 0$, we have,

$$R_e \leq \frac{1}{4} \quad (\text{A.26})$$

Thus,

$$\frac{V_2^2 g_2^2 - 4P_{trip}(g_2 + G_L)}{4P_{trip}(g_2 + G_L)^2} \leq \frac{1}{4} \quad (\text{A.27})$$

Leading to,

$$P_{trip} > \frac{V_2^2 g_2^2}{4(g_2 + G_L) + (g_2 + G_L)^2} \approx V_2^2 \approx 200W \quad (\text{A.28})$$

Which is a very high value for a typical PPTC. Higher side values go to about 35W. Thus, we presume that P_{trip} would be close to about 200W and not more. Thus, $R_e \approx \frac{1}{4}$. Therefore, Eq. A.9 leads to the condition

$$\frac{V_1 R g_1 - V_T}{V_T(g_1 + G_1)} > R + R_e \quad (\text{A.29})$$

or,

$$\frac{V_1 R g_1 - V_T}{V_T(g_1 + G_1)} > \frac{1}{4} \quad (\text{A.30})$$

Simplifying, we get,

$$R > \frac{V_{trip}(1 + \frac{1}{4}(g_1 + G_1))}{V_1 g_1} \quad (A.31)$$

Therefore, we can get an accurate idea of what values to choose for P_{trip} and R for the PPTC. Thus, we can design a PPTC keeping all these design criteria in mind and design a Zener keeping Eq. (A.22) in mind.

A.5 Problem With The PPTC

We have put forward a scheme of designing the PPTC which would work in the way we want it to. However, we need the PPTC to operate in the tripped state in the unaffected condition and to return to conducting state only in the event of the fault. But the present PPTC commercially available does not allow it to stay in a tripped state for such a long time. The present architecture only allows a tripped state time of 4 hours. Thus, the present strategy cannot be implemented with commercially available PPTC's. However, were a PPTC with suitable specifications to be designed, we could apply it to the circuit and bring it to safety in the event of the $42V - 14V$ fault.

A.6 Conclusion

We see that the with the above modifications, we can make the circuit to work in the way we want it to. However, the only road block is the inability of the commercial PPTC to work in the tripped state for a adequately long time.

Appendix-Analyzing A Constant Power Load Driven By A Constant Voltage Source

We discuss the issues related to a constant power load and its behavior when driven by a constant voltage source such as the voltage bus in our model. The assumption made is that the constant power load has been designed for the given voltage source and the voltage source will be able to supply the required amount of power to the load. For a detailed discussion of a load design for a voltage source see Appendix C. For our discussion, the circuit considered is shown in Fig. B.1.

B.1 Circuit Analysis

We will start with the basic constant power relation

$$IV = P \tag{B.1}$$

where,

I=current flowing in the constant power load

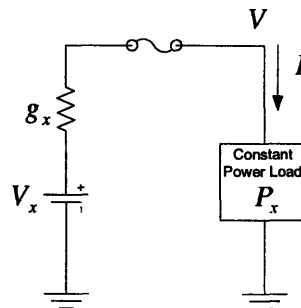


Figure B.1: Circuit Considered In The Analysis Of A Constant Power Load

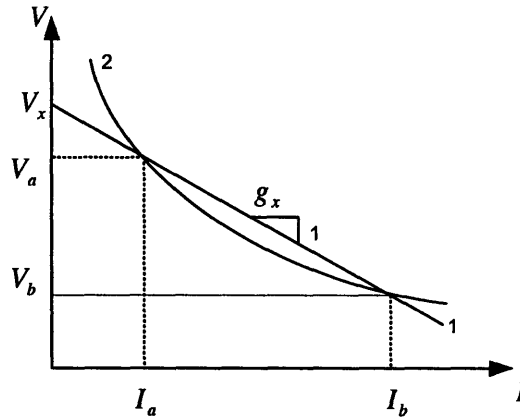


Figure B.2: Load Line Analysis Of A Constant Power Load

V=voltage across the constant power load

P=power rating of the load

This translates into,

$$(V_x - V)g_x V = P \tag{B.2}$$

This is a quadratic equation with two solutions as shown in Fig. B.2 which represents the load line analysis of the constant power load. We can see that there can be two solutions for the voltage across the load. However, a properly designed load will operate at the high voltage solution since the other solution drives a lot of current through the wire harness resulting in very high losses. Thus, the solution to the quadratic in Eq. (B.2) is

$$V = \frac{V_x + \sqrt{V_x^2 - 4\frac{P_x}{g_x}}}{2} \tag{B.3}$$

And the current is

$$I = \frac{g_x(V_x - \sqrt{V_x^2 - 4\frac{P_x}{g_x}})}{2} \tag{B.4}$$

Also note that the voltage source modelled this way cannot serve a power load with rating above $V_x^2 g_x / 4$, because this leads to a complex solution to both the current and the voltage and implies that the load line and the source lines never intersect.

Appendix-Simulation Code

C.1 Code 1

This code is for the load cases $G_{42} - G_{14}$, $G_{42} - I_{14}$, $I_{42} - G_{14}$ and $I_{42} - I_{14}$

```
function LoadCase=LoadCase(L1,L2);

V1max=48; V2max=16; V1=32; V2=16; F1=1.5; F2=1.5;

Ng1=20; Ng2=20; g1=linspace(0.001,30,Ng1);
g2=linspace(0.001,30,Ng2); NG1=20; NG2=20; NI1=20; NI2=20; NP1=20;
NP2=20; hold on; vgx_ratio=6;

%%%%%%%%%% 42 G 14 I

for ig1=1:Ng1
    for ig2=1:Ng2
        G1max=g1(ig1)/(vgx_ratio-1);
        I2max=V2max*g2(ig2)/(vgx_ratio);
        G1=linspace(0.001,G1max,NG1);
        I2=linspace(0.001,I2max,NI2);
        IF1o=V1max*g1(ig1)/(vgx_ratio);
        IF2o=V2max*g2(ig2)/(vgx_ratio);
        for iG1=1:NG1
            for iI2=1:NI2
                %V=solve('(V1-V)*g1(ig1)+(V2-V)*g2(ig2)=V*G1(iG1)+I2(NI2)', 'V')
                V=(g1(ig1)*V1+g2(ig2)*V2-I2(iI2))/(g1(ig1)+g2(ig2)+G1(iG1));
                i1=(V1-V)*g1(ig1);
                i2=(V-V2)*g2(ig2);
                if abs(i1)<F1*IF1o & abs(i2)<F2*IF2o
                    plot(g1(ig1),g2(ig2),'g*');
                end
            end
        end
    end
end
```

```

end
end

%%%%%%%%%% 42 I 14 G

for ig1=1:Ng1
    for ig2=1:Ng2
        G2max=g2(ig2)/(vgx_ratio-1);
        I1max=V1max*g1(ig1)/(vgx_ratio);
        G2=linspace(0.001,G2max,NG2);
        I1=linspace(0.001,I1max,NI1);
        IF1o=V1max*g1(ig1)/(vgx_ratio);
        IF2o=V2max*g2(ig2)/(vgx_ratio);
        for iG2=1:NG2
            for iI1=1:NI1
                %V=solve('(V1-V)*g1(ig1)+(V2-V)*g2(ig2)=V*G2(iG2)+I1(NI1)', 'V')
                V=(g1(ig1)*V1+g2(ig2)*V2-I1(iI1))/(g1(ig1)+g2(ig2)+G2(iG2));
                i1=(V1-V)*g1(ig1);
                i2=(V-V2)*g2(ig2);
                if abs(i1)<F1*IF1o & abs(i2)<F2*IF2o
                    plot(g1(ig1),g2(ig2),'g*');
                end
            end
        end
    end
end
end
end

```

```

%%%%%%%%%% 42 I 14 I

for ig1=1:Ng1
    for ig2=1:Ng2
        I1max=V1max*g1(ig1)/(vgx_ratio);
        I2max=V2max*g2(ig2)/(vgx_ratio);
        I1=linspace(0.001,I1max,NI1);
        I2=linspace(0.001,I2max,NI2);
        IF1o=V1max*g1(ig1)/(vgx_ratio);
        IF2o=V2max*g2(ig2)/(vgx_ratio);
        for iI1=1:NI1
            for iI2=1:NI2

```



```

                                %V=solve('(V1-V)*g1(ig1)+(V2-V)*g2(ig2)=I1(iI1)+I2(iI2)', 'V')
                                V= (g1(ig1)*V1+g2(ig2)*V2-I1(iI1)-I2(iI2))/(g2(ig2)+g1(ig1));
                                i1=(V1-V)*g1(ig1);
                                i2=(V-V2)*g2(ig2);
                                if abs(i1)<F1*IF1o & abs(i2)<F2*IF2o
                                    plot(g1(ig1),g2(ig2),'g*');
                                end
                            end
                        end
                    end
                end
            end

%%%%%%%%%% 42 G 14 G

for ig1=1:Ng1
    for ig2=1:Ng2
        G1max=g1(ig1)/(vgx_ratio-1);
        G2max=g2(ig2)/(vgx_ratio-1);
        G1=linspace(0.001,G1max,NG1);
        G2=linspace(0.001,G2max,NG2);
        IF1o=V1max*g1(ig1)/(vgx_ratio);
        IF2o=V2max*g2(ig2)/(vgx_ratio);
        for iG1=1:NG1
            for iG2=1:NG2
                %V=solve('(V1-V)*g1(ig1)+(V2-V)*g2(ig2)=V*G1(iG1)+V*G2(iG2)', 'V')
                V= (g1(ig1)*V1+g2(ig2)*V2)/(g1(ig1)+g2(ig2)+G1(iG1)+G2(iG2));
                i1=(V1-V)*g1(ig1);
                i2=(V-V2)*g2(ig2);
                if abs(i1)<F1*IF1o & abs(i2)<F2*IF2o
                    plot(g1(ig1),g2(ig2),'g*');
                end
            end
        end
    end
end
end
end
end

```

C.2 Code 2

The next code is for the other load cases viz. $I_{42} - P_{14}$, $G_{42} - P_{14}$, $P_{42} - G_{14}$, $P_{42} - I_{14}$ and $P_{42} - P_{14}$

```
function LoadCase_2=LoadCase_2(L1,L2);

V1max=48; V2max=16; V1min=32; V2min=9; F1=1.5; F2=1.5;

Ng1=15; Ng2=15; NV1=10; NV2=10; V1=linspace(V1min,V1max,NV1);
V2=linspace(V2min,V2max,NV2); g1=linspace(0.001,15,Ng1);
g2=linspace(0.001,15,Ng2); NG1=10; NG2=10; NI1=10; NI2=10; NP1=10;
NP2=10; hold on; sym V; vgx_ratio=8;

%%%%%%%%%% 42 P 14 G
for iV1=1:NV1
    for iV2=1:NV2
        for ig1=1:Ng1
            for ig2=1:Ng2
                G2max=g2(ig2)/(vgx_ratio-1);
                P1max=V1max^2*g1(ig1)*(1-(vgx_ratio-2)^2/(vgx_ratio)^2)/4;
                G2=linspace(0.001,G2max,NG2);
                P1=linspace(0.001,P1max,NP1);
                IF1o=V1max*g1(ig1)/(vgx_ratio);
                IF2o=V2max*g2(ig2)/(vgx_ratio);
                for iG2=1:NG2
                    for iP1=1:NP1
                        %V=solve('(V1(iV1)-V)*g1(ig1)+(V2(iV2)-V)*g2(ig2)=V*G2(iG2)
                        %+P1(iP1)/V','V')
                        V=1/2/(g1(ig1)+g2(ig2)+G2(iG2))*(g1(ig1)*V1(iV1)+g2(ig2)...
                        *V2(iV2)+(g1(ig1)^2*V1(iV1)^2+2*g1(ig1)*V1(iV1)*g2(ig2)...
                        *V2(iV2)+g2(ig2)^2*V2(iV2)^2-4*P1(iP1)*g1(ig1)-4*P1(iP1)...
                        *g2(ig2)-4*P1(iP1)*G2(iG2))^(1/2));
                        i1=(V1(iV1)-V)*g1(ig1);
                        i2=(V-V2(iV2))*g2(ig2);
                        if abs(i1)<F1*IF1o & abs(i2)<F2*IF2o

                                plot(g1(ig1),g2(ig2),'g*'); xlabel('g1');
                                ylabel('g2');
                            end
                        end
                    end
                end
            end
        end
    end
end
```

```

    end
end
end
end
%%%%%% 42 P 14 I
for iV1=1:NV1
    for iV2=1:NV2
for ig1=1:Ng1
    for ig2=1:Ng2
        I2max=V2max*g2(ig2)/(vgx_ratio);
        P1max=V1max^2*g1(ig1)*(1-(vgx_ratio-2)^2/(vgx_ratio)^2)/4;
        I2=linspace(0.001,I2max,NI2);
        P1=linspace(0.001,P1max,NP1);
        IF1o=V1max*g1(ig1)/(vgx_ratio);
        IF2o=V2max*g2(ig2)/(vgx_ratio);
        for iI2=1:NI2
            for iP1=1:NP1
                %V=solve('(V1(iV1)-V)*g1(ig1)+(V2(iV2)-V)*g2(ig2)=I2(iI2)
                %+P1(iP1)/V','V')
                V=1/2/(g2(ig2)+g1(ig1))*(g1(ig1)*V1(iV1)+g2(ig2)*V2(iV2)...
                -I2(iI2)+(g1(ig1)^2*V1(iV1)^2+2*g1(ig1)*V1(iV1)*g2(ig2)...
                *V2(iV2)-2*g1(ig1)*V1(iV1)*I2(iI2)+g2(ig2)^2*V2(iV2)^2 ...
                -2*g2(ig2)*V2(iV2)*I2(iI2)+I2(iI2)^2-4*P1(iP1)*g2(ig2)...
                -4*P1(iP1)*g1(ig1))^(1/2));
                i1=(V1(iV1)-V)*g1(ig1);
                i2=(V-V2(iV2))*g2(ig2);
                if abs(i1)<F1*IF1o & abs(i2)<F2*IF2o

                    plot(g1(ig1),g2(ig2),'g*'); xlabel('g1');
                    ylabel('g2');
                end
            end
        end
    end
end
end
end
end
%%%%%% 42 P 14 P
for iV1=1:NV1
    for iV2=1:NV2
for ig1=1:Ng1
    for ig2=1:Ng2
        P1max=V1max^2*g1(ig1)*(1-(vgx_ratio-2)^2/(vgx_ratio)^2)/4;

```

```

P2max=V2max^2*g2(ig2)*(1-(vgx_ratio-2)^2/(vgx_ratio)^2)/4;
P1=linspace(0.001,P1max,NP1);
P2=linspace(0.001,P2max,NP2);
IF1o=V1max*g1(ig1)/(vgx_ratio);
IF2o=V2max*g2(ig2)/(vgx_ratio);
for iP1=1:NP1
    for iP2=1:NP2
        %V=solve('(V1(iV1)-V)*g1(ig1)+(V2(iV2)-V)*g2(ig2)=P1(iP1)/V
        %+P2(iP2)/V','V')
        V=1/2/(g2(ig2)+g1(ig1))*(g1(ig1)*V1(iV1)+g2(ig2)*V2(iV2)...
        +(g1(ig1)^2*V1(iV1)^2+2*g1(ig1)*V1(iV1)*g2(ig2)*V2(iV2)...
        +g2(ig2)^2*V2(iV2)^2-4*P1(iP1)*g2(ig2)-4*P2(iP2)*g2(ig2)...
        -4*P1(iP1)*g1(ig1)-4*P2(iP2)*g1(ig1))^(1/2));
        i1=(V1(iV1)-V)*g1(ig1);
        i2=(V-V2(iV2))*g2(ig2);
        if abs(i1)<F1*IF1o & abs(i2)<F2*IF2o

            plot(g1(ig1),g2(ig2),'g*'); xlabel('g1');
            ylabel('g2');
        end
    end
end
end
end
end

%%%%%%%%%% 42 G 14 P
for iV1=1:NV1
    for iV2=1:NV2
        for ig1=1:Ng1
            for ig2=1:Ng2
                G1max=g1(ig1)/(vgx_ratio-1);
                P2max=V2max^2*g2(ig2)*(1-(vgx_ratio-2)^2/(vgx_ratio)^2)/4;
                G1=linspace(0.001,G1max,NG1);
                P2=linspace(0.001,P2max,NP2);
                IF1o=V1max*g1(ig1)/(vgx_ratio);
                IF2o=V2max*g2(ig2)/(vgx_ratio);
                for iG1=1:NG1
                    for iP2=1:NP2
                        %V=solve('(V1(iV1)-V)*g1(ig1)+(V2(iV2)-V)*g2(ig2)=V*G1(iG1)
                        %+P2(iP2)/V','V')
                        V=1/2/(g1(ig1)+g2(ig2)+G1(iG1))*(g1(ig1)*V1(iV1)+g2(ig2)...
                        *V2(iV2)+(g1(ig1)^2*V1(iV1)^2+2*g1(ig1)*V1(iV1)*g2(ig2)*...

```

```

        V2(iV2)+g2(ig2)^2*V2(iV2)^2-4*P2(iP2)*g1(ig1)-4*P2(iP2)...
        *g2(ig2)-4*P2(iP2)*G1(ig1))^(1/2));
        i1=(V1(iV1)-V)*g1(ig1);
        i2=(V-V2(iV2))*g2(ig2);
        if abs(i1)<F1*IF1o & abs(i2)<F2*IF2o
            plot(g1(ig1),g2(ig2),'g*');
        end
    end
end
end
end
end

%%%%%% 42 I 14 P
for iV1=1:NV1
    for iV2=1:NV2
        for ig1=1:Ng1
            for ig2=1:Ng2
                I1max=V1max*g1(ig1)/(vgx_ratio);
                P2max=V2max^2*g2(ig2)*(1-(vgx_ratio-2)^2/(vgx_ratio)^2)/4;
                I1=linspace(0.001,I1max,NI1);
                P2=linspace(0.001,P2max,NP2);
                IF1o=V1max*g1(ig1)/(vgx_ratio);
                IF2o=V2max*g2(ig2)/(vgx_ratio);
                for iI1=1:NI1
                    for iP2=1:NP2
                        %V=solve('(V1(iV1)-V)*g1(ig1)+(V2(iV2)-V)*g2(ig2)=I1(iI1)
                        %+P2(iP2)/V','V')
                        V=1/2/(g2(ig2)+g1(ig1))*(g1(ig1)*V1(iV1)+g2(ig2)...
                        *V2(iV2)-I1(iI1)+(g1(ig1)^2*V1(iV1)^2+2*g1(ig1)*V1(iV1)...
                        *g2(ig2)*V2(iV2)-2*g1(ig1)*V1(iV1)*I1(iI1)+g2(ig2)^2*...
                        V2(iV2)^2-2*g2(ig2)*V2(iV2)*I1(iI1)+I1(iI1)^2-4*P2(iP2)*g2(ig2)-...
                        4*P2(iP2)*g1(ig1))^(1/2));
                        i1=(V1(iV1)-V)*g1(ig1);
                        i2=(V-V2(iV2))*g2(ig2);
                        if abs(i1)<F1*IF1o & abs(i2)<F2*IF2o
                            plot(g1(ig1),g2(ig2),'g*');
                        end
                    end
                end
            end
        end
    end
end
end
end
end

```

end
end

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