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DYNAMIC WEAPON-TARGET ASSIGNMENT

PROBLEMS WITH VULNERABLE C^2 NODES

by

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ABSTRACT

In this paper we present a progress report on our work on the dynamic version of the Weapon to Target Assignment (WTA) problem and on the static version of the WTA problem in which vulnerable C^3 nodes are included in the formulation.

In the static WTA problem, weapons must be assigned to targets with the objective of minimizing the total expected number (or value) of the surviving targets. In the dynamic version, this allocation is done in time stages so that the outcomes of previous engagements can be used in making future assignments. We will show that, for the simple cases studied, there is a significant cost advantage in using the dynamic strategy. We believe that similar results will hold for the more general problem.

In the static defense-asset problem with vulnerable C^3 nodes the offense is allowed to either attack the assets themselves or to first attack the command and control system, and then the assets; if the C^3 nodes are destroyed then the defensive interceptors are assumed unusable. We first consider simple cases where assumptions are made as to offensive and defensive states of knowledge and kill probabilities. Strategies are then developed and optimal weapon allocations identified. These assumptions are then relaxed, and further examples demonstrate the ensuing complexity.

1. INTRODUCTION

Our long-range research objective is the quantitative study of the theory of distributed C^3 organizations. Our present research has been concentrated on certain aspects of situation assessment and resource commitment.

Situation assessment entails the use of sensors to detect and track the enemy and its weapons (i.e missiles, tanks etc.). These sensors are usually geographically distributed so that distributed algorithms are desirable. This problem can be formulated as a distributed hypothesis testing problem. Results on this research can be found in the paper by Papastavrou and Athans [1] in these proceedings.

The resource commitment problem deals with the optimal assignment of the defense's resources against the offense's forces so as to minimize the damage done to the defense's assets. If the battle is such that the defense has a single opportunity to engage the enemy then the problem can be formulated as a static resource allocation problem. If multiple engagements are possible (as for example in the Strategic Defense System (SDS) scenario) then better use can be made of the defense's resources by assigning them dynamically (i.e. observe the outcomes of some engagements before making further assignments). This is sometimes called a shoot-look-shoot strategy in the literature. In this paper we will provide some results on simple cases of the dynamic problem and make comparisons with the corresponding static problem.

The above resource allocation problem will typically be solved at a C^3 node and the results transmitted to the relevant resources. These C^3 nodes will therefore be of vital importance to the defense since their destruction will in effect paralyze the resources over which they have control. One approach, that can be used to increase the reliability of the system, is to replicate the C^3 nodes. In this way destruction of the primary C^3 node does not affect the defense's system since its function can then be performed by one of the "backup" C^3 nodes. We have formulated a model which includes these replicated nodes and will provide results on simple cases of the problem.

This paper is in effect a progress report of our work on the resource allocation problem and on the survivability issue mentioned in the previous paragraph. The models being used are rich enough to capture the nature of the mission (e.g. defense of assets), enemy strength (number and effectiveness of the enemy's weapons), defense strength (number and effectiveness of the defense's weapons) and strategy and tactics (preferential defense, shoot-look-shoot, etc.) It should be noted that basic research studies on these topics are virtually non-existent.

Our work is motivated by military defense problems, two examples of which are as follows. The first example involves the Anti-Aircraft Weapon (AAW) defense of the Naval battle group or battle force platforms. The assets being defended are aircraft carrier(s), escort warships and support ships each of which is of some intrinsic value to the defense. The threat to these assets are enemy missiles launched from submarines, surface ships and aircraft. These missiles may have different damage probabilities which depend on the missile type, target type, etc. The defense's weapons are different types of AAW interceptors launched from Aegis and other AAW ships. The kill probability of these weapons may also depend on the specific target-interceptor pair. The objective of the defense is to maximize the expected surviving value of the assets. The problem is to find which AAW interceptors should be assigned to each of the enemy missiles, when should they be launched and why. This formulation allows for a preferential defense where, in a heavy attack, it may be optimal for the defense to leave "low" valued assets undefended and concentrate its resources on saving the "high" valued assets.

The second motivating example for our work is the midcourse phase of the Strategic Defense System. In this case the assets are our (the defense's) Inter-Continental Ballistic Missile (ICBM) silos, military installations, C^3 nodes, populations centers, etc. The threat to these assets are enemy re-entry vehicles (RV's), surrounded by decoys. The defense's weapons are Space-based kinetic-kill vehicles (SBKKV's) and ERIS interceptors. The objective of the defense is the maximization of the expected surviving value of the assets. The problem is the determination of the optimal weapon-target assignments and the timing of the interceptor launches.

2. THE STATIC TARGET-BASED WTA PROBLEM

In this section we will present the static version of the target-based WTA problem. In this model, a number of missiles (the targets) are launched by the offense. The defense has a number of interceptors (the weapons) with which to destroy these missiles. The defense assigns a value to each of the targets based on factors such as target type, point of impact, etc. Associated with each weapon-target pair is a kill probability which is the probability that the weapon will destroy the target if it is fired at it. We will make the assumption that the action of a weapon on a target is independent of all other weapons and targets. The problem faced by the defense is the assignment of weapons to targets with the objective of minimizing the total expected surviving target value.

2.1 Mathematical Statement of the Static WTA Problem

The following notation will be used:

N = the number of enemy targets
M = the number of defense weapons
V_i = the value of target i
p_{ij} = probability that weapon j kills target i if shot at it

The solution will be represented by:

$$x_{ij} = \begin{cases} 1 & \text{if weapon j is assigned to target i} \\ 0 & \text{otherwise} \end{cases}$$

The optimization problem can now be stated as:

$$\min_{\{x_{ij} \in \{0,1\}\}} \quad J = \sum_{i=1}^{N} V_i \prod_{j=1}^{M} (1 - p_{ij})^{x_{ij}} \qquad (2.1)$$

$$\text{subject to:} \quad \sum_{i=1}^{N} x_{ij} = 1, \quad j = 1,2,\ldots,M.$$

The objective function is the total expected value of the surviving targets while the constraint is due to the fact that each weapon can attack only one target.

2.2 Comments on the Solution of the Static WTA Problem

This problem has been proven, by Lloyd and Witsenhausen [2], to be NP-Complete in general. This means that polynomial time algorithms for obtaining the optimal solution do not exist. One must therefore resort to sub-optimal algorithms.

In the special case in which the kill probabilities are independent of the weapons, denBroeder et al. [3], have proposed an optimal algorithm for the problem which runs in polynomial time. This algorithm, which is usually called the Maximum Marginal Return algorithm, works by assigning the weapons sequentially with each weapon being assigned to the target which results in the maximum marginal return in the objective function.

In the special case in which the kill probability is the same for all weapon-target pairs and all targets have the same value, the optimal assignment is obtained by dividing the weapons as evenly as possible among the targets.

3. THE DYNAMIC WTA PROBLEM

The battle scenario for the dynamic problem is the same as for the static problem except that the weapon assignments are done in time stages each of which consists of the following steps:

(a) Determine which targets have survived the last engagement,
(b) Assign and fire a subset of the remaining weapons with the objective of minimizing the total expected value of the surviving targets at the end of the final stage.

We have looked at some simple cases of this problem to gain insight into the general problem and to help bolster our intuition.

3.1 The Two-Target Case

In this case we will assume that there are two targets (N=2), the kill probability is the same for all weapon-target pairs, the defense has M weapons and there are K time stages. With these assumptions we have proved the following theorem:

Theorem 3.1 *Under the assumptions given above, an optimal strategy for the present stage can be found as follows. Let* $T = \lceil M/K \rceil$*. Solve the corresponding static problem with* T *weapons and denote the solution by* $\{x_i\}$ *where* x_i *is the optimal number of weapons to be assigned to target* i*. The optimal assignment for the present stage of the dynamic problem is to assign* x_i *weapons to target* i.

If we further assume that V_i=1 and that M is divisible by 2K then it can be shown that the optimal target leakage J(K) is given by:

$$J(K) = 2K(1-p)^{M(1-\frac{1}{2K})} - 2(K-1)(1-p)^M. \qquad (3.1)$$

Note that if K is large then the optimal leakage $J(K) \approx 2(1-p)^M$ while the optimal leakage for the corresponding static problem is $2(1-p)^{M/2}$. In other words, roughly *half* as many weapons are required for the dynamic strategy to obtain the same expected leakage as that of the static strategy. This expresses, in a convenient form, the value of the battlespace to the defense.

In figure 1 we have plotted the ratio of the K-stage leakage to the static leakage versus the kill probability p for the case of two targets and 16 weapons. We have plotted the cases K = 2, 4 and 8. First note that the leakage advantage of the dynamic strategy over the static one increases with the kill probability. This is due to the fact that as the kill probability increases, the information gained from the first stage increases. This implies that more effective use will be made of (costly) high accuracy weapons if the dynamic strategy is used. Next, note that the leakage advantage of the dynamic strategy increases with the number of stages. This is due to the fact that the information gained increases with the number of stages. Note, however, that most of the improvement is obtained after only a small number of stages. In other words, the curves rapidly converge as the number of stages increases. This implies that, even if the number of stages is small, the dynamic strategy offers a significant leakage advantage.

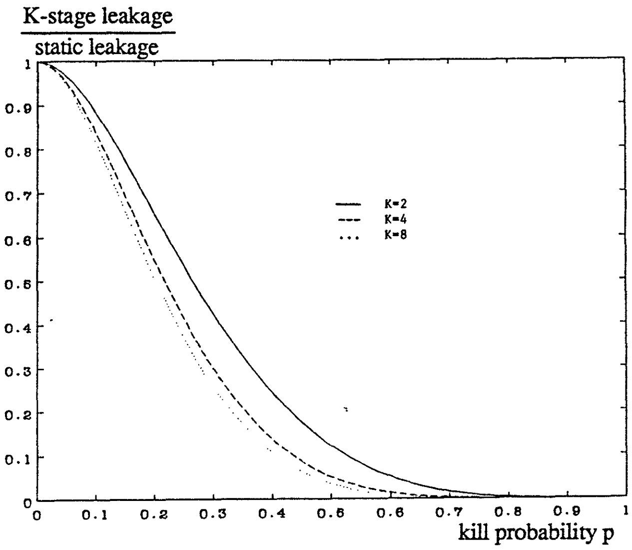

Figure 1 : J(K)/J(1) vs. p for the case M=16 and N=2

Figure 2 contains plots of the ratio of the two-stage leakage to the static leakage versus the kill probability p for the case of two targets. We have plotted the cases for which M = 4,8,12,16 and 20. Note that the leakage advantage of the dynamic strategy increases with the number of weapons used. Note also that the same leakage advantage can be obtained by either using a few high accuracy weapons or many low accuracy weapons.

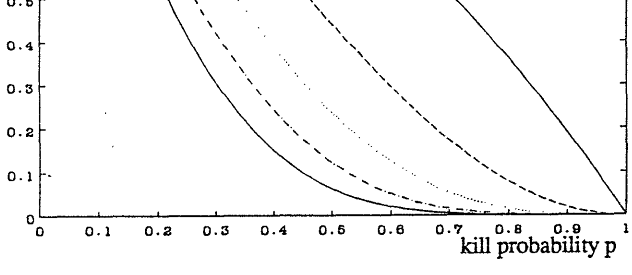

Figure 2 : J(2)/J(1) vs. p for the case K=2 and N=2

3.2 The Two-Stage, N-Target Case

In this section we will consider the two-stage dynamic problem with N equally valued targets (each of value 1), a single kill probability p (for all stages and weapon-target pairs), and M weapons. Unlike the two target problem, we were unable to find an analytical solution to this problem for the case N>2. However, we were able to derive useful properties of the optimal solution. The first property, which holds for the more general problem of K stages, is given as theorem 3.2.

Theorem 3.2 *The optimal solution to the problem given above has the property that the weapons to be used at each stage are spread evenly among the surviving targets.*

The above result simplifies the problem to be solved. Let us consider the two-stage problem. Let M_2 denote the number of weapons used in the first stage (i.e. 2 stages to go). The number of weapons that will be used in the last stage will then be $M-M_2$. Denote the corresponding cost of the assignment, in which weapons are spread evenly at each stage, by $J(M_2)$. The optimal solution can then be obtained by minimizing $J(M_2)$ over the set $\{0,1,...,M\}$.

If $J(M_2)$ happened to have a "convex" shape (i.e have the property that $J(M_2+1) - J(M_2) \geq J(M_2) - J(M_2-1)$) then the above minimization could be done efficiently. Unfortunately, we can show (by example) that this is not the case.

Let us now consider the case of K stages, N equally valued targets, a single kill probability and M weapons with the constraint that the number of weapons is less than the number of targets. Our intuition tells us that a dynamic allocation should not perform any better than a static allocation. This is in fact the case. This result is given in the following theorem.

Theorem 3.3 *Under the assumptions given above, a dynamic allocation cannot perform any better than a static allocation. Hence it is optimal to assign all of the weapons in the first stage.*

The above theorem is not particularly enlightening but it allows us to concentrate on the cases in which M>N. Let us now consider the case in which there are two stages, N targets each of unit value, a single kill probability p and M weapons with M > N. As before, let M_2 denote the number of weapons used in the first stage of a strategy.

Theorem 3.4 *Under the assumptions given above, the optimal assignment has the property that* $M_2 \geq N$.

We conjecture that the above theorem can be extended to the case of more than two stages.

Using the properties given above we computed the optimal solution to the two stage dynamic problem for various numbers of weapons and targets using a kill probability of p = 0.9. The optimal values of M_2 are given in table 1. The number of targets varies from 1 to 10 (the columns) and the number of weapons varies from 1 to 25.(the rows). In the cases where the solution is non-unique we have chosen values of M_2 which exemplify any patterns. Note that for the cases $N \leq M \leq 2N-1$ the optimal value of M_2 is N. We conjecture that this result holds in general but have so far failed to find a proof.

M	N = 1	N = 2	N = 3	N = 4	N = 5	N = 6	N = 7	N = 8	N = 9	N =10
1	1	1	1	1	1	1	1	1	1	1
2	1	2	2	2	2	2	2	2	2	2
3	1	2	3	3	3	3	3	3	3	3
4	1	2	3	4	4	4	4	4	4	4
5	1	2	3	4	5	5	5	5	5	5
6	3	3	3	4	5	6	6	6	6	6
7	4	4	3	4	5	6	7	7	7	7
8	4	4	4	4	5	6	7	8	8	8
9	5	5	5	5	5	6	7	8	9	9
10	5	5	6	4	5	6	7	8	9	10
11	6	6	6	7	5	6	7	8	9	10
12	6	6	6	8	6	6	7	8	9	10
13	7	7	7	8	7	7	7	8	9	10
14	7	7	8	8	10	6	8	8	9	10
15	8	8	9	9	9	6	7	8	9	10
16	8	8	9	8	10	12	7	8	9	10
17	9	9	9	8	10	11	8	8	9	10
18	9	9	9	12	10	12	14	8	9	10
19	10	10	11	12	10	12	13	10	9	10
20	10	10	12	12	10	12	14	8	9	10
21	11	11	12	12	11	12	14	15	9	10
22	11	11	12	12	10	12	14	16	10	10
23	12	12	13	13	15	12	14	16	11	11
24	12	12	14	12	14	12	14	16	18	12
25	13	13	15	15	15	13	14	16	18	10

Table 1 : Optimal values of M_2 for N=1:10, M=1:25, p=0.9

Figure 3 contains a plot of the ratio of the 2-stage leakage to the static leakage versus the kill probability p with a 2:1 weapon to target ratio (i.e. M = 2N). We have plotted the cases N = 2,4,6,8 and 10. Here we see that as the size of the problem increases the leakage advantage of the dynamic model increases. This implies that, for the problems that concern us (i.e. large scale problems), the dynamic model has a significant leakage advantage over the static model.

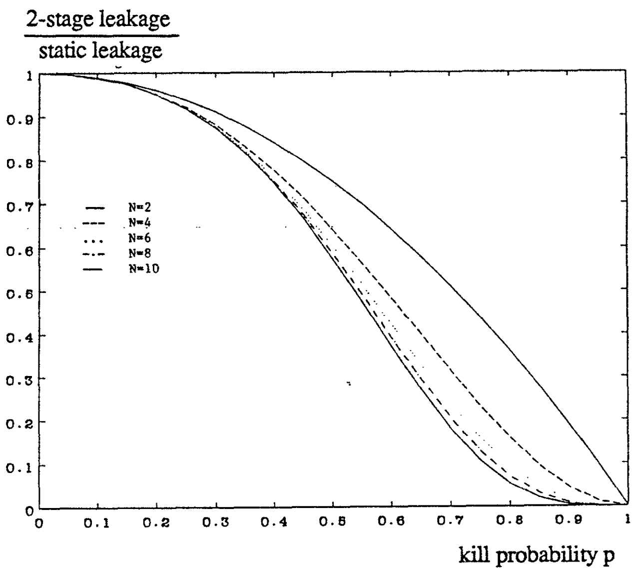

Figure 3 : J(2)/J(1) vs. p for the case K=2 with M=2N

Figure 4 contains a plot of the ratio of the 2-stage leakage to the static leakage versus the number of weapons M with a kill probability p = 0.5. We have plotted the cases N = 2,4,6,8 and 10. Note that the leakage advantage of the dynamic model increases roughly exponentially with the number of weapons. This implies that the dynamic model is significantly better even for relatively small weapon to target ratios.

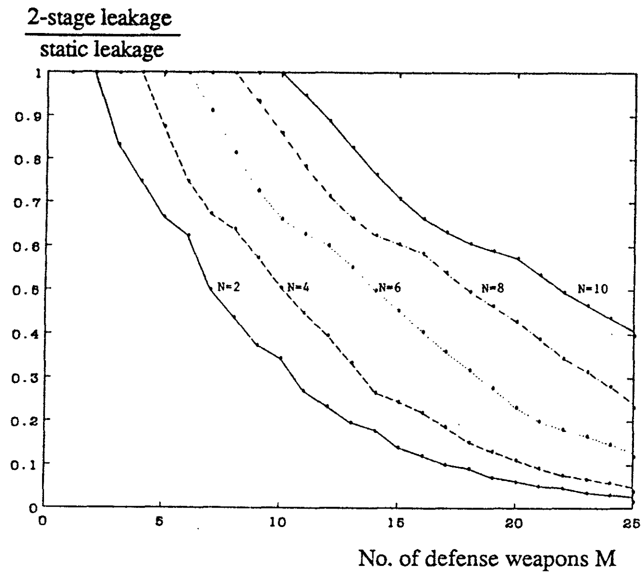

Figure 4 : J(2)/J(1) vs. M for the case K=2 with p = 0.5

4. DISTRIBUTION OF THE C³ FUNCTION

4.1 Introduction

Although distribution of C^3 functions is generally regarded as desirable, little quantitative insight exists as to the resulting gains. Enhanced survivability of the BM/C^3 system is typically cited as the reason for such distribution. Here we develop a concept of vulnerability for the WTA function and consider its effect on allocation strategies. The underlying paradigm is the defense asset model, in which the defense defends a collection of differently valued assets against attack.

In such an engagement the objective of the offense is to minimize expected surviving defense asset value. This can be accomplished by either attacking the assets themselves or by first attacking the defense system, and then the assets. Although various components of the defense system are vulnerable we assume here that the objective is the command and control centers, the destruction of which renders the defensive stockpile unusable. A precursor attack increases the likelihood that the assets will be undefended and thus increases their vulnerability, making this a potentially attractive offensive strategy. Increased vulnerability, however, comes at the cost of a reduction in the offensive stockpile available to attack assets.

The overall objective of the defense is to maximize expected surviving asset value; BM/C^3 nodes exist and are defended only insofar as they further this. Unless some redundancy is planned, a destroyed BM/C^3 node causes the defensive weapons it controls to be useless. For the purposes of initial analysis, we assume that command and control is centralized, and replication of function will occur at this level. The defense has the potential to increase expected surviving value by allocating some portion of its stockpile to BM/C^3 defense, but in so doing reduces the number of weapons available for defending assets.

4.2 Problem Definition

We consider a set of Q defense assets each of value v_q to be defended, and a set of T identical BM/C^3 nodes to do so. Assets and C^3 nodes are far enough apart so that a successful attack on one does not affect any other. The defense can see which assets or nodes are being engaged in time to intercept the attack, if desired, but cannot adaptively change weapon assignments based on damage assessment.

The defense has a stockpile of M interceptors and the offense a stockpile of N missiles. Each side knows these quantities. Interceptors are in a central stockpile, and are thus capable of defending any C^3 node or asset. Initially we assume that all attacking missiles have the same probability of kill, p, and defending interceptors r. Thus the miss probability for an unintercepted target (e.g. attacking missile) is $(1\text{-}p)$, if intercepted by j defenders $(1\text{-}p(1\text{-}r)^j)$ and for i such intercepted targets directed at the same asset $(1\text{-}p(1\text{-}r)^j)^i$. Were kill probabilities dependent on weapons, targets and assets this would become (for a given asset q)

$$\prod_{i=1}^{N_q}[1-p_{qi}\prod_{j=1}^{M}(1-r_{ij})^{x_{ij}}] \tag{4.1}$$

where $x_{ij} = 1$ if weapon j is assigned to target i and 0 otherwise, r_{ij} = kill probability for interceptor j fired at target i, p_{qi} = kill probability for target i against asset q and N_q indexes the set of targets aimed at asset q. The simpler version is

$$\prod_{i=1}^{N_q}[1-p(1-r)^{x_i}] \tag{4.2}$$

where x_i is the number of defensive weapons assigned to the i-th target.

The objective function used is

$$U = \xi V_u + (1-\xi)V_d \tag{4.3}$$

where ξ = probability that all C^3 nodes are destroyed, V_u = expected surviving asset value if undefended and V_d = expected surviving asset value if defended. The offense, then, seeks to minimize V and the defense to maximize it. In general,

$$V_d = \sum_{q=1}^{Q} V_q \prod_{i=1}^{N_q}[1-p_{qi}\prod_{j=1}^{M}(1-r_{ij})^{x_{ij}}] \tag{4.4}$$

With the above assumptions about kill probabilities, this reduces to

$$V_d = \sum_{q=1}^{Q} V_q \prod_{i=1}^{N_q}[1-p(1-r)^{x_i}] \tag{4.5}$$

The expected number of surviving assets, if undefended, is

$$V_u = \sum_{q=1}^{Q} V_q (1-p)^{N_q} \tag{4.6}$$

The probability that all BM/C^3 nodes are destroyed, using simplified probabilities, is

$$\xi = \prod_{t=1}^{T}\{1 - \prod_{s=1}^{a_t}[1-p_c(1-r)^{y_s}]\} \tag{4.7}$$

where a_t and y_s are offensive and defensive allocations for the precursor attack, t indexes the C^3 nodes and p_c is the offensive kill probability for C^3 nodes. We assume that all BM/C^3 nodes must be destroyed to eliminate the defense.

The offense wants to minimize U, while the defense seeks to maximize it:

$$\max_{\{y_s, x_i\}} \ \min_{\{a_t, n_q\}} \ U \tag{4.8}$$

A formal statement of the problem also includes constraints reflecting stockpile size and shot integrality. The offense allocation constraint

$$\sum_{t=1}^{T} a_t + \sum_{q=1}^{Q} N_q = N \tag{4.9}$$

requires that the sum of targets directed at C^3 nodes and at assets sum to the offensive stockpile. The defense allocation constraint

$$\sum_{t=1}^{T}\sum_{s=1}^{a_t} y_s + \sum_{q=1}^{Q}\sum_{i=1}^{N_q} x_i = M \tag{4.10}$$

similarly requires that defensive weapons sum to the interceptor stockpile size.

The utility function, however, is nonlinear, nonconvex and nonconcave. Minimal analytical insight has been developed, and initially we discuss small numerical examples.

4.3 Discussion

The best way to demonstrate the complexity of strategies for such problems is to first consider simple cases. Assume that the defense has the last move, and also that kill probabilities for targets against BM/C^3 nodes (p) and interceptors against targets (r) are unity; while targets against assets are less than one. Assets are taken to be of equal value and hence normalized to one. The offense, then, can either attack just the assets, or first the BM/C^3 nodes and then the assets. If the control nodes are attacked the defense will defend just one by matching each incoming missile, obviously choosing the most lightly attacked. As long as there are interceptors left they will be so used in the first stage, for otherwise they are useless in the second. If there are any left for the second stage, they will similarly be employed to successively defend the least attacked targets. Thus the offense will attack all C^3 nodes evenly, as it will also do for assets (though not necessarily at the same level for both). Note that this is true only when C^3 nodes are of equal value; the same applies to the asset attacks. The purpose of a precursor attack, with these kill probabilities, is to deplete the defender's stockpile rather than to actually destroy the control nodes.

An example from [4], with M=12, N=36, Q=12, p=1/3 (targets against assets), p_c=r=1, v_q=1 for all q and 1 radar (rather than C^3 node), shows that the offense minimizes expected surviving value by exhausting the defensive stockpile in the first stage and then attacking the assets. [4] also give expected surviving values for different strategies for the same parameters when there are 2 radars, and find that the optimal offensive strategy is to ignore the C^3 nodes and attack the assets with the entire stockpile. But with the unity kill probabilities used, it can be observed that the offense will never choose a two stage attack (regardless of the value of p) if there is more than one radar. Since the offense attacks radars evenly, the defense receives a target-to-interceptor exchange ratio equal to the number of radars, and this makes a precursor attack undesirable.

When the assumption of unity kill probabilities is relaxed, strategies become much more complex. Interceptors might no longer be singly matched to targets, and the destruction of C^3 nodes loses its binary character. The offense and defense are able only to modulate the probability that nodes and targets are destroyed. The defense last-move case becomes a two-stage allocation problem, with two different defensive objectives: BM/C^3 allocation strategies seek to maximize the probability of at least one node surviving, while asset defense tries to maximize the expected surviving value.

In [4]'s example the offense would not attack the BM/C^3 system if there were more than one node. Another example shows that this no longer need hold when weapons are imperfect. For the case of Q=4, N=12, p=.9 (both assets and C^3 nodes) and r ranging from .7 to 1.0, figures 5-8 show optimal strategies for M=4, 6, 8 and 10. Note that the optimal strategies were obtained by an intelligent enumeration: against candidate offensive strategies the defense employs that allocation which maximizes U; the offense then selects the strategy corresponding to the minima of these maxima. Only by simplifications regarding p, r and the v_i is such an enumeration feasible. In figure 5 (M=4) the offense allocates 4 of its 12 weapons to attack the C^3 nodes when r is low, but increases to 8 when r is high; this corresponds to defensive allocations of at first 2 and then 3 interceptors in defense of the C^3 nodes. Not surprisingly, the number of expected survivors is quite low.

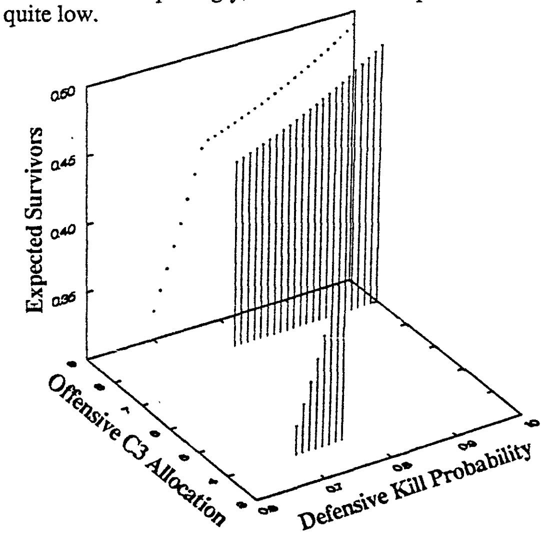

Figure 5 : P_c = .9, P_a = .9, Q = 4, N = 12, M = 4

In figure 6 (M=6), however, increasingly effective interceptors prompts the offense to decrease its C^3 attack allocation from 4 to 0; expected survivors have increased significantly. This pattern continues in figure 7 for M=8, with the offense switching to an asset only attack at an even lower value of r. In figure 8, the offense will not attack the BM/C^3 system even for r=.7. We conclude that more and more effective defensive interceptors discourage attacks on the BM/C^3 system.

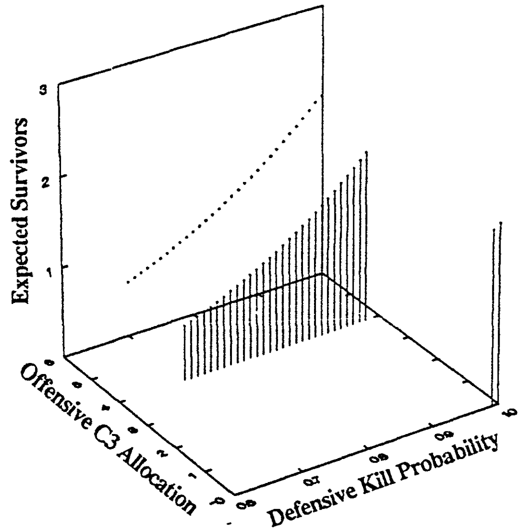

Figure 6 : P_c = .9, P_a = .9, Q = 4, N = 12, M = 6

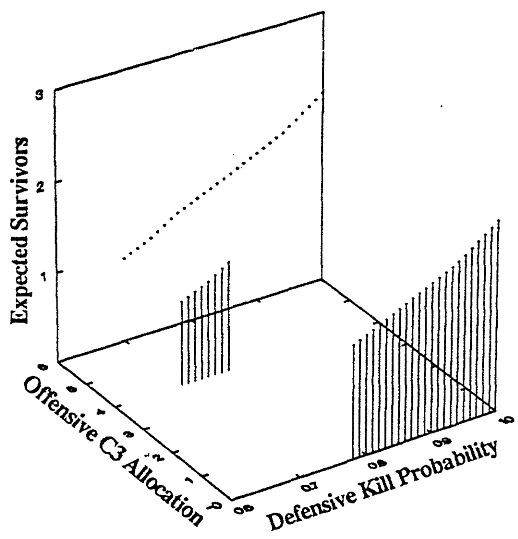

Figure 7 : P_c = .9, P_a = .9, Q = 4, N = 12, M = 8

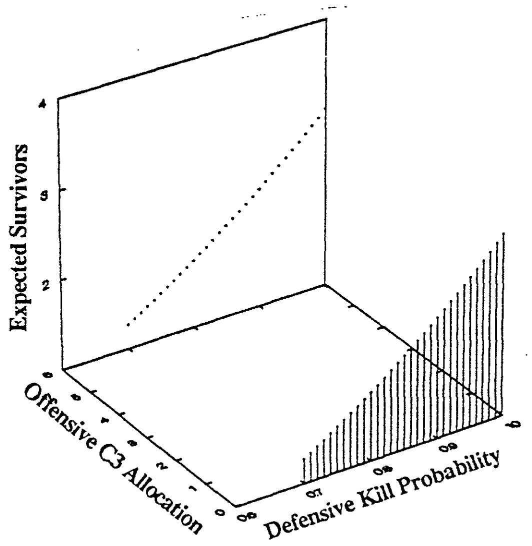

Figure 8 : $P_c = .9$, $P_a = .9$, Q = 4, N = 12, *M = 10*

In figures 9-12 we present the results when the C^3 nodes are softer targets than the assets: $p_c = .95$ and p_a (assets) = .9; other parameters are Q=2, N=12 and r from .5 to .7. When M=12 (figure 9) the offense attacks the C^3 nodes with 6 weapons for all values of r. When the defensive stockpile increases to 14, the C^3 attack level drops to 4 only for r close to 1.0 (figure 10). If increased to 16 or 18 (figures 11 and 12), however, the offensive C^3 allocation drops to as low as 2 for high r, and then switches to 4 and then 6 as r drops. It can be seen that softer C^3 nodes encourage attack, even for relatively large defensive stockpiles.

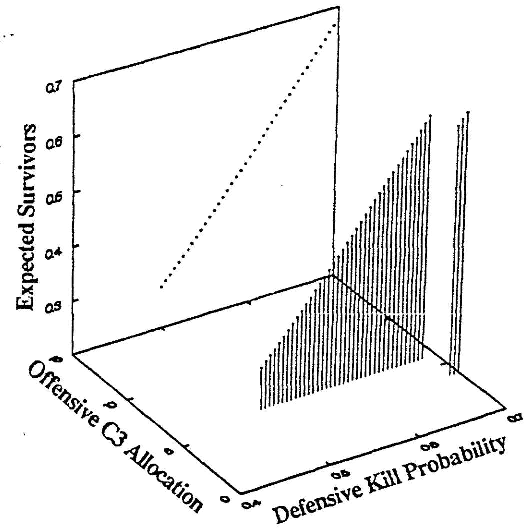

Figure 10 : $P_c = .95$, $P_a = .9$, Q = 2, N = 12, *M = 14*

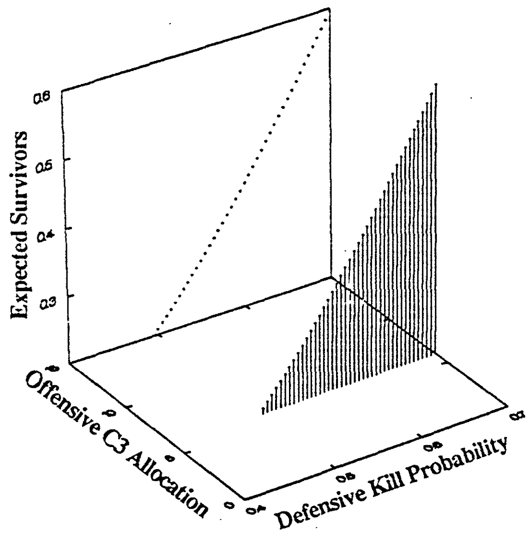

Figure 9 : $P_c = .95$, $P_a = .9$, Q = 2, N = 12, *M = 12*

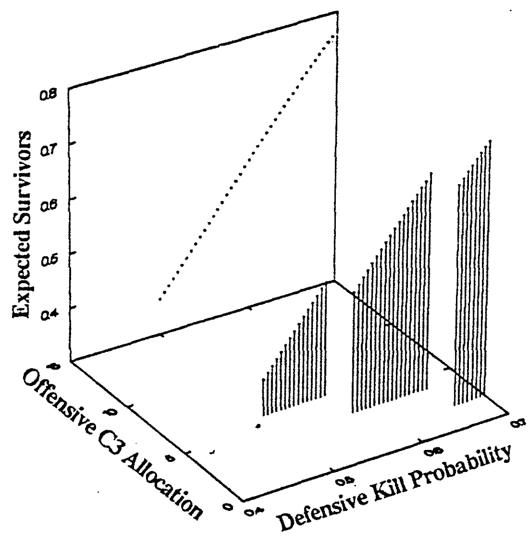

Figure 11 : $P_c = .95$, $P_a = .9$, Q = 4, N = 12, *M = 16*

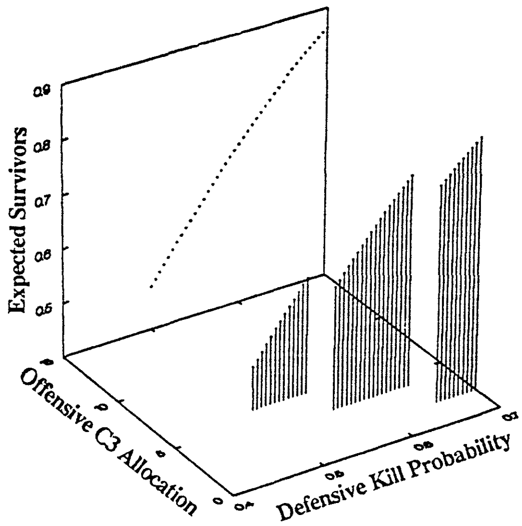

Figure 12 : $P_c = .95$, $P_a = .9$, $Q = 2$, $N = 12$, $M = 18$

Figure 13 shows the gain that replication of the command and control function provides. The ratio of the payoff for 2 C^3 nodes to that of 1 C^3 node is plotted against values of r that range from .70 to 1.00 and for defensive stockpiles of 4, 8 and 12.

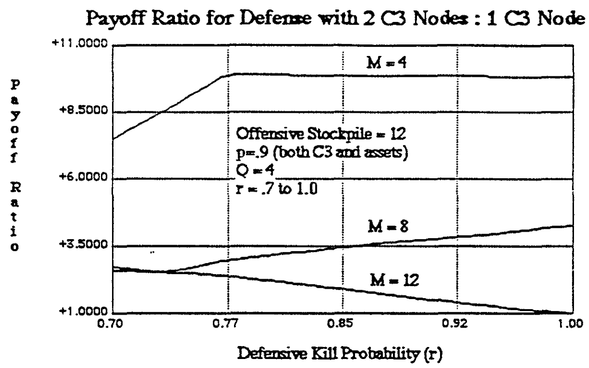

Figure 13

For small M (4), the gain is quite marked: it climbs from 7 to over 10. The plateau occurs at the point at which the offense switches from 2 to 4 and the defense from 2 to 3 weapons for the 2 C^3 node case. When M=8 the gain at first drops and then rises for increasing r, with the minima occurring where strategies for the one C^3 node case change. Strategies remain the same for all values of r when M=12, and the gain decreases monotonically - equalling one when r equals one. This is to be expected, since when the defense has 12 perfect weapons, it doesn't matter whether there are 1 or 2 C^3 nodes: all of the offense's weapons will be successfully intercepted. Figures 14 and 15 detail the strategies that lead to this result, both one and two C^3 nodes resulting in the same number of expected survivors when r=1, but with two C^3 nodes being more effective for lower values of r.

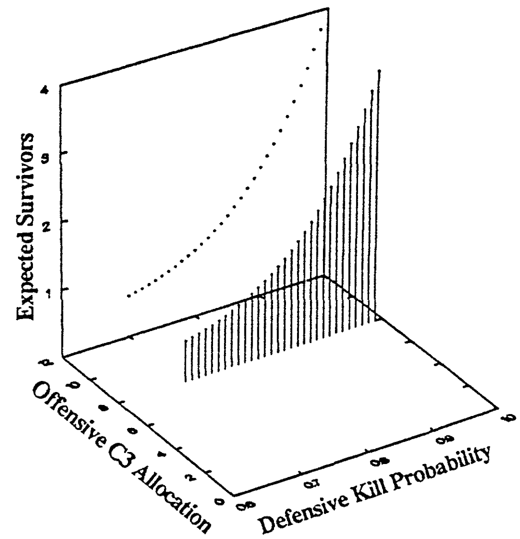

Figure 14 : P_c=.9, P_a=.9, Q=4, N=12, M=12, *1 C^3 Node*

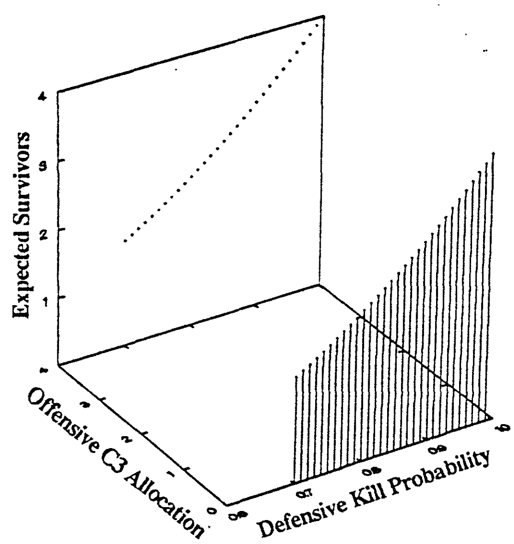

Figure 15 : P_c=.9, P_a=.9, Q=4, N=12, M=12, *2 C^3 Nodes*

5. CONCLUSIONS

The introduction of feedback, i.e. optimal "shoot-look-shoot-look-shoot-..." strategies, can significantly improve defense effectiveness. Some extensions to the present model, which we plan to consider, include stage-dependent target values and kill probabilities, as well as consideration of the dynamic version of the asset-defense problem. Unfortunately, this will mean dealing with substantially increased complexity.

In completing the study of the impact of vulnerable C^3 nodes, a limited domain of control for each node will be introduced. This begins to consider the effect that a distributed organizational structure has on the implementation of WTA strategies, and raises such questions as how many BM/C^3 nodes and of what type there should be. The issue of differing values for both C^3 nodes and assets must be addressed, since valuation will better enable us to quantify performance, and evaluate the tradeoffs that occur as distribution increases.

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