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COMMAND AND CONTROL EXPERIMENT DESIGN USING DIMENSIONAL ANALYSIS*

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ABSTRACT

Dimensional analysis is a method used in the design and analysis of experiments in the physical and engineering sciences. When a functional relation between variables is hypothesized, dimensional analysis can be used to check the completeness of the relation and to reduce the number of experimental variables. The approach is extended to include dimensions pertinent to experiments containing cognitive aspects so that it can be used in the design of multi-person experiments. The proposed extension is demonstrated by applying it to a single decisionmaker experiment already completed. New results from that experiment are described.

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INTRODUCTION

In the last few years, a mathematical theory for the analysis and design of organizations supported by Command, Control, and Communications (C3) systems has been developed based on the model of interacting human decisionmakers (DMs) with bounded rationality [1], [2]. While this model was motivated by empirical evidence from a variety of experiments, and by the concept of bounded rationality [3], there were no direct experimental data to support it. An experimental program has been undertaken to test the theory and obtain values for the model parameters [4].

One of the major difficulties in developing a model-driven experimental program is the specification of the large number of parameters that have to be specified and varied. The resulting problem has two aspects: (a) The parameterization of the experimental conditions leads to a very large number of trials, a situation that is not really feasible when human subjects are to be used, and (b) Not all experimental variables can be set at the values required by the experimental design because of the lack of direct controls on the cognitive variables.

Consequently, some orderly procedure is needed that will allow the reduction of the number of experimental variables and, more importantly, that will lead to variables that are easier to manipulate. Such an approach, called dimensional analysis, has been in use in the physical and engineering sciences [5], [6].

The purpose of this paper is to extend the approach to problems that have cognitive aspects so that it can be used for the design and analysis of experiments. The class of problems we are interested in are those that relate organizational structure directly to performance, as measured by accuracy and timeliness, and, more indirectly, to cognitive workload.

A special class of organizations will be considered - a team of well-trained decisionmakers executing repetitively a set of well-defined cognitive tasks under severe time pressure. The cognitive limitations of decisionmakers imposes a constraint on the organizational performance. Performance, in this case, is assumed to depend mainly on the time available to perform a task and on the cognitive workload associated with the task. When the time available to perform a task is very short (time pressure is very high), decisionmakers are likely to make mistakes so that performance will degrade.

This class of organizations is a reasonable model for tactical distributed decisionmaking such as that in the Command Information Center (CIC) of a battle group; a team of well trained individuals receive information from a variety of sources, process the information to develop the situation assessment, generate courses of action (COA), select a COA, and produce the set of commands or orders that will implement the chosen COA.

Dimensional analysis will be introduced briefly in the next section. The approach is then extended to include cognitive variables and a completed experiment will be used as an example to demonstrate the approach. Then, the application of dimensional analysis to the design of experiments for the analysis and evaluation of distributed tactical decisionmaking organizations will be described.

DIMENSIONAL ANALYSIS

Dimensional analysis is a method for reducing the number and complexity of experimental variables which affect a given physical phenomenon. A detailed introduction to dimensional analysis can be found in [5], [6].

Dimensions and Units. A dimension is the measure which expresses a physical variable qualitatively. A unit is a particular way to express a physical quantity, that is, to relate a number to a dimension. The dimension of a physical variable exists independently of the units in which it is measured. For example, length is a dimension associated to physical quantities such as distance, height, depth, etc., while foot, meter,... are different units for expressing length.

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Fundamental Dimensions. Fundamental dimensions are the basic dimensions which characterizes all variables in a physical system. For example, length, mass, and time are fundamental dimensions in mechanical systems. A dimension such as length per time is a secondary or derived dimension.

Dimensionally independent variables. If the dimension of a physical variable cannot be expressed by the dimensions of others in the same equation, this dimension is independent. For example, distance, velocity and time are three physical quantities which are not dimensionally independent because the dimensions of any two variables can form the dimension of the third. They are, however, pair-wise dimensionally independent.

The foundation of dimensional analysis is the Principle of Dimensional Homogeneity, which states that if an equation truly describes a physical phenomenon, it must be dimensionally homogeneous, i.e., each of its additive terms should have the same dimension.

For example, consider a moving vehicle with initial velocity v_o and constant acceleration a. During time t, the distance traveled s can be described by the following equation:

$$s = v_o t + at^2/2 \qquad (1)$$

where s has dimension of length, v_o has dimension of length per unit time, t has dimension of time, a has dimension of length per unit time per unit time, and the constant 1/2 is a pure number which has no dimension. Expressing the terms of this equation dimensionally, we obtain:

$$[s] = L$$
$$[v_o t] = LT^{-1}T = L$$
$$[at^2/2] = LT^{-2}T^2 = L$$

This shows all additive terms have dimension of length, therefore, Eq. 1 is dimensionally homogeneous.

The basic theorem of dimensional analysis is the π theorem, also called Buckingham's theorem.

π theorem: If a physical process is described by a dimensionally homogeneous relation involving n dimensional variables, such as

$$x_1 = f(x_2, x_3,..., x_n) \qquad (2)$$

then there exists an equivalent relation involving (n-k) dimensionless variables, such as

$$\pi_1 = F(\pi_2, \pi_3, ..., \pi_{n-k}) \qquad (3)$$

where k is usually equal to, but never greater than, the number of fundamental dimensions involved in the x's.

Each of the π's in Eq. 3 is formed by combining (k+1) x's to form dimensionless variables. Comparing Eqs. 2 and 3, it is clear that the number of independent variables is reduced by k, where k is the maximum number of dimensionally independent variables in the relation. The proof of the π theorem can be found in [5].

The π theorem provides a more efficient way to organize and manage the variables in a specific problem and guarantees a reduction of the number of independent variables in a relation. Dimensionless variables, also called dimensionless groups, are formed by grouping primary variables with each one of the secondary variables. The procedure for applying dimensional analysis will be described now through an example:

Step 1 Write a dimensional expression.

Let the dependent physical variable be denoted by q and the set of independent variables on which q depends be represented by w, x, y, and z. Since all the variables represent physical quantities, the have appropriate dimensions.

Then, a dimensional expression can be written as

$$q = f(w, x, y, z) \qquad (4)$$

There are five dimensional variables in Eq. 4, that is, n = 5.

Step 2 Determine the number of dimensionless groups.

To illustrate this step, a physical system and real physical quantities have to be assumed. Assume q is energy, w is time, x is a mass, y is acceleration, and z is distance in some mechanical system. One set of fundamental dimensions of a mechanical system are mass (M), length (L), and time (T), i.e., there are three dimensionally independent variables, k = 3. The dimensions of the variables in Eq. 4 are shown in Table1.

TABLE 1 Dimensions of variables in Eq. 4

Variable		Dimension	Notation
energy	q:	force × length	$[q] = ML^2T^{-2}$
time	w:	time	$[w] = T$;
mass	x:	mass	$[x] = M$;
acceleration	y:	length per time per time	$[y] = LT^{-2}$;
distance	z:	length	$[z] = L$

Since n = 5 from Step 1, there are,

$$n - k = 5 - 3 = 2,$$

so that three primary variables should be selected and two dimensionless groups can be constructed.

Step 3 Construct dimensionless groups.

While the choice of primary variables is essentially arbitrary, consideration should be given that the dimensionless groups be meaningful. If w, x, y are chosen as the three (k = 3) primary variables, two dimensionless groups are constructed on the basis of the remaining variables q and z. The first dimensionless group π_1 is formed by the combination of q, w, x, and y. Using the power-product method, π_1 can be determined by the following procedure. Write π_1 as

$$\pi_1 = q^a w^b x^c y^d$$

where a, b, c, and d are constants which make the right hand side of the equation dimensionless so that the equation is dimensionally homogeneous. In terms of dimensions of q, w, x, and y, we have

$$[M^0 L^0 T^0] = [ML^2T^{-2}]^a[T]^b[M]^c[LT^{-2}]^d$$

$$= [M^{a+c} L^{2a+d} T^{-2a+b-2d}]$$

By the Principle of Dimensional Homogeneity, the following set of simultaneous algebraic equations must be satisfied.

For M: $a + c = 0$
For T: $-2a + b - 2d = 0$
For L: $2a + d = 0$

There are three equations but four unknowns. The solution is not unique. In general, it is convenient for the secondary variables, in this example q and z, to appear in the first power, that is, a is set equal to unity. Thus, by solving the set of algebraic equations, we obtain:

$a = 1, \quad b = -2,$
$c = -1, \quad d = -2.$

then

$$\pi_1 = q / (w^2xy^2).$$

Similarly,

$$\pi_2 = zw^2 / y.$$

The dimensionless form of Eq. 4 is

$$q / (w^2xy^2) = \Psi(zw^2/y),$$

or in terms of the dimensionless groups,

$$\pi_1 = \Psi(\pi_2) \qquad (5)$$

This is the result obtained by the application of dimensional analysis. The function Ψ is unknown and needs to be determined by experiments. The dimensional analysis reduces Equation 4, which has four (4) independent dimensional variables, to Equation 5 which has only one independent dimensionless variable. The complexity of the equation is reduced dramatically. Furthermore, in designing an experiment, it is only necessary to specify a sequence of values for the independent variable π_2; these values can be achieved by many combinations of w, y, and z.

APPLICATION OF DIMENSIONAL ANALYSIS TO PROBLEMS IN COMMAND AND CONTROL

To apply dimensional analysis to decisionmaking organizations, the fundamental dimensions of the variables that describe their behavior must be determined. A system of three dimensions is shown in Table 2 that is considered adequate for modeling cognitive workload and bounded rationality. An experiment conducted in 1987 [4] is used to demonstrate the application of dimensional analysis to Command and Control problems. The purpose of the single-person experiment was to investigate the bounded rationality constraint. The experimental task was to select the smallest ratio from a sequence of comparisons of ratios consisting of two two-digit integers. Two ratios were presented to a subject at each time. The subject needed to decide the smaller one and compare it with the next incoming ratio until all ratios were compared and the smallest one was found. The controlled variable (or manipulated variable) was the amount of time allowed to perform the task. The measured variable was the accuracy of the response, i.e., whether the corect ratio was selected.

TABLE 2: DIMENSIONS FOR C^2 PROBLEMS

Dimension	Symbol	Units
Time	T	sec
Information (uncertainty)	I	bit
Alphabet	S	symbol

The controlled variables were the number of comparisons in a sequence, denoted by N, and the allotted time to do the task, denoted by T_w. For each value of N, where N could take the value of 3 or 6, T_w took twelve values with constant increment in the following way:

$$T_w = \{ 2.25, 3, 3.75, ..., 10.5 \} \qquad \text{for } N = 3;$$
$$T_w = \{ 4.50, 6, 7.50, ..., 20.1 \} \qquad \text{for } N = 6.$$

The performance was considered to be accurate or correct if the sequence of comparisons was completed and if the smallest ratio selected was correct. The details of the experiment can be found in [4].

The hypothesis is that there exists a maximum processing rate for human decision makers. When the allotted time is decreased, there will be a time beyond which the time spent doing the task will have to be reduced if the execution of the task is to be completed. This will result in an increase in the information processing rate F, if the workload is kept constant. However, the bounded rationality constraint limits the increase of F to a maximum value F_{max}. When the allotted time for a particular task becomes so small that the processing rate reaches F_{max}, further decrease of the allotted time will cause performance to degrade. The performance drops either because all comparisons were not made or because errors were made.. It was hypothesized that the bounded rationality constraint F_{max} is constant for each individual DM, but varies from individual to individual. The bounded rationality constraint can be expressed as

$$F_{max} = G / T_w^* \qquad (6)$$

where T_w^* is the minimum allotted time before performance degrades significantly. G and T_w^* vary for different tasks, but F_{max} is constant for a decision maker, no matter what kind of tasks he does. Therefore, significant degradation of performance indicates that the allotted time approaches T_w^*. Observation of this degradation during the experiment allows the determination of the time threshold and, therefore, the maximum processing rate, provided the workload associated with a specific task can be estimated or calculated [4].

The retroactive application of dimensional analysis to this experiment will be shown step by step.

Step 1 Write a dimensional expression.

In the experiment, accuracy, J, of information processing and decisionmaking is defined as the number of correct decisions, that is, the number of correct results in a sequence of comparisons. Therefore, J has the dimension of symbol and depends on the following variables:

- N: number of comparisons in each trial;
- T_w: allotted time to do N comparisons;
- H: uncertainty of input, that is, the uncertainty of the ratios to be compared in a trial;

Then, the dimensional expression is

$$J = f(T_w, N, H) \quad (7)$$

First, dimensional analysis checks whether this functional relation could describe the relation between J and other variables. The dimensions of the variables in Eq. 7 are the following:

$[J] = S$
$[T_w] = T$
$[N] = S$
$[H] = I$

Since the dimension of J is S, the right hand side of Eq. 7 has to be of the same dimension regardless of what the functional relation f is. However, all three fundamental dimensions are represented by the three independent variables. There is no way to combine these variables to obtain a term of dimension S only. Therefore, according to Principle of Dimensional Homogeneity, this functional relation is not a correct expression of the relation under the investigation.

There are two approaches to obtain the correct relation. The first is to delete T_w and H from the relation. This is not acceptable because the allotted time is a critical factor in this experiment. The other approach is to add some variables or dimensional constants to satisfy the requirement for dimensional homogeneity. Dimensional constants are physical constant such as gravity, the universal gas constant, and so on. No such dimensional constant has been identified in C^2 system as yet, therefore, some variables which have dimensions of time and information should be added to the relation. Moreover, the additional variables have to be relevant to the measurement of accuracy. Consideration of the nature of the tasks subjects performed and the data collected led to the observation that the entire allotted time period was not used to process information. This consideration led to a new variable: the actual processing time, T_f. Cognitive workload, denoted by G_a, is another significant variable affecting accuracy. Therefore, two variables are introduced to Eq. 7. The equation describing accuracy becomes

$$J = f(T_w, T_f, N, H, G_a) \quad (8)$$

This equation is dimensionally homogeneous. There are six dimensional variables in Eq. 8, that is, n = 6.

Step 2 Determine the number of dimensionless groups.

The number of dimensionless variables is equal to n-k, where k is the maximum number of dimensionally independent variables in Eq. 8. Dimensions of the variables are

$[J] = S$, $\quad [N] = S$
$[T_w] = T$, $\quad [T_f] = T$
$[H] = I$, $\quad [G_a] = I$

The maximum number of dimensionally independent variables is three. Therefore, k is equal to three. Then, the number of dimensionless groups is

$$n - k = 6 - 3 = 3.$$

There will be three dimensionless groups in the equivalent dimensionless equation.

Step 3 Construct the dimensionless groups.

The selection of primary variables is arbitrary as long as they are dimensionally independent. In this case, T_w, N, and H are selected as the primary variables. Using the power-product method, the π's are found to be

$$\pi_1 = J/N$$
$$\pi_2 = T_f/T_w$$

and

$$\pi_3 = G_a/H.$$

Now, we can write Equation 8 in a dimensionless form

$$J/N = \phi(T_f/ T_w, G_a/H) \quad (9)$$

or, in terms of the π's

$$\pi_1 = \phi(\pi_2, \pi_3) \quad (10)$$

In Eq. 10, π_1 is the percentage of correct decisions; π_2 indicates that portion of the time window used to process information and make decisions; and π_3 represents the ratio of actual workload and input uncertainty. Equation 10 represents a model driven experiment in which π_1, π_2 and π_3 are the experimental variables to be measured or controlled. The function ϕ needs to be determined experimentally.

Comparing Equations 8 and 10, one finds that the number of independent variables is reduced from five to two. This reduction reduces the complexity of the equation and facilitates experiment design and analysis. Properly designed experiments using dimensional analysis provide similitude of experimental condition for different combinations of dimensional variables which result in the same value of π's. Similitude reduces the number of trials needed to be run in order to define ϕ. This is a major advantage when the physical (dimensional) experimental variables cannot be set at arbitrary values.

The experiment that has been described was not designed using dimensional analysis. The independent variables that were manipulated were not π_2 and π_3. Therefore, ϕ cannot be determined from the experimental data. The purpose of using this experiment is to illustrate the dimensional analysis procedure for the design and analysis of model driven experiments. Therefore, only new results from dimensional analysis will be shown.

The model developed by applying dimensional analysis allows for more thorough analysis of the experimental data. In the original experiment, the allotted time was used to find the time threshold which was taken to correspond to the maximum processing rate. However, since the most obvious manipulated variable was the allotted time, the first priority of subjects seemed to be the completion of the comparisons within that time. The results from the experiment all reflect this observation. The rational expectation that the larger time window would result in better performance does not apply here. Instead, actual processing time to complete a task was increasing with increase of the allotted time, but was close to a constant when the allotted time became larger than a certain value. Knowing the allotted time, subjects tried to finish the task as soon as possible. The experimental data show that in most case, subjects either used a portion of the allotted time to finish the task, or could not finish the task within the allotted time. The ratio of actual processing time and the allotted time is always less than one. Therefore, calculation of the processing rate using allotted time led to underestimating the actual value. The use of the actual processing time leads to a new time threshold that yields a more accurate estimate of the maximum processing rate.

To find the critical value of T_f, the relation between the allotted time T_w and the actual processing time T_f has been studied. Figure 1 shows scatter plots of T_f versus T_w for two subjects.

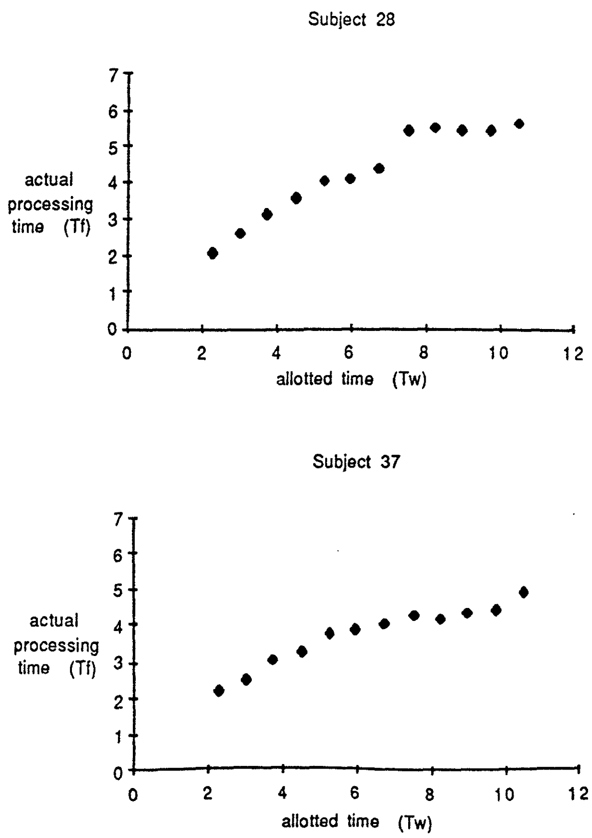

Fig. 1 Scatter Plot of T_f versus T_w for Two Subjects

The study of this relation results in postulating the following functional relation between T_f and T_w :

$$T_f = a\, e^{-b/T_w} \qquad (11)$$

where a and b are constant for each subject and vary among subjects. A least-squares fit was performed to determine the coefficients for each subject. Fig. 2 shows the results of curve fitting for the same two subjects.

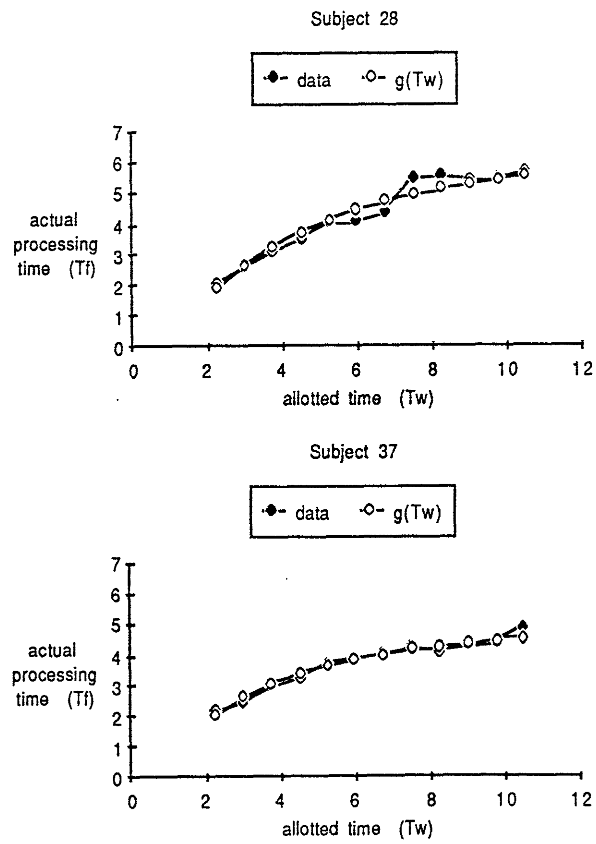

Fig. 2 Exponential Fit for Two Subjects

Since the critical value, T_w*, of T_w has been found from the original analysis [7], the critical value of T_f, T_f*, can be calculated using Eq. 11. Table 3 shows the statistics of the time threshold corresponding to T_f and T_w.

TABLE 3 Summary of Time Threshold for T_f and T_w over Subjects

	MEAN	ST. DEV	MAX.	MIN.
T_f*	4.50	1.12	7.21	2.06
T_w*	6.38	2.11	9.88	2.73

From Table 3, it is clear that the mean value of T_f* is smaller than that of T_w*. The standard deviation of T_f* is less than that of $T_w{}^*$ because T_f has to be less than or equal to T_w. Consequently, the calculation of information processing rate using $T_f{}^*$ gives a higher value because F^* is computed by

$$F^* = G/T^*.$$

For a particular subject, G does not change regardless whether $T_f{}^*$ or $T_w{}^*$ is used.

The procedure for designing experiments to study the effect of organizational structure on performance measures using dimensional analysis is:

1. According to dimensional considerations, determine independent variables which may affect the physical phenomenon, then form a general expression with an unknown function or functions;

2. Apply dimensional analysis to the expression to derive dimensionless groups and check for completeness;

3. Design experiments in which the values of the independent dimensionless groups are manipulated.

4. Run experiments to check the choice of independent variables and determine the hypothesized functional relation.

In the case of tactical decisionmaking organizations supported by C^3 systems, it is assumed that accuracy of a n-DM organization depends on the tempo of operations (which determines the allotted time to perform different tasks) and the cognitive workload of the individual decisionmakers, that is:

$$J = f(T, G^1, G^2, ..., G^n) \qquad (12)$$

where J is accuracy, T is a measure of time, and G^i is the workload of the i-th DM.The experimental model is established by augmenting Eq. 12. The measure of time is decomposed into the response times of individual DMs. The number of tasks is considered as a variable which affects accuracy. Uncertainty of the input can be controlled, and will also affect accuracy. For a particular task, cognitive activity varies among human decision makers because each DM may use a different approach to do the task. Let T^i the denote response time of the i-th DM, N denote the number of tasks, and H denote the input uncertainty. Equation 12 becomes

$$J = f (H, N, T^1, T^2,..., T^n, G^1, G^2,..., G^n) \qquad (13)$$

Equation 13 is an experimental model for an organization with n DMs. The unknown function f needs to be determined by experiment. There are (2n+2) independent variables in Eq. 12. Dimensional analysis will be used to reduce the complexity of the equation and organize the variables into groups amenable to manipulation in the context of experiments with human subjects.

CONCLUSIONS

Dimensional analysis has been introduced to the design of experiments that have cognitive aspects. An extension has been presented that makes it possible to include variables such as cognitive workload and bounded rationality of human decision makers. An existing single-person experiment has been used as an example to show how the methodology can be applied. A new result from the existing experiment has been presented to illustrate the possible advantages of using dimensional analysis. Note that dimensional analysis only determines possible relations between relevant variables; the actual functional expression has to be found from experimental data.

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