OPTIMUM CONFIGURATION FOR DISTRIBUTED TEAMS OF TWO DECISION-MAKERS

by

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ABSTRACT

This paper deals with problems of quantitative organizational design. We show that the optimal architecture of even a very simple team of two decision makers (DMs) performing binary hypothesis testing depends on variables external to the team. On the other hand, there exist particular probability distributions for the observations which lead to unambiguous optimal architectures in which the "better" decision maker makes the final team decision based upon finite-bit messages from the "worse" decision maker. But, even in these cases the results are difficult to generalize for teams with three or more DMs, because of the complexity of the problem. A heuristic algorithm for organization design is presented.

1. INTRODUCTION AND MOTIVATION

Our main research goal is to develop basic understanding of decision making in distributed organizations. As we shall see such problems can become very complicated because of the distributed (decentralized) decision process. In order to gain understanding into the basic fundamental issues we need a paradigm which represents simple decision making, and whose centralized version is easy to formulate, solve, and compute. We have adopted the problem of hypothesis testing as our basic paradigm; see references [1] to [4] for related prior research in this area.

The classic decision problem in this setting relates to the design a team to perform target detection (no target vs target present) using several distributed sensors. Suppose that each sensor has significant computational capability to process his raw returns and can perform local target detection. Because of the unreliability and uncertainty of the observations there is a high probability of error associated with each sensor's decision when he operates in isolation. Thus, it is desirable to have many sensors to operate together as a team to decrease the error probability. In order to achieve this, we have to define the architecture of the organization (i.e. which sensor communicates with whom) and derive a decision protocol to fuse the "tentative" sensor decisions into a global team decision.

We employ a binary hypothesis testing model, which can be generalized to more general hypothesis testing problems. These are indeed generic in the situation assessment C^2 function. We would like to develop a quantitative design methodology to deal with them.

We examine several important issues of these problems, starting with real-time decision making rules for distributed hypothesis testing. The team architecture is the way the DMs of the team are set-up. We want to obtain the performance of a given team architecture (say the probability of error) and compare the performance of alternative architectures. We also seek to design an organization to meet some global team performance specifications and study the effect of adding a new DM to the team. Finally, we would like to understand and develop the theoretical aspects and computational complexity associated with this class of problems.

Suppose that a team consists of N DMs. Evidently, the team may have many alternative architectures and communication protocols. For example, if $N = 3$, we can see two different architectures in Figure 1a and 1b. The environment consists of several hypotheses. Each DM receives a conditionally independent observation and makes a tentative decision, based upon his own measurement and the decisions of the other processors which have been transmitted to him, according to some specified communication protocols. The final team decision has some costs associated with it. We would like to determine somehow which configuration results into superior performance; moreover, given three DMs and a particular configuration, we would like to determine which DM should be employed in each position. We would also like to test the effects of different communication protocols. Another type of problem is illustrated in Figure 1c. Given a team of DMs which does not meet certain specifications, we would like to determine what DM should be introduced to the team and in what position, for the team to meet the posed specifications.

In this paper all the hypothesis testing performed is assumed binary. In Section 2, we will discuss the optimum configuration of a team consisting of two DMs. In Section 3, we examine the same problem for the special case where the observations of the DMs are described by Gaussian distributions with different variances under each hypothesis. In Section 4, we present a problem of organizational design and an algorithm to solve it. Finally in Section 5, we present some concluding remarks and suggestions for future research.

2. TWO DM ORGANIZATIONS

2.1 General Remarks

Since organizations with two DMs are key building blocks for larger organizations, our objective is to study them extensively and analyze them completely. There are two alternative architectures for this type of teams: fusion and tandem (Figure 2). Since the DMs in the tandem architecture can always employ the decision rules of the DMs in the fusion architecture (hence even the optimal decision rules for the fusion architecture), the performance of the tandem architecture is always at least as good as the performance of the fusion architecture. Thus, we will restrict ourselves to the study of the tandem architecture.

FIGURE 1
LONG RANGE OBJECTIVES

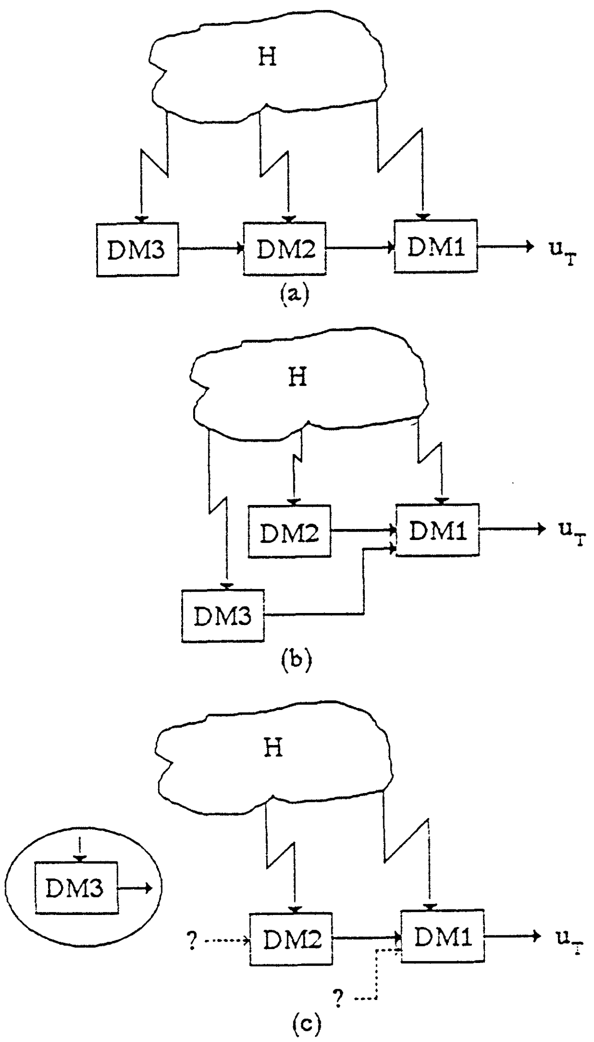
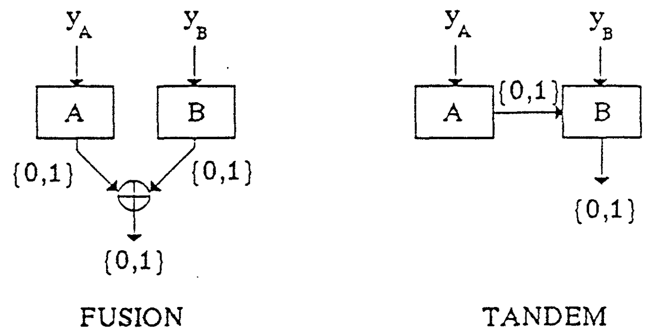


FIGURE 2
TWO DM ORGANIZATIONS



2.2 The ROC Curve

In the binary hypothesis testing problem each DM can be characterized by his Receiver Operating Characteristic (ROC) Curve. This curve plots the probability of detection as a function of the probability of false alarm.

The probability of detection, P_D, is the probability that the DM decides $u = 1$ (indicating that H_1 is the true hypothesis) when H_1 is indeed true and is defined by

$$P_D = \int_n^{\infty} P_{\Lambda|H_1}(\Lambda \mid H_1)\, d\Lambda \qquad (1)$$

where

$$\Lambda(y) = \frac{P(y \mid H_1)}{P(y \mid H_0)}$$

is the likelihood ratio and n represents the decision threshold.

The probability of false alarm, P_F, is the probability that the DM decides $u = 1$ when H_0 is the true hypothesis and is defined by

$$P_F = \int_n^{\infty} P_{\Lambda|H_0}(\Lambda \mid H_0)\, d\Lambda \qquad (2)$$

Thus, the ROC curve is expressed by two parametric equations, with the threshold parameter n varying from zero to infinity; in general, can not be expressed in a closed form. The ROC curve is concave and it has another useful property; suppose that by substituting n^* in equations (1) and (2), the point (P_F^*, P_D^*) of the ROC curve is obtained. Then, the slope of the tangent to the ROC curve at (P_F^*, P_D^*) is n^* (Figure 3). Consequently, if a DM performs detection with some given n^*, his optimal operating point is the point of the ROC curve where the slope of the tangent is n^*.

In our research, we use the ROC curve to quantify the relative expertise of different DMs. Moreover, since the team of DMs also performs binary hypothesis testing, team performance can also be quantified by the team ROC curve. If the ROC curve of DM A is higher than the ROC curve of

DM B, then we say that A is a better DM than B, because for the same level of probability of false alarm, A will have a higher probability of detection (Figure 4a). But, the DMs can not always be ranked globally because sometimes their ROC curves intersect (Figure 4b).

FIGURE 3
THE ROC CURVE

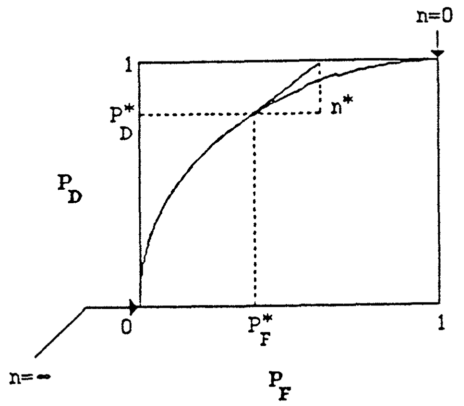
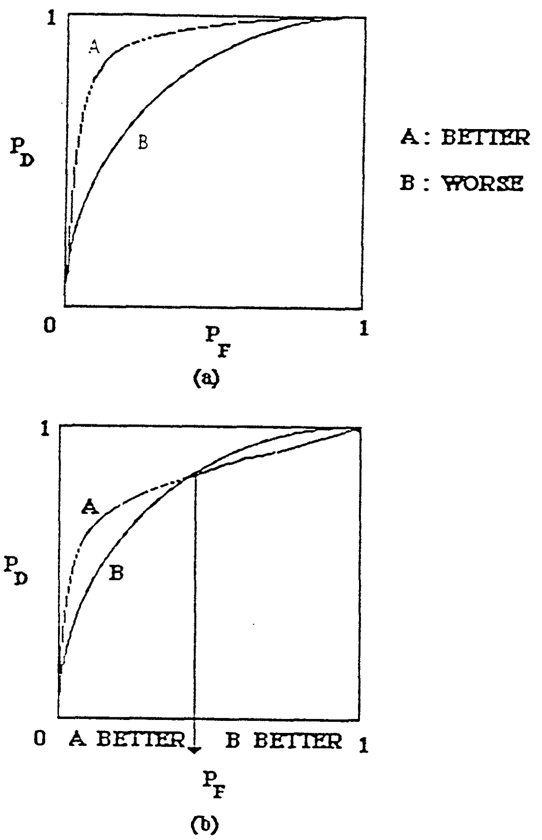


FIGURE 4
RANKING DECISION MAKERS



2.3 The Problem

Consider a team consisting of two DMs in tandem architecture, which performs binary hypothesis testing (Figure 5a). The prior probabilities ($P_i = P(H_i)$ for i = 0,1) are assumed known, as well as the costs J(u,H) which are incurred by the team when it decides u and H is the true hypothesis. It is assumed that it is more costly for the team to err than to be correct. The team objective is to minimize the expected cost incurred by the team.

Each DM receives a conditionally independent observation. One DM, called the consultant DM, makes a binary decision ($u_c = 0$ or $u_c = 1$) based on his measurement, y_c, and transmits it to the othe DM, called the primary DM. Then, the primary DM has to make the team decision (based upon his own measurement y_p and the message from the consultant) which has to be either $u_p = 0$ or $u_p = 1$ indicating that the corresponding hypothesis is considered to be true.

The optimal solution for the decision rules of the two DMs is given by likelihood ratio test with constant thresholds [3]. For the primary DM:

$$\text{If } u_c = 0: \qquad \Lambda(y_p) \underset{u_p=0}{\overset{u_p=1}{\gtrless}} \frac{1 - P_F^c}{1 - P_D^c}\, n \tag{3}$$

$$\text{If } u_c = 1: \qquad \Lambda(y_p) \underset{u_p=0}{\overset{u_p=1}{\gtrless}} \frac{P_F^c}{P_D^c}\, n \tag{4}$$

For the consultant DM:

$$\Lambda(y_c) \underset{u_c=0}{\overset{u_c=1}{\gtrless}} \frac{P_F^1 - P_F^0}{P_D^1 - P_D^0}\, n \tag{5}$$

where

$$n = \frac{P_0}{P_1} \; \frac{J(1,H_0) - J(0,H_0)}{J(0,H_1) - J(1,H_1)}$$

and P_D^i (P_F^i) is the probability of detection (probability of false alarm) for the primary DM when $u_c = i$ was received by the consultant (i = 0,1) and P_D^c (P_F^c) is the probability of detection (probability of false alarm) for the consultant DM, when both DMs are operated according to the optimal decision rules of eqs.(3)-(5). For example,

$$P_F^0 = \Pr\left(\Lambda(y_p) \geq \frac{1 - P_F^c}{1 - P_D^c}\, n \mid H_0\right) \tag{6}$$

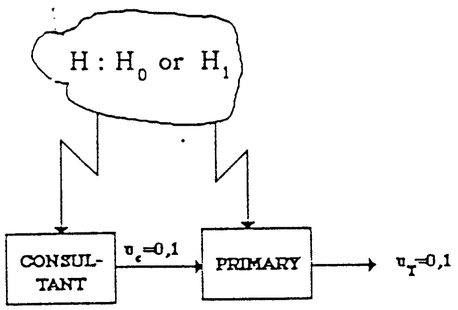
Figure 5b demonstrates the form of the operating points.

The ROC curve of the team as a whole can be computed and is given by:

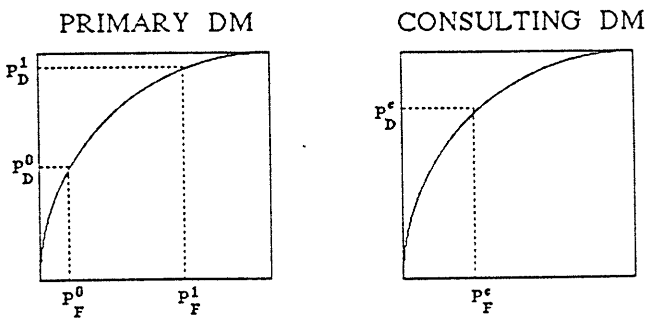
$$P_F^T = (1 - P_F^c)\, P_F^0 + P_F^c\, P_F^1 \tag{7}$$
$$P_D^T = (1 - P_D^c) P_D^0 + P_D^c\, P_D^1 \tag{8}$$

FIGURE 5

THE PROBLEM AND ITS SOLUTION



(a)



(b)

Note that the team ROC depends not only upon the characteristics ("expertise") of the individual DMs, but also on the particular way that they have been constrained to interact (the team or organization architecture).

2.4 Architecture Comparisons

Suppose that one of the two DMs is "better" than the other, i.e. his ROC curve is higher than the ROC curve of the other DM. There exist two candidate architectures for the team; either make the "better" DM the primary DM or make the "better" DM the consultant DM. Recall that the primary DM makes the final team decision. We would like to determine which of the two architectures yields better performance than the other for all values of n, that is whether the optimal architecture is independent of the external parameters of the problem (details of cost function, prior probabilities) which determine the value of n.

The architecture with the better DM as the primary DM was conjectured [3] to be better. This conjecture is appealing from an intuitive point of view; given two DMs one would like to have the "better" DM make the final decision, independent of the prior probabilities and the cost assignments. If this were the case, then the optimal way of organizing two DMs would not change, say, as the prior probabilities of the underlying hypotheses vary. Unfortunately, as we show below, this conjecture can be false.

2.5 A Counterexample to the Conjecture

In Figure 6, we present the ROC curves of two DMs, one better than the other according to our prior definition. Table 1 contains the discrete distributions of their observations; the elements in the matrix denote probabilities. For example, the "worse" DM will observe y = 1 with probability 0.1 if H_0 is true and with probability 0.5 if H_1 is true. From Table 1 we can then see that the better DM has as good or better discrimination of the two hypotheses, and of course this is reflected in the dominance of his ROC curve in Figure 6.

In order to establish the counterexample we compared the two architectures using tedious, albeit straight forward calculations of the probability of error. The results are illustrated in Table 2, which contains the probability of error for two different values of n for each architecture -- B denotes the "better" DM, while W denotes the "worse" one. For n = 1.0 having the better DM as the consultant is

FIGURE 6
THE ROC CURVES

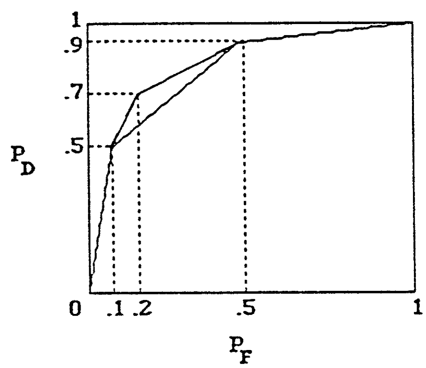


TABLE 1
DESCRIPTION OF DMs

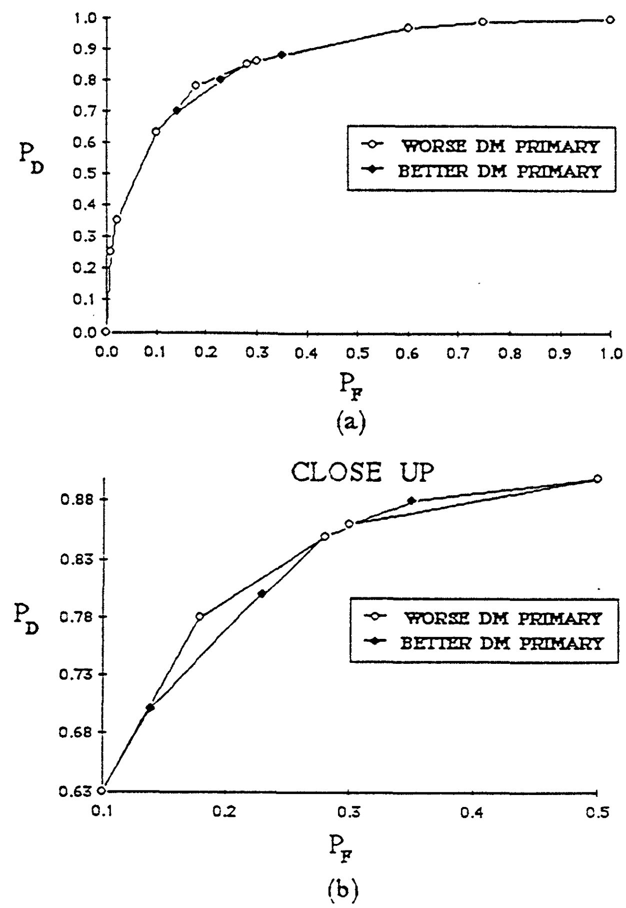
y \ H	"WORSE" DM H_0	"WORSE" DM H_1	"BETTER" DM H_0	"BETTER" DM H_1
1	0.1	0.5	0.1	0.5
2	0.4	0.4	0.1	0.2
3	0.5	0.1	0.3	0.2
4	0	0	0.5	0.1

optimal while for n = 0.38 having the better DM as the primary is optimal. This can be also verified by deriving the team ROC curves for each architecture (Figure 7a). As the close-up of Figure 7b shows the two ROC curves intersect near P_F= 0.3. Thus, in this special example, the optimal team

TABLE 2
COMPARISONS OF
PROB. OF ERROR

	B → W →	W → B →
n = 1.00	0.200 (optimal)	0.215
n = 0.38	0.1840	0.1833 (optimal)

FIGURE 7
TEAM ROC CURVES



architecture depends on the value of n (i.e. the numerical values of the prior probabilities and costs). On the other hand, for this example, both architectures have very similar performance, since their ROC curves are quite close (Figure 7a).

3. COMPARING GAUSSIAN VARIANCES

3.1 General Remarks

Consider special case of the problem presented in Section 2.3 above, where each DM receives two independent observations distributed with the Gaussian distribution with different variance under each hypothesis. The ROC curves in this case are simple and given in a closed form [1]. A summary of this case is given in Table 3.

3.2 The First Architecture

Suppose that the better DM is made the primary. Then, from the solution of the problem and the property of the ROC curve

$$n^c = \frac{P_F^1 - P_F^0}{P_D^1 - P_D^0}\, n = \left.\frac{dP_D^B}{dP_F}\right|_{(P_F^c, P_D^c)} = \frac{\sigma_0^2}{N\sigma_1^2}\,\frac{P_D^c}{P_F^c} \qquad (9)$$

$$n^0 = \frac{1 - P_F^c}{1 - P_D^c}\, n = \left.\frac{dP_D^W}{dP_F}\right|_{(P_F^0, P_D^0)} = \frac{\sigma_0^2}{\sigma_1^2}\,\frac{P_D^0}{P_F^0} \qquad (10)$$

$$n^1 = \frac{P_F^c}{P_D^c}\, n = \left.\frac{dP_D^W}{dP_F}\right|_{(P_F^1, P_D^1)} = \frac{\sigma_0^2}{\sigma_1^2}\,\frac{P_D^1}{P_F^1} \qquad (11)$$

where the superscripts B (better) and W (worse) indicate which DMs ROC curve is being differentiated. Solving the system of eqs.(9)-(11) and recalling the concavity of the ROC curve, we obtain that in this case

$$(P_F^1, P_D^1) = (1,1)$$

which implies that whenever $u_c = 1$ is received from the consultant, the primary decides $u_p = 1$ independent of his own observation. Substituting into (7) and (8), we obtain that the team ROC curve in this case is given by:

$$P_F^T = P_F^0 + P_F^c - P_F^0 P_F^1 \qquad (12)$$

$$P_D^T = P_D^0 + P_D^c - P_D^0 P_D^1 \qquad (13)$$

for some (P_F^0, P_D^0) $[(P_F^c, P_D^c)]$ in the ROC curve of the worse [better] DM.

TABLE 3

COMPARING GAUSSIAN VARIANCES

WORSE DM:	BETTER DM:
$Y_1, Y_2 \sim N(0, \sigma^2)$	$Y_1, Y_2 \sim N(0, \sigma^2)$
$H_0 : \sigma^2 = \sigma_0^2$	$H_0 : \sigma^2 = \sigma_0^2$
$H_1 : \sigma^2 = \sigma_1^2$	$H_1 : \sigma^2 = N\sigma_1^2$
$P_D = P_F^{\frac{\sigma_0^2}{\sigma_1^2}}$	$P_D = P_F^{\frac{\sigma_0^2}{N\sigma_1^2}}$

with: $\sigma_0^2 < \sigma_1^2$; $N > 1$

3.3 The Second Architecture

Suppose that now the better DM is made the primary. Then, we can arbitrarily assign to the DMs the following operating points:

(P_F^0, P_D^0): to the Consultant (Worse) DM
(P_F^c, P_D^c): to the Primary (Better) DM when $u_c = 0$ is received
$(1,1)$: to the Primary (Better) DM when $u_c = 1$ is received

Substituting into eqs. (7) and (8), we obtain eqs. (12) and (13) again. Since for this arbitrary assignment of operating points, the architecture with the better DM as the primary can achieve performance equal to the optimal performance of the other architecture, the <u>better DM should always be the primary DM.</u>

3.3 Obtaining the Team ROC Curve

Suppose that the better DM is the consultant. Then, from the system of eqs. (9)-(11), we can solve for P_F^c to obtain:

$$P_F^c = \left[N\sigma_1^2 (1-P_D^c)\, n \; \frac{[\sigma_0^2(1-P_D^c)]^{-\frac{\sigma_1^2}{\sigma_1^2-\sigma_0^2}} - [\sigma_1^2(1-P_F^c)n]^{-\frac{\sigma_1^2}{\sigma_1^2-\sigma_0^2}}}{[\sigma_0^2(1-P_D^c)]^{-\frac{\sigma_0^2}{\sigma_1^2-\sigma_0^2}} - [\sigma_1^2(1-P_F^c)n]^{-\frac{\sigma_0^2}{\sigma_1^2-\sigma_0^2}}} \right]^{-\frac{N\sigma_1^2}{N\sigma_1^2-\sigma_0^2}}$$

This is an equation of just P_F^c. We could have substituted for P_D^c from the equation of the ROC curve of the consultant (better) DM, but did not do it because of space limitations. If the equation is solved P_F^c is obtained. Moreover:

$$P_F^0 = \left[\frac{\sigma_1^2}{\sigma_0^2}\,\frac{1-P_F^c}{1-P_D^c}\,n\right]^{-\frac{\sigma_1^2}{\sigma_1^2-\sigma_0^2}}$$

By substituting into the equation of the ROC curve of the primary (worse) DM, P_D^0 is obtained. Finally by substituting for all the probabilities into equations (7) and (8), the team ROC curve is obtained as a function of n, the variances of the DMs and N.

It should be clear that the team ROC curve will not be of the same form as the ROC curves of the individual DMs. In fact, it is not even given by a closed form expression. Thus, we cannot easily extend the result to the case of three DMs in a tandem architecture.

4. DESIGNING ORGANIZATIONS

4.1 General Remarks

Suppose that we are given a team of DMs and a set of requirements on the team performance, which are not met. We could perform several changes in the team, such as adding or deleting a DM or changing the team interconnections, or redesigning the communication protocols, to make the team meet posed performance requirements. Presently, we are employing a trial and error approach because of the mathematical complexity of the problems; we hope for analytical insight from our future research.

4.2 Adding a New DM

By introducing the "perfect" DM to the team, that is a DM who always knows which is the true hypothesis (i.e. his ROC curve goes through $(P_F, P_D) = (0,1)$), the team probability of error will be reduced to zero. Hence, specifications no matter how strict can always be met.

We would like to introduce a trade off between the team performance and the quality of the DM to be introduced. To measure quality we need to rank the DMs even in cases of ambiguity (Figure 4b). The measure we will employ is the area under the ROC curve. This measure is scalar and preserves the ranking of unambiguous situations (Figure 4a); the "perfect" DM has a measure of 1 and the "worst" DM (the DM who is equally likely to choose between either hypothesis independent of his observation) has a measure of 0.5.

The design problem will now be to find the "cheapest" DM which will enable the team to meet the requirements; by cheapest meaning the DM with the smallest area under the ROC curve.

4.3 A Sample Problem

Suppose we are given a DM ("old") with ROC curve:

$$P_D = P_F^{\frac{\sigma_0^2}{\sigma_1^2}}$$

and a set of requirements for team performance (i.e. minimum levels of probability of detection for specified levels of probability of false alarm). We want to find the "cheapest" DM ("new") with omiomorphic ROC curve to the old DM, that is:

$$P_D = P_F^{\frac{\sigma_0^2}{K\sigma_1^2}} \quad ; K \geq 0$$

which will make the team satisfy the requirements. In this case, the <u>smaller</u> the value of the constant K the <u>cheaper</u> the DM.

The problem is the same as the one described in Section 2.3 above. The two possible architectures are to use the new DM as the consultant or to use the new DM as the primary.

In the following algorithm we use our theoretical analysis which suggests that <u>the better DM should be the primary</u> to avoid a completely our trial and error approach.

4.4 The Algorithm

STEP 0: Start with two identical "old" DMs

STEP 1: If the requirements are met, then the team is too good. Thus, the new DM can be worse than what he is, which implies that the K of the new DM can and should decrease. From our theoretical analysis we know that the new DM should be the consultant Thus, we decrease the consultant's K and go to STEP 3.

STEP 2: If the requirements are not met, then the team is too weak. Thus, the new DM should be better than what he is, which implies that the K of the new DM can and should increase. From our theoretical analysis we know that the new DM should be the primary. Thus, we decrease the primary's K and go to STEP 4.

STEP 3: If all the requirements are met and one is met exactly, we stop. If the requirements are met then we decrease the K of the consultant. If the requirements are not met we increase the K of the consultant. We then repeat STEP 3.

STEP 4: If all the requirements are met and one is met exactly, we stop. If the requirements are met then we decrease the K of the primary. If the requirements are not met we increase the K of the primary. We then repeat STEP 4.

Our theoretical analysis indicated whether the new DM should be the primary or the consultant. Using educated choises for the values of K in our trial and error approach our problem will be solved efficiently.

5. CONCLUSIONS

By a counterexample we have shown that the optimal team architecture may depend on parameters external to the team (prior probabilities, cost structure etc). Hence, we can have ambiguity of whether a particular architecture is optimal for all values of the external parameters. It is possible, however, to use the area under the team ROC curve to remove the ambiguity.

Special distributions lead to architectural comparisons that are unambiguous. We demonstrated this in the case of comparing gaussian variances, in which the better DM should always be the primary DM. Computer simulations (not reported here) indicated that this result holds true for comparisons of means of gaussian distributions, but the inherent complexity of the equations prohibited us from obtaining analytical results.

Even if the individual DM ROC curves are analytical, the team ROC curve is not. Thus, it is hard to generalize our results to teams with more than two DMs. We hope to obtain some novel results to help us design more complex organizations; but, it is not clear whether such results exist.

Finally, we plan to study the effects on the team performance of different communication protocols as well as of more complex (non-binary) hypotheses.

ACKNOWLEDGMENT

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