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A REGULATORY ADJUSTMENT PROCESS FOR OPTIMAL  
PRICING BY MULTIPRODUCT MONOPOLY FIRMS

by

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ABSTRACT

This paper describes an incentive mechanism that is shown to enforce the use of Ramsey prices by multiproduct monopolies. The constraint given is simple. It limits information requirements on the regulatory agency to bookkeeping data of the firm. Its implementation could be easily controlled by outside courts or auditors. The process, therefore, makes use of invisible hand properties shifting the workload of welfare optimization from the regulatory agency to the regulated firm. This may lead to the ironical conclusion that regulatory commissions should fire their economists. It, however, becomes both profitable and socially beneficial for the regulated firms to employ them.

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## 1. INTRODUCTION

Following a survey by Baumol and Bradford (1970) of the by now classical literature on second-best pricing for public enterprises a series of articles has focused on this topic. Conditions for prices to achieve a constrained welfare maximum are well-known. They are named after Ramsey, who first (1927) derived them as the solution to an optimal taxation problem. Welfare maximizing firms should inflate all demand elasticities for their products by a common factor and otherwise behave like an unconstrained monopolist (Drèze and Marchand, 1975).

However, there remains the task of translating this rule into an incentive scheme for the firms so that it becomes operational for the firm management and the regulators. This question has been raised but left open by Bawa and Sibley (1975) within the scheme of rate-of-return regulation.

If regulators had to decide on efficient price structures, they would have to know demand elasticities and cost functions within some range of the current prevailing prices and current costs. In general, the firm's staff and managers will know price elasticities and cost functions for their products better than do regulators. Hence, the firm management may be in a superior position to calculate and implement welfare maximizing prices. But why should they? We see three basic reasons for them to do so. First, they may hold a professional<sup>1</sup> or humanitarian interest in pursuing a welfare-maximizing strategy. This cannot generally be

expected from managers, and may even be an undesirable feature of somebody running an enterprise. Second, the survival of the firm may be in danger because of potential competition by newcomers.

Welfare-maximizing prices can then be a limit-pricing strategy in the sense that they are best sustainable.<sup>2</sup> The empirical significance of this hypothesis remains largely unexplored. Third, the regulatory agency could try to force the firm to convey the necessary information or to compute welfare-maximizing prices. But without duplicating company management and staff, how could the regulators evaluate the information they receive? Even the firm is not sure of its demands and costs. Discrepancies between ex post figures and projections previously filed with a regulatory agency are not necessarily evidence of cheating. The solution of this problem could be a well-defined rule that motivates firms to charge economically efficient prices.

Regulatory agencies may be excessively influenced or even corrupted by special interest groups. For this reason, an additional advantage of a predetermined price-setting rule is precisely that it prevents continuous direct intervention by an agency in the price structure used by a regulated firm.

As a first step in this direction we suggest a simple incentive mechanism which leads firm management to improve the price structure step by step and which, under certain conditions, results in an optimum. This is described in Section 2.2 after Section 2.1 has outlined the concept of

constrained welfare maximization used throughout in this paper. Sections 2.3 and 2.4, being the heart, give proof to our main proposition and the economic rationale behind it. Then, Section 2.5 and the Conclusion turn to the policy issues that might follow from our reasoning.

Two complications which our rule may have to face are dealt with in Section 2.6. One is that the firm tries to pass inside the effect of the short-term-oriented rule by using a long-term strategy. The other is that the firm does not face decreasing ray average costs.

## 2. THE MODEL

### 2.1 Characterization of Budget-Constrained Welfare Maximization

We consider a regulated private or public enterprise in a natural monopoly setting.

Assumption 1: (Objective of the regulatory agency) The firm's social objective as pursued by the regulatory agency is the maximization of welfare subject to a budget constraint:

$$\begin{aligned} & \max W(p) \\ & \text{s.t. } \Pi(p) \geq 0 \end{aligned}$$

(p) is the firm's profit function

$$\Pi(p) = x(p)p - C(x(p))$$

with  $C(x)$  the cost function.

Assumption 2: (Welfare function) Welfare is supposed to be given by consumers' surplus  $W(p)$  with the following properties:

- (a)  $W(p)$  is continuously differentiable and convex<sup>3</sup>
- (b)  $\text{grad } W(p) = -x(p)$ , where  $p = (p^1, \dots, p^n)$  denotes the price vector corresponding to the demand vector  $x(p)$  for the  $n$  commodities produced by the firm.

It is Assumption 2(b) which characterizes  $W(p)$  as consumers' surplus. This is restrictive insofar as income effects are taken to be insignificant. They could in fact play a major role, if a substantial part of the economy were regulated in the suggested way.<sup>4</sup> Then, however, regulation as a market-oriented policy becomes questionable. Profits of the regulated firm have been eliminated from the welfare function because in the limit they will be shown to vanish. Also, welfare effects on commodities not supplied by the regulated firm are neglected. As long as the firm's input/output decisions do not substantially affect prices on other markets, such welfare effects will be small.<sup>5</sup>

The nontrivial first order condition for an optimum with  $\Pi(p) \leq 0$  is

Condition 1

$$\left(p - \frac{\partial C}{\partial x}\right) \frac{\partial x}{\partial p} = -\lambda x$$

where  $0 \leq \lambda \leq 1$  and

$$\frac{\partial x}{\partial p} = \begin{pmatrix} \frac{\partial x_1}{\partial p_1} & \cdots & \frac{\partial x_1}{\partial p_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial x_n}{\partial p_1} & \cdots & \frac{\partial x_n}{\partial p_n} \end{pmatrix}$$

Condition 1 is the generalized Ramsey formula. It is implied by the optimal taxation result in Diamond and Mirrlees (1971, p. 262).<sup>6</sup> An unconstrained profit-maximizing monopolist in equilibrium would satisfy Condition 1 with  $\lambda = 1$ , but at a constrained second best solution  $\lambda$  will be just high enough to allow the firm to break even.

Geometrically, as seen in Fig. 1, Condition 1 means that at the optimum<sup>7</sup>, the normal to the surface  $\{p | \Pi(p) = 0\}$  and the demand

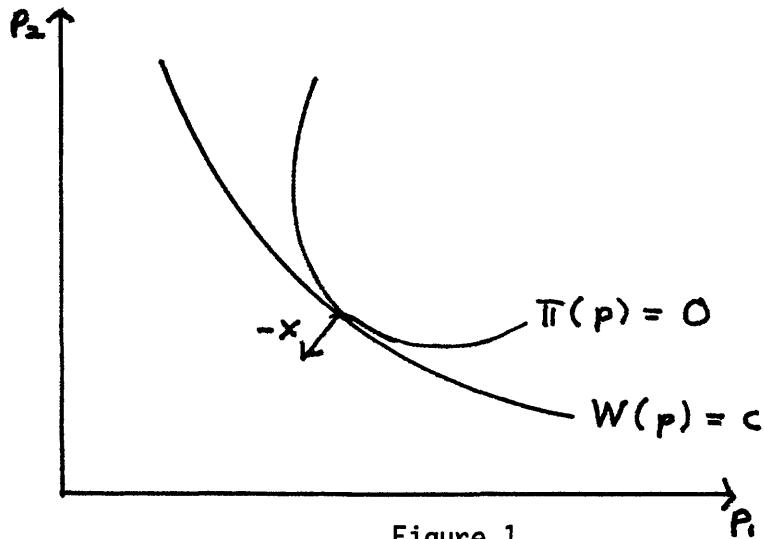


Figure 1

vector  $x$  are collinear. This result is plausible once we recall that the normal to the isowelfare surfaces  $\{p | W(p) = c\}$  is  $-x$  (Assumption 2(b)). Hence, at the optimum the isowelfare surface is tangent to the zero profit surface.

## 2.2 The Regulatory Algorithm

Assumption 3: (The firm and its markets) (a) We suppose a regulated private or public multiproduct enterprise whose objective is to maximize profits in each consecutive period  $j$ ,  $j \in \mathbb{N}$ . It faces demand and cost functions which do not change over time. There are no intertemporal cost effects. The firm management is assumed to know these functions.

(b) The inverse demand function  $p(x)$  shall be continuous and nonnegative for all  $x \in \mathbb{R}_+$  and  $\lim_{x \rightarrow \infty} p(x) \rightarrow 0$ .



This is compatible with the assumed welfare function  $W(p)$ .

(c) The profit function  $\Pi(p)$  shall have the following properties:

- (1)  $\Pi(p)$  is continuously differentiable
- (2) In all neighborhoods of  $p$  with  $\Pi(p) = 0$ , there exists  $p^+$  with  $\Pi(p^+) > 0$
- (3)  $\{p \mid \Pi(p) \geq 0\}$  is a compact set.

(d) The cost function  $C(x)$  exhibits decreasing ray average costs:

$$\forall r \in \mathbb{R} \text{ with } r \geq 1 \quad C(rx) \leq r(C(x))$$

Assumption 4: (Regulatory constraint) (a) The regulatory agency knows only actual costs, prices, and quantities which have been realized by the firm up to the present. The firm is required to serve all demand at current prices. Hence the data observed by the regulatory agency at time  $j$  is  $p_j$ , resulting in

$$x_j = x(p_j), C(x_j), \text{ and } \Pi(p_j).$$

$p_j$  is the price vector at time  $j$ .

(b) The regulatory agency defines the set of feasible prices for each consecutive period  $j + 1$ ,  $j \in \mathbb{N}$ , by

$$R_j = \{p \mid x_j p - C(x_j) \leq 0\}$$

Assumption 4(b) means that in period  $j + 1$  the firm may ask for prices which at best would produce no profit, if applied to the quantities sold in period  $j$ . Taking quantities of period  $j$  as weights the firm on average has to reduce its prices by the previous profit margin.

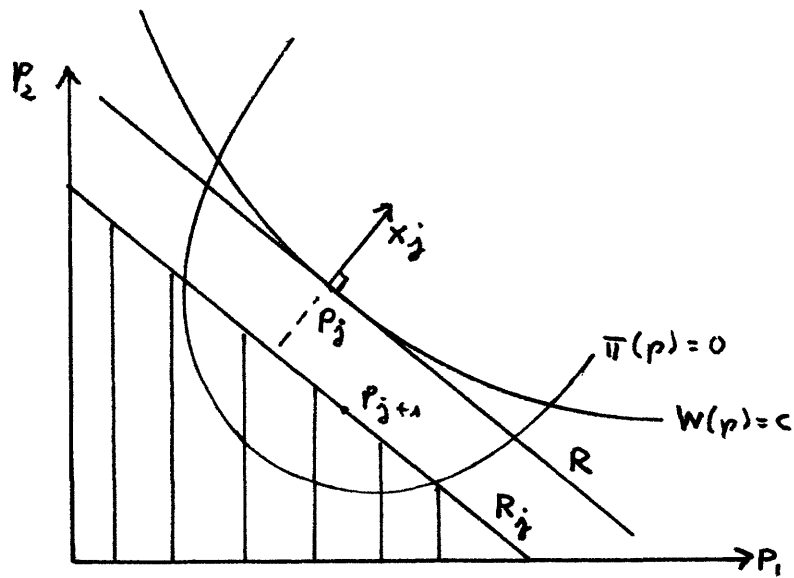


Figure 2.

In Fig. 2, the shaded area corresponds to  $R_j$ . The firm is allowed to maximize profit constrained by  $R_j$ . Indeed, the convergence to the optimal point requires just that.<sup>8</sup> In spite of managerial theories of the firm and the satisficing literature we think it plausible that the

management can be induced to conform with this objective to the extent that its income depends on profits and losses. Thus, if sufficiently motivated by a strong profit sharing scheme, the management will try to maximize  $(p)$  subject to  $R_j$ .  $R_j$  has been constructed by moving the tangent hyperplane  $R$  at  $p_j$  into the direction of the largest welfare increase  $\text{grad } W(p_j) = -x_j$ . Now, assume the firm chooses the price vector  $p_{j+1}$  in period  $j + 1$ . In terms of welfare the firm cannot do worse than choose some point on the boundary of  $R_j$ . Then Fig. 2 suggests that  $W(p_{j+1}) \geq W(p_j)$ .  $R_j$  can therefore be interpreted as a linear approximation of a minimum welfare constraint. The firm thus solves a problem quite similar to the dual of constrained welfare maximization. There welfare is the objective and profit the constraint. Here profit is the objective and welfare the constraint.

The procedure can be repeated at the end of period  $j + 1$  if  $\Pi(p_{j+1})$  is nonnegative. It gives rise to an iterative process, which is described by the flow chart in Figure 3.

The process can only work in the described manner, if the firm can always find a constrained price vector, which yields nonnegative profit  $\Pi_j$ . This condition depends on the nature of the firm's cost function. If the firm is a natural monopoly by conventional standards, it is fulfilled. This means that its cost function exhibits decreasing ray average costs, i.e.  $\forall r \geq 1 \ C(rx) \leq r \ C(x)$ . Decreasing ray average costs represent the natural extension of decreasing average costs to the

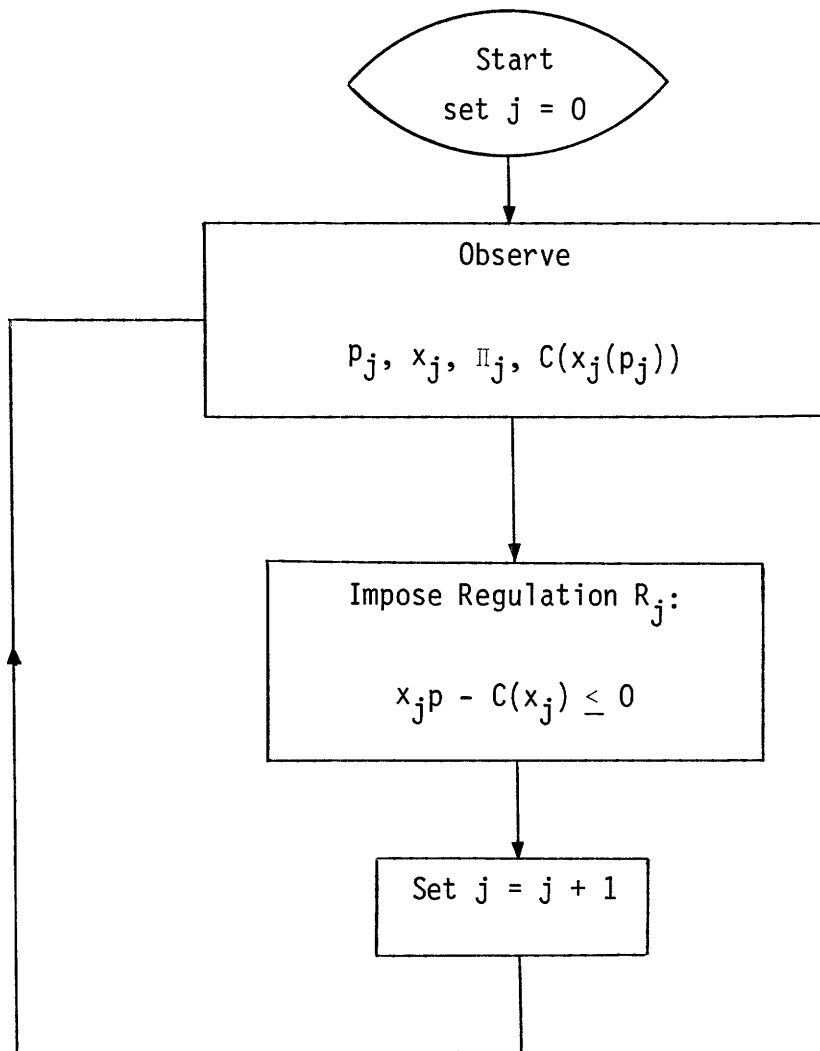


Figure 3, Flow Chart I.

multiproduct firm.<sup>9</sup> In a local context they are equivalent with the unprofitability of marginal cost prices. In precisely this situation Ramsey prices are relevant, for they maximize welfare under the constraint of the financial viability of the firm.

Our aim was to arrive at a mechanism that sets a limit on the firm's ability to influence its price level but at the same time gives it enough freedom of choice regarding the price structure. The method employed has some similarity with the construction of a price index. According to our suggestion the firm should have the freedom to choose prices in such a way that a Laspeyres price index for its products would not exceed a constrained level. This constraint reduces the level of the base period by at least the firm's profit margin in that period. This can be verified by considering the constraint  $R_j$ :

$$0 = x_j p_j - C(x_j) - \Pi_j \geq x_j p - C(x_j).$$

Thus  $1 - \frac{\Pi_j}{x_j p_j} \geq \frac{x_j p}{x_j p_j}$ . Within the price level defined by this

Laspeyres index the relative prices are allowed to vary. In the following period we therefore expect to get new quantities giving a new base for the index. The result is a monotonically decreasing Laspeyres chain index.

The proposed algorithm makes explicit use of a regulatory lag. In the literature, the benefits of such a lag have been stressed with respect to

lowering production costs. It has been shown that a lag can help to reduce factor distortion caused by regulation. If a breakeven constraint is imposed, a lag will force a profit-maximizing firm to produce at a cost-minimizing point (Bailey, 1973). This result is also implied by our procedure. The breakeven constraint prevents a factor distortion of the Averch-Johnson kind to occur, because there is no asymmetric treatment of capital and the other factors of production. During the lag period of the process the firm is allowed to benefit from reducing its costs and changing its price structure vis-a-vis a maximum price level. Obeying  $R_j$  it will move into a profit-increasing direction. In this direction, surplus must increase. At worst this additional surplus goes entirely to the firm. But in the next period the new regulatory constraint,  $R_{j+1}$ , absorbs the additional profit and hands it over to the consumers. Thus the firm always moves into the right direction both for itself and the public at large.

Our proposition is that the process described by Flow Chart I will converge to a constrained welfare maximum.

### 2.3 Convergence of the Regulatory Algorithm

Proposition 1: Under Assumptions 2 to 4  $W(p_j)$  converges to a point  $W^*$  with the following property: there exists  $\bar{p}$  such that  $W(\bar{p}) = W^*$ ,  $\Pi(\bar{p}) = 0$ , and at  $\bar{p}$  the necessary optimality Condition 1 holds.

Remark: Although the sequence  $W(p_j)$  converges, the sequence of the  $p_j$  need not converge. There may be many  $p$  with  $W^* = W(p)$  and  $\Pi(p) = 0$ , between which  $p_j$  may oscillate.

Proof: Step 1: We show that  $\pi_j \geq 0 \quad \forall j \in \mathbb{N}$ . Without loss of generality  $\pi_0 \geq 0$ .<sup>10</sup> We denote the inverse demand function by  $p(x)$ .  $\forall j \in \mathbb{N}$  with  $\pi_j \geq 0$ , there exists  $r \geq 1$  such that  $p(rx_j)$  exactly meets the regulatory constraint i.e.  $p(rx_j)x_j - C(x_j) = 0$ . This is so, because Assumption 3(b) implies that  $p(rx_j)x_j$  is a continuous function of  $r$  and approaches 0 as  $r$  gets large. Hence the intermediate value theorem is applicable. Now, we have  $p(rx_j)rx_j - C(rx_j) \geq p(rx_j)rx_j - rC(x_j) = 0$ . Clearly,  $\pi_{j+1} \geq p(rx_j)rx_j - C(rx_j)$  and thus  $\pi_{j+1} \geq 0$ . By induction  $\pi_j \geq 0$  holds for all  $j \in \mathbb{N}$ .

Step 2: We show that  $W(p_j)$  converges. By step 1  $\pi_j \geq 0 \quad \forall j \in \mathbb{N}$ . Fig. 2 suggests that welfare increases at each step. Indeed, we have

$$W(p_{j+1}) \geq W(p_j) + \pi_j.$$

To see this, note that the convexity of  $W(p)$  implies

$$W(p_{j+1}) \geq W(p_j) + \text{grad } W(p_j)(p_{j+1} - p_j).$$

But  $\text{grad } W(p_j) = -x(p_j)$  and also the regulatory constraint  $R_j$  can be written as

$$R_j = \left\{ p \mid x_j p - C_j \leq 0 \right\} = \left\{ p \mid x_j(p_j - p) \geq \pi_j \right\}.$$

Hence

$$W(p_{j+1}) \geq W(p_j) + \Pi_j.$$

At each step welfare increases by at least the amount of the previous profit. Thus,  $\{W(p_j)\}_{j=1}^{\infty}$  is a monotonically increasing sequence. It is bounded by the constrained welfare optimum. Because of the continuity of  $W(p)$  and the compactness of  $\{p | \Pi(p) \geq 0\}$  such an optimum must exist. Hence, the limit  $W^* = \lim_{j \rightarrow \infty} W(p_j)$  exists. It is clear that there are points  $\bar{p}$  with  $W(\bar{p}) = W^*$  and  $\Pi(\bar{p}) = 0$  because  $0 \leq \lim \Pi_j \leq \lim(W_{j+1} - W_j) = 0$ . If there is only one such point, then  $p_j \rightarrow \bar{p}$ . Otherwise, there exists a subsequence  $p_{j_k}$  converging to one of the  $\bar{p}$ , because the  $p_j$  lie in the compact set  $\{p | \Pi(p) \geq 0\}$ .

Step 3: We show that at such a  $\bar{p}$  the necessary Condition 1 holds. The necessary condition for the constrained profit maximization of the regulated firm is

$$\text{grad } \Pi(p_j) = \lambda_j x_{j-1}, \quad \lambda_j \geq 0.$$

Here  $x_{j-1}$  are market demands at prices  $p_{j-1}$ . At any limit point  $\bar{p}$  we have  $\text{grad } \Pi(\bar{p}) = \bar{\lambda} \bar{x}$ , where  $\bar{x}$  are market demands at  $\bar{p}$ . Now, also  $\Pi(\bar{p}) = 0$ . Together with Assumption 2 this means that at  $p$  the three surfaces  $R$ ,  $\{p | W(p) = W(\bar{p})\}$ , and the breakeven constraint of the welfare-maximizing problem,  $\{p | \Pi(p) = 0\}$ , must be tangent to each other.



This completes the proof.

We want to give an intuitively more appealing explanation to Proposition I. First, why with decreasing ray average cost can the firm always generate nonnegative profit under constraint  $R_j$ ? The constraint would allow the firm to break even at quantities  $x_j$  of the previous period. This is not in general feasible for the firm, for the demanded quantities change with prices. But it can always exactly fulfill the constraint with some multiple  $r \geq 1$  of  $x_j$ . Along this ray, average costs are decreasing, while average prices called for by the constraint are allowed to stay constant. Hence, there always exists some  $r \geq 1$  and corresponding prices fulfilling  $R_j$  which enable the firm to make profit. Thus, decreasing ray average costs prevent the constraint from overshooting.

Second, why does welfare increase in each period by at least the profit of the last period? Here a revealed preference argument helps. The constraint  $R_j$  has been set in such a way that the consumers in period  $j$  could acquire the quantities they bought in period  $j - 1$  at just  $\Pi_{j-1}$  less than what they actually paid during period  $j - 1$ . If, however, in period  $j$  they choose to buy  $x_j \neq x_{j-1}$  they must be at least as well off with  $x_j$  as with  $x_{j-1}$ .

Third, why does the necessary optimum Condition 1 hold at a point  $\bar{p}$  of convergence of the algorithm?  $\bar{p}$  lies on the zero profit surface.

Otherwise profit could be increased which contradicts the limit property of  $W(\bar{p})$ . The constraint  $\bar{R}$  at  $\bar{p}$  is tangent to the welfare surface  $\{p | W(p) = W(\bar{p})\}$ . The necessary Condition 1 means that  $\bar{R}$  is tangent to the zero profit surface. Suppose this were not so. Then, as demonstrated in Figure 4,  $\bar{R}$  intersects the set  $\{p | \Pi(p) \geq 0\}$ . The firm now is allowed to charge any price to the southwest of  $\bar{R}$ . Prices in the shaded area will yield positive profit due to Property (2) of Assumption 3(d). Hence, welfare could be increased, which contradicts the limit property of  $W(\bar{p})$ .

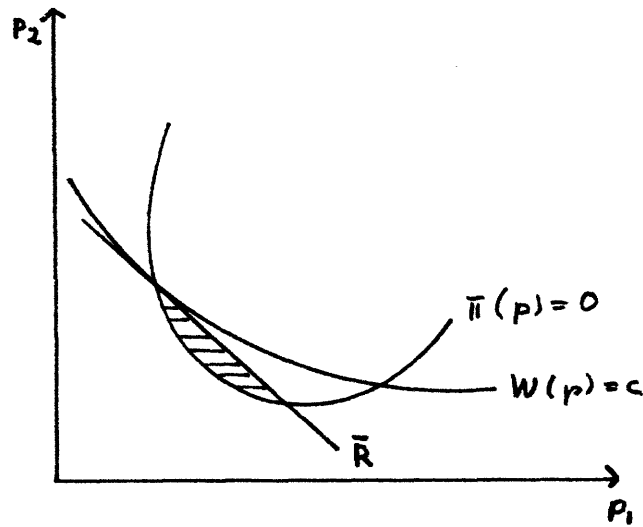


Figure 4

Corollary:

At each step of the algorithm, the welfare indicator  $S(p) = W(p) + \Pi(p)$  increases.  $S(p)$  can be interpreted as the sum of the consumers' and producers' surplus.

Proof:

From  $W(p_j) \geq W(p_{j-1}) + \Pi_{j-1}$  and from  $\Pi_j \geq 0 \forall j \in \mathbb{N}$  we obtain

$$W(p_j) + \Pi_j \geq W(p_{j-1}) + \Pi_{j-1} \quad \text{Q.e.d.}$$

This Corollary shows that under Assumptions 2 to 4 constrained welfare optimization never decreases total surplus  $S(p)$  over the unregulated situation, in particular in comparison to the unconstrained profit-maximizing monopolist. In case of increasing ray average cost this result will not necessarily hold.

2.4 Characterization of the Point of Convergence

Proposition 1 establishes the convergence of the regulatory process to a welfare level  $W^* = W(\bar{p})$  such that at  $\bar{p}$  the first-order optimality Condition 1 holds. Of course, there may be many points  $p$  fulfilling Condition 1. Some of these  $p$  represent constrained local welfare maxima. But  $\bar{p}$  need not be such a maximum.

Fortunately, our process does not choose  $\bar{p}$  at random amongst the  $p$  fulfilling Condition 1. In fact, we can be highly certain that  $\bar{p}$  is a constrained local welfare maximum. First, we know from Step 2 of the proof to Proposition 1 that in each period the algorithm increases welfare by at least the amount of the previous profit. Second,  $W^* = W(\bar{p})$  never is a local minimum. The only type of situation where  $W^* = W(\bar{p})$  is not a constrained local maximum, is shown in Figure 5. There  $\bar{p}$  fulfills the tangency Condition 1. But the constrained local welfare maximum is at  $p^*$ . Without formally introducing probability arguments, we can see: The algorithm would only by "extreme mischief" (measure zero) arrive at such a point. Additionally, this point may be called unstable. A small change in the environmental conditions  $x()$  and  $C()$  will cause the firm (and the regulatory agency to induce the firm) to move into a welfare-increasing direction between  $\{p | W(p) = C\}$ , and  $\{p | \Pi(p) = 0\}$ . cf. Fig. 5.

Imagine that in spite of this the situation shown in Figure 5 occurs and persists. Then in a one-consumer economy, the firm could induce the regulatory agency to run an experiment. The firm would calculate  $p^*$  of the constrained welfare maximum from its knowledge of costs and demand. It could then offer the consumer a choice between  $\bar{p}$  and a price vector  $\tilde{p}$  close to  $p^*$ , but profitable. If the consumer reveals to prefer  $\tilde{p}$  to  $\bar{p}$ , the process could continue from  $\tilde{p}$ . In an economy consisting of many consumers such a choice is in general only feasible if unanimity is assured through compensation payments. This involves both transaction costs and free rider problems.

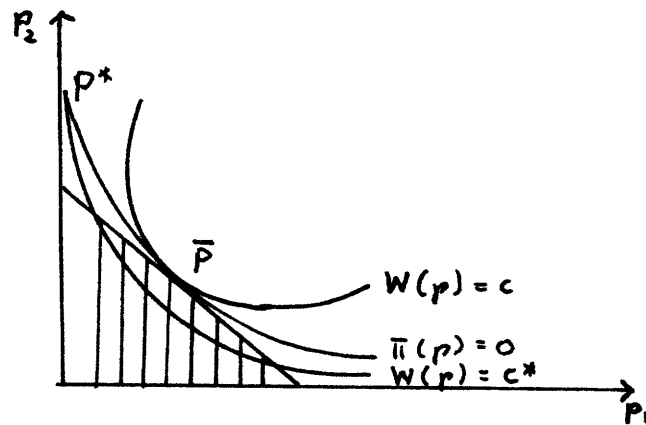


Figure 5

## 2.5 How Does This Algorithm Differ from Conventional Regulation of Price Structures?

Historically, under regulatory routine price structures have tended to become rigid and/or internally subsidized. These phenomena can be attributed to political and judicial influences on price structures. This observation has led economists to convince regulators that such a policy has its costs in terms of allocational efficiency. They have hence tediously demonstrated the positive effect of explicitly considering demand elasticities. Against this background our paper seems ironical, as it frankly asks regulators to forget what they learned from economists whenever conditions are sufficiently stable.<sup>11</sup>

The fair rate of return on capital or the balanced budget rule are "easily" implementable regulatory measures to constrain price levels. So far there has been no comparable indicator for efficient price structures. The strength of the algorithm described above lies in the

incentives it gives to the regulated firm to find the efficient price structure itself. The firm management may, for instance, use peak-load pricing or two-part pricing techniques. The management may split demand into demand components at different time periods and may introduce license fees. For such tariffs a necessary optimality condition analogous to Condition 1 can be derived. With two-part tariffs, for instance,  $x$  does not only contain the demands  $x_i$  for good  $i$  but also the demand  $\hat{x}_i$  for licensing/subscription to good  $i$ . Similarly,  $p$  does not only contain the prices  $p_i$  for good  $i$  but also the license fee  $\hat{p}_i$  for good  $i$ .

Within the limits of bookkeeping and auditing, the information requirements with respect to the regulatory agency are low. The agency is supposed to have some general knowledge about the structure of the regulated industry, especially whether it produces at decreasing ray average costs. Furthermore, it has to know what has actually happened in the past, but not the full possibility set of the firm. This contrasts for instance with the problem solution of finding the optimal rate of return in the Averch-Johnson model (Klevorick, 1971). Even with some knowledge of demand elasticities and costs in the neighborhood of the status quo of the firm, a regulatory agency can hardly hope to do better than approximate first-order conditions. To the best of our knowledge, all previous efforts to implement constrained welfare optimal prices have therefore taken the first-order conditions to be sufficient. Compared to this our procedure ensures that  $W(\bar{p})$  is no local minimum and that the

point reached by the procedure is preferred to the status quo. Only in a small class of cases, to which we give little empirical significance, will the process stop short of a local welfare maximum.

The agency in our model has little discretionary power. It is, however, obliged to control the quality of the firm's output because the firm may want to reduce costs through hidden quality deterioration. This, of course, is a standard regulatory issue.

## 2.6 Some Qualifications and Extensions

### 2.6.1 Myopic Management Behavior

So far we have assumed that the firm maximizes profit in every period  $j$  subject to constraint  $R_{j-1}$ . However, such myopic profit maximization may deviate from the long-run interest of the management, which is to maximize the discounted flow of future profits. Does the algorithm converge in this case?

The convergence of the algorithm depends on the simple inequality  $W(p_{j+1}) \geq W(p_j) + \Pi_j$ . From this inequality it can be seen that the convergence is not affected as long as  $\Pi_j \geq 0$  for all  $j$ . The process, however, could converge to a suboptimal point, if the firm keeps profits sufficiently low. But this would mean that in total it foregoes profits which would be allowed and feasible. It can thus only be inferred that the convergence rate may be slower in earlier periods and faster in later periods compared to the rate corresponding to the myopic profit maximization case. Management may want to produce losses for some periods in order to recoup higher profits later. With decreasing ray average costs losses are an indication of either strategic

behavior or mismanagement. Thus, if losses occur, the regulatory agency can either fire the management<sup>12</sup> or keep the previous regulatory constraint until profits turn up.

If profits occur, the regulatory constraint for the next period becomes more binding and therefore narrows the firm's discretion for the future. Eventually, the firm will be forced by its profits to follow the convergence path. The higher the discount rate employed by the firm the more it will want to make higher profits early. Thus, a high discount rate may speed up the process.

#### 2.6.2 Nondecreasing Ray Average Costs

Assumption 3(d), which postulates decreasing ray average cost may be violated even in case of a natural monopoly.<sup>13</sup> Therefore, the procedure described above should be adapted to the regulation of monopoly firms with nondecreasing ray average costs. As is illustrated in Figs. 6 and 7 for the simple case of a one-product firm, there can be a striking difference in applying the algorithm for a decreasing or nondecreasing (ray) average cost firm. With decreasing (ray) average cost profits always stay nonnegative, while with increasing (ray) average costs losses may be inevitable. For a one-product firm profits and losses will follow each other in a hog cycle manner. If, furthermore, the average cost curve is absolutely steeper than the demand curve the process will explode.



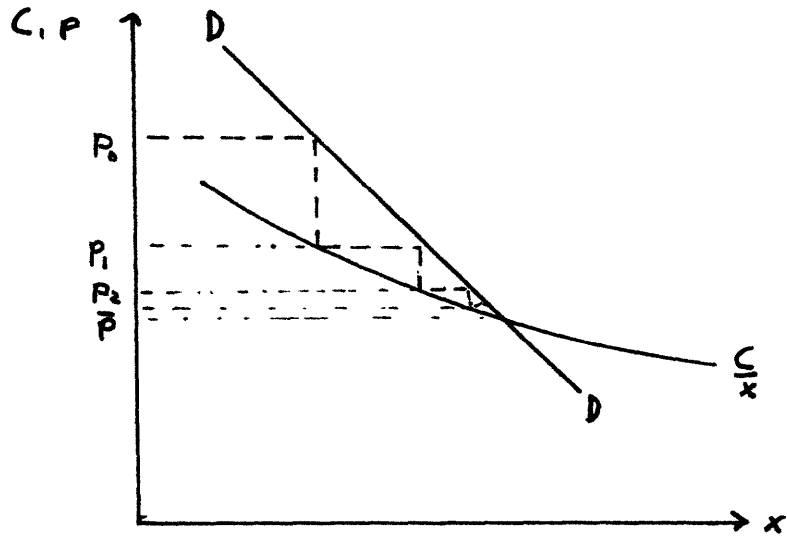


Figure 6

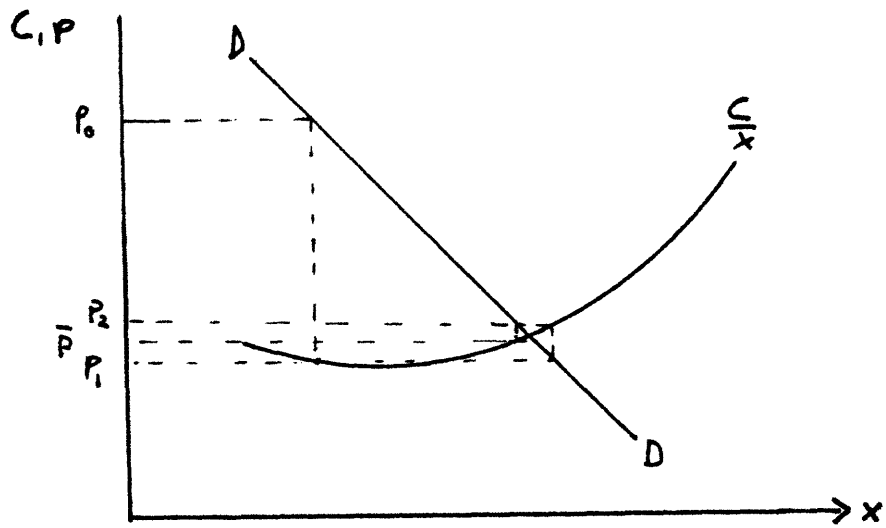


Figure 7

Thus, in order to preserve the incentive structure of the algorithm and in order to limit the informational requirements of the agency to bookkeeping data, we insert a second loop into the algorithm. Loop II deals with the case where the regulatory constraint was too strong. At each iteration in loop II in Flow Chart II the regulatory constraint is relaxed until nonnegative profit is again possible.

### Proposition II

Under the assumptions 2, 3(a) - (c) and 4(a), the algorithm described in Fig. 8 below converges as in Proposition I.

### Proof:

Step 1: We show that loop II is finite. We have  $\Pi_{j-1} = 0$  implies  $\Pi'_j \geq 0$ . Hence,  $\Pi'_j < 0$  implies  $\Pi_{j-1} > 0$ . But if  $\Pi_{j-1} > 0$ , then there exists a neighborhood of  $p_{j-1}$  such that for all  $p \in U_\epsilon(p_{j-1})$   $\Pi(p) \geq 0$ . Hence, there exists  $n \in \mathbb{N}$  such that  $\Pi'_j \geq 0$ .

Step 2: Coming out of loop II we have  $x_{j-1}p_j - C(x_{j-1}) \leq x_{j-1}p_{j-1} - C(x_{j-1})$  by construction of the constraint. Therefore  $x_{j-1}(p_j - p_{j-1}) \leq 0$ . This means that at prices  $p_j$  consumers could have bought  $x_{j-1}$  cheaper than at prices  $p_{j-1}$ . They, however, decided to buy  $x_j$ . Hence welfare must have increased:  $W(p_j) \geq W(p_{j-1})$ . This means that  $\{W(p_j)\}_{j=1}^{\infty}$  is a monotonically increasing

## FLOW CHART OF ALGORITHM II

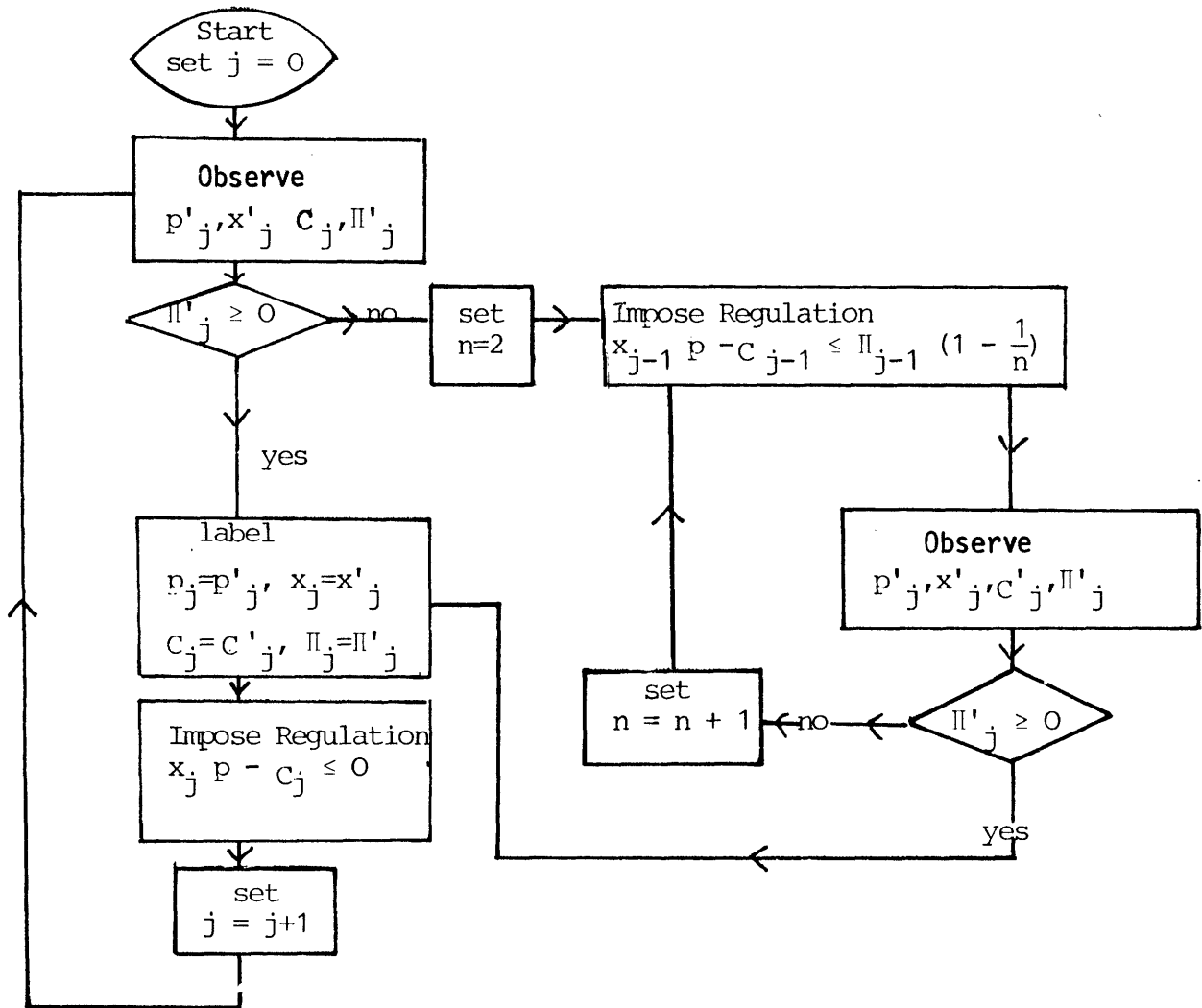


Figure 8 Flow Chart II

Note, we assume  $\Pi_0 \geq 0$ .

sequence. From here on the proof follows steps 2 and 3 of the proof of Proposition 1 q.e.d.

The case of nondecreasing ray average cost is less straightforward than that of decreasing ray average cost. Although the arguments of section 2.4 carry over, the number of periods necessary to come close to the optimum may increase considerably due to loop II. Current profit is no longer a lower bound for next period's welfare increase. Furthermore, strategic firm behavior producing high losses in order to relax the constraint may be attractive. On the other hand it can be hoped that the fear of cream-skimming competition will in this case limit the firm's discretion.

### 3. CONCLUSION

The process described rests on some general principles, which are not dependent on their regulatory application. As an equilibrium, letting the firm maximize profits subject to constraint on the welfare level of consumers generates an optimum. Convexity of the welfare function permits substitution of the tangent hyperplane for the indifference surface. Decreasing ray average costs or the mechanism of Flow Chart II prevent overshooting. Hence we can converge to a local optimum by raising the allowed welfare level in each step.

The regulatory algorithm can be interpreted as an incentive pricing mechanism in the sense of Cross (1970). The regulated firm constrained by  $R_j$  is encouraged to exploit both the potential for cost decreases and the consumers' willingness to pay. The firm converts these into profits. But both these advantages are turned over to the consumers in the next period.

Our process does not differ substantially from the regulatory procedure which outside inflation periods traditionally has been used in the United

States. Here the rate level is set by applying a rate of return constraint on past cost and quantity data of the regulated monopoly firm. In theory, regulated firms are free then to adjust the rate structure subject to the proof that they stay within the overall rate of return constraint again based on past data. In practice, however, they meet many obstacles in doing so. Due to issues of discrimination rate structures show some inherent rigidity. As a policy recommendation this paper hence suggests that once the rate level has been established the actual freedom of the firms to alter their rate structure on profit-maximizing grounds should be increased. Basically the argument used for this recommendation is similar to the one in favor of a regulatory lag.

Enforcement of the regulatory constraint  $R_j$  developed in this paper could be supervised by auditing. Hence, its application could be prescribed to regulatory agencies by law. Once they have accepted the philosophy behind this approach regulatory agencies could go back to using their beloved concept of historic costs.

Appendix

Let the direct utility function of an individual be given by  $\hat{U}(x, p, M) = \hat{U}(x) - px + M$  where  $M$  represents income (wealth) and  $\hat{U}(x)$  represents the willingness to pay for  $x$ . We want to show the convexity of the corresponding indirect utility function  $V(p, M) = \hat{U}(x(p), p, M)$  i.e.

$$\forall 0 \leq \lambda \leq 1$$

$$V(\lambda p_1 + (1 - \lambda)p_2) \leq \lambda V(p_1) + (1 - \lambda)V(p_2) \quad (1)$$

First, observe that

$$U(\bar{x}, p_1, M) \leq U(x(p_1), p_1, M) \quad (2)$$

and

$$U(\bar{x}, p_2, M) \leq U(x(p_2), p_2, M) \quad (3)$$

for any  $\bar{x}$  and in particular for  $\bar{x} = x(\lambda p_1 + (1 - \lambda)p_2)$ . Multiply (2) and (3) by  $\lambda$  and  $(1 - \lambda)$ , respectively. Then, addition yields

$$\begin{aligned} \hat{U}(\bar{x}) - (\lambda p_1 + (1 - \lambda)p_2)\bar{x} + M &\leq \\ &\leq \lambda [\hat{U}(x(p_1)) - p_1 x(p_1)] \\ &\quad + (1 - \lambda) [U(x(p_2)) - p_2 x(p_2)] + M \end{aligned}$$

Rewriting this inequality in terms of  $V(p, M)$  yields (1).

Since  $W(p)$  is the sum of the individual indirect utility functions  $V(p, M)$ ,  $W(p)$  must be convex.



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Footnotes

1. A professional interest in finding optimal price structures and in proving that they can be implemented may be assumed at the Electricité de France under the leadership of Boiteux who first solved the problem of constrained welfare optimization for public firms.
2. For this see Baumol, Bailey, and Willig (1977), or Panzar and Willig (1977).
3. The convexity assumption (a) follows from a revealed preference argument and the assumed absence of income effects. This argument we owe to C.C. von Weizsäcker. It is given in the appendix.
4. The degree of approximation of more general welfare measures by consumer's surplus is given by Willig (1976).
5. See Finsinger (1978).
6. Diamond and Mirrlees use a much less restrictive framework. See also Baumol and Bradford (1970).
7. For simplicity corner solutions meaning zero prices are ignored here and throughout.
8. For a qualification see 2.6 below.
9. See Baumol, Bailey, and Willig (1977), Panzar and Willig (1977), or Baumol (1977) for a discussion of the concept with the view of sustainability of public utility prices.
10. By not constraining the firm in the first period the regulatory agency can always induce the firm to start the process with profits.
11. We owe the basis of this point to an anonymous referee. The sustainability argument can lead to the same kind of policy conclusion. Indeed, in Germany peak-load pricing some decades ago was introduced by high-cost electric utilities threatened by competitive pressure and takeovers.
12. In the context of U.S. regulation the agency can threaten to withdraw the firm's license.
13. Panzar and Willig (1977) show that subadditivity of the cost function does not imply decreasing ray average cost.