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THE DEMAND FOR ELECTRICITY:

COMMENT AND FURTHER RESULTS

by

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I. INTRODUCTION

The analysis of factors affecting demand for electricity has become an important topic of econometric research recently, due partly to rapidly increasing electricity prices since 1973, smaller than expected growth rates of demand, and continuing controversies regarding pricing policy, future capital needs and construction plans within the electric utility industry. Econometric research has focussed particular attention on estimates of electricity price elasticities. This econometric literature has been surveyed recently by Lester Taylor [1975].

Taylor's survey is striking, for in addition to castigating almost all of the existing empirical literature, it outlines clear directions for future research. More specifically, Taylor shows that on the basis of economic theory, both average intramarginal and marginal price should appear as regressors in the demand equation. He then goes on to assert (somewhat mistakenly) that up to 1975 not one study had included both these variables as regressors, and that "... no amount of econometric virtuosity can overcome the problems caused by the failure to correctly specify the price of electricity in the demand function."[1] Taylor recommends as the first order of business, the rather costly construction of a data set for prices based on actual rate schedules. Results of such data construction and model estimation have since been published by Taylor et al. [1977].

In Section II of this note I show analytically that the quantitative, empirical significance of Taylor's omitted variable argument is negligible, since least squares estimates with the average intramarginal price variable included as a regressor will typically be virtually identical to least

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[1] Taylor [1975], p. 106

squares estimates with that variable excluded. In Section III I comment on problems of simultaneity and stochastic specification. In Section IV I discuss the interesting model developed by Robert Halvarsen [1975], in which price elasticity estimates based on average electricity prices are numerically equal equal to those based on marginal electricity prices, and show that this invariance result is due to the highly restrictive functional form specification. Then in Section V I illustrate my analytical remarks empirically using data from H. S. Houthakker's pioneering [1951b] study. In this section I also amend considerably Taylor's interpretation of Houthakker's classic study. In section VI I present brief concluding remarks.

II. ECONOMETRIC CONSEQUENCES OF INCORRECTLY OMITTING INTRAMARGINAL-AVERAGE

VARIABLE

A basic thrust of Taylor's survey is that it is necessary to include as regressors in the residential electricity demand equation both the marginal ("tailing block" or "running") price of electricity faced by the "typical customer" and the average price per kWh of electricity consumed up to but not including the final block. Alternatively, in place of this average price one can employ total expenditure on electricity up to the final intramarginal block. The qualitative effect of incorrectly omitting the average price or intramarginal expenditure variable is, according to Taylor, as follows:

> "If average and marginal prices are positively correlated
> (as is likely to be the case), then use of one of the
> prices in absence of the other will lead, in general, to
> an upward bias in the estimate of the price elasticity. That
> this is so follows from the theorem on the impact of an
> omitted variable."[1]

Taylor does not speculate on the empirical significance of such a misspecification. To that issue I now focus attention.

Let the correct demand equation be of the form

$$(1) \qquad y_i = \beta_1 x_{1i} + \beta_2 x_{2i} + \beta_3 x_{3i} + \ldots + \beta_k x_{ki} + u_i \quad ,$$

$$i = 1, \ldots, m$$

where for the ith observation y_i is the consumption of electricity per household in kWh, x_{1i} is the marginal price of electricity, x_{2i} is the intramarginal average price of electricity (i.e., the average price up to but not including the final block of the typical customer), $x_{3i}, x_{4i}, \ldots, x_{ki}$

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[1]Taylor [1975], p. 80.

are other explanatory variables such as income, cooling and/or heating degree days, female labor force participation, lagged electricity consumption per household, etc. and u_i is the random disturbance. Without loss of generality, all variables are measured in terms of deviations from their means; this implies that no intercept term appears in (1).

Let the estimated least squares regression equation when k regressors are included be (deleting i subscripts)

$$(2) \qquad \hat{y}_1 = b_{y1 \cdot k} x_1 + b_{y2 \cdot k} x_2 + b_{y3 \cdot k} x_3 + \ldots + b_{yk \cdot k} x_k$$

where $\hat{y}$ is the least squares "fitted" or "predicted" value of y, and where $b_{y1 \cdot k}, b_{y2 \cdot k}, \ldots, b_{yk \cdot k}$ are the least squares coefficients on $x_1, \ldots, x_k$ (in the presence of all k regressors). Hereafter (2) is called the "correct" regression equation.

The misspecification noted by Taylor occurs when x_2 is incorrectly omitted from the regression equation (2). Let the misspecified least squares regression equation with x_2 omitted be

$$(3) \qquad \hat{y} = b_{y1} x_1 + b_{y3} x_3 + \ldots + b_{yk} x_k$$

where $b_{y1}, b_{y3}, \ldots, b_{yk}$ are the least squares coefficients on $x_1, x_3, \ldots, x_k$ in the misspecified regression equations. Taylor notes that (3) is the equation fitted by H. S. Houthakker [1951b]; even though he correctly included the marginal price variable x_1, according to Taylor, Houthakker incorrectly excluded x_2.

The econometric consequences of this misspecification on estimates of the marginal price elasticity can be assessed by determining the difference between $b_{y1 \cdot k}$ and b_{y1}. Arthur S. Goldberger[1] [1968, Chapter 3] has derived the analytical relationship between the least squares estimates $b_{y1 \cdot k}$ and b_{y1}:

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[1] Also see Zvi Griliches [1957] and Henri Theil [1957].

(4) $b_{y1 \cdot k} = b_{y1} - b_{21} b_{y2 \cdot k}$

where b_{21} is the least squares coefficient in the "auxiliary" regression equation

(5) $\hat{x}_2 = b_{21} x_1 + b_{23} x_3 + \ldots + b_{2k} x_k$.

In the present context, b_{21} is the least squares coefficient when intramarginal average price is regressed on marginal price and the other regressors in (3).

Equation (4) allows easy determination of the qualitative consequences of incorrectly omitting x_2. Assuming that b_{y1}, $b_{y1 \cdot k}$ and $b_{y2 \cdot k}$ are negative (as suggested by economic theory) and that b_{21} is positive, then b_{y1} is larger in absolute value than $b_{y1 \cdot k}$ and b_{y1} provides an absolute upward biased estimate of the marginal price elasticity. Notice, however, that from (4) the sign of the bias depends not only on the sign of b_{21}, but also on the sign of $b_{y2 \cdot k}$.

A more precise quantitative assessment of this bias is also possible. Since Taylor's preferred specifications typically involve double-logarithmic regressions, let y and each of the x's in (1), (2), and (3) be logarithmic transforms of the original variables. Under such logarithmic transformations, $b_{y1 \cdot k}$ is an estimate of the marginal price elasticity of demand for electricity, and $b_{y2 \cdot k}$ is the least squares estimate of the intramarginal average price elasticity of demand. Fortunately, microeconomic theory provides a clear interpretation of $b_{y2 \cdot k}$. Since a change in x_2 represents only an income effect and no substitution effect (see Taylor [1975, pp. 75-80] for discussion), $b_{y2 \cdot k}$ is the negative of the income elasticity of demand for electricity times the budget share of intramarginal electricity expenditure in the total income of the typical residential customer. Taylor [1975] and Taylor et al.

[1977] suggest that a reasonable estimate of the income elasticity is unity, while in Taylor et al. [1977] it is reported that the mean intramarginal budget share by state over the 1961-72 time period is about 0.01.[1] Hence, if the theory and specification were correct, a reasonable value for

$b_{y2 \cdot k}$ = -(1.0)(.01) = -.01. Such a value can be inserted into (4).

In order to complete a quantitative assessment of misspecification, estimates of b_{21} must be obtained and inserted into (4). Recall that in the logarithmic context, b_{21} in (5) represents an elasticity of average intra-marginal price with respect to marginal price. Although b_{21} is likely to be sample dependent, it would seem reasonable to expect that utilities with relatively high marginal prices also have relatively large fixed and intra-marginal charges in their rate structure, and thus that b_{21} might be positive and perhaps in the range of 0.25 to 1.0. Data for four Alberta electric utilities over the 1962-76 time period (see Data Metrics, Ltd. [1978]) suggest a mean value of b_{21} of around 0.50. If $b_{y2 \cdot k}$ = -0.01 and b_{21} = 0.50 is inserted into (4), one finds that the bias due to misspecification is

$$(6) \qquad b_{y1 \cdot k} - b_{y1} = 0.005 \qquad .$$

This is a rather strong and significant result, for it suggests that if one incorrectly omits the intramarginal average price variable then the "correct" and the "misspecified" least squares estimates of the marginal price elasticity are virtually identical. For example, using the "correct" Taylor et al. [1977] $b_{y1 \cdot k}$ estimate of -0.8, one finds that the "incorrect" estimate would be virtually identical at -0.805. Hence the "cost" of this

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[1] See Taylor et al. [1977], p. 7-4.

misspecification appears to be very small--especially if viewed relative to typical standard error estimates and to costs of collecting intramarginal expenditure data.

The actual difference between $b_{y1 \cdot k}$ and b_{y1} will of course depend on the sample values of $b_{y2 \cdot k}$ and b_{21}. Assuming that income elasticities of demand for electricity equal unity, in Table 1 I present differences between the "correct" estimates $b_{y1 \cdot k}$ and the "misspecified" estimates b_{y1} under alternative values for the intramarginal budget shares and b_{21}. The essential point is clear: the empirical significance of incorrectly omitting x_2 appears to be negligible.

TABLE 1

Differences Between "Correct" Estimates $b_{y1 \cdot k}$ and

"Misspecified" Estimates b_{y1} Under Alternative Assumptions

Intramarginal Budget Shares	Values of b_{21}			
	.25	.50	.75	1.00
.005	.00125	.00250	.00375	.00500
.010	.00250	.00500	.00750	.01000
.025	.00625	.01250	.01875	.02500
.050	.01250	.02500	.03750	.05000

The above results are not affected substantially if average intramarginal price were replaced by intramarginal expenditure; the principal alteration would be the new interpretation of b_{21} as the elasticity of intramarginal expenditure with respect to marginal price.[1]

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[1] Alberta data 1962-1976 for four utilities (see Data Metrics Ltd. [1978]) suggests that a reasonable value of b_{21} in such a context is about 0.75. If one inserts this value into (4), assumes an intramarginal budget share of .01 and an income elasticity of 1.0, then one obtains a bias of .0075.

Although the above discussion has focussed attention on the omitted

variable bias affecting the marginal price elasticity estimate, it is clear that

omitting x_2 also affects other coefficients. Fortunately, the bias on other

coefficients (such as income elasticities) will also be small. To see this,

note that the difference between the "correct" estimate of the coefficient on

x_j (denoted by $b_{yj \cdot k}$) and the "misspecified" estimate on x_j due to incorrectly

omitting x_2 (denoted by b_{yj}) is

$$(7) \qquad b_{yj \cdot k} - b_{yj} = -b_{2j} b_{y2 \cdot k} \ , \qquad j = 1,3,4,\ldots,k$$

where b_{2j} is the least squares coefficient in the regression equation

$$(8) \qquad \hat{x}_2 = \sum_j b_{2j} x_j \ , \qquad j = 1,3,4,\ldots,k \ .$$

Note that $b_{y2 \cdot k}$ always appears in (7) regardless of which regression coefficient

is being checked for bias. Since it has been shown above that it is

reasonable to expect $b_{y2 \cdot k}$ to be small (around -0.01 if income elasticities

are 1.0 and mean intramarginal budget shares are 0.01), it follows that

unless the b_{2j} elasticity is very large, in general the difference between

$b_{yj \cdot k}$ and b_{yj} will be rather small.

Taylor's discussion of this last point tends to be less precise and

at times mistaken, since he focusses only on b_{2j} and ignores the important

role of the small and negative $b_{y2 \cdot k}$ coefficient. For example,

> "The extent to which biases will exist depends on the
> correlation between the variable that is left out and
> the variables that are included. If, for example,
> marginal and average price are positively correlated
> (which is likely) and both are positively correlated with
> income (which is less likely), then the exclusion of
> one of the prices will lead to an upward bias in the
> coefficient for the other price and also to an upward bias
> in the coefficients for income."[1]

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[1] Taylor [1975], p. 102.

Taylor's comments on the upward bias of the income coefficient is a mistaken conjecture. If one denotes the income variable as x_3 and follows Taylor's assumptions that $b_{y3 \cdot k}$, b_{y3} and b_{23} are positive, then from (7) it can be seen that $b_{y3 \cdot k} - b_{y3}$ is positive, which implies that omitting x_2 leads to a downward (not upward) bias in the coefficient for income. Note that since $b_{y2 \cdot k}$ is relatively small, the downward bias on the income coefficient is likely to be rather minor.

In conclusion, the above analysis suggests that the empirical consequences of incorrectly omitting the average intramarginal price (or intramarginal expenditure) variable from the residential electricity demand equations appear to be very small and negligible. If the regression analyst chooses to omit x_2 (say, because of substantial data gathering costs), he can still obtain estimates very close to the "correct" estimates by estimating the misspecified equation and then inserting reasonable estimates of b_{2j} into (7).

III. SIMULTANEITY ISSUES

The above discussion on empirical consequences of incorrectly omitting the intramarginal average price or expenditure variable has implicitly assumed that the stochastic specification is appropriate. Sigificantly, the econometric literature dealing with the effects of declining block rate schedules on demand for electricity contains extensive discussions of simultaneity problems. Although most of this literature has focussed on demand equations using ex post average price as a regressor,[1] similar issues arise with the use of marginal price.

To see this, consider the following simple model. Let the quantity of electricity demanded in the residential sector (Q_D) be a function of income (Y), the marginal electricity price (MP), a vector of other exogenous variables affecting electricity demand (X_D) and a random disturbance term (u), i.e.

$$(9) \quad Q_D = f(Y, MP, X_D, u) \quad .$$

Taylor [1975] has noted, however, that virtually all empirical studies have substituted the ex post average price (AP) for the marginal price in (9) so that

$$(10) \quad Q_D = f^*(Y, AP, X_D, u^*) \quad .$$

Assume that the electric utility is regulated so that economic profits are zero, which implies that AP equals average cost (AC) per kWh,[2] i.e.

$$(11) \quad AP = AC \quad .$$

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[1] Typically the ex post average price is computed as total expenditure on electricity divided by kWh consumption.

[2] The average cost includes, of course, a normal rate of return on capital.

Define the average cost function of the electric utility as

(12) $AC = g(Q_S, X_C, v)$

where Q_S is the quantity of kWh supplied, X_C is a vector of other variables affecting average cost, and v is a disturbance term. Finally, since in general electricity cannot be stored as inventory, let quantity demanded equal quantity supplied, i.e.

(13) $Q_D = Q_S$.

The classic simultaneity problem in this model arises as follows. Suppose that the random disturbance term in (10) takes on a positive value, thereby increasing observed Q_D which by (13) increases Q_S. If the utility operates a production technology with increasing returns to scale, then the increase in Q_S reduces AC which by (11) results in a reduction in AP. In (10) it is then seen that (i) AP and u are negatively correlated, and (ii) a spurious negative correlation exists between Q_D and AP. Result (i), a non-zero covariance between a regressor and the disturbance term, implies that estimation of (10) by ordinary least squares will generally yield inconsistent and asymptotically biased estimates of the "true" parameters, while result (ii), the negative covariance, suggests that the bias will likely be upward in absolute value, i.e. estimation of (10) by ordinary least squares will generally result in negative own-price elasticity estimates that are "too large" in absolute value. This observation that such demand elasticity estimates capture the declining block rate effect in addition to the price effect has led some electricity demand analysts to discard the resulting econometric estimates as reliable measures of demand elasticities.

These absolute upward bias will still occur when ex post marginal price is used in the demand equation instead of ex post average price. To see this,

it is sufficient to note that if economies of scale were present, both

marginal cost (MC) and average cost would fall with increases in Q_S, and

as long as MP declines with reductions in MC and AC, in (9) MP and u would

be negatively correlated, implying again that a spurious negative correlation

would occur between Q_D and the price variable MP.

The extent of this negative spurious correlation will fall, of course,

as returns to scale approach unity and/or as the time lag between realized

cost variations and changes in utility rate structures increase.

The above discussion implies then that simultaneity problems may be

present regardless of whether one uses ex post average or ex post marginal

price as a regressor. The obvious remedy is to devise an appropriate

simultaneous equations estimation procedure. Particularly in the context

of micro data, however, traditional simultaneous equation estimation

procedures may not be appropriate. Basically the problem is that utility rate

structures typically feature discrete price-quantity breaks rather than

smooth, continuous and differentiable price-quantity relationships. Not

only might there be a traditional simultaneity problem, but because of the

discrete blocks the observed marginal price of a consumer might also be

different from that marginal price corresponding with the consumer's utility

maximizing level of electricity consumption. In a rather different context--

that of an individual's labour supply--estimation methods have been proposed

by Gary Burtless and Jerry Hausman [1977] and by Terence Wales and

Alan Woodland [1977] that explicitly take into account the endogeneity

of a discrete price variable--in their case, the after-tax wage rate.[1] A

closely related instrumental variable estimation procedure has recently

been applied to micro electricity consumption data by Jerry A. Hausman et al.

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[1]Also see J. A. Hausman and D. A. Wise [1978].

[1978]. Although the computational costs associated with these estimation procedures appear to be substantial, especially when the number of micro units is large, the usefulness of these procedures in the electricity demand context merits additional attention. At the present time relatively little is known regarding the extent to which estimates and inference based on the more sophisticated procedures would differ from that based simpler estimation methods.[1]

At the aggregate level, aggregation over a large number of individual utility block rate schedules can produce a "reasonably smooth" aggregate price-quantity demand schedule. Taylor [1975] suggests and implements empirically (Taylor et al. [1977]) a procedure whereby he collects marginal price data for the typical consumers from actual rate schedules of the individual utilities, aggregates these into larger geographical observations (states), and then estimates (9) by ordinary or generalized least squares. Such a procedure is particularly attractive if the rate schedule is reasonably "flat" in the region faced by the typical consumer and if the time lag is substantial between utilities' realized cost variations and alterations in their rate schedules. An important result obtained by Taylor et al. [1977] is that elasticity estimates using marginal price data based on actual rate schedules of utilities are very similar to estimates using the ex post marginal price data based on the widely available and less expensive Typical Electric Bills (TEB) data.

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[1] Monte Carlo evidence in the labour supply model is discussed by Wales-Woodland [1977].

IV. AVERAGE AND MARGINAL PRICES: FURTHER ANALYSIS

In the previous section it has been shown that due to simultaneity issues estimation of demand equations may produce biased parameter estimates regardless of whether one uses marginal and/or average price as a regressor. A related issue which has received some attention in the econometric literature concerns the numerical change in elasticity estimates resulting if one uses the ex post average price of electricity instead of the marginal price. In the model developed by Robert Halvorsen [1975], for example, it is noted that "... the elasticities of demand estimated with average price data are equal to those that would be obtained with marginal price data".[1] A priori, this result is somewhat surprising; a demand elasticity with respect to an average price is generally conceptually different from a demand elasticity with respect to a marginal price, since in general a given percentage change in the marginal price does not necessarily imply an equal percentage change in the average price. Thus the simple concept of price elasticity can be ambiguous in multi-block rate schedules.

In the model of Halvorsen [1975], both the demand equation (10) and the cost equation (12) are specified as linear in the logarithms. More specifically, with the AC = AP condition (11) and the $Q_S = Q_D$ condition (13) incorporated Halvorsen employs an average price equation of the form

$$(14) \quad \ln AP = \ln \alpha_0 + b \ln Q_D + \sum_i d_i \ln X_i + u$$

where α_0, b and d_i are parameters to be estimated, and X_i are exogenous

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[1] Robert Halvorsen [1975], p. 12.

cost variables affecting the shape and location of the rate schedule. The parameter b is of particular interest. Recalling that AP = AC, one can substitute AC into (14), multiply by Q_S (= Q_D) and then compute the elasticity of total cost with respect to output. If this is done, one finds that the degree of returns to scale is simply $1/(1+b)$, and by implication, that

$$(15) \qquad \frac{AC}{MC} = \frac{1}{1+b} \qquad ,$$

i.e., the ratio of average cost to marginal cost is constant and equal to returns to scale. If one assumes in addition that $\frac{AC}{MC} = \frac{AP}{MP}$, then

$$(16) \qquad \frac{AP}{MP} = \frac{1}{1+b} \qquad .$$

Since (16) implies that $\ln AP = \ln MP - \ln(1+b)$, one can insert either AP or MP into the log-linear demand equations. Because of Halvorsen's logarithmic specification, only the estimated constant term will be affected; the estimated coefficient on the price variable will be numerically invariant to the choice of AP or MP.[1] This analysis discloses, then, why Halvorsen obtains his curious result: Because of the logarithmic specifica-tion with the degree of returns to scale equal to a constant $\frac{1}{(1+b)}$, a given percentage change in marginal price implies an equal percentage change in the average price.

Unfortunately, Halvorsen's logarithmic specification is very restrictive. First, on the cost or supply side the log-linear specification implies an

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[1]Halvorsen circumvents simultaneity problems by estimating his aggregate model using two stage least squares.

underlying Cobb-Douglas type cost or production function. L. R. Christensen and William H. Greene [1976] have estimated cost functions for steam generating electric utilities in the U.S., and report that the parameter restrictions corresponding with the homogeneous Cobb-Douglas specification are decisively rejected at normal significance levels. Instead, Christensen-Greene find that Hicks-Allen substitution elasticities among the capital, labour and fuel inputs are less than unity, and that returns to scale are consistent with a U-shaped nonhomothetic unit cost function instead of a monotonically increasing or decreasing homogeneous unit cost function. The nonhomotheticity implies that the ratio of average cost to marginal cost is not constant, but instead depends on the level of output and the prices of the capital, labour and fuel inputs.[1]

In terms of the demand side, Halvorsen's log-linear demand equation specification is also restrictive; the log-linear demand equations are consistent with underlying Cobb-Douglas utility functions. In summary, then, although Halvorsen's log-linear specification of cost and demand functions is convenient and provides elasticity estimates numerically invariant to the choice of AP or MP, unfortunately this functional form is also very restrictive. With more general specifications, price elasticity measures and estimates based on AP would differ from those based on MP.

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[1] A most interesting and apparently unnoticed implication of the Christensen-Greene econometric findings is that under their estimated nonhomothetic technology (see their Tables 3 and 4, pp. 664-6), the optimal scale of output --that level associated with minimum average cost--is predicted to decrease substantially with increases in relative fuel prices. In particular, the implied elasticity of optimal kWh output with respect to an increase in fuel prices is -0.316, -0.407 and -0.429 based on their 1955I, 1955II and 1970 data, respectively. I am not aware of any industry or engineering-economic literature which considers whether recent fuel price increases have in fact reduced the optimal scale of electricity output from steam-generated power plants.

V. EMPIRICAL ILLUSTRATION

The arguments presented above regarding the consequences of misspecification are largely analytical, with occasional reference to "reasonable" parameter values. The practical student of the demand for electricity might well wonder how some of the classic published results would have changed had the regressions been specified in different ways. Thus I now briefly present results of such an analysis. Fortunately, in the seminal study by Houthakker [1951b], data are presented which allows one to run regressions using various combinations of marginal price, intramarginal expenditure and ex post average price for 42 provincial towns in Great Britain, 1937-1938. Houthakker's study is pioneering in two other aspects:

(i) it appears to be the first published econometric article which reports least squares regression results obtained using an electronic computer. Readers interested in this aspect of intellectual history might consult J. A. C. Brown, H. S. Houthakker and S. J. Prais [1953] for an enlightening and prescient discussion of experiences with the EDSAC (the electronic delay storage automatic calculater at University Mathematical Laboratory at Cambridge University).

(ii) Houthakker's study clearly recognized the implications of a two-part tariff for electricity, and reports estimates using the marginal price. Moreover, Houthakker states [1951b, Paragraph 1.4.1, p. 367] that he initially ran regressions with marginal price and average fixed charge per customer included as regressors; he chose not to report final results with the latter variable included since "its influence was not

statistically significant."[1] Houthakker's analysis thus implemented

empirically already in 1951 the theoretical framework discussed

by Taylor in his 1975 survey.[2]

In Column 1 of Table 2 I reproduce the results reported by Houthakker

[1951b]. The dependent variable is (the logarithm of) average electricity

consumption per domestic customer on the two-part tariff. To adjust for hetero-

skedasticity, Houthakker transformed his data, multiplying all observations

on each variable by the square root of the number of customers in the town.

The notation is as follows: M is money income, P_{-2} and P_{-0} are the marginal

("running") prices of electricity lagged two and zero years, respectively,

AP_{-0} is the current ex post average price of electricity, G_{-2} is the two-

year lagged marginal price of natural gas, H is the average potential

electricity consumption (in kilowatts) of appliances operated by domestic

two-part customers in 1937-1938, and F is average fixed charge for domestic

two-part tariff customers in 1937-38.[3] As seen in column 1, Houthakker's

estimates of the income and price elasticities were 1.166 and -0.893,

respectively. Although the potential electricity consumption in kilowatts

is a regressor in Houthakker's equation, the fact that the marginal price of

electricity is lagged two years prompts Houthakker to interpret these

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[1]Houthakker, [1951b], p. 36. The analytical results of Section II above
suggest of course that this coefficient should be very small.

[2]Taylor appears to have overlooked this aspect of Houthakker's study, for
in his empirical survey Taylor states: "... not one study has recognized
that completely proper treatment or price of this context requires that
intramarginal prices as well as the marginal price be represented in the
demand function." [1975, p. 102].

[3]The F variable is computed as average total expenditure on electricity by
two-part consumers in 1937-38 (in) minus the product of the running
charge on domestic two-part tariffs in 1937-38 (in pence per kWh) times
average consumption per two-part tariff consumer (in kWh) divided by 240
(since at that time there were 12 pence per shilling and 20 shilling per
pound sterling).

TABLE 2

ALTERNATIVE DEMAND FOR ELECTRICITY EQUATIONS
USING HOUTHAKKER'S DATA FOR 42 PROVINCIAL TOWNS

Great Britain, 1937-38

(Standard Errors in Parentheses)

Regressor (In Logarithms)	1	2	3	4	5	6	7
Constant	na	-1.053 (.533)	-0.204 (.614)	-0.067 (.679)	-0.230 (.695)	-0.202 (.793)	1.891 (.492)
M	1.166 (.088)	1.160 (.088)	1.065 (.082)	1.041 (.096)	1.066 (.088)	1.062 (.104)	0.786 (.081)
P_{-2}	-0.893 (.190)	-0.879 (.189)	-0.902 (.212)	-0.892 (.215)			
P_{-0}					-0.758 (.238)	-0.754 (.248)	
AP_{-0}							-0.912 (.140)
G_{-2}	0.211 (.116)	0.208 (.118)	0.051 (.165)	0.039 (.161)	0.079 (.189)	0.076 (.196)	0.127 (.138)
H	0.177 (.033)	0.176 (.033)	0.184 (.035)	0.185 (.036)	0.194 (.038)	0.194 (.039)	0.096 (.034)
F				0.056 (.113)		0.010 (.125)	
R^2	.934	.998	.839	.840	.812	.812	.889

Notes to Table 2:

M is money income, P_{-2} is the marginal electricity price lagged two years, P_{-0} is the current electricity marginal price, AP_{-0} is the ex post average electricity price in the current period, G_{-2} is the marginal natural gas price lagged two years. H is the average holdings (in kilowatts) of heavy electric equipment bought on hire purchase by domestic two-part customers in 1937-38, and F is average fixed charge for domestic two-part customers in 1937-38.

Column 1 reproduces results as reported by Houthakker [1951b, p. 367], while Column 2 presents my attempt at replication based on Houthakker's published data [1951b, pp. 364-365]. Both columns represent estimates adjusted for heteroscedasticity (i.e., both are "weighted" regressions). Columns 3-7 report various "unweighted" estimates (not adjusted for heteroscedasticity).

na = not available from Houthakker's reported results.

elasticities as long-run estimates.[1]

In column 2 of Table 2 I present results of my attempt to replicate Houthakker's empirical findings. As can be seen, the results I obtained using the National Bureau of Economic Research TROLL [1975] program on an IBM 370-Model 168 are very close to the classic ones obtained by Houthakker in 1951 using his own subroutines on the EDSAC electronic computer. This close agreement is remarkable given the small working space available on the EDSAC.[2]

In column 3 of Table 2 I report results using an unweighted regression. The point estimates of the income and own price elasticities are not affected greatly, although the coefficient estimate and statistical significance of the gas cross-price elasticity is reduced considerably when compared to results of the weighted regression. In column 4 I add the average intra-marginal price (average fixed charge) variable F. The coefficient on F is positive and quite large, although a 95% confidence interval based on the rather large standard error estimate would include a theoretically plausible

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[1]Taylor's discussion of Houthakker's results on this issue is somewhat confusing. In particular, Taylor [1975, p. 84] states "Houthakker is silent as to whether the elasticities he has estimated refer to the short run or long run." In Houthakker [1951a], however, reference is made to the Houthakker [1951b] study. Houthakker states clearly [1951a, pp. 18-19] that: "From pre-war information the long-term elasticity of demand with respect to the running charge in a two-part tariff has been estimated at about 0.9" Taylor instead states "... in view of the presence of the holdings of heavy electrical equipment as a predictor ... they should be interpreted ... as short-run elasticities" [1975, p. 84]. While Taylor's short-run interpretation is probably preferable a priori (although the price variable is lagged two years), Houthakker unambiguously interpreted his results as long-run estimates. In any case, the 0.9 price elasticity estimate is closer to conventional econometric estimates of the long run.

[2]In informal discussion, Houthakker has informed me that he was particularly worried about the accuracy of his matrix inversion subroutine, since the EDSAC computer had only 480 locations, each location comprised 35 bytes, and the inversion subroutine for a 10 × 10 matrix already occupied 200 locations.

estimate of about -.004 to -.005 (the mean intramarginal expenditure budget share for the 42 towns was about .0044, and the estimated income elasticity is slightly greater than unity). Of particular interest is the fact that the difference between electricity own-price elasticity estimates in the "correct" (column 4) and the "misspecified" (column 3) equations is quite small--0.010; similar small differences appear for the other regressors.[1]

In columns 5 and 6 of Table 2 I report estimates with the lagged price P_{-2} replaced by the current price P_{-0}; the resulting electricity price elasticities might then be interpreted as short-run elasticities, especially since the electricity consuming stock variable H is included as a regressor. These price elasticity estimates are smaller (in absolute value) than those in columns 3 and 4; however, the values of -0.758 (column 5) and -0.754 (column 6) seem rather large for short-run elasticities. The differences between the electricity own-price elasticity estimates in the "correct" (column 6) and "misspecified" (column 5) equations again is small -- .004.[2] These two comparisons--columns 3 with 4 and 5 with 6--adequately illustrate the analytical argument presented in Section II above, namely, that the econometric consequences of incorrectly omitting the average intramarginal prive variable are negligible.

Finally, in column 7 of Table 2 I report estimates using the current ex post average electricity price variable instead of a marginal price. The

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[1]The unweighted auxiliary regression equation was (in logarithms)
$$\hat{F} = -2.461 + .429M - .173P_{-2} + .224G_{-2} - .010H.$$

[2]The estimated auxiliary regression equation here is (in logarithms)
$$\hat{F} = -2.916 + .426M - .460P_{-0} + .347G_{-2} - .014H.$$

resulting average electricity own-price elasticity estimate of -0.912 is about 20% larger in absolute value than those based on the current marginal price (-0.754 and -0.758), but coincidentally is very close to the marginal price elasticity estimates using the two-year lagged marginal price (-0.902 and -0.892). However, the estimated income elasticity using average ex post electricity price (0.786) is considerably smaller than the 1.041 - 1.165 income elasticity estimates using marginal price data. Thus the price elasticity estimate based on average ex post price appears to capture a portion of the income effect. Coefficient differences based on the average ex post and marginal price data, however, are much greater than the budget share of electricity.[1]

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[1] It might be noted that in the regression results reported in columns 4 and 6 of Table 2, the coefficient on F has not been constrained to equal the theoretical value of the negative of the average fixed charge budget share times the estimated income elasticity. A very simply way of imposing this restriction is to redefine the income variable as income minus the intra-marginal expenditure, and then to regress electricity on this redefined income variable and the other regressors (but excluding F).

VI. CONCLUDING REMARKS

This paper has made several principal points. First, the quantitative, empirical significance of Taylor's extensive argument in defense of including both average intramarginal and marginal price variables in the demand equation is negligible, since least squares estimates with the average intramarginal price variable excluded will typically differ very little from regression estimates with that variable included. This conclusion holds for all the regression coefficients, not just the price coefficient. This point has also been illustrated empirically using Houthakker's [1951b] data. Hence, if the regression analyst chooses to omit the intramarginal price variable due say, to substantial data gathering costs, he can still obtain estimates very close to the "correct" estimates by using (7).

Second, in general econometric results using only average price will differ from those using only marginal price. Halvorsen's curious invariance result is due to his choice of Cobb-Douglas specifications for production and utility functions; these specifications are highly restrictive and are unlikely to be supported empirically.

Third, future work at the micro level might best be focussed on the specification and estimation of models that explicitly recognize the discrete endogenous nature of the marginal electricity price. Procedures recently introduced by Wales-Woodland and Burtless-Hausman in the context of effects of after-tax wage rates on labour supply could be adapted to the electricity demand context. At the present time relatively little is known regarding the extent to which estimates and inference based on these more costly and sophisticated estimation procedures would differ from that based on simpler estimation methods.

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