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STRATEGIES FOR ASSET DEFENSE WITH PRECURSOR ATTACKS ON THE COMMAND AND CONTROL SYSTEM

James T. Walton and Michael Athans

Laboratory for Information and Decision Systems
Massachusetts Institute of Technology
Cambridge, MA 02139

ABSTRACT

In this paper we present a progress report on our work on the static version of the Weapon to Target Assignment (WTA) problem in which vulnerable, distributed C^3 nodes are included in the formulation.

In the static defense asset problem with vulnerable C^3 nodes the offense is allowed to attack either just the assets or first the command and control system and then the assets. In previous work we considered only command and control nodes that were capable of assigning any defensive weapon; now a C^3 node might control only a subset of weapons, with overlap in the spans of control permitted. A problem formulation that permits such distribution of the assignment responsibility is presented, as is a representative architecture. We show that such duplication of C^3 unambiguously improves the capability of the defense. Approximations for determining both the attacker's and defender's stockpile partition for the two stages of attack and defense are given.

1. INTRODUCTION

Our overall objective is to develop quantitative insight into the vulnerability of distributed C^3 organizations. In this work we use the allocation of weapons to targets as a paradigm for the command and control function of resource commitment. In a previous paper the basic problem was introduced, and numerical results for certain special cases were given [1]. For completeness we will review the basic problem and motivation.

The weapon-to-target assignment (WTA) problem quite simply entails optimally assigning weapons to targets so as to maximize (or minimize) one of several cost functions, and it is a hard problem [2]. A variant on the basic problem is to allocate defensive interceptors against offensive forces so as to minimize the damage done to defensive assets. Such a decision making process is traditionally assumed to be performed by a single, global C^3 node that is itself not subject to attack, with decision outcomes being communicated to the appropriate resources. The survival of such control nodes is essential for the defense to perform effectively; their destruction would render defensive weapons unusable. The vulnerability of this process can be reduced by hardening the nodes, replicating their functions, altering their orbits/positions (if they are mobile), or by allowing them to shoot back if attacked. We have formulated a model that includes redundant, vulnerable C^3 nodes that will be defended if attacked. While considedrable work has been done on the asset-based WTA problem [3-14], virtually no consideration has been given to system vulnerability [1,8].

This work is motivated by military defense problems, a couple of examples of which are the mid-course phase of a strategic defense and the defense of a naval battle group. In the former example the decision making capability would be represented by the battle management computers that prioritize targets and allocate the various elements of the defensive arsenal to incoming targets. Also vulnerable are the space and ground based satellite tracking systems. When engaging targets, he defense will be able to see which C^3 nodes and defensive assets have been selected by the offense prior to having to allocate its own resources. Defensive assets would be such things as military installations, population centers, the defense's own ICBM force, etc. The defense can increase the survivability of its C^3 system by the means noted above: hardening, replication, orbital selection or by defending itself.

The defense of a naval battle group against air attack presents another relevant scenario. Defensive assets would be an aircraft carrier, troop or material transports, and escort or support ships. The objective of the defense would be to maximize the expected surviving value of these assets. Elements of centralized command and control might be performed in an aircraft based radar platform such as an E-2C; were it destroyed or out of position, however, its' functions would need to be performed at the individual weapon platforms. These distributed decision makers would then face the problem of access to fewer and less reliable sources of track information, which would effectively serve to degrade the kill probabilities for their weapons.

2. THE ASSET-BASED WTA PROBLEM

We consider the following basic defense asset problem. A collection of separated point targets is to be defended against an attack in which offensive weapons arrive simultaneously. The defense has area interceptors, each of which is capable of defending any asset. In addition, the defense has "last move" status, i.e. the defense can observe the intended targets of the offensive weapons prior to allocating it's interceptors. The offense seeks to minimize, and the defense to maximize, the expected surviving defense asset value. The result is a min-max problem.

Our interest in this problem is a version in which vulnerable C^3 nodes are included. The defensive weapons are controlled by a set of command and control nodes that are themselves subject to a precursor attack. Redundancy and overlap in the spans of control of the C^3 nodes reduce the likelihood that interceptors will be rendered useless by a successful attack on the battle management system. Both the offense and defense may choose to assign some of their resources to such a first phase engagement.

We introduce the following notation:

- M = Number of defensive weapons
- N = Number of targets (offensive weapons)
- K = Number of assets
- w_k = Value of asset k
- T = Number of C^3 nodes
- πt_t = Probability that a target kills C^3 node t

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πk_k = Probability that a target kills asset k
p_{ij} = Probability that weapon j kills target i
m_T = Set of weapons used to defend C^3 nodes
M_T = Number of weapons in m_T
M_K = $M-M_T$= Number of weapons left to defend assets
nt_t = Number of targets aimed at C^3 node t; $\Sigma_t nt_t=N_T$
nk_k = Number of targets aimed at asset k; $\Sigma_k nk_k=N_K$
x_{ij} = 1 if weapon j is assigned to target i, 0 otherwise

This yields the following basic defense asset problem:

$$\min_{A_K}\max_{X} \sum_{k=1}^{K} w_k \prod_{i=1}^{nk_k}[1-\pi k_k \prod_{j=1}^{M}(1-p_{ij})^{x_{ij}}] \quad (1)$$

subject to (stockpile constraints):

$$A_K^{tr}e = N \quad (2)$$

$$e^{tr}Xe = M \quad (3)$$

$$x_{ij} \in \{0,1\}\ \forall\ i,j \text{ and } nk_k \in Z_+ \quad (4)$$

A_K is the offensive strategy vector consisting of the nk_k's; X is a defensive strategy matrix consisting of the x_{ij}'s. e is a column vector of ones of conformable dimension. ^{tr} denotes transpose.

3. THE ASSET-BASED WTA PROBLEM WITH REDUNDANT C^3

We first considered [1] a generalization of the basic problem (1)-(4) in which defensive weapons are controlled by T C^3 nodes, any surviving one of which can assign any defensive weapon. This represents replication of C^3 functions in "global" C^3 nodes. The offense can then either attack only the assets, or first attack the C^3 nodes and then the assets. The defense, in turn, has a choice on how to split its weapons in defending the C^3 nodes and assets - after the offense has revealed its strategy. In a weapons-poor (or target-rich) environment, preferential defense can be employed for both the C^3 nodes and assets.

This results in a new objective function: expected asset surviving value

$$U = \xi V_u + (1-\xi)V_D \quad (5)$$

where ξ, the probability that all C^3 nodes are destroyed, is given by

$$\xi=\prod_{t=1}^{T}\{1-\prod_{i=1}^{nt_t}[1-\pi t_t \prod_{j=1}^{M}(1-p_{ij})^{x_{ij}}]\} \quad (6)$$

and V_U, the expecting surviving asset value if the assets are undefended, is

$$V_u = \sum_{k=1}^{K} w_k \prod_{i=1}^{nk_k}(1-\pi k_k) \quad (7)$$

V_D is the surviving defended asset value, which remains the same as in the basic problem. The defense seeks to maximize U and the offense seeks to minimize it, subject to their respective stockpile constraints. In [1], optimal offensive and defensive allocations for the special case of identically valued assets, equal kill probabilities and uniform attack were computed using an enumerative marginal return algorithm, and then plots were made of the resultant strategies. Some conclusions were drawn from the computational work, such as that greater numbers and more effective defensive weapons discourage attack of the C^3 nodes, as well as that whether or not the offense attacks C^3 nodes depends on problem parameters. Because of combinatorial explosion in the number of feasible strategies, however, such computations can only be made for small scale examples with such simplifying assumptions.

For the special case described above, approximate formulas that reflect how strategies change with kill probabilities and stockpile size have been developed. The objective is to try to calculate the offensive and defensive reaction sets, i.e. the set of responses (strategies) for opposing moves. Even approximations for this scenario are difficult to develop, however, since V_D is neither concave nor convex in defensive weapon assignments. Consider the continuous variable version of the probability of survival for an asset, $(1-\pi k(1-p)^{x_k})^{nk}$, where $nk \in Z_+$ is the number of attackers and $x_k \geq 0$ is the number of defenders per attacker at asset k. The second derivative of this quantity is:

$$nk[1-\pi k(1-p)^{x_k}]^{nk-2}[\pi k(1-p)^{x_k}][\ln(1-p)]^2[nk\times\pi k(1-p)^{x_k}-1].$$

This is convex if the final term is positive, concave if it is negative; the change occurs at $x_k^*=-\ln(nk\times\pi k)/\ln(1-p)$. Between values of x_k that are integral multiples of nk, this probability is strictly convex.

Because the objective function is not concave, a greedy algorithm cannot be optimal for the defender. This is an algorithm that calls for the defender to assign his next weapon, or clump of weapons, to thet target that provides him with the greatest incremental contribution to expected surviving asset value. This approach does perform well in general for the basic defense asset problem, and is optimal for some cases. We can use it to derive an approximation for the size of a preferential defense in a weapons-poor or target-rich environment: a randomly selected subset of assets the heavy defense of which yields greater surviving value than if all assets were more lightly defended.

$$\text{Number of Defended Assets} \approx \frac{M_K}{(N_K/K)\gamma} \quad (8)$$

$$\text{where } \gamma = \lceil -\frac{\ln(\pi N_K/K)}{\ln(1-p)} \rceil$$

The partition of the defensive stockpile between asset and C^3 defense when each C^3 node is capable of assigning any weapon is given by

$$M_K \approx \frac{(1-p)^{-M_T/(N_T/T)}-\pi}{-\pi\ln(1-p)} \quad (9)$$

$$\text{subject to: } M_T + M_K = M \quad (10)$$

The defense defends only one randomly selected C^3 node with M_T weapons. Not knowing which one will be defended, the offense must attack them all evenly. The principal assumption needed for this result is that undefended assets /nodes are almost surely destroyed, allowing us to neglect those terms. (9)-(10) are then produced using a Lagrange multiplier technique.

This approach is extendable to the offensive partition. For a uniform attack (within stages), the offensive partition between C^3 and asset attacks is given by

$$\frac{N_T}{T} \approx \left\{ M_T \ln(1-p) \times \left[\frac{-(p + p \times \ln(\frac{N_K}{K}\pi))}{\frac{N_K}{K}\pi \ln(\frac{N_K}{K}\pi)} + \ln(1-(N_K/K)^{-1}) + ((N_K/K)-1)^{-1} \right] \right\}^{-1/2} \tag{11}$$

$$\text{subject to:} \quad N_T + N_K = N \tag{12}$$

Solution of (9)-(12) gives local optima for N_T & N_K and M_T & M_K in the case of equal valued assets, uniform kill probabilities, global command and control and preferential defense. These provide good agreement with the numerical results obtained in [1].

4. THE ASSET-BASED WTA PROBLEM WITH DISTRIBUTED C^3

We now generalize the asset-based WTA problem with vulnerable C^3 further, to include command and control nodes that may not be capable of assigning all weapons. Thus we speak of the "distribution" of control. A particular weapon could be controlled by many (but not necessarily all) C^3 nodes. A node with a limited span of control might correspond to a reserve capability for the allocation of a particular weapon cluster or type. A manned fire unit will almost certainly have the capability to make weapon assignments for terminal engagements in defense of itself.

4.1 Problem Formulation

Additional notation is needed. Let Ω be the set of all T-fold combinations of C^3 nodes that survive the precursor attack. Ω is of cardinality 2^T, and each element is a T dimensional binary column vector denoted by ω. If $\omega_t = 1$, then C^3 node t survived; if $\omega_t = 0$, then it did not. If T = 3, for example, then Ω consists of the following:

$$\left\{ \begin{bmatrix}0\\0\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\end{bmatrix}, \begin{bmatrix}0\\1\\0\end{bmatrix}, \begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\1\\1\end{bmatrix}, \begin{bmatrix}1\\0\\1\end{bmatrix}, \begin{bmatrix}1\\1\\0\end{bmatrix}, \begin{bmatrix}1\\1\\1\end{bmatrix} \right\}$$

The probability that a particular ω is realized will be denoted by ξ_ω, and is given by:

$$\xi_\omega = \left\{ \prod_{t|\omega_t=1} \left\{ \prod_{i=1}^{nt_t} \left[1 - \pi t_t \prod_{j=1}^{M} (1-p_{ij})^{x_{ij}}\right] \right\} \right\} \times$$

$$\left\{ \prod_{t|\omega_t=0} \left\{ 1 - \prod_{i=1}^{nt_t} \left[1 - \pi t_t \prod_{j=1}^{M} (1-p_{ij})^{x_{ij}}\right] \right\} \right\} \tag{13}$$

The first product term taken over t computes the probability of the specified C^3 nodes surviving, the second the probability of their complement being destroyed. $\Sigma_{\omega\in\Omega}\xi_\omega = 1$, and hence $\xi_{(\cdot)}$ represents a probability density function over the sample space Ω.

Each combination of surviving C^3 nodes will induce a set of "available" weapons for asset defense. These resulting sets of weapons need be neither unique nor mutually exclusive, and for most C^3 architectures won't be. The function g: $\Omega \rightarrow \Theta$ will be used to represent the mapping from C^3 survival modes to sets of indices of available or "active" weapons (Θ is the index set of the defensive weapons). Each possible combination of surviving C^3 nodes produces a set of weapons that can be assigned to targets in the asset defense stage, and g specifies what that set is. Thus g is a point-to-set map from Ω to Θ. g captures the character of different organizational architectures – a control node of limited scope would "map" back only to those weapons that it can assign.

Surviving asset value, for a given ω, is:

$$V_\omega = \sum_{k=1}^{K} w_k \prod_{i=1}^{nk_k} \left\{1 - \pi k_k \prod_{\substack{j\in g(\omega)\\ j\notin m_T}} (1-p_{ij})^{x_{ij}}\right\} \tag{14}$$

Thus at asset k the defense can employ interceptors controlled by surviving C^3 nodes ($j \in g(\omega)$) that were not used for BM/C^3 defense ($j \notin m_T$).

The resulting optimization problem is:

$$\min_{A_T,\, A_K} \max_{X} \sum_{\omega\in\Omega} \xi_\omega V_\omega \tag{15}$$

subject to:

$$A_T^{tr} e + A_K^{tr} e = N \tag{16}$$

$$e_{tr} X e = M \tag{17}$$

$$x_{ij} \in \{0,1\} \ \forall\ i,j \ \text{ and } nt_t, nk_k \in Z_+ \tag{18}$$

where A_T is the strategy vector consisting of the nt_t's. Constraints (16)-(17) are the "shot conservation" constraints.

It is instructive to see the form that the objective function takes for a specific command and control architecture that doesn't consist exclusively of global decision makers. Consider the case where there is a single global C^3 node, and T-1 additional nodes that each control an equal number of defensive weapons. The T-1 sets of weapons are mutually exclusive and collectively exhaustive. There will then be 2^T different failure modes for the C^3 nodes, 2^{T-1} of which contain the global node. The objective function will take this form:

$$\text{Surviving Value} = 2^{T-1} V_G \xi_G + \sum_{b=0}^{T-1} \binom{T-1}{b} V_b \xi_b \tag{19}$$

where

$$V_{G,b} = \sum_{k=1}^{K} w_k \prod_{i=1}^{nk_k} \left(1 - \pi k_k \prod_{j=1}^{y_{G,b}} (1-p_{ij})^{x_{ij}}\right) \tag{20}$$

$$y_G = M - M_T \tag{21}$$

$$y_b = \frac{b}{T}(M - M_T) \tag{22}$$

$$\xi_G = \prod_{i=1}^{nt_G}(1 - \pi t_G \prod_{j=1}^{M}(1 - p_{ij})^{x_{ij}}) \qquad (23)$$

$$\xi_b = \{\prod_{i=1}^{nt}\{1 - \pi t \prod_{j=1}^{M}(1 - p_{ij})^{x_{ij}}\}\}^{b} \times$$
$$\{1 - \prod_{i=1}^{nt}\{1 - \pi t \prod_{j=1}^{M}(1 - p_{ij})^{x_{ij}}\}\}^{T-b-1} \times$$
$$\{1 - \prod_{i=1}^{nt_G}\{1 - \pi t_G \prod_{j=1}^{M}(1 - p_{ij})^{x_{ij}}\}\} \qquad (24)$$

G indexes the global C^3 node; πt_t is taken to be the same for the non-global C^3 nodes and thus the corresponding nt_t's are the same by symmetry. b is the number of them that survive the first stage attack. y_b reflects that some of the weapons for which viable command and control exists in the second stage will have been fired in defense of the C^3 system in the first stage.

It can readily be shown that this distributed formulation reduces to the problem previously considered for the case of redundant, centralized command and control (equations (5)-(7) plus stockpile constraints). Distribution yields an inherently more complex system, however. If the defensive stockpile is split equally, with each half controlled by two nodes, we do not get a separable problem: system performance depends on the performance of *both* weapon sets.

4.2 Discussion

Is the presence of additional decision makers of benefit? As has been noted, the distribution or replication of C^3 functions is considered desirable, although there is little quantification of the advantages. In the case of the weapon-target assignment function we have the following theorem.

Theorem 4.1: *The defense will always perform better with two or more global C^3 nodes than with one.*

Proof: Let z^1, A^1 and X^1 be the optimal game and decision variable values for the vulnerable defense asset problem with one global C^3 node, which we will call problem P^1. (A represents the concatenation of A_T and A_K). Similarly, z^*, A^* and X^* are the optimal values for problem P^2, reflecting the same problem with two or more C^3 nodes (we will assume there to be two without loss of generality). We must show that there exist defensive strategies for P^2 such that for all possible offensive allocations, $z^*>z^1$ results.

The probability, ξ_0, that both C^3 nodes fail in P^1, given $N_T^{\,1}$, is:

$$\xi_0^1 = 1 - \prod_{i=1}^{N_T^1}[1 - \pi t \prod_{j=1}^{M}(1 - p_{ij})^{x_{ij}}]$$
$$> 1 - \prod_{i=1}^{\lceil N_T/2 \rceil}[1 - \pi t \prod_{j=1}^{M}(1 - p_{ij})^{x_{ij}}]$$
$$> \{1 - \prod_{i=1}^{\lceil N_T/2 \rceil}[1 - \pi t \prod_{j=1}^{M}(1 - p_{ij})^{x_{ij}^1}]\} \times$$
$$\{1 - \prod_{i=1}^{\lfloor N_T/2 \rfloor}[1 - \pi t \prod_{j=1}^{M}(1 - p_{ij})^{x_{ij}^1}]\}$$
$$= \xi_0^2 \qquad (25)$$

The final quantity in (3.16) is ξ_0 for the optimal P^2 offensive strategy against the defensive strategy X^1 (given this partition of N^1 into $N_T^{\,1}$ and $N_Q^{\,1}$). That splitting $N_T^{\,1}$ as equally as possible between C^3 nodes is optimal for the offense (if faced with X^1) follows from

$$(1 - a^{\lceil b/2 \rceil})(1 - a^{\lfloor b/2 \rfloor}) \geq (1 - a^{\lceil b/2 \rceil - 1})(1 - a^{\lfloor b/2 \rfloor + 1})$$

for $0 \leq a<1$ and $b \in Z_+$. Neither V_u nor V_d will have changed, thus the expected surviving value has increased, since $\xi_0^{\,1} > \xi_0^{\,2}$ $\Rightarrow \xi_0^{\,1}V_u + \xi_1^{\,1}V_d > \xi_0^{\,2}V_u + \xi_1^{\,2}V_d$.

We now show that there exists no $N_T^* \neq N_T^{\,1}$, $N_Q^* \neq N_Q^{\,1}$ that can result in a value of z^* that is less than or equal to z^1. $N_T^* < N_T^{\,1}$ contradicts the optimality of $[N^1, X^1]$ for P^1, since a lower $\xi_0^{\,1}$ could have been obtained in P^1 by reallocating offensive weapons from C^3 nodes to assets. Now consider $N_T^* > N_T^{\,1}$. The defensive strategy employed in response is as follows: the partition of defensive weapons between C^3 and assets remains the same, and *only* the least attacked C^3 node is defended. The offense will then split its C^3 attack as evenly as possible between the control nodes. An attack level as high as $N_T^*=2N_T^1$ (i.e. attacking one of the C^3 nodes in a fashion identical to the optimal solution to P^1) will still not achieve an ξ_0^* as high as ξ_0^1, yet will reduce both V_u and V_d, producing $z^*<z^1$. Although further shifts of offensive weapons to the C^3 attack might produce an $\xi_0^*>\xi_0^1$, this will again contradict the optimality of $[N^1, X^1]$ for P^1, since an ξ_0^* greater than ξ_0^1 could have been obtained with $2N_T^1$ fewer offensive weapons allocated to the C^3 attack. //

Similarly, we have the following two theorems for distributed command and control architectures:

Theorem 3.2: *A global C^3 node with a distributed alternative command and control capability is better for the defense than is a single, global C^3 node.*

Theorem 3.3: *Distributed command and control is less vulnerable than a single, global C^3 capability.*

The proof of these propositions follows that of proposition 3.1.

5. SUMMARY

The preferential defense of differently valued assets is a complex problem in its own right; incorporating C^3 nodes in the preferential defense paradigm yields yet greater complexity. This is due in part to the coupling between offensive and defensive strategies, which implies that a min-max problem must be formulated and solved.

Optimal strategies, such as a decision by the offense on whether to ignore or attack the C3 system, and at what level,

depend on the numerical values of problem parameters (stockpile size, kill probabilities). For special cases it is possible to obtain approximations to the truly optimal solution. These provide analytical insights as well as initial starting conditions for more elaborate computational algorithms.
This is an ongoing research progress report. Analytical and algorithmic solutions for more complex problem formulations are, as yet, not availabe.

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