Application of Modern Control Theory on a Model of the X29 Aircraft

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Abstract

In this report, two different control strategies for a model of the X29 are presented. The first is based on the l^1 design methodology and the second is based on the H_∞ design methodology. The purpose here is to show that these methodologies make the design process quite systematic, and that the resulting controllers conform with our intiuition.

1 X29 Model

The model for the X29 was taken directly out of [8]. A diagram of a forward swept wing aircraft is shown in Fig. 1. The model describes the use of the canard and flaperon to control the pitch angle and angle of attack. The pitch angle θ is the angle of the nose of the aircraft with respect to horizontal, and the angle of attack α is the angle of the nose with respect to the direction of the aircraft's velocity. The flight path angle γ is $\theta - \alpha$ and is the angle of the velocity with respect to horizontal. These quantities are all shown in Fig. 2.

Three different systems are presented in [8]. The first has the canard deflection as input and the angle of attack as output. The second has pitch attitude as output, and the third system uses both the canard and the flaperon to control the pitch attitude and the angle of attack. In this work, only the first case is treated.

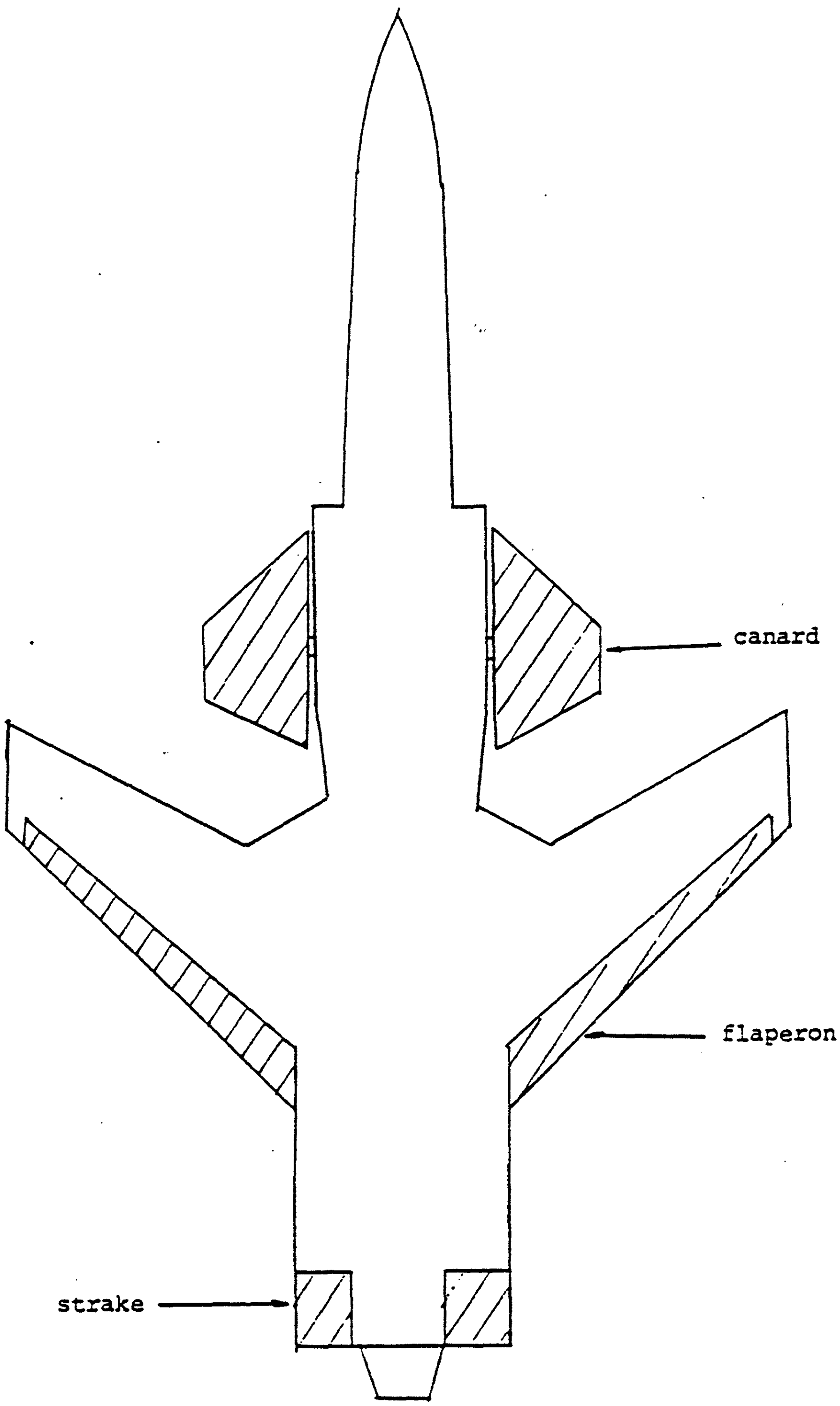

Figure 1: Diagram of the X29 Control Surfaces

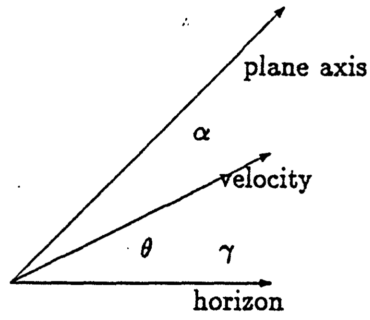

Figure 2: Pitch, Flight Path, Angle of Attack

2 l^1 Control Design

The challenge of the control design in the case of the X29 is to meet the various design specifications. For the first design case with the canard controlling the angle of attack, the specifications are that 10% error or less is required in command following for $0.01 < \omega < 1$ rad/sec, disturbances must be attenuated by at least a factor of 1.5 for $\omega < 1$ rad/sec, and the system must be robust in the face of multiplicative errors. Multiplicative errors are introduced from ignoring the high frequency wing torsion mode and from the scaling.

A final objective is to avoid exciting the wing bending mode at about 60 rad/sec. To accomplish this, it is desired to have the system bandwidth less than 60 rad/sec. This will cause attenuation of any excitation of the bending mode. In the design, this spec is approximated by requiring the crossover frequency of GK to be below 60 rad/sec.

All together, these specs require that the sensitivity be small and that the closed loop transfer function rolls off sufficiently at high frequency to guarantee robust performance. We will employ the l^1 methodology to achieve this. For details on the l^1 problem consult [1,3,4,5,6,9,10]. To meet these requirements, a good strategy is to minimize the sensitivity function, particularly in the range $\omega < 1$.

With no weighting of the sensitivity, the design specifications were not quite met. The results are shown in Figs. 3-4. The sensitivity was sufficiently small in the appropriate range at low frequencies, but the bandwidth of the system was too high and did not satisfy the

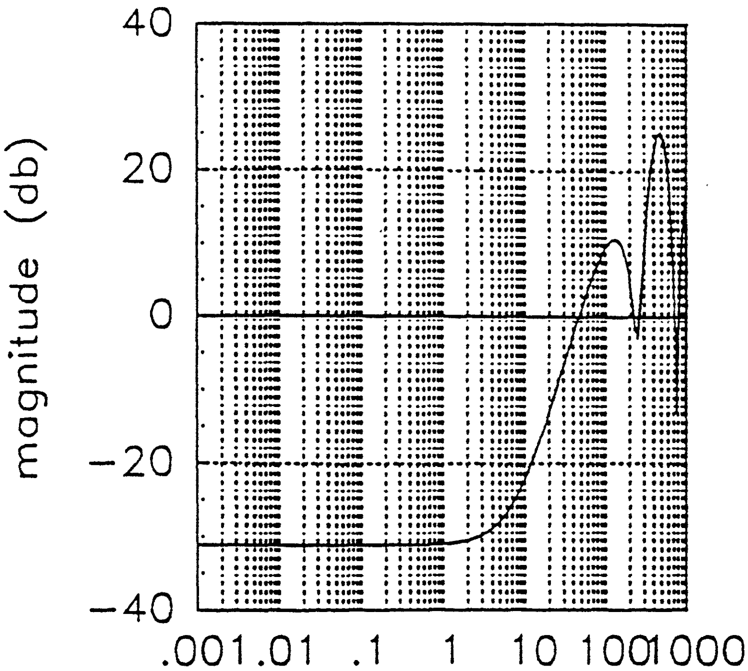

Figure 3: Sensitivity for Unweighted Optimization

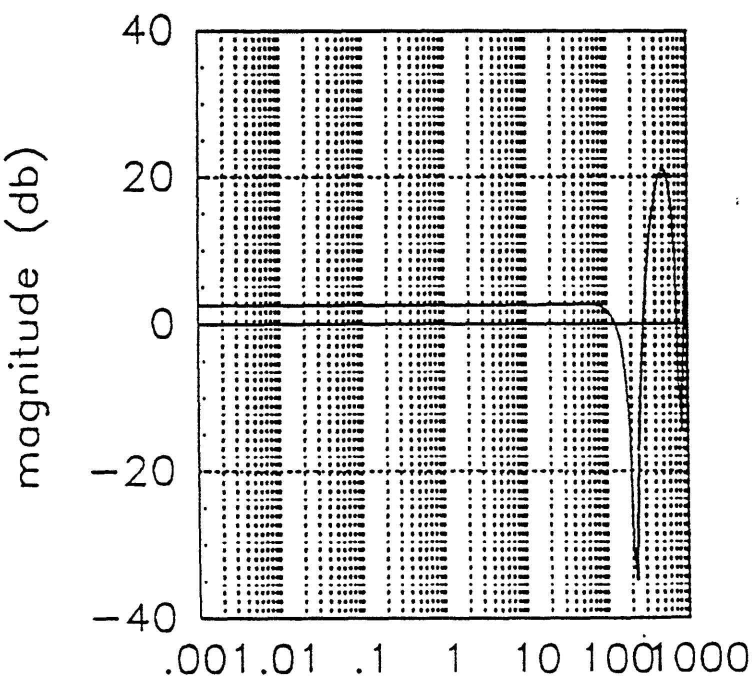

Figure 4: $GK(I - GK)^{-1}$ for Unweighted Optimization

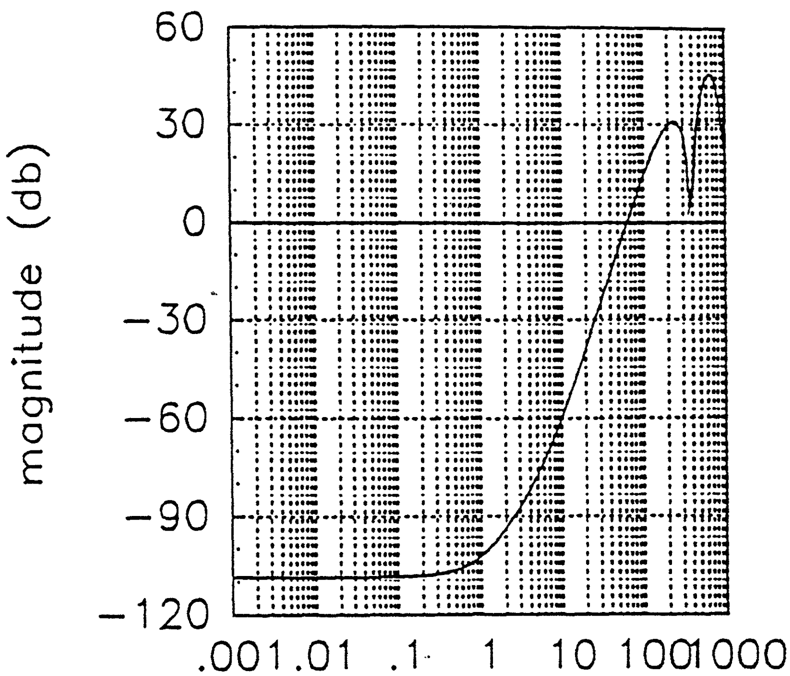

Figure 5: Optimal Sensitivity With Weight $\frac{1}{s+1}$

robustness requirements.

In fact the requirements were not met even with several weights which were used, an example of which is depicted in Fig. 5. The weighting was the discretized equivalent of $\frac{1}{s+1}$. Compared to Fig. 3, Fig. 5 demonstrates the effect of the higher weighting at low frequency. The low frequency sensitivity has been decreased at the expense of the high frequency sensitivity as expected.

After using several weighting functions, it was determined that the requirements could not be met (or would be difficult to meet) using only the sensitivity function. The next step was to use the complementary sensitivity $\mathbf{GK}(\mathbf{I}-\mathbf{GK})^{-1}$ in the objective function. The sum of the sensitivity and the complementary sensitivity was used as the objective.

The results of the new optimization are shown in Figs. 6-8. The low frequency sensitivity has increased slightly and **GK** crosses the 0 db point at about 40 rad/sec. This new system meets the design specs. Various weights were used to try to tune the system better,

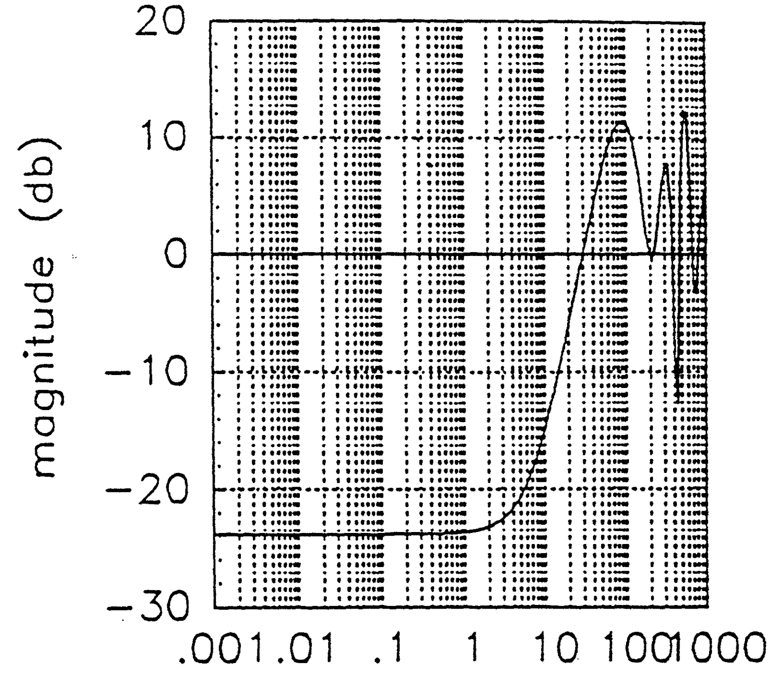

Figure 6: New Sensitivity

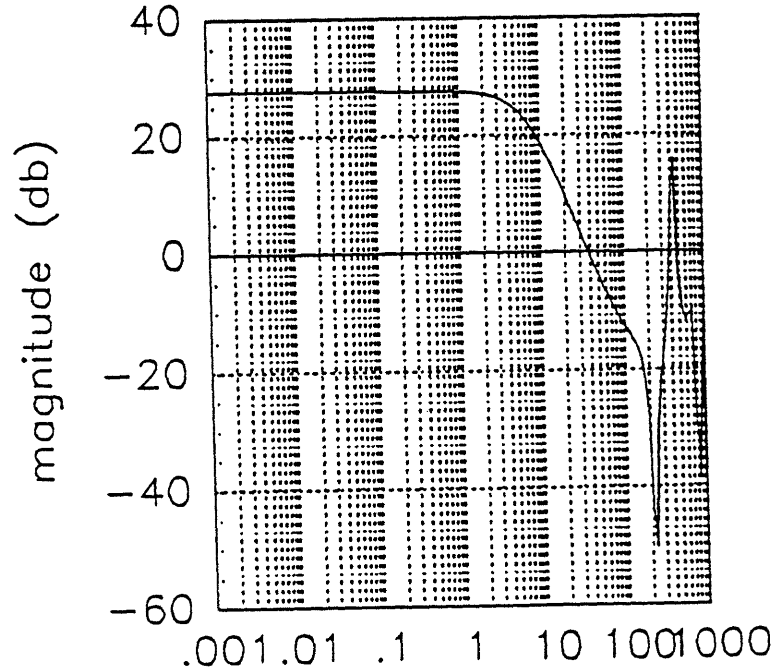

Figure 7: GK

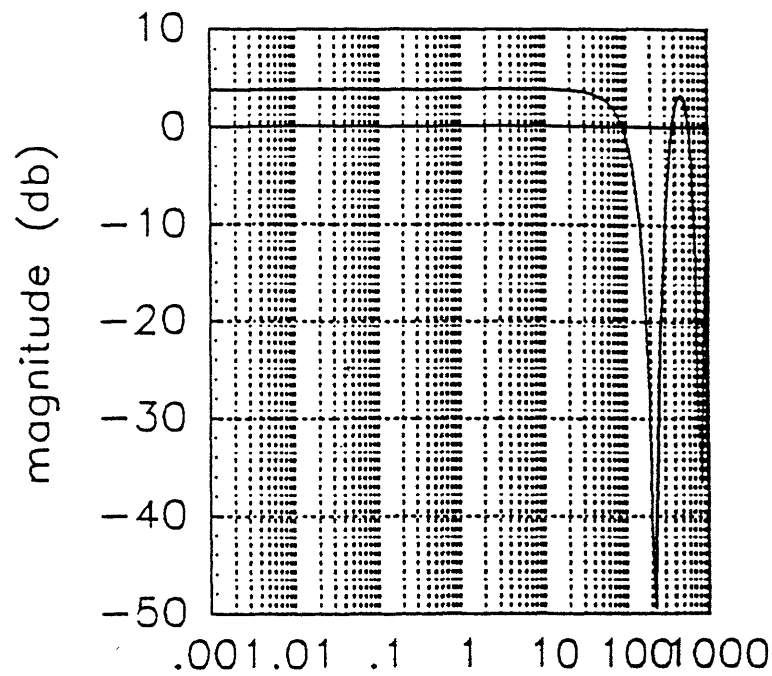

Figure 8: Complelentary Sensitivity

but none were found which produced much improved results.

3 H_∞ Control Design

To meet the above specification, we considered the mixed sensitivity problem directly. Basically, we require the sensitivity to be small at low frequency ($0 \leq w \leq 1$), and the complementary sensitivity to be small at high frequency. After several iterations, the weight on the sensitivity function was choosen as

$$W = \frac{100}{(s+1)^2}.$$

To use the recent algorithms that involve solving two Riccati equations [7], it is necessary to include the control transfer function in the objective. Hence the final objective is to minimize the H_∞ norm of

$$\begin{bmatrix} WS \\ T \\ \rho KS \end{bmatrix}$$

where ρ is a small number. The resulting closed loop transfer functions are shown in Figures 9-10-11 below. It is apparent that the specifications are met.

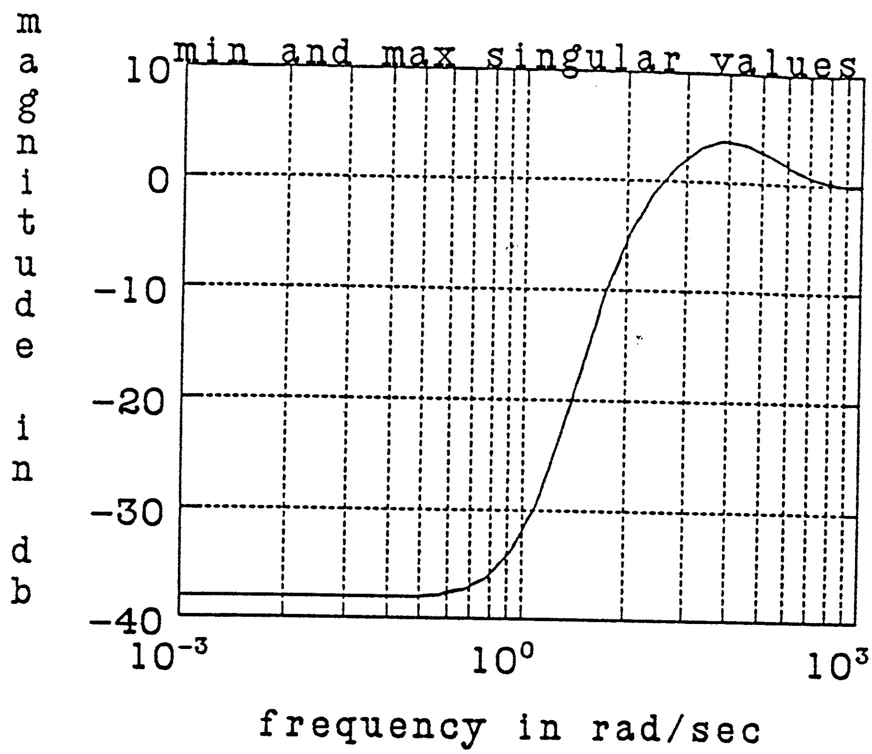

Figure 9: Sensitivity

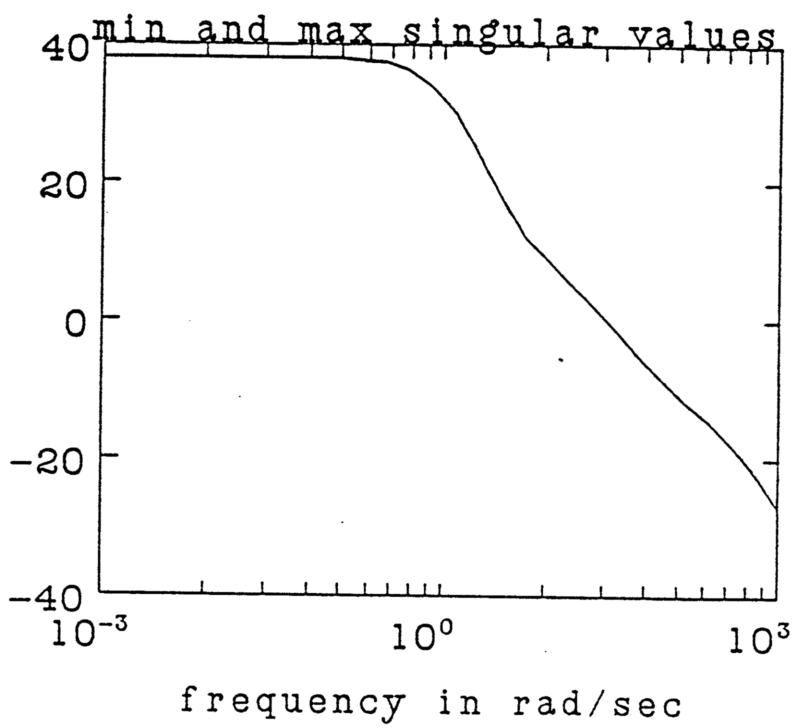

Figure 10: GK

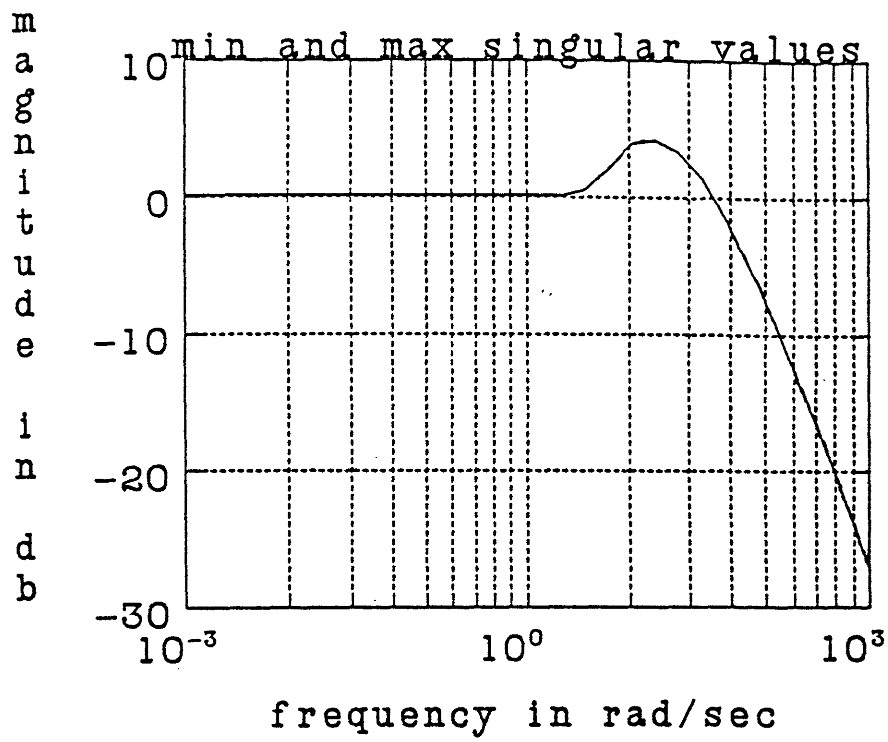

Figure 11: Complelentary Sensitivity

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