MECHANICS OF BENDING OF NONWOVEN FABRICS

by

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ABSTRACT

The aim of this investigation is to explore the micro- and macro-mechanics of nonwoven fabrics in bending. The central approach is to model the nonwoven fabrics as composite materials in which different phases correspond to different portions of the fabric structure contributing in different ways to the bending stiffness. Mathematical models, built to evaluate the bending behavior of these materials, have been verified with experiments. The relative significance of bending mechanisms has therefore been identified.

Four different fabrics from two families of nonwovens are studied individually: spunbonded nonwovens and print-bonded nonwovens. The thinnest self-bonded spunbonded fabric, of only two fiber diameters thickness, is modeled as a heterogeneous plate where fiber junctions are considered as through-thickness inclusions embedded in a continuum matrix representing a free fiber network. Estimations of bending stiffness, made with a self-consistent scheme, agree well with experimental results. In the other extreme, a spunbonded fabric with a thickness of about 26 fiber diameters is found to act as a uniform plate closely described by the model of no-freedom-of-relative-fiber-motion. But fabrics with thicknesses of about 7 fiber diameters behave with certain deviations from this model—a situation attributed to the presence of a large proportion of less constrained fibers near the fabric surface.

Fibers in the unbonded region of the thermal spot-bonded (spunbonded) nonwoven fabric are found to be more or less wavy. The effect of the degree of this waviness to the bending stiffness of the fabric is analyzed. Further, assuming fibers to be pinned together occasionally through frictional contacts, the effective fiber waviness becomes a function of the fiber packing density and the stiffness is thus also influenced by the frictional bond distribution.

(continued)
A print-bonded nonwoven fabric with herring-bone bonds is modeled as a two-phase composite: a model of no-freedom-of-relative-fiber-motion for the bonded phase and a bulky fiber assembly model for the unbonded phase. By examining the "unit cell," the bending stiffness of the fabric is estimated. The possible influence of the penetration depth of the applied binder on the macroscopic bending stiffness is also discussed and is considered as an important parameter affecting the overall bending stiffness.

Finally, a print-bonded nonwoven that is examined is observed to be composed of groups of straight and curved fibers around a predetermined pattern of elliptic holes. When this fabric is bent, the major bending moment is carried by the straight fibers. The bulky fiber assembly developed for the print-bonded nonwoven is also employed here to describe the unbonded area of the fabric. The theoretical estimations of the bending response of the fabric agree with the experimental results.

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INTRODUCTION

Textile fabrics are made of a large variety of fibers. The properties of fabrics are mainly governed by the fiber properties, fiber arrangements, and, if applicable, the binder used. Woven fabrics which are produced by first twisting fibers into yarns and then interlacing yarns into fabrics usually allow for considerable degrees of freedom in design and in fabric structure, but they require expensive processing operations. Nonwoven fabrics are manufactured in unconventional processes that depart from operations for woven or knitted fabrics through elimination of many laborious procedures such as yarn preparatory processes, spinning, twisting, weaving, knitting, etc., thus permitting considerable cost reduction in the final product. However, in contrast with woven fabrics, the process of preparing nonwovens, which consists simply of forming fiber webs, then bonding them by mechanical or chemical means, offers less control over the details of the fiber arrangement in the fabric and hence less capability of altering fabric properties.

It is recognized that since they are comprised of twisted yarns, woven or knitted fabrics have the capacity of allowing for easy bending and shearing under the light load of apparel usage but develop into strong self-locked fiber assemblies under heavy load. Such low bending and shearing stiffnesses are due primarily to the fine diameters of the component fibers and particularly to the relative freedom of
movement between fibers and between yarns. In the case of nonwoven fabrics, these desired advantages are not altogether easy to acquire. For the purpose of improving the integrity of the nonwoven fiber assembly and giving it some desirable elastic properties, a certain density of bonds between fibers is indispensable. These bonds can be adhesive, junctions of self-bonding fibers, or fibers heat-fused together at some areas in the fabric. The existence of the bonds, however, limits the degrees of freedom of fibers under imposed distortions of the fabric. Hence, the characteristic difference between nonwoven and woven fabrics is that the former possess a higher bending stiffness than the latter for equivalent basis weights while their tensile stiffnesses are usually comparable.

A wide range of textile applications demand the bending stiffnesses of fabrics to be satisfactorily low, particularly for clothing purposes. The need to better their quality in bending, and to develop controllable factors in nonwoven fabric designing, calls for a close study of macro and micro mechanics of nonwoven fabrics under deformation. Although the tensile stiffness and strength of nonwoven materials have been understood for some time, the bending stiffness and its relation to the fabric structure has remained unclear. The obstacles lie in the fact that nonwovens usually respond to bending like complicated composite systems in which many mechanisms of comparable importance coexist simultaneously. These mechanisms are, in the first place, not easy to identify, especially because the fiber arrangement in the fabric is often a matter of chance,
not to mention the complexities introduced by the presence of binding agents. Moreover, even if certain operative mechanisms can be distinguished, to account for their contributions to the bending stiffnesses will remain difficult since they usually interact with or are interdependent upon each other.

Perhaps for the above reasons, not many investigators in the field have dealt with the questions of mechanical behavior of nonwovens under bending. Among those who did, notably, Backer and Petterson\(^1\) have discussed the possible principal bending deformation mechanisms of nonwoven fabrics. Freeston and Platt\(^2\) have built two extreme models of complete freedom and no freedom of relative fiber motion to analyze the bending stiffnesses of nonwoven fabrics. Their experimental results have provided an interesting comparison with those calculated from the models and suggested that the bonded nonwoven fabric under their examination has limited freedom of relative fiber motion. Efforts have also been made by others\(^3-7\) in exploring the bending behavior of nonwoven fabrics.

The objective of this research is essentially an attempt to treat the nonwoven fabrics as composite materials in which different phases correspond to different portions of structure in the fabrics with different mechanisms in bending. The strategy of our approach has been to develop physical models which furnish the details of the constituent phases for each material under investigation on the basis of extensive morphological studies. The modes of deformation associated with these
individual phases are by no means the only possible ones. They are chosen and emphasized for their relative importance and/or for the simple reason that our ability to deal with very complex composites is, unfortunately, still quite limited. Second, based on the physical model the bending stiffnesses of individual phases are evaluated. Then a suitable mathematical model of composite material (designed to consider the interaction between constituent phases and reflect the shapes of individual phases) is proposed to compute the effective, or macroscopic, bending stiffness of the fabric. At the last stage, experiments with a specially-designed device and a commercially available testing machine are performed to measure the bending stiffnesses of the fabrics. The techniques developed in testing the bending stiffnesses of small regions may be valuable in evaluating very localized bending stiffness of relatively flat fabrics. Through the comparison of computed and measured results, the validity of theoretical models can be verified and hence the relative significance of mechanisms involved in bending can be determined.

Nonwovens cover a wide range of textile fabrics which can vary from a fibrous structure with deliberately entangled but totally un-bonded fibers to a highly bonded paper-like material. Valid description of the term "nonwoven fabric" perhaps cannot be made in a simple manner on account of their extremely broad coverage. It is, however, not the intention of this study to involve itself with the full range of these materials. Rather, the attention will be directed towards two important families of nonwovens, namely, spunbonded nonwoven and
print-bonded nonwoven fabrics. Spunbonded nonwovens, which are marked by their simplicity in converting polymer to fabric into one process, are normally made of continuous filaments. Print-bonded nonwovens are generally adhesive bonded fabrics made by means of printing the binder in predetermined patterns on unbonded webs consisting of staple (finite length) fibers. These fabrics are selected because they are particularly representative among those nonwovens whose bending behavior is critical to meet definite requirements for their end uses.

The thesis is divided into four chapters and each chapter will explore one specific material. For the first two chapters, two spunbonded nonwovens, Cerex and Fibretex, the former a self-bonded fabric and the latter a thermal spot-bonded fiber assembly, will be handled separately. Although there are some fundamental structural differences between them, they can still be modeled to share the same theoretical treatment, via a self-consistent scheme, to execute the calculation of their bending stiffnesses. As for the third and fourth chapters, two print-bonded nonwovens, Masslinn, a preferentially oriented fiber web, and Keybak, a fabric with predetermined pattern of elliptic openings, will be investigated. There are also certain identical analytic methods available that can be applied to both fabrics; for instance, a bulky fiber assembly model will be proposed for their unbonded regions.

Only a few materials are presented in the study, but the mechanisms of bending revealed will be meaningful in explaining other kinds of nonwovens. In addition, the concept of composite materials will probably
also be of some assistance in opening up a new avenue in dealing with nonwovens which are usually of complicated structure. While the results of the research provide answers to only a fraction of the problems concerning the bending of nonwovens, it is hoped that they will serve the purpose of providing some new understanding of nonwoven technology and can be employed by others to do further research in this field.
1.1.1 Description of the Material

Cerex (Monsanto product), a spunbonded, nonwoven material composed of quasi-straight, continuous Nylon 66 fibers, with a high fraction of junction bonds at fiber cross-overs, has been chosen for study. The lightest-weight Cerex (0.01 kg/m²), shown in Fig. 1-1, with the least complicated planar structure, is particularly suitable for a first step model. Although it does not concern itself with the many complexities of heavier-weight material through more involved topology, the results of this study nevertheless point out the relative importance of some structural parameters that control the flexibility of nonwovens. As Fig. 1-1 indicates clearly, on a scale larger than the distance between cross-overs of fibers (free fiber length), the density of the fabric is not uniform. Scanning electron microscopy has described, as shown in Fig. 1-2, that the fabric is almost entirely two-dimensional, has a micrometer thickness of only two fiber diameters, and that there are junction bonds at nearly every fiber cross-over, of varying area. The fabric has considerable anisotropy as found in Fig. 1-1, with the machine direction being outlined with a higher density of fibers. Although the effects of such anisotropy has been considered, they are not of profound
Fig. 1-1 Two light micrographs showing variations of areal densities of a light spunbonded nonwoven. The bars on the micrographs represent a length of 1 mm.

Fig. 1-2 Micrograph showing size of junctions in the fiber network such as that of Fig. 1-1. We are grateful to Dr. R. W. Dent of Monsanto Co. for this micrograph. The bar on the micrograph represents a length of 100 μm.
importance and will be ignored here in favor of a simpler isotropic model. The histogram of the free fiber length between junctions shown in Fig. 1-3 illustrates that the linear density of junctions on fibers is large enough to contribute significantly to the overall fabric bending stiffness.

1.1.2 Physical Model for the Bent Fabric

Exploratory experiments on bending of the fabric in the scanning electron microscope (SEM) pointed out that no important change took place in the crimp of the fibers in the bent fabric and that they remained substantially within the bent surface. This justified a continuum representation for even this lightest-weight material. Since the fabric demonstrated important variations in uniformity from point to point, it was decided for experimental and theoretical ease to represent it as an assembly of through-thickness disk-shaped heterogeneities as illustrated in Fig. 1-4. The diameter (4.75 mm) of these disks is taken to be large on the scale of the free fiber length but small on the scale of the wave length of the density variations shown in Fig. 1-1. The individual disks themselves were considered, on a small scale, to be made up of a "matrix" phase of a single layer of free fibers as a continuum, containing disk-shaped, stiff, through-thickness heterogeneities representing the junctions. In this manner, viewed at two levels of heterogeneity, the stiffness of the individual disks could be related to the local density of fibers and junctions, and the large scale stiffness
Fig. 1-3 Histogram of free fiber length $L$ from measurements on 32 disks.

Fig. 1-4 Part of disk as a heterogeneous plate of free fiber "matrix" and junctions also showing co-ordinate axes and applied moments. Micrograph showing a disk of 4.75 mm diameter.
variations in the fabric could be perceived as a result of a particular spectral distribution of measured or computed stiffnesses of individual disks. As already mentioned earlier, fabric is treated here as locally isotropic in fiber orientation distribution. At both the level of individual disks, and at the fabric level of a large collection of disks, the stiffness of a plate is determined by a self-consistent model adapted to bending.

1.1.3 Self-Consistent Model

The estimation of the overall mechanical properties, elastic or inelastic, of composite materials or polycrystals has long been of technological interest. The essential issue is, in general, how the properties of the separate phases (or crystals for polycrystal) and their volume fractions, dimensions, and shapes determine the properties of the overall composite. Usually the properties of a precise distribution of individual phases cannot be easily determined. Even if it is identified, an exact calculation of the macroscopic properties is still very difficult. Hence, various approximate approaches have been proposed to calculate composite properties. Among these the self-consistent model (SCM), which was conceived to consider the effect of randomly dispersed constituents of a composite, has many advantages over other methods.

For convenience, a composite material is taken here as a heterogeneous mixture of two or more phases with at least one continuous solid phase such that the material can be considered as solid. Generally the
material comprises a matrix phase in which other phase(s) in the form of inclusions is(are) embedded. In the extreme of a vanishing matrix, the material is a polycrystal which can be treated as a special case of a composite material.

The concepts of SCM were introduced independently by Hershey\(^8\) and Kröner\(^9\) in their analyses of the elastic moduli of polycrystalline aggregates. Basically, the elasticity of a polycrystalline aggregate is expressible in terms of the elasticity of individual grains. The overall stress, or strain, of the polycrystal is a volume average over stresses, or strains, in individual grains. For self-consistency, the overall elastic moduli of the overall material must satisfy the relation between overall stress and strain that are determined from estimated stresses and strains of individual grains. Following similar methods, Kröner,\(^10\) Budiansky and Wu,\(^11\) and Hutchinson\(^12\) continued researches on the plastic deformation problem of polycrystals. It was perhaps Hill\(^13,14\) who justified the term "self-consistent method" and laid the foundation of the method.

Basically, when a composite is uniformly loaded, SCM approximates the stress or the strain state within an inclusion after the entire surroundings of the inclusion are substituted by a matrix whose moduli coincide with the overall moduli. This situation produces an auxiliary problem: a heterogeneous inclusion embedded in otherwise homogeneous surroundings is subject to a uniform remote stress. SCM possibly would not have advanced too far from here had it not been for the famous
Eshelby solution\textsuperscript{15,16} to the problem of an ellipsoidal inclusion. The central feature of Eshelby's results is that the stress or the strain field within an ellipsoidal inclusion embedded in a uniform distant field is uniform, but not necessarily with principal axes parallel to those of the distant uniform field. Although a formulation of the solution for inclusion with arbitrary shape can be derived (e.g., Kinoshita and Mura\textsuperscript{17}) and in very few cases, results can be found [e.g., Eq. (2.26) of Eshelby\textsuperscript{16}], at present, a reasonable assumption of ellipsoidal inclusions is still necessary for a possible calculation of the elastic moduli of three-dimensional composite systems. For a heterogeneous plate with through-thickness inclusions subject to bending, the stress and the moment distribution in the inclusion can usually be solved from the plate theory for circular or elliptic\textsuperscript{18} inclusions.

To establish a generalized self-consistent theory for a composite material, a two-phase composite with distinct matrix and inclusion phases is considered here. The inclusion phase with inclusions of the same shape and orientation, though not necessarily the same size, is randomly distributed in the matrix phase. Let the respective phase properties be distinguished by subscript 1 for the matrix, and 2 for the inclusion, and let $C_1$ and $C_2$ be the fractional concentration such that

\[ C_1 + C_2 = 1. \]  \hspace{1cm} (1.1)

If $\sigma, \varepsilon, L, M$ represent generalized stress, strain, moduli, and
compliance tensors\(^1\) respectively, to constitute the relation between stress and strain in tension or that between bending moment and curvature in bending, the individual phases will have

\[
\sigma_1 = L_1\varepsilon_1 \tag{1.2}
\]

\[
\sigma_2 = L_2\varepsilon_2 \tag{1.3}
\]

For a uniform stress \(\tilde{\sigma}\) applied on the composite which produces uniform strain \(\tilde{\varepsilon}\), the tensor of effective moduli \(\tilde{L}\) of the composite is defined to be

\[
\tilde{\sigma} = \tilde{L}\tilde{\varepsilon} , \tag{1.4}
\]

and the effective compliance \(\tilde{M}\)

\[
\tilde{\varepsilon} = \tilde{M}\tilde{\sigma} . \tag{1.5}
\]

The tensors \(\tilde{\sigma}\) and \(\tilde{\varepsilon}\) are, however, the volume averages (or the area averages for the plate bending problem) over all \(\sigma_i, \varepsilon_i\) in the composite; i.e.,

\[
\tilde{\sigma} = \frac{1}{V} \int \sigma \, dV = \{\sigma\} \tag{1.6}
\]

\[
\tilde{\varepsilon} = \frac{1}{V} \int \varepsilon \, dV = \{\varepsilon\} . \tag{1.7}
\]

\(^1\)The tensor notation and symbolism given by Hutchinson\(^1\) of a self-consistent method for determining composite properties has been adopted here. In this set of symbols, a tilde (\(\sim\)) inscribed under a symbol signifies a tensor quantity, while a bar (\(\bar{\text{\_\_}}\)) inscribed above a symbol represents a composite average. In addition, local averages of a commonly variable field are represented by a hat (\(\hat{\text{\_\_}}\)), and all volume (or area) averages are given by braces \(\{\}\).
If "\( \wedge \)" stands for the average over the individual phases, \( \hat{\sigma}_1, \hat{\sigma}_2, \) and \( \hat{\varepsilon}_1, \hat{\varepsilon}_2 \) can be expressed as

\[
\hat{\sigma}_1 = \hat{B}_1 \bar{\sigma} \quad \hat{\sigma}_2 = \hat{B}_2 \bar{\sigma} \quad (1.8)
\]

\[
\hat{\varepsilon}_1 = \hat{A}_1 \bar{\varepsilon} \quad \hat{\varepsilon}_2 = \hat{A}_2 \bar{\varepsilon} \quad (1.9)
\]

where \( \hat{B}_i, \hat{A}_i \) are defined respectively as the stress and the strain concentration tensors for the \( i \)th phase. Equations (1.6) and (1.7) can be rewritten as

\[
\bar{\sigma} = c_1 \hat{\sigma}_1 + c_2 \hat{\sigma}_2 = c_1 \hat{B}_1 \bar{\sigma} + c_2 \hat{B}_2 \bar{\sigma} \quad (1.10)
\]

\[
\bar{\varepsilon} = c_1 \hat{\varepsilon}_1 + c_2 \hat{\varepsilon}_2 = c_1 \hat{A}_1 \bar{\varepsilon} + c_2 \hat{A}_2 \bar{\varepsilon} \quad (1.11)
\]

These relations can also be reduced to

\[
\bar{I} = c_1 \hat{A}_1 + c_2 \hat{A}_2 \quad (1.12)
\]

\[
\bar{\bar{I}} = c_1 \hat{B}_1 + c_2 \hat{B}_2 \quad (1.13)
\]

where \( \bar{I} \) is the identity matrix. From Eq. (1.4), a relation is found with the aid of the above equations:

\[
\bar{\bar{\lambda}} \bar{\varepsilon} = \bar{\sigma} = c_1 \hat{\sigma}_1 + c_2 \hat{\sigma}_2 = c_1 \hat{L}_1 \bar{\varepsilon}_1 + c_2 \hat{L}_2 \bar{\varepsilon}_2 = (c_1 \hat{L}_1 \hat{A}_1 + c_2 \hat{L}_2 \hat{A}_2) \bar{\varepsilon} \quad (1.14)
\]

It can be concluded that

\[
\bar{\lambda} = c_1 \hat{L}_1 \hat{A}_1 + c_2 \hat{L}_2 \hat{A}_2 = \{ L_i \hat{A}_i \} \quad (1.15)
\]

and similarly,
\[
M = c_1 M_1 \hat{B}_1 + c_2 M_2 \hat{B}_2 = \{ M_i \hat{B}_i \}. \tag{1.16}
\]

According to Hill, Budiansky, and Walpole, when all the inclusions in a two-phase composite are of the same shape and the same alignment, \( L \) and \( M \) can be determined without differentiating the inclusion phase from the matrix phase. In other words, either phase can be taken as having the same shape and orientation and the individual \( \hat{A}_i, \hat{B}_i \) can be solved from the auxiliary problem.

Equations (1.15) and (1.16) can be further reduced to

\[
L = c_1 L_1 \hat{A}_1 + c_2 L_2 \hat{A}_2 + c_2 L_1 \hat{A}_2 - c_2 L_1 \hat{A}_2
= L_1 (c_1 \hat{A}_1 + c_2 \hat{A}_2) + c_2 (L_2 - L_1) \hat{A}_2
= L_1 + c_2 (L_2 - L_1) \hat{A}_2 \tag{1.17}
\]

and

\[
M = M_1 + c_1 (M_2 - M_1) \hat{B}_2. \tag{1.18}
\]

Likewise, the generalized N-phase composite with phase 1 as matrix will have \( L \) and \( M \) as follows:

\[
L = L_1 + \sum_{i=2}^{N} c_i (L_i \hat{A}_i - L_1 \hat{A}_1) \tag{1.19}
\]

\[
M = M_1 + \sum_{i=2}^{N} c_i (M_i \hat{B}_i - M_1 \hat{B}_1). \tag{1.20}
\]

Or for a polycrystalline material, a composite without matrix phase, phase 1, \( L \) and \( M \) are found as
-28-

\[ L \sim = \sum_{i=2}^{N} c_i L_{\sim i} \hat{A}_{i} \quad (1.21) \]

\[ M \sim = \sum_{i=2}^{N} c_i M_{\sim i} \hat{B}_{i} \quad . \quad (1.22) \]

In general,

\[ L \sim = \{ L_{\sim i} \hat{A}_{i} \} \quad (1.23) \]

\[ M \sim = \{ M_{\sim i} \hat{B}_{i} \} \quad . \quad (1.24) \]

1.1.4 Self-Consistent Model for Bending of a Plate

In relation to a bent plate a moment tensor can be defined

\[ \mathbf{\mu} = \left[ \begin{array}{c} g_x \\ g_y \\ h_x \\ h_y \end{array} \right] \quad , \quad (1.25) \]

where \( g_x, g_y \) are edge bending moments per unit length; \( h_x, h_y \) are edge twisting moments per unit length acting on any section of plate parallel to the \( xz \) and \( yz \) planes, as seen in Fig. 1.4, while the plate is in the \( xy \) plane. In addition, a curvature tensor is also defined:

\[ \mathbf{\kappa} = \left[ \begin{array}{c} \frac{\partial^2 w}{\partial x^2} \\ \frac{\partial^2 w}{\partial y^2} \\ \frac{\partial^2 w}{\partial x \partial y} \\ \frac{\partial^2 w}{\partial y \partial x} \end{array} \right] \quad (1.26) \]
where $w$ is the deflection of the plate in the $z$-direction. The moment and curvature tensors are related to each other by a flexibility tensor $M$:

$$\kappa = M_{ij},$$  \quad (1.27)

where

$$M = \begin{bmatrix}
-\frac{1}{D(1-v^2)} & \frac{\nu}{D(1-v^2)} & 0 & 0 \\
\frac{\nu}{D(1-v^2)} & -\frac{1}{D(1-v^2)} & 0 & 0 \\
0 & 0 & \frac{1}{D(1-v)} & 0 \\
0 & 0 & 0 & -\frac{1}{D(1-v)}
\end{bmatrix}$$  \quad (1.28)

where $D$ is the bending stiffness of the plate and $\nu$ is its Poisson's ratio. In the self-consistent model, each element of the composite is regarded as a through-thickness inclusion in a uniform plate having the stiffness properties of the overall composite plate that is sought. For the $i$th element of included heterogeneity we have individually

$$\kappa^i = M^i_{ij} \kappa^i,$$  \quad (1.29)

$$M^i = M^i(D^i, V^i).$$  \quad (1.30)

Overall "average" elements of curvature of the composite plate are area averages over all elements in the plate, i.e.,

$$\kappa^i = \frac{1}{A} \int \frac{\partial^2 w}{\partial x^2} \, dA \equiv \left\{ \frac{\partial^2 w}{\partial x^2} \right\},$$  \quad etc.  \quad (1.31)
Therefore,

\[ \tilde{\kappa} = \{ \tilde{\kappa}^i \} . \] (1.32)

Similarly, equilibrium requires that

\[ \tilde{\mu} = \{ \tilde{\mu}^i \} . \] (1.33)

The overall average flexibility tensor \( \tilde{M} \) is defined as

\[ \tilde{\kappa} = \tilde{M} \tilde{\mu} \] (1.34)

\[ \tilde{M} = \tilde{M}(\tilde{D}, \tilde{\nu}) . \] (1.35)

From the solution of a boundary value problem of an included disk with stiffness \( D^i \) in an infinite plate having average composite stiffness \( \tilde{D} \) and being subjected to an external moment \( \tilde{\mu} \), the local edge moment distribution \( \tilde{\mu}^i \) on the \( i \)th heterogeneity are assumed determinable\(^{18} \)

and representable as

\[ \tilde{\mu}^i = B^i \tilde{\mu} \] (1.36)

\[ B^i = B^i(D^i, \nu^i, \tilde{D}, \tilde{\nu}) . \] (1.37)

Unlike the case of ellipsoidal inclusions in a uniform distant field solved by Eshelby,\(^{15,16} \) where the local field is constant, in the case of the bent plate \( \tilde{\mu}^i \) will vary over the boundary of the heterogeneity.\(^{18} \)

If "\( \tilde{\cdot} \)" stands for the average just of the heterogeneity, then

\[ \hat{\mu}^i = \hat{B}^i \tilde{\mu} \] (1.38)

Also from Eq. (1.29) it can be written as

\[ \hat{\kappa}^i = \hat{M}^i \hat{\mu}^i . \] (1.39)
Finally, by manipulating the above equations, a relation is reached:

\[ \bar{\kappa} = \{ \hat{\kappa} \} = \{ \hat{\kappa} \} = \{ \hat{M}^i \bar{\mu}^i \} = \{ \hat{M}^i \bar{B}^i \bar{\mu} \} = \{ M^i \bar{B}^i \} \bar{\mu} = M \bar{\mu} \]. \hspace{1cm} (1.40)

By comparing the last two terms in Eq. (1.40), it is concluded that

\[ \bar{M} = \{ M^i \bar{B}^i \} \]. \hspace{1cm} (1.41)

For operational ease, the junctions and free fiber "matrix" all are considered in the form of circular heterogeneities. Also in self-consistent models, each inclusion is regarded as lying in a matrix which already has bending stiffness of overall value. The moment distribution on the edge of circular inclusions in a plate under a uniform distribution of edge moments \( M \bar{\mu} \) at a distance are obtained from Goland. For a plate subjected to pure bending at a distance, it can be written as

\[ \bar{\mu} = \begin{bmatrix} -G(\bar{v}^2 - \bar{v}) \\ G(1-\bar{v}^2) \\ 0 \\ 0 \end{bmatrix} \], \hspace{1cm} (1.42)

where \( G \) is the edge moment length. The effective Poisson's ratio of the fabric in bending is chosen to be zero because of its open fibrous structure. Hence, it can be found

\[ \bar{\mu} = \begin{bmatrix} 0 \\ G \\ 0 \\ 0 \end{bmatrix} \]. \hspace{1cm} (1.43)
In the inclusion, 

\[
\begin{align*}
{\hat{\mu}_c}^{-1} &= \begin{bmatrix} G^c_c & \omega^c \omega \end{bmatrix} = \hat{B}^c_{\sim\sim}^{-1} \\
\hat{B}^c_{\sim\sim} &= \begin{bmatrix} \hat{B}_{11} & \hat{B}_{12} & \hat{B}_{13} & \hat{B}_{14} \\
\hat{B}_{21} & \hat{B}_{22} & \hat{B}_{23} & \hat{B}_{23} \\
\hat{B}_{31} & \hat{B}_{32} & \hat{B}_{33} & \hat{B}_{34} \\
\hat{B}_{41} & \hat{B}_{42} & \hat{B}_{43} & \hat{B}_{44} \end{bmatrix};
\end{align*}
\]  

of all the elements of this matrix only the following are necessary: 

\[
\begin{align*}
\hat{B}_{12} &= \frac{G_1^c}{G} & \hat{B}_{22} &= \frac{G_2^c}{G} \\
\hat{B}_{32} &= 0 & \hat{B}_{42} &= 0 .
\end{align*}
\]  

All other \( \hat{B}_{ij} \)'s are not needed for the present computation. Since 

\[
\tilde{\mathbf{M}} = \{ M^c \hat{B}^c \},
\]

\[
\begin{bmatrix}
-1/\bar{D} & 0 & 0 & 0 \\
0 & -1/\bar{D} & 0 & 0 \\
0 & 0 & 1/\bar{D} & 0 \\
0 & 0 & 0 & -1/\bar{D}
\end{bmatrix}
\]

and

\[
\begin{bmatrix}
\frac{-1}{D_c(1-\nu_c^2)} & \frac{\nu_c}{D_c(1-\nu_c^2)} & 0 & 0 \\
\frac{\nu_c}{D_c(1-\nu_c^2)} & -\frac{1}{D_c(1-\nu_c^2)} & 0 & 0 \\
0 & 0 & \frac{1}{D_c(1-\nu_c)} & 0 \\
0 & 0 & 0 & -\frac{1}{D_c(1-\nu_c)}
\end{bmatrix}
\]

\[
\begin{bmatrix}
\hat{B}_{11} & \hat{B}_{12} & \hat{B}_{13} & \hat{B}_{14} \\
\hat{B}_{21} & \hat{B}_{22} & \hat{B}_{23} & \hat{B}_{24} \\
\hat{B}_{31} & \hat{B}_{32} & \hat{B}_{33} & \hat{B}_{34} \\
\hat{B}_{41} & \hat{B}_{42} & \hat{B}_{43} & \hat{B}_{44}
\end{bmatrix}
\]  

(1.48)
Concentrating on the second term in the diagonal of \( \tilde{M} \) gives

\[
- \frac{1}{D} = \left\{ \frac{\nu_c}{D_c(1 - \nu_c^2)} \hat{B}_{12} - \frac{1}{D_c(1 - \nu_c^2)} \hat{B}_{22} \right\} .
\]  

(1.49)

At the disk level, based on Fig. 1-2, the junctions are taken as cylinders of diameter \( d_J \) and a height of 2 fiber diameters, and are modeled in the same height but appropriately larger modulus. In view of the varying junction diameters shown in Fig. 1-2, several junction diameters ranging from one fiber diameter to \( \sqrt{2} \) times a fiber diameter have been covered in the computations. The number of junctions in a disk of the chosen diameter of 4.75 mm is decided by measuring the disk weight \( W \) in a microbalance and the average free fiber length \( L \) in the disk directly through a microscope. Then, if \( \rho \) is the weight of fiber per unit length, the total fiber length \( L \) contained in a disk is

\[
L = \frac{W}{\rho} .
\]

(1.50)

The total number of junctions \( n_J \) in a disk, the total projected area \( A_J \) of junctions appearing in a disk, and the area fractions \( C_J \) of junctions in a disk are

\[
n_J = \frac{\rho}{2L} = \frac{W}{2\rho L} \]

(1.51)

\[
A_J = \frac{\pi d_J^2}{4} \left( \frac{W}{2\rho L} \right) = \frac{\pi d_J^2}{8\rho} \left( \frac{W}{L} \right) \]

(1.52)

\[
C_J = \frac{A_J}{A} = \frac{\pi d_J^2}{8A\rho} \left( \frac{W}{L} \right) ,
\]

(1.53)
where $A$ is the area of the disk. The specific bending stiffness of each junction is taken from the junction modeled as if it were a plate with a thickness twice the fiber diameter, i.e.,

$$D_J = \frac{E(2d)^3}{12(1-\nu^2)}, \quad (1.54)$$

where $E$ is the Young's modulus of the fiber, $\nu$ its Poisson's ratio, and $d$ its diameter. As discussed already, the junction of this stiffness is then considered as a disk of the same thickness as that of the free fiber matrix. The specific bending stiffness $D_F$ of the reference free fiber "continuum" is obtained from Freeston and Platt's\(^2\) model as

$$D_F = \frac{1}{8} N_f E \left( \frac{\pi d^4}{64} \right) \left( 3 + \frac{1}{1+\nu} \right) \approx \frac{E}{8L} \left( \frac{\pi d^4}{64} \right) \left( 3 + \frac{1}{1+\nu} \right), \quad (1.55)$$

where $N_f$, of dimension (length\(^{-1}\)), is the total number of fibers in a circle with unit length diameter (see Appendix A). Although the Poisson's ratio $\nu_F$ of the free fiber matrix is taken to be zero, that of the junctions $\nu_J$ is taken to be equal to the fiber material. Hence, from Eq. (1.49) the reciprocal of the disk stiffness is found to be

$$\frac{1}{D} = C_J \left[ -\frac{\nu_J}{D_J(1-\nu_J^2)} \hat{B}_{12}(D_J,\bar{D},\nu_J) + \frac{1}{D_J(1-\nu_J^2)} \hat{B}_{22}(D_J,\bar{D},\nu_J) \right] + (1 - C_J) \frac{1}{D_F} \hat{B}_{22}(D_F,\bar{D}). \quad (1.56)$$
With this equation, the overall bending stiffness $\tilde{D}$ can be solved. A computer program was developed to carry out an iteration scheme for calculating $\tilde{D}$ which consists of estimating a value of $\tilde{D}$ for insertion in the right-hand side of Eq. (1.56) to determine a better value through the use of Eq. (1.56), which is then taken a second iteration, and so on, until the values in succeeding iterations coincide. Similarly, the overall bending stiffness of the fabric judged as an assembly of many disks of graded stiffness can also be evaluated from the stiffnesses of these individual disks by the self-consistent model, i.e.,

$$\frac{1}{D_{\text{fabric}}} = \left\{ \frac{1}{D_{\text{disk}}} \right\} \tilde{D} $$

where it is assumed that the Poisson's ratios of the fabric and the individual disks are all zero.

1.1.5 Upper and Lower Bound Models, and a Bulbous Free Fiber Model

For the purpose of comparison, upper bound and lower bound models have also been computed for the disk stiffnesses. In the lower bound model the bending moment is taken to be constant in the composite plate while in the upper bound model the curvature is taken to be constant. These models also amount to alternatively grouping the stiff and flexible phases in series and in parallel, respectively. Thus, taking the two phases as plates of constant curvature in parallel, the upper bound to the disk stiffness is obtained as
\[ \tilde{D} = C_J D_J + (1 - C_J) D_F . \] (1.58)

Taking the two phases as plates in series sharing the same bending moment, the lower bound to the disk stiffness is found as

\[ \frac{1}{\tilde{D}} = \frac{C_J}{D_J} + \frac{(1 - C_J)}{D_F} . \] (1.59)

Note that these two models are completely analogous to those given by Paul, who dealt with the tensile moduli of the composite material.

As a third estimate, Freeston and Platt's free fiber model is modified to consider the fibers and junctions as a set of bulbous free fibers where the free fiber population is taken as stretches of regular fiber and the junctions as stretches of stiffer fiber. It should be noted that junctions are specifically assumed to be equally spaced along each fiber at a distance \( L \), equal to the free fiber length, and that every junction represented as a length equal to a fiber diameter \( d \) has a stiffness \( \alpha \) times that of the free fiber. Thus, every free fiber section between junctions would become \( (L - d) \) long. And if the bending stiffness of free fiber and junctions are \( D_{ff} \) and \( D_J \), respectively, and \( D_J = \alpha D_{ff} \), the equivalent bending stiffness \( D_{eq} \) of the bulbous fiber becomes

\[ \frac{1}{D_{eq}} = \frac{1}{D_{ff}} \left[ 1 + \left( \frac{1}{\alpha} - 1 \right) \left( \frac{d}{L} \right) \right], \] (1.60)

where

\[ D_{ff} = E \left( \frac{\pi d^4}{64} \right) . \] (1.61)
The disk stiffness, therefore, is a certain multiple of that of the free fiber model given by Eq. (1.55).

1.1.6 Experiments

(1) Disks of 4.75 mm diameter were punched out randomly from a large section of the fabric. The weight \( W \) of the individual disks was obtained with a microbalance. Carefully, the average free fiber length \( L \) of each disk was measured directly with a microscope. In measuring \( L \), at least 200-300 junctions for every disk were counted. The histograms of \( W \) and \( L \) (for all disks) are given in Figs. 1-3 and 1-5, respectively.

(2) The bending stiffnesses of individual disks were determined by measuring the center deflection of the clamped disks under uniform air pressure on a special microscope stage device, as shown in Fig. 1-6. The functions of the major components of this device are described as follows.

(a) Specimen Holder. The specimen holder, made of a cylindrical block of acrylrite, was employed to accommodate the clamped disk set. One hole was drilled axially at the center of the holder to join the other hole drilled radially inward from the periphery of the cylinder to form an L-shaped tunnel. The tunnel provided a passage to the vacuum system, which induced a uniform air pressure on the top of the disk.
FIGURE 1-5. Histogram of disk weight $W$ from measurements on 32 disks.
FIGURE 1-6. Illustration of the special microscope stage device for bending test.
(b) **Clamped Disk Set.** The disks were covered by a very thin film of Parlodion to support the applied air pressure. A 5% Parlodion solution in butyl acetate was spread thinly on a microscope slide. The edge of another slide was dragged over the fluid with slight pressure to remove excess fluid to form reproducible, very thin layers of Parlodion film upon drying. Very cautiously, this film was removed from the glass slide by first applying, then tearing off, a piece of Scotch tape with a pre-punched hole. The part of the film that covered the hole on the tape remained free and was used to cover the disk specimen. The thin films were estimated to be about 1 to 2 \( \mu \text{m} \) in thickness. Their contributions to bending were considered negligible when the disks were bent.

Two thin steel clamping rings with the same ID of 3.56 mm and OD of 6.38 mm were used to clamp the disk specimen and the thin film to form a clamped disk set. Two pieces of double-stick Scotch tape in the shape of a ring were applied to fasten both faces of the disk edge between the rings. The set was then put at the center hole of the specimen holder. Another piece of Scotch tape was placed on the ring and the holder to cover the gap between them while revealing the disk through a circular hole (4.75 mm in diameter) on the tape.

(c) **Vacuum System.** The vacuum system, shown in Fig. 1-7, produced a transverse pressure difference on the disk. Gas was supplied from a bottled compressed gas to a venturi nozzle which would induce a slight vacuum at its neck. A tube was then connected from there to the specimen holder to complete the system. The pressure was controlled by a
FIGURE 1-7. Illustration of bending tests equipment.
regulator together with a pressure gage. In this experiment, the maximum pressure was about $1.24 \times 10^2$ N/m$^2$.

(3) The center deflections of disks corresponding to different levels of transverse air pressure were measured by means of the fine focusing micrometer on a Zeiss Ultraphoto Microscope. The bending stiffnesses of individual disks were then computed from the initial rate of change of pressure with center deflection.

$$D = \frac{R^4}{64} \left( \frac{\Delta p}{\Delta w} \right), \quad (1.62)$$

where $R$ is the radius of the disk between clamps, $\Delta p$ is the pressure change producing the change $\Delta w$ in the deflection of the center of the disk.

(4) The bending stiffness of the overall fabric was calculated from a cantilever beam sample sagging under its own weight. The radius of curvature $r$ of a 2.5 cm wide strip of fabric hung as a cantilever beam in vacuum was measured near the clamped end. The bending moment $M_s$ on the strip is then obtained from the dimensions and the average areal density (0.01 kg/m$^2$) of the fabric strip. The bending stiffness of the strip is then

$$D = M_s r. \quad (1.63)$$

Figures 1-8 and 1-9 show bending stiffnesses of the fabric estimated by this method in the longitudinal and the transverse directions.
FIGURE 1-9. Cantilever beam tests for specimens in the transverse direction of the fabric.
1.1.7 Comparison of Theoretical and Experimental Results

Figure 1-10 shows the experimental results of the measured individual disk stiffnesses $D$ plotted as a function of $W/L$, which is a direct measure of the areal density of junctions as can be seen from Eq. (1.51).

Considering various junction sizes between zero and $\sqrt{2}d$, several sets of disk stiffnesses were computed by means of the self-consistent model. Theoretical values of $D$ were computed for each disk by the self-consistent model, the upper bound model and the lower bound model for two junction diameters $d_J = d$ and $d_J = \sqrt{2}d$, and are shown in Figs. 1-11 and 1-12, respectively. When $d_J = 0$, all junctions vanish; the self-consistent model gives the same result as that of Freeston and Platt for which computed values are shown in Fig. 1-12.

The experimental values lie between the two sets of values computed by the self-consistent model for $d_J = d$ and $d_J = \sqrt{2}d$. Freeston and Platt's free fiber model, overlooking effects of junctions, gives a rather low estimate. The upper bound model, not unexpectedly, gives rather high estimates for the disk stiffnesses in comparison with the experimental data. The lower bound model gives disk stiffnesses well below those actually measured, but closer to them than those of the upper bound model.

Figure 1-13 gives the results of the modified free fiber model that considers bulbous fibers. It indicates that for infinitely stiff junctions ($D_J = \infty$), this model gives bending stiffnesses of disks that
Experiments on Individual Disks

FIGURE 1-10. Experimentally measured bending stiffnesses D vs. W/L.
FIGURE 1-11. Bending stiffnesses $D$ computed from theoretical models with $d_j = d$. 
FIGURE 1-12. Bending stiffnesses $D$ computed from theoretical models with $d_j = \sqrt{2}d$, and from Freeston and Platt's free fiber model with $d_j = 0$. 

- $d_j = d$
- Self-Consistent Model
- Upper Bound Model
- Lower Bound Model

$D$, Bending Stiffness (N·m)

W/L, Disk Weight/Disk Free Fiber Length (µg/µm)

Average Experiments

$2.5 \times 10^6$
FIGURE 1-13. Average lines of bending stiffnesses for different $\alpha$'s calculated from modified free fiber model.
are close to, but still smaller than, the experimental values. When \( D_j \) equals four to eight times the fiber stiffness, which was what was taken for the junctions in the self-consistent composite plate model, the estimates are generally smaller than the measured values shown in Fig. 1-10.

Table 1-1 lists the various fabric stiffnesses computed by employing the self-consistent model in which either the experimentally measured or theoretically computed stiffnesses of the individual disks were used, together with the fabric stiffness measured by the cantilver beam method. It should be noted here that actual measurements at the fabric level reveal the anisotropic nature of the material which had been smoothed out in the analysis. The consideration of an "equivalent" isotropic material in the self-consistent model thus generates a fabric stiffness, from measured disk stiffnesses, that lies between the two extreme values measured.

1.1.8 Discussion

In what has been presented above, the significant role of junctions as "stiffness raisers" in a fabric seems clear. The presence of junctions has given a rational explanation for the difference between the bending stiffness obtained from experiments and the bending stiffness estimated from the free fiber model. The relatively high stiffnesses of junctions make them play an important role in the bending of the very thin fabric in spite of their relatively small areal fraction in
<table>
<thead>
<tr>
<th>Disk Level</th>
<th>Fabric Level</th>
<th>Fabric Bending Stiffness (10^7 N-m)</th>
<th>U.B.</th>
<th>L.B.</th>
<th>S.C.M.</th>
<th>S.C.M.</th>
<th>S.C.M.</th>
<th>S.C.M.</th>
<th>U.B.</th>
<th>L.B.</th>
<th>S.C.M.</th>
</tr>
</thead>
<tbody>
<tr>
<td>EXP</td>
<td>S.C.M.</td>
<td>6.61</td>
<td>5.51</td>
<td>3.86</td>
<td>4.74</td>
<td>6.82</td>
<td>23.60</td>
<td>5.32</td>
<td>2.48</td>
<td>10.73</td>
<td>Long</td>
</tr>
<tr>
<td>F.F.</td>
<td>S.C.M.</td>
<td>F.F.</td>
<td>Self-Consistent Model</td>
<td>L.B.</td>
<td>Lower Bound Model</td>
<td>EXP</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
the disk. The self-consistent model, among all the models discussed, gives the closest approximation to the bending stiffness of a nonwoven fabric. The effect of the interactions between the component phases of a composite material, as a nonwoven fabric has been modeled, can be best accounted for by the self-consistent model. The outlined approach provides an accurate method for the evaluation of the bending stiffnesses, at least for very light nonwovens, from simple and measurable parameters. Only the disk weight $W$, which is a measure of the local areal density, and the disk free fiber length $L$ are required in all calculations.
CHAPTER 1:
SPUNBONDED (SELF-BONDED) NONWOVEN FABRICS

PART 2: THICK FABRIC

1.2.1 Description of the Materials

The relative fiber motion in Cerex is usually limited because bonds, or junctions, are formed almost at all contacts between fibers. For this reason, as the thickness of the self-bonded nonwoven is increased, the fabric resembles more closely a paper-like material. The fabrics studied here are heavier Cerex with rather complicated three-dimensional structures.

Two samples of Cerex, Fabric A and Fabric B, with basis weights 0.05 kg/m² (micrometer thickness 0.14 mm, about 7 fiber diameters) and 0.27 kg/m² (micrometer thickness 0.52 mm, about 26 fiber diameters), respectively, were chosen for study. These two fabrics are all composed of randomly distributed continuous filaments of Nylon 66. SEM observations have shown that most of the fibers in the fabrics are quite parallel to the plane of the fabric, resulting in layer structures. Almost all fibers appear quasi-straight with the average free fiber length estimated approximately to be only 3 to 5 fiber diameters. Although the preferred orientation of these thick fabrics cannot be clearly distinguished because of the high packing density of fibers, the anisotropic nature of the fabrics was indeed reflected through the results of bending and tension experiments. Each material has tensile modulus measured
in the machine direction about two times higher than that in the transverse direction. Similar results were obtained for the bending stiffnesses in these two principal directions. This probably suggests that the fibers oriented in the machine direction outnumber those in the transverse direction.

1.2.2 Physical Model for the Bent Fabrics

In building physical models for these thick materials, certain advantages of modeling a thin Cerex are missing. Most significantly, the free fiber matrix approximation introduced to thin material is not applicable here since the fibers in thick materials are subject to both tension and compression during bending. Moreover, because junctions in thick materials are usually shorter than the thickness, the concept of a plate with through-thickness heterogeneous inclusions representing junctions is no longer appropriate.

The model of no-freedom-of-relative-fiber-motion\(^2\) was found to serve Fabric B reasonably well in interpreting its bending response. Under pure bending, this material would possibly follow the condition that planes perpendicular to the fabric but parallel to the bending axis remain planar. The model, however, cannot very accurately describe the fiber assembly of Fabric A under bending. A new model has been proposed for it. Since its bending response is probably too difficult to be solved mathematically, this model could only be conceptually described. The actual evaluation of the model is therefore absent.
Nevertheless, the concepts mentioned here would possibly be helpful to the further exploration of the problem.

A. A Proposed Model

To consider the effect of a free fiber upon the fabric structure, fiber segments between centers of adjacent junctions are taken to be components which constitute the fiber network. The fabric viewed as a composite plate is comprised of these fiber segments together with empty space. The space encircles the fiber segments so as to reveal free surface on the fibers.

For the problem of a fabric subject to the remote uniform pure bending moment $M_0$, the strain energy $\varepsilon_0$ per unit area of this fabric imagined as a continuum plate is the sum total of the strain energies of all fiber segments per unit area of the fabric; i.e.,

$$
\varepsilon_0 = \sum_{i=1}^{n} V_i ,
$$

where $n$ is the number of the fiber segments per unit area of the fabric, and $V_i$ is the strain energy of the $i$th fiber segment including both bending and tension (or compression) energies. The strain energy density $\varepsilon_0$ of the fabric actually is

$$
\varepsilon_0 = \frac{1}{2} M_0 \kappa_0 ,
$$

where $\kappa_0$ is the average curvature of the bent fabric. Since the effective bending stiffness $D$ of the fabric is defined as
\[ D = \frac{M_0}{K_0} ; \]  
\( \varepsilon_0 \) can also be expressed as
\[ \varepsilon_0 = \frac{M_0^2}{2D} . \]  
Therefore, Eq. (1.64) can be written as
\[ \frac{M_0^2}{2D} = \sum_{i=1}^{n} V_i . \]  
Individual \( V_i \) have to be evaluated to solve \( D \) from this equation.

Every fiber segment can be viewed as being attached to its surrounding material only through its two ends. The surface of the segment is free from lateral traction. With a self-consistent way, \( V_i \) is thus obtained by considering the \( i \)th fiber segment still as a part of a continuous fiber. This fiber is embedded in a continuum with overall material properties, while the \( i \)th segment is not embedded but passes through a cavity (\( i \)th cavity) in the continuum matrix, illustrated in Fig. 1-14. The ratio between the volumes of the \( i \)th fiber segment and the \( i \)th cavity should be equal to that between the macroscopic volume fractions of fiber and empty space. This arrangement will maintain the stress-free state on the periphery of the \( i \)th fiber segment and in the meantime reflects that every segment is a portion of a continuous fiber. When the fabric is bent, ideally, the stress state in the \( i \)th fiber segment is thus solved to give the strain energy \( V_i \). Obviously, \( V_i \) is a function of \( D \) and the mechanical properties of the fiber, the length and the geometric arrangement of the \( i \)th segment,
FIGURE 1-14. Illustration of a continuous fiber embedded in an infinite plate with a portion of the fiber, \( i \)th fiber segment, passing through a cavity.
and the shape and size of the cavity. Therefore, Eq. (1.68) can be rewritten as

$$\frac{M_0^2}{2D} = \sum_{i=1}^{n} V_i$$

(M₀, D: parameters relevant to ith segment, cavity shape)

and D can be self-consistently determined.

Actually, this auxiliary problem of a continuous fiber embedded in an infinite plate with a portion of the fiber passing through a cavity is almost unsolvable, not to mention that some assumption has to be made on the shape of the cavity. At least for the time being, this model cannot be developed further, in spite of its reasonable descriptions.

B. The Model of No-Freedom-of-Relative-Fiber-Motion (MNF)

Proposed by Freeston and Platt,² the model of no-freedom-of-relative-fiber-motion assumes that planes perpendicular to the fabric surface and parallel to the bending axis remain planar during pure bending. Consequently, the strain $\varepsilon$ in the direction perpendicular to the bending axis, or $\theta = 0$, in any fiber in the fabric is given by

$$\varepsilon = \frac{y}{r},$$

(1.70)

where $y$ is the distance of any point in a fiber from the midplane (neutral plane of bending) of the fabric and $r$ is the radius of curvature of the bent fabric. If a fiber is aligned in the direction of $\theta$, the strain $\varepsilon_f$ in that direction is
\[
\varepsilon_f = \frac{y \cos^2\theta}{r}.
\]

The axial force \( F_f \) in the fiber at a distance \( y \) from its center to the midplane of the fabric is

\[
F_f = \varepsilon_f (EA)_f = \frac{(EA)_f y \cos^2\theta}{r},
\]

(1.72)

where \( (EA)_f \) is Young's modulus multiplied by cross-sectional area of the fiber. The moment \( M_f \) of this force about the bending axis is

\[
M_f = F_f y \cos \theta = \frac{(EA)_f y^2 \cos^3\theta}{r}.
\]

(1.73)

Assuming equal fiber distribution throughout the thickness of the fabric, the average value of \( M_f \) for fibers in the \( \theta \)-direction is found as

\[
\bar{M}_f = \frac{1}{t} \int_{-t/2}^{t/2} M_f \, dy = \frac{(EA)_f}{tr} \int_{-t/2}^{t/2} y^2 \cos^3\theta \, dy.
\]

(1.74)

The moment \( M^\theta \) per unit width parallel to the bending axis contributed by fibers lying within \( \pm \frac{\Delta \theta}{2} \) of the angle \( \theta \) throughout the thickness of the fabric is

\[
M^\theta = N(\theta) \cos \theta \bar{M}_f
\]

(1.75)

\[
= \frac{N(\theta)(EA)_f}{tr} \int_{-t/2}^{t/2} y^2 \cos^4\theta \, dy.
\]

(1.76)
where \( N(\theta) \) is the number of these fibers per unit width perpendicular to \( \theta \), as shown in Appendix A. The sum of the moments per unit width parallel to the bending axis of all fibers about the bending axis is given by

\[
M = \sum M^\theta = \sum \frac{N(\theta)(EA)_f t}{2 \Delta \theta} \int_{-t/2}^{t/2} y^2 \cos^4 \theta \, dy \, \Delta \theta . \quad (1.77)
\]

In the limit of \( \Delta \theta \to 0 \), from Eq. (A.7) of Appendix A, the above equation becomes

\[
M = \frac{(EA)_f}{2 \Delta \theta} \int_{-\pi/2}^{\pi/2} \int_{-\pi/2}^{\pi/2} \frac{dN(\theta)}{d\theta} y^2 \cos^4 \theta \, d\theta \quad (1.78)
\]

\[
= \frac{(EA)_f}{2 \Delta \theta} \int_{-\pi/2}^{\pi/2} \int_{-\pi/2}^{\pi/2} N_f \phi(\theta) y^2 \cos^4 \theta \, d\theta \quad (1.79)
\]

\[
= \frac{N_f(EA)_f t^2}{12r} \int_{-\pi/2}^{\pi/2} \phi(\theta) \cos^4 \theta \, d\theta , \quad (1.80)
\]

where \( N_f \) is the total number of fibers through the thickness of the fabric in a circle of unit diameter, and \( \phi(\theta) \) is the fiber orientation distribution function as shown in Appendix A. The bending stiffness \( D \) of the fabric is therefore given by
D = \frac{N_f(EA) t^2}{12} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \phi(\theta) \cos^4 \theta \, d\theta \quad \text{(1.81)}

Since \( N_f \) also represents the length of the fibers per unit area, as shown in Appendix A, it can be written as

\[ N_f = \frac{W'}{\rho} \quad \text{(1.82)} \]

where \( W' \) is the weight per unit area of the fabric and \( \rho \) the weight of fiber per unit length. The bending stiffness \( D \) in Eq. (1.81) can be rewritten as

\[ D = \frac{W'(EA) t^2}{12 \rho} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \phi(\theta) \cos^4 \theta \, d\theta \quad \text{(1.83)} \]

For an isotropic material with \( \phi(\theta) = 1/\pi \), the bending stiffness \( D \) is then

\[ D = \frac{N_f(EA) t^2}{32} \quad \text{(1.84)} \]

\[ = \frac{W'(EA) t^2}{32 \rho} \quad \text{(1.85)} \]

C. The Relation Between the Fiber Web Theory and MNF

With the fiber web theory developed by Petterson,\(^2^8\) the modulus of the spunbonded nonwoven fabric can be predicted under the assumptions of straight fibers, with rigid bonds between fibers, and without fiber buckling. Petterson has applied this theory in his studies successfully to predict the tensile responses of the bonded fabrics. For small strain
and negligible Poisson's ratio, the tensile modulus $E$ in the direction of $\theta = 0$ is given by

$$E = \frac{N_f(EA)_f \pi/2}{t} \int_{-\pi/2}^{\pi/2} \phi(\theta) \cos^4 \theta \, d\theta.$$  

(1.86)

The elastic modulus $E$ can be related to $D$, the bending stiffness in Eq. (1.84), by

$$D = E \left( \frac{t^3}{12} \right),$$  

(1.87)

where $t^3/12$ is the moment of inertia per unit width of the fabric.

In other words, the bending stiffness obtained from the model of no-freedom-of-relative-fiber-motion is exactly equal to tensile modulus--found from the fiber web theory--multiplied by the moment of inertia of the fabric. This coincides with the relation between the bending stiffness and the tensile modulus of a continuum plate, and indicates that the behavior of the very thick fabric approaches that of a continuum.

1.2.3 Experiments

A. Fabric A

The microscope bending stage device employed to test the thin Cerex was adopted here again to measure the bending stiffnesses of disk specimens with diameter 4.75 mm. These specimens were punched out randomly from a large piece of fabric. Disk weights of these specimens were also weighed with a microbalance.
In addition, a MITEX bending tester, an IDR Company product, was employed to measure the bending stiffnesses of the strip samples. A top view of the machine, as shown in Fig. 1-15, displays two pairs of pins that are constructed to create a four-point loading system. The strip sample set between these sets of pins will be bent by the moment exerted through them. The portion of the sample between these two sets of pins can then be subject to pure bending. One such pair mounted on the moving carriage translates and rotates to produce bending. Meanwhile, it maintains a constant length of a circular arc of the sample. One pin in the other pair is attached to the tip of a cantilever spring that is connected to a transducer. The force in one of the couples applied on the sample can therefore be measured and converted to the bending moment on the center portion of the sample. The bending moment and the curvature can be plotted on an x-y recorder to determine the bending stiffness of the tested sample.

Several samples with width 20 mm and length of about 50 mm were cut out from the fabric in the machine direction and the transverse direction to be tested. The average bending stiffnesses were measured to be $D_m = 4.77 \times 10^{-5} \text{ N-m}$ in the machine direction and $D_t = 2.21 \times 10^{-5} \text{ N-m}$ in the transverse direction.

Another group of specimens obtained by cutting along the machine

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†For details on this kind of device, see Poper,25 and Poper and Backer.26
FIGURE 1-15. A top view of the pure bending tester.
and the transverse directions of the fabric were tensile tested on an Instron testing machine. The average values of Young's modulus multiplied by thickness $E_t$ were found to be $(E_t)_m = 4.26 \times 10^4 \text{ N/m}$ in the machine direction and $(E_t)_t = 2.00 \times 10^4 \text{ N/m}$ in the transverse direction.

B. Fabric B

On the bending machine, specimens with width 5 mm and length of about 50 mm were tested to find the bending stiffnesses to be $D_m = 5.45 \times 10^{-3} \text{ N-m}$ in the machine direction and $D_t = 3.09 \times 10^{-3} \text{ N-m}$ in the transverse direction.

Tensile tests performed on an Instron testing machine showed that the average values of Young's modulus multiplied by the fabric thickness, i.e., $E_t$, were $(E_t)_m = 2.28 \times 10^5 \text{ N/m}$ in the machine direction and $(E_t)_t = 1.13 \times 10^5 \text{ N/m}$ in the transverse direction.

1.2.4 Comparison of Experimental and Theoretical (MNF) Results

A. Fabric A

The bending stiffnesses of previously mentioned disks measured with a microscope bending stage device are plotted in Fig. 1-16 as a function of $W'$, the weight per unit area of individual disks. Note that $W'$ is proportional to $N_f$, as shown in Eq. (1.82). Since these measurements cannot reflect the anisotropy of the specimen, in order to make a compatible comparison, the theoretical bending stiffnesses from Eq. (1.85) were obtained by assuming isotropic fiber distribution in
FIGURE 1-16. Experimentally measured bending stiffnesses $D$ versus $W'$. 
the fabric. As a result of the linear relation between $D$ and $W'$ in Eq. (1.85), the calculated bending stiffnesses $D$ form a straight line, as shown in Fig. 1-16. The ratio of the theoretical bending stiffnesses to the experimental ones is about 2 to 1.

With the fiber web theory and the idealized assumption that the fiber distribution in this Cerex is isotropic, the value of $E_t$ was estimated to be approximately $E_t = 3.64 \times 10^4$ N/m. This value is found between the experimental values measured in the machine and the transverse directions, shown in Table 1-2. This probably suggests that the theory is able to describe well the tensile response of this fiber assembly if a proper $\phi(\theta)$, i.e., the fiber orientation distribution function, is available. Once this point is recognized, whether the fabric during pure bending acts like an assembly of no relative fiber motion can now be verified by checking whether the relation $D = E_t^2/12$ of Eq. (1.87) can be established between the measured bending stiffness $D$ and the measured $E_t$.

Given also in Table 1-2 are the average bending stiffnesses measured on the MITEX bend-testing machine and the values of measured $E_t$ multiplied by $t^2/12$. Clearly, the values of $E_t \times t^2/12$ are higher than the corresponding bending stiffnesses measured in different directions.

B. Fabric B

Calculated with Petterson's fiber web theory and the idealized assumption of isotropic fiber distribution, the value of $E_t$ was also
<table>
<thead>
<tr>
<th></th>
<th>Machine Direction</th>
<th>Transverse Direction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measured D (N·m)</td>
<td>4.77 × 10⁻⁵</td>
<td>2.21 × 10⁻⁵</td>
</tr>
<tr>
<td>Measured Et (t_2) (N·m)</td>
<td>6.96 × 10⁻⁵</td>
<td>3.27 × 10⁻⁵</td>
</tr>
<tr>
<td>Measured Et (N/m)</td>
<td>4.26 × 10⁴</td>
<td>2.00 × 10⁴</td>
</tr>
<tr>
<td>Calculated Et (N/m)</td>
<td>3.64 × 10⁴</td>
<td></td>
</tr>
</tbody>
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**TABLE 1.2.** Bending Stiffness of Fabric A
found to be between the tested values, shown in Table 1-3. The bending stiffnesses $D$, determined by experiments, together with the values of $(t^2/12)$ multiplied by measured $E_t$, are also listed in this table. Since both approaches yield fairly similar results, when under pure bending this extremely thick material seems to follow the condition of no relative fiber motion.

1.2.5 Discussion

It appears that the thicker the self-bonded nonwovens are, the more accurately the model of no-freedom-of-relative-fiber-motion describes the fabrics under pure bending. Fabric B behaves like a continuum plate, i.e., its bending stiffness equals the product of its tensile modulus and the moment of inertia of the fabric. However, as fabrics get thinner, their discrete structural characteristics are more pronounced.

It can be concluded that for a very thick self-bonded nonwoven, the mechanisms involved in bending and tension are alike. No freedom of relative fiber motion is, therefore, a rational description of its structural response. The medium-thick fabric clearly will allow some relative fiber motion, and therefore the fabric will be more flexible than the prediction of the plate theory.

Qualitatively speaking, the greater the number of fibers per unit area of fabric, the more effective the constraints between the fibers will be. Possibly, fibers near the surfaces of the fabric lack good
<table>
<thead>
<tr>
<th>Table 1-3: Bending Stiffness of Fabric B.</th>
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<tbody>
<tr>
<td>Et, Calculated From Fiber Web Theory (N/m)</td>
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<tr>
<td>------------------------------------------</td>
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<tr>
<td>1.90 $\times 10^5$</td>
</tr>
<tr>
<td></td>
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<tr>
<td>Transverse Direction</td>
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</tbody>
</table>
constraints. Therefore, they are likely to have some relative fiber motion when the fabric is bent. As the layer occupied by these surface fibers is a sizable fraction of the thickness of these fabrics, the relative fiber motion effect will be pronounced. This explains the difference between the bending stiffness of Fabric A and that computed from the model of no-freedom-of-relative-fiber-motion. This fabric has a thickness of only 7 fiber diameters, so that the less constrained surface fibers make up a significant fraction of the total thickness. However, the kind of fibers in the much thicker Fabric B have relatively less influence on the bending stiffness.

The (self-consistent) model proposed earlier can provide similar arguments about the possible lack of constraints for the fiber segments near the surface. For fibers far from the fabric surface, the surroundings of the fiber segments can be imagined to be a three-dimensional infinite medium. Fiber segments near the fabric surface, however, will be strongly influenced by the free boundary. They will be less stressed than if they were in an infinite medium. These surface fibers on the other hand contribute a significant fraction to the overall bending stiffness.

Therefore, the self-bonded fabric can be viewed as a three-layer sandwich plate with its two outer layers softer than its inner layer. The thickness of these outer layers should be fixed for fabric of any thickness. Hence the thicker the fabric, the less significant the contribution to the bending stiffness of these surface fibers to the fabric.
For these self-bonded nonwoven fabrics, the bending stiffness is primarily controlled by fiber members through tension and compression. Contrary to its importance to the thin Cerex, the junction bending mode is of negligible influence here. By limiting the relative motion between fibers, however, junctions do indirectly contribute to the bending stiffness of the fabric. Of course, as the number of junctions in the fabric decreases, the degree of freedom of relative fiber motion increases. The fabric thus becomes more flexible. The bending stiffness is expected to change just slightly when junction sizes are varied but not to cause debonding during bending.
CHAPTER 2:
THERMAL SPOT-BONDED NONWOVEN FABRICS

2.1 Description of the Material

Thermal spot-bonded nonwovens composed of thermoplastic fibers are generally bonded, without a binder, by heat and pressure in discrete spots scattered through the area of unbonded fibers. The structure of the unbonded region allows for great freedom of relative fiber motions which leads to good bending flexibility. Bonds, meanwhile, provide integrity and strength to the fabric. As a result, these fabrics can be manufactured with wide ranges of strength and flexibility.

The subject chosen for study is Fibretex, a Crown Zellerbach Co. product, made of randomly distributed continuous polypropylene filaments of about 18 μm diameter. The fiber web is spot-bonded by passing it between two heated rollers, one of which has a raised pattern of bosses that fuse the fibers together to exhibit square bonds, as shown in Fig. 2-1. With sides \( s \approx 0.89 \text{ mm} \), these bonds are equally spaced with a distance \( f \approx 1.13 \text{ mm} \) apart. In comparison, the bonded region is much thinner than the unbonded region. At the sites of bonds, the original structure of fiber network has largely fused together to transform it into a compact thin sheet, as shown in Fig. 2-2.

The unbonded region has been observed to be a rather bulky fiber assembly. Fibers within it are ordinarily not straight; rather, they follow some winding and curved paths in the plane of the fabric. In
Figure 2-1. Micrograph showing square bond spots in the fabric.

Figure 2-2. Micrograph showing sheet like bond spots formed by compressed and clustered fibers.
addition, because of the drastic thickness difference between the bonded and the unbonded areas (about 60 \( \mu \text{m} \) and 250 \( \mu \text{m} \), respectively), most fibers between bonds unavoidably curve out of the plane of the fabric. Fibers thus trace three-dimensional curved trajectories in the fabric. Once under load fibers will be unevenly stressed, and only those readily stretched will offer resistance immediately. As for fibers that are somewhat relaxed in the beginning, they will defer picking up the load.

Although the fibers may not be isotropically oriented in the fabric, micrographs of the fabric do not indicate clearly preferred fiber orientation. Besides, the tests in tension and bending detect no obviously different elastic responses in various directions of the fabric. The fabric can be reasonably treated as an isotropic material.

When the fabric was bent, prominent in-plane motions were observed under the microscope in a large number of fibers, especially those under compression. This is further evidenced from the bending tests: the bending moment-curvature curves revealed the effect of hysteresis. This means that relative fiber motions did exist and overcame the friction between the fibers.

### 2.2 Physical Model for the Bent Fabric

Bonds in this spot-bonded fabric can be regarded as second-phase inclusions embedded in the matrix of an unbonded fiber network. With the self-consistent theory, the effective bending stiffness of this composite system can be evaluated. Here the matrix phase is considered
as a continuum with the same bending stiffness as that of the real fiber assembly. The bonds are represented by through-thickness heterogeneities. Moreover, the unbonded region is thought of as a homogeneous isotropic medium. For ease of theoretical calculations, square bonds are taken to be in the shape of through-thickness circular inclusions; otherwise, the necessary plate bending solution would be quite difficult to obtain. It may be assumed further that in the composite plate, the effect of circular bonds is not too different from that of square ones as long as their area fractions remain the same.

A. Bonded Region

The values of measured bond thicknesses are higher than those estimated by assuming solid polymer bonds. This suggests that some space between fibers is retained during the forming of the bonds to create voids in the bond areas. These voids, which are most likely flat, resemble flat cracks with planes elongated and parallel to the plane of the bonds. With this geometry, when the fabric is stretched or bent, the flat voids will not produce serious stress concentrations. The in-plane modulus of the material containing the above structure will be roughly proportional to the fraction of solid at the cross-section normal to the plane of the plate. This means that the rule of mixtures, or the upper bound model, will account for the modulus $E_b$ of the bonds, i.e.,

$$E_b = \frac{t_s}{t} E,$$  \hspace{1cm} (2.1)
where $E$ is the modulus of the polymer (polypropylene), $t$ is the measured thickness, and $t_s$ is the estimated thickness for a completely dense bond. Note that $t_s/t$ gives the fraction of the solid in the whole plate. $t_s$ can be estimated from the weight $W'$ per unit area of the fabric and the weight $\gamma$ per unit volume of the bulk polymer. It is given as

$$t_s = \frac{W'}{\gamma}.$$  \hfill (2.2)

The bending stiffness $D^{(1)}$ of the bonds is determined from the plate bending theory with the bonds being treated as homogeneous continuum plates with Young's modulus $E_b$; i.e.,

$$D^{(1)} = \frac{E_b t^3}{12(1-v^2)} = \frac{Et^2 W'}{12\gamma(1-v^2)},$$  \hfill (2.3)

where $v$, the Poisson's ratio of the polymer, is chosen to be 0.3.

**B. Unbonded Region and a Modified Model of No-Freedom-of-Relative-Fiber-Motion**

Based on the observation that fibers in the unbonded region are commonly wavy, one possible model is advanced to explore the bending response of the region. The model considers that fibers in a curved state have a smaller spring constant than if they were in straight form. Based on the assumption that the wavy fibers in the whole unbonded region are replaced by straight fibers with elastic modulus corresponding to the reduced tensile response of curved fibers, the model of no-freedom-of-relative-fiber-motion$^2$ can be applied to estimate the bending
stiffness of this imagined system.

When collinear forces are applied at the two ends of a curved fiber segment, the relative displacement \( \delta \) of the ends as a function of the applied forces \( P \) is as plotted in Fig. 2-3. The spring constant \( k \) of the fiber, i.e., \( k = \frac{dP}{d\delta} \), will increase as the fiber is stretched and decrease as the distance between ends is shortened, as shown in Fig. 2-4. Once the fiber is straightened, a fixed value of \( k \) will be reached. In general, even a slightly curved fiber has a much smaller \( k \) than the straight fiber.

Although there may be severely curved fibers in the fabric, through contacts with crossing fibers, the mobility of the fibers in fact is not unlimited. Therefore, fibers are modeled as stretches of less wavy segments corresponding to their shape between pinning points. Rather than being physically bonded to crossing fibers, these segments are merely pinned by friction at some contact points. The pinned segment lengths are estimated to be of the order of a few free fiber lengths. Between the pinned points, minimum constraints are expected. The number of fibers per unit area, together with the distance between spot bonds, determines the lengths of these segments. If bonds are arranged closer to each other, the fibers will be pinned more readily because bonds tighten the fiber assembly resulting in denser packing. As the area density of the fabric is increased, the free fiber lengths of the fiber will decrease to lead to shorter segments. Ideally, these segments will respond to the applied load through their pinned ends.
FIGURE 2-3. Load $P$ vs. end displacement $\delta$ for a curved fiber.

FIGURE 2-4. Spring constant $k$ plotted as a function of $\delta$ for a curved fiber.
As a modification of the model of no-freedom-of-relative-fiber-motion, it is assumed that pinned points that are on a plane perpendicular to the plane of the fabric but parallel to the bending axis remain on the same plane during bending. It is equivalent to apply the model of no-freedom-of-relative-fiber-motion on the unbonded region by replacing curved fiber segments with straight ones whose $(EA)_f'$ correspond to the curved segments. The bending stiffness $D^{(2)}$ of the unbonded region now can be found as

$$D^{(2)} = \frac{N_f (EA)'_f \bar{t}^2}{32}, \quad (2.4)$$

where $N_f$ is the number of the fibers in a circle of unit diameter, and $\bar{t}$ is the average thickness of the unbonded region. The fiber shapes are usually not easy to define since they are all in three-dimensionally curved form. This model, therefore, shall be used to estimate $(EA)_f'$ from the experimental results to examine if the actual fiber shapes are compatible with this $(EA)_f'$.

2.3 Self-Consistent Model

The self-consistent model developed in the first chapter for a heterogeneous plate under bending is employed here. The two distinct phases just modeled, of course, are substituted with equivalent continuum phases in accordance with the plate theory. Goland's method$^{18}$ is used to solve $\hat{B}_{12}, \hat{B}_{22}$, shown in Eq. (1.56). The equation of overall bending
stiffness $D$ adopted from Chapter 1 is therefore

$$\frac{1}{D} = C_1 \left\{ -\frac{V}{D(1)(1-v^2)} \hat{B}_{12}(D(1),D,v) + \frac{1}{D(1)(1-v^2)} \hat{B}_{22}(D(1),D,v) \right\}$$

$$+ (1-C_1) \frac{1}{D(2)} \hat{B}_2(D(2),D).$$

(2.5)

2.4 Experiments

The polypropylene fibers from which the fabric was spun were measured to have a diameter $d = 0.018$ mm, and a Young's modulus $E = 1.38 \times 10^9$ N/m². The microscope stage bending method for measuring the stiffness becomes unusable for this fabric because the bulky fiber distribution in the unbonded region makes the fabric deflection measurement almost impossible. The pure bending tester, mentioned in the second part of Chapter 1, was adopted here for the task.

Samples in strip shapes of 20 mm width and 50 mm length taken from a large piece of fabric were tested in the MITEX bending tester. No important difference was found among the bending stiffnesses of the specimens obtained from the nearby regions with different orientations. The areas and the weights of samples were also carefully measured to give their areal densities $W'$, as shown in the histogram in Fig. 2-5. The average $W'$ is about 0.034 kg/m² and the coefficient of variation is 26%. Each sample was then severed with a pair of sharp scissors to gain a clear cross-sectional view. Thicknesses of the bonds and the unbonded areas were measured under the microscope and averaged for each sample.
2.5 Comparison of Theoretical and Experimental Results

Experimental results of the bending stiffness $D$ are plotted in Fig. 2-6 as a function of the areal density $W'$. Although the measured $D$ are somewhat scattered, they do rise with increasing $W'$.

Based on theoretical models, the bending stiffnesses $D$ were calculated for different $(EA)_f'$ of fibers in the unbonded area, shown in Fig. 2-7. The values of $(EA)_f'$ were chosen to be

$$ (EA)_f' = \alpha(EA)_f , $$

where $\alpha$ is a crimp factor (less than 1) representing the degree of coiling of the fibers, and $EA$ is the plastic constant of straightened fibers. Note that theoretical $D$ is not a linear function of $W$ because the bond thickness $t$ is also a variable as indicated in Eq. (2.3).

When $(EA)' \approx 0.3(EA)$, theoretical results match best with the experimental results. If fiber segments are in sinusoidally waved form with wavelength $\lambda$ and amplitude $a_0$, from the method given in Appendix B, if $\lambda/a_0 = 5$, $a_0$ should be equal to about 0.45 times the fiber diameter $d$, and if $\lambda/a_0 = 20$, $a_0 = 0.5d$ to achieve this value of $(EA)'$. All these seem to be of reasonable fiber shapes. With this model, segments do not have to be severely curved to be accounted for; hence, all fibers become less stiff because of their waviness.

A rough estimation with the fiber web theory gives $Et$ of this material to be $Et = 1.91 \times 10^4$ N/m. The tensile tests performed on short but wide specimens—to prevent lateral contraction—gives a typical
FIGURE 2-6. Experimentally measured bending stiffnesses D vs. W′.
FIGURE 2-7. Theoretically calculated $D$ vs. $W'$. 

$$(EA)' = \alpha (EA)$$

- $\triangle \alpha = 0.4$
- $\square \alpha = 0.3$
- $\triangledown \alpha = 0.2$

$D$, Bending Stiffness $(10^{-5} \text{ N-m})$

$W'$, Areal Density $(10^{-2} \text{ kg/m}^2)$
nonlinear elastic stress-strain curve as shown in Fig. 2-8. The measured values of $E_t$ are about $E_t = 3.9 \times 10^3$ N/m for strain from 0 to 0.025 m/m, and $E_t = 9.76 \times 10^3$ N/m for strain from 0.025 to 0.05 m/m and up. 

Since fiber web theory assumes that fibers in the fabric are straight, the discrepancy should be due to the waviness of fibers. Referring to Eq. (2.6) that the curved fibers have $(EA)' = \alpha(EA)$, $\alpha$ is estimated from the results of tensile experiments to be 0.2 - 0.5 as the strain increased. This is consistent with the previously estimated value from the bending tests. Therefore, the existence of curved fibers is manifested from another viewpoint.

2.6 Discussion

Qualitatively speaking, the modified model of no-freedom-of-relative-fiber-motion points out some significant factors related to the bending stiffness of the fabric. It may be true that many fibers within this particular fabric are more or less wavy. The unbonded fiber assembly also seems to respond to bending under the influence of the waviness of the fibers. Since frictional contacts are found between fibers, the effective waviness is determined by the waviness of the pinned segments along the fiber. Obviously the shorter the segments are (or, the denser the fiber is packed), the less severe the effective waviness will be.

It appears that the bending stiffnesses can be determined by the condition of the fibers, the areal density of the fabric, and the area fraction and arrangement of the bonds. If fibers are considerably
FIGURE 2-8. A typical nonlinear elastic stress-strain curve of Fibretex.
crimped, the fabric will be relatively flexible. On the other hand, if the fabric is dense, the number of contact points along the fibers will increase to limit the fiber motion; consequently, the fabric will be stiffer. The bending stiffness of the fabric is influenced directly by the area fraction of the bonds and indirectly by the space between the bonds because of its effect on the fiber packing condition in the unbonded region. For fabrics made of similar fibers, if the pattern of bonds of a heavier material remains the same as that of a lighter one, the effective waviness will be decreased. The introduction of bonds will tighten the fiber assembly of the unbonded region because the bonds are much thinner than the unbonded region. The space between bonds should be wider in a heavier material to maintain comparable effective waviness of the fibers.

In this random fiber system, mechanisms involved during bending are hard to distinguish. One feature pointed out by this study is that curved fibers definitely make the fabric more flexible. However, since bonds also contribute to the packing condition of the unbonded fibers, the arrangement of bonds will also determine how effectively advantage can be taken of the curved fibers.
3.1 Description of the Material

Print-bonded nonwovens constitute a prominent branch of nonwoven fabrics which are manufactured by an adhesive-bonding process. This is one of the most commonly employed methods in nonwoven technology. Generally, these fabrics are formed by printing unbonded fiber webs at chosen spots with chemicals which serve as both coloring agents and binder. This technique simultaneously provides the fabrics with a woven appearance and a bonding mechanism.

One of the print-bonded nonwovens that was studied is Masslinn, a Chicopee Company product, with a basis weight of 0.043 kg/m², which contains red herring-bone bonded regions in the fabric, shown in Fig. 3-1. It is made up of staple fibers, composed of 75% rayon and 25% polyester, with a length of about 3.80 cm (1.5 in.). The density of the fabric is generally uniform as micrographs and histograms (shown in Fig. 3-3) of areal density measurement readily prove. Fibers are overwhelmingly oriented in one direction, as shown in Fig. 3-2, so as to produce a fabric with extreme anisotropy. For example, tensile tests of the fabric indicate that Young's modulus in the machine direction is about 50 times that in the transverse direction. The product $E_t$, i.e., Young's modulus multiplied by fabric thickness, in these two directions are about $3.84 \times 10^4$ N/m and $8.01 \times 10^2$ N/m, respectively. Furthermore,
Figure 3-1. Micrograph showing herring-bone bonds, dark areas on the micrograph, distributed in the fabric.

Figure 3-2. Micrograph showing that fibers are predominantly oriented in one direction.
FIGURE 3-3. Histogram of areal density $W'$. 

$W'$, Areal Density ($10^{-2} \text{ kg/m}^2$)
bending tests show that bending stiffnesses in two principal directions are also drastically different.

Since it is reasonable to assume that all fibers are unidirectionally oriented, attention in this research will be centered on the bending response of the fabric about an axis perpendicular to the machine direction only. Consistent with the assumption about fiber orientation, the bending stiffness in the transverse direction is considered negligible and is set to be zero. This is realistic as evidenced by experiments. The fabric then can be treated as an orthotropic plate which is stiff in one principal direction and perfectly flexible in the other.

Acrylic latex is used as binder to form regular patterned dark areas, red in practice, on fabric shown in Fig. 3-1. The white unbonded region is relatively bulky compared with the adhesive-added region. A cross-sectional view of the fabric is shown in Fig. 3-4. Interestingly enough, one can easily separate the fabric into two identical layers in the thickness direction—a phenomenon that indicates that fibers are generally parallel to each other and that the binder has not penetrated deeply enough to reach the midplane of the fabric.

3.2 Physical Model of the Fabric In Bending

The region with added adhesive and the unbonded region respond differently to bending deformation and hence form a two-phase composite material. Herring-bone bonded regions are viewed as inclusions embedded in the matrix of the unbonded phase. These two phases are modeled separately as follows.
Figure 3-4. Micrograph showing the cross-sectional view of flat bonded regions and bulky unbonded regions of the fabric.

Figure 3-5. Micrograph taken with transmitted light showing the cross-sectional view of the bonded region with dark binder distributed near the surfaces of the fabric.
A. Bonded Region

In the bonded region phase, relative fiber motion is hindered considerably by the adhesive that spreads between the fibers. This phase is then considered as one that can be approximated by Freeston and Platt's model of no-freedom-of-relative-fiber-motion.

Fibers are assumed to be covered uniformly by adhesive to form bi-component fibers. These fibers are taken to be straight and parallel to the surface of the fabric. By the assumed condition of no freedom of relative fiber motion in pure bending, any plane in the bonded phase perpendicular to the surface of the fabric but parallel to the bending axis stays plane. The bending stiffness $D$ in the direction $\theta = 0$ (i.e., the machine direction) of this totally bonded fabric with arbitrary fiber distribution is obtained from Freeston and Platt as

$$D = \frac{N_f (EA)_f t^2}{12} \int_{-\pi/2}^{\pi/2} \phi(\theta) \cos^4 \theta \, d\theta,$$

where $N_f$ is the number of fibers in a circle with diameter of unit length, $(EA)_f$ is Young's modulus of fiber multiplied by fiber cross-sectional area, $t$ is the thickness of the fabric, and $\phi(\theta)$ is the fiber orientation distribution function. Since all fibers in the non-woven are parallel to each other, $N_f$ means the number of fibers per unit length in the direction perpendicular to the fibers. The derivation of $N_f$ is shown in Appendix C. It is given by

$$N_f = \frac{W_z'}{\rho},$$
where \( W_2' \) is the weight per unit area of the originally unbonded fiber web and \( \rho \) is the fiber weight per unit length. As previously modeled, fibers are covered with binder; \((EA)_f\) is thus the total of \((EA)_c\) of the core fiber and \((EA)_b\) of the outer layer binder. It is written as

\[
(EA)_f = (EA)_c + (EA)_b. \tag{3.3}
\]

The core fiber, which is either rayon or polyester, has the average value of \((EA)_c\) expressed as

\[
(EA)_c = C_R(EA)_R + C_P(EA)_P, \tag{3.4}
\]

where \( C_R, C_P \) are the fractions, i.e., 0.75 and 0.25, of rayon and polyester, respectively, and \((EA)_R, (EA)_P\) are the values of \((EA)\) of rayon and polyester fibers, respectively. Thus, Eq. (3.3) can be rewritten as

\[
(EA)_f = C_R(EA)_R + C_P(EA)_P + (EA)_b. \tag{3.5}
\]

Because all fibers are oriented in the same direction, \( \theta = 0 \), the distribution function \( \phi(0) \) can be conveniently expressed as a (Dirac) delta function \( \delta \), i.e.,

\[
\phi(0) = \delta(0). \tag{3.6}
\]

By substituting Eqs. (3.2), (3.5), (3.6) into (3.1), the bending stiffness can be found as

\[
D(1) = \frac{N_f(EA)_f t^2}{12} = \frac{W[C_R(EA)_R + C_P(EA)_P + (EA)_b] t^2}{12\rho}, \tag{3.7}
\]
where superscript 1 represents the bonded phase. Note that since Young's modulus and modeled cross-sectional area of the binder are much smaller than those of the fibers, \((EA)_b\) in Eq. (3.8) is negligible in comparison with other terms. Actually, the function of the binder in the unbonded region is primarily to prohibit relative fiber motion. In other words, it does not present itself as a stiff material.

B. Unbonded Region and the Model of Bulky Fiber Assembly

Fibers in the unbonded regions are usually not quite straight, but slightly curved and bulged out of the midplane of the fabric, as illustrated in Fig. 3-4. Based on this, a bulky fiber assembly model is built to describe the unbonded fiber system. All fibers in this model are assumed to be in circular curves lying in the planes which are perpendicular to the surface of the fabric but parallel to the machine direction of the fabric. These fibers are built in to the neighboring continuum phases, representing the bonded regions, at the interphase boundary plane perpendicular to the surface of the fabric. Furthermore, fibers are considered to form mirror images about the midplane of the fabric, as a side view of the nonwoven in Fig. 3-6 shows.

For this study, all fibers are modeled to be discretely distributed in fixed layers. As shown in Fig. 3-6, when \(t_2\) is the largest thickness of the unbonded region in the middle of two bonds, the number \(n\) of fiber layers is

\[
 n = \frac{t_2}{d} , \quad (3.9)
\]
FIGURE 3-6. Illustration of the discrete layers in which fibers of unbonded region are distributed.
where \( d \) is the diameter of the fiber.

As the sketch in Fig. 3-6 depicts, the bottom outer layer (layer 1) and the top outer layer (layer n) are taken to be circular segments built into the interface of the bonded region at a distance \( t_1 \) apart (the thickness of the bonded region phase). If the circular curves representing these two layers are extended, they would finally meet at points 0 and 0' on the line representing the midplane of the fabric. Although any two adjacent layers are one fiber diameter \( d \) apart at the thickest section of the unbonded region, for modeling ease it is suggested that all layers be ideally arranged such that the extensions of the circular curves representing them can all interpenetrate and meet at 0 and 0' as shown in Fig. 3-6. In these layers, fibers are in reality offset in the transverse direction of the fabric to avoid possible interference. The fiber arrangement in this phase hence can be identified from the modeling. An investigation of the bending response of this model can now be carried out.

When the unbonded phase is bent, the bending moment is built up by the forces and the moments transmitted to the fiber through the interphase boundaries. The interaction between fibers within the unbonded region is taken to be nonexistent for a first-order approximation. Every fiber then is only subject to the end force \( P \) and the end moment \( M \), as shown in Fig. 3-7. This loading condition produces the end displacement \( \delta \) and the end rotation \( \phi \) of the fiber. The derivation of \( \delta \) and \( \phi \) is given in Appendix D. It is found that \( \delta \) and \( \phi \) are expressible in terms of linear combinations of \( P \) and \( M \); i.e.,
FIGURE 3-7. Illustration of a fiber in the unbonded region subject to tension and bending at its built-in ends at interphase boundaries.
\[ \delta = AP + BM \quad (3.10) \]
\[ \phi = CP + DM \quad , \quad (3.11) \]

where \( A, B, C, D \) are constants determined by the geometry and Young's modulus of the fiber, shown in Appendix D.

For a set of \( n \) fibers to occupy all \( n \) layers, with the \( i \)th fiber in the \( i \)th layer, the end displacement \( \delta_i \) and the end rotation \( \phi_i \) of the \( i \)th fiber are

\[ \delta_i = A_i P_i + B_i M_i \quad (i = 1, 2, ..., n) \quad (3.12) \]
\[ \phi_i = C_i P_i + D_i M_i \quad , \quad (3.13) \]

where \( P_i, M_i \) are the force and the moment exerted on the built-in ends of the \( i \)th fiber segment and \( A_i, B_i, C_i, D_i \) are constants related to the \( i \)th fiber, sharing similar meanings with those appearing in Eqs. (3.10) and (3.11).

A constant angle of rotation will be induced at the built-in ends of all fibers as bending occurs, i.e.,

\[ \phi_i = \phi \quad \text{for all } i = 1, 2, ..., n \quad . \quad (3.14) \]

The interphase boundary which is originally a plane is assumed to remain a plane during bending. The end extension, or contraction, of an individual fiber is then related to the end rotation by

\[ \delta_i = y_i \phi \quad \text{for all } i \quad , \quad (3.15) \]

where \( y_i \) is the distance from the midplane, a neutral plane of bending,
to the built-in end of the \( i \)th fiber. The value of \( y_i \) is either positive or negative, depending on whether the \( i \)th fiber is above or below the midplane. Note here that since every layer follows a prescribed surface as already mentioned, \( y_i \) is readily obtained.

Equation (3.13), in reference to Eq. (3.14), can be rewritten as

\[
\phi = C_i P_i + D_i M_i .
\]

This relation, together with Eqs. (3.12) and (3.15), leads to the following expression:

\[
A_i P_i + B_i M_i = C_i y_i P_i + D_i y_i M_i .
\]

A rearrangement can reduce Eq. (3.17) to the form

\[
P_i = E_i M_i ,
\]

where

\[
E_i = \frac{D_i y_i - B_i}{A_i - C_i y_i} .
\]

Alternatively, \( M_i \) can be easily derived to be

\[
M_i = F_i P_i ,
\]

where

\[
F_i = \frac{1}{E_i} = \frac{A_i - C_i y_i}{D_i y_i - B_i} .
\]

Equations (3.18) and (3.20) point out that \( M_i \) and \( P_i \) are interdependent. By substituting them into Eq. (3.16), new expressions of \( M_i \) and \( P_i \) in terms of \( \phi \) can be obtained as
In other words, as angle $\phi$ of end rotation is known, the loading condition of an individual fiber is decided. The net moment $M_0$ applied to the system of $n$ fibers of the unbonded region resulting in a rotation $\phi$ at the edge of this phase is the total of all the moments that result from the forces and the moments at built-in ends of the fibers, i.e.,

$$M_0 = \sum_{i=1}^{n} M_i + \sum_{i=1}^{n} y_i P_i = \phi \left[ \sum_{i=1}^{n} \left( \frac{1}{C_i E_i + D_i} + \frac{y_i}{C_i + D_i F_i} \right) \right].$$

(3.24)

Not all fiber segments which fall into a specific unbonded region share the load in the bending process. Only those that span the entire region contribute to the bending. Their number in one unbonded region of unit width is $N_1'$ which is derived in Appendix C:

$$N_1' = \frac{W_2'}{\rho} \left( 1 - \frac{S_2}{\lambda} \right),$$

(3.25)

where $W_2'$, $\rho$, and $\lambda$ each stand for, respectively, the weight of the unbonded region per unit area, the fiber weight per unit length, and the fiber length; $S_2$ is the distance across one unbonded region in the machine direction. Since the moment $M_0$ shown in Eq. (3.24) is associated with a set of $n$ fibers in $n$ layers, the moment $M$ for the
The unbonded region of unit width is

\[ M = \frac{N_i'}{n} M_0 = \frac{w_2}{n} \left( 1 - \frac{S_2}{\lambda} \right) \phi \left[ \sum_{i=1}^{n} \left( \frac{1}{C_i E_i + D_i} + \frac{y_i}{C_i + D_i F_i} \right) \right] \]

\[ = \frac{w_2 d}{2 \rho t_2} \left( 1 - \frac{S_2}{\lambda} \right) \phi \left[ \sum_{i=1}^{n} \frac{1 + E_i y_i}{C_i E_i + D_i} \right] \]

(3.26)

The bending stiffness reduced from Eq. (3.26) is given by

\[ D^{(2)} = \frac{M}{\frac{S_2/2}{\phi}} = \frac{w_2 d S_2}{2 \rho t_2} \left( 1 - \frac{S_2}{\lambda} \right) \left[ \sum_{i=1}^{n} \frac{1 + E_i y_i}{C_i E_i + D_i} \right] \]

(3.27)

where the superscript 2 represents the phase of the unbonded region.

Interestingly enough, when all fibers are straight and parallel to the midplane of the fabric, this model would yield exactly the same result as that of the model of no-freedom-of-relative-fiber-motion.

In view of the fact that the fabric can be delaminated without difficulty, the binder which is printed on the surface of the fabric does not seem to penetrate too far to effectively bond the fibers near the midplane of the bonded region. Figure 3-5 shows the cross-sectional view of the bonded region with dark binder distributed near the surfaces of the fabric. The bulky fiber assembly system just modeled, therefore, does not have all the fibers subject to the assumed condition of end constraints. Outer layer fibers can be assumed to have better chances than inner layer fibers of reaching the rigid portions of the bonded phase, i.e., the regions near the surface. This situation is especially
pronounced near the interphase boundary where binder is expected to be thin. Another estimation of the bending stiffness of the unbonded area is made by considering that only a number of layers, layers 1 to \( n' \) and layers \( (n - n' + 1) \) to \( n \), actively respond to bending in the mode described by the model. The bending stiffness contributed by these layers hence is found as

\[
D'(2) = \frac{W'dS_2}{2\rho t_2} \left(1 - \frac{S_2}{\lambda}\right) \left[ \sum_{i=1}^{n'} \left( \frac{1 + E_iy_i}{C_iE_i + D_i} \right) + \sum_{i=n-n'+1}^{n} \left( \frac{1 + E_iy_i}{C_iE_i + D_i} \right) \right]. \tag{3.28}
\]

The remaining layers are considered to follow the free fiber model. The bending stiffness contributed by them is

\[
d''(2) = \frac{(n - 2n')W'd}{\rho t_2} \left(1 - \frac{S_2}{\lambda}\right) (EI). \tag{3.29}
\]

Hence, the bending stiffness of the unbonded region is written as

\[
D(2) = D'(2) + d''(2). \tag{3.30}
\]

Measurements under the microscope gave \( t_1 \approx 230 \mu \text{m} \), \( t_2 \approx 890 \mu \text{m} \), and \( S_2 \approx 4900 \mu \text{m} \), from which \( n \) and \( S_1 \), the distance between \( 0 \) and \( 0' \), can be determined. From these, the geometries of the fibers in individual layers are found.

3.3 Composite Material Model

The overall bending response of the fabric is governed by the bending stiffnesses of bonded and unbonded phases and their geometrical
arrangement. Plate bending theory is applied here to analyze the composite plate that is subject to the remote uniform bending moment. The solutions of the deflections in individual constituent phases can be used to evaluate the average plate curvature. The overall bending stiffness of the plate can hence be determined. In doing so, the two virtually discrete phases are regarded as equivalent continuum phases.

The fabric shown in Fig. 3-9 illustrates that herring-bone inclusions of parallelogram-shaped bonded phase are embedded in the unbonded matrix phase. The pattern of inclusions appears symmetric about the machine direction. A row of inclusions with the same orientation occupies a fabric strip that extends in the machine direction. In reality, adjacent rows are found on slightly overlapped strips to provide, no doubt, some lateral integrity. Yet, for ease of study, it is assumed here that all strips are connected without any overlap. Because of the periodic nature of the inclusion distribution, it is perhaps sufficient to examine just two sorts of unit cells, A and B, shown in Fig. 3-8. Their bending responses would be duplicated all over the fabric, among all identical cells. These two, with different orientations of the bonded phase, are mirror images of one another. They also have area fractions of two constituent phases equal to those of macroscopic fabric.

When the fabric is subject to a uniform bending moment $M_0$, as shown in Fig. 3-8, by symmetry the bending responses of these two unit cells are also mirror images of one another. Therefore, only one of the two cells, A, is chosen for study, while the presence of cell B is
FIGURE 3-8. Illustration of the fabric composed of unit cells A and B subject to remote uniform bending moment $M_o$. 

Machine Direction

Transverse Direction

Unit Cell A

$M_o$

Unit Cell B

$M_o$
reflected by imposing suitable boundary conditions upon cell A. The unit cell is defined in such a way that the parallelogram-shaped bonded phase divides its unbonded phase into two physically alike parts.

3.4 Bending of the Unit Cell

An x-y coordinate system, whose origin is at the center of the unit cell (also the center of the bonded region) is set at the midplane as shown in Fig. 3-9. The axis of y is in the fiber direction, or the machine direction of the fabric, and the axis of z is perpendicular to the xy plane. The unit cell has width \( b_1 \) (7.4 mm) and length \( c_1 \) (8.4 mm). The bonded phase in it has \( a_1 \) (3.5 mm) as its width in the y direction. Its two longer sides are inclined to the x axis with an angle \( \theta \), shown in Fig. 3-9.

During bending, the unit cell is taken to obey the condition of a thin plate undergoing small deflection. Therefore, all stress patterns in the plate are determined by the deflection \( w \) of the plate in the z-direction. Since this material is highly anisotropic, or more precisely, orthotropic, the partial differential equation to be solved for a generalized orthotropic plate\(^2\) is given by

\[
D_1 \frac{\partial^2 w}{\partial x^4} + 2D_3 \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_2 \frac{\partial^4 w}{\partial y^4} = 0, \quad (3.31)
\]

where \( D_1 \) and \( D_2 \) are bending stiffnesses in the principal \( x,y \) directions and \( D_3 \) is given by
FIGURE 3-9. Illustration of unit cell subject to bending.
\[ D_3 = D_1 \nu_2 + D_k, \quad (3.32) \]

where \( \nu_2 \) is Poisson's ratio found from the deformation in the x-direction, resulting from loading in the y-direction, and \( D_k \) is the twisting stiffness of the plate in the principal directions. The moments and the shear forces of the orthotropic plate\(^2\) are also given as

\[ g_x = -D_1 \left( \frac{\partial^2 w}{\partial x^2} + \nu_2 \frac{\partial^2 w}{\partial y^2} \right) \quad (3.33) \]

\[ g_y = -D_2 \left( \frac{\partial^2 w}{\partial y^2} + \nu_1 \frac{\partial^2 w}{\partial x^2} \right) \quad (3.34) \]

\[ h_x = -h_y = -2D_k \frac{\partial^2 w}{\partial x \partial y} \quad (3.35) \]

\[ N_x = -\frac{\partial}{\partial x} \left( D_1 \frac{\partial^2 w}{\partial x^2} + D_3 \frac{\partial^2 w}{\partial y^2} \right) \quad (3.36) \]

\[ N_y = -\frac{\partial}{\partial y} \left( D_3 \frac{\partial^2 w}{\partial x^2} + D_2 \frac{\partial^2 w}{\partial y^2} \right), \quad (3.37) \]

where \( g_x, g_y \) are bending moments on the plane with normals parallel to the x and y-directions, \( h_x, h_y \) are twisting moments, and \( N_x, N_y \) are transverse shear forces.

By considering certain structural characteristics of these two phases, some simplified approximations are made in the following ways. For the sake of convenience, superscripts, 1 to represent the bonded phase and 2 to represent the unbonded phase, are used in what follows.
(1) Since fibers are almost unidirectionally aligned, $D_1$ is negligible in comparison with $D_2$. Therefore,

$$D_1(i) = 0,$$  \hspace{1cm} (3.38)

where $i = 1, 2, \ldots n$.

(2) The unbonded phase, as modeled before with no interaction taking place between parallel fibers, is unable to resist twisting in the principal directions. It thus has zero torsional rigidity, i.e.,

$$D_k^{(2)} = 0.$$  \hspace{1cm} (3.39)

The above relation, together with Eqs. (3.32) and (3.38), yields

$$D_2^{(2)} = 0.$$  \hspace{1cm} (3.40)

(3) The fibers in the bonded region are enclosed in a loosely-packed binder matrix. In this study, the binder is much softer than the fibers. When this phase is subject to twisting in the principal directions, it is the binder, scattered between fibers, that is deformed mostly. Rogers and Pipkin\textsuperscript{29} have assumed that for a composite material with parallel fibers lying in a weak matrix, the shear modulus would be negligible compared with the tensile modulus in the fiber direction. Since the moduli of fibers and latex binder are about $6.75 \times 10^9$ N/m$^2$ and $5.8 \times 10^5$ N/m$^2$ respectively, it may be analogously hypothesized here that in the fiber direction of the bonded phase of the nonwoven fabric, the torsional stiffness would be much smaller than the bending stiffness, i.e.,

$$D_k^{(1)} \ll D_2^{(1)}.$$  \hspace{1cm} (3.41)
Another inequality can be easily obtained by substituting Eq. (3.41) into Eq. (3.32) to obtain

\[ D_3^{(1)} \ll D_2^{(1)} . \]  \hspace{1cm} (3.42)

It is probably safe to argue further that

\[ D_3^{(1)} \approx 0 . \]  \hspace{1cm} (3.43)

The above simplified assumptions greatly help to analyze the problem of the unit cell in bending, that otherwise would be very difficult to solve. Now the partial differential equation in Eq. (3.31) can be reduced to an ordinary differential equation and adapted for the present problem, i.e.,

\[ D_2^{(1)} \frac{\partial^4 w^{(1)}}{\partial y^4} = 0 \] \hspace{1cm} (3.44)

\[ D_2^{(2)} \frac{\partial^4 w^{(2)}}{\partial y^4} = 0 , \] \hspace{1cm} (3.45)

where \( w^{(2)} \) only represents the deflection of the unbonded phase in the region above the bonded phase shown in Fig. 3-9, and the deflection in the remaining area of the unbonded phase is an antisymmetric function of \( w^{(2)} \).

There are several boundary conditions left to be satisfied by \( w^{(1)} \) and \( w^{(2)} \): \hspace{1cm} (1) The total amount of the bending moments applied at \( y = \pm c_1/2 \), the edges of the unit cell, is \( bM_0 \), when the remote uniform bending
moment per unit length is \( M_0 \); i.e.,
\[
-D_2^{(2)} \int_{-b_1/2}^{b_1/2} \frac{\partial^2 w^{(2)}}{\partial y^2} \left( x, \pm \frac{C_1}{2} \right) \, dx = b_1 M_0. \tag{3.46}
\]

If the end deflections are not symmetric about the \( y \)-axis when one unit cell joins others of the same kind to form a strip, twisting in addition to bending will take place in the strip. However, the presence of adjacent strips of opposite kind of cells prevents any twisting, which inevitably will cause two adjacent strips to twist in different directions, to occur for the reason of compatibility.

(3) \( w^{(1)}(x,y) = w^{(1)}(-x,-y) \),
by antisymmetry.

(4) At the interphase boundary, where \( y = x \tan \theta + \frac{C_1}{2} \), the equality of deflections, slopes, and tractions yields the following conditions:

(i) If the \( x-y \) coordinates are rotated counterclockwise by an angle \( \theta \) to a new \( x'-y' \) coordinate system, as shown in Fig. 3-9, the requirement for slope continuity brings about the relation
\[
\frac{\partial w^{(1)}}{\partial y'} = \frac{\partial w^{(2)}}{\partial y'} \quad . \tag{3.50}
\]
Equation (3.50) can be transformed into the \( x-y \) coordinate
system as
\[-\sin \theta \frac{\partial w(1)}{\partial x} + \cos \theta \frac{\partial w(1)}{\partial y} = -\sin \theta \frac{\partial w(2)}{\partial x} + \cos \theta \frac{\partial w(2)}{\partial y}.\] (3.51)

(iii) The bending moments of the bonded phase and the unbonded phase at the interphase boundary are \(g_y^{(1)}\) and \(g_y^{(2)}\), respectively, while

\[g_y^{(1)} = g_y^{(2)}.\] (3.52)

By tensor transformation, \(g_y^{(i)}\) can be found as

\[g_y^{(i)} = \sin^2 \theta g_x^{(i)} + 2 \sin \theta \cos \theta h_x^{(i)} + \cos^2 \theta g_y^{(i)},\] (3.53)

where \(i = 1,2\).

From Eqs. (3.33), (3.34), (3.35), (3.38), (3.40), and (3.43), it also can be concluded that

\[g_x^{(i)} = h_x^{(i)} = -h_y^{(i)} = 0\] (3.54)

and

\[g_y^{(i)} = -D_2 \frac{\partial^2 w^{(i)}}{\partial y^2}.\] (3.55)

Therefore, the continuity of bending moment will give the relation

\[-D_1(1) \frac{\partial^2 w^{(1)}}{\partial y^2} = -D_2(2) \frac{\partial^2 w^{(2)}}{\partial y^2}\] (3.56)

on the boundary.

(iv) \[N_y^{(1)} + \frac{\partial h_y^{(1)}}{\partial x'} = N_y^{(2)} + \frac{\partial h_y^{(2)}}{\partial x'} .\] (3.57)

This relation represents that, at the boundary, the statically
equivalent vertical forces of both phases are equal. Equation (3.57) can be transformed into x-y coordinates to yield

\[-D_2^{(1)} \left\{ \frac{\partial^3 w^{(1)}}{\partial y^3} \cos \theta + 2 \sin \theta \cos \theta \left[ \frac{\partial^3 w^{(1)}}{\partial x \partial y^2} \cos \theta + \frac{\partial^3 w^{(1)}}{\partial y^3} \sin \theta \right] \right\} \]

\[= -D_2^{(2)} \left\{ \frac{\partial^3 w^{(2)}}{\partial y^3} \cos \theta + 2 \sin \theta \cos \theta \left[ \frac{\partial^3 w^{(2)}}{\partial x \partial y^2} \cos \theta + \frac{\partial^3 w^{(2)}}{\partial y^3} \sin \theta \right] \right\} \]  \hspace{1cm} (3.58)

Thus, Eqs. (3.46), (3.47), (3.48), (3.49), (3.51), (3.56), and (3.58) furnish seven boundary conditions to find \( w^{(1)} \) and \( w^{(2)} \). From Eqs. (3.44) and (3.45), it is clear that \( w^{(1)} \) and \( w^{(2)} \) must be in the forms of

\[ w^{(1)} = A^{(1)}(x)y^3 + B^{(1)}(x)y^2 + C^{(1)}(x)y + D^{(1)}(x) \]  \hspace{1cm} (3.59)

\[ w^{(2)} = A^{(2)}(x)y^3 + B^{(2)}(x)y^2 + C^{(2)}(x)y + D^{(2)}(x) \]  \hspace{1cm} (3.60)

where \( A^{(1)}(x) \), \( B^{(1)}(x) \), \( C^{(1)}(x) \), and \( D^{(1)}(x) \) are functions of \( x \).

For convenience, these functions are chosen in the polynomial form of \( x \). By Eq. (3.48), \( A^{(1)} \), \( C^{(1)} \) must be polynomials of odd powers and \( B^{(1)} \), \( D^{(1)} \) of even powers. It has been found by trial and error that if

\[ w^{(1)} = A_1^{(1)}xy^3 + \left[ B_2^{(1)}x^2 + B_0^{(1)} \right]y^2 + \left[ C_3^{(1)}x^3 + C_1^{(1)}x \right]y + D_2^{(1)}x^2 \]  \hspace{1cm} (3.61)

\[ w^{(2)} = A_1^{(2)}xy^3 + \left[ B_2^{(2)}x^2 + B_0^{(2)} \right]y^2 + \left[ C_3^{(2)}x^3 + C_1^{(2)}x + C_0^{(2)} \right]y \]

\[ + D_3^{(2)}x^3 + D_1^{(2)}x + D_0^{(2)} \]  \hspace{1cm} (3.62)
there are just enough equations to solve all the unknown coefficients by satisfying all the boundary conditions. The simultaneous equations used to solve the unknown constants can be reduced to an equation involving the matrix product as

$$Ax = b, \quad (3.63)$$

where

$$x = \begin{bmatrix}
A_0^{(1)} \\
B_2^{(1)} \\
B_0^{(1)} \\
C_3^{(1)} \\
C_1^{(1)} \\
C_2^{(1)} \\
D_2^{(1)} \\
A_1^{(2)} \\
B_2^{(2)} \\
B_0^{(2)} \\
C_3^{(2)} \\
C_1^{(2)} \\
C_0^{(2)} \\
D_3^{(2)} \\
D_1^{(2)} \\
D_0^{(2)}
\end{bmatrix}.$$

The 15×15 matrix $A$ and the 15×1 column matrix $b$ are given in Appendix E. The column matrix $x$, the unknown coefficients vector, is found.
readily by multiplying both sides of Eq. (3.63) by the inverse of \( A \), \( A^{-1} \); i.e.,

\[
x = A^{-1}b
\]  

(3.65)

The area average of the curvature (also an average over the macroscopic fabric), \( \bar{\partial^2 w/\partial y^2} \), over the unit cell is found as

\[
\bar{\partial^2 w/\partial y^2} = \left[ \frac{b_1}{2} \int \int \frac{\partial^2 w(2)}{\partial y^2} \, dy \, dx + \frac{b_1}{2} \int \int \frac{\partial^2 w(1)}{\partial y^2} \, dy \, dx \right] \frac{1}{b_1 c_1}
\]

\[
= \left[ -\frac{A_1(2)}{2} \tan \theta \left( \frac{a_1}{c_1} \right) + \frac{B_2(2)}{b_1} \left( 1 - \frac{a_1}{c_1} \right) \right] b_1^2 + 2B_0(2) \left( 1 - \frac{a_1}{c_1} \right)
\]

\[
+ \left[ A_1(1) \frac{1}{2} \tan \theta \left( \frac{a_1}{c_1} \right) + B_2(1) \left( \frac{a_1}{c_1} \right) \right] b_1^2 + 2B_0(1) \frac{a_1}{c_1}
\]  

(3.66)

Since the remote uniform bending moment applied on the fabric is \( M_0 \), the average bending moment \( \bar{g}_y \) of the unit cell is also

\[
\bar{g}_y = M_0
\]  

(3.67)

which is an average over large-scale fabric as well. With the average bending moment and the average curvature given in Eqs. (3.67) and (3.66), the effective bending stiffness \( D \) now can be defined as

\[
D = -\frac{\bar{g}_y}{\partial^2 w/\partial y^2}
\]  

(3.68)
The bending response of an orthotropic material also can be estimated in directions other than the principal ones. The new $x'-y'$ coordinates can be obtained by rotating the $x-y$ coordinates with axes in the principal directions counterclockwise through an angle $\theta$. The equation for the bent plate in the new coordinates can be written in generalized tensor notation as

$$\mu_{i'j'} = D_{i'j'k'l'} \kappa_{k'l'} ,$$

where $\mu_{i'j'}$ is the moment tensor, $\kappa_{k'l'}$ is the curvature tensor, and $D_{i'j'k'l'}$ is the stiffness tensor. The stiffness tensor can be expressed in terms of the stiffness tensor $D_{ijkl}$ in $x-y$ coordinates:

$$D_{i'j'k'l'} = \ell_{i'i} \ell_{j'j} \ell_{k'k} \ell_{l'l} D_{ijkl} ,$$

where $\ell_{i'i}$, $\ell_{j'j}$, $\ell_{k'k}$, $\ell_{l'l}$ are direction cosines. For example, $D_{2'2'2'2'}$ can be evaluated from $D_1$, $D_2$, and $D_3$

$$D_{2'2'2'2'} = D_1 \sin^4 \theta + 2D_3 \cos^2 \theta \sin^2 \theta + D_2 \cos^4 \theta$$

3.5 Experiments

Single fibers, composed of rayon and polyester, supplied by Chicopee Co., were tested in tension in an Instron testing machine. The testing strain rate was in the range 0.02-0.05 min$^{-1}$. This was comparable to the strain rate that would be experienced by the fibers in specimens to be tested in bending. The average values of modulus multiplied by
cross-sectional area of the fibers were found to be \((EA)_R = 1.08 \text{ N for rayon fibers and } (EA)_P = 0.65 \text{ N for polyester fibers. These two kinds of fibers were measured from SEM micrographs to have diameters of } d \approx 0.013 \text{ mm for rayon, } d \approx 0.012 \text{ mm for polyester. The average areal density of the fabric was measured to be } W' = 0.043 \text{ kg/m}^2 \text{ while that of the bonded region is } W_1' = 0.048 \text{ kg/m}^2 \text{ and the unbonded region is } W_2' = 0.038 \text{ kg/m}^2. The area fractions of these two phases are approximately } C_1 \approx 0.4 \text{ and } C_2 \approx 0.6.\)

Acrylic latex, a National Starch Co. product (NACRYLIC X-4260) used to bond the fabric, was cured in an oven at 100°C for 24 hours to form films of 0.60-1.50 mm in thickness. These films were then tested in tension in an Instron machine with strain rates of 0.07 and 0.15 min\(^{-1}\). The average Young's modulus of this rubbery material was found to be \(E_b = 5.8 \times 10^5 \text{ N/m}^2\). By weighing a piece of processed latex in air and in water separately, its specific gravity was found to be \(\gamma = 1.11\).

Several strips of 20 mm wide and roughly 50 mm long were taken from the fabric along the directions \(\theta = 0^\circ, 30^\circ, 60^\circ, \text{ and } 90^\circ\), where \(\theta\) is the angle to the machine direction. They were tested in the MITEX bending tester, described in Chapter 1, with bending axes perpendicular to their longer sides.

3.6 Comparison of Theoretical and Experimental Results

Experimental results of the bending stiffnesses \(D\) are plotted in Fig. 3-10 as a function of the areal density \(W'\) of individual tested
FIGURE 3-10. Experimentally measured bending stiffness \( D \) vs. \( W' \).
specimens. The areal density $W'$, proportional to $W'_f$, is directly proportional to $N_f$, the number of the fibers per unit width of the fabric in the transverse direction, as indicated in Eq. (3.2). The ratio of the measured bending stiffnesses in the machine direction to those in the transverse direction is roughly 10 to 1. This more or less supports the previous assumption that the fabric can be treated as an orthotropic material with negligible bending stiffness in the transverse direction. Note that the measured bending stiffnesses for $\theta = 30^\circ$ and $60^\circ$ are approximate values. As can be seen from Eq. (3.69), the bending moment $\mu_{2',2'}$ (the $y'$ axis in the $\theta$-direction) applied to bend the specimens is in the form

$$\mu_{2',2'} = D_{2',2',2',2'} k_{2',2'} + 2D_{2',2',1',2'} k_{1',2'} + D_{2',2',1',1'} k_{1',1'} \quad (3.72)$$

where $D_{2',2',2',2'}$ is the bending stiffness $D$ in the $\theta$-direction. The four pin setup of the pure bending tester should prevent $K_{1',2'}$, and $K_{1',1'}$ from having substantial values to yield

$$\mu_{2',2'} \approx D_{2',2',2',2'} k_{2',2'} \quad . \quad (3.73)$$

Therefore, from the readings of $\mu_{2',2'}$ and $K_{2',2'}$ from the MITEX bending tester, $D_{2',2',2',2'}$ can be estimated.

The values of measured $D$ averaged in each direction are plotted in Fig. 3-11 as a function of $\theta$. Alternative estimations for the bending stiffnesses in $\theta = 30^\circ$ and $60^\circ$ can be made with Eq. (3.71). In this equation, measured $D$ in $\theta = 0^\circ$ and $90^\circ$ are taken to be $D_1$, $D_2$.
FIGURE 3-11. Average experimental results of bending stiffnesses \( D \) plotted as a function of \( \theta \). Also shown is the curve estimated from Eq. (3.71).
while $D_3$ is chosen to be zero. The measured and the estimated values are different but close, as shown in Fig. 3-11. The discrepancy is likely due to the neglect of the torsional stiffness $D_3$. Moreover, measured $D$ in $\theta = 30^\circ$ and $60^\circ$ are perhaps not the real values, for $\kappa_1$, $\kappa_1'$, and $\kappa_1''$ in Eq. (3.72) may not be negligible.

The calculations based on the theoretical models showed that the bending stiffness of the bonded region is almost two orders of magnitude higher than that of the unbonded region, i.e., $1.00 \times 10^{-3}$ N-m to $2.0 \times 10^{-5}$ N-m. This is consistent with the assumption that the rigid end constraint is imposed upon fibers in the unbonded region. Theoretical values of $D$ were computed by considering that the bulky fiber assembly model was followed by only some outer layers, i.e., a fraction $\beta$ of the total layers in the unbonded region. Figure 3-12 shows the calculated $D$ for $\beta = 1, 5/6, 2/3, 1/2, 1/3, 1/6, 0$. As the theoretical models imply that the bending stiffnesses of both phases are linear functions of $W'$, the bending stiffnesses form straight lines for different $\beta$'s on the diagram.

As $\beta = 1$, i.e., the whole unbonded region behaves in accordance with the bulky fiber assembly model, the theoretical values are much higher than the experimental results. The tested values stay between two sets of computed values of $\beta = 2/3$ and $\beta = 1/2$.

On the basis of all the simplified assumptions, the result of the mathematical calculation of the bending stiffness altered only slightly with the alignment variation of bonds. The orientation of the bonds,
FIGURE 3-12. Bending stiffnesses $D$ in the longitudinal direction of the fabric computed from theoretical models.
nevertheless, may not make too much difference to the fabric bending stiffness in the machine direction, especially since fibers are predominantly oriented. But the lateral integrity of the fabric still depends largely on the bond arrangement.

3.7 Discussion

Although the computations based on the proposed bulky fiber assembly model may not be totally reliable owing to the existing structural complexities, the model qualitatively points out a few significant mechanisms and provides ideas to the improvement of fabric design.

The experimental results can be partially explained by considering that only some of the surface layers of the bonded region are effectively bonded. The ineffectiveness of adhesive bonding, however, is really not so pronounced, as shown in Fig. 3-12. The discrepancy between the experimental and the theoretical bending stiffnesses of the fabric should be accounted for also by the actual fiber conditions. Fibers virtually are never all perfectly unidirectionally oriented. Besides, some of the fibers in the fiber assembly are cramped in the plane of the fabric and do not carry too much load at the initial loading stages. The major load is shared only by those readily-stretched fibers. Under the influences of all these factors, the bulky fiber assembly model is bound to overestimate the real bending stiffness of the unbonded region.

Examining what has been presented so far, the binder distribution in the bonded region is likely to influence the bending of the fabric.
A shallow penetration of the binder in the fabric results in the decrease of the bending stiffness, as can be seen in Fig. 3-12. In the meantime, tensile strength may be weakened and delamination may be induced. An optimum application of binder, however, should keep all the properties in the desired range. In addition, as shown by Eq. (3.25), $N_\perp'$ is decreasing with increasing $S_2$ (the distance between two bonds in the machine direction) and decreasing $\lambda$ (the fiber length). By proper control of these factors, the bending stiffness, a function of $N_\perp'$, can be altered.
4.1 Description of Material

The second kind of print-bonded nonwoven fabric that was studied is also a Chicopee Company product (trade name "Keybak"), made of staple rayon fibers of about 3.80 cm in length. Fiber arrangement in the fabric determines a pattern of elliptic openings, as shown in Figs. 4-1 and 4-2. These openings are formed by applying water jets on an initially parallel lay-up of unbonded fiber web. Some interlocking between fibers is introduced by the forming process of the openings so as to keep them from being separated easily. Acrylic latex, used as binder, is printed on the fabric to form parallel dark lines, shown in Fig. 4-1 as being perpendicular to the machine direction of the fabric.

One way to learn more about fiber arrangement in the fabric is by examining the fiber web structures, before and after the elliptic openings are produced, so sketched in Fig. 4-3. Originally, the totally unbonded fiber web, as illustrated in Fig. 4-3(a), is uniform in fiber distribution. All fibers are more or less straight and unidirectionally oriented. For convenience of study, this fiber web can be partitioned in the machine direction into strips of two alternative fiber groups, A and B. The group A with width $a$ equal to the minor axis of the elliptic opening inhabits the site where a row of openings would
Figure 4-1. Micrograph showing the appearance of the fabric "Keybak".

Figure 4-2. Micrograph showing the elliptic openings surrounded by fibers.
FIGURE 4-3. Illustrations of the groups A and B of fibers (a) before and (b) after the formation of openings.
locate. When elliptic openings emerges, group A is expanded to span a final width $a'$, as shown in Figs. 4-2 and 4-3(b). Fibers in group A then have migrated to border a predetermined pattern of openings. In addition, they form cross-laid fiber regions between adjacent elliptic openings with collinear long axes. These fibers thus can hardly maintain their original straightness, especially when most every fiber is stretched around at least several openings. In general, under the influence of applied water force, fibers in group A would be curved both in the plane and in the thickness directions of the fabric. The group B, with initial width $b$, being pushed aside and narrowed to span a final width $b'$ during the forming process, nevertheless largely manages to preserve the original straight state of the fibers. These relatively straight fiber groups appear as dividers between rows of holes.

Macroscopically, the fabric has the appearance of a plate that is weakened by arrays of elliptic holes. At the microscopic level, these openings are circled by fibers—in group A—of considerably large curvature, particularly those found in cross-laid regions. These curved fibers in the unbonded regions should be easier to deform than relatively straight ones that stay in the parallel groups. When the fabric is under load, the majority of the load falls on the streams of relatively straight fibers of group B, to cause them to be stressed immediately.

The curved ones (fibers in group A) between these streams exert much less resistance to the applied load. This situation was evidenced by stretching the fabric in the machine direction. At least at early
stages of loading, curved fibers in group A were not stretched to significantly change their orientations. Since most fibers, even the curved ones, are spread in the machine direction under bending or tension, the material is much stiffer in this direction than in the transverse direction. Attention, therefore, is paid here only to the bending response of the fabric in the machine direction. The fabric can be treated as an orthotropic material: stiff in one principal direction and very flexible in the other.

4.2 Lower Bound Model to Evaluate the Bending Stiffness of the Fabric

Clearly, the fabric can be divided distinctly into bonded and unbounded phases. These two phases are alternately arranged in parallel strips which extend in the transverse direction of the fabric. When evaluating the bending stiffness $D$ of the fabric in the machine direction, the lower bound model that can well describe this kind of composite material is readily applied. If $C_1, C_2$ are the area fractions and $D^{(1)}, D^{(2)}$ are the bending stiffnesses in the machine direction of the bonded and the unbounded phases respectively, the effective bending stiffness $D$ of the fabric in the machine direction would be found as

$$
\frac{1}{D} = \frac{C_1}{D^{(1)}} + \frac{C_2}{D^{(2)}}.
$$

(4.1)
4.3 Physical Model of the Fabric in Bending

The fiber distribution within the two phases of the fabric are basically similar. Nevertheless, in the adhesive-bonded phase, fibers almost lie in the plane of the fabric, while in the unbonded phase, fibers slightly curve out of the plane of the fabric, as shown in Fig. 4-4. The analyses of these two phases under bending are given below.

A. Bonded Region

The relative fiber motion in the bonded phase is greatly restricted owing to the presence of binder. A hand examination of the fabric would reveal that the bending stiffness of the bonded phase is much higher than that of the unbonded phase; i.e.,

$$D^{(1)} \gg D^{(2)}.$$  \hspace{1cm} (4.2)

From this relation, the effective bending stiffness $D$ of the fabric in Eq. (4.1) can be reduced to

$$\frac{1}{D} \approx \frac{C_2}{D^{(2)}},$$  \hspace{1cm} (4.3)

since $C_1$ and $C_2$ are comparable. It is therefore unnecessary to evaluate $D^{(1)}$, the bending stiffness of the bonded phase. The model of no-freedom-of-relative-fiber-motion, however, is expected to describe this phase rather accurately. The effective bending stiffness of the fabric can be estimated solely from that of the unbonded phase.
Figure 4-4. Micrograph showing the cross-sectional view of flat bonded regions and bulky unbonded regions of the fabric.
B. Unbonded Phase

Since the fibers in the fabric roughly form two alternate groups A and B, as shown in Fig. 4-4, the bending stiffness $D^{(2)}$ of the unbonded phase becomes the sum of the bending stiffnesses, $D_A^{(2)}$ and $D_B^{(2)}$, contributed by groups A and B respectively; i.e.,

$$D^{(2)} = D_A^{(2)} + D_B^{(2)} .$$  \hfill (4.4)

It seems that the relatively straight fibers in group B of the unbonded region closely resemble, in certain aspects, those in the unbonded region of Masslinn (the print-bonded nonwoven mentioned in Chapter 3). For instance, they both bridge between bonded phases. In addition, they both slightly bulge out of the plane of the fabric to form bulky structures. The cross-sectional appearance of the unbonded phase similar to Fig. 3-4, as shown in Fig. 4-4, suggests that perhaps the bulky fiber assembly model of Chapter 3 could be adopted here. The fibers in group B thus are modeled to be circular curved beams. As described in the physical modeling of Masslinn, these beams are distributed in discrete layers. The bending stiffness $D_B^{(2)}$ of the unbonded phase, contributed by group B, can then be found from Eq. (3.27) to be

$$D_B^{(2)} = \frac{2dn_B}{s_2^2} \left(1 - \frac{s_2}{s}\right) \left(\sum_{i=1}^{n} \frac{1+e_i y_i}{c_i e_i + d_i}\right) .$$  \hfill (4.5)

where $n_B$ is the number of fibers of group B per unit width of the fabric. All other terms are as defined in Chapter 3. By examining
Fig. 4-3, \( N_B \) can be obtained. Before the formation of elliptic openings, the number \( N_f \) of fibers per unit width of the unbonded fiber web is

\[
N_f = \frac{W'}{\rho} \quad , \tag{4.6}
\]

where \( W' \) is the weight per unit area of the unbonded web and \( \rho \) is the weight of the fiber per unit length, shown in Appendix C. At this stage, \( N_B \) is found as

\[
N_B = \left( \frac{b}{\alpha + b} \right) N_f = \frac{W'_2}{\rho} \left( \frac{b}{\alpha + b} \right) \quad . \tag{4.7}
\]

As the openings are formed, the width of group B shrinks to \( b' \). The number \( N_B \) of fibers of B per unit width of the fabric, however, remains the same since the distance between the centers of two adjacent B groups is unchanged. Thus, \( D_B^{(2)} \) is written as

\[
D_B^{(2)} = \frac{2W'_2 d}{\rho t_2 S_2} \left( \frac{b}{\alpha + b} \right) \left( 1 - \frac{S_2}{\lambda} \right) \sum_{i=1}^{n} \frac{1 + E_i y_i}{C_i E_i + D_i} \quad . \tag{4.8}
\]

Some parameters relevant to this model, as shown in Fig. 3-6, were measured. They were found as: the thickness of the bonded phase \( t_1 = 0.26 \text{ mm} \), the largest thickness of the unbonded region \( t_2 = 0.48 \text{ mm} \), and the distance between the two adjacent bonded phases \( S_2 = 3.78 \text{ mm} \). SEM micrographs—for instance, Fig. 4-2—reveal that the real fiber arrangement more or less follows the pattern of alternate groups of A and B. Actually, well-defined regions cannot be easily distinguished. Approximate values of \( \alpha \) and \( b \), however, as shown in Fig. 4-3, were
estimated to be \( a \approx 0.7 \text{ mm} \) and \( b \approx 0.3 \text{ mm} \).

When the unbonded region is bent about an axis in the transverse direction, fibers in group A, which are curved in the plane of the fabric in order to accommodate the patterned holes, are stretched. Since these fibers have much larger curvatures than those in group B, the force and the moment needed to deform them are comparatively negligible. Ideally, all fibers in group A are assumed to be in sinusoidally waved shapes, and all wavelengths are about equal to twice the distance between the centers of two ellipses with collinear major axes. The amplitudes are all approximately half the minor axis of the ellipses, as shown in Fig. 4-5.

For the purpose of applying the bulky fiber assembly model, the fibers of group A are thought of as straight. However, the imagined straight fibers have reduced tensile moduli corresponding to their tensile responses in the actual curved shape.

In fact, when reduced modulus \( E' \) of sinusoidal-shaped fibers, as described above, was calculated by the method given in Appendix B, it was found that

\[ E' \approx 0.07E, \quad (4.9) \]

where \( E \) is the Young's modulus of straight fibers. Hence, \( D_A^{(2)} \) is taken as negligible compared with \( D_B^{(2)} \), i.e.,

\[ D_A^{(1)} \ll D_B^{(2)}. \quad (4.10) \]

Thus, the bending stiffness of the unbonded phase is approximately
FIGURE 4-5. Illustration of fibers in A in sinusoidally waved shape with wavelength $\lambda \approx 3.6$ mm and amplitude $a_0 \approx 0.35$ mm.
contributed by the fibers in group B only, i.e.,

\[ D(2) \approx D_B^{(2)} \]  

(4.11)

4.4 Experiments

The rayon fibers (the same as those in Masslinn) which constitute the fabric have \( EA = 1.08 \text{N} \), or \( E = 6.07 \times 10^9 \text{N/m}^2 \), and diameter \( d = 0.013 \text{mm} \). Acrylic latex (National Starch Company product NACRYLIC X-4280) was molded into films of thicknesses between 0.62 to 1.75 mm and oven cured at 100°C for 24 hours. Tensile tests of these films were performed on an Instron machine with strain rates of 0.07 and 0.15 \text{min}^{-1}. The average Young's modulus of the latex was found to be \( E_b = 2.32 \times 10^6 \text{N/m}^2 \). By weighing the same piece of latex in air and in water, the specific gravity of this latex was estimated to be \( \gamma = 1.11 \).

The average basis weight of the fabric was measured to be \( W' = 0.044 \text{kg/m}^2 \). Bonded and unbonded regions were cut out then weighed in a microbalance. Their individual average areal densities were determined to be \( W_1' = 0.051 \text{kg/m} \) for the former, and \( W_2' = 0.039 \text{kg/m}^2 \) for the latter. The areal fractions of these two regions are roughly \( C_1 = 0.4 \) and \( C_2 = 0.6 \), respectively.

Specimens cut out from the fabric in the machine and the transverse directions were tensile-tested to demonstrate the anisotropy of the fabric. The value of the Young's modulus multiplied by the fabric thickness in the machine direction was \( (E_t) = 3.76 \times 10^4 \text{N-m} \) and in
the transverse direction \( (E_t) = 3.32 \times 10^2 \text{ N-m} \).

Specimens in strips of 20 mm width and 50 mm length were also cut out from the fabric with their longer sides making an angle \( \theta \) with the machine direction such that \( \theta = 0^\circ, 30^\circ, 60^\circ, \) and \( 90^\circ \). The strips were then tested on the pure bending tester with bending axis perpendicular to the \( \theta \) direction to determine their bending stiffnesses. In order to determine the areal density, information on the area and the weight of each specimen was carefully collected. This gives a histogram of the areal density for the fabric shown in Fig. 4-6. The coefficient of variation is about 10%.

To verify the theoretical modeling that fibers in group A play minor roles during bending, specimens with \( \theta = 0^\circ \) were bend-tested, then severed with a sharp razor in the unbonded A regions along the machine direction, as shown in Fig. 4-7. The processed specimens were further tested to check if their bending stiffnesses had varied in any significant way. The bending stiffnesses \( D' \) and \( D'' \), before and after such segmentation, were measured for three specimens. It was found that \( D' = 2.38 \times 10^{-5} \text{ N-m}, D'' = 2.17 \times 10^{-5} \text{ N-m} \) for specimen No. 1. For specimen No. 2, \( D' = 1.98 \times 10^{-5} \text{ N-m}, D'' = 1.77 \times 10^{-5} \text{ N-m} \). For specimen No. 3, \( D' = 2.02 \times 10^{-5} \text{ N-m} \) and \( D'' = 1.93 \times 10^{-5} \text{ N-m} \). The results qualitatively agree with the above modeling.
FIGURE 4-6. Histogram of areal density $W'$. 

$W'$, Areal Density ($10^{-2}$ kg/m$^2$)
FIGURE 4-7. Illustration of the specimen in which cuts were made in unbonded regions A along the dotted lines.
4.5 Comparison of Theoretical and Experimental Results

Measured bending stiffnesses \( D \) are plotted in Fig. 4-8 as functions of \( W' \), the areal density, for the angles of orientation \( \theta = 0^\circ, 30^\circ, 60^\circ, \) and \( 90^\circ \).† These bending stiffnesses were averaged at every angle of orientation and plotted as a function of \( \theta \), as shown in Fig. 4-9. On the same diagram, \( D \) for \( \theta = 30^\circ \) and \( 60^\circ \) were estimated from Eq. (3.72) by neglecting twisting stiffness \( D_3 \) and by taking the tested \( D \) in two principal directions, i.e., \( \theta = 0^\circ \) and \( 90^\circ \), to be \( D_1, D_2 \). Since the estimated and the measured values are close, it suggests that the twisting stiffness is low.

These experiments have confirmed that the bending stiffness \( D_A^{(2)} \) of group A in the unbonded region is nearly negligible compared with \( D_B^{(2)} \) of group B. The bending stiffness \( D^{(2)} \) of the unbonded region is thus almost totally governed by the fibers in group B. Bending stiffness \( D \) of the fabric was computed with theoretical models for the different values of the effective geometrical term \( C_B = b/(a+b) \), appearing in Eq. (4.8), and are plotted in Fig. 4-10.

Note that \( C_B \) is the fraction of the fibers in group B among all fibers in the fabric. When \( C_B = 0.225 \), the experimental values match well with the theoretical values. If this value of \( C_B \) is taken to be the real fraction of the fibers in group B, the validity of the theoretical

†Note that the bending stiffnesses measured for \( \theta = 30^\circ \) and \( 60^\circ \) are approximate values, as discussed in Section 3.5.
FIGURE 4-8. Experimentally measured bending stiffness D versus $W'$. 

- $0^\circ$ 
- $30^\circ$ 
- $60^\circ$ 
- $90^\circ$ 

$D$, Bending Stiffness ($10^{-5}$ N-m) 

$W'$, Areal Density ($10^{-2}$ kg/m$^2$)
FIGURE 4-9. Average experimental results of bending stiffness $D$ plotted as a function of angle $\theta$ of rotation. Also shown is the curve estimated from $D$ at $\theta = 0^\circ$ and $90^\circ$ by the use of Eq. (3.71).
FIGURE 4-10. Bending stiffnesses $D$ in the machine direction computed from theoretical models.
model can be tested in another way. Since the fibers in group B are relatively straight, when the fabric is tensioned in the machine direction, these fibers would respond immediately. They would almost exclusively determine the tensile modulus of the fabric, at least in the early stages of loading. An estimation from $W_2 = 0.039 \text{ kg/m}^2$, $(EA)_f = 1.076 \text{ N}$, $\rho = 1.8 \times 10^{-6} \text{ g/cm}$, and $\sigma = 0.225$ will give the value of the Young's modulus $E$ multiplied by thickness $t$ of the fabric as

$$E_t \approx \frac{W}{\rho} (EA)f \sigma = 4.38 \times 10^4 \text{ N/m}.$$ Considering all the other possible uncertainties, this is a reasonably good estimation in comparison with the tested $E_t = 3.76 \times 10^4 \text{ N/m}$ in the machine direction. Also from Fig. 4-2, this value of $\sigma$ seems to be reasonably estimated.

4.6 Discussion

The relatively straight fibers, or the fibers in group B as modeled, are the prime contributors to the bending stiffness of the unbonded region. The bulky fiber assembly model introduced in Chapter 3 describes in general the fiber behavior in bending. The adhesive-added region appears to be bonded so effectively that delamination of the fabric cannot be easily achieved. Therefore, the built-in end condition at the interphase boundary for the fibers in the unbonded region approximates the end condition assumed in this model.

By making the fibers in group A wavier, the elliptic openings in fact have weakened their resistance to the applied load. Meanwhile, the openings provide space to allow some relative fiber motions. The
parallel fiber groups simulate the yarns of a woven fabric in the sense that they are stressed under tension immediately. In addition, on account of the structural characteristics, when the unbonded region is bent, some relative motions between fibers are still allowed.

According to the theoretical model, more flexibility in the fabric can be effectively obtained by reducing the number of straight fibers. In doing so, the tensile strength of the fabric would probably be reduced. However, as the amount of the straight fibers—decided by the arrangement of the predetermined pattern of openings—is reduced to an optimum, flexible but still strong fabrics can be obtained.
CONCLUSIONS

The investigations of the bending of nonwoven fabrics have revealed certain important parameters affecting the fabric bending stiffness. Although the fabrics studied here have different structures, there are some common principles in controlling their bending flexibilities. It should be mentioned that the parameters will often be interactive, so that a fully quantitative assessment may not be possible. However, their relative quantitative influences in governing the fabric bending stiffness can still be found. Since the fabric properties are determined by (1) fiber arrangement, (2) fiber properties, and (3) bonding condition, the parameters can be characterized in three categories.

1. Parameters Related to the Fiber Arrangement

1.A Areal Density (or Basis Weight) of the Fabric

This is not an absolutely independent variable to alter the fabric bending stiffness. But an increase of the areal density for the same kind of fabric usually means an increase in the thickness and the moment of inertia, to result in a stiffer material. Concerning thin Cerex, bending stiffnesses \( D \) were found approximately proportional to \( W/L \). Since \( 1/L \), the inverse of the free fiber length, monotonically increases with disk weight \( W \), which is proportional to the areal density, \( D \) becomes a power function of \( W \). Thick Cerex yields somewhat linear relations between \( D \) and areal density \( W' \), as long as the fabric
thickness remains the same.

The bending stiffnesses of Fibretex roughly exhibit a rising trend only, with increasing $W'$ in the range of the fabric areal densities. As for Masslinn and Keybak, since their areal densities are uniform, $D$ were found, experimentally and theoretically, to be linear in a small interval of $W'$, if the fiber length $\lambda$ and bond spacing remain unchanged.

1.B Packing Density (or Weight Per Unit Volume) of the Fabric

For the same fabric areal density, various packing densities will lead to various fabric bending stiffnesses. A higher packing density indicates more load carrying fiber members per unit volume and a higher density of fiber contact points. In addition, the degree of freedom of relative fiber motion, desired to increase the fabric flexibility, will be more limited to stiffen the fabric.

1.C Fibers Bundling Into Groups

One way to reduce the fabric bending stiffness and maintain some tensile strength at the same time is to let only a portion of the fibers take the major load. This can be effectively achieved by forming groups of straight and curved fibers. The former group responds to the applied load immediately; a good example of this structural nature is Keybak. A linear relation between the fabric stiffness and the fraction of straight fibers among all fibers is possible.
2. **Parameters Related to Fiber Properties**

2.A **Fiber Crimp**

Even as fibers are slightly crimped, they have a much smaller effective elastic constant than if they were straight. The introduction of high fiber crimp would bring about better fabric flexibility. For instance, Fibretex, composed of such fibers, produces easy deformation for fibers in the bent fabric.

2.B **Fiber Length, i.e., Staple vs. Continuous Fibers**

When fiber webs are print-bonded or thermal spot-bonded, the fiber assemblies are tightened in the unbonded regions. In the case of staple fiber webs, fibers can generally adjust their relative positions to more or less free themselves from the dense packing. In contrast, continuous fibers will be more readily restrained and tightened to result in a higher bending stiffness. Further, the end effects of staple fibers cause them to carry load less effectively than continuous fibers; therefore, more flexible fabrics are produced. The bending stiffnesses of unbonded areas, for Masslinn and Keybak, are proportional to \(1 - \frac{S_2}{\lambda}\) (where \(\lambda\) is the fiber length and \(S_2\) the bond distance) provided all other parameters remain the same.

3. **Parameters Related to the Bonding Condition**

3.A **Junctions of Self-Bonded Nonwovens**

The junction bending mode is rather important to thin fabrics. The junction size—or the size of the contact area between fibers—has a
predominant effect on the fabric bending stiffness. The reduction in the number of junctions $n_J$ will increase $L$, as seen from Eq. (1.51). The bending stiffness, proportional to $W/L$, should vary linearly with $n_J$. But for thick fabrics, this bending mode is negligible. Inevitably, the reduction in the number of junctions will reduce constraints between fibers as well as the bending stiffness of the thick fabric.

3.B **Separate Thermal Spot Bonding of Generally Loose Fibers**

Since spot bonding fuses fibers together, the spot bonds become stiffness raisers in the fabric. Whenever the areal fraction of bonds, dictated by the fabric strength requirement, are reduced, the fabric bending stiffness can be reduced.

3.C **Separate Zone Bonding of Loose Fibers by Latex**

The influence of distance $S_2$ between parallel bonds upon the bending stiffness of unbonded areas, for Masslinn and Keybak, has been mentioned in 2.B. The penetration depth of latex in the bonds is also expected to influence the bending stiffness of the unbonded areas. Approximately, areal fraction $C_2$ of the unbonded area, or $(1-C_1)$, as reflected through the theoretical calculations, is inversely proportional to $D$.

All these discussed parameters are, in fact, interdependent or interactive. The quantitative dependence of the fabric bending stiffness upon individual parameters is by no means as simple as stated, particularly when several parameters are altered simultaneously. For
example, the fabric packing density is a function of the bond spacing, and the wider the bonds are spaced, the less dense the packing will be. Moreover, the packing density will also affect the effective fiber crimp. Among these parameters, the fabric areal density, the fiber length, and the bonds can be more or less individually controlled. The fiber bundling, packing density, and effective fiber crimp, however, are strongly interactive and are influenced by other parameters. As far as the fabric bending flexibility is concerned, almost all parameters are constructively interactive except that bundling of fibers may cause high local packing density and loss of fiber crimp.
SUGGESTIONS FOR FUTURE RESEARCH

1. Spunbonded Nonwovens

From the proposed (self-consistent) model, a reasonable calculation is called for to estimate the bending stiffness of the thick Cerex. The question of a continuous fiber embedded in a bent continuum plate with the \( i \)th segment passing through a cavity can be solved by considering the cavity and the continuous fiber as two free bodies. First, the cavity, in the absence of fiber, can be imagined to be embedded in the bent continuum plate. The displacement field of the cavity, of course, ought to be solved. The fiber introduces a pair of collinear forces \( F_1 \) on opposite sides of the cavity walls to result in a net relative displacement \( \delta_1 \) at the points of the forces. Then, the continuous fiber itself will be assumed to be embedded in the bent plate and is subject to a pair of forces \( F_2 \) at the ends of the \( i \)th segment. Therefore, another net relative end displacement \( \delta_2 \) is obtained. Considering that the segment ends locate on the boundary of the cavity, the requirement for equal displacements and forces yields that

\[
\delta_1 = \delta_2 ; \quad F_1 = F_2 = F_i .
\]

The force \( F_i \) can be solved to give the strain energy \( V_i \) in Eq. (1.68) as

\[
V_i = \frac{F_i^2 L_i}{2(EA)_f} ,
\]

where \( L_i \) is the length of the \( i \)th segment.
2. **Thermal Spot-Bonded Nonwovens**

Obviously, fabrics with fibers of various extent of waviness are worthy of attention. Once the examination is done, the assumption about curved fibers softening the fabric can be tested. Future investigations of the fabric with different bond concentrations and/or arrangement as well as the influence of bonds on the unbonded fibers' packing conditions are highly desirable.

3. **Print-Bonded Nonwovens**

Experimental techniques designed to measure the bending stiffnesses of the bonded and the unbonded regions separately are desirable, in order to verify closely the employed models. Fabrics with various bond distributions can also be investigated to show if there is any significant difference between their bending stiffnesses. A mathematical derivation of the macroscopic bending stiffness, more rigorous than the one presented here, may include the consideration of the twisting stiffness of the bonded region. Further, the study of the effect of bonds with different degree of binder penetration appears to be important.

4. **Print-Bonded Nonwovens with Predetermined Patterns of Openings**

Examinations of fabrics with diverse opening patterns are most desirable, in order to understand more complicated mechanisms involved in bending. In view of the importance of straight fibers in sharing the load, further investigation can be directed to find how the variation of their number influences the bending stiffness of the fabric.
BIBLIOGRAPHY


APPENDIX A

If \( N(\theta) \) is the number of continuous fibers through the entire thickness of the fabric which lie within \( \pm(\Delta \theta/2) \) of any prescribed angle \( \theta \) per unit width perpendicular to the direction of the fibers, the sum total of \( N(\theta) \) over all angles is

\[
N_f = \sum N(\theta) .
\]

Note that \( N_f \) gives the total number of fibers in a circle of unit diameter (unit circle) drawn on the fabric, as shown in Fig. A-1(a). With dimension of \((\text{length}^{-1})\), \( N_f \) is obtained by redistributing all the fibers in the unit circle such that they parallel each other in a two-dimensional plane, shown in Fig. A-1(b). Every fiber maintains its original length in the circle. The number of these fibers per unit width perpendicular to the fibers equals exactly \( N_f \). In a statistical sense, these fibers can be taken as equally spaced with a distance \( 1/N_f \) apart. If the \( i \)th fiber of the \( N_f \) fiber set has a length \( \ell_i \), the area \( A_i \) between the \( i \)th and the \((i+1)\)th fiber is approximately

\[
A_i \approx \ell_i \left( \frac{1}{N_f} \right) .
\]

The summation of \( A_i \) over all the \( N_f \) fibers gives approximately the total area \( A_u \) of the unit circle, i.e.,

\[
A_u = \sum_{i=1}^{N_f} A_i \approx \frac{1}{N_f} \sum_{i=1}^{N_f} \ell_i .
\]
FIGURE A-1. Illustrations of (a) $N_f$ fibers through the thickness of the fabric in a circle of unit diameter; and (b) these $N_f$ fibers being redistributed in the circle to parallel each other in a two-dimensional plane.
Therefore, \( N_f \) is found as

\[
N_f \approx \frac{N_f}{A_u} \sum_{i=1}^{N_f} \lambda_i.
\]  

(A.4)

Since \( \sum \lambda_i \) represents the total length of the fibers in \( A_u \), \( N_f \) in fact means the total fiber length per unit area.

The fiber orientation distribution function \( \phi(\theta) \) (the fraction of total fibers that lie in the direction of \( \theta \)) is defined as

\[
\phi(\theta) = \frac{N(\theta)}{\Delta \theta N_f}.
\]  

(A.5)

As \( \Delta \theta \) approaches zero, \( \phi(\theta) \) becomes

\[
\phi(\theta) = \frac{1}{N_f} \frac{dN}{d\theta}.
\]  

(A.6)

Therefore, it can be concluded that

\[
\frac{dN}{d\theta} = N_f \phi(\theta).
\]  

(A.7)
APPENDIX B

This is a discussion about a fiber in sinusoidally waved form, i.e., \( y = a_0 \sin 2\pi x/\lambda \), subject to tension \( P \) (or compression) along its axis of symmetry, as shown in Fig. B-1. For ease of study, only a segment of the fiber in one wavelength \( \lambda \) interval is considered here, shown in Fig. B-2. In addition to the applied force \( P \), there will be a moment \( M \) at the end of this fiber segment to prevent the ends from rotating. The end displacement \( \delta \) of this segment, together with \( M \), can all be found from Castigliano's theorem.

The strain energy \( \varepsilon_b \) of the bending stress and the strain energy \( \varepsilon_t \) of the tensile stress can be found as

\[
\varepsilon_b = \int_{-\lambda/4}^{3\lambda/4} \left[ \frac{M - P\alpha_0 (1 + \sin \frac{2\pi x}{\lambda})}{2EI} \right]^2 \sqrt{1 + \left( \frac{2\pi \alpha_0}{\lambda} \cos \frac{2\pi x}{\lambda} \right)^2} \, dx
\]

\[
= \frac{2}{EI} (M_0^2 - 2MP\alpha_0 + P_0^2\alpha_0^2) \sqrt{1 + \left( \frac{2\pi \alpha_0}{\lambda} \right)^2} \frac{\lambda}{2\pi} \int_0^{\pi/2} \sqrt{1 - \kappa^2 \sin^2 z} \, dz
\]

\[
+ \frac{2}{EI} P^2\alpha_0^2 \sqrt{1 + \left( \frac{2\pi \alpha_0}{\lambda} \right)^2} \frac{\lambda}{2\pi} \int_0^{\pi/2} \sin^2 z \sqrt{1 - \kappa^2 \sin^2 z} \, dz \quad (B.1)
\]

\[
\varepsilon_t = \int_{-\lambda/4}^{3\lambda/4} \left[ \frac{P}{A} \frac{1}{\sqrt{1 + \left( \frac{2\pi \alpha_0}{\lambda} \cos \frac{2\pi x}{\lambda} \right)^2}} \right]^2 \sqrt{1 + \left( \frac{2\pi \alpha_0}{\lambda} \cos \frac{2\pi x}{\lambda} \right)^2} \, dx
\]

\[
= \frac{2P^2}{EA} \frac{(\lambda/2\pi)}{\sqrt{1 + (2\pi \alpha_0/\lambda)^2}} \int_0^{\pi/2} \frac{1}{\sqrt{1 - \kappa^2 \sin^2 z}} \, dz \quad , \quad (B.2)
\]
FIGURE B-1. Illustration of a fiber in sinusoidally waved form subject to tension P (or compression) along its axis of symmetry.

FIGURE B-2. Illustration of a segment of the waved fiber in one wavelength \( \lambda \) interval.
where

\[ \kappa^2 = \frac{(2\pi\alpha_0/\lambda)^2}{1 + \left(\frac{2\pi\alpha_0}{\lambda}\right)^2} \quad \text{(B.3)} \]

The total strain energy \( \varepsilon \) therefore is

\[ \varepsilon = \varepsilon_b + \varepsilon_t \quad \text{(B.4)} \]

The moment \( M \) can be determined from the condition of no end rotation \( \phi \), i.e.,

\[ \phi = \frac{\partial \varepsilon}{\partial M} = 0 \quad \text{(B.5)} \]

From Eq. (B.1)-(B.5), \( M \) can be found as

\[ M = P\alpha_0 \quad \text{(B.6)} \]

The displacement \( \delta \) can also be obtained as

\[ \delta = \frac{d\varepsilon}{dP} = P \left( \frac{4\alpha_0^2}{EI} \lambda B_1 + \frac{4\lambda}{AE} B_2 \right) \quad \text{(B.7)} \]

where

\[ B_1 = \sqrt{1 + \left(\frac{2\pi\alpha_0}{\lambda}\right)^2} \frac{1}{2\pi} \int_0^{\pi/2} \sin^2 z \sqrt{1 - \kappa^2 \sin^2 z} \, dz \quad \text{(B.8)} \]

\[ B_2 = \frac{1/2\pi}{\sqrt{1 + \left(\frac{2\pi\alpha_0}{\lambda}\right)^2}} \int_0^{\pi/2} \frac{dz}{\sqrt{1 - \kappa^2 \sin^2 z}} \quad \text{(B.9)} \]

This curved fiber segment with extension \( \delta \) given in Eq. (B.7) under tension \( P \) can also be considered as the fiber in a straight state.
under the same load, while its elastic constant $EA$ is replaced by an effective Young's modulus $(EA)'$; i.e.,

\[
(EA)' = \frac{P}{\delta/\lambda} = \frac{EA}{\frac{4A}{I} \alpha_0^2 B_1 + 4B_2}.
\]  

(B.10)

For circular fibers, $A = \pi d^2/4$, $I = \pi d^4/64$, the above equation can be reduced to

\[
(EA)' = \frac{EA}{64 \left( \frac{\alpha_0}{d} \right)^2 B_1 + 4B_2}.
\]  

(B.11)
APPENDIX C

If \( \rho \) is the weight of a fiber per unit length and \( W' \) is the weight of fabric per unit area, the total fiber length \( \lambda \) per unit area is found to be

\[
\lambda = \frac{W'}{\rho} \quad \text{(C.1)}
\]

If the fiber length is \( \lambda \), the total number \( n_f \) of fiber per unit area is

\[
n_f = \frac{\lambda}{\lambda} = \frac{W'}{\lambda \rho} \quad \text{(C.2)}
\]

The number \( n_e \) of fiber ends is

\[
n_e = 2n_f = \frac{2\lambda}{\lambda} = \frac{2W'}{\lambda \rho} \quad \text{(C.3)}
\]

Considering a rectangular area \( A \), as shown in Fig. C-1, with one side of unit length in the transverse direction of the fabric while the other side with length \( S \) in the machine direction, the number \( n_e^{(A)} \) of fiber ends in this area is

\[
n_e^{(A)} = n_e S = \frac{2W'S}{\lambda \rho} \quad \text{(C.4)}
\]

The fibers with ends falling within \( A \) have a statistical average length \( L_e^{(A)} \) in \( A \) given by

\[
L_e^{(A)} = \frac{S}{2} \quad \text{(C.5)}
\]

Therefore, the total length \( \lambda_s^{(A)} \) of the fibers that do not end in \( A \) is given by
\[ \xi_s(A) = \xi_s - L_e n_e(A) = \frac{W'S}{\rho} - \left( \frac{2W'S}{\lambda \rho} \right) \left( \frac{S}{2} \right) = \frac{W'S}{\rho} \left( 1 - \frac{S}{\lambda} \right) S \quad (C.6) \]

The number \( N' \) of fibers without ends in \( A \) is found to be

\[ N' = \frac{\xi_s(A)}{S} = \frac{W'}{\rho} \left( 1 - \frac{S}{\lambda} \right) . \quad (C.7) \]

The number \( N_f \) of fibers going through one or the other of the two sides of \( A \), perpendicular to the fibers, is obtained as

\[ N_f = N' + \frac{\xi_e(A)}{S} = \frac{W'}{\rho} \left( 1 - \frac{S}{\lambda} \right) + \frac{W'S}{\rho \lambda} = \frac{W'}{\rho} \quad (C.8) \]

**FIGURE C-1.** Illustration of fibers with ends in a rectangular area \( A \).
The problem of a curved beam of circular arc subject to end forces \( P \) and end moments \( M \) which produce an end displacement \( \delta \) in the direction of \( P \) and an end rotation \( \phi \) is analyzed here, shown in Fig. D-1.

By symmetry, under the load the center of the arc is taken as being clamped to a rigid wall so that only the half, e.g., right half, of the beam should be considered, as shown in Fig. D-2. As shown in Fig. D-1, the angle swept by the arc with radius \( R \) is \( 2\theta \), and the distance from the center of the chord to the arc in the radial direction is \( f \). This problem can be solved by the energy method. Assuming that the angle \( \alpha \) is measured from the side of the sector shown in Fig. D-2, the strain energy \( \varepsilon_b \) of bending and \( \varepsilon_t \) of tension can be obtained as

\[
\varepsilon_b = \int_0^\theta \frac{\{P\left[ R \sin\left( \frac{\pi}{2} + \alpha - \theta \right) - (R - f) \right] - M\}^2}{2EI} R \, d\alpha \quad (D.1)
\]

\[
\varepsilon_t = \int_0^\theta \frac{\left[ P \cos (\theta - \alpha) \right]^2}{2AE} R \, d\alpha \quad , \quad (D.2)
\]

where \( E,I,A \) are Young's modulus, the moment of inertia, and the cross-sectional area of the fiber, respectively. The total energy \( \varepsilon \) is thus

\[
\varepsilon = \varepsilon_b + \varepsilon_t = \frac{R}{2EI} \left[ PR^2 \left( \frac{\theta}{2} + \frac{\sin 2\theta}{4} \right) - 2P^2R(R - f)\sin \theta + P^2(R - f)^2 \theta 
+ M^2 \theta - 2MPR \sin \theta + 2MP(R - f) \theta \right] + \frac{RP^2}{2AE} \left( \frac{\theta}{2} + \frac{\sin^2 \theta}{4} \right) . \quad (D.3)
\]
FIGURE D-1. Illustration of a curved beam of circular arc subjected to end forces P and moments M.

FIGURE D-2. Illustration of the half of the curved beam.
The end displacement \( \delta \) and the rotation \( \phi \) can be derived from Castigliano's theorem and are given by

\[
\delta = \frac{\partial \epsilon}{\partial P} = P \left\{ \frac{R}{2EI} \left[ R^2 \left( \theta + \frac{\sin 2\theta}{2} \right) - 4R(R-f)\sin \theta + 2(R-f)^2 \theta \right] \right. \\
+ \left. \frac{R}{2AE} \left( \theta + \frac{1}{2} \sin 2\theta \right) \right\} + \left\{ \frac{T}{EI} \left[ -R \sin \theta + (R-f)\theta \right] \right\} M \tag{D.4}
\]

\[
\phi = \frac{\partial \epsilon}{\partial M} = P \left\{ \frac{R}{EI} \left[ -R \sin \theta + (R-f)\theta \right] \right\} + M \left( \frac{R\theta}{EI} \right) \tag{D.5}
\]

The first and second brackets on the right-hand side of Eq. (D.4) can be represented by \( A \) and \( B \), respectively, and the first and second brackets in Eq. (D.5) can be represented by \( C \) and \( D \), respectively. Note that \( A, B, C, \) and \( D \) are all constants determined from the geometry and Young's modulus of the beam. Equations (D.4) and (D.5) can be rewritten as

\[
\delta = AP + BM \tag{D.6}
\]

\[
\phi = CP + DM \tag{D.7}
\]
For $q = \frac{a_1}{2}$, $R = \tan \theta$, $T = \cos \theta$, and $S = \sin \theta$, $A$ is found as

\[
\begin{bmatrix}
-R^3 & -R^2 & 0 & -R & 0 & 0 & R^3 & R^2 & 0 & R & 0 & 0 & 0 & 0 \\
-3R^2q & -2rq & 0 & -q & 0 & 0 & 3R^2q & 2Rq & 0 & q & 0 & 0 & 1 & 0 \\
-3Rq^2 & -q^2 & -R^2 & 0 & -R & -1 & 3Rq^2 & q^2 & R^2 & 0 & R & 0 & 0 & 0 \\
-q^3 & 0 & -2Rq & 0 & -q & 0 & q^3 & 0 & 2Rq & 0 & q & R & 0 & 1 \end{bmatrix}
= A
\]

(E.1)
The matrix $b$ is given by

$$b = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ b_1 M_0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

(E.2)
BIOGRAPHICAL NOTE

The author was born on February 22, 1951 in Taichung, Taiwan, where he attended elementary and high schools. In 1972 he received the Bachelor of Science degree in Mechanical Engineering from National Taiwan University, Taipei, Taiwan. He then served two years in the Chinese Army as a Mechanical Engineer at the 60th Governmental Arsenal. In September 1974, he came to the United States to study applied mechanics in the Department of Engineering and Applied Science of Yale University. A year later he received his Master of Science degree and entered the Department of Mechanical Engineering at MIT to work toward a Ph.D. in the field of mechanical behavior of materials.

He is married to Tsou-ming D. Liu.