November 1991 LIDS-P-2079

STABILITY AND PERFORMANCE IN THE PRESENCE OF MAGNITUDE BOUNDED REAL UNCERTAINTY: RICCATI EQUATION BASED STATE SPACE APPROACHES[1]

by

D. Obradovic
L. Valavani

[1] This research was conducted at the M.I.T. Laboratory for Information and Decision Systems with support provided by AFOSR/Eglin AFB Contract #FO 8635-87-K-0031 and, in part, by a gift from the Boeing Company and by the NASA Ames and Langley Research Centers under grant NASA/NAG 2-297.

STABILITY AND PERFORMANCE IN THE PRESENCE OF MAGNITUDE BOUNDED REAL UNCERTAINTY : RICCATI EQUATION BASED STATE SPACE APPROACHES

DRAGAN OBRADOVIC* and LENA VALAVANI**

Abstract

This paper is concerned with Riccati-equation-based analysis methods for determining stability and performance of perturbed feedback systems. The elements of the state-space representation of the systems are assumed to be linearly perturbed with real, magnitude-bounded uncertainties. Performance is defined as the value of the H_∞ norm of the transfer function of interest. An analysis method based on checking the existence of a common solution to a finite set of Riccati inequalities is proposed; it represents a nonconvex optimization problem. This problem is then transformed into a convex optimization problem by the use of properties of certain matrix inequalities.

1. Introduction

In this paper we discuss stability and performance robustness analysis of the closed-loop system where the elements of its state-space representation contain real uncertainty. The real uncertainty is assumed to be magnitude-bounded and to enter all elements of the state space representation linearly. Furthermore, the performance specification is defined as the bound on the H_∞ norm of a certain transfer function, a bound that should remain

---

* Laboratory for Information and Decision Systems, Massachusetts Institute of Technology, Cambridge, MA 02139, U.S.A.

** Department of Aeronautics and Astronautics, Massachusetts Institute of Technology, Cambridge, MA 02139, U.S.A.

satisfied in the presence of uncertainty.

It is known that in such a case the stability and performance condition for the nominal closed-loop system is equivalent to the existence of a solution to a particular Riccati equation (Doyle et al,1988). In the presence of real uncertainty in the state space representation of the system, the resulting Riccati equation will contain additional perturbation terms. Thus, the stability and performance of the perturbed system becomes equivalent to the existence of individual solutions to the infinite set of Riccati equations, each of which corresponds to one possible combination of values of uncertain parameters. A commonly used approach in dealing with such a problem is to construct a so-called "quadratic" function (Peterson and Hollot,1986), that bounds from above the perturbed terms within the Riccati equation. The performance and stability criterion for the perturbed system then becomes equivalent to the existence of a solution to a single Riccati equation that contains the quadratic bound. Unfortunately, this approach has been shown (Khargonekar, Peterson, and Zhou,1990), (Obradovic,1990) to be as conservative as the "Small Gain Theorem" condition (Zames,1965) for the same perturbed system. This conservatism implies that real parameter perturbations are treated as frequency dependent ones .

In this paper we present a less conservative stability and performance criterion, one that requires no quadratic bounding functions. This criterion is based on the existence of a common solution P for a set of Riccati-type inequalities. The criterion is obtained by extending Horisberger and Belanger's (1976) results concerning Lyapunov stability (with respect to linear perturbation in the "A" matrix only), to the case where the performance specification is included and where perturbations enter all parts of the state space representation of the system. In Horisberger and Belanger (1976), the search for a common solution to a set of Lyapunov equations is posed as a convex optimization problem. An extension of this result to the set of Riccati inequalities introduces optimization which is nonconvex in P due to the nonlinear term in the Riccati expression.

The further contribution of this paper is in the construction of an equivalent optimization problem, which is convex in P, but of a higher dimension. This construction is achieved without introducing an auxiliary passive system as it was done in (Boyd and Yang,1988).

2. Stability and Performance in the Presence of Magnitude Bounded Real Uncertainty

Let the closed loop system $M(s,\mathbf{q})$ be defined as follows :

$$M(s,\mathbf{q}) = C(\mathbf{q})\,[\,sI - A(\mathbf{q})\,]^{-1}\,B(\mathbf{q}) \qquad A(\mathbf{q}) \in R^{nxn} \tag{2.1}$$

where $\mathbf{q}$ is the vector of the real parameter perturbations which belong to the set Ω,

$$\Omega = \{\ \mathbf{q} \quad \text{s.t.} \quad \mathbf{q} = [\,q_1, \ldots, q_m\,]' \in R^m \quad |\,q_i\,| \le r_i\ , \quad r_i \in R_+\ \} \tag{2.2}$$

Therefore, the uncertainty belongs to a "hyperbox" Ω in parameter space. In the case of m = 2, the "hyperbox" is depicted in Figure 1.

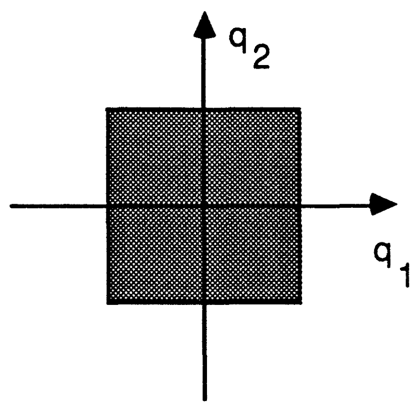

Figure 1. Parameter Box in Two-Dimensional Parameter Space

Furthermore, let us assume that the uncertain parameters enter the elements of the state space representation of the system linearly, i.e.

$$A(\mathbf{q}) = A_o + \sum_{1}^{m} q_i E_i = A_o + \Delta A \qquad (2.3)$$

$$B(\mathbf{q}) = B_o + \sum_{1}^{m} q_i F_i = B_o + \Delta B \qquad (2.4)$$

$$C(\mathbf{q}) = C_o + \sum_{1}^{m} q_i G_i = C_o + \Delta C \qquad (2.5)$$

where $q_i \in \Omega$ is defined in (2.2) and $[A_o, B_o, C_o, 0]$ is the state space representation of the nominal system $M(s,\mathbf{q}=0)$. The matrices E_i, F_i, and G_i define the structure of uncertainty with respect to each parameter perturbation q_i. They can always be scaled in such a way that $r_i = 1$ for $i = 1, 2, ..., m$.

The nominal system, obtained for $\mathbf{q} = 0$, is assumed to be stable and with the infinity norm $\| M(s,\mathbf{q}=0) \|_\infty < \gamma$ where $\| M(s,\mathbf{q}=0) \|_\infty := \sup_\omega \bar{\sigma}(M(j\omega,\mathbf{q}=0))$. This is satisfied if the following holds:

Lemma 2.1

Let $M(s,\mathbf{q}=0) = Co\,[\,sI - Ao\,]^{-1}\,Bo$, (Ao, Bo) stabilizable and (Co, Ao) detectable. Then, the necessary and sufficient condition for $M(s,\mathbf{q}=0) \in RH_\infty$ and $\|M(s,\mathbf{q}=0)\|_\infty < \gamma$ is the existence of $P=P'$ s.t.

i) $P \geq 0$

ii) $Ao'P + PAo + (1/\gamma)^2 PBoBo'P + Co'Co = 0$

iii) $[\,Ao + (1/\gamma)^2 BoBo'P\,]$ is stable $\qquad$ (2.6)

Proof : It is presented in (Doyle et al,1988).

■

The first two conditions i) and ii) are sufficient for $\| M(s,\mathbf{q}=0)\|_\infty \le \gamma$. The third condition is needed for the strict inequality i.e. $\| M(s,\mathbf{q}=0)\|_\infty < \gamma$.

Corollary 2.1.1

Let $M(s,\mathbf{q}=0) = Co[sI-Ao]^{-1}Bo$. Then, a sufficient condition for $M(s,\mathbf{q}=0) \in RH_\infty$ and $\|M(s,\mathbf{q}=0)\|_\infty < \gamma$ is the existence of $P=P'>0$ s.t.

$$Ao'P + PAo + \left(\frac{1+\varepsilon}{\gamma}\right)^2 PBoBo'P + Co'Co < 0 \tag{2.7}$$

where ε is an arbitrarily small positive number.

Proof :

The strict inequality in (2.7) implies that $\exists\, Q > 0$ s.t.

$$Ao'P + PAo + \left(\frac{1+\varepsilon}{\gamma}\right)^2 PBoBo'P + Co'Co + Q = 0 \tag{2.8}$$

This guarantees that the system $M_1 = [\, Co \;\; Q^{1/2}\,]\,[\, sI - Ao\,]^{-1}\,[(1+\varepsilon)Bo]$ has the following properties :

i) $(\, Ao, [\, Co \;\; Q^{1/2}\,]\,)$ is observable since $(\, Ao, \;\; Q^{1/2}\,)$ is. The latter is true because $Q^{1/2}$ is a square matrix with full rank.

ii) Ao is a stable matrix. This follows from the Lyapunov stability criterion since $P>0$ and (2.8) can be rewritten as the following Lyapunov equation :

$$Ao'P + PAo + Q_1 = 0 \tag{2.9}$$

where

$$Q_1 = \left(\frac{1+\varepsilon}{\gamma}\right)^2 PBoBo'P + Co'Co + Q > 0 \tag{2.10}$$

iii) $\| M_1(jw) \|_\infty \le \gamma$. This result follows from (2.8) and Lemma 2.1 since

Ao is stable. On the other hand, it implies the following :

$$\| M(jw) \|_\infty < | 1 + \varepsilon | \ \| M(jw) \|_\infty \le \| M_1(jw) \|_\infty \le \gamma \qquad ; \ \varepsilon > 0 \qquad (2.11)$$

This completes the proof.

■

Our goal is to derive conditions under which the system remains stable and with norm smaller than γ for all $\mathbf{q} \in \Omega$. Once this is done, their conservatism will be studied and the connections with already existing results established.

According to Lemma 2.1 and its corollary, a sufficient condition for stability and norm boundedness of the system $M(\mathbf{q})$ in the hyperbox is given in the following lemma.

Lemma 2.2

Let $M(\mathbf{q})$ be defined as in (2.1) : (2.5). If $\exists$ $P=P' \in \mathbb{R}^{nxn}$ s.t. $P > 0$, $\varepsilon > 0$, and

$$R(\mathbf{q}) := A(\mathbf{q})'P + PA(\mathbf{q}) + \left(\frac{1+\varepsilon}{\gamma}\right)^2 PB(\mathbf{q})B(\mathbf{q})'P + C(\mathbf{q})'C(\mathbf{q}) < 0 \qquad \forall \ \mathbf{q} \in \Omega \qquad (2.12)$$

then $\| M(\mathbf{q}) \|_\infty < \gamma \quad \forall \ \mathbf{q} \in \Omega$.

Proof :

This statement is the extension of the result in Corollary 2.1.1 to all the points in the hyperbox Ω with a common matrix $P=P'$.

■

The condition in (2.12) represents an infinite dimensional problem because it requires a common $P=P'$ for all the points in the hyperbox Ω which is an uncountable set. To make this approach useful we have to try to reduce its dimensionality, i.e. to make it finite dimensional. We now show that the existence of a matrix $P=P'$ s.t. $R(\mathbf{q})$ is affine with respect to $\mathbf{q} \in \Omega$ and negative definite at all the vertices of the hyperbox is sufficient to achieve condition (2.12) for all $\mathbf{q} \in \Omega$. This can be seen as an extension of the result for Lyapunov functions presented in (Horisberger and Belanger,1976).

Lemma 2.3

Let the function $R(\mathbf{q})$ defined in (2.12) be affine with respect to $\mathbf{q} \in \Omega$ for every $P=P'>0$. Then, $R(\mathbf{q}) < 0 \quad \forall\, \mathbf{q} \in \Omega$ if $\exists\ P=P'>0$ s.t. $R(\mathbf{q}_j) < 0$, $j = 1, 2, \dots, 2^m$, where $\mathbf{q}_j$ corresponds to the "j'th" vertex of the hyperbox Ω.

Proof:

The hyperbox Ω is a convex set in parameter space. The function $R(\mathbf{q})$ is affine over the same set. Therefore, for any $\mathbf{q} \in \Omega$ we have:

$$R(\mathbf{q}) \le \sum_{1}^{2^m} \alpha_j R(\mathbf{q}_j) ; \qquad \sum_{1}^{2^m} \alpha_j = 1 ; \quad \alpha_j \ge 0 \tag{2.13}$$

Hence, $R(\mathbf{q}) < 0$ for all $\mathbf{q} \in \Omega$ since $R(\mathbf{q}_j) < 0$. This completes the proof.

■

The previous lemma requires that $R(\mathbf{q})$ be a linear affine function for every $P=P'>0$. The overall expression for $R(\mathbf{q})$, with the perturbation defined as in (2.3) - (2.5), has the following form :

$$R(\mathbf{q}) = R_o + \sum_{1}^{m} q_i \left[(E_i'P + PE_i) + \left(\frac{1+\varepsilon}{\gamma}\right)^2 P(F_i B_o' + B_o F_i')P + (G_i'C_o + C_o'G_i) \right] +$$

$$+ \left(\frac{1+\varepsilon}{\gamma}\right)^2 P\left(\sum_{1}^{m} q_i F_i\right) \left(\sum_{1}^{m} q_i F_i'\right)P + \left(\sum_{1}^{m} q_i G_i'\right) \left(\sum_{1}^{m} q_i G_i\right) \tag{2.14}$$

where $R_o := A_o'P + PA_o + \left(\frac{1+\varepsilon}{\gamma}\right)^2 PB_oB_o'P + C_o'C_o$. The last two elements in the above expression are the only ones where the parameter perturbation appears in nonlinear form. If they could be bounded from above with a function linear in q_i, the resulting $R(\mathbf{q})$ would be linear affine in $\mathbf{q}$. Possible linear bounds are presented as follows :

$$\Delta C'\,\Delta C = \left(\sum_{1}^{m} q_i\, G_i'\right)\left(\sum_{1}^{m} q_i\, G_i\right) \le \left(\sum_{1}^{m} \bar{\sigma}(G_i)\right)^2 I = \alpha_C^2\, I \tag{2.15}$$

where $|\, q_i \,| \le 1$. Furthermore,

$$\alpha_B^2\, I \ge \Delta B\, \Delta B' \tag{2.16}$$

can be obtained in an analogous way. There are possibly different methods that can be used for bounding quadratic perturbation terms. An alternative method based on properties of positive matrices was used in (Yeh et al,1989) where all the quadratic and linear terms in ΔB and ΔC were bounded from above.

The function $R(\mathbf{q})$ can now be bounded from above with some $R_1(\mathbf{q})$ defined as :

$$R_1(\mathbf{q}) = R_o + \sum_{1}^{m} q_i\left[\, (E_i'P+PE_i) + \left(\frac{1+\varepsilon}{\gamma}\right)^2 P(F_i\, B_o' + B_oF_i')P + (G_i'C_o + C_o'G_i)\,\right] +$$

$$+ \left(\frac{1+\varepsilon}{\gamma}\right)^2 \alpha_B^2\, P\, P + \alpha_C^2\, I = R_o + \sum_{1}^{m} q_i\, \Delta R_i + \left(\frac{1+\varepsilon}{\gamma}\right)^2 \alpha_B^2\, P\, P + \alpha_C^2\, I \ge R(\mathbf{q}) \tag{2.17}$$

This function is linear affine in $\mathbf{q}$ for every $P=P'$. Therefore, we have managed to construct a function that bounds $R(\mathbf{q})$ from above and is affine in $\mathbf{q}$. By using the result of Lemma 2.3, we can formulate the following.

Corollary 2.3.1

Let the system be described as in Lemma 2.3. If $\exists\ P=P'>0$ s.t. $R_1(\mathbf{q},P) < 0$ at every vertex of the hyperbox in parameter space. Then the perturbed system $M(\mathbf{q})$ is stable and $\|M(\mathbf{q})\|_\infty < \gamma$ for all $\mathbf{q} \in \Omega$.

Proof:

Stability of the system in the hyperbox is guaranteed from a Lyapunov type argument. For every $\mathbf{q} \in \Omega$ the $R_1(\mathbf{q},P)$ can be rewritten as the Lyapunov equation

$$A(\mathbf{q})'P + P\,A(\mathbf{q}) + Q_1(\mathbf{q}) = 0 \qquad P,\, Q_1(\mathbf{q}) > 0 \tag{2.18}$$

as was done in (2.9). This implies stability of $A(\mathbf{q})$.

It is important to notice that, by having the strict inequalities in $P>0$ and $R_1(\mathbf{q},P)<0$, there was no need to require stabilizability and detectability of $(A(\mathbf{q}),B(\mathbf{q}))$ and $(C(\mathbf{q}),A(\mathbf{q}))$ for every $\mathbf{q} \in \Omega$ in order to check stability of $A(\mathbf{q})$.

The above conditions also guarantee that $\|M(s,\mathbf{q})\|_\infty < (1+\varepsilon)\,\|M(s,\mathbf{q})\|_\infty \le \gamma \quad \forall\ \mathbf{q} \in \Omega$. This completes the proof.

■

The previous statement establishes the condition for the norm boundedness of the system in the parameter box in terms of the existence of a symmetric P that simultaneously satisfies inequalities at vertices. This represents an important result because it reduces an infinite dimensional problem to a finite dimensional one.

Let $R_1(\mathbf{q})$ at the vertex "j" be given as :

$$\begin{aligned} R_1(\mathbf{q}_j) = A'(\mathbf{q}_j)P + PA(\mathbf{q}_j) + \left(\frac{1+\varepsilon}{\gamma}\right)^2 PB_oB_o'P + C_o'C_o + \left(\frac{1+\varepsilon}{\gamma}\right)^2 PW(\mathbf{q}_j)P\ + \\ + Y(\mathbf{q}_j) + \left(\frac{1+\varepsilon}{\gamma}\right)^2 \alpha_B^2\, P\,P + \alpha_C^2\, I \end{aligned} \tag{2.19}$$

The expression in (2.19) can be rewritten in the form of an ordinary Riccati equation as :

$$R_1(\mathbf{q}_j) = A'(\mathbf{q}_j)P + PA(\mathbf{q}_j) + \left(\frac{1+\varepsilon}{\gamma}\right)^2 P\tilde{B}(\mathbf{q}_j)\tilde{B}(\mathbf{q}_j)'P + \tilde{C}(\mathbf{q}_j)'\tilde{C}(\mathbf{q}_j) \qquad (2.20)$$

where $\tilde{B}(\mathbf{q}_j)$ and $\tilde{C}(\mathbf{q}_j)$ are augmented input and output matrices. They are defined as follows:

$$\tilde{B}(\mathbf{q}_j) = [\, B_o \quad W(\mathbf{q}_j)^{0.5} \quad \alpha_B I \,] \quad \text{and} \quad \tilde{C}(\mathbf{q}_j) = \begin{bmatrix} C_o \\ Y(\mathbf{q}_j)^{0.5} \\ \alpha_C I \end{bmatrix} . \qquad (2.21)$$

The condition defined in Corollary 2.3.1 requires a single matrix P=P' which simultaneously satisfies 2^m Riccati inequalities. However, there are some situations where the requirement can be further simplified.

Remark

It is easy to see that, if the matrices ΔR_i as defined in (2.17) are definite in sign for the given P, then there is a single Riccati equation corresponding to one of the vertices whose solution would simultaneously satisfy the remaining inequalities.

A possible way to come up with a matrix P that satisfies inequalities $R_1(\mathbf{q}_j) < 0$, $j=1, \ldots, 2^m$ for a fixed magnitude of perturbation, is to formulate the following minimization problem

$$\min_{P=P'} \quad \lambda_{max}[\, \text{diag} \{ R_1(\mathbf{q}_j), -P \}] \quad j = 1, 2, \ldots, 2^m \qquad (2.22)$$

where λ_{max} is the largest eigenvalue. This approach was originally presented in (Horisberger and Belanger,1976) for the Lyapunov inequalities. If λ_{max} is negative, then the system will remain stable and with infinity norm smaller than γ.

Unfortunately, the function (2.22) is not convex in P as it was originally obtained in

(Horisberger and Belanger,1976) by using Lyapunov inequalities. We now transform the original problem into a convex one with respect to the matrix P. This is shown in the following lemma.

Lemma 2.4

A matrix $P = P' > 0$ satisfies

$$A'P + PA + \left(\frac{1+\varepsilon}{\gamma}\right)^2 P\tilde{B}\tilde{B}'P + \tilde{C}'\tilde{C} < 0$$

iff the following holds for all $\alpha \in R_+$

$$T = \begin{bmatrix} A'P + PA + \alpha^{-1}\tilde{C}'\tilde{C} & \left(\frac{1+\varepsilon}{\gamma}\right) P\tilde{B} \\ \tilde{B}'P\left(\frac{1+\varepsilon}{\gamma}\right) & -\alpha^{-1} I \end{bmatrix} < 0 \tag{2.23}$$

Proof :

The matrix T can be rewritten as :

$$\begin{bmatrix} A'P + PA + \alpha\left(\frac{1+\varepsilon}{\gamma}\right)^2 P\tilde{B}\tilde{B}'P + \alpha^{-1}\tilde{C}'\tilde{C} & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} -\alpha\left(\frac{1+\varepsilon}{\gamma}\right)^2 P\tilde{B}\tilde{B}'P & \left(\frac{1+\varepsilon}{\gamma}\right) P\tilde{B} \\ \tilde{B}'P\left(\frac{1+\varepsilon}{\gamma}\right) & -\alpha^{-1} I \end{bmatrix} < 0 \tag{2.24}$$

Let the first matrix be T1 and the second T2, s.t. $T = T1 + T2$. It is easy to show that T2 is always negative semidefinite whenever $P > 0$.

The sufficiency is proved by assuming $T < 0$. At the same time $T2 \leq 0$ with the null space defined as $[\, x' \quad P\tilde{B}\, x' \,]'$ where $x \in R^n$. Therefore, for every nonzero vector in the null space of T2 and, thus, for all $x \in R^n / x \neq 0$ we require

$$x' \, [A'P + PA + \alpha \left(\frac{1+\varepsilon}{\gamma}\right)^2 P\tilde{B}\tilde{B}'P + \alpha^{-1} \tilde{C}'\tilde{C}] \, x < 0. \quad \forall x \in R^n / \, x \neq 0 \qquad (2.25)$$

Hence, the Riccati expression in T1 is negative definite if $T<0$.

On the other hand, the negative definiteness of the Riccati equation (2.25) results in $T1 \leq 0$ which, together with the negative semidefiniteness of T2 for $P>0$, guarantees that $T \leq 0$. We now show that this is a strict inequality, i.e. that $y'Ty < 0$ for every nonzero $y \in R^{2n}$.

For every vector $y \in R^{2n}$ that doesn't belong to the null space of T2, we have $y'T2y < 0$ and, therefore, $T<0$. At the same time, for every vector y belonging to the null space of T2 i.e., $y=[\, x' \quad P\tilde{B}\, x' \,]'$ where $x \in R^n$, we have $y'Ty = x'[Ricc]x < 0$ which implies $T<0$. Therefore, negative definiteness of the Riccati equation in (2.25) guarantees $y'Ty<0$ for every nonzero vector $y \in R^{2n}$.

This completes the proof.

■

By applying Lemma 2.4, the convexity in P is achieved without constructing an auxiliary passive system as it was done in (Boyd and Yang,1988).

Remark

The matrix T being negative definite implies the following :

$$[\, x' \, u'] \, T \, [\, x' \, u']' < 0 \qquad \text{for every} \qquad x \in R^n \;, \quad u \in R^s \qquad (2.26)$$

The expression in (2.26) is equivalent to:

$$2\, x'\, P\, [\, Ax + \left(\frac{1+\varepsilon}{\gamma}\right) \tilde{B}u\,] < u'u - y'y \tag{2.27}$$

where $\qquad dx/dt = Ax + \left(\frac{1+\varepsilon}{\gamma}\right) \tilde{B}u \quad ; \qquad y = \tilde{C}x.$

If the former inequality is integrated from $to = 0$ to $t = \tau$ with $x(to) = 0$ and $u(to) = y(to) = 0$, we have :

$$0 \le V(x) = x'\, P\, x < \int_0^\tau u'u\, dt - \int_0^\tau y'y\, dt \tag{2.28}$$

where V(x) is a Quadratic Lyapunov function associated with the system M_a=[A, $\left(\frac{1+\varepsilon}{\gamma}\right) \tilde{B}$, $\tilde{C}$, 0]. This system has its infinity norm strictly smaller than one since, at any point in time, the energy at its output is smaller than the energy at the input. Therefore, ε=0 can be used in this case.

■

According to the previous lemma, the minimization of the maximal eigenvalue of the matrix $T(\mathbf{q}_j)$, corresponding to a vertex "j", implies minimization of the associated Riccati inequality. Therefore, the minimization process in (2.22) is transformed into the following:

$$\min_{P=P'} \quad \lambda_{max}[\, diag\, \{\, T(\mathbf{q}_j), -P\, \}] \quad j = 1, 2,..., 2^m \tag{2.29}$$

where $\lambda_{max}[\, diag\, \{\, T(\mathbf{q}_j), -P\, \}]$ is a convex function. This transformation is done at the expense of dimensionality of the matrix inequality being minimized. Thus, if the dimension of a Riccati inequality in (2.22) is nxn, the dimension of the corresponding matrix T will be 2nx2n.

The minimization in (2.29) is done by searching for individual entries of the matrix P.

The set of all symmetric matrices P is a convex set. Furthermore, it can be scaled without loss of generality, to the set of all symmetric matrices P whose entries have bounded magnitude, i.e. each $|P_{kl}| \leq 1$, k= 1,..., n, l = 1,..., n. This scaling is possible since, for the fixed vertex "j" and with P>0, the inequality $R_1(q_j, A, \tilde{B}, \tilde{C}, P) < 0$ in (2.20) holds if and only if $R_1(q_j, A, \beta^{1/2}\tilde{B}, \beta^{-1/2}\tilde{C}, \beta^{-1}P) < 0$ for all scalar $\beta \in R_+$. Then, Lemma 2.4 implies that $T(q_j, A, \tilde{B}, \tilde{C}, P) < 0$ if and only if $T(q_j, A, \beta^{1/2}\tilde{B}, \beta^{-1/2}\tilde{C}, \beta^{-1}P) < 0$. Furthermore, the H_∞ norms of the systems $[A,\tilde{B},\tilde{C},0]$ and $[A, \beta^{1/2}\tilde{B}, \beta^{-1/2}\tilde{C}, 0]$ are the same implying that the performance condition is not violated by the scaling of P. A very good and detailed survey of different optimization methods for solving the type of problems which include the one defined in (2.29) can be found in (Boyd and Yang,1988).

According to (2.29), a system with MacMillan degree equal to n will require determining n(1+n)/2 different elements of P. Therefore, a system of the fifth order and 4 uncertain parameters will give rise to j=16 matrix inequalities in (2.29) while the number of unknown elements in P=P' will be 15. This illustrates the complexity of this type of approach even for relatively low order systems with few uncertain parameters. On the other hand, this method may produce less conservative results than the "quadratic" bounding function approach that has been shown to be as conservative as the "Small Gain Theorem" condition for the same system.

3. Conclusions

An analysis method based on the Riccati-type condition for stability and infinity norm boundedness of a system perturbed with real uncertainty was discussed. The perturbation was assumed to be magnitude-bounded and to enter linearly all the elements of the state space description of the system. By linearizing the nonlinear "perturbation" terms in the

Riccati expression, a condition based on simultaneous satisfaction of a finite set of Riccati inequalities was established. The search for a common solution to the set of Riccati inequalities corresponding to the vertices of the hyperbox in the parameter space was then posed as a nonconvex optimization problem. In order to achieve convexity with respect to the matrix P=P', the latter was transformed into an equivalent convex optimization by the use of particular matrix inequality.

References

Boyd S. and Q. Yang (1988). Structured and Simultaneous Lyapunov Functions for System Stability Problems, *Techn. Report No. L-104-88-1,* Stanford University.

Doyle J.C., K. Glover, P. Khargonekar, and B. Francis (1988). State Space Solutions to Standard H_2 and H_∞ Control Problems, *Proc. Amer. Cont. Conf.*, pp. 1691-1700, Atlanta, GA.

Horisberger H.P. and P.R. Belanger (1976). Regulators for Linear, Time Invariant Plants with Uncertain Parameters, *IEEE Trans. Auto. Cont.,* AC-21, pp. 705-708.

Khargonekar P.P., I.R. Petersen, and K. Zhou (1990). Robust Stabilization of Uncertain Systems and H_∞ Optimal Control, *IEEE Trans. Auto. Cont.*, vol. 35, No. 3, pp. 256-261.

Obradovic D. (1990). *Design of a Control System for Postfailure Operation*, Ph.D. Thesis, Mech. Eng. Dept., MIT, Cambridge, MA.

Petersen I.R. and C.V. Hollot (1986). A Riccati Equation Approach to the Stabilization of Uncertain Linear Systems, *Automatica*, vol. 22, No. 4, pp. 397-411.

Yeh H., S.S. Banda, A.C. Bartlett, and S.A. Heise (1989). Robust Design of Multivariable Feedback Systems with Real Parameter Uncertainty and Unmodelled Dynamics, *Proc. Amer. Cont. Conf.*, Pittsburgh, PA.

Zames G. (1965). Nonlinear Time-Varying Feedback Systems - Condition for L_∞ Boundedness Derived Using Conic Operators on Exponentially Weighted Spaces, *Proc. 3rd. Allerton Conf.*