A GENERALIZED REAL OPTIONS METHODOLOGY FOR EVALUATING INVESTMENTS UNDER UNCERTAINTY WITH APPLICATION TO AIR TRANSPORTATION

by

BRUNO MILLER

S. B. Aeronautics and Astronautics – Massachusetts Institute of Technology, 1998
M.S. Aeronautics and Astronautics – Massachusetts Institute of Technology, 2000
M.S. Technology and Policy – Massachusetts Institute of Technology, 2001

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Author: ______________________
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Certified by: ______________________
John-Paul Clarke, Associate Professor of Aeronautics and Astronautics
Thesis Supervisor

Certified by: ______________________
Amedeo Odoni, Professor of Aeronautics and Astronautics
& Civil and Environmental Engineering
Thesis Committee Member

Certified by: ______________________
R. John Hansman, Professor of Aeronautics and Astronautics
Thesis Committee Member

Certified by: ______________________
Eric K. Clemons, Professor of Operations and Information Management
The Wharton School of the University of Pennsylvania
Thesis Committee Member

Accepted by: ______________________
Jaime Peraire, Professor of Aeronautics and Astronautics
Chair, Committee on Graduate Students

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ABSTRACT

Real options analysis is being increasingly used as a tool to evaluate investments under uncertainty; however, traditional real options methodologies have some shortcomings that limit their utility, such as the use of the geometric Brownian motion to model the value of the underlying asset and the assumption of a fixed cost to exercise the option. In this thesis, an alternative real options methodology is developed that overcomes some of the difficulties of traditional approaches. In particular, the methodology proposed here presents an analytical framework that allows the value of completion and the strategy-enabling completion cost (commonly referred to as stock price and strike price in the real options literature, respectively) to be represented by any probability distribution. If these probability distributions can be described analytically, an exact solution to the real options valuation problem can be found. Otherwise, the probability distributions can be generated with numerical simulation (e.g. Monte Carlo simulation), and the answer can then be found numerically. This generalized methodology combines the simplicity of analytical approaches with the flexibility to represent completion costs and the value of completion with any probability distribution.

The generalized real options methodology is illustrated with an example from aviation: the decision to launch a new aircraft development program. This type of investment is suitable for real options analysis because of the many uncertainties involved, the long-term nature of the project, and the ability of management to act and influence the project as uncertainties are resolved during its evolution. The analysis shows that investors can use the numerical results of the real options evaluation to determine the investment limits on the different stages of the aircraft program, that managers can use insights from the real options approach to restructure the program to improve the financial feasibility of the project, and that both investors and managers can use the output of derivative analyses to define minimum requirements (in terms of aircraft orders) to ensure program success.

Thesis Supervisor: John-Paul Clarke
Title: Associate Professor of Aeronautics and Astronautics

This thesis is dedicated to:

José Luis Esquivel Cooper

Bruno Miller Lacher

Aida María Esquivel de Miller

For their joy of living, their service to others, their strength of character, and their exceptional wisdom have guided and nurtured my spirit.

May their example be an inspiration for us all.

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Abbreviations

AA: American Airlines

ASM: Available-seat-mile

ATC: Air Traffic Control

CAPM: Capital Asset Pricing Model

CEQ: Certainty Equivalent

CNS: Communication, Navigation, and Surveillance

DA: Decision Analysis

DCF: Discounted Cash Flow

GBM: Geometric Brownian Motion

GDP: Gross Domestic Product

MEANS: MIT Extensible Air Network Simulation

MIT: Massachusetts Institute of Technology

MU: Monetary Units

NAS: National Airspace System

NPV: Net Present Value

R&D: Research and Development

RPM: Revenue-passenger-mile

RO: Real Options Analysis

SATS: Small Aircraft Transportation System

UA: United Airlines

UPS: United Parcel Service

VLA: Very Large Aircraft

WACC: Weighted Average Cost of Capital

1. Introduction

1.1 Introduction to real options

The owner of a real option has the right, but not the obligation, to take an action involving the purchase or sale of a real asset in the future, at a price. Real options are therefore enablers of phased investments: a partial expenditure today gives an investor the opportunity to complete the investment at a later date if conditions are favorable, or cancel it otherwise. Thus, by purchasing the option, the investor can wait and learn more about the state of the world before fully committing. In addition, the investor can, by virtue of the development afforded by the initial investment, react quickly if and when the decision is made to continue with the investment. The flexibility and the improved speed of reaction provided by real options can be valuable in uncertain operating environments. Indeed, acquiring this flexibility is the reason why real options are purchased and it is the source of their value.

For example, consider a city that is interested in building a new airport. Assume further that current levels of demand require only one runway, but there are indications that future demand will grow to levels where a second runway will be necessary. A prudent strategy would be to build one runway and, at the same time, acquire the land for the second runway so that the city can readily build the second runway should the traffic levels require it.

Ownership of the land for the second runway gives the airport developers the right, but not the obligation, to expand capacity if and when it is needed. In this manner, capacity can be provided sooner than in a case where one runway was built but no land was purchased for the second runway, thus increasing the likelihood that the second runway would be better timed with the needs of the market. Another approach would be to build both runways now; however, given uncertainties in demand, there is a risk that the second runway may not be needed. The option to build the second runway offers protection against this situation.

The same general flexible structure can be used for many other projects. For example, an aircraft manufacturer may use a development process as a means of testing the viability of a new aircraft model. The development process provides a real option: if at the end of development there are positive signals from the market with respect to the new aircraft, the manufacturer can start production of the new model; however, if the outcome is not promising, the manufacturer can cancel the project and the losses would be limited to the resources invested in development. By investing in development, the aircraft manufacturer puts itself in a position where it can produce the aircraft that the market desires and thereby protect or perhaps increase its market share by outpacing its competitors.

Real options analysis (RO) has been used for many years to evaluate investments under uncertainty. The first mention of the concept of real options can be traced back to a paper on corporate borrowing written in 1977 by Stewart Myers [Myers, 1977]. Since then, RO has been used in many fields, including the oil industry and other tradable commodities [Kulatilaka, 1993; Tufano and Moel, 1997; Paddock et al., 1988], the airline industry

[Markish and Willcox, 2002; Stonier, 1999], transportation regulation [Hausman and Myers, 2002], real estate [Childs et al., 1996; Geltner, 1989], functional uses of space in buildings [Greden and Glicksman, 2005], and business strategy [Amram and Kulatilaka, 1999; Dixit and Pindyck, 1994; Trigeorgis, 1996].

Traditional real options analysis techniques are based on the theory of financial options. In financial options, the holder of the option has the right, but not the obligation, to purchase an asset (call option) or to sell an asset (put option). This asset is called the underlying asset and it is typically the stock upon which the option is written. Its value is given by the price of the stock, as determined by its valuation in the stock market. In a call option, the investment required to buy the underlying asset is called the strike price. Similarly, in a put option, the strike price is the benefit that the owner of the underlying asset receives when the underlying asset is sold.

The terminology of financial options, i.e., underlying asset, strike price, and stock price, is typically used in the real options literature to refer to the different components of real options. In this thesis, however, a different terminology that is more intuitive to the physical meaning of the elements that constitute a real option has been adopted. This terminology is similar to the one used by Clemons and Gu (2003). For example, the underlying asset is the project that is the purpose of the investment. It can be many things, tangible or intangible, e.g., a new product, a capital investment project, or a business strategy. One can think of the underlying asset as an underlying objective function, which the real options analysis is trying to maximize. The value of the underlying asset that the holder of the real option acquires by exercising the option, i.e., the stock price in financial options, is called "expected value of completion" or "value of completion." The cost associated with exercising the option, i.e., the strike price in financial options, is called "strategy-enabling completion cost" or "completion cost." Finally, the cost of acquiring the option in the first place is the "strategy-enabling partial investment" or "initial investment."

1.2 Real options analysis: the need for a generalized approach

Even though RO has been increasingly accepted as an investment evaluation tool, traditional real options methodologies have some shortcomings that limit their utility. For example, because most real projects are not traded, there is usually not enough information to identify and model an underlying asset that is well correlated with the value of the project being considered and that can be used with traditional real options techniques [Copeland and Antikarov, 2001]. In some cases, when the real option is on projects related to tradable commodities, like copper or oil, for example, the tendency is to use the price of the commodity as the value of completion and to model it as a stochastic process, such as a geometric Brownian motion (GBM) [Kulatilaka, 1993; Trigeorgis, 1996]. While this may be a reasonable approximation in some cases, it may not capture all of the risks and uncertainties associated with the project. For example, the value of an option to exploit a copper mine not only depends on the price of copper, but also on the difficulties of developing the mine, processing the ore, and bringing the final product to the market including the political risks associated with the country in which the mine is located. Thus, if the value of completion does not fully represent the value of the project, the valuation may be distorted.

Another major limitation of existing RO approaches is the assumption that the behavior of the completion cost, i.e.,the expenditure required to exercise the option, is known a priori. In some cases, the completion cost is assumed to be fixed, which is not necessarily true for real projects, because the completion cost of real options (generally taken to be a cost related to the project, such as capital investments and/or operational or maintenance expenditures) can vary over time. In other cases, the evolution of thc complction cost is assumed to follow a given function or stochastic process, such as a GBM [Fisher, 1978; Dixit and Pindyck, 1994]; however, as in the case of the value of completion, a stochastic process such as a GBM may not capture all the important dynamics of the completion cost and it may be difficult to find enough information to model the completion cost.

In this thesis, an alternative real options methodology is developed that overcomes some of the difficulties of traditional approaches. In particular, the methodology proposed here presents an analytical framework that allows the value of completion and the completion cost to be represented by any probability distribution. If these probability distributions can be described analytically, an exact solution to the real options valuation problem can be found. Otherwise, the probability distributions can be generated with numerical simulation (e.g. Monte Carlo simulation), and the answer can then be found numerically. This generalized methodology combines the simplicity of analytical approaches with the flexibility to represent the value of completion and completion costs with any probability distribution.

The methodology is illustrated with an example from the aviation industry: the decision to launch a new aircraft development program. This type of investment is suitable for real options analysis because of the many uncertainties involved, the long-term nature of the project, and the ability of management to act and influence the project as uncertainties are resolved during its evolution. This analysis is performed on a representative aircraft program with values that are based on actual aircraft manufacturer's data; however, in order to maintain the confidentiality of the information, the name of the manufacturer has been withheld and the scale of monetary values has been changed.

1.3 Thesis objectives

The generalized real options methodology that is presented in this thesis provides decision-makers with a means of evaluating flexible investment strategies in uncertain environments. This methodology provides an analytical framework that allows the value of completion and the completion cost to be represented by any probability distribution. Thus, it offers an alternative to traditional real options approaches that may be difficult to implement because of the difficulty of modeling the value of completion and the completion cost analytically.

The following steps represent the path through which the methodology will be presented:

1. Develop a valuation formula based on real options concepts to support investment decisions under uncertainty without assuming that the value of completion or the completion cost follow any particular stochastic processes.
2. Find analytical solutions for this valuation formula for cases were the value of completion and the completion cost can be described analytically.
3. For cases were the value of completion or the completion cost cannot be described analytically, use simulation, e.g. system dynamics and Monte Carlos simulation, to determine the value of the real option.
4. Illustrate the applicability of the real options valuation formula by analyzing a new aircraft development program at a major aircraft manufacturer.
5. Demonstrate the use of the real options methodology to investigate the possible impact on expected project value of alternative investment strategies and indicate how this knowledge can be used to draw guidelines useful for decision-making.
6. Discuss the application of the methodology to other projects.

1.4 Thesis contributions

There are a number of contributions of this thesis to the fields of decision-making under uncertainty and investments in air transportation. The main contributions are summarized below:

1) A real options methodology to evaluate investments under uncertainty was developed. Unlike existing approaches, this real options approach allows both the value of completion and the completion cost to be described by any probability distribution. Thus, it is not necessary to force the representation of the value of

completion into known stochastic processes, such as the commonly-used geometric Brownian motion, or to assume that the completion cost is fixed a-priori. This can allow a better portrayal of the true value of the real option without sacrificing computational convenience. Furthermore, the flexibility to represent the probability distributions of the value of completion and the completion cost allows the methodology to be applied to many types of examples. In particular, the real options methodology allows investors to represent the value of projects for which little or no historical data exists by using numerical simulation that can model the value of the project based on historical or behavioral relationships for which data is available.

2) Insights useful for strategic decision-making were obtained through the application of the generalized real options methodology to a new aircraft development program at a major aircraft manufacturer. Numerical results from the real options valuation indicate the investment limits at each stage of the aircraft development process. Furthermore, they demonstrate that the risk of program failure is high in the early stages of the program but that this risk decreases markedly after a certain point in the development process. Thus, this result suggests that outside intervention in the early stages of the program may be justified to ensure its success. Moreover, results from the real options valuation were analyzed to give managers insights into how to restructure the program to improve the financial feasibility of the project. In particular, it was shown that postponing or restructuring investments can have a positive effect on the expected value of the project. Thus, the generalized real options methodology can be used to systematically explore trade-offs in project structure and

determine the optimal investment strategy to maximize expected value. Finally, it was demonstrated how derivative analyses could be used by both investors and managers to define minimum requirements (in terms of aircraft orders) to ensure program success.

1.5 Thesis overview

The thesis is structured as follows:

- *Chapter 1*: The motivation, objective, and structure of the thesis are introduced.
- *Chapter 2:* A review of the relevant literature is presented. In this review, basic concepts of financial risk and of financial options theory are explained. Next, existing methodologies for evaluating real options are described, and their main limitations are discussed. Then, a comparison of RO to other investment techniques is presented. This is followed by a review of previous applications of real options in aeronautics and astronautics. Finally, some fundamentals of system dynamics are discussed.
- *Chapter 3:* The valuation methodology proposed in this thesis is presented. First, a real options formula for any probability distribution of the value of completion and a fixed completion cost is derived. Second, this formula is generalized to cases where the completion cost can also be described by any probability distribution. Third, solutions for cases when the probability distribution of the value of completion and the cost of completion can be described analytically are shown. Finally, a methodology that combines system dynamics and Monte Carlo simulation for instances when the value of completion and the completion cost must be calculated numerically is explained.

- *Chapter 4:* The valuation methodology is demonstrated by evaluating a new aircraft development program at a major aircraft manufacturer. Numerical results from the real options calculation are presented and analyzed.
- *Chapter 5:* The ability that the generalized real options methodology gives investors to evaluate flexibility in projects where limited or no historical data exists and to systematically explore alternative investment strategies is illustrated by re-visiting the aircraft development program. It is then shown how this knowledge can be used to provide managers with strategic insights to improve the performance of the project as uncertainties are resolved. Then, sensitivities of numerical results to model and data assumptions are presented. Finally, implications and recommendations for change in the aircraft manufacturing industry derived from insights from numerical results are discussed.
- *Chapter 6:* The application of the generalized methodology to other projects within and outside air transportation are discussed.
- *Chapter 7:* Conclusions and contributions of this thesis are summarized. Finally, recommendations for future work are presented.

2. Literature review

The relevant background for the methodology proposed in this thesis is presented in this chapter. First, some terms related to financial risk are explained. Second, basic concepts of financial options theory, the basis for traditional real options analysis (RO), are discussed. Third, the fundamentals of traditional RO methodologies are reviewed. Through this discussion, weaknesses of traditional RO approaches are identified and the need for the methodology proposed in this thesis is made manifest. Fourth, a comparison of RO to other investment techniques is presented. Fifth, previous applications of RO in the field of aeronautics and astronautics are presented. Sixth, a brief description of the general characteristics of system dynamics as a modeling tool are described. Finally, the main points of this literature review are summarized.

2.1 Basic financial risk concepts

Before exploring techniques to deal with uncertainty and risk in investment projects, it is useful to review some of the common concepts associated with financial risk. In any type of investment, an investor is typically concerned with two types of risk: a) technical or unsytematic risk, and b) market or systematic risk [Brealey and Myers, 1996]. Technical risk is internal to the project and can be influenced by its developers. For example, technical risk can include uncertainty in research and development (R&D) or unavailability of skilled personnel. Market risk is due to fluctuations in the market and it affects all projects and assets in the economy. An individual investor can generally not influence it.

Investors can protect themselves against technical risk by diversification, i.e., by holding more than one asset in their portfolio of investments. The idea is to hold assets whose risk is not correlated with each other, thus, when one is suffering losses, the other(s) may not. Conversely, market risk can not be diversified away because it affects all assets in the economy. Even the most diversified portfolios can not eliminate market risk [Brealey and Myers, 1996].

A general assumption in finance is that investors are risk-averse and, therefore, they demand some compensation for putting their money in risky ventures. A commonly used tool to calculate this risk premium is the Capital Asset Pricing Model (CAPM), which was developed in the mid 1960s by William Sharpe, John Lintner and Jack Treynor [Sharpe and Alexander, 1991]. The risk premium is the difference between the expected return on a risky asset and the risk-free rate of return. The risk premium can be computed as follows (see Equation 2-1):

$$\begin{aligned} \textit{Expected risk premium} &= \textit{Expected return of risky asset} - \textit{risk-free return} \\ &= \beta \cdot (r_m - r_f) \end{aligned} \quad \text{(Eq. 2-1)}$$

where r_m is the expected return of a portfolio including all the assets in the economy (also known as the market portfolio), r_f is the risk-free interest rate, and β measures the sensitivity of the asset to movements in the market [Brealey and Myers, 1996]. The term β can be defined as follows (see Equation 2-2):

$$\beta = \frac{\sigma_{i,m}}{\sigma_m^2} \quad \text{(Eq. 2-2)}$$

where σ_{im} is the covariance between the returns of asset i and the market and σ_m is the variance of the market returns. Thus, with CAPM, it is possible to determine the appropriate discount rate, r, to calculate the present value of a series of risky cash flows (see Equation 2-3):

$$\textit{Risk-adjusted discount rate} = r_f + \beta \cdot (r_m - r_f) \qquad \text{(Eq. 2-3)}$$

The present value of a string of risky cash flows over time T is found by discounting future cash flows, CF, with the risk-adjusted discount rate, r (see Equation 2-4):

$$PV = CF_0 + \frac{CF_1}{(1+r)^1} + \ldots + \frac{CF_T}{(1+r)^T} \qquad \text{(Eq. 2-4)}$$

In using the risk-adjusted discount rate, two steps are performed in one: the present value of the string of risky cash flows is calculated by simultaneously accounting for the time value of money and for market risk [Brealey and Myers, 1996] (see upper branch of Figure 2-1):

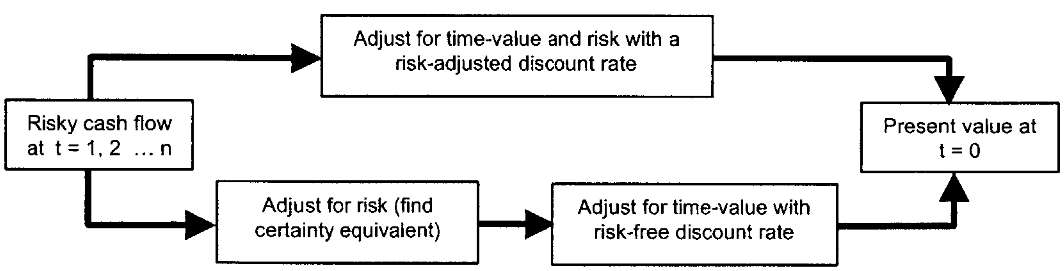


Figure 2-1: Two alternatives to calculate present values. ***Source: Brealey and Myers, (1996).***

Present values can also be computed by first transforming the cash flows into their so-called "certainty equivalents" and then discounting these certainty equivalents with the risk-free rate [Brealey and Myers, 1996]. The certainty equivalent (CEQ) is the certain cash flow that a risk-averse investor would be willing to exchange for the risky cash flow. It can be computed with CAPM, for example [Brealey and Myers, 1996; Trigeorgis, 1996]. Because these cash flows are now certain, the risk-free discount rate is the appropriate discount rate to find their present value. Thus, with the use of certainty equivalents, the adjustment for risk and for the time value of money is disaggregated (see the lower branch of Figure 2-1).

The present value calculated with either a risk-adjusted discount rate or with CEQs and the risk-free discount rate is the same. In this thesis, CEQs are used because they simplify the calculation of the value of the real option, as will be shown in Chapter 3.

2.2 Basic financial options theory concepts

2.2.1 General ideas and terminology of simple financial options

Financial options are securities that give you the right, but not the obligation, to buy or sell an asset, at a pre-determined price, within a specified period of time [Black and Scholes, 1973]. The price paid for the asset when the option is exercised is called the "exercise price" or "strike price." The last day on which the option may be exercised is called the "expiration date" or "maturity date." A "European option" can only be exercised on the expiration date; an "American option" can be exercised at any time up to and including the maturity date.

By owning an option, investors are able to defer the decision to finish the investment until they have more information about the state of the world. Thus, investors can protect their downside losses by not investing if conditions are not favorable, while maintaining the right to invest and reap benefits if conditions are favorable.

The payoff of a call option on a non-dividend paying stock, S, is shown in Figure 2-2.[1] If the stock price, S, is less than the strike price, X, the option does not get exercised and the payoff is zero; however, if S is larger than X, the option holder has the option of buying the stock for X and then selling it for S, thus, making an instantaneous profit of S – X. Mathematically, the payoff of a call option can be expressed as the maximum of S-X or zero, i.e., max[S-X, 0]. This profit must be compared to the cost of obtaining the option to determine the net profit.

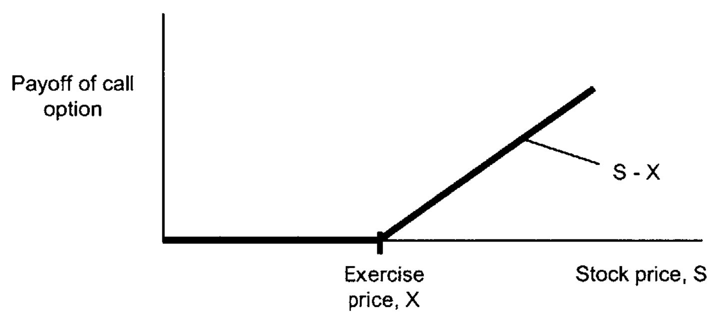


Figure 2-2: Payoff of a call option. ***Source: Brealey and Myers (1996).***

Options are said to be "in the money," "at the money," or "out of the money" depending on the cash flows that the option holder would obtain if the option would be exercised immediately [Hull, 1995]. If exercising the option results in positive cash flow, the option

[1] This discussion is based on [Brealey and Myers, 1996].

is in the money; if it results in a zero cash flow, it is at the money; and if it yields a negative cash flow, it is out of the money. For example, a call option is in the money if S > X, at the money if S = X, and out of the money if S < X.

Options are valuable because the future stock price is uncertain (see Figure 2-3). In fact, the value of an option increases with the volatility of the stock, because this means that the stock can reach higher prices (it can also reach lower prices, but investors would not be concerned because the option protects them from downside movements).

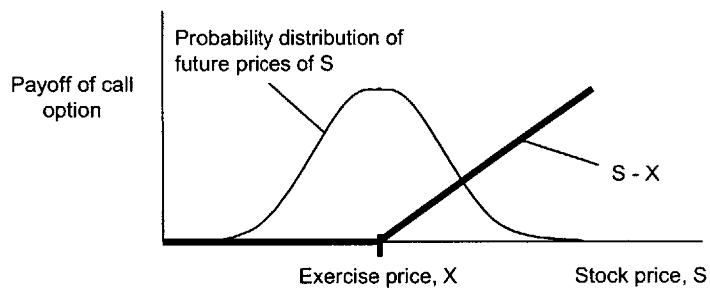


Figure 2-3: The stochastic nature of stock prices make options valuable. ***Source: Brealey and Myers (1996).***

The total value of an option can be considered as the sum of two parts: the intrinsic value and the extrinsic or time value [Hull, 1995]. The intrinsic value is the payoff from exercising the option immediately. For a call option, the intrinsic value is max[S-X, 0] (see Figure 2-4). The extrinsic or time value is the portion of the option price that is not the intrinsic value [Summa and Lubow, 2002]. It arises from the probability that, with time, the intrinsic value of an option will increase. For example, the intrinsic value of an out of the money option is zero, but its price is not zero because it has some time value.

The person buying that option has the expectation that the option will get in the money eventually and thus, gain some intrinsic value.

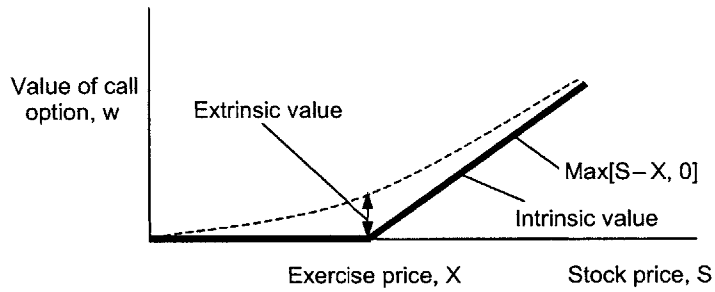


Figure 2-4: The value of an option consists of an intrinsic and an extrinsic part. The extrinsic or time value is highest when the option is at the money. ***Source: Brealey and Myers (1996) and Summa and Lubow (2002).***

The time value of an option is highest when the option is at the money [Summa and Lubow, 2002]. To see the reason for this consider the following: if the option is deep out of the money, the probability that over time it may get in the money is very small. If the option is deep in the money, there is already great certainty that it will be exercised. Therefore, there is not much value in waiting. In both cases, the price of the option approaches its intrinsic value. If the option is at the money, however, its intrinsic value is zero but, because there is a high probability that it may expire in the money, the time value is very high.

2.2.2 Evaluation of financial options

Financial options are part of a class of securities called *derivative securities*, whose value depends on (is derived from) the value of other basic underlying variables. For example, the value of a stock option is contingent on the price of a given stock [Hull, 2000]. A powerful approach to value derivative securities, such as stock options, is to build a replicating portfolio of existing traded assets that match the risk and return characteristics of the new asset. This approach rests on two fundamental assumptions [Dixit and Pindyck, 1994]:

1) Complete markets: It is possible to find a traded asset or a combination of traded assets that exactly track or span the risk and return characteristics of the asset being valued [Dixit and Pindyck, 1994].

2) No-arbitrage: In well-functioning capital markets, it is impossible for anyone to make a profit by buying an asset at a given price and then selling it immediately at a higher price. If such an opportunity were to arise, it would almost immediately disappear as many investors would try to take advantage of it [Brealey and Myers, 1996].

The first assumption implies that there is enough information available to all investors to find a replicating portfolio for each new asset being valued and that the market is complete enough for such assets to exist. The second assumption indicates that the price of this new asset must equal the market value of the replicating portfolio [Dixit and Pindyck, 1994].

Financial options can be evaluated with different methods, including continuous time approaches such as the Black-Scholes formula and contingent claims analysis, and discrete-time techniques such as binomial lattices and the use of risk-neutral probabilities. These methods vary in complexity and accuracy, but they are all based on the previous two assumptions of complete markets and no arbitrage to justify the use of a risk-free discount rate. The Black-Scholes formula will be presented next because it is a common reference when discussing option valuation approaches and because it is found in many real options applications. Discrete-time approaches are also used frequently in real options and they are explained in Appendix A.

2.2.3 The continuous case: The Black-Scholes formula

Before discussing the details of the Black-Scholes formula, it is helpful to explain the modeling of stock prices as a geometric Brownian motion (GBM). This is a common means of representing the evolution of the stock price, and it is an integral assumption of the Black-Scholes formula and other financial and real options valuation techniques.

Modeling stock prices: Geometric Brownian Motion

The stochastic process known as the Wiener Process or Brownian Motion is widely used as the basis to model stock prices [Dixit and Pindyck, 1994; Black and Scholes, 1973; McDonald and Siegel, 1986; Trigeorgis, 1996]. The Wiener process has three characteristics that are important for modeling stock prices:

1) Markov process: The probability distribution of all future values depend only on its current value. Therefore, past information is not useful to create any forecasts. This property is important because it is generally assumed that current stock prices incorporate all publicly available information, and thus, past price patterns have no forecasting value [Dixit and Pindyck, 1994].

2) Independent increments: The probability distribution for changes in the process is independent of any other (non-overlapping) interval [Dixit and Pindyck, 1994].

3) Increments are normally distributed: The changes in the process over any finite interval of time are normally distributed with mean equal to zero and a variance that increases linearly with the time interval [Dixit and Pindyck, 1994; Trigeorgis, 1996]. A caveat in this respect is that since stock prices can not go negative, it is usually assumed that stock prices are lognormally distributed. Therefore, it is the natural logarithm of price which follows a Wiener process [Trigeorgis, 1996].

In addition to the stochastic element represented by the Wiener process, stock prices also typically have a non-zero drift and some volatility; therefore, a better representation of stock prices is a geometric Brownian motion [Trigeorgis, 1996], which is also called a random walk. A mathematical representation of this process is shown in Equation 2-5:

$$\frac{dS}{S} = \alpha \cdot dt + \sigma \cdot dz \qquad \text{(Eq. 2-5)}$$

where dS is the increment in stock price S during the interval dt, α is the expected rate of return on the stock, σ is the standard deviation of stock returns, and dz is the increment of a standard Wiener process. The increment of the Wiener process, dz, is given by Equation 2-6:

$$dz = \varepsilon_t \cdot \sqrt{dt} \qquad \text{(Eq. 2-6)}$$

where ε_t is a normally distributed random variable with zero mean and unit standard deviation [Dixit and Pindyck, 1994].

The Black-Scholes formula

In the early 1970s, Fischer Black and Myron Scholes, together with Robert Merton, developed the theory that has greatly influenced how financial options are valued today [Hull, 2000]. The formula developed by Black and Scholes for the evaluation of European options rests on the assumptions of complete markets and no arbitrage mentioned above. Black and Scholes assume that markets are complete and that, therefore, it is possible to set up a hedged position of stocks and options, called the replicating portfolio. In equilibrium, and in the absence of arbitrage, the expected return of the replicating portfolio must be equal to the return of a riskless asset [Black and Scholes, 1973]. Thus, the correct interest rate to be used in the valuation of options using the Black-Scholes formula is the risk-free interest rate. Another important assumption in Black-Scholes is the modeling of stock prices as a geometric Brownian motion, as explained above.

In the derivation of their formula, Black and Scholes assume a hedged position composed of buying some shares of the stock and selling a certain amount of options on the same stock [Black and Scholes, 1973]. The number of options that must be sold against one share of stock is given by the option delta or hedge ratio. Black and Scholes assume that, as time passes, the hedged position can be maintained by adjusting the hedge ratio. In the limit where the position in the hedged portfolio is adjusted continuously, the risk in the hedged position becomes zero, therefore, the expected return in the hedged position must be at the risk-free interest rate.

The formula derived by Black and Scholes for the valuation of European call options with no dividends is presented below[2] (see Equation 2-7):

$$w(S_0, t) = S_0 \cdot N(d_1) - X\, e^{-r_f (t-T)} \cdot N(d_2) \qquad \text{(Eq. 2-7)}$$

where N(d) is the cumulative normal density function, S_0 is the stock price at time zero, X is the strike price, r_f is the risk-free interest rate, σ^2 is the stock price volatility, T is the maturity date (thus, T - t is the time to maturity), and d_1 and d_2 are as given below:

$$d_1 = \frac{\ln(S_0 / X) + (r_f + 0.5\ \sigma^2) \cdot (T - t)}{\sigma \cdot \sqrt{T - t}} \qquad \text{(Eq. 2-8)}$$

$$d_2 = \frac{\ln(S_0 / X) + (r_f - 0.5\ \sigma^2) \cdot (T - t)}{\sigma \cdot \sqrt{T - t}} \qquad \text{(Eq. 2-9)}$$

The Black-Scholes formula is attractive because it is compact and has a closed-form solution. In addition, all the information required to determine the value of a financial option is typically available or can typically be deduced from historical market data.

2.3 Real options theory

In the past decades, real options analysis (RO) has emerged as an alternative project valuation technique. It is based on financial options theory, but, instead of finding the

[2] For details on the derivation of this formula see [Black and Scholes, 1973], [Hull, 2000] or [Trigeorgis, 1996].

value of holding an option on a financial asset, it is applied to "real" projects to estimate the value of flexibility in the face of uncertainty [Dixit and Pindyck, 1994]. In a traditional sense, real options are about actions with physical assets: expand an airport, switch from fuel oil to natural gas in a dual-fuel combustor, develop an oil field, or abandon a copper mine, for example. There are also "strategic options" which, rather than implying an action with a physical asset, frequently denote an investment to develop a capability needed for rapid and flexible response at a later time [Clemons and Gu, 2003].

As was mentioned in Chapter 1, it is common to find the terminology of financial options in the real options literature. In this thesis, however, a terminology that is more intuitive to the physical meaning of the elements that constitute a real option has been adopted. Thus, the value of the underlying asset that the holder of the real option acquires by exercising the option, i.e., the stock price in financial options, is called expected value of completion or value of completion. The cost associated with exercising the option, i.e., the strike price in financial options, is called strategy-enabling completion cost or completion cost. Finally, the cost of acquiring the option in the first place is the strategy-enabling partial investment or initial investment. This terminology will also be used when describing traditional real options techniques below.

2.3.1 Traditional methodologies for evaluating real options

The financial options theory described earlier and in Appendix A is the basis for many of the traditional RO methodologies. Three of the main categories of solution approaches for real options commonly found in the literature are presented below:

a) Discrete-time contingent claims analysis

This approach is based on the risk-neutral probabilities approach developed for evaluating financial options (See Appendix A). The main idea is to build a recombining binomial tree to model the risk-neutral evolution of the value of completion. The value of the real option is calculated by solving backwards through the decision tree according to the specified decision rules for the option (e.g., European, American, etc.) being evaluated. Examples of this valuation approach can be found in [Copeland and Antikarov, 2001], [Markish and Willcox, 2002], and [Stonier, 1999].

b) Continuous-time contingent claims analysis

The main idea behind this technique is to set up a replicating portfolio using a continuous-time representation of the value of completion (typically a GBM or another stochastic process, such as a mean-reverting model). Generally, Ito calculus is then used to obtain a partial differential equation of the expected return. By specifying the appropriate boundary conditions for the option being analyzed, the partial differential equation can be solved to find the value of the real option [Dixit and Pindyck, 1994]. The formula derived by Black and Scholes is an example of a continuous-time contingent claims solution. Other examples of this technique can be found in [Dixit and Pindyck, 1994], [McDonald and Siegel, 1986], and [Paddock et al., 1998].

c) Simulation:

A third way of evaluating options on real projects is the use of numerical simulation. One approach, proposed by Robert Tufano and Alberto Moel (1997) is to simulate the evolution of the value of completion until the end of the life of the project assuming that the real option is always exercised. Then, using different assumptions for the discount rate, the net present value of the project is calculated. This process is repeated thousands of time using Monte Carlo simulation to incorporate multiple sources of uncertainty on revenues as well as on costs. In this manner, a distribution of net present values for the project with its associated mean is obtained (see Figure 2-5, left).

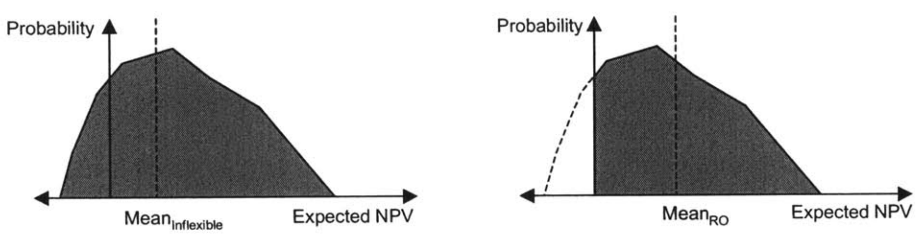


Figure 2-5: The approach proposed by Tufano and Moel consists of using simulation to determine the distribution of net present values without flexibility and its associated mean (left). Flexibility is simulated by substituting negative NPV values with zero (right). The mean of the truncated distribution is the value of the project with flexibility. ***Source: Modified from Tufano and Moel (1997).***

The power of real options lies in the ability of managers to adjust the evolution of projects to take advantage of uncertainties as they are resolved. In the Tufano and Moel model, the authors argue that managers can exercise this capability by canceling projects with negative net present values. The authors represent this ability by substituting negative NPVs with zero, thus, essentially truncating the distribution of net present values (see Figure 2-5, right). The mean of this truncated distribution is assumed to be the value of the project with flexibility. Consequently, the value of the real option is the difference between the means of the projects with and without flexibility [Tufano and Moel, 1997]. This approach can be used to evaluate European-like real options.

Vinay Datar and Scott Matthews (2004) have developed an intuitive method that is algebraically equivalent to the Black-Scholes formula for European call options and that is similar to the approach proposed by Tufano and Moel. The first step in the Datar-Matthews method is to find the probability distribution of the discounted revenues of a given project at time zero. Next, the distribution of revenues is truncated using the strike price as the lower threshold. The value of the option is calculated as the expected value of the revenues for revenues larger than the strike price minus the present value of the strike price [Datar and Matthews, 2004].

Another simulation-based approach, which can be used for American and more complex types of options, uses least squares regression to determine the optimal exercise rule by calculating the expected payoff from continuation (see [Longstaff and Schwartz, 2001] for a complete description). First, m paths of the value of completion (the state variable) over n time periods are generated using Monte Carlo simulation. Then, for each time period, the payoffs from continuation (i.e., if the option is not exercised) realized in the next period are regressed against the state variables in the current period to determine a conditional expectation function (this is essentially a forecasting function). Thus, given the current value of the state variable, this function is used to determine the value that the state variable could take in the next period if the option is not exercised now. This conditional expectation is compared against the payoff given that the option is exercised immediately. By selecting the maximum of the two, the algorithm determines whether to exercise immediately or keep the option alive. The process continues until the end of the n periods. The realized cash flows are then discounted back to time zero and averaged to find the value of the real option. An example of this relatively new technique applied to real options can be found in [Gamba, 2002].

The discrete-time contingent claims approach is convenient because it is easy to implement. In addition, it is flexible enough to handle different types of options including European and American options with or without dividends. Among its disadvantages is the fact that since it is a discrete-time approach, the solution may not be as accurate as the alternative continuous time solution; however, this can be solved to a certain extent by increasing the number of time periods [Brealey and Myers, 1996]. Another disadvantage is that as the number of uncertainties, states for each uncertainty, or periods increase, the number of branches in the decision tree grows geometrically, making the analysis more difficult to implement [Longstaff and Schwartz, 2001]; however, this hurdle could be overcome by increasing computer power and by utilizing specialized software.

The continuous-time contingent claims methodology has almost the opposite attributes of the discrete-time version. It is generally considered to be more exact, but it is also considerably more difficult to implement. One of the main difficulties of this approach is solving the partial differential equation of the expected returns. This involves finding suitable boundary conditions that reflect the type of option being modeled and, in many circumstances, the solution must be found numerically. A further disadvantage is that, similar to the discrete-time case, it is difficult to capture all the uncertainties in the analytical representation of the value of completion.

Simulation techniques are in general easy to implement and more transparent than continuous-time contingent claims approaches [Longstaff and Schwartz, 2001]. In addition, multiple sources of uncertainty may be considered at the same time, and the value of completion does not necessarily have to be represented by an analytical stochastic model. Therefore, the simulated value of completion may come closer to reflecting the true value of the project. A main difficulty with simulation techniques is that since they are forward-looking, it is difficult to find the optimal solution; however, recent advances in the literature, such as the least squares methodology proposed by Longstaff and Schwartz (2001) for the evaluation of American options, show that finding the optimal solution with simulation may be possible.

2.3.2 Key questions with traditional RO approaches

Two fundamental questions must be answered before using the financial options concepts and the techniques described above to evaluate real options. These questions are:

1. What is the appropriate underlying asset and how can the value of completion be represented?
2. What is the appropriate discount rate?

The answer to these questions have important implications regarding the suitability of methods based on financial options to evaluate real options. Thus, the answer to these questions will be considered next.

a. Identifying an underlying asset and modeling the value of completion for RO

In order to use traditional tools of financial options to evaluate real options, such as discrete-time and continuous-time contingent claims analysis, it is necessary to identify an underlying asset and to model the value of completion analytically. For simulation approaches, the value of completion does not necessarily have to be represented analytically, but it is necessary to identify one that captures the value of the project being evaluated. In financial options, the underlying asset is very clear: it is the stock upon which the option is written, and, as discussed before, its value is usually modeled as a geometric Brownian motion.

The choice of underlying asset for real options is not so clear, nor is it clear how to represent the value of completion. For projects related to traded commodities such as copper, natural gas or oil, the trend is to use the commodity as the underlying asset and to use its price as the value of completion [Tufano and Moel, 1997; Kulatilaka, 1993; Paddock et al., 1988]. In the case of real options on manufactured goods where the price of the product is believed to be highly correlated to the manufacturer's stock price, this stock price is usually taken as the value of the underlying asset [de Neufville and Neely, 1998; Dixit and Pindyck, 1994; Markish and Willcox, 2002]. If the project is not related to a traded commodity nor is its value reflected on a stock price, Copeland and Antikarov

(2001) suggest that " ...the NPV of the project without flexibility [...] is the best unbiased estimator of the market value of the project were it a traded asset" and this could be used as the value of completion [Copeland and Antikarov, 2001]. These alternatives are attractive because they can be typically modeled as GBMs or other stochastic processes; however, as will be discussed shortly, these simplified approaches may not capture all of the relevant uncertainties and dynamics of the project.

Assuming that an appropriate underlying asset has been identified, the next step in real options approaches based on contingent claims analysis is to model the value of completion analytically. In some cases, e.g., when the underlying asset is a stock price, the GBM assumption may be suitable. In other circumstances, the value of the underlying asset may be best approximated by other stochastic process. Two commonly used alternative stochastic models are mean-reverting processes and jump processes. Mean-reversion models are generally used to represent the evolution of quantities that show a tendency to return to a long-run equilibrium value such as the price of commodities (e.g., copper or oil [Dixit and Pindyck, 1994] or products in cyclical industries (e.g., aircraft [Stonier, 1999]). A simple mean-reverting model is the *Ornstein-Uhlenbeck* process shown in Equation 2-10 [Dixit and Pindyck, 1994]:

$$dx = \eta \cdot (X - x)dt + \sigma \cdot dz \qquad \text{(Eq. 2-10)}$$

where η is the speed of reversion, i.e., a metric that represents how fast the process returns to its long-term trend, X, σ is the standard deviation of the process, and dz is the increment of a standard Wiener process (see Equation 2-6).

Another alternative stochastic model describes jump processes. Jump processes are generally used to reflect sudden changes in the evolution of the value of the underlying asset because of the arrival of some unexpected information. Some examples are entry of competitors, natural disasters or the outbreak of armed conflicts [Trigeorgis, 1996]. An example of a jump process is given in Equation 2-11 [Trigeorgis, 1996]:

$$dx = \alpha \cdot x + (1-k)dN \qquad \text{(Eq. 2-11)}$$

where α is the expected rate of return, (1-k) is the proportion of the project's loss when the unexpected information arrives and dN is described by a Poisson-jump process with arrival rate λ:

$$dN = \begin{cases} \mathit{1\ with\ probability}\ \lambda dt \\ \mathit{0\ with\ probability\ 1-}\ \lambda dt \end{cases}$$

Under the assumption that the NPV of the project without flexibility can be used as the underlying asset, Copeland and Antikarov (2001) propose using Samuelson's theorem that properly anticipated prices fluctuate randomly [Samuelson, 1965] to model the change in the present value of the project as a geometric Brownian motion [Copeland and Antikarov, 2001]. Thus, they suggest calculating the present value of the project in a spreadsheet and, by applying Monte Carlo techniques to account for multiple sources of uncertainty, determine the standard deviation of the returns of the project. This standard deviation can then be used to model the value of the underlying asset as a GBM.

All these efforts to model the value of underlying assets may be straightforward and relatively easy to implement, but they may not capture all of the risks and uncertainties associated with the project. For example, the value of an option to exploit a copper mine not only depends on the price of copper, but also on the difficulties of developing the mine, processing the ore, and bringing the final product to market. Thus, taking the price of copper as the value of the underlying asset and modeling the value of completion as a GBM will only reflect the uncertainty in the price of the metal once it reaches the market, but it does not give any information about the risks associated with bringing it to the market. Furthermore, GBM processes assume that volatility remains constant, which is not necessarily true for real projects. Thus, a valuation based on such a value of the underlying asset may be distorted.

Another major difficulty in finding an appropriate underlying asset applies to projects with ill-defined structure, as there may not be adequate data to develop a stochastic model for its value. For example, the application of real options to R&D in the pharmaceutical industry has been facilitated by the fact that developing new drugs is a highly regulated and, therefore, structured process [Myers, 2003; Myers and Howe, 1997]. Thus, there is a fair amount of data in the public domain and it is therefore possible to determine the probability of success of drugs as they proceed along the R&D process and into the marketplace. On the other hand, it would be more difficult to find enough data to model the underlying asset in a less structured environment, such as software development [Myers, 2003].

The modeling of the value of underlying assets that more closely represent the returns of the project may be easier using simulation. In this case, the value of the underlying asset can be modeled directly without the need to approximate analytical stochastic processes. The simulation can take into consideration multiple sources of uncertainty, feedback loops, and other dynamics to provide a better approximation of project value. However, it must be remembered that all models have limitations and that care must be taken to choose the one that best represents the value of the project, be it analytical or simulation-based.

b. Finding the appropriate discount-rate

The fact that most real projects are not traded in open markets has a further implication which is closely related to the difficulty of finding a suitable underlying asset. As was mentioned above, a crucial assumption in financial options theory is the existence of complete markets so that a replicating portfolio that exactly matches the payoffs of the option can be found. The existence of complete markets and the no-arbitrage assumptions are the basis for justifying the use of the risk-free discount rate in financial options. Because many real projects are not traded publicly, it is difficult (if not impossible) to find the right assets in the market, or a combination thereof, to construct the replicating portfolio necessary to apply the concepts of financial options theory [Copeland and Antikarov, 2001]. Thus, the use of the risk-free rate may not be appropriate for the evaluation of real options since market risk may not be completely hedged away.

In order to use the risk-neutral approaches of financial options on real projects, it is necessary to adjust the evolution of the value of the project, i.e., the stochastic process, for systematic (market) risk. Specifically, a risk-premium, δ, associated with the project should be subtracted from the expected return of capital gain to determine the certainty equivalent expected return [Trigeorgis, 1996]; consequently, because the expected return is now risk neutral, risk-neutral valuation can be used and the risk-free interest rate is the appropriate discount rate. The risk-premium, δ, can be calculated using an equilibrium model such as the CAPM [Dixit and Pindyck, 1994; Trigeorgis, 1996] or it could be estimated using expert judgment.

In the discrete-time case (see Appendix A), the adjustment consists of subtracting δ from the risk-free interest rate in the risk-neutral probability, q, to obtain the adjusted probability, q', [Amram and Kulatilaka, 1999] (see Equation 2-12):

$$q' = \frac{(1 + r_f - \delta) - d}{u - d} \qquad \text{(Eq. 2-12)}$$

where u and d are as defined in Appendix A.

In the continuous case, the adjustment is made by subtracting δ from the expected return, α, to determine the certainty equivalent expected return α' [Dixit and Pindyck, 1994; Trigeorgis, 1996] (see Equation 2-13):

$$\alpha' = \alpha - \delta \qquad \text{(Eq. 2-13)}$$

In cases where the value of the underlying asset is modeled with simulation, there may be cases where it can be assumed that the risk in the value of completion can be hedged away and, therefore, the risk-free discount rate can be justified (see, for example, [Tufano and Moel, 1997]). In most cases, however, such assumptions are difficult to justify and, thus, a risk-adjusted discount rate should be used. The choice of the discount rate will depend on the type of project being analyzed. For example, if the market risk characteristics of the investment under evaluation are similar to those of other projects in the investor's portfolio, a discount rate similar to that used in the other projects could be assumed. If the market risk characteristics of the investment are different, CAPM or the Weighted Average Cost of Capital (WACC) of the company, and/or expert judgment may be necessary to estimate an appropriate discount rate [Myers and Shyam-Sunder, 1996]. In terms of costs, the risk-free discount rate could be used to discount the strike price if risks associated with expenditures are primarily technical and if a diversified investor can be assumed [Myers and Shyam-Sunder, 1996]. The assumption in this case is that costs are not affected by market risk and, thus, the discount rate only needs to adjust for the time value of money. Otherwise, a risk-adjusted discount rate should also be identified to discount the cost of completion.

2.3.3 The need to advance the real options frontier

Traditional real options approaches based on analytical methodologies, such as discrete-time or continuous-time contingent claims analysis, have the advantages of being simple to implement and relatively familiar (at least to those that have been exposed to financial options theory); however, these tools have two main shortcomings. First, they usually assume that the value of the underlying asset can be represented as a geometric Brownian motion; however, as was mentioned before, the GBM may not capture all of the relevant dynamics and uncertainties of the project. In addition, it may be difficult to find enough information to describe the probability distribution of the value of completion analytically. Second, the completion cost is typically assumed to be known a priori or it is assumed to follow a GBM [Fischer, 1978; McDonald and Siegel, 1985; Dixit and Pindyck, 1994]. In a real project, the completion cost can be thought of an expenditure to obtain the underlying asset. This expenditure can be, for example, the construction of infrastructure or a production facility. This cost is also subject to uncertainties and, thus, it is to be expected that the cost will change over time. Furthermore, as in the case of the underlying asset, representing the evolution of cost as a GBM may not capture all of the relevant dynamics and risks.

Simulation approaches for solving real options can circumvent some of the shortcomings of analytical methodologies. For example, Monte Carlo simulation can be used to account for multiple sources of uncertainty in the value of completion. Therefore, instead of modeling the value of completion analytically, it can be calculated numerically. In addition, the use of simulation does not imply the need for a fixed completion cost. Nevertheless, existing simulation-based real options techniques have some disadvantages. For example, even though the approaches suggested by Datar and Matthews (2004) and Longstaff and Schwartz (2001) use simulation to find the distribution of the value of completion, they still assume a fixed completion cost. The Tufano and Moel approach can incorporate a variable completion cost because its value of completion is the net present value of the project; however, this approach does not provide any intuition regarding the relative magnitude or the probability distributions of the value of completion and the completion cost. Thus, while this model gives the solution to the real options valuation, it does not readily provide information about how the performance of the project may be affected by changes in the distribution of costs and revenues. This information can be useful to project managers who are trying to improve the attractiveness of the investment.

The real options methodology proposed in this thesis builds on the advantages of both analytical and simulation approaches and addresses their main shortcomings. First, this methodology is based on a simple analytical evaluation formula with only three inputs: the probability distributions of the value of completion and of the completion cost, and the risk-free discount rate. Second, the completion cost may be fixed but it is not necessary to assume so. Third, there is no constraint on the shape of the probability distribution of the value of completion or the completion cost. These distributions may be GBMs or completely random processes. Fourth, if such distributions can be described analytically, the methodology can be used to find an exact solution. Otherwise, if there is no analytical formulation that adequately captures the uncertainties and particular characteristics of the processes being analyzed, simulation can be used to obtain the probability distributions of the value of completion, the completion cost, or both. Fifth, because the methodology considers the value of completion and the completion cost separately, project managers can explore how changes in the probability distribution of either one would affect the value of the real option without having to simulate the entire process numerous times. Thus, the methodology developed in this thesis goes beyond the

Black-Scholes formula when addressing real options and, likewise, provides improvements over traditional real options approaches.

2.4 Comparison of RO to other investment evaluation techniques

Real options analysis is one of several investment evaluation techniques available to decision-makers. Other popular methodologies are the net present value (NPV) rule and decision analysis (DA). Each approach has its advantages and disadvantages, and the selection of one over the others should depend on the nature of the problem being analyzed. In this section, these three evaluation techniques are discussed and their relative strengths and weaknesses are highlighted.

A fundamental objective of any investment evaluation is to inform the investor if a project is worth undertaking and, furthermore, how much such a project is worth today. Discounted Cash Flow (DCF) is normally used to find the present value of the future cash flows that constitute the investment [Brealey and Myers, 1996]. Each of the three evaluation techniques mentioned above use DCF, but they differ mostly in the assumptions about when cash flows occur and how information about uncertainties is incorporated into the evaluation.

In the Net Present Value rule, a string of future cash flows (positive and negative) is assumed, and DCF is used to find their present value. The resulting present values of the cash flows are added or subtracted and the result is the net present value of the investment. Monte Carlo simulation can be used, for example, to incorporate multiple sources of uncertainty and determine the expected cash flows and, consequently, the expected net present value. A key distinction of the NPV rule is that the cash flows occur at fixed points in time. Thus, the evaluation is done entirely with information available today. There is no explicit consideration of the ability of managers to change the course of the investment in the future as they receive more information about the state of the world. Alternatively, managers can assume different project structures to determine the payment schedules that would result in the highest expected NPV to find the preferred strategy. The expected value of the investment using this approach would be the maximum among a series of mutually exclusive alternatives that are fixed from the beginning of the project [Copeland and Antikarov, 2001].

Decision Analysis also uses DCF as an input to find the value of an investment today. A main difference with respect to the NPV rule is the explicit incorporation of managerial flexibility via decision trees. Thus, as time passes and the process moves along the decision tree, the investor can choose the optimal path based on how uncertainties are resolved. The structure of DA assumes that information will be incorporated in the future and that this information can be used to enhance the value of the project. Thus, the value of the investment today can be interpreted as the expected value of following the optimal strategy along the decision tree.

Real options analysis is similar to DA in its ability to incorporate information about the future state of the world into the investment evaluation. A real options valuation can be structured as a decision tree to allow project managers to take different actions depending on how uncertainties are resolved. In fact, the discrete-time contingent claims solution approach for real options is a decision tree in which the risk-neutral probabilities have been substituted for the real probabilities (see Appendix A); however, not all real options approaches necessarily have the structure of a decision tree. This is an important distinction because it indicates two potential advantages of RO over DA.

In DA, in order to use decision trees, it is necessary to discretize probabilities and time. This may limit the number of end states that would otherwise be possible using continuous probabilities and continuous time as is the case in continuous-time contingent claims techniques for real options, for instance. Thus, Decision Analysis may not be able to represent all possible states of the value of the underlying asset. This should not necessarily rule out DA as an evaluation tool because the error introduced by the discretization of probabilities and time may be small compared to the end result. Furthermore, the number of states for each uncertainty at any given time period and the number of time periods can be increased in order to approximate the continuous case. There are practical limits to this approach, and herein lies a second potential advantage of some RO techniques over DA. As the number of states or periods in the decision tree increase, the number of branches grows geometrically, making the analysis more computationally intensive. Furthermore, this problem is compounded if several sources of uncertainty are considered. Thus, the decision tree can become very large and, therefore, onerous to manage. Some RO techniques, especially simulation-based ones, can incorporate multiple sources of uncertainty without significantly increasing the computational burden. Therefore, from a practical point of view, RO may be preferable to DA depending on the size of the problem.

The choice of one evaluation technique approach over the other should depend on the nature of the project being evaluated. If there is reason to believe that there is no room for managerial flexibility, the NPV rule approach would provide an accurate valuation; however, if it is possible for management to influence the outcome of the project as more information is obtained, an approach that explicitly accounts for this flexibility, such as Decision Analysis or Real Options Analysis, should be used.

In investments under uncertainty, the ability to wait and obtain more information can be very valuable. Thus, a technique that allows the incorporation of managerial action into the valuation is warranted. If the investment problem can be described with a decision tree of a manageable size, DA or discrete-time RO approaches can be appropriate; however, in cases where the structure of the problem is difficult to represent in a decision tree, or in cases where the size of the tree would be very large due to several sources of uncertainty or the discretization of probabilities and/or time periods, a continuous time or simulation-based real options method would be preferred.

2.5 Real options in air transportation

The real options methodology developed in this thesis is illustrated with an example from the air transportation industry; therefore, in order to place this application in the context of previous work in this field, a brief review of the literature of real options in air transportation is given.

There has been a growing number of publications on real options in air transportation in recent years. For example, John E. Stonier (1999), from the Airbus Company, presents the valuation of aircraft purchase options using discrete-time contingent claims analysis. He uses the present value of all operating cash flows over the economic life of a given aircraft as the value of completion. Then, he finds the value of the option with risk-neutral probabilities in a binomial decision tree. Real options are also being used by Airbus' main competitor in the large aircraft market, The Boeing Company. The real options method by Datar and Matthews discussed above is being applied throughout the Boeing company to analyze strategic technology investment projects [Matthews, 2004]. Shackleton, Tsekrekos and Wojakowski (2004) analyzed entry decisions in competitive markets with a two-player real options framework. They assume net operating profitability as the value of completion for each firm, which is modeled as a geometric Brownian motion. Furthermore, the cost to enter the market (the completion cost) is assumed to be fixed. This framework is applied to understand Boeing's optimal response to Airbus' launch of the A380 very-large aircraft.

On the academic front of commercial air transportation studies, Grayson (2001) used real options to analyze corporate restructuring in the U.S. aerospace and defense industry. Using the approach outlined in Copeland and Antikarov (2001) combined with game theory, Grayson investigated the competition of Boeing and Airbus in the large aircraft market, two big mergers (Boeing with McDonnell Douglas and Lockheed with Martin Marietta), and the award of major contracts by the U.S. Department of Defense. Markish and Willcox (2002) developed an algorithm to determine the value of a flexible aircraft design program using a dynamic programming algorithm based on discrete-time contingent claims analysis. They use historic aircraft demand data as the value of completion, which they suggest follows a geometric Brownian motion. In another example, Mavris and Fernández (2003) present an engine development support system that links technical design parameters with market needs and other non-technical considerations. They propose the use of real options as a means of capturing the value of managerial flexibility in the engine development process. Although the particular real options aspect has not been fully developed yet, this approach is an application of real options as a component of a larger strategic management tool.

The space sector has also being a fertile field for real options applications. Saleh, Lamassoure and Hastings (2002) developed a framework based on the Black-Scholes formula to find the value of the real option to service an on-orbit satellite to extend its operational life. The framework considers a compound option because the satellite can be serviced at multiple discrete points. The value of completion is revenues from selling communication services, which are modeled as a GBM. The completion cost is the sum of repair costs and extended operating costs which are assumed fixed. In a companion paper, Lamassoure, Saleh and Hastings (2002) apply the same framework to consider the compound real option to abandon the mission if operational costs exceed a given threshold. This framework has since been used by Joppin and Hastings (2003) to evaluate the options to upgrade an on-orbit satellite by either servicing or replacing it. Again, revenues are taken to behave as a random walk and the completion cost is assumed to be fixed. De Weck, de Neufville and Chaize (2003) explore the value of the option to stage the deployment and on-orbit reconfiguration of a constellation of communication satellites. As in the previous cases, revenues from communication services are considered as the value of completion and are modeled as a GBM. The model assumes that a new stage of the constellation is deployed only when demands exceeds the capacity of the system, thus, the development path does not follow a pre-existing plan but evolves according to market needs. The total cost of the development path is calculated to obtain the life cycle cost (LCC) of the project. The difference between the LCC of the flexible path and a pre-determined path is considered to be the value of flexibility. Finally, Weigel and Hastings (2004) use real options to determine the value of transitioning to alternative architectures for space projects that face annual budget uncertainties. As annual budget cuts in space programs are lowered, their schedules are stretched out, which typically implies a cost increase. Thus, the authors use real options to determine the value of the option to switch to an alternative architecture if the transition costs are less than the extra costs due to the schedule increase. The value of the underlying asset is the initial projected cost of the given space system architecture. The authors follow the analysis framework suggested by Amram and Kulatilaka (1999) to find the value of the real option.

2.6 System dynamics in air transportation

System dynamics is a qualitative and quantitative methodology to understand how delays, feedback, and interrelations between different components affect the dynamics of a managed system [Galvin, 2002]. It is a simulation-based approach that grew out of engineering feedback control theory and uses nonlinear difference equations to describe and model the behavior of complex systems [Skinner et al., 1996]. System dynamics is very versatile and it has been applied to study very different issues, ranging from corporate strategy to global climate change to the spreading of diseases [Sterman, 2000].

The use of system dynamics to describe, model and analyze systems in air transportation is appropriate because of the strong inter-relationships, feedbacks and delays within the elements of this type of systems. In addition, system dynamics can be combined with Monte Carlo simulation to incorporate multiple sources of uncertainty. Thus, by explicitly taking into account all these factors and uncertainties, a system dynamics and Monte Carlo simulation can provide a better representation of project value than modeling the value of completion as a GBM. Therefore, system dynamics is used in this thesis to model the evolution of the value of completion to analyze real options for investment in air transportation projects.

There are several examples of applications of system dynamics in air transportation. Skinner et al. (1996) developed a model of the boom and bust cycles of the airline industry based on exogenous macro-economic fluctuations and the mismatch between demand for aircraft and delays in aircraft delivery. They argue that if decision-makers were to understand the basic feedback mechanisms and dynamic structures of the industry, a strategy to mitigate the impacts of the cycles could be developed. Liehr et al. (2002) conceived a similar model consisting of two delays (aircraft manufacturer lead-time and delayed recognition of surplus passenger capacity) with one main feedback loop (reduction of passenger capacity when surplus is positive) [Liehr et al., 2002]. Galvin

(2002) uses system dynamics to analyze the future behavior of the principle components of the air traffic control (ATC) system over time and determine what resource management strategy could support the Small Aircraft Transportation System (SATS) in the United States. This model is governed by demand for air services and the supply for those services provided by airports, controllers, facilities and equipment in the ATC system. The feedback within the system determines the adjustments in the resources to the different components of the system.

2.7 Summary of the literature review

This literature review covers the relevant background for the generalized real options methodology proposed in this thesis. The main points are summarized below:

1) Systematic (market) risk and the time value of money can be taken into account simultaneously with a risk-adjusted discount rate. Alternatively, a certainty equivalent of the risky cash flows can be found to account for systematic risk and the risk-free rate can be used to discount the cash flows. Both approaches yield the same results.

2) The evaluation of financial options is based on the assumptions of complete markets and no-arbitrage.

3) Real options analysis, which is based on financial options theory, can be used to determine the value of managerial flexibility in a given project if a suitable underlying asset is found and proper adjustments for risk are made.

4) Traditional analytical real options techniques are limited in how they model the value of completion. In addition, most approaches assume a fixed completion cost, even though costs are also subject to uncertainty.

5) Existing real options simulation approaches can be applied to evaluate many different projects but they may not provide enough insight to managers about the performance of the investment.

6) The Net Present Value rule, Decision Analysis and Real Option Analysis use Discounted Cash Flow to find the present value of future cash flows. The choice of one evaluation technique over the others should depend on the nature of the project being evaluated.

7) Real options analysis has been applied to several examples in the air transportation industry. Most of the earlier work is based on traditional RO approaches in which the value of completion is typically assumed to follow a GBM and the completion cost is fixed.

8) System dynamics is a useful tool to model the underlying asset of projects in air transportation because it can combine the delays, feedbacks, and strong inter-

relations common in investments in this field. Thus, by explicitly taking into account all these factors, a system dynamics model can provide a better representation of the evolution of project value than modeling the value of completion as a GBM.

3. Generalized real options valuation formula

The methodology to evaluate real options presented in this chapter resolves two shortcomings of traditional analytical approaches to real options analysis. First, it does not constrain the probability distribution of the value of completion to a geometric Brownian motion or any other particular stochastic process. The value of completion can be described by any probability distribution. Second, it does not assume that the behavior of the completion cost is fixed or that it can be described with a particular stochastic process. As in the case of the value of completion, the completion cost can be described by any probability distribution. With respect to simulation RO techniques, this methodology offers the advantage of combining the power of simulation with the simplicity of analytical solutions to expedite calculations and provide insights into how the probability distribution of costs and revenues affect the value of the real option.

A. 3.1 Real options formula for any distribution of the value of completion and fixed completion cost

The derivation of a real options formula where the value of completion is described with any probability distribution and the completion cost is fixed is presented below. The formula is derived using two different approaches: the first is based on Hull (2000) and Chriss (1997), and the second utilizes the work by McDonald and Siegel (1985). Even though both derivations lead to the same result, it is helpful to show both for a better understanding of the intuition behind the derived formula.

3.1.1 Certainty equivalents (CEQs) and the risk-free discount rate

To simplify the calculation of the value of the real option and the treatment of the discount rate, it is helpful to express the value of completion and the completion cost in terms of their certainty equivalents [Myers, 2004]. Therefore, the risk-free discount rate becomes the appropriate rate to discount the value of the real option. In the following paragraphs, a procedure to find the CEQs for the value of completion and the completion cost is presented.

As was explained in Section 2.1, there are two fundamental alternatives to calculate the present value of any cash flow. One is by discounting future values with a risk-adjusted discount rate. This takes into account the market risk of the future values as well as the time value of money in a single step. The second alternative is to find the certainty equivalent of the cash flow and then discount with the risk-free discount rate. Since both alternatives yield the same result, the following relationship must always hold:

$$CF\big|_{t=0} = \frac{CF\big|_{t=T}}{(1+r)^T} = \frac{CF_{CEQ}\big|_{t=T}}{(1+r_f)^T} \qquad \text{(Eq. 3-1)}$$

where $CF|_{t=0}$ is the present value of the cash flow at time zero, r_f is the risk-free discount rate, $CF|_{t=T}$ is the present value of the risky cash flow, CF, at time T, r is the risk-adjusted discount rate appropriate to discount CF, and $CF_{CEQ}|_{t=T}$ is the present value of the certainty equivalent of CF at time T. The value $CF|_{t=T}$ can be found by discounting the risky cash flows of CF for $t > T$ with the appropriate discount rate, r. Rearranging the terms in Equation 3-1, the following expression for the present value of the certainty equivalent at time T is found:

$$CF_{CEQ}\Big|_{t=T} = CF\Big|_{t=T} \cdot \frac{(1+r_f)^T}{(1+r)^T} \qquad \text{(Eq. 3-2)}$$

The certainty equivalent of the value of completion and of the completion cost at time T can be found with Equation 3-2. Now, since the market risk has been accounted for in $CF_{CEQ}|_{t=T}$, the risk-free discount rate can be used to calculate the value of the cash flow at time zero.

A key step in the calculation of the certainty equivalents is to find a suitable risk-adjusted discount rate, r, to calculate the present value of the value of completion and of the completion cost at time T. This is similar to the task of finding a risk-adjusted discount rate to discount revenues using simulation-based real options approaches, as discussed at the end of Section 2.3.2. Again, if the market risk of the investment is similar to those of other projects in the investor's portfolio, a discount rate similar to that used in the other projects could be assumed. Otherwise, the risk-adjusted discount rate could be estimated with CAPM, the WACC, and/or expert judgment. Furthermore, if the entity undertaking the project is a diversified investor and the costs are primarily subject to technical risk, the risk-free discount rate could be applied to the completion cost. The assumption in this case is that costs are not affected by market risk and, thus, the discount rate only needs to adjust for the time value of money.

3.1.2 Derivation of the real options formula for any distribution of value of completion and fixed completion cost

With the explanation of the calculation of certainty equivalents completed, the derivation of the real options formula for any probability distribution of the value of completion and a fixed completion cost can be presented. This first approach is based on Hull (2000) and Chriss (1997).

Let $f_v(v)$ be the probability density function of the certainty equivalent of the value of completion, V, at time T. Similarly, let C be the certainty equivalent of the completion cost at time T. Both quantities are shown in Figure 3-1:

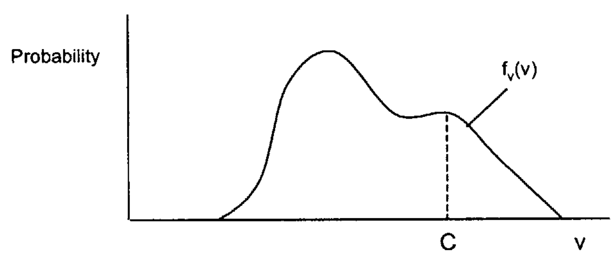


Figure 3-1: Probability distribution of the certainty equivalent of the value of completion, V, and the certainty equivalent of the completion cost, C, at time T.

According to the principle of options, i.e., of taking an action only when it is favorable to do so, the decision-maker would only exercise in those instances when the value of completion is greater than the completion cost. The value of an option at time T, $O|_{t=T}$, can be calculated as the difference of two terms. The first term is the expected value of revenues given that the revenues are realized, i.e., given that the option is exercised. Since the option would only be exercised if the value of completion is higher than the completion cost, this expected value can be represented as the expected value of V for values of v > C (see first term in Equation 3-3) [Hull, 2000]. The second term represents the costs associated with exercising the option. It can be computed as the completion cost, C, weighted by the probability that it is realized, i.e., the likelihood that the option is exercised. This can be expressed as C times the probability that C will be incurred, i.e., the probability that v > C (second term in Equation 3-3) [Chriss, 1997]:

$$O\big|_{t=T} = \int_{v=C}^{\infty} v\, f_v(v)dv - C\int_{v=C}^{\infty} f_v(v)dv \qquad \text{(Eq. 3-3)}$$

where $f_v(v)$ and C are the certainty equivalents of the probability distribution of the value of completion and of the completion cost at maturity time, T, respectively. The value of the real option at time zero can be found by discounting $O|_{t=T}$ with the risk-free discount rate:

$$O = e^{-r_f T}\left(\int_{v=C}^{\infty} v\, f_v(v)dv - C\int_{v=C}^{\infty} f_v(v)dv\right) \qquad \text{(Eq. 3-4)}$$

Notice that Equation 3-4 does not make any assumptions about the probability distribution of the value of completion, but it requires that the completion cost be fixed. Formulations similar to this have been explored recently by others. The methodology developed by Datar and Matthews (2004) mentioned in Chapter 2 rests on a formula similar to Equation 3-4. Datar and Matthews' derivation is based on [Hull, 2000], but instead of using certainty equivalents, they use a risk-adjusted discount rate to find the present value of the value of completion and the risk-free rate to discount the completion cost. They apply this methodology to cases where the value of completion follows a binomial or a jump-diffusion process.

3.1.3 Alternative derivation of the formula for any distribution of value of completion and fixed completion cost

An alternative, more intuitive, derivation of Equation 3-4 can be obtained following the approach outlined in McDonald and Siegel (1985). Here, the authors define the value of a European call option at time T, $O|_{t=T}$, as the expected value of the cash flow given that the option is only exercised if profits are equal or greater to zero:

$$O_{t=T} = E[\max(V_T - C_T, 0)] \qquad \text{(Eq. 3-5)}$$

where V_T and C_T are the value of completion and the completion cost at maturity, T, respectively. The value of this option today, O, is found by discounting the value of the option at time T to the present with a suitable discount rate, r:

$$O = e^{-rT} E[\max(V_T - C_T, 0)] \qquad \text{(Eq. 3-6)}$$

Assuming that the value of completion at time T can be described with a probability density function, $f_v(v_T)$, and using the definition of expected value for continuous random variables, Equation 3-6 becomes:

$$O = e^{-rT} \int_{v=C_T}^{\infty} (v_T - C_T)\ f_v(v_T) dv_T \qquad \text{(Eq. 3-7)}$$

Notice that the lower limit of the integral must be the completion cost, C_T, in order for $(v_T - C_T) \geq 0$. Rearranging terms, Equation 3-7 can be written as:

$$O = e^{-rT}\left(\int_{v=C_T}^{\infty} v_T f(v_T) dv_T - C_T \int_{v=C_T}^{\infty} f(v_T) dv_T \right) \qquad \text{(Eq. 3-8)}$$

Finally, substitute $f_v(v_T)$ and C_T for their certainty equivalent at time T, $f_v(v)$ and C, respectively. In addition, replace the discount rate, r, with the risk-free discount rate, r_f, since all terms in the Equation are certainty equivalent:

$$O = e^{-r_f T}\left(\int_{v=C}^{\infty} v\, f_v(v) dv - C \int_{v=C}^{\infty} f_v(v) dv \right) \qquad \text{(Eq. 3-9)}$$

This equation is the same as the one derived based on Hull (2000) and Chriss (1997), i.e., Equation 3-4. It is important to note that McDonald and Siegel do not take the step to reach Equation 3-9. Rather, they continue the derivation of Equation 3-8 assuming that the value of completion follows a random walk and eventually come to an expression that, depending on their assumptions on risk, is identical to Black and Scholes's (1973) formula for European call options, or Merton's (1973) European call option formula on stocks that pay a proportional dividend.

3.2 Expanding the real options formula to any distribution of the value of completion and of the completion cost

The real options formula derived thus far can be used for any distribution of the value of completion; however, it still assumes that the completion cost is fixed. In reality, exercise costs can also be uncertain and may not follow a GBM or any other known stochastic processes. Thus, in order to evaluate real projects, it is of interest to find a formula that can take any type of probability distribution for the value of completion and for the completion cost.

The formula to evaluate real options when both the value of completion and the completion cost are uncertain can be derived from Equation 3-9. Let $f_v(v)$ and $f_c(c)$ be the probability distributions of the certainty equivalent of the value of completion and of the completion cost, respectively, at maturity time T. Furthermore, assume that these distributions can be of any shape (see Figure 3-2):

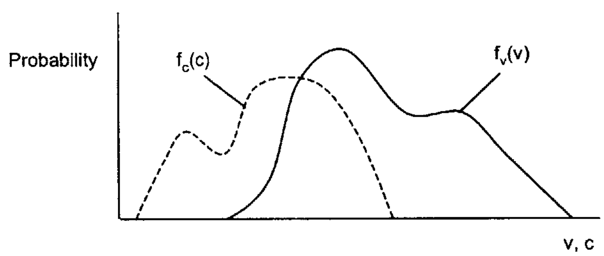


Figure 3-2: The certainty equivalent of the value of completion, $f_v(v)$, and of the completion cost, $f_c(c)$, can be described with general probability distributions.

With a random completion cost, c, the value of the option is now dependent on c. Thus, substituting c for C in Equation 3-9 yields:

$$O(c) = e^{-r_f T}\left(\int_{v=c}^{\infty} v \; f_v(v)dv - c \int_{v=c}^{\infty} f_v(v)dv \right) \qquad \text{(Eq. 3-10)}$$

The expected value of the option, w, can be determined by applying the definition of expected value for continuous random variables to Equation 3-10:

$$w = E[O(c)] = \int_{c=-\infty}^{c=\infty} O(c)\ f_c(c)dc = e^{-r_f T}\left(\int_{c=0}^{\infty} f_c(c) \int_{v=c}^{\infty} v\ f_v(v)dvdc - \int_{c=0}^{\infty} c\ f_c(c) \int_{v=c}^{\infty} f_v(v)dvdc \right)$$

(Eq. 3-11)

Similarly, the variance of the option value can be found by applying the definition of variance for continuous random variables to Equation 3-10:

$$Var(O(c)) = E\left[O(c)^2\right] - \left(E[O(c)]\right)^2$$

$$= e^{-2r_f T}\left[\int_{c=-\infty}^{c=\infty} \left[\left(\int_{v=c}^{\infty} v\ f_v(v)dv \right)^2 - 2\ c \int_{v=c}^{\infty} v\ f_v(v)dv \int_{v=c}^{\infty} f_v(v)dv + c^2 \left(\int_{v=c}^{\infty} f_v(v)dv \right)^2 \right] f_c(c)dc \right.$$

$$- \left(\int_{c=0}^{\infty} f_c(c) \int_{v=c}^{\infty} v\ f_v(v)dvdc \right)^2 + 2 \left(\int_{c=0}^{\infty} f_c(c) \int_{v=c}^{\infty} v\ f_v(v)dvdc \right)\left(\int_{c=0}^{\infty} c\ f_c(c) \int_{v=c}^{\infty} f_v(v)dvdc \right)$$

$$\left. - \left(\int_{c=0}^{\infty} c\ f_c(c) \int_{v=c}^{\infty} f_v(v)dvdc \right)^2 \right]$$

(Eq. 3-12)

Equations 3-11 and 3-12 give the expected value and the variance, respectively, of a European call option on an asset with a random value of completion and a random completion cost. These formulae can be used to evaluate European-like real options on any projects for which a probability distribution for the value of completion and the completion cost can be determined. These distributions can be completely arbitrary as the formulae do not constrain them to any particular type of stochastic process.

An important assumption in the derivation of Equations 3-11 and 3-12 is that the value of completion and the completion cost are independent. This can be a reasonable assumption for real projects where, for example, the couplings between revenues and costs are non-existent or very weak. In the case where these couplings may be significant, the distributions of the value of completion and of the completion cost would have to be conditional on each other.

An example of another generalized methodology to evaluate real options can be found in Clemons and Gu (2003). In this paper, the authors first determine an analytical expression for the value of a strategic real option as a function of several uncertain variables. Then, the value of the real option is calculated by integrating this analytical expression over the bounds of the distributions of the uncertainties. Similar to the methodology developed in this thesis, this approach is very general because it does not constrain the probability distribution for the different uncertainties nor does it limit the type of function for the value of the strategic option. In addition, this approach can be used with simulation when no analytical expression for the value of the option as a function of the uncertain variables exists. A main difference of this approach with the one developed in this thesis, however, is that Clemons and Gu analyzed the value of an option assuming that exercise was required when certain competitive circumstances were met, but they did not consider the probability distribution of these circumstances being realized. Thus, the treatment of risk is not as explicit as it is in the methodology developed here.

3.4 Multiple time periods (American-like options)

The evaluation formula developed thus far can be used to evaluate European-like options, i.e., those that can only be exercised at maturity; however, it is not difficult to expand the methodology to incorporate the possibility of exercising not only at maturity but also at other times up to and including maturity, similar to American options. Consequently, the formula could be used to evaluate more complex (and more realistic) situations in which exercise would be possible at several points in time.

Consider that the decision-making process can be divided in a binary decision-tree with multiple time periods, as shown in Figure 3-3. Furthermore, assume that at the end of each period two alternatives are possible: immediate exercise or keeping the option alive by not exercising. The option can be kept alive up to the final period (maturity), at which point if it is not exercised, it expires and the payoff is zero. The value of the option if exercised can be calculated at each time period with the single-period evaluation formula shown above.

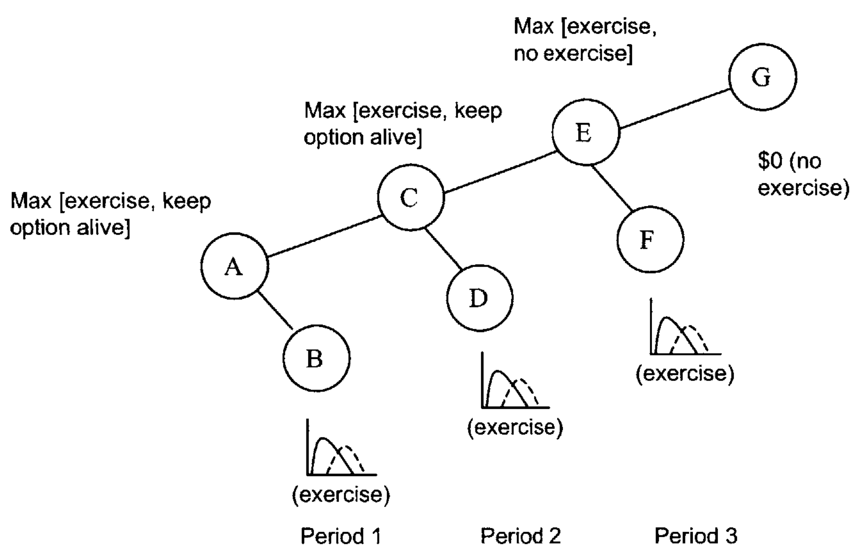


Figure 3-3: Evaluation of real options that can be exercised at several points up to maturity.

The value of the option with several decision points can be found by solving the decision-tree backwards. The value at the last decision node (Node E) is the maximum between the value of exercising the option (Node F) and zero (Node G). Similarly, the value at node C is the maximum between exercising the option (Node D) and keeping the option alive (Node E). Finally, the value of the multiple-period decision at Node A is, again, the maximum between exercising (Node B) and keeping the option alive (Node C). The values of exercising the option at nodes B, D and F can be calculated with Equation 3-11.

3.5 Evaluation of real options with the generalized methodology

The process to evaluate real options with the generalized methodology consists of a few steps, as shown in Figure 3-4. First, it is necessary to determine the probability distributions of the value of completion, V, and of the completion cost, C. Each of these distributions can be specified either analytically or numerically. Analytical models may include stochastic processes, such as mean-reverting or jump-diffusion processes. If a numerical approach is preferred, a numerical model can be combined with Monte Carlo simulation, for example, to obtain the probability distributions of the variables of interest. Once the probability distributions of the value of completion and the completion cost are known, they can be substituted in the generalized real options equations 3-11 and 3-12 to determine the expected value and the variance, respectively, of the real option.

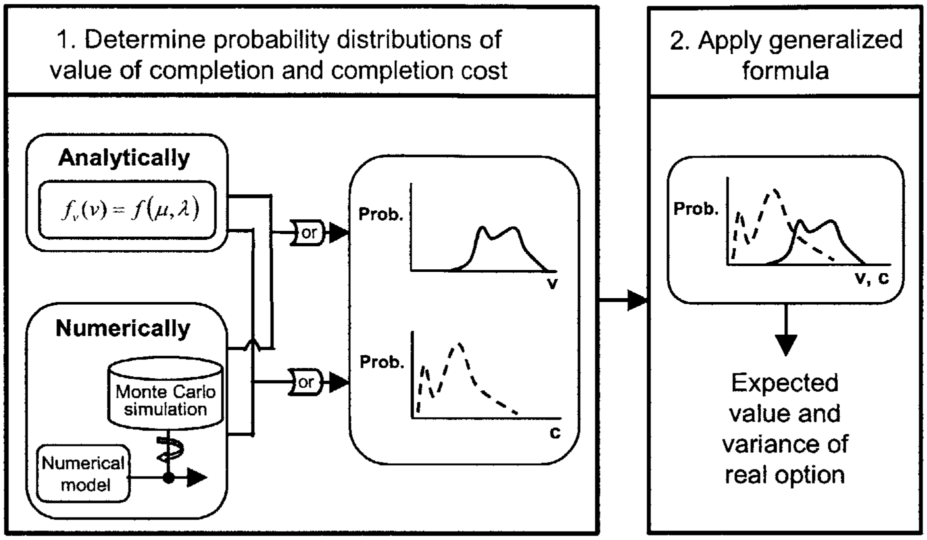


Figure 3-4: Schematic of the process to evaluate real options with the generalized real options approach.

In the next section, two examples of solutions to the generalized real options equations when both the probability distributions of V and C are given analytically are shown. Following this, the case where both distributions are calculated numerically is discussed.

3.5.1 Analytical solutions

Analytical solutions for the expected value and the variance of the value of the European-like real option can be found with Equations 3-11 and 3-12, respectively, if the probability distributions of the value of completion and the completion cost can be expressed analytically, and if their integral exists in closed-form. Two examples are explored in this section. The first assumes uniform probability distributions for both the value of completion and the completion cost. The second assumes an exponential distribution for the value of completion and a uniform distribution for the completion cost.

Example 1: Two uniform distributions

In this case, both the value of completion and the completion cost are described by uniform probability distributions. As an example for this situation, consider a group of people deciding to invest in the development of a new consumer product such as cars, monkey-wrenches, or a new lines of clothing, for example. Given the seasonality in consumer goods, they need to introduce the product by a certain date, T, otherwise the opportunity may be gone forever. Because of uncertainties in demand and in the cost of producing the product, the investors have decided to develop a prototype first.

Developing the prototype gives them a real option: once the prototype is finished, they will know with more certainty the potential revenue and the cost of mass production, thus, if potential revenue (the value of completion) exceed production costs (the completion cost), they would exercise the option and proceed with mass production. Otherwise, they would not spend any more money on the project and their losses would be limited to what they spent on the prototype.

The investors need to decide how much to spend on the prototype today. They estimate that the value of completion for this product could be anywhere between v_1 and v_2 with equal likelihood u_v (see Figure 3-5). Furthermore, they assume that completion costs for mass production fall with equal probability u_c between c_1 and c_2.

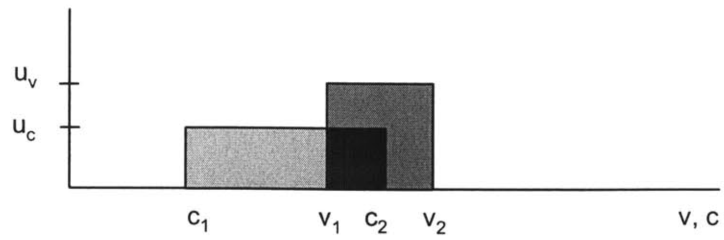


Figure 3-5: Hypothetical example of a real option with uniform probability distributions for the value of completion and the cost of completion.

The investors can use this information and the real options formula derived in this thesis to calculate the maximum amount that they should spend in the prototype. This is arguably a very simple situation, but it is useful to explain the subtleties of the integration to find the exact analytical solution. The details of this integration are shown in Appendix B.

Example 2: An exponential and a uniform distribution

A second example of an analytical solution to the real options equations developed in this thesis is the combination of an exponential and a uniform probability distribution. To illustrate this situation, recall the decision to build a prototype for a consumer product mentioned above. In the previous example, the distribution of potential revenue was assumed uniform between v_1 and v_2. Now, the investors believe that there is a high probability that potential revenue may be low, but there is also some probability that it may be high. Thus, assume that the probability distribution of the potential revenue can be described with an exponential function, $f_{v\_exp}(v)$ (see Figure 3-6).

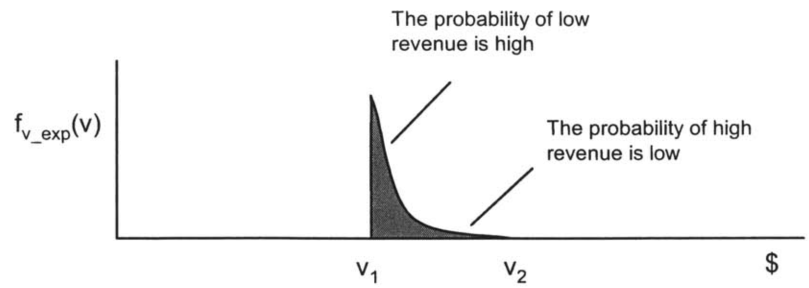


Figure 3-6: Assumed probability distribution of potential revenue for the consumer product.

In terms of production costs, the investors still assume that they fall with equal probability u_c between c_1 and c_2, as shown in Figure 3-7:

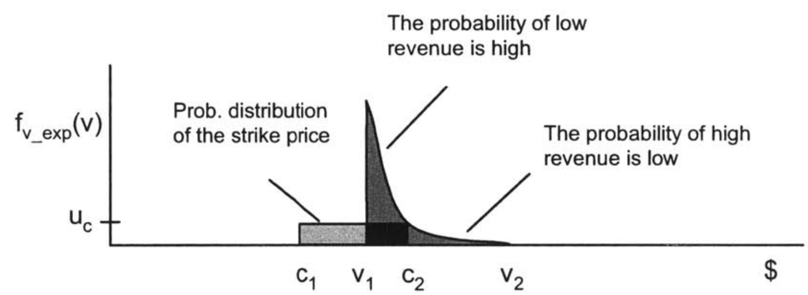


Figure 3-7: The exercise cost is still assumed uniform.

An analytical expression for the expected value and variance of this real option can be determined by substituting the probability distributions for the value of completion and the completion cost in Equations 3-11 and 3-12, respectively. The derivation of the analytical expression follows the same steps as in the previous example of two uniform distributions. Complete derivation results are shown in Appendix B.

3.5.2 Numerical solutions

In cases where the probability distribution of the value of completion and/or of the completion cost can not be described analytically, such distributions must be found numerically. Similarly, the solution to the real options formula developed in this thesis must be found numerically in these situations.

There are several alternatives for finding the probability distribution of the value of completion and of the completion cost. In this thesis, a combination of system dynamics and Monte Carlo simulation is used. System dynamics is a powerful tool to model complex systems where variables are inter-related and where they vary over time. In addition, system dynamics is very flexible and allows the modeling of many different types of problems. Furthermore, Monte Carlo simulations can be run by using the system dynamics model as the evaluation function, thus, the system dynamics model is run repeatedly with different values for the exogenous variables, which are drawn from specified probability distributions for each of them. Consequently, multiple sources of uncertainty can be taken into account in the calculation of the probability distributions of the value of completion and the completion cost. This adds more realism to the simulation because it allows the user to incorporate estimates of the uncertainties into the simulation. System dynamics is often not exact in the answers it produces; however, with the use of Monte Carlo simulation, and given appropriate values for the distribution of the uncertainties, the combination of Monte Carlo simulation and system dynamics is useful to determine the probability distribution of the variables of interest in uncertain environments.

Once the probability distributions of the value of completion and the completion cost have been calculated, the expected value and variance of the real option can be found by solving Equations 3-11 and 3-12 numerically. An algorithm for the numerical solution of both equations is shown in Appendix C. This algorithm is simple and it can be implemented in electronic spreadsheets and Matlab®.

3.7 Chapter summary

In this chapter, the derivation of a generalized real options methodology has been presented. The main points of this chapter can be summarized as follows:

1) A generalized methodology to calculate the expected value and the variance of European-like real options has been developed. This methodology can be used to evaluate real options where the value of completion and the completion cost are described by any probability distribution.

2) The methodology offers the advantage over existing analytical approaches that it is not necessary to force the representation of the value of the underlying asset into known stochastic processes, such as the commonly-used geometric Brownian motion, or to assume that the completion cost is fixed a-priori. This can allow a better portrayal of the true nature of the underlying asset and the completion cost. If the probability distribution of the value of completion and the completion cost can be expressed analytically, and if their integrals exist in closed form, an exact solution can be found.

3) The methodology offers the advantage over simulation approaches in that if the probability distribution of the value of completion and the completion cost are given analytically, an exact solution can be found without the computational complexity of simulation. In cases were no analytical expressions exist, the analytical framework of this methodology can be used to generate insights into how the probability distribution of costs and revenues affect the value of the real option without the need to simulate the entire process numerous times.

4) The real options methodology can be extended to evaluate American-like real options by framing the problem in a binary decision tree. This increases the number of possible applications of this methodology.

5) Exact solutions for two examples where the probability distributions of the value of completion and the completion cost are given analytically have been derived. The first assumes uniform probability distributions for both the value of completion and the completion cost. The second assumes an exponential distribution for the value of completion and a uniform distribution for the completion cost.

6) If the probability distribution of the completion cost or the value of completion can not be described analytically, the solution to the real options formula must be found numerically. A simple algorithm for solving the real options valuation numerically has been developed.

4. Application of the generalized real options methodology to a new aircraft development program

A new aircraft program is a typical example of the type of investments that are made in air transportation: it requires large capital expenditures, it is a multi-year project, and its success depends on many market and technical uncertainties. Under these circumstances, a flexible investment strategy, i.e., one that allows investors to wait until more information is available, can be of great value. The value of such a strategy can be found with the real options methodology proposed here.

The real options explored in this chapter are on the development of the new airplane model by the aircraft manufacturer. These are different from options for purchase of the finished product by airlines, which are also found in the literature (see, for example, Stonier (1999)) . The analysis presented here differs from previous studies of options in the development of aircraft programs because the value of completion and the completion cost are modeled with a bottom-up approach using a combination of system dynamics and Monte Carlo simulation. Previous work, such as the one by Markish and Willcox (2002), use a top-bottom model that assumes the value of completion to follow a geometric Brownian motion.

The example explored in this thesis is a case study of a real-world aircraft program where revenues and costs are derived from data provided by a major aircraft manufacturer. The project described here does not correspond to a specific aircraft but is rather grounded in the manufacturer's experience with different programs; however, because of confidentiality concerns, the name of the company cannot be divulged. In addition, the scale of the numbers has been altered and monetary results are given in terms of monetary units (MU). Despite these modifications, the analysis shows that investors can use the generalized real options methodology to evaluate flexible investment strategies in new aircraft programs and, thereby, determine how much they should spend on such projects.

This chapter is structured as follows: first, an overview of some challenges of the airline transportation industry as they relate to investments are given; second, a description of the new aircraft development program is presented; third, a simple example of launching a derivative aircraft is described to illustrate the basic mechanics of the real options methodology; fourth, the real options valuation is applied to the entire development program; finally, a summary of the main points of the application is provided.

4.1 Challenges for investments in the airline industry

Air transportation is a cyclical industry characterized by periods of high growth followed by periods of deep traffic reductions [Skinner et al., 1999; Stonier, 1999]. For example, in the United States, demand for aviation services, measured in terms of revenue-passenger miles (RPMs)[3], had robust growth from 1982 to 1987 (see Figure 4-1). It then experienced a decline, which was exacerbated by the Gulf War in 1991. After this, the industry entered a period of strong recovery throughout the 1990s. This bonanza waned

[3] RPM is a standard measure of airline traffic. It represents one paying passenger flown one mile.

in early 2001 and demand plummeted after the attacks of September 11th of the same year. Since then, the path to recovery has been slow but steady.

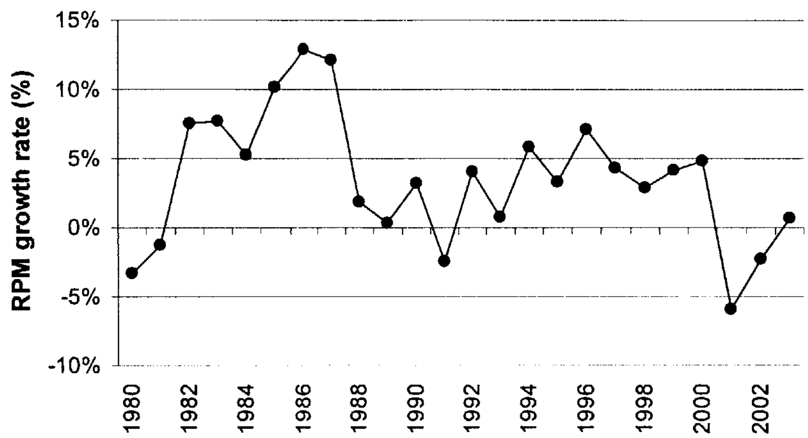


Figure 4-1: The demand growth rate for air transportation in the United States domestic market is cyclical. Airline demand is measured in terms of revenue-passenger miles (RPMs). ***Source: ATA (2004).***

Planning in the face of this volatile traffic demand is a major challenge for all stakeholders, in particular airlines, aircraft manufacturers, and airports. The long lead times associated with delivery of new aircraft, construction of new production lines, or new passenger buildings may result in these investments not arriving at the appropriate time: a premature investment may result in unused capacity that sits idle without generating any returns whereas a tardy investment may miss the potential market completely.

For example, consider the impact of a cyclical market on airlines' fleet planning in the context of the orders and deliveries of Boeing 737 aircraft (all series) for United Airlines (UA) and American Airlines (AA) from 1980 to 2004 that are shown in Figure 4-2 along with the annual growth rate of US domestic market demand (measured in revenue-passenger miles, or RPMs). United placed a large order for 101 airplanes in 1985 and a second order for 57 aircraft in 1989. Although United began receiving some of these aircraft in 1986, the majority of the aircraft were delivered during the four-year period between 1988 and 1992. At the time that the large order of 101 planes was placed in the mid 1980s, traffic was growing rapidly; however, by the time aircraft began to be delivered in large numbers in 1988, traffic growth was substantially less. In fact, in one year (1999) during the aforementioned four-year period in which UA received most of its new 737s, the year-over-year change in traffic was negative.

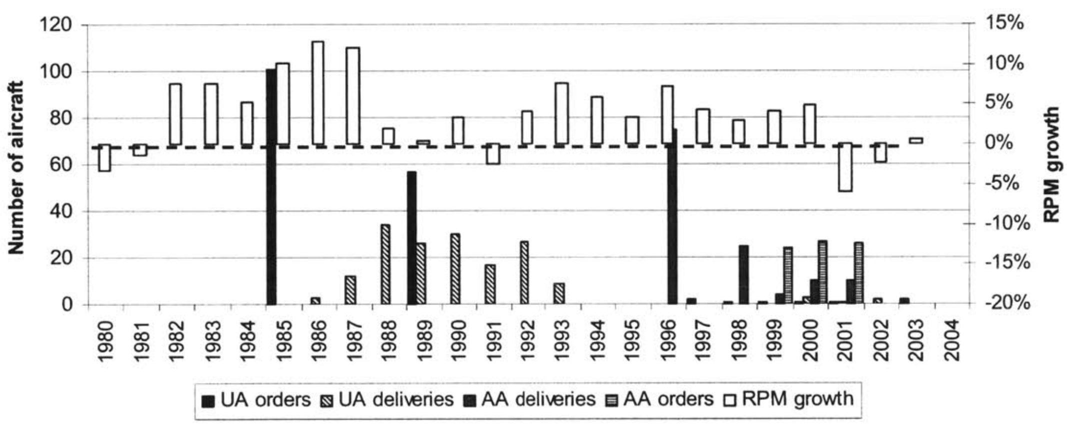


Figure 4-2: Orders and deliveries of Boeing 737 aircraft (all series) for United Airlines (UA) and American Airlines (AA) from 1980 to 2004. The blank bars represent traffic demand in the US domestic market in terms of RPM growth rate. ***Source: ATA (2004) and Boeing (2004).***

A similar situation occurred to American Airlines in the second half of the 1990s and early 2000s. In 1996 and 1998, AA ordered 75 and 25 aircraft, respectively. This was another period of sustained traffic growth in the domestic US airline industry; however, as in the case of United, American started receiving the aircraft in 1999, three years after the initial order. Traffic growth remained strong in 1999 and 2000, but it plummeted in the ensuing years as a result of a weak economy and the terrorist attacks of September 11th, 2001. Of the 124 aircraft ordered between 1996 and 2001, AA had only taken 77 by mid 2004. Receiving this extra capacity at the time when the market is decreasing may not be in the interest of the airline.

Another potential pitfall of long lead times in the delivery of air transportation infrastructure is the possibility of paradigm shifts in the industry in the mean time. These may render the ordered, yet unavailable investment, obsolete before it is even deployed. Between 1998 and 2001, United Parcel Service (UPS) ordered 90 A300-600 cargo aircraft from Airbus; however, in March of 2004, there were indications that UPS may cancel 20 of them [Dow Jones Business News, 2004]. During the past few years, UPS and FedEx have shifted part of their business from air shipments to the less expensive ground deliveries in response to customer preferences prompted by the economic slump of recent years. Therefore, the need for all the originally ordered A300-600s is apparently no longer there.

4.2 Overview of the new aircraft development program

Aircraft manufacturers, in their position as key players in the air transportation system, are subjected to the many uncertainties in the industry. Launching a new aircraft program is a no small undertaking as the cost of new aircraft programs is typically in the billion of dollars and a failed project may seriously compromise the future of the company [Esty and Ghemawat, 2002; Newhouse, 1982]. Given the fierce competition among aircraft manufacturers, the large capital expenditures required to develop a new product, and uncertainties in the market, a flexible investment strategy is essential to enhancing the profitability and the likelihood of success of any new aircraft program.

A typical new aircraft development program consists of a number of phases in sequence. A highly simplified sketch of a typical aircraft program based on the data provided by a major aircraft manufacturer is shown in Figure 4-3. For the purposes of this thesis, it is assumed that the process starts with preliminary design, i.e., it is assumed that all preliminary work in terms of market research, preliminary trades studies, etc. has already been completed. Once preliminary design is finished, the next phase is product development. Product development is divided in three steps. The first step is initial development of the aircraft. In the second step, the aircraft for the first test flights are built. The third phase of product development is flight certification. Once the aircraft has been certified, serial production of the baseline aircraft can begin. In the particular example considered here, it is assumed that after a year of serial production, the manufacturer has two alternatives to continue production: a) maintain production of the baseline aircraft, or b) maintain production of the baseline aircraft and launch a derivative aircraft.

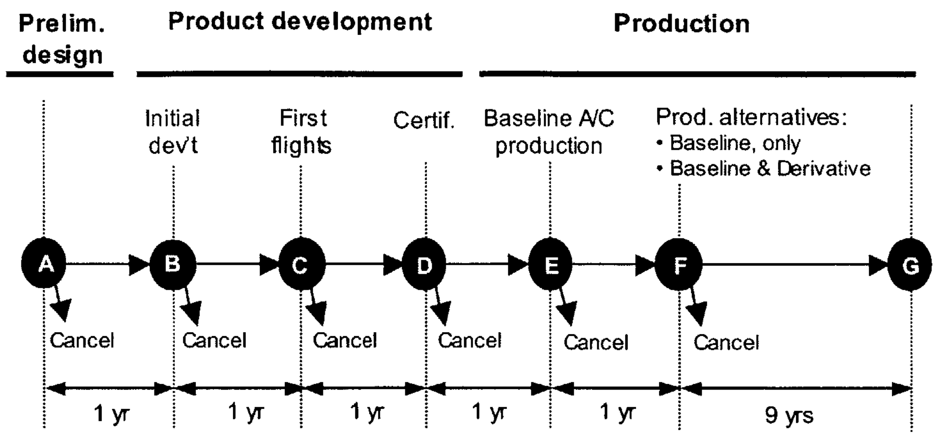


Figure 4-3: Main steps in a new aircraft development process.

A project structure like the one shown in Figure 4-3 has several real options. For example, at the end of each phase, the project manager has the option of continuing or canceling the process. Each phase gives the development team the opportunity to spend a relatively small incremental amount of resources to gather more information about the product before fully committing to a large investment. The real options methodology developed here can inform managers if the project should be executed and, if so, how much should be spent in each phase.

Notice that there may be more real options in this process that the ones shown in Figure 4-3. For example, in Steps B, C, and D, in addition to the options of continuing and canceling the project, the aircraft manufacturer could have the option to alter the development process to spend more resources to solve particular technical problems that may arise. Furthermore, there could be the option of stopping the project for a number of years to wait for better market conditions. For the purposes of the discussion in this chapter, however, only the options shown in Figure 4-3 have been considered.

4.3 Simple example: value of the real option to launch a derivative aircraft

Before showing the application of the real options methodology to the entire new aircraft development program, it is useful to analyze a simple example first to become familiar with the details of this methodology. The example presented here is the evaluation of the real option to launch a derivative aircraft. A derivative aircraft has many characteristics in common with the baseline product but it offers certain modifications that make it attractive for other market needs. For example, typical attributes of derivative aircraft include different seat capacity and/or range than the baseline model. Because the development and construction of the derivative aircraft can share many of the resources devoted to the development and construction of the baseline aircraft, the launch of a derivative can occur faster and at a fraction of the cost than if the manufacturer had not developed the baseline aircraft in the first place. Thus, the real option to launch a derivative aircraft can be very valuable.

The taxonomy of the real option to launch a derivative aircraft will be presented next. This is followed by a description of the system dynamics and Monte Carlo simulation used to model the value of completion and the completion cost of this real option. Then, numerical results for this simple example are given.

4.3.1 Taxonomy of the real option to launch a derivative aircraft

To begin this discussion, consider a group of investors standing on step E in Figure 4-3 and assume for the moment that there are two alternatives to continue production in Step F: the investors can continue producing the baseline aircraft, only, or they can decide to produce the baseline aircraft and launch a derivative. In any case, the investors have the right to cancel the project at Step F if conditions are not favorable.

In order to have the option to launch the derivative in Step F, the investors must spend some resources in the development of the derivative aircraft, which are additional to the resources already spent in the development of the baseline product. Once this additional development effort is completed, the investors will have a real option: they will have the right to produce and sell the derivative product by building the production facilities to produce the derivative aircraft. The investors would only do so if the expected revenues from the derivative aircraft sales would outweigh the expected cost of its production facilities. Thus, the structure of the real option to launch a derivative aircraft can be defined as follows:

Value of completion of launching a derivative, $V_{launch\ derivative}$: This is the real asset that the investors want to acquire by exercising the real option. Here, it is assumed to be the expected net present value of income from sales of the derivative aircraft. Income is defined as the difference between revenues from sales minus production costs of the derivative aircraft. It is further assumed that sales of the derivative aircraft will not affect sales of the baseline aircraft.

Completion cost of launching a derivative, $C_{launch\ derivative}$: This is the cost that the investors must pay in order to acquire the real asset. It is defined as the expected present value of costs for the production facilities (hangars, tooling, etc) for the derivative aircraft. The completion cost is assumed to be 20% of the development costs for the derivative aircraft.

Maturity of the option to launch a derivative, $M_{\text{launch derivative}}$: This is the amount of time that the real option is alive. According to the assumptions in this example, development of the derivative aircraft starts after the baseline aircraft enters production in Step E and the launch of the derivative occurs a year later in Step F. Thus, the option is alive between Steps E and F and has a maturity of one year.

Value of the real option to launch a derivative, $O_{\text{launch derivative}}$: This is the maximum amount of money that the investors should invest in the development of the derivative aircraft. This development occurs between Steps E and F. Once the probability distributions of the value of completion and the completion cost are known, the expected value and the variance of this real option can be calculated with equations 3-11 and 3-12, respectively.

It is worth to point out that $V_{\text{launch derivative}}$ and $C_{\text{launch derivative}}$ are defined as explained above to make them independent from each other, which is necessary to apply the generalized real options methodology without calculating conditional probabilities. If $V_{\text{launch derivative}}$ was defined in terms of total expected revenues, for example, completion costs would have been total expected production costs (including the cost of production facilities and unit production costs), which would clearly depend on how many units are sold and, therefore, the value of completion and the completion cost would be dependent.

4.3.2 System dynamics and Monte Carlo simulation for the simple example

To evaluate the real option to launch a derivative with the generalized methodology, it is necessary to determine the probability distribution of the value of completion, $V_{launch\ derivative}$, and of the completion cost, $C_{launch\ derivative}$. Once these distributions are known, they can be substituted in equations 3-11 and 3-12 to calculate the expected value and variance, respectively, of the real option.

For the particular example considered here, the probability distribution of the value of completion can be calculated with a combination of system dynamics and Monte Carlo simulation, where the system dynamics model is run many times with different values for the exogenous variables which are drawn from probability distributions specified for each one of them (see Figure 4-4). The probability distribution of the completion cost can be calculated directly from data provided by the aircraft manufacturer. The procedure to determine both quantities is explained next.

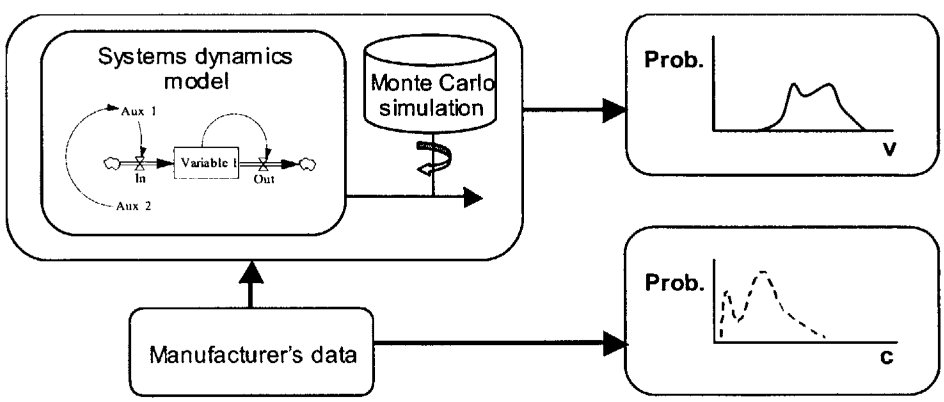


Figure 4-4: Schematic of the process to obtain the probability distributions of the value of completion, $V_{launch\ derivative}$, and of the completion cost, $C_{launch\ derivative}$, for the real option to launch a derivative aircraft.

System dynamics model for the value of completion of the real option to launch a derivative aircraft

A system dynamics model was developed with input from the aircraft manufacturer to determine the value of completion for the real option to launch a derivative (see Figure 4-5).

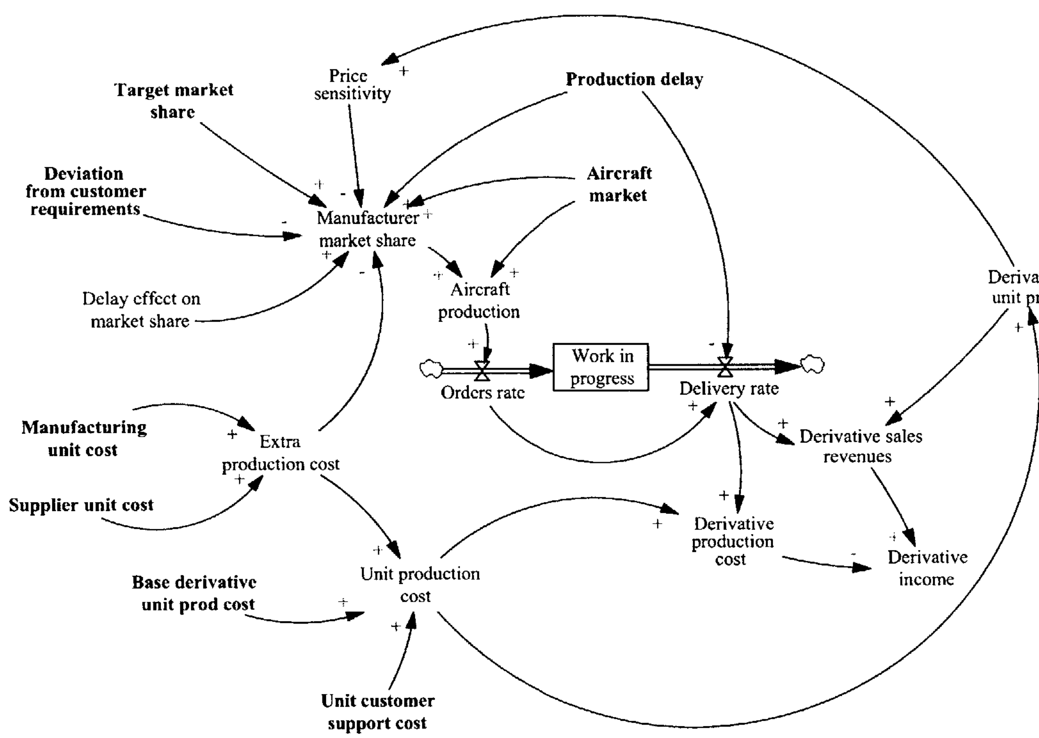


Figure 4-5: System dynamics model to determine the value of completion of the real option to produce a derivative aircraft.

The main accumulating stock in this model is Work In Progress, which indicates how many derivative aircraft are in production at any point in time. The number of aircraft in production increases with the inflow of orders (Orders Rate) and decreases with the outflow of finished aircraft (Delivery Rate). Orders Rate depends on Aircraft Production, which is the product of the size of the Aircraft Market times Manufacturer Market Share.[4] The magnitude of Manufacturer Market Share depends on Target Market Share, Production Delay, Deviation from Customer Requirements, and Price Sensitivity, as will be explained below. The Delivery Rate is equal to Orders Rate unless Production Delay is positive, in which case the finished aircraft are delivered with a delay specified by Production Delay. Derivative Sales Revenues is the product of Delivery Rate times Derivative Unit Price, and Derivate Production Cost is the product of Delivery Rate times Unit Production Cost. The value of completion of the real option to launch a derivative is the present value of Derivative Income, which is the difference between Derivative Sales Revenues and Derivative Production Cost.

The variables in bold in Figure 4-5 are exogenous and their value is determined by probability distributions in the Monte Carlo simulation (see below). Price Sensitivity is also exogenous, but its value is fixed and does not change in the Monte Carlo simulation. A summary of the type of variables (i.e., input or exogenous, intermediate, and output) of the system dynamics model is shown in Table 4-1:

[4] The minus and plus signs of the arrows in the model indicate the polarity of the effect of the originating variable on the destination variable. Thus, if the polarity is positive, the change in the destination variable is in the same direction as in the originating variable. For example, an increase in Aircraft Market increases the value of Aircraft Production. Otherwise, if the polarity of the arrow is negative, the change in the destination variable is in the opposite direction as in the originating variable. For example, an increase in Derivative Production Cost decreases Derivative Income.

Table 4-1: Summary of variables in the system dynamics model used to determine the value of completion of the real option to launch a derivative aircraft.

Input (exogenous) variables	**Intermediate variables**	**Output variable**
Aircraft Market	Aircraft Production	Derivative Income
Base Derivative Unit Production Cost	Delay Effect on Market Share	
Deviation From Customer Requirements	Delivery Rate	
Manufacturing Unit Cost	Derivative Unit Price	
Price Sensitivity	Derivative Production Cost	
Production Delay	Derivative Sales Revenues	
Supplier Unit Cost	Extra Production Cost	
Target Market Share	Manufacturer Market Share	
Unit Customer Support Cost	Orders Rate	
	Unit Production Cost	
	Work in Progress	

The principal variables and relationships in the model are explained in more detail next. There are three main variables that affect the inflow and outflow of orders in this model: Manufacturer Market Share, Derivative Unit Production Costs, and Derivative Unit Price:

Manufacturer Market Share is key in this model because it determines the number of aircraft orders per year (Orders Rate). There are four variables that determine Manufacturer Market Share:

a) Target Market Share: This is an exogenous variable that determines the maximum share of the total market for this particular aircraft that the manufacturer expects to capture. This parameter is assumed uncertain because it depends on the actions by competitors. To capture this risk, a probability distribution for this variable is assigned in the Monte Carlos simulation (see below).

b) Production Delay: This is another exogenous variable. Delays in the delivery of aircraft are assumed to have a negative impact on Manufacturer Market Share. The assumption is that delays in delivering the finished aircraft result in some customers placing orders with other manufacturers. Production Delay is an uncertain parameter that is accounted for in the Monte Carlo simulation.

c) Deviation from Customer Requirements: The new aircraft model is developed to achieve a defined performance to satisfy customer needs. It is assumed that deviations from the performance targets promised to the customer results in a loss of market share. Deviation from Customer Requirements is another exogenous variable and its magnitude is determined by a probability distribution in the Monte Carlo simulation.

d) Price Sensitivity: Price sensitivity is a fixed variable that reflects the negative effect of increases in Derivative Unit Price on Manufacturer Market Share. Derivative Unit Price is determined so that a certain margin over Unit Production Costs is achieved (see below). Uncertainties in Unit Supplier Cost and Unit Manufacturing Cost can lead to Extra Production Costs, which increase Unit Production Cost and, in turn, lead to higher Derivative Unit Price. Through Price Sensitivity, Increases in Derivative Unit Price have a negative impact on Manufacturer Market Share because as the price of the product increases, some customers prefer to purchase aircraft from other manufacturers or they do not purchase any aircraft at all. Price Sensitivity for this example is estimated at approximately 0.6% loss of Manufacturer Market Share for each 1% increase in Derivative Unit Price. Unit Supplier Cost and Unit Manufacturing Cost are exogenous variables whose values are drawn from probability distributions specified in the Monte Carlo simulation.

Derivative Unit Production Cost is the cost of producing each individual derivative aircraft. It is the sum of the following variables:

a) Base Derivative Unit Production Cost: This is an exogenous variable that is accounted for in the Monte Carlo simulation. It is the baseline production cost of the derivative aircraft.

b) Unit Customer Support Cost: This variable reflects the expenditure that the manufacturer expects to spend per aircraft because of warranties and it is calculated as a percentage of Derivative Unit Price. This particular aircraft manufacturer adds Customer Support Costs to Unit Production Cost, although warranty costs could also be treated as after-sales expenditures. Unit Customer Support Cost is an exogenous variable that is determined by a probability distribution in the Monte Carlo simulation.

c) Unit Supplier Cost: This variable reflects uncertainty in production costs because of higher than anticipated costs from suppliers. This is an exogenous variable whose value is determined by a probability distribution in the Monte Carlo simulation.

d) Unit Manufacturing Cost: This variable incorporates uncertainty in production costs because of higher than expected manufacturing costs at the manufacturer's site. It is an exogenous variable whose value is determined by a probability distribution in the Monte Carlo simulation.

Derivative Unit Price is the sale price for each derivative aircraft. It is determined so that a certain margin over Unit Production Cost is achieved. The aircraft manufacturer assumes a 12% margin. If there are Extra Production Costs, Unit Production Cost increases, and, thus, Derivative Unit Price also increases to maintain the 12% margin over Unit Production Cost. As Derivative Unit Price increases, Price Sensitivity increases and Manufacturer Market Share decreases, as explained above.

Other important variable in the system dynamics model are Aircraft Market and Production Delay. Aircraft Market is an exogenous variable with a probability distribution provided by the aircraft manufacturer. It reflects the manufacturer's expectation of total market size for the particular aircraft model. Production Delay is used to model delays in the production process. It is assumed that all airplanes ordered in year n will be delivered in year n+1. If Production Delay is positive, aircraft are delivered in year n+1+Production Delay.

The value of completion for the real options valuation of the derivative aircraft is calculated with the variable Derivative Income. Derivative Income is the difference between Derivative Sales Revenues and Derivative Production Cost. The certainty equivalent of the value of completion is calculated with Equation 3-2 assuming a risk-adjusted discount rate of 18% and a risk-free discount rate of 5%. The value for the risk-adjusted discount rate was suggested by the aircraft manufacturer as representative of its experience with these type of projects.

The model running time is in years. The first year, i.e., the time to go from Step E to Step F in Figure 4-3, corresponds to development of the derivative aircraft. Production of the derivative aircraft is launched at the beginning of the second year of the simulation and deliveries start a year later. The data from the aircraft manufacturer indicates that the derivative aircraft is produced for 10 years.

Monte Carlo simulation for the value of completion of the real option to launch a derivative aircraft

With a Monte Carlo simulation, the system dynamics model is run repeatedly with different values for the exogenous variables, which are drawn from probability distributions specified for each one of them. In this way, many values of the output variable Derivative Income are calculated, and a probability distribution for the value of completion can be obtained. The exogenous variables selected for this study and their associated probability distributions are shown in Table 4-2. These values are based on data provided by the aircraft manufacturer. They illustrate a representative new aircraft development program at this manufacturer but note that these numbers are not necessarily representative of other programs in the industry. The purpose of this example is to illustrate the use of the generalized real options methodology to evaluate investments under uncertainty in aircraft development programs. Thus, the emphasis of the example is on the framework, which could be used to analyze other aircraft programs if the data was available.

Table 4-2: Variables selected for the Monte Carlo simulation to calculate the probability distribution of the value of completion of the derivative aircraft and their associated probability distributions.

Variable	Probability distribution						
	Unit	**Value**	**P(value)**	**Value**	**P(value)**	**Value**	**P(value)**
Aircraft Market	Aircraft/year	100	0.6	140	0.2	180	0.2
Deviation from Customer Requirements	%	5	0.6	4	0.2	1	0.2
Production Delay	Year	1	0.6	0.75	0.2	0.5	0.2
Target Market Share	%	30	0.5	40	0.3	50	0.2
Unit Customer Support Cost	% of Unit price	5	0.5	4	0.4	3	0.1
Unit Manufacturing Cost	MU Million/year	10	0.5	5	0.3	0	0.2
Unit Supplier Cost	MU Million/year	10	0.5	5	0.3	0	0.2

The probability distributions provided by the aircraft manufacturer shown in Table 4-2 are not very smooth. This may be a reflection of the small amount of historical data on aircraft programs at this manufacturer, which is not unexpected, since it typically takes aircraft manufacturers several years to launch new or derivative aircraft programs. In addition, notice that these probability distributions are rather conservative as evidenced by the higher probabilities given to outcomes that would decrease income from derivative aircraft sales. For example, the probability distributions of costs, such as Customer Support Cost, Unit Manufacturing Cost, and Unit Supplier Cost, are skewed towards the higher values. Furthermore, the highest value in the distribution of Deviation from Customer Requirements has a probability of 60%, while the lowest has a probability of only 20%. Finally, notice that Target Market Share has a high probability of being 30% (50% chance) and a lower probability of being 40% (30% chance), or 50% (20% chance).

For this first part of the study, the variable Aircraft Market was assumed to take a value that remained constant throughout each simulation run. An alternative for introducing more realism into the simulation is to assume a time-varying stochastic behavior of the aircraft market through the simulation. This will be explored in Chapter 5.

Calculation of the completion cost for the real option to launch a derivative aircraft

According to the data provided by the aircraft manufacturer, it is assumed that the completion cost of the real option to launch the derivative aircraft is 20% of Derivative Development Cost. This expenditure covers the building of the production facilities (hangars, tooling, etc.) for the derivative aircraft. The value of Derivative Development Cost is given by a probability distribution supplied by the manufacturer (see Table 4-3). As in the case of the value of completion, the certainty equivalent of this quantity is calculated with Equation 3-2 assuming a risk-adjusted discount rate of 18% and a risk-free discount rate of 5%.

Table 4-3: Probability distribution of Derivative Development Cost based on the data provided by the aircraft manufacturer.

Variable	Probability distribution						
	Unit	Value	P(value)	Value	P(value)	Value	P(value)
Derivative Development Cost	MU Million	2,500	0.6	2,000	0.2	3,000	0.2

4.3.3 Numerical results for the real option to launch a derivative aircraft

The probability distributions for the value of completion and the completion cost for the real option to launch the derivative calculated with the system dynamics model and the Monte Carlo simulation are shown in Figure 4-6. The graph shows the certainty equivalent of the value of completion and the completion cost at the maturity of the real option (Step F in Figure 4-3).

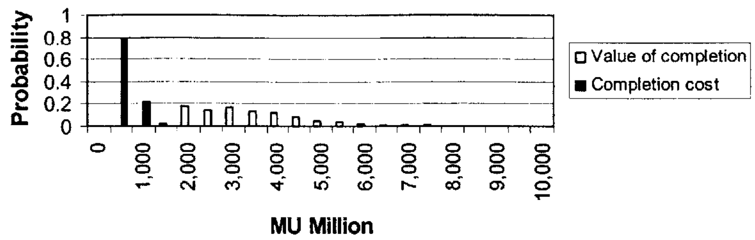


Figure 4-6: Probability distribution of the value of completion and the completion cost for the real option to launch a derivative aircraft as determined by the system dynamics model and the Monte Carlo simulation.

A series of financial performance parameters can be calculated with this data. These quantities are explained below and they are shown in Table 4-4:

- Value Flexible: This is the expected value of the real option to launch the derivative aircraft. Here, the investor exercises the real option to produce aircraft at maturity only if $V_{launch\ derivative}$ is larger than $C_{launch\ derivative}$. The expected value of the flexible strategy and its variance are calculated by substituting the probability distributions of $V_{launch\ derivative}$ and $C_{launch\ derivative}$ in Equations 3-11 and 3-12, respectively, and by assuming a risk-free discount rate of 5%. The standard deviation is calculated by taking the square root of the variance.

- Value Inflexible: This is the expected value of the strategy in which the real option to launch the derivative aircraft is always exercised at maturity regardless of the relative values of $V_{launch\ derivative}$ and $C_{launch\ derivative}$. The expected value and the variance of Value Inflexible can be calculated with Equations 3-11 and 3-12, respectively, by modifying the integration limit for the integrals over the value of completion. The modification consists in dropping the requirement that v > c. Consequently, the integration limits for the expected value and the variance of Value Inflexible are as shown in Equations 4-1 and 4-2, respectively:

$$Expecte\_value_{Inflex.} = e^{-r_f T}\left(\int_{c=0}^{\infty} f_c(c) \int_{v=0}^{\infty} v \cdot f_v(v) dv dc - \int_{c=0}^{\infty} c \cdot f_c(c) \cdot \int_{v=0}^{\infty} f_v(v) dv dc \right)$$

(Eq. 4-1)

$$Variance_{Infelx.} = e^{-2r_f T}\left[\int_{c=0}^{\infty} \left[\left(\int_{v=0}^{\infty} v \cdot f_v(v) dv \right)^2 - 2 \cdot c \cdot \int_{v=0}^{\infty} v \cdot f_v(v) dv \cdot \int_{v=0}^{\infty} f_v(v) dv + c^2 \cdot \left(\int_{v=0}^{\infty} f_v(v) dv \right)^2 \right] \cdot f_c(c) dc \right.$$

$$- \left(\int_{c=0}^{\infty} f_c(c) \int_{v=0}^{\infty} v \cdot f_v(v) dv dc \right)^2 + 2 \cdot \left(\int_{c=0}^{\infty} f_c(c) \int_{v=0}^{\infty} v \cdot f_v(v) dv dc \right) \left(\int_{c=0}^{\infty} c \cdot f_c(c) \cdot \int_{v=0}^{\infty} f_v(v) dv dc \right)$$

$$\left. - \left(\int_{c=0}^{\infty} c \cdot f_c(c) \cdot \int_{v=0}^{\infty} f_v(v) dv dc \right)^2 \right]$$

(Eq. 4-2)

- Engineering Cost: This is the initial investment required to obtain the option, i.e., this is the cost (premium) of the option. Engineering Cost is a surrogate metric that reflects the resources spent during development on engineering, testing, final design, etc. of the derivative airplane. According to the data given by the aircraft manufacturer, Engineering Cost was estimated to be 80% of Development Cost of the derivative aircraft. This is the reference against which the value of the inflexible and flexible strategies should be compared.

- Value Project: This is the net present value of the project. It is calculated by subtracting Engineering Cost from Value Flexible or Value Inflexible to find Value Project Flexible or Value Project Inflexible, respectively. This is the metric that management should consider when making the decision of investing in the project. If it is positive, it means that the value of launching and selling the derivative aircraft is higher than the cost of developing the program and, therefore, it should be undertaken. On the contrary, if Value Project is negative, it means that the development costs are higher than the value of the project and it should be dropped.

- Value of Flexibility: This is the difference between Value Project Flexible and Value Project Inflexible. It determines the relative value of the flexible strategy (the one that considers the real option) against the inflexible strategy (the one that always exercises at maturity). This definition of the value of flexibility can be found elsewhere, e.g., in Tufano and Moel (1997), Clemons and Gu (2003), and Greden et al. (2005).

The data shown in Table 4-4 are the present values of the different quantities at Step E in the aircraft development process shown in Figure 4-3. Step E is the time when management should decide whether to invest in development of the derivative aircraft to have the right of launching the program in Step F.

Table 4-4: Expected values (in MU million) of different quantities of interest for the real option to launch a derivative.

	Value Flexible (Real Option)	**Value Inflexible**	**Engineering Cost**	**Value Project**		**Value of flexibility**
				Flexible	Inflexible	
Expected value	2,637.0	2,637.0	2,000.0	637.0	637.0	0.0
Standard deviation	104.0	104.0	316.0	332.0	332.0	0.0

The expected values of the inflexible and of the flexible strategies are both MU 2,637 million. As shown in Figure 4-6, all values of the probability distribution of the value of completion are higher than the values of the distribution of the completion cost. This means that the real option is in the money and it will be exercised with certainty. Thus, because the investors know that they will launch the derivative aircraft, the flexibility provided by the real option does not improve the expected value of the project. In this case, the value of flexibility is zero.

A reason for this situation is that the completion cost to launch the derivative is small compared to the value of the value of completion. By the time the project comes to the maturity of the real option, only a small investment remains to be executed. Thus, if the development program has survived thus far, the manufacturer should proceed with launching the derivative aircraft.

The net present value of the project, following either the flexible or the inflexible strategy, is MU 637 million. Therefore, the aircraft manufacturer should invest the MU 2,000 million of Engineering Cost to obtain this real option. Furthermore, the manufacturer should be willing to spend up to MU 2,637 million in Engineering Cost to develop the derivative aircraft.

The results of this example are arguably very simple but the main purpose of the example is to illustrate the mechanics of the generalized real options methodology. In the next section, a more complex situation will be analyzed.

4.4 Advanced example: evaluating several real options in an aircraft development program

In the previous example, the basics of the real options evaluation methodology developed in this thesis were explained. A more advanced case is considered below to demonstrate how this methodology can be applied to situations where more than one real option is present. First, an overview of the several real options in the aircraft development program considered here are identified. Next, the value of completion, completion cost, and maturity of each option are explained. This is followed by a description of the system dynamics and Monte Carlo simulation used to obtain numerical values for these real options. Finally, numerical results are presented and discussed.

4.4.1 Overview of real options in the aircraft development program

Several real options can be identified in the aircraft development program shown in Figure 4-7. In this example, it is assumed that the investors are at Step A and they are considering how much to spend on preliminary design. By spending resources in preliminary design, the investors will obtain the real option to start development in Step B. This real option creates subsequent options that must be considered when evaluating this investment opportunity. The value of the real option to start development is the maximum that the investors should spend on preliminary design.

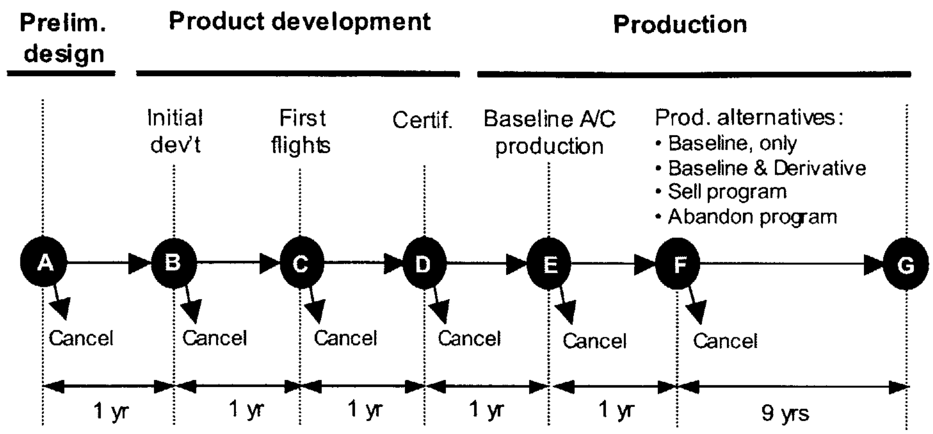


Figure 4-7: Real options in a new aircraft development program.

The real options in the aircraft development process analyzed in this chapter are described below. They can be identified by the step in which they are exercised:

B. Real option to start development: By engaging in initial development, the investor will have the option of building a few test aircraft to enter first flight tests in Step C.

C. Real option to do first test flights: The first flights of the new aircraft will provide information about product performance and it will open the option to certify the airplane in Step D.

D. Real option to certify the aircraft: By certifying the airplane, the investor can start production of the baseline aircraft in Step E.

E. Real option to start production: The beginning of serial production of the baseline airplane creates the option to follow one of four different alternatives in Step F.

F. Real option to continue a production alternative: In Step F, the manufacturer has the option to pursue one of the following mutually exclusive alternatives:

i. Production of the baseline aircraft, only: the manufacturer dedicates all its resources to producing and selling the baseline aircraft.

ii. Production of the baseline and a derivative aircraft: the manufacturer has the option of launching a derivative in addition to producing the baseline aircraft.

iii. Sale of the aircraft program: the manufacturer can sell the aircraft program to an interested investor.

iv. Abandon the program: the manufacturer can abandon the program and recuperate its salvage value, if any.

Notice that options B through E are compound options. This means that the value of each option is dependent on the value of subsequent options. For example, the value of the real option to certify the aircraft depends on the value of the real options to produce and to continue a production alternative. Consequently, in order to find the value of options earlier in the process, it is necessary to start the evaluation at the end of the program and work backwards.

Before explaining the structure of each option in detail, it is helpful to explain the cash flow of the project. As in the example of the derivative aircraft considered before, the development costs of the aircraft program can be divided in Engineering Cost and Production Facilities. Engineering Cost includes resources spent in the final design, blueprints, wind tunnel testing, etc. and corresponds to 80% of the development costs.

Production Facilities covers hangars, tooling, etc. and comprises the remaining 20% of the development costs.

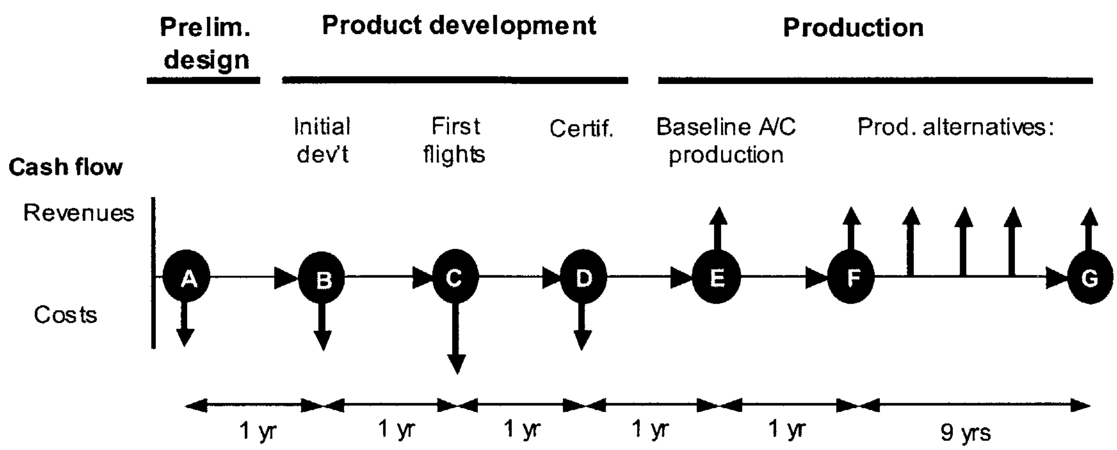


Figure 4-8: Expected cash flow in the new aircraft development program.

Major investments are expected to occur in Steps B, C and D (see Figure 4-8). The expenditure necessary to begin initial development in Step B is 30% of Engineering Cost. The cost of starting first test flights in Step C includes 40% of Engineering Cost, Production Facilities and the production cost of 5 aircraft. In Step D, the remaining 30% of Engineering Cost would have to be incurred to start certification. Revenues are realized in Step E if production of the baseline aircraft starts. Revenues between Step F and Step G depend on which production alternative is chosen. Finally, determining the size of the expenditure in Step A is the goal of the real options analysis. This value should be no larger than the expected value of the option to start development in Step B as calculated by an investor standing at Step A because the previous step, i.e., preliminary design, may or may not be successful.

4.4.2 Taxonomy of the real options in the new aircraft development program

The structure of each of the real options identified above is presented below. The discussion starts with the last real option in the aircraft development process. Note that all quantities, both revenues and costs, must be given in terms of their certainty equivalents to use the generalized real options methodology.

Starting with the last, the structure of each real option is as follows:

F. Real option to continue a production alternative:

- Value of completion of continuing production, $V_{continue\ production}$: since the four production alternatives considered here are mutually exclusive, the value of completion of this real option is the maximum of the value of completion from of each of the alternatives in each simulation run. The value of completion for each alternative is explained below:
  - Value of completion of producing the baseline aircraft, $V_{baseline\ aircraft}$: In this alternative, the baseline aircraft remains in production. The value of completion is income from baseline aircraft sales.
  - Value of completion of producing the baseline and a derivative aircraft, $V_{baseline\ \&\ derivative\ aircraft}$: Here, the manufacturer produces the baseline aircraft and launches a derivative. The value of completion is the sum of income from baseline aircraft sales plus the net value of the option to launch the derivative, i.e., the value of the option to launch the derivative minus the cost of obtaining this option. The value and the cost of the

option to launch a derivative are calculated using the procedure explained in Section 4.3.

  - Value of completion of selling the aircraft program, $V_{sell\ aircraft\ program}$: It is assumed that after one year of baseline aircraft production, the aircraft program can be sold. The value of the sold program is estimated at 70% of development costs.
  - Value of completion of abandoning the program, $V_{abandon\ program}$: The investor decides to abandon the aircraft development project and obtain a salvage value equal to 15% of development costs.

- Completion cost of continuing production, $C_{continue\ production}$: The completion cost of this real option is zero. If production of the baseline aircraft continues, the completion cost is zero because the production facilities are already in place and no more investments are necessary. Similarly, it is assumed that there are no expenditures associated with the exercise of the alternatives to sell or abandon the program. If the alternative to produce the baseline aircraft and launch the derivative is chosen, $C_{continue\ production}$ is also zero because the cost of exercising this option is already included in the value of completion of this production alternative.

- Maturity of the option to continue production, $M_{continue\ production}$: This real option has a maturity of one year. It is alive between Steps E and F in Figure 4-7.

E. Real option to start production:

- Value of completion of starting production, $V_{start\ production}$: The value of completion is the maximum between income from the first year of baseline aircraft sales plus the discounted value of the payoff of the real option to start production, and zero. The payoff of the option to start production is discounted from Step F (exercise date of the option to continue production) to Step E (exercise date of the option to start production). The risk-free rate is used because all payoffs are given in terms of their certainty equivalent.

  For each run i in the Monte Carlo simulation, $V_{start\ production}$ can be expressed mathematically as:

$$V_{start\ production_i} = \max\left[Baseline\_sales_{1st\_year_i} + e^{-r_f} Payoff_{cont\_production_i}, 0\right]$$
$$= \max\left[Baseline\_sales_{1st\_year_i} + e^{-r_f}\left(V_{cont\_production_i} - C_{cont\_production_i}\right), 0\right]$$

(Eq. 4-3)

- Completion cost of starting production, $C_{start\ production}$: The completion cost to start production is zero, because it is assumed that serial production of the baseline aircraft can use the same facilities built for constructing the test aircraft. Therefore, no extra expenditures to enter production are required.

- Maturity of the option to start production, $M_{start\ production}$: This real option has a maturity of one year. It is alive between Steps D and E in Figure 4-7.

C. Real option to certify the aircraft:

- Value of completion of certification, $V_{certification}$: The value of completion is the maximum between the discounted value of the payoff of the real option to start production and zero. The payoff of the option to start production is discounted from Step E (exercise date of the option to start production) to Step D (exercise date of the option to certify the aircraft). Again, the risk-free rate is used because all payoffs are given in terms of their certainty equivalent.

  For each run i in the Monte Carlo simulation, $V_{certification}$ can be expressed mathematically as:

$$V_{certification_i} = \max\left[e^{-r_f} Payoff_{start\_production_i}, 0\right]$$

$$= \max\left[e^{-r_f}\left(V_{start\_production_i} - C_{start\_production_i}\right), 0\right] \qquad \text{(Eq. 4-4)}$$

- Completion cost of certification, $C_{certification}$: The completion cost to enter the certification phase is estimated to be 30% of Engineering Cost.

- Maturity of the option to do certification, $M_{certification}$: This real option has a maturity of one year. It is alive between Steps C and D in Figure 4-7.

C. Real option to do first test flights:

- Value of completion of doing first test flights, $V_{1st\ flights}$: The value of completion is the maximum between the payoff of the real option to certify and zero. The payoff of the option to certify is discounted from Step D (exercise date of the option to certify) to Step C (exercise date of the option to do first test flights). Again, the risk-free rate is used because all payoffs are given in terms of their certainty equivalent.

For each run i in the Monte Carlo simulation, $V_{1st\ flights}$ can be expressed mathematically as:

$$V_{1st\_flights_i} = \max\left[e^{-r_f} Payoff_{certification_i}, 0\right]$$

$$= \max\left[e^{-r_f}\left(V_{certification_i} - C_{certification_i}\right), 0\right] \quad \text{(Eq. 4-5)}$$

- Completion cost of doing first test flights, $C_{1st\ flight}$: The completion cost to do the first test flights is substantial. It includes 40% of Engineering Cost, Production Facilities estimated to be 20% of Development cost plus the cost of building five baseline aircraft.

- Maturity of the option to do first test flights, $M_{1st\ flight}$: This real option has a maturity of one year. It is alive between Steps B and C in Figure 4-7.

B. Real option to start development:

- Value of completion of starting development, $V_{start\ development}$: The value of completion is the maximum between the payoff of the real option to do first flights and zero. The payoff of the option to do first flights is discounted from Step C (exercise date of the option to certify) to Step B (exercise date of the option to start development). Again, the risk-free rate is used because all payoffs are given in terms of their certainty equivalent.

For each run i in the Monte Carlo simulation, $V_{start\ development}$ can be expressed mathematically as:

$$V_{start\_development_i} = \max\left[e^{-r_f} Payoff_{1st\_flights_i}, 0\right]$$

$$= \max\left[e^{-r_f}\left(V_{1st\_flights_i} - C_{1st\_flights_i}\right), 0\right] \quad \text{(Eq. 4-6)}$$

- Completion cost of starting development, $C_{start\ development}$: The completion cost to start development is 30% of Engineering Cost.

- Maturity of the option to start development, $M_{start\ development}$: this real option has a maturity of one year. It is alive between Steps B and A in Figure 4-7.

The expected value and the standard deviation of the real option to start development can be calculated by substituting the distributions of $V_{start\ development}$ and $C_{start\ development}$ in Equations 3-11 and 3-12, respectively. This is the maximum amount that the manufacturer should spend in the preliminary design of the new aircraft program.

4.4.3 System dynamics and Monte Carlo simulation for the advanced example

The structure of the system dynamics model used to obtain the numerical values of the value of completion for the real options in the new aircraft development program is similar to the model used to calculate the value of completion of the real option to launch a derivative aircraft described Section 4.3.2. The main differences between the models correspond to the timing of the aircraft production process and the calibration of the variables in the Monte Carlo simulation.

In the advanced example, orders for the baseline aircraft are first taken in the third year of the simulation, which corresponds to Step E in Figure 4-7. Production starts in the same year and deliveries begin in the fourth year. As in the case of the derivative aircraft, the baseline aircraft is produced for 10 years.

The probability distributions for the exogenous variables in the Monte Carlo simulation to calculate the probability distribution of the value of completion for the real options in the advanced example are shown in Table 4-5. As in the simple example of the derivative discussed in Section 4.3, these values are based on data provided by an aircraft manufacturer. They illustrate a representative new aircraft development program at this manufacturer but these numbers are not necessarily representative of other programs in the industry.

Table 4-5: Variables selected for the Monte Carlo simulation to calculate the probability distribution of the value of completion of the real options in the advanced example and their associated probability distributions.

Variable		Probability distribution					
	Unit	Value	P(value)	Value	P(value)	Value	P(value)
Aircraft Market	Aircraft/year	500	0.6	750	0.2	1000	0.2
Deviation from Customer Requirement	%	5	0.6	4	0.2	1	0.2
Production Delay	Year	1	0.6	0.75	0.2	0.5	0.2
Target Market Share	%	20	0.5	30	0.3	40	0.2
Unit Customer Support Cost	% of Unit price	5	0.5	4	0.4	3	0.1
Unit Manufacturing Cost	MU Million/year	10	0.5	5	0.3	0	0.2
Unit Supplier Cost	MU Million/year	10	0.5	5	0.3	0	0.2

The completion costs for the real options in the advanced example are based on the Development Cost of the baseline aircraft and the production cost of five baseline aircraft, as described in Section 4.4.2. The probability distribution for Development Cost was provided directly by the aircraft manufacturer and is shown in Table 4-6.

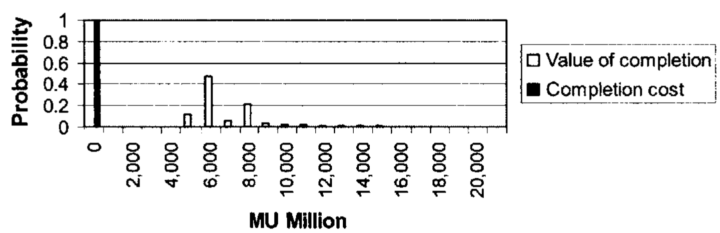
Table 4-6: Probability distribution of Development Cost for the advanced example provided by the aircraft manufacturer.

Variable		Probability distribution					
	Unit	Value	P(value)	Value	P(value)	Value	P(value)
Development Cost	MU Million	15,000	0.6	19,000	0.2	12,000	0.2

As in the case for the real option to launch the derivative aircraft, the certainty equivalent of the value of completion and the completion cost for all real options in the advanced example where calculated with Equation 3-2 assuming a risk-adjusted discount rate of 18% and a risk-free discount rate of 5%.

4.4.4 Numerical results for the real options in the advanced example

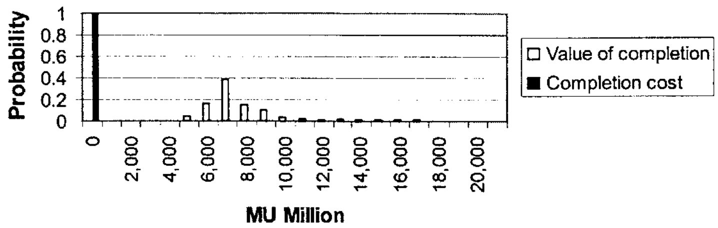
Numerical results for the different real options in the new aircraft development program are presented below. The graphs show the probability distribution for the value of completion and the completion cost for each real option. In addition, the expected value and the standard deviation of each real option as calculated with Equations 3-11 and 3-12, respectively, are given. The figures also include the expected value and standard deviation of an inflexible strategy, i.e., one in which the option is always exercised regardless of the relative values of the value of completion and the completion cost at maturity of the option. These values are calculated with Equations 4-1 and 4-2, respectively. All values have been discounted to Step A in the aircraft development process:



Real Option Expected Value = MU 5,267.0 million
Std. Deviation = MU 0.0 million

Inflexible strategy Expected Value = MU 5,267.0 million
Std. Deviation = MU 0.0 million

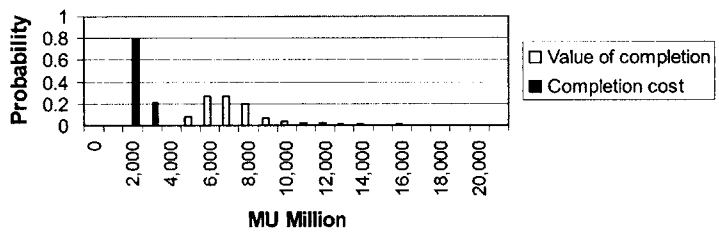
Figure 4-9: Numerical results for the real option to continue a production alternative (Option F).



Real Option Expected Value = MU 6,113.0 million
Std. Deviation = MU 0.0 million

Inflexible strategy Expected Value = MU 6,113.0 million
Std. Deviation = MU 0.0 million

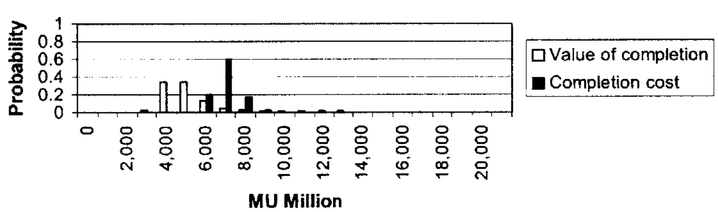
Figure 4-10: Numerical results for the real option to start production (Option E).



Real Option Expected Value = MU 4,491.8 million
Std. Deviation = MU 233.7 million

Inflexible strategy Expected Value = MU 4,491.0 million
Std. Deviation = MU 233.7million

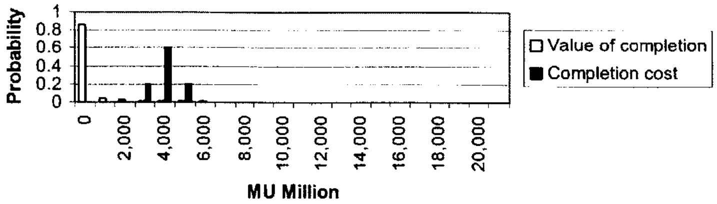
Figure 4-11: Numerical results for the real option to certify the aircraft (Option D).



Real Option Expected Value = MU 415.5 million
Std. Deviation = MU 112.3 million

Inflexible strategy Expected Value = -MU 1,369.0 million
Std. Deviation = MU 723 million

Figure 4-12: Numerical results for the real option to do first flight tests (Option C).



Real Option	Expected Value = MU 115.4 million Std. Deviation = MU 24.9 million
Inflexible strategy million	Expected Value = -MU 4,503.0 Std. Deviation = MU 455.0 million

Figure 4-13: Numerical results for the real option to start development (Option B).

The expected value and standard deviation for the real option, the inflexible strategy, and the value of flexibility for each case presented above are summarized in Figure 4-14 and in Table 4.7. The value of flexibility is defined as in the simple example of the derivative aircraft presented above (see Section 4.3.3), i.e., it is the difference between the expected value of the real option and the expected value of the inflexible strategy.

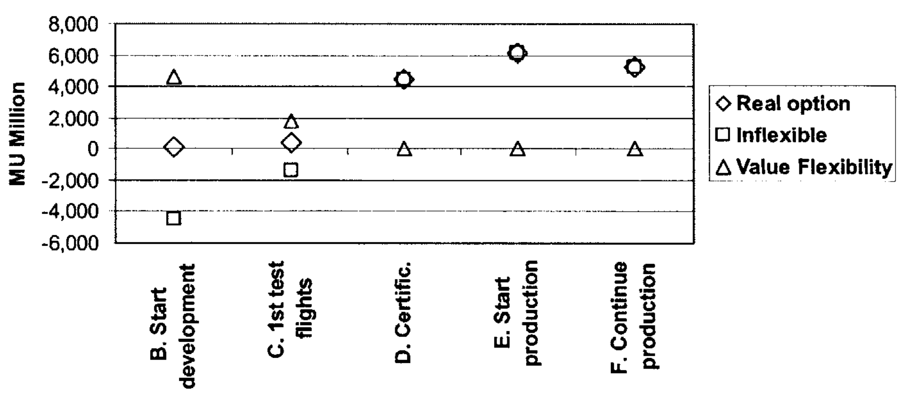


Figure 4-14: Summary of the numerical results of the real options in the new aircraft development process. All figures are in MU million.

Table 4-7: Summary of the numerical results of the real options in the new aircraft development process. All figures are in MU million.

		Real option				
		B. Start development	C. 1st test flights	D. Certification	E. Start production	F. Cont. production
Real option strategy	Expected Value	115.4	415.5	4,491.8	6,113.0	5,267.0
	Std. Deviation	24.9	112.3	233.7	0.0	0.0
Inflexible strategy	Expected Value	-4,503.0	-1,369.0	4,490.0	6,113.0	5,267.0
	Std. Deviation	455.0	723.0	234.0	0.0	0.0
Value of flexibility	Expected Value	4,618.4	1,784.5	1.8	0.0	0.0
	Std. Deviation	455.7	731.7	330.7	0.0	0.0

There are a number of observations from the above results that are worth highlighting. First, the expected value of the strategies with or without the real option tends to increase as the process moves forward. For example, the expected value at Step B (start development) is MU 115.4 million for the strategy with the real option and –MU 4,503.0 for the inflexible strategy, while the expected value at Step E (continue production) is MU 6,113.0 million for both strategies. Typical aircraft development programs are structured such that large expenditures occur in earlier rather than in later stages. [5] At the same time, manufacturers generally do not receive revenues until they deliver the finished aircraft. In some cases, manufacturers may receive advance payments for firm orders and aircraft order options, but these tend to be small compared to the full price of the aircraft. In fact, the manufacturer that provided the data for the analysis presented here suggested the assumption that no revenues are received until the aircraft are delivered. Thus, the further the manufacturer moves along the process, past expenditures

[5] These are characteristics of aircraft development programs in general, not only the one considered here. See, for example, Jenkinson et al. (1999) and Schaufele (2000).

become sunk costs, less investments remain outstanding, and the time to receive revenues approaches. Therefore, from the viewpoint of an investor standing at point A, the expected value of the project with either strategy is higher towards the end of the process.

An exception to the behavior noticed above is the drop in the expected value of the option as the process moves from E to F. This can be explained by considering the timing of the cash flow in the development process (see Figure 4-8). At E, the value of completion of the real option includes revenues from the first year of baseline aircraft sales plus revenues from one of the production alternatives chosen at F. At F, the value of completion is less than the value of completion at E because it no longer includes revenues for the first year of sales. Therefore, the value of the real option at F is less than at E.

At this point, it is important to make a distinction between ex ante and ex post expected values for the real options and how they may change over time. In this analysis, it is assumed that investors in the new aircraft program are standing at point A in Figure 4-7 and that the numerical results shown in Table 4-7 are based on information available to them at that time, i.e., these are ex ante expected values for the real options. The calculations that led to the ex ante expected values were made with certain assumptions about the possible future behavior of a number of variables. As the aircraft development program moves forward and time passes, the investors will realize to what extent their ex ante assumptions are confirmed. Therefore, the value of the real options ex post may be different than the expected value calculated ex ante. For example, as the process reaches

Step B, the technical development of the aircraft may have gone astray or the market may have deteriorated and, thus, the value of the option at B may be less that was originally calculated. Keeping in mind the difference between ex ante and ex post expected values is important for at least two reasons. First, this indicates the need for revising ex ante assumptions as new information becomes available and for re-calculating the value of the options with the new data. Second, the ex ante calculations of the expected value of the real options are not meant to forecast what the value of those options will be in the future. What the ex ante expected values denote is the maximum amount that investors should be willing to pay for the real option, given what is known about the state of the world at that point.

The second observation regarding the numerical results shown in Table 4-7 pertains to the value of flexibility, i.e., the value of the strategy with a real option compared to the value of an inflexible strategy. Given the assumptions in the structure of the project and in the numerical data used in this analysis, the value of flexibility decreases as the process moves forward. Again, this behavior can be explained with the assumed schedule of expenditures shown in Figure 4-8. As the process moves forward, less expenditure remain outstanding and, thus, options further in the process tend to be in the money and, consequently, will be exercised with great certainty. For example, consider the option to continue a production alternative (option E): all values of the value of completion are higher than all values of the completion cost (see Figure 4-9), thus, the option will be exercised always. Under these circumstances, there is no added value in having a real option. The same observation can be made of the option to continue a production alternative (option F). Notice that these comments are valid given the ex ante assumptions. Since steps E and F are so far in the future from the point of view of investors standing at A, there is plenty of opportunity for conditions to change as time passes. Thus, investors should revise their assumptions periodically and re-calculate the expected value of the remaining real options with the new information.

Another aspect worth highlighting in the context of the value of flexibility is that, in many other examples of real options, the value of flexibility tends to increase, not decrease, throughout the life of the option. A fundamental feature of real options is the phasing of investments until more information is available. Thus, in general, the holder of the option pays a small price at the beginning of the investment to purchase the option and the large expenditure, i.e., the completion cost, comes at a later time when, ideally, a significant portion of the uncertainties have been resolved. In the particular case of aircraft manufacturing, however, this does not seem to be the case. As mentioned above, in typical aircraft development programs much of the expenditures occur in the early stages of the process and they dwindle as the project advances. Therefore, the value of the ability of waiting to invest decreases because less expenditures remain outstanding. This point will be discussed further in the next chapter.

The third observation regarding the results in Table 4-7 is that the importance of real options can be seen in the earlier stages of the program. For example, the real option to do first flight tests (option C) has an expected value of MU 415.5 million while an inflexible strategy at this point would result in an expected value of –MU 1,369.0 million.

Thus, the ability that the option gives the investor to cancel the investment at the maturity of this option if conditions are not favorable is very valuable. Similarly, the value of flexibility provided by the real option to start development (option B) is significant: the expected value of the real option is MU 115.4 million while the expected value of an inflexible strategy is –MU 4,503.0 million.

This observation highlights an important characteristic of options that was mentioned in Chapter 2 (see Section 2.2.1): options are most valuable in uncertain situations. For the particular assumptions in the aircraft development program used here, the option is deep in the money in the later stages of the process and, thus, it will be exercised with great certainty. The value of flexibility is low in that case. In earlier stages, however, there is great uncertainty about the fate of the program. The project is at the money and it is uncertain whether its financial performance will be positive or negative. It is here that options are valuable, because they can make the difference in the financial viability of the project.

There is at least one implication of this observation for policymaking. The results in Table 4-7 suggest that the new aircraft program is very risky in earlier stages but, after a certain point (Step C in this example), the project becomes profitable. This suggests that if there are reasons other than profit maximization for having such an aircraft program, such as national security, job creation, or maintenance of a high tech capability, outside intervention in the early stages of the project may be justified to guarantee its viability until it reaches a point of self-sufficiency.

The last observation regarding the numerical results in Table 4-7 is that the value of the real option to start development is also the maximum amount that the investor should spend during the preliminary design phase. Thus, the aircraft manufacturer should not pay more than an expected MU 115.4 million with a standard deviation of MU 24.9 million for the first phase of the project.

Finally, it should be noted that the numerical results may not be representative of project returns of other new aircraft programs. The values in the data provided by the aircraft manufacturer are based on its own experience but they are not necessarily representative of other programs in the industry; however, the emphasis of the thesis is to demonstrate an evaluation methodology that can be used with different data sets and not necessarily on obtaining specific numerical results.

4.5 Chapter summary

A practical application of the generalized real options methodology developed in this thesis was demonstrated with the analysis of flexible investment strategies in a new aircraft development program. The following points were made in this chapter:

1) Basic characteristics of capital projects in air transportation, such as large capital expenditures, long delivery times, and multiple technical and market uncertainties, indicate that a flexible investment strategy, i.e., one that allows investors to wait until more information is available, can significantly improve the financial performance of investments in this sector.

2) Flexible investment strategies in a new aircraft development program at a major aircraft manufacturer were analyzed and evaluated with the generalized real options methodology. A typical new aircraft development program is structured as a series of sequential phases. Each phase can be considered as a real option, as managers have the flexibility to continue or stop the process after each stage depending on the most currently available information. The analysis presented here differs from previous studies because the value of completion and the completion cost have been modeled with a bottom-up approach as opposed to top-bottom models.

3) A system dynamics model of the new aircraft production process was created to calculate revenues and costs of the program. The system dynamics model was used as the evaluation function in a Monte Carlo simulation to take into account different sources of market and technical risk. The results of the system dynamics and Monte Carlo simulations were used to determine the probability distributions of the value of completion and the completion cost used in the real options valuation. The model and the Monte Carlo simulation were calibrated with information provided by a major aircraft manufacturer.

4) Numerical results using the assumptions and data described above indicate that the value of the real option to launch a derivative aircraft is approximately MU 2,637 million with a standard deviation of MU 104.0 million. Furthermore, the value of the real option to start the new aircraft development program is on the order of MU 115.4 million with a standard deviation of MU 24.9 million. These are the maximum

amounts that investors should spend in the development of the derivative aircraft and the preliminary design of the baseline aircraft, respectively. Note that these results apply to the particular example considered here and are not representative of other programs in the industry.

5) Calculations indicate that options are more valuable in earlier stages of the aircraft development process. Since most expenditures occur in the first phases, by the time the process reaches the certification step, small or no expenditures are outstanding. Thus, according to the assumptions and the data in this particular examples, the investor should always proceed with the program as the option is always in the money. Under these circumstances, having the option to stop the process if conditions are not favorable has little value.

6) In many other examples of real options, the value of flexibility tends to increase throughout the life of the option. In the particular case of aircraft manufacturing, however, this does not seem to be the case. As mentioned above, in typical aircraft development programs much of the expenditures occur in the early stages of the process and they dwindle as the project advances. Therefore, the value of the ability of waiting to invest decreases because less expenditures remain outstanding.

7) Numerical results presented in this chapter suggest that the new aircraft program is very risky in earlier stages but, after a certain point (Step C in this example), the project becomes profitable. This suggests that if there are reasons other than profit

maximization for having such an aircraft program, such as national security, job creation, or maintenance of a high tech capability, outside intervention in the early stages of the project may be justified to guarantee its viability until it reaches a point of self-sufficiency.

5. Using the generalized methodology to explore flexibility

A main attribute of the generalized real options methodology is the ability to represent the value of completion and the completion cost with any probability distribution. Therefore, the methodology can be used to explore flexibility in projects where limited or no historical data to justify a particular probability distribution of the value of completion or the completion cost exists. In addition, the methodology provides a framework that can be used to systematically explore alternative investment strategies.

In the previous chapter, the mechanics of the real options methodology were illustrated with the evaluation of a new aircraft development program. In this chapter, this example is re-visited to show how the generalized methodology can be used to relax some of the fundamental data and structural assumptions of a project, explore how the numerical results can be used to gain insights useful for strategic decision-making, and quantify the sensitivity of the results to the data and other assumptions. In the last section of the chapter, some of the behavioral implications of this analysis for the aircraft manufacturing industry are discussed.

5.1 Evaluating real options with different data assumptions

A fundamental assumption in traditional analytical real options methodologies is the existence of historical data to model the value of completion as a geometric Brownian motion or other stochastic processes; however, there are two difficulties with this approach. First, there is usually not enough information available on real projects to calibrate a GBM or another stochastic process. Many projects are unique or so infrequent that it is difficult to find enough data points to have a statistically significant sample. This is particularly true if the investment is on new products for which there is no precedent. Second, the use of historical data to predict future outcomes assumes that the future will behave like the past, which may or may not be accurate depending on the investment under evaluation.

With the methodology developed in this thesis, it is possible to avoid these difficulties because there is no specification on the shape of the probability distribution of the value of completion or the completion cost. Investors can use numerical models in which the variables of interest for which no data exists (e.g., market for new aircraft) are calculated based on auxiliary variables for which historical and/or behavioral data exists (e.g., gross domestic product growth rate, typical market share, etc.). Therefore, the predictions of possible future values of the variables of interest can be informed by historical data without making the assumption that the future will necessarily behave like the past. This is particularly important for new projects were there are few or no precedents and, thus, little or no historical data, but for which certain assumptions of past behavior (e.g., market acceptance, technological feasibility, production capability, etc.) may still hold. Thus, the generalized real options offers investors the flexibility to represent the value of completion and the completion cost without the compromise of conforming to known stochastic processes calibrated with historical data. At the same time, investors can use the generalized methodology to systematically compare how different assumptions in the data or the probability distributions of the value of completion or completion cost affect the expected value of the real options.

To illustrate this capability of the generalized methodology, recall the system dynamics model developed in Chapter 4 to determine the value of completion of the real options in the aircraft development program. The relationships in the system dynamics model have been calibrated based on the aircraft manufacturer's knowledge of previous programs and their expectations of the performance of the new program. Notice, however, that there are some simplistic assumption in this model that could be refined by taking into account historical data. For example, in the model discussed in Chapter 4, a non-time-varying market for aircraft was assumed, i.e., the size of the aircraft market varies from one simulation run to the next according to the probability distribution provided by the aircraft manufacturer, but it remains unchanged during each simulation run. A more realistic representation of the aircraft market would be a time-varying model that allows the market to vary over time in each simulation run. Such a market model would be more representative of observed dynamics in the real world and, together with the other relationships already defined in the model, would add more realism to the simulation.

A time-varying stochastic market model will be described next, followed by numerical results of the real option to start development using the time-varying market assumption. Then, this enhanced system dynamics model will be used to explore other capabilities of the generalized methodology, such as the ability to evaluate alternative investment strategies.

5.1.1 Mean-reverting stochastic model of the aircraft market

As was mentioned at the beginning of Chapter 4, the air transportation industry is highly cyclical. Thus, a mean-reverting stochastic process for the growth rate of the aircraft market was assumed to reflect the cyclical nature of demand in air transportation. A mean-reverting stochastic process fluctuates randomly around a long-term trend. Here, an Ornstein-Uhlenbeck process of the form shown in Equation 5-1 was used:

$$dx = \eta \cdot \left(X - x \right) dt + \sigma \cdot dz \qquad \text{(Eq. 5-1)}$$

where dx is the change in the size of the aircraft market x over a time interval dt, η is the speed of reversion, i.e., a metric that represents how fast the process returns to its long-term trend, X is the level to which x tends to revert, σ is the variance parameter, and dz represents a Wiener process, i.e., a normally distributed random process (for more details, please consult [Dixit and Pindyck, 1994]). This equation was calibrated using the method outlined in Dixit and Pindyck (1994) with historical airline industry capacity[6] data contained in the Form 41 database (DOT, 1979-2001) for the United States domestic market between 1979 and 2001. The long-term annual change in the aircraft market was estimated to be 3.31%, η was determined to be 2.02, and σ was 0.29. The historical data and two sample simulation runs are shown in Figure 5-1 for illustration purposes:

[6] Measured in terms of available-seat-miles (ASM).

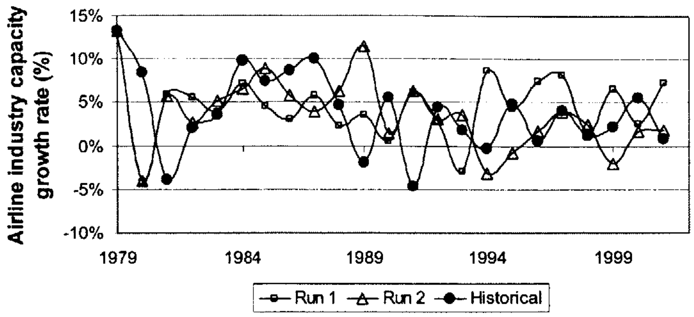


Figure 5-1: Historical airline industry capacity growth rate in the United States domestic market (solid dots) and two sample simulation runs as determined by the mean-reverting stochastic process. ***Source: DOT, 1979-2001.***

From data provided by the manufacturer, it was assumed that the market for the new aircraft over ten years can be 5,000 units with 60% probability, 7,500 units with 20%, or 10,000 units with 20% probability. The mean-reverting stochastic model was calibrated to achieve a probability distribution at the end of the life of the aircraft program in the system dynamics and Monte Carlo simulation roughly approximate to this assumed probability distribution of market size (see Figure 5-2):

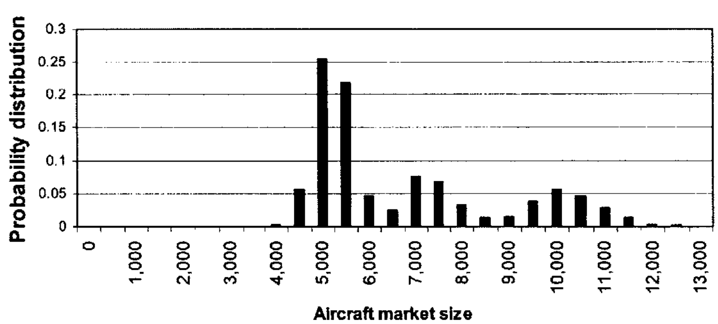


Figure 5-2: Probability distribution of the size of the aircraft market at the end of the program in the system dynamics and Monte Carlo simulation assuming a mean-reverting stochastic process.

5.1.2 Value of the real option start development with a time-varying stochastic market assumption

The introduction of the time-varying model of the aircraft market affects the distribution of program revenues but it does not change the structure of the calculation of the real options. The value of the real option to start development assuming mean-reverting process for the aircraft market is MU 14.5 million with a standard deviation of MU 4.9 million (see Table 5-1).

Table 5-1: Summary of the numerical results of the real options in the new aircraft development process assuming a time-varying market. All figures are in MU million.

		Real option				
		B. Start development	C. 1st test flights	D. Certification	E. Start production	F. Cont. production
Real option strategy	Expected Value	14.5	104.4	3,957.0	5,575.0	4,874.0
	Std. Deviation	4.9	45.3	239.0	0.0	0.0
Inflexible strategy	Expected Value	-5,016.0	-1,893.0	3,956.0	5,575.0	4,874.0
	Std. Deviation	466.0	743.0	239.0	0.0	0.0
Value of flexibility	Expected Value	5,030.5	1,997.4	1.0	0.0	0.0
	Std. Deviation	466.0	744.4	338.3	0.0	0.0

Notice that previously, the value of the real option to start development was estimated at MU 115.4 million. The reduction in the value of the option is due to the asymmetric distribution of aircraft orders in the mean-reverting stochastic market model. As was mentioned in Chapter 4, under the original scenario provided by the manufacturer, market demand is distributed uniformly throughout the ten years of production. Thus, a market size of 7,500 units means 750 aircraft per year for 10 years. The mean-reverting model assumes that aircraft demand has a long-term growth rate. Therefore, in order to achieve the same distribution at the end of the 10 years of production as in the original data, production in the first years would be less than in the non-time-varying model. Eventually, at the end of the 10 years in the simulation, both models would result in similar market size distribution; however, because of the time value of money, the timing of the cash flows is important: revenues further in time are discounted more and, therefore, have a smaller present value. In the mean-reverting model, less orders are placed in earlier years and more occur in later years. Because of discounting, this leads to smaller present values for aircraft revenues. Thus, the value of the real option using the mean-reverting market assumption is lower than in the case where the market is assumed constant throughout the simulation run.

The difference in option value indicates the importance of choosing a market assumption that best approximates reality. The time-varying market model is arguable a better representation of real-world events than the non-time-varying case. Thus, the aircraft manufacturer may revise the simple assumption that aircraft orders are distributed evenly through the production years to obtain a better estimate of project value.

With the real options generalized methodology, changes to the fundamental assumptions of the numerical models are easy to incorporate, because the calculation of the expected value of the option is not sensitive to the shape of the probability distribution of the value of completion or the completion cost. Therefore, the methodology allows the user to adjust the model to determine the value of completion without compromising the real options calculation. Furthermore, with the generalized methodology, investors can compare how the value of the real options vary depending on the assumptions in the calculations of the probability distributions of the value of completion and the completion cost. In this way, investors can obtain an idea of the sensitivity of the results to different data and modeling assumptions.

5.2 Evaluating real options with different project structure assumptions

As was shown above, the generalized methodology can be used to evaluate real options with any probability distributions of the value of completion or completion cost. This feature can be exploited to investigate how changes in the structure of the project affect the expected value of the real options. Investors are interested in knowing the expected value and the variance of real options; however, they may also be interested in investigating how the process can be modified to improve its financial performance. For example, investors may want to explore how changes in the timing and the size of the investments may affect the expected value of the real options. By systematically analyzing changes to the process, investors may be able to draw insights useful to modify their projects. The generalized real options methodology developed in this thesis provides a framework to explore different investment strategies because it is not sensitive to changes in the probability distributions of the value of completion or completion cost that may arise from changing assumptions regarding the structure of the project.

Until now, the expected value of the real options in the aircraft development process were calculated using a fixed set of assumptions regarding the timing and the size of the expenditures in the process. In this section, the use of the generalized methodology to determine the expected value of real options with different project structure assumptions is illustrated by analyzing alternative project investment strategies in the aircraft development process. One strategy consists of postponing investment decisions until more information is available. A second strategy corresponds to restructuring capital expenditures so that major investments occur later in the program as opposed to in earlier stages, as it is typically the case in traditional aircraft projects. The effects of these strategies on the expected value of the program are investigated below.

5.2.1 Exploring value of postponing investments

To explore the value of postponing capital investments, it is assumed that the decision to do initial flights at Step C can be postponed by a year(see Figure 5-3). This particular step was chosen because the expenditure at Step C is the single-largest in the aircraft development program analyzed here. Thus, the ability to postpone the investment at this stage until more information is available is likely to be a significant lever for project managers to increase the profitability of the project.

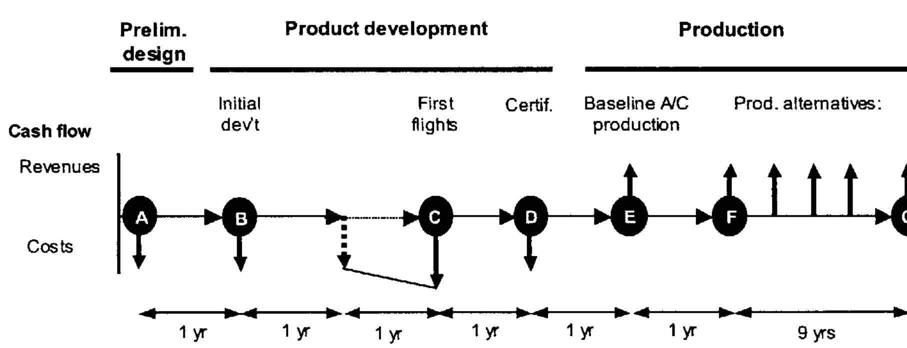


Figure 5-3: Schematic of changes to the cash flow in the new aircraft process if investments are postponed at Step C.

There are several advantages and disadvantages related to the ability to delay investments in the development of new products. A main benefit is the possibility of gathering more information to reduce market and technical uncertainty. For example, waiting to invest allows the investor to observe the evolution of the aircraft market during the waiting period. In addition, the waiting time gives the investor the opportunity to spend more resources to reduce technical uncertainties and ensure that the performance targets and delivery dates are met. Aircraft purchase contracts typically include performance warranties and on-time delivery clauses, which if violated, could mean fines for the aircraft manufacturer and loss of market share. Thus, the ability to ensure performance targets and delivery dates may be valuable; however, these benefits must be waged against the potential for loss of market share because if the aircraft manufacturer is late in introducing its product relative to its competitors, there may be significant losses in the number of orders. A further disadvantage of postponing the investment is that there may be extra Engineering Costs because more resources may be required during the extra year to work on reducing the technical uncertainties. Therefore, the total Completion Cost at Step C may be higher assuming the investment can be postponed by a year compared to the original investment scenario, as illustrated in Figure 5-3. Finally, another drawback of postponing the investment is that revenues are pushed further into the future. Because of discounting, these revenues will have a lower present value and, thus, lower the present value of the project.

Robinson and Min (2002) analyzed the probability of survival for market pioneers and early followers in markets for industrial products. They show that market pioneers have a higher probability of survival. Market pioneers enjoy the benefits of short-term monopoly until there is a second entrant. In addition, market pioneers have first-mover advantages such as brand loyalty, switching costs, and scale economies. Nevertheless, Robinson and Min (2002) also determine that early followers can benefit from some delay in entering a market because this delay allows them to resolve market and technical uncertainties. For industrial goods, Robinson and Min (2002) estimate a 2 year market entry delay with respect to the market leader to be optimal. A longer delay is found to actually hurt an early follower's probability of survival.

According to the aircraft manufacturer that supplied the numerical data, the aircraft example in this thesis is that of an early follower. Thus, given the results in Robinson and Min (2002), it is possible that delaying market entry by postponing investments at Step C for a year may lead to improvements in the expected value of the new aircraft program because of reduced uncertainties. These possible effects were incorporated into the analysis by assuming that waiting to invest would affect the probability distribution of the following variables in the system dynamics model and Monte Carlo simulation: Deviation from Customer Requirements, Production Delay, and Manufacturer's Market Share. The assumed changes in these probability distributions are given below. Notice that these changes are meant to illustrate a possible scenario of perturbation around the original values given by the aircraft manufacturer and that further analysis would be required to determine the exact change in the probability distributions from a one year delay at Step C:

1) Deviation from Customer Requirements: As explained above, postponing the investment may give project engineers more time to work on the design and, thereby, increase the likelihood that performance targets are met. To incorporate this effect, it is assumed that the magnitude of the estimated Deviation from Customer Requirements in the case where the investment is postponed is reduced by half compared to the case with the original investment schedule (see Figure 5-4). Thus, if with the original investment schedule there was a 20% probability of 1% Deviation from Customer Requirements, by postponing the investment there is a 20% probability of 0.5% Deviation from Customer Requirements.

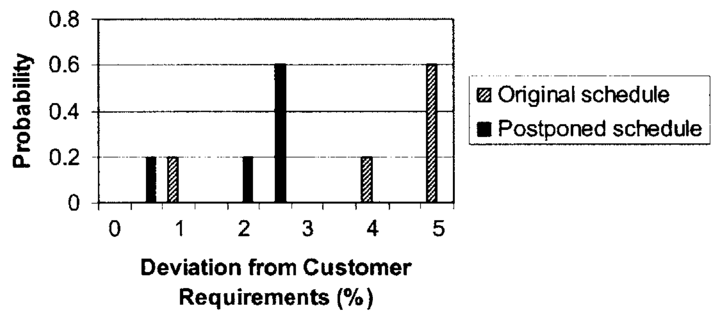


Figure 5-4: Assumed probability distribution of the Deviation from Customer Requirements in the original and postponed investment schedules.

2) Production Delay: Delaying the overall process by postponing the decision to do first test flights may give the aircraft manufacturer more time to organize production logistics and increase the likelihood of on-time product deliveries relative to the time of the actual program launch. As in the previous case, it is assumed that the probability distribution remains unchanged from the original

case, but the magnitude of the estimated Production Delay is reduced by half (see Figure 5-5):

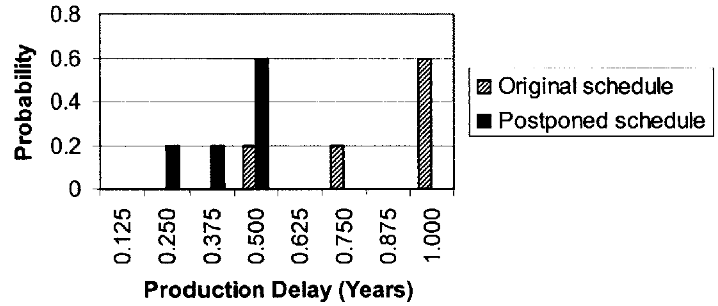


Figure 5-5: Assumed probability distribution of Production Delay in the original and postponed investment schedules.

3) Manufacturer's Market Share: By postponing the development program, market entry of the new aircraft may suffer from early competitor action. To account for this effect, it is assumed that the distribution of Manufacturer's Market Share is altered such that the lowest market share value has a probability of 60% and the highest has a probability of 10% as opposed to 50% and 20%, respectively, under the original investment schedule (see Figure 5-6).

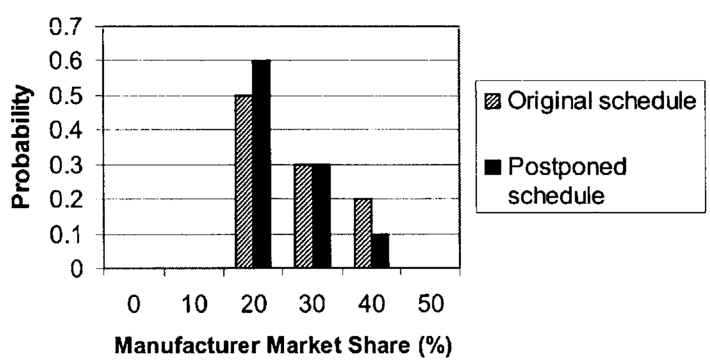


Figure 5-6: Assumed the probability distribution of Manufacturer Market Share in the original and postponed investment schedules.

In addition to the perturbation of the probability distributions shown above, an additional cost was added to Engineering Costs at Step C. This reflects extra resources that would be spent during the year of postponement to reduce some of the technical uncertainties. For the purposes of this exercise, the additional costs were assumed at 5% of Development Cost, bringing the total cost of Engineering for this stage to 45% of Development Cost, compared to 40% of Development Cost in the original case. Finally, the negative effects of pushing revenues further into the future are taken into account through the discounting of the cash flow.

The impact postponing the investment at Step C by one year on the overall value of the real option to start development design may be significant. Using the assumptions above, the value of the option is MU 19.0 million, a 31% improvement over the value of the option with the original investment schedule of MU 14.5 million. This indicates that according to the assumptions used in this analysis, the benefits of waiting to invest in terms of reducing delivery delays and deviation from customer requirements are larger than the combination of potential losses in market share and extra Engineering Costs. The insight for the aircraft manufacturer is that delaying the investment in Step C for a year may lead to a higher expected payoffs if there is reason to believe that the extra time can lead to a product with better performance and on-time delivery schedule without significantly compromising market share or increasing development costs. More research is needed to better estimate the effects of postponement on the different model variables, but this example shows how the analysis could be conducted.

5.2.2 Exploring the value of restructuring capital expenditures

As was discussed above, new aircraft development programs are typically very capital intensive in the early stages of the project. This has at least two important implications for project managers. First, it means that large investments are spent on products that still have a high degree of technical and market uncertainty and, thus, a high probability of negative expected payoffs. Second, large expenditures early in the project reduce the ability of managers to influence the project as uncertainties are resolved further along the development process. As the numerical results in Sections 4.4.4 and 5.1.2 indicate, once the project advances beyond first test flights, the value of terminating the project in later stages is zero. Because the amount of outstanding investments at that point is small compared to potential revenues, the manufacturer should always continue with the project.

Restructuring capital investments so that major expenditures occur in later as opposed to earlier stages may increase the ability of managers to influence the outcome of the project and, thus, increase its expected payoffs. The single major expenditure in the aircraft development program analyzed in this thesis occurs at Step C. Part of the investment at this stage includes building the assembly line for the new product to produce the test aircraft; however, if the test aircraft could be built using existing facilities, the new assembly line could be delivered later in the program, after the new aircraft has gone through test flights and certification. Restructuring the project in this fashion would give managers the ability to increase the expected payoff of the project by avoiding capital expenditures if the program does not progress as desired. In addition, payments further into the future are discounted more which, if nothing else, increases the present value of the expected payoffs.

To analyze the value of restructuring the investment schedule, it is assumed that the delivery of the production facilities for the new aircraft is shifted from Step C to Step E (start of production), as shown in Figure 5-7. Furthermore, it is assumed that the test aircraft can be produced using existing assembly lines. The unit cost of the test aircraft is increased 25% over the unit cost of the production aircraft to take into account extra costs associated with building the test aircraft using facilities designed for another product.

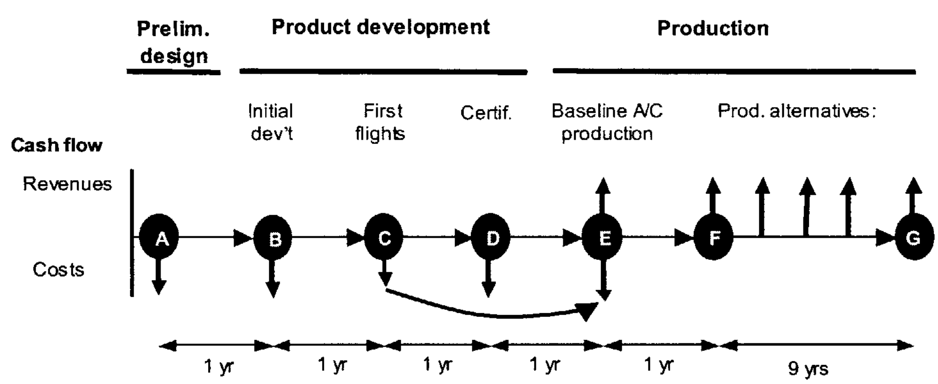


Figure 5-7: Schematic of changes to the cash flow in the new aircraft process if investments are restructure at Step C.

With the restructuring of the investment schedule, the expected value of the option to start development is MU 22.1 million, a 52% increase from the expected payoff of MU 14.5 million of the flexible strategy with the original capital expenditures. An analysis of the numerical results indicates that even with the new investment schedule, the option to start production at Step E is always exercised. Thus, in this particular case, the benefits of restructuring capital expenditures come from more heavily discounted expenditures as opposed to the ability of managers to cancel the project.

5.2.3 Summary of expected project payoffs with alternative investment strategies

The ideas of postponing the investment at Step C and restructuring the capital expenditures are not mutually exclusive. In fact, combining the positive effects of both increases the expected payoff of the project. Calculations show that a strategy that combines both alternatives results in an expected value of the option to start development of MU 28.2 million, a 93% increase over the original flexible investment strategy (see Table 5-2). Notice also that the payoff from the combined strategy is higher than the expected payoff that would result from implementing either alternative separately.

Table 5-2: Summary of the expected payoff of the new aircraft program given different alternative investment strategies. All values in MU million.

Strategy	Real options strategy with original investment schedule	Real options with 1yr postponement at C	Real options with restructuring of production facilities to Step E	Real options with 1yr postponement at C and restructuring of production facilities to Step E	Inflexible strategy
Exp. Value	14.5	19.0	22.1	28.2	-5,016.0
Std. Dev.	4.9	6.8	7.0	9.1	466.0

The expected payoffs of each alternatively in isolation, i.e., only postponing investments at 2.1 or only restructuring capital expenditures, are similar to each other. The expected payoff of postponing is MU 19.0 million whereas for restructuring it is MU 22.1 million. These values are still higher than the expected payoff of the real options strategy with the original investment schedule of MU 14.5 million, therefore, pursuing either one would be in the interest of the aircraft manufacturer. Finally, notice that the expected payoff of any strategy with real options is higher than the expected payoff of the inflexible strategy of -MU 5,016.0 million. This indicates that the aircraft manufacturer should always follow a strategy where managers are allowed to react as uncertainties are resolved as opposed to following an investment plan fixed from the beginning of the project. This may appear obvious, but it requires a cultural commitment to backing down, revisions, and adaptation that may not always be present.

This example shows how the generalized real options methodology can be used to systematically investigate real options with different project structure assumptions. Therefore, investors can test different investment strategies, compare their impact on the expected value of the real options and, thereby, gain deeper insights into the optimal project structure.

5.3 Insights for strategic decision-making

Knowing the value of the real options is useful information for management to make its budgeting plan, but it does not provide strategic guidance about how to proceed as uncertainties get resolved. In multi-stage projects like the aircraft development program considered here, the real options valuation alone does not give management any indication as to how to proceed as the project moves forward and more information is obtained; however, further analysis of the numerical data from the valuation using the generalized real options methodology can be performed to uncover insights that are useful for strategic decision-making.

An important metric of progress in aircraft programs is the number of aircraft orders. Typically, decisions of the board of directors and of managers are based on the number of firm orders at the time the decision is to be made (see, for example, [Esty and Ghemawat, 2002]). Thus, a practical approach to using the generalized methodology developed in this thesis is to create decision rules based on the number of firm orders. Indeed, the information obtained from numerical simulations may be used to determine the probability of program success and the expected project value. This can provide managers and board members with strategic guidance as the aircraft development process advances.

To illustrate the use of number of aircraft orders as a metric to guide decision-making, assume that the aircraft program is at Step C and management is deciding how to proceed next. The project has already advanced past preliminary design and the first phase of development. According to the strategies discussed in the previous section, there are several alternatives available:

1) Proceed with the original investment schedule with real options
2) Combine real options with a postponement of first test flights by a year
3) combine real options with restructuring of capital investments so that production facilities for the new aircraft are delivered in Step E
4) Combine real options with a postponement of first test flights by a year and a restructuring of capital investments so that production facilities for the new aircraft are delivered in Step E

In addition to these four strategies, the inflexible strategy is also considered, i.e., one in which capital expenditures occur as planned from the beginning of the project without the ability to react as uncertainties are resolved.

Using the numerical results from the system dynamics and Monte Carlo simulation and the real options analysis, it is possible to calculate the probability of positive program expected payoff from Step C on as a function of aircraft orders at that point (see Figure 5-8). In other words, the information in Figure 5-8 indicates the probability that the aircraft program from Step C until the end of the project will result in a positive expected payoff, given the number of orders at that point, expectations about further orders as a function of firm orders in hand, and given an investment strategy.

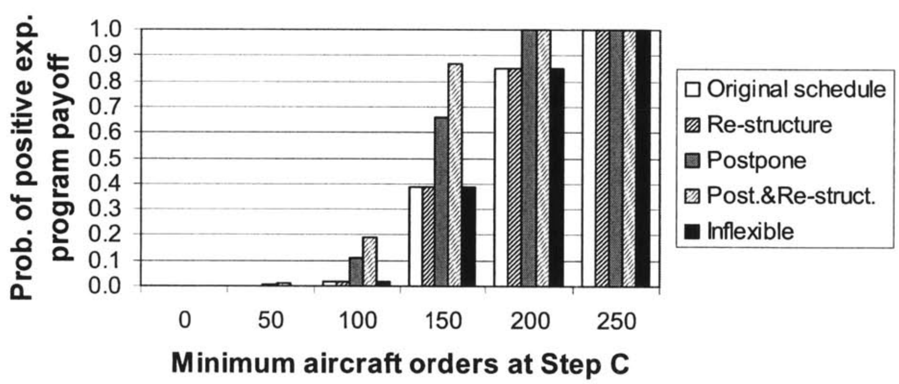


Figure 5-8: Probability of positive expected program payoff as a function of minimum aircraft orders and investment strategy at Step C.

The data in Figure 5-8 can be relevant for strategic decision-making. For example, it indicates that the program will always have a positive expected payoff if there are at least 250 aircraft orders at Step C, regardless of the strategy followed. On the other extreme, the data shows a practically zero probability of program success if aircraft orders are less than 50. As the minimum number of orders increases, so does the probability of program success for all investment strategies.

The strategies to postpone first test flights and the strategy that combines postponement of first test flights and restructuring of capital investment lead to the highest probability of program success for a given number of aircraft orders, with the latter strategy offering a considerable advantage where there are only 150 orders.

Based on the information provided in Figure 5-8, and assuming that investors are willing to accept a minimum probability of success of 80%, the following guidelines for strategic decision-making can be established (see Table 5-3):

Table 5-3: Guidelines for strategic decision-making to maximize the probability of project success as a function of aircraft orders at Step C and assuming a minimum allowable probability of success of 80%.

Minimum aircraft orders at Step C	Suggested strategy	Probability of success
0	Cancel project	N/A
50	Cancel project	N/A
100	Cancel project	N/A
150	Postpone & Restructure	87%
200	Postpone & Restructure	100%
	Postpone, only	100%
250	Any	100%

Besides knowing the probability of program success, managers might also be interested in the average value of the project as a function of aircraft orders. The average value of the aircraft program for a given number of orders and investment strategy is shown in Figure 5-9. This information indicates the average value of the project that can be expected from following a specified investment strategy for a given number of aircraft orders at Step C.

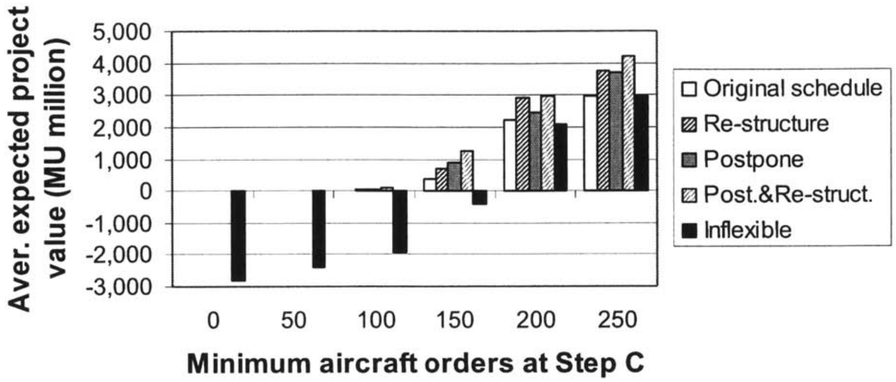


Figure 5-9: Average expected program value as a function of minimum aircraft orders at Step C.

As expected, the average value of the project increases with the number orders for a given investment strategy. The maximum average project value of MU 4,224.0 million corresponds to a situation in which 250 minimum orders have been received at Step C and a postponement and restructuring strategy has been followed. Notice that even the inflexible strategy achieves positive average project values for minimum orders above 200 aircraft.

Guidelines for strategic decision-making can also be made based on maximizing the average project values in Figure 5-9 (see Table 5-4):

Table 5-4: Guidelines for strategic decision-making based on maximizing average project value as a function of aircraft orders at Step C. Average project value given in MU million.

Minimum aircraft orders at Step C	Suggested strategy	Average project value
0	Cancel project	0.0
50	Postpone & Restructure	5.4
100	Postpone & Restructure	118.3
150	Postpone & Restructure	1,267.5
200	Postpone & Restructure	2,948.5
250	Postpone & Restructure	4,224.0

The strategy to postpone and restructure achieves the maximum average project value when there are more than 50 aircraft orders. For the case when there are 200 minimum aircraft orders, this is still the preferred strategy, although by a small margin because the average expected value of restructuring is MU 2,897.0 million, very close to the MU 2,948.5 million of postponing and restructuring.

The guidelines based on average project value in Table 5-4 can be combined with those drafted using the probability of project success shown in Table 5-3 to give managers more elements of judgment when making decisions to proceed with the project. Assuming that managers still require a minimum probability of project success of 80% and that they want to maximize average project value, the suggested strategy based on both metrics is shown in Table 5-5. According to these results, managers would cancel the project if aircraft orders at Step C are less than 150 units. If the number of orders is higher than 150 aircraft, the preferred strategy would be to postpone the investment at Step C for a year and build the production facilities at Step E.

Table 5-5: Guidelines for strategic decision-making based on the probability of success and average project value as a function of aircraft orders at Step C.

Min. aircraft orders at Step C	Suggested strategy based on probability of success	Suggested strategy based on average project value	Suggested strategy based on both metrics
0	Cancel project	Cancel project	**Cancel project**
50	Cancel project	Postpone & Restructure	**Cancel project**
100	Cancel project	Postpone & Restructure	**Cancel project**
150	Postpone & Restructure Postpone, only	Postpone & Restructure	**Postpone & Restruct.**
200	Postpone & Restructure Restructure, only	Postpone & Restructure	**Postpone & Restruct.**
250	Any	Postpone & Restructure	**Postpone & Restruct.**

5.4 Challenging the assumptions

As with any numerical model, many simplifying assumptions were used in the development of the system dynamics and Monte Carlo simulation to calculate the value of real options in the new aircraft development program. Several of these assumptions are investigated here to provide an indication of the limitations of the results presented in this study.

The baseline case for the sensitivity analysis is the strategy that postpones the exercise of the option to do first test flights at Step C by one year. Recall that the expected value of the option to start development with this strategy is MU 19 million.

5.4.1 Cancellation costs

In the real options framework, investors have the ability to cancel projects if continuing with them would result in negative expected payoffs. A typical assumption in these type of analysis is the absence of penalties for canceling the investment. In reality, however, canceling a program may come at a cost to the investor. For example, there may be "reputation costs" associated with canceling the project. Termination of the new aircraft project can erode the credibility of the manufacturer in the marketplace and, thereby, its ability to secure sales of future aircraft. Furthermore, depending on when the project is terminated, the manufacturer may be liable to pay compensation costs to customers that may have already placed orders, and/or to suppliers that may have started to procure materials or to produce parts. Thus, there may be some resistance from managers to cancel a project even when its expected payoff may be negative.

The possible effects of these cancellation costs on the expected value of the project are considered individually below:

- *Reputation costs:*

The potential effect of reputation costs on the expected value of an aircraft program is difficult to estimate because the impact of reputation costs are likely to be reflected in relation to other, future products in the manufacturer's portfolio. Thus, to determine the cancellation cost for the current program, it may be necessary to analyze future programs in the manufacturer's horizon, which is outside the scope of this thesis. Nevertheless, some intuition about the effect of reputation costs on a firm's value can be obtained by considering the reaction of capital markets to a company's decision to terminate a capital investment program.

In general, the stock market rewards decisions by management if they increase shareholder value. Thus, if analysts consider a new aircraft model to be a good investment, the market valuation of the aircraft company is likely to increase. For example, during the late 1990s, Airbus decided to launch a very large aircraft (VLA), the A380. Boeing's response to Airbus was to announce the development of a stretched version of the 747 jumbo jet. Analysts were skeptical about the rationale for a stretched 747 since it would be expensive to develop (about $7 billion compared to an estimated $10-15 billion for the A380) and there was the impression that it would cannibalize sales of the established 747-400 [Esty and Ghemawat, 2002]. Esty and Ghemawat (2002) show that Boeing's stock had abnormal negative returns on the day that Boeing announced that it would pursue the stretched 747 and that it had abnormal positive returns when this program was dropped. Canceling the 747 derivative was the rational decision that analysts were expecting from Boeing's management and, thus, when the company finally decided to cancel this project, it was rewarded by the market.

In the context of real options, an investment is canceled when cancellation is the value-maximizing alternative. Thus, if the conclusion of a real options analysis was to cancel the aircraft program, this would be the rational course of action for managers because otherwise the company would be investing in a project with negative expected payoff. In the same way that the market rewarded Boeing for dropping the stretched 747 idea, if the real options analysis determines that canceling the project is the rational, value-maximizing decision, the market should reward this managerial action. Consequently, if the real options approach suggests canceling a project, the impact on the reputation of the company should be minimal since this is in the best interest of the firm.

It is important to note that canceling a project is not a good idea because it is rewarded in the marketplace, but, rather, it is rewarded because it is probably a good idea. Consequently, the point is not that managers should try to cancel projects to increase shareholder value, but that they should cancel them when it is the best possible course of action.

There may be some limitations to this intuition, however. For example, if a company is known for canceling many projects, even if that is the rational alternative, markets may doubt the ability of the firm to undertake those type of investments and that may be reflected negatively in the company's stock market valuation.

- *Penalties and compensations costs*

The assumption of zero terminating costs can be relaxed by establishing a penalty in case the aircraft manufacturer decides to cancel the project. This penalty would be paid to aircraft customers that may have placed orders or to suppliers that may have been already in a contractual relationship with the aircraft manufacturer. In this case, the managerial decision becomes choosing between keeping the project alive or paying a cancellation cost, F, if the project is terminated. In the example investigated in this thesis, the ability to cancel the program is relevant only at Step B (Start Development) and Step C (First Test Flights), because if the program has advanced beyond Step C, it is always in the money and, thus, it is carried to completion in all occasions.

The effect of termination costs in Step C can be accounted for by incorporating them in the expression for the value of completion of the option to do first test flights with cancellation costs, $V'_{1st\_flights}$. Recall that the formula for the value of completion at Step C without cancellation costs, $V_{1st\_flights}$, is:

$$V_{1st\_flights_i} = \max\left\lfloor e^{-r_f} Payoff_{certification_i}, 0\right\rfloor$$

$$= \max\left\lfloor e^{-r_f}\left(V_{certification_i} - C_{certification_i}\right), 0\right\rfloor \qquad \text{(Eq. 4-5)}$$

Substituting cancellation cost, $F_{1st\ test\ flights}$, for zero in Equation 4-5 yields Equation 5-1:

$$V'_{1st\_flights_i} = \max\left[e^{-r_f} Payoff_{certification_i}, F_{1st\_flights_i}\right]$$

$$= \max\left[e^{-r_f}\left(V_{certification_i} - C_{certification_i}\right), F_{1st\_flights_i}\right] \quad \text{(Eq. 5-1)}$$

Now, program managers must weigh the decision of canceling the program against the requirement of paying $F_{1st\ test\ flights}$ if they choose to terminate the project. Contrary to the situation before where the minimum value for $V_{1st\ flights}$ was zero, with Equation 5-1 there can be negative values for $V_{1st\ flights}$ (notice that $F_{1st\ test\ flights} < 0$). Furthermore, notice that if reputation costs could be quantified, they could be incorporated into the calculations in the same fashion.

Recall the expression for the value of completion to start development at Step B, $V_{start\ development}$ (Equation 4-6):

$$V_{start\_development_i} = \max\left[e^{-r_f} Payoff_{1st\_flights_i}, 0\right]$$

$$= \max\left[e^{-r_f}\left(V_{1st\_flights_i} - C_{1st\_flights_i}\right), 0\right] \quad \text{(Eq. 4-6)}$$

The value of completion for the option to start development at Step B with cancellation costs, $V'_{start\ development}$ is obtained by substituting Equation 5-1 for $Payoff_{1st\_flights}$ in Equation 4-6:

$$V'_{start\_development_i} = \max\left[e^{-r_f}\left(V'_{1st\_flights_i} - C_{1st\_flights_i}\right), 0\right] \quad \text{(Eq. 5-2)}$$

The expected value of the option to start development at Step B with cancellation costs is found by first substituting the distribution of $V'_{start\ development}$ and $C_{Start\ development}$ (same as in the case with no cancellation costs) in the generalized valuation formula 3-11 and then subtracting the expected value of cancellation costs at Step B, $E_{cancellation\ cost}$:

$$E_{cancellation\ cost_{start\_development}} = E\left[\min\left[\max\left[V'_{1st\_flights_i} - C_{1st\_flights_i}, F_{start\_development}\right], 0\right]\right]$$

(Eq. 5-3)

where $F_{start\ development}$ is the cost of terminating the project in Step B. The expression $\max[V'_{first\ flights} - C_{first\ flights}, F_{start\ development}]$ is the value of the optimal strategy given the values of the three variables. For example, for a given value of the value of completion, if $V'_{first\ flights}$, is larger than the completion cost, $C_{first\ flights}$, then the payoff is positive. This implies that the project is continued and, therefore, there are no cancellation costs. In this case, $E_{cancellation\ cost}$ is zero. If the reverse is true, i.e., $V'_{first\ flights}$ is less than $C_{first\ flights}$, the payoff is negative. The manager can decide to continue with the project if doing so is less costly than terminating it (i.e., $(V'_{first\ flights} - C_{first\ flights}) > F_{start\ development}$) or, conversely, the manager can cancel the project if $(V'_{first\ flights} - C_{first\ flights}) < F_{start\ development}$. In this case, $E_{cancellation\ cost}$ is the maximum of either strategy.

The expected value of the real option to start development at Step B with cancellation costs, $w'_{Step\ B}$ is given by Equation 5-4:

$$w'_{Step\ B} = e^{-r_f T}\left(\int_{c=0}^{\infty} f_c(c) \int_{v=c}^{\infty} v \cdot f_v(v)\,dv\,dc - \int_{c=0}^{\infty} c \cdot f_c(c) \cdot \int_{v=c}^{\infty} f_v(v)\,dv\,dc\right) - E_{cancellation\ cost_{start\_development}}$$

(Eq. 5-4)

The sensitivity of the expected value of the real option to start development to cancellation costs at Step B is shown in Figure 5-10. Cancellation costs are normalized by total development costs, i.e., they are given in terms of percentage of total development costs.

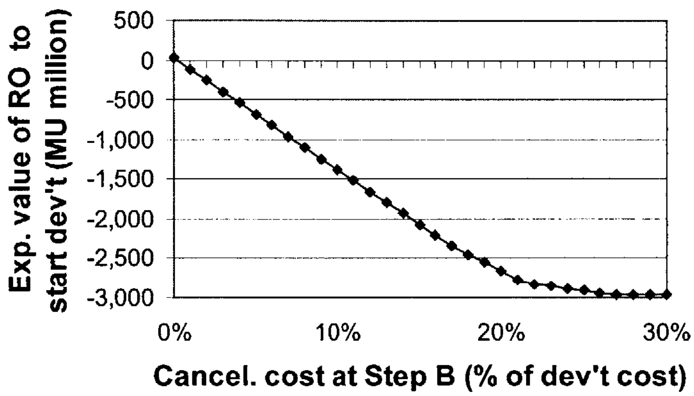


Figure 5-10: Expected value of the real option to start development as a function of cancellation costs at Step B. Cancellation costs are specified as a percentage of total development cost.

The result in Figure 5-10 shows that the expected value of the real option to start development is very sensitive to cancellation costs at Step B. With a cancellation cost of just 0.15% of development cost, the expected value of the real option is zero, compared to MU 19.0 million with no cancellation costs.

Similarly, the cancellation costs at Step C have a significant impact on the value of the real option to do first test flights. The expected value of this real option becomes negative if cancellation costs at Step C are more than 1.35% of development costs (see Figure 5-11).

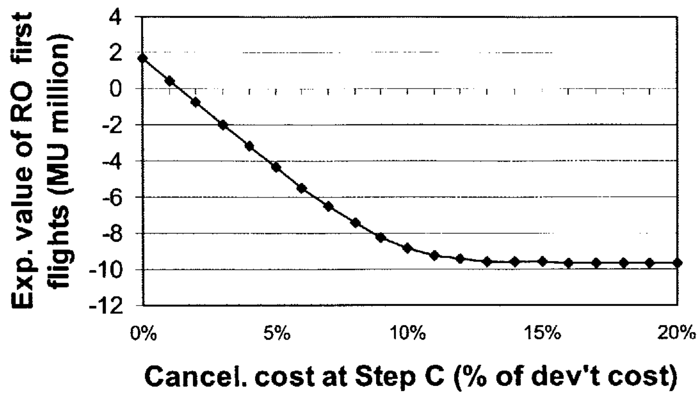


Figure 5-11: Sensitivity of the expected value of the option to do first test flights (Step C) to cancellation costs at Step C. Cancellation costs are specified as a percentage of average development cost.

To guarantee the profitability of the project, the manufacturer must ensure that termination costs at Steps B and C are kept below 0.15% and 1.35% of total development costs, respectively. Otherwise, the expected value of the options to start development and do first test flights would be negative.

5.4.2 Unit price

In the system dynamics model of the new aircraft program, unit price is determined by establishing a profit margin over the base unit production cost, according to input from the aircraft manufacturer. In reality, however, unit price can be affected by many factors outside the manufacturer's control. Thus, the manufacturer cannot always achieve a desired profit margin for each unit sold; therefore, the sensitivity of the expected value of the program to variability in the profit margin must be investigated.

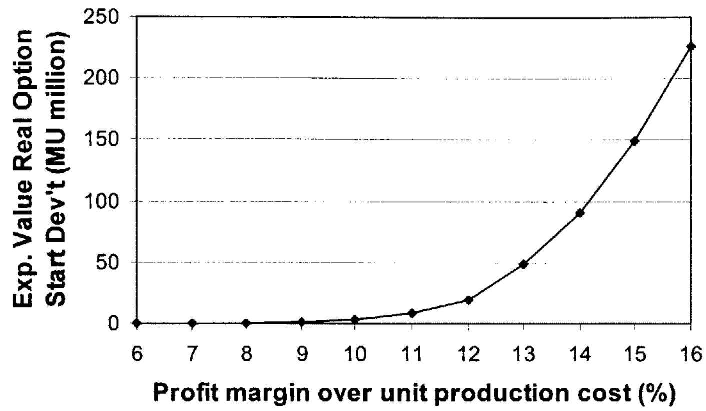


Figure 5-12: Sensitivity of the expected value of the option to start development to the profit margin specified in the system dynamics model.

The expected value of the real option to start development is very sensitive to changes in the profit margin over unit production costs (see Figure 5-12). For the baseline value specified for the profit margin (12%), the expected value of the real option is MU 19.0 million. If the profit margin is 11%, the expected value of the option decreases by 53% to MU 9 million. For a profit margin of 13%, the expected value of the option increases 158% to MU 49 million. Notice that if the profit margin is less than 9%, the expected value of the real option to start development is zero.

This sensitivity analysis shows that the magnitude of profit margin for each unit sold is critical to determine the value of the real option to start development. It indicates that this is a variable that deserves close attention by managers to ensure the financial success of the project. In addition, this sensitivity analysis can be used to inform negotiations with customers.

5.4.3 Discount rate

The choice of the discount rate for any project is typically not clear and there is always some uncertainty about which rate should be used. Thus, the sensitivity of the numerical results to the risk-adjusted discount rate used to calculate the present value of all cash flows is another effect that must be explored.

The expected value of the real option to start development is sensitive to the choice of the risk-adjusted discount rate used to determine the certainty-equivalent values of the value of completion and the completion cost. Using a discount rate of 18%, as suggested by the data from the aircraft manufacturer, the expected value of the real option is MU 19 million. If the discount rate is 17%, the expected value of the option increases 53% to MU 29.1million (see Figure 5-13). For a discount rate of 19%, the expected value of the option decreases 34% to MU 12.5 million.

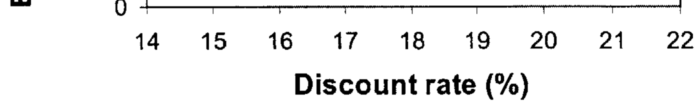


Figure 5-13: Sensitivity of the expected value of the option to start development to the discount rate.

The sensitivity of the expected value of the project to the discount rate is less than the sensitivity to the profit margin, but it is still significant. Thus, the discount rate is another variable that must be chosen carefully by project managers. The discount rate can be the firm's Weighted Average Cost of Capital (WACC) or there may be internal company guidelines for choosing the discount rate for new aircraft development projects.

5.5 Implications of insights from the real options analysis

A series of insights into the particular characteristics and challenges of the aircraft manufacturing industry have been obtained through the application of the real options methodology and derivative analyses. These insights indicate that changes in new aircraft development projects may lead to significant improvements in their financial performance; however, change is typically not easy to implement because it requires a cultural commitment to modifications, revisions, and adaptation that may not always be present. Thus, in order to institute change, it is often necessary to systematically evaluate trade-offs and alternative courses of action so that managers can make informed decisions. The generalized real options methodology is a tool that allows decision-makers to make these types of analysis. In this section, the implications of the insights for decision-making and behavioral change obtained with the generalized real options methodology are discussed.

5.5.1 Share production resources

As was mentioned in the overview of the cash flow in new aircraft development program in Chapter 4, this type of projects are very resource intensive in their earlier stages and significant revenues are typically not realized until later in the program. This implies that many expenditures are spent on highly uncertain products and that companies incur heavy costs early in the program before obtaining significant payments for their products. As the analysis of the strategies to postpone and restructure the aircraft development program earlier in this chapter show, there may be advantages by shifting some of the earlier costs to later stages in the program.

An alternative to delay development costs to later stages is to share production facilities with existing programs. For example, if the new aircraft is designed so that it can be assembled in an existing production line, the manufacturer would have the ability to wait before it builds the new production line. Delaying this investment may bring several benefits. First, from a cash flow perspective, since the investment happens later in the future, the present value of the expenditure is less and the manufacturer can earn returns on that money by investing it elsewhere. Second, by using existing facilities to build prototypes of the new aircraft, the manufacturer can optimize the design of the production line for the new model before it is built. In addition, building the prototype on an existing production line gives the manufacturer the opportunity to test if the existing line could be used for both aircraft types. If this is the case, the manufacturing process would likely enjoy economies of scale because instead of requiring a whole new production line, expanding the existing one could accommodate the expected volume of orders. Furthermore, building two or more aircraft types with the same facilities offers economies of scope, which may be valuable if orders from one aircraft type dwindles but orders for the other(s) remain strong. Finally, sharing facilities may prolong the life of the production line because even if one of the aircraft types goes out of production, the other(s) model(s) can keep it open. The socioeconomic implications of this may be enormous. Production facilities at major aircraft manufacturers employ thousands of people and are typically key economic drivers for the regions where they are located. Thus, sharing production facilities for different aircraft types may not only bring benefits in terms of reducing costs for new aircraft models, but also in terms of smoothing shocks in production and maintaining a stable workforce and economic base.

Even though there may be many advantages by sharing production facilities, there are some disadvantages that should be considered, too. For example, the need to comply with requirements for the existing production line may add some design constraints to the new aircraft. Moreover, the cost of producing the prototype aircraft in a shared production line may be significantly more costly than in a fully dedicated facility. Furthermore, if there is a problem in the production line and it has to be temporarily stopped, the interruption would affect deliveries of all the aircraft types produced there.

To make an informed decision with respect to sharing facilities to produce different aircraft models, the advantages and disadvantages mentioned above must be carefully considered and trade-off studies must be conducted. Given its flexibility to represent different project structure assumptions, the generalized real options methodology can be used to systematically analyze and determine the value of shared production lines compared to the value of separate dedicated facilities. Thus, managers will have more information to evaluate the trade-offs and come to a decision regarding production facilities.

5.5.2 Diversification of production through derivative aircraft

Based on the analysis in Chapter 4 and the assumptions made in this thesis, investors should always exercise the option of launching a derivative aircraft. This is consistent with the observation in the outside world that derivative aircraft are always part of major aircraft programs. The existence of derivatives is not surprising given the many advantages they offer to manufacturers. For example, derivatives can be developed at a fraction of the cost of a completely new model. Thus, the manufacturer can offer another product without having to spend all the resources required for a new development program. Moreover, through sales of the baseline aircraft, the manufacturer can explore the market and make a better-informed decision about the specific qualities that customers would value in a derivative aircraft. Furthermore, the concept of several closely-related aircraft, i.e., a family of aircraft, is a powerful selling point because of the many advantages it provides to airlines, such as reduced crew training requirements and savings in spare parts inventories.

In addition to all these reasons to launch a derivative, there is another rationale why aircraft manufacturers should think of derivatives when structuring their aircraft development programs: derivative aircraft offer the advantage of diversification. As was made evident in Chapter 4, new aircraft development programs are very risky, especially because of the high costs required to develop new models. Thus, if the manufacturer can use the baseline aircraft as a platform to launch several derivatives, it can expand the revenue base to amortize the fixed costs of development and, at the same time, it can diversify the revenue base by offering a portfolio of products. A diversified portfolio of products is attractive because it can allow the manufacturer to cater to a wider range of customers and it can help to hedge against fluctuations in individual markets.

The idea of diversification through a portfolio of derivative aircraft can be an additional argument to justify a new aircraft development program. In the example analyzed in this thesis, it is assumed that only one derivative aircraft is launched after the baseline aircraft enters production. In reality, however, aircraft manufacturers can launch several derivative aircraft. The development of a new aircraft should not be evaluated only in terms of sales of that one product, but also in terms of the options that it opens to produce a number of derivatives. Thus, by recognizing the strategic value of a new aircraft program in terms of the options for diversification that it offers, manufacturers could be more aggressive in the design and development of the baseline aircraft to ensure that it could be easily modified to cater to different market needs. Adding this type of flexibility into a baseline product may put some constraints with respect to obtaining an optimal design for a particular mission. Therefore, the advantages of being able to optimally fulfill a specific market niche must be weighed against the options of reasonably meeting the requirements for several different missions.

The generalized real options methodology can be used to perform the comparisons and trade studies required to inform the decision regarding the characteristics of the baseline aircraft as a platform for subsequent derivative models. The analysis developed in this thesis considers only one derivative aircraft but it can be easily expanded to include two or more derivative models. In this way, decision-makers can explore the value of a new aircraft program not only in terms of the sales of the baseline aircraft, but also in terms of the potential sales of several derivative aircraft. Thus, similar to the car industry, aircraft manufacturers could evaluate the possibility of creating a common platform upon which to provide several options to potential customers.

5.5.3 Supporting the cost of flexibility

The analysis of the new aircraft development program presented in this thesis shows that a flexible strategy, i.e., a strategy with real options, can lead to higher expected project values in the early stages of the investment than an inflexible strategy, i.e., a strategy that always continues the investment regardless of the relative magnitudes of the value of completion and the completion cost; however, even with the real options, the expected value of the project at Step B (Start development) or Step C (First test flights) are very sensitive to the assumption of zero cancellation costs. In fact, if this assumption is relaxed, the expected value of the project turns negative with very small non-zero cancellation costs.

These observations highlight the fact that new aircraft development programs are very risky in their earlier stages but, as they move forward, the chances of success increase greatly. In the particular example analyzed in this thesis, once the process advances past first test flights, the expected value of the project is always positive with either the flexible or the inflexible strategies.

From a purely financial return perspective, the results of the analysis performed here suggest that aircraft development programs are not very attractive; however, there may be other reasons for desiring an aircraft production capability. As was mentioned before, aircraft manufacturers employ many people and are typically key local economic drivers. In addition, aircraft sales to foreign customers constitute a significant portion of exports in countries like the United States. Furthermore, aircraft production provides high-qualifying jobs and contribute to maintaining a high-technology knowledge-base. Thus, there may be many reasons why a country or a region may be interested in keeping an aircraft production capability. Under these circumstances, the intervention of outside parties (e.g., the government) to reduce risks in early stages of aircraft development programs with the expectation that this will help sustain a viable commercial program later may be justified.

The idea of outside intervention to improve the viability of an aircraft manufacturing industry is a sensitive issue because it can lead to market distortions and, subsequently, confrontations over international trade agreements; however, there may be an opportunity to compromise. Rather than advocating for outside help to cover their development expenditures in early stages of the program, aircraft manufacturers could instead request assistance to cover expected losses from non-zero cancellation costs. This approach has several advantages. First, the main feature of this approach is that it just provides insurance in case the project fails and it must be terminated and, thus, it does not imply that the aircraft manufacturer will always receive outside help. Second, the cost of this insurance is smaller than the cost of development and the aircraft manufacturer is still spending all the required development costs. Therefore, the outside assistance protects the manufacturer if conditions are not favorable but it does not reduce the burden of development costs.

The magnitude of the expected cancellation costs can be computed using the modifications to the generalized methodology discussed in Section 5.4.1. This approach can be used to explore different assumptions regarding cancellation costs and, thus, try different outside assistance policies. Eventually, these analysis could be incorporated into the discussions leading to agreements for outside intervention in aircraft manufacturing.

5.6 Chapter summary

In this chapter, the evaluation of a new aircraft development program was re-visited to show how the generalized methodology can be used to relax some of the fundamental data and structural assumptions of a project, explore how the numerical results can be used to gain insights useful for strategic decision-making, and quantify the sensitivity of the results to the data and other assumptions. In addition, some of the behavioral implications of this analysis for the air transportation industry were discussed. The main points of this chapter are summarized below:

1) With the generalized real options methodology developed in this thesis, investors can use numerical models based on historical and/or behavioral data to determine probability distributions even for projects with few or no precedents. Therefore, their predictions of possible future values can be informed by historical data without making the assumption that the future will necessarily behave like the past. This capability was illustrated by introducing a time-varying mean-reverting stochastic

model of the aircraft market based on historical data in the system dynamics model used to determine the value of completion of the real options in the new aircraft development program. Results show that the value of the option to start development with the time-varying market is less than when a constant market is assumed. This difference in option value indicates the importance of choosing a market representation that best approximates the situation being analyzed.

2) With the time-varying market model, it was possible to explore how changes in the structure of the project would affect the expected value of the real options. In particular, the potential effect of postponing and restructuring investments on the value of the program were investigated. Compared to the originally assumed flexible strategy and given the assumptions in the analysis, numerical results suggest that these alternative investment strategies can significantly increase the value of the real option to start development. For example, postponing capital expenditures at Step C by a year increases the value of the option to start development by 31%. Furthermore, restructuring the investment schedule so that production facilities are delivered at Step E at opposed to Step C increases the value of the option by 52%. Combining both strategies results in a 93% increase in the value of the real option to start development.

3) Guidelines useful for strategic decision-making can be derived from the numerical data. These guidelines combine the probability of program success and expected program value as a function of minimum number of aircraft orders to indicate the best

strategy given the available data. Results show that, given the assumptions in the analysis performed here, if at Step C aircraft orders are less than 150, the program should be cancelled, but if the number of orders are more than 150, a strategy that combines postponing and restructuring the investment at Step C should be followed.

4) The expected value of the project is sensitive to cancellation costs at Steps C and Step B. The manufacturer should not launch the project if termination costs at Steps B and C are more than 0.15% or 1.35% of development costs, respectively.

5) The expected value of the project is sensitivity to the profit margin used to determine unit price. Thus, project managers should pay close attention to this parameter to ensure the financial success of the project. In addition, this sensitivity analysis can be used to inform negotiations with customers..

6) The sensitivity of the expected value of the project to the discount rate is less than the sensitivity to the profit margin, but it is still significant. Thus, the discount rate is another variable that must be chosen carefully. The discount rate can be the firm's Weighted Average Cost of Capital (WACC) or there may be internal company guidelines for choosing the discount rate for new aircraft development projects.

7) Insights from the analysis of the new aircraft development program indicate that changes in this type of projects may lead to improvements in their financial performance. In particular, the use of shared facilities to produce several aircraft

types, diversification of production through the launch of a portfolio of derivative aircraft, and outside assistance to cover non-zero cancellation costs may greatly improve the viability of aircraft manufacturing projects and, thereby, contribute to their continued existence; however, in order to institute changes like these, it is often necessary to systematically evaluate trade-offs and alternative courses of action so that managers can make informed decisions. The generalized real options methodology is a tool that allows decision-makers to make these types of analysis.

6. Other applications of the generalized methodology

The generalized real options methodology developed in this thesis is, of course, not specific to the evaluation of investments in aircraft development programs. It can be applied to many other questions within and outside the air transportation field; however, this methodology does not necessarily apply to all investment evaluation questions. There are circumstances where the real options approach may not be suitable and other evaluation techniques should be used. In this chapter, guidelines to identify circumstances where the generalized real options approach would be justified are presented. Possible applications of this generalized methodology in other aviation examples are then identified. Next, the use of the methodology to evaluate investments in other domains is briefly discussed. Finally, some limitations of the real options methodology are highlighted.

6.1 Guidelines to apply the generalized methodology

The generalized real options methodology developed in this thesis can be used to determine the value of projects where the ability to incorporate information as time progresses can affect the performance of the investment. The following conditions may be used to identify projects where this approach could be applied:

1) The investment can be structured in a way that it allows managers to affect the fate of the project as uncertainties are resolved.
2) A suitable underlying asset can be identified.

3) The probability distribution for the value of completion and for the completion cost can be determined.

The first condition may seem obvious but it is key for the real options valuation. Even though many projects under uncertainty may theoretically allow room for managers to influence its progress as uncertainties evolve, there are a number of factors that may inhibit this managerial capability. For example, contractual agreements, competitive forces, or government policy may oblige projects to be completed according to original plans even if it entails financial losses, thus, negating managers of the real option to alter the course of the investment. Therefore, before performing a real options evaluation, it is important to verify that managers will indeed realistically have options available to act as more information about the project becomes available. Otherwise, the evaluation of the project could be performed with simpler tools, such as the Net Present Value rule, in which it is assumed that the investment follows the plan devised from the beginning.

The second guideline, being able to identify an underlying asset, is evident: the whole premise of real options analysis is to find the value of the initial investment required to have the right to acquire the underlying asset at a later time; however, as was mentioned in the literature review in Chapter 2, the choice of the underlying asset is not always easy. In traditional real options methodologies, the choice of the underlying asset is typically influenced by a desire of analytical tractability and, thus, sometimes there is a trade-off between closely capturing the value of the project and being able to model it analytically. With the generalized real options methodology developed in this thesis, there is no need for this compromise because the value of the underlying asset does not necessarily have to be described with a stochastic process such as a geometric Brownian motion. With the generalized real options methodology, the value of the underlying asset can be described analytically or numerically.

The third guideline, being able to represent the probability distributions of the value of completion and the completion cost, is the last general condition that must be met to apply the generalized real options methodology. These probability distributions can be specified analytically or they can be calculated using numerical simulation. Thus, the generalized methodology gives the user more alternatives to represent the value of completion and the completion cost than traditional real options approaches. This is an important consideration because oftentimes with traditional techniques, there is a trade-off between choosing an analytically/numerically convenient method and being able to closely represent the value of completion and the completion cost.

Finally, there should be mention of the discount rate, an important input in any investment evaluation. In financial options theory, the assumptions of complete markets and no arbitrage are used to justify the use of the risk-free discount rate to evaluate financial options. In real options applications, those assumptions are not necessarily valid, therefore, in many circumstances adjustments for risk must be made. The generalized real options methodology presented here uses certainty equivalents as inputs into the evaluation formula to justify the use of the risk-free discount rate; however, in order to find the certainty equivalents, it is necessary to make certain assumptions with respect to the attitude of investors towards risk. This may imply, for example, identifying a risk-adjusted discount rate with the Weighted Average Cost of Capital (WACC) or the Capital Asset Pricing Model (CAPM), or finding utility functions. The generalized real options methodology does not necessarily solve the question of the discount rate, but it allows the user to make his own adjustments.

6.2 Application of the generalized methodology to other air transportation examples

A new aircraft development program was used to illustrate the generalized real options methodology developed in this thesis; however, this methodology is not restricted to this specific problem. Two examples of projects in air transportation where this methodology could be used are described below. The first project involves capacity expansion at an airport. The second example corresponds to investments in air traffic control (ATC) infrastructure. The discussion highlights the extent to which the guidelines specified above are met and, thus, it indicates how appropriate the generalized real options analysis would be to evaluate these investments.

6.2.1 Capacity expansion at an airport

Airport capacity expansion projects, such as the construction of runways or passenger buildings, are examples of investments where managers may have ample room to act as uncertainties are resolved. These investments are typically large, take many years for completion, and are subject to many uncertainties. There are technical risks related to the ability of contractors to meet project specifications on time and within budget. Moreover, the cyclical nature of the air transportation industry, as discussed in Chapter 4, introduces significant demand risk to the investment. Given these circumstances, a flexible project structure can give managers the ability to make rational decisions as uncertainties are resolved to improve the profitability of the investment. For example, the construction of new passenger buildings can be programmed in phases that are built as demand requires them, instead of completing the entire investment at once, which can result in over-capacity that sits idle for some time.

There may be circumstances, however, where the flexibility inherent to project managers may be reduced or even taken away. Airport expansion projects often face opposition from neighbors and other interest groups that object to the increased noise and pollution from more activity at the airport. Managers may be inhibited from acting quickly as negotiations with neighbors and legal requirements, such as environmental impact studies, can take years. Furthermore, once and if all legal hurdles have been cleared, it may be in the best interest for managers to finish the project before more delays occur, at the expense of a better timing with the market. Political pressures may also drive the schedule of the investment rather than considerations about the financing of the project or travel demand, for example. These concerns must be taken into account when evaluating the project because they can stifle much of the flexibility available, in theory, to project managers.

The choice of the underlying asset for a real options analysis of an airport capacity expansion is straightforward: it is the piece of infrastructure put in place to extend the airport's capacity. Thus, for example, the underlying asset would be the new runway or the new passenger building.

The calculation of the value of completion and the cost of completion in this case depends on the stakeholders involved in the investment decision. If the airport is privately owned, and if the owner can be assumed to be a profit-maximizing entity, the value of completion can be a metric of revenue directly related to the capacity expansion project, such as increases in landing fees, passenger facility charges, concession fees, or a combination of these, that result from the extra traffic because of the new investment. This metric would capture the benefits of the project to the investor and it would be appropriate for the investment analysis. To determine the probability distribution of the value of completion, a model of revenues as a function of airport capacity can be developed (see, for example, [Miller and Clarke, 2004]). Such a model can be the basis for calculating the probability distribution of the value of completion with Monte Carlo simulation, for example.

In case that the airport is not a profit-maximizing entity, as it can be the case for publicly-owned airports, for instance, the value of completion may be more difficult to identify because maximizing revenue may not capture the whole value of the project for the investor. A publicly-owned airport must be considered as another element in a wider socio-economic context where the system boundaries go beyond the enclosure of the airport and capture all beneficiaries of the facility. This typically includes determining the socio-economic benefits to the region that is serviced by the airport as well as negative externalities such as noise and emissions. Calculating these effects may require the use of sophisticated regional economics tools, e.g., input-output models, which require large amounts of data. Thus, while not easy, a numerical model of the value of completion that captures at least some of the main regional economic effects and externalities may be feasible. Such a model could be combined with Monte Carlo simulation to obtain the probability distribution of the value of completion.

The probability distribution of the completion cost of capacity expansion projects can be calculated with a numerical cost model and Monte Carlo simulation. Completion costs in this type of projects are subject to many uncertainties and the Monte Carlo simulation provides a means to include these risks in the computation. This approach could be used for both the profit-maximizing and the publicly-owned airport investor.

Finally, a word about the discount rate. To use the generalized real options methodology, it is necessary to express the value of completion and the completion cost in terms of their certainty equivalents. Thus, if the approach for finding the CEs outlined in Section 3.1.1 is chosen, a risk-adjusted discount rate must be found. If the owner of the airport is a private entity whose shares are traded in open markets, such as the British Airport Authority (BAA), there may be enough information to determine the risk-adjusted discount rate using CAPM. Otherwise, the WACC may be used to identify the discount rate. If the airport is publicly held, the discount rate could be the rate specified by the government body that owns the airport for projects with similar risk and return profiles.

In summary, the generalized real options methodology can be an appropriate approach to find the value of investments in airport capacity expansion projects. There are several aspects that must be given careful consideration, such as the value of completion, the freedom of managers to act as uncertainties are resolved, and the risk-adjusted discount rate, to accurately determine the value of the investment.

6.2.2 Infrastructure investments in air traffic control

Infrastructure investments in air traffic control (ATC) share similar characteristics to capacity expansion projects at airports: capital expenditures may be significant, projects may take many years to be delivered, and there are many technical and demand uncertainties. Thus, investments in ATC occur in an environment where managers can potentially play an important role to affect the outcome of projects. For example, new communication, navigation and surveillance (CNS) technologies for air traffic control, including hardware and automation tools, may take years to be developed and tested before being deployed on the field. At the same time, aircraft technologies, ranging from aircraft operational performance to cockpit automation, are continuously improved and brought into the fleet mix with new or upgraded aircraft models. Thus, investments in ATC must be actively managed to ensure that once they are fielded, the market will be ready to benefit from them. A strategy that includes alternative courses of action as technical and market uncertainties are resolved is a way to mitigate the risks inherent in these type of investments.

As in the case of airports, there may be circumstances that could take away some of the inherent project flexibility available to managers. Unlike the case of airports, where there are many privately owned facilities, air traffic control is a service provided publicly in many parts of the world. In addition, ATC is key to ensuring aviation safety both en-route and around terminal areas. Thus, governments have the responsibility to ensure that ATC facilities are capable of providing a minimum level of service. This level of service can be provided in many countries with the existing infrastructure, but as air travel increases, aircraft technology improves, and current facilities get older, investments in ATC are required. Given the critical role played by ATC in the air transportation system, some of these investments may be mandated and specified by executive orders or laws, limiting the ability of managers to vary the scale, timing, and nature of projects as more information becomes available. Paradoxically, because of the possibility of strict rules, managers should pre-emptively build flexibility into their proposed projects. At the same time, rulemakers should consider allowing for less stringent timelines and project specifications to give managers the leeway to react given new information about their specific projects.

Similar to the case of capacity expansion projects at airports, the choice of the underlying asset for investments in ATC infrastructure is straightforward: it is the piece of infrastructure that is put in place. Thus, the underlying asset could be ground-based radars, navigation aids, or communications satellites, for example.

Given that air traffic control is typically a public service, determining the value of completion for these projects is not obvious because there are usually objectives other than profit maximization; therefore, to proceed with the investment evaluation, it is necessary to identify metrics that reflect the selected objective(s) and to find monetary values for them. For example, it was mentioned earlier that guaranteeing aviation safety is considered one of the main objectives of air traffic control. Number of lives saved could be used as a metric for aviation safety, but a significant hurdle with this metric is that putting a value on human life is controversial. Thus, a surrogate metric such as number of aircraft per ATC sector could be used although the difficulty of assigning monetary values remains. In addition to safety, ATC investments may also be intended to increase aircraft throughput in the air transportation system and reduce delays. These objectives could be quantified in terms of aircraft per hour or minutes of delay, respectively, and monetary values for each of them would have to be assigned.

Determining the probability distribution of the value of completion will depend on the specific selection of the investment objective. If the problem is of a manageable size and it is tractable, queuing models could be used to determine the number of aircraft per sector, aircraft per hour, or delays. Then, a monetary value would have to be assigned to these metrics to determine the value of completion. If queuing models are not feasible, numerical simulations may be necessary. There are different simulators of the U.S. National Airspace System (NAS) that could be used to model the value of completion. One of them is the MIT Extensible Air Network Simulation (MEANS), an event-based simulator of the NAS that has been developed at the Massachusetts Institute of Technology (MIT) to evaluate new concepts in air traffic management and control, and airline scheduling and recovery [Clarke et al., 2005]. MEANS can be used to analyze the individual and the combined effect of different investment projects (e.g., new runways in specific airports, better weather prediction, new ATC automation tools) on the metrics highlighted above. In addition, MEANS can be combined with Monte Carlo simulation to incorporate different sources of uncertainty. As in the case with the queuing models, monetary values would have to be assigned to these metrics to determine the value of completion.

As in the case of investments in airport capacity expansion, completion costs for projects in ATC infrastructure can be determined with numerical cost models; however, since many of these projects tend to be large-scale and infrequent, there are usually many uncertainties associated with estimating their associated parameters and metrics. Therefore, Monte Carlo simulation may be necessary to determine the probability distribution of completion costs.

The identification of a risk-adjusted discount rate to calculate the certainty equivalents of the value of completion and of completion costs as suggested in Section 3.1.1 may be difficult in this type of projects. If the investment under consideration is similar to previous projects in the portfolio of the investor, the discount rate used in the evaluation of those projects could be used. Furthermore, if a public entity is doing the investment, the discount rate could be the rate specified by the government for projects with similar risk and return profiles. Note, however, that if the value of completion results from a complex combination of factors, such as capacity metrics and the quality of life, it may not be possible to express it in monetary terms and, thus, it may be impossible to identify a discount factor in the financial sense.

The generalized real options methodology may be appropriate to find the value of investments in air traffic control if the value of completion can be expressed in monetary terms, which may be difficult depending on the metrics used to measure it. Furthermore, finding an appropriate financial discount rate may be also be problematic, especially if the value of completion can not be given in monetary terms.

6.3 Application of the generalized methodology to other domains

Real options are present in many domains other than air transportation. For example, projects that are subject to market or technical uncertainties, that can be executed in several phases, and that can be implemented with different managerial approaches and/or technologies, are likely to have real options embedded in them. The generalized methodology developed in this thesis is not particular to aviation and, therefore, could be used to evaluate real options in these situations.

Typical examples of projects with real options are investments in power generation plants, roads and bridges, manufacturing plants, research and development (R&D), as well as almost any strategic project that will require rapid response if needed, but that may indeed not be required at a known future time, or at all (see [Clemons and Gu, 2003]). Generally, these undertakings occur over several years and can be separated in stages, are subject to technical and market risks, and require large capital investments, and, thus, are likely to include real options. Possible value of completion for these examples could be expected revenues from user fees (electricity bills for power plants, tolls for roads and bridges), revenues from selling products (for manufacturing plants), or revenues from possible new product lines (for R&D). The probability distributions for the value of completion and the completion cost can be determined with Monte Carlo simulation to include different sources of risk. The generalized formula developed in this thesis can then be used to find the value of these real options regardless of the shape of the probability distributions.

Real options can be found not only in large and complex projects. Options may be present almost anywhere where uncertainties and alternative courses of action exist. Booking a hotel room in advance when plans are still unclear offers the option of securing a better rate than if the reservation is made closer to the arrival date, and since there is usually no penalty for canceling hotel rooms up to a few days before arrival, the option can be very valuable; offering free samples of any given product at a grocery store or a stadium to elicit customer feedback creates the real option to test possible market demand before fully committing to launching a new product. In some of these cases, identifying a suitable underlying asset, the value of completion, or completion cost may be difficult, but if they can be determined, and if the probability distributions for the value of completion and the completion cost can be calculated, the generalized methodology could be used to determine the value of the real option.

6.4 Advancing the real options frontier

As was mentioned above, the presence of a real option does not necessarily mean that it can be evaluated. The generalized methodology developed in this thesis makes a modest contribution to the field of real options by providing an analytical framework to find the value of options for any probability distribution of the value of completion and the cost of completion. Thus, with this methodology, it is not necessary to force the representation of the value of completion into known stochastic processes, such as the commonly-used geometric Brownian motion, or to assume that the completion cost is fixed a-priori. This can allow a better portrayal of the true nature of the real option without sacrificing computational convenience.

Even with these improvements, the generalized methodology is still not enough to find the value of certain type of real options that can be of significant political and socioeconomic importance. The discussion of investments in airport and air traffic control alluded to the fact that when the government is the holder of the option, determining the value of completion may be difficult because it may include elements that are hard to quantify, such as happiness, equity, and general well-being. This is clearly not a problem unique to real options. Any other investment evaluation tool, be it the net present value rule, decision analysis, or any other that is based on quantifying these attributes of human perception, faces the same difficulty. Thus, to the extent that the quantitative representation of these qualities advances, more applications of the generalized real options and other project evaluation methodologies will be possible.

6.5 Chapter summary

The generalized real options methodology developed in this thesis is not specific to the evaluation of investments in aircraft development programs. It can be applied to many other questions within and outside the air transportation field. The main points of this chapter are summarized below:

1) Guidelines were presented to determine when to apply the generalized the real options methodology. These guidelines include determining the presence of managerial flexibility to affect project performance, identifying a suitable underlying asset, and the ability to represent the probability distribution of the value of completion and the completion cost numerically.

2) The generalized methodology gives the user more alternatives to represent the value of completion and the completion cost than traditional real options approaches. This is an important consideration because oftentimes with traditional techniques, there is a trade-off between choosing an analytically/numerically convenient method and being able to closely represent the value of the value of completion and the completion cost.

3) The generalized real options methodology can be used to evaluate investments in airport capacity expansion and air traffic control; however, there are several aspects that must be given careful consideration, such as value of completion and the freedom of managers to act as uncertainties are resolved, to accurately determine the value of the investment.

4) The generalized real options methodology can also be used to evaluate investments in many other domains. In general, projects that are subject to market

or technical uncertainties, that can be executed in several phases, and that can be implemented with different managerial approaches and/or technologies, are likely to have real options embedded in them.

5) There are still practical and theoretical limitations to the use of real options, even with the improvements provided by the generalized methodology presented in this thesis. For example, when the government is the holder of the option, determining the value of completion may be difficult because it may include elements that are hard to quantify, such as happiness, equity, and general well-being.

7. Conclusion

7.1 Summary of main findings and contributions

In this thesis, a generalized real options methodology has been developed to evaluate investments under uncertainty. Given the many technical and market uncertainties affecting the outcome of projects in air transportation and many other domains, a flexible strategy that allows managers to react as new information about the state of the world becomes available is recommended to reduce the risk of such ventures; however, having the real options that offer this flexibility usually comes at a cost. For example, the aircraft development process analyzed in this thesis is the cost that must be paid to have the flexibility to launch a new product at a future date. Thus, if the cost of the flexibility is high or the probability of using it is low, valuation is both more complex and more necessary.

In the last few decades, real options analysis has emerged as a technique to calculate the value of flexibility. The methodology presented in Chapter 3 builds on previous work in this field and is an improvement over existing approaches. Specifically, this methodology allows both the value of completion and the completion cost to be described by any probability distribution. Thus, with this methodology, it is not necessary to force the representation of the underlying asset into known stochastic processes, such as the commonly-used geometric Brownian motion, or to assume that the cost of completing the investment (the completion cost) is fixed a-priori.

If the probability distributions of the value of completion and the completion cost can be given analytically, the generalized methodology can be used to find an exact solution to the problem of options valuation. Otherwise, if the necessary distributions are not known, numerical simulation, such as a combination of system dynamics and Monte Carlo modeling, can be used to determine the probability distributions. This flexibility allows the user to specify the distributions that best represent the value and the costs of the project under investigation.

A practical application of the evaluation methodology was illustrated in Chapter 4 with a new aircraft development program at a major aircraft manufacturer. The analysis presented here differs from previous studies because the value of completion and the completion cost have been calculated with a bottom-up approach as opposed to top-bottom models. To perform the evaluation, the taxonomy of the different real options embedded in the project was determined first. Then, a system dynamics model of the aircraft development process was created. Next, this model was used as the evaluation function within a Monte Carlo simulation to obtain the probability distributions of the value of completion and of the completion cost. The model was calibrated with data provided by a major aircraft manufacturer which, even though it does not correspond to an actual project, it is based on the manufacturer's experience with many programs. Finally, the value of a flexible investment strategy with real options that allows managers to react as uncertainties are resolved was determined using the generalized real options valuation formula.

Numerical results show that a flexible strategy is preferred to one that follows the plan devised from the beginning without consideration of new information that becomes available as the project unfolds. In addition, calculations indicate that options are more valuable in earlier stages of the aircraft development process. Since most expenditures occur in the first phases, by the time the process reaches the certification step, small or no expenditures are outstanding. Thus, according to the assumptions and the data in this particular example, the investor should always proceed with the program once certification is reached as the option is always in the money. Under these circumstances, having the option to stop the process if conditions are not favorable has little value.

Another conclusion from this analysis is that the value of waiting to invest in aircraft development projects tends to decrease as the project moves forward, which is unlike what is observed in typical real options. This observation can be explained by the structure of the options in aircraft manufacturing processes. In typical real options, by spending a small amount to purchase the option, investors are able to defer the majority of the investment to a later time. In these case, the value of waiting to invest is high and, consequently, the value of flexibility tends to increase throughout the life of the option. In the particular case of aircraft manufacturing, however, this does not seem to be the case. As mentioned above, in typical aircraft development programs, much of the expenditures occur in the early stages of the process and they dwindle as the project advances. Therefore, the value of the ability of waiting to invest decreases because less expenditures remain outstanding.

A final comment based on the analysis in Chapter 4 is that the new aircraft program is very risky in earlier stages but, after a certain point (Step C in this example), the project becomes profitable. This suggests that if there are reasons other than profit maximization for having such an aircraft program, such as national security, job creation, or maintenance of a high tech capability, outside intervention in the early stages of the project may be justified to guarantee its viability until it reaches a point of self-sufficiency.

In Chapter 5, the evaluation of a new aircraft development program was re-visited to show how the generalized methodology can be used to evaluate the effect of changing or relaxing some of the fundamental data and structural assumptions on the expected value of a project. Because the methodology does not constraint the probability distributions of the value of completion or the completion cost, investors can use numerical models in which the variables of interest for which no data exists (e.g., market for new aircraft) are calculated based on auxiliary variables for which historical and/or behavioral data exists (e.g., gross domestic product growth rate, typical market share, etc.). Therefore, the generalized methodology can be used to evaluate options even for projects with little or not available historical precedents. This capability was illustrated by introducing a time-varying mean-reverting stochastic representation of the aircraft market based on historical data in the system dynamics model used to determine the value of completion of the real options in the new aircraft development program. Furthermore, the methodology provides a framework to explore the possible effect of alternative investment strategies on expected project value. This was demonstrated by analyzing the following three strategies: one in which investments could be postponed, one in which investments could be restructured, and one that combined both strategies. Numerical results show that, according to the assumptions made in the analysis, any of these strategies can improve the financial performance of the project.

In addition to determining the expected value of the real options, numerical results obtained with the generalized real options methodology can be used in derivative analysis to uncover insights that are useful for strategic decision-making. This information can help managers in the administration of the project as uncertainties are resolved. A typical metric of progress in aircraft development programs is the number of orders received by certain stages. Thus, to demonstrate a practical application of the results from the methodology to guide decision-making, decision rules based on the probability of program success and the expected project value as a function of the number of firm orders and investment strategy were established. As uncertainties are resolved and depending on observed developments in the aircraft market, managers can use these decision rules to guide the project by choosing the recommended strategies for each particular set of conditions.

As with any numerical model, many simplifying assumptions were used in the development of the system dynamics and Monte Carlo simulation to calculate the value of real options in the new aircraft development program. The effect of several of these assumptions were investigated to provide an indication of the limitations of the results presented in this study. The sensitivity of the expected value of the project to non-zero cancellation costs, price mar gin, and the discount rate were tested. Results show that the expected value of the project is very sensitive to these assumptions. In particular, the sensitivity to the assumption of non-zero cancellation costs means that managers have little margin for error if canceling the project comes at a cost.

The results summarized above indicate that changes in new aircraft development projects, such as postponing or restructuring investments, may lead to improvements in their financial performance. Based on the insight from this analysis, a series of recommendations for change in the structure of typical aircraft development programs have been derived. In particular, the use of shared production resources, the diversification of production through derivative aircraft, and outside intervention to support cancellation costs are suggested as they may strengthen the financial viability of these projects; however, change is typically not easy to implement because it requires a cultural commitment to modifications, revisions, and adaptation that may not always be present. Thus, in order to institute change, it is often necessary to systematically evaluate trade-offs and alternative courses of action so that managers can make informed decisions. As discussed in the last section of Chapter 5, the generalized real options methodology is a tool that allows decision-makers to make these types of analysis.

Finally, in Chapter 6, the applicability of the generalized real options methodology to other examples was briefly discussed. First, conditions to test the applicability of the methodology to evaluate investments were presented. Then, two examples from the air transportation field (investments in airport expansion and air traffic control infrastructure) were analyzed in view of these conditions. Next, the application of the methodology to examples in other domains was highlighted, and difficulties that inhibit the widespread use of this methodology were briefly discussed. A main point in this chapter is that the generalized methodology gives the user more alternatives to represent the value of completion and the completion cost than traditional real options approaches. This is important because frequently with traditional techniques, there is a trade-off between choosing an analytically/numerically convenient method and being able to closely represent the value of completion and the completion cost. The methodology in this thesis was developed to render this trade-off unnecessary.

7.2 Recommendations for future work

The analysis of the new aircraft development program analyzed in this thesis could be enhanced in a number of ways. For example, a technical uncertainty reduction process could be added to the system dynamics model to simulate the effect of resources spent in development on the technical evolution of the product. This could lead to insights that enable better decisions with respect to the investment schedule in development. In particular, this capability could be used to estimate the extra cost of development to ensure that performance targets and delivery schedules are met. This information could be used to explore the trade-off between higher expenditures in development and reductions in time-to-market to reduce the risk of competitors entering the market earlier.

Another suggestion is to expand the analysis by considering the aircraft program as an asset in the manufacturer's portfolio of other programs. Typically, aircraft manufacturers have a number of different programs in different stages of development. Thus, the analysis of one program could be considered within this context to determine, from a portfolio perspective, the optimal investment schedule that results in highest portfolio returns per unit of risk.

Furthermore, more work is required to evaluate real options in projects where the underlying asset may be tied to elements that are hard to quantify, such as happiness, equity, and general well-being. These includes investments in the public sector, such as infrastructure, education, and social security, which are of great importance and can impact many people.

Finally, a major motivation for writing this thesis was the desire to investigate deeper some common assumptions in real options analysis. The use of real options is just another investment evaluation tool like the Net Present Value (NPV) rule and Decision Analysis (DA). Each of these techniques has its advantages and disadvantages but the selection of one over the other should be driven by the applicability to the problem under consideration. The work presented in this thesis is expected to contribute to a better understanding of the potential and limitations of real options to evaluate investments under uncertainty.

8. References

Amram, Martha and Nalin Kulatilaka (1999). *Real Options: Managing Strategic Investment in an Uncertain World.* Harvard Business School Press, Boston, Massachusetts.

Air Transportation Association (2004). "Annual Traffic and Capacity: US airlines – scheduled traffic." Available at: http://www.airlines.org/econ/d.aspx?nid=1032

Black, Fischer and Scholes, Myron (1973). "The Pricing of Options and Corporate Liabilities," *Journal of Political Economy.* Vol. 81.

The Boeing Co. (2004). "Orders & Deliveries." Available at: http://active.boeing.com/commercial/orders/index.cfm

Brealey, Richard A. and Myers, Stewart C. (1996). *Principles of Corporate Finance.* 5th Edition. McGraw-Hill, New York.

Childs, Paul D., Riddiough, Timothy J. and Triantis, Alexander J. (1996). "Mixed Uses and the Redevelopment Option." *Real Estate Economics.* Vol. 24, No. 3, pp. 317-339.

Chriss, Neil A. (1997). *Black-Scholes and Beyond: Option Pricing Models.* Irwin, Chicago, Illinois.

Clarke, J.-P., Melconian, T., Bly, E., and Rabbani, F. (2005) "MEANS - The MIT Extensible Air Network Simulation," *Simulation: Transactions of the Society International for Computer Simulation.* Accepted for Publication.

Clemons, Eric K. and Gu, Bin (2003). "Justifying Information Technology Investments: Balancing the Need for Speed of Action With Certainty Before Action." *Proceedings of the 36th Hawaii International Conference on System Sciences.* Honolulu, Hawaii.

Copeland, Tom and Vladimir Antikarov (2001). *Real Options: A Practitioner's Guide.* Texere, New York.

Cox, John, Ross, Stephen A. and Rubinstein, Mark (1979). "Option Pricing: A Simplified Approach," *Journal of Financial Economics.* Vol. 7, pp. 229-263.

Datar, Vinay and Matthews, Scott (2004). "European Real Options: An Intuitive Algorithm for the Black-Scholes Formula," *Journal of Applied Finance.* Spring/Summer 2004.

de Neufville, Richard and Neely, James (2001). "Hybrid Real Options Valuation of Risky Product Development Projects." *International Journal of Technology, Policy and Management,* Vol. 1, No. 1, pp. 29-46.

de Weck, Olivier, de Neufville, Richard and Chaize, Mathieu (2003). "Enhancing the Economics of Communication Satellites via Orbital Reconfigurations and Staged Deployment," presented at the *Space 2003* conference, Long Beach, CA, Sep. 23-25, AIAA Paper 2003-6317.

Dixit, Avinash K. and Pindyck, Robert S. (1994). *Investment under Uncertainty.* Princeton University Press, Princeton, New Jersey.

Dow Jones Business News (2004). "UPS enters talks to cancel $1.6 billion in Airbus orders." On-line edition, March 1st, 2004.

Esty, Benjamin C. and Ghemawat, Pankaj (2002). *Airbus vs. Boeing in Superjumbos: A Case of Failed Preemption.* Harvard Business School working paper No. 02-061. Available at: http://www.people.hbs.edu/pghemawat/pubs/Airbus_vs_Boeing.pdf

Fisher, Stanley (1978). "Call Option Pricing when the Exercise Price is Uncertain, and the Valuation of Index Bonds," *Journal of Finance.* Vol. 33, No. 1.

Gamba, Andrea (2002) *Real Options Valuation: A Monte Carlo Approach.* Working Paper Series 2002/2003, Faculty of Management, University of Calgary.

Geltner, David (1989). "On the Use of the Financial Option Price Model to Value and Explain Vacant Land." *AREUEA Journal,* Vol. 17, No. 2, pp. 142-158.

Grayson, Richard Andrew (2001). *Corporate Restructuring and Real Option in the U.S. Aerospace and Defense Industry.* Doctoral dissertation, University of Georgia, United States of America.

Greden, L. and Glicksman, L. (2005). "A Real Options Model for Valuing Flexible Space." *Journal of Corporate Real Estate.* Vol. 7, No. 1, pp. 34-48.

Greden, L., Glicksman, L., and Lopez-Betanzos, G. (2005). A Real Options Methodology for Evaluating Risk and Opportunity of Natural Ventilation. *Proceedings of 2005 International Solar Energy Conference.* August 6-12, 2005, Orlando, Florida. pp. 1-10.

Hausman, Jerry and Myers, Stewart (2002). "Regulating the United States Railroads: The Effects of Sunk Costs and Asymmetric Risk." *Journal of Regulatory Economics,* Vol. 22, No. 3, pp. 287-310.

Hull, John (1995). *Introduction to Futures and Options Markets.* New Jersey: Prentice Hall. 2nd Edition.

Hull, John C. (2000). *Options, Futures and Other Derivatives.* Upper Saddle River, New Jersey: Prentice Hall. 4th Edition.

Jenkinson, Lloyd R., Simpkin, Paul and Rhodes, Darren (1999). *Civil Jet Aircraft Design.* AIAA Educational Series. American Institute of Aeronautics and Astronautics, Inc., Reston, Virginia.

Joppin, Carole and Hastings, Daniel (2003). "Evaluation of the Value of the Flexibility Offered by On-orbit Servicing - Case of Satellite Upgrade," presented at the *Space 2003* conference, Long Beach, CA, Sep. 23-25, AIAA Paper 2003-6366.

Kulatilaka, Nalin (1993). "The Value of Flexibility: The Case of a Dual-Fuel Industrial Steam Boiler." *Financial Management.* Autumn issue.

Lamassoure, Elisabeth, Saleh, Joseph H. and Hastings, Daniel (2002). "Space Systems Flexibility Provided by On-Orbit Servicing: Part 2," *Journal of Spacecraft and Rockets.* Vol. 39, no. 4, July-August 2002, pp. 561-570.

Longstaff, Francis A. and Schwartz, Eduardo S. (2001). "Valuing American Options by Simulation: A Simple Least-Squares Approach," *Journal of Financial Studies.* Spring 2001, Vol. 14, No. 1, pp. 113-147.

Markish, Jacob and Willcox, Karen (2002). "Multidisciplinary Techniques for Commercial Aircraft System Design," presented at the *9th AIAA/ISSMO Symposium on Multidisciplinary Analysis and Optimization.* AIAA Paper 2002-5612

Matthews, Scott (2004). "Valuing Real Options at Boeing." Presented at the *8th Annual International Conference on Real Options* in Montreal, Canada.

Mavris, Dimitri N. and Fernández, Ismael (2003). "Integration of Real Options Analysis Into an Engine Development Decision Support System," presented at the *AIAA's 3rd Annual Aviation Technology, Integration and Operations* (ATIO) Forum, Denver, Colorado. AIAA Paper 2003-6734.

McDonald, Robert and Siegel, Daniel (1985). "Investment and the Valuation of Firms When There is an Option to Shut Down," *International Economic Review.* Vol. 26, No. 2, pp. 331-349.

McDonald, Robert and Siegel, Daniel (1986). "The Value of Waiting to Invest." *The Quarterly Journal of Economics,* Vol. 1, No. 4, pp. 707-728.

Merton, Robert C. (1973). "Theory of Rational Option Pricing," *Bell Journal of Economics and Management Science.* Vol. 29, Spring 1973, pp. 141-183.

Miller, Bruno and Clarke, John-Paul (2005) "Investments Under Uncertainty in Air Transportation: A Real Options Perspective." *Journal of the Transportation Research Forum.* Accepted for publication.

Myers, Stewart (1977). "Determinants of Corporate Borrowing." *Journal of Financial Economics,* Vol. 5, pp. 147-75.

Myers, Stewart and Shyam-Sunder, Laskshmi (1996). "Measuring Pharmaceutical Risk and the Cost of Capital," Robert B. Helms ed. *Competitive Strategies in the Pharmaceutical Industry.* AEI Press, Washington, D.C.

Myers, Stewart C. (2003). *Real Options Seminar*, taught at the Massachusetts Institute of Technology. Spring semester.

Myers, Stewart, G. Y. Billiard Professor of Finance, Sloan School of Management, Massachusetts Institute of Technology (2004). Personal communication.

Newhouse, John (1982). *The Sporty Game.* New York: Alfred A. Knopf.

Paddock, J.L., Siegel, D. R. and Smith, J. L. (1988). "Option Valuation of Claims on Real Assets: The Case of Offshore Petroleum Leases," *Quarterly Journal of Economics,* August 1988, pp. 479-508.

Pavlova, Anna and Vayanos, Dimitri (2002). *Finance Theory I – Lecture Notes.* Massachusetts Institute of Technology. Cambridge, Massachusetts.

Robinson, William T. and Min, Sungwook (2002). "Is the First to Market the First to Fail? Empirical Evidence for Industrial Goods Business," *Journal of Marketing Research,* February, 2002, Vol. XXXIX, pp. 120-128.

Saleh, Joseph H., Lamassoure, Elisabeth and Hastings, Daniel (2002). "Space Systems Flexibility Provided by On-Orbit Servicing: Part 1," *Journal of Spacecraft and Rockets.* Vol. 39, no. 4, July-August 2002, pp. 551-560.

Shackleton, Mark B., Tsekrekos, Andrianos E. and Wojakowski, Rafal (2004) "Strategic entry and market leadership in a two-player real options game," *Journal of Banking and Finance.* Vol. 28, no. 1, January 2004, pp. 179-201.

Schaufele, Roger (2000). *The Elements of Aircraft Preliminary Design.* Aries Publications, Santa Ana, California.

Sharpe, William F. and Alexander, Gordon J. (1991). *Investments.* Prentice Hall, New Jersey. 4th edition.

Skinner, Steve, Dichter, Alex, Langley, Paul and Sabert, Hendrik (1999). "Managing Growth and Profitability across Peaks and Troughs of the Airline Industry Cycle – An

Industry Dynamics Approach," G. Butler and M. Keller eds. *Handbook of Airline Finance.* MacGraw-Hill, New York.

Summa, John F. and Lubow, Jonathan W. (2002). *Options on Futures: New Trading Strategies.* John Wiley and Sons, New York.

Stonier, John E. (1999). "What is an Aircraft Purchase Option Worth? Quantifying Asset Flexibility Created through Manufacturer Lead-Time Reductions and Product Commonality." G. Butler and M. Keller eds. *Handbook of Airline Finance.* MacGraw-Hill, New York.

Trigeorgis, Lenos (1996). *Managerial Flexibility and Strategy in Resource Allocation.* MIT Press, Cambridge, Massachusetts.

Tufano, Peter and Moel, Alberto (1997). *Bidding for Antamina*, Harvard Business School Case No. 9-297-054. Rev. Sept. 15.

USDOT (1979-2001), *Form 41*, US Department of Transportation, Bureau of Transportation Statistics, Office of Airline Information.

Weigel, Annalisa and Hastings, Daniel (2004). "Measuring the value of designing for uncertain future downward budget instabilities," *Journal of Spacecraft and Rockets.* Vol. 41, no. 1, Jan. 2004, p. 111-119.

9. Appendix A: Discrete-time contingent claims analysis

A popular and mathematically convenient approach for evaluating financial options is discrete-time is contingent claims analysis. In this section, the technique of contingent claims analysis is presented, starting with a simple one-period example and continuing with a multiple period valuation. Then, the use of risk-neutral probabilities is presented. Finally, the relationship between the discrete and continuous contingent-claims analysis is discussed briefly.

9.1 A one-period example[7]

It is useful to start with a one-period example to understand the valuation of options using discrete-time contingent claims analysis. Assume that the price, S, of a stock follows a multiplicative binomial process over discrete periods [Cox et al., 1979]. At the end of each period, the stock price has a probability p of going "up" with a rate of return u-1, or it has a probability (1-p) of going "down" with a rate of return d-1 (see Figure 9-1). Thus, at the end of the period, the stock price is either Su = S*u or Sd = S*d, with probabilities p and (1-p), respectively. For example, consider a share, S, of company ABC that is worth $36 today. If u = 1.2 and d = 0.9, the value of S at the end of the period can be either Su = $36 * 1.2 = $43.2 or Sd = $36 * 0.9 = $32.4 (see Figure 9-1).

[7] The discussion of the single-period and multiple-period examples are based on [Cox, Ross and Rubinstein, 1979] and [Pavlova and Vayanos, 2002]

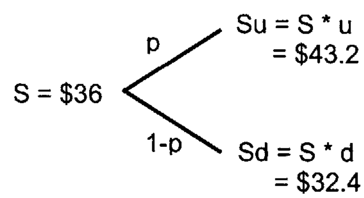


Figure 9-1: One-period, multiplicative binomial process of the stock price, S, of a share of company ABC.

Assume now that European call options on this stock have a strike price of X = $35 and a maturity of one period. Remember that the payoff of a call option at maturity is the maximum of the difference between the stock price minus the strike price or zero. Consequently, if the stock price is "up," the payoff is given by Cu = max[Su-X, 0] and if the stock price is "down," the payoff is Cd = max [Sd – X, 0] (see Figure 9-2). In the example of ABC's call option, the results for Cu and Cd are: Cu = $43.2 - $35 = $8.2 and Cd = $0.

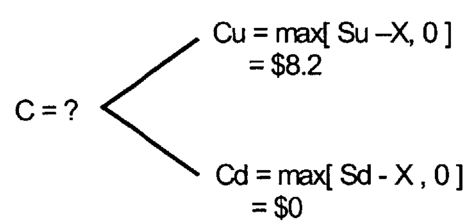


Figure 9-2: Payoffs of a European call option on ABC's stock with strike price X = $35.

Now, the only unknown left to determine is the value, C, of the call option. To do so, it is assumed that markets are complete and, therefore, it is possible to set up a replicating portfolio that replicates the payoffs of the call option on ABC's stock. Then, in the absence of arbitrage, the market value of the replicating portfolio must equal the price of the option.

A replicating portfolio consisting of a certain amount Δ of shares (also known as the hedge ratio or option delta) and a certain amount B of government bonds that pay \$100 at maturity[8] is constructed (see Table 9-1):

Table 9-1: Replicating portfolio for the call option on ABC shares.

	Replicating portfolio	Call option
Stock price goes up	Δ Su + B \$100 = Δ \$43.2 + B \$100	Cu = max [Su – X,0] Cu = max [\$8.2, 0] Cu = \$8.2
Stock price goes down	Δ Sd + B \$100 = Δ \$32.4 + B \$100	Cd = max [Sd – X,0] Cd = max [-\$2.6, 0] Cd = \$0

If the stock price goes up, the option gets exercised for a payoff of \$8.2. If the stock price goes down, the option does not get exercised and the payoff is \$0. In order to find the amount of shares and bonds (Δ and B, respectively) needed to establish this replicating portfolio, the payoff of the replicating portfolio and the call option must match in both cases (see Equations A-1 and A-2):

$$\Delta\ Su + B\ \$100 = Cu \qquad \text{(Eq. A-1)}$$

$$\Delta\ Sd + B\ \$100 = Cd \qquad \text{(Eq. A-2)}$$

[8] Investing in government bonds is similar to putting money in the bank. When you buy a bond, you are promised a given return at the bond's maturity. Since the risk of default of the U.S. government is practically non-existent, bonds are considered a risk-free investment, and, consequently, the bond's rate of return is considered the risk-free interest rate, r_f (technically, this is true only for short-term maturities, since inflation and other factors may affect the *real* return of longer-maturity bonds) [Brealey and Myers, 1996].

Equations A-1 and A-2 can be re-arranged to express Δ and B in terms of the current stock price, S, the up and down stock movements, u and d, respectively, the option payoffs, Cu and Cd, and the face value of the bond (see Equations A-3 and A-4):

$$\Delta = \frac{Cu - Cd}{(u-d)\cdot S} \qquad \text{(Eq. A-3)}$$

$$B = \frac{uCd - dCu}{(u-d)\cdot 100} \qquad \text{(Eq. A-4)}$$

Substituting in the values for Cu, Cd, u, d and S, the following results are obtained: Δ = 0.76 and B = -0.246, i.e., the replicating portfolio consists of buying 0.76 shares of the stock and selling –0.246 units of $100 government bonds. The intuition behind this is that the investor is borrowing money (selling bonds is the same as borrowing) to partially finance the purchase of shares and create a protected or *hedged* position [Brealey and Myers, 1996]:

- If the stock price goes up to Su = $43.2, the investors sells her shares to obtain 0.76*$43.2 = $32.8, out of which she pays her debt of 0.246*$100 = $24.6 and keeps a profit of $8.2! (the net profit is $8.2 minus the cost of the option, C).

- If the stock price goes down to Sd = $32.4, the investor sells her shares to obtain 0.76*$32.4 = $24.6, which is enough to pay the debt of 0.246*$100 = $24.6. In this case, the only loss the investor incurs is whatever amount C she paid for the call option in the first place.

In the absence of arbitrage, the value of the replicating portfolio must be the same as the price of the call option. The price of the replicating portfolio is the sum of the present value of the position in stocks and in bonds (see Equation A-5):

$$C = \Delta \cdot S + B \cdot \frac{100}{1 + r_f} \qquad \text{(Eq. A-5)}$$

The present value of the bond is calculated by discounting its face value at maturity ($100) with the 1-period risk-free interest rate, r_f. Assuming $r_f = 5\%$, the price of the replicating portfolio and, therefore, the price of the call option, C, is:

$$C = 0.76 \cdot \$36 - 0.246 \cdot \frac{100}{1 + 5\%} = \$3.9$$

On a technical note, it is important to point out that to avoid arbitrage, it must be true that $d\text{-}1 < r_f < u\text{-}1$ [Pavlova and Vayanos, 2002]. If this were not the case, it would always be possible to obtain an arbitrage profit. For example, if $r_f < d\text{-}1$, one would sell bonds and buy stock. At maturity, the return on the stock (d-1) would always be greater than the return on the bonds (r_f), thus, one could always have a riskless return. The importance of this point will become apparent when discussing risk-neutral probabilities.

9.2 Binomial process and multiple periods

The one-period example is useful to illustrate the concept of replicating portfolios, but it is somewhat limited because of the assumption that the stock price can only take two possible end values. This can be expanded to multiple periods so that more final states for the final payoffs are possible. Assuming that the stock price follows a binomial process, i.e., at each time interval its value can go up or down by u and d, respectively, a *binomial tree* as shown in Figure 9-3 can be built to model the behavior of stock S over time:

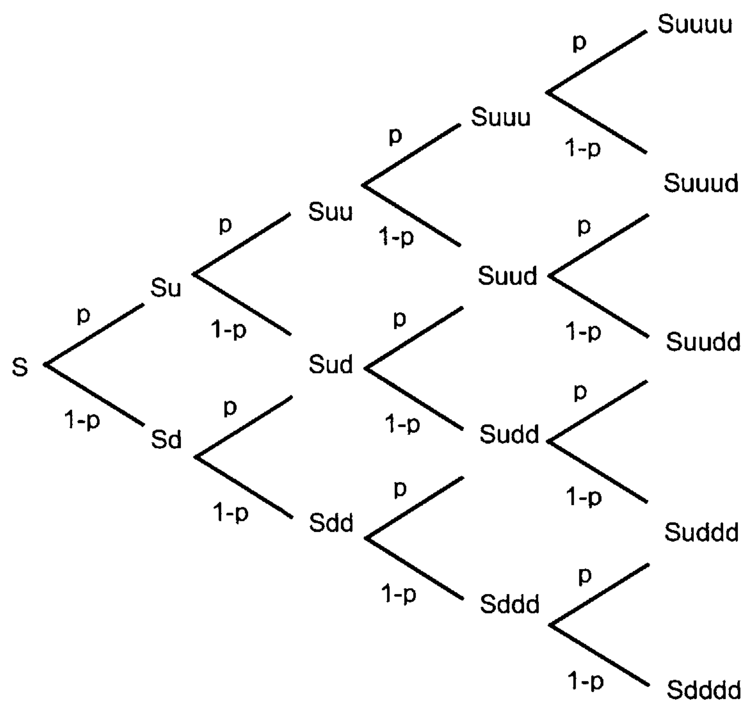


Figure 9-3: Multiple-period binomial tree model of stock price S.

Note that the tree shown in Figure 9-3 is recombining, i.e., the branches come back to the same point. For example, state Sud can be reached by first going up to Su and then down to Sud. Alternatively, Sud can be reached by first going down to Sd and then up to Sud. An important feature of a recombining tree is that as the number of periods increases, the distribution of outcomes at the end branches approach a continuous distribution [Copeland and Antikarov, 2001]. In fact, if u and d are carefully selected, this distribution would approach a lognormal distribution, which is a reasonable approximation for the distribution of stock prices as they can not go negative [Dixit and Pindyck, 1994] (more details are given below). Cox, Ross and Rubinstein (1979) show that as the number of periods, n, approach infinity, a continuous-time lognormal distribution can be approximated if u and d are given by Equations 9 and 10 [Cox et al., 1979] (see below):

$$u = e^{\sigma \cdot \sqrt{t/n}} \quad \text{(Eq. A-6)}$$

$$d = \frac{1}{u} = e^{-\sigma \cdot \sqrt{t/n}} \quad \text{(Eq. A-7)}$$

where t is a fixed period of time to maturity and σ is the standard deviation of stock returns.

The price of the option in the multiple-period example can be determined with the same replicating portfolio approach used in the single-period example. To illustrate this, consider a European call option on a stock, S, of company DEF with strike price X = $28 and maturity of 2 years.

Figure 9-4 shows a two-period binomial tree model of S. The starting price is $30 and at each interval, it can go up by u = 1.2 or down by d = 0.9. At the end of the two periods, the possible end-states for S are three: Suu = $43.2, Sud = $32.4 and Sdd = $ 24.3. The 1-year interest rate, r_f, is again assumed to be 5%.

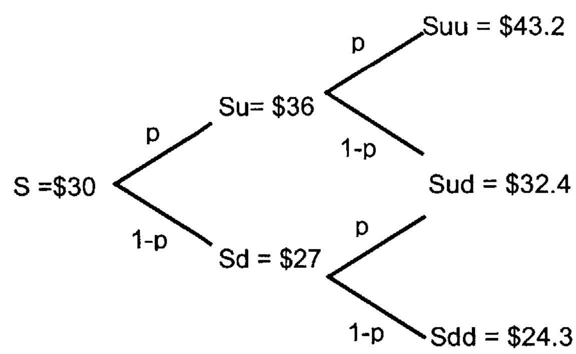


Figure 9-4: Two-period model for the stock price, S, of company DEF.

Once the end values of the stock price are known, it is possible to calculate the possible option payoffs. These are the maximum of the difference between the stock price minus the strike price or zero. For this example, the possible option payoffs are Cuu = $15.2, Cud = $4.4 and Cdd = $0 (see Figure 9-5). Notice that if the stock price reaches the lowest state Sdd = $24.3, the option is not exercised because doing so would result in a financial loss.

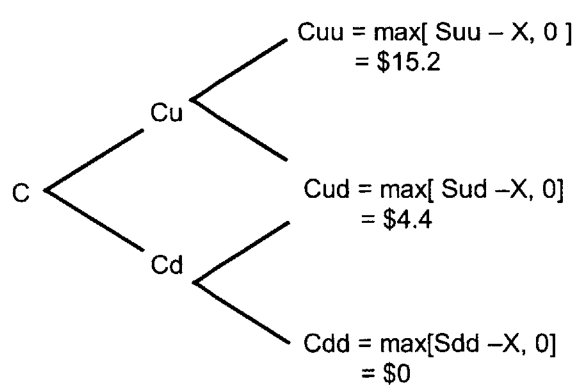


Figure 9-5: Payoffs of a European call option on DEF's stock with strike price X = $28 at maturity.

To find the price, C, of this call option, it is necessary to find the value of the option at each of the nodes by starting at the end of the tree and moving backwards. Thus, in this example, we need to calculate Cu and Cd to determine C:

1) Determine Cu. First, find a replicating portfolio of Δ_u shares and B_u bonds that replicate the option at this node. Then, calculate Cu:

$$\Delta_u = \frac{Cuu - Cud}{(u-d)\cdot Su} = \frac{15.2 - 4.4}{(1.2-0.9)\cdot 36} = 1$$

$$B_u = \frac{uCud - dCuu}{(u-d)\cdot 100} = \frac{1.2\cdot 4.4 - 0.9\cdot 15.2}{(1.2-0.9)\cdot 100} = -0.28$$

$$Cu = \Delta_u \cdot S_u + B_u \cdot \frac{100}{1+r_f} = 1\cdot 36 - 0.28\cdot \frac{100}{1+5\%} = \$9.33$$

2) Determine Cd. Again, find a replicating portfolio of Δ_d shares and B_d bonds that replicate the option at this node. Then, calculate Cd:

$$\Delta_d = \frac{Cud - Cdd}{(u-d)\cdot Sd} = \frac{4.4 - 0}{(1.2-0.9)\cdot 27} = 0.54$$

$$B_d = \frac{uCdd - dCud}{(u-d)\cdot 100} = \frac{1.2\cdot 0 - 0.9\cdot 4.4}{(1.2-0.9)\cdot 100} = -0.13$$

$$Cd = \Delta_d \cdot S_d + B_d \cdot \frac{100}{1+r} = 0.54 \cdot 27 - 0.13 \cdot \frac{100}{1+5\%} = \$2.2$$

3) Determine C. With Cu and Cd known, find a replicating portfolio of Δ shares and B bonds that replicate the call option at this node. Then, calculate C:

$$\Delta = \frac{Cu - Cd}{(u-d) \cdot S} = \frac{9.33 - 2.2}{(1.2 - 0.9) \cdot 30} = 0.80$$

$$B = \frac{uCd - dCu}{(u-d) \cdot 100} = \frac{1.2 \cdot 2.2 - 0.9 \cdot 9.33}{(1.2 - 0.9) \cdot 100} = -0.19$$

$$C = \Delta \cdot S + B \cdot \frac{100}{1+r_f} = 0.80 \cdot 30 - 0.19 \cdot \frac{100}{1+5\%} = \$5.91$$

Notice that the hedge ratio, i.e., the amount of shares held (Δ, Δ_u, Δ_d), changes as the process moves along the tree. This is necessary in order to assure that the replicating portfolio matches the risk and return profile of the stock, which also changes as we traverse the tree. If the hedge ratio would not be adjusted continuously, the replicating portfolio would no longer be hedged appropriately and the valuation of the stock option would not be correct.

9.3 Evaluation of options with risk-neutral probabilities

Notice that in neither the single-period nor in the multiple-period examples shown above is there any mention of risk or risk preferences. The probabilities of an "up" or a "down" jump in the stock price, i.e., p or (1-p), respectively, do not appear in the option valuation equation (Equation A-5). Because of the no-arbitrage condition, the investor is indifferent between holding the replicating portfolio or the call option: she or he knows exactly what the price of the call should be and needs no premium to compensate for risk [Brealey and Myers, 1996]. Therefore, if risk preferences do not matter, a world where investors are risk-neutral and the expected return on all securities is the risk-free interest rate, r_f [Hull, 2000] can be assumed. In such a world, the correct discount rate to calculate any present value would be r_f.

In order to illustrate these concepts, it is helpful to re-arrange some of the terms in Equation A-5:

$$C = \Delta \cdot S + B \cdot \frac{100}{1+r_f} = \frac{Cu - Cd}{(u-d)\cdot S} \cdot S + \frac{uCd - dCu}{(u-d)\cdot 100} \frac{100}{(1+r_f)}$$

$$C = \frac{(1+r)-d}{u-d} \cdot \frac{Cu}{1+r_f} + \frac{u-(1+r)}{u-d} \cdot \frac{Cd}{1+r_f}$$

Thus, Equations A-8 and A-9 are obtained:

$$C = \frac{q \cdot Cu + (1-q) \cdot Cd}{1+r_f} \qquad \text{(Eq. A-8)}$$

where,

$$q = \frac{(1 + r_f) - d}{u - d} \qquad \text{(Eq. A-9)}$$

As was stated above, in order to avoid arbitrage opportunities, it must be true that $d-1 < r_f < u-1$; therefore, q must always be between 0 and 1, which are properties of a probability. The interpretation is that q is the value that the probability p of an "up" movement in a risk-averse world would have, in equilibrium, in a risk-neutral world [Cox et al., 1979]. Because of this property, q is usually called the "risk neutral probability" (RNP) [Trigeorgis, 1996]. Thus, if a risk-neutral world is assumed, the value of the call can be interpreted as the expected value of its future cash flows (Cu and Cd), weighted by the RNPs (q and 1-q, respectively), and discounted with the risk-free rate (r_f) [Cox et al., 1979; Trigeorgis, 1996] (see Equation A-8, above).

Note that value of the call option obtained with RNPs must be the same as the value obtained with the replicating-portfolio approach. For example, consider the one-period example presented in Section 5.1. There, we had u=1.2, d=0.9, Cu = $8.2, Cd = $0, r_f = 5% and the value of the call was C = $3.9. The first step with RNPs is to calculate the value of q with Equation A-9:

$$q = \frac{(1 + r_f) - d}{u - d} = \frac{(1 + 5\%) - 0.9}{1.2 - 0.9} = 0.5$$

The second step is to use Equation A-8 to determine the value of the call C_{RNP}, which is, indeed, equal to C = $3.9:

$$C_{RNP} = \frac{q \cdot Cu + (1-q) \cdot Cd}{1 + r_f} = \frac{0.5 \cdot \$8.2}{1 + 5\%} = \$3.9 = C$$

The use of risk-neutral probabilities is a convenient approach for calculating the value of options. The important points to remember are:

a) The actual probabilities of stock movements in the risk-averse world must be transformed to risk-neutral probabilities that are valid in a risk-neutral world.

b) The appropriate discount rate in the risk-neutral world is the risk-free discount rate, r_f.

9.4 Relationship between the discrete and continuous cases

Both the discrete-time and continuous-time approaches for evaluating financial options are based on the same assumptions of market completeness and no-arbitrage. In addition, both use the stock price as the underlying asset and both model its evolution as a lognormal distribution using geometric Brownian motion. Thus, as should be expected, both techniques are intimately related. In fact, the discrete-time methodology approaches the continuous-time approach as the number of time intervals go to infinity [Cox et al., 1979; Brealey and Myers, 1996].

10 Appendix B: Analytical solution of the generalized real options formula

In this Appendix, the derivation of analytical solutions for the generalized real options formula are presented. Two examples are considered: first, the value of completion and the completion cost are given by uniform probability distributions, and, second, the value of completion is given by an exponential distribution and the completion cost is described by a uniform distribution.

Before presenting the details of the analytical solutions, re-write Equations 3-12 and 3-13 as follows:

$$w = E[O(c)] = e^{-r_f T}\left(\int_{c=0}^{\infty} f_c(c)\int_{v=c}^{\infty} v\cdot f_v(v)dvdc - \int_{c=0}^{\infty} c\cdot f_c(c)\cdot\int_{v=c}^{\infty} f_v(v)dvdc\right) = T_1 - T_2$$

where,

$$T_1 = e^{-r_f T}\int_{c=0}^{\infty} f_c(c)\int_{v=c}^{\infty} v\cdot f_v(v)dvdc$$

$$T_2 = e^{-r_f T}\int_{c=0}^{\infty} c\cdot f_c(c)\cdot\int_{v=c}^{\infty} f_v(v)dvdc$$

and

$$Var(O(c)) = E[O(c)^2] - (E[O(c)])^2 = e^{-2r_f T}\left(\int_{c=0}^{\infty} f_c(c)\left[\int_{v=c}^{\infty} v\cdot f_v(v)dv - c\cdot\int_{v=c}^{\infty} f_v(v)dv\right]^2 dc\right) - (E[O(c)])^2$$

$$= e^{-2r_f T}\left(\int_{c=0}^{\infty} f_c(c)\left(\int_{v=c}^{\infty} v\cdot f_v(v)dv\right)^2 dc - 2\int_{c=0}^{\infty} c\cdot f_c(c)\left(\int_{v=c}^{\infty} v\cdot f_v(v)dv\cdot\int_{v=c}^{\infty} f_v(v)dv\right)dc + \int_{c=0}^{\infty} f_c(c)\cdot\left(c\int_{v=c}^{\infty} f_v(v)dv\right)^2\right) - (E[O(c)])^2$$

$$= VT_1 - 2VT_2 + VT_3 - (T_1 - T_2)^2$$

where,

$$VT_1 = e^{-2r_fT} \int_{c=0}^{\infty} f_c(c) \left(\int_{v=c}^{\infty} v \cdot f_v(v) dv \right)^2 dc$$

$$VT_2 = e^{-2r_fT} \int_{c=0}^{\infty} c \cdot f_c(c) \left(\int_{v=c}^{\infty} v \cdot f_v(v) dv \cdot \int_{v=c}^{\infty} f_v(v) dv \right) dc$$

$$VT_3 = e^{-2r_fT} \int_{c=0}^{\infty} f_c(c) \cdot \left(c \int_{v=c}^{\infty} f_v(v) dv \right)^2$$

and T_1 and T_2 are as given above.

10.1 Example 1: Two uniform distributions

Let the probability distribution of the certainty equivalent of value of completion, $f_s(s)$, and of the completion cost, $f_x(x)$, be defined as follows:

$$f_v(v) = \begin{cases} 0 & v < v_1 \\ u_v & \text{for } v = [v_1; v_2] \\ 0 & v > v_2 \end{cases} \qquad f_c(c) = \begin{cases} 0 & c < c_1 \\ u_c & \text{for } c = [c_1; c_2] \\ 0 & c > c_2 \end{cases}$$

where u_v and u_c are constants.

The real options valuation formula rests on the principle that the value of completion must be higher than the completion cost, otherwise the option is not exercised and its value is zero. Thus, careful attention must be paid to the limits of integration to ensure that the running variable s is larger than x. In order to facilitate the integration, it is helpful to consider 5 cases:

- Case 1: $v_1 > c_2$

Here, all values of v are larger than all values of c (see Figure 10-1):

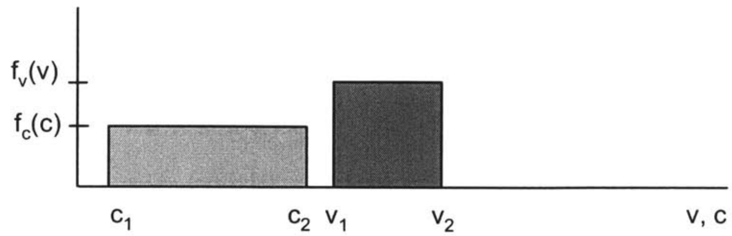


Figure 10-1: Integration Case 1: all values of v are larger than all values of c.

The limits of integration are $[c_1;c_2]$ for c and $[v_1;v_2]$ for v. The integration results are:

$$T_1\big|_{Case1} = e^{-r_f T}\left(\int_{c_1}^{c_2} f_c(c)\int_{v_1}^{v_2} v\cdot f_v(v)dvdc\right) = e^{-r_f T}\left(u_c u_v\left(\frac{v_2^2 - v_1^2}{2}\right)(c_2 - c_1)\right)$$

$$T_2\big|_{Case1} = e^{-r_f T}\left(\int_{c_1}^{c_2} c\cdot f_c(c)\int_{v_1}^{v_2} f_v(v)dvdc\right) = e^{-r_f T}\left(u_v\left(\frac{c_2^2 - c_1^2}{2}\right)(v_2 - v_1)\right)$$

$$VT_1\big|_{Case1} = e^{-2r_f T}\left(\int_{c_1}^{c_2} f_c(c)\left(\int_{v_1}^{v_2} v\cdot f_v(v)dv\right)^2 dc\right) = e^{-2r_f T}\left(u_c u_v^2\left(\frac{v_2^2 - v_1^2}{2}\right)^2(c_2 - c_1)\right)$$

$$VT_2\big|_{Case1} = e^{-2r_f T}\left(\int_{x_1}^{x_2} x\cdot f_x(x)\left(\int_{s_1}^{s_2} s\cdot f_s(s)ds\cdot\int_{s_1}^{s_2} f_s(s)ds\right)dx\right) = e^{-2r_f T}\left(u_x u_s\left(\frac{s_2^2 - s_1^2}{2}\right)\left(\frac{x_2^2 - x_1^2}{2}\right)\right)$$

$$VT_3\big|_{Case1} = e^{-2r_f T}\left(\int_{x_1}^{x_2} f_x(x)\cdot\left(x\int_{s_1}^{s_2} f_s(s)ds\right)^2\right) = e^{-2r_f T}\left(u_x\left(\frac{x_2^3 - x_1^3}{3}\right)\right)$$

- Case 2: $v_1 < c_2 < v_2$, $c_1 < v_1$

In this case, the distribution of the completion cost overlaps partially the distribution of the value of completion from the right-hand side (see Figure 10-2):

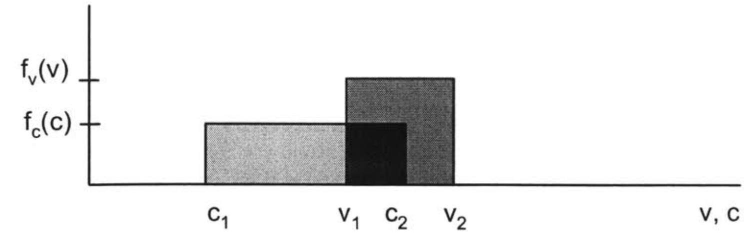


Figure 10-2: Integration Case 2: the distribution of completion costs overlaps partially the distribution of the value of completion from the right-hand side.

The integration needs to be divided in two parts. The first, similar to Case 1, includes the interval for which all values of c are less than v. Here, the integration limits for c are [c_1;v_1] and for v they are [v_1;v_2]. The second part covers the values of c greater than the lower limit of v, i.e., v_1. The integration limits are [v_1;c_2] for c and [c;v_2] for v:

$$T_1\big|_{Case2} = e^{-r_f T}\left(\int_{c_1}^{v_1} f_c(c)\int_{v_1}^{v_2} v\cdot f_v(v)\,dv\,dc + \int_{v_1}^{c_2} f_c(c)\int_{c}^{v_2} v\cdot f_v(v)\,dv\,dc\right)$$

$$= e^{-r_f T}\left(\frac{u_c u_v}{2}\left(v_2^2 c_2 - \frac{c_2^3}{3} - v_2^2 c_1 + v_1^2 c_1 - \frac{2}{3}v_1^3\right)\right)$$

$$T_2\big|_{Case2} = e^{-r_f T}\left(\int_{c_1}^{v_1} c\cdot f_c(c)\int_{v_1}^{v_2} f_v(v)\,dv\,dc + \int_{v_1}^{c_2} c\cdot f_c(c)\int_{c}^{v_2} f_v(v)\,dv\,dc\right)$$

$$= e^{-r_f T}\left(u_c\left(\frac{v_1^2 - c_1^2}{2}\right) + u_c u_v\left(\frac{v_2 c_2^2}{2} - \frac{c_2^3}{3} - \frac{v_2 v_1^2}{2} + \frac{v_1^3}{3}\right)\right)$$

$$VT_1\big|_{Case2} = e^{-2r_f T}\left(\int_{c_1}^{v_1} f_c(c)\left(\int_{v_1}^{v_2} v\cdot f_v(v)\,dv\right)^2 dc + \int_{v_1}^{c_2} f_c(c)\left(\int_{c}^{v_2} v\cdot f_v(v)\,dv\right)^2 dc\right)$$

$$= e^{-2r_f T} u_c u_v^2\left(\left(\frac{v_2^2 - v_1^2}{2}\right)^2 (v_1 - c_1) + \frac{c_2^5 - v_1^5}{20} - v_2^2\,\frac{c_2^3 - v_1^3}{6} + v_2^4\,\frac{c_2 - v_1}{4}\right)$$

$$VT_2\big|_{Case2} = e^{-2r_f T}\left(\int_{c_1}^{v_1} c\cdot f_c(c)\left(\int_{v_1}^{v_2} v\cdot f_v(v)\,dv\cdot\int_{v_1}^{v_2} f_v(v)\,dv\right)dc + \int_{v_1}^{c_2} c\cdot f_c(c)\left(\int_{c}^{v_2} v\cdot f_v(v)\,dv\cdot\int_{c}^{v_2} f_v(v)\,dv\right)dc\right)$$

$$= e^{-2r_f T} u_c u_v^2 \left(\left(\frac{v_2^2 - v_1^2}{2} \right) \left(\frac{v_1^2 - c_1^2}{2} \right) (v_2 - v_1) + \frac{c_2^5 - v_1^5}{10} - v_2 \frac{c_2^4 - v_1^4}{8} - v_2^2 \frac{c_2^3 - v_1^3}{6} + v_2^3 \frac{c_2^2 - v_1^2}{4} \right)$$

$$VT_3 \big|_{Case2} = e^{-2r_f T} \left(\int_{c_1}^{v_1} f_c(c) \cdot \left(c \int_{v_1}^{v_2} f_v(v) dv \right)^2 + \int_{v_1}^{c_2} f_c(c) \cdot \left(c \int_{c}^{v_2} f_v(v) dv \right)^2 \right)$$

$$= e^{-2r_f T} u_c u_v^2 \left(\left(\frac{v_1^3 - c_1^3}{3} \right) (v_2 - v_1)^2 + \frac{c_2^5 - v_1^5}{5} - v_2 \frac{c_2^4 - v_1^4}{2} + v_2^2 \frac{c_2^3 - v_1^3}{3} \right)$$

- Case 3: $v_2 < c_2$, $c_1 < v_1$

Here, the distribution of completion cost overlaps completely the distribution of the value of completion but there is still an interval for which values of c are smaller than the minimum value of v (see Figure 10-3):

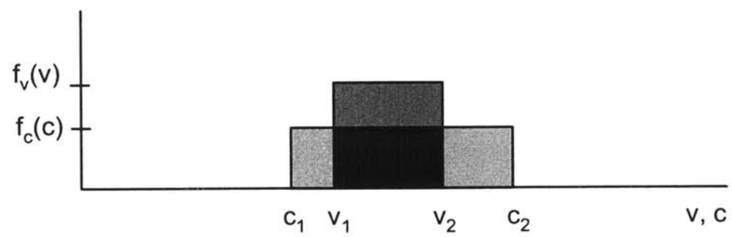


Figure 10-3: Integration Case 3: the distribution of completion costs overlaps completely the distribution of the value of completion but there is still an interval for which values of c are smaller than the minimum value of v.

As in the previous case, the integration needs to be divided in two parts. The first, similar to Case 1, includes the interval for which all values of c are less than v. Here, the integration limits for c are $[c_1;v_1]$ and for v they are $[v_1;v_2]$. The second part covers the values of c between v_1 and v_2. The integration limits are $[v_1;v_2]$ for c and $[c;v_2]$ for v:

$$T_1\big|_{Case3} = e^{-r_f T}\left(\int_{c_1}^{v_1} f_c(c)\int_{v_1}^{v_2} v\cdot f_v(v)dvdc + \int_{v_1}^{v_2} f_c(c)\int_{c}^{v_2} v\cdot f_v(v)dvdc\right)$$

$$= e^{-r_f T}u_c u_v\left(\frac{v_2^3}{3} - \frac{v_2^2 v_1}{2} + \frac{v_1^3}{6} + \frac{1}{2}\left(v_2^2 - v_1^2\right)\left(v_1 - c_1\right)\right)$$

$$T_2\big|_{Case3} = e^{-r_f T}\left(\int_{c_1}^{v_1} c\cdot f_c(c)\int_{v_1}^{v_2} f_v(v)dvdc + \int_{v_1}^{v_2} c\cdot f_c(c)\int_{c}^{v_2} f_v(v)dvdc\right)$$

$$= e^{-r_f T}\left(u_c\left(\frac{v_1^2 - v_1^2}{2}\right) + u_c u_v\left(\frac{v_2^3}{6} - \frac{v_2 v_1^2}{2} + \frac{v_1^3}{3}\right)\right)$$

$$VT_1\big|_{Case3} = e^{-2r_f T}\left(\int_{c_1}^{v_1} f_c(c)\left(\int_{v_1}^{v_2} v\cdot f_v(v)dv\right)^2 dc + \int_{v_1}^{v_2} f_c(c)\left(\int_{c}^{v_2} v\cdot f_v(v)dv\right)^2 dc\right)$$

$$= e^{-2r_f T}u_c u_v^2\left(\left(\frac{v_2^2 - v_1^2}{2}\right)^2\left(v_1 - c_1\right) + \frac{v_2^5 - v_1^5}{20} - v_2^2\frac{v_2^3 - v_1^3}{6} + v_2^4\frac{v_2 - v_1}{4}\right)$$

$$VT_2\big|_{Case3} = e^{-2r_f T}\left(\int_{c_1}^{v_1} c\cdot f_c(c)\left(\int_{v_1}^{v_2} v\cdot f_v(v)dv\cdot\int_{v_1}^{v_2} f_v(v)dv\right)dc + \int_{v_1}^{v_2} c\cdot f_c(c)\left(\int_{c}^{v_2} v\cdot f_v(v)dv\cdot\int_{c}^{v_2} f_v(v)dv\right)dc\right)$$

$$= e^{-2r_f T}u_c u_v^2\left(\left(\frac{v_2^2 - v_1^2}{2}\right)\left(\frac{v_1^2 - c_1^2}{2}\right)\left(v_2 - v_1\right) + \frac{v_2^5 - v_1^5}{10} - v_2\frac{v_2^4 - v_1^4}{8} - v_2^2\frac{v_2^3 - v_1^3}{6} + v_2^3\frac{v_2^2 - v_1^2}{4}\right)$$

$$VT_3\big|_{Case3} = e^{-2r_f T}\left(\int_{c_1}^{v_1} f_c(c)\cdot\left(c\int_{v_1}^{v_2} f_v(v)dv\right)^2 + \int_{v_1}^{v_2} f_c(c)\cdot\left(c\int_{c}^{v_2} f_v(v)dv\right)^2\right)$$

$$= e^{-2r_f T}u_c u_v^2\left(\left(\frac{v_1^3 - c_1^3}{3}\right)\left(v_2 - v_1\right)^2 + \frac{v_2^5 - v_1^5}{5} - v_2\frac{v_2^4 - v_1^4}{2} + v_2^2\frac{v_2^3 - v_1^3}{3}\right)$$

- Case 4: $v_2 < c_2 < v_2$, $v_1 < c_1 < v_2$

In this case, the distribution of completion cost overlaps partially the distribution of value of completion from the left-hand side (see Figure 10-4):

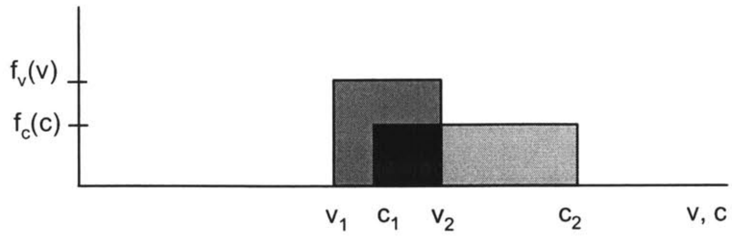


Figure 10-4: Integration Case 4: the distribution of completion cost overlaps partially the distribution of value of completion from the left-hand side.

Here, the integration occurs only over one interval, namely that for which values of c are between the lower and upper bounds of v. The integration limits for c are $[c_1;v_2]$ and for v they are $[c;\, v_2]$:

$$T_1\big|_{Case4} = e^{-r_f T}\left(\int_{c_1}^{v_2} f_c(c)\int_{c}^{v_2} v\cdot f_v(v)\,dv\,dc\right)$$

$$= e^{-r_f T} u_c u_v\left(\frac{v_2^3}{3} - \frac{v_2^2 c_1}{6} + \frac{c_1^3}{6}\right)$$

$$T_2\big|_{Case4} = e^{-r_f T}\left(\int_{c_1}^{v_2} c\cdot f_c(c)\int_{c}^{v_2} f_v(v)\,dv\,dc\right)$$

$$= e^{-r_f T} u_c u_v\left(\frac{v_2^3}{6} - \frac{c_1^2 v_2}{2} + \frac{c_1^3}{3}\right)$$

$$VT_1\big|_{Case4} = e^{-2r_f T}\left(\int_{c_1}^{v_2} f_c(c)\left(\int_{c}^{v_2} v\cdot f_v(v)\,dv\right)^2 dc\right)$$

$$= e^{-2r_f T} u_c u_v^2\left(\frac{v_2^5 - c_1^5}{20} - v_2^2\,\frac{v_2^3 - c_1^3}{6} + v_2^4\,\frac{v_2 - c_1}{4}\right)$$

$$VT_2\big|_{Case4} = e^{-2r_f T}\left(\int_{c_1}^{v_2} c\cdot f_c(c)\left(\int_{c}^{v_2} v\cdot f_v(v)\,dv\cdot\int_{c}^{v_2} f_v(v)\,dv\right)dc\right)$$

$$= \mathrm{e}^{-2r_f T} u_c u_v^2 \left(\frac{v_2^5 - c_1^5}{10} - v_2 \frac{v_2^4 - c_1^4}{8} - v_2^2 \frac{v_2^3 - c_1^3}{6} + v_2^3 \frac{v_2^2 - c_1^2}{4} \right)$$

$$VT_3\Big|_{Case4} = \mathrm{e}^{-2r_f T} \left(\int_{c_1}^{v_2} f_c(c) \cdot \left(c \int_{c}^{v_2} f_v(v) dv \right)^2 \right)$$

$$= \mathrm{e}^{-2r_f T} u_c u_v^2 \left(\frac{v_2^5 - c_1^5}{5} - v_2 \frac{v_2^4 - c_1^4}{2} + v_2^2 \frac{v_2^3 - c_1^3}{3} \right)$$

- Case 5: $v_2 < c_1$

Here, all values of c are greater than all values of v (Figure 10-5):

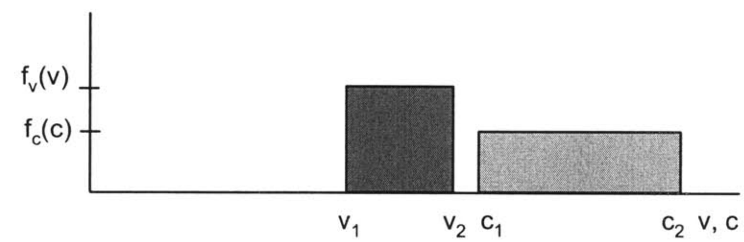


Figure 10-5: Integration Case 5: all values of c are greater than all values of v.

In this case, since there is no value of v greater than c, the option would never be exercised. Thus, the expected value and the variance of the value of the real option are zero.

10.2 Example 2: An exponential distribution and a uniform distribution

Let the probability distribution of the certainty equivalent of stock price, $f_v(v)$, and of the strike price, $f_c(c)$, be defined as follows:

$$f_v(v) = \begin{cases} 0 & v < v_1 \\ \lambda\, e^{-\lambda(x-\mu)} & \text{for } = [v_1;v_2] \\ 0 & v > v_2 \end{cases} \qquad f_c(c) = \begin{cases} 0 & c < c_1 \\ u_c & \text{for } c = [c_1;c_2] \\ 0 & c > c_2 \end{cases}$$

where λ, μ, and u_c are constants.

Again, close attention to the limits of integration must be paid, therefore, consider the following 5 cases:

- Case 1: $v_1 > c_2$

Here, all values of v are larger than all values of c (see Figure 10-6):

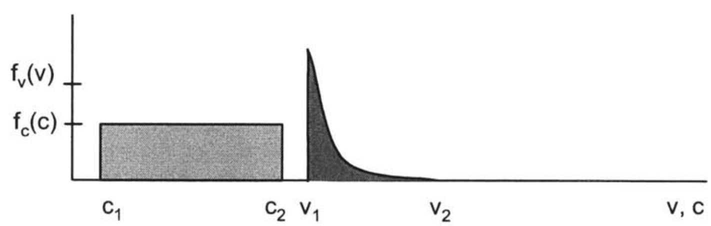


Figure 10-6: Integration Case 1: all values of v are larger than all values of c.

The limits of integration are $[c_1;c_2]$ for c and $[v_1;v_2]$ for v. The integration results are:

$$T_1\big|_{Case1} = e^{-r_f T}\left(\int_{c_1}^{c_2} f_c(c) \int_{v_1}^{v_2} v\, f_v(v)\,dv\,dc\right)$$

$$= e^{-r_f T}\left[\frac{u_c(c_2-c_1)}{\lambda}\left((v_1\lambda+1)\, e^{-\lambda(v_1-\mu)} - (v_2\lambda+1)\, e^{-\lambda(v_2-\mu)}\right)\right]$$

$$T_2\big|_{Case1} = e^{-r_f T}\left(\int_{c_1}^{c_2} c\ f_c(c) \int_{v_1}^{v_2} f_v(v)dvdc\right) = \mathrm{e}^{-r_f T}\left[\frac{u_c\left(c_2^2 - c_1^2\right)}{2}\left(e^{-\lambda(v_1-\mu)} - e^{-\lambda(v_2-\mu)}\right)\right]$$

$$VT_1\big|_{Case1} = e^{-2r_f T}\left(\int_{c_1}^{c_2} f_c(c)\left(\int_{v_1}^{v_2} v\ f_v(v)dv\right)^2 dc\right)$$

$$= \mathrm{e}^{-2r_f T}\left[\frac{u_c\left(c_2 - c_1\right)}{\lambda^2}\left(\left(v_1\lambda + 1\right) e^{-\lambda(v_1-\mu)} - \left(v_2\lambda + 1\right) e^{-\lambda(v_2-\mu)}\right)^2\right]$$

$$VT_2\big|_{Case1} = e^{-2r_f T}\left(\int_{c_1}^{c_2} c\ f_c(c)\left(\int_{v_1}^{v_2} v\ f_v(v)dv \int_{v_1}^{v_2} f_v(v)dv\right)dc\right)$$

$$= \mathrm{e}^{-2r_f T}\left[\frac{u_c\left(c_2^2 - c_1^2\right)}{2\lambda}\left(\left(v_1\lambda + 1\right) e^{-\lambda(v_1-\mu)} - \left(v_2\lambda + 1\right) e^{-\lambda(v_2-\mu)}\right)\left(e^{-\lambda(v_1-\mu)} - e^{-\lambda(v_2-\mu)}\right)\right]$$

$$VT_3\big|_{Case1} = e^{-2r_f T}\left(\int_{c_1}^{c_2} f_c(c)\cdot\left(c\int_{v_1}^{v_2} f_v(v)dv\right)^2\right) = \mathrm{e}^{-2r_f T}\left[\frac{u_c\left(c_2^3 - c_1^3\right)}{3}\left(e^{-\lambda(v_1-\mu)} - e^{-\lambda(v_2-\mu)}\right)^2\right]$$

- Case 2: $v_1 < c_2 < v_2$, $c_1 < v_1$

The distribution of completion costs overlaps partially the distribution of the value fo completion from the right-hand side (see Figure 10-2):

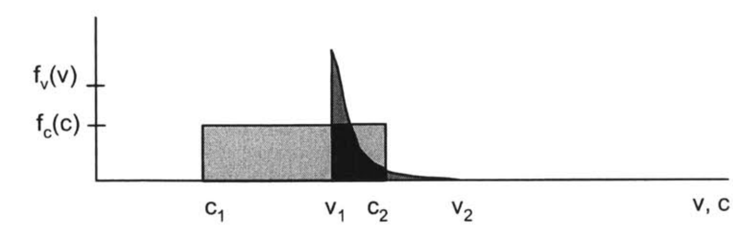


Figure 10-7: Integration Case 2: the distribution of completion costs overlaps partially the distribution of the value of completion from the right-hand side.

The integration needs to be divided in two parts. The first, similar to Case 1, includes the interval for which all values of c are less than v. Here, the integration limits for c are $[c_1;v_1]$ and for v they are $[v_1;v_2]$. The second part covers the values of c greater than the lower limit of v, i.e., v_1. The integration limits are $[v_1;c_2]$ for c and $[c;v_2]$ for v:

$$T_1\big|_{Case2} = e^{-r_f T}\left(\int_{c_1}^{v_1} f_c(c)\int_{v_1}^{v_2} v\ f_v(v)dvdc + \int_{v_1}^{c_2} f_c(c)\int_{c}^{v_2} v\ f_v(v)dvdc\right)$$

$$= \mathrm{e}^{-r_f T}\left[\frac{u_c(v_1 - c_1)}{\lambda}\left((v_1\lambda + 1)\ e^{-\lambda(v_1-\mu)} - (v_2\lambda + 1)\ e^{-\lambda(v_2-\mu)}\right)\right.$$

$$\left. + \frac{u_c}{\lambda^2}\left((v_1\lambda + 2)\ e^{-\lambda(v_1-\mu)} + (v_1v_2\lambda^2 + v_1\lambda - c_2v_2\lambda^2 - c_2\lambda)\ e^{-\lambda(v_2-\mu)} - (c_2\lambda + 2)\ e^{-\lambda(c_2-\mu)}\right)\right]$$

$$T_2\big|_{Case2} = e^{-r_f T}\left(\int_{c_1}^{v_1} c\ f_c(c)\int_{v_1}^{v_2} f_v(v)dvdc + \int_{v_1}^{c_2} c\ f_c(c)\int_{c}^{v_2} f_v(v)dvdc\right)$$

$$= \mathrm{e}^{-2r_f T}\left[\frac{u_c(v_1^2 - c_1^2)}{2}\left(e^{-\lambda(v_1-\mu)} - e^{-\lambda(v_2-\mu)}\right)\right.$$

$$\left. + \frac{u_c e^{-v_2\lambda}}{2\lambda^2}\left((2v_1\lambda + 2)\ e^{-\lambda(v_1-\mu-v_2)} + (v_1^2\lambda^2 - c_2^2\lambda^2)\ e^{\lambda\mu} - (2c_2\lambda + 2)\ e^{-\lambda(c_2-\mu-\lambda v_2)}\right)\right]$$

$$VT_1\big|_{Case2} = e^{-2r_f T}\left(\int_{c_1}^{v_1} f_c(c)\left(\int_{v_1}^{v_2} v\ f_v(v)dv\right)^2 dc + \int_{v_1}^{c_2} f_c(c)\left(\int_{c}^{v_2} v\ f_v(v)dv\right)^2 dc\right)$$

$$= \mathrm{e}^{-r_f T}\left[\frac{u_c(v_1 - c_1)}{\lambda^2}\left((v_1\lambda + 1)\ e^{-\lambda(v_1-\mu)} - (v_2\lambda + 1)\ e^{-\lambda(v_2-\mu)}\right)^2\right.$$

$$+ \frac{u_c e^{-2v_2\lambda}}{4\lambda^3}\left((2\ v_1^2\lambda^2 + 6v_1\lambda + 5)\ e^{-2\lambda(v_1-v_2-\mu)} - 8(2\ v_2\lambda + v_1\lambda + v_1v_2\lambda^2 + 2)\ e^{-\lambda(v_1-v_2-2\mu)}\right.$$

$$+ (8\ c_2v_2\lambda^2 + 8\ c_2\lambda + 16\ v_2\lambda + 16)\ e^{-\lambda(c_2-v_2-2\mu)} - (2c_2^2\lambda^2 + 6c_2\lambda + 5)\ e^{-2\lambda(c_2-v_2-\mu)}$$

$$\left.\left. - 4(v_1v_2^2\lambda^3 + v_1\lambda + 2v_2v_1\lambda^2 - v_2^2c_2\lambda^3 - 2v_2c_2\lambda^2 - c_2\lambda)\ e^{2\lambda\mu}\right)\right]$$

$$VT_2\big|_{Case2} = e^{-2r_fT}\left(\int_{c_1}^{v_1} c\ f_c(c)\left(\int_{v_1}^{v_2} v\ f_v(v)dv\ \int_{v_1}^{v_2} f_v(v)dv\right)dc + \int_{v_1}^{c_2} c\ f_c(c)\left(\int_{c}^{v_2} v\ f_v(v)dv\ \int_{c}^{v_2} f_v(v)dv\right)dc\right)$$

$$= e^{-2r_fT}\Bigg[\frac{u_c\left(v_1^2-c_1^2\right)}{2\lambda}\left(\left(v_1\lambda+1\right)e^{-\lambda(v_1-\mu)}-\left(v_2\lambda+1\right)e^{-\lambda(v_2-\mu)}\right)\left(e^{-\lambda(v_1-\mu)}-e^{-\lambda(v_2-\mu)}\right)$$

$$-\frac{u_c}{2\lambda^3}e^{-2\lambda\left(v_1+v_2+c_2-\mu\right)}\Big(\left(-v_1^2\lambda^2-2\lambda v_1-1\right)e^{2\lambda(c_2+v_2)}+\left(2v_1^2\lambda^2+8\lambda v_1+8+2v_1v_2\lambda^2+2v_2\lambda\right)e^{\lambda(2c_2+v_1+v_2)}$$

$$+\left(v_1^2v_2\lambda^3+\lambda^2v_1^2-v_2c_2^2\lambda^3-c_2^2\lambda^2\right)e^{2\lambda(c_2+v_1)}+\left(c_2^2\lambda^2+2\lambda c_2+1\right)e^{2\lambda(v_1+v_2)}$$

$$-\left(2c_2^2\lambda^2+8\lambda c_2+8+2v_2\lambda^2c_2+2v_2\lambda\right)e^{\lambda(c_2+2v_1+v_2)}\Big)\Bigg]$$

$$VT_3\big|_{Case2} = e^{-2r_fT}\left(\int_{c_1}^{v_1} f_c(c)\cdot\left(c\int_{v_1}^{v_2} f_v(v)dv\right)^2 + \int_{v_1}^{c_2} f_c(c)\cdot\left(c\int_{c}^{v_2} f_v(v)dv\right)^2\right)$$

$$= e^{-2r_fT}\Bigg[\frac{u_c\left(v_1^3-c_1^3\right)}{3}\left(e^{-\lambda(v_1-\mu)}-e^{-\lambda(v_2-\mu)}\right)^2-\frac{u_c}{12\lambda^3}e^{-2\lambda v_2}\Big(\left(-6\lambda^2v_1^2-6\lambda v_1-3\right)e^{-2\lambda(v_1-v_2-\mu)}$$

$$+\left(24\lambda^2v_1^2+48\lambda v_1+48\right)e^{-\lambda(v_1-v_2-2\mu)}+\left(6\lambda^2c_2^2+6\lambda c_2+3\right)e^{-2\lambda(c_2-v_2-\mu)}+4\lambda^3\left(v_1^3-c_2^3\right)e^{2\lambda\mu}$$

$$-48\left(\lambda c_2-0.5\lambda^2c_2^2+1\right)e^{-\lambda(c_2-v_2-2\mu)}\Big)\Bigg]$$

- Case 3: $v_2 < c_2$, $c_1 < v_1$

The distribution of completion costs overlaps completely the distribution of the value of completion but there is still an interval for which values of c are smaller than the minimum value of v (see Figure 10-8):

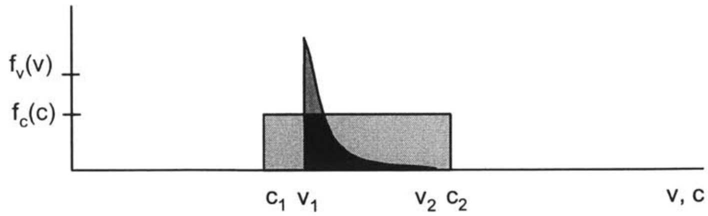


Figure 10-8: Integration Case 3: the distribution of completion costs overlaps completely the distribution of the value of completion but there is still an interval for which values of c are smaller than the minimum value of v.

As in the previous case, the integration needs to be divided in two parts. The first, similar to Case 1, includes the interval for which all values of c are less than v. Here, the integration limits for c are $[c_1;v_1]$ and for v they are $[v_1;v_2]$. The second part covers the values of c between v_1 and v_2. The integration limits are $[v_1;v_2]$ for c and $[c;v_2]$ for v:

$$T_1\big|_{Case3} = e^{-r_f T}\left(\int_{c_1}^{v_1} f_c(c)\int_{v_1}^{v_2} v\ f_v(v)dvdc + \int_{v_1}^{v_2} f_c(c)\int_{c}^{v_2} v\ f_v(v)dvdc\right)$$

$$= e^{-r_f T}\left[\frac{u_c(v_1 - c_1)}{\lambda}\left((v_1\lambda + 1)\ e^{-\lambda(v_1-\mu)} - (v_2\lambda + 1)\ e^{-\lambda(v_2-\mu)}\right)\right.$$

$$\left. + \frac{u_c}{\lambda^2}\left((v_1\lambda + 2)\ e^{-\lambda(v_1-\mu)} + \left(v_1 v_2\lambda^2 + v_1\lambda - v_2^2\lambda^2 - 2v_2\lambda - 2\right)\ e^{-\lambda(v_2-\mu)}\right)\right]$$

$$T_2\big|_{Case3} = e^{-r_f T}\left(\int_{c_1}^{v_1} c\ f_c(c)\int_{v_1}^{v_2} f_v(v)dvdc + \int_{v_1}^{v_2} c\ f_c(c)\int_{c}^{v_2} f_v(v)dvdc\right)$$

$$= e^{-r_f T}\left[\frac{u_c\left(v_1^2 - c_1^2\right)}{2}\left(e^{-\lambda(v_1-\mu)} - e^{-\lambda(v_2-\mu)}\right)\right.$$

$$\left. + \frac{u_c e^{-v_2\lambda}}{2\lambda^2}\left((2v_1\lambda + 2)\ e^{-\lambda(v_1-\mu-v_2)} + \left(v_1^2\lambda^2 - 2\lambda v_2 - v_2^2\lambda^2 - 2\right)\ e^{\lambda\mu}\right)\right]$$

$$VT_1\big|_{Case3} = e^{-2r_f T}\left(\int_{c_1}^{v_1} f_c(c)\left(\int_{v_1}^{v_2} v\ f_v(v)dv\right)^2 dc + \int_{v_1}^{v_2} f_c(c)\left(\int_{c}^{v_2} v\ f_v(v)dv\right)^2 dc\right)$$

$$= \mathrm{e}^{-2r_f T}\left[\frac{u_c(v_1 - c_1)}{\lambda^2}\left((v_1\lambda\ +1)\ e^{-\lambda(v_1-\mu)} - (v_2\lambda\ +1)\ e^{-\lambda(v_2-\mu)}\right)^2\right.$$

$$+\frac{u_c e^{-2v_2\lambda}}{4\lambda^3}\left(\left(2v_1^2\lambda^2 + 6v_1\lambda + 5\right)\ e^{-2\lambda(v_1-v_2-\mu)} - 8\left(2v_2\lambda + v_1\lambda + v_1 v_2\lambda^2 + 2\right)\ e^{-\lambda(v_1-v_2-2\mu)}\right.$$

$$\left.\left. + \left(-4v_1 v_2^2\lambda^3 - 8\lambda^2 v_1 v_2 + 22v_2\lambda - 4\lambda v_1 + 4v_2^3\lambda^3 + 14v_2^2\lambda^2 + 11\right)\ e^{2\lambda\mu}\right)\right]$$

$$VT_2\big|_{Case3} = e^{-2r_f T}\left(\int_{c_1}^{v_1} c\ f_c(c)\left(\int_{v_1}^{v_2} v\ f_v(v)dv\ \int_{v_1}^{v_2} f_v(v)dv\right)dc + \int_{v_1}^{v_2} c\ f_c(c)\left(\int_{c}^{v_2} v\ f_v(v)dv\ \int_{c}^{v_2} f_v(v)dv\right)dc\right)$$

$$= \mathrm{e}^{-2r_f T}\left[\left(\ \frac{u_c\left(v_1^2 - c_1^2\right)}{2\lambda}\left((v_1\lambda\ +1)\ e^{-\lambda(v_1-\mu)} - (v_2\lambda\ +1)\ e^{-\lambda(v_2-\mu)}\right)\left(e^{-\lambda(v_1-\mu)} - e^{-\lambda(v_2-\mu)}\right)\right.\right.$$

$$-\frac{u_c}{2\lambda^3} e^{-2\lambda(v_1+v_2-\mu)}\left(\left(-v_1^2\lambda^2 - 2\lambda v_1 - 1\right)\ e^{2\lambda v_2} + \left(2v_1^2\lambda^2 + 8\lambda v_1 + 8 + 2v_1 v_2\lambda^2 + 2v_2\lambda\right)\ e^{\lambda(v_1+v_2)}\right.$$

$$\left.\left.\left. + \left(v_1^2 v_2\lambda^3 + \lambda^2 v_1^2 - v_2^3\lambda^3 - 4v_2^2\lambda^2 - 8\lambda v_2 - 7\right)\ e^{2\lambda v_1}\right)\right)\right]$$

$$VT_3\big|_{Case3} = e^{-2r_f T}\left(\int_{c_1}^{v_1} f_c(c)\cdot\left(c\int_{v_1}^{v_2} f_c(c)dc\right)^2 + \int_{v_1}^{v_2} f_c(c)\cdot\left(c\int_{c}^{v_2} f_v(v)dv\right)^2\right)$$

$$= \mathrm{e}^{-2r_f T}\left[\frac{u_c\left(v_1^3 - c_1^3\right)}{3}\left(e^{-\lambda(v_1-\mu)} - e^{-\lambda(v_2-\mu)}\right)^2 - \frac{u_c}{12\lambda^3} e^{-2\lambda v_2}\left(\left(-6\lambda^2 v_1^2 - 6\lambda v_1 - 3\right)\ e^{-2\lambda(v_1-v_2-\mu)}\right.\right.$$

$$\left.\left. + \left(24\lambda^2 v_1^2 + 48\lambda v_1 + 48\right)\ e^{-\lambda(v_1-v_2-2\mu)} - \left(18\lambda^2 v_2^2 + 42\lambda v_2 + 4\lambda^3 v_2^3 + 4\lambda^3 v_1^3 + 45\right)\ e^{2\lambda\mu}\right)\right]$$

- Case 4: $v_2 < c_2 < v_2$, $v_1 < c_1 < v_2$

The distribution of completion costs overlap partially the distribution of the value of completion from the left-hand side (see Figure 10-9):

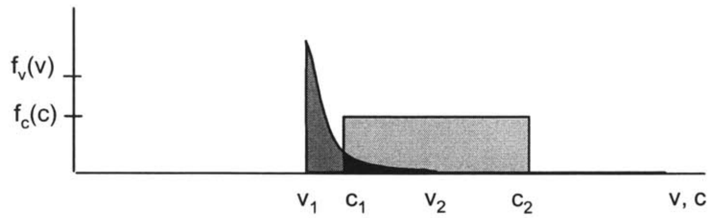


Figure 10-9: Integration Case 4: the distribution of completion costs overlaps partially the distribution of the value of completion from the left-hand side.

Here, the integration occurs only over one interval, namely that for which values of c are between the lower and upper bounds of v. The integration limits for c are $[c_1;v_2]$ and for v they are $[c_1;v_2]$:

$$T_1\big|_{Case4} = e^{-r_f T}\left(\int_{c_1}^{v_2} f_c(c)\int_{c}^{v_2} v\cdot f_v(v)\,dv\,dc\right)$$

$$= e^{-r_f T}\left[-\frac{u_c}{\lambda^2}\left(\left(-c_1\lambda-2\right)e^{-\lambda(c_1-\mu)}+\left(-c_1v_2\lambda^2-c_1\lambda+v_2^2\lambda^2+2v_2\lambda+2\right)e^{-\lambda(v_2-\mu)}\right)\right]$$

$$T_2\big|_{Case4} = e^{-r_f T}\left(\int_{c_1}^{v_2} c\cdot f_c(c)\int_{c}^{v_2} f_v(v)\,dv\,dc\right)$$

$$= e^{-r_f T}\left[-\frac{u_c e^{-v_2\lambda}}{2\lambda^2}\left(\left(-2c_1\lambda-2\right)e^{-\lambda(c_1-\mu-v_2)}+\left(-c_1^2\lambda^2+2\lambda v_2+v_2^2\lambda^2+2\right)e^{\lambda\mu}\right)\right]$$

$$VT_1\big|_{Case4} = e^{-2r_f T}\left(\int_{c_1}^{v2} f_c(c)\left(\int_{c}^{v_2} v\cdot f_v(v)\,dv\right)^2 dc\right)$$

$$= e^{-2r_f T}\left[\frac{u_c e^{-2v_2\lambda}}{4\lambda^3}\left(\left(-4\lambda c_1-4c_1v_2^2\lambda^3-8\lambda^2c_1v_2+22v_2\lambda+4v_2^3\lambda^3+14v_2^2\lambda^2+11\right)e^{2\lambda\mu}\right.\right.$$

$$\left.\left.+\left(\left(2c_1^2\lambda^2+6c_1\lambda+5\right)e^{-2\lambda(c_1-v_2-\mu)}-8\left(2v_2\lambda+c_1\lambda+c_1v_2\lambda^2+2\right)e^{-\lambda(c_1-v_2-2\mu)}\right)\right)\right]$$

$$VT_2\big|_{Case4} = \mathrm{e}^{-2r_f T}\left(\int_{c_1}^{v_2} c \cdot f_c(c)\left(\int_{c}^{v_2} v \cdot f_v(v)dv \cdot \int_{c}^{v_2} f_v(v)dv\right)dc\right)$$

$$= \mathrm{e}^{-2r_f T}\left[\frac{u_c}{2\lambda^3} e^{-2\lambda(c_1+v_2-\mu)}\left(\left(-c_1^2 v_2 \lambda^3 - \lambda^2 c_1^2 + v_2^3\lambda^3 + 4v_2^2\lambda^2 + 8\lambda v_2 + 7\right) e^{2\lambda c_1}\right.\right.$$

$$\left.\left. + \left(c_1^2\lambda^2 + 2\lambda c_1 + 1\right) e^{2\lambda v_2} + \left(-2c_1^2\lambda^2 - 8\lambda c_1 - 8 - 2c_1 v_2 \lambda^2 - 2v_2\lambda\right) e^{\lambda(c_1+v_2)}\right)\right]$$

$$VT_3\big|_{Case4} = \mathrm{e}^{-2r_f T}\left(\int_{c_1}^{v_2} f_c(c) \cdot \left(c\int_{c}^{v_2} f_v(v)dv\right)^2\right)$$

$$= \mathrm{e}^{-2r_f T}\left[\frac{u_c}{12\lambda^3} e^{-2\lambda v_2}\left(\left(+6\lambda^2 c_1^2 + 6\lambda c_1 + 3\right) e^{-2\lambda(c_1-v_2-\mu)}\right.\right.$$

$$\left.\left. + \left(-24\lambda^2 c_1^2 - 48\lambda c_1 - 48\right) e^{-\lambda(c_1-v_2-2\mu)} + \left(18\lambda^2 v_2^2 + 42\lambda v_2 + 4\lambda^3 v_2^3 - 4\lambda^3 c_1^3 + 45\right) e^{2\lambda\mu}\right)\right]$$

- Case 5: $s_2 < x_1$

Here, all values of c are greater than all values of v (see Figure 10-10):

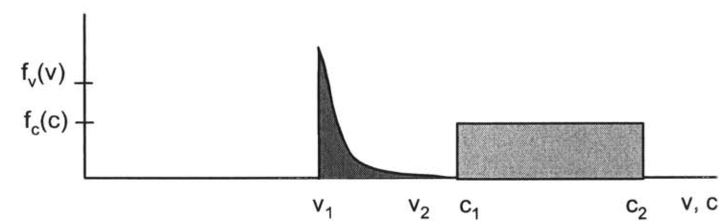


Figure 10-10: Integration Case 5: all values of c are greater than all values of v.

In this case, since there is no value of v greater than c, the option would never be exercised. Thus, the expected value and the variance of the value of real option are zero.

11 Appendix C: Algorithm for solving the generalized real options equations numerically

The following algorithm can be used to calculate the expected value of a real option using the generalized real options methodology developed in this thesis numerically. It can be implemented in Matlab® or a spreadsheet program.

1. **Prepare the data of the probability distribution of the value of completion, V, and the completion cost, C:**

   1.1. Determine the histograms of the certainty equivalents of V and C.

   1.2. Organize the data for each variable in an array of three columns (see below):

pdf_V

v	$f_v(v)$	$v^* f_v(v)$

pdf_C =

c	$f_c(c)$	$c^* f_c(c)$

2. **Define some intermediate variables:**

   Based on Equation 3-11 in Section 3.2, define the following intermediate variables:

$$w = E[O(c)] = \int_{c=-\infty}^{c=\infty} O(c) \cdot f_c(c)dc = e^{-r_f T}\left(\int_{c=0}^{\infty} f_c(c) \int_{v=c}^{\infty} v \cdot f_v(v)dvdc - \int_{c=0}^{\infty} c \cdot f_c(c) \cdot \int_{v=c}^{\infty} f_v(v)dvdc \right)$$

$$V_1 = \int_{v=c}^{\infty} v \cdot f_v(v)dvdc$$

$$V_2 = \int_{v=c}^{\infty} f_v(v)dvdc$$

$$C_1 = \int_{c=0}^{\infty} f_c(c) \cdot V_1$$

$$C_2 = \int_{c=0}^{\infty} c \cdot f_c(c) \cdot V_2$$

N_rows_V: number of rows in the array of the histogram of V

N_rows_C: number of rows in the array of the histogram of C

3. Calculate value of the option using the following pseudo-code:

```
Rf;                                    % risk-free discount rate
T;                                     % maturity of the option
V1 = 0;                                %initialize variables
V2 = 0;
C1 = 0;
C2 = 0;

    for j=1:N_rows_C                        %loop over c

        c = pdf_C(j,1);                     %completion cost

        for i=1:N_rows_V                    % loop over v

        if(pdf_V(i,1)>c)
          V1 = V1 + pdf_V(i,1)*pdf_V(i,3);
```

```
  V2 = V2 + pdf_V(i,3);
 end
 end

 C1 = C1 + pdf_C(j,3)*V1;
 C2 = C2 + c*pdf_C(j,3)*V2;

 V1 =0;
 V2 =0;

end

W = exp(-rf*T)*(C1 - C2)                    %Expected value of the real option
```