# GEOSTROPHIC DEVIATIONS IN RELATION TO THE STRENGTH OF THE CIRCUMPOLAR VORTEX

by

William Eric Hardy

A.B., Furman University
(1935)

S.B., Southwestern Louisiana Institute
(1947)

SUBMITTED IN PARTIAL FULFILLMENT OF THE
REQUIREMENTS FOR THE DEGREE OF

MASTER OF SCIENCE

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
(1949)

Signature of Author . . . . . . . . . . . . . . . . . .
Department of Meteorology, February 28, 1949

Certified by . . . . . . . . . . . . . . . . . . . . .
Thesis Supervisor

. . . . . . . . . . . . . . . . . . . . . . . . . . . .
Chairman, Department Committee on Graduate Students

# TABLE OF CONTENTS

CHAPTER I

CHAPTER II

The author wishes to express his appreciation to Professor Thomas F. Malone for his helpful suggestions in the preparation of this thesis.

facts and relationships can be found which will add to the over-all attempts to explain the phenomena, and which may in turn lead to more accurate forecasting techniques.

This paper represents such an attempt, which, in its essence, is an empirical study of certain particular relations between the strength of the zonal westerly wind flow of the northern hemispheric middle-latitudes, and the components of the geostrophic wind deviations.

In an attempt to build a clear picture of the physical and theoretical concepts of that portion of the northern hemispheric air circulations which is being studied, it was deemed advisable to review briefly such concepts as a preface to the main phase of the investigation. Consequently, this review appears first, followed by the empirical study which is the essential feature of this paper.

It is hoped that the results to be set forth presently will serve to clarify somewhat the existing picture of the zonal westerlies particularly at the 3 kilometer level with respect to the accelerational fields which accompany changes in their intensity.

# CHAPTER I

# THE MAXIMUM ZONAL WESTERLY WINDS OF THE NORTHERN HEMISPHERE

## A. Qualitative Aspects of the Circumpolar Vortex

The recent advent of improved instruments for making more accurate upper-air observations to great heights, and the increased hemispheric coverage of such observations has focused the attention of Meteorologists on some defects of the classical concepts of the basic planetary atmospheric circulation which seek to explain the existing conditions. This is particularly emphasized by the more recent verification (1) of the existence of a comparatively narrow zone of strong westerly winds between 30 and 40 degrees north latitude in winter, and up to 55 degrees in summer at levels between 5 and 15 kilometers above sea level, and having its maximum speed located at about 10 to 12 kilometers, the approximate level of the tropopause.

The zone of maximum west wind intensity is referred to by Rossby as the jet stream, and may reach speeds in excess of 250 miles per hour at lower latitudes where it attains maximum intensity. This rather extensive zone of westerly wind flow of which the jet stream represents the core, takes the form of a circumpolar vortex and has been described as a continuous,

# CHAPTER I

# THE MAXIMUM ZONAL WESTERLY WINDS OF THE NORTHERN HEMISPHERE

## A. Qualitative Aspects of the Circumpolar Vortex

The recent advent of improved instruments for making more accurate upper-air observations to great heights, and the increased hemispheric coverage of such observations has focused the attention of Meteorologists on some defects of the classical concepts of the basic planetary atmospheric circulation which seek to explain the existing conditions. This is particularly emphasized by the more recent verification (1) of the existence of a comparatively narrow zone of strong westerly winds between 30 and 40 degrees north latitude in winter, and up to 55 degrees in summer at levels between 5 and 15 kilometers above sea level, and having its maximum speed located at about 10 to 12 kilometers, the approximate level of the tropopause.

The zone of maximum west wind intensity is referred to by Rossby as the jet stream, and may reach speeds in excess of 250 miles per hour at lower latitudes where it attains maximum intensity. This rather extensive zone of westerly wind flow of which the jet stream represents the core, takes the form of a circumpolar vortex and has been described as a continuous, meandering river of air completely surrounding the globe, and flowing through more or less stagnant air mass surroundings to the north and south. Its flow is seldom a straight eastward course, but is characterized by a wave-like pattern with wave-lengths varying from around 50 to 120 degrees of longitude.

The jet stream is sharply defined with the winter about twice the summer speeds, and the zonal wind decreasing extremely rapidly with increasing and decreasing latitude. Just to the north and beneath it is concentrated the strongest solenoid field existing between the equatorial and polar regions. A low-level tropopause is also located just north of the jet stream, while to the south a double tropopause is often observed. There is also a break in the tropopause in the zone of the jet stream.

It is probable that there exists a definite relation between the upper west wind belt and the smaller, true frontal waves, since it has often been observed that a well-defined frontal zone is located below the jet stream which intersects the surface to the south and the tropopause to the north, and which is characterized by maximum concentration of the horizontal temperature gradient. The relationship between the upper, long wave flow and the movement of the shorter surface disturbances is emphasized by the acceptance of the "steering" principle as a good forecast tool. However, there is little or no evidence to indicate that the lower surface disturbances have any important influence upon the formation of the upper long waves.

Rossby and Willett (2) have indicated that the circumpolar vortex with its jet stream core fluctuates rather irregularly from day to day and from week to week. These fluctuations are characterized by an expansion or contraction of the circumpolar vortex, a variation of the speed of the jet stream as it fluctuates, the speed being greater to the south, and a wave-like character of the flow above the 700 millibar level as it moves around the hemisphere. This wave-like motion of the circumpolar vortex varies in wave length, the wave length increasing with the speed of the westerlies, and moving through a cycle, the period of which varies from 3 to 8 weeks. The amplitude increases as the vortex moves to lower latitudes until the wave-like pattern of the flow breaks down so that cyclonic vortices of cold air sometimes become sequestered, closed systems to the south of the jet stream, and anticyclonic systems of warm air occur in similar fashion to the north of the maximum flow. Due to the interaction of the secluded warm and cold air cells the latitudinal temperature contrast is diminished at all levels and the vortex is thus in a relaxed phase, to be reactivated when a latitudinal temperature contrast is again built up.

## B. Some Recent Theories on the Possible Mechanism of the Jet Stream

The classical theory of the general planetary atmospheric circulation fails to explain the observed jet stream characteristics of the westerly wind flow in mid-latitudes. This theory holds that the atmosphere will tend towards a circulation which is thermal in character, and which results in circulation from cold to warm areas at the ground, from warm to cold areas at high altitudes. It further holds that due to the principle of the conservation of angular momentum the over-all hemispherical thermal flow tendency will be broken down into two principle, thermally driven cells one lying between the thermal equator and about 30 degrees north latitude, and the other from about 60 degrees north latitude to the pole, both with generally northeasterly winds at the surface and southwesterly winds aloft. The theory further calls for ascending motion at the thermal equator and 60 degrees north latitude, and descending motion at the pole and at 30 degrees north latitude. The middle latitudes should be characterized by a third thermally, indirect cell which is driven by the two thermally direct cells flanking it, and with west winds at sea level. Surface friction requires a component of flow to the north so that aloft there would result a relatively westward flow in the northern portion of the zone, and easterly flow in the southern portion (2).

Rossby's (3) first attempt in 1941 became nullified as a result of more recent upper air observations. It held that the upper westerlies of the middle latitudes were frictionally driven at a supergradient velocity by the flanking westerly currents of the northern and southern thermally direct cells through horizontal, turbulent exchange. The momentum principle and the existence of the maximum solenoid field between equator and pole which is just beneath the jet stream tend to disqualify this explanation.

A more recent attempt by Rossby (4) was to make a study of the effect of lateral transfer of vertical vorticity on the distribution of zonal motion in thin atmospheric envelopes on the assumption that such lateral mixing should effect an equilization of absolute vorticity rather than one of angular momentum. This would result in the transport of warm air and anticyclonic relative vorticity northward, and cold air and cyclonic relative vorticity southward. On the basis that the vertical component of the vorticity of the absolute motion should be independent of latitude and equal to that of the planetary rotation at the pole, he computed a zonal wind speed profile for such a thin shell and found that in the upper troposphere when the jet stream is strong there exists a similarity of the zonal velocity profile to this theoretical wind profile of constant vorticity down to the point of maximum velocity of the wind flow. From this point to the equator the observed velocity profile did not as faithfully follow a theoretical profile based upon constant vorticity transfer. He further holds that the inertial stability of the atmosphere limits the development of the jet stream such that when the zonal wind shear on the outer periphery of the stream exceeds that of the constant angular momentum, an indirect, forced meridional circulation is developed. This lifts the tropopause upward and consequently concentrates the strong solenoid field beneath the jet stream, thus leading to the conclusion that the strength of the jet stream is responsible for the strong solenoid field, or that such concentration of solenoids normal to the zonal wind current is a dynamic consequence of the zonal motion itself. He concluded that the meridional solenoid field could be dynamically maintained by forced circulation, which is stronger than those found in the driving cells themselves, and that the jet stream is driven from the higher rather than from the lower latitudes. Since Starr (5) finds a poleward transport of angular momentum in middle and lower latitudes, it is not yet known which of the driving cells contributes more to the westerly winds of the middle latitudes, or how much they may be influenced by the strong solenoid field which is found therein.

When a period of low surface index is reached the interaction between the warm and cold air cells, which had previously been cut off as a result of the increasing amplitudes and shortening wave lengths of the zonal westerlies, causes a diminution of latitudinal temperature contrasts at all levels. The jet stream is thus deteoriorated and in a relaxed state, and is activated again only after radiational cooling of the polar troposphere increases the poleward temperature gradient.

Since it appears that the exchange of air between high and low latitudes occurs as a result of mixing through vertical axes, lateral exchange of this type would tend to transport warm air and anticyclonic relative vorticity northward, and cold air and cyclonic relative vorticity southward, with the resulting establishment of a zonal wind profile with constant vertical absolute vorticity. Rossby shows that a polar cap which has acquired such constant vertical absolute vorticity due to lateral mixing would have an absolute zonal angular velocity

$$1. \quad \omega = \frac{\zeta}{1 + \sin\phi}, \quad \zeta = 2\omega_p, \quad \omega_p = \frac{2\pi}{T_p}$$

where $\omega_p$ is the absolute angular velocity of the cap at the pole and $\phi$ the latitude. $T_p$ is the period of absolute rotation of the atmospheric shell at the pole. The relative linear velocity of the cap would be

2. $$u = a\Omega \frac{\frac{2\omega_p - \Omega}{\Omega} - \sin\phi}{1 + \sin\phi} \cos\phi$$

where $a$ is the earth's radius and $\Omega$ its angular velocity. The earth's linear equatorial velocity is

3. $$C_E = a\Omega$$

The non-dimensional ratio for zonal wind distribution is

4. $$\frac{u}{C_E} = \frac{\frac{2\omega_p - \Omega}{\Omega} - \sin\phi}{1 + \sin\phi} \cos\phi$$

and the absolute vertical vorticity of the polar cap is

5. $$\zeta = 2\omega_p$$

If $\zeta = 2\omega_p < 2\Omega$ there would be a belt of easterly winds at high latitudes. So the profile for $\zeta = 2\omega_p = 2\Omega$ is considered where

6. $$\frac{u}{C_E} = \frac{1 - \sin\phi}{1 + \sin\phi} \cos\phi$$

as a limit of maximum vorticity. In this case equation 6 above is the abscissa, with latitude the ordinate of the profile.

As to the determination of the constant vorticity profile with which the observed westerly wind flow of the northern hemisphere agrees rather well down to the region of maximum velocity, Rossby, Willett, and the Staff Members of the Department of Meteorology of the University of

Chicago are essentially in agreement.

The constant vorticity transfer portion of the profile which Rossby assumes to continue southward from the latitude of the jet stream through the equatorial regions is not followed very closely by the observed wind profile. The Staff Members of the Department of Meteorology of the University of Chicago frequently found that south of the zonal-wind maximum the observed wind profiles showed a decreasing rate similar to a constant absolute angular momentum per unit of mass. However, the few observations which exist indicate rather complicated wind systems about which very little is as yet known.

It must be sufficient to say that the observed wind profiles are in fair agreement with the theoretical profile computed on the basis of constant vorticity down to the latitude of the jet stream maximum. From this point toward the equator the observed flow-profiles tend to equal or exceed the profile based upon constant angular momentum, the agreement in this case being considerably better than in the case of the profile of constant vorticity transport over the same region.

Another school of thought as represented by Namias holds that the jet stream results from a confluence of warm and cold air currents which effect the concentration of the strong solenoid field found near the center of such maximum westerly flow. This being a comparatively recent concept, not much detailed information regarding it has been made available.

# CHAPTER II

## GEOSTROPHIC DEVIATIONS AND THE CHANGING INDEX PATTERN

### A. Introduction

It has been pointed out by Rossby and Willett (2) that the jet stream varies in intensity and latitude in a normal manner from season to season in such a way as to parallel the major wave pattern of the westerlies at the lower 500-or 700-millibar levels. However, superimposed upon this normal, seasonal variation of intensity and latitude, there are irregular variations which are not as yet satisfactorily explained, but which very definitely exist. This irregular cycle of change of the general circulation pattern is best represented by the winter season conditions, and varies from three to eight week periods, and in amplitude, and regularity. These variations are best shown by the patterns of high and low index, high index patterns being those which are essentially zonal in character, and low index patterns being those which are primarily cellular in character. High index patterns are further characterized by a minimum of storminess and meridional air transport in the middle latitudes, while low index patterns are usually associated with maximum storminess conditions and transfer of air meridionally in middle and lower latitudes. Essentially, the value of the index will be a measure of the strength of the zonal westerlies. That there is a striking basic similarity between the long period variations of the world weather patterns and the week-to-week cycle of high-and low-index patterns has been suggested by Willett (6). Rossby and Willett (2) further showed that on the basis of the existing observational data, the zonal index cycle is definitely related to the fluctuations of the circumpolar vortex and the jet stream which represents its maximum strength in the region of the tropopause. Concluding that this relationship rests upon a substantial observational foundation, it was decided to make use of existing wind reports at the upper air level at which the zonal index is computed, the 700 millibar level, in making an investigation of the relationship between geostrophic wind deviations and the changing index pattern or zonal westerly strength at that level. It was further decided to extend the investigation to include not only the geostrophic deviations at the latitude of the index value being considered, but also those geostrophic deviations 5 degrees of latitude on either side of the latitude of the maximum flow velocity. Thus comparisons between the geostrophic deviations on the north and south sides of the wind stream, and those at the point of maximum strength of the zonal westerly wind flow can be made, with the possibility of gaining a greater insight into the ramifications of this important atmospheric current.

## B. Some Considerations of Latitudinal Wind Shear

It should be mentioned at this point that

Namias (7) made a somewhat related investigation regarding the accelerating and retarding forces which result from the shearing stresses due to lateral mixing in a broad, westerly current of varying velocities which is flowing along an isentropic surface. In such a current the shearing forces tend to distribute the momentum uniformly, since a retarding force will be setup at the point of maximum eastward velocity and an accelerating force at points of less than maximum eastward velocity. He further showed that since such forces must be balanced by Coriolis forces normal to the flow axes, such eastward accelerating forces will produce a southward motion. Consequently, at points above and below the point of maximum flow velocity he obtained eastward accelerations parallel to the flow axes, and southward accelerations normal to such axes. At the point of maximum flow velocity the accelerations were opposite those above, and of greater magnitude. One should be cautious in attempting to compare this profile-picture of Namias and those resulting from this investigation since any comparison must be predicated by the realization that the mixing processes along isentropic surfaces are of a different nature from those taking place along constant pressure surfaces.

## C. Sources and Development of the Data

The two principle methods of presentation of the relationships between the geostrophic deviations and the strength of the zonal westerlies are as follows:

(1) The relationship between low, intermediate, and high five-day mean zonal index values (8), and the components of the geostrophic deviation normal and parallel to geostrophic wind flow are indicated in Table I. This presentation not only shows these relationships at the latitude of the mean zonal index ($LL_0$), but simultaneously at latitudes approximately five degrees below ($LL_{-5}$), and five degrees above ($LL_{+5}$), this latitude. The differences between the absolute values of the observed and the geostrophic winds ($V-V_{gs}$) are also shown for each latitude.

In each case the five-day mean zonal index values (8) were obtained for the two year period, 1946 and 1947, there being 207 such values and ranging from a minimum of 5.6 meters per second to a maximum of 14.93 meters per second. These values were next grouped into seasons as indicated in Table II and their index frequencies tallied. Table III shows how this frequency distribution was in turn broken down into the low, intermediate and high index categories.

The components of the geostrophic deviations were computed for each index value in the three categories wherever the data were available for stations chosen at the appropriate latitudes. The stations whose observations were employed were generally distributed throughout the central part of the United States and south-central and western Canada. Their geographical distribution would be approximately centered by a line running from Florida to southeastern Alaska, and generally within the region between 30 degrees and 60 degrees north latitude. The final results were not computed for the maximum number of station observations existing in the two year period, since all such observations were not available due to missing upper winds observations and unanalyzed 700 millibar maps. The number of observations used here comprise approximately 50 per cent of the total possible number assuming two observations per station per day. The observations were taken each day at 0400 and 1600 Greenwich mean times such that a maximum of ten observations for each five day index value was possible at each latitude. For each such time the wind direction and speed were recorded from the appropriate pilot balloon and rawinsonde observations, using the observations of the rawinsonde stations whenever possible in order to eliminate the fair weather bias. The corresponding geostrophic wind directions and speeds were measured from the appropriate 700 millibar constant pressure contour charts as analyzed daily by the Department of Meteorology. The directions were taken parallel to the map contours, and the speeds measured with an appropriate geostrophic wind scale. Machta (9) showed that the geostrophic winds computed by a simple, objective technique were in close enough agreement with those measured from the contours so that it could be concluded no systematic error was present. These vectors were in turn laid out on polar coordinate paper and the components of the geostrophic deviation normal, and parallel to the geostrophic wind flow were determined by graphical means. These two components will be referred to hereinafter respectively as the accelerational component, and the tangential component of the geostrophic deviation. Cross-gradient flow as represented by the component normal to geostrophic flow, the accelerational component, is toward low pressure if of positive sign, and toward high pressure if of negative sign. The flow parallel to geostrophic flow as represented by the tangential component is sub-geostrophic if the sign of the component is negative and super-geostrophic if the sign is positive. The geostrophic deviations are understood to be the vector quantity resulting from the difference between the observed wind vector and the geostrophic wind vector.

Since the values of the geostrophic deviation components and the velocity differences were either positive, negative or zero, the algebraic means of each group were computed in each of the three main categories, and for all three latitudes. Table I gives these results, and Table IV shows the number of positive, negative and zero values which went into the computations of the algebraic means in each case.

(2) The relationship between cases of increasing and decreasing index values, each group in turn being subdivided into small, intermediate,and large increasing and decreasing index, and the components of the geostrophic deviations is given in Table V.

An increasing or decreasing index was determined as one where the immediately preceding and following index values were respectively, smaller and larger, or larger and smaller than the middle value.

Table VI indicates the method whereby the increasing and decreasing index values were in turn subdivided each into small, intermediate, and large groups. The frequencies of the differences between the smaller and the larger of each group of three consecutive index values were tabulated within the limits indicated in Table V, and from these frequencies the criteria which determine the small, intermediate, and large increases or decreases were chosen.

As previously, on the basis of these groups, the accelerational and tangential components of the geostrophic deviations were graphically computed, the difference between the observed and geostrophic wind speeds taken, and the algebraic means of each determined at the three latitudes. The results appear in Table V.

Table VII indicates the number of positive, negative and zero values used in each individual computation found in Table V. All mean values are given in miles per hour.

## D. Representativeness of the Data

Some indication as to the representativeness of the data used in arriving at the final values can be deduced from a review of the technique employed in making the observed and the geostrophic measurements, and by noting the number of values involved in each final result.

The individual observed values of wind speed taken from the pilot-balloon reports may possibly be in error from ± 2.2 to ± 4.5 miles per hour (10) at the 700 millibar level, while the direction error may be from ± 2 to ± 4 degrees. Studies mentioned by Neiburger (11) indicate that the standard deviation of single-theodolite observations from double-theodolite observations is from approximately 2 to 3 meters per second, and that errors of RAWIN observations using the SCR 658 radio direction-finding equipment are about the same percentage. With a large number of cases such error should cancel. This should be the case also as regards the measurements of the geostrophic wind directions and speeds from the 700 millibar charts. Over the two-year period the maps used were analyzed by a number of different persons, and the subjective error which arises in making the measurements of the geostrophic values should also cancel when a relatively large number of figures are used in computing the final mean values. Table IV shows that the smallest number of terms used in computing an algebraic mean was 234 in the case of the intermediate indices at latitude $LL_{+5}$, while the largest was 355 terms in the case of the low indices at latitude $LL_{-5}$. Altogether, a total of 2661 complete observations were used and 7983 computations involved in arriving at the final values in Table I.

Table VII shows that the values determined in Table V, although computed from fewer terms, can still be considered adequately representative. In this case there were only 4 mean values out of 18 where the number of terms used fell between 40 and 50, the average being over 60 for each mean value computed. Over the two year period a total of 1186 observations were used and 3558 computations involved in arriving at the final values in Table V.

Consequently, it is believed that the number of terms used in the determination of the mean values was sufficiently large to effectively eliminate the subjective errors of analysis and measurements so that the final results are representative.

The Student's "t" test (12) was applied to a number of the computed mean values in order to determine their significance, the results of which are found in Table V.

## E. Presentation of Results

### 1. The comparison of high, intermediate, and low index.

Table I contains the results of the first phase of this investigation. It shows the break-down of the two-year period into low, intermediate, and high values of the indices, and the corresponding values of the geostrophic deviation components, (the accelerational and tangential components), and the differences between the absolute values of the observed velocities and the geostrophic velocities. These values are indicated at the latitude of the index and at latitudes approximately 5 degrees on either side of this mid-latitude.

Some conclusions can be drawn from the data in Table I.

(a) In all cases the wind flow is indicated as being sub-geostrophic. This is shown by the negative signs of the tangential components of the geostrophic deviations, and by the fact that the signs of the third columns ($V-V_{gs}$) which denote the difference between the absolute values of the observed wind speeds and the geostrophic wind speeds are also predominately negative.

## TABLE 1

Mean Values of Geostrophic Deviation Components and Observed Less Geostrophic Wind Speeds. All Seasons 1946-47, 3 kilometer level (in m.p.h.)

| | $LL_0$ | | | $LL_{-5}$ | | | $LL_{+5}$ | | |
|---|---|---|---|---|---|---|---|---|---|
| INDICES | Accel. Comp. | Tan. Comp. | $V-V_{gs}$ | Accel. Comp. | Tan. Comp. | $V-V_{gs}$ | Accel. Comp. | Tan. Comp. | $V-V_{gs}$ |
| Low | -0.7 | -3.5 | -1.1 | -0.4 | -4.9 | -2.7 | -0.1 | -1.8 | -0.2 |
| Intermediate | -1.0 | -4.8 | -2.7 | -0.1 | -2.4 | -0.5 | 0.0 | -1.7 | +0.5 |
| High | -0.2 | -3.9 | -2.5 | -0.5 | -5.1 | -3.6 | -0.6 | -2.8 | -0.7 |
| Averages by latitudes | -0.6 | -4.1 | -2.1 | -0.3 | -4.1 | -2.3 | -0.2 | -2.1 | -0.1 |
| Averages all latitudes | -0.4 | -3.4 | -1.5 | | | | | | |

## TABLE II

### Seasonal Frequencies of Indices
### 1946-47

| Indices (m.p.s.) | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
|---|---|---|---|---|---|---|---|---|---|---|
| Fall Sept., Oct., Nov. |  | 5 | 5 | 11 | 11 | 12 | 6 | 1 | 1 |  |
| Winter Dec., Jan., Feb. |  |  |  |  | 2 | 6 | 15 | 16 | 8 | 4 |
| Spring Mar., Apr., May | 1 | 4 | 5 | 12 | 4 | 7 | 10 | 5 | 4 | 1 |
| Summer June, July, Aug. | 1 | 12 | 15 | 14 | 8 | 1 |  |  |  |  |

## TABLE III

### Grouping of Indices

|  |  | Low | Inter-mediate | High |
|---|---|---|---|---|
| Fall | Indices | 6,7,8 | 9 | 10,11,12,13 |
|  | Cases | 21 | 11 | 20 |
| Winter | Indices | 9,10, 11 | 12 | 13,14 |
|  | Cases | 23 | 16 | 12 |
| Spring | Indices | 5,6,7,8 | 9,10 | 11,12,13,14 |
|  | Cases | 22 | 11 | 20 |
| Summer | Indices | 5,6 | 7 | 8,9,10,11 |
|  | Cases | 13 | 15 | 23 |

## TABLE IV

Number of Computations Used in Table I

| INDICES | | $LL_0$ | | | $LL_{-5}$ | | | $LL_{+5}$ | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | Accel. Comp. | Tan. Comp. | $V-V_{gs}$ | Accel. Comp. | Tan. Comp. | $V-V_{gs}$ | Accel. Comp. | Tan. Comp. | $V-V_{gs}$ |
| Low | (-) | 126 | 205 | 126 | 153 | 243 | 224 | 118 | 168 | 149 |
| | (+) | 118 | 107 | 177 | 134 | 102 | 111 | 115 | 120 | 133 |
| | (0) | 76 | 8 | 17 | 68 | 10 | 20 | 68 | 13 | 19 |
| | Totals | 320 | 320 | 320 | 355 | 355 | 355 | 301 | 301 | 301 |
| Inter-mediate | (-) | 110 | 152 | 142 | 110 | 161 | 142 | 104 | 130 | 110 |
| | (+) | 90 | 78 | 88 | 114 | 112 | 129 | 88 | 89 | 110 |
| | (0) | 42 | 12 | 12 | 63 | 14 | 16 | 42 | 15 | 14 |
| | Totals | 242 | 242 | 242 | 287 | 287 | 287 | 234 | 234 | 234 |
| High | (-) | 114 | 181 | 174 | 144 | 221 | 203 | 133 | 178 | 165 |
| | (+) | 112 | 101 | 110 | 117 | 94 | 112 | 117 | 112 | 124 |
| | (0) | 68 | 12 | 10 | 69 | 15 | 15 | 48 | 8 | 9 |
| | Totals | 294 | 294 | 294 | 330 | 330 | 330 | 298 | 298 | 298 |

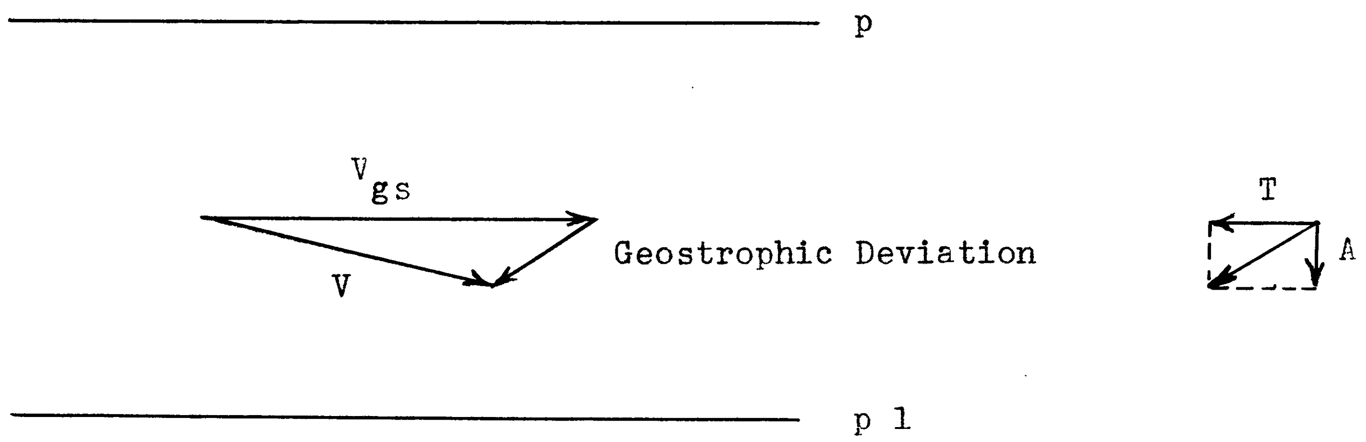
(b) Another characteristic of this data is the fact that all signs of the accelerational components of the geostrophic deviations are negative. This indicates a component of flow of air across the pressure gradient from low to high pressure.

Table I also gives the averages of low, intermediate, and high index values by latitude, and the over-all average for the two-year period, all seasons, for the components of the geostrophic deviation and the $V-V_{gs}$ values. A model of the over-all condition indicated in Table I would be as shown in Figure I, (not to scale), where $V_{gs}$ and V represent geostrophic flow and observed flow respectively.

Figures II, III, and IV are velocity profiles which indicate the variations of the accelerational and tangential components for low, intermediate and high index classifications, and at all three latitudes. As in Figure I (b) T and A represent respectively the tangential and the accelerational components of the geostrophic deviation.

It may be a significant point that the over-all average value of the accelerational component for low, intermediate, and high indices, all latitudes, for the two year period is -0.4 miles per hour, a relatively small cross-pressure gradient component toward increasing elevation. Figure V shows the average picture of the

FIGURE I



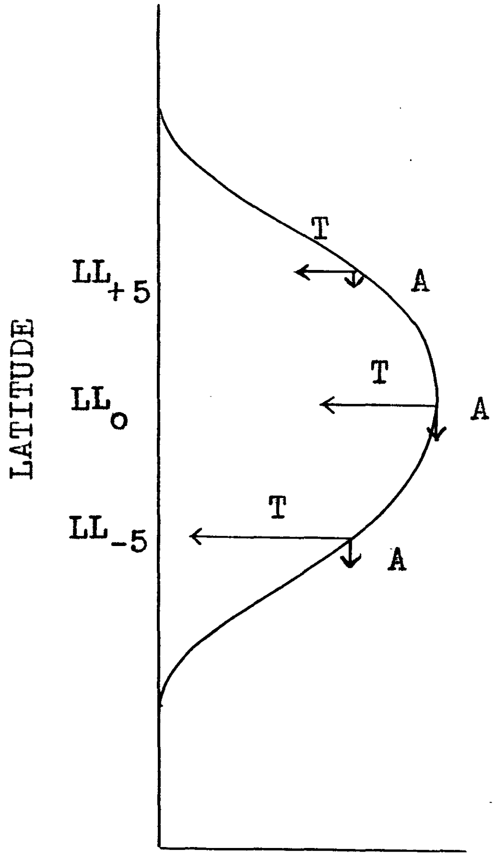
(a)

OBSERVED AND GEOSTROPHIC FLOW
AT 3 KILOMETER LEVEL
(NOT TO SCALE)
PERIOD I946-47

(b)

COMPONENTS OF THE
GEOSTROPHIC DEVIATION
T-TANGENTIAL
A-ACCELERATIONAL

FIGURE II



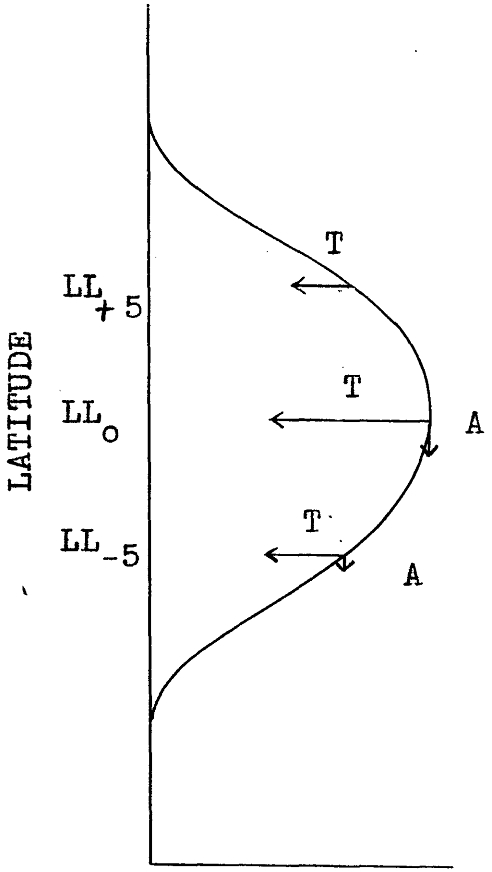
WESTERLY FLOW PROFILE

LOW INDEX

PERIOD I946-47

3 KILOMETER LEVEL

FIGURE III



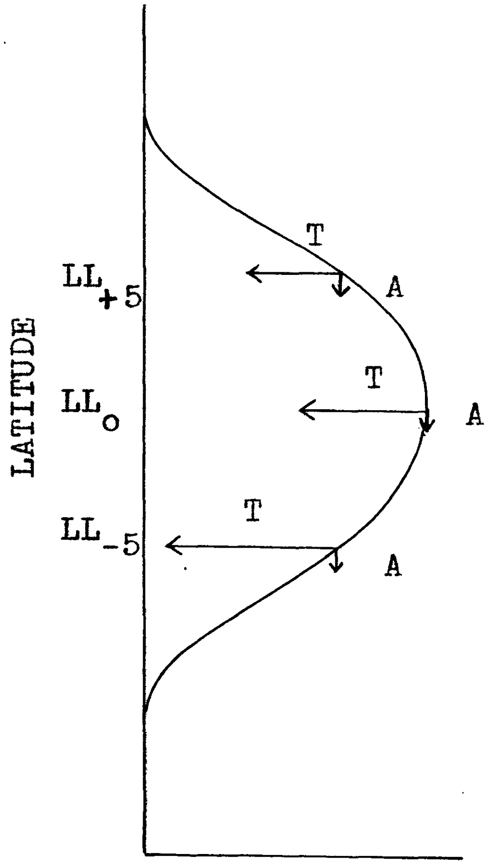
WESTERLY FLOW PROFILE

INTERMEDIATE INDEX

PERIOD 1946-47

3 KILOMETER LEVEL

FIGURE IV



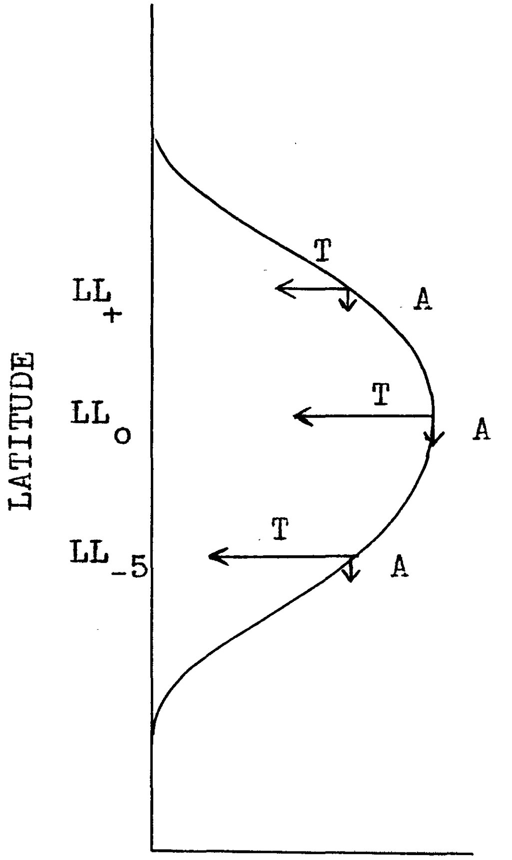
WESTERLY FLOW PROFILE

HIGH INDEX

PERIOD 1946-47

3 KILOMETER LEVEL

FIGURE V



WESTERLY FLOW PROFILE

ALL INDICES

PERIOD 1946-47

3 KILOMETER LEVEL

geostrophic deviation components for all index values at the significant latitudes.

2. The comparison between increasing and decreasing index.

In Table V are found the relationships existing between increasing and decreasing index values and the components of the geostrophic deviations.

(a) If the larger increases or decreases are considered as most representative of their respective groups, it is seen that for the increasing index, all latitudes, the accelerational components are negative, indicating cross-gradient flow from low to higher elevation, while for the decreasing index these components have positive signs with consequent flow from high to lower elevation. The Student's "t" test (12), which shows the probability of accidental occurence, was applied to the mean values of the accelerational components for large, increasing and decreasing index, at each latitude, the results being shown in Table V beneath each value thus tested.

(b) The wind flow for both increasing and decreasing index is predominately sub-geostrophic as indicated by the preponderance of negative, tangential component values.

Figures VI and VII are velocity profiles which indicate the variations of the geostrophic deviation components for large increasing and for large decreasing index.

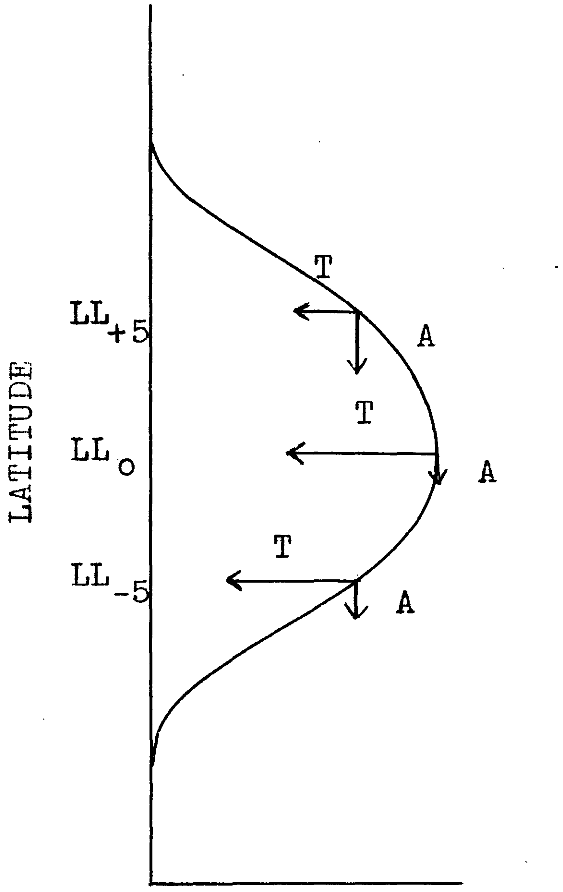
It is of interest to note here that whereas the

# TABLE V

Mean Values of Geostrophic Deviation Components and
Observed Less Geostrophic Wind Speeds
All seasons 1946-47 (in m.p.h.)

| INCREASING INDEX | $LL_0$ | | | $LL_{-5}$ | | | $LL_{+5}$ | | |
|---|---|---|---|---|---|---|---|---|---|
| Indices | Accel. Comp. | Tan. Comp. | $V-V_{gs}$ | Accel. Comp. | Tan. Comp. | $V-V_{gs}$ | Accel. Comp. | Tan. Comp. | $V-V_{gs}$ |
| Small | -4.0 | -4.3 | -1.9 | -1.0 | -4.3 | -2.7 | -2.0 | -3.2 | -1.1 |
| Inter-mediate | -0.1 | -1.0 | +1.0 | -1.0 | -6.1 | -4.2 | -0.2 | -4.2 | -2.3 |
| Large | -0.5 | -5.4 | -1.9 | -0.9 | -3.9 | -1.9 | -1.7 | -1.9 | -4.7 |
| Student's "t" Test (large) | 30% | | | 20% | | | 6% | | |
| DECREASING INDEX | | | | | | | | | |
| Small | -0.05 | -4.0 | -2.1 | 0.0 | -4.4 | -3.0 | -1.1 | -1.7 | +0.7 |
| Inter-mediate | -2.9 | -3.6 | -1.5 | +1.3 | -5.5 | -3.2 | +2.3 | -0.2 | +1.3 |
| Large | +1.9 | -8.7 | -6.8 | +1.1 | -3.8 | -1.8 | +0.1 | -0.1 | +1.6 |
| Student's "t" Test (large) | 10% | | | 24% | | | 45% | | |

FIGURE VI



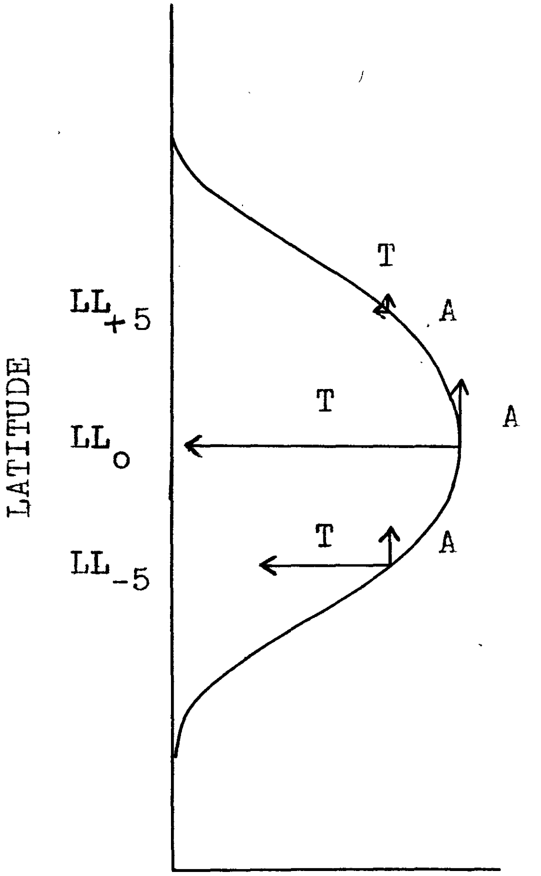
WESTERLY FLOW PROFILE

LARGE, INCREASING INDEX

PERIOD 1946-47

3 KILOMETER LEVEL

FIGURE VII



WESTERLY FLOW PROFILE

LARGE, DECREASING INDEX

PERIOD 1946-47

3 KILOMETER LEVEL

## TABLE VI

### Determination of Degrees of Increasing and Decreasing Indices

| | Small<br>0-- 1.49 | | | Intermediate<br>1.5 - 2.49 | | Large<br>2.5 - 5.49 | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | 0-0.49 | 0.5-0.99 | 1.0-1.49 | 1.5-1.99 | 2.0-2.49 | 2.5-2.99 | 3.0-3.49 | 3.5-3.99 | 4.0-4.49 | 4.5-4.99 | 5.0-5.49 |
| Increasing indices | 2 | 6 | 8 | 7 | 11 | 7 | 2 | 2 | 3 | 3 | 0 |
| Decreasing indices | 2 | 7 | 7 | 8 | 9 | 3 | 2 | 1 | 2 | 0 | 2 |

# TABLE VII

## Number of Computations Used in Table V

| INCREASING INDEX | | $LL_0$ | | | $LL_{-5}$ | | | $LL_{+5}$ | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Indices | | Accel. Comp. | Tan. Comp. | $V-V_{gs}$ | Accel. Comp. | Tan. Comp. | $V-V_{gs}$ | Accel. Comp. | Tan. Comp. | $V-V_{gs}$ |
| Small | (-) | 26 | 30 | 27 | 31 | 48 | 43 | 26 | 31 | 29 |
| | (+) | 11 | 15 | 16 | 30 | 26 | 29 | 18 | 19 | 22 |
| | (0) | 10 | 2 | 4 | 16 | 3 | 5 | 9 | 3 | 2 |
| | Totals | 47 | 47 | 47 | 77 | 77 | 77 | 53 | 53 | 53 |
| Inter-mediate | (-) | 21 | 27 | 24 | 26 | 46 | 45 | 19 | 32 | 32 |
| | (+) | 22 | 26 | 29 | 22 | 18 | 18 | 22 | 21 | 21 |
| | (0) | 12 | 2 | 2 | 16 | 0 | 1 | 12 | 0 | 0 |
| | Totals | 55 | 55 | 55 | 64 | 64 | 64 | 53 | 53 | 53 |
| Large | (-) | 36 | 61 | 54 | 41 | 66 | 61 | 44 | 48 | 38 |
| | (+) | 36 | 27 | 32 | 33 | 32 | 36 | 24 | 26 | 33 |
| | (0) | 19 | 3 | 5 | 26 | 2 | 3 | 11 | 5 | 8 |
| | Totals | 91 | 91 | 91 | 100 | 100 | 100 | 79 | 79 | 79 |
| DECREASING INDEX | | | | | | | | | | |
| Small | (-) | 31 | 49 | 40 | 34 | 57 | 53 | 35 | 40 | 38 |
| | (+) | 32 | 28 | 33 | 34 | 24 | 27 | 25 | 32 | 33 |
| | (0) | 17 | 3 | 7 | 15 | 2 | 3 | 15 | 3 | 4 |
| | Totals | 80 | 80 | 80 | 83 | 83 | 83 | 75 | 75 | 75 |
| Inter-mediate | (-) | 32 | 37 | 33 | 25 | 43 | 38 | 24 | 35 | 30 |
| | (+) | 18 | 22 | 29 | 33 | 23 | 28 | 26 | 27 | 29 |
| | (0) | 15 | 6 | 3 | 11 | 3 | 3 | 13 | 1 | 4 |
| | Totals | 65 | 65 | 65 | 69 | 69 | 69 | 63 | 63 | 63 |
| Large | (-) | 17 | 34 | 35 | 21 | 25 | 24 | 17 | 22 | 21 |
| | (+) | 19 | 10 | 9 | 17 | 20 | 20 | 16 | 19 | 21 |
| | (0) | 8 | 0 | 0 | 8 | 1 | 2 | 9 | 1 | 0 |
| | Totals | 44 | 44 | 44 | 46 | 46 | 46 | 42 | 42 | 42 |

over-all picture for low, intermediate, and high indices as indicated in Table I illustrates a cross-gradient flow toward higher pressure at the 3 kilometer level, Table V shows cross-gradient flow toward high pressure for cases of increasing index, and cross-gradient flow toward low pressure for cases of decreasing index.

It will be remembered that Namias found for westerly flow along isentropic surfaces that at points above and below the point of maximum flow velocity there were eastward accelerations parallel to the flow axes, and southward accelerations normal to such axes. At the point of maximum velocity of the eastward flow he found accelerations opposite and of larger magnitude than those on either side.

The application of the Student's "t" test to the difference between the means of the accelerational components for large, increasing and large, decreasing index at $LL_O$ showed a probability of accidental occurrence of 7%.

Table VIII indicates the mean values of the components of the geostrophic deviations for the summer months (June, July, August, and September) and for the winter months (December, January, and February) for the years 1946-47. N indicates the number of cases used in the computations. It is seen that for both seasons the cross-contour flow is directed toward higher elevations,

# TABLE VIII

## Mean Values of Components of Geostrophic Deviations, and Velocity Differences

Period 1946-47

3 kilometer level

| | $LL_0$ | | | $LL_{-5}$ | | | $LL_{+5}$ | | |
|---|---|---|---|---|---|---|---|---|---|
| | Accel. Comp. | Tan. Comp. | $V-V_{gs}$ | Accel. Comp. | Tan. Comp. | $V-V_{gs}$ | Accel. Comp. | Tan. Comp. | $V-V_{gs}$ |
| Summer (June, July, Aug., Sept.) | -0.9 | -4.0 | -1.2 | -0.2 | -3.5 | -1.4 | +0.2 | -2.7 | -0.5 |
| N | 218 | 218 | 218 | 257 | 257 | 257 | 213 | 213 | 213 |
| Average, all latitudes | -0.3 | -3.4 | | | | | | | |
| Winter (Dec., Jan., February) | -1.6 | -4.6 | -3.7 | -1.0 | -3.0 | -1.6 | -1.3 | -1.6 | 0.0 |
| N | 190 | 190 | 190 | 226 | 226 | 226 | 187 | 187 | 187 |
| Average, all latitudes | -1.3 | -3.0 | | | | | | | |

the value for the winter season being substantially a larger negative number than that for the summer season, while the wind flow in all cases is indicated as being sub-geostrophic.

Machta (9) made a similar study of the geostrophic deviation components for winter and summer. He used data for the months of June, July, August, and September of 1947 for the summer classification, and data for the months of January, 1946, December, 1946, and January, 1947 for the winter classification. At the 700 millibar level he found a mean value of the accelerational component of + 0.4 miles per hour for the winter period, and +0.5 miles per hour for the summer months indicating cross-contour flow down-gradient. The tangential component means were negative for both seasons indicating sub-geostrophic flow. It is seen that the cross-gradient flow as found in this study disagrees with that found by Machta, while in both cases sub-geostrophic winds were indicated.

## F. Summary and Conclusions

The preceding information can be summarized generally as follows:

(1) The average of the components of the geostrophic deviations along the gesotrophic flow is not of the same magnitude as that of the components normal to such flow. This emphasizes the relative importance of changes in speed as compared to direction changes.

(2) The components parallel to geostrophic flow are predominately negative indicating sub-geostrophic flow, the magnitude of the components averaging somewhat larger in summer than in winter.

(3) The flow normal to the contours is predominately negative or toward higher elevations for the entire period. This is also true for both winter and summer seasons, the magnitude for the winter season being considerably larger than that for the summer season.

(4) For increasing and decreasing index the flow along contours is predominately negative or sub-geostrophic.

(5) The cross-contour flow is negative for increasing index, and positive for decreasing index. Considering the $LL_0$ values only for the increasing and decreasing index, it is seen that the magnitude for the decreasing case is considerably larger than that for the increasing case. The average values of the accelerational components for large, increasing and decreasing index, all latitudes, are -1.0 for increasing index and +1.0 for decreasing index, while for all indices, all latitudes the average values were -1.3 for increasing index, and + 0.9 for decreasing index.

It should be remembered at this point that the above conditions pertain to the zonal flow at approximately 3 kilometers above sea level, and should not be construed as necessarily representative of the zonal wind flow at any other level or levels.

If it can be assumed that the results found in the $LL_O$ column for large increasing and decreasing index values are best representative of what is the true picture of the zonal wind flow at the 700 millibar level, it appears that the most important over-all conclusion to be drawn is that apparently the pressure field is a consequence of the wind field. It would seem that any increase in the value of the index would be indicative of an increasing pressure gradient, which in turn would result in an accelerational force normal to the flow and down-gradient, with consequent down-gradient flow or flow toward lower pressure. Conversely, any loosening or decrease of the pressure gradient should result in cross-gradient flow up-gradient or flow toward higher pressure. The initiating force in each case would be that of the pressure gradient. For increasing index or increasing north-south pressure gradient the value of the accelerational component is relatively small negative indicating cross-contour flow up-gradient, while the tangential component indicates sub-geostrophic flow. For decreasing index or decreasing north-south pressure gradient, the value of the accelerational component is relatively large, positive, indicating cross-contour flow down gradient, while the tangential component is relatively more sub-geostrophic. It seems this would indicate that essentially, the wind field precedes the pressure field. The same picture is somewhat less emphatically indicated when all three latitudes for the large, increasing and large, decreasing indices are averaged and compared, the magnitudes being approximately equal, with opposite sign.

It is recommended that more extensive studies should be made with respect to the geostrophic deviations, at other levels as well as the one chosen in this investigation, and using daily indices instead of 5 day mean indices. Such studies should become more revealing as the upper wind observational techniques are improved with possibly more precise results being obtained in efforts to relate the intensity of the circumpolar vortex to the accelerational field in the vicinity of the maximum westerlies.

# BIBLIOGRAPHY

1. Staff Members of the Department of Meteorology of the University of Chicago, "On the General Circulation of the Atmosphere in Middle Latitudes", Bulletin of the American Meteorological Society, Volume 28, June, 1947, Number 6.

2. Rossby, C. G., and Willett, H. C., "The Circulation of the Upper Troposphere and Lower Stratosphere", Reprinted from SCIENCE, December 10, 1948, Volume 108, Number 2815, pages 643-652.

3. Rossby, C. G., "Yearbook of Agriculture", Washington, D. C., U. S. Department of Agriculture, 1941, pages 599-655.

4. Rossby, C. G., "On the Distribution of Angular Velocity in Gaseous Envelopes Under the Influence of Large-Scale Horizontal Mixing Processes", Bulletin of the American Meteorological Society, February, 1947, Volume 28, Number 2.

5. Starr, Victor P., Journal of Meteorology, 1948, Volume 5, Number 2, pages 39-43.

6. Willett, H. C., "Final Report of the Weather Bureau-Massachusetts Institute of Technology Extended Forecasting Project for the Fiscal Year July 1, 1946-June 30, 1947. Cambridge, Massachusetts, M. I. T., 1947.

7. Namias, J., "Isentropic Analysis", from Petterssen, WEATHER ANALYSIS AND FORECASTING, Chapter VIII.

8. Weather Bureau, Extended Forecast Section, Massachusetts Institute of Technology, 1948.

9. Machta, "A Study of the Observed Deviations from the Geostrophic Wind", Doctor's Thesis, Massachusetts Institute of Technology, 1948, (Unpublished).

10. Middleton, METEOROLOGICAL INSTRUMENTS, page 176.

11. Neiburger, Sherman, Kellogg and Gustafson, "On the Computation of Wind from Pressure Data", Journal of Meteorology, Volume 5, Number 3, June, 1948, pages 87-92.

12. Hoel, P. G., INTRODUCTION TO MATHEMATICAL STATISTICS, pages 140-149.