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On the Number of Wavelengths Needed in WDM Networks

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Abstract

We present bounds on the minimum number of wavelengths needed in an all optical WDM network using passive wavelength routing, frequency changing devices, and switching. Networks with near optimal wavelength re-use efficiency are presented.

Bounds on the Number of Wavelengths Needed in WDM Networks [1]

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1 Introduction

Consider the all optical network (AON) shown in Fig. 1a. The λ-nodes selectively route the signals on the input links to the output links based on wavelength only. We assume that the λ-nodes cannot be reconfigured, thus if there is a path from transmitter n to receiver m on wavelength λ, there is always such a path even if there is no traffic between n and m. Paths for three wavelengths, Red, Green, and Blue are shown in Fig. 1b. The only freedom, after the network topology (fig. 1a) and wavelength paths (fig. 1b) have been determined, is in the tuning of the transmitters and receivers. Note that this network can support any matching of the transmitters to two of the receivers (with no multi-casting) except the matching $T = \{(1, B), (2, C)\}$. The reason being that $(1, B)$ and $(2, C)$ must both use Red simultaneously. Since there is a path from 2 to B on Red, the two sessions collide at receiver B. This does not happen for any other pair.

In section 2, we present a lower bound on the number of wavelengths required to avoid collisions. This bound holds for any network topology with λ-routing and wavelength changing devices. We do not restrict ourselves, as we did in the example above, to traffics without multicasting.

We will see that for very large networks, $> 10^6$ users, λ-routing cannot provide full connectivity without some possibility of collisions. λ-routing can be combined with conventional circuit switching to produce networks of the type proposed by Stern [3]. In these networks, the λ-paths are reconfigurable. In section 3, we present a lower bound on the number of switching states required for a network with M users and F wavelengths. We show that unless $F \approx \sqrt{M}$, the number of switching states cannot be dramatically reduced from that of a conventional circuit switched network (F=1).

2 A Bound on the Required Number of Wavelengths

Let $s = (n, m)$ be a *session*, where n is a transmitter and m is a receiver. A *traffic*, T , is defined as a set of sessions. A receiver can be active in at most one session of a Traffic. A transmitter is permitted to be active in more than one session (multicasting). A *traffic set*, $\mathcal{T}$, is a set of traffics. The traffics, $T \in \mathcal{T}$ are the *allowable* traffics. Let $F(\mathcal{T})$ be the minimum number of wavelengths for any switchless AON needed to support $\mathcal{T}$ without contention. Then, we will show that

$$F(\mathcal{T}) \quad \geq \quad \left[\binom{M}{\rho M}^{-1} \sqrt{|\mathcal{T}|} \right]^{\frac{1}{\rho M}} - 1 \tag{1}$$

where ρM is the maximum number of sessions that can be on at any time. $0 \leq \rho \leq 1$ is called the maximum utilization of the network.

Using eqn. 1, or variations of it, the number of wavelengths required for different $\mathcal{T}$ can be bounded. Three examples, all with maximum utilization, $\rho = 1$, are listed below. Upper bounds are obtained by construction. In each example, the best bounds known are presented. The last example shows that the lower bound can be very tight.

Arbitrary: $\sqrt{M} \leq F(\mathcal{T}) \leq M$

Each receiver can be paired with any transmitter.

Permutations: $\sqrt{\frac{M}{e}} \leq F(\mathcal{T}) \leq \lceil \frac{M}{2} \rceil + 2$

Arbitrary connections between transmitters and receivers without multicasting [2].

Blocking: $\frac{\sqrt{M}}{e} \leq F(\mathcal{T}) \leq \sqrt{M}$

Group the transmitters into disjoint sets, T-LANS, of size $\sqrt{M}$. Similarly group the receivers into R-LANS. Allow at most one active session between any [T-LAN,R-LAN] pair. A typical traffic is shown in Fig. 2. The upper bound construction can be implemented by making each T-LAN (R-LAN) a broadcast star.

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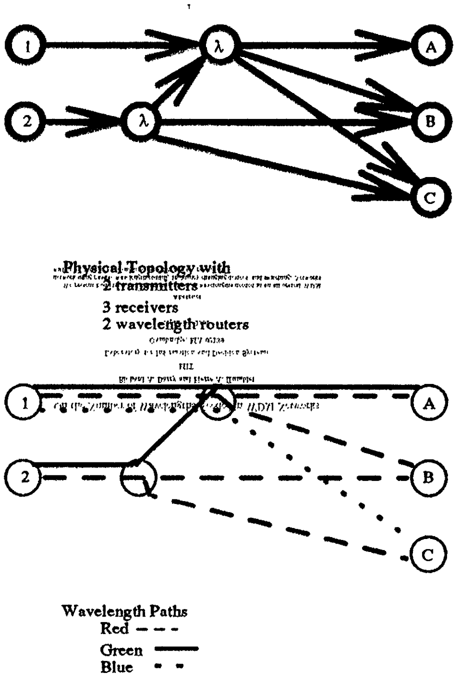

Fig. 1a,b

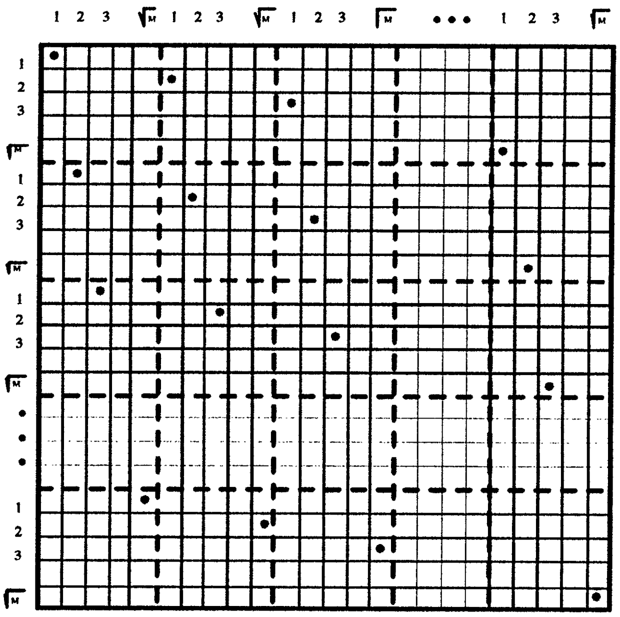

Fig. 2

3 Switching States

Now consider an AON with λ-routing, wavelength changers, and switches. Let S be the number of switching states. Then for permutation routing, the number of states is bounded below by

$$\log_2 S \quad \geq \quad M \log_2 M - 2M \log_2 F - 1.44M \tag{2}$$

where F is the number of wavelengths. When $F = 1$, this agrees with the well known formula for the minimum complexity of an MxM non-blocking switch [1]. It can be seen that λ-routing cannot dramatically reduce the number of switching states unless $F \approx \sqrt{M}$.

4 Conclusions

Without switches, about F^2 users can be supported with F wavelengths with minimal blocking. For instance, dividing the fiber into 1000, 1 *Gbps* channels and using λ-routing, we can design a partially blocking network (Fig. 2) with 10^6 users. The maximum throughput is 1000 *Tbps*. Instead of blocking, wavelengths can be shared by using TDMA. In the worst case, 1000 users from one LAN connecting to 1000 users from another LAN, the user data rate is limited to 1 *Mbps*. It may be possible to design a switchless non-blocking fully connected network with F^2 users. This is an open question.

For networks with $\approx 10^8$ users, switches will be necessary. The main advantage here of combining λ-routing and switching is to reduce the frequency of changing states. This is because each state can handle many traffics (up to $\approx 10^6$!). This may greatly simplify control of the network.

References

[1] Joseph Y. Hui. *Switching and Traffic Theory for Integrated Broadband Networks.* Kluwer Academic Publishers, 1990.

[2] Thomas F. Leighton. *Introduction to Parallel Algorithms and Architectures: Arrays, Trees, Hypercubes.* Morgan Kaufmann Publishers, 1992.

[3]