DYNAMICS OF THE INDUCTION SYSTEM OF AN INTERNAL COMBUSTION ENGINE

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I. INTRODUCTION

The power output of an internal combustion engine is directly proportional to the amount of air that can be forced into the cylinder per cycle. This amount is most effectively increased by means of a mechanical supercharger. It may also be increased by the selection of a suitable length for the intake pipe. Previous investigators have found that the amount of air forced into the cylinder can be increased almost twenty per cent by this method. The value of such an increase with no increase in the mechanical complexity of the engine is easily recognized.

The previous work that has been done on the effect of the intake pipe has not been very complete, nor has it resulted in a satisfactory picture of the events taking place within the pipe. For this reason the author decided that an investigation of the pressures in the intake pipe was of paramount importance. The first step in the research was to secure a pressure-time recording instrument and mount it in the valve port of an engine. This, apparently, was the first time that any such record had been taken.

The engine selected for the research was the N.A.C.A. Universal test engine. It was selected as it had adjustments for varying the valve lift and timing and the compression ratio. The pressure-time recording instrument available was the M.I.T. High Speed Engine Indicator which was developed by Professors C. S. Draper and E. S. Taylor.

Typical records of the pressure time variation in the valve port are shown in Fig. 1. The ordinate represents the pressure in pounds per square inch and the abscissa is the time for one revolution of the engine. The two traces occur on the record as the time for a complete valve cycle is twice that for one engine revolution. As several hundred valve cycles are completed in the course of making this record, it is apparent that some sort of steady state pressure wave phenomena is occurring in the intake pipe.

A little consideration of the valve-piston mechanism of the engine shows that its action is conducive to the generation of pressure waves. A schematic diagram of the engine and the variation of the air velocity through the intake port is shown in Fig. 2. The effect of the valve-piston mechanism is evidently to send a pressure pulse out toward the open end of the intake pipe. This pulse is sent out periodically each time that the valve is opened. Just as the valve opens the velocity of the air in the inlet port is zero. An instant later air has started flowing into the cylinder; and its velocity increases to a maximum and again becomes zero when the valve closes. There is no velocity in the port until the valve reopens again. This periodic "driving mechanism" will set up forced standing waves of sound in the intake pipe.

TYPICAL INDICATOR CARDS

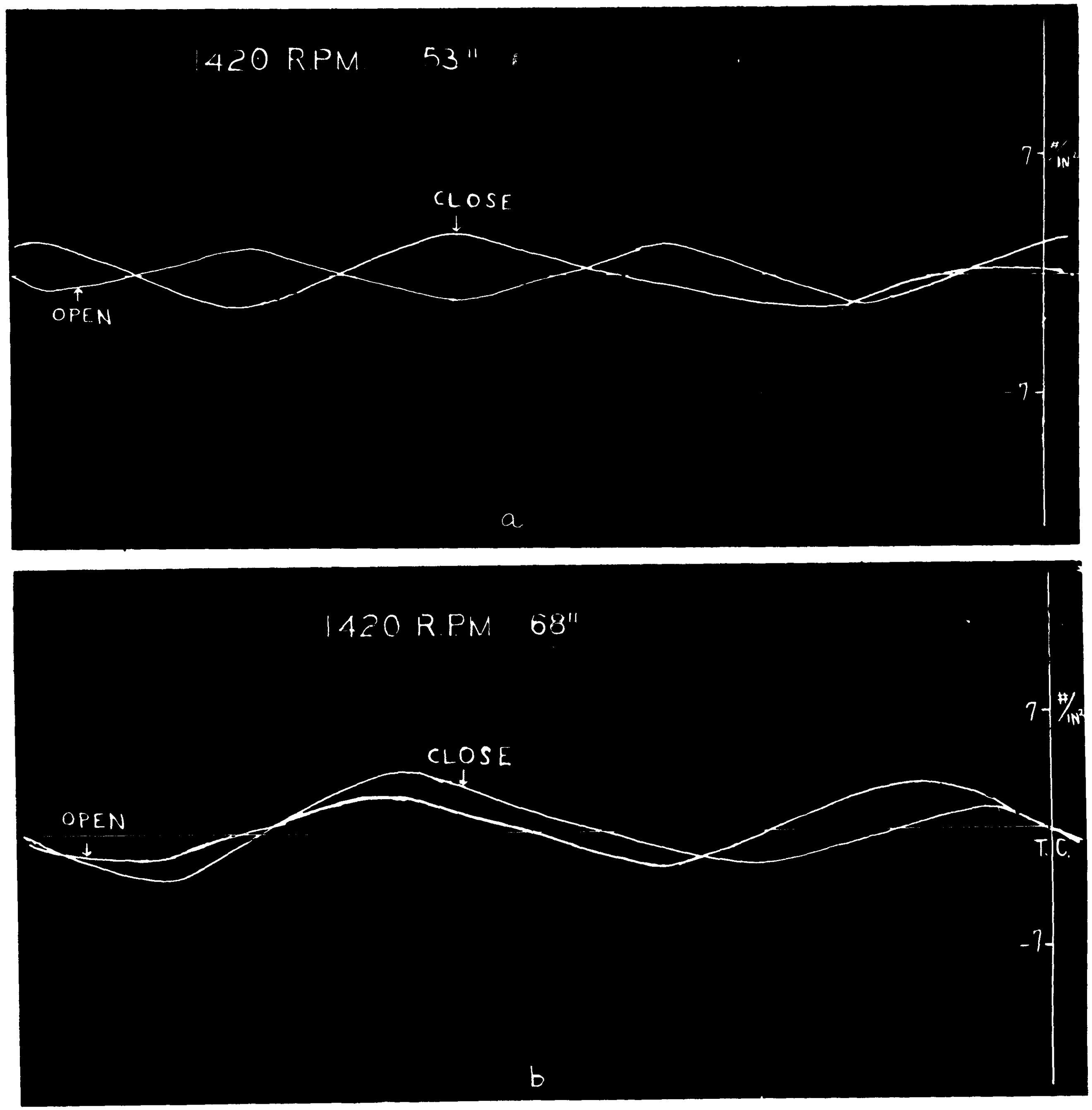


FIG. 1

VALVE ACTION & AIR VELOCITY DURING INTAKE STROKE

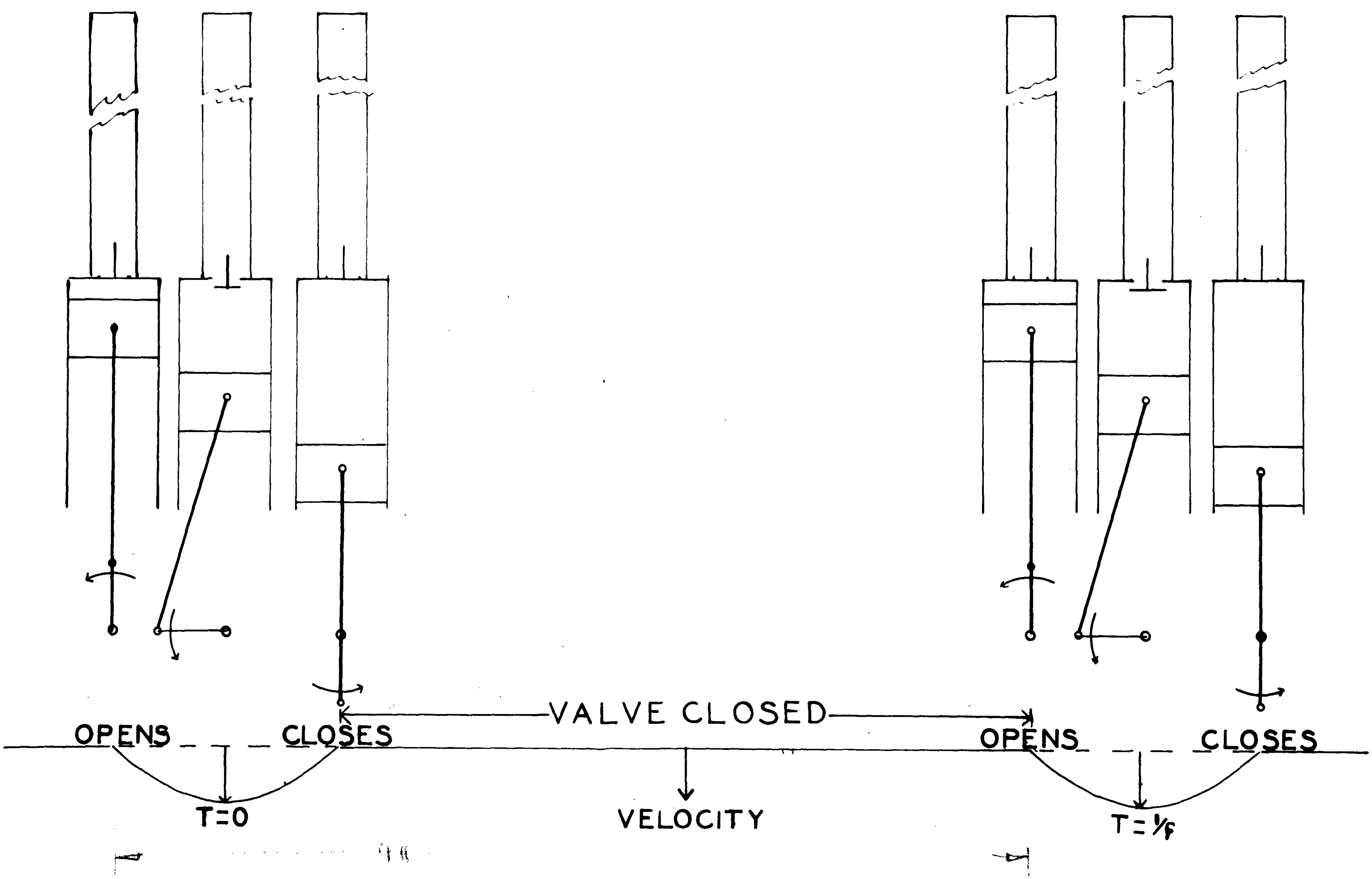


FIG. 2

Sound theory is developed on the assumption that the space variation of the velocity and of the condensation due to any excess pressure is small. This is not exactly true in the case in question, but it is nevertheless a good approximation. The lengths of pipe for which supercharging occurs are long. The pressure at the intake port may be considerably above or below atmospheric pressure, but at the open end of the pipe the pressure is always atmospheric. For this reason the rate of variation of the condensation along the pipe due to any excess pressure is small. Therefore it seemed advisable to use the simple sound theory to study the progress of the pulse of excess pressure through one valve cycle for several resonance cases.

At the open end of the pipe there is no excess pressure, and a positive pressure is reflected as a negative pressure, while a velocity pulse is not changed in phase. At the valve end, when it is closed, a pressure pulse is reflected with no change in phase, but a positive velocity pulse is reflected with change of phase.

The velocity at the valve end is assumed to be a simple sinusoidal function when the valve is open. The assumption is also made that its maximum velocity will occur when the piston velocity is a maximum. This is not exactly correct, as will be discussed later. In this case the velocity may be expressed as the following function of time:

$$u_0 = U_0 \cos 4\pi ft \quad \text{when the valve is open,}$$

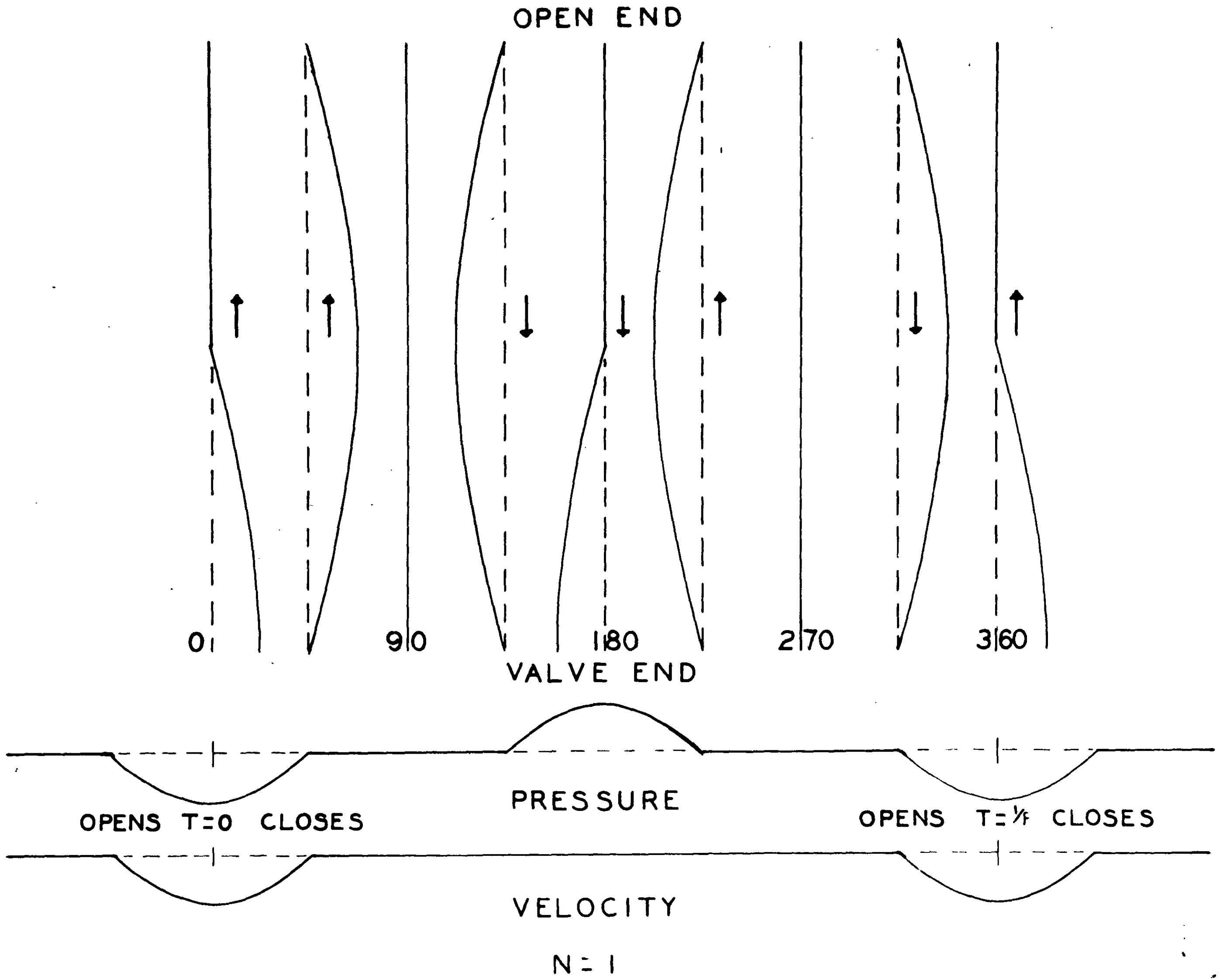
$$= 0 \quad \text{when the valve is closed.}$$

f = valve frequency.

The time origin is then at the time when the piston velocity is a maximum. The simplest case that can be discussed is that in which the valve opens and closes at top center and bottom center of the engine, respectively. This case may also be easily experimentally realized on the N.A.C.A. engine.

Fig. 3 (pipe freq. = f) is a schematic diagram of the pressure distribution in the intake pipe at time intervals which are one-eighth the period of the valve mechanism. In the case shown the frequency of the pipe is equal to the frequency of the valve mechanism. The assumed velocity variation with time at the intake port is shown at the bottom of the figure and the resulting pressure-time variation at the valve end is indicated immediately above. The direction in which the pulse is traveling along the pipe is indicated by the arrows. The dotted lines represent the equilibrium atmospheric pressure.

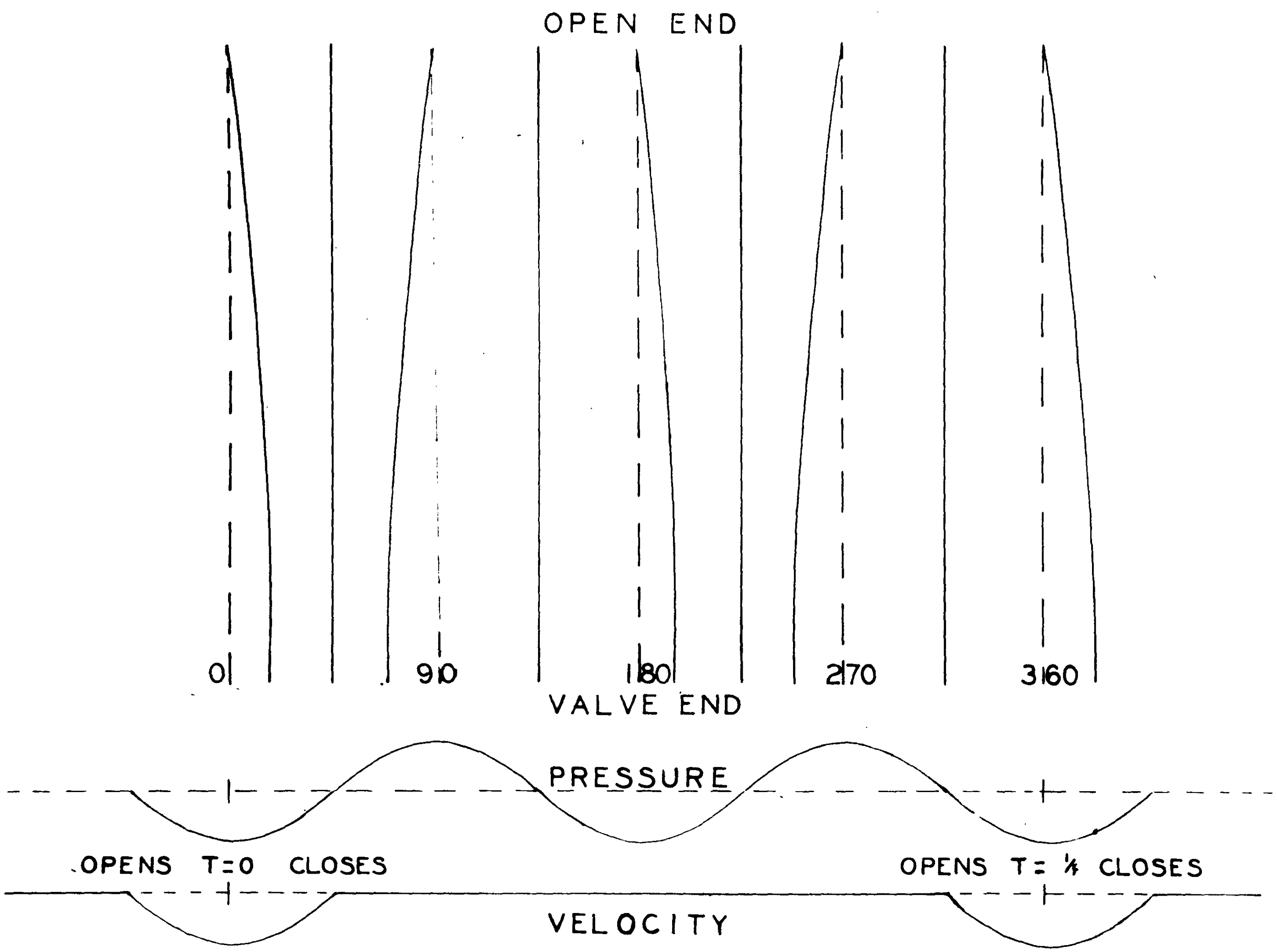
When the valve opens the velocity at the port end of the pipe is zero. An eighth period later the pulse front has traveled half the length of the pipe and the pressure has a maximum negative value at the port. At the end of the next eighth period the pressure pulse is approaching the open end of the pipe where it will be reflected as a positive



TRAVELING WAVE IN INTAKE PIPE

FIG. 3

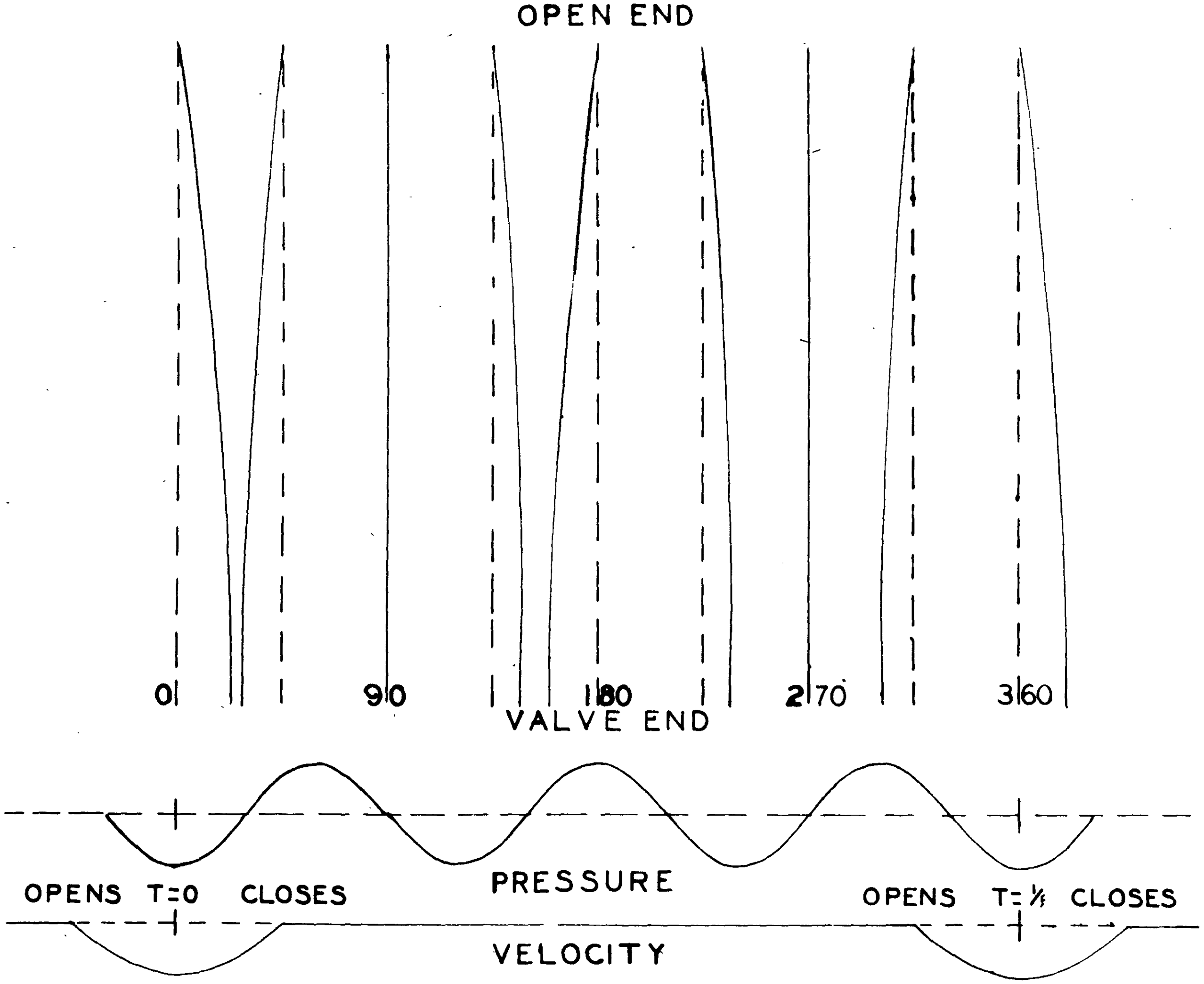
pressure. The velocity of the air in the port is zero, and remains so until the valve opens again. At the quarter period the front half of the pressure pulse which has been reflected from the open end as a positive pulse and the last half which is still negative nullify each other so that the resultant excess pressure is zero. The end of the next 1/8 period finds the pulse completely reflected and approaching the closed valve. There it is reflected without change of phase, with the result that at the half period the two halves of the pulse reinforce to build the pressure to twice its original magnitude. In the figure the amplitude has been reduced to half its value for the sake of symmetry. During the next 1/8 period the pulse is approaching the open end of the pipe again. This time the positive pulse is reflected as a negative pulse, and at the three-quarter period the reflected and the incident halves of the pulse are again found to just nullify each other. After seven-eighths of the cycle have passed, the pulse, now negative, is again approaching the valve which is just beginning to open. The action of the piston is then to reinforce this negative pulse during the time the valve is open. The cycle of events in the intake pipe is then repeated. Obviously this is a resonance case. However, supercharging is not to be expectedas the pressure at the time the valve closes is just atmospheric so the density of the charge in the cylinder is the same or somewhat less than in the surrounding atmosphere.



N = 2

STANDING WAVES IN INTAKE PIPE

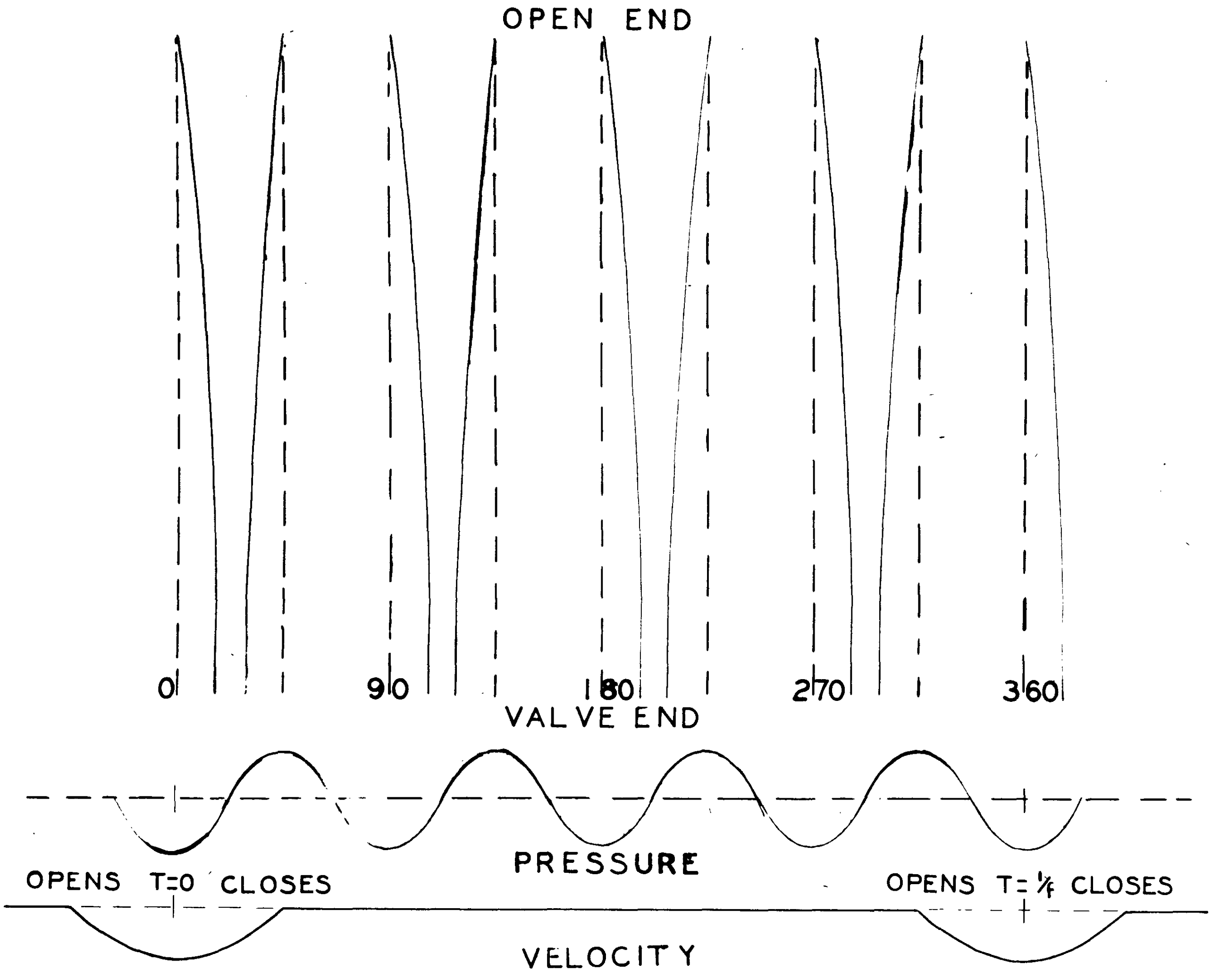
FIG. 4



N = 3

STANDING WAVES IN INTAKE PIPE

FIG. 5



N=4

STANDING WAVES IN INTAKE PIPE

FIG. 6

Measurement of the power output of the N.A.C.A. engine, so adjusted as to fulfill as closely as possible the conditions assumed above, verified the above statements. Fig. 1a is the pressure time record in the valve port when the pipe frequency is approximately five times the frequency of the valve. A slight peak in the power output occurred. Fig. 1b is the pressure time record for a pipe frequency four times that of the valve. The power output was greatly increased with this intake pipe. A slightly lessened output was found for the case when the frequency of the pipe was three times the valve frequency. The pressure-time record is shown in Fig. 7a. Fig. 7b is the pressure-time record when the frequency of the pipe is twice the frequency of the valves. In this case the power output of the engine was a minimum. The maximum total power output at the speed at which these records were taken was thirty-one per cent greater than the minimum output.

The general form of these records is the same for each power peak at whatever speed the engine is run.

A quantitative treatment for the prediction of the phenomena in the intake pipe and the power output requires a more general application of the theory of sound. The general theory accounts for the effects of the dissipation of energy in the intake pipe, the effect of valve timing, frictional dissipation in the valve port and the residual pressure existing in the cylinder when the valve opens.

TYPICAL INDICATOR CARDS

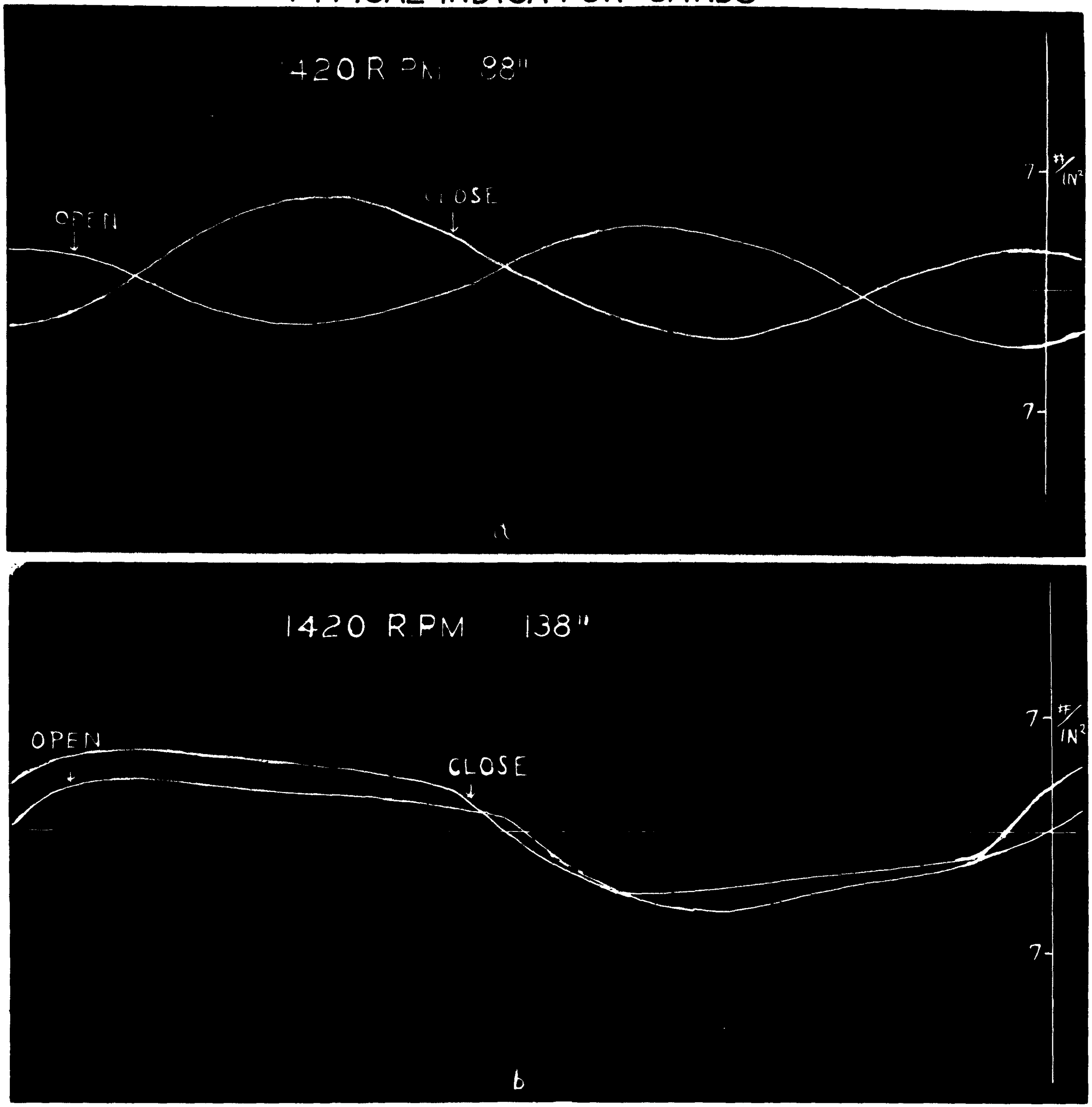


FIG. 7

The effect of viscosity in pipes has been treated by Kirchoff. Although the attenuation of the waves in the pipe is due not only to viscosity but to the vibrations set up in the pipe itself and dissipation from the end, Kirchoff's treatment indicates how this dissipation should be handled. The solution of the equation of motion of the air in a pipe is expressed as a Fourier series of the normal modes of vibration of the pipe. The value of the coefficients of the terms of the series depend upon the valve timing, piston speed, friction in the valve, and the residual pressure in the cylinder. The attenuation factor for various pipes has been measured by Eckhardt of the Bureau of Standards. His measurements indicate that the attenuation is nearly independent of the frequency. This has been assumed in the calculations. The attenuation factor finally decided upon is ten per cent less than the factor given by Eckhardt for a pipe of approximately the same diameter and half the wall thickness of the pipe used in the experiments.

The pressure in the pipe at any instant may be derived from the solution of the equation of motion. A comparison of the calculated and the experimentally determined pressures is made in Fig. 8. The pressure is in pounds per square inch and the time is measured along the abscissa in terms of the engine crank angle. The engine speed is 1220 revolutions per minute. The pressure variation is shown

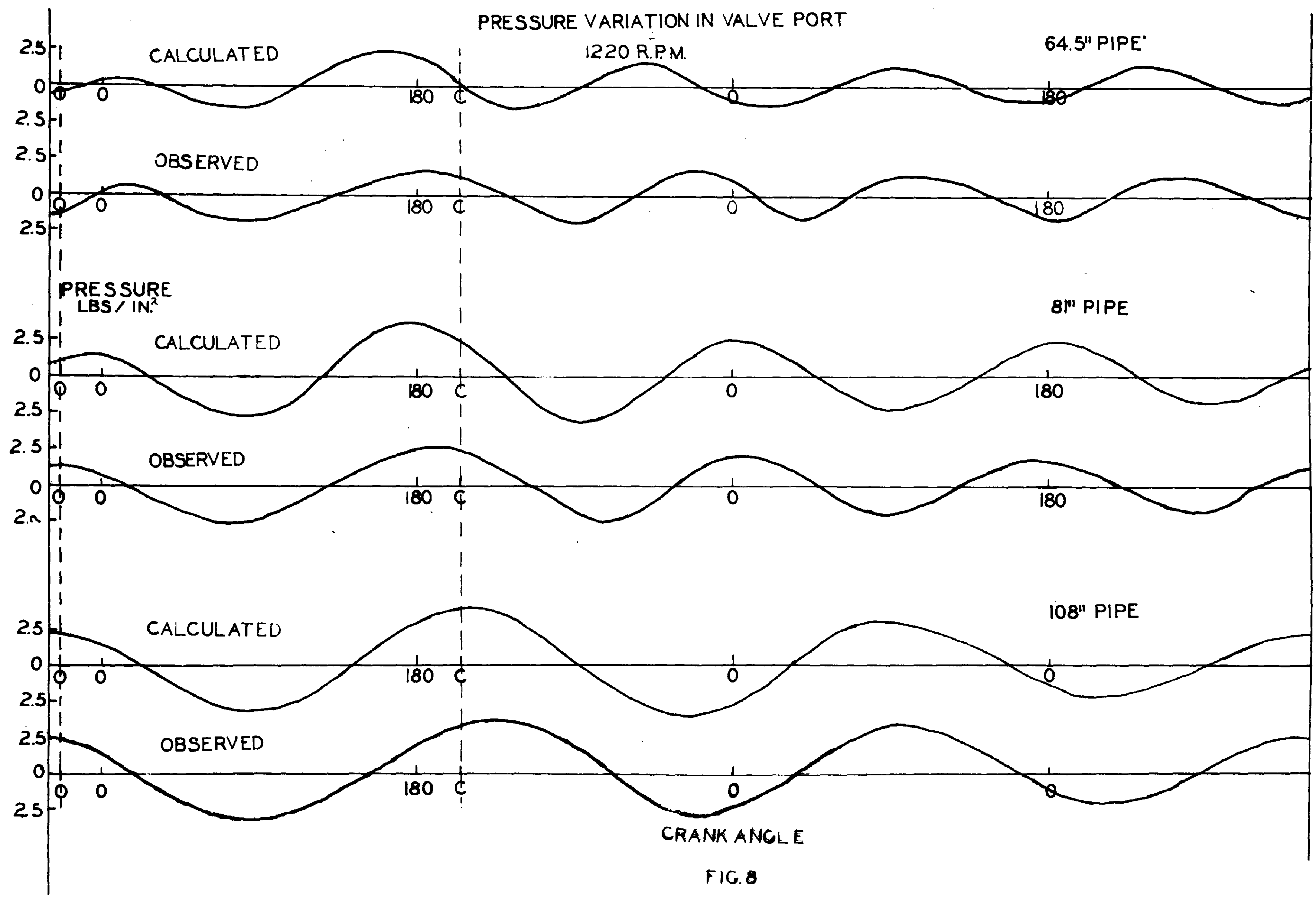

PRESSURE VARIATION IN VALVE PORT
1220 R.P.M.
64.5" PIPE
81" PIPE
108" PIPE
CALCULATED
OBSERVED
PRESSURE LBS/IN.^{2}
2.5
0
180
C
CRANK ANGLE

FIG. 8

over one valve cycle which requires two complete revolutions of the engine before completion. The valve is open from the dotted line marked "O" to the dotted line marked "C". The frequency of the pipe is five times the frequency of the valves in the first set of curves, four times the valve frequency in the second and three times the valve frequency in the third.

Fig. 9 shows the comparison of the pressure time records for an engine speed of 1630 revolutions per minute corresponding to those of Fig. 8.

The first set of curves in Fig. 10 shows the comparison of the calculated and observed pressures in the valve port for a pipe whose frequency is approximately 3.6 that of the valve frequency. The engine speed is 1220 revolutions per minute. The second set is a comparison of the records when the pipe frequency is twice that of the valves and the engine speed is 1630 r.p.m. The flattening of the experimental curve indicates that the amplitude of the pressure waves are so great that the assumptions made in the development of the theory of sound no longer hold.

The power output of the engine is of primary importance. Some function is desired which will indicate just how the output will vary for different engine speeds and lengths of intake pipe. The Volumetric efficiency, which is defined as the ratio of the volume of air actually swept into the engine per cycle, reckoned at the temperature and pressure of the

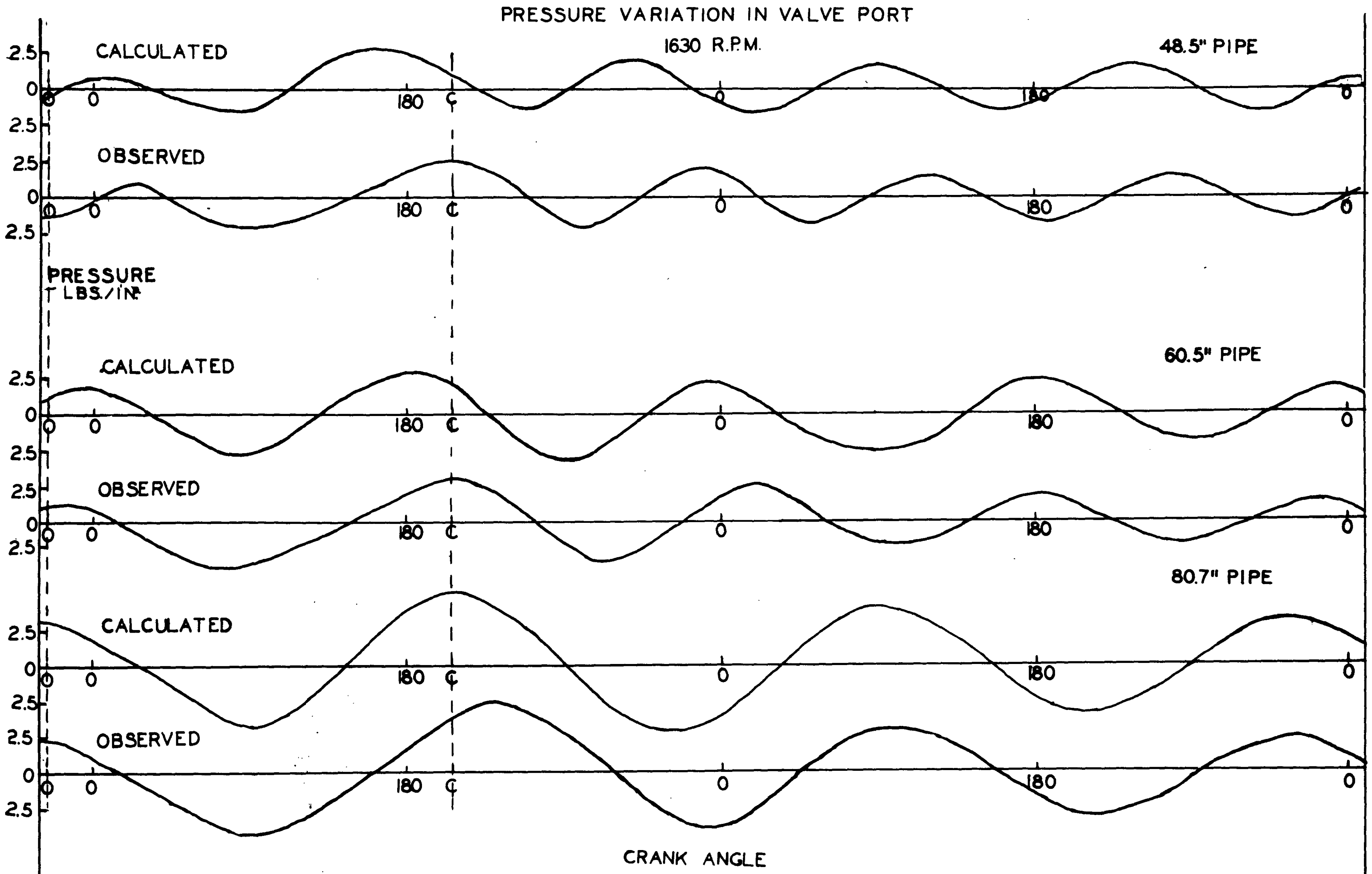

PRESSURE VARIATION IN VALVE PORT
1630 R.P.M.
48.5" PIPE
CALCULATED
OBSERVED
PRESSURE LBS./IN²
60.5" PIPE
CALCULATED
OBSERVED
80.7" PIPE
CALCULATED
OBSERVED
2.5
0
180
CRANK ANGLE

FIG. 9

PRESSURE VARIATION IN VALVE PORT

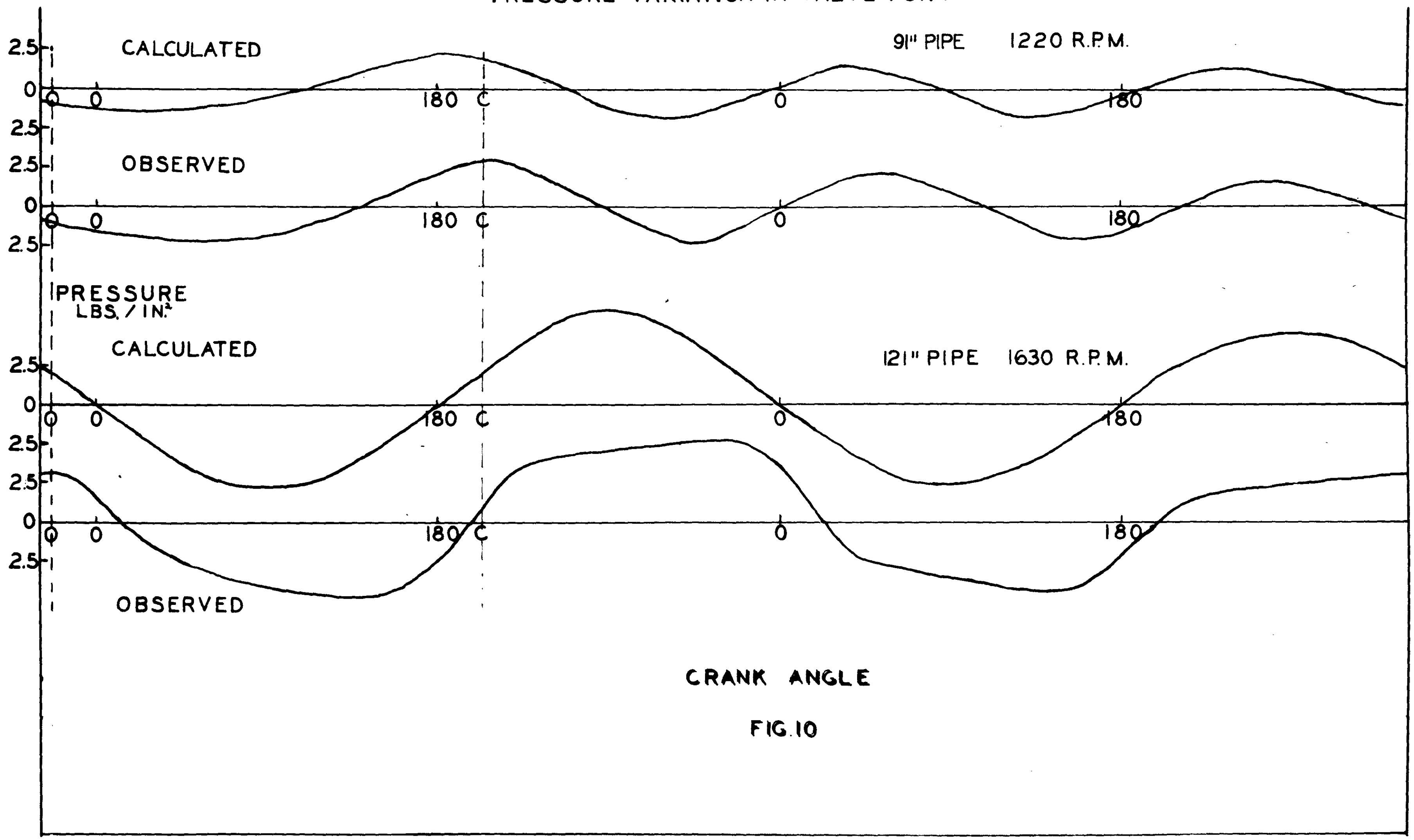


FIG.10

COMPARISON OF EXPERIMENTALLY DETERMINED INDICATED MEAN EFFECTIVE PRESSURE AND THE CALCULATED VOLUMETRIC EFFICIENCY
Diameter of pipe: 2 9/16 inches
1220 R. P. M.

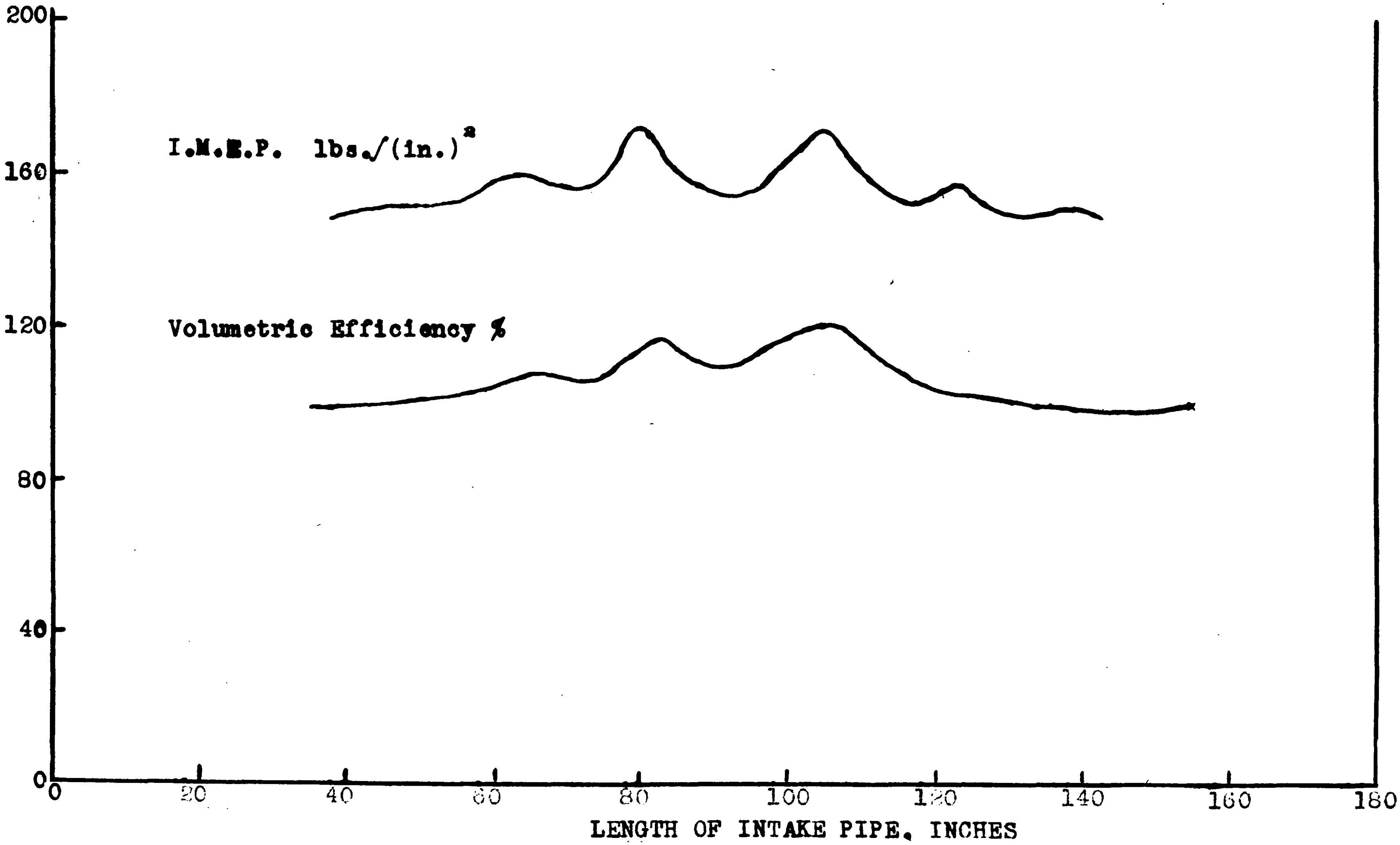


Figure 11.

COMPARISON OF EXPERIMENTALLY DETERMINED INDICATED MEAN EFFECTIVE PRESSURE
AND THE CALCULATED VOLUMETRIC EFFICIENCY
Diameter of pipe: 2 9/16 inches
1630 R. P. M.

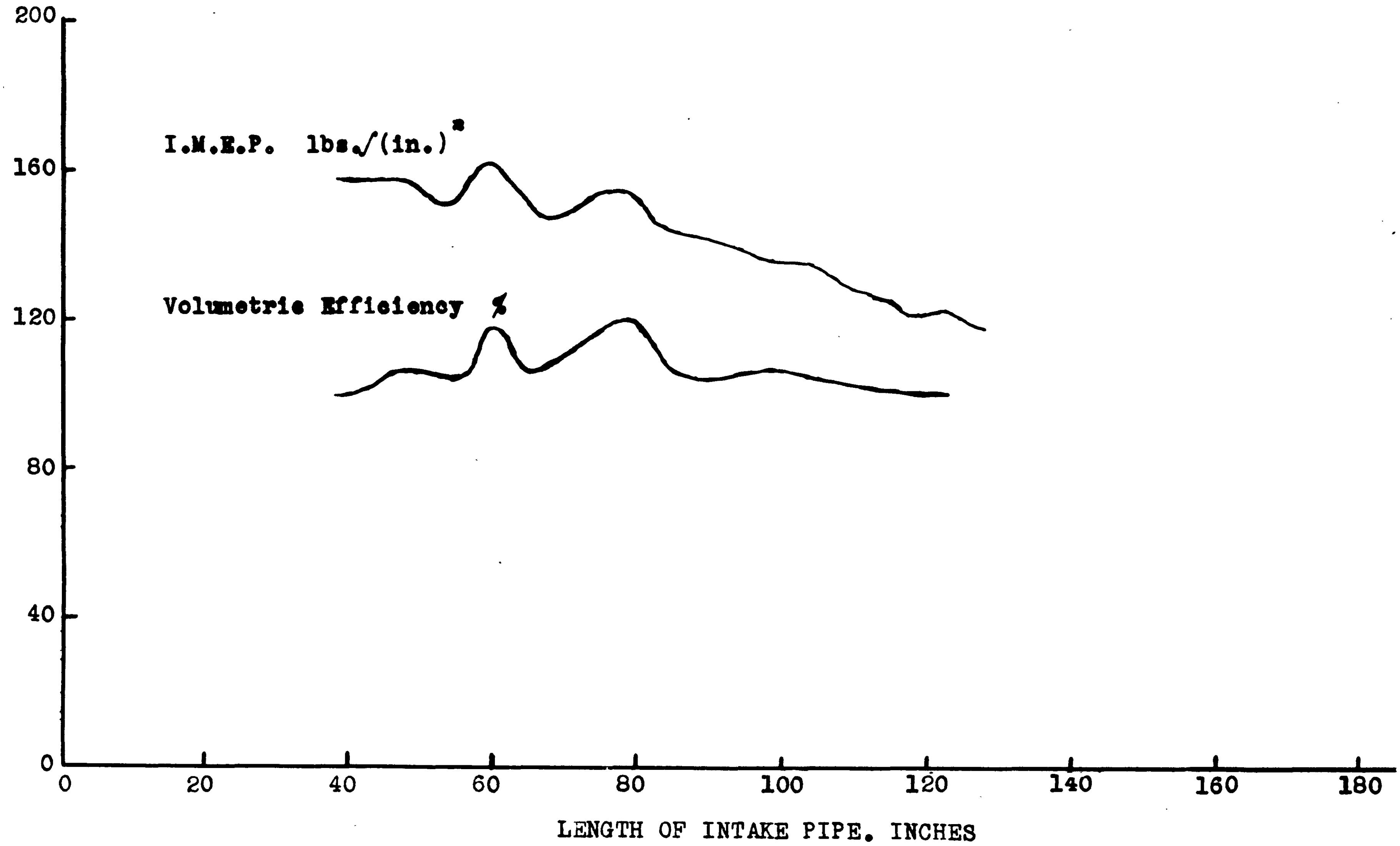
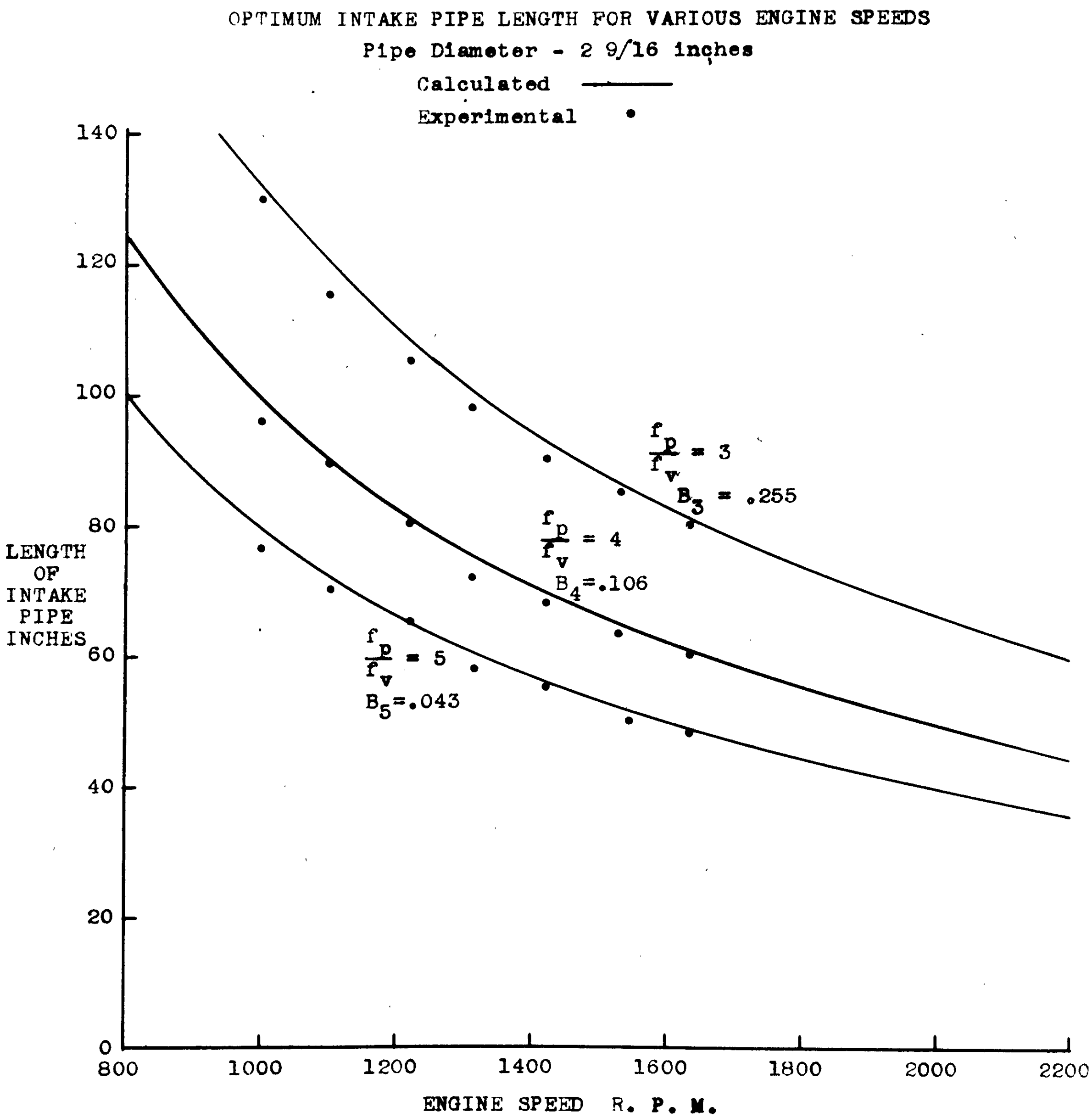


Figure 12.



OPTIMUM INTAKE PIPE LENGTH FOR VARIOUS ENGINE SPEEDS
Pipe Diameter - 2 9/16 inches
Calculated
Experimental
140
120
100
80
60
40
20
0
LENGTH OF INTAKE PIPE INCHES
800
1000
1200
1400
1600
1800
2000
2200
ENGINE SPEED R. P. M.
fp/fv = 3
B3 = .255
fp/fv = 4
B4=.106
fp/fv = 5
B5=.043

Fig. 13

the simplified theory given earlier. The solid lines of Fig. 13 indicate the best length of intake pipe for different engine speeds as predicted by the simple sound theory. The three curves are for the cases when the fundamental pipe frequency is three, four or five times the frequency of the valve mechanism respectively. The points indicate the best lengths of intake pipe as determined from experiment. It is to be particularly noticed that as the engine speed becomes greater the experimental points tend to approach more and more closely the curve determined from the simple theory. The treatment of the viscous effects occurring in the pipe predicts that such an approach is to be expected. The frequencies excited in the pipe which are higher than the fundamental suffer a greater attenuation. When the valve frequency becomes high, the components made up of the higher harmonics are almost completely damped out. The coefficient of the predominant term of the Fourier series is given with each curve, and indicates the relative amount of supercharging.

Comparison of Figs. 11, 12 and 13 indicates that the effect of the higher harmonics which are excited in the pipe by the pressure pulse is to shorten the length of pipe at which maximum supercharging will occur as predicted by the simple theory.

The general theory clearly indicates that the length of pipe for which the maximum supercharge will occur may be varied to some extent by varying the valve timing. Physically, this means that the velocity-time record at the intake port will be changed by the change in valve timing.

The very good agreement of the general form of the calculated and the observed pressure-time records and of the calculated volumetric efficiency and the observed I.M.E.P. indicates that over the range of speeds studied the theory of sound adequately describes the phenomena taking place in the intake pipe. The length of pipe for the best supercharging may be predicted as accurately as desired provided the attenuation factor of the intake pipe, the velocity of sound in the pipe and the variation of the air velocity in the intake port is known. The air velocity in the intake port is determined by the valve timing, the residual pressure in the cylinder when the valve opens, and the piston velocity and the frictional resistance of the intake port. The simple sound theory is certainly sufficient for any usual design problem which might arise.

The previous investigations of the intake pipe problem certainly show that the phenomena occurring in the intake pipe were not at all well understood. List[1] alone

(1) List - Increasing the Volumetric Efficiency of Diesel Engines - N.A.C.A. Technical Memo. No. 700

recognizes that the air velocity in the intake port plays an important part in the generation of the vibrations in the intake pipe. His work does not consider the effect of the valves. The lack of understanding is most clearly shown in the methods of attack on the experimental problem. In the first place, previous investigators have not recognized that the peaks in the power output occur over a relatively short range of intake pipe length. The broadest peaks which occur for the longer lengths of pipe are within a range of eight or ten inches of pipe length. Thus a change of length of one foot will completely miss a power peak, and will miss more than one peak when the pipe length is short. These investigations have stopped at lengths of intake pipe which are certainly much shorter than those for which the greatest maxima in power output occur.

While the volumetric efficiency is the most easily calculated quantity, it is not the most easily and accurately measured one on account of the difficulty of metering the air flowing into the engine. For this reason any measurements of volumetric efficiency are unreliable. The total power output or the I.M.E.P. which is directly proportional to the power output is the best measure of the air forced into the cylinder per stroke, provided the fuel air ratio is correct for best combustion.

Dennison[1] and List state that the best results were gotten for smaller diameter pipes. This is true, as far as their investigations went. However, their experiments stopped at relatively short lengths of pipe. In the narrower tubes the velocity of sound is reduced and the resonant pipe length is shorter than for the larger diameter pipe. Evidently, in their experiments they approached more closely a resonance peak for the smaller diameter pipe. It is certainly to be expected that if they had carried their investigations through for longer lengths of pipe, they would have found the larger diameter pipe giving the greatest effect at resonance.

In the following pages the general theory of the forced waves will be developed, and the statements made above will be justified.

II. THE WAVE EQUATION

The discussion of the intake pipe problem of the internal combustion engine requires a knowledge of the fundamental principles of the dynamics of fluid flow. These principles are incorporated in the equation of continuity and the hydrodynamical equations of motion. A complete derivation of these two equations is given in Lamb's Hydrodynamics, Chapter I. The equation of continuity describes the mass flor of fluid through a space element in the fluid. It is expressed mathematically as:

$$1) \qquad \frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0$$

ρ is the density of the fluid, and u, v, and w are the components of velocity of the fluid along the x, y, and z axes, respectively. The equations of motion which describe the forces acting on the element of fluid are:

$$2) \qquad \begin{aligned} \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} &= X - \frac{1}{\rho}\frac{\partial p'}{\partial x} \\ \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} &= Y - \frac{1}{\rho}\frac{\partial p'}{\partial y} \\ \frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} &= Z - \frac{1}{\rho}\frac{\partial p'}{\partial z} \end{aligned}$$

p' is the pressure in the fluid and X, Y, and Z are the components of the external forces acting on the fluid element.

In the discussion of the behaviour of a compressible fluid it is customary to introduce the condensation, s, which is defined as the fractional change in the density of the fluid. The change in the density is due to the forces which are acting on the fluid. The equilibrium density and the instantaneous density are then related by the equation,

3) $\rho = \rho_0(1 + s)$.

This expression is introduced into the equation of continuity which becomes

$$\rho_0 \frac{\partial s}{\partial t} + \rho_0(1+s)\left[\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right] + \rho_0\left[u\frac{\partial s}{\partial x} + v\frac{\partial s}{\partial y} + w\frac{\partial s}{\partial z}\right] = 0.$$

The assumption is made that the condensation and the products of the condensation and the space rates of change of the velocity components are small compared to unity. The equation of continuity reduces to

4) $\frac{\partial s}{\partial t} + \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$.

Then the assumption is made that the velocities may be derived from a velocity potential, φ. The velocities in terms of the velocity potential are then

5) $u = -\frac{\partial \varphi}{\partial x}$; $v = -\frac{\partial \varphi}{\partial y}$; $w = -\frac{\partial \varphi}{\partial z}$.

This last assumption reduces the equation of continuity to the form:

6) $\frac{\partial s}{\partial t} = \frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} + \frac{\partial^2 \varphi}{\partial z^2} = \nabla^2 \varphi$.

In their general form the equations of motion are difficult to integrate. In the theory of sound the assumption is made that the product of the velocities and their space rates of change are small and can be neglected in comparison with the other terms of the equations. The simplified equations of motion are

$$7)\quad \frac{\partial u}{\partial t} = X - \frac{1}{\rho}\frac{\partial p'}{\partial x} , \qquad u\frac{\partial u}{\partial x} \text{ neglected.}$$

$$\frac{\partial v}{\partial t} = Y - \frac{1}{\rho}\frac{\partial p'}{\partial y} ,$$

$$\frac{\partial w}{\partial t} = Z - \frac{1}{\rho}\frac{\partial p'}{\partial z} .$$

These equations are multiplied by dx, dy and dz, and summed. The result is a total derivative,

$$8)\quad -d\left[\frac{\partial \varphi}{\partial t}\right] = -dW - \frac{1}{\rho}\, dp;$$

if the external forces can be derived from a potential function, W.

$$-dW = Xdx + Ydy + Zdz.$$

The components of the force are then:

$$X = -\frac{\partial W}{\partial x};\quad Y = -\frac{\partial W}{\partial y};\quad Z = -\frac{\partial W}{\partial z} .$$

When equation (8) is integrated, it becomes

$$9)\quad \frac{\partial \varphi}{\partial t} - \left(\frac{\partial \varphi}{\partial t}\right)_0 = W - W_0 + \frac{p' - p_0'}{\rho} .$$

The density variation has been previously assumed to be small and so has been considered as a constant in this integration. The zero subscripts refer to the equilibrium state of the fluid. When the main interest is in the variation of the quantities from equilibrium, the equilibrium values may be set equal to zero.

The compression and the expansion occur rapidly enough so that very little heat transfer takes place. The fluid is assumed to obey the adiabatic gas law.

10) $\frac{p}{\rho^{\gamma}} = \text{const.}$

γ is the ratio of the specific heats of the gas at constant pressure and at constant volume. To a first approximation the small excess pressures may be derived from the gas law by differentiation.

11) $\frac{p'}{\Delta\rho} = \frac{dp'}{d\rho} = \frac{\gamma p'}{\rho}$.

The quotient, p'/ρ , may be written

$$\frac{p'}{\rho} = \frac{p_o'}{\rho_o}\frac{\left(1 + \frac{\Delta p'}{p_o'}\right)}{\left(1 + \frac{\Delta\rho}{\rho_o}\right)} = \frac{p_o'}{\rho_o}\left(1 + \frac{\Delta p'}{p_o'} - \frac{\Delta\rho}{\rho_o} - \frac{\Delta p'}{p_o'}\frac{\Delta\rho}{\rho_o}\right).$$

The sum of the last terms in the bracket is small and may be safely neglected. Equation (11) then becomes

$$\frac{\Delta p'}{\Delta\rho} = p_o'/\rho_o = c^2 = \text{constant.}$$

The condensation is just $\frac{\Delta\rho}{\rho_o}$ so that the excess pressure, p, may be written as

12) $p = \Delta p' = \rho_o c^2 s$.

The equation of continuity then reduces from the form (9) to

(9a) $\frac{1}{c^2}\frac{\partial^2\varphi}{\partial t^2} = W + c^2 s.$

This latter form of the equation of continuity is substituted into the equation of motion, (6), to give the general wave equation:

$$13) \quad \frac{1}{c^2}\frac{\partial^2\varphi}{\partial t^2} - \frac{1}{c^2}\frac{\partial W}{\partial t} = \nabla^2\varphi .$$

When no forces are acting except those due to a pressure gradient, this last equation reduces to the ordinary wave equation.

$$14) \quad \frac{1}{c^2}\frac{\partial^2\varphi}{\partial t^2} = \nabla^2\varphi .$$

The main interest in this discussion is the case for plane waves. The solution of the equation for the case of plane waves,

If $u\frac{\partial u}{\partial x}$ was not neglected, this equation would be

$$15) \quad \frac{1}{c^2}\frac{\partial^2\varphi}{\partial t^2} = \frac{\partial^2\varphi}{\partial x^2} , \qquad c^2\frac{\partial^2\varphi}{\partial x^2} - \frac{\partial^2\varphi}{\partial t^2} = 0 . \qquad \left[c^2 - \left(\frac{\partial\varphi}{\partial x}\right)^2\right]\frac{\partial^2\varphi}{\partial x^2} - 2\frac{\partial\varphi}{\partial x}\frac{\partial^2\varphi}{\partial x\partial t} - \frac{\partial^2\varphi}{\partial t^2} =$$

is achieved by the introduction of the arbitrary transformation, due to D'Alembert,

$$x_1 = x + a_1 t \ ; \ x_2 = x + a_2 t$$

a_1 and a_2 are arbitrarily selected constants. Upon making the transformation the various partial derivatives become:

$$\frac{\partial\varphi}{\partial t} = \frac{\partial\varphi}{\partial x_1}\frac{\partial x_1}{\partial t} + \frac{\partial\varphi}{\partial x_2}\frac{\partial x_2}{\partial t} = a_1\frac{\partial\varphi}{\partial x_1} + a_2\frac{\partial\varphi}{\partial x_2} \ ;$$

$$\frac{\partial^2\varphi}{\partial t^2} = a_1^2\frac{\partial^2\varphi}{\partial x_1^2} + a_2^2\frac{\partial^2\varphi}{\partial x_2^2} + 2a_1a_2\frac{\partial^2\varphi}{\partial x_1\partial x_2} \ ;$$

$$\frac{\partial^2\varphi}{\partial x^2} = \frac{\partial^2\varphi}{\partial x_1^2} + \frac{\partial^2\varphi}{\partial x_2^2} .$$

The equation of motion then assumes the form

$$(a_1^2 - c^2)\frac{\partial^2\varphi}{\partial x_1^2} + (a_2^2 - c^2)\frac{\partial^2\varphi}{\partial x_2^2} + 2a_1a_2\frac{\partial^2\varphi}{\partial x_1\partial x_2} = 0 .$$

As a_1 and a_2 are arbitrary constants, they can be selected as the roots of the arbitrary equation

$$(a^2 - c^2) = 0.$$

In this case $a_1 = c$; and $a_2 = -c$, and the differential equation that must be solved is

$$\frac{\partial^2\varphi}{\partial x_1\partial x_2} = 0$$

Its solution is simply

16) $\varphi = F(x_1) + G(x_2) = F(x + ct) + G(x - ct)$

which is the familiar expression for the velocity potential of a wave propagated with the phase velocity c. F describes the wave traveling in the negative direction, and G the wave in the positive direction.

The velocity has been assumed to obey the relation

$$u = -\frac{\partial\varphi}{\partial x}$$

The sinusoidal solution which satisfies the equation is

17) $\varphi = Ae^{-ik(ct - x)} + Be^{-ik(ct + x)}$

in which $k = \omega/c = 2\pi f/c = 2\pi/\lambda$

f is the frequency of the wave, and λ is its wavelength. The velocity u is then

$$u = -\frac{\partial\varphi}{\partial x} = -ik[Ae^{ikx} - Be^{-ikx}]e^{-i\omega t}$$

Integration of the equation of motion gives the expression for the total pressure, p'. $\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial p'}{\partial x}$.

$$p' = \rho k\omega \int [Ae^{ikx} - Be^{-ikx}]\, e^{-i\omega t}\, dx$$

The excess pressure, $p = p' - p'_o$, is immediately obtained on the integration of this last equation.

18) $$p = -i\rho_o\omega[Ae^{ikx} + Be^{-ikx}]\, e^{-i\omega t}$$

This last expression is just the product of the density and the time derivative of the velocity potential.

19) $$p = -i\rho_o\omega\varphi = \rho_o c^2 s.$$

The displacement from equilibrium, y, is

20) $$y = \frac{1}{c}[Ae^{ikx} - Be^{-ikx}]e^{-i\omega t}$$

The space rate of variation of the displacement is then just

21) $$\frac{\partial y}{\partial x} = i\,\frac{k}{c}\,[Ae^{ikx} + Be^{-ikx}]e^{-i\omega t} = i\,\frac{\omega}{c^2}\,\varphi.$$

By comparing this equation with (19) it is immediately seen that the condensation is

22) $$s = -\frac{\partial y}{\partial x}$$

In general this relation holds for the solution of any equation whose solution can be put in the form of equation (20).

This general theory holds for the simplified picture of the intake pipe phenomena. The effects of viscosity and various forms of dissipation must be considered in order to get a quantitative view of the subject. These effects are dealt with in the next section.

III. THE EFFECT OF VISCOSITY

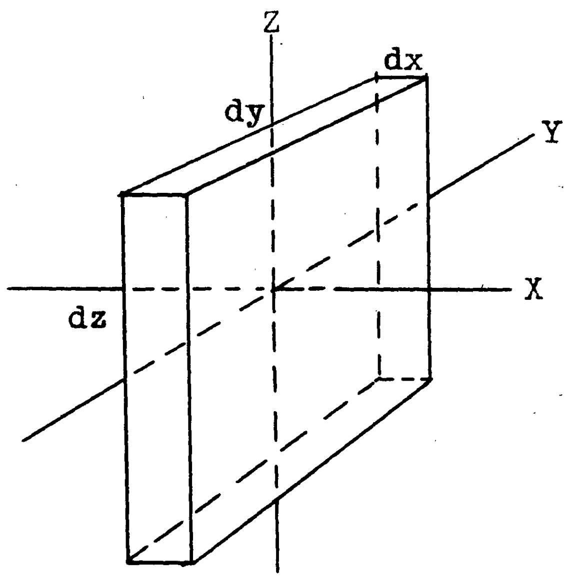
The intake pipe of an internal combustion engine is often quite long and of quite small diameter. On this account the pressure waves occurring in the pipe suffer considerable attenuation. The attenuation is not due entirely to the viscosity of the air alone, but energy is also dissipated by the vibrations induced in the walls of the pipe and from the open end. The diameter of the pipe is usually quite large enough so the conduction of heat has little effect on the waves. The problem of heat conduction is thoroughly treated by Lord Rayleigh.[1]

The dissipation of energy due to vibrations set up in the walls of the intake pipe becomes more and more marked the thinner the tube walls and the greater the amplitude of the pressure waves. The mathematical treatment of these losses becomes so involved, especially for a jointed pipe, that itwill not be treated in this discussion.

The analytical treatment of the effect of viscosity was originally carried out by Kirchoff, and his results have been checked very closely in experimental investigations in which suitable precautions were taken to prevent any losses due to the pipe itself. This treatment indicates how the effects of mechanical losses influence the vibrations and will be carried out in detail.

(1) Rayleigh - Theory of Sound, Vol. II.

Viscosity has its greatest effects in the neighborhood of the walls of the tube. An estimate of the thickness of the fluid layer in which the major part of the effect will be observed can be made by the following analysis. A plane is assumed to have an oscillating motion relative to the fluid. Its velocity is u, and it has an angular frequency ω. An element of the plane has an area (dxdz).



The forces acting upon an element of fluid of thickness, dx, to the wall are those of inertia and viscosity. The inertia force is simply

1) $\rho \frac{du}{dt}\, dxdydz$.

If the coefficient of viscosity is μ, the viscous force at the wall, x = 0, is

$$F = -\mu \frac{\partial u}{\partial x}\Big|_{x=0} dydz$$

On a plane a distance dx out in the fluid, the viscous force is

2) $\left\{ -\mu \frac{\partial u}{\partial x}\Big|_{x=0} + \frac{\partial}{\partial x}\left[-\mu \frac{\partial u}{\partial x} \right] dx \right\} dydz$

so the resultant force due to viscosity is

3) $-\frac{\partial}{\partial x}[-\mu\frac{\partial u}{\partial x}]\, dxdydz$,

The coefficient of viscosity is assumed to be a constant so the equation of motion becomes

4) $\frac{\partial u}{\partial t} = \frac{\mu}{\rho}\frac{\partial^2 u}{\partial x^2}$.

As the discussion is about a plane having a frequency of oscillation ω relative to the fluid, the solution of the equation of motion is assumed to be

5) $u = Ae^{-i(\omega t - k'x)}$.

When this is substituted into the differential equation, an expression involving the constants of the system and k' results.

$-i\omega u = -\frac{\mu}{\rho}uk'^2$.

From the theory of complex variables it is known that

$\sqrt{i} = e^{\frac{2\pi}{4}} = \frac{1 \pm i}{\sqrt{2}}$; $i = \sqrt{-1}$,

so that k' reduces to the form

6) $k' = \pm\sqrt{\frac{i\rho\omega}{\mu}} = \pm\sqrt{\frac{\rho\omega}{2\mu}}(1 + i) = \pm b\,(1 + i)$;

$b = \sqrt{\frac{\rho\omega}{2\mu}}$.

The solution of the equation of motion is then

7) $u(x) = Ae^{-i[\omega t \pm b(1 + i)x]}$

This solution describes a damped velocity wave travelling normally to the wall. The expression for a wave travelling outwardly from the wall is

$$8)\ u(x) = Ae^{-bx}e^{-i(\omega t - bx)}$$

Now let $b = 2\pi/\lambda$. Then at a distance λ from the plane the velocity has reduced to $e^{-2\pi} = 1/540$ th of its value at the plane. The value of λ can be easily calculated from the relation

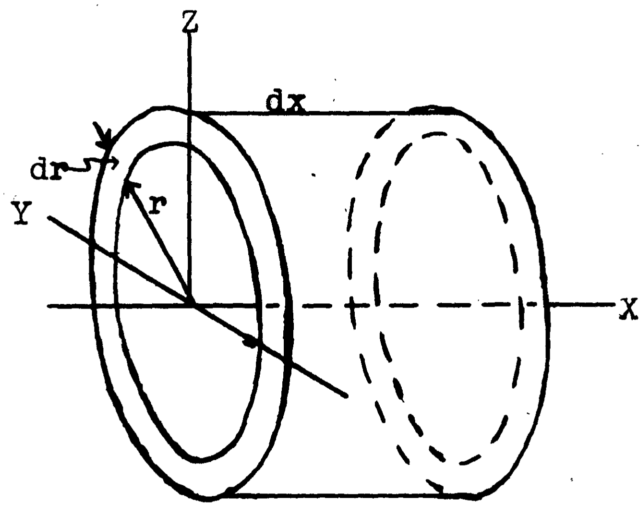
$$\lambda = \frac{2\pi}{b} = 2\pi \sqrt{\frac{2\mu}{\rho\omega}}$$

The ratio, $\frac{\mu}{\rho}$, is the kinematic coefficient of viscosity and is approximately equal to .13 at room temperature and one atmosphere pressure. If the frequency of oscillation is f cycles per second, the value of λ is about

$$= \sqrt{\frac{.52\pi}{f}}$$

At ten cycles per second this wavelength is about four millimeters and reduces to two millimeters at 40 cycles per second. It may be safely assumed that the effect of viscosity is negligible over a half centimeter from the wall. This means that in a tube of over a centimeter radius there will be a column down its center in which plane waves will be propagated with essentially no effects from the side walls.

The important question is the attenuation of the wave as it travels the length of the intake pipe. This is developed from the consideration of the forces acting upon an annular ring of radius r and length dx which is concentric with the tube as shown in the following figure.



On the inner surface of the ring the viscous force is

$$-2\pi r \mu \frac{\partial u}{\partial r} dx$$

The force on the outer surface is

$$-2\pi r \mu \frac{\partial u}{\partial r} dx + \frac{\partial}{\partial r} [-2\pi r \, dx \frac{\partial u}{\partial r}] \, dr$$

The resultant retarding viscous force is then

9) $2\pi \mu \frac{\partial}{\partial r}(-r \frac{\partial u}{\partial r}) \, dxdr$

The external force X of the hydrodynamical equations of motion is just the ratio of this force to the mass of the annular ring of fluid.

10) $X = -\frac{\mu}{\rho r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right).$

Thus the hydrodynamical equation of motion becomes

11) $\frac{\partial u}{\partial t} = -\frac{\mu}{\rho r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) - \frac{1}{\rho}\frac{\partial p}{\partial x}.$

The pressure gradient is not a function of the radius of the tube, and the velocity is assumed to be an harmonic function of time. The solution of the equation of motion is

12) $u = u_0 e^{-i\omega t}\ ;\ \frac{\partial u}{\partial t} = -i\omega u\ .$

The equation of motion can then be placed in the form

13) $\left[-i\omega\rho + \frac{\mu}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)\right] u = -\frac{\partial p}{\partial x}$

$\left[\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + k^2\right] u = \frac{1}{\mu}\frac{\partial p}{\partial x};\ k^2 = \frac{i\rho\omega}{\mu}\ .$

Since the pressure gradient is not a function of r, and the operator is a Bessel's operator of zero'th order, the solution in terms of r is

14) $u = \frac{1}{\mu k^2}\frac{\partial p}{\partial x} + AJ_0(Kr)\ .$

The velocity must be finite at the center of the tube and zero at $r = a$, the radius of the tube. Thus the constant A is just

$$A = -\frac{1}{\mu k^2}\frac{1}{J_0(ka)}\frac{\partial p}{\partial x}\ .$$

The complete solution of the equation in terms of r is then

$$15) \quad u = \frac{1}{\mu k^2}\frac{\partial p}{\partial x}\left[1 - \frac{J_0(kr)}{J_0(ka)}\right]$$

This is Crandall' solution of the equation of motion.
The average fluid velocity, u, over the cross section of the tube is

$$16) \quad u = \frac{2}{a^2}\int_0^a u r\,dr = \frac{1}{\mu k^2}\frac{\partial p}{\partial x}\left[1 - \frac{2}{k^2a^2 J_0(ka)}\int_0^a J_0(kr)(kr)dr\right]$$

$$= \frac{1}{k^2}\frac{\partial p}{\partial x}\left[1 - \frac{2}{ka}\frac{J_1(ka)}{J_0(ka)}\right].$$

The average velocity depends upon the radius of the tube, the frequency of the vibrations existing in the tube and the kinematic coefficient of viscosity, $\frac{\mu}{\rho}$, of the fluid.

In the case of a tube having a diameter of five centimeters in which vibrations having a frequency of two cycles per second exist the magnitude of (ka) for air at atmospheric pressure and room temperature is approximately twenty-five. In the intake pipe problem the value of this product will always be larger than this. By expansion of the Bessel's functions (Jahnke and Emde, page 301) it may be shown that if the magnitude of their argument is greater than ten, the ratio of the two functions is just equal to -i.

$$\frac{J_1(x\sqrt{-i})}{J_0(x\sqrt{-i})} = -i; \quad x\sqrt{-i} = ka = a\sqrt{\frac{i\rho\omega}{\mu}}\ ; \quad x > 10 .$$

The average velocity across a large diameter tube is then

$$17) \quad u = -\frac{1}{\mu k^2}\frac{\partial p}{\partial x}\left[1 + \frac{2i}{ka}\right] = -\frac{i}{\rho\omega}\frac{\partial p}{\partial x}\left[1 + \frac{1}{a}\sqrt{\frac{2\mu}{\rho\omega}}(1-i)\right],$$

since $\sqrt{-i} = \frac{1-i}{\sqrt{2}}$ and $\mu k^2 = -i\rho\omega$

The last term in the brackets arises from the effect of viscosity. Its influence becomes less the larger the diameter of the pipe and the higher the frequency of the vibrations in the pipe.

Equation (12) states that

$$\frac{du}{dt} = -i\omega u.$$

With this in mind it is seen immediately that (17) is the differential equation describing the average fluid displacement ,y, over the cross-section of the pipe.

$$18) \quad \rho\left(1+\frac{1}{a}\sqrt{\frac{2\mu}{\rho\omega}}\right)\frac{\partial^2 y}{\partial t^2} + \rho\frac{\omega}{a}\sqrt{\frac{2\mu}{\rho\omega}}\frac{\partial y}{\partial t} = -\frac{\partial p}{\partial x}$$

In the previous section the excess pressure was in a sound wave shown to be equal to $-\rho c^2\frac{\partial y}{\partial x}$. Thus equation (18) reduces to a form in which y is the dependent variable.

$$19) \quad \rho\left(1 + \frac{1}{a}\sqrt{\frac{2\mu}{\rho\omega}}\right)\frac{\partial^2 y}{\partial t^2} + \rho\frac{\omega}{a}\sqrt{\frac{2\mu}{\rho\omega}}\frac{\partial y}{\partial t} = \rho c^2\frac{\partial^2 y}{\partial x^2}$$

The solution of this equation is

20) $y = Ae^{i(mx - \omega t)}$

in which

21) $m = \pm \frac{\omega}{c}[(1 + \frac{1}{a}\sqrt{\frac{2\mu}{\rho\omega}})(1 + i\frac{1}{a}\sqrt{\frac{2\mu}{\rho\omega}})]^{1/2}$.

The quantity $\frac{1}{a}\sqrt{\frac{2\mu}{\rho\omega}}$ is small. Then if the value of c' is defined as

22) $c' = \frac{c}{[1+\frac{1}{a}\sqrt{\frac{2\mu}{\rho\omega}}]^{1/2}} = c(1 - \frac{1}{a}\sqrt{\frac{\mu}{2\rho\omega}})$,

the value of m reduces to

23) $m = \pm [\frac{\omega}{c'} + \frac{i}{ac'}\sqrt{\frac{\mu\omega}{2\rho}}]$.

The complete solution of the wave equation is

24) $y = Ae^{-\frac{x}{ac'}\sqrt{\frac{\mu\omega}{2\rho}}}\, e^{i\omega(t - \frac{x}{c'})} + Be^{\frac{x}{ac'}\sqrt{\frac{\mu\omega}{2\rho}}}\, e^{-i\omega(t + \frac{x}{c'})}$.

The phase velocity is c' and it is seen to be less than in the open air. Many investigators have shown that the decreases in the phase velocity varies inversely as the radius of the tube and that the attenuation factor increases inversely as the radius of the tube. (Rayleigh - Theory of Sound Vol. II - Section 350.)

The effect of heat conduction has been shown to add a constant to the coefficient of $1/a\sqrt{\omega}$ (Rayleigh, Section 350).

The dissipation of energy from the vibrations set up in the walls of the jointed tube is so complex that no analytical treatment will be attempted. The assumption will be made that over the small range of frequencies to be studied, the attenuation coefficient is practically a constant, and that the solution of the wave equation can be written as

$$25)\quad y = Ae^{-\alpha\, a_n x}\, e^{-i\omega_n(t - \frac{x}{c'})} + Be^{\alpha\, a_n x}\, e^{-i\omega_n(t + \frac{x}{c'})}$$

For a given set of conditions this is a perfectly general expression describing the wave.

Eckhardt, Chrisler, Quayle and Evans (U. S. Bureau of Standards Technologic Papers, Vol. 21, page 163) have measured the attenuation in voice tubes of various diameters. The conditions under which their measurements were made are very similar to those existing for the engine intake pipe. They find that the attenuation factor is almost a constant over a wide range of frequencies. Their results are used in the calculations which are made later in this discussion.

IV. THE ACOUSTIC IMPEDANCE

In the later sections of this discussion the concept of the acoustic impedance will be found to be most helpful. For this reason its development is carried out quite fully in this section.

A general form of the wave equation is

$$1)\quad P\frac{\partial^2 y}{\partial t^2} + Q\frac{\partial y}{\partial t} + Ry = F$$

y is the fluid displacement and F is a force which is a harmonic function of time. In general the coefficients P, Q, and R are functions of the space coordinates. If F is

$$2)\quad F = F_o e^{-i\omega t},$$

the equation may be put into the form

$$3)\quad [i(\omega P - \frac{R}{\omega}) + Q]u = F$$

since the displacement may be expressed as $y = y_o e^{-i\omega t}$. The ratio Z_1 of the force to the displacement velocity,

$$4)\quad Z_1 = \frac{F}{u} = Q + i\ (\omega P - \frac{R}{\omega})\ ,$$

is a constant at a given point in space, or if, P,Q, and R are all constants, Z is constant everywhere in space. The motion which is described in equation (1) is fixed everywhere in space if the value of Z is given and the magnitude of y or u is specified at some particular point.

In the analysis of electrical circuits the coefficients P, Q and R are the inductance, resistance and capacitance of the circuit, respectively. The force F is the applied voltage and u is the current. In this case the ratio Z_1 is defined as the electrical impedance. If the inductance, resistance and capacitance are constants, the electrical impedance is distributed.

As an analogue to the electrical impedance the acoustic impedance is defined as the ratio of the force acting on the fluid to the fluid velocity. The specific acoustic impedance Z is defined as the ratio of the excess pressure to the velocity.

$$5) \quad Z = \frac{F}{Au} = \frac{p}{u}$$

A is the area over which the force is acting.

In general the force is not a simple single term harmonic function, but more often is described by a sum of terms having different frequencies. An example is that of a force described by a Fourier series.

$$6) \quad F = \sum_{n=-\infty}^{\infty} F_n e^{-in\omega t}$$

In this case there is a displacement velocity, u_n, for <u>each</u> term of the series describing the forcing function. Thus the ratio of the force to the velocity is not a function of space alone. However, each harmonic term, F_n, of the series has an associated velocity, u_n, and the ratio

$$7) \quad Z_n = \frac{F_n}{u_n}$$

is a function of the space coordinates alone. Since the total velocity is the sum of all the component velocities,

$$8) \quad u_n = \sum_{n=-\infty}^{\infty} \frac{F_n}{Z_{1n}} e^{-in\omega t} = \sum_{n=-\infty}^{\infty} \frac{p_n}{Z_n} e^{-in\omega t},$$

The force may be written

$$9) \quad F = \sum_{n=-\infty}^{\infty} u_n Z_{1n} e^{-in\omega t}.$$

The pressure is

$$p = \sum_{n=-\infty}^{\infty} u_n Z_n e^{-in\omega t}.$$

V. THE ACOUSTIC IMPEDANCE OF A PLANE WAVE IN A TUBE.

Each time the intake valves of an internal combustion engine opens, a plane wave pulse is sent down the tube, being superimposed upon any vibrations which exist in the pipe. It is convenient to discuss the phase relations of the velocity due to this pulse and the pressure by means of the specific acoustic impedance.

The solution of the equation of motion which describes a plane pulse traveling the length of the tube is

1) $y = Be^{-i\omega t - \varkappa (u - ib)x}; \quad \varkappa b = \frac{\omega}{c'}$

The excess pressure of the wave is

2) $p = -\rho c^2 \frac{y}{x} = \varkappa \rho c^2 (a - ib)\, y$

and the fluid velocity is

3) $u = -i\omega y.$

The specific acoustic impedance for any given frequency is then

4) $Z = \frac{i\varkappa \rho c^2}{\omega}(a - ib)$

The damping coefficient per unit length of the tube ,a, is small and can be neglected. The impedance reduces to

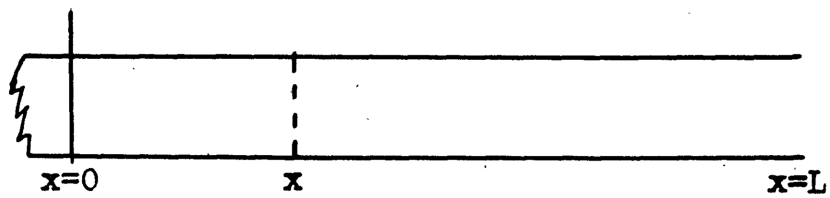
5) $Z = \rho \frac{c^2}{c'} \cong \rho c.$

in which c is the velocity of sound in the open air, and c' is the velocity of sound in the tube. The impedance for a plane wave travelling in one direction along a tube is real, positive or negative, depending upon whether the wave travels on the positive or negative direction, and is almost independent of the frequency of vibration. It is independent of the time and position. Physically, this means that if a pulse is started from the valve end of the intake pipe the pressure and velocity of the pulse remain in phase until a boundary is reached. Whether they remain in phase afterward, depends upon the effect of the boundary. If the specific acoustic impedance of the standing waves in the pipe has the same sign as the impedance of the pulse, the standing waves will be maintained, and resonance will exist in the intake pipe.

The next step in the analysis is to investigate the acoustic impedance of a pipe open at one end, and discover whether a resonance condition can exist.

VI. SPECIFIC ACOUSTIC IMPEDANCE IN A TUBE WITH AN OPEN END.

The specific acoustic impedance of waves in a finite length of tube is complicated by the reflections from the ends. An engine intake pipe is open to the atmosphere, so the impedance of a tube open at one end will be derived. A simple diagram of the tube is shown in the figure.



The impedance will be derived for any point x along the tube. The open end is at L.

Since pressure waves are reflected from the end of the tube the general solution of the equation of motion must be utilized to derive the impedance. The general solution is:

$$1) \quad y = (Ae^{\kappa(a - ib)x} + Be^{-\kappa(a - ib)x})e^{-i\omega t} .$$

At an open end the pressure is always that of the surrounding atmosphere so there is no excess pressure. Since the excess pressure is directly proportional to the space rate of change of the displacement,

$$2) \quad \left(\frac{\partial y}{\partial x}\right)_{x=L} = \kappa(a - ib)\,(Ae^{\kappa(a - ib)L} - Be^{-\kappa(a - ib)L}) = 0.$$

The equation is solved for A.

3) $A = Be^{-2\varkappa(a - ib)L}$.

The displacement becomes

4) $y = Ae^{-\varkappa(a - ib)L}\left[e^{\varkappa(a - ib)(x - L)} + e^{-\varkappa(a-ib)(x-L)}\right] \cdot e^{-i\omega t}$.

The bracketed term is just twice the hyperbolic cosine so the displacement reduces to

5) $y = 2Ae^{-\varkappa(a - ib)L} \cosh \varkappa(a - ib)(x - L)e^{-i\omega t}$.

The pressure at any point in the pipe is

6) $p = -\rho c^2 \frac{\partial y}{\partial x} = 2\varkappa\rho c^2 A(a - ib)e^{-\varkappa(a - ib)L} \sinh \varkappa(a - ib) \cdot (x - L)e^{-i\omega t}$,

and the velocity is

7) $u = -2i\omega Ae^{-\varkappa(a - ib)L} \cosh \varkappa(a - ib)(x - L)e^{-i\omega t}$.

The ratio of the pressure to the velocity is the specific acoustic impedance for a given frequency.

8) $Z_n = \frac{p}{u} = -\frac{i\varkappa\rho c^2}{\omega}(a - ib) \tanh \varkappa(a - ib)(x - L)$.

Since a is very small, it may be neglected in the coefficient of the hyperbolic tangent. From the preceding section $\varkappa b = \omega/c'$ so the impedance reduces to

9) $Z_n = -\rho c \tanh \varkappa(a - ib)(x - L)$.

An investigation of the properties of the hyperbolic tangent reveals that, when the product, $b(x - L)$ is negative and equal to an odd, positive half integer, it is real and negative in sign. In this case the impedance in the tube is in phase with the impedance of the plane pulse, and it is to be expected that the succession of plane pulses produced by the valve-piston mechanism of the engine will maintain standing waves in the induction system.

For very small damping the impedance in the tube approaches more and more closely the value for the non-viscous case.

10) $Z_n = -i\rho c \tan \frac{\omega}{c} (x - L).$

THE HYPERBOLIC TANGENT.

The simplest method of obtaining a particular value of the hyperbolic tangent for the evaluation of the specific acoustic impedance in the tube is a graphical one. The hyperbolic tangent is expanded into its real and imaginary parts, designated as X and Y, respectively. These expressions may be solved for the arguments of the hyperbolic tangent, and the results for constant values of the argument are plotted as functions of X and Y which are considered as ordinary Cartesian coordinates.

The hyperbolic tangent is first written

1) $\tanh \pi(a-ib)(x-L) = \tanh \pi(\alpha - i\beta) = X + iY = \left|\frac{Z_n}{\rho c}\right| e^{-i\varphi_n},$

$\alpha = a(x - L); \beta = b(x - L).$

The function is expanded by means of trigonometric identities.

2) $\tanh \pi(\alpha - i\beta) = \dfrac{-i \sin \pi(\beta + i\alpha) \cos \pi(\beta - i\alpha)}{\cos \pi(\beta + i\alpha) \cos \pi(\beta - i\alpha)}$

$= \dfrac{-i(\sin 2\pi\beta + \sin 2\pi i\alpha)}{\cos 2\pi\beta + \cos 2\pi i\alpha}.$

3) $X = \dfrac{\sinh 2\pi\alpha}{\cosh 2\pi\alpha + \cos 2\pi\beta}$; $Y = \dfrac{-\sin 2\pi\beta}{\cosh 2\pi\alpha + \cos 2\pi\beta}$.

When β is eliminated from these expressions, α is reduced to the function of X and Y.

4) $\coth 2\pi\alpha = \dfrac{X^2 + Y^2 + 1}{2X}$.

This is just the equation of a circle with a radius equal to the csch $2\pi\alpha$ with its center lying on the X axis a distance from the origin equal to coth $2\pi\alpha$.

5) $(X - \coth 2\pi\alpha)^2 + Y^2 = \operatorname{csch}^2 2\pi\alpha$.

When α is eliminated from equation (3), β is expressed by the relation

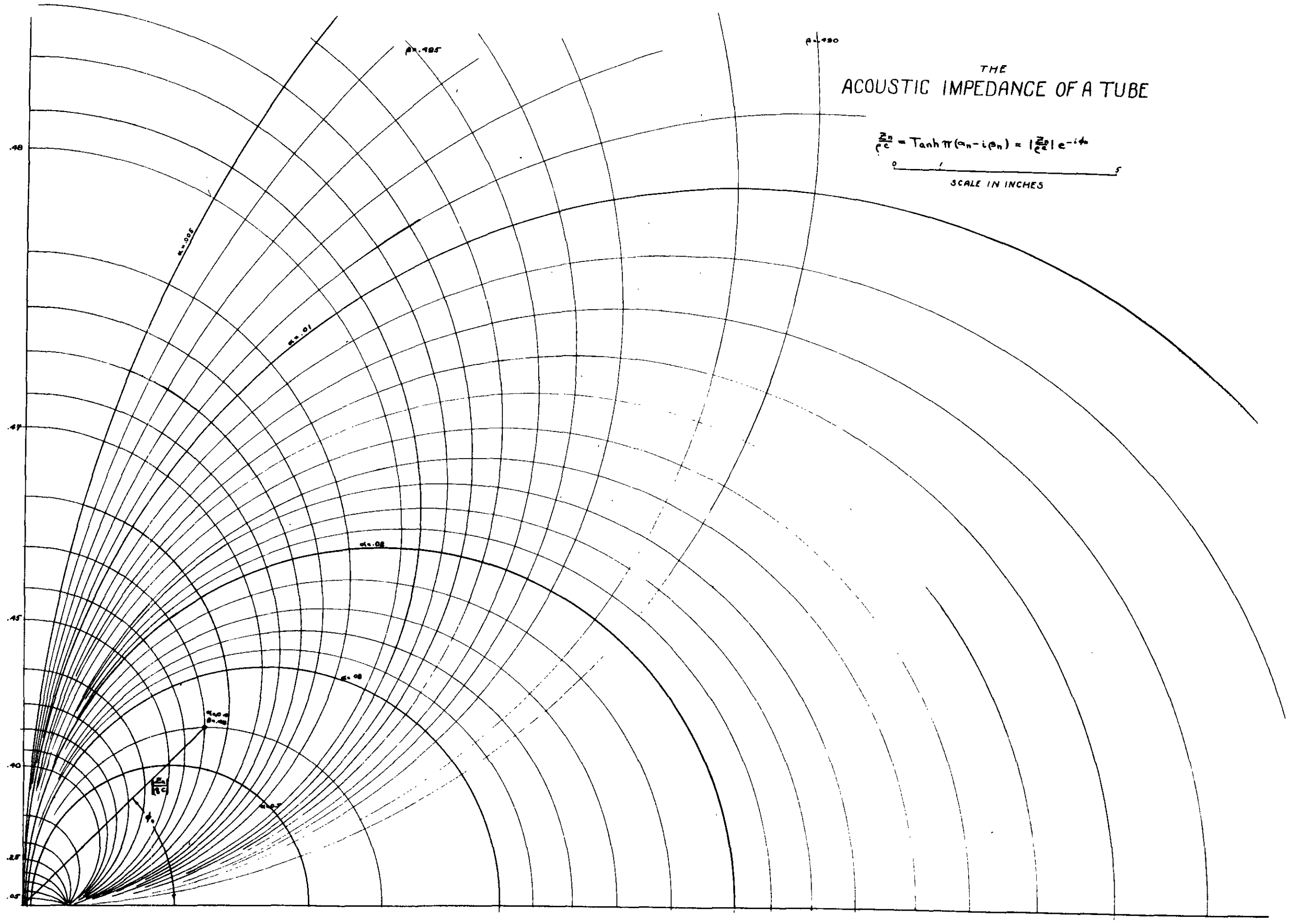
6) $\cot 2\pi\beta = \dfrac{X^2 + Y^2 - 1}{2Y}$.

which is the equation of the circle

7) $X^2 + (Y - \cot 2\pi\beta)^2 = \csc^2 2\pi\beta$.

This circle has a radius equal to the csc $2\pi\beta$, and its center lies on the Y axis at a distance equal from the origin to cot $2\pi\beta$.

These circles have been plotted in Figs. 14 and 15. The values of β have been indicated where the associated circles cut the Y axis. The values of α are indicated on the proper circles. The magnitude, $\left|\frac{Z_n}{\rho c}\right|$, is measured from the origin to the intersection of the two circles which are associated with the desired values of α and β. The phase angle, φ_n, is the angle between the real axis and the measured line. This angle is zero for half integral values of β. φ_n is positive for values of β between n and $(n + \frac{1}{2})$,



THE
ACOUSTIC IMPEDANCE OF A TUBE
$\frac{Z_n}{\rho c} = \tanh \pi(\alpha_n - i\beta_n) = |\frac{Z_n}{\rho c}| e^{-i\phi_n}$
0 1 5
SCALE IN INCHES

FIG. 14.

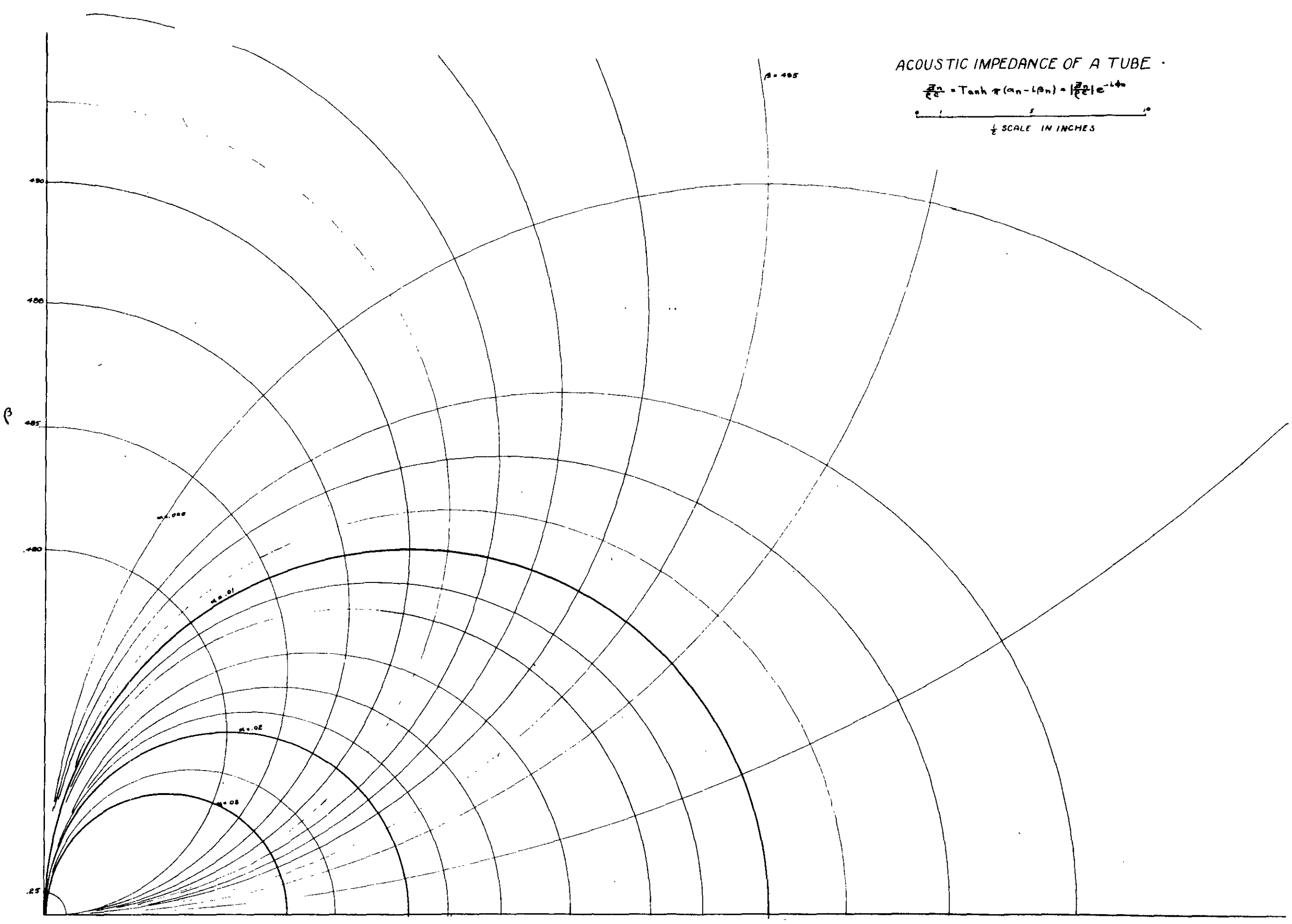

ACOUSTIC IMPEDANCE OF A TUBE.
$\frac{\bar{Z}_n}{\rho c} = \text{Tanh}\ \pi(\alpha_n - i\beta_n) = \left|\frac{\bar{Z}_n}{\rho c}\right| e^{-i\phi_n}$
½ SCALE IN INCHES
β

FIG. 15

n being a positive interger varying from zero to infinity φ_n is negative when β has a value between $n + \frac{1}{2}$ and $(n + 1)$.

The maximum values of the hyperbolic tangent are seen to occur when the phase angle is zero and

8) $b(x-L) = \frac{\omega}{c}\tau\ (x-L = \frac{n + 1}{2}$; $n = 0, 1, 2, 3, 4, 5,$ ---.

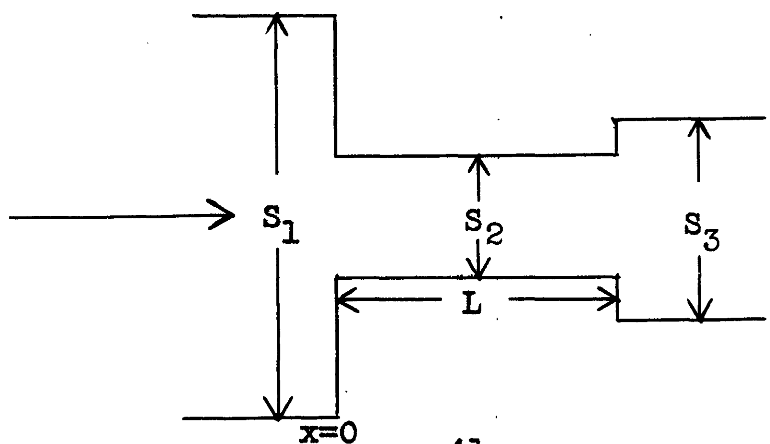
The method of measurement is clearly shown in Fig. 15.

VII. TRANSMISSION.

1. Change in phase at a constriction.

When the intake valves are open during the suction stroke of the engine, a plane wave pulse is sent through the valve opening, In general the area of the valve opening is considerably less than the area of the intake pipe. The friction in the valve port is quite large, and the pressure on the intake pipe side of the valve can be expected to lag the pressure just within the cylinder. The following discussion shows that the lag is too small to be detected, although the change in magnitude of the pressure across the port may be appreciable.

The pressure pulse is traveling in the direction indicated by the arrow in the accompanying figure. The area of the cylinder cross-section is S_1; the effective area of the valve opening is S_2; and the area of the intake pipe cross-section is S_3. The effective length of the valve constriction is l.



The incident and the reflected displacements in the cylinder are

1) $y_1 = A_1 e^{-i(\omega t - k^1 x)}$,

and

2) $y_r = B_1 e^{-i(\omega t + k^1 x)}$; $\omega = \frac{k^1}{c}$,

respectively. In the constriction the displacements are

3) $A_2 e^{-i(\omega t - kx)}$ and $B_2 e^{-i(\omega t + kx)}$,

respectively.

The pulse which emerges into the intake pipe and serves to build up the standing waves is described by the relation

4) $A_3 e^{-i[\omega t - k''(x - L)]}$.

In each case the coefficients of the exponentials are complex functions. The problem is to express the amplitude of the pulse within the cylinder which is incident upon the valve in terms of the amplitude of the emergent pulse and the dimensions of the system.

The pressure must be continuous across each joint, and the equation of continuity must be satisfied. The following four relations express the satisfaction of these conditions in terms of the complex amplitudes and the dimensions.

5) $$\left.\begin{aligned} A_1 - B_1 &= A_2 - B_2 \\ S_1(A_1 + B_1) &= S_2(A_2 + B_2) \end{aligned}\right\} \text{at } x = 0 .$$

$$6)\quad \left.\begin{aligned} A_2e^{ikL} - B_2e^{-ikL} &= A_3 \\ S_2(A_2e^{ikL} + B_2e^{-ikL}) &= S_3A_3 \end{aligned}\right\} \text{ at } x = L.$$

The ratios S_1/S_2 and S_2/S_3, are set equal to m and n. In terms of the amplitudes of the waves in the constriction the amplitude of the incident wave in the cylinder is

$$7)\quad A_1 = \frac{1}{2}(m+1)A_2 + \frac{1}{2}(m-1)B_2 ,$$

and the amplitudes of the waves in the constriction are expressed in terms of the amplitude of the pulse emerging from the constriction into the intake pipe.

$$8)\quad A_2 = \frac{(n+1)}{2}e^{-ikL}A_3;\ B_2 = \frac{(n-1)}{2}e^{ikL}A_3 .$$

Equations 7 and 8 are solved for the amplitude of the wave in the cylinder in terms of the emergent pulse.

$$9)\quad A_1 = \frac{A_3}{4}[(m+1)(n+1)e^{-ikL} + (m-1)(n-1)e^{ikL}] =$$

$$\frac{A_3}{2}[mn+1)\cos kL - i(m+n)\sin kL].$$

The phase angle Θ, of the coefficient of A_3 is just

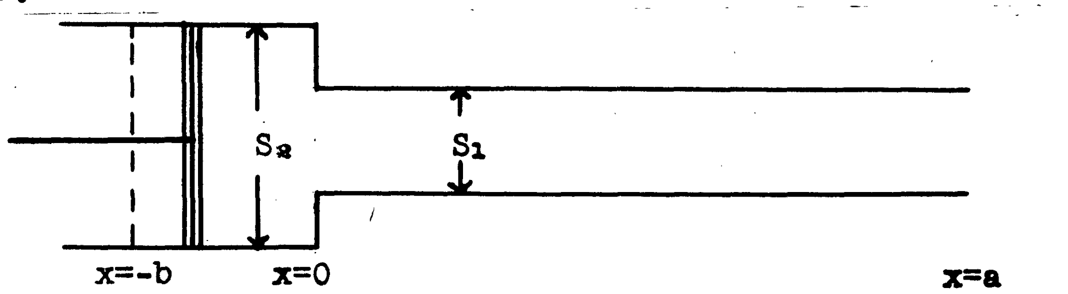
$$10)\quad \Theta = \tan^{-1}\left(-\frac{m+n}{mn+1}\tan kL\right);\ k = \frac{\omega}{c'} .$$

The effective length of the valve constriction is but a few centimeters in most engines, and the ratio of the angular frequency of the engine to the velocity of sound c', in the constriction will be quite small if the valve opening is sufficiently large that Kirchoff's equation of Motion will hold. In this case the phase lag of the pulse in the intake pipe is very small and can be neglected. No detectible phase lag was found on pressure-time records taken simultaneously within the cylinder and in the valve port. A measurable drop in pressure across the port was, however, found.

2. The Effect of a Small Volume at the End of a Pipe.

The dimensions of the cylinder of an internal combustion engine are small compared to the dimensions of the intake pipe for which supercharging effects are obtained. The analysis is considerably simplified if the effect of the cylinder volume can be neglected. This section presents the justification for such an assumption.

A simplified diagram of the piston-cylinder-intake pipe system is shown in the accompanying figure. The cylinder area is S_1 and the intake pipe area is S_2. The position of the piston when the crank is turned 90° from top center is at $x = -b$. The valve port is at $x = 0$, and the open end of the intake pipe at $x = a$.



The displacement within the cylinder is

1) $$y_2 = A_2 e^{-i(\omega t - kx)} + B_2 e^{-i(\omega t + kx)} ,$$

and that in the intake pipe is

2) $$y_1 = A_1 e^{-i(\omega t - kx)} + B_1 e^{-i(\omega t + kx)} .$$

At the open end of the pipe there is no excess pressure, and the amplitudes of the displacements in the pipe must satisfy the relation

3) $\left(\frac{\partial y_1}{\partial x}\right)_{x=a} = k(A_1 e^{ika} - B_1 e^{-ika}) = 0.$

Thus

4) $y_1 = 2A_1 e^{-ika} \cos k(x-a) e^{i\omega t}$.

The air follows the motion of the piston so at $x = -b$, the air displacement is equal to the piston displacement.

5) $A = A_2 e^{ikb} + B_2 e^{-ikb}$.

A is the amplitude of the piston displacement.

Across the joint at $x = 0$ the pressure must be continuous and the equation of continuity must be satisfied. These conditions produce the two relations

6) $2S_1 A_1 e^{-ika} \cos ka = S_2(A_2 + B_2) = S_1 A_1' \cos ka,$

and

7) $-2A_1 e^{-ika} \sin ka = A_2 - B_2 = -A_1' \sin ka.$

The amplitudes of the displacement within the cylinder are then

8) $A_2 = A_1' \frac{m \cos ka - \sin ka}{2}$,

$B_2 = A_1' \frac{m \cos ka + \sin ka}{2}$; $m = \frac{S_1}{S_2}$

These coefficients are substituted in equation (5) which is then solved for the coefficient A_1'.

$$9)\quad A_1' = \frac{\frac{A}{2} e^{ika}}{\left[\frac{S_1}{S_2} \cos ka \cos kb - i \sin ka \sin kb\right]} \quad .$$

The displacement in the intake pipe is then

$$10)\quad y_1 = \frac{A \cos k\,(x + a)\, e^{-i\omega t}}{\frac{S_1}{S_2} \cos ka \cos kb - i \sin ka \sin kb} \; ; \; k = \frac{\omega}{c} \quad .$$

If the dimensions of the cylinder are small, so the product of kb is small, the term with the sin kb may be neglected, and the cos kb is approximately unity. Then at the valve port, $x = 0$, the displacement is approximately

$$11)\quad y_1 = \frac{S_2}{S_1} A e^{-i\omega t} \quad .$$

When the frequency of the engine is in the neighborhood of the resonance frequency of the intake pipe, this relation is even more closely true since the sin (ka) is then nearly zero. Thus the displacement and the velocity just inside the valve port may be calculated from the piston stroke and speed just as if the air in the cylinder were an incompressible fluid. The effect of the engine piston is the same as an equivalent piston situated in the valve port whose velocity is equal to the product of the piston velocity and the ratio of the cross sectional areas of the cylinder and the intake pipe.

The resistance in the valve port decreases the amount of the displacement and hence the velocity at the intake side of the valve. The specific acoustic impedance of a plane wave has been shown to be approximately equal to the product of the equilibrium density of the air and the velocity of sound. This affords a method for estimating the decrease in velocity across the valve

$$\text{Velocity decrease} = \frac{\text{Pressure decrease}}{\rho c} .$$

This decrease must be kept in mind for consideration in the following analysis. Rough measurements of the pressure decrease indicate that the decrease in velocity may be predicted quite accurately from this relation.

VIII. THE PROBLEM OF THE INTAKE PIPE

The vibrations in the induction system of an internal combustion engine are considered to be the same as those generated by a piston located in the valve port of the engine. This piston is driven and controlled by the valve-piston mechanism of the engine. The velocity of this equivalent piston determines the velocity along the pipe which may be calculated from the theory of sound. Since the specific acoustic impedance of the pipe is known, the pressures in the pipe may be calculated from the velocity. The variation of the pressure in the valve port gives a qualitative picture of the density variations in the port and hence some idea of supercharging effects. The density in the port is directly related to the pressure, and its calculation can be performed quantitatively. The density at the instant the valves close is directly proportional to the power output. The object of this particular part of the discussion is to investigate these pressures and densities and draw some conclusions about the best design for the induction system of an internal combustion engine.

The velocity of the equivalent piston is determined by the engine piston velocity, U_o; the time of opening, t_1, and of closing, t_2, of the valves; the residual pressure, P_o, in the clearance volume of the cylinder; and the pressure drop across the valves, D. In general the equivalent piston velocity, u_o, can be expressed as

1) $u_o = u_o(t_1, t_2, U_o, P_o, D, t)$.

$u_o = 0$ the rest of the valve cycle.

The time origin is taken when the displacement of the engine piston is zero, i.e., the crank is at an angle 90° after top center.

When the valve opens and closes the velocity of the air in the valve port is definitely zero and remains so while the valve is closed. The air receives considerable kinetic energy during the time the valve is open. This energy must be transformed entirely to potential energy by the time the valve is closed. For this reason a tendency toward a high pressure can be expected when the valve closes. This is born out by the pressure-time records in the valve port.

The residual pressure in the cylinder when the valve opens may be either above or below the equilibrium pressure in the intake pipe, depending upon the efficiency of the scavenging of the burned gases from the clearance volume during the previous stroke and the time of opening of the valve. This pressure determines whether the velocity in the port will increase or decrease immediately after the valve is opened. If the residual pressure is just that of the air in the intake port, the velocity will follow the engine piston velocity as discussed in the previous section.

The velocity variation in the intake port is repeated during each valve cycle so the frequency of the impulses is the frequency, f, of the valve mechanism. This permits the velocity to be expressed as a Fourier series of the normal modes of vibration of the intake pipe. (MacRobert - Functions of a Complex Variable, page 87).

$$2)\quad u_o = -\sum_{n=-\infty}^{\infty} U_n\, e^{-2\pi i n f t} \;.$$

The coefficients of the series are

$$3)\quad U_n = f\int_{t_1}^{t_2} u_o(t)\; e^{-2\pi i n f t}\, dt.$$

The section on the specific acoustic impedance has shown that in an open end pipe the velocity of the air is given by the relation

$$4)\quad u_n = A_n' \cosh \pi(a_n - ib_n)(x - L)\, e^{-2\pi i n f t}$$

for each mode of vibration that is excited in the pipe. The velocity in the port must be that given in equation (2). Therefore, for each mode of vibration

$$5)\quad U_n e^{-2\pi i n f t} = A_n' \cosh \pi(a_n - ib_n)L e^{-2\pi i n f t}$$

$$6)\quad A_n' = \frac{U_n}{\cosh \pi(a_n - ib_n)L}$$

Since the resultant velocity in the pipe is made up of the sum of the component velocities, the fluid velocity in the pipe is

$$7)\quad u = -\sum_{n=-\infty}^{\infty} U_n \frac{\cosh \pi(a_n - ib_n)(x - L)}{\cosh \pi(a_n - ib_n)L} e^{-2\pi inft} \quad .$$

By inspection it is seen to reduce to the velocity u_o at the valve port.

The specific acoustic impedance in the pipe for each harmonic component has been shown to be

$$Z_n = -\rho c \tanh \pi(a_n - ib_n)(x - L) \quad ,$$

at any point. Therefore the excess pressure can be calculated immediately.

$$8)\quad p = \rho c \sum_{n=-\infty}^{\infty} U_n \frac{\sinh \pi(a_n - ib_n)(x - L)}{\cosh \pi(a_n - ib_n)L} e^{-2\pi inft} \quad .$$

This is the pressure on the intake pipe side of the valve port. Just inside the cylinder the pressure is either more or less than the above pressure, depending upon the direction of flow in the port, since there is the pressure decrease ,D, across the valve due to its resistance. Thus the excess pressure within the cylinder is

$$9)\quad p = \rho c \sum_{n=-\infty}^{\infty} U_n \frac{\sinh \pi(a_n - ib_n)(x - L)}{\cosh \pi(a_n - ib_n)L} e^{-2\pi inft} \pm D.$$

The fractional change in the density of the air in the cylinder from the pressure vibrations existing in the pipe can be calculated from this pressure at once. This change is simply the condensation.

$$10)\quad s = \frac{p}{\rho c^2} = -\frac{1}{c}\sum_{n=-\infty}^{\infty} U_n \frac{\sinh \pi(a_n - ib_n)(x - L)}{\cosh \pi (a_n - ib_n)/L} e^{-2\pi inft} + \frac{D}{\rho c^2}$$

If this condensation is greater than zero at the time the valves close the mass of air actually forced into the cylinder is more than the mass in the displaced volume would be if reckoned at the pressure of the surrounding atmosphere. Under these circumstances a greater power output will be obtained.

The quantity generally used in the discussion of the power output of an engine is the volumetric efficiency ,E, since it is directly proportional to the power output. It has been defined in the introduction as the ratio of the volume of air swept into the cylinder per valve cycle, reckoned at the temperature and pressure of the surrounding atmosphere, to the displacement volume of the engine. Actually, this ratio is a "mass efficiency" and is equal to the ratio of the mass of the air swept into the cylinder per valve cycle to the mass in the displacement volume in the open air, which in turn is equal to the ratio of the density in the outside air. This density ratio is simply the sum of the real part of the condensation at $t = t_2$ and unity.

$$11) \quad E = \frac{\rho}{\rho_0} = 1 + s = 1 - \frac{1}{c}\sum_{n=-\infty}^{\infty} U_n \tanh{}^{*}(a_n - ib_n)L e^{-2\pi intt_2}.$$

There is no pressure drop when the valve closes since the velocity through the port is then zero, according to our assumption of the form of u_0.

Each coefficient of the Fourier series, U_n, may be expressed in terms of its magnitude and angle.

$$U_n = |U_n| e^{-i\theta_n} \quad ,$$

and the hyperbolic tangent has been expressed in the form

$$\left|\frac{Z_n}{\rho c}\right| e^{-i\varphi_n}$$

Thus the volumetric efficiency is

$$12) \quad E = 1 - \frac{1}{c}\sum_{n=-\infty}^{\infty}|U_n| \left|\frac{Z_n}{\rho c}\right| \cos\,(2\pi nft_2 + \varphi_n + \theta_n) \quad .$$

To have a large volumetric efficiency the pressure decrease, D, must be as small as possible since the magnitude of the coefficients of the Fourier series depend directly upon the velocity in the port which is decreased by the pressure drop. For this reason the valves should be so designed as to present as little resistance to the flow of air as possible.

The hyperbolic tangent is found to have its maximum values when the product $(b_n)L$ is a positive half integer. The term b_n was defined in the section on the specific acoustic impedance as $\frac{\omega_n}{\pi c}r$. The length of pipe for which the hyperbolic tangent is a maximum is then given by the relation

$$13) \quad L = \frac{(2m+1)}{2b_n} = \frac{(2m+1)c'}{4nf} ; \quad \begin{array}{l} m = 0,1,2,3, \text{ etc.} \\ n = 0,1,2,3, \text{ etc.} \end{array}$$

Since $\omega_n = 2\pi nft$. From inspection of figure 15 it is seen that for small attenuation the magnitude of the hyperbolic tangent becomes small if the product $b_n L$ has a value a very little different from an odd positive half integer. If the intake pipe length L satisfies equation (13) for a given value of n and the product of $|U_n| \cos(2\pi nft_2 + \varphi_n + \theta_n)$ is large, the n'th term of the series will predominate and it may be expected that the series will have nearly a maximum amplitude for this length of pipe. Whether the volumetric efficiency will have nearly a maximum or a minimum value depends upon the sign of this product (i.e., whether U_n is positive or negative and whether $\varphi_n + \theta_n$ is less than or greater than $\pi/2$).

The magnitude and the sign of the product U_n

$$U_n \cos(2\pi nft_2 + \varphi_n + \theta_n),$$

depend primarily upon the design of the engine, being particularly sensitive to the timing of the valves. Thus by suitable timing the length of pipe for which maximum supercharging will occur can be varied at will.

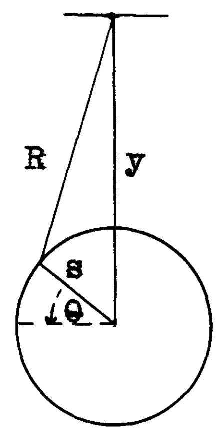
Further discussion of the maxima and minima becomes too complicated to treat except in relation to a specific example.

1. A Particular Example of the Intake Pipe.

The calculations for the pressure-time records and the volumetric efficiencies for an engine which is operating under a given set of test conditions are carried out in this section. The test conditions have been selected to represent the simplest feasible ones under which an engine will operate satisfactorily. The N.A.C.A. Universal test engine which is easily adjusted to satisfy the operating conditions is used for the experimental check of the results of this section.

The determination of the velocity variation of the air in the valve port is the difficult part of the problem. The velocity in the port when the valve is closed is zero. The conditions in the cylinder are assumed to be such that the residual pressure in the cylinder when the valve opens is just atmospheric. In this case the motion of the air in the port will follow that of the engine piston as closely as possible.

The motion of the engine piston can be determined from the simple geometry of the system. In the accompanying figure the position of the piston in respect to the crank shaft of the engine is y, half the stroke is s, and the length of the piston rod is R. The piston position is then given by the equation



$$y^2 = R^2 + s^2 + 2sy \sin \theta$$

1) $Y = s \cdot \sin\theta + R(1 - \frac{s^2}{2R^2} \cos^2 \theta)$,

since ratio, $(s/R)^2$, is small.

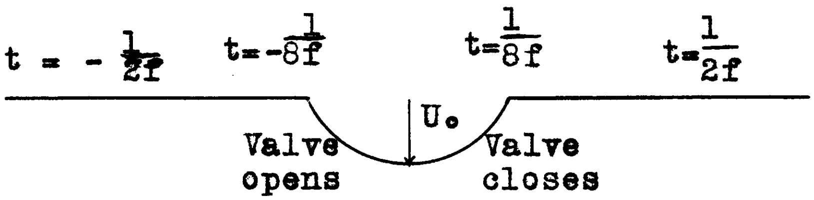
The piston speed is

2) $U_p = s(R.P.M.)\frac{\pi}{30}[1+\frac{s}{R}\sin \theta] \cos \theta$.

The velocity on the cylinder side of the intake valves can be calculated from Equation 11, section VII, part 2. On the intake pipe side of the valves the velocity is less because of the pressure decrease across the mv. The decrease in velocity is not directly proportional to the velocity of the air in the port, but to a higher power of the port velocity. The errors caused by the neglect of the effect of the length of the crank arm of the engine and the assumption that the velocity drop is proportional to the velocity in the valve port are opposite in sign and of the same order of magnitude. For this reason the velocity at the end of the intake pipe is assumed to have a velocity varying sinusoidally with time while the valve is open and the frequency of this variation is that of the engine. Under these assumptions the velocity of the equivalent piston in the valve port is described by the relations

3) $u_o = U_o \cos 4\pi nft, \quad \frac{-1}{8f} < t < \frac{+1}{8f}$;
$\quad\quad 0$, the rest of the valve cycle.

if the valves open at top center and close at bottom center of the engine and the frequency of the valve mechanism is f. The variation of the port velocity with the time is illustrated in the figure below.



The Fourier series expansion of the velocity in the intake port is

4) $$u_o = -U_o \sum_{n=-\infty}^{\infty} A_n e^{-2\pi inft} .$$

The coefficients of the series are (MacRobert - Functions of a Complex Variable, p. 87).

5) $$A_n = f \int_{-\frac{1}{8f}}^{\frac{1}{8f}} \cos 4\pi nft \; e^{-2\pi inft} \, dt .$$

The velocity can be separated into its real and imaginary parts which are the familiar Fourier series involving cosines and sines, respectively.

6) $$u_o = -U_o A_o - U_o \sum_{n=1}^{\infty} (A_n + A_n) \cos 2\pi nft - U_o \sum_{n=1}^{\infty} (A_n - A_{-n}) \cdot i \sin 2\pi nft$$

By multiplying both sides of equation (6) by the sine or the cosine of $2\pi nft$ and integrating over the valve cycle, the coefficients of the real and the imaginary parts of the series are evaluated in the usual manner.

$$7) \quad A_n + A_{-n} = 2f \int_{-\frac{1}{8f}}^{\frac{1}{8f}} \cos 4\pi ft \cos 2\pi nft \, dt \quad .$$

$$8) \quad A_n - A_{-n} = -2if \int_{-\frac{1}{8f}}^{\frac{1}{8f}} \cos 4\pi ft \sin 2\pi nft \, dt \; .$$

Since the cosine $4\pi ft$ is an even function, equation (8) is equal to zero, and A_n and A_{-n} are equal. Equation (7) shows at once that A_n is a real quantity.

The excess pressure in the valve port is

$$9) \quad p = -\rho c U_o \sum_{n=-\infty}^{+\infty} A_n \tanh \pi(a_n - ib_n)L e^{-2\pi inft} \quad .$$

The real part of this function is plotted.

$$10) \quad p = -\rho c U_o \sum_{n=1}^{\infty} B_n \cdot \left|\frac{Z_n}{\rho c}\right| \cos\left(2\pi nft + \varphi_n\right).$$

Where the values of the quantities Z_n are obtained from Figs. 14 and 15.

In this equation $B_n = 2A_n$ and the summation is taken from n unity to n infinity since the magnitude $\left|\frac{Z_n}{\rho c}\right| = \left|\frac{Z_{-n}}{\rho c}\right|$, $Z_o = 0$, and the cosine of a negative angle is equal to the cosine of a positive angle, Thus

$$11)\quad B_n = 2f \int_{-\frac{1}{8f}}^{\frac{1}{8f}} \cos 4\pi ft \cos 2\pi nft \, dt = \frac{1}{\pi} \cdot \left(\frac{\sin (2-n)\pi/4}{2-n} + \frac{\sin (2+n)\pi/4}{2+n} \right)$$

$$= \frac{4}{(n^2-4)} \sin (n-2)\pi/4;$$

$$B_0 = A_0 = 1/2\pi \quad .$$

The first twenty Fourier coefficients of the series follow:

$B_0 = \frac{1}{2\pi} = .159$; $B_1 = \frac{2\sqrt{2}}{\pi} = .300$; $B_2 = \frac{1}{2} = .5$; $B_3 = \frac{2\sqrt{2}}{5\pi} = .255$;

$B_4 = \frac{1}{3\pi} = .106$; $B_5 = \frac{2\sqrt{2}}{21\pi} = .0428$; $B_6 = 0$; $B_7 = -\frac{2\sqrt{2}}{45\pi} = -.0200$;

$B_8 = -\frac{1}{15\pi} = -.0242$; $B_9 = -\frac{2\sqrt{2}}{77\pi} = -.0117$; $B_{10} = 0$;

$B_{11} = \frac{2\sqrt{2}}{221\pi} = .0077$; $B_{12} = \frac{1}{35\pi} = .0091$; $B_{13} = \frac{2\sqrt{2}}{165\pi} = .00545$;

$B_{14} = 0$; $B_{15} = \frac{-2\sqrt{2}}{221\pi} = -.0041$; $B_{16} = \frac{-4}{256} = -.0050$;

$B_{17} = \frac{-2\sqrt{2}}{289\pi} = -.0030$; $B_{18} = 0$; $B_{19} = \frac{2\sqrt{2}}{361\pi} = .0025$;

$B_{20} = \frac{1}{99\pi} = .0025$; $B_{21} = \frac{2\sqrt{2}}{441\pi} = .0020$; $B_{22} = 0$.

Since the values of the coefficients for the sixth and higher harmonics are either zero or very small, the terms in the series higher than the sixth are neglected.

Measurements of the attenuation factor in pipes of various dimensions have been made by Eckhardt of the Bureau of Standards. The measurements were made in pipes which were placed in the open air so the external conditions were very similar to those under which the measurements were made on the intake pipe. The attenuation factor, which was measured by Eckhardt is for the intensities and for a one foot length of pipe. Since the intensity of sound is proportional to the velocity squared, his value is twice the factor for the velocity or the pressure. Thus the value of a_n is

$$12) \quad a_n = \frac{d}{24x}$$

for a one inch length of pipe. The measurements show that the attenuation is practically independent of the frequency. This is assumed to hold in the present calculations. The attenuation factor, a_n, calculated from Eckhardt's data, for a pipe with an inside diameter of 2 7/8 inches and a wall thickness of 0.051 inches is .00023 for one inch. The value of a_n estimated for the intake pipe which had a diameter of 2 9/16 inches and a wall thickness of 0.10 inches is estimated to be .0002 for one inch.

Rough estimates of the velocity of the air through the intake port when the valves were open indicated that the value of the product $\rho c U_0$ was approximately equal to unity. Comparison of the amplitudes of the calculated and observed pressures show that this product is approximately 1.2 when the engine speed is 1220 revolutions per minute, and 0.95 when the engine speed is 1630 revolutions per minute. These values are used in the calculations.

The calculated and the observed pressure-time records are plotted in Figs. 8, 9 and 10 for several different cases. Time is expressed in terms of the crank angle of the engine. The intake valves start opening at O which is about twenty-five degrees before the piston is at top center and close at C which is about twenty-five degrees after bottom center. The general agreement in form of the two curves is good.

Since the valves are assumed to close at bottom center when $t = 1/8f$, the volumetric efficiency, E, is

$$13)\quad E = 1 - \frac{U_o}{c}\left[\sum_{n=1}^{\infty} B_n \left|\frac{Z_n}{\rho c}\right| \cos\left(\frac{n\pi}{4} + \varphi_n\right)\right].$$

The value of $\frac{U_o}{c}$ used in the calculation of the volumetric efficiency is determined from the product of cU_o which is given above from the comparison of the calculated and observed pressure amplitudes.

The hyperbolic tangent for the n'th harmonic has been shown to have maxima when the relation

$$\frac{f_p}{f} = \frac{n}{2m+1}\ ;\ m = 0,1,2,3,----,$$

where f_p is the natural frequency of a pipe with one end closed, and f is the valve frequency, and its value decreases quite rapidly when the ratio of the frequencies becomes only a little different from this value. Thus a maximum volumetric efficiency may be predicted for a length of intake pipe which has a frequency very close to a value which satisfies this relation. For harmonics higher than the sixth the Fourier coefficients are so small that the corresponding series terms can be neglected. For this reason it is necessary to consider only that case in which $m = 0$, and large values of the sum can be expected when the ratio of pipe frequency to valve frequency is approximately 1, 2, 3, 4, or 5.

The following table shows at a glance the approximate ratios of pipe to valve frequency for which the greatest increase in the volumetric efficiency can be expected.

f_p/f	n	B_n	$\cos(\frac{\pi}{4}n + {}_n)$	Sign of Summation.
1	1	.300	.707	-
2	2	.500	0	0
3	3	.255	-.707	+
4	4	.106	-1.00	+
5	5	.043	- .707	+
6	6	0	0	0

From the table it is seen that the volumetric efficiency is actually less than unity if the pipe frequency is the same as the valve frequency, and there is no increase if the pipe frequency is twice that of the valves. There is a definite increase to be expected for n equal to 3, 4 or 5.

The relative magnitudes of the increase are indicated by the product of the cosine and the Fourier coefficient.

The exact determination of the maxima in the volumetric efficiency curve must be determined from a complete calculation of the sum. Figs. 11 and 12 show the results of the complete calculation. The curves for the calculated volumetric efficiency agree quite well with those of the measured Indicated Mean Effective Bressure, which is proportional to the total power output. The further discussion of the I.M.E.P. will be left to the section on the experimental procedure.

Fig. 13 shows the length of pipe which will give an increased volumetric efficiency as a function of the speed of the engine, computed using just the largest Fourier coefficient. The values of the Fourier coefficients, B_n, and the frequency ratios are given. The points are the lengths of pipe for which a maximum power output was observed. From this figure it is evident that the effect of the higher harmonics which are excited in the pipe is to decrease the length of pipe for which a maximum power output is observed. The experimental points show a definite trend toward closer agreement with the pipe lengths which are predicted approximately. Such an effect is to be expected according to the theory of the viscous effects since the attenuation factor varies directly as the square root of the frequency which is excited.

IX. EXPERIMENTAL PROCEDURE

The National Advisory Committee for Aeronautics Universal test engine is designed especially for research on internal combustion engine problems, and was selected for use in the experimental program. It is a conventional four-stroke, single cylinder gasoline engine. The general details of its construction are given in the N.A.C.A. report No. 250. Figs. 16a and 16b are photographs of the engine which was used.

An injection type of fuel system was used instead of a carburetor. This system has the advantage from the standpoint of this research of eliminating the venturi constriction in the intake pipe which would undoubtedly effect the pressure variations in the pipe. It also eliminates the heat and change of density effects due to the vaporization of the fuel. The fuel feed was controlled by a lever and graduated semicircle arrangement which was connected to the control on the fuel pump by a system of spools and cables so the motion of the lever was directly proportional to the motion of the pump control. The fuel was fed by gravity from calibrated tanks. In Fig. 16a one of the tanks is shown in the upper left-hand corner, and the fuel control is at the bottom of the instrument panel.

The valve system of the engine is arranged so that the lift and the timing of the valves can be varied. The lift was set so the intake valves were opened 0.312 inches and the

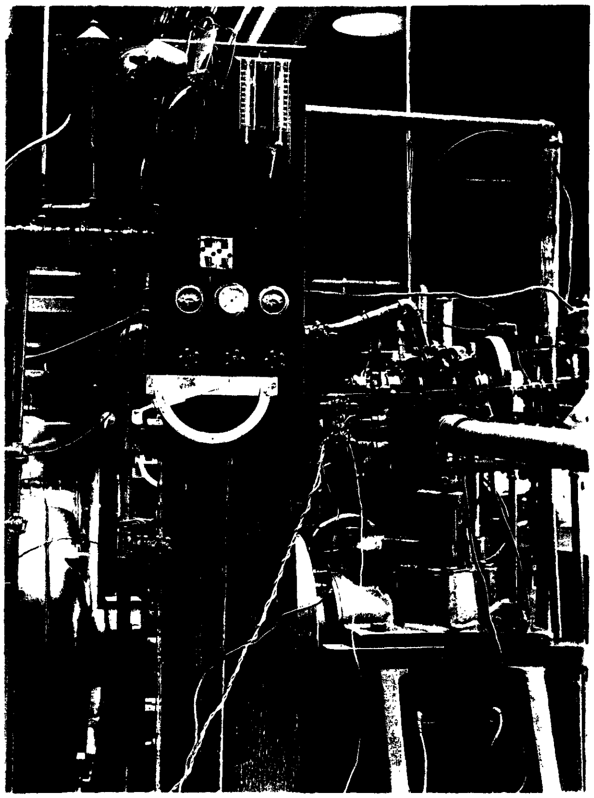


Fig. 16a.

The N.A.C.A. Universal Test Engine

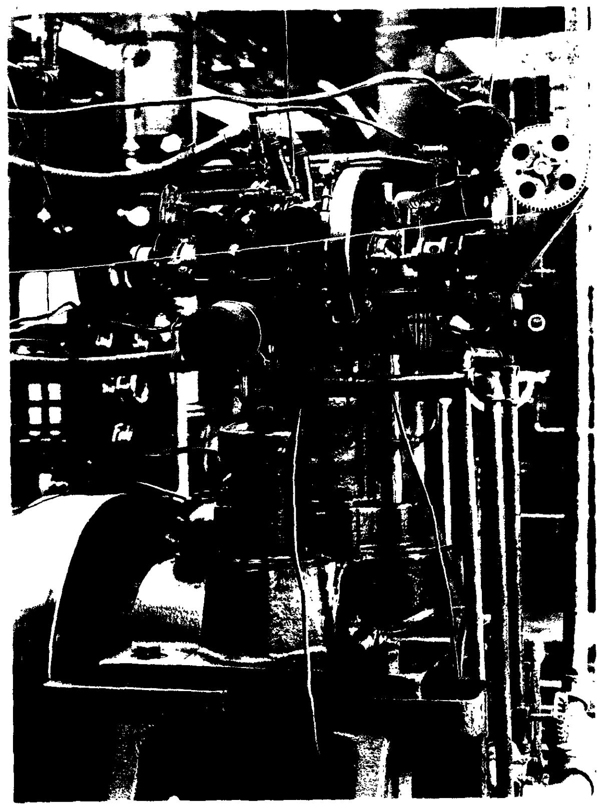


Fig. 16b.

The N.A.C.A. Universal Test Engine

exhaust valves 0.275 inches. About seventy degrees of crank travel were required to either open or close both the intake and the exhaust valves. The intake valves were estimated to be effectively wide open when the lift had reached 42 per cent of the maximum. Twenty-five degrees of crank travel were required to open the valve this amount after it had "cracked" and the same amount of travel was required to completely close the valves from the estimated lift. Thus the intake valve was set to open twenty-five degrees before the piston reached top center and to close twenty-five degrees after bottom center. The exhaust valves were set to open twenty-five degrees before bottom center and close twenty-five degrees after top center. The lift when the piston was at top or bottom center was 47 per cent of the total. The overlap of the intake and the exhaust valves was fifty degrees.

The compression ratio was 5:1 in all of the tests.

The power output of the engine was measured with a standard type of electrical dynamometer which had a constant equal to 4000. The brake horse power is calculated from the relation

$$1)\quad \text{B.H.P.} = \frac{(\text{Brake Load})(\text{R.P.M.})}{4000}$$

The frictional and mechanical power losses within the engine were obtained by driving the engine with the dynamometer and measuring the brake load. The total power output of the engine or indicated horse power I.H.P. is then

$$2)\quad I.H.P. = \frac{(\text{Brake load} + \text{Friction load})(R.P.M.)}{4000} .$$

A more convenient quantity for this research is the Indicated Mean Effective Pressure which is defined as the ratio of the Indicated Horse Power and the rate of displacement calculated from the displacement volume of the engine. Since the displacement volume per valve cycle has been defined as D, the I.M.E.P. is

$$3)\quad I.M.E.P. = \frac{(I.H.P.) \cdot 12 \cdot 33000}{D \cdot (R.P.M./2)} .$$

For this particular engine the

$$4)\quad I.M.E.P. = 1.44\ (\text{Brake Load} + \text{Friction Load}).$$

This quantity is directly proportional to the total power output for a given engine speed, and hence is directly proportional to the volumetric efficiency.

The intake pipe was standard seamless tubing which had an inside diameter of 2 9/16 inches and an average wall thickness of 0.10 inches. The tubing was attached to the engine in the condition that it was received with no smoothing of the inside. Sections of the tubing were cut so five inch variations in the intake pipe length could be obtained. The sections were joined with heavy four-ply rubber hose which was clamped with heavy pieces of wire. The method of connecting the pipe is clearly shown in Fig. 16a. Care was taken to have the ends of two sections of pipe which were joined as closely together as possible since the pressure vibrations were so large that, if any of the hose were exposed, it immediately started vibrating, heavy as it was. A sliding trombone-like intake pipe was not used as the pressure waves were so large that a considerable blast of air cameout from between the sliding walls of the two pipes.

The length of the intake pipe is measured from the intake valves to the open end along the center line. The sharp bend of the pipe in the valve port is a very short distance from the valves compared to the wavelengths of the pressure waves which are generated and has no noticeable effect. The cross sectional area of the pipe remains constant up to the valves. The shape of the cross section becomes ovular where the pipe enters the cylinder head. The ends of the oval are semi-circles of inch radius and the major axis is three inches long. This change in shape has little effect as it is small and does not change the cross sectional area.

The M.I.T. point-to-point pressure[1] indicator was used to measure the pressure variations in the intake pipe and in the cylinder. Fig. 16d shows the instrument as it was used in this investigation. The rotating cylinder upon which the pressure-time records are made is coupled directly to the dynamometer shaft. Directly above are the calibrated spring and stylus which are connected to a piston operated

(1) E.S.Taylor and C. S. Draper - A New High Speed Engine Indicator. Mechanical Engineering, Nov. 1933.

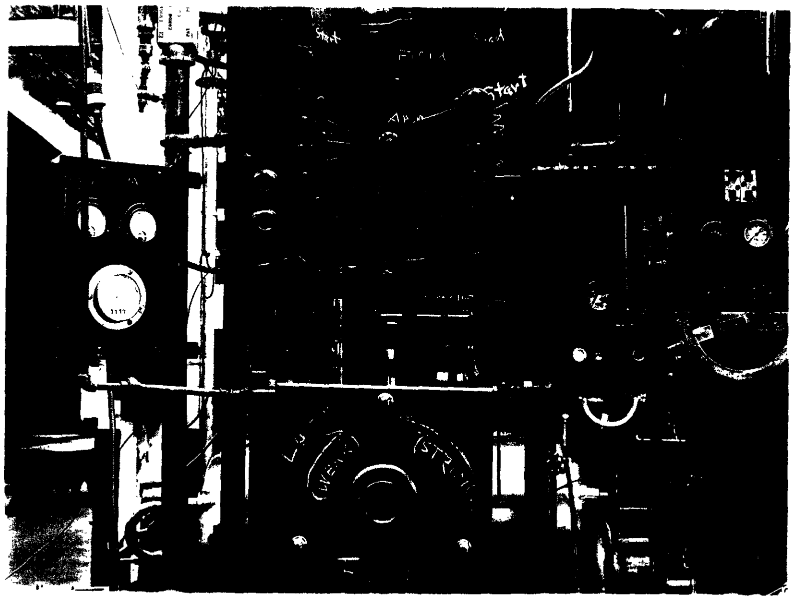


Fig. 16c.

Dynamometer Control Panel

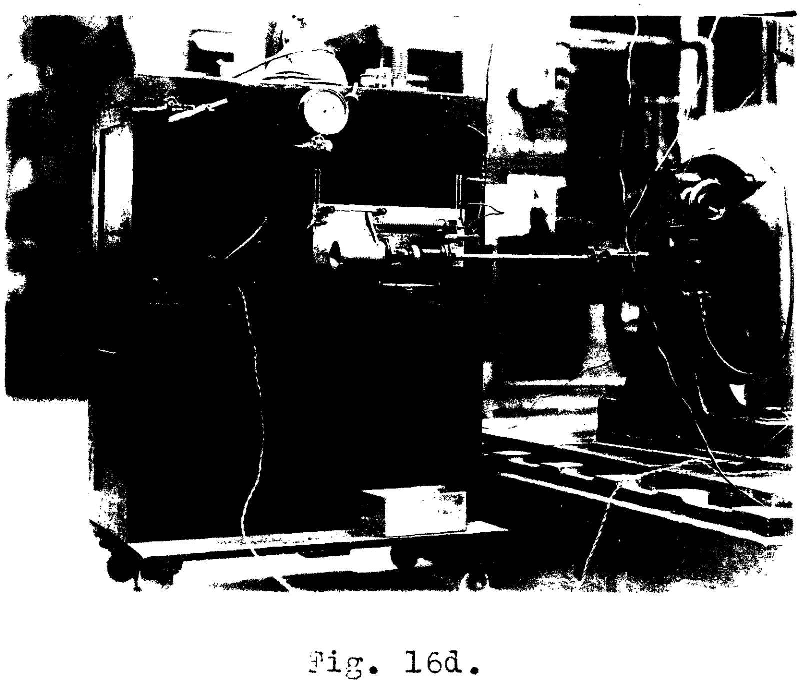


Fig. 16d.

M.I.T. High Speed Indicator

by the balancing pressure of the indicator elements. A one inch deflection of the stylus represents a pressure of 7 pounds per square inch. The pressure supply is obtained from the large tank of nitrogen. In these experiments it was convenient to have a small auxiliary pressure tank between the high pressure supply and the instrument. The auxiliary tank was maintained at about 40 pounds pressure. The thyratron trip circuit which furnishes the spark for marking the pressure record cards is on the left.

The indicator elements were fitted with a .0015 inches thick spring steel diaphragms. These elements are small balanced pressure diaphragms which act as switches to make and break the thyratron circuit. Care was taken to slightly loosen, then tighten the clamps holding the diaphragm after the engine had warmed in order that the diaphragm would not buckle due to the change in temperature. The installation of the elements is clearly shown in Figs. 16a and 16b. One is mounted on the under side of the intake pipe as close to the valve ports as possible, and another is mounted in the side of the cylinder head. A schematic diagram of the arrangement of the apparatus and the indicator is shown in Fig. 17. The pressure lines from the elements are run to two needle valves which are shown mounted together on the lower right-hand side of the engine instrument board in Fig. 16a. One pressure line was run from these valves to the indicator. These valves were found to be of the greatest convenience.

SCHEMATIC ARRANGEMENT OF APPARATUS

1. Indicator Element in Valve Port
2. Indicator Element in Cylinder
3. Pressure-Vacuum Pump
4. Calibrated Spring
5. Recording Cylinder

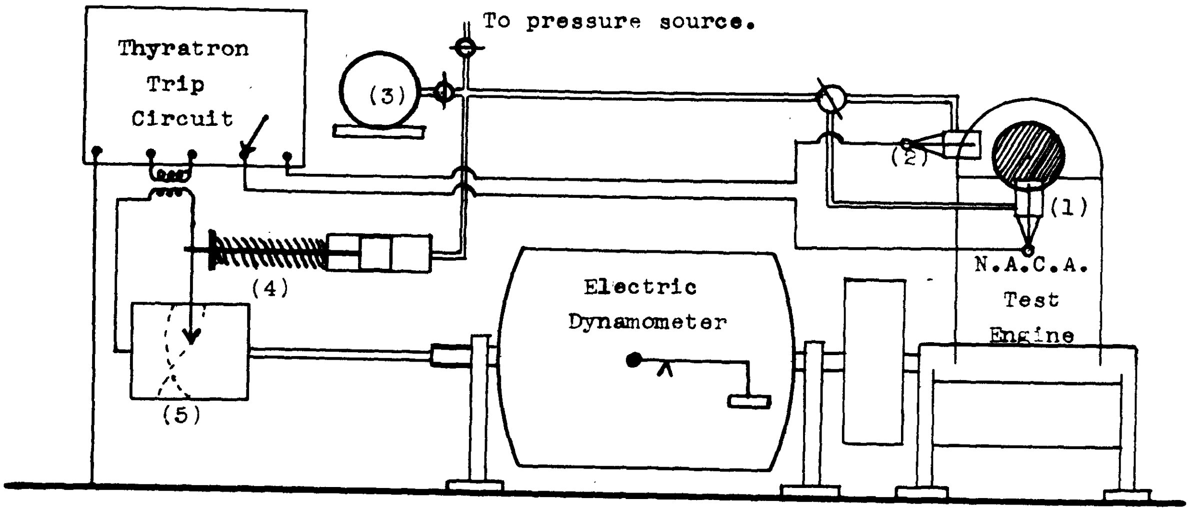


Figure 17

The dynamometer controls, tachometer and part of the engine controls are shown in Fig. 16c. A wet and dry bulb thermometer is mounted on the engine instrument board.

Before any tests were run, the engine was warmed until the oil pressure and temperature and the water temperature remained within a degree of suitable constant values. The criterion for good test conditions was that the brake load should remain constant for a period of approximately five minutes.

The throttle was removed from the intake pipe as soon as the engine had warmed sufficiently and all tests were made at full throttle. The spark advance was set to give the maximum power output with the shortest length of intake pipe attached to the engine. This adjustment was not changed during a run at given speed.

A test consisted of running the engine at constant speed while varying the length of the intake pipe over the desired range. For a given length of pipe the fuel setting was adjusted for the maximum power output. The engine was allowed to settle down to a steady running condition, and then the desired measurements were recorded. At the same time light spring pressure time records were taken. Immediately after all measurements and records were completed, the fuel supply was stopped and a friction brake reading was taken. This measurement was accomplished rapidly enough so there was no measurable drop in the temperature of water temperature. Immediately after this measurement had been accomplished, the fuel supply was reopened and the engine was made ready for the next run. An additional piece of intake pipe was connected, and the test cycle was repeated. With the care taken the operating conditions remained very constant during the time required to obtain the desired pressure-time records.

Fig. 1 and 7 show typical pressure-time records as they are taken from the recording cylinder of the indicator. Since two revolutions of the engine are required before a valve cycle is completed so that two traces appear on the pressure-time records, the records are opened out before comparison with the calculated pressure-time records of Figs. 8, 9 and 10. When records of the general form of Fig. 7 occur for any engine speed, a large increase in power output can be expected with the engine operating under the above conditions.

The Indicated Mean Effective Pressures calculated from the friction and brake load measurements are compared with the calculated volumetric efficiencies in Fig. 11 and 12. Additional measured I.M.E.P.'s are plotted in Figs. 18, 19 and 20. The experimental points plotted in Fig. 13 are taken from the maxima occurring in these plots of I.M.E.P. vs. intake pipe length.

VARIATION OF THE INDICATED MEAN EFFECTIVE PRESSURE WITH THE INTAKE PIPE LENGTH
Diameter of pipe: 2 9/16 inches

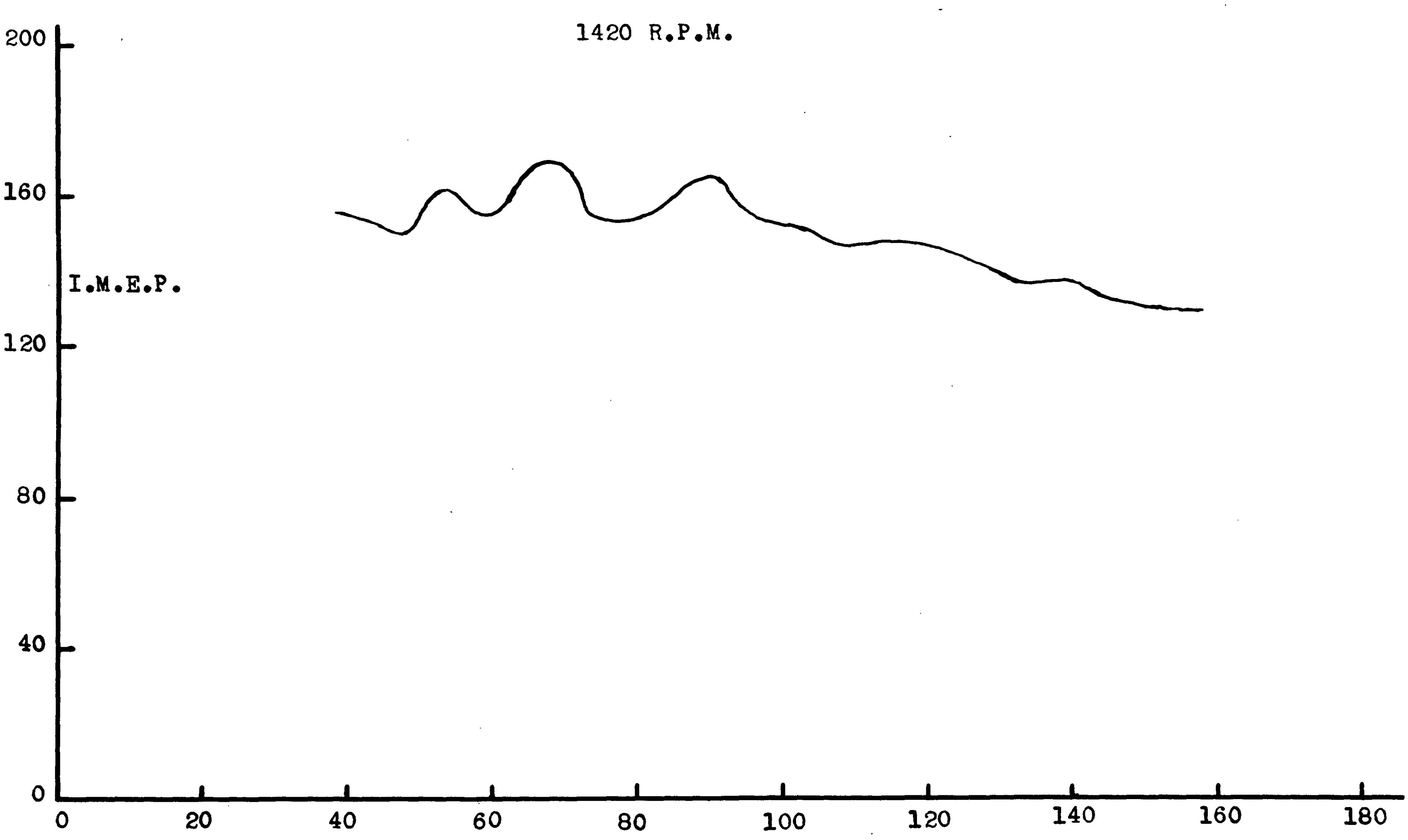
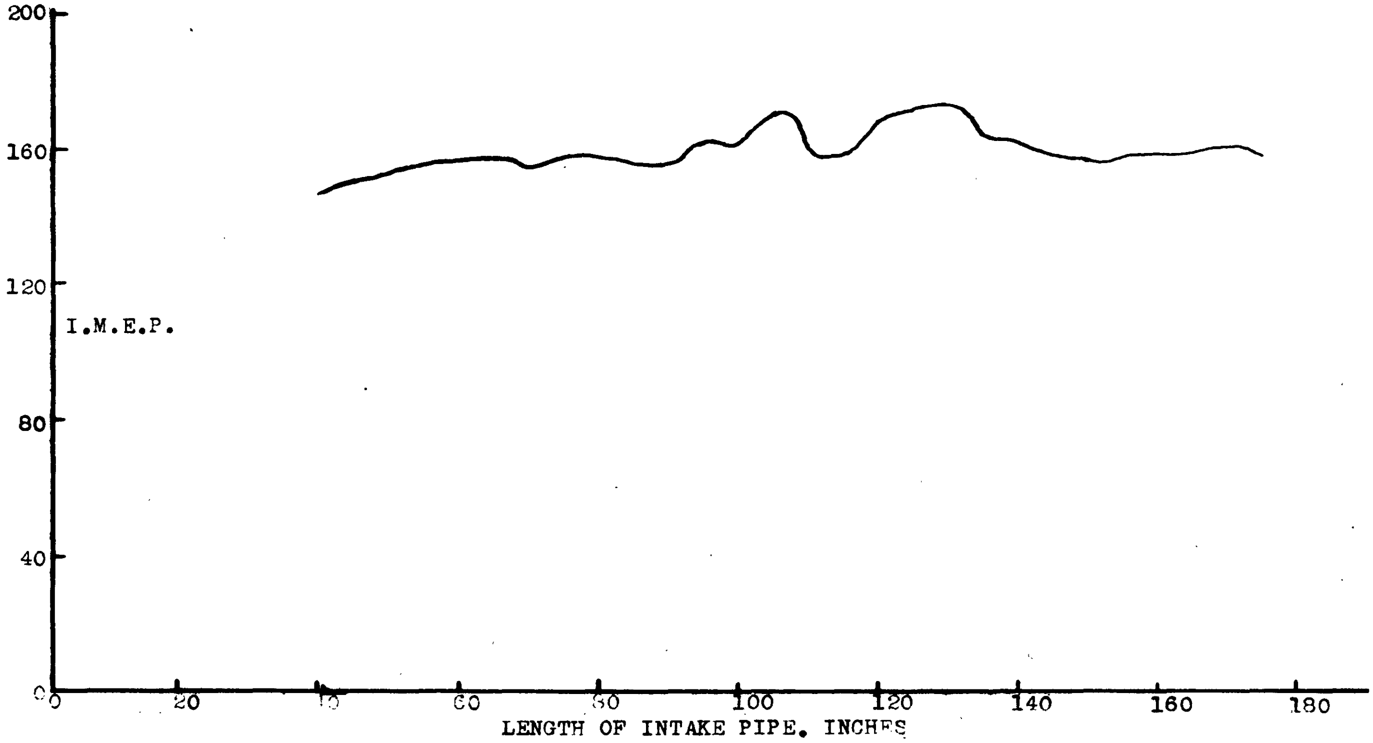


Figure 18.

VARIATION OF THE INDICATED MEAN EFFECTIVE PRESSURE WITH THE INTAKE PIPE LENGTH
Diameter of pipe: 2 9/16 inches
1000 R.P.M.



VARIATION OF THE INDICATED MEAN EFFECTIVE PRESSURE WITH THE INTAKE PIPE LENGTH
Diameter of pipe: 2 9/16 inches
1100 R. P. M.

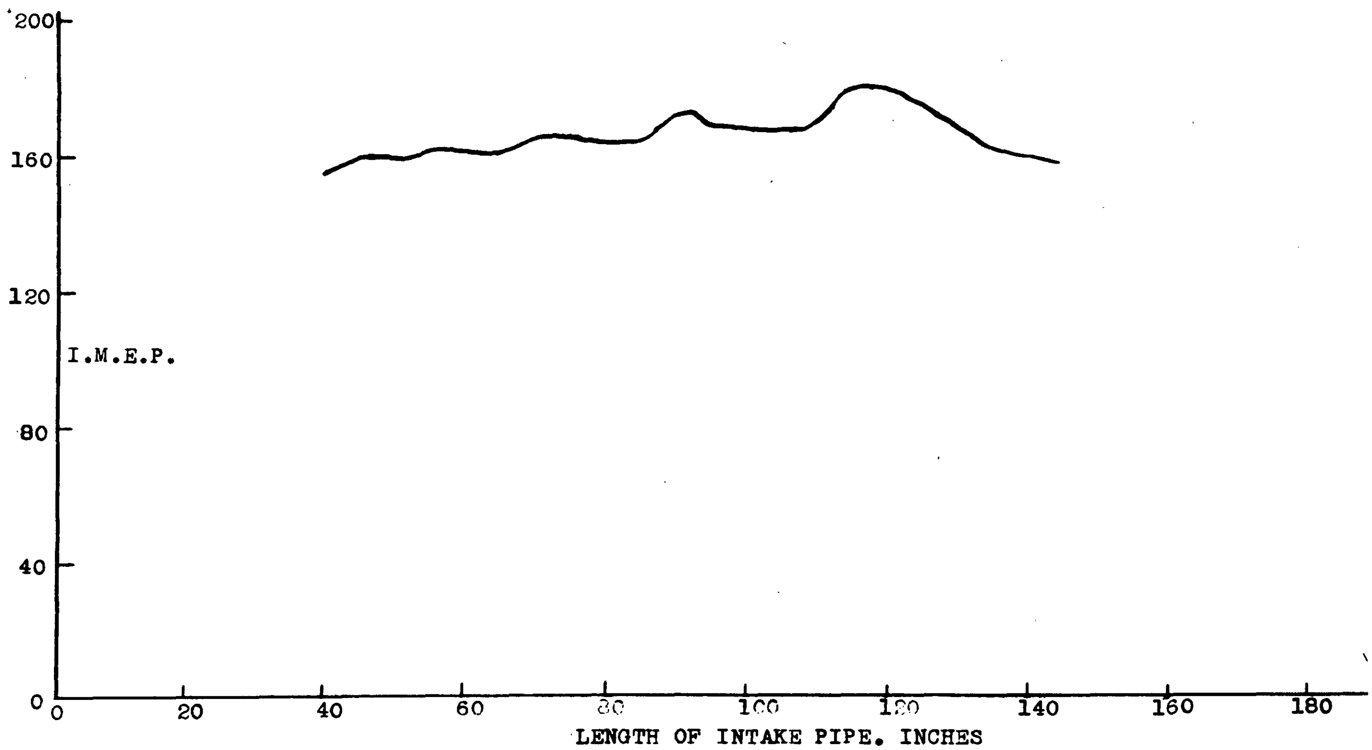
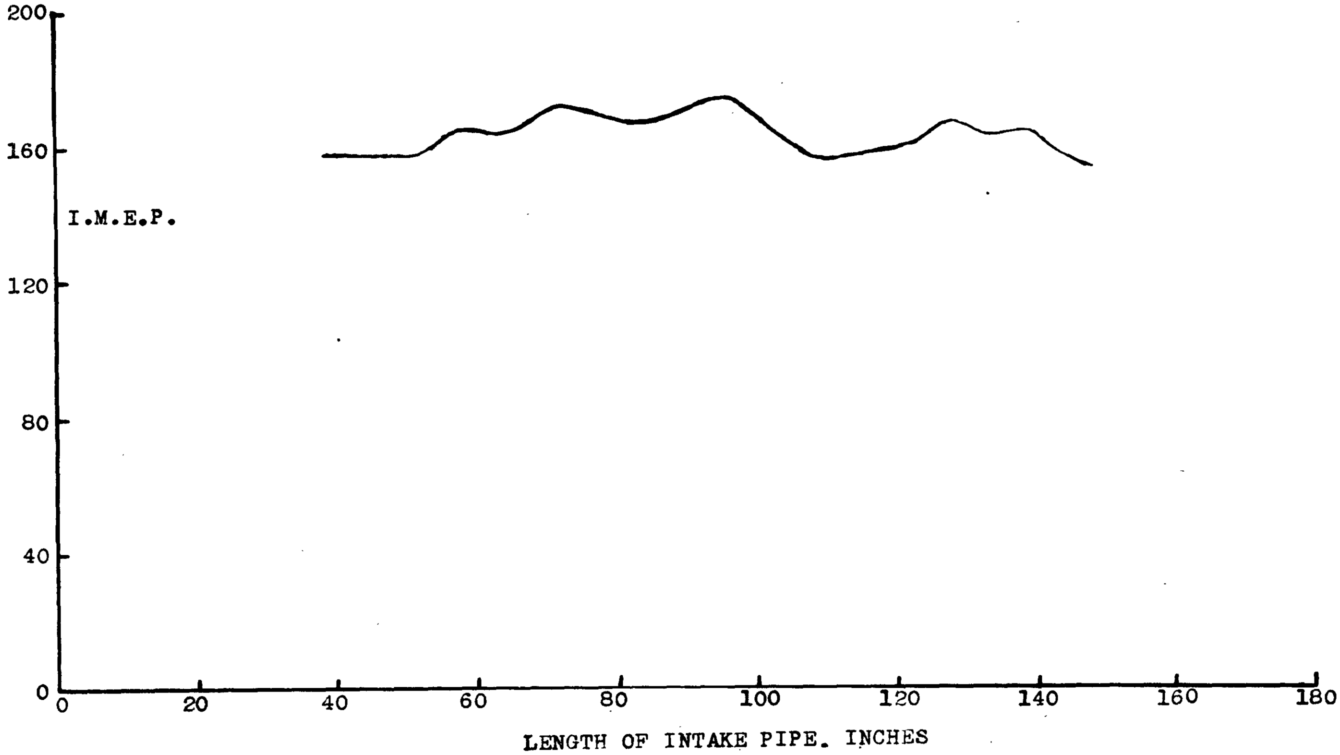


FIG. 19

VARIATION OF THE INDICATED MEAN EFFECTIVE PRESSURE WITH THE INTAKE PIPE LENGTH
Diameter of Pipe: 2 9/16 inches

1320 R.P.M.



VARIATION OF THE INDICATED MEAN EFFECTIVE PRESSURE WITH THE INTAKE PIPE LENGTH
Diameter of pipe: 2 9/16 inches

1530 R.P.M.

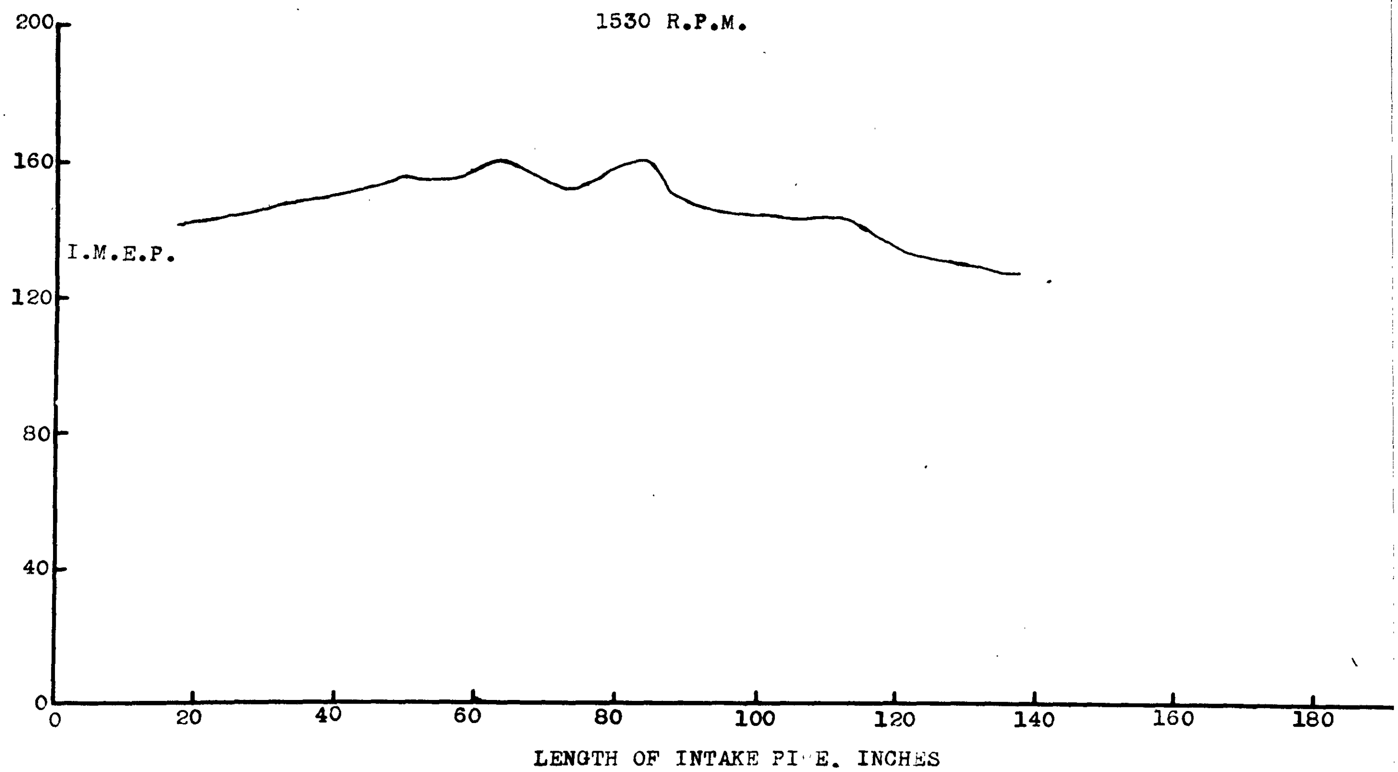


FIG. 20

The velocity of sound in the pipe was measured by means of a calibrated oscillator and a loudspeaker. A plug with a one-eighth inch piece of rubber tubing through it was made for one end of the pipe. The speaker was set up before the open end of the pipe, the plug with the listening tube was inserted in the other end and the resonant frequency for the fundamental and five harmonics of a given length of pipe were measured. The velocity is calculated from the relation

5) $c' = 4Lf_p(2m+1)$. $m = 0, 1, 2, 3$-----,

in which m is the harmonic of the pipe which is excited. Three lengths of pipe were used. The average velocity of sound was found to be 334 $\pm$ 1 meters per second.

X. CONCLUSION

The good agreement of the general form of the calculated and the observed functions which have been plotted in the figures of the preceding section show conclusively that the methods of sound theory satisfactorily serve to describe accurately the phenomena which take place in the induction system of an internal combustion engine over the range of engine speeds investigated.

From the standpoint of design the exact theory is not quite as useful as the simplified analysis which predicts that the maximum supercharge will occur when the frequency of the intake pipe, considered open at one end, is approximately three or four times the frequency of the valve mechanism. In general an internal combustion engine is not a constant speed device and the length of pipe, predicted by the simple analysis, which gives maximum supercharging for the average speed of the motor should be very satisfactory.

The problem of the exhaust system is analogous to that of the intake system. The pulse which is sent down the pipe in this case is positive instead of negative. Since the positive pulse undergoes the same type of reflections, a maximum pressure amplitude will be obtained when the frequency of the exhaust pipe is an integral number of times the frequency of the valve mechanism. Which pipe frequency will produce the best scavenging of the exhaust gases is difficult to predict as the velocity-time variation at the exhaust port is probably not described by a simple sinusoidal function. The velocity probably becomes very great just after the valve has opened and then decreases to a much lower value by the time the valve closes since the hot gases in the cylinder expand very rapidly.

Since the increase in the supercharging which occurs depends directly upon the valve frequency of the engine, this analysis will hold for the case of the two cycle engine. In this case the frequency of the impulses due to the valve-piston mechanism is the same as the engine frequency. In general this type of analysis will solve the problems of the intake systems of mechanical devices which are similar to the internal combustion engine, such as gas compressors, steam engines, etc.

This analysis is a good basis from which to consider the vibrations in the intake or exhaust manifold of a multi-cylinder engine. Further consideration of the effect of junctions in the pipe must be made in this case. Suitable tuning of the various components of a manifold may possibly accomplish a considerable increase in the power output of a multi-cylinder engine, especially when used in conjunction with a mechanical supercharger.

XI. SUMMARY

The phenomena occurring in the induction system of an internal combustion engine are analyzed by the use of the theory of sound. First, a simple picture is developed and the lengths of intake pipe for best power output are qualitatively predicted. A quantitative development follows in which the lengths of pipe for which a maximum volumetric efficiency can be secured are given accurately. The analysis is checked experimentally by measuring the power output of a four stroke gasoline engine. The analytical results are used to predict the best design of the induction system to secure the maximum power output of the engine.

XII. BIBLIOGRAPHY

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XIII. EXPERIMENTAL DATA

Experiment No. I — Date: February 6, 1936

Temperatures: Wet Bulb ; Dry Bulb 23°C

Barometer: 767 mm. Hg. — Fuel: Aviation. Sp. Gr. .73

Cooling Water Temperature: 180°_4. — Oil Pressure: 25 lbs/in.^{2}

Spark Advance: 22° A. T. C.

Run	R. P. M.	Brake Load	Friction Brake	Total Br.Load	I.M.E.P. Lbs/in.	Intake Length Inches	Secs. for 235 cc. Fuel
1	1000	93.4	8.7	102.1	147	38	102.6
2	"	96.4	8.5	104.9	151	43	94.6
3	"	98.1	8.3	106.4	153	48	89.0
4	"	99.8	8.3	108.1	156	53	85.2
5	"	100.8	8.3	109.1	157	58	84.2
6	"	101.5	8.4	109.9	158	63	84.4
7	"	99.5	8.4	107.9	155	68	84.6
8	"	101.8	8.4	110.2	159	73	83.2
9	"	102.2	8.4	110.6	159	78	83.0
10	"	100.4	8.4	108.8	157	83	85.4
11	"	100.0	8.4	108.4	156	88	85.2
12	"	105.5	8.4	113.8	164	93	81.4
13	"	104.3	8.4	112.7	162	98	84.2
14	"	103.6	8.5	112.1	161	103	84.4
15	"	102.8	8.4	111.2	160	108	84.2
16	"	102.3	8.4	110.7	159	113	84.4
17	"	95.4	8.7	104.1	150	17	98.2

Experiment No. I a　　　　　　　　Date: February 7, 1936

Temperatures: Wet Bulb　　　　; Dry Bulb 19°C

Barometer: 767 mm. Hg.　　　　　Fuel: 87 Octane. Sp. Gr. .73

Cooling Water Temperature: 180° Oil Pressure: 22

Spark Advance: 22° A. T. C.

Run	R. P. M.	Brake Load	Friction Brake	Total Br.Load	I.M.E.P. Lbs/in.^{2}	Intake Length Inches	Seconds for 235 cc. Fuel
1	1000	103.9	9.0	112.9	163	113	87.2
2	"	108.9	8.9	117.8	170	118	85.2
3	"	110.2	9.6	119.8	173	123	85.6
4	"	110.2	10.2	120.4	174	128	96.5
5	"	103.8	9.9	113.7	164	133	90.0
6	"	104.1	9.1	113.2	163	138	85.4
7	"	102.6	8.6	111.2	160	143	84.2
8	"	101.3	7.9	109.2	157	148	87.2
9	"	102.0	8.2	110.2	159	153	83.6
10	"	102.7	8.1	110.8	160	158	84.6
11	"	102.1	8.7	110.8	160	163	86.0
12	"	104.2	8.6	112.8	162	168	86.0
13	"	102.6	8.6	111.2	160	173	85.0

Experiment No. II Date: Bebruary 7, 1936

Temperatures: Wet Bulb ; Dry Bulb 19°C

Barometer: 767.2 mm. Hg. Fuel: 87 Octane Sp. Gr. .73

Temperature of Cooling Water: 180° ±2. Oil Pressure: 25 lbs/in.

Spark Advance: 29° A. T. C.

Run	R. P. M.	Brake Load	Friction Br. Load	Total Br. Load	I.M.E.P. Lbs/in.^{2}	Intake Length Inches	Sec. for 235 cc. Fuel
1	1100	98.1	9.4	107.5	155.0	38	79
2	"	101.1	9.5	110.6	159.3	43	75.4
3	"	101.0	9.4	110.4	159.1	48	75.6
4	"	102.6	9.5	112.1	161.7	53	76.0
5	"	102.3	9.5	111.8	161.0	58	76.6
6	"	101.5	9.3	110.8	159.5	63	75.8
7	"	105.0	9.7	114.7	165.1	68	76.4
8	"	104.4	9.7	114.1	164.5	73	76.0
9	"	104.2	9.6	113.8	164.0	78	75.8
10	"	104.4	9.6	114.0	164.2	83	76.2
11	"	110.0	9.6	119.6	172.2	88	76.6
12	"	107.8	9.6	117.4	169.1	93	76.0
13	"	107.4	9.5	116.9	168.2	98	77.4
14	"	106.6	9.5	116.1	167.3	103	77.2
15	"	107.5	9.6	117.1	168.9	108	78.0
16	"	114.5	10.4	124.9	180.0	113	76.5
17	"	111.2	10.7	121.9	175.5	118	76.0
18	"	106.5	10.6	117.1	168.8	123	78.2
19	"	102.7	9.6	112.3	162.0	128	75.4
20	"	101.4	9.4	110.8	159.6	133	75.2
21	"	100.0	9.3	109.3	157.7	138	76.8

Experiment No. III — Date: February 10, 1936

Temperatures: Wet Bulb 53° F ; Dry Bulb 77° F

Barometer: 752 mm. Hg. — Fuel: 87 Octane. Sp. Gr. .73

Cooling Water Temperature: 182±2 — Oil Pressure: 24 lbs/in.^{2}

Spark Advance: 31° A.T.C.

Run	R. P. M.	Brake Load	Friction Brake	Total Br.Load	I.M.E.P. Lbs/in.^{2}	Intake Length Inches	Seconds for 235 cc. Fuel
1	1220	92.3	10.4	102.7	148	38	70
2	1220	94.4	10.4	104.8	151	43	70.6
3	1210	95.0	10.5	105.5	152	48	71.0
4	1220	94.6	10.4	105.0	151.5	53	71.0
5	1230	96.9	10.4	107.3	155	58	71.2
6	1220	100.8	10.4	111.2	160	63	71.0
7	1220	95.7	10.5	106.2	158	68	70.6
8	1220	98.7	10.5	109.2	157.5	73	71.4
9	1210	107.2	10.7	117.9	170	78	71.0
10	1210	104.3	10.6	114.9	165.5	83	71.0
11	1220	98.8	10.4	109.2	157.5	88	70.6
12	1220	97.1	10.3	107.4	155	93	71.0
13	1220	100.2	10.7	110.7	159.5	98	69.8
14	1220	106.8	10.7	117.5	169	103	69.8
15	1220	104.8	10.6	115.4	166	108	70.2
16	1220	97.6	10.5	108.1	156	113	70.0
17	1220	95.5	10.4	105.9	152.5	118	69.8
18	1220	96.0	10.4	106.4	158.5	123	70.0
19	1220	94.5	10.3	104.8	151	128	70.0
20	1220	94.1	9.6	103.7	149.2	133	69.6
21	1220	95.2	10.6	105.8	152.8	138	70.2
22	1210	92.8	10.7	103.5	149	143	70.8

Experiment No. IV Date: February 11, 1936

Temperatures: Wet Bulb 55° F ; Dry Bulb 77° F

Barometer: 751 mm. Hg. Fuel: 87 Octane. Sp. Gr. .73

Cooling Water Temperature: 180±2 Oil Pressure: 23 lbs./in.^{2}

Spark Advance: 33° A. T. C.

Run	R. P. M.	Brake Load	Friction Brake	Total Br.Load	I.M.E.P. Lbs/in.^{2}	Intake Length Inches	Seconds for 235 cc. Fuel
1	1310	99.0	10.7	109.7	158	38	71.6
2	1320	98.9	10.7	109.6	158	43	71.6
3	1310	99.0	10.9	109.9	158	48	73.0
4	1310	99.8	10.8	110.6	159	53	73.2
5	1310	104.1	10.8	114.9	165	58	72.6
6	1310	101.9	11.9	113.8	164	63	73.0
7	1320	105.2	11.4	116.6	168	68	73.2
8	1320	107.8	11.2	119.0	172	73	73.0
9	1310	106.4	11.0	117.4	169	78	73.2
10	"	105.5	10.8	116.3	168	83	72.8
11	"	107.2	10.9	118.1	170	88	72.4
12	"	109.9	10.9	120.8	174	93	72.2
13	"	108.4	11.4	119.8	173	98	72.4
14	"	102.2	10.9	113.1	163	103	72.2
15	"	98.5	10.8	109.3	157	108	73.4
16	"	98.8	10.7	109.2	157	113	73.2
17	"	100.1	10.7	110.8	159	118	73.0
18	"	100.8	10.9	111.7	161	123	73.2
19	"	104.9	10.9	115.8	167	128	72.8
20	1320	102.8	10.9	113.7	164	133	73.0
21	1310	102.3	12.0	114.3	165	138	73.2
22	1310	99.2	11.7	110.9	159	143	73.0
23	1310	95.9	11.7	107.6	155	148	73.2

Experiment No. V. Date: February 13, 1936

Temperatures: Wet Bulb 54.5°F; Dry Bulb 75.5°F

Barometer: 768 mm. Hg. Fuel: 87 Octane. Sp. Gr. 173

Cooling Water Temperature: 180°±2. Oil Pressure: 24 lbs/in.^{2}

Spark Advance: 34° A. T. C.

Run	R.P.M.	Brake Load	Friction Brake	Total Br. Load	I.M.E.P. Lbs/in.^{2}	Intake Length Inches	Seconds for 235 cc. Fuel
1	1420	97.1	11.0	108.1	156	38	67.4
2	"	96.9	11.1	107.0	154	43	67.4
3	"	93.3	11.4	104.7	151	48	67.2
4	"	99.7	12.4	112.1	162	53	66.8
5	1410	96.7	11.8	108.5	156	58	67.6
6	1420	100.8	12.0	112.8	162	63	67.4
7	"	105.6	12.3	117.9	170	68	67.0
8	"	97.5	11.7	109.2	157	73	67.2
9	"	94.7	12.0	106.7	154	78	67.2
10	"	97.0	12.2	109.2	157	83	67.2
11	"	101.0	12.7	113.7	164	88	67.0
12	"	100.2	11.8	112.0	161	93	67.2
13	1410	95.1	11.8	106.9	154	98	67.8
14	1420	93.8	11.3	105.1	152	103	67.6
15	"	91.0	11.2	102.2	147	108	68.0
16	"	90.5	12.2	102.7	148	113	68.0
17	"	90.2	12.8	103.0	149	118	68.2
18	"	87.8	13.2	101.0	146	123	69.0
19	"	84.7	13.6	98.3	142	128	70.8
20	"	81.6	13.7	95.3	137	133	71.2
21	"	82.5	13.3	95.8	138	138	71.0
22	"	81.0	12.8	93.8	135	143	71.0
23	"	78.9	12.3	91.2	131	148	71.2
24	"	77.9	12.2	90.1	130	153	71.2
25	"	86.9	13.0	99.9	144	183	70.8
26	"	86.9	13.0	99.9	144	193	71.0
27	"	75.7	13.3	89.0	128	253	71.4

Experiment No. VI | Date: February 14, 1936

Temperatures: Wet Bulb 58°F; | Dry Bulb 75°F

Barometer: 754 mm. Hg. | Fuel: 87 Octane Sp. Gr. .73

Temperature of Cooling Water: 180° ± 2. | Oil Pressure: 26 lbs/in.^{2}

Spark Advance: 36° A. T. C.

Run	R.P.M.	Brake Load	Friction Brake	Total Br. Load	I.M.E.P. Lbs/in.^{2}	Intake Length Inches	Sec. for 235 cc. Fuel
1	1530	91.3	12.4	103.7	149	38	63.2
2	"	93.0	12.6	105.6	152	43	64.0
3	"	95.0	12.8	107.8	155	48	64.0
4	"	95.0	12.9	107.9	155	53	64.2
5	"	95.6	13.4	108.4	156	58	63.8
6	"	98.9	12.8	111.7	161	63	63.6
7	"	95.5	12.6	108.1	156	68	64.2
8	"	92.0	13.1	105.1	152	73	64.6
9	"	93.8	14.4	108.2	156	78	63.8
10	1520	97.8	13.9	111.7	161	83	63.6
11	1530	91.7	13.3	105.0	151	88	63.6
12	"	89.8	12.2	102.0	147	93	64.0
13	"	89.5	12.2	101.7	146	98	64.0
14	"	86.2	12.2	98.4	143	103	64.2
15	"	87.0	12.9	99.9	144	108	64.4
16	"	85.7	14.5	100.2	144	113	64.4
17	"	80.9	14.2	95.1	137	118	64.4
18	"	77.8	14.3	92.1	133	123	64.6
19	"	77.0	14.2	91.2	131	128	64.8
20	"	76.0	13.9	89.9	130	133	64.8
21	"	74.9	12.6	88.5	128	138	64.8
22	"	81.8	12.4	94.2	136	168	64.4
23	"	86.2	11.5	97.7	141	17	64.2

Experiment No. VII | Date: February 16, 1936

Temperatures: Wet Bulb 58°F; | Dry Bulb 78°F

Barometer: 768 mm. Hg. | Fuel: 87 Octane Sp. Gr. .73

Cooling Water Temperature: 180°_2. | Oil Pressure: Lbs/in.^{2}

Spark Advance: 38°A. T. C.

Run	R.P.M.	Brake Load	Friction Brake	Total Br.Load	I.M.E.P. Lbs/in.^{2}	Intake Length Inches	Sec. for 235 cc. Fuel
1	1630	96.3	13.3	109.6	158	38	59.6
2	"	95.7	13.4	109.1	158	43	59.2
3	"	96.1	13.6	109.7	158	48	59.0
4	"	91.1	13.3	104.4	150	53	59.2
5	"	98.1	13.8	111.9	161	58	59.6
6	"	94.6	12.9	108.5	156	63	60.0
7	"	88.6	13.6	102.2	147	68	59.8
8	"	90.9	14.4	105.3	152	73	59.6
9	"	92.4	14.8	107.2	155	78	59.6
10	"	86.8	13.7	100.5	145	83	59.8
11	"	85.7	13.7	99.4	143	88	59.8
12	"	83.7	13.4	97.1	140	93	59.6
13	"	80.8	13.7	94.5	136	98	60.2
14	"	80.4	13.8	94.2	136	103	60.2
15	"	76.2	14.7	90.9	131	108	60.4
16	"	72.8	15.7	88.5	127	113	61.0
17	"	68.2	15.6	83.8	121	118	61.0
18	"	69.7	15.8	85.5	123	123	60.8
19	"	67.2	15.1	82.3	119	128	61.2
20	"	74.5	13.6	88.1	127	158	60.8

DYNAMICS OF THE INDUCTION SYSTEM OF AN INTERNAL COMBUSTION ENGINE

ABSTRACT

The power output of an internal combustion engine for given operating conditions can be made to have a maximum value by selecting the proper length of the intake pipe. The experimental results show that large amplitude pressure waves occur in the intake manifold for these conditions. Maximum, or minimum, power output occurs for approximately resonant lengths of the intake pipe.

Sound theory is used to account for the physical phenomena of the intake pipe and the engine cylinder, and for the changes in the power output resulting from varying the intake pipe length. Corresponding experimental results are obtained from the operation of a conventional four-cycle gasoline engine.

BIOGRAPHICAL SKETCH.

The author was born in Glendive, Montana, November 14, 1910. He attended the grade schools and high schools at Missoula, Montana. In June 1932 he was graduated from the State University of Montana with honors. The following September he entered the Massachusetts Institute of Technology, from which he received his Master of Science degree in June 1934. While at the Institute he was elected an associate member of Sigma Xi. He left the Institute in June 1935 and entered the Bureau of Standards. In December 1935 he returned to the Institute as a Research Assistant in the Mechanical Engineering Department.