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# High Speed Imaging of Transient Non-Newtonian Fluid Phenomena

by

Benjamin H Gallup

Submitted to the Department of Mechanical Engineering  
in partial fulfillment of the requirements for the degree of

Bachelor of Science in Mechanical Engineering

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

June 2004

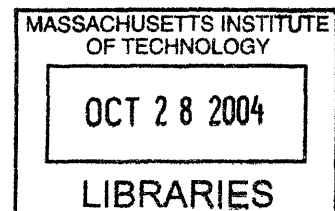
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## Abstract

In this thesis, I investigate the utility of high speed imaging for gaining scientific insight into the nature of short-duration transient fluid phenomena, specifically applied to the Kaye effect. The Kaye effect, noted by A. Kaye in the March 9, 1963 issue of *Nature*, is the deflection and rebound of a free-falling non-Newtonian fluid stream incident on a pool of the same fluid. The effect was successfully reproduced using Suave™ shampoo, and imaged using the Phantom™ High Speed Video system. This task involved developing a knowledge of the photographic process as applied to high speed imaging, and of non-Newtonian fluid mechanics. No precisely reproducible method for producing rebounding streams was found, and behavior contrary to the existing body of observation were noted. In conclusion, areas that merited further investigation and potential variables of interest to future Kaye effect research are discussed.

Thesis Supervisor: Anette E Hosoi  
Title: Assistant Professor



# Acknowledgments

This is everybody's fault but mine.

That said, this would not have been possible without the limitless patience and help of my thesis advisor Professor Peko Hosoi, the advice and guidance of Professor Gareth McKinley, the lab assistance of Tim Scott, the resources of the MIT Edgerton Center, and the kind, kind folks in the SIPB office for their untappable reserves of  $\text{\LaTeX}$  advice. And, of course, this would have been even more impossible without Mom, Dad, Kim and Rich being Mom, Dad, Kim and Rich, respectively.



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# Chapter 1

## Introduction

Imaging technology in scientific research is typically limited to a documentary role. However, with various imaging techniques it is possible to utilize imagery as a unique source of data. Particularly in the regime of high-speed transient phenomena, scientific imaging can provide measurements that are otherwise impossible to obtain. Such phenomena may elude standard measurement techniques - they may be extraordinarily sensitive to interference from instrumentation, be too unpredictable to reliably distinguish the moment of interest, or be otherwise excessively sensitive. The passive nature of photography and perpetual nature of high speed videography are uniquely suited to address these problems. One such phenomena is the Kaye effect, in which a thin stream of highly non-Newtonian fluid falling under the influence of gravity rebounds from a pool of the same fluid, redirecting itself upwards and effectively 'bouncing' off of the fluid pool. This thesis seeks to investigate the Kaye effect with high speed videography.

The second chapter provides background to basic photography. It briefly covers the basic theory and current technology driving modern photography. It defines standard photographic vocabulary, a necessary prerequisite for understanding and intelligently utilizing imaging equipment. It outlines the features of standard imaging equipment, and discusses the nuances of high-speed imaging.

The third chapter outlines the principles of non-Newtonian fluids - focusing on how such fluids deviate from standard Newtonian theory. It establishes some of the

vocabulary associated with non-Newtonian fluids, and gives a brief overview of models commonly used for considering non-Newtonian fluid.

The fourth chapter covers the results of investigating the Kaye effect with high speed videography. It contains and describes a time-evolution sequence of a set of leaping fluid streams, and describes general behavior trends noted while trying to produce the Kaye effect. This chapter also covers how these observations deviate from previously published observations regarding the Kaye effect.

The fifth and final chapter addresses areas needing further work and focuses principally on variables that should be considered in future studies of the Kaye effect.

## Chapter 2

# Photography and High Speed Imaging

Photography is essentially the process of capturing a single still frame of visual space - a replication of the instantaneous viewpoint of a given recording device. While the utility of photography for documentary purposes or as an aesthetic medium is readily apparent, its less obvious worth as a scientific instrument is no less significant. Photography can establish the position of moving objects that elude other measurement techniques and record delicate phenomena that would be fundamentally altered by the presence of contact instrumentation. With appropriate equipment and techniques, standard photography can be used as scientific imaging, providing a means to capture transient phenomena and yield data that could not be acquired through other means. Through careful calibration it is possible to derive numerical values of displacement and elapsed time leading to a wide range of derived values: including, but not limited to, position, strain, velocity, momentum, kinetic energy, acceleration, and net applied force.

### 2.1 Background to Basic Photography

Photography is essentially the controlled exposure of some light-sensitive media to appropriately focused light. This media comes in many forms - any number of traditional



silver-emulsion photographic films, or one of the available digital sensor technologies. Given the versatility of digital photography, the high portability of a digital image, and the effort associated with photochemical developing and printing, this paper will focus solely on digital imaging - though many concepts discussed apply correctly to film photography. Digital imaging devices come in two principal forms; complementary metal oxide semiconductor (CMOS) sensors and charge coupled devices (CCDs). Each technology has different specific characteristics - for example, CCDs typically consume more power, and CMOS sensors are more easily manufactured - but both operate in a fundamentally similar manner, and the difference in photographic output is transparent. A third and fundamentally different imaging sensor - the Foveon X3 chip - has been released within the past few years, but has not become a dominant technology, and again its functional difference is transparent to the camera user.

Manually taking a photograph is an underconstrained process - there are more free variables available to the user than are necessary to completely define the process. There exist multiple unique sets of camera settings, all of which will produce what is referred to as a well exposed final image. To understand the subtle tradeoffs between the available options, the user must understand a fair amount of vocabulary. The idealized goal - a *well exposed image* - is one that utilizes the greatest dynamic range of the imaging media. It is neither *overexposed* (also referred to as *saturated* or *blown out*) from receiving too much light, nor *underexposed* from receiving too little. These different scenarios are shown in Figure 2-1, below.

This maximizes the *contrast* of the image, the variation in intensity between the light and dark areas of an image. This property is usually evaluated qualitatively, but Equation 2.1 gives the quantitative definition.

$$C = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} \quad (2.1)$$

Where  $C$  is the contrast, and  $I_{max}$  and  $I_{min}$  are the maximum and minimum intensities, respectively.  $C = 1$  would be the ideal, maximum contrast resulting from  $I_{min} = 0$ , and  $C = 0$  would be a flat, featureless image with no information content.

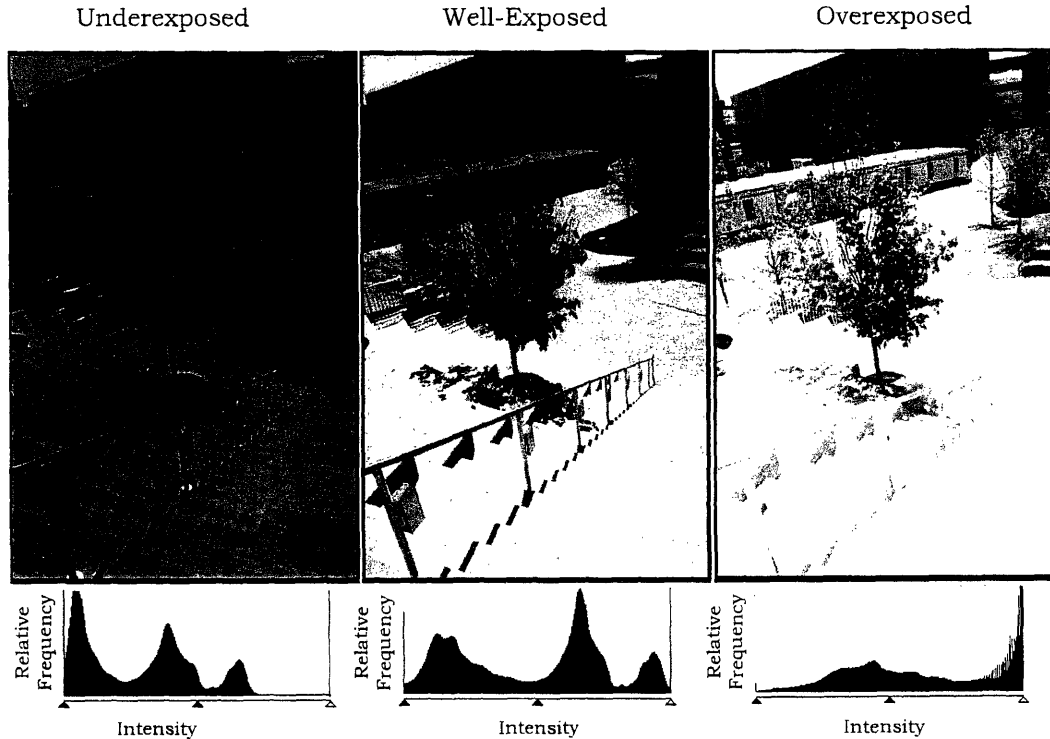


Figure 2-1: Examples of underexposure, proper exposure, and overexposure

The distributions below each picture, generated with the Adobe Photoshop 7.0 Levels command, show the normalized frequency of each intensity level from unexposed (black) to fully saturated (white). Note how the underexposed image distribution is shifted left, towards lower intensity, and the overexposed image distribution is shifted right, towards higher intensity. Also note that the intensity distribution for the well-exposed image covers the entire range.

## 2.2 Basic Practical Photography

The principal variables available to the user are *shutter speed*, *aperture* and *film speed*. The photography profession is rife with non-technical and non-standard units, especially when dealing with these variables.

The *shutter speed* is simply an expression for exposure time - the duration that the mechanical shutter is physically open, exposing the imaging device to incoming light. Fast shutter speeds (less than half of a second) are typically expressed on-camera in inverse seconds - the denominator of exposure time when it expressed as a fraction

with one in the numerator. A shutter speed of 500 corresponds to an exposure of 1/500 of a second. Slow shutter speeds (half of a second to thirty seconds) are expressed directly in seconds. *Bulb mode* - named for the antique pneumatic squeeze-bulb style actuator - is the highest setting for shutter speed. Under bulb mode, the shutter is opened for as long as the trigger is depressed. Faster shutter speeds (smaller exposure times) lead to darker images, and can be used to image brightly lit phenomena. Slower shutter speeds (greater exposure) times lead to brighter images - allowing the capture of dimly lit objects. However, these slower shutter speeds introduce the first of two types of blur - *motion blur*. If the object (or the imaging device) being imaged moves appreciably while the shutter is open, the resulting image will be blurred and of dubious scientific merit. Thus, faster shutter speeds are typically desired for scientific photography.

The *aperture* corresponds to physical size of a light-constricting element within the lens system of the imaging device. It is typically expressed as an *f-stop*, or dimensionless number corresponding to the ratio of aperture diameter to imaging focal length. For example, an aperture setting of 6.3 (also written f/6.3) sets the diameter of the aperture to 1/6.3 of the 35mm film equivalent<sup>1</sup> focal length of the current lens. Lower aperture values (larger physical aperture) and are referred to as 'fast', as they let in more light, thus allowing faster shutter speeds. Conversely, higher, or slower, aperture settings (smaller physical aperture) let in less light, necessitating slower shutter speeds in environments that are not fully lit. Minimum aperture settings range from 4-5.6 in medium quality, medium range or high quality long-range optics. Medium quality short range and high quality medium range optics reach a minimum

---

<sup>1</sup>While digital sensing technology has revolutionized image detection, optical systems have remained fundamentally similar to the days of 35mm photography. Commercial lenses are universally marked with the physical focal length of the contained optical system, which produces an image of a given size on a 35mm frame of film. However, only the most expensive digital cameras (the \$8399 Canon 1ds or the \$4799 Kodak DCS) have sensors the size of a full frame of 35mm film; a typical digital camera has a smaller sensor that only captures some central subsection of the 35mm equivalent image. This is expressed as the *cropping factor*, a published statistic that expresses the ratio of a 35mm frame to a smaller digital sensor. Typical cropping factor values range from 1.6 in a quality digital camera, to 5 and larger in cheaper, compact cameras. This smaller image corresponds to a high zoom factor, which is a direct product of focal length. Thus, digital imaging devices have a focal length that is effectively longer - the original, 35mm equivalent focal length multiplied by the cropping factor. This image focal length is not included in aperture calculation.

aperture of 2.8, and extreme speciality short and medium range lens reach a minimum aperture of 1.0. Maximum aperture does not pose a significant engineering challenge, and is 22 or greater in almost any lens.

Aperture settings drive the second of two types of blur - *focal blur*. A pinhole camera - a theoretically infinite aperture value - is a perfect imaging device in the optical sense - all objects are in perfect focus. As the aperture deviates from this ideal, objects very close or very far from the point of focus begin to blur. This gives rise to the concept of *depth of field* - the range of objects in view that are in focus. Low aperture values have very poor depth of field, and objects outside this depth are blurred. This is typically used to isolate the object of focus and blur the background so that it makes an indistinct image that does not detract from elements of interest. However, when an entire scene must be in focus, this demands a high aperture value and longer exposure times. This induces motion blur, and if this is unacceptable, alternative photographic techniques must be sought.

The final variable is *film speed*, or sensor sensitivity. This is typically measured in units of ISO (International Organization for Standardization) or ASA (American Standards Association), an abstract quantity originally used for expressing the relative sensitivities of different films. The two designations, ASA and ISO, are wholly indistinguishable and different only in name. ASA 400 film behaves precisely the same as ISO 400 film. ISO 100 is typically the 'slowest' film - it is relatively insensitive, which yields low noise images that require longer exposure times or very bright lighting. ISO 1600 film is typically the 'fastest' film, and its high sensitivity yields noisy images but permits faster shutter speed. Digital cameras vary the sensitivity of their imaging chips such that they correspond well to standard film ratings, and enumerate these values with the same units. One of the great strengths of digital imagery is the ability to change the ISO rating on-camera, without having to exchange an entire roll of film.

All three of these variables control how well the resulting image is exposed, and can be exchanged freely. The selectable levels of all three variables are designed to facilitate such a tradeoff. Shutter speeds are a series of rough powers of two (called

shutter stops), as doubling the shutter speed would halve the total incident light on the sensor, and halving the shutter speed would double the incident light. Aperture f-stops are in approximate multiples of 1.4 - a rough value for  $\sqrt{2}$ . Increasing the aperture diameter (f-stop) by a factor of  $\sqrt{2}$  increases the aperture area by a factor of 2. This doubles the total incident light, which is a function of aperture area, not diameter. A given well exposed image will still be essentially well exposed if one shutter stop is traded for one aperture stop. Either type of stop is also interchangeable with a doubling or halving of film sensitivity. More expensive digital cameras offer half and third stop increments, allowing for greater exposure control. These tradeoffs, and their associated differences allow for a broad range of technically well exposed images, each with their own aforementioned benefits or handicaps.

## 2.3 High Speed Photography

The strength of photography as a scientific tool lies in its ability to effectively stop time. However, no photographic medium can truly do so - any image can only approximate frozen time. Every photographic process is characterized by an exposure time, during which the recording medium is sensitive to light given off by the phenomena of interest, and an image is formed. For standard camera photography, this exposure time is the shutter speed of the camera - the duration, typically in fractions of a second, over which the physical shutter is open, allowing light to strike the film or other light-sensitive media. The actuation of this shutter is accomplished by various mechanical means, but regardless of specific mechanism, there is a practical lower limit to shutter speed. The most expensive professional cameras have maximum shutter speeds of 1/8000 second ( $125\mu\text{s}$ ) - beyond which friction and lag in the shutter device are insurmountable. 125 microseconds may seem like a short time interval but for truly high speed phenomena, it is inadequate. A 1cm long .22 caliber bullet moving at sonic speeds in air will travel over 3cm in  $125\mu\text{s}$ , and such an image would be highly motion-blurred, and most likely unacceptable for scientific evaluation. Also, successfully illuminating the bullet such that it would be well exposed in  $125\mu\text{s}$  would

require prohibitively intense lighting.

The exposure time of film or digital sensor can be alternatively considered not as the time elapsed while the shutter is open, but as the time elapsed while the subject is illuminated. Harold 'Doc' Edgerton, an MIT professor during the 1940's drew such a conclusion, and this idea gave rise to strobe illumination - extremely high intensity pulses of light over very small times. By continually exposing film in a darkened environment and only instantaneously illuminating the subject, Edgerton could capture images that were otherwise photographically impossible. Edgerton went on to form EG&G, Incorporated and commercially produce strobe lighting. Perhaps the most impressive product was the Microflash, a high intensity lamp with a sub-microsecond flash duration. The same sonic .22 caliber bullet would only be blurred an imperceptible .24mm given microsecond-duration illumination. Photography with a single discharge strobe is typically referred to as *synch and delay* photography, where some trigger is detected, and a controlled amount of delay elapses before the strobe discharges. An image taken via synch and delay microflash photography taken by the author in the MIT Strobe Lab is shown below in Figure 2-2. The camera shutter was held open in a darkened room, a rifle was sighted and discharge at the egg, and a microphone beneath the egg detected the shockwave of the barely supersonic rifle round, triggering the microflash.



Figure 2-2: A .22 caliber bullet is captured puncturing an egg using microflash photography

Edgerton's research was one of the first steps into the realm of high speed imaging. With strobe photography, the new dimension of microsecond resolution analysis was

revealed, Nigh-instantaneous processes or crucial moments of essentially continuous processes could be pinpointed and investigated - impacts and collisions, explosions, oscillations - the array of subjects for high-speed analysis was vast, and has remained so. Technology has advanced the state of high-speed imaging, and the contemporary pinnacle of instrumentation is the high speed video camera. The Vision Research, Inc. Phantom™ 5.0 High Speed Video (HSV) system used in this investigation is typical of modern HSV systems.

The Phantom™ system is a self-contained digital imaging and mass memory storage device. It accepts typical professional photographic optics, and consequently uses the same aperture concept as standard photography. Shutter speed is emulated as the interval over which light incident on the photosensor is integrated, ranging from  $10\mu\text{s}$  to the largest value permitted by the selected framerate. The photosensor itself is a CMOS sensor with an effective resolution of  $1024 \times 1024$  pixels, capable of recording up to 1000 full frame images per second. Higher framerates may be achieved at the cost of incrementally reduced resolution, up to 95,000 frames per second at  $64 \times 32$  pixels. The technologically novel aspect of the camera is that it can be triggered at any moment, and it will record, to the limits of its memory, the events immediately following the trigger, the events immediately preceding the trigger, or an arbitrarily portioned fraction of events both before and after the trigger. The camera does so by circularly buffering 1 gigabyte of onboard high speed solid state memory - continuously recording the incoming imagery, looping back to the beginning of its available memory when it reaches the end.

While HSV systems do not match the resolution of readily available digital cameras, such cameras do not match the versatility and continuous recording capabilities of HSV. The two photographic systems are both very suitable for high speed imagery, and both have the potential to very useful under different circumstances. HSV requires very large amounts of lighting - a one kilowatt lamp (20 times more powerful than an ordinary 50 watt lightbulb) proves insufficient to effectively light a scene being recorded at 1000 frames per second, as less than one joule of light energy will be incident on the target per frame, and still less will be reflected into the camera.

Such intense lighting is not suitable for subjects sensitive to heat, and synch and delay photography would prove more suitable. If high resolution is required, HSV may be inadequate. Synch and delay photography's principal weakness is that it can only develop time histories of highly repeatable phenomena, where the delay can be increased slightly and the precise process repeated. HSV suffers no such drawback, and is the optimal tool for investigating transient, poorly repeatable phenomena such as the Kaye effect.





## Chapter 3

# Non-Newtonian Fluid Mechanics

The study of many engineering disciplines rests heavily on idealized, well-behaved scenarios that adhere to reasonable, predictable equations. Fundamental dynamics relies heavily on the postulated existence of linear elements, and the well behaved nature of the resulting linear, constant coefficient second order differential equations. Analog electronics rely on similar idealized linear components, and the same mathematics. Solid mechanics has its Hookean elastic solid, where displacement (strain) is directly proportional to force (stress), expressed by the linear constitutive relation

$$\sigma = E\epsilon, \tag{3.1}$$

where  $\sigma$  is the stress in force per unit area,  $E$  is the elastic modulus in force per unit area, and  $\epsilon$  is the dimensionless strain. The elastic modulus is a product of the simplifications inherent in the model of elastic solids. Describing how a differential element of a solid responds to loading originally requires a 81-element four-dimensional tensor, but through symmetries and the assumption of isotropy, the governing mechanics can be reduced to constitutive laws like equation 3.1 and two material properties - the elastic modulus  $E$  for actions normal to the surface, and the Poisson ratio  $\nu$  for transverse actions.

Fluid mechanics rests on an analogous idealization: the Newtonian fluid. Isaac Newton hypothesized the existence of a linear relationship between shear forces in a

moving fluid and the velocity gradient perpendicular to the plane of shear. Analogous in function and derivation to the elastic modulus for solid mechanics, this proportionality constant is viscosity, and the relationship can be expressed as

$$\tau = \eta \frac{dv}{dy}, \quad (3.2)$$

where  $\tau$  is the shear in force per unit area,  $\eta$  is shear viscosity,  $v$  is the velocity of the fluid parallel to the shear, and  $y$  is the the coordinate perpendicular to the shear plane. Physically, this is most easily pictured by considering two parallel plates, separated by a viscous fluid-filled gap. When the lower plate is fixed and the upper translated, a linear velocity profile develops as a direct consequence of Equation 3.2, as shown below in Figure 3-1. This assumption holds true for all gases and liquids with simple molecular formula and low molecular weight such as water, thin oils, and glycerol ( $\text{CH}_2\text{OHCHOHCH}_2\text{OH}$ )[4]. Combined with Newton's second law of motion, Equation 3.2 is a direct precursor to the Navier-Stokes equation - one of the fundamental equations of fluid mechanics.

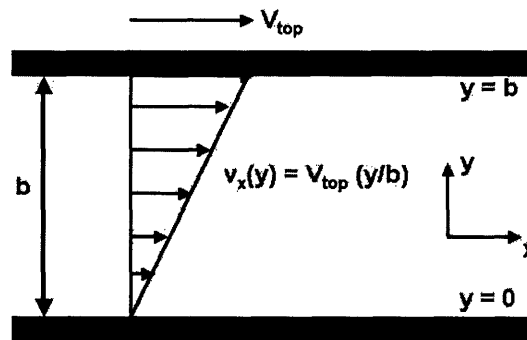


Figure 3-1: Velocity profile in a fluid-filled gap between two relatively moving parallel plates.[3]

## 3.1 Classes of Non-Newtonian Fluids

Most real world fluids invariably fall outside the province of classical Newtonian mechanics. Such fluids are grouped together under the title *non-Newtonian fluids*. This is a catch-all term, as a fluid can deviate from the Newtonian model of material constant, time-invariant viscosity in many ways. Essentially, if given a constant temperature the viscosity is anything other than constant, the fluid is non-Newtonian. The principle classes are nonlinear stress-strain rate relationships, time variant viscosities, and deformation memory, or viscoelasticity.

### 3.1.1 Nonlinear stress-strain rate relationships

The simplest deviation from the Newtonian fluid model is a nonlinear, but still direct relationship between strain rate and shear stress. The viscosity of such fluids, while not constant, is independent of time and deformation history. Figure 3-2 below compares the principle time-independent types of non-Newtonian fluids. Figure 3-2

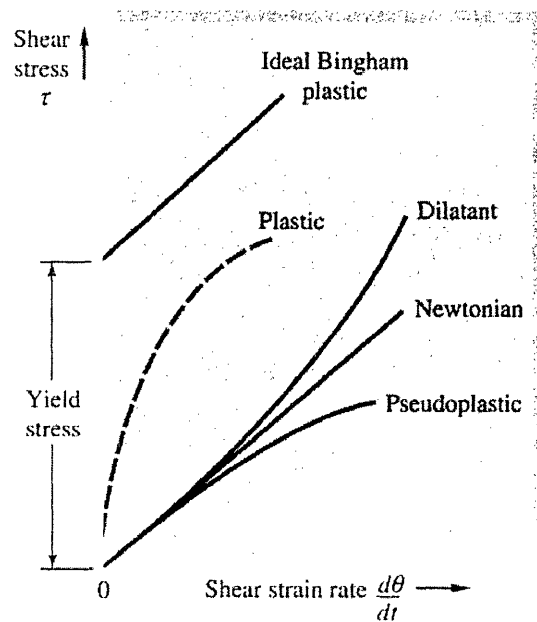


Figure 3-2: Relative stress-strain rate curves for various Non-newtonain fluids.[5]

shows the shear stress,  $\tau$ , behaviors of various fluid classes as a function of strain rate,

$\frac{d\theta}{dt}$ , which is dimensionally equivalent to both the  $\frac{dv}{dy}$  of equation 3.2, and  $\dot{\epsilon}$ , which shall be used for the rest of this paper.

The ideal Newtonian fluid shows a direct, linear relationship between shear stress and strain rate, as shown previously. If the viscosity (also considered as the first derivative of shear stress versus strain rate) increases with strain rate, the fluid is *dilatant*. Dilatant, or *shear-thickening* fluids are relatively uncommon, but solutions of titanium dioxide (commonly found in paint) and those of sand and starch behave in this manner.

If the shear stress decreases with increasing shear rate the fluid is *pseudoplastic*, or *shear-thinning*. A fluid that is ‘slimy’ or ‘slippery’ to the touch is most likely a shear-thinning fluid. Most non-newtonian fluids are pseudoplastic, such as polymer solutions, blood, or shampoo. If a fluid is originally highly viscous, yet the shear-thinning effect is highly pronounced, the fluid is highly analagous to a plastic solid, and consequently is referred to as a *plastic* fluid, as shown by the dotted line in figure 3-2.

The extremum of this trend of increasing plasticity is the *ideal Bingham plastic*, which can withstand a finite yield stress before beginning to flow. Figure 3-2 depicts the linear-flow idealized Bingham plastic, but actual Bingham plastic fluids may shear-thin or shear-thicken after yielding. Toothpaste is the classic example of a Bingham plastic, but the class also includes certain jellies and slurries.

### 3.1.2 Time-dependent viscosities

If the viscosity varies with time, the fluid is certainly non-Newtonian. This can be a product of accruing aggregate particles within a fluid or mechanically degrading the components of a solution responsible for certain fluid behaviors. Figure 3-3 shows the two major classes of such a fluid.

If the viscosity a fluid under constant strain rate decreases with time, the fluid is *thixotropic*, or *time-thinning*. Such behavior exists in many forms of muds and jellies, and commerical products expressly designed for such behavior.

Conversely, if the viscosity increases with time, the fluid is *rheopectic*, or *time-*

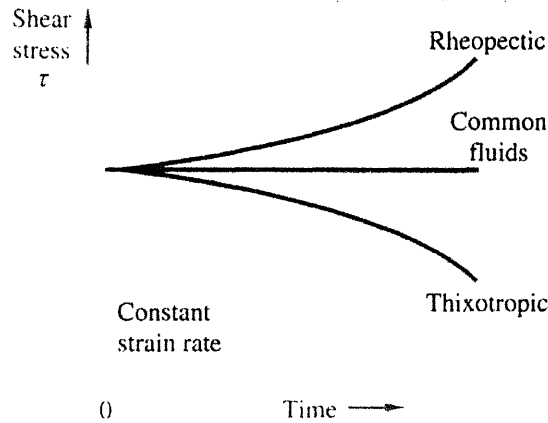


Figure 3-3: Thixotropic and rheopectic fluids over time under constant strain rate.[5]

*thickening*. Rheopectic fluids are rare, but known examples include gypsum paste and specialty printer's ink.

### 3.1.3 Viscoelasticity

Fluids essentially differ from solids in that they are permanently deformed by the application of any small shear force, where most solids will deform elastically and return to their original shape when the stress is removed. One category of non-Newtonian fluids responds elastically to small deformations like a solid, yet still flow viscously. These fluids - with velocity-dependent (viscous) and position-dependent (elastic) behavior are referred to as *viscoelastic*. Many gels and polymer solutions are viscoelastic.

## 3.2 The Root of some Non-Newtonian behavior

Much non-Newtonian behavior arises from the existence of some sort of internal structure to the fluid. Most non-Newtonian fluids are complex mixtures, solutions or suspensions. The constituent components of these mixtures give rise to complicated phenomena, and the macroscopic result of these phenomena is the suite of aforementioned non-Newtonian characteristics.

In a relatively concentrated solution of sand - that of near-tidal beach - there exists a some semblance of a grid of sand particles. That is to say, the size of the sand particles is larger than that of the mean free space between sand particles. When a load is applied - when a beachgoer steps onto an area of sand solution - the sand grains slide relative to each other, but do not flow freely past each other. Any beachcomber has found the appropriately concentrated sand solution that apparently 'dries' underfoot. The sand solution immediately under load seems to dry out, thickens, and effectly increases the viscosity greatly. This is not, however, a product of forcing water out of the sand solution. With the free upper surface, the air occupying the free space in the suddenly 'dry' sand would not be able to act upon any water and displace it. Loading the sand has shifted the sand particles as to effectively increase the porosity of the solution - and yeilding a 'drier', thicker solution with the same amount of water, but with more open space. The internal structure of the sand, and not readily apparent fluid properties of the sand solution, have given rise to non-Newtonian behavior.

Polymer solutions are also highly non-Newtonian. High molecular weight polymers like polyethylene oxide (PEO) can weigh up to 2,000,000 grams per mol due to its extremely long polymer chains. In solution these long chains of PEO interfere with each other, giving an internal structure to the fluid. This internal structure of interlinked polymers gives rise to first order non-Newtonian behavior, such as shear-thickening viscous effects as polymer chains are stretched. The presence of PEO or other long chain polymers also has secondary non-Newtonian effects: as these polymer solutions undergo shear, the polymers mechanically degrade as they are physically broken up. Ostensibly, the now-thixotropic fluid time-thins, and potentially loses its non-Newtonian character.

# Chapter 4

## The Kaye Effect

### 4.1 Background

In a March 9th, 1963 article in *Nature*[2] A. Kaye issued a short report on what he called ‘an unexpected effect observed when a thin stream of solution of polyisobutylene in ‘Decalin’ (an industrial solvent) is poured into solution in an open dish.’ Kaye’s innocuous description opened the first article published on the phenomena that now bears his name - the Kaye effect. When Kaye pouring a thin (1mm diameter) stream of his solution into a 10cm diameter evaporation tray filled with the same fluid from a height of 25cm, he noted the stream building up in a small pile at the point of impact, which was itself nothing revolutionary - simply a product of the high viscosity of the fluid. However, there was some critical condition that the growing pile would reach, at which the stream would deflect and apparently leap off of the pile, momentarily airborne. The pile would shrink while the stream was deflecting and no longer adding to it, and eventually the deflection effect would cease, as the pile size decreased below some minimum threshold. The pile would then again start to grow, and the entire process would repeat.

Kaye offered no explanation for the behavior, but noted that the solution was ‘markedly non-Newtonian and visco-elastic’ as demonstrated by previous research. He mentions that the phenomena was observed by chance, and that he had done no research into the conditions necessary to precipitate the event. He mentioned that



it was ‘tempting’ to speculate that pile at the base of the stream was the key, but having no scientific ground to base this on, he pushed the issue no further. Kaye provided some of the fluid properties of his solution, mentioning that it was 5.8g of L100 grade ‘Vistanex’ (Esso Petroleum Company Co, Ltd.) in 100cc of ‘Decalin’. The solution yielded a viscosity of 3.3 poise at a shear rate of  $10\text{sec}^{-1}$  in a room temperature concentric cylinder viscometer.

A. A. Collyer and P. J. Fisher advanced Kaye’s work 13 years later, with an article in the June 24, 1976 issue of *Nature*[1]. Collyer and Fisher discovered that the deflected streams were actually closed loops of fluid. Collyer and Fisher noted the same pile-dependent behavior, and echoed Kaye’s observations that the effect was triggered by the fluid pile reaching a certain height, that the pile decayed while the stream (or loop) deflected, and the cycle was renewed after the pile fell below some threshold and began to build up again. Collyer and Fisher went beyond Kaye’s work, and proposed a mechanism by which to describe the effect.

Collyer and Fisher noted that the slow-moving pile must have a high viscosity, and must present a rigid body to the incoming stream, especially when in a glass dish or on a glass plate. The falling fluid stream has been unsheared while in free-fall, so also must have a high viscosity. However, upon striking the pile, high changes in velocity must relate back to high changes in viscosity over short timescales - consequently at high shear rates. Hence, the fluid in question must be highly shear thinning.

Phrased differently, Collyer and Fisher hypothesized that to meet the necessary requirements for producing the Kaye effect the fluid must have high viscosities at low shear rates - yielding a near-rigid pile, and low viscosities at high shear rates, as the stream deflects and forms a loop. This means the fluid must be a shear-thinning, or plastic, fluid. Additionally, in order to create the rebound effect, the fluid must have significant elastic character - that is to say, it must be viscoelastic.

Collyer and Fisher achieved results similar to those of Kaye, with slightly different parameters. They used 4.4g of Vistanex L140 grade,  $2 * 10^6$  molecular weight polyisobutylene in 100ml of dekaline, and a drop height of 400mm onto a flat glass plate. Collyer and Fisher gave no specific viscosity measurements, but claimed to find the

fluid a power law fluid <sup>1</sup> of index 0.4 over a shear rate range of 40sec<sup>-1</sup> to 120sec<sup>-1</sup>,

## 4.2 Procedure

The fluid of choice was Suave® Naturals® Vanilla Floral Shampoo, chosen for its noted viscoelasticity, and presumed shear-thinning properties. Experts familiar with the Kaye effect also recommended shampoo - Suave® specifically - for Kaye effect experimentation. The first step was to characterize the viscosity of the shampoo. This was accomplished using a TA Instruments AR1000-N Rheolyst cone and plate constant shear viscometer.

For a specific fluid with time independent mechanical properties, the aforementioned literature on the Kaye effect clearly indicates two principle variables; the Kaye effect should be a function of drop height and volumetric flow rate, or analagous properties. Analogs to drop height are impact velocity or impact shear rate, and analogs to volumetric flow rate could be mass flow rate or stream diameter. Other potential variables could include impact surface composition, or impact surface inclination. To test these variables, the experimental setup shown in figure 4-1 was devised.

The camera, lighting, and impact surface were positioned as close together as allowed by the physical space of the lab. This maximized the intensity of light incident on the subject, which in turn maximized the intensity of light incident on the camera. For this setup, the HSV system was 80cm from the impact point, and the 600 watt light source was 110cm. The laptop was configured to view the camera output in real time. A stationary object was placed at the impact point, and while viewing the output on the laptop the HSV system was manually focused using the focus ring on the lens, a Nikon AF-S 28-70mm zoom lens set at a focal length of 70mm.

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<sup>1</sup>The term *power law fluid* refers to a fluid in which the shear stress,  $\tau$ , is proportional to the shear rate,  $\dot{\epsilon}$ , raised to some power  $n$ , expressed as

$$\tau = K * \dot{\epsilon}^n. \quad (4.1)$$

A power law fluid of index 1.0 would be Newtonian, and  $K$  would be the viscosity,  $\eta$ . A power law fluid of index greater than one would shear-thicken, and a power law fluid of index less than 1.0 would shear thin. An index of 0.4 corresponds to an aggressively shear-thinning fluid.

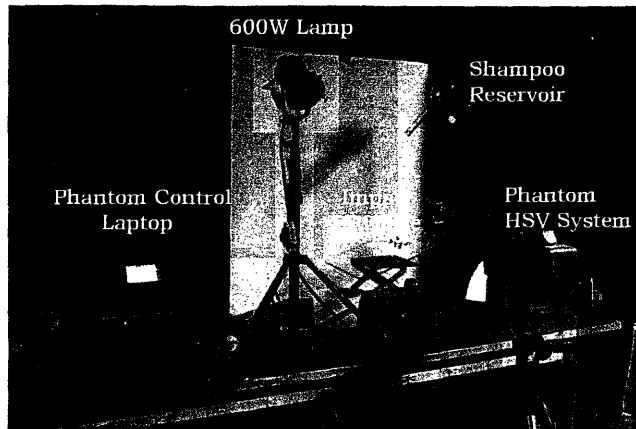


Figure 4-1: Experimental setup

The experimental setup for investigating the Kaye effect had five major features - the Phantom HSV system, the laptop with associated driving software, a lab stand holding a long-necked flask of shampoo, and a high-intensity light source. The flask could be placed at an arbitrary height or angle, producing differing impact velocities or flow rates. The impact surface could be raised, lowered, or inclined.

The flask of shampoo was pre-clamped in a lab stand right angle bracket, to allow a repeatable angular position. With the camera focused, the flask and adapter were attached to the lab stand, and the fluid began to flow. The camera was set to 1000 frames per second (noting that the literature had mentioned the effect occurred on a one second timescale) and minimum shutter speed (to maximize overall exposure). The corresponding HSV system settings were a recording interval of  $929\mu\text{s}$  and an exposure interval of  $918\mu\text{s}$ . The HSV recording loop was halted with a post-sequence trigger upon the visual confirmation of Kaye effect behavior.

The original experimental procedure was to parameterize the Kaye effect process. This would have entailed bracketing the range of drop heights, flow rates, and impact inclines that led to the Kaye effect, finding some reasonable quantity to characterize the magnitude - such as leap height, leap radius, leap frequency, or some unpredicted quantity - and discover the optimal conditions. Unfortunately, this was not possible, for reasons explained below. The Kaye effect was highly non-repeatable and very chaotic. No reliable trend or consistent measure could be established at constant height or angle, let alone across a spectrum. The complete and total lack of

consistency or reproducibility made the aforementioned procedural method useless.

The Kaye effect is, in a sense, an instability in the fluid-piling process where the stream hits the plate. In an attempt to induce this instability, experiments deflecting the stream to initiate a Kaye-effect style bounce were attempted. Initial, non-methodical trials blowing on the stream produced fantastic results, but subsequent trials of mechanical stream deflection with high- and low-speed fans failed, as did translating the impact sheet and pile below the stream.

### 4.3 Results

The results of testing the shampoo viscosity were promising. The fluid was markedly shear thinning, as shown below in figure 4-2. This showed that the fluid - if it were viscoelastic - would be a candidate for the Kaye effect. This also offers some insight as to why the experimental results differed in consistency from Kaye's results. Kaye reported a fluid viscosity of 3.3 poise, or 0.33 pascal-seconds at a shear rate of  $10\text{sec}^{-1}$ , where the shampoo has a viscosity of 10.5 pascal seconds.

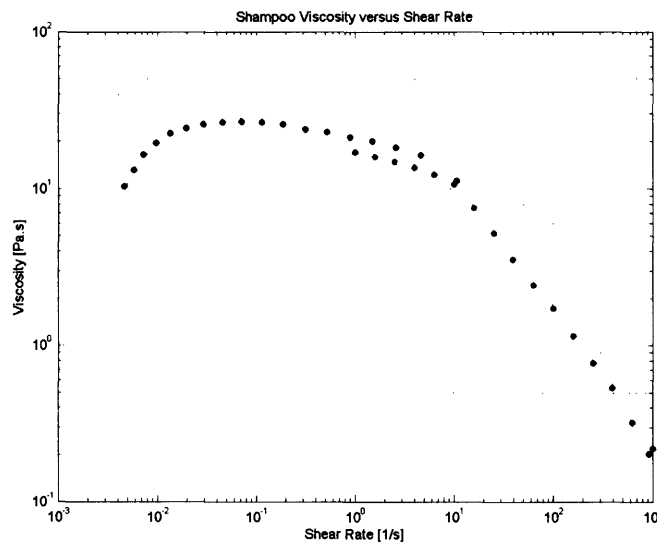


Figure 4-2: Shampoo Viscosity versus Shear Rate.

The visual results from the first stream-deflection test were highly compelling, albeit ultimately irreproducible. The following image sequence shows the evolution of a leaping loop of fluid as it the deflected stream tracks away from the pile. Figures 4-5 through 4-13 show the instantaneous velocity of a trackable kink in the fluid stream, from formation until it unfolds, rendering it untrackable. Also of note, the stream crosses its own rebounding loop, beginning at Figure 4-11.

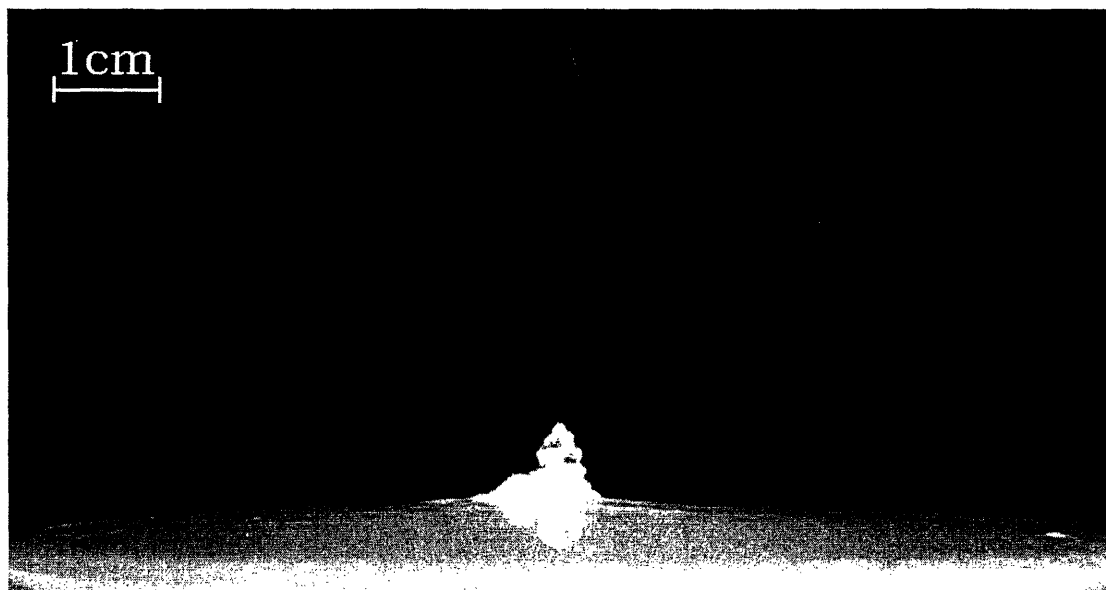


Figure 4-3: The Kaye Effect: Frame 0, time 0ms

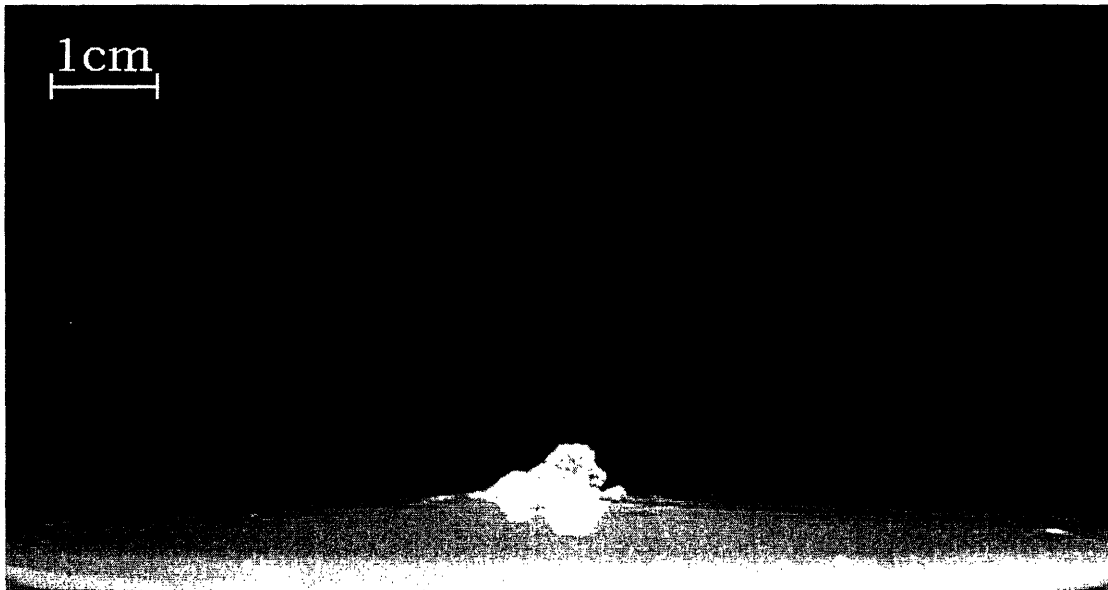


Figure 4-4: The Kaye Effect: Frame 38, time 35.2ms

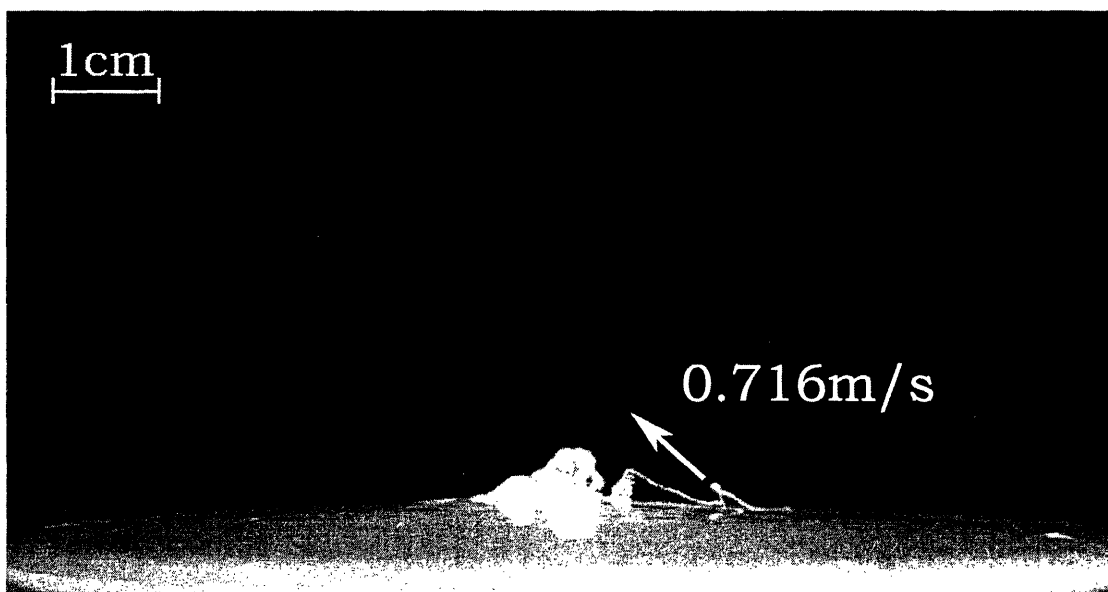


Figure 4-5: The Kaye Effect: Frame 57, time 52.9ms

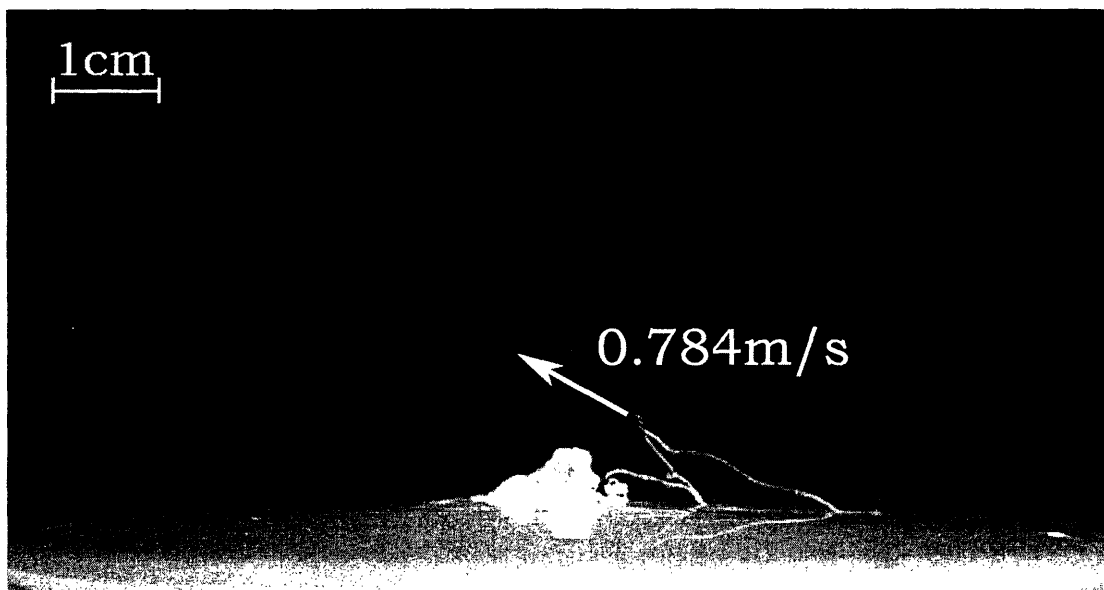


Figure 4-6: The Kaye Effect: Frame 71, time 65.9ms

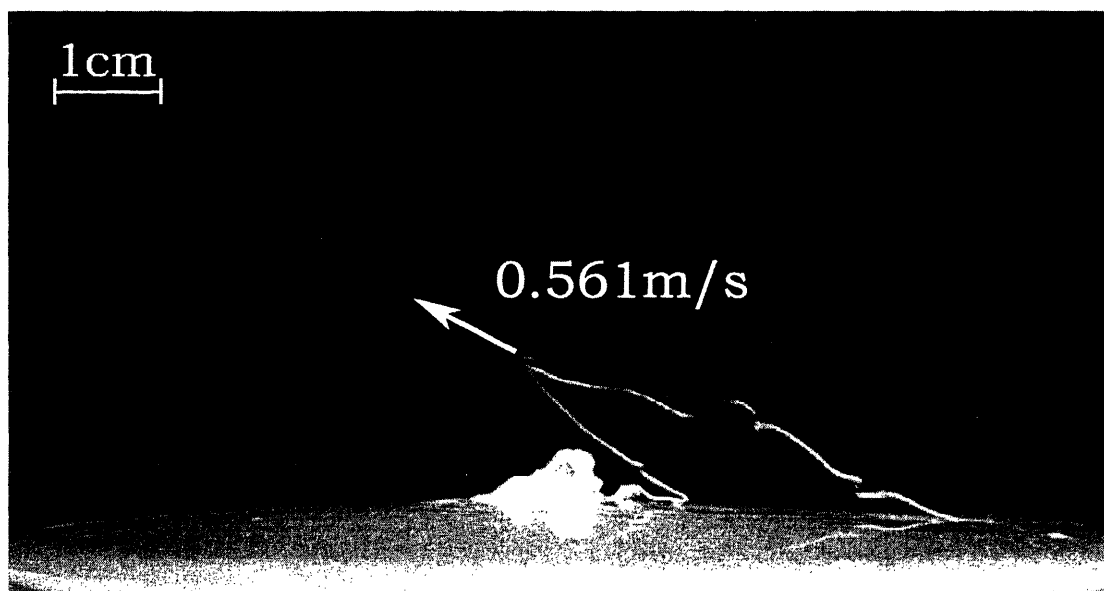


Figure 4-7: The Kaye Effect: Frame 87, time 80.7ms

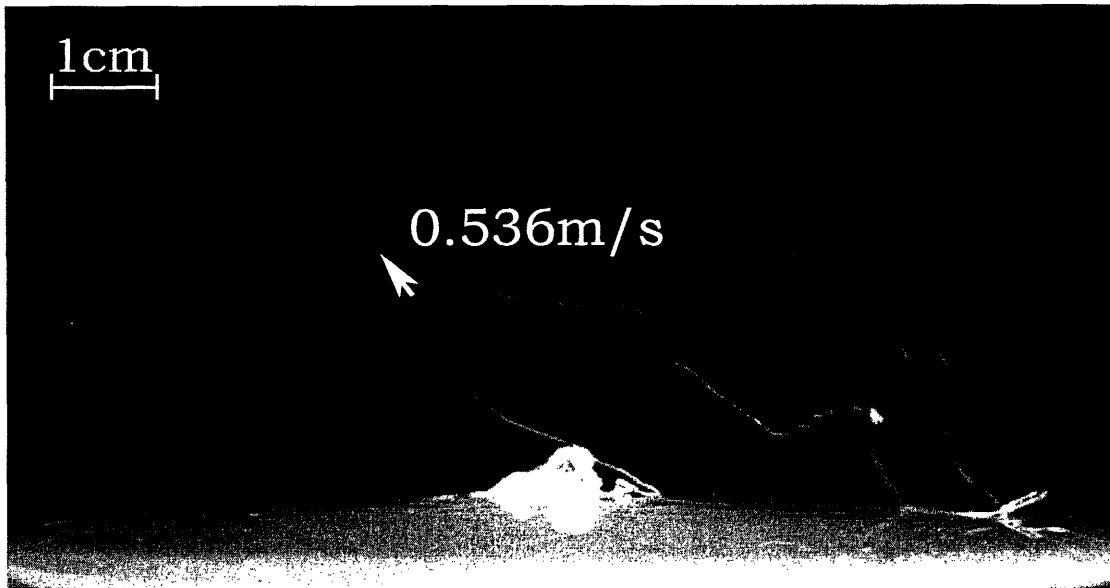


Figure 4-8: The Kaye Effect: Frame 108, time 100.2ms

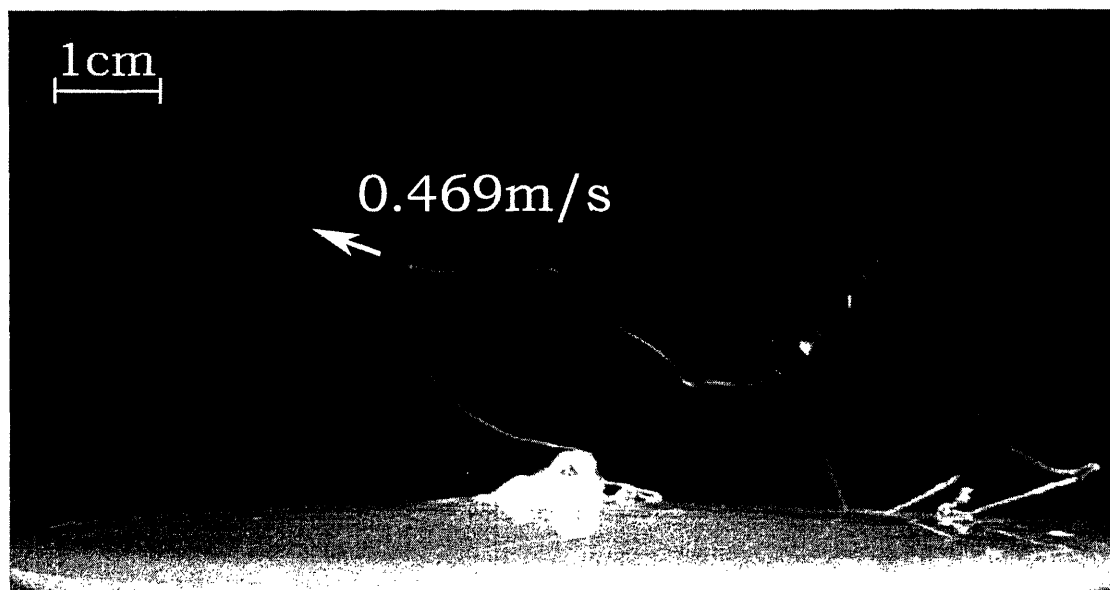


Figure 4-9: The Kaye Effect: Frame 118, time 109.5ms



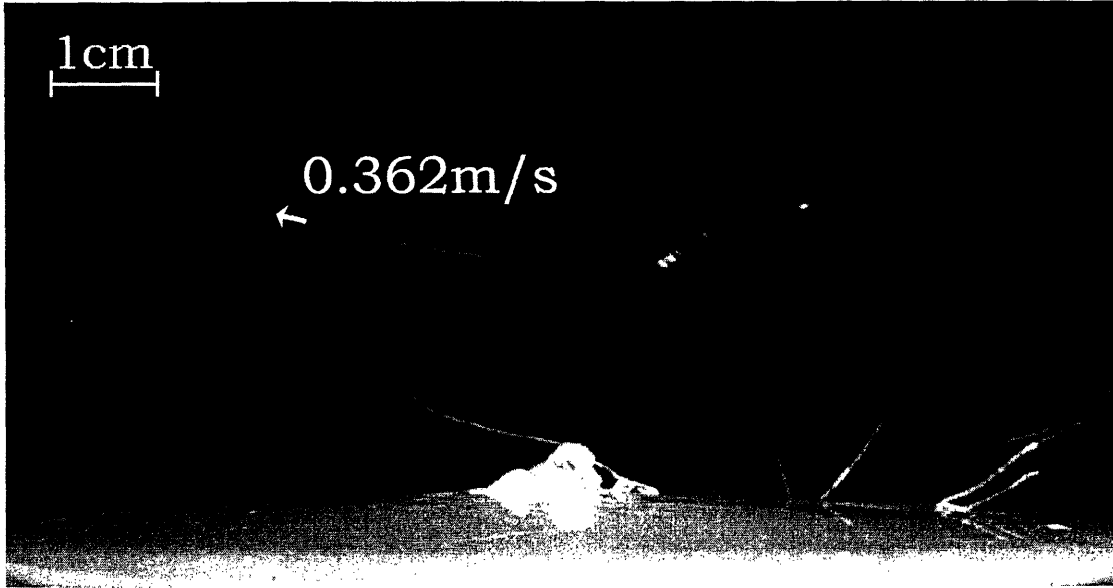


Figure 4-10: The Kaye Effect: Frame 134, time 124.4ms

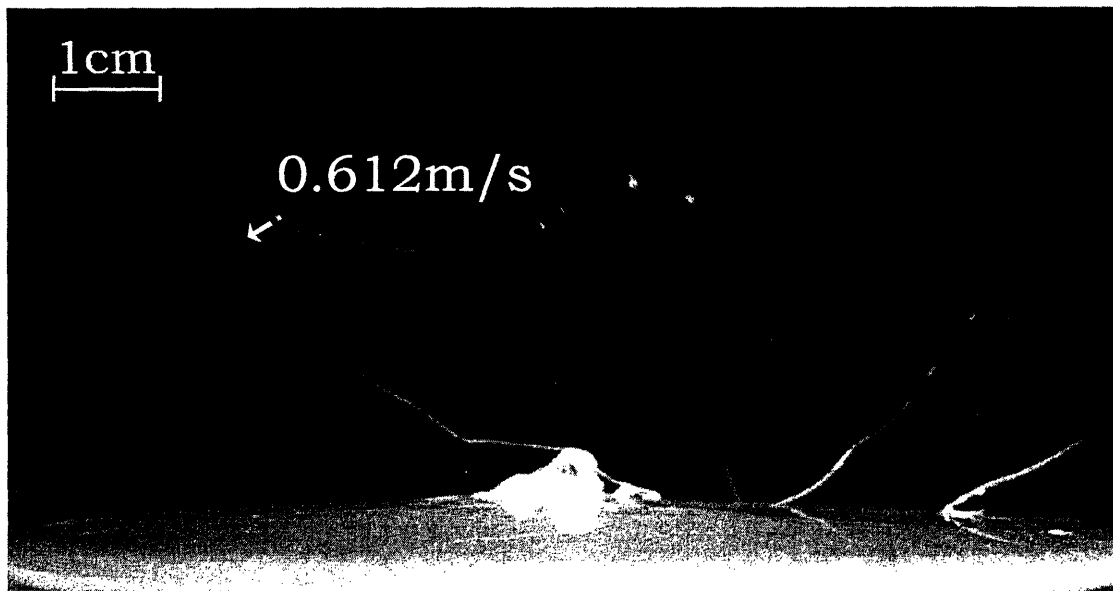


Figure 4-11: The Kaye Effect: Frame 145, time 134.6ms

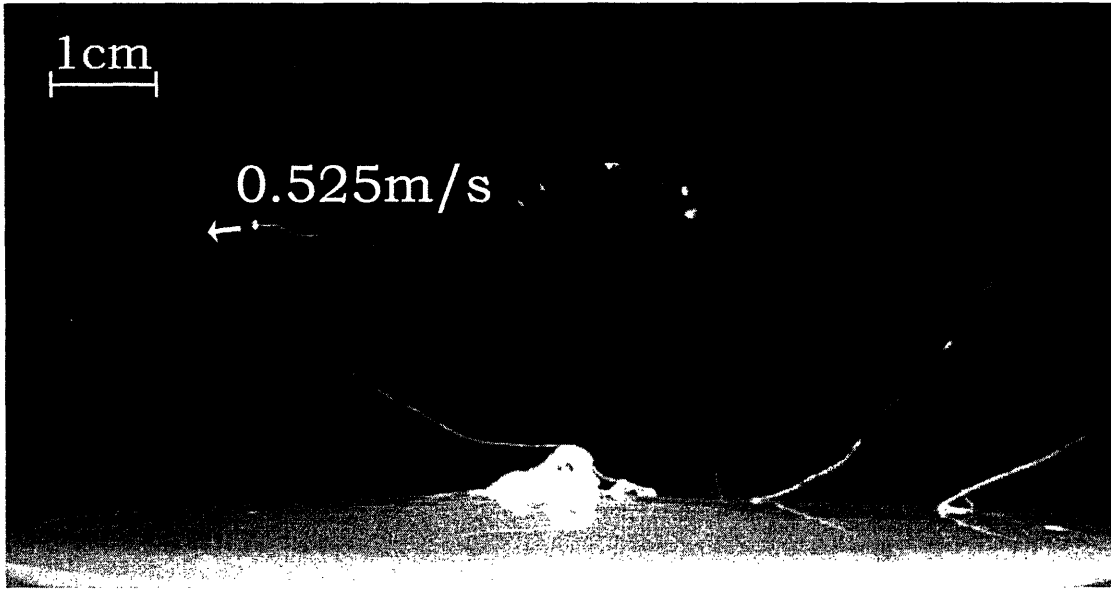


Figure 4-12: The Kaye Effect: Frame 148, time 137.3ms

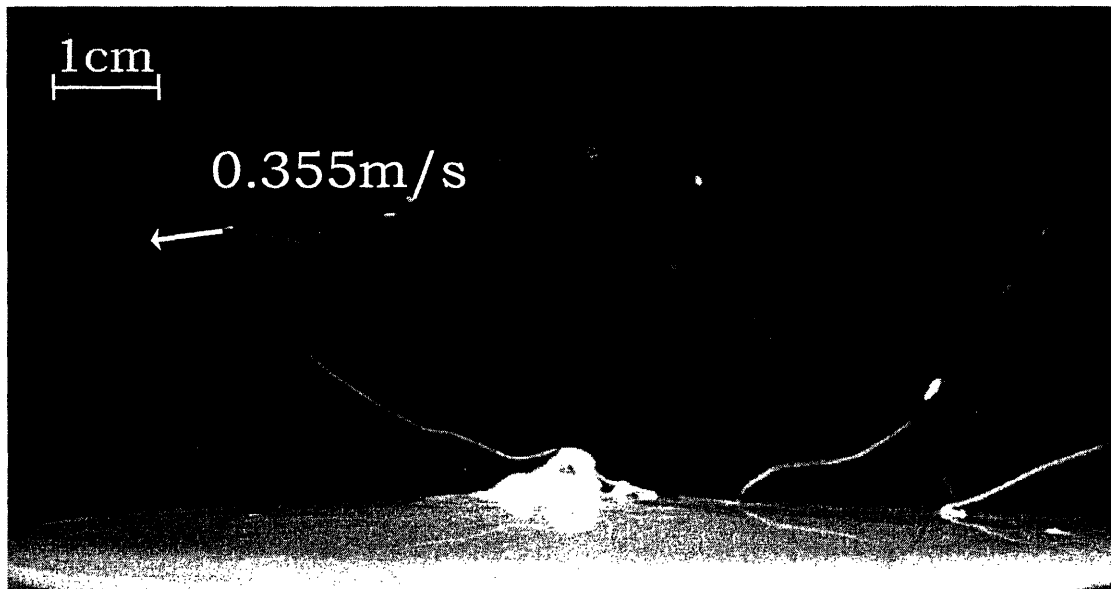


Figure 4-13: The Kaye Effect: Frame 155, time 143.8ms

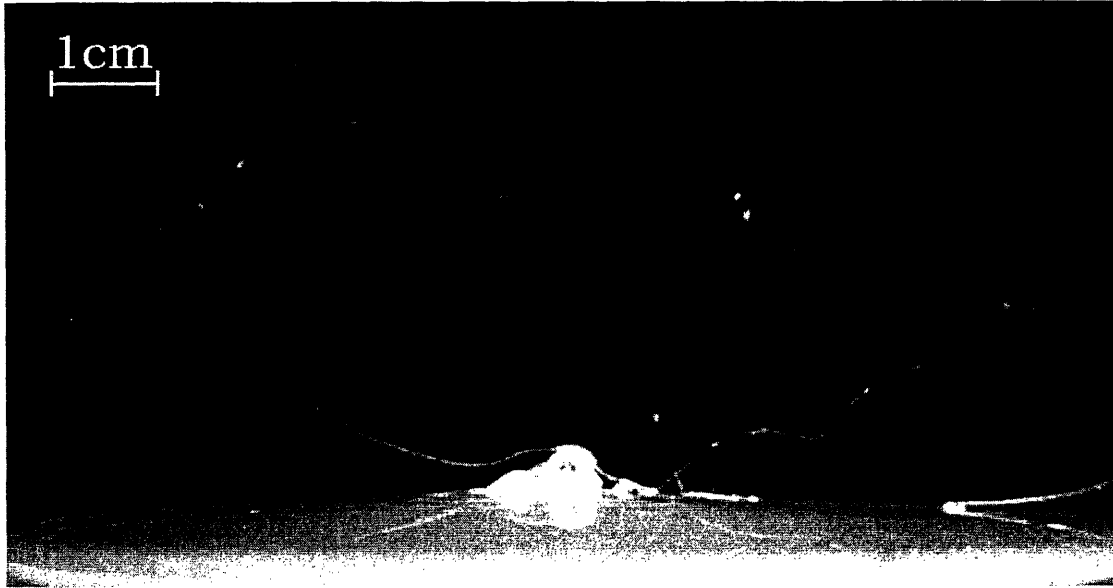


Figure 4-14: The Kaye Effect: Frame 175, time 162.4ms



Figure 4-15: The Kaye Effect: Frame 189, time 175.4ms

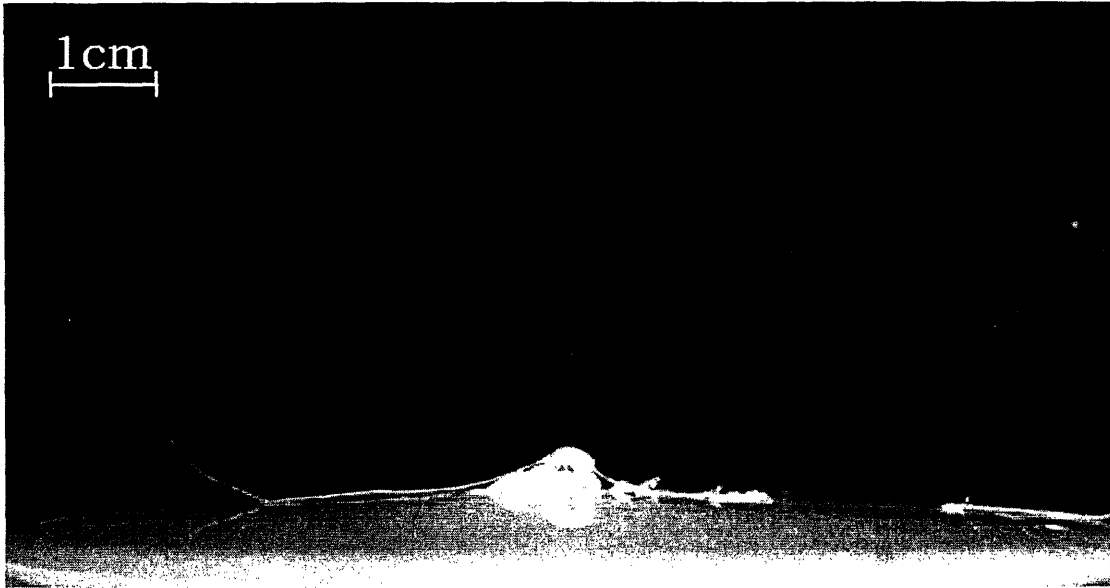


Figure 4-16: The Kaye Effect: Frame 202, time 187.5ms

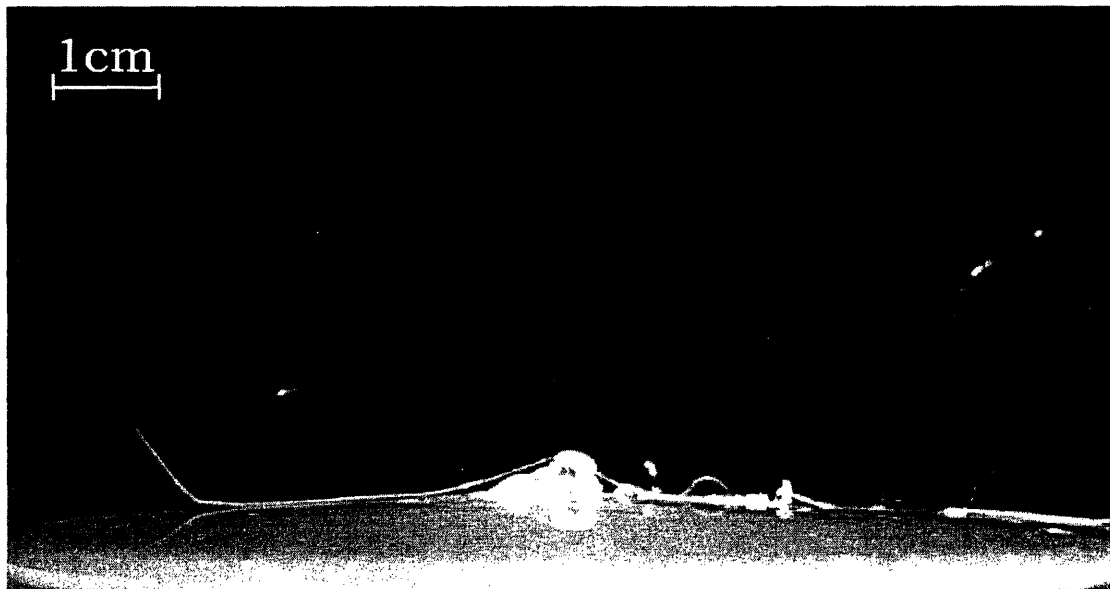


Figure 4-17: The Kaye Effect: Frame 214, time 198.6ms

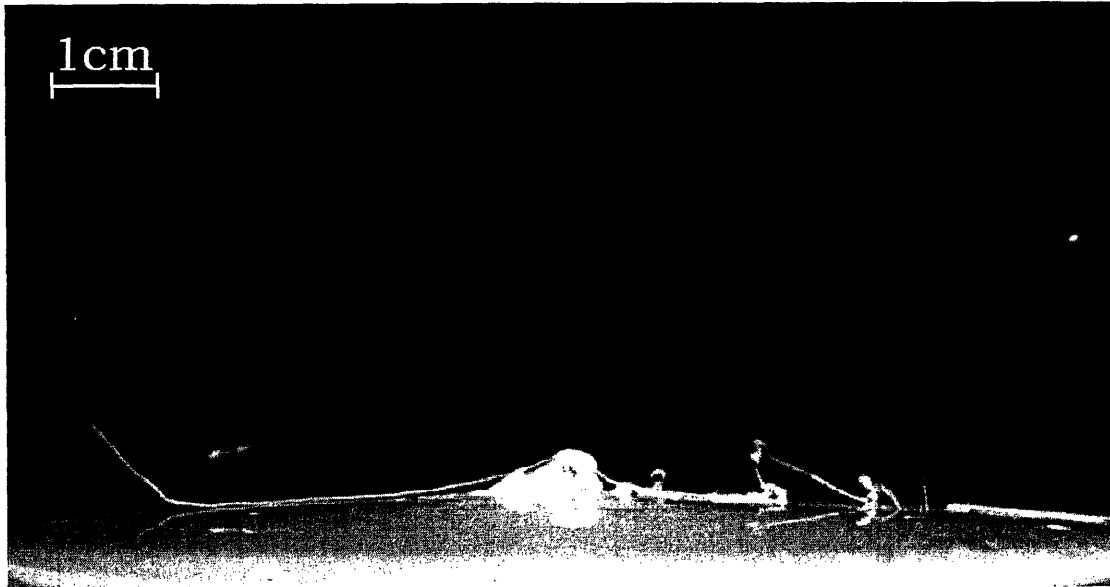


Figure 4-18: The Kaye Effect: Frame 225, time 208.8ms

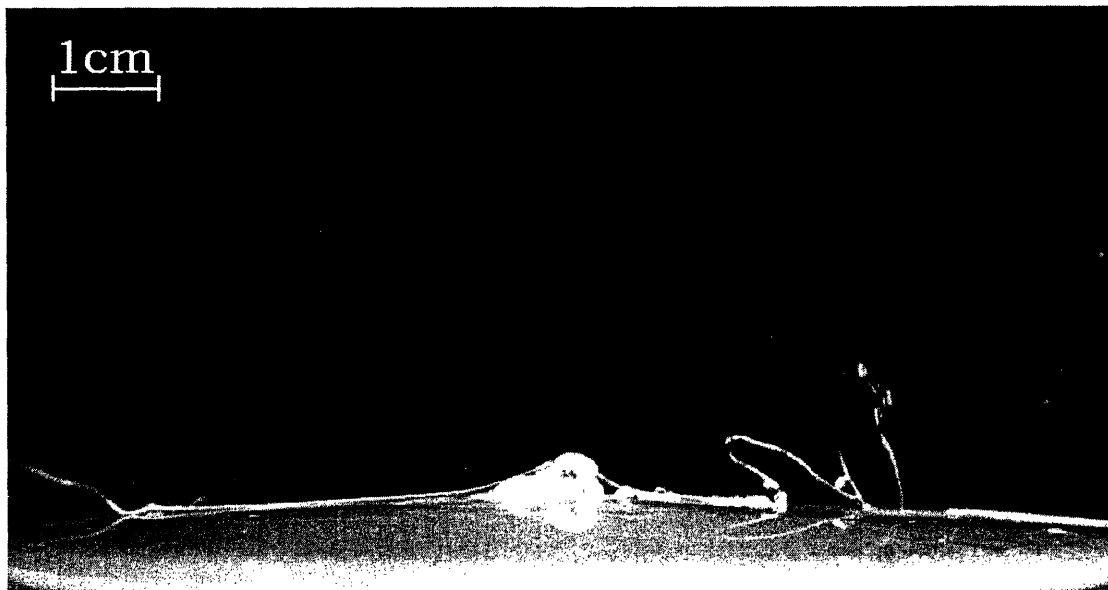


Figure 4-19: The Kaye Effect: Frame 236, time 219.0ms

## 4.4 Discussion and Conclusions

The Kaye effect, as observed through experimentation, was inconsistent in height, range and frequency for a constant drop height and effectively constant flow rate. The greatest source of inconsistency was the falling stream itself. In order to achieve the Kaye effect, it was found that thin streams of fluid were required. If the streams were too large, inertial effects dominate and the incoming stream simply impacts the pile, adding to it. Also, the stream must fall from a height great enough to impart sufficient impact velocity. However, with a highly elastic fluid like the shampoo used, a long, thin stream as required for the Kaye effect is prone to high extensional stresses. These stresses would become much too great for the stream to support, and the stream would fragment in midair. The stream from the preceding image sequence was uncharacteristically long, and contributed to the overall reproducibility.

When the stream did break, however, the falling end of the intact stream still flowing from the flask would occasionally curl somewhat, forming a J-like structure of falling fluid. This structure would have a highly oblique angle to the pile at the moment of impact, and would almost always deflect nearly horizontally. This is not entirely unexpected in a highly viscous fluid, so the effect was not investigated thoroughly.

There is some subtle parameter, obscured by the high viscosity of the shampoo at Kaye-effect range shear rates, dominating traditional Kaye effect behavior. Ostensibly, some product of impact speed, flow diameter, and fluid viscosity would dictate the presence, or lack thereof of the Kaye effect. When considering such variables, the Reynolds number is of readily apparent importance. For the flow detailed in the results, the corresponding Reynolds number falls between  $3.4 * 10^{-7}$  and  $1.7 * 10^{-5}$  depending on the applicable shear rate. The Deborah for this flow (using pre-rebound stream-pool contact time for characteristic relaxation timescale, and stream diameter over stream velocity for characteristic flow timescale) is approximately 6.7. These number impart no insight to this specific experiment, but may be of use to future researchers.

Inclining the plate yielded no readily apparent interesting behavior. Previous Kaye effect anecdotes from Detlef Lohse of the Netherlands University of Twente tell of a cascading Kaye effect - a chain of piles built by downslope deflected streams forming new piles, and deflecting new streams downslope. While not noted quantitatively, the only effect of inclining the plane even in excess of 35 degrees was to bias the distribution of deflected streams upslope, contrary to Lohse's observations.

The previous literature on the Kaye effect focused heavily on the interaction between the incoming stream and the pile - yet the detailed image sequence is wholly independent of the accumulated pile of fluid. This is fundamentally different than the traditional Kaye effect, which as mentioned previously is driven by the size, slope, and decay of the fluid pile. This nontraditional Kaye-style effect observed here is driven more by the elasticity of the fluid, and its ability to resist merging of the impacted stream with the fluid pool on a timescale greater than the elastic response of the pool. As the fluid does not merge like a Newtonian fluid, elastic forces capture and return the kinetic energy of the falling stream, sending it upwards.

However, it does not seem to be simply a matter of not hitting the pile. Attempts to recreate the effect more methodically - by translating the impact sheet - failed. While the high viscosity of the fluid may admittedly obscure any more sensitive processes present, it appears the fluid stream needs some velocity component perpendicular not just to the fluid sheet, but also to gravity to effectively bounce.

The fluid used had a viscosity 30 times greater than that observed by Kaye at the same shear rate, and this was more likely responsible for the apparent inconsistency of the Kaye effect. However, the fluid was also highly elastic, and gave rise to new phenomena of interest - that of a fluid stream reflecting off of a flat pool of fluid as opposed to deflecting off of a pile of fluid. This phenomena, while it differs significantly from the traditional Kaye effect, is probably rooted in the same fundamental mechanical processes.

## Chapter 5

# Recommendations for Future Work

As detailed in the preceding discussion section, this investigation did not succeed in detailing the parameters driving the Kaye effect. The Reynolds number of the falling stream is most likely an important variable - but it does not account in any way for the elastic properties of the fluid, which are clearly core requirements of the Kaye effect. The fluid used in this report, shampoo, was 30 times more viscous than the fluid used by Kaye at the shear rate he quoted. While this shear rate may not be the shear rate most pertinent to the Kaye effect, the difficulties experienced in this experiment could be attributed to excessive velocity, and this factor of 30 is in some way indicative of these problems. Finding a fluid of lower viscosity that retains high elastic character would facilitate future work.

In regards to this new effect, some mechanism to induce a deflection in the incident stream would probably be of great utility. The high and low speed fans used in this experiment proved to be too powerful, as the incident stream has very little mass. A more gentle, methodical means of inducing a deflection would vastly improve the understanding of both this phenomena, and the traditional Kaye phenomena.





# Appendix A

## Shampoo Ingredients

Table A.1: Suave® Naturals® Vanilla Floral Shampoo Ingredients

Water
Ammonium Lauryl Sulfate
Ammonium Laureth Sulfate
Ammonium Chloride
Cocamide MEA
PEG-5 Cocamide
Fragrance
Hydroxypropyl Methylcellulose
Tetrasodium EDTA
DMDM Hydantoin
Citric Acid
Silk Protein
Methylchloroisothiazolinone
Propylene Glycol
Methylisothiazolinone
Polysorbate 20
Vanilla Planifolia Extract
Prunus Amygdalus (Sweet Almond) Extract
FD&C Red No. 40 (CI 16035)
D&C Twllow No. 10 (CI 47005)



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