SEPTEMBER 14, 1992

LIDS-P-2132

On Synthesizing Partially Decentralized Controllers *

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Abstract

Quite often, in the control of physical systems, structural constraints are placed on the feedback controller. Issues of complexity, computation, ease of implementation and physical dimensions play a role in the decision to select structurally constrained controllers. This paper examines controllers which are constrained by the amount of information sharing occurring in the feedback channels. A fully decentralized controller is characterized by no sharing of information among the feedback channels. Three other types of partially decentralized controllers are characterized by the way in which the feedback channels locally share information between adjacent channels. It is shown in this paper via left and right unimodular transformations, that from the set of stabilizing decentralized controllers associated with a transformed plant operator a set of stabilizing partially decentralized controllers can be recovered. This serves to identify a potentially useful methodology for the synthesis of partially decentralized controllers by formulating the problem within a framework which can take advantage of the stable factors approach to the parameterization of the set of all stabilizing fully decentralized controllers.

*This research was conducted at the MIT Laboratory for Information and Decision Systems with support provided by IRAD funds from the C.S. Draper Laboratory Inc., Dr. K.W. Flueckiger and Dr. J.R. Dowdle of CSDL are the technical monitors.

This paper has been submitted to the 1993 American Control Conference.

1 Introduction

Due to practical engineering issues of complexity, computation, ease of implementation, spatial topology etc. feedback controllers with specific structural constraints play an important role in the control of many physical systems. Perhaps the best known structurally constrained controller is the completely decentralized controller. A completely decentralized controller imposes a partitioning and pairing of the systems controls and outputs. This constrains the controller structure to be block diagonal thereby providing an individual controller for each channel of the partitioned system. The plant operated on by such a controller is effectively partitioned into multiple channels and for completely decentralized control no information is shared between the feedback channels [1].

Partially decentralized control structures can be characterized by the way in which local information between channels is shared. In this paper three types of partially decentralized controllers will be identified. The characterization is limited to the natural order in which local information between channels would most likely be shared, however the method used to generate controllers with such structure can be applied to any particular combinations of local information sharing between feedback channels. The use of partially decentralized controllers usually arises out of physical systems where strong local interactions of subsystems exist. For example, reference [2] demonstrates the benefits of using a partially decentralized controller over a fully decentralized controller in terms of the performance obtainable in the simulated closed loop systems. These controllers were used in the design of the Laser Demonstration Facility (LDF) laser alignment control system at the Lawrence Livermore National

Laboratory. The subsystems of laser transport system comprised a chain-like structure. The partially decentralized controller used in this system employs a local information sharing structure which consists of individual subsystems sharing the feedback channel information with the adjacent subsequent subsystem in the chain-like system structure.

In this paper theoretical issues associated with developing partially decentralized controllers by stable factor methods [3] are examined. The methodology relies on transforming the plant operator into a form suitable for direct application of the recently developed [4] parameterization of the class of all stabilizing decentralized compensators. The appropriate stabilizing partially decentralized controller is then recovered from the fully decentralized controller via a transformation by left and right unimodular operators.

1.1 Notation

H	principle ideal domain
$U \subset H$	is the group of units of H
G	is the set of fractions associated with H
$m(H)$	set of matrices with elements in H
$m(G)$	set of matrices with elements in G
$m(0)$	set of matrices whose elements are 0
$\|\cdot\|$	refers to the H_∞ norm of enclosed operator

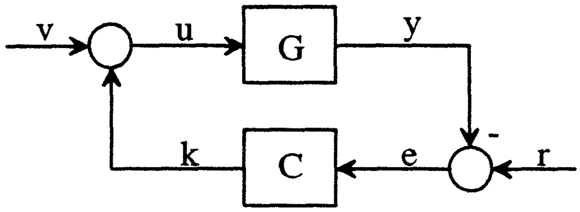

Figure 1: Two Block Control Problem

2 Partially Decentralized Controller Structures

Figure 1 gives the representation of the standard two block problem. G is an element of $m(G)^{p\times q}$ with $y, e, r \in \Re^p$ and $k, u, v \in \Re^q$. For a plant G partitioned into m channels the associated fully decentralized controller has the following structure:

$$\begin{bmatrix} k_1 \\ k_2 \\ \vdots \\ k_m \end{bmatrix} = \begin{bmatrix} C_1 & & & \\ & C_2 & & \\ & & \ddots & \\ & & & C_m \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_m \end{bmatrix} \tag{1}$$

Where $e_i \in \Re^{p_i}$ and $k_i \in \Re^{q_i}$ with $\sum_i p_i = p$ and $\sum_i q_i = q$. The feedback channels $e_1, e_2 \cdots e_m$ are independent of one another or in other words the channels share no information between one another. Three partially decentralized controller structures based on the local sharing of information among the feedback channel are characterized in the following manner. A

Type 1 partially decentralized controller is defined to have the following structure:

$$\text{Type 1} \quad \begin{bmatrix} k_1 \\ k_2 \\ k_3 \\ \vdots \\ k_m \end{bmatrix} = \begin{bmatrix} C_{11} & 0 & \cdots & & & 0 \\ C_{21} & C_{22} & 0 & \cdots & & 0 \\ 0 & C_{32} & C_{33} & 0 & \cdots & 0 \\ \vdots & & \ddots & & & \vdots \\ 0 & \cdots & & 0 & C_{m,m-1} & C_{mm} \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ \vdots \\ e_m \end{bmatrix} \tag{2}$$

Where the local sharing of information in the feedback channel with respect to the k_i output channel of the controller consist of information in channels e_{i-1} and e_i. A Type 2 partially decentralized controller is defined to have the following structure:

$$\text{Type 2} \quad \begin{bmatrix} k_1 \\ k_2 \\ \vdots \\ k_m \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & 0 & \cdots & & 0 \\ 0 & C_{22} & C_{23} & 0 & \cdots & 0 \\ \vdots & & & \ddots & & \vdots \\ 0 & \cdots & & & 0 & C_{mm} \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_m \end{bmatrix} \tag{3}$$

Where the local sharing of information in the feedback channel with respect to the k_i output channel of the controller consist of information in channels e_i and e_{i+1}. And finally a Type

3 partially decentralized controller is defined to have the following structure:

$$\text{Type 3} \quad \begin{bmatrix} k_1 \\ k_2 \\ k_3 \\ \vdots \\ k_{m-1} \\ k_m \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & 0 & \cdots & & 0 \\ C_{21} & C_{22} & C_{23} & 0 & \cdots & 0 \\ 0 & C_{32} & C_{33} & C_{34} & \cdots & 0 \\ \vdots & & & \ddots & & \vdots \\ 0 & \cdots & & C_{m-1,m-2} & C_{m-1,m-1} & C_{m-1,m} \\ 0 & \cdots & & 0 & C_{m,m-1} & C_{m,m} \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ \vdots \\ e_{m-1} \\ e_m \end{bmatrix} \tag{4}$$

Where the local sharing of information in the feedback channel with respect to the k_i output channel of the controller consist of information in channels e_{i-1}, e_i and e_{i+1}.

In the case of the fully decentralized compensator the structure of the compensator factorization into stable factors is readily apparent. For example, one factorization of the block diagonal compensator could be $C_d = V^{-1}U$ where V and U are coprime and also block diagonal [4]. However, the complex structure of the partially decentralized controllers (as exhibited by eqs. (2)-(3)) do not simplify into a readily recognizably stable factors structure and hence the parameterization of partially decentralized controllers using stable factors directly becomes difficult. The method developed in this paper takes advantage of the stable factor parameterization indirectly by transforming the original plant operator via left and right unimodular transformations and then lifting or effectively repartitioning the resulting operator into a multichannel operator which can be stabilized by the set of parameterized fully decentralized controllers as given in reference [4]. The desired partial decentralized

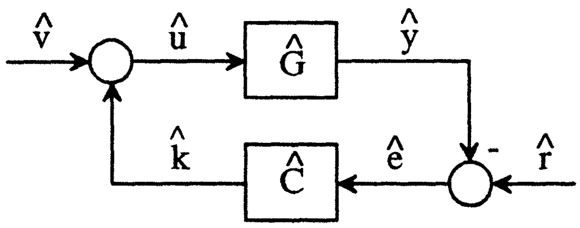

Figure 2: Two Block Control Problem for $\hat{G}$

controller will then be recovered from the fully decentralized controllers by the reciprocal left and right unimodular transformations. Before detailing the precise steps involved in this method the following section presents some needed definitions and theorems.

3 Unimodular Transformations

A Left unimodular operator is defined as follows:

Definition 1 (Left Unimodular) *An operator N an element of $m(H)$ is left unimodular if there exists an operator $Z \in m(H)$ such that $ZN = I$.*

Likewise a right unimodular operator is defined as follows:

Definition 2 (Right Unimodular) *An operator M an element of $m(H)$ is right unimodular if there exists an operator $W \in m(H)$ such that $MW = I$.*

Synthesis of structurally constrained controllers from say fully decentralized controllers is dependent on establishing a relation between the original plant operator G and an operator $\hat{G}$. For example figure 2 shows the two block problem for operator $\hat{G}$ and stabilizing controller $\hat{C}$. By requiring that the relation holds $G = M\hat{G}N$, where M is right unimodular with

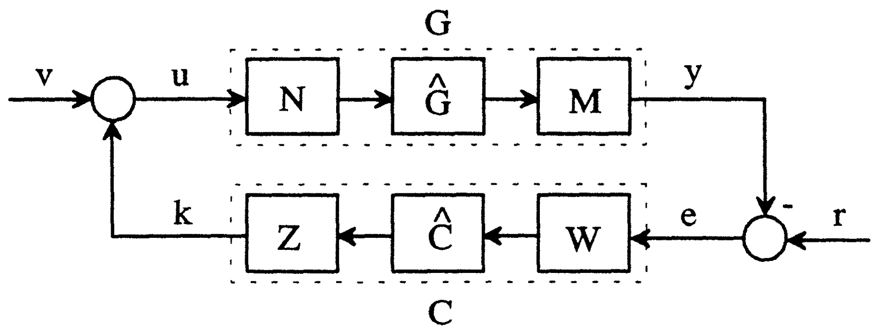

Figure 3: Two Block Problem Transformed Using Left and Right Unimodular Operators

$MW = I$ and N is left unimodular with $ZN = I$, a two block diagram can be written as shown in figure 3. Where $M : \hat{r} \mapsto r$ and $Z : \hat{v} \mapsto v$. This leads to the following theorem which will be instrumental in recovering partially decentralized controllers from fully decentralized controllers.

Theorem 1 *Given $\hat{C}$ stabilizes $\hat{G}$ and $G = M\hat{G}N$ where M is right unimodular with $MW = I$ and N is left unimodular with $ZN = I$, $C \stackrel{def}{=} Z\hat{C}W$ stabilizes G.*

Proof

$$H(\hat{C},\hat{G}) : \quad \begin{bmatrix} \hat{r} \\ \hat{v} \end{bmatrix} \longrightarrow \begin{bmatrix} \hat{e} \\ \hat{u} \end{bmatrix} \quad \text{where}$$

$$H(\hat{C},\hat{G}) = \begin{bmatrix} (I+\hat{G}\hat{C})^{-1} & -(I+\hat{G}\hat{C})^{-1}\hat{G} \\ (I+\hat{C}\hat{G})^{-1}\hat{C} & (I+\hat{C}\hat{G})^{-1} \end{bmatrix} = \begin{bmatrix} \hat{H}_{11} & \hat{H}_{12} \\ \hat{H}_{21} & \hat{H}_{22} \end{bmatrix}$$

$$\hat{G} \quad \text{stabilizes} \quad \hat{C} \Longrightarrow \hat{H}_{ij} \in m(H) \quad \forall\, i,j$$

$G = M\hat{G}N$ where M is right unimodular with $MW = I$ and N is left unimodular with $ZN = I$, and with C defined as $C \stackrel{def}{=} Z\hat{C}W$ the following maps are defined:

$$
\begin{aligned}
W: &\ r \mapsto \hat{r} \\
N: &\ v \mapsto \hat{v} \\
M: &\ \hat{e} \mapsto e \\
Z: &\ \hat{u} \mapsto u
\end{aligned}
$$

Using these above maps the mapping corresponding to $H(\hat{C}, \hat{G})$ can be rewritten as

$$
\begin{aligned}
\begin{bmatrix} \hat{e} \\ \hat{u} \end{bmatrix} &= H(\hat{C}, \hat{G}) \begin{bmatrix} \hat{r} \\ \hat{v} \end{bmatrix} \\
\begin{bmatrix} M & 0 \\ 0 & Z \end{bmatrix} \begin{bmatrix} \hat{e} \\ \hat{u} \end{bmatrix} &= \begin{bmatrix} M & 0 \\ 0 & Z \end{bmatrix} H(\hat{C}, \hat{G}) \begin{bmatrix} W & 0 \\ 0 & N \end{bmatrix} \begin{bmatrix} r \\ v \end{bmatrix} \\
\begin{bmatrix} e \\ u \end{bmatrix} &= \begin{bmatrix} M\hat{H}_{11}W & M\hat{H}_{12}N \\ Z\hat{H}_{21}W & Z\hat{H}_{22}N \end{bmatrix} \begin{bmatrix} r \\ v \end{bmatrix}
\end{aligned}
$$

Note however that the map from $\begin{bmatrix} r \\ v \end{bmatrix}$ to $\begin{bmatrix} e \\ u \end{bmatrix}$ is the closed loop map H(C,G).

Therefore $H(C, G) = \begin{bmatrix} M\hat{H}_{11}W & M\hat{H}_{12}N \\ Z\hat{H}_{21}W & Z\hat{H}_{22}N \end{bmatrix}$

Since $M, W, N, Z \in m(H)$ this implies $M\hat{H}_{11}W$, $M\hat{H}_{12}N$, $Z\hat{H}_{21}W$, $Z\hat{H}_{22}N \in m(H)$ and that C stabilizes G.

$\square$

Using Theorem 1, in general a number of structurally constrained controller can be synthesized. The focus here will be on synthesizing partially decentralized controls as given in eqs. (2)-(4). To illustrate this method the synthesis of a three channel, type 3 controller will be developed since the type 3 structure is more complex then the other two controller types. Extensions to the multichannel case proceeds directly along the lines outlined in the next section for the three channel case.

4 Synthesizing Type 3 Controllers

For the following three channel plant, using the notation described above and in Figure 3:

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} G_{11} & G_{12} & G_{13} \\ G_{21} & G_{22} & G_{23} \\ G_{31} & G_{32} & G_{33} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \tag{5}$$

The structure corresponding to a three channel, type 3 controller takes the form:

$$\begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & 0 \\ C_{21} & C_{22} & C_{23} \\ 0 & C_{32} & C_{33} \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} \tag{6}$$

As will be demonstrated a m-channel, type 3 controller can be recovered from a $(m-1)$-channel fully decentralized controller where the channel dimension has been appropriately increased. For the 3-channel, type 3 controller, it will be recovered from a 2-channel fully decentralized controller in the following manner:

$$C = Z\hat{C}W \tag{7}$$

Where $\hat{C}$ has the following 2-channel structure

$$\begin{bmatrix} \hat{C}^{(1)} & 0 \\ 0 & \hat{C}^{(2)} \end{bmatrix} = \begin{bmatrix} \hat{C}_{11}^{(1)} & \hat{C}_{12}^{(1)} & 0 & 0 \\ \hat{C}_{21}^{(1)} & \hat{C}_{22}^{(2)} & 0 & 0 \\ 0 & 0 & \hat{C}_{11}^{(2)} & \hat{C}_{12}^{(2)} \\ 0 & 0 & \hat{C}_{21}^{(2)} & \hat{C}_{22}^{(2)} \end{bmatrix} \tag{8}$$

The type 3 controller C can be recovered from $\hat{C}$ by using the following right and left unimodular operators:

$$Z = \begin{bmatrix} I_{i1} & 0 & 0 & 0 \\ 0 & I_{i2} & I_{i2} & 0 \\ 0 & 0 & 0 & I_{i3} \end{bmatrix} \tag{9}$$

$$W = \begin{bmatrix} I_{o1} & 0 & 0 \\ 0 & I_{o2} & 0 \\ 0 & I_{o2} & 0 \\ 0 & 0 & I_{o3} \end{bmatrix} \tag{10}$$

The identity operator I_{ij} is compatible with the input dimension of the plant operator G_{ij} and likewise I_{oi} is compatible with the output dimension of G_{ij}. Applying Z and W to $\hat{C}$ as given in eq. (7) gives the following:

$$C = Z\hat{C}W = \begin{bmatrix} \hat{C}_{11}^{(1)} & \hat{C}_{12}^{(1)} & 0 \\ \hat{C}_{21}^{(1)} & \hat{C}_{22}^{(2)} + \hat{C}_{11}^{(2)} & \hat{C}_{12}^{(2)} \\ 0 & \hat{C}_{21}^{(2)} & \hat{C}_{22}^{(2)} \end{bmatrix} \tag{11}$$

Which has the desired structure of a type 3, 3-channel controller. C will be stabilizing, according to theorem 1 as long as $G = M\hat{G}N$ where M is right unimodular with $MW = I$ and N is left unimodular with $ZN = I$. Based on Z and W as given in eqs. (10)-(10), M and N can have the following form:

$$M = \begin{bmatrix} I_{o1} & 0 & 0 & 0 \\ 0 & .5I_{o2} & .5I_{o2} & 0 \\ 0 & 0 & 0 & I_{i3} \end{bmatrix} \tag{12}$$

$$N = \begin{bmatrix} I_{i1} & 0 & 0 \\ 0 & .5I_{i2} & 0 \\ 0 & .5I_{i2} & 0 \\ 0 & 0 & I_{i3} \end{bmatrix} \tag{13}$$

What needs to be determined is the structure of $\hat{G}$.

Since $\hat{G}$ must satisfy

$$G = M\hat{G}N \tag{14}$$

$\hat{G}$ can be obtained in the following fashion

$$\hat{G} = WGZ + S \tag{15}$$

Applying eq. (14) we obtain:

$$\begin{aligned} \hat{G} &= M(WGZ + S)N && (16) \\ &= MWGZN + MSN && (17) \\ &= G + MSN && (18) \end{aligned}$$

Which is satisfied if $MSN \in m(0)$ where $m(0)$ is the matrix ring whose elements are all zero. For the three channel case WGZ has the following form:

$$WGZ = \begin{bmatrix} G_{11} & G_{12} & G_{12} & G_{13} \\ G_{21} & G_{22} & G_{22} & G_{23} \\ G_{21} & G_{22} & G_{22} & G_{23} \\ G_{31} & G_{32} & G_{32} & G_{33} \end{bmatrix} \tag{19}$$

Before lifting WGZ to a two channel plant for which a set of parameterized fully decentralized controllers can be developed using stable factors, an S operator can be added which will minimize the size of the coupling operator, [5], for $\hat{G}$. An operator S having the following form will achieve this:

$$S = \begin{bmatrix} 0 & G_{12} & -G_{12} & 0 \\ G_{21} & G_{22} & -G_{22} & -G_{23} \\ -G_{21} & -G_{22} & G_{22} & G_{23} \\ 0 & -G_{32} & G_{32} & G_{33} \end{bmatrix} \tag{20}$$

And $\hat{G}$ takes the following form:

$$\hat{G} = WGZ + S = \begin{bmatrix} G_{11} & 2G_{12} & 0 & G_{13} \\ 2G_{21} & 2G_{22} & 0 & 0 \\ 0 & 0 & 2G_{22} & 2G_{23} \\ G_{31} & 0 & 2G_{32} & G_{33} \end{bmatrix} \tag{21}$$

Now $\hat{G}$ can be lifted or equivalently repartitioned into a two channel plant having the structure:

$$\hat{G} = \begin{bmatrix} \hat{G}_{11} & \hat{G}_{12} \\ \hat{G}_{21} & \hat{G}_{22} \end{bmatrix} \tag{22}$$

where

$$\hat{G}_{11} = \begin{bmatrix} G_{11} & 2G_{12} \\ 2G_{21} & 2G_{22} \end{bmatrix} \tag{23}$$

$$\hat{G}_{12} = \begin{bmatrix} 0 & G_{13} \\ 0 & 0 \end{bmatrix} \tag{24}$$

$$\hat{G}_{21} = \begin{bmatrix} 0 & 0 \\ G_{13} & 0 \end{bmatrix} \tag{25}$$

$$\hat{G}_{22} = \begin{bmatrix} 2G_{22} & 2G_{23} \\ 2G_{32} & 2G_{33} \end{bmatrix} \tag{26}$$

Now if $\hat{G}$ has no unstable fixed modes [6] a parameterized set of two channel fully decentralized controllers exists based on the stable factors method presented in [4]. And from each two channel fully decentralized controller a type 3 partially decentralized controller can be recovered as given in eq. (11).

5 Weak Coupling When Using Partially Decentralized Controllers

As demonstrated in reference [5] weak coupling can be quantified in terms of the norm of the off-diagonal elements of a stable plant operator for which fully decentralized controllers are to be designed. The effect as the coupling goes to zero is that the unimodular constraint which restricts the design parameters used in the selection of fully decentralized controllers disappears [4]. These notions serve to provide a measure of the improvement obtain via the use of partially decentralized controllers versus applying a fully decentralized control strategy directly to the plant operator G. For example, a stable three channel plant as given in eq. (5) would have the following coupling operator norm if a three channel fully decentralized controller where to be designed:

$$||G_c|| = \left\| \begin{matrix} 0 & G_{12} & G_{13} \\ G_{21} & 0 & G_{23} \\ G_{31} & G_{32} & 0 \end{matrix} \right\| \tag{27}$$

However, when a three channel, type 3 controller is designed for G, a fully decentralized controller is designed using the associated $\hat{G}$ operator as given by eq. (22). The coupling operator norm for a stable $\hat{G}$ is

$$\left\|\hat{G}_c\right\| = \left\| \begin{matrix} 0 & \hat{G}_{12} \\ \hat{G}_{21} & 0 \end{matrix} \right\| = \max\left[\|\hat{G}_{21}\|, \|\hat{G}_{12}\|\right] \tag{28}$$

Due to eq. (25) and eq. (24), it follows that $\|\hat{G}_{21}\| = \|G_{31}\|$ and $\|\hat{G}_{12}\| = \|G_{13}\|$. Hence the coupling norm when using a type 3 controller is given by $\|\hat{G}_c\| = \max[\|G_{31}\|, \|G_{13}\|]$. This implies (as outlined in [5] for the general two channel fully decentralized case) a quantification of weak coupling can be developed using only the norms of the G_{13} and G_{31} operators of the three channel plant when controlled by a type 3 controller. This simplification with respect to the more complicated coupling operator for the fully decentralized 3 channel controller (see eq. (27)) is not unexpected considering information sharing occurs between adjacent channels in the type 3 controller (see eq. (6)) unlike the fully decentralized 3 channel controller where there is no sharing of information among the feedback channels.

6 Synthesizing Type 1 and Type 2 Controllers

Synthesizing Type 1 and Type 2 controllers follows the same basic pattern as performed for the type 3 controller. To demonstrate this a 3 channel type 1 controller will be developed. The type 2 controllers are developed in a complementary fashion. In general a type 1 or type 2, m-channel controller can be synthesized from a m-channel fully decentralized controller where $(m-1)$ of the fully decentralized channels have an increased dimension.

The structure of the 3 channel, type 1 controller is

$$C = \begin{bmatrix} C_{11} & 0 & 0 \\ C_{21} & C_{22} & 0 \\ 0 & C_{32} & C_{33} \end{bmatrix} \tag{29}$$

Such a controller can be recovered from a three block fully decentralized controller with the following form:

$$\hat{C} = \begin{bmatrix} \hat{C}^{(1)} & 0 & 0 \\ 0 & \hat{C}^{(2)} & 0 \\ 0 & 0 & \hat{C}^{(3)} \end{bmatrix} = \begin{bmatrix} \hat{C}^{(1)} & 0 & 0 & 0 & 0 \\ 0 & \hat{C}_1^{(2)} & \hat{C}_2^{(2)} & 0 & 0 \\ 0 & 0 & 0 & \hat{C}_1^{(3)} & \hat{C}_2^{(3)} \end{bmatrix} \tag{30}$$

The right and left unimodular operators Z and W have the following form

$$Z = I \tag{31}$$

$$W = \begin{bmatrix} I_{o1} & 0 & 0 \\ I_{o1} & 0 & 0 \\ 0 & I_{o2} & 0 \\ 0 & I_{o2} & 0 \\ 0 & 0 & I_{o3} \end{bmatrix} \tag{32}$$

And the recovered type 1 controller has the following form:

$$C = \begin{bmatrix} \hat{C}^{(1)} & 0 & 0 \\ \hat{C}_1^{(2)} & \hat{C}_2^{(2)} & 0 \\ 0 & \hat{C}_1^{(3)} & \hat{C}_2^{(3)} \end{bmatrix} \tag{33}$$

Once again $\hat{G}$ is found from $\hat{G} = WGZ + S$ where

$$WGZ = \begin{bmatrix} G_{11} & G_{12} & G_{13} \\ G_{11} & G_{12} & G_{13} \\ G_{21} & G_{22} & G_{23} \\ G_{21} & G_{22} & G_{23} \\ G_{31} & G_{32} & G_{33} \end{bmatrix} \tag{34}$$

$$S = \begin{bmatrix} G_{11} & -G_{12} & G_{13} \\ -G_{11} & G_{12} & -G_{13} \\ -G_{21} & G_{22} & -G_{23} \\ G_{21} & -G_{22} & G_{23} \\ 0 & 0 & 0 \end{bmatrix} \tag{35}$$

Resulting in a $\hat{G}$ of the following form:

$$\hat{G} = \begin{bmatrix} 2G_{11} & 0 & 2G_{13} \\ 0 & 2G_{12} & 0 \\ 0 & 2G_{22} & 0 \\ 2G_{21} & 0 & 2G_{23} \\ G_{31} & G_{32} & G_{33} \end{bmatrix} \tag{36}$$

Now $\hat{G}$ can be repartitioned into a three channel plant having the structure:

$$\hat{G} = \begin{bmatrix} \hat{G}_{11} & \hat{G}_{12} & \hat{G}_{13} \\ \hat{G}_{21} & \hat{G}_{22} & \hat{G}_{23} \\ \hat{G}_{31} & \hat{G}_{32} & \hat{G}_{33} \end{bmatrix} \tag{37}$$

where

$$\hat{G}_{11} = 2G_{11} \tag{38}$$

$$\hat{G}_{13} = 2G_{13} \tag{39}$$

$$\hat{G}_{22} = \begin{bmatrix} 2G_{12} \\ 2G_{22} \end{bmatrix} \tag{40}$$

$$\hat{G}_{31} = \begin{bmatrix} 2G_{21} \\ G_{31} \end{bmatrix} \tag{41}$$

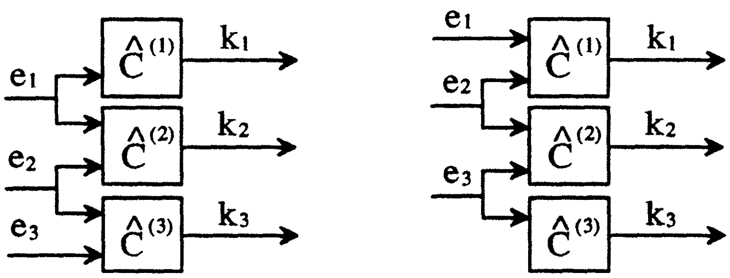

Figure 4: Type 1 and Type 2 Controller Structure

$$\hat{G}_{32} = \begin{bmatrix} 0 \\ \\ G_{32} \end{bmatrix} \tag{42}$$

$$\hat{G}_{33} = \begin{bmatrix} 2G_{23} \\ \\ G_{33} \end{bmatrix} \tag{43}$$

And $\hat{G}_{12}, \hat{G}_{21}, \hat{G}_{23} \in m(0)$. Now once again if $\hat{G}$ has no unstable fixed modes [6] a parameterized set of three channel fully decentralized controllers exists based on the stable factors method presented in [4]. And from each three channel fully decentralized controller a type 1 partially decentralized controller can be recovered as given in eq. (33).

Finally figures 4 and 5 illustrate how the recovered partially decentralized controllers maintain the desirable property of parallel processing with (in the case of a type 3 controller) the small additional overhead of output channel summations.

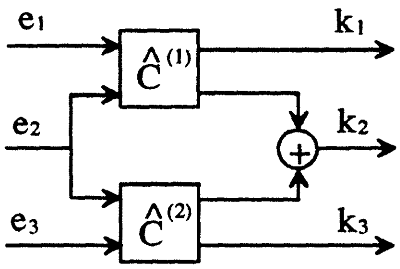

Figure 5: Type 3 Controller Structure

7 Conclusion

The use of partially decentralized controllers can be beneficial for physical systems where strong local interactions of subsystems exist. An example cited in the introduction was the improved performance of the LDF laser alignment control system [2] through the use of a partially decentralized control. This paper classifies three types of partially decentralized controllers (eqs. (2)-(3)) and presents a method of controller synthesis linked to stable factor methods [4]. The method develop in this paper is sufficiently broad enough that other structurally constrained controllers not specifically contained in the three classifications can also be synthesized using these methods. The general notion of left and right unimodular transformations are develop for the recovery of a stabilizing compensator C for the plant G as given by theorem 1 in section 3. Using these ideas a method of synthesizing partially decentralized controllers from the parameterized set of fully decentralized controllers is developed. Section 4 illustrates the method for type 3 partially decentralized controllers and section 6 illustrates the method for type 1 and type 2 partially decentralized controllers. Finally a discussion quantifing the concept of weak plant coupling under partial decentralized control is examined in section 5 and insight is gained by contrasting this against weak plant coupling in the more familiar sense [5] under fully decentralized control.

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