Mechanical Design of a Simple Bipedal Robot

by

Ming-fai Fong

Submitted to the Department of Mechanical Engineering
in partial fulfillment of the requirements for the degree of

Bachelor of Science in Mechanical Engineering

at the

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Abstract

The thesis objective was to design a walking robot appropriate for testing a machine learning control system. As a robot that would learn how to walk, it was pre-maturely named Toddler. The passive dynamic walker, a class of simple robots driven by gravity and inertia, was selected as the model for the Toddler robot. Two preliminary passive walkers were explored: a three-dimensional, toy-like biped with one degree of freedom at the hip; and a two-dimensional, four-legged kneed walker with degrees of freedom at the hip and knees. Bench level and computer modeled analyses were conducted during the design and construction of these robots. The simpler, toy-like walker was selected as the unactuated model for actuated Toddler. Two degrees of freedom, controlling step size and frequency, were introduced at the ankle. Toddler was able to walk with a dynamically-stable gait either passively down a ramp, or actively on a flat or slightly uphill surface. With its simple design and robust performance, Toddler provides an excellent platform for testing machine learning control in walking robotics.

Thesis Supervisor: H. Sebastian Seung
Title: Professor, Department of Brain and Cognitive Sciences and Department of Physics

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Chapter 1

Introduction

Designing controllers for walking robots presents many challenges. While many researchers have been able to build functional robots, there is little consensus on how to control them. There is yet to be a walking robot that comes close to matching human skill. Meanwhile, robotic arms have been perfected to a stage where they can match or even surpass human skill for trajectory following tasks. In order for walking robotics to reach this stage, perhaps a new approach to legged robotics is necessary.

1.1 Background

Although nearly every human understands what it feels like to walk, it is difficult to comprehend all the mechanisms that make walking possible. Many researchers have tackled this question, and some of their efforts have produced remarkable results. Two of today's premiere walking robots are ASIMO and Qrio, made by Honda and Sony, respectively. Both of these humanoid robots use a position control algorithm to move and rotate various limbs and joints similar to those used in robotic arms.

1.1.1 The Walking Robotics Control Problem

Although ASIMO and Qrio appear to be the most human-like robots, the position controller requires an exorbitant amount of energy, far beyond that required by real human walking. Furthermore their movements are tense and shaky, and high precision is necessary in all motion. For these reasons, controlling a walking robot is a more complex problem than simple position control systems have been able to solve. A solution to this control problem requires exploring alternative controller designs.

1.1.2 The Machine Learning Hypothesis

One hypothesis is that machine learning may be used to control walking robots. Machine learning techniques are derived from a set of algorithms used to train neural networks in the human brain. The idea is that the robot can essentially learn how to walk given a simple set of initial conditions and a reward-based feedback controller. The possibility of using machine learning algorithms opens up the possibility of having an intelligent walking robot that learns how to walk in the same way that human toddlers learn how to walk. By exploring this possibility, walking robotics is moving in a new direction.

1.2 Thesis Objective

Walking robotics and machine learning theory provide the backdrop for my mechanical engineering thesis. The goal of my thesis is to design and build a robot that is ideal for exploring the machine learning hypothesis.

1.3 Thesis Organization

In the following chapters I will discuss the steps I took towards fulfillment of my thesis objective. In Chapter 2, I will explain the selection of the passive dynamic walker as a model for the final robot. In Chapter 3, I will discuss two passive walkers I built. In Chapter 4, I will outline the design process I used in creating Toddler, the robot for testing machine learning. Finally, in Chapter 5, I will summarize my accomplishments and discuss the future of the Toddler robot.

Chapter 2

Selection of the Passive Dynamic Walker

After studying many different walking robots, the Passive Dynamic Walker was selected as a model for the robot to be used in machine learning experiments. Although ASIMO and Qrio are remarkable humanoid robots, the passive dynamic walker was most appropriate choice for my thesis project.

2.1 Description

The passive dynamic walker is a type of bipedal robot that was introduced by Tad McGeer [1]. It is called "dynamic" because its movement is characterized by a dynamic stability. The robot is not stable at any one point in its motion. However, it is balanced in time so that the gait is steady and smooth. All bipedal walkers, including humans, must maintain dynamic stability in order to walk without falling over. The passive dynamic walker achieves this dynamic stability through a steady rocking cycle.

"Passive" refers to the robot's ability to generate locomotive movement without motor input. Instead, the passive dynamic walker can produce a steady gait using only gravity and inertia. The robot will walk stably when placed on a shallow downhill walking surface.

2.2 Advantages

The passive dynamic walker is an ideal robot for testing learning algorithms [6]. One main advantage of the walker is its simplicity. This makes the robot easier to build and modify. In addition, with fewer degrees of freedom than other robots, problems can be identified more quickly and with more certainty. Also, since the passive dynamic walker can operate without relying on motors, it is possible to study the complete dynamics of the robot by only considering a small, finite set of sources of energy gain and loss.

Another advantage of using the passive dynamic walker is its exploitation of physics to induce locomotive movement. As stated previously, the passive dynamic walker relies only on the natural gait created by gravity to drive it. The walker naturally settles into a steady oscillation that stunningly resembles walking. In addition to appearing very realistic, the passive dynamic walker is very energy efficient since it does not require any motor inputs. Other robots that are based on position control have only limited stability. Controlling these robots requires a large amount of energy that far exceeds the energy expelled by a human during locomotion. Furthermore, the large amount of energy required by position control necessitates the incorporation of heavy battery packs into the robot. The batteries dramatically alter the inertial properies of the robot, such as center of mass and moment of inertia. The passive dynamic walker is able to avoid these difficulties because gravity is its only source of energy.

Finally, and perhaps most importantly, the passive dynamic walker is a robot that almost walks by itself, making it an ideal choice for testing machine learning. In order to walk forward, it needs a downhill slope to provide energy. It essentially "falls" down the incline. For the walker to move on a flat surface, the energy provided by gravity can be replaced by minimal actuation and simple control. This simple control system is ideal for testing learning algorithms. The differences between the unactuated case and controlled learning case should reveal whether machine learning is an effective method for approaching the walking robotics problem.

Chapter 3

Preliminary Robots

3.1 Passive Dynamic Walking Toy

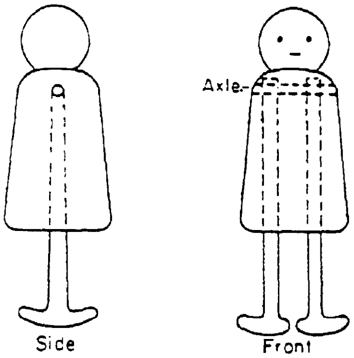

Figure 3-1: McGeer's inspiration. Walking toy schematic reprinted from McMahon 1984 [3].

McGeer's original passive dynamic walker was in part inspired by a simple walking toy [1], shown in Figure 3-1, that could move by teetering forward and sideways. In order to fully understand passive-dynamic walking and create a model for future robots, I built a toy-like robot of my own. I hoped to explore different mechanical design strategies that relied solely on exploitation of physics. I will explain the design process of this passive dynamic walking toy, or simply, the Passive Toy.

3.1.1 Design Parameters

The overarching design goal was to create a simple robot that could walk down a shallow incline with a dynamically stable gait in a repeatable, predictable fashion without the aid of motors. I began the design process by defining the parameters and sub-parameters that would help to satisfy this goal.

The first parameter for the robot was a hip joint with a single degree of freedom. An important sub-parameter was to minimize friction in the hip so that each leg could swing forward in a smooth, predictable manner. Another important sub-parameter was to minimize vibrations in the hip to reduce energy loss at the joint. The second parameter was a roll oscillation to control step frequency and allow ground clearance for the swinging leg. It was important for oscillation to be fluid, and for energy loss to be minimized. The final design parameter was a pitch oscillation to control step size.

After defining the desired parameters, I performed several levels of analysis to choose the best implementation for the design. In the next few sections, I will describe how each of the three design parameters were addressed.

3.1.2 Bench Level Analysis

I will begin by discussing strategies for satisfying each design parameter, and constraints associated with each one. I will elaborate on the implementation of the selected strategies, and how these implementations were developed and modified. When relevant, I will cite evidence from simple experiments.

Hip Joint

A hip joint with a single degree of freedom was be created by press fitting one leg onto a cylindrical shaft, and using a bushing to couple the other leg to the shaft. I chose to use thin aluminum bars for the legs and precision hardened steel for the hip shaft. However, the lightweight aluminum legs could not sufficiently counterbalance the friction in the shaft interface, and the leg did not rotate smoothly. I explored more options for satisfying the low friction sub-parameter. Several different types of bushings were tested, including brass and delrin, and rulon bushings were determined to provide the lowest friction. Unfortunately, the rulon bushings did not provide sufficiently low friction unless they were slightly worn down, contributing to undesired vibration in the shaft. This tradeoff was resolved by replacing the bushings with stainless steel ball bearings. While vibration was significantly reduced, there was still marginal play in the joint that caused one leg to wobble during motion. To eliminate this play, I introduced an aluminum bearing block to help better distribute the axial load across the shaft. The block was rigidly bolted to the inside of the rotating leg, with ball bearings on either side of the block-leg module. An identical aluminum block was symmetrically placed on the inside of the other leg to provide balance and to help solidify the press-fit.

Roll Oscillation

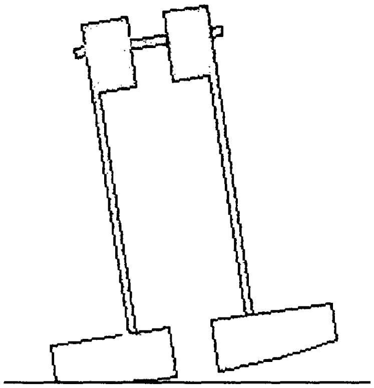

Figure 3-2: Frontal plane of Passive Toy [4].

Our second design parameter was to create a smooth roll oscillation to control step frequency and allow ground clearance. I accomplished this by curving the bottoms of the feet in the frontal plane, as shown in Figure 3-2. The precise curvature was selected such that the arcs were swept by the same circle when the robot oscillated in the two-dimensional frontal plane.

When attempting to satisfy the sub-parameters of minimizing energy losses and providing ground clearance, I found that the implementations conflicted. In order to minimize energy losses, I wanted to move the legs close together. The ultimate implementation of this would look like a semicircle rocking on a flat surface. In order to allow sufficient ground clearance during roll oscillation, I wanted to move the legs further apart. To resolve this conflict, I chose to use another level of analysis to optimize distance between feet for both sub-parameters. I could also use this analysis to find the appropriate radius of foot curvature for providing a steady oscillation that satisfied all of the roll oscillation design parameters. This analysis will be discussed in a later section. For my bench level analysis, I picked an arbitrary foot displacement and tested the roll oscillation of the robot on a variety of feet with different curvature until I found a set of feet that oscillated in a visibly satisfactory manner. I used this information in our initial guesses during modeling analysis.

Pitch Oscillation

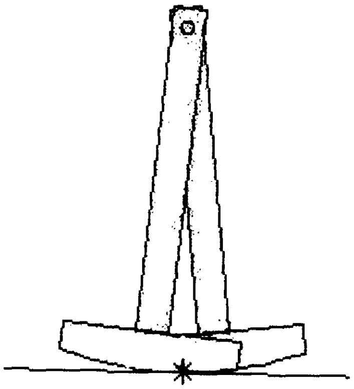

Figure 3-3: Frontal plane of Passive Toy [4].

Our final design parameter was to create a pitch oscillation to control step size. I created this oscillation by implementing curved feet into the design, as shown in Figure 3-3. I wanted step size to be relatively small, and the radius of curvature was selected arbitrarily to be slightly larger than the frontal plane radius. The angle of incline of the walking surface could be altered to adjust step size as well. The foot was made sufficiently long to allow the robot to oscillate at high amplitude, and to increase pitch moment.

3.1.3 Modeling Analysis

In order to bolster our bench level decisions, I performed another level of analysis. I created computer models of the Passive Toy to analyze its static physical properties and dynamic stability. These models provided insight for adjusting and finalizing our design implementations.

Computer Aided Design

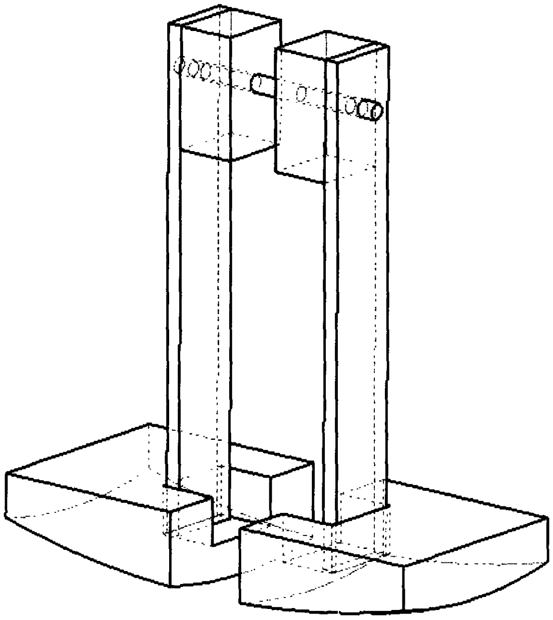

Figure 3-4: CAD model of Passive Toy created in SolidWorks.

I created a three-dimensional CAD model of the Passive Toy using SolidWorks. The model is shown in Figure 3-4 and a detailed drawing can be found in Appendix A. Using this model, I could collect information about the robot's inertial properties such as center of mass and moment of inertia. When generating these values, I assumed uniform density in each of the components. This assumption was validated by benchtop tests on stock material. I also assumed that the mass of machine screws was negligible. To confirm this, I measured the mass of the assembled robot and the mass of the robot parts with screws, and the difference was less than 0.1%. I used the values generated for total mass, center of mass, and moment of inertia, in the animated simulations.

In addition, SolidWorks allowed me to analyze how changes in component parameters could affect the physical properties of the robot. For example, I could make alterations in the foot material and shape, and I could view the changes in inertial parameters. Using this tool, I found the combinations of feet and legs that produced the center of masses and moments of inertia within the range that were most desirable.

Animated Simulation

I also used a two-dimentional MATLAB animation, developed in my thesis mentor [4], of the robot's movement. These models were based on the frontal and sagittal plane dynamics, available in Appendix B. The animations revealed how changes in physical properties affected the motion of the robot. For example, I was able to change the radius of curvature of the feet in the frontal plane, and determine which values caused the robot to succeed or fail.

To begin my analysis, I tested robot performance with random initial conditions. The initial conditions included frontal tilt angle and sagittal angle of incline. I chose several different parameters which I wanted to analyzed beyond the bench level. These parameters included foot curvature, distance between feet, and leg length. I was able to optimize all of these parameters to be robust for a variety of initial tilt angles. The angle of incline and sagittal curvature were directly related and chosen in conjunction.

3.1.4 Additional Design Notes

There were a few other important factors that I considered during the design process of the Passive Toy. First and foremost, I wanted to divide the robot into distinct modules. During the design process, I relied heavily on the bench level and modeling analyses to develop improvements. A highly modular design facilitated the implementation of these improvements. For example, I experimented with several different sets of feet, some of which are shown in Figure 3-5. The modular design made the

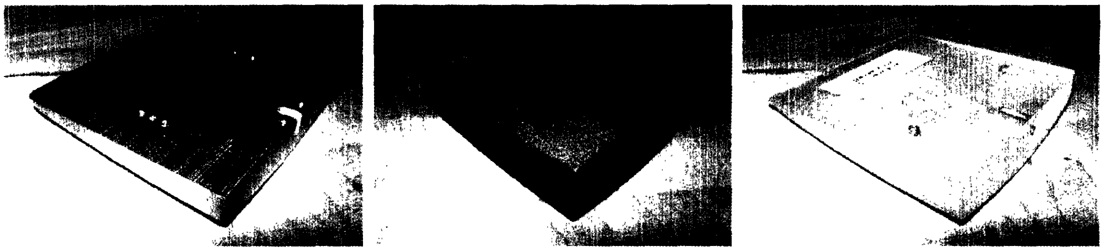

Figure 3-5: Alternate foot materials rejected during production of Passive Toy. Materials (from left to right) include aluminum, rapid prototyping foam, and acrylic.

robot robust across many implementations of each design parameter.

Another important factor in design was feasibility of production. The curved feet presented the greatest challenge in production capabilities. Attempt to test aluminum as a material for feet was hindered by the precision and power of available machine tools. While I was able to produce aluminum feet with satisfactory curvature, it was clear that future production of aluminum feet would continue to be challenging. The decision to use wood was partially based on its precision machinability and significantly lower production time.

3.1.5 Performance

The Passive Toy was tested on a stiff ramp made of a 5 x 1.5 ft wooden plank with a discreetly adjustable angle of incline. Walking was initialized by tilting the Passive Toy onto one foot, and then letting go of it. I observed a significant yaw in the robot gait across all angles of incline. In most cases, the robot could not recover from this effect. I attributed this yaw to the moment created by the swinging leg about the point of ground contact. I addressed the yaw effect by placing a thin layer of rubber on top of the wood. Rubber's high compliance helped to create a larger surface of ground contact. The increased ground contact friction dramatically decreased the yaw such that the robot was able to walk down the entire length of the ramp with a dynamically-stable gait.

Although I compensated for the majority of yaw rotation, I still observed a minor effect. Differences in the two leg-hip interfaces created an assymmetry in the horizontal distribution of mass. Furthermore, the wood screws might not be holding the metal leg completely perpendicular to the surface. Wood does not allow for high-precision bolts because the material is not stiff enough to maintain threads. The dynamics of the robot do not actively stabilize the yaw direction, so tuning can minimize but not remove the effect.

In addition, I observed that the robot leaned slightly up the ramp when placed in a static position with parallel feet. I discovered the source of this tilt to be a slight off-perpendicular angle of the feet in the sagittal plane, causing a backward shift in the center of curvature. I also noticed that this up-ramp lean actually helped the robot perform successfully, especially for high angles of ramp inclination.

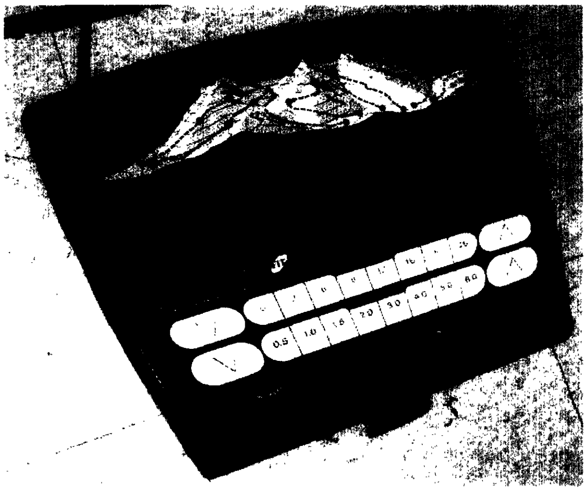

Figure 3-6: Treadmill controller. Adjusts for speed and angle of inclination.

After successful performance on the rubber-covered ramp, I tested the repeatability and predictability of the Passive Toy's gait by placing it on a moving treadmill. The treadmill had a fairly compliant walking surface and was equipped with a control unit, shown in Figure 3-6, for fine adjustments of speed and angle of incline. After some brief tinkering, I found a speed and angle of incline at which the robot could walk indefinitely with constant roll and pitch oscillation frequencies. Video of the Passive Toy's performance can viewed at our robots website [4].

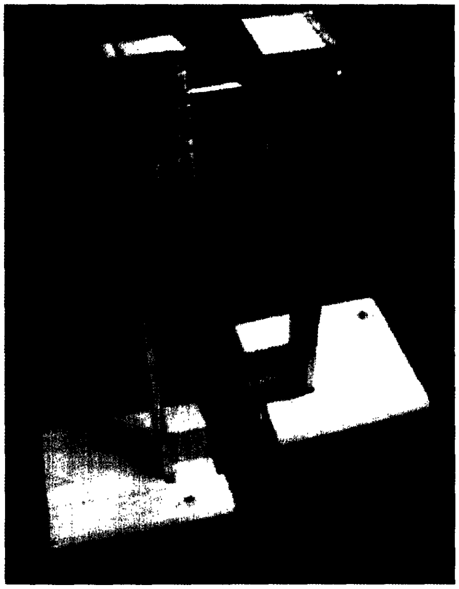

Figure 3-7: Completed Passive Toy.

3.1.6 Conclusions

I satisfied the design objective to create a simple robot that could walk down a shallow incline with a dynamically stable gait in a repeatable, predictable fashion without the aid of motors. Performance testing revealed significant yaw effects. Brief analysis showed that the main source of yaw could be counteracted by re-defining the testing paradigm; meanwhile, other sources of yaw were inherent to bipedal passive walking and production limitations. A minor production flaw revealed the benefit in shifting the feet or the foot curvature such that the robot leans up the ramp. Finally, I concluded that an actuated, controllable version of the Passive Toy could be a good robot for testing machine learning.

3.2 Kneed Walker

I also decided to explore kneed passive walking as a possibility for a robot to test machine learning. Whereas the Passive Toy used curved feet and pitch oscillation, knees offer a more anthropomorphically-accurate solution for providing ground clearance and controlling step frequency. A second motivation for exploring kneed passive walking is McGeer's evidence that bipeds with knees can, in some cases, walk more stably than those with straight legs [2].

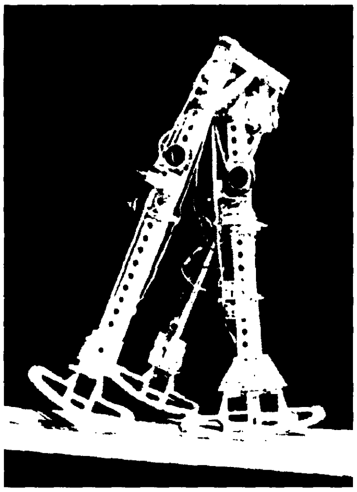

Figure 3-8: McGeer's kneed walker. Reprinted from McGeer's *Passive Dynamic Walking* [1].

In order to explore passive walking with knees, I began by constructing a robot resembling the walker built by McGeer, shown in Figure 3-8. McGeer's robot is a two-dimensional passive walker with two pairs of two legs, with one pair located centrally and the other pair located laterally. For both pairs, the two legs are rigidly-connected and move synchronously, as though the robot is bipedal. The purpose of this unusual design is to horizontally balance the robot in the frontal plane. By limiting the design of this "four-legged biped" to two dimensions, the yaw effects experienced by the Passive Toy are eliminated.

Ultimately, I was unable to pursue the Kneed Walker as a model for the actuated Toddler robot. Although I will still explain the design process, I will devote less text to specific analyses like those described in the Passive Toy. While I have not mentioned other failed robot designs, I elected to include the Kneed Walker in my thesis report for two reasons. First of all, the Walker appeared to be on the verge of a dynamically stable gait. Another kneed robot might easily be built using many of the same strategies.

The second reason that I have included this section is that knees are an important design strategy in walking robots. The use of knees is a key component to human walking, and a machine learning control system may eventually want to try controlling knees. While the combination of strategies implemented in the Kneed Walker were not successful in the creating a walking robot, I believe that with some design tweaking, a dynamically-stable solution can be reached.

3.2.1 Design Parameters

The design goal was to create a two-dimensional kneed passive walker that could walk down a shallow incline with a dynamically stable gait in a repeatable, predictable fashion without the aid of motors. Some of the parameters were similar to the Passive Toy.

The first parameter was a hip joint with a single degree of freedom. As in the Passive Toy, relevant sub-parameters included minimizing friction and vibration in the hip. The second parameter was a knee joint with a single degree of freedom to allow ground clearance of the swinging leg. The third design parameter was to create a sagittal plane oscillation to control step size. The fourth parameter was a leg coupling mechanism, necessary for created the bipedal properties described earlier. The fifth and final parameter was a knee locking mechanism to control step frequency.

In the next section I will discuss the implementation of design strategies chosen to satisfy each design parameter.

3.2.2 Design Implementation

As I stated earlier, I was unable to use the Kneed Walker as the model for the actuated robot, and thus I will focus less on design analysis and more on chosen strategies. Below I will briefly outline the chosen strategies used to satisfy the five design parameters.

Hip Joint

I used a high-precision hardened steel shaft for the hip and bar aluminum for the legs. The set of outer legs was press fit onto the hip shaft, and the set of inner legs was interfaced to the shaft using low-friction stainless steel ball bearings. Bearing blocks, like those used in Passive Toy, were unnecessary. The ball bearings and coupled legs sufficiently eliminated all noticable vibrations from the hip axis.

Knee Joint

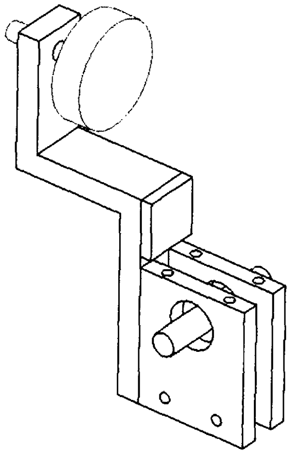

Figure 3-9: CAD model of Kneed Walker's knee with magnetic knee lock.

In order to create single degree of freedom knee joint, we press fit a precision hardened steel dowel pin through a hole at the bottom of the upper leg. We then interfaced this shaft to the lower leg using stainless steel ball bearings. Ball bearings press fit into the lower leg were placed on both sides of the upper leg to eliminate play.

We restricted the lower leg of the robot from rotating past a 180° angle with the upper leg to be consistent with knees in mammalian creatures. We accomplished this by rigidly attaching piece of rubber to the lower leg that would block further rotation upon contacting the upper leg.

Pitch Oscillation

Like in the Passive Toy, the foot curvature in the sagittal plane helped the robot to rock forwards when placed on a shallow incline. In order to insure the identical curvature of all feet, they were clamped together and machined simultaneously. A preliminary set of feet was machined out of aluminum bars, with the intention of machining new set of feet after higher-level analysis in SolidWorks and MATLAB.

Leg Coupling

Each pair of legs we rigidly coupled with an aluminum bar both above and below the knee. Movement of the outer legs was identical in the sagittal plane. Likewise, movement of the two inner legs was identical in the sagittal plane.

Knee Lock

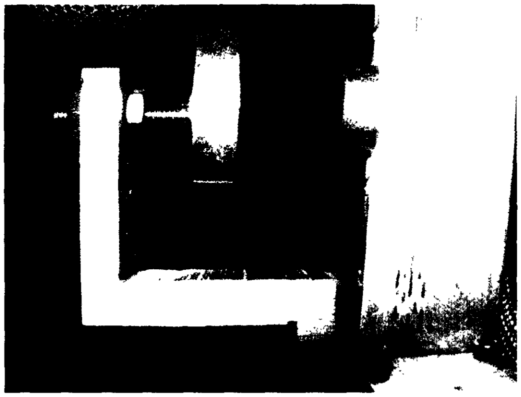

Figure 3-10: Knee locking mechanism.

While the Kneed Walker could exploit the momentum from the forward movement of the lower leg, it required some additional force to help temparily hold it in place until the ground collision. Normally, this done using suction cups. Leakage from the suction cups can be tuned such that the knee lock breaks at the appropriate time. Instead, we decided to use rare-earth magnets to impose this force, speculating that the curved foot ground collision would provide sufficient force to actively break the knee lock for another step be taken. As shown in Figure 3-10, a bolt could be used for quick manual adjustments of the distance between the magnets. These adjustments altered the force between the magnets.

3.2.3 Modeling Analysis

I conducted analyses in SolidWorks and in MATLAB to finalize the design implementations.

Computer Aided Design

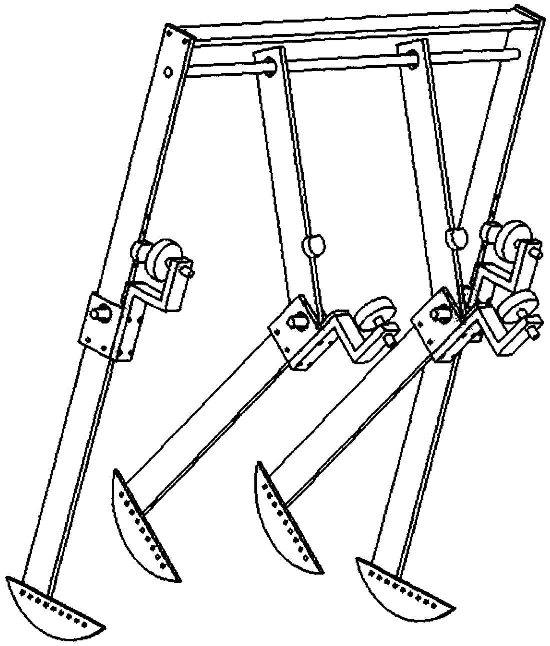

Figure 3-11: CAD model of Kneed Walker.

As in the Passive Toy, I made solid models of the robot to determine center of mass and moment of inertia. I verified these predictions experimentally by weighing parts of the robot and calculating these values by hand. Depending on the distance between the magnets in the knee-locking module, the center of mass could be shifted forward or backward. This function was particularly important when searching for the ideal distance between the magnets. A detailed drawing of the leg is located in Appendix A.

Animated Simulation

Since the robot was horizontally balanced in the frontal plane, I created an animated simulation for the sagittal plane alone. In this simulation, I could adjust inertial and physical properties of all components of the robot. I could also adjust the distance between the magnets to alter the force controlling the knee-locking mechanism.

The simulated performance of the robot changed drastically for alterations in leg length, foot curvature, foot placement, magnet distance, incline angle, and initial positions and velocities of the leg parts. In the simulation, I tested thousands of different variable value combinations. None of these combinations provided a dynamically-stable solution. In its best performances, the robot took two steps before falling. Because I was not able to find a stable solution in simulation, I did not expect to find a stable solution in reality.

3.2.4 Additional Design Notes

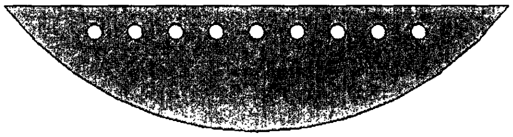

Figure 3-12: Sagittal plane view of foot for Kneed Walker.

Like in the Passive Toy, I made the design of the Kneed Walker modular and gave specific attention to foot adjustments. In addition, I incorporated the knowledge I gained about the benefits of shifting the foot angle so that the robot leans up the ramp. As shown in Figure 3-12, I placed several holes in the foot so that they could be shifted forward or backward.

3.2.5 Performance

The experiments with the final product were consistent with the simulation. I tested the robot several times, altering all the same variables as in the simulation. Unfortunately, I could not quantitatively change the initial conditions of the robot. In its best performance, the kneed walker failed in the middle of its second step. Again, the source of the problem appeared to be a lack of dynamic stability. I also speculated the performance of individual modules could be improved, though the design made it difficult to detect specific flaws.

3.2.6 Conclusions

As predicted by my analyses, the kneed walker did not meet the design objective of creating a two-dimensional kneed passive walker that could walk down a shallow incline with a dynamically stable gait in a repeatable, predictable fashion without the aid of motors. Namely, I was unable to achieve a dynamically stable gait with the Kneed Walker. The failure of the robot was due to the inability of the knees to break at the appropriate time. In the future, a different knee locking mechanism, perhaps using the more standard suction cup design, might introduce full functionality.

For the time being, however, I conclude that the kneed walker is not an appropriate first robot for testing learning. It is has too many degrees of freedom and the dynamics are overly complex. Since the effectiveness of the machine learning control system is unknown, it would be wiser to choose a robot with simpler, more managable dynamics.

Chapter 4

Toddler

I chose the Passive Toy as a model for the actuated, controllable robot. I decided that it would be an ideal model for a robot for testing maching learning because of its simple dynamics and robust performance.

A human is equipped with the necessary muscle and bone structure for bipedal locomotion; however, it requires a controller (the brain) to help it use this machinery for walking. Similarly, the structure of the Passive Toy has been shown to be capable of bipedal locomotion. In order to complete the analogy, the Passive Toy must be equipped with a controller (a computer) that helps it utilize this machinery for walking. In both cases, the control system for determining gait is learned. In the spirit of our robot's parallels to a human learning to walk, we named our robot "Toddler".

Toddler was designed by actuating and controlling the Passive Toy. The control system allowed me to manipulate the robot's gait. Eventually this manipulation will be carried out by machine learning.

4.1 Design Parameters

The design objective was to create a simple robot that could walk down a shallow incline with a dynamically stable gait in a repeatable, predictable fashion; and can walk actively on a flat surface with a dynamically stable gait in a repeatable, controllable way. The design parameters are similar to the Passive Toy, with a few key changes for the active scenario.

The first parameter was a passive hip joint with a single degree of freedom. Like the Passive Toy, sub-parameters included low friction and low vibrations in the joint. The second parameter was a roll oscillation to control step frequency and allow ground clearance. Whereas Passive Toy utilizes potential energy from the manually-applied sagittal tilt angle, I wanted Toddler's roll oscillation be self-initializing, controllable, and measurable. The third parameter was a pitch oscillation to control step size. While the Passive Toy walks down a ramp, Toddler must be able to walk without using gravity from the downhill slope as a source of energy. A sub-parameter of this roll oscillation is that it must be self-initializing, controllable, and measurable. The final parameter was to a mechanism for interfacing between the robot and the computer controller.

With my design parameters well-defined, I performed bench level and computer modeling analyses, drawing upon methods used in the previous robot design processes. In many cases, I was able to replicate strategies used in the Passive Toy. However, in some cases, the actuated robot required drastic changes. In the next few sections, I will describe, through analysis, how the design parameters were realized.

4.2 Bench Level Analysis

In this analysis, I will discuss the chosen design strategies, the constraints associated with each one, and the implementations of each strategy. In some cases, I will discuss simple experiments performed on a specific implementation, and how the results of these experiments altered the design.

4.2.1 Hip Joint

The hip joint design was identical to that of the Passive Toy. It contained a precision hardened steel shaft with aluminum legs and bearing blocks. One leg-block module was press fit onto the shaft, and the other was coupled to the shaft using two ball bearings. For more details, see Section 3.1.2.

4.2.2 Active Roll and Pitch Oscillations

The second and third design parameters will be addressed together. The roll and pitch oscillations for the active case were implemented using an ankle joint with two degrees of freedom. The joint connected the leg to a curved foot. The radii of curvature of the feet were kept identical to the Passive Toy. The height of the legs and feet were reduced such that Toddler's height was equivalent to that of the Passive Toy. Aluminum was chosen as the material for the joint because of its sturdiness and precision machining capabilities.

Each foot was constrained to rotate in the roll and pitch directions. To satisfy the roll rotation, I tested several bushings and ball bearings. I determined that rulon bushing's rotation on the precision steel dowel pin was sufficient for providing low friction rotation without play. The bushings, separated by slightly more than 1 inch, were contained by a bracket that was rigidly connected to the foot.

To satisfy the pitch rotation, I tested the bushings and bearings again. Play was an issue for pitch rotation, but not roll, because the area of contact was the leg surface with minimal. This time, I determined that stainless steel ball bearings were necessary for maintaining low friction and for reducing play. The leg fit into a slot in the ankle joint, and a precision steel dowel pin passed through each layer.

The final test showed that feet could smoothly roll and pitch when the leg was swung gently in both the frontal and sagittal planes.

Active foot movement was controlled by servo motors mounted on the leg. The lower servo controlled foot roll, and was coupled to the foot using thin threaded rod. The threaded rod was attached to the motor on a small plastic wheel, shown in the lower left portion of Figure 4-1. A bench level experiment showed that this was effective for smooth control of the roll rotation without visible losses in energy.

The upper servo controlled the foot pitch, and was coupled to the foot using steel cable on an acrylic wheel. A bench level experiment showed significant energy losses for this design. Therefore, I replaced the coupling with a plastic servo horn, shown the upper right portion of Figure 4-1, and the apparent energy losses were eliminated.

Figure 4-1: Servo motors mounted on Toddler's legs control roll and pitch of feet. Views from inside and outside of leg. Cable ties were used to hold down motor wires.

The motors were wired to an external power supply and controller, via a circuit board mounted on one of the hip joint bearing blocks. Motor wires were fastened to the legs using cable ties.

The wood used in the Passive Toy's feet was a difficult material to machine the finely-threaded taps needed for the motor interfaces. Thus, the wooden feet were replaced by acrylic equivalents. In the Passive Toy, wood had been favored because of its easy machinability and light weight. Acrylic was reasonably easy to machine, and the weight was no longer relevant because Toddler's overall mass had increased significantly.

Sensors were used to gather measurements pertaining to the oscillations. An potentiometer was used to measure the hip rotation during sagittal oscillation, and was mounted on the hip axis, as shown in Figure 4-2. A tilt sensor measured the frontal and sagittal plane oscillations and was mounted on top of a bearing block.

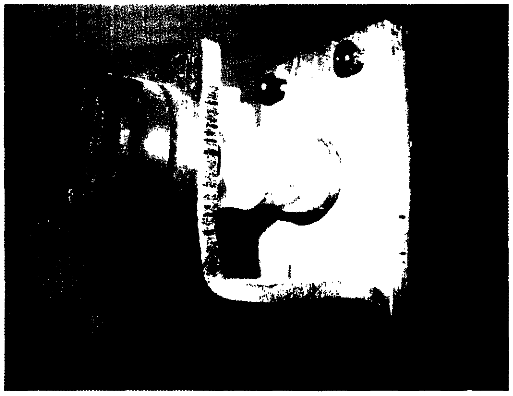

Figure 4-2: Potentiometer mounted on hip joint of Toddler used to measure hip angle.

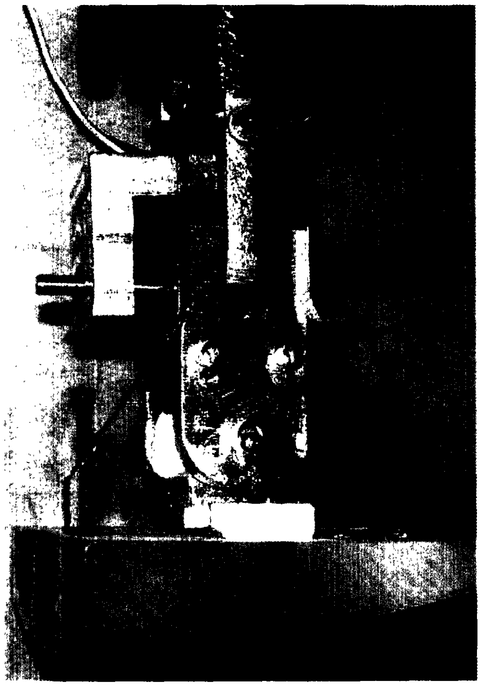

Figure 4-3: Passive Toddler ankle bracket.

4.2.3 Passive Roll and Pitch Oscillation

Passive Roll and Pitch Oscillations were achieved by rigidly binding the leg to the foot, such that its motion resembled that of the Passive Toy. I created a bracket, shown in Figure 4-3 to bolt between the leg and the foot, holding the ankle joint in a perpendicular position. Acrylic was chosen as the material for the bracket due to its sturdy and lightweight properties, such that the effect of the additional mass on the robot's inertial parameters would be negligible.

4.2.4 Robot-Controller Connector

The motors and sensors were wired to an external power source and computer controller. The bundle of wires was consolidated into a single 10-pin connector. The connector could be attached to the external hardware via a multi-wire tether.

The connector was mounted to the back side of a bearing block. To determine the effect of the connection on the passive robot dynamics, I turned off the power supply and bolted on the ankle restraints. I tested the passive case both with and without the tether from the connector. This simple bench level experiment showed a significant yaw about the connection for the tethered case over the untethered case. When the tether was manually held with slack, the moment was reduced, though exact results were inconsistent. The tether's external force on the rigid and asymmetric connector was determined to be the source of the yaw moment. The friction from tether dragging along the ground amplified this moment.

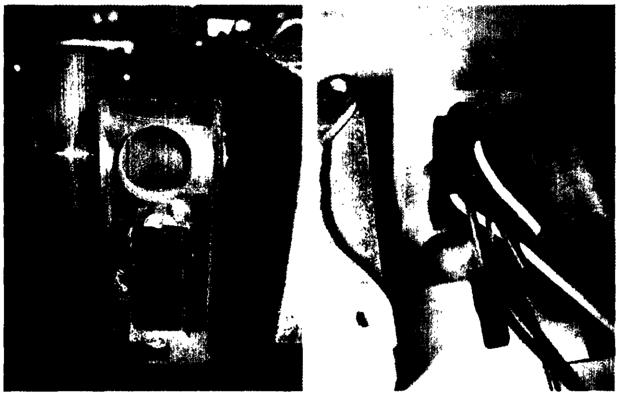

Figure 4-4: Tether connector bracket on Toddler designed to reduce yaw. Connector interfaces robot motors and sensors with power supply and controller. View from above and below shown.

I re-designed the tether connection to eliminate the yaw. I machined an acrylic bracket for the 10-pin connector, shown on Toddler in Figure 4-4. I placed the bracket at the center of the hip shaft to address the asymmetry problem. I coupled the bracket to the shaft using a rulon bushing so that it was free to rotate and move laterally, thus eliminating the rigidness of the previous design. To address the tether drag issue, I installed an apparatus behind the robot, along which the tether could slide. The apparatus was an aluminum rod bolted to the wall such that it protruded in the direction of the robot's walking at the height of the hip shaft. I affixed the tether to a rulon bushing that was free to rotate and slide along the aluminum rod. This design addressed the tether drag issue. I repeated the simple bench level experiment performed earlier and showed that these design changes significantly reduced yaw, although they did not eliminate it entirely.

4.3 Additional Design Notes

In terms of inertial proporties, Toddler differed greatly from the Passive Toy. Because of the added hardware, Toddler was no longer horizontally balanced. When the passive Toddler was set on a flat surface, it leaned more on one leg. I manually balanced the robot by clamping weights to the opposite leg. Later on using computer modeling, I determined the most appropriate places to apply these weights.

4.4 Modeling Analysis

Once again, I used CAD and animated simulations to improve and finalize the robot design. The modifications were based on Toddler's static physical properties and dynamic stability.

4.4.1 Computer Aided Design

The three-dimensional solid model of Toddler is shown in Figure 4-5, and a detailed drawing is available in Appendix B. For all machined parts, I entered the geometry and density of each component to determine its inertial parameters. I obtained mass properties of the sensors and motors from product specification sheets. I modeled these items as point masses at their centers of mass. I measured the mass of the circuit and tether connector, and also modeled this as a point mass. From these values, I generated an estimate for total mass, center of mass, and moment of inertia.

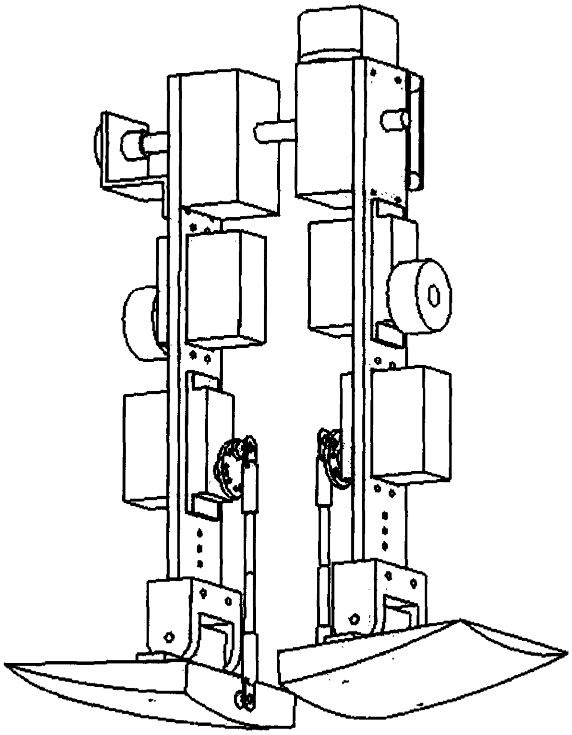

Figure 4-5: CAD model of Toddler

The center of mass of the Toddler in the frontal plane was not horizontally balanced. The imbalance creates inconsistencies in roll oscillation. By adding point masses to one leg of the robot, I was able to balance the robot. On the actual robot, I implemented this mass in the form of a small metal clamp.

In addition, SolidWorks allowed me to analyze how changes in component parameters could affect the physical properties of the robot. For example, I could make alterations in the feet's material and shape, and then view the changes in inertial parameters. Using this tool, I found the combinations of feet and legs that produced the centers of mass and moments of inertia within the range that I found most desirable.

4.4.2 Animated Simulation

The MATLAB simulation of the passive Toddler was identical to Passive Toy's with new input values for mass, center of mass, and moment of inertia. Foot curvature, distance between feet, and leg length remained the same.

Variable initial conditions included frontal tilt angle and angle of incline. I tested robot performance at random initial conditions. Tilt angle was confined to the values allowed by the length of the foot's frontal arc. I found a small range of angles of incline across which Toddler could walk down a ramp for most initial tilt angles.

4.5 Performance

Toddler's gait was tested on the treadmill, varying speed and tilt angle. For both the passive and the active cases, Toddler's performance was good. It was able to maintain a dynamically-stable gait while walking passively down a ramp, and actively on a flat surface. Below I will discuss the passive and active cases in more detail.

4.5.1 Passive Movement

For the passive case, Toddler wore the ankle restraints shown in Figure 4-3. Initially, the passive roll oscillation was tested on a flat surface, with a random initial tilt angle. Similar to the case of the Passive Toy, oscillations lasted for a longer duration when placed on more compliant surfaces such as rubber because of the increased ground contact friction.

Next, Toddler was tested on a slightly inclined treadmill, beginning with a random frontal tilt angle. When untethered, Toddler performed well on both the moving and unmoving treadmill. When tethered, with the power supply off, Toddler experienced a slight but noticable yaw. For the moving treadmill, this yaw caused the robot to fail on approximately 10% of trials. For the unmoving treadmill, the yaw caused the robot to fail on approximately 20% of trials. In both cases, the failures could be reduced to zero with careful initial positioning and low amplitude oscillations. Video footage of the robot's performance is available at our robots website [4].

4.5.2 Active Locomotion

For the active case, Toddler's ankles were controlled by both feed forward and feedback systems. In all cases, the tether was used to power and control the robot. Initially, the active roll oscillation was tested on a flat surface. The motors restrained the ankle's pitch degree of freedom. Toddler was set on a flat surface and an initial roll movement in one foot initialized oscillation. The goal of the control loop was to create a perfect sinusoid oscillation. The amplitude of the sinusoidal needed to be confined to the range of possible rotation for the curved feet. Additional ankle roll movements compensated for ground contact energy losses experienced by the passive robots. The feed forward and feedback systems both achieved steady, though imperfect, sinusoidal oscillations on all flat surfaces.

Next the robot was tested in walking on a flat surface, using motors to control pitch oscillation. Initially, Toddler was allowed to settle into a sinusoidal roll oscillation with fixed zero pitch. Afterwards, the feet were rotated in the sagittal plane, causing the robot to rock forward on one foot. The robot was able to swing its legs and make several steps. If the controller was stopped and restarted, the robot could also walk backwards. In some cases, a yaw moment from the tether manifested itself as the robot veering to one side. While it sometimes affected the path of motion, the robot continued to walk with ease. The robot successfully walked on wood, rubber, plastic, and the static treadmill.

Finally, the robot was tested on a moving treadmill. Toddler was tested first on a flat surface and treadmill speed was adjusted such that the robot remained the same place. Next the treadmill surface was tilted slightly, and with online adjustment of pitch. These manual adjustments enabled the robot to keep up to the speed of the treadmill, and even walk up the inclined plane! For both the feed forward and feedback systems, the robot performed well. Video footage of the robot's performance is available at our robots website [4].

In all cases, the robot experienced some difficulties. None of these difficulties inhibited movement, but they were sources of error in the gait. First, in all cases, Toddler experienced a yaw effect due to the tether. Secondly, Toddler would periodically "jump" due to an erroneous signal sent by the controller. I attribute these controller imperfections to system noise. Overall, however, Toddler's performance was excellent.

4.6 Conclusions

Toddler's performance showed that all design criteria were properly met. The robot was able to walk passively down a shallow incline with a dynamically stable gait in a repeatable, predictable fashion; and walk actively on a flat surface with a dynamically stable gait in a repeatable, controllable way. There were some important lessons to be learned from the errors witnessed during performance. A prime source of error was the tether's connection to the robot. Although an improved connector was designed, it was not sufficient to undo the yaw moment. In the future, a wireless robot would eliminate this problem. The other main source of error was system noise. A well-insulated coaxial tether, or a wireless robot, would partially address this problem. The bulk of the error is likely from noisy sensory input. Noisy input can be addressed using parameter estimation techniques.

Toddler is a simple three-dimensional walking robot that can walk passively with small gravitational input as a source of energy. It also contains two controllable degrees of freedom. With the design process completed for Toddler, we have developed a platform for testing machine learning on walking robotics.

Chapter 5

Conclusion

Through the course of this project, I learned a great deal about strategies for designing walking robots. I will close my thesis report by summarizing my accomplishments and discussing the appropriate next steps in this project.

5.1 Accomplishments

In this section I will discuss the progression of robots I built and how the final robot satisfied my thesis objective.

5.1.1 Robots Built

While trying to satisfy my thesis objective, I built three robots. Each robot had fairly similar design parameters, but utilized different strategies in achieving them. Below I will summarize the chosen strategies and the legacy left by each robot.

Passive Toy

The Passive Toy is a simple, three-dimensional biped. It contains one passive degree of freedom at the hip. The Passive Toy does not have any motors–instead it utilizes gravity and curved feet to maintain roll and pitch oscillations that control step size and frequency. The Toy is able to walk down a 5ft-long shallow incline. This is consistent with two-dimensional animated simulations in the frontal and sagittal planes. The Toy provides a simple, robust model for passive dynamic walking, and was thus selected as the model for the first Toddler robot.

Kneed Walker

The Kneed Walker is a two-dimensional, four-legged biped with knees. It contains three passive degrees of freedom–one at the hip and one at each knee pair–and no motors. The Kneed Walker utilizes curved feet to achieve pitch oscillation necessary to control step size. The Walker also utilizes rare earth magnets in a knee locking mechanism to control pitch frequency. In both simulation and in reality, it can take no more than 1.5 steps down a shallow incline.

Although the Kneed Walker was not an appropriate choice for the first Toddler robot, I hope that knees are explored as an option for future versions of Toddler. Knees provide an elegant and anthromorphical solution to the ground clearance problem, and the extra degrees of freedom may reveal limitations on different types of control systems. I suggest that a future Kneed Walker use the more standard suction cup method for knee locking. I have included the simulation in Appendix C for reference.

Toddler

Toddler is a simple, three-dimensional biped. It contains one passive degree of freedom at the hip, and two active degrees of freedom at the ankles. Toddler can walk passively down an shallow incline, or actively on a flat or uphill surface. The two active degrees of freedom at the ankles can be controlled by feed forward and feedback systems. In the future, a machine learning controller can be implemented. Toddler provides a good base for testing machine learning because it is a robot that can almost walk on its own. As stated in Section 2.2, the differences between the passive case and the controlled learning case should reveal whether machine learning is an effective method for the walking robotics problem.

5.1.2 Fulfillment of Thesis Objective

My thesis objective was to design and build a robot that is ideal for exploring the machine learning hypothesis. Having designed and built Toddler, I believe that I have satisfied this goal.

5.2 Extensions of Work

5.2.1 Machine Learning

The next logical step in this project was to test machine learning on Toddler. My thesis mentor has implemented a machine learning-based controller on Toddler and tested it on various terrains. His results show that learning provides a more adaptive control system while non-learning systems require more constant attention and parameter tweaking [5].

5.2.2 Past and Current Work

More recent version of the robot have been created since I completed the first actuated Toddler. The first important set of design alterations was making the robot wireless and internalizing the hardware, eliminating the need for the problematic tether and enabling more flexibility in terrain testing. To support this infrastructure, the robot was enlarged. Additionally, arms were added to the robot to help provide balance and counter the unwanted yaw.

In order to address the noisy sensor issues, I have developed a parameter estimation script to help filter out noise from signals. The script is based on least squares estimation and I have performed preliminary tests to affirm that it is effectively both offline and online. The next step will be to incorporate this parameter estimation script into the controller.

Other current projects include putting knees on Toddler and developing a comprehensive three-dimensional model of the dynamics.

5.3 The Future of Toddler

The success of machine learning on the first version of Toddler shows promise for future walking robots with learning-based controllers. The next step is to test the effectiveness of the machine learning control system on other walking robots. As for Toddler, it is continuing to grow and change. As the dynamics become increasingly complex and energy requirements increase, it will be interesting to see how the controller will adapt to these changes. Today, Toddler continues to make strides in the field of walking robotics.

Appendix A

Solid Model Drawings

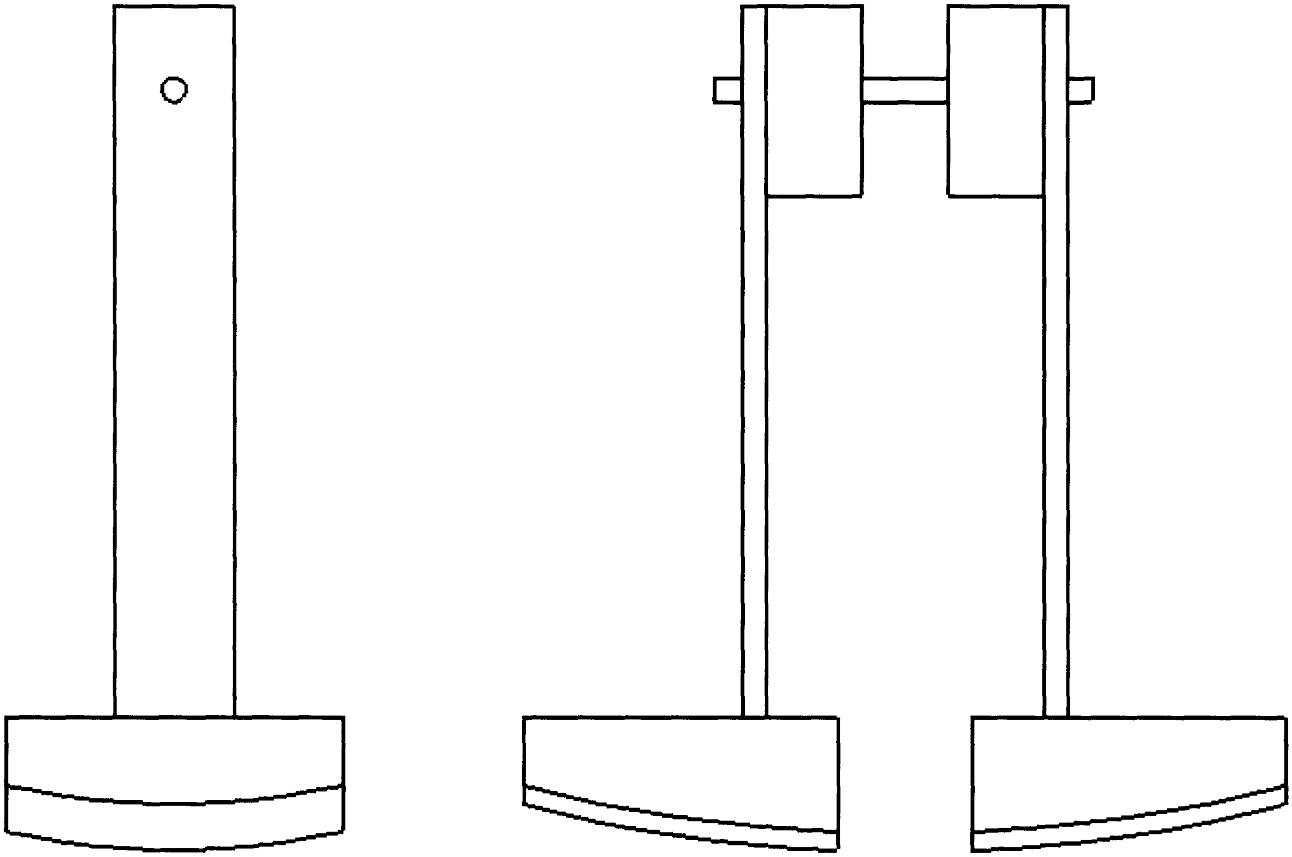

Figure A-1: Passive Toy Frontal and Sagittal Views

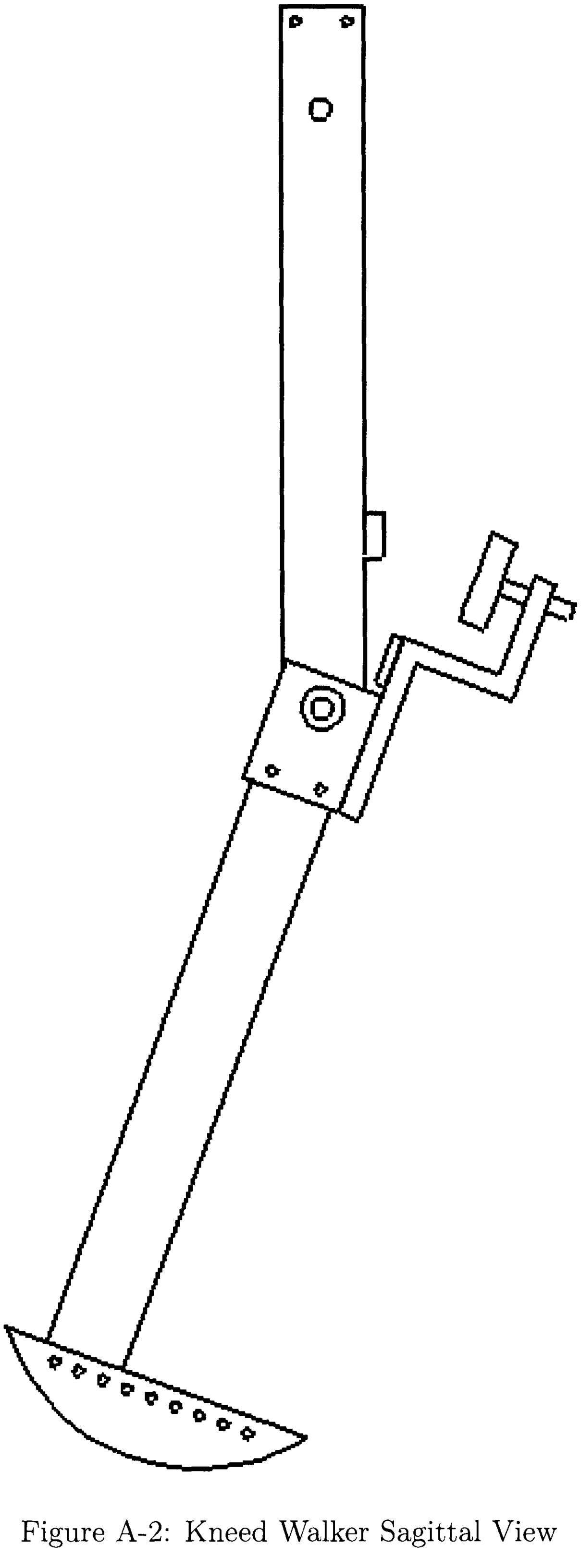

Figure A-2: Kneed Walker Sagittal View

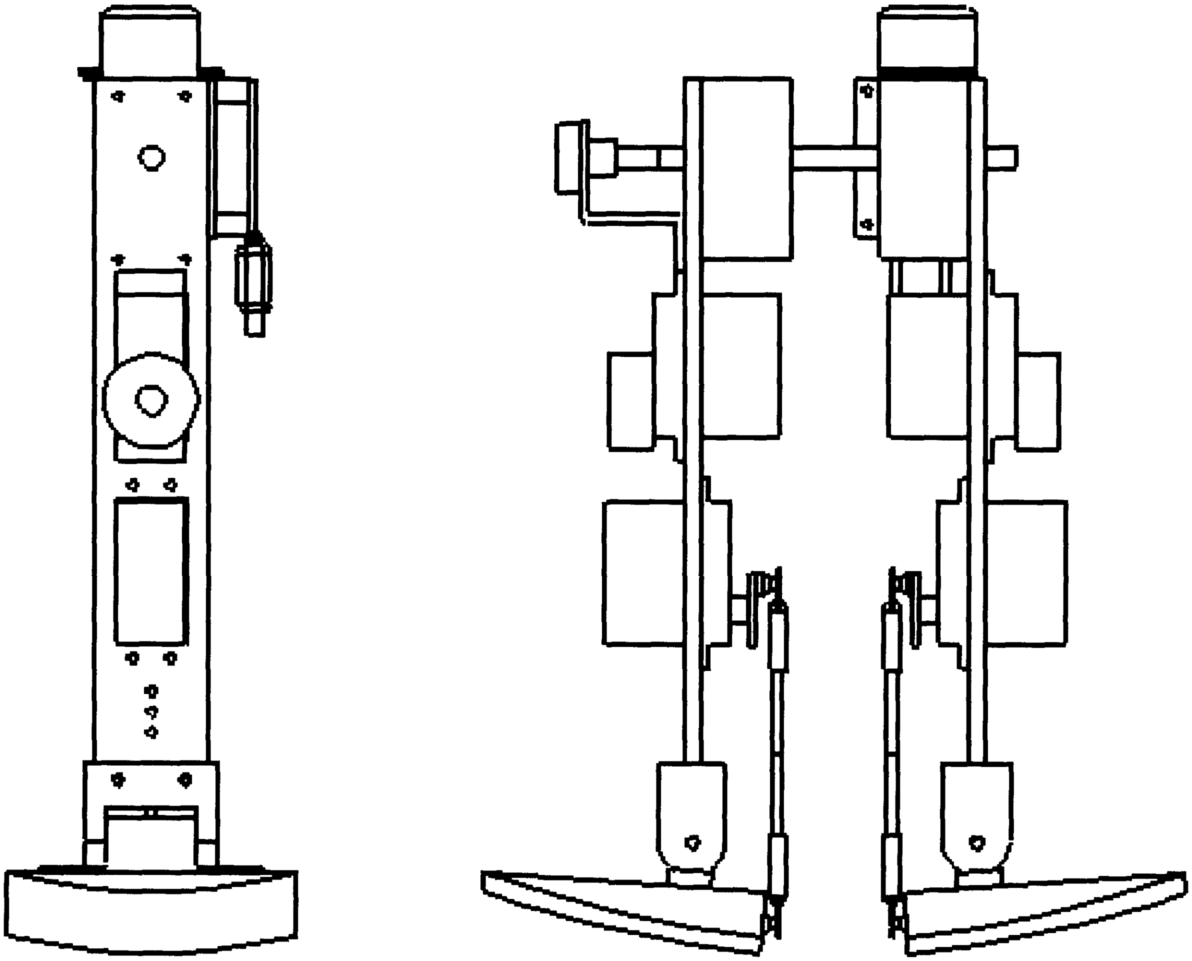

Figure A-3: Toddler Frontal and Sagittal Views

Appendix B

Passive Toy Dynamics

Robot dynamics were calculated and previously published [6]. They are reprinted below.

B.1 Frontal Plane Dynamics

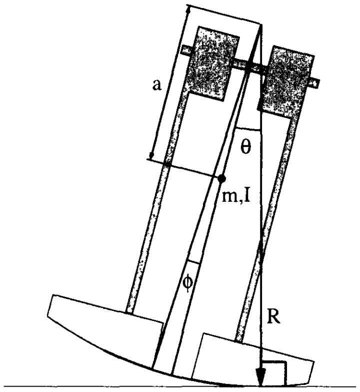

Figure B-1: Frontal Plane Model

To model the dynamics in the frontal plane, we assume that the robot is always in contact with the ground at exactly one point and that the foot rolls without slipping. The equations of motion, in terms of body angle θ, for this planar model are given in three parts using the form

$$H(\theta)\ddot{\theta} + C(\theta, \dot{\theta})\dot{\theta} + G(\theta) = 0.$$

When $|\theta| > \phi$, the ground contact point is in the curved portion one of the feet (the boundary condition on the outside of the foot is not modeled), and the dynamics are:

$$\begin{aligned} H(\theta) &= I + ma^2 + mR_f^2 - 2mR_f a \cos\theta, \\ C(\theta, \dot{\theta}) &= mR_f a\dot{\theta}\sin\theta, \\ G(\theta) &= mga\sin\theta. \end{aligned}$$

When $|\theta| \leq \phi$, the ground contact is along the inside edge of the foot. In this case, the dynamics are:

$$\begin{aligned} H(\theta) &= I + ma^2 + mR_f^2 - 2mR_f a \cos(\theta - \alpha), \\ C(\theta, \dot{\theta}) &= 0, \\ G(\theta) &= mg(a\sin\theta - R_f\sin\alpha). \end{aligned}$$

where $\alpha = \theta - \phi$ if $\theta > 0$, otherwise $\alpha = \theta + \phi$.

Finally, the collision of the swing leg with the ground is modeled as an inelastic (angular momentum conserving) impulse,

$$\dot{\theta}^{+} = \dot{\theta}^{-} \cos\left[2\tan^{-1}\left(\frac{R_f \sin\phi}{R_f\cos\phi - a}\right)\right],$$

which occurs when $\theta = 0$.

B.2 Sagittal Plane Model

In the sagittal plane, the dynamics are a slightly modified version of the well-studied compass gait. The parameters of this model can be found in Figure B-2, and the dynamics are given by:

$$\mathbf{H}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{G}(\mathbf{q}) = \mathbf{0},$$

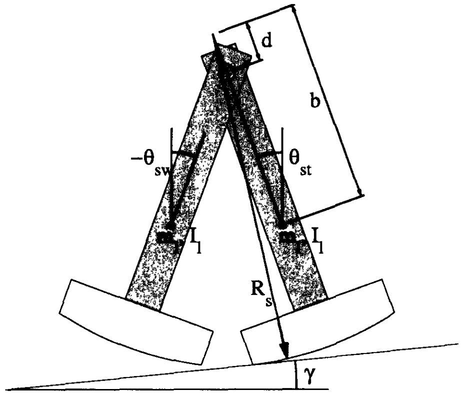

Figure B-2: Sagittal Plane Model

where $\mathbf{q} = [\theta_{st}, \theta_{sw}]^T$, and:

$$\begin{aligned}
H_{11} =& I_l + m_l b^2 + m_l d^2 + 2m_l R_s^2 - 2m_l R_s (b+d)\cos(\theta_{st}-\gamma)\\
H_{12} =& H_{21} = m_l(b-d)[d\cos(\theta_{st}-\theta_{sw}) - R_s\cos(\theta_{sw}-\gamma)]\\
H_{22} =& I_l + m_l(b-d)^2,\\
C_{11} =& m_l R_s(b+d)\sin(\theta_{st}-\gamma)\dot{\theta}_{st} + \frac{1}{2}m_l a(b-d)\sin(\theta_{st}-\theta_{sw})\dot{\theta}_{sw}\\
C_{12} =& m_l(b-d)[d\sin(\theta_{st}-\theta_{sw})(\dot{\theta}_{sw} - \frac{1}{2}\dot{\theta}_{st}) + R_s\sin(\theta_{sw}-\gamma)\dot{\theta}_{sw}]\\
C_{21} =& m_l(b-d)[d\sin(\theta_{st}-\theta_{sw})(\dot{\theta}_{st} - \frac{1}{2}\dot{\theta}_{sw}) - \frac{1}{2}R_s\sin(\theta_{sw}-\gamma)\dot{\theta}_{sw}]\\
C_{22} =& \frac{1}{2}m_l(b-d)[d\sin(\theta_{st}-\gamma) + R_s\sin(\theta_{sw}-\gamma)]\dot{\theta}_{st}\\
G_1 =& m_l g(b+d)\sin\theta_{st} - 2m_l g R_s\sin\gamma,\\
G_2 =& m_l g(b-d)\sin\theta_{sw}.
\end{aligned}$$

The abbreviation *st* is shorthand for the stance leg and *sw* for the swing leg.

Using the output of the frontal plane model, we estimate that a collision occurs once every half period of the roll oscillations. At this moment, the collision is again modeled as an angular momentum conserving impulse:

$$\mathbf{\Omega}^+(\mathbf{q})\dot{\mathbf{q}}^+ = \mathbf{\Omega}^-(\mathbf{q})\dot{\mathbf{q}}^-,$$

where

$$
\begin{aligned}
\Omega_{11}^{-} =& 2bd\cos(\theta_{sw}-\theta_{st}) - (b+d)R_s\cos(\theta_{sw}-\gamma) \\
& - 2bR_s\cos(\theta_{st}-\gamma) + 2R_s^2 + b^2 - bd \\
\Omega_{12}^{-} =& \Omega_{21}^{-} = (b-d)(b - R_s\cos(\theta_{sw}-\gamma)) \\
\Omega_{22}^{-} =& 0 \\
\Omega_{11}^{+} =& (b-d)[d\cos(\theta_{st}-\theta_{sw}) - R_s\cos(\theta_{st}-\gamma) + (b-d)] \\
\Omega_{12}^{+} =& - R_s(b-d)\cos(\theta_{st}-\gamma) - R_s(b+2d)\cos(\theta_{sw}-\gamma) \\
& + d^2 + 2R_s^2 + R_s b\cos(\theta_{sw}+\gamma) - b^2\cos(2\theta_{sw}) \\
& + d(b-d)\cos(\theta_{st}-\theta_{sw}) \\
\Omega_{21}^{+} =& (b-d)^2 \\
\Omega_{22}^{+} =& (b-d)(d\cos(\theta_{st}-\theta_{sw}) - R_s\cos(\theta_{st}-\gamma)
\end{aligned}
$$

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