Valuing Project Financing Using Monte Carlo Simulation, With Application to the Expansion of a Chemical Company

by

Meghan Gibbons

Submitted to the Department of Mechanical Engineering
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ABSTRACT

The purpose of this thesis is to analyze the specific project value of a chemical company that undertook a major expansion in the late 1990s. The actual details of the investment were unique and conditional on the level of cumulative internal rate of return for the project. This thesis evaluates the project value and the terms of the project financing using Monte Carlo simulation, which is able to take into account the path-dependent nature of the cash flow model. The results of this research are of interest to investors facing similar financing terms or similar investment opportunities in the future.

Acknowledgements

I would like to extend a special thanks to Professor Stewart Myers, who provided guidance and instruction throughout this process. I truly value all of his support and patience. I am also extremely grateful to The Brattle Group – especially Lynda Borucki, who allowed me access to critical background information and also spent time going through extensive documentation with me.

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1. Introduction

This thesis investigates the unique project financing of a specific manufacturing project using Monte Carlo simulation. Due to the path-dependent nature of this particular investment, and the complication of many critical underlying variables, it was necessary to use simulation in order to generate a probability distribution of possible outcomes. Through the careful inspection of company documents, a model was set up to represent the possible cash flows over a given time horizon. This thesis details the setup of the model and further explains the incorporation of the model into the Monte Carlo procedure. Finally, results of repeated simulations facilitate analysis of the degree to which certain unique terms of the financial agreement affect value to outside investors.

Moreover, this thesis emphasizes the complications that arise as a result of a path-dependent project. The investment opportunity is described and analyzed in terms of its path-dependent nature. A clear need is established for a more sophisticated valuation method in order to accurately judge the investment opportunity, and Monte Carlo simulation is presented as a very viable solution. Furthermore, simulations facilitated the valuation and analysis of this investment by allowing the alteration of many key variables. This type of sensitivity analysis would be of particular interest to investors who are faced with the challenge of trying to evaluate the potential profit or loss of a similar type of investment.

2. The Investment Opportunity

2.1 Use of Documents

Documents and contracts detailing the project were fortunately available and easily accessible during the writing of this thesis. However, these documents obviously contain a large amount of private information relevant to the company and investors who undertook this project. In order to respect the privacy rights of their agreement, this thesis relies on an altered version of most quantities mentioned in the actual signed documents. Overall, the focus of this thesis is not to use exact numbers but to demonstrate a methodology for valuation.

2.2 Overview of Project Details

In the late 1990s, a large chemical company was interested in financing a major expansion of their chemical plant. The company had to consider several key aspects regarding the potential expansion. Beyond the costs of completing the project itself, the expanded plant would require additional fixed costs, which would include energy costs as well as costs of inputs necessary in the production of standard chemicals. Another important issue was the degree to which the expansion would increase overall production efficiency. The company also had to bear in mind taxes and the possible benefit of tax shields from incurred debt. At the same time, the company lacked a comprehensive model to help value the investment.

One fundamental issue was the type and source of funding. The company decided that the expansion would be financed with $400 million of bank debt as well at $100 million in equity. Of the total equity, the company itself contributed $50 million, and outside investors contributed the remaining $50 million. The debt was incurred in the initial year and paid off annually through the tenth year. Interest payments were a fixed percentage of the remaining principal each year. This information provides the necessary details to understand the underlying financial structure of the project. Moreover, it is important to take this financial structure into account when making assumptions in the cash flow model.

Another fundamental issue was the determination of how the outside investors would receive value for investing in the project. In this situation, the contract specified that the net cash flows from the project would be divided equally between the outside investors and the company. This condition would hold until the outside investors hit a cumulative target internal rate of return of 16%. Once this target IRR was reached or surpassed once, the investors would no longer receive any cash flows from that point on, thus terminating their investment in the project. The company would receive all subsequent cash flows. The sequence of events was especially important in this situation, as the outside investors would still receive the cash flow that put them up to or beyond their target IRR; they were just restricted from obtaining any subsequent amount after that point.

This rule demonstrates the challenging issue of path-dependence. Outside investors were critically dependent on their cumulative IRR, since they would only receive future cash flows if the target IRR rule were satisfied. For example, with a $50 million investment and 16% target, the IRR rule would begin with the conditional statement: If $-50(1.16) + C_1 \geq 0$, then stop, and otherwise continue. The next level of branching would reference "$-50(1.16) + C_1$" as X_1, such that the new condition would be: If $X_1(1.16) + C_2 \geq$

0, then stop, and otherwise continue. The future cash flow stream is thus directly dependent on all previous cash flows. The decision whether or not to continue in the project must be evaluated at every node in the tree. The specific challenge is that the exact stream of cash flows must be stored and used to evaluate the cumulative IRR with each new move in the tree. This path-dependent nature presents a critical problem that must be overcome to determine the NPV obtained by outside investors.

Several key calculations were performed in order to determine the profitability of the investment. The project model in this case focuses on the net present value of the project as a whole, as well as the net present value of the outside investors. Obviously the outside investors prefer a project that will give them the largest possible net present value. This thesis looks at several main input values that affect the net present value to outside investors under the financial terms of this project. These quantities include standard deviation, target IRR, fixed costs, as well as the percentage of project cash flows going to the outside investors. Simulations were rerun with each of these quantities varied and comparisons were then made to an initial base case. This repeated procedure revealed the impact on the value obtained by outside investors due to changes in any of the above inputs.

Overall, the terms of the original agreement specified many key variables that had to be incorporated into the project model. The type of funding and the specific terms critically established the basic financial structure of the project. This information facilitates realistic calculations and analysis regarding how project financing affects the value obtained by outside investors.

3. Application of Monte Carlo Simulation

Monte Carlo simulation is a useful method in cases in which varying cash flows extend over many years into the future. Project financing often involves variables and inputs that are not well understood, thus making a valuation very tedious and difficult. A detailed model of the project facilitates a more comprehensive evaluation of the investment opportunity. Monte Carlo simulation can use such a detailed project model to generate a probability distribution for the overall value of a path-dependent project that extends over many years.

Path-dependence is difficult to value because the stream of future cash flows relies directly on knowledge of all prior cash flows. Certain criteria are evaluated with each forthcoming cash flow, and this evaluation directly involves the series of previous cash flows. At each point in time, the criteria determine whether or not the series is continued based on the current cash flow, as well as the previous stream of cash flows. Thus, a critical aspect of the expansion project is that it establishes a path-dependent investment opportunity. As specified above, the investors are critically dependent on their cumulative internal rate of return, earned from year zero onward. The path-dependent nature is a crucial issue that must be addressed in order to determine the net present value obtained by outside investors. Since the rule, establishing when the investors are "out," is predefined as a certain cumulative IRR, the Monte Carlo method can be used to simulate and value the path-dependent nature of this investment.

In general, the use of Monte Carlo simulation provides the means to avoid difficulties that would arise in a complicated valuation, such as a path-dependent project.

Indeed, the specification of a particular cash flow stream over the next 30 years would allow a straightforward calculation of present value. However, in cases that involve multiple variables or extend over many years, it is often a difficult and time-consuming process to come up with one project value. By using Monte Carlo simulations, it is possible to establish multiple variables and forecast calculations that extend far into the future. Rather than guessing specific cash flows and calculating one possibly inaccurate value, computer simulations take samples for each variable based on specified probability distributions and subsequently come up with probability distributions for quantities chosen to be forecasted. These forecasted values, for example, can be the present value of the project overall or even the cash flow from a certain year during the life of the project.

Monte Carlo simulation involves a number of distinct steps. It is necessary to start with a model of the project that establishes a structure of possible cash flows over a certain time. For example, this type of cash flow model can be conveniently represented as an expanding binomial tree. At each node in the tree, probabilities are specified for the next two branches that indicate a movement upward or downward in the cash flow tree. Once the model and probabilities are set up, simulations can be run in which the computer generates possible cash flows of the project in each year based on the probability distributions specified for the unknown quantities. Many iterations of the simulation result in probability distributions of cash flows for each year of the project. From these results, further present value calculations and analysis can be performed.

4. Project Model

4.1 Structure and Input Variables

Before running Monte Carlo simulations, the project model was set up to portray the series of cash flows as a typical binomial tree extending from year zero to year 30. Every branching step is the passage of one year, and each cash flow node is a simple estimation of revenue minus fixed costs.

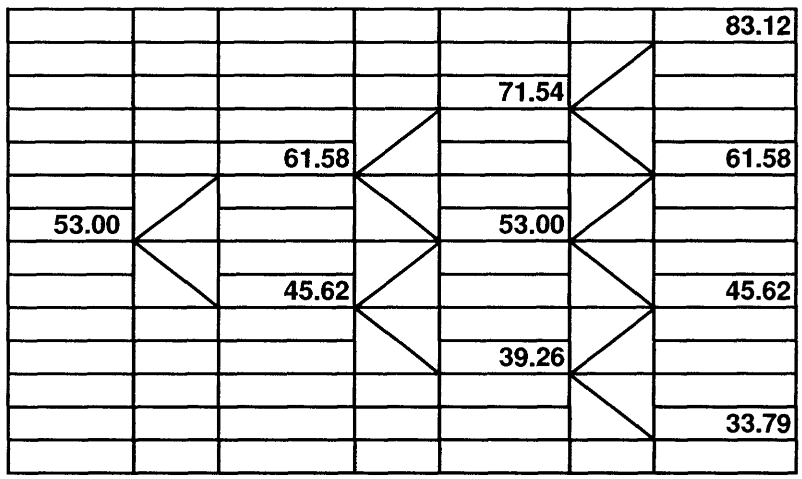


Figure 1: Initial Branching of a Binomial Tree

As a model of the real project, the binomial tree references key input variables that affect project valuation. The risk-free rate was assumed to be 6%, indicating the safe return of a U.S. treasury bond. The standard deviation, representing the degree of uncertainty of cash flows, was initially set at 15%, somewhat less than the stock market variability of around 20%. In other words, the expansion project was assumed to be relatively safe. The probabilities of upward and downward movements are calculated automatically using the entered standard deviation in order to obtain an expected rate of return that is equivalent to the risk-free rate. This concept is more commonly referred to as the risk-neutral rate of growth over any given year. The binomial tree uses these main input values, along with the initial revenue amount, to calculate cash flows over the 30-year life of the project.

4.2 Base Case Assumptions

In order to start with a simple base case, certain assumptions were made regarding initial terms of the project. The total investment in the project was kept at $500 million. However, the base case assumed that the project was completely equity-financed and included no portion of debt. The equity contribution was thus the total $500 million, with the outside investors and the company each contributing $250 million. Furthermore, fixed costs were initially assumed to be zero, and consequently the binomial tree only relied on the beginning revenue value, estimated at $53 million, which gave a project NPV of $45 million.

In the base case, the outside investors received 50% of the project cash flows, and their initial target IRR was assumed to be 11%. These input values were chosen in order to establish a realistic level from which further comparisons could be made. The overall terms of the agreement remained the same: the outside investors received their percentage of project cash flows until they reached or surpassed their cumulative target IRR, in which case they were out of the investment and all cash flows went to the company.

4.3 How The Model Generates Output

Overall, the simulations are path-dependent due to the fact that they must start at year zero and go forward in time. Again, the path-dependent nature of this problem is what dictates the need for Monte Carlo simulations. As the binomial tree shows, each subsequent cash flow branches off of the previously-calculated entry. In other words, each cell's calculation directly stems from the previous calculated cash flow. Monte Carlo simulations also step forward through the tree in such a path-dependent fashion. Moreover, running thousands of iterations provides a probability distribution of possible cash flows for each year. The model was set up as a binomial tree that started with the spreadsheet shown in *Principles of Corporate Finance* (Brealey and Myers, 2003, p. 626). All necessary calculations, such as the routine for calculating the IRR, were written into the model. Crystal Ball Software, produced by Decisioneering Inc., was used as a macro.

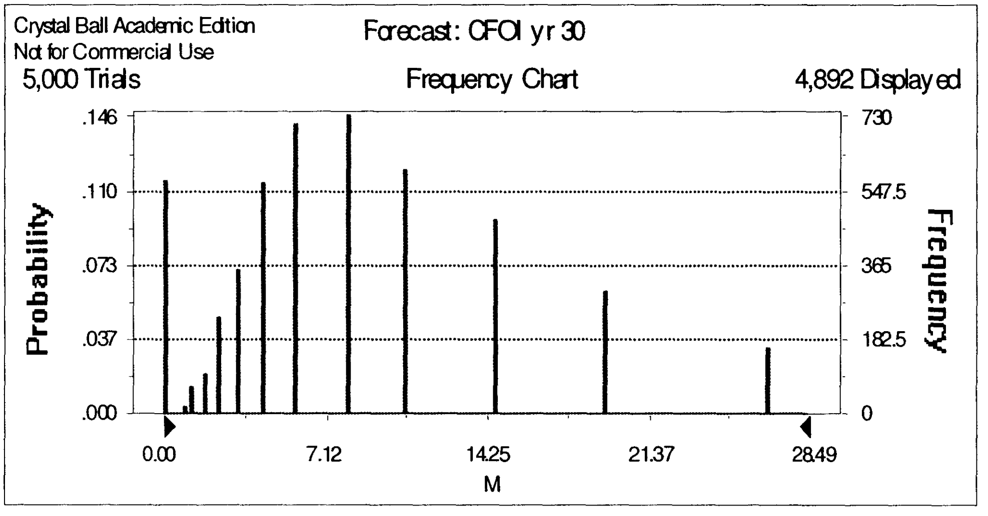
For each iteration, the program runs through the tree one time. A random number is generated in order to specify an upward or downward movement at each consecutive node in the cash flow model. The random numbers are generated from a continuous uniform distribution, ranging from zero to one. This specification allows there to be a clear cut-off level between zero and one that establishes which random numbers constitute an upward-move or a downward-move at any given node. Thus, the generated string of random numbers actually indicates a string of "up" and "down" moves in the binomial tree. The program generates a random number for every year and then stops. Once this series of up and down moves is established, the project's cash flows are calculated for each of 30 years.

5. Simulation Results and Interpretation

5.1 The Base Case

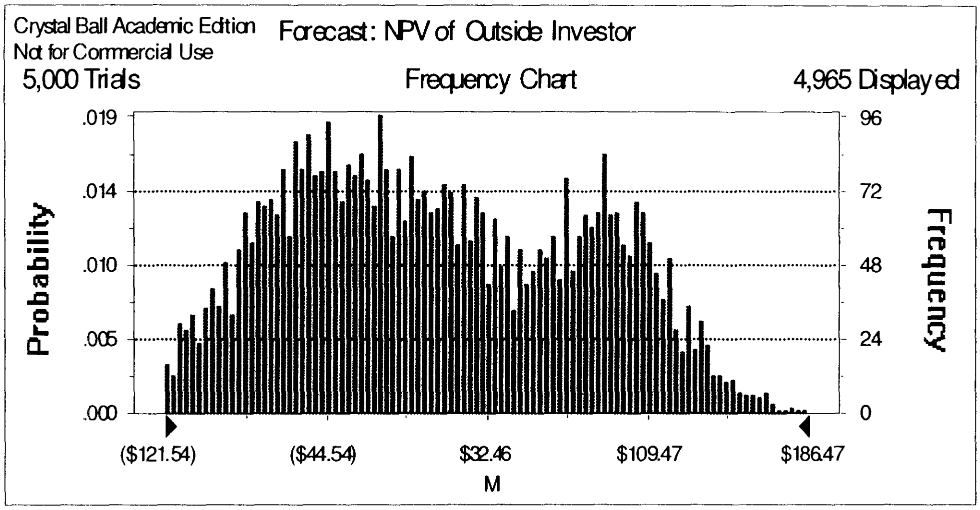
The base case detailed above provides a useful starting point. As shown in Table 1, the key input values and investor terms are stated. Three graphs show the probability distributions of cash flows to outside investors in years five, 15, and 30, respectively. The fourth graph in the table is the NPV to outside investors, and the final graph shows the

NPV of the project. The average value in each graph is also specifically noted, and all subsequent tables follow this same format. In the base case, due to the fact that there are no fixed costs or debt repayments, all cash flows to the outside investors are actually positive. These assumptions, for example, allow outside investors to earn an average cash flow of $8.5 million, even in the 30th year:

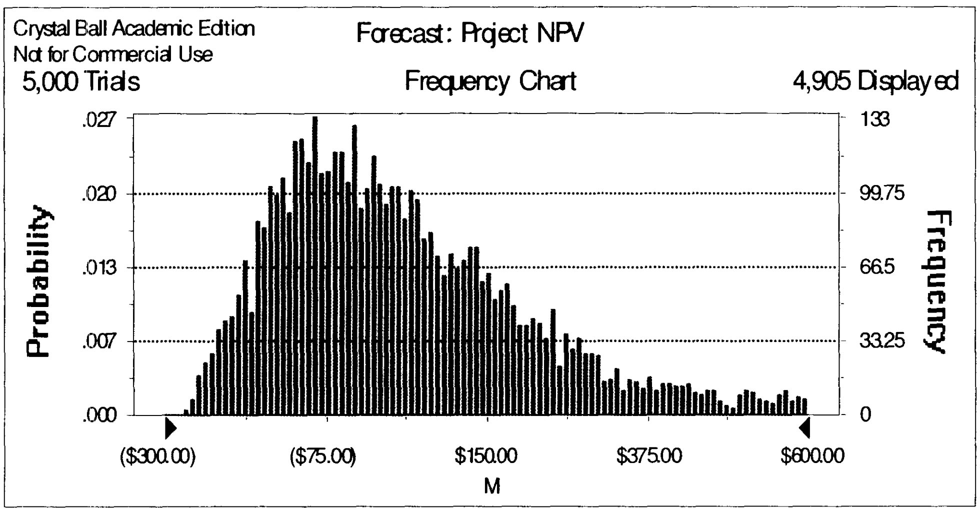


From Table 1: Base Case, Cash Flow to Outside Investors Year 30

However, the varying magnitude over the 30 years of the project causes there to be cases in which investors realize a negative net present value for their investment. As the graphs indicate, the outside investors earned an average net present value of $8.4 million in the base case. The corresponding mean NPV of the project was $45 million. These quantities provide a realistic starting point since the outside investors would earn between a three to four percent return on their assumed investment of $250 million. The graphs of the NPV to outside investors and the project NPV reveal similar shapes, although it is worthwhile to note that the first shows more of a bimodal distribution. Resulting shape changes were studied in more detail as the key input values were varied.



From Table 1: Base Case, NPV to Outside Investors

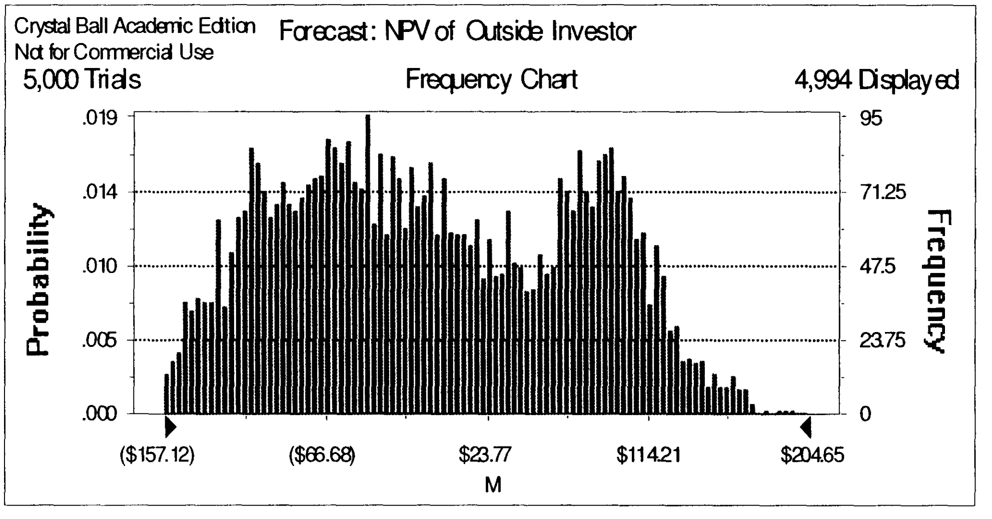


From Table 1: Base Case, NPV of Project

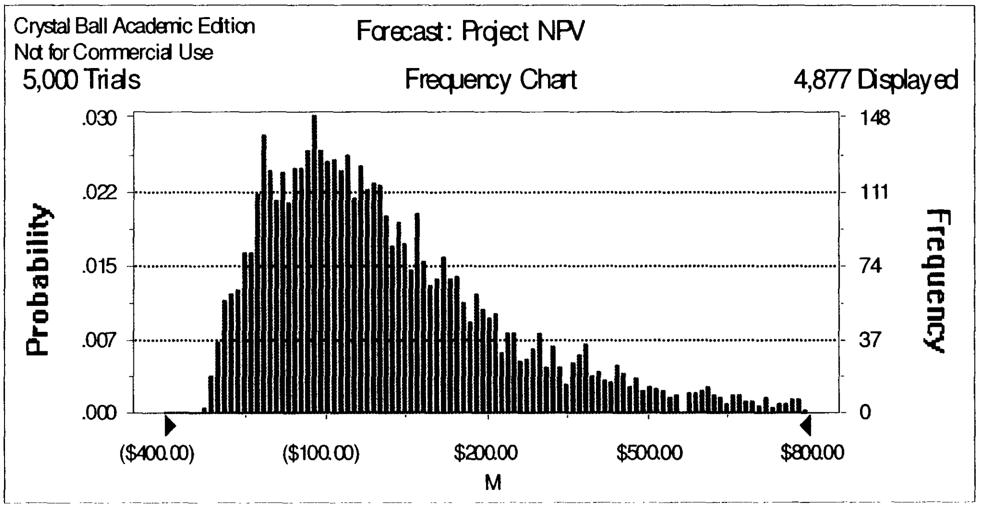
5.2 Varying the Standard Deviation

One of the fundamental input variables in the cash flow model is the standard deviation. This quantity determines the magnitude of each up-move or down-move in the tree. The base case involved a standard deviation of 15%, which is reflected in the variability of cash flows as the binomial tree branches out over the 30 years. However, by changing this quantity, it is possible to gain insight regarding the affect to the net present value of outside investors as well as to the present value of the project as a whole.

The standard deviation was first increased from 15% to 20%. Table 2 includes graphs showing selected years of cash flows received by outside investors. The last two graphs depict the overall net present value obtained by outside investors and the net present value of the whole project, respectively, as reproduced below.



From Table 2: NPV to Outside Investors, St. Dev. 20%



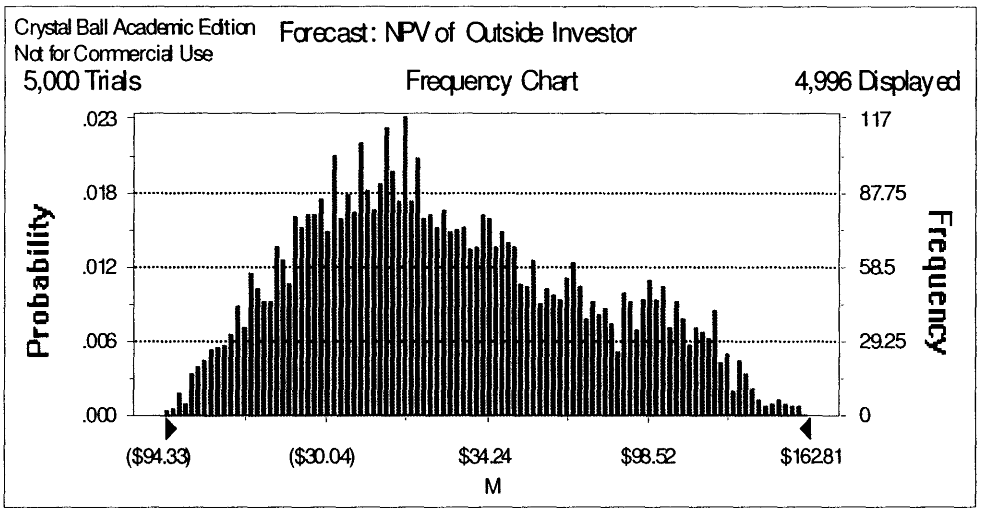
From Table 2: NPV of Project, St. Dev. 20%

Increasing the standard deviation had the effect of decreasing the average net present value of outside investors to -$5.8 million, demonstrating an unprofitable investment. A comparison of probability distributions with the base case revealed that outside investors received around the same average cash flow over the first few years. However, their cash flows over the second half of the project declined relative to the base case. This change contributed to the lower net present value due to the increased standard deviation. Overall, the value to outside investors was affected more than the value of the whole project. The net present value of the project remained relatively close to its value in the base case.

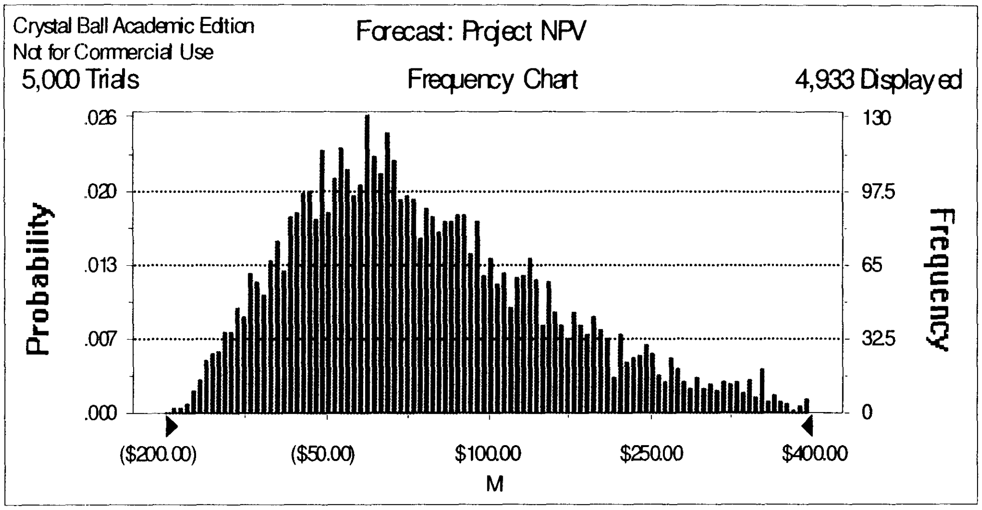
An increase in standard deviation is essentially increasing the variability of the upward and downward movements of cash flows over the 30 years. This effect causes larger variability over a longer time horizon and explains why outside investors saw a larger impact further out in the project life. This variability is further enforced by comparing the ranges of net present values obtained by the outside investors. For example, the base case provided a net present value ranging from -$134 to $186 million. With the increased standard deviation, the range became -$160 to $238 million. The higher standard deviation also resulted in a graph of investors' NPV that revealed a more pronounced bimodal structure. There was also an increased likelihood of being out of the investment at an earlier stage but with a lower NPV. As the simulations will show, opposite results occurred when the standard deviation was lowered.

The standard deviation was subsequently decreased from the base case of 15% to 10%. Decreasing the standard deviation had the effect of increasing the mean net present value obtained by the outside investors to around $19.2 million, representing roughly an

8% return on the invested $250 million. A comparison of probability distributions with the base case revealed that outside investors received, as before, around the same average cash flow over the first few years. The decrease in standard deviation caused cash flows over the second half of the project to increase in comparison with the base case. This change contributed to the higher net present value obtained by outside investors. Moreover, the value to outside investors was once again affected more than the value of the project as a whole. The net present value of the project increased slightly to $45.9 million. Again, a comparison of the following graphs reveals that the shape of outside-investor NPV begins to approach the shape of the project NPV with the lower standard deviation. The first graph also reveals less of a bimodal shape compared to previous version.



From Table 3: NPV to Outside Investor, St. Dev. 10%



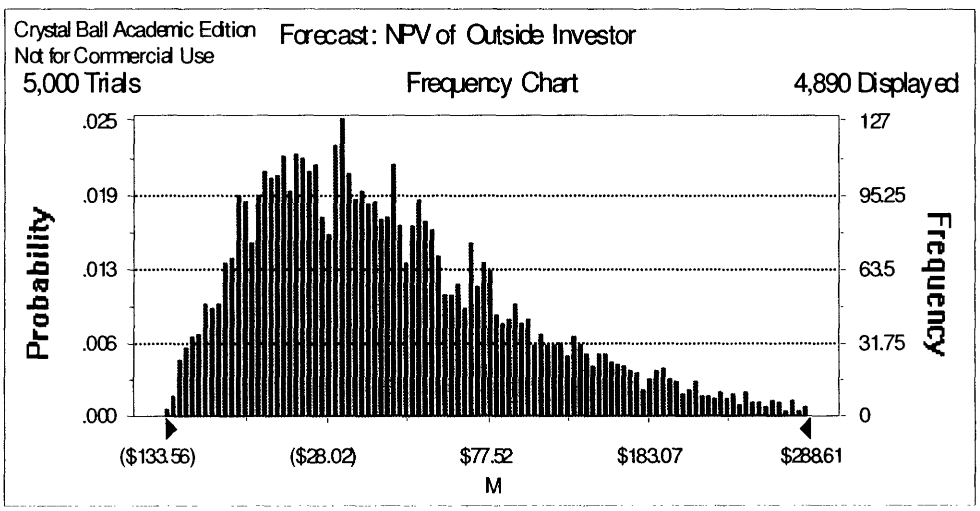
From Table 3: NPV of Project, St. Dev. 10%

In summary, the 10% standard deviation provided the case that was the most beneficial to both the outside investors as well as to the overall project value. The simulations revealed that the value of outside investors was more sensitive to the change in standard deviation than was the project value itself.

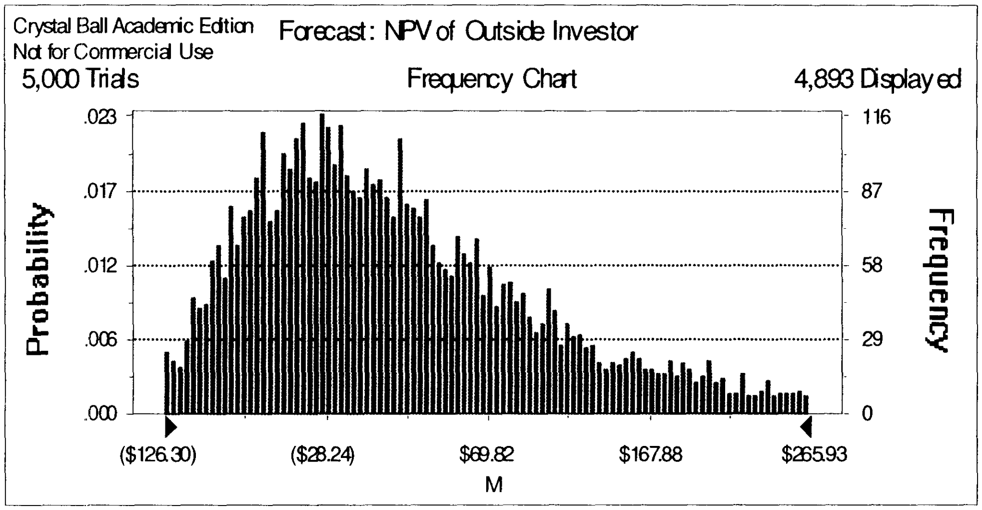
5.3 Varying the Target Rate

One of the key determinants of value to outside investors was the specified cumulative IRR target, which the base case assumes to be 11%. By altering this percentage, it was possible to study the sensitivity the outside investors' value to the target IRR. This rate was especially important in this type of investment because it determined the cut-off point after which investors no longer received any value from the project. The outside investors would want large payoffs without surpassing the limit set by the IRR. The ideal outcome would be an IRR of 10.99% over 30 years. To study the effects of this target rate, the initial 11% was increased and decreased by different amounts, and subsequently Monte Carlo simulations were rerun each time in order to compare the resulting probability distributions.

The target IRR was initially increased to 16% and then to 21%, while keeping the other assumptions of the base case the same. These increases should effectively allow the outside investors to obtain more cash flows before surpassing the target rate. The mean net present value of outside investors did increase to $19.5 and $21.3 million, in the 16% and 21% cases, respectively. Furthermore, the graph displaying the probability distribution of net present values to outside investors reveals that the higher target rate significantly increased the range maximum. The range minimum was roughly the same as it was in the base case. However the range maximum extended to almost $585 and then $680 million, as the IRR was increased each time. The graphs demonstrate that the higher target rate effectively increases the range maximum while only somewhat increasing the mean NPV obtained by investors. As expected, the net present value of the project remained around the same value due to the fact that the new target rates did not affect the overall project cash flows. In both sequential increases, the graphs of outside investors' NPV closely resemble each other as well as the graph of the project NPV. The following two graphs of cash flows to investors correspond to the 21% IRR case, and to the 16% IRR case:



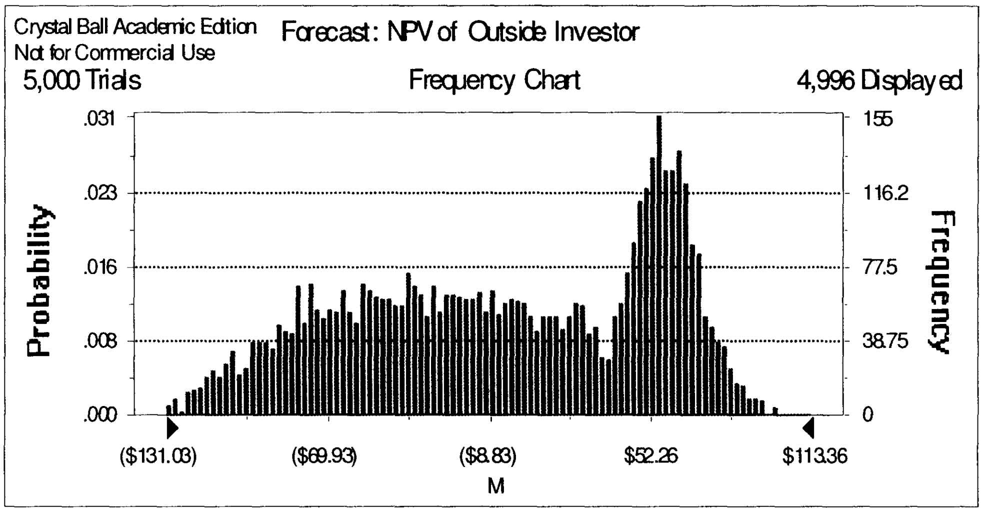
From Table 4: NPV to Outside Investors, 21% IRR



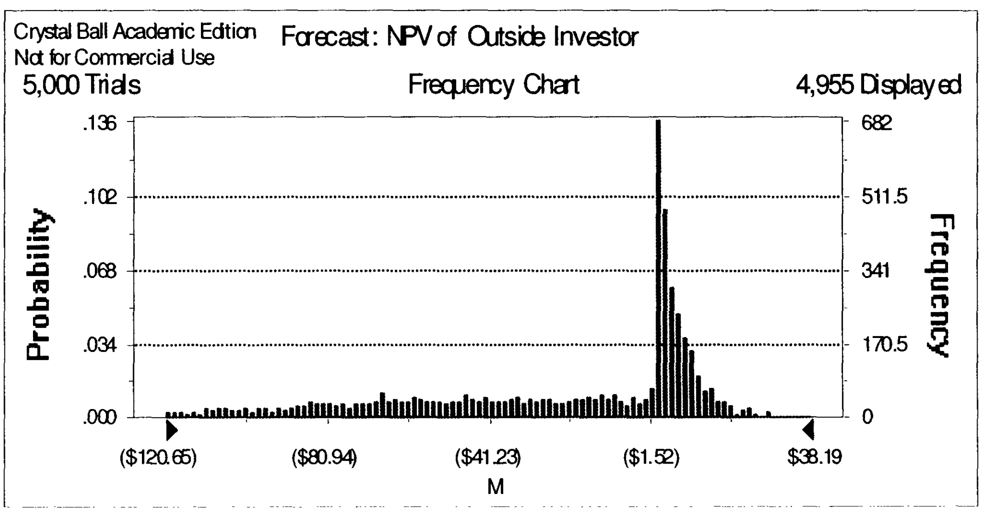
From Table 5: NPV to Outside Investors, 16% IRR

The target rate was subsequently decreased to 9% and to 6%, and results were compared with the previous cases. A lower target should essentially cause investors to reach their ending point sooner in the life of the project. The mean net present value of the outside investors decreased to -$3.2 million and -$23.8 million, respectively. The graphs indicate a larger change from the base case compared to the previous example in which the IRR was increased. Results indicate that the lower rate significantly decreases the range maximum of net present value for investors. The range minimum again remained the same as in the base case, while the range maximum decreased to $38 million with the 6% target rate. Overall, these results reveal that the outside investors are more negatively affected by a lowered target rate than they are positively affected by an increased target rate. Moreover, the mean NPV is shown to increase at a decreasing rate as the target IRR is raised.

The two cases of lowered IRR targets reveal interesting graphs of the NPV to outside investors. As was previously the case when the IRR was increased, the same graphs more closely resemble the project NPV graph. In the cases of lowered IRR, the graphs become flatter except for a spike that develops toward the right of the distribution. In the case when the IRR is equal to the risk-free rate of 6%, the graph becomes extremely flat with a spike at around $7 million. The following two graphs demonstrate the results of lowering the IRR:



From Table 6: NPV of Outside Investors with 9% IRR



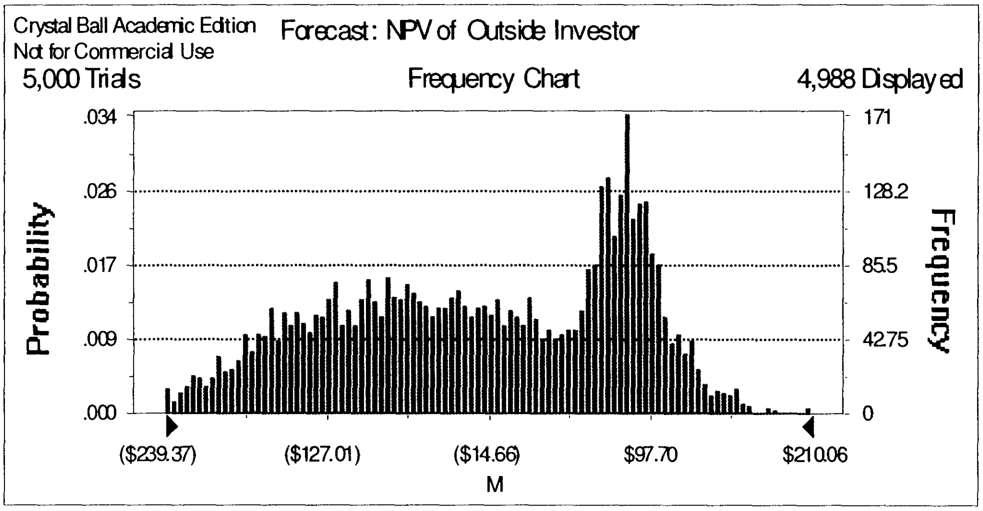
From Table 7: NPV of Outside Investors with 6% IRR

5.4 The Effect of Including Fixed Costs

The addition of fixed costs to the model allows further study of the main inputs affecting investors' value. To start, a fixed amount of $30 million was subtracted from each node in the tree. To facilitate comparison with the base case, the revenue was increased from $53 to $93.195 million. This alteration provided the same base-case NPV calculated by simply extending the amount of revenue minus fixed costs over 30 years and discounting back to year zero.

The graphs in Table 8 show several key differences resulting from the inclusion of fixed costs. The average cash flows to outside investors were larger in the initial years of the project. However, as shown in the 15th and 30th years, the average cash flow decreased significantly. For example, year 30 of the base case revealed an average cash flow to outside investors of $8.5 million. However, when fixed costs were added, this average cash flow sunk to -$0.09 million in the thirtieth year. The addition of fixed costs caused investors' average cash flows to be lower in later years since more nodes in the tree generate negative cash flows later in the project. Thus, larger negative cash flows are possible later in the life of the project as the binomial tree extends its branches over time. Overall the mean net present value obtained by outside investors diminished to -$19.9 million, representing a situation any investor would want to avoid. The mean net present value of the project remained in area of $42 million. This result reinforces the fact that the revenue increase helped offset the increase in fixed costs in terms of overall project value.

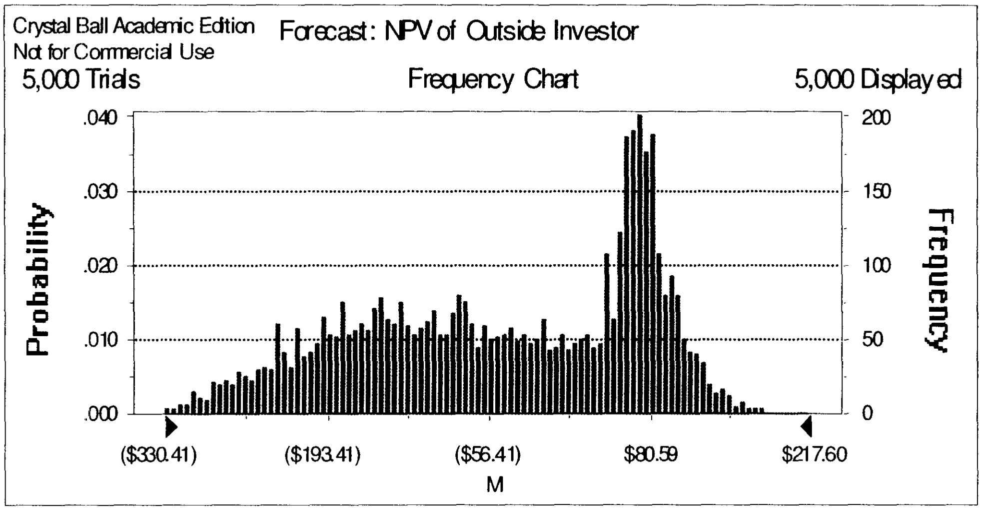
The following graph, taken from Table 8, shows the resulting shape of the NPV to outside investors in the case that included fixed costs of $30 million. Similar to the case in which the IRR was decreased, this graph also reveals a flattened shape with a spike.



From Table 8: NPV to outside investors with fixed costs of $30 million

For further comparison, the simulations were run again using a different fixed cost. Keeping the overall net present value of the project the same as before, the new fixed cost was increased to $50 million and the initial revenue was set at $119.99 million. Since revenue was increased by slightly more than fixed costs were increased, the input may initially imply larger payoffs for the outside investors. However, the simulations demonstrate that the higher revenue and higher fixed costs actually have the opposite effect. In this case, average cash flows to outside investors were even higher in the beginning years of the project. However, especially in the second half of the project life, cash flows declined sharply. In year 30, investors obtained an average of -$5 million. The graph of investors' net present value shows the negative impact resulting from fixed costs of $50 million. The outside investors obtained a mean net present value of -$43.19 million, revealing an even worse investment opportunity than the previous case. The graph also demonstrates a similar shape to the previous case with fixed costs of $30 million.

However, this graph has become slightly flatter with a narrow spike. This graph is also similar in shape to the situation involving a lowered IRR target.



From Table 9: NPV to Outside Investors with fixed costs of $50 million

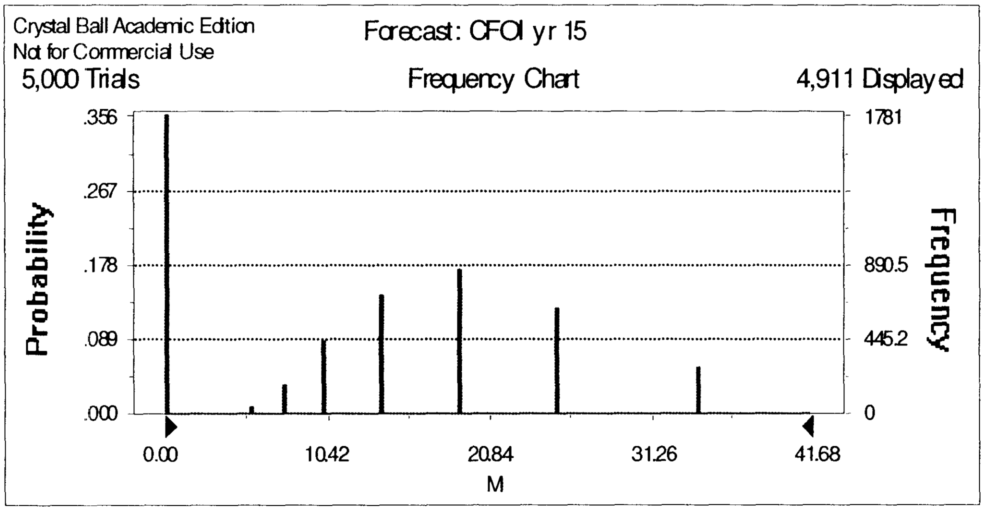
Overall, these two examples involving different levels of fixed costs emphasize several important issues. The value that outside investors expect to gain from their investment is extremely dependent not just on the expected NPV of the project but on the actual level of revenue relative to fixed costs, as well. As simulations demonstrated, the project could be valued at the same net present value but the value obtained by outside investors depended critically on initial estimates of revenue and fixed costs. A project with higher initial revenue may imply a more profitable investment. However, a project with the same NPV but higher estimated revenue and a higher estimate of fixed costs actually negatively impacts value to outside investors. In general, investors may have the tendency to focus on the resulting amount of revenue minus fixed costs. These simulations demonstrate the importance of the actual individual levels of revenue and fixed costs. In the case with fixed costs of $30 million, net income was initially $93.2 - $30 = $63.2.

When fixed costs were $50 million, net income was $119.99 - $50 = $70 million. The inclination may be to place higher value on the situation with the higher net income. On the other hand, the equal estimate of net present value may seem to imply that the two situations would result in similar outcomes for investors. However the graphs demonstrate that the magnitude of fixed costs has a negative impact that outweighs the small increase in initial revenue. Thus, the case in which there is higher revenue and higher fixed costs actually results in a much worse outcome to investors' net present value.

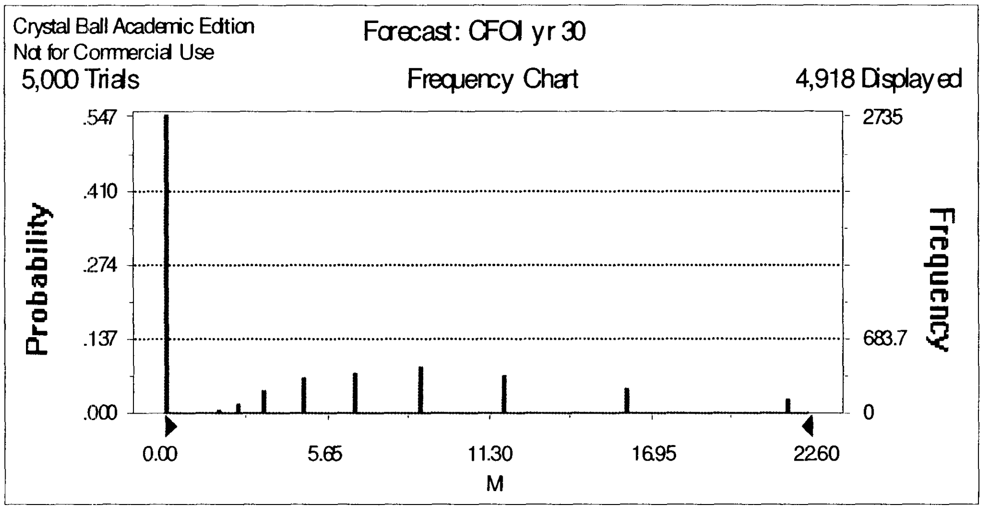
5.5 Changing the Distribution Percentage

An essential part of the contract was the percent of cash flows that outside investors would get each year. Previously kept at the specified 50%, this distribution percentage was altered in order to determine its overall effect on investors' value. This particular input variable provides an interesting situation in which a tradeoff exists concerning receiving larger cash flows earlier but increasing the risk of reaching the target IRR sooner. Likewise, a decreased distribution percentage lowers the chance of reaching the target rate but spreads lower cash flows over a longer stretch of time. Without further examination through simulations, neither case presents itself as the obvious choice.

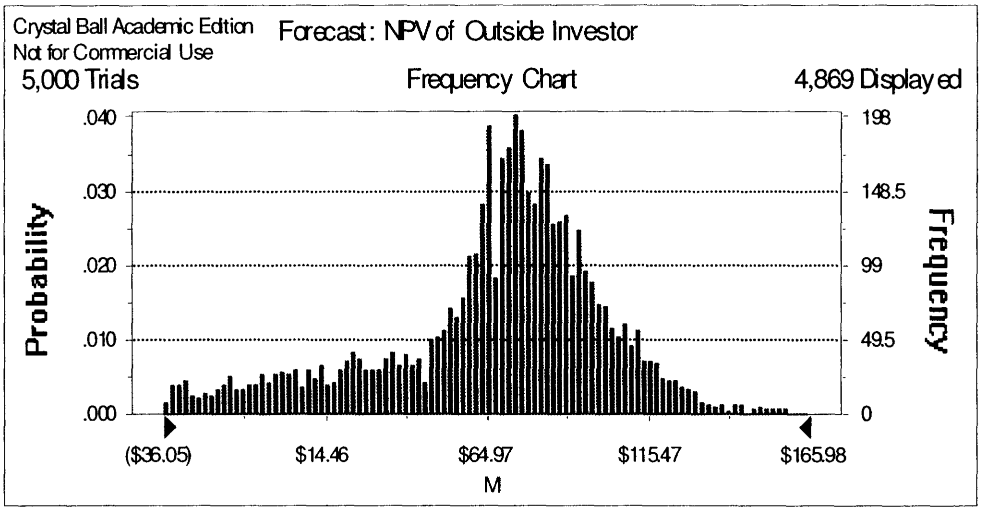
The distribution percentage was initially increased from 50% to 75%. As expected, graphs of the initial years revealed a higher average cash flow to outside investors. Graphs of years 15 and 30 also reveal much higher probabilities of the investors being "out" by that time (Table 10). The resulting mean NPV to outside investors rose to $64 million. Moreover, the shape of the graph reveals a flattened lower tail, with an upsurge in value that is pushed toward the front of the distribution.



From Table 10: Cash Flow to Outside Investors in Year 15, 75% Distribution



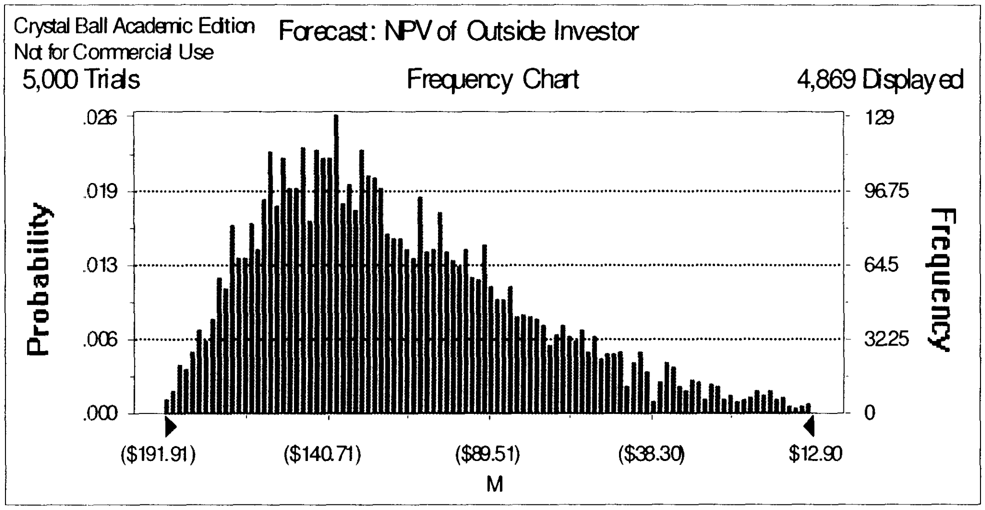
From Table 10: Cash Flow to Outside Investors in Year 30, 75% Distribution



From Table 10: NPV to Outside Investors, 75% Distribution

For further comparison, the distribution percentage was also decreased to 25%. This change presents the case of receiving lower cash flows over a longer period of time. As expected, outside investors received lower average cash flows in the initial years of the project, and had a much lower probability of being "out" of the investment in years 15 and 30. As the graph below indicates, the mean NPV of outside investors decreased to -$114 million. Compared to the previous case, the shape of investor NPV more closely resembles the graph of the project NPV with more of a lognormal distribution.

Regarding the tradeoff between percentage of cash flow and likelihood of surpassing the target IRR, simulations revealed that outside investors would definitely prefer the higher distribution percentage. In this case, they obtained a significantly more profitable investment by receiving larger sums of cash in the initial years, while giving up a longer life of their investment.



From Table 11: NPV to Outside Investors, 25% Distribution

6. Conclusion

6.1 Summary of Results

This thesis looks at the use of Monte Carlo simulation to evaluate project financing by studying a detailed example involving a financing deal for an expanding chemical company. Although a seemingly simple problem, the expansion project actually revealed an investment opportunity that necessitated a more sophisticated method of valuation than was available at the time. Despite the confidentiality of exact numerical values of the project, a carefully-chosen base case was presented to represent the same type of investment opportunity that was part of the original project financing. This thesis specifically focuses on the sensitivity of value obtained by outside investors to several key input values detailed in the terms of the project.

Simulations facilitated the valuation and analysis of this investment by allowing the alteration of many key variables, including the cumulative target IRR as well as to the distribution percentage of cash flows. Key tradeoffs were examined such as the inclusion of fixed costs, accompanied by higher revenue. In this particular example, results demonstrated that larger fixed costs, despite an offsetting increase in revenue, created a much worse investment opportunity for outside investors. Another key tradeoff studied was the balance of receiving larger sums of cash with a higher risk of reaching the cumulative IRR limit as opposed to receiving smaller amounts of cash with a higher likelihood of remaining in the investment longer. Although not immediately apparent, simulations clearly demonstrated the beneficial outcome of the larger distribution percentage for a shorter time horizon. The graph of investors' NPV with the higher distribution percentage was of similar shape to the previous graphs involving a decreased IRR. Both situations forced investors out of their investment earlier. On the other hand, the graphs that involved increasing the IRR, and the graph that involved lowering the distribution percentage, were also similar in shape to each other as well as to the graph of project NPV. These two cases allowed investors to remain in the project for a longer period of time. Thus, the particular shapes of investors' payoff distributions revealed a critical dependence on the length of time of the investment. At the same time, similar shapes of payoffs, indicating roughly the same time horizon, did not necessarily indicate anything about the actual value of the investment itself. For example, the lowered IRR provided a negative mean NPV to outside investors, while the similarly-shaped graph of a 75% distribution percentage showed high levels of profit. In summary, the ability to simulate and value important variations of the investment provides critical knowledge concerning the sensitivity of investors' value to these variations. Moreover, Monte Carlo simulations provide the essential method to examine the path-dependent nature of this investment.

6.2 Areas of Further Work

There are many interesting related issues to address stemming from this thesis. With this particular project, the expansion can be further studied by experimenting with different time horizons. In this thesis, the life of the project was estimated to be 30 years. However, similar calculations could be performed to compare the same project over different time horizons and the subsequent effect it would have on value to outside investors. In addition, further studies could be done that include a financing structure involving debt to be paid off over any number of years. One could even go into details concerning the effects of default at different point in the binomial tree.

In addition, many creative situations could be set up to encourage further study. For example, the valuation could include the option of project abandonment as part of the financial strategy. Broadening possibilities further, experiments could be done with varying risk-free rates. These areas of further study are obviously a mere sampling of the possible situations of project financing to address. Fortunately, with the help of Monte Carlo methods, many projects and conditional events can be simulated and valued.

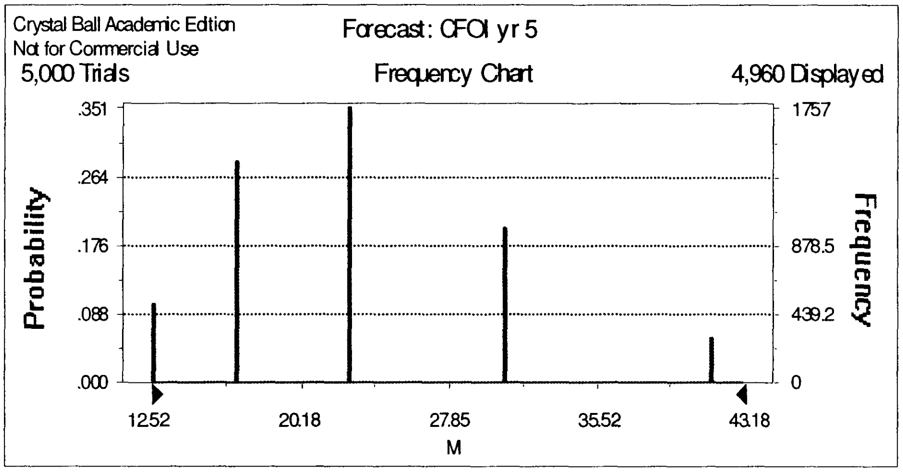
Table 1: Base Case

Key Input Values:

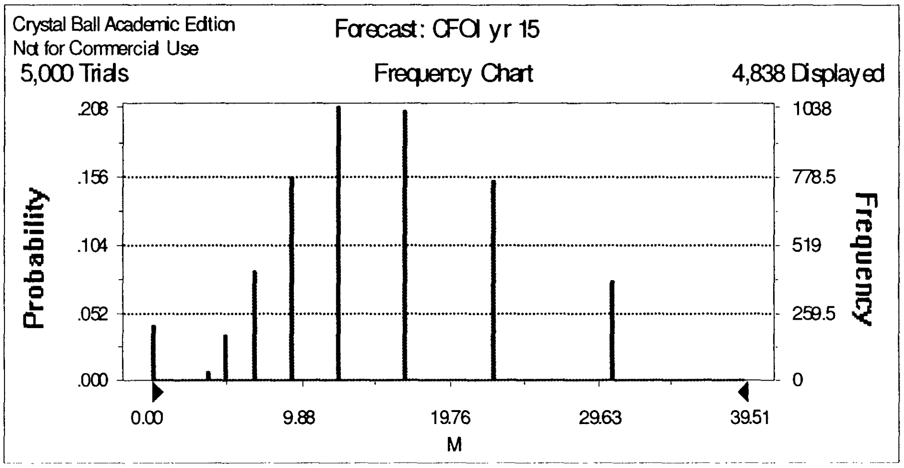
INVEST	C1	FC	RF	St. Dev.	UP	DOWN	PROB	1 - PROB
500	53	0	6.00%	15.0%	1.161834	0.861	0.371	0.629

Investor Terms:

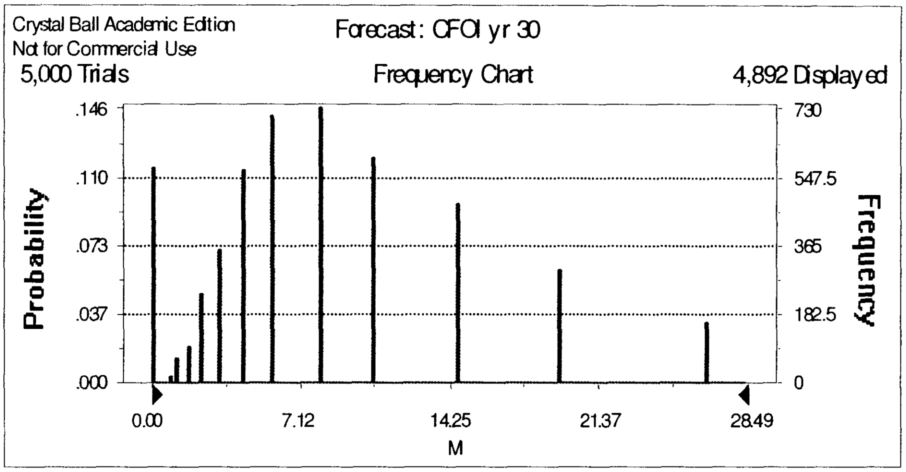
% to Outside Investors	Target IRR
50%	**11%**



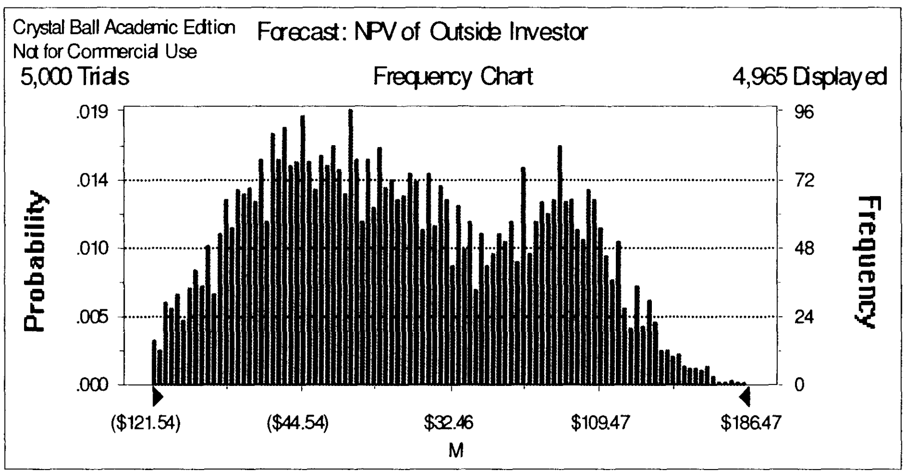
Average Cash Flow, Year 5: $23 million



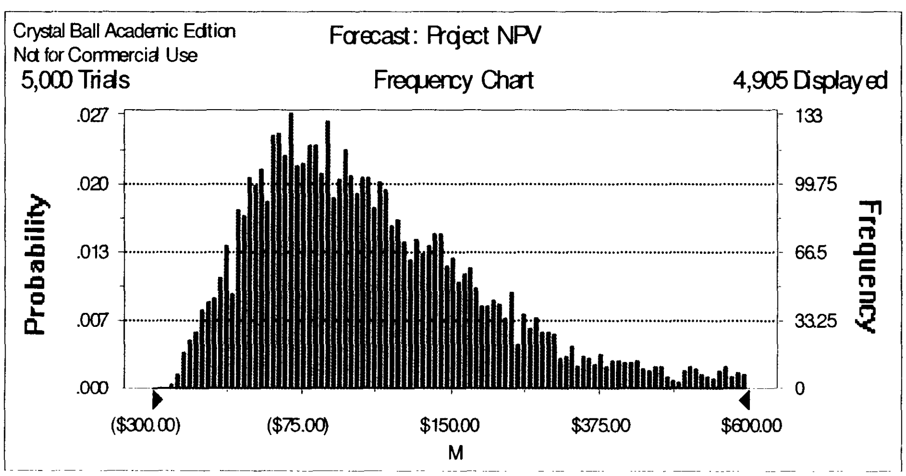
Average Cash Flow, Year 15: $16 million



Average Cash Flow, Year 30: $8.5 million



Average NPV to Outside Investors: $8.4 million



Average NPV of Project: $45 million

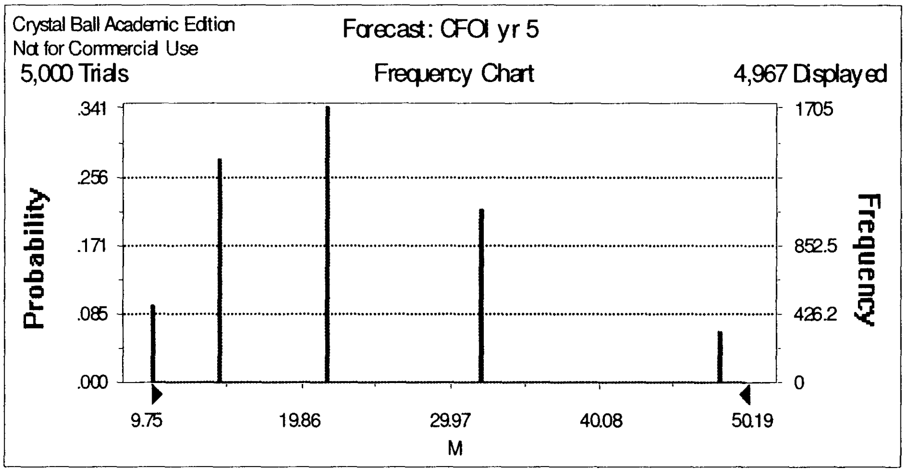
Table 2: Standard Deviation 20%

Key Input Values:

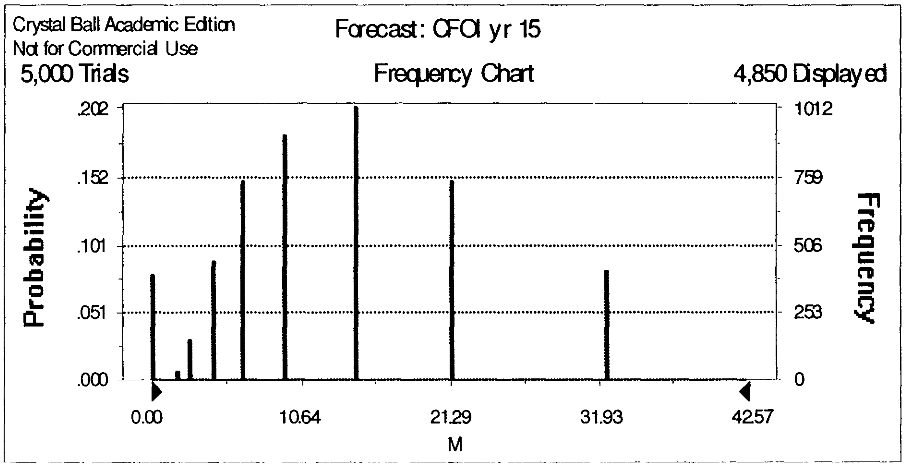
INVEST	C1	FC	RF	St. Dev.	UP	DOWN	PROB	1 - PROB
500	53	0	6.00%	**20.0%**	1.221403	0.819	0.382	0.618

Investor Terms:

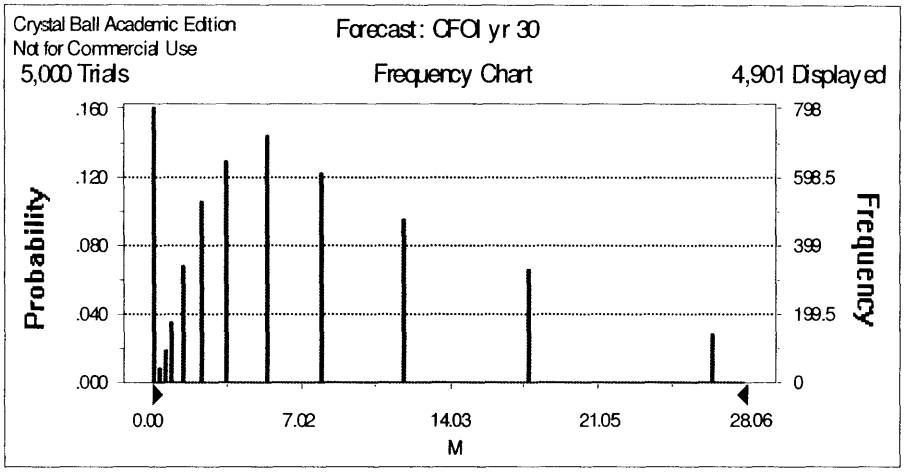
% to Outside Investors	Target IRR
50%	11%



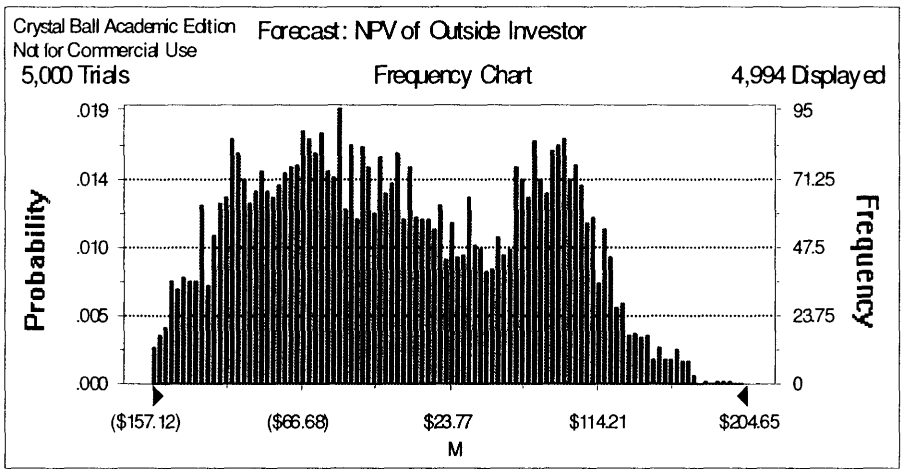
Average Cash Flow, Year 5: $23 million



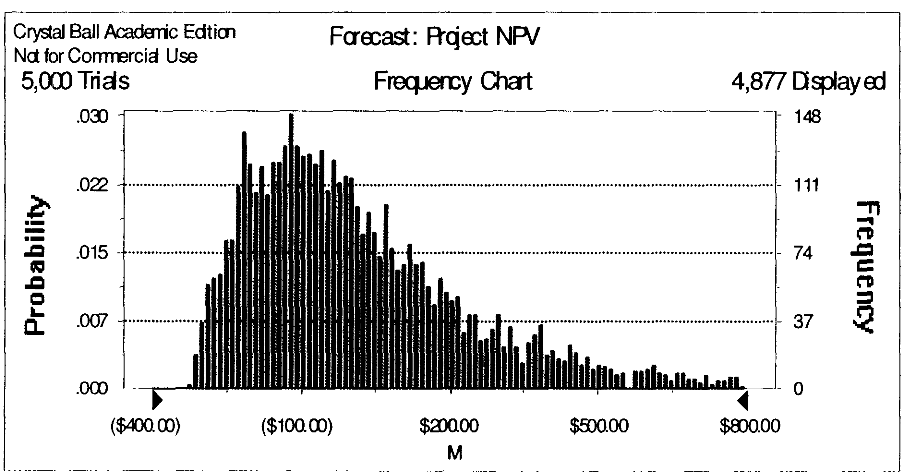
Average Cash Flow, Year 15: $13.6 million



Average Cash Flow, Year 30: $6.55 million



Average NPV of Outside Investors: -$5.77 million



Average NPV of Project: $42 million

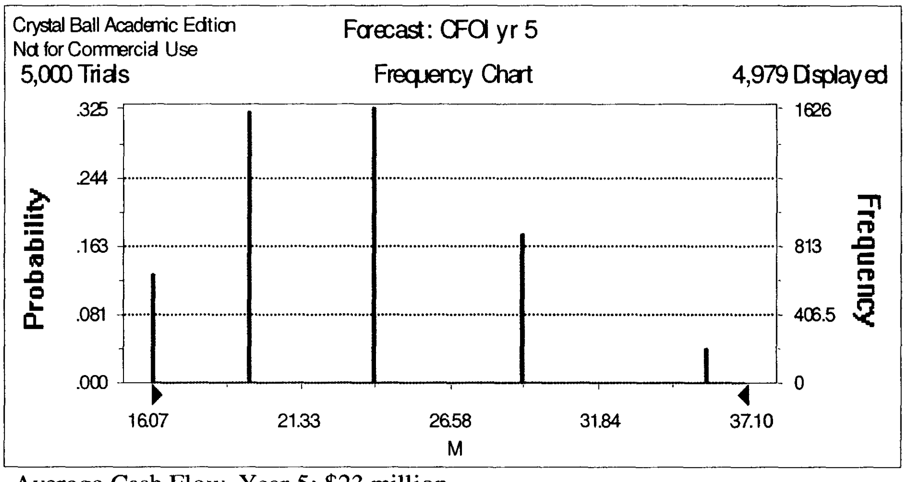
Table 3: Standard Deviation 10%

Key Input Values:

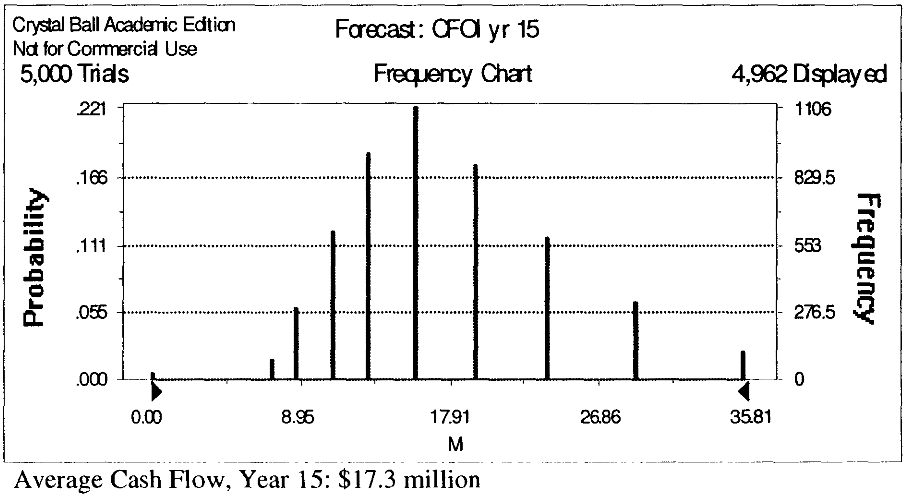
INVEST	C1	FC	RF	St. Dev.	UP	DOWN	PROB	1 - PROB
500	53	0	6.00%	**10.0%**	1.105171	0.905	0.338	0.662

Investor Terms:

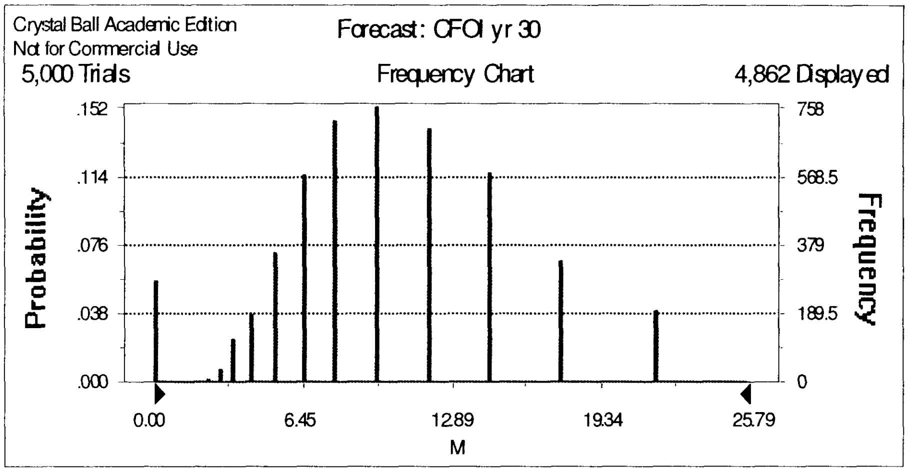
% to Outside Investors	Target IRR
50%	11%



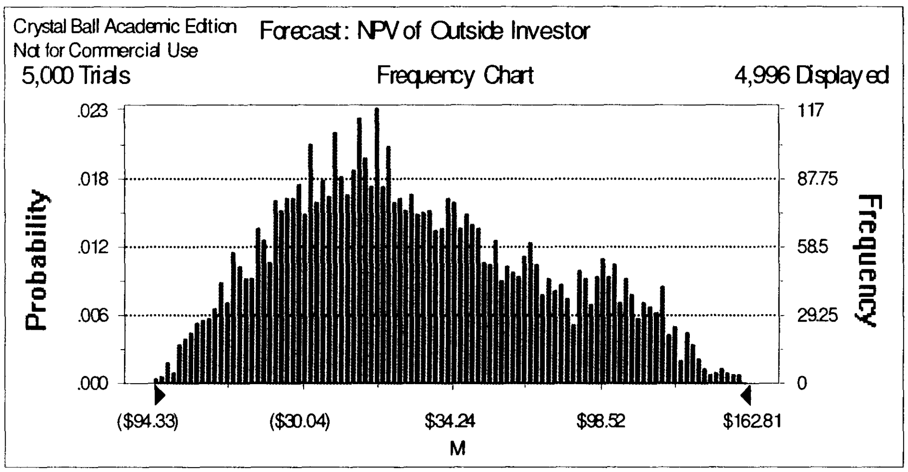
Average Cash Flow, Year 5: $23 million



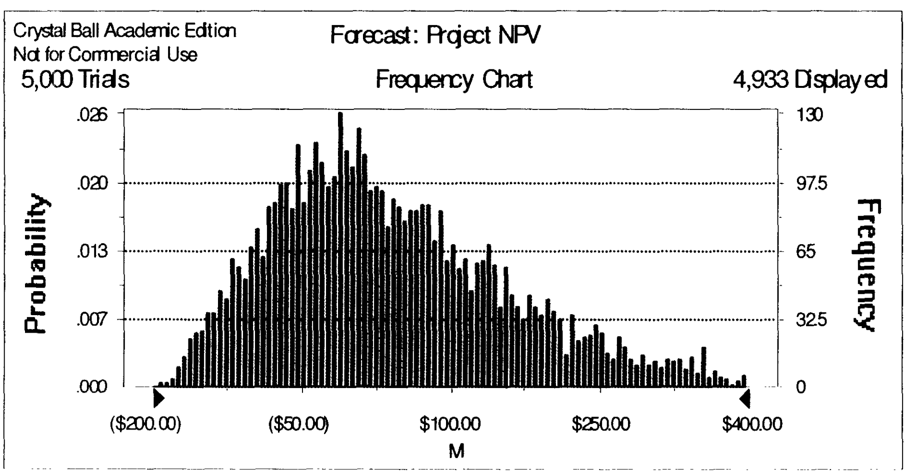
Average Cash Flow, Year 15: $17.3 million



Average Cash Flow, Year 30: $10.25 million



Average NPV of Outside Investors: $19.22 million



Average NPV of Project: $45.9 million

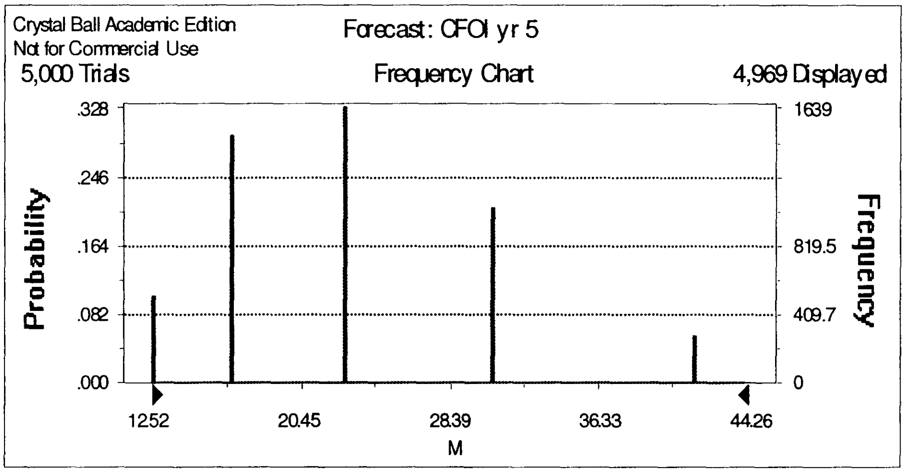
Table 4: IRR 21%

Key Input Values:

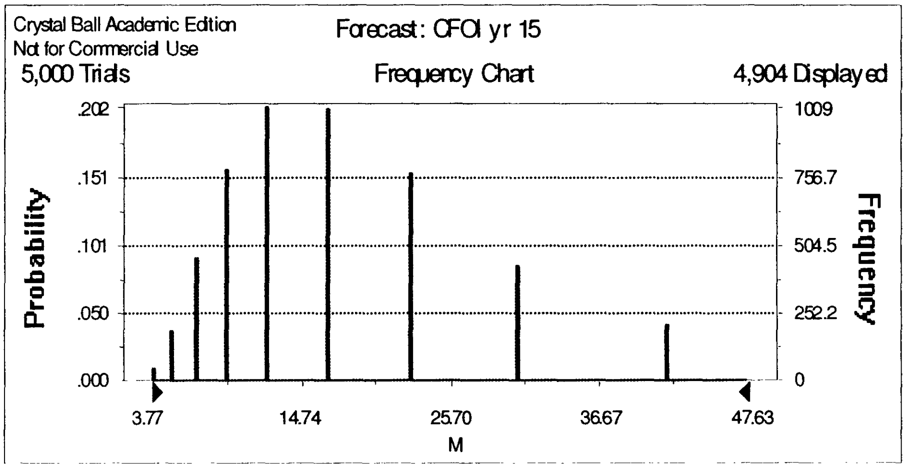
INVEST	C1	FC	RF	St. Dev.	UP	DOWN	PROB	1 - PROB
500	53	0	6.00%	15.0%	1.161834	0.861	0.371	0.629

Investor Terms:

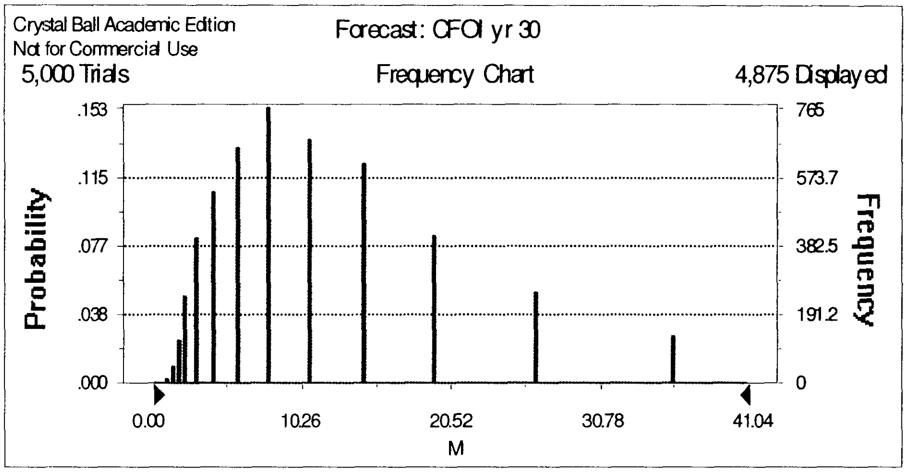
% to Outside Investors	Target IRR
50%	**21%**



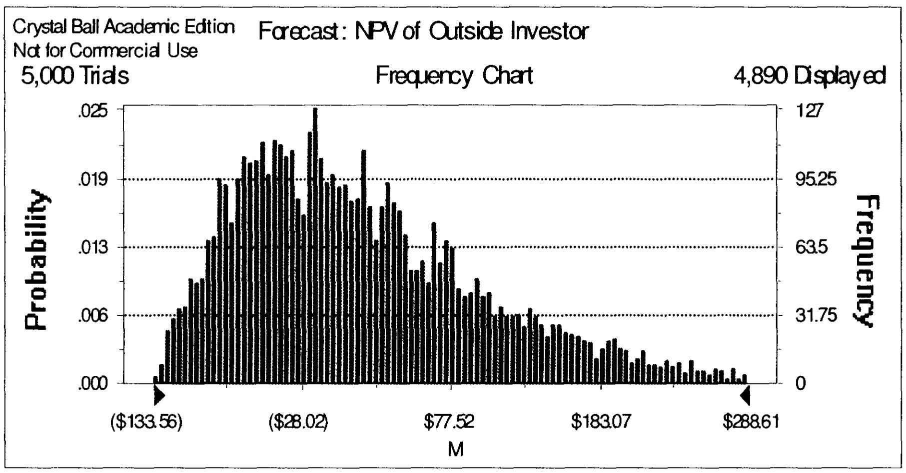
Average Cash Flow, Year 5: $22.9 million



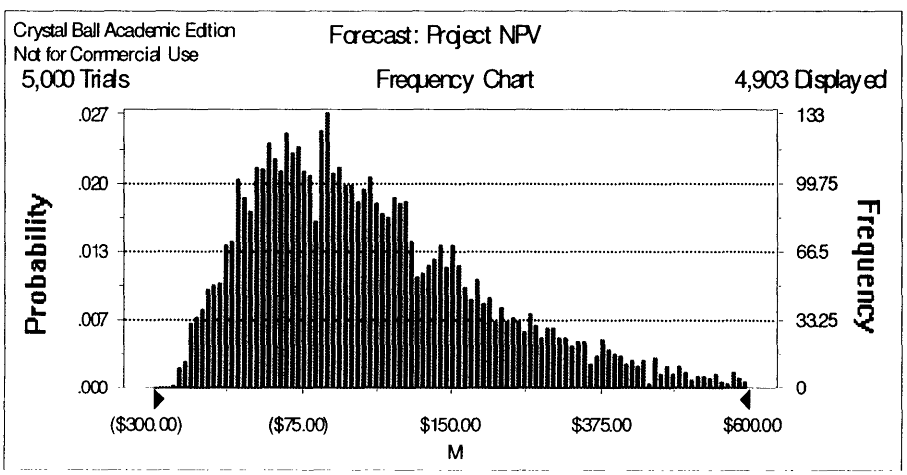
Average Cash Flow, Year 15: $17.1 million



Average Cash Flow, Year 30: $10.94 million



Average NPV of Outside Investors: $19.47 million



Average NPV of Project: $42.1 million

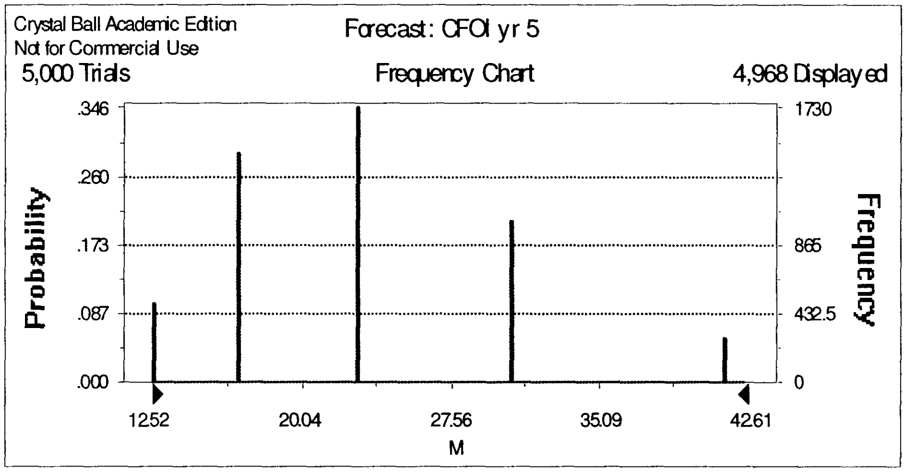
Table 5: IRR 16%

Key Input Values:

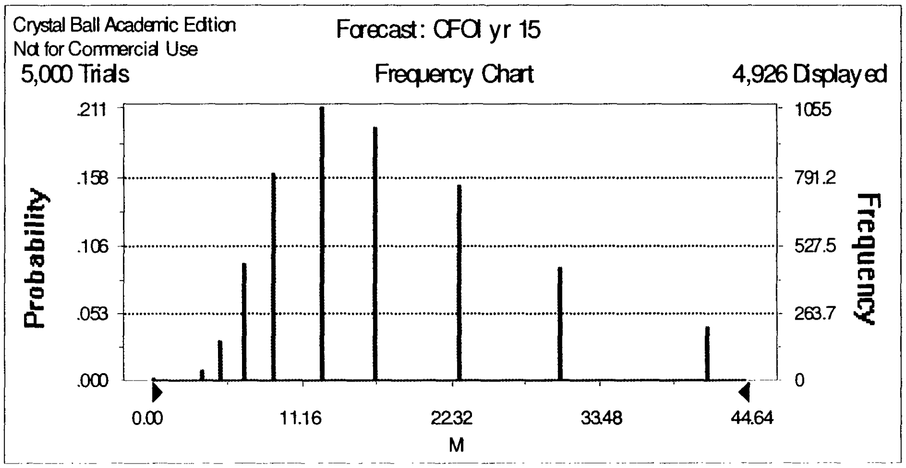
INVEST	C1	FC	RF	St. Dev.	UP	DOWN	PROB	1 - PROB
500	53	0	6.00%	15.0%	1.161834	0.861	0.371	0.629

Investor Terms:

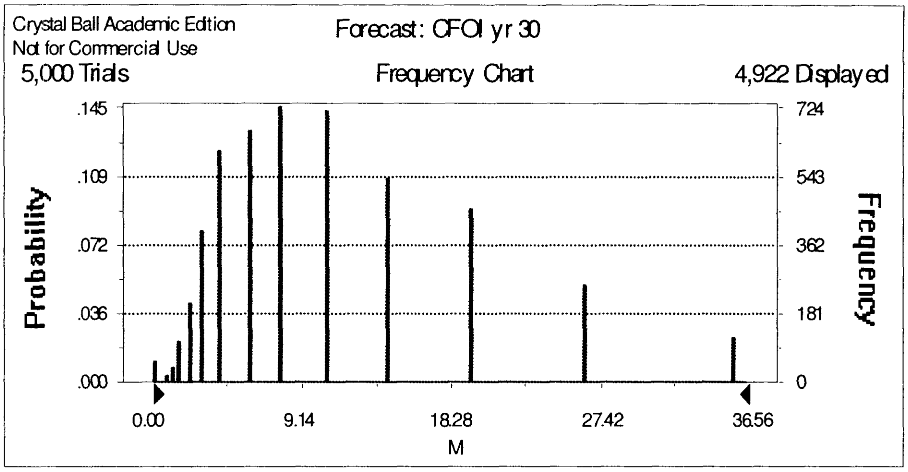
% to Outside Investors	Target IRR
50%	**16%**



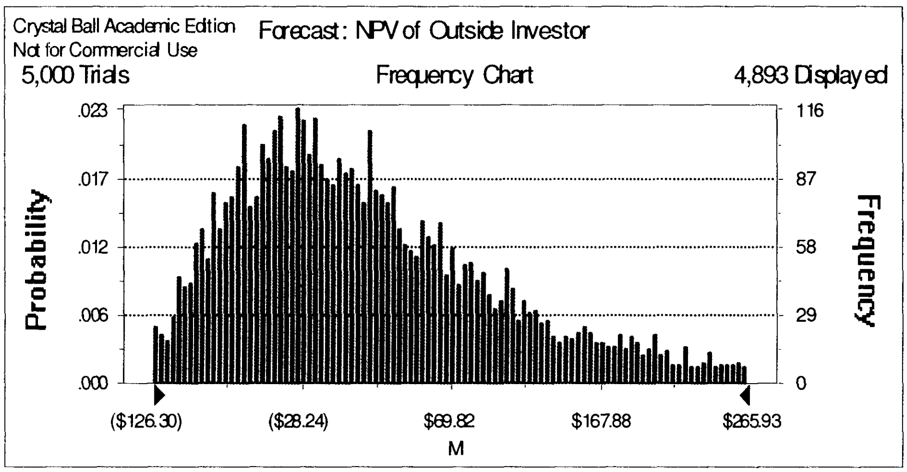
Average Cash Flow, Year 5: $23 million



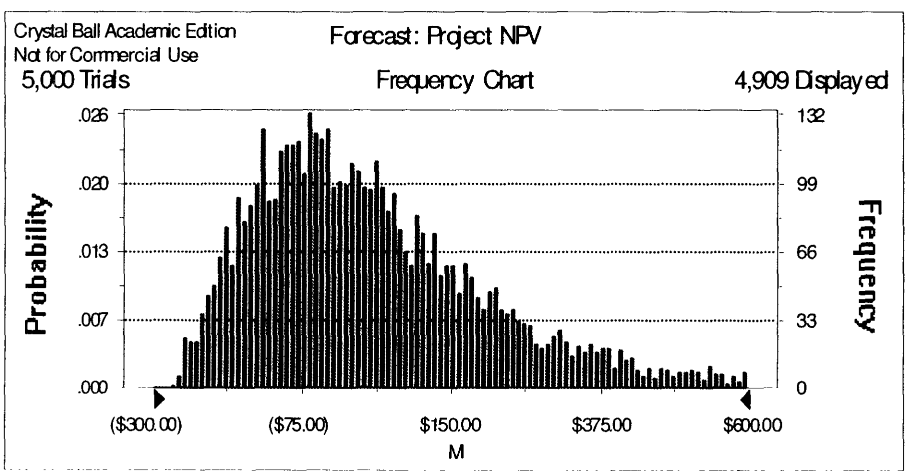
Average Cash Flow, Year 15: $17.3 million



Average Cash Flow, Year 30: $11.6 million



Average NPV of Outside Investors: $21.28 million



Average NPV of Project: $43 million

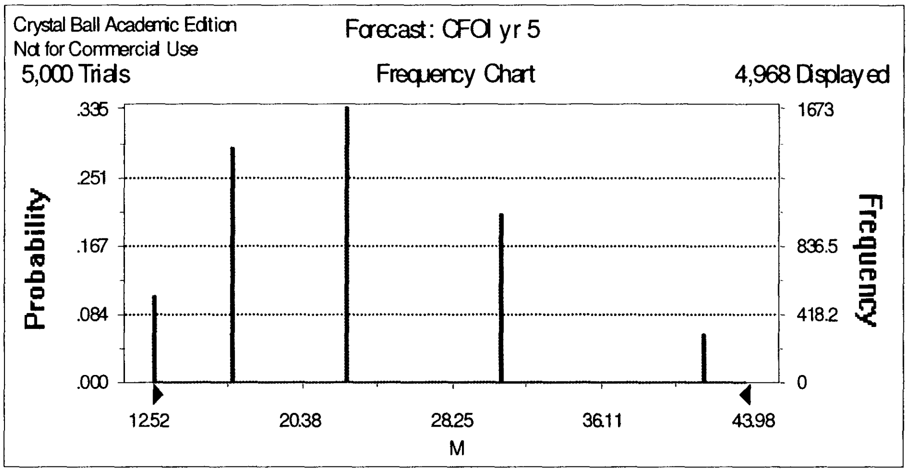
Table 6: IRR 9%

Key Input Values:

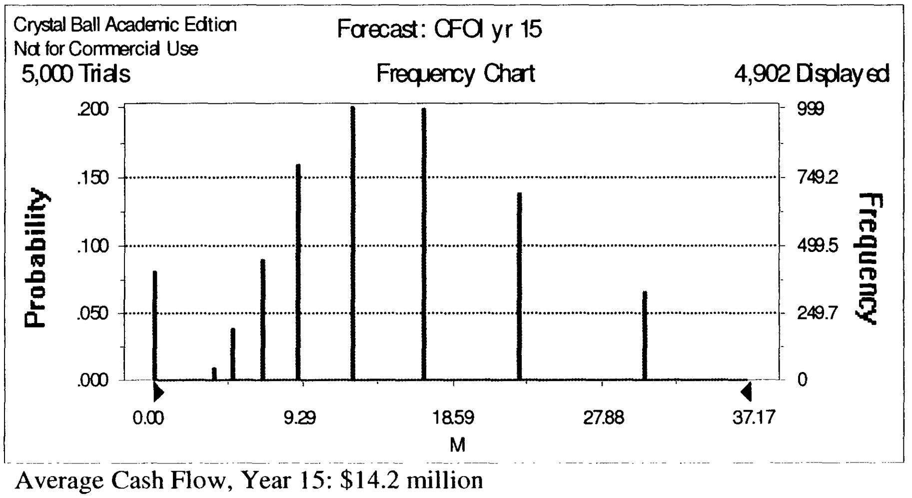
INVEST	C1	FC	RF	St. Dev.	UP	DOWN	PROB	1 - PROB
500	53	0	6.00%	15.0%	1.161834	0.861	0.371	0.629

Investor Terms:

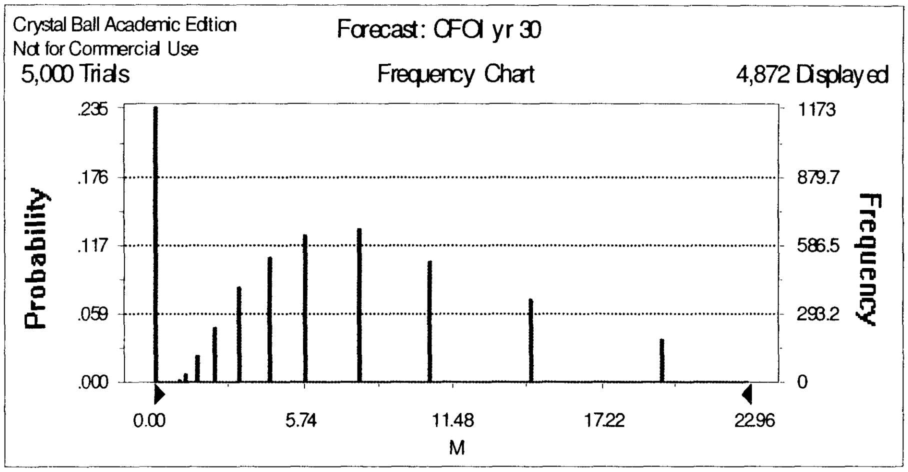
% to Outside Investors	Target IRR
50%	**9%**



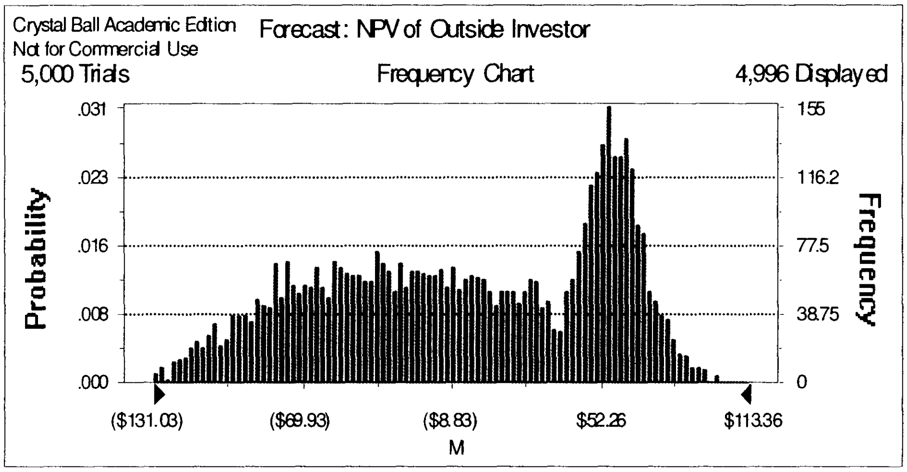
Average Cash Flow, Year 5: $23 million



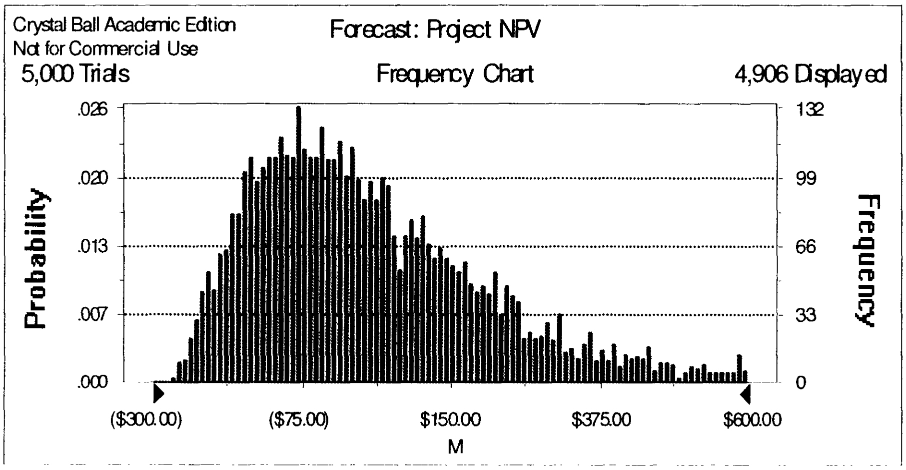
Average Cash Flow, Year 15: $14.2 million



Average Cash Flow, Year 30: $6.4 million



Average NPV of Outside Investors: -$3.2 million



Average NPV of Project: $41.2 million

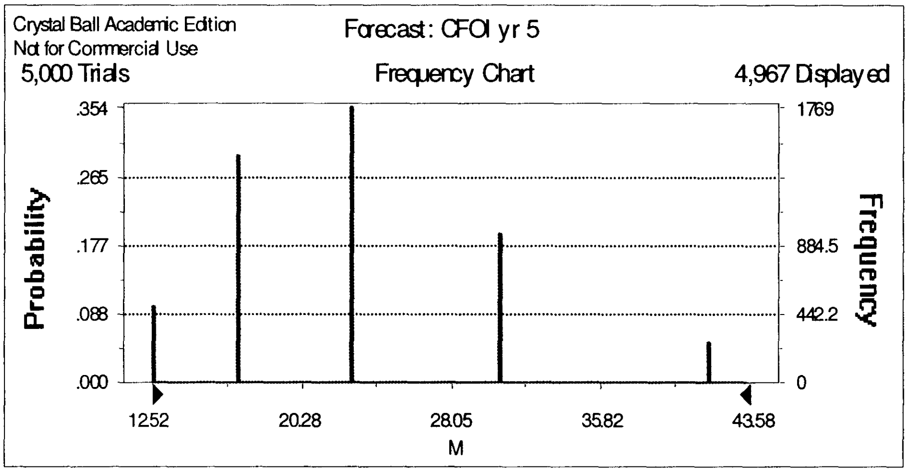
Table 7: IRR 6%

Key Input Values:

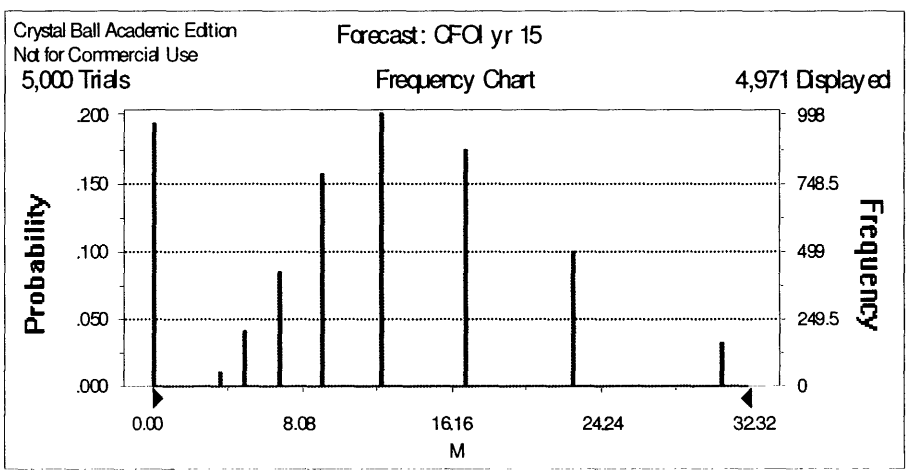
INVEST	C1	FC	RF	St. Dev.	UP	DOWN	PROB	1 - PROB
500	53	0	6.00%	15.0%	1.161834	0.861	0.371	0.629

Investor Terms:

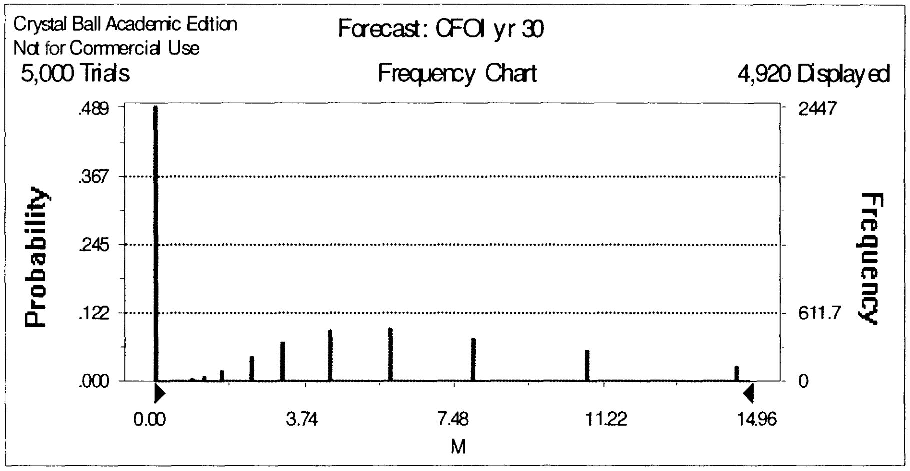
% to Outside Investors	Target IRR
50%	**6%**



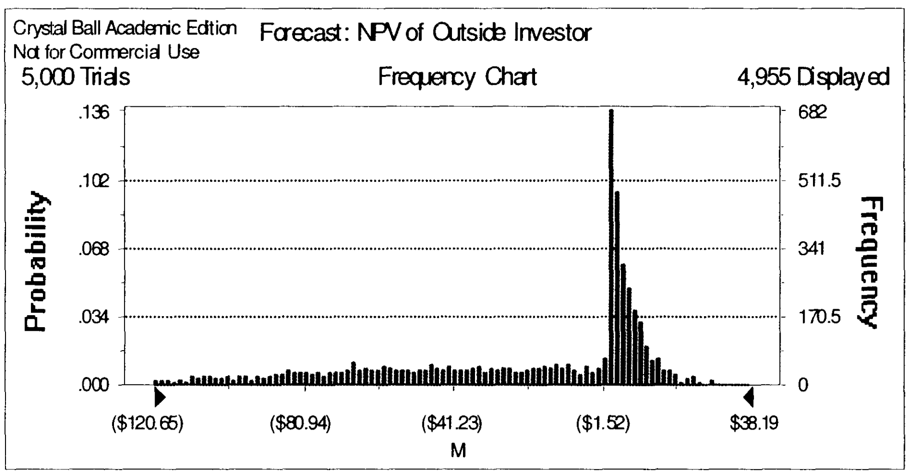
Average Cash Flow, Year 5: $22.8 million



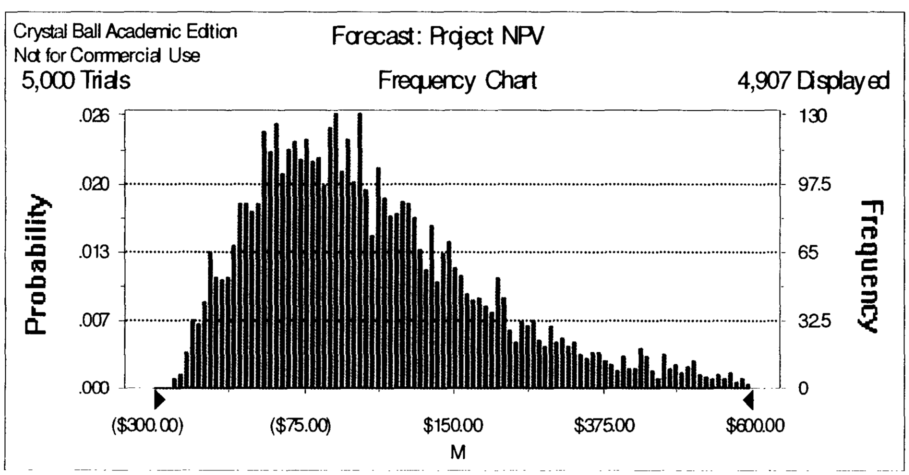
Average Cash Flow, Year 15: $11.24 million



Average Cash Flow, Year 30: $3.28 million



Average NPV of Outside Investors: -$23.78 million



Average NPV of Project: $41 million

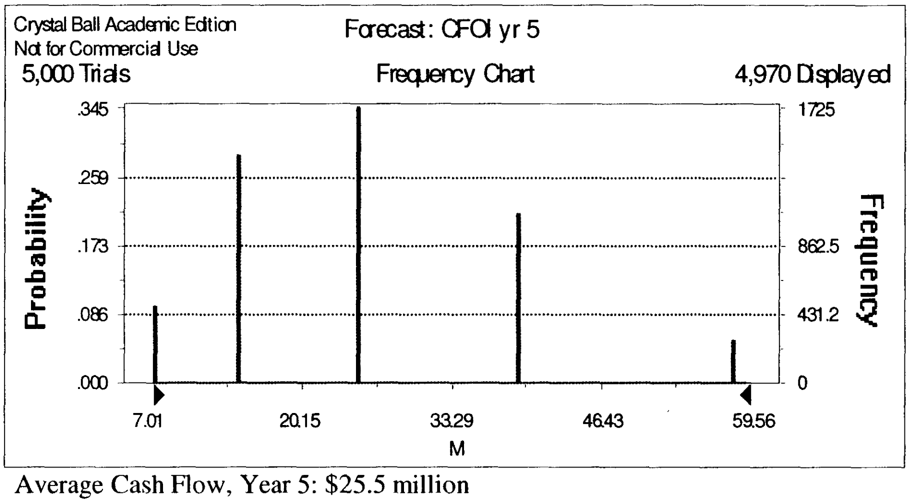
Table 8: Fixed Costs $30 Million

Key Input Values:

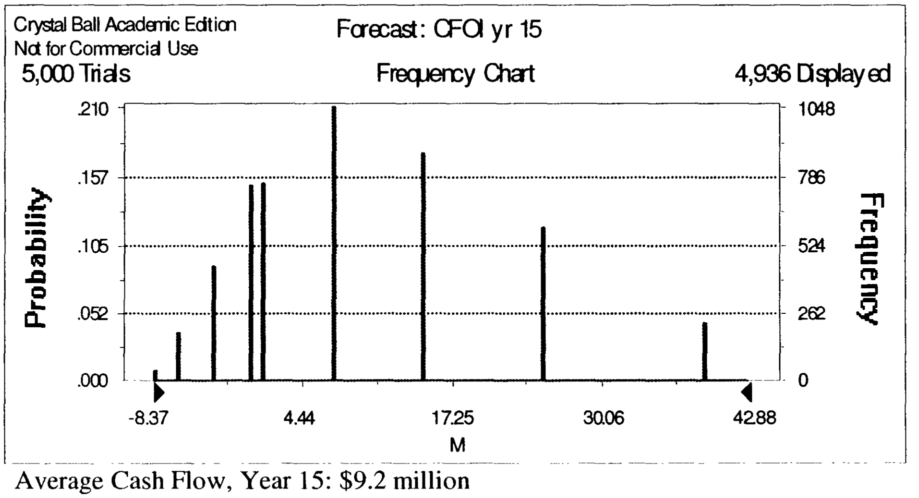
INVEST	C1	FC	RF	St. Dev.	UP	DOWN	PROB	1 - PROB
500	93.195	**30**	6.00%	15.0%	1.161834	0.861	0.371	0.629

Investor Terms:

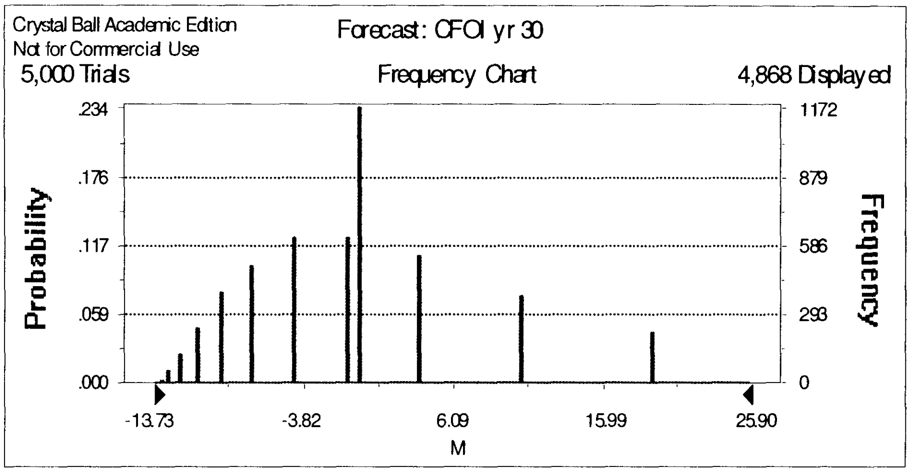
% to Outside Investors	Target IRR
50%	11%



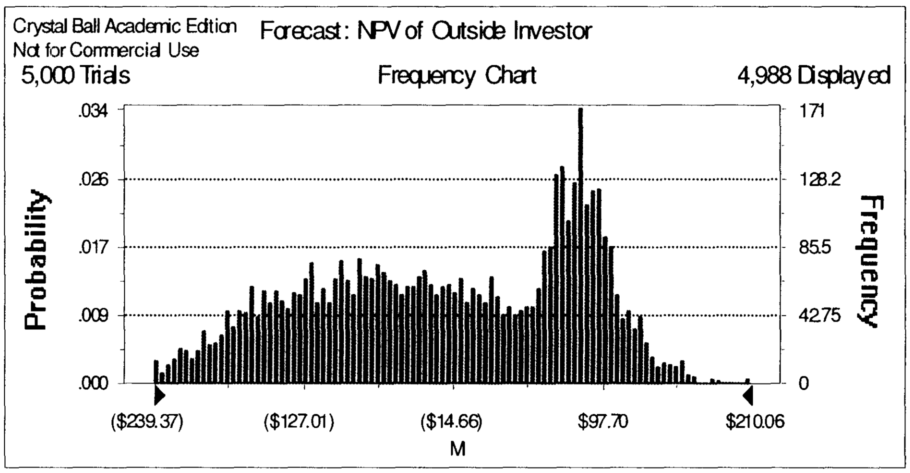
Average Cash Flow, Year 5: $25.5 million



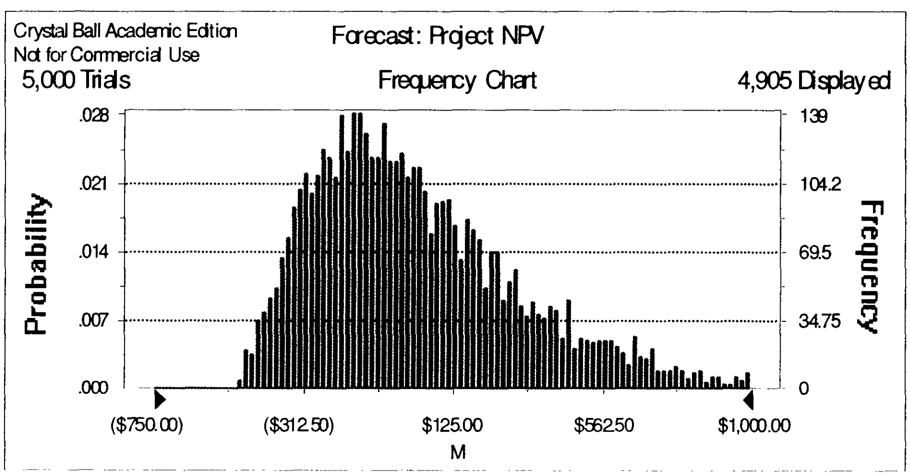
Average Cash Flow, Year 15: $9.2 million



Average Cash Flow, Year 30: -$0.1 million



Average NPV of Outside Investors: -$19.9 million



Average NPV of Project: $42.1 million

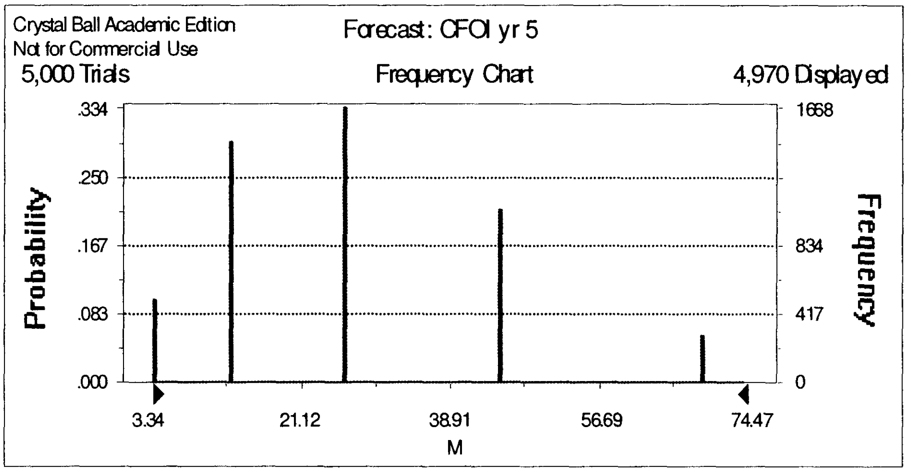
Table 9: Fixed Costs $50 Million

Key Input Values:

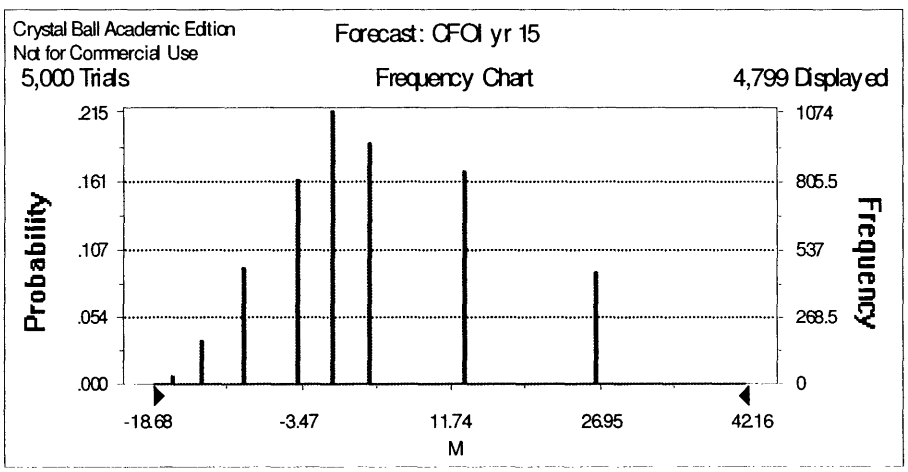
INVEST	C1	FC	RF	St. Dev.	UP	DOWN	PROB	1 - PROB
500	119.991	**50**	6.00%	15.0%	1.161834	0.861	0.371	0.629

Investor Terms:

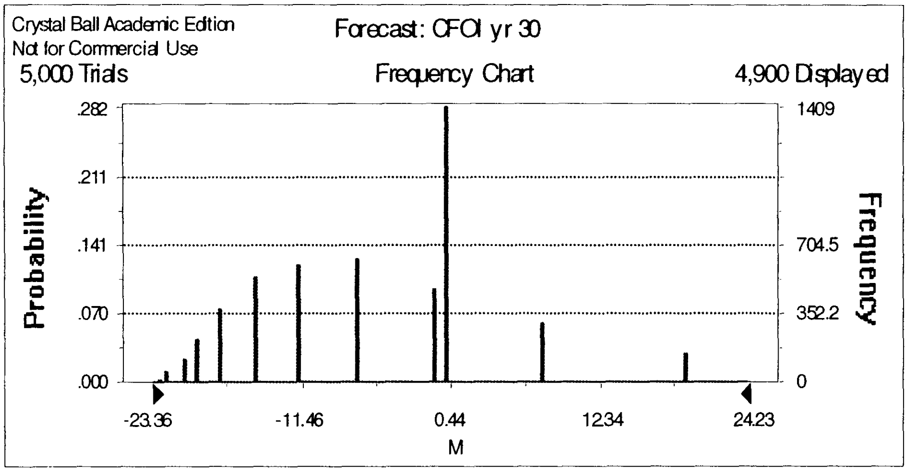
% to Outside Investors	Target IRR
50%	11%



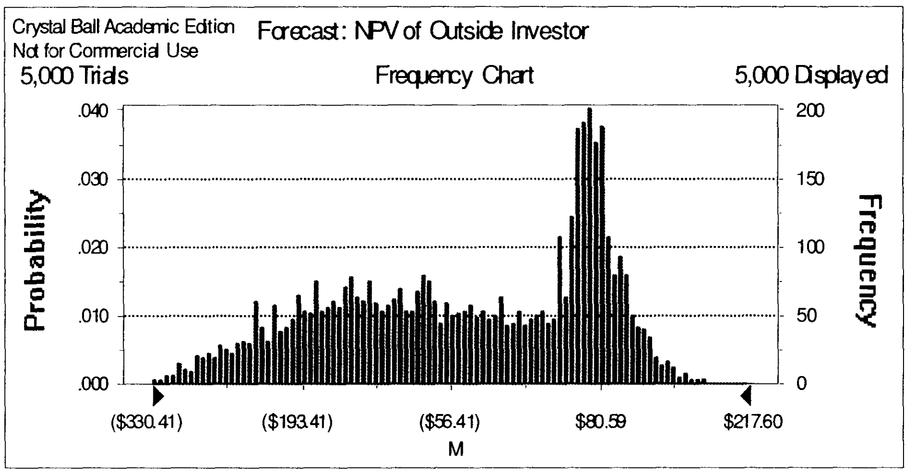
Average Cash Flow, Year 5: $27.1 million



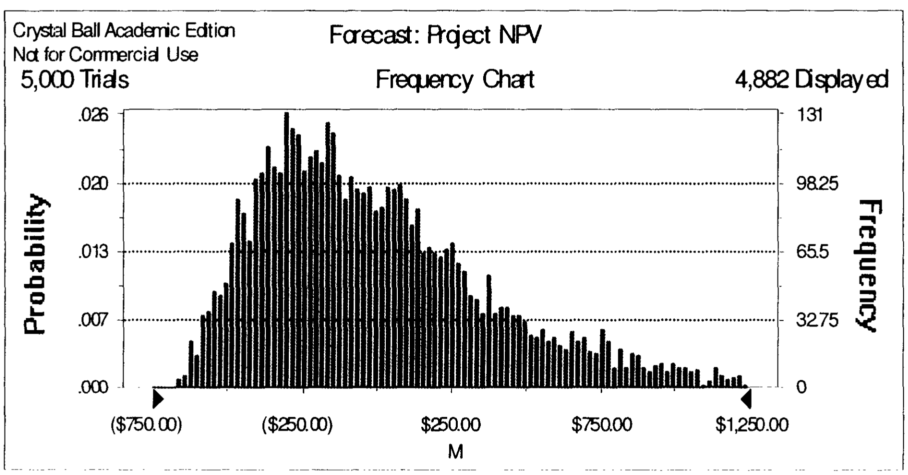
Average Cash Flow, Year 15: $5.12 million



Average Cash Flow, Year 30: -$5.05 million



Average NPV of Outside Investors: -$43.19 million



Average NPV of Project: $36 million

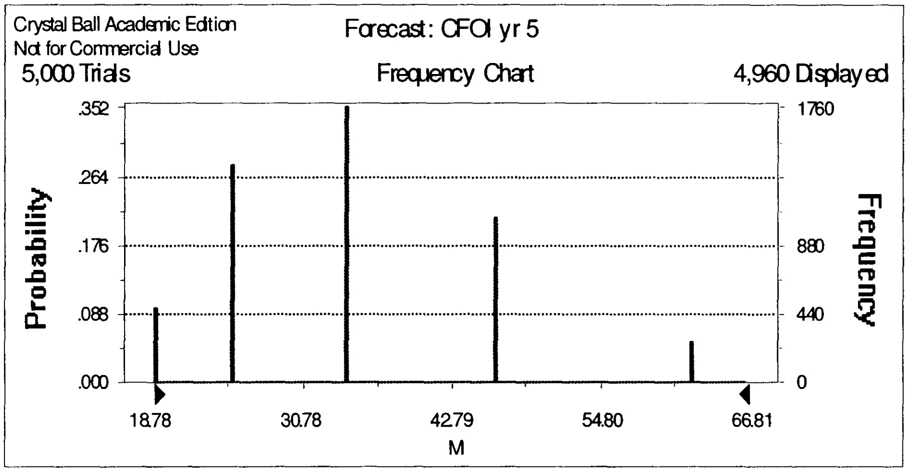
Table 10: Distribution to Outside Investor 75%

Key Input Values:

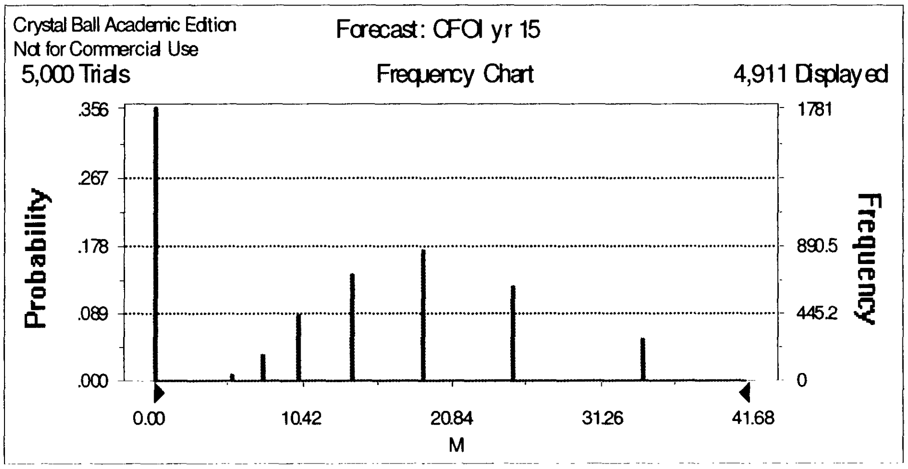
INVEST	C1	FC	RF	St. Dev.	UP	DOWN	PROB	1 - PROB
500	53	0	6.00%	15.0%	1.161834	0.861	0.371	0.629

Investor Terms:

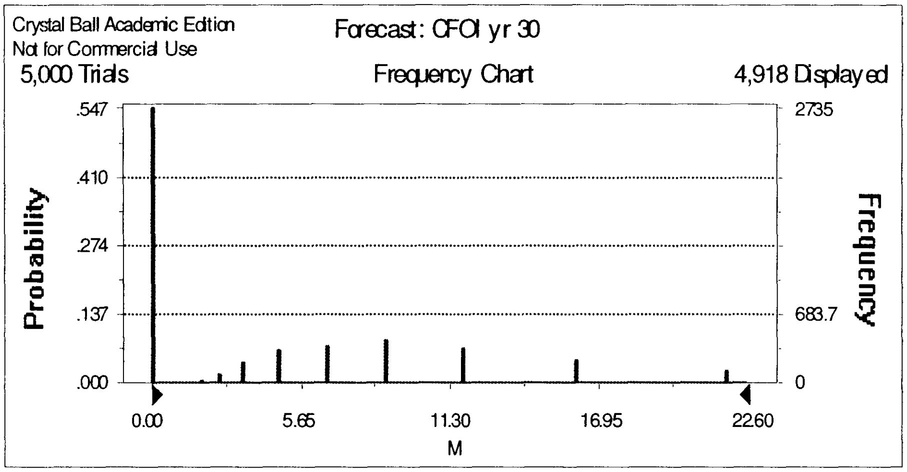
% to Outside Investors	Target IRR
75%	11%



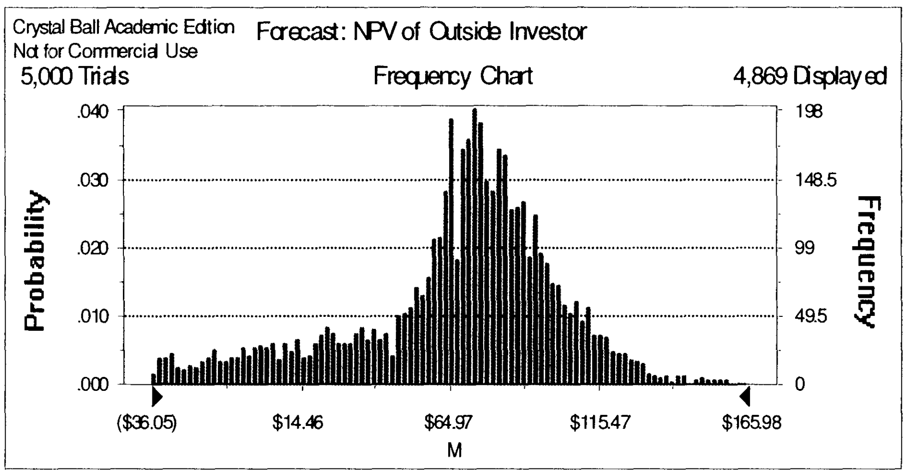
Average Cash Flow, Year 5: $34.7 million



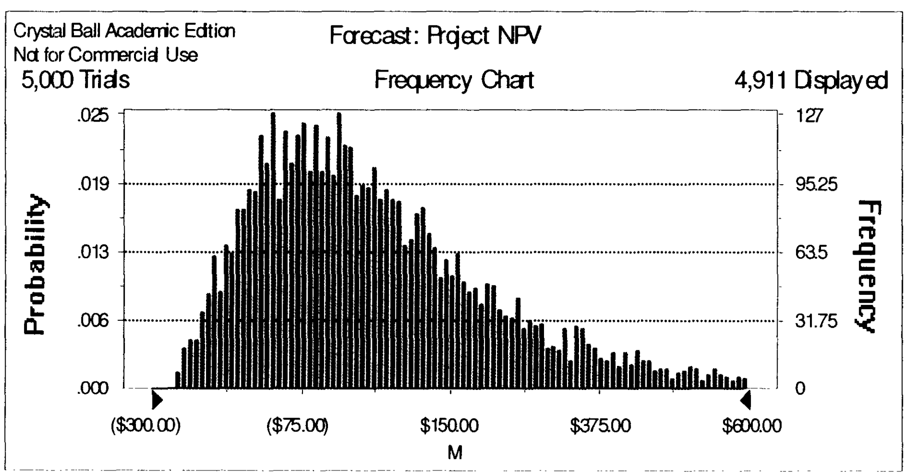
Average Cash Flow, Year 15: $12.4 million



Average Cash Flow, Year 30: $4.5 million



Average NPV of Outside Investors: $64.44 million



Average NPV of Project: $45.9 million

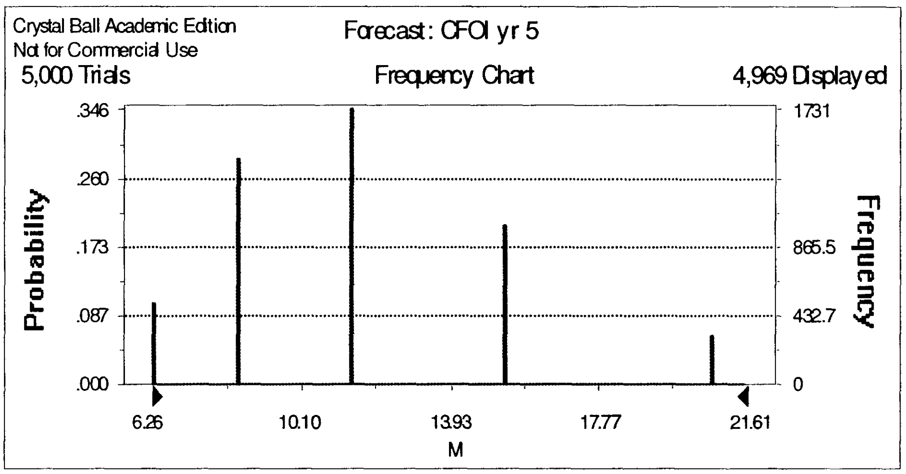
Table 11: Distribution to Outside Investor 25%

Key Input Values:

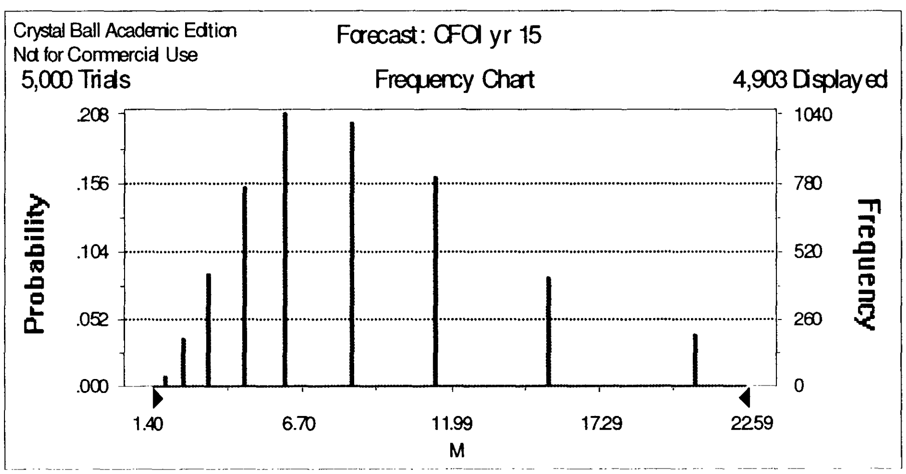
INVEST	C1	FC	RF	St. Dev.	UP	DOWN	PROB	1 - PROB
500	53	0	6.00%	15.0%	1.161834	0.861	0.371	0.629

Investor Terms:

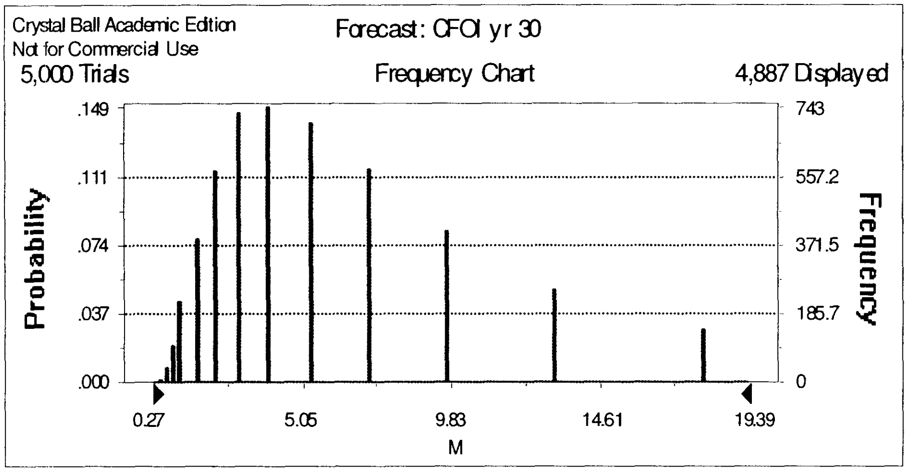
% to Outside Investors	Target IRR
25%	11%



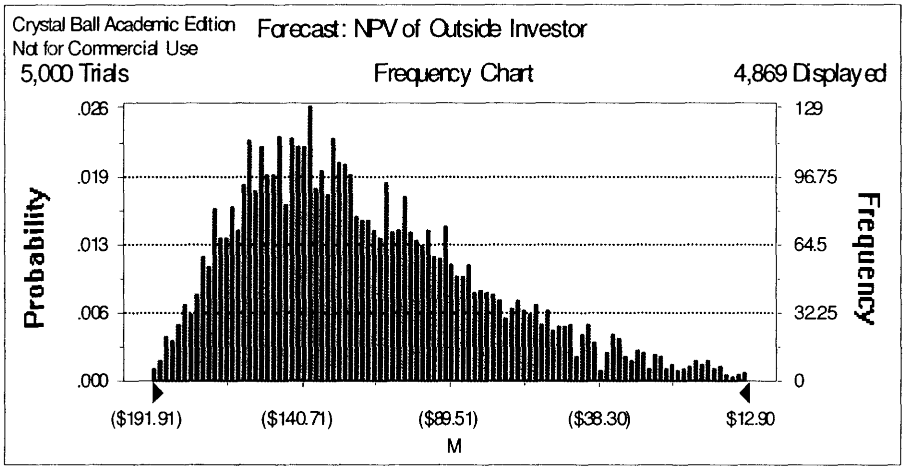
Average Cash Flow, Year 5: $11.5 million



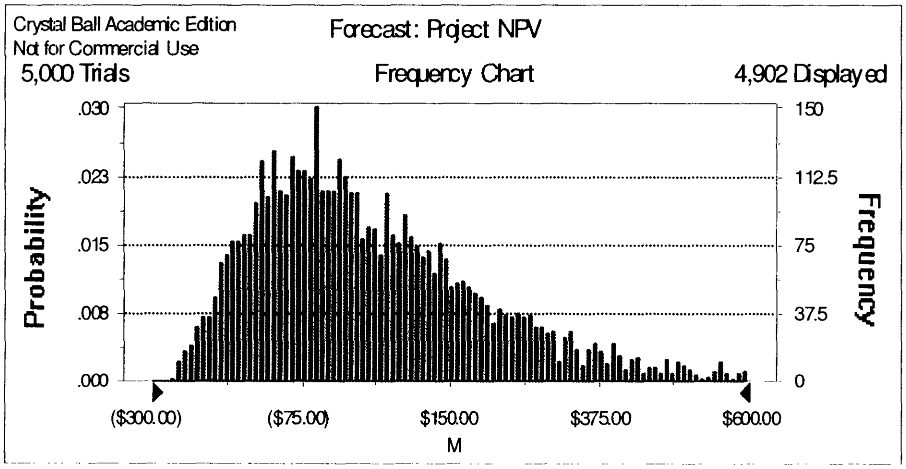
Average Cash Flow, Year 15: $8.7 million



Average Cash Flow, Year 30: $5.72 million



Average NPV of Outside Investors: -$114.24 million



Average NPV of Project: $43 million

List of References

Bertsimas, Dimitris, and Robert M. Freund. *Data, Models, and Decisions: The Fundamentals of Management Science.* Cincinnati: South-Western College Publishing, 2000.

Brealey, R.A., and S. C. Myers. *Principles of Corporate Finance 7^{th} Ed.* New York: McGraw Hill Book Company, 2003.

Myers, Stewart, ed. *Modern Developments in Financial Management.* New York: Praeger, 1976.