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SPACE-TIME CHARACTERISTICS OF ALOHA PROTOCOLS IN HIGH SPEED BIDIRECTIONAL BUS NETWORKS*

by

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Abstract

We study the space-time characteristics of the ALOHA multiple access protocol in a high speed bidirectional bus network where transmissions are in the form of packets of constant length. For point-to-point communications, slotted ALOHA is generally considered to have better throughput performance than unslotted ALOHA, whose maximum throughput is known to be 1/(2e), independent of station configuration. We show that, with a probabilistic station configuration, the maximum point-to-point throughput of slotted ALOHA degrades to nearly 1/(3e), when a, the end-to-end propagation delay normalized with respect to the packet transmission time, is much greater than 1. However, with a deterministic station configuration, the point-to-point throughput of slotted ALOHA can be as high as 1/2. For broadcast communications, the maximum throughput for slotted ALOHA is well known to be 1/{e(1+a)}. For unslotted ALOHA, we show that, if the offered load density is uniform along the bus, the maximum broadcast throughput achievable by a station varies along the bus and is maximized at its center. We also derive the optimal profile of the offered load density for achieving a uniform throughput density. In any case, the maximum broadcast throughput is greater than that derived by conventional analysis.

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1 INTRODUCTION

The ALOHA protocol, which is the simplest contention-based multiple access protocol, may either be completely asynchronous or require packet transmissions starting only at the beginning of fixed-length time slots [1], [2]. The former version is known as the unslotted ALOHA protocol, and the latter the slotted ALOHA protocol. In this paper, we present the space-time characteristics of both slotted and unslotted ALOHA protocols in high speed bidirectional bus networks where transmissions are in the form of packets of unit length. We assume that the bus is of unit length and has perfectly non-reflecting terminations at both ends. We consider both point-to-point and broadcast communications. For *point-to-point communications*, each transmission is designated for successful reception by exactly one station. For *broadcast communications*, each transmission must be successfully received by all stations.

The speed of a bus network is often specified by the parameter a, which denotes the end-to-end propagation delay normalized with respect to the packet transmission time. In this paper, we are interested in high speed (i.e. $a>1$) bidirectional bus networks. We will not consider carrier sensing since it is known to be inefficient for contention-based multiple access protocols for this case [3].

When propagation delay is negligible, as conventionally assumed, there is no difference in the performance of the ALOHA protocol between point-to-point and broadcast communications. The vulnerability of a transmission is simply characterized by the time interval over which any other packet transmitted could cause a collision. During this time interval, which is known as the vulnerable period, the given transmission is vulnerable everywhere on the bus.

Due to propagation delay, packets transmitted simultaneously may overlap non-destructively, as shown in the space-time diagram in Figure 1. When this occurs, we say that there is *channel reuse* [4], [5]. Space-time diagrams have been widely used in the literature for the analysis of multiple access protocols in bus networks: e.g. [6], [7]. Allowing for channel reuse, point-to-point transmissions are generally less demanding on channel resources than broadcast transmissions, and one expects the former to have better throughput performance. Such distinction is seldom emphasized in the literature because the difference is insignificant in networks with small propagation delays.

In a bidirectional bus network where channel reuse is possible, vulnerable periods do not adequately characterize the vulnerability of transmissions. We need to consider space-time vulnerable regions instead. A *vulnerable region* associated with a transmission is the space-time region over which any other packet arriving at the network could cause a collision. The size of the vulnerable regions is a limiting factor on the performance of a contention-based protocol. In general, for a given protocol, the larger the size of the vulnerable regions, the smaller is the probability of success of each transmission.

The spatial properties of the ALOHA protocol were first studied by Abramson, who analyzed the spatial densities of throughput and offered load in a packet radio broadcasting network with capture [8]. It was only recently that the space-time behavior of the ALOHA protocol on bus networks was studied [4], [9], [10]. Gonsalves and Tobagi have conducted a simulation study of the effects of station locations on the broadcast performance of Ethernet type bus networks using the CSMA/CD protocol [11]. They observed that, with stations uniformly distributed along the bus, those near the center of the bus obtain better performance than those near the ends. We confirm the above behavior analytically for the unslotted ALOHA protocol.

In [9], Maxemchuk showed that for unidirectional bus networks, slotted protocols are always more efficient than unslotted protocols. We show a different result for bidirectional bus networks. With a probabilistic station configuration, the maximum point-to-point throughput of slotted ALOHA must degrade to nearly 1/(3e), when $a >> 1$. On the other hand, the maximum point-to-point throughput of unslotted ALOHA is known to remain as 1/(2e) for $a >> 1$. For a bidirectional bus network with N evenly spaced stations, and $a = (N-1)$, Levy and Kleinrock showed that, provided there is no carrier sensing, the maximum throughput of any slotted contention-based multiple access protocol in such a network approaches 1/e as the number of stations becomes very large [10], [12]. We show that the maximum throughput of slotted ALOHA in such a network is at least 1/2. Our definition of a slot includes propagation delay, and is thus different from that of Maxemchuk, and that of Levy and Kleinrock.

We discuss slotted ALOHA and unslotted ALOHA in Section 2 and Section 3 respectively. In each case, we first specify our ALOHA model, and review the basic results from conventional throughput analysis. We then examine the space-time characteristics of the protocol, and present our analysis of maximum throughput. For slotted ALOHA, we offer some new results in point-to-point communications. For unslotted ALOHA, we offer some new results in broadcast communications.

2 SLOTTED ALOHA

In this section, we study the slotted ALOHA protocol in a bidirectional bus network supporting N stations. Time is divided into slots of length $(1+a)$ units of packet transmission time. A packet arriving at a station during a slot is transmitted at the beginning of the following slot, and is completely received by the designated station before the end of the same slot. We summarize below our slotted ALOHA model.

- Large but finite population of users;
- Synchronous transmissions at discrete points in time with period $(1+a)$ units of packet transmission time;
- Offered traffic including retransmissions is a memoryless random process;
- Symmetric traffic configuration;
- Statistical equilibrium.

2.1 Conventional Analysis

Conventional analysis of the slotted ALOHA protocol without channel reuse is based on the assumption that a transmission in a given slot is successful only if there are no other transmissions within the same slot. Let G be the average offered traffic per slot, in packets per packet transmission time. The offered traffic is assumed to be uniform across all stations. By symmetry, each station is active during a slot with probability $(1+a)G/N$. Thus, the probability of success is

$$P_a(G) = \left\{1 - (1+a)\frac{G}{N}\right\}^{N-1} \qquad \text{for } a \geq 0 \qquad (1)$$

The throughput is then $S_a(G) = G.P_a(G)$. For large N,

$$S_a(G) = G\, e^{-(1+a)G} \qquad \text{for } a \geq 0 \qquad (2)$$

whose maximum with respect to G is

$$S_a{}^* = \left(\frac{1}{1+a}\right)\frac{1}{e} \qquad \text{for } a \geq 0 \qquad (3)$$

The above analysis applies to both point-to-point and broadcast communications. Note that $S_a{}^*$ vanishes as a becomes very large. When channel reuse is taken into consideration, we reach a different conclusion for the slotted ALOHA protocol with point-to-point communications.

2.2 Space-Time Characteristics

For broadcast communications with $a \geq 0$, and point-to-point communications with $0 \leq a \leq 1$, the entire previous slot is the vulnerable region. Hence, the maximum throughput is the same as that derived by conventional analysis. For point-to-point communications with $a > 1$, a vulnerable region may be considerably smaller than a whole slot. We show that the point-to-point throughput of slotted ALOHA does not degrade indefinitely as a becomes very large.

In Figures 2 and 3, we show how two simultaneously transmitted packets may collide destructively in the same time slot. We call the inverted V-shaped space-time region covered by a transmission a *transmission region*. We examine these two cases separately.

In the case of Figure 2, two transmitting stations are within $(1/a)^-$ units of distance from each other, where

$$(*)^- = \min(*,1) \tag{4}$$

There is a *totally destructive collision* since no station can successfully receive the transmission. The spatial interval, in which no other transmission may originate without causing a totally destructive collision, is called a *totally vulnerable interval.*

In the case of Figure 3, two transmission regions cross each other, and there is a *potentially destructive collision.* The collision is non-destructive if neither of the two designated receivers is located within the spatial interval where the transmission regions cross each other. This spatial interval is $(1/a)^-$ units long. The spatial interval in which no other transmission may originate without causing a potentially destructive collision is called a *potentially vulnerable interval.* Note that this spatial interval does not exist if the position of the receiving station, $y \in [0,1]$, falls outside the following range.

$$Y_a(x) \equiv \left[\frac{1}{2}\left(x - \frac{1}{a}\right), \frac{1}{2}\left(x + \frac{1}{a} + 1\right) \right] \cap [0,1] \quad \text{for } x \in [0,1] \tag{5}$$

where x is the position of the transmitting station (See Figure 3).

In Figure 4, we show a typical transmission, and its corresponding totally and potentially vulnerable regions, which are respectively specified by the totally vulnerable interval, $[x_L, x_R]$, and the potentially vulnerable interval, $[z_L, z_R]$, where

$$x_R = \left(x + \frac{1}{a}\right)^- \tag{6}$$

$$x_L = \max\left\{ \left(x - \frac{1}{a}\right) , 0 \right\} \quad (7)$$

$$z_R = \max\left\{ \left(z + \frac{1}{a}\right)^- , 0 \right\} \quad (8)$$

$$z_L = \max\left\{ \left(z - \frac{1}{a}\right)^- , 0 \right\} \quad (9)$$

and $z = 2y - x$.

Let $X_a(x)$ denote the length of the totally vulnerable interval, and $Z_a(x,y)$ that of the potentially vulnerable interval. Then, it can be verified that

$$X_a(x) = x_R - x_L \le \left(\frac{2}{a}\right)^- \qquad \text{for } x \in [0,1] \quad (10)$$

and

$$Z_a(x,y) = z_R - z_L \le \left(\frac{2}{a}\right)^- \qquad \text{for } x \in [0,1] \text{ and } y \in Y_a(x) \quad (11)$$

Note that for $a >> 1$, end effects are negligible, and the above relations hold with equality (See Figure 5). From (5), we see that for $a >> 1$, $Y_a(x) \cong [x/2,(x+1)/2]$ and the length of $Y_a(x)$ is approximately 1/2.

We refer to the union of the vulnerable intervals as the *vulnerable interval-set*, and the union of the vulnerable regions as the *vulnerable union*. The larger the size of the vulnerable union, the smaller is the maximum throughput of the slotted ALOHA protocol.

2.3 Maximum Point-to-Point Throughput

We consider two kinds of station configurations. In the *probabilistic station configuration*, each station, except for the end-stations, is independently located at a uniformly distributed point on the bus. In the *deterministic station configuration*, the positions of the stations on the bus are fixed. A special case of the deterministic configuration is the regular bus network, in which the distance between any two adjacent stations is a constant.

2.3.1 Probabilistic Station Configuration

We first examine a bidirectional bus network with probabilistic station configuration. We show below that unslotted ALOHA is more efficient than slotted ALOHA for point-to-point communications in a very high speed bidirectional bus network.

Theorem 1

Consider a bidirectional bus network with a probabilistic station configuration. Given a large number of stations, the point-to-point throughput of the slotted ALOHA protocol is given by

$$S_a(G) = G \int_0^1 dx \int_0^1 dy \left\{ e^{-(1+a)F_a(x,y)G} \right\} \qquad \text{for } a \geq 0 \qquad (12)$$

where

$$F_a(x,y) = \left(\max(x_R, z_R) - \min(x_L, z_L) \right)$$
$$- \max\left(\left(\max(x_L, z_L) - \min(x_R, z_R) \right), 0 \right) \qquad \text{for } a \geq 0 \qquad (13)$$

The maximum throughput, S_a^*, obtained from maximizing $S_a(G)$ with respect to G, decreases with a to an asymptotic value of 0.1304, which is greater than 1/(3e) (See Figure 6).

■

Proof of Theorem 1

Consider a transmission from a station located at $x \in [0,1]$ to a station located at $y \in [0,1]$. Let $F_a(x,y)$ be the fraction of the bus that is within the vulnerable interval-set. The potentially and totally vulnerable intervals may overlap each other or be disjoint. Moreover, one of them may be on either side of the other. It follows that there are four cases to consider in order to obtain $F_a(x,y)$.

Let's refer to the location x=0 as the left end of the bus, and the location x=1 the right end of the bus. The right-most edge of the vulnerable interval-set is $\max(x_R,z_R)$, and the left-most edge is $\min(x_L,z_L)$. Thus, the length of the vulnerable interval-set including possibly a gap in between is $\{\max(x_R,z_R) - \min(x_L,z_L)\}$. It can be verified that the length of the gap is given by $\max\{(\max(x_L,z_L) - \min(x_R,z_R)), 0\}$. Hence, we obtain $F_a(x,y)$.

When there are N stations, the probability of success is

$$P_a(G) = \int_0^1 dx \int_0^1 dy \left\{1 - (1+a)F_a(x,y)\,\frac{G}{N}\right\}^{N-1} \tag{14}$$

For large N,

$$P_a(G) = \int_0^1 dx \int_0^1 dy \left\{e^{-(1+a)F_a(x,y)G}\right\} \tag{15}$$

The throughput, $S_a(G)$, is simply $G.P_a(G)$. $S_a{}^*$, can be derived numerically from (12), and its asymptotic behavior can be verified.

As S_a* decreases monotonically with **a**, its asymptotic value is a lower-bound. We recall from Section 2.2 that when **a**>>1, $F_a(x,y) = 2/a$ in half of the relevant space of x and y, and $F_a(x,y) = 4/a$ in other half.

For a>>1, (12) simplifies to

$$\lim_{a>>1} \{S_a(G)\} = \frac{G}{2}\left(e^{-2G} + e^{-4G}\right) \quad (16)$$

whose maximum with respect to G is 0.1304.

On the average, the space-time area of the vulnerable union tends to 3 when **a**>>1. From (12) and by Jensen's Inequality, we obtain the following bound.

$$\lim_{a>>1} \{S_a(G)\} \geq G\, e^{-3G} \quad (17)$$

It follows that the maximum throughput cannot degrade below 1/(3e).

Q.E.D.

Note that for $0 \leq a \leq 1$, there is no channel reuse, and the maximum throughput of slotted ALOHA is the same as in (3). For **a** ≥1, $S_a* \leq S_1* = 1/(2e)$. It is well known that the maximum throughput of unslotted ALOHA is 1/(2e). We have thus shown that slotted ALOHA is less efficient than unslotted ALOHA for point-to-point communications in a very high speed bidirectional bus network.

2.3.2 Deterministic Station Configuration

We now examine a bidirectional bus network with deterministic station configuration. In Lemma 1 and Lemma 2, we derive bounds on the maximum point-to-point throughput achievable by the slotted ALOHA protocol. These bounds are valid for any deterministic station configuration. In Theorem 2, we present bounds on the same maximum throughput in a bidirectional bus network with regularly spaced stations.

Lemma 1

The maximum point-to-point throughput, S_a*, achievable with slotted transmissions, is bounded as follows.

$$S_a* \leq \left(\frac{N}{1+a}\right)^- \leq 1 \qquad \text{for } a \geq 0 \qquad (18)$$

■

Proof of Lemma 1

The bound is derived by considering only the totally vulnerable regions. For successful synchronous transmissions, no two packets may originate from stations that are less than $(1/a)^-$ units of distance apart. Otherwise, there is a totally destructive collision. Let $M \geq 1$ be the maximum possible number of successful synchronous transmissions on the bus.

For $0 \leq a < 1$, the totally vulnerable region covers the entire slot, and thus $M \leq 1$. For $a \geq 1$, we have

$$(M-1)\,\frac{1}{a} \leq 1 \qquad (19)$$

It follows that

$$M \leq (1+a) \qquad \text{for } a \geq 0 \qquad (20)$$

And, we have

$$S_a* \leq \frac{M}{1+a} \leq 1 \qquad \text{for } a \geq 0 \qquad (21)$$

Since there are N stations, and $M \leq N$, we obtain the upper bound on S_a^*.

Q.E.D.

Note that the above upper bound on S_a^* is valid for any slotted protocol without carrier sensing, and for any station configuration.

Lemma 2

The maximum point-to-point throughput of slotted ALOHA in a bidirectional bus network with a deterministic station configuration satisfies the following bound.

$$S_a^* \geq \left\{ \frac{N}{C_a(N)+1} \right\} \left(\frac{1}{1+a} \right) \frac{1}{e} \qquad \text{for } a \geq 0 \qquad (22)$$

where $C_a(N) \leq (N-1)$ is the maximum number of stations which can be located within the vulnerable interval-set associated with any given transmission.

■

Proof of Lemma 2

The probability of success is lower bounded as follows.

$$P_a(G) \geq \left\{ 1 - (1+a) \frac{G}{N} \right\}^{C_a(N)} \qquad \text{for } a \geq 0 \qquad (23)$$

The throughput is then $S_a(G) = G.P_a(G)$. Maximizing $S_a(G)$ with respect to G, we obtain

$$S_a^* \geq \left(\frac{1}{1+a} \right) \left\{ \frac{N}{C_a(N)+1} \right\} \left\{ 1 - \frac{1}{C_a(N)+1} \right\}^{C_a(N)} \qquad \text{for } a \geq 0 \qquad (24)$$

It can be verified that

$$\left\{1 - \frac{1}{C_a(N)+1}\right\}^{C_a(N)} \geq \frac{1}{e} \qquad \text{for } C_a(N) \geq 0 \qquad (25)$$

Hence, we have

$$S_a^* \geq \left\{ \frac{N}{C_a(N)+1} \right\} \left(\frac{1}{1+a} \right) \frac{1}{e} \qquad \text{for } a \geq 0 \qquad (26)$$

Q.E.D.

For $0 \leq a \leq 1$, there is no channel reuse. Hence, $C_a(N) = (N-1)$, and the bound in (26) is satisfied with equality.

For a bus network with regularly spaced stations, one can verify that

$$C_a(N) = \min \left\{ (N-1) , \left(4 \left\lceil \frac{N-1}{a} \right\rceil - 3 \right) \right\} \qquad \text{for } a \geq 0 \qquad (27)$$

$C_a(N)$ is a monotonically decreasing function of a. For $0 \leq a < (N-1)$, it decreases from (N-1) to 1.

For $a \geq (N-1)$, no stations are sufficiently close to each other to allow any simultaneous transmissions that are totally destructive. Moreover, there can be at most one station located within the potentially destructive interval of any transmission. Hence, $C_a(N) = 1$ for $a \geq (N-1)$. An example of such a very high speed bidirectional bus network with N=8 stations is shown in Figure 7.

Theorem 2

In a bidirectional bus network with N equally spaced stations, the maximum point-to-point throughput of slotted ALOHA is bounded as follows.

For $0 \le a < (N-1)$,

$$\left(\frac{N}{C_a(N)+1}\right)\left(\frac{1}{1+a}\right)\frac{1}{e} \le S_a^* \le \left(\frac{N}{N-1}\right)\left(\frac{a}{2}\right)^+\left(\frac{1}{1+a}\right)\frac{1}{e} \qquad (28)$$

where

$$(*)^+ = \max(*,1) \qquad (29)$$

For $a \ge (N-1)$,

$$\frac{1}{2}\left(\frac{N}{1+a}\right) \le S_a^* \le \left(\frac{N}{1+a}\right) \le 1 \qquad (30)$$

■

Note that the first factor in the lower bound for $0 \le a < (N-1)$ represents the gain of maximum throughput over that of slotted ALOHA without channel reuse. The gain increases from 1 at $a=0$ to N/2 at $a=(N-1)$, and then remains to be N/2 for greater values of a. For $a \ge 2$, the upper bound in (28) remains below 1/(2e). Also note that the lower bound on S_a^* for $a \ge (N-1)$ can be as large as 1/2. This is the case when $a=(N-1)$.

Proof of Theorem 2

From Lemma 1 and Lemma 2, we have

$$\left(\frac{N}{C_a(N)+1}\right)\left(\frac{1}{1+a}\right)\frac{1}{e} \le S_a^* \le \left(\frac{N}{1+a}\right)^- \le 1 \qquad \text{for } a \ge 0 \qquad (31)$$

These bounds are valid for any station configuration. When the stations are regularly spaced, we can tighten the bounds as follows.

The upper bound in (28), for $0 \le a < (N-1)$, is derived as follows. As in Lemma 1, we consider only the totally vulnerable regions. Let $M_a(N)$ be the number of stations located in the totally vulnerable region of a given transmitting station, not including the transmitting station itself.

Without accounting for end effects, the length of a totally vulnerable interval is $(2/a)^-$. Given that adjacent stations are separated by a distance of 1/(N-1), we have

$$M_a(N) \geq (N-1)\left(\frac{2}{a}\right)^- -1 \qquad (32)$$

Then, the probability of success is bounded as follows.

$$P_a(G) \leq \left\{1-(1+a)\frac{G}{N}\right\}^{M_a(N)} \qquad \text{for } 0 \leq a < (N-1) \qquad (33)$$

For fixed a and large N, we obtain the following bound on throughput.

$$S_a(G) \leq G\, e^{-\{(2/a)^-\}(1+a)G(N-1)/N} \qquad \text{for } 0 \leq a < (N-1) \qquad (34)$$

whose maximum with respect to G gives the upper bound in (28).

The lower bound in (30), for $a \geq (N-1)$, is derived, as follows, by taking into consideration the absence of the potentially destructive regions for some transmissions. For large a, many stations fall outside the vulnerable interval-set. In the network of interest, a transmission can result in a destructive collision only if it is overlapped by another transmission at the receiving point in space and time. For each transmission, there are (N-1) potential receiving points. For each transmission, at least half of the potential receiving points are not vulnerable to any collision (See Figure 7). Any of the remaining receiving points will suffer a destructive collision only if the one and only one station within the corresponding potentially vulnerable region is also transmitting a packet during the slot. By symmetry, each station is active during a slot with probability $(1+a)G/N$. Thus, the probability of success is

$$P_a(G) \geq \frac{1}{2} + \frac{1}{2}\left\{1 - (1+a)\,\frac{G}{N}\right\} \qquad \text{for } a \geq (N-1) \qquad (35)$$

It follows that

$$S_a(G) \geq \left\{1 - (1+a)\,\frac{G}{2N}\right\} G \qquad \text{for } a \geq (N-1) \qquad (36)$$

Maximizing with respect to G, we obtain the lower bound in (30).

Q.E.D.

When the stations are not regularly spaced, the maximum throughput for $a \geq (N-1)$ may be even higher because a smaller fraction of the potential receiving points are vulnerable to collisions. In Figure 8, we show a network, in which the fraction of potentially destructive collisions is small. It is obvious from the figure that the maximum throughput for slotted ALOHA in this network is very close to the upper bound in (30). It is natural to wonder, for a given number of stations on the bus, which station configuration offers the maximum throughput. We leave this open question for further research.

3 UNSLOTTED ALOHA

In this section, we study the unslotted ALOHA protocol in a bidirectional bus network. Let positions on the bus be specified with respect to the center of the bus, so that any position x must fall within the range [-1/2, 1/2]. We let all functions of distance be defined for this range. We summarize below our unslotted ALOHA model.

- Asynchronous transmissions;

- Offered traffic including retransmissions is a Poisson process;
- Statistical equilibrium;

3.1 Conventional Analysis

We review in this section the conventional analysis of the unslotted ALOHA protocol for both broadcast and point-to-point communications.

3.1.1 Broadcast Communications

Conventional analysis of the unslotted ALOHA protocol without channel reuse is based on the assumption that a transmission is successful only if there are no other transmissions within a vulnerable period of $2(1+a)$. This time interval is chosen for the worst case in which an end-station broadcasts a packet to every other station. The conventional vulnerable region for unslotted ALOHA with broadcast communications is shown in Figure 9.

Let G be the constant offered traffic rate, in packets per second, including retransmissions. Then, the probability of success is

$$P_a = e^{-2(1+a)G} \tag{37}$$

The broadcast throughput is given by

$$S_a = G\,e^{-2(1+a)G} \tag{38}$$

whose maximum with respect to G is

$$S_a{}^* = \left(\frac{1}{1+a}\right)\frac{1}{2e} \tag{39}$$

Note that $S_a{}^*$ vanishes as a increases to infinity, with a factor of 2 faster than that of unslotted ALOHA.

When channel reuse is taken into consideration, we obtain different results.

3.1.2 Point-to-Point Communications

Allowing for channel reuse, the vulnerable region for point-to-point communications is actually smaller than that for broadcast communications. As shown in Figure 10, the space-time area of a point-to-point vulnerable region is always equal to 2, independently of $\mathbf{a}$. It is well known that the throughput of unslotted ALOHA for point-to-point communications is Ge^{-2G}, whose maximum is 1/(2e).

3.2 Space-Time Characteristics for Broadcast Communications

For broadcast communications, the vulnerable region for a transmission is shown in Figure 11. Let $V_a(x)$ be its area. It is easy to verify that

$$V_a(x) = 2 + \mathbf{a}/2 + 2\mathbf{a}x^2 \tag{40}$$

$V_a(x)$ is symmetric about, and minimized at, x=0. Hence, we could expect the throughput performance to be a function of x, and to be largest in the middle of the bus. Since $V_a(x)$ increases with $\mathbf{a}$, and is less than $(2+\mathbf{a}) \leq 2(1+\mathbf{a})$, the broadcast throughput of the unslotted ALOHA protocol indeed degrades as $\mathbf{a}$ increases, but more slowly than that under the conventional assumption.

3.3 Maximum Broadcast Throughput

We show that, if the offered load density is uniform along the bus, the maximum throughput density depends on the location along the bus. To achieve a uniform throughput density, the offered load density has to vary along the bus. In any case, the maximum aggregate throughput degrades with the ratio of end-to-end propagation delay to packet transmission time.

In Theorem 3, we show a differential equation relating the throughput density to the offered traffic rate density. We then obtain complete solutions for two special cases.

Theorem 3

Consider unslotted ALOHA in a bidirectional bus network. Let $g(x)$ be the offered traffic rate density at location $x \in [-1/2, 1/2]$, in packets per second. The throughput density at location x for broadcast communications is

$$S_a(x) = g(x)\, P_a(x) \qquad \text{for } x \in [-1/2,\ 1/2] \tag{41}$$

where $P_a(x)$ is the spatial density of the probability of success.

$S_a(x)$ is the solution to the following differential equation.

$$S_a'(x) g(x) = S_a(x) \left\{ g'(x) - g(x) h_a(x) \right\} \tag{42}$$

where

$$h_a(x) = 2a \left\{ \int_{-1/2}^{x} g(z)\, dz - \int_{x}^{1/2} g(z)\, dz \right\} \tag{43}$$

and $f'(x)$ denotes the derivative of a function, $f(x)$, with respect to x.

■

Proof of Theorem 3

Let $k_a(x,z)$ be the temporal length of the vulnerable region at location z when the transmission originates at location x. For broadcast communications, as shown in Figure 11,

$$k_a(x,z) = 2(1+a|z-x|) \tag{44}$$

The spatial density for the probability of success is

$$P_a(x) = \exp\left\{- \int_{-1/2}^{1/2} k_a(x,z)\, g(z)\, dz\right\} \tag{45}$$

Taking the derivative of (41), multiplying each side by g(x), and using (45), we obtain

$$S_a'(x)g(x) - g'(x)S_a(x) = g(x)S_a(x) \int_{-1/2}^{1/2} k_a'(x,z)g(z)\, dz \tag{46}$$

It is easy to verify that

$$k_a'(x,z) = \begin{cases} -2a & \text{if } x \leq z \\ +2a & \text{if } x > z \end{cases} \tag{47}$$

It follows that (42) holds with $h_a(x)$ defined below.

$$h_a(x) = \int_{-1/2}^{1/2} k_a'(x,z)\, g(z)\, dz = 2a \left\{ \int_{-1/2}^{x} g(z)\, dz - \int_{x}^{1/2} g(z)\, dz \right\} \tag{48}$$

Q.E.D.

We apply the above theorem to two special cases, as Abramson did in [8] for a packet radio broadcasting network with capture.

3.3.1 Case #1: Constant Offered Load Density

Suppose that g(x) is constant, such that

$$g(x) = G \qquad \text{for } x \in [-1/2,\ 1/2] \tag{49}$$

From (43), we have

$$h_a(x) = 4aGx \tag{50}$$

From (42), (49), and (50), we obtain the following differential equation.

$$S_a'(x) = -4aGxS_a(x) \tag{51}$$

Solving (51), we obtain the broadcast throughput density of the unslotted ALOHA as follows.

$$S_a(x) = G\, e^{-(2+a)G} \left\{ e^{-2aG(x^2-1/4)} \right\} \tag{52}$$

Note that for a given G, $S_a(x)$ is minimized at the ends and maximized at the center of the bus. It follows from (52) that

$$G\, e^{-(2+a)G} \le S_a(x) \le G\, e^{-(2+a/2)G} \tag{53}$$

The aggregate throughput is

$$S_a(G) = \int_{-1/2}^{1/2} S_a(x)\, dx \tag{54}$$

Note that we have explicitly indicated in (54) the dependence of the aggregate throughput on G.

We can write

$$S_a(G) = G\, e^{-(2+a/2)G} \left\{ \frac{\pi}{2aG} \right\}^{1/2} \operatorname{erf}\left(\left\{ \frac{aG}{2} \right\}^{1/2} \right) \tag{55}$$

where erf(*) is the following standard error function:

$$\operatorname{erf}(y) = \frac{2}{\sqrt{\pi}} \int_0^y e^{-w^2}\, dw \tag{56}$$

For any given $a \ge 0$, we can determine the maximum throughput $S_a{*}$, defined as follows.

$$S_a^* = \max_G \{S_a(G)\} \quad (57)$$

From (53) and (54), we obtain the following bounds.

$$\left(\frac{1}{1+a/2}\right)\frac{1}{2e} \le S_a^* \le \left(\frac{1}{1+a/4}\right)\frac{1}{2e} \quad (58)$$

Note that for large a, the lower bound is twice the maximum broadcast throughput derived by conventional analysis.

In Figure 12, we show S_a^* and its bounds. We have included the result of Case #2 and that of conventional analysis for comparison.

3.3.2 Case #2: Constant Throughput Density

Suppose that $S_a(x)$ is independent of location, such that

$$S_a(x) = Q_a \qquad \text{for } x \in [-1/2,\ 1/2] \quad (59)$$

This corresponds to the interesting case where all stations have the same throughput. We assume that g(x) is symmetric about the center of the bus. Thus,

$$g(x) = g(-x) \quad (60)$$

Taking the derivative of (43) and using (60), we obtain

$$h_a'(x) = 4a\,g(x) \quad (61)$$

From (42) and (59), we have

$$g'(x) = h_a(x)g(x) \quad (62)$$

Taking the derivative of (62), multiplying each side by g(x), and using (61), we obtain

$$g''(x)g(x) = 4a\,g^3(x) + \{g'(x)\}^2 \quad (63)$$

Solving the above differential equation, using (60), we obtain

$$g(x) = \frac{b^2}{2a\cos^2(bx)} = \frac{b^2}{2a}\sec^2(bx) \quad (64)$$

for some constant b.

Define R as follows.

$$R = \pi/(2b) \tag{65}$$

Note that g(x) is unbounded if

$$|x| \geq R \tag{66}$$

If $b > \pi$, then $R < 1/2$, and (45) implies that $P_a(x) = 0$ for $x \in [-1/2, 1/2]$. It follows that S_a can only be zero. For a given b, [8] defines the *Sisyphus Distance* as the value of x with which g(x) in (64) becomes unbounded. It does not appear to have any physical meaning in this case (without capture), as b is an arbitrary parameter. In the analysis below, b is always smaller than π, so that $R > 1/2 \geq x$.

To evaluate $P_a(x)$, we make use of the following indefinite integral, which can be derived by means of integration by parts.

$$\int x \sec^2(bx)\, dx = \frac{x}{b}\tan(bx) + \frac{1}{b^2}\ln\left\{\cos(bx)\right\} \tag{67}$$

Using (44), (45), (64), and (67), we obtain

$$P_a(x) = \exp\left\{-b\{\tan(b/2)\}\left(1 + \frac{2}{a}\right) - \ln\{\sec^2(bx)\}\right\} \tag{68}$$

From (41), (64), and (68), one obtain

$$S_a(b) = Q_a = \frac{b^2}{2a}\exp\left\{-\frac{b}{a}\{\tan(b/2)\}(a+2)\right\} \tag{69}$$

where $S_a(b)$ is the aggregate throughput as a function of b.

For any given $a \geq 0$, we can determine the maximum aggregate throughput, S_a^*, defined as follows.

$$S_a^* = \max_b \left\{S_a(b)\right\} \tag{70}$$

Taking the derivative of (70) with respect to b, and setting it to zero, we obtain

$$\left(\frac{b}{2}\right)^2 \tan^2\left(\frac{b}{2}\right) + \left(\frac{b}{2}\right) \tan\left(\frac{b}{2}\right) + \left(\frac{b}{2}\right)^2 = \left(\frac{a}{a+2}\right) \tag{71}$$

Equation (71) can be solved numerically to determine the value of b which maximizes $S_a(b)$ in (69).

Making use of the fact that tan(b/2) > (b/2), we obtain from (69) the upper-bound in the following expression.

$$\left(\frac{1}{1+a}\right)\frac{1}{2e} \leq S_a^* \leq \left(\frac{1}{1+a/2}\right)\frac{1}{2e} \tag{72}$$

The lower-bound in (72) follows from the fact that the vulnerable region considered in the analysis is smaller than that assumed in the conventional analysis.

We show in Figure 12 the behavior of S_a^* as a function of a. The optimal offered load density, g*(x), which is obtained from (64) with the optimal value of b, is shown in Figure 13. Note that g*(x) decreases with increasing value of a. As g*(x) is proportional to the number of retransmissions, this confirms the observation in [11].

4 CONCLUSION

Channel reuse is part and parcel of many contention-based multiple access protocols. In this paper, we have evaluated the throughput performance of slotted and unslotted ALOHA in a bidirectional bus network by giving special attention to the inherent channel reuse characteristics of the protocols. We have particularly examined the behavior of the ALOHA protocols when propagation delays are much larger than the packet transmission time. We have shown that conventional analysis sometimes overestimates the maximum throughput by neglecting the effect of propagation delay, and sometimes underestimates the maximum throughput by not considering channel reuse.

For point-to-point communications in a bidirectional bus network with probabilistic station configuration, the maximum throughput for slotted ALOHA degrades below that of unslotted ALOHA when propagation delay is large, but remains above 1/(3e). When the station configuration is deterministic, the maximum throughput for slotted ALOHA can exceed the classic limit of 1/e.

For unslotted ALOHA with broadcast communications, we have shown that, if the offered load density is uniform along the bus, the maximum throughput achievable by a station varies along the bus, and is maximized at its center. To achieve a uniform throughput density, the offered load density has to vary along the bus. We have derived the optimal profile of the offered load density. The maximum throughput of unslotted ALOHA is higher for the case with constant offered load density than for the case with constant throughput density. The latter case is the one of practical interest.

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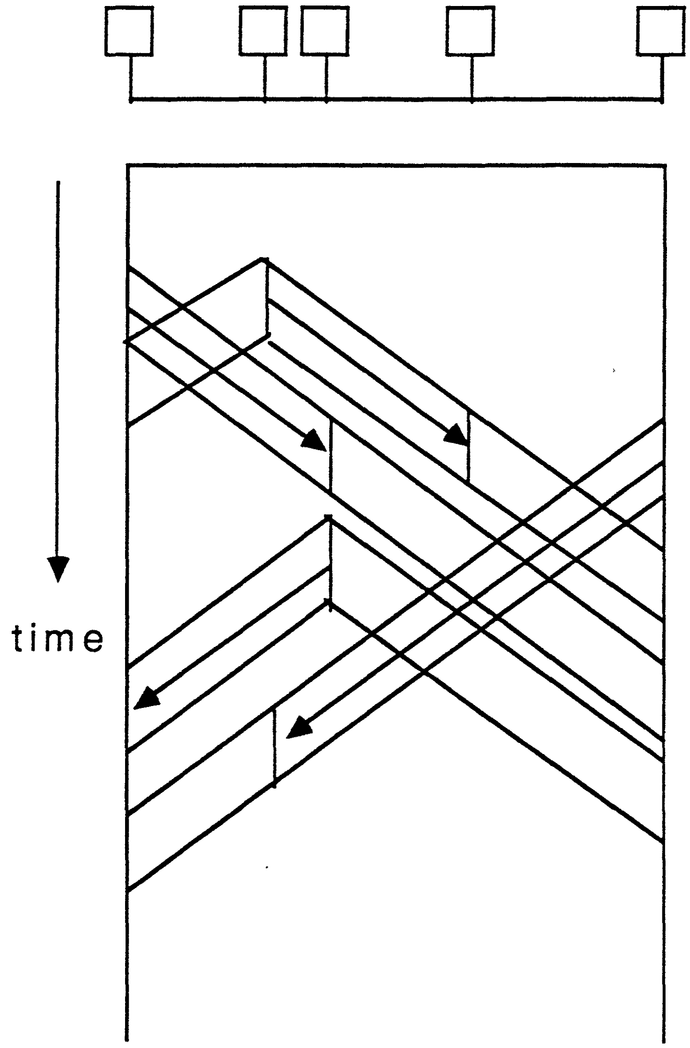

Figure 1: Channel Reuse

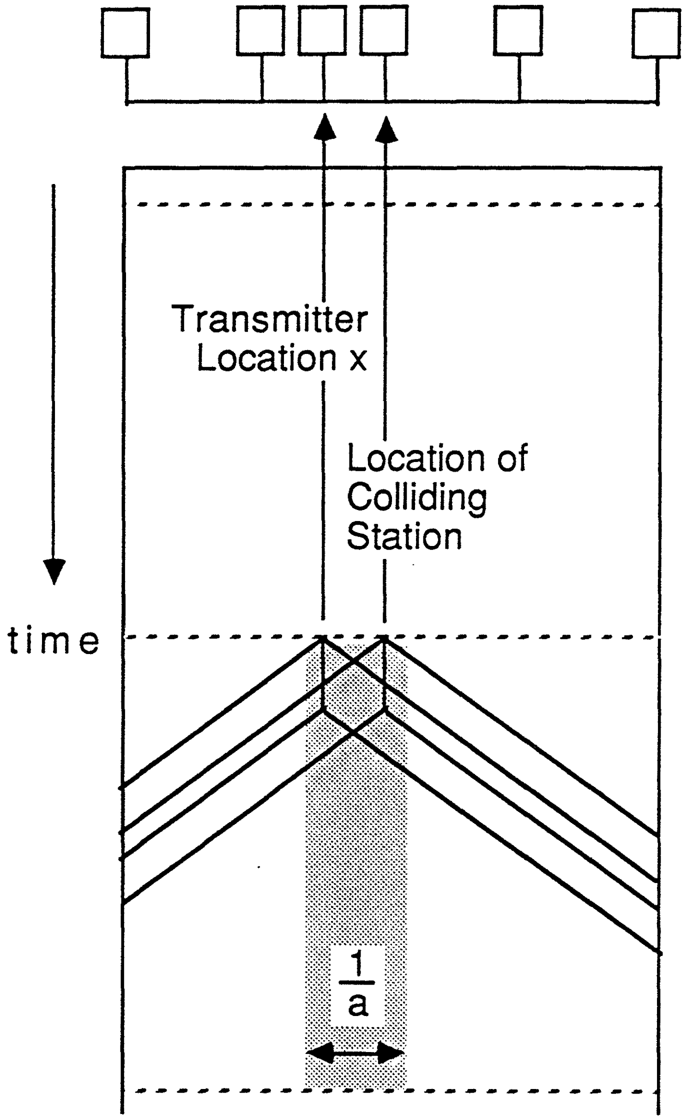

Figure 2: Totally Destructive Collisions

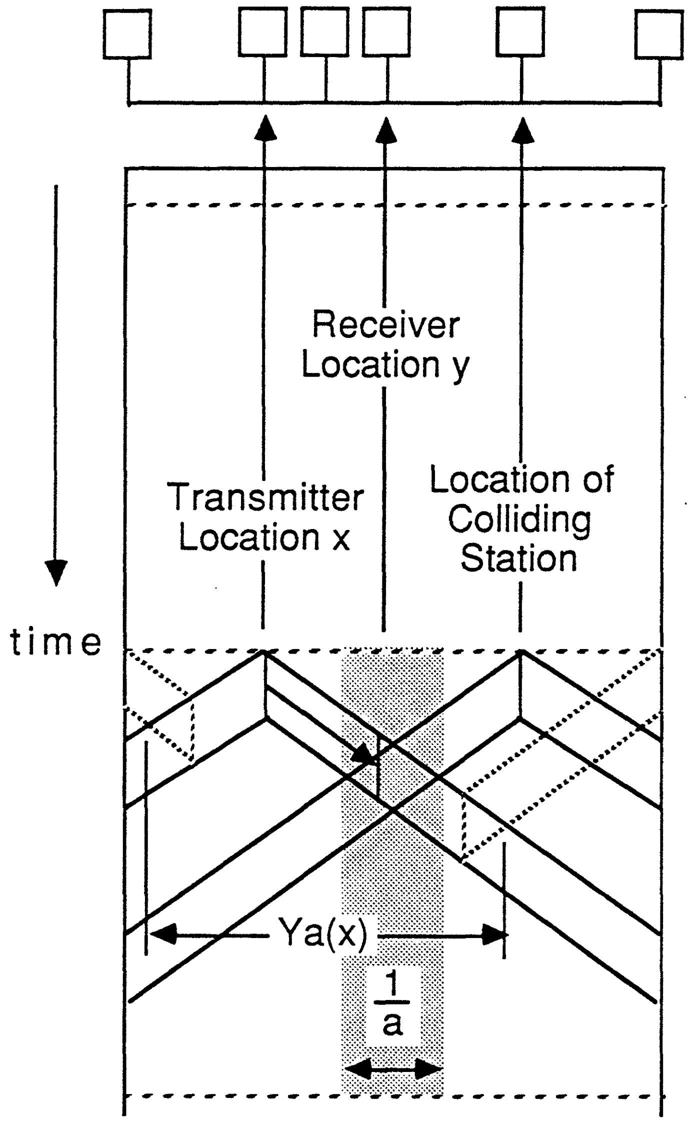

Figure 3: Potentially Destructive Collisions

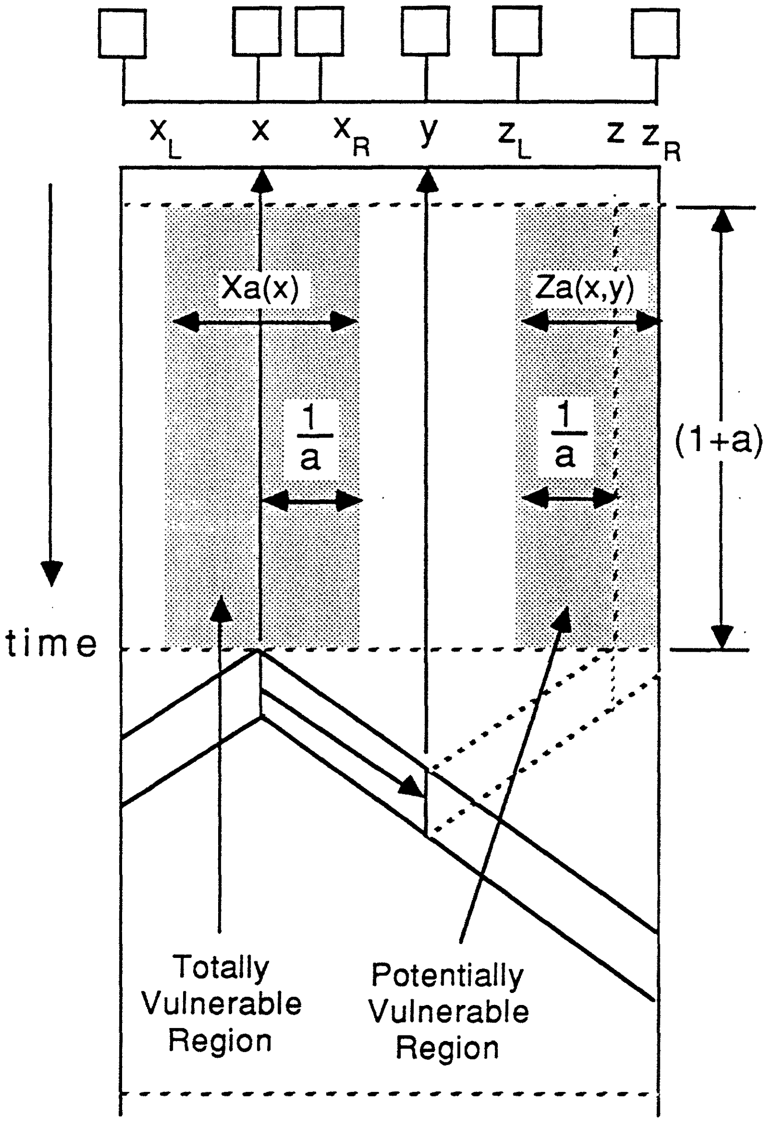

Figure 4: Vulnerable Regions for Slotted ALOHA

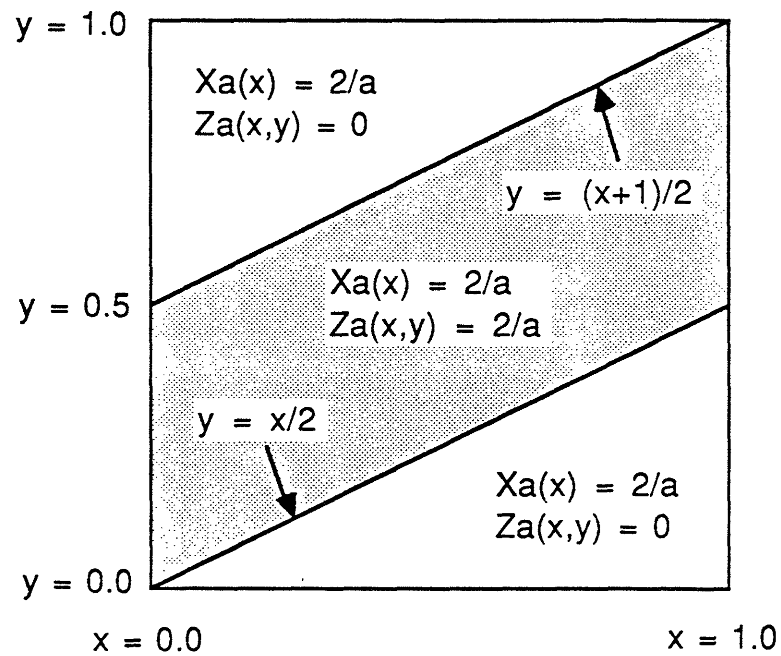

Figure 5: Vulnerable Intervals for a >> 1

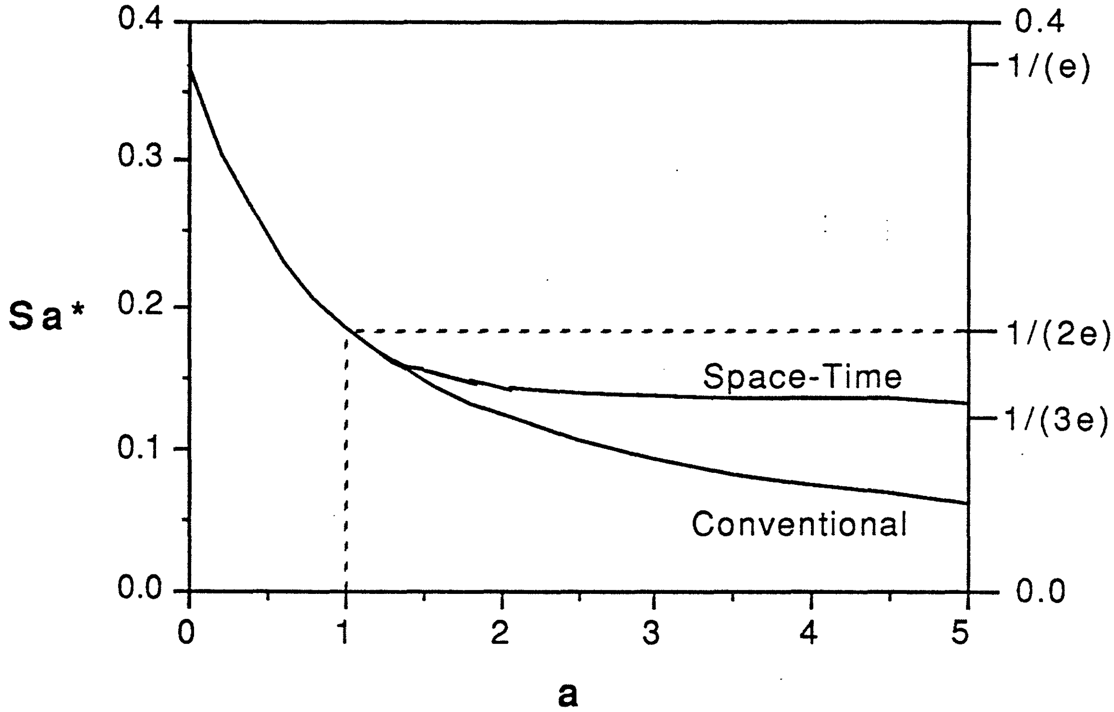

Figure 6: Maximum Point-to-Point Throughput for Slotted ALOHA

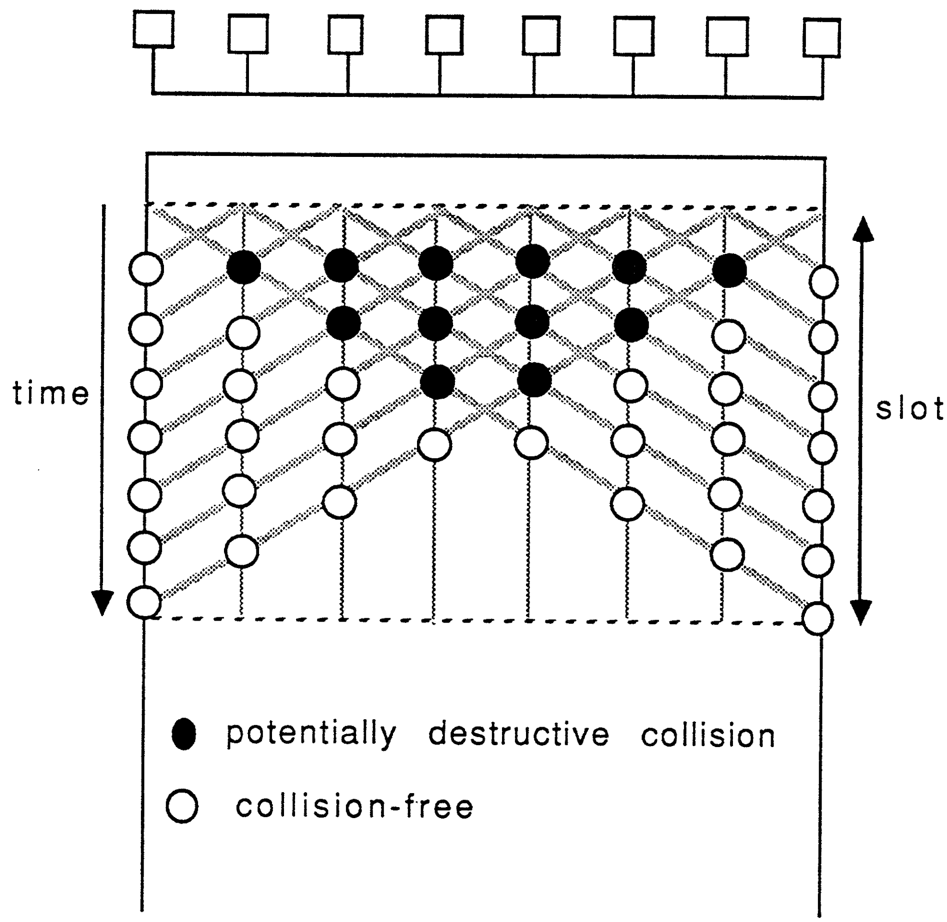

Figure 7: A Very High Speed Regular Bus Network

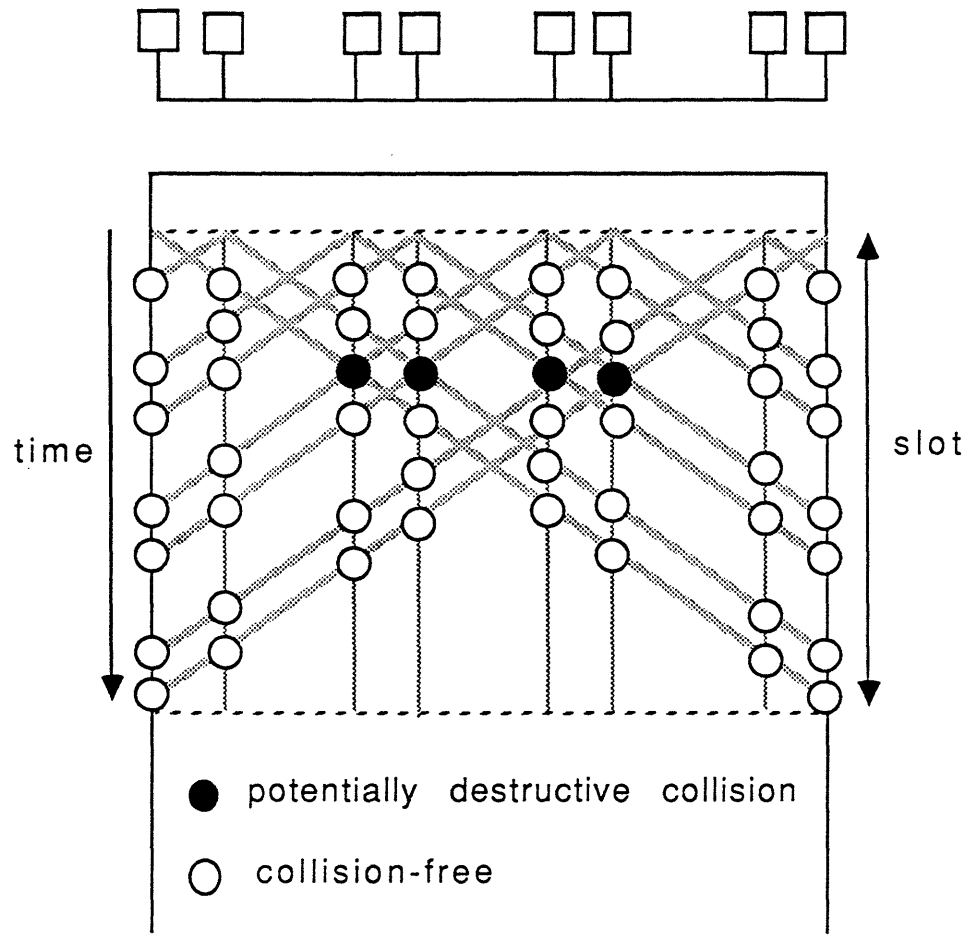

Figure 8: A Network with Few Destructive Collisions

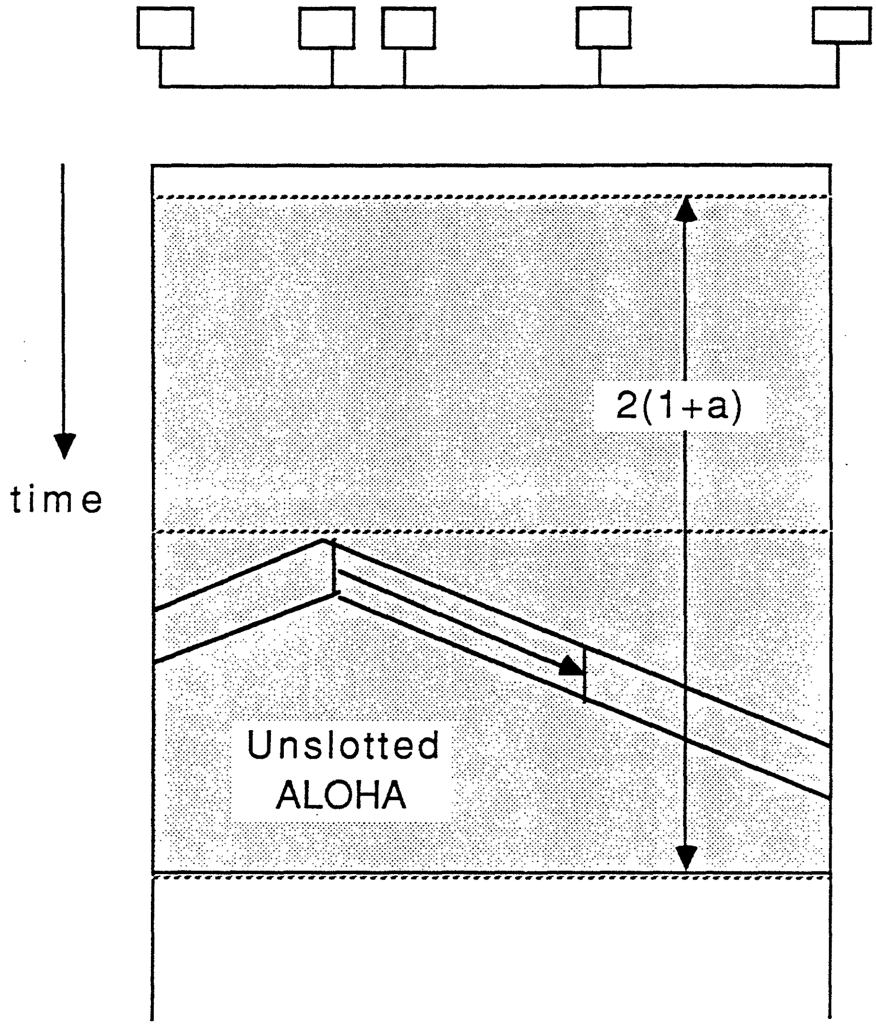

Figure 9: Conventional Vulnerable Region

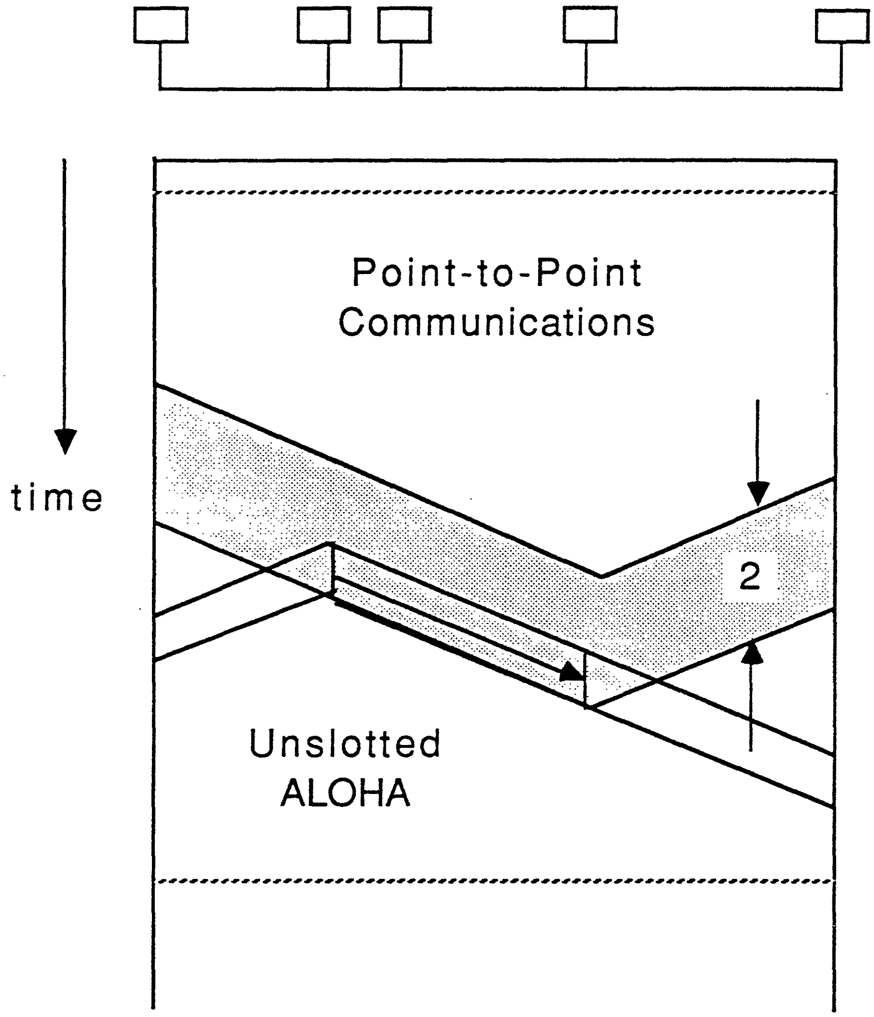

Figure 10: Vulnerable Region for Point-to-Point Communications

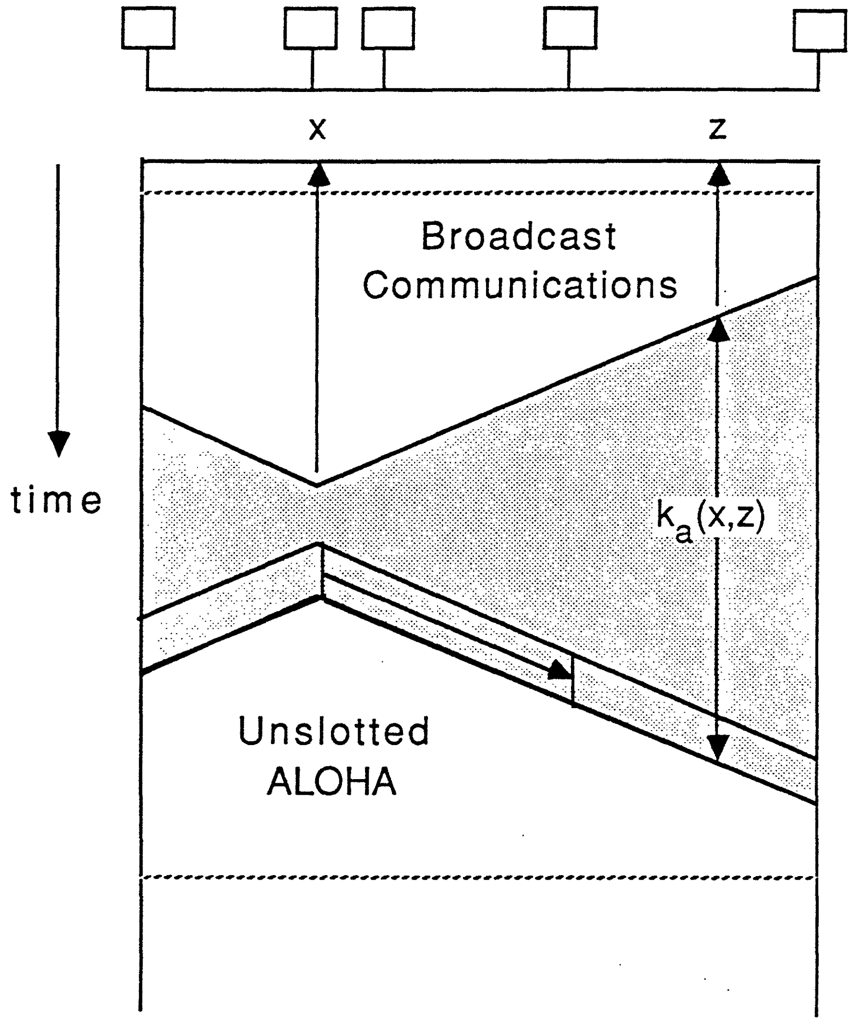

Figure 11: Vulnerable Region for Broadcast Communications

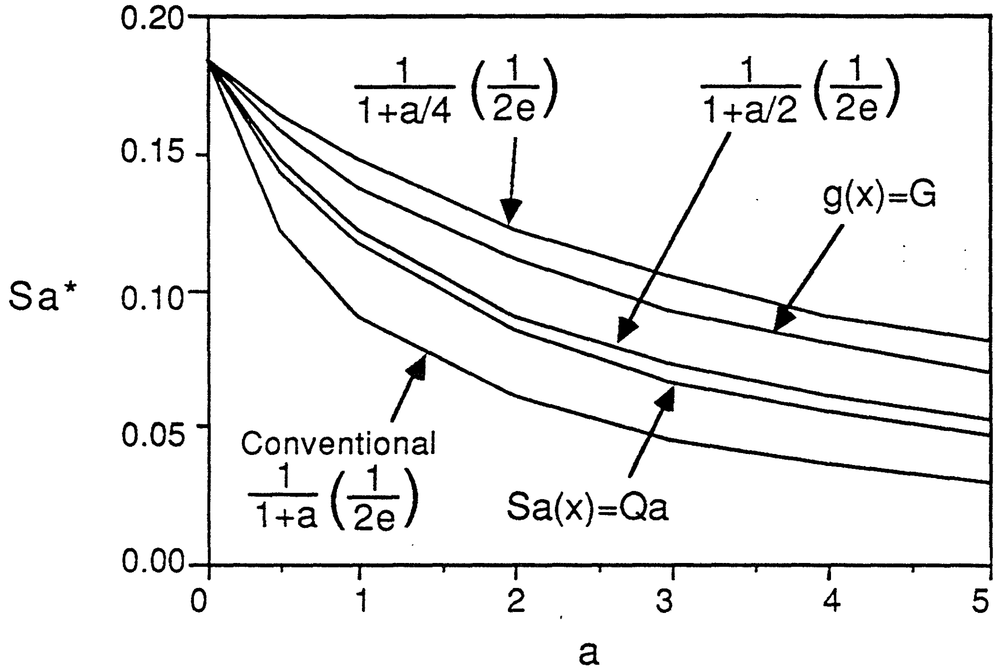

Figure 12: Maximum Broadcast Throughput for Unslotted ALOHA

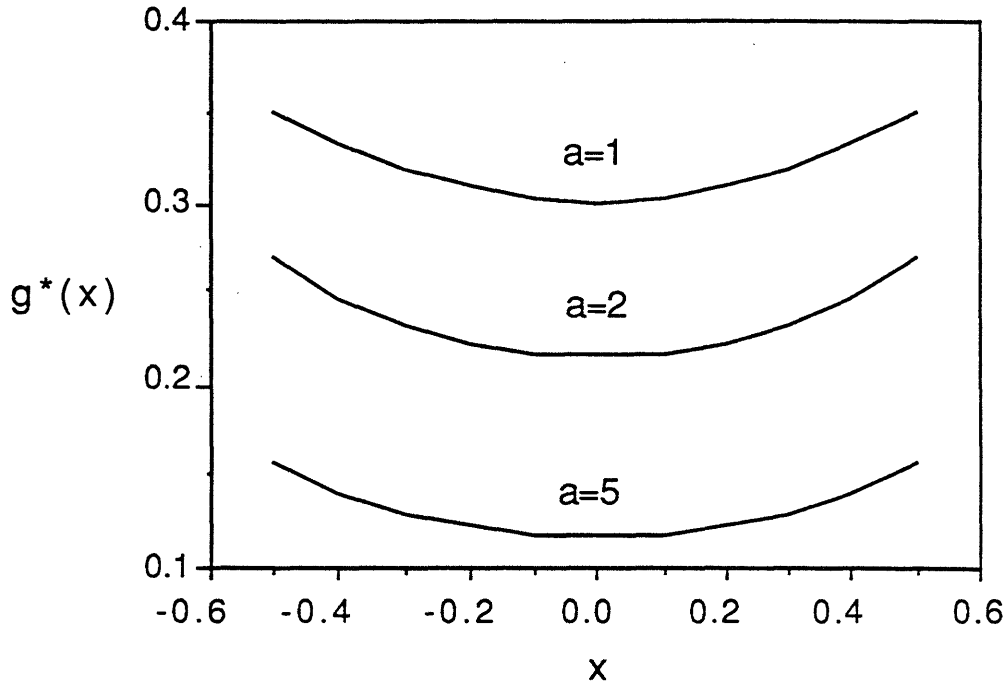

Figure 13: Optimal Offered Load Density