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PREDICTING THE TIME RATE OF SUPPLY FROM A PETROLEUM PLAY*

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1. Introduction

The time rate of additions to petroleum supply generated by exploratory effort is the principal focus of this study. Forecasting supply from discovery of new deposits differs from that drawn from known deposits both in the type of data available for generating predictions and in the substantially greater uncertainties characteristic of petroleum exploration as opposed to petroleum production from known deposits.

Our objective is to build an intertemporal model of exploration in a petroleum "play" which links together geostatistical data, probabilistic models of exploration, discovery, and economics, a model which can be used to generate probabilistic predictions of the time rate of additions to supply from new discoveries as a function of policy choice; i.e. of future prices, costs, fiscal and regulatory regimes.

A schematic representation of such a model is shown in Figure 1[†]. The model is composed of two distinct types of submodels: first, submodels describing the generation of physical observables -- drilling successes and failures and sizes of discovered deposits -- as a response to inputs of exploration effort - predrilling exploratory effort and exploratory wells drilled; and second, submodels of the economic processes which influence decisions as to how many wells to drill and when, and as to what known deposits are to be developed and produced at what rate. The output of the drilling and of the

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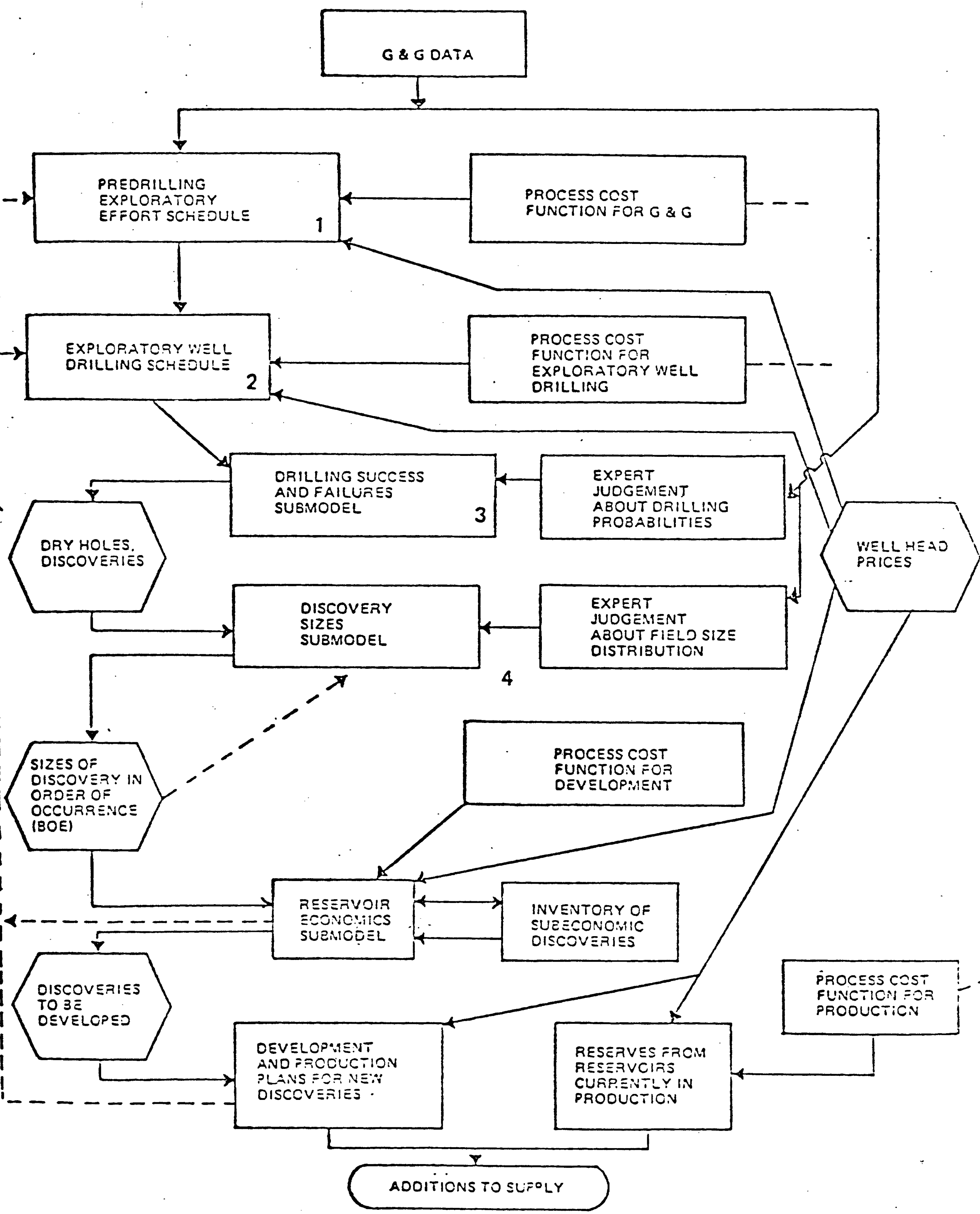
[†] From "The Interface Between Geostatistical Modelling of Oil and Gas Discovery and Economics" (forthcoming in Mathematical Geology).

Figure 1


G & G DATA
PREDRILLING EXPLORATORY EFFORT SCHEDULE
1
PROCESS COST FUNCTION FOR G & G
EXPLORATORY WELL DRILLING SCHEDULE
2
PROCESS COST FUNCTION FOR EXPLORATORY WELL DRILLING
DRILLING SUCCESS AND FAILURES SUBMODEL
3
EXPERT JUDGEMENT ABOUT DRILLING PROBABILITIES
DRY HOLES, DISCOVERIES
WELL HEAD PRICES
DISCOVERY SIZES SUBMODEL
EXPERT JUDGEMENT ABOUT FIELD SIZE DISTRIBUTION
4
SIZES OF DISCOVERY IN ORDER OF OCCURRENCE (BOE)
PROCESS COST FUNCTION FOR DEVELOPMENT
RESERVOIR ECONOMICS SUBMODEL
INVENTORY OF SUBECONOMIC DISCOVERIES
DISCOVERIES TO BE DEVELOPED
PROCESS COST FUNCTION FOR PRODUCTION
DEVELOPMENT AND PRODUCTION PLANS FOR NEW DISCOVERIES
RESERVES FROM RESERVOIRS CURRENTLY IN PRODUCTION
ADDITIONS TO SUPPLY

discovery sizes submodels is the desired probabilistic forecast of future returns to exploratory effort in the form of drilling successes and failures and sizes of discoveries in order of occurrence.

The reservoir economics submodel brings the output of the drilling and discovery sizes submodels together with the economics of production. Once a normative economic criterion for choice is specified, a minimum economic reservoir size (MERS) can be computed -- the smallest deposit that is economical to produce at a given point in time. The MERS is an economic gate, monitoring the flow of new discoveries into development and production. Discoveries which are currently uneconomical enter an inventory of subeconomic discoveries; a "favorable" change in prices, costs, and/or fiscal regime may induce members of this inventory to be developed and produced.

Essential attributes of exploration and discovery are captured by use of a probabilistic model of discovery studied by Barouch and Kaufman [2,3]. The setting is the North Sea and we use cost functions constructed by Eckbo [5] for this province. Exploratory drilling data and field size data are drawn from Beall [4]. We examine the behavior of drilling strategies and the consequent time rate of supply from an individual petroleum play in the North Sea under specific assumptions about future price and cost patterns and fiscal regimes, which are assumed known with certainty. The North Sea is considered here as a "price-taker" province, so exploration outcomes do not influence present or future prices; i.e. there is no feedback from the results of exploration to price. Uncertainty enters via the submodels describing exploration. The two key uncertain quantities are the outcomes of drilling exploratory wells --successes and dry holes--and the sizes, measured in barrels of hydrocarbons in place, of discoveries.

In our formulation, the number of exploratory wells drilled at each time-period t _is_ exploratory effort and this variable drives all succeeding submodels. _A priori_ the rate of exploratory effort at each time-period is an uncertain quantity and we specify the joint probability law for rates of effort by forward-looking dynamic optimization; i.e., probabilistic dynamic programming. Once computed, this joint probability law for numbers of wells drilled at each time-period is used together with that for drilling successes and failures to compute the probability law for the times at which the first, second, third, ... discoveries occur. The submodel for discovery sizes is the probability law for discovery sizes in order of discovery, so upon combining it with that for "waiting times" to the first, second, third, ... discoveries, we have a probabilistic description of both sizes of discoveries and the time-periods at which they are made. Each field or reservoir of a given size possesses a prespecified production profile and a superpositioning of these profiles according to times of discoveries of each possible size as dictated by the joint probability law for time and size of the n^{th} discovery, n = 1, 2, ... generates a probabilistic description of the future rate of production at each time-period.

This _is_ the supply function. Its properties depend jointly on the physical attributes of the play,

--the size distribution of fields as deposed by Nature,

--the number of prospects and the number of fields among these prospects,

on how

--drilling successes and failures occur,

--sizes of discoveries unfold,

--production profiles for discoveries are determined,

and on economic attributes, among which are

--a projection of (future) prices per barrel,

--exploratory drilling, development and production costs,

--the fiscal regime in force (taxes, amortization, debt service, royalties, etc.),

--a normative criterion for making exploratory well drilling decisions and development and production decisions.

While our approach to discovery and supply is superficially similar to that adopted by Eckbo, Jacoby and Smith [6], it differs in essential ways; perhaps the most important difference is that ours is an intertemporally dynamic analysis while theirs is static. In particular they assume that the rate of exploratory drilling is fixed for each future time-period and known with certainty, and that drilling successes and failures are Bernoulli-like with known probability of success. In our approach, drilling rates are uncertain quantities with a probability law determined by dynamic optimization, and drilling successes and failures are adaptive as described in section 2.

Hnyilicza and Wang [7]* study a model for intertemporal supply formally almost identical to that presented here. Their analysis is based on a computation scheme developed by the present authors and described in subsequent sections. However, their treatment of supply in the North Sea Jurassic Central play differs: it incorporates uncertainty about drilling successes and failures alone. Uncertainty about sizes of fields discovered and consequent effects on probabilities for future discoveries are ignored and marginal expectations of discovery sizes in order of occurrence (cf. formula (2.6) of section 2) are adopted as certainty equivalents; i.e., sizes of all discoveries in order of occurrence are assumed to be known with certainty at the outset

*Revised and extended version by J.W. Wang.

of the planning horizon. The only uncertainty remaining is at what well and in what time period they will be discovered. By contrast, a discretized version of the probability law for discovery sizes presented in [2] is employed here, so predictive probabilities for future discoveries are explicitly dependent on past observations. Replacing the joint probability law for discoveries with certainty equivalents greatly reduces computation time at the expense of ignoring an essential feature of the discovery process. This approach may, however, be useful as an approximation. How robust an approximation it is, is a topic for future research.

The economics of discovery and production enters the analysis through a projection of price per barrel for each time period in the planning horizon of time periods $1, 2, \ldots, T$ and corresponding projections of exploratory drilling cost functions, reservoir development and production cost functions and a specification of the tax regime and production profiles for fields of sizes S_1, S_2, and S_3.

The time horizon T is defined to be the last time period at which a wildcat drilling decision can be made. That is, a decision to drill or not can be made at $t=1,2,\ldots T$, but no wildcat drilling may occur at time periods $t > T$. Production decisions are allowed at any time period $t=1,2,\ldots,T$ but not beyond. Hence, the decision to produce a field of size S_i discovered at $t \leq T$ must be made at time period T or earlier. Since there is a time lag between time of discovery and the time when production begins, even when a decision to produce is made at the time of discovery, production may continue past time period T. Production ceases at a time period $T + \tau^*$ or earlier, where τ^* is the time period length of the longest production profile among all possible profiles (plus built-in lag). A finite time horizon introduces unwanted end effects. However, numerical analysis shows that T may be selected large enough to render these end effects negligible.

Table 4 (from Eckbo [5])

Fraction of Total Exploration/Delineation Expenditures, Investment Expenditures, and Recoverable Reserves Occurring In Each Year Following Discovery

Year	Exploration/ Delineation Profile All Fields	Fields < 300 MM Bbl		300 MM Bbl ≤ Fields ≤ 1500 MM Bbl		1500 MM Bbl < Fields	
		Investment Profile	Production Profile	Investment Profile	Production Profile	Investment Profile	Production Profile
1	0.1	0	0	0	0	0	0
2	0.2	0	0	0	0	0	0
3	0.2	.04	0	.04	0	.04	0
4	0.2	.44	0	.12	0	.12	0
5	0.2	.27	.09	.20	.03	.20	.01
6	0.1	.11	.13	.24	.08	.24	.04
7		.08	.15	.16	.10	.16	.06
8		.06	.13	.07	.10	.07	.09
9			.13	.06	.10	.06	.10
10			.11	.06	.10	.06	.10
11			.08	.05	.10	.05	.10
12			.07		.10		.10
13			.06		.08		.10
14			.05		.06		.08
15					.05		.07
16					.04		.05
17					.03		.03
18					.03		.03
19							.02
20							.01
21							.01

Eckbo [5] used Wood-MacKenzie North Sea data to construct typical investment and production profiles for North Sea fields as shown in his Table 14. We use these profiles in our calculations and assume that they remain fixed; i.e., do not change with the time period of discovery of a field. Then the fraction $\delta_{i,t+\tau}$ of a discovery at time period t of size S_i that is produced at time period $t + \tau$ is independent of the time t of discovery and $\delta_{i,t+\tau} = \delta_{i\tau}$ for all t.* If τ_i denotes the number of time periods to termination of production from a field of size S_i, the production profile for it is $S_i\delta_{i0}$, $S_i\delta_{i1}$, ..., $S_i\delta_{i\tau_i}$ for any t.

The process cost formula $C(S_i, t)$ for development and production from a field of size S_i discovered at time period t is expressed in net present value dollars at time period t and is a composite of fixed drilling costs C_0, variable drilling and production costs $a_i + b_iS_i$, production profile δ_{it}, $\delta_{i,t+1}$, ..., $\delta_{i,t+\tau_i}$, and capital investment costs $A_i + B_iS_i$. In place of straightforward use of the investment schedule detailed in Eckbo's Table 14,

*This independence assumption is discussed in more detail in Section 3.

we allocate capital investment costs to each time period t, t + 1, ..., $t + \tau_i$ for a production profile of length τ_i by averaging: $(A_i + B_i S_i)/\tau_i$ per period. A cost inflation factor β, distinguished from the net present value factor α, is introduced and in terms of α, β, and the abovementioned cost and production variables, total net present value of cost at time-period t associated with a discovery of size S_i at t that begins development at $t+\tau$ is

$$C(S_i, t) = \beta^{t-1}\left[C_0 + (\alpha\beta)\{a_i + b_i S_i\}\{\sum_{j=0}^{\tau_i} \delta_{ij}(\alpha\beta)^j\} + \frac{1}{\tau_i}\{A_i + B_i S_i\}\left(\frac{1-(\alpha\beta)^{\tau_i}}{1-(\alpha\beta)}\right)\right]$$

This function is a slightly simplified version of that suggested by Eckbo [5].

While any future price sequence is allowable within the framework of our analysis, we assume that an initial price per barrel p_1 is subject to geometric price inflation by a price inflation factor γ, so that price p_t at t is $p_t = p_1\gamma^{t-1}$. Per-period tax payments are normally computed as a function of production rate and the per-period annual net reservoir operator profit. To simplify computation, we introduce a tax and fiscal cost factor $\gamma < 1$ and represent the net present value of revenue flow at t from a field of size S_i commencing development at t as

$$R(S_i, t) = p_t S_i \sum_{\tau=t}^{t+\tau_i} \delta_{i,\tau-t}(\alpha\gamma)^{\tau-t}$$

The total net present value of a discovery of size S_i made at time period t is thus

$$R(S_i, t) - C(S_i, t) \equiv v(S_i, t).$$

While this model for the net present value of cash-flow from discovery of S_i at t is a reasonable approximation to reality, any non-anticipative model for $v(S_i,t)$, no matter how complicated in accounting detail, fits within the framework for dynamic optimization of drilling effort described subsequently.

We adopt maximization of expected net present value as a normative criterion and assume that once a field of size S_i is discovered, the decision as to when--if at all--to develop and produce it is separable from future drilling aimed at new discoveries and from the past history of exploration. If the interaction between the new discovery, fields already discovered, and possible future discoveries imposed by pipeline network considerations are ignored, this is reasonable. Given this assumption, commencing development at t of a discovery of size S_i made at t is desirable only if $v(S_i, t) > 0$. Optimal times for development to begin are at t*s such that $v(S_i, t^*) \geq \max \{v(S_i, t), v(S_i, t + 1), \ldots, v(S_i, T)\}$ provided that at least one $v(S_i, t)$ in this set is positive; otherwise never. Holding t fixed, a value $S^0(t)$ of field size such that $v(S^0(t), t) = 0$ is the minimum size for which development commencing at t is economically justifiable. In order for it to be optimal to begin development of a discovery of size S at t, it is necessary but not sufficient that $S > S^0(t)$.

An optimal sequential drilling strategy is computed by use of dynamic programming, and probabilities for number of wells drilled at time-period t given the state history for drilling successes and failures and sizes of discoveries prior to time-period t are a by-product.

Figure 2 is an outline of major steps in the computation of probability laws for cumulative amount discovered by time-period t, for cumulative amount produced by time-period t, and for the corresponding rates of discovery and of production when a sequentially optimal drilling strategy is employed. Output consists of

--mean amount and mean of cumulative amount discovered at each time period (Table 2 and Figures 4, 7, and 8)

--mean amount and mean of cumulative amount produced at each time period (Table 1 and Figures 3 and 9)

--mean and standard deviation of number of wells drilled at each time period (Table 3 and Figures 5 and 6)

--marginal probability distribution for number of wells drilled at $t=1,2,...T$ (Table 4)

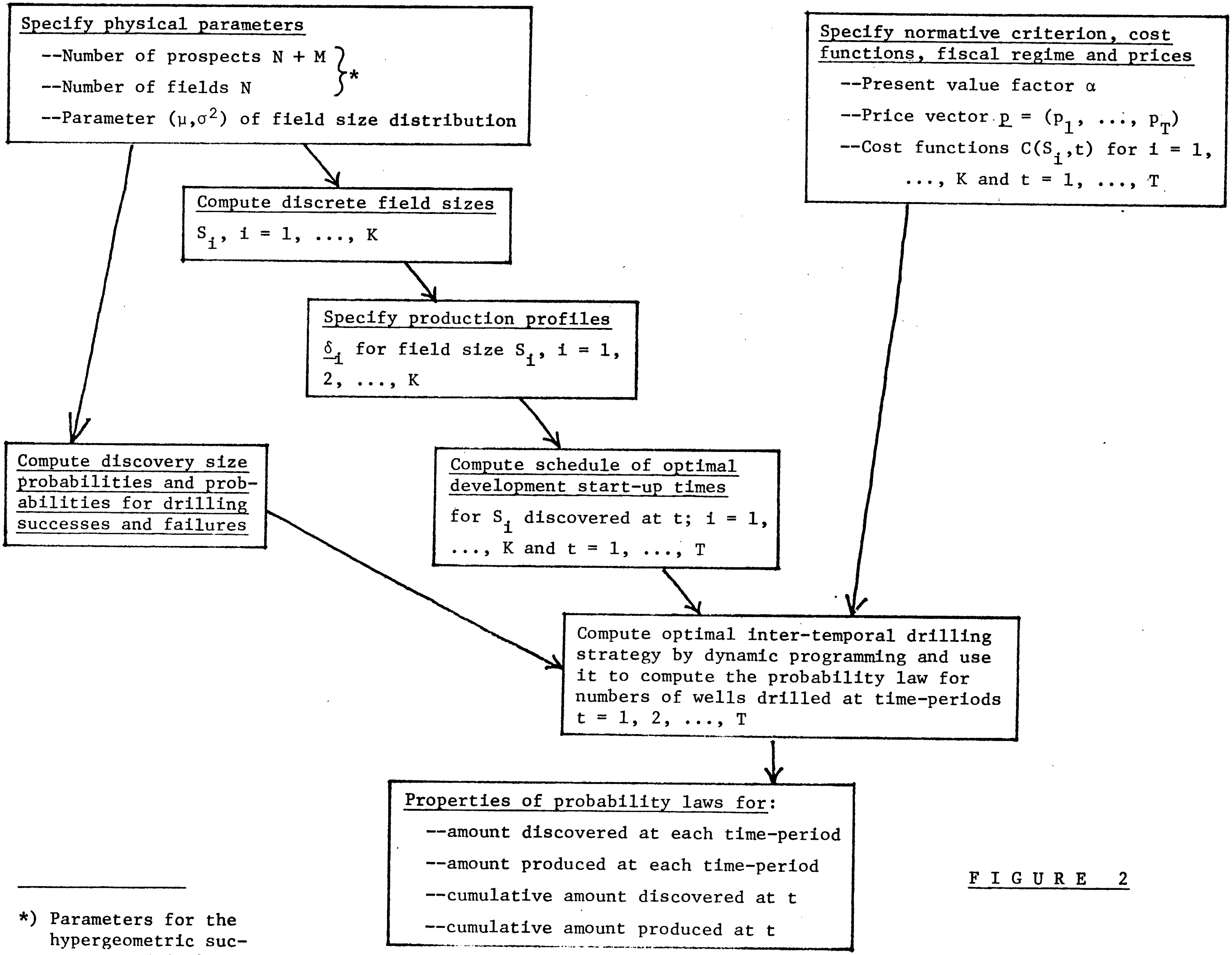
--joint probability distribution for number of wells drilled at t and cumulative number of wells drilled prior to t (Table 5)

--probability distribution for cumulative number of discoveries up to and including t (Table 6).

Graphs displaying the means of amount discovered at time period t, of cumulative amount discovered at t, of amount produced at t, and of cumulative amount produced at t appear in Figures 3, 4, 7, 8 and 9.

Table 5 is generated and displayed because expectations of rate of discovery at each time-period and of the amounts produced at each time-period are functions of the joint probability law for cumulative number $\tilde{w}(t-1)$ of wells drilled at and prior to time-period $t-1$ and number $\tilde{d}(t)$ of wells drilled at time-period t.*

*Henceforth we distinguish a random variable (rv) from a value assumed by it with a tilde; e.g., the rv $\tilde{d}(t)$ takes on a value of $d(t)$.



F I G U R E   2

*) Parameters for the hypergeometric successes and failures.

Description of the numerical output for one of the cases presented in section 7 will set the stage for the mathematical results of sections 2, 3, and 4.

Consider a play with twenty prospects, four of which are fields; which prospects are in fact fields not known with certainty a priori. Drilling successes and failures are generated by hypergeometric sampling from $N + M = 20$ prospects of which $N = 4$ are fields.* The sizes of the N fields as deposed by Nature are generated by drawing independent sample values from a crude discrete approximation to a lognormal distribution with the mean μ of the log of size equal to 5.78 and the variance of σ^2 of log size equal to 6.38. These values of μ and σ^2 are maximum likelihood estimates computed using the discovery process model of Barouch and Kaufman [2,3] applied to sizes of ten discoveries in the North Sea's Jurassic Central play as reported by Beall [4]. The discrete approximation is a partition of the range $(0,\infty)$ for size into three intervals of equal probability. The geometric mean for each of these three intervals is $S_1 = 100$ million barrels, $S_2 = 450$ million barrels, and $S_3 = 1500$ million barrels.

The planning horizon is $T = 20$ years (periods). (Exploratory drilling in fact drops to zero by the ninth period.) Expected discoveries and rates of production are computed for two initial prices per barrel: \$5 and \$12. Price is assumed to increase by 6.6% per year ($\gamma = 1.066$). Eckbo's cost formulation (1.1) is used and costs are assumed to increase by 6.6% per year also ($\beta = 1.066$). Individual components of the cost function (1.1) are in 10^6\$:

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*With no change in the size of the state space, drilling successes and failures can be assumed to be realizations drawn from an infinite sequence of dichotomous exchangeable uncertain quantities if the mixing cdf is indexed by a parameter of dimension 2. Modelling drilling successes and failures as hypergeometric sampling is proposed by Jacoby, Eckbo, and Smith (1977), p. 232.

Exploratory Drilling Cost = \$5

Total Operating Costs[†] = \$18.87 + .004*(Size of field)

Total Investment Costs (not including exploratory drilling)
= \$296.10 + 1.12*(Size of field)

Investment costs are incurred according to the investment profiles shown in Eckbo's Table 14. For each discovery size, total operating costs are spread evenly over the time periods in which production takes place.

Figure 3 displays the expectation of amount produced in each period if an optimal sequential drilling strategy is employed (numerical values appear in Tables 3 and 4 of section 7).At 12\$/BBL all discovered fields of size S_1 = 100 million barrels, S_2 = 450 million barrels, and S_3 = 1500 million barrels are put into production, while at 5\$/BBL fields of size 100 million barrels, even if discovered, are not put into production. Hence expected rates of production at 5\$/BBL are smaller than at 12\$/BBL. Cumulative expected rates of production and cumulative expected discoveries are shown in Figure 4.

While cumulative production behaves differently at 5\$/BBL than at 12\$/BBL, the mean and variance of the underlying size distribution of fields is sufficiently large to induce virtually identical optimal drilling policies. Hence only one graph of expectation of cumulative discoveries is displayed in Figure 4. That is, given the costs, prices, and size distribution employed in this run, production policy is price sensitive, but drilling policy is not.

[†]Operating costs consist of platform operating costs, administrative costs, transportation costs and harbor and terminal operating costs. (See Table 5, in Eckbo.) Investment costs include platform costs, drilling costs, pipeline costs, terminal costs and miscellaneous costs.

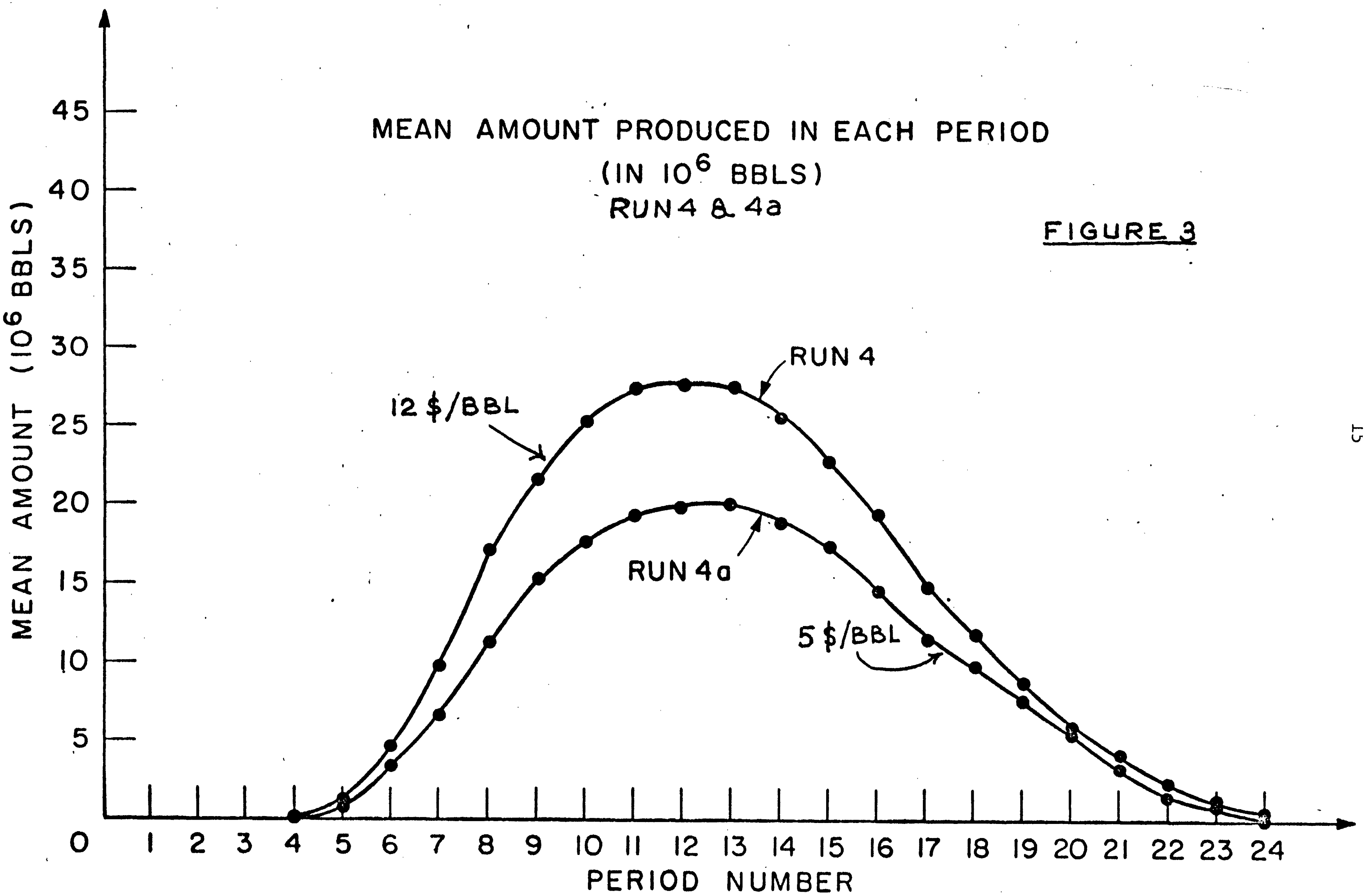

MEAN AMOUNT PRODUCED IN EACH PERIOD
(IN 10^6 BBLS)
RUN 4 & 4a
MEAN AMOUNT (10^6 BBLS)
45
40
35
30
25
20
15
10
5
0
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24
PERIOD NUMBER
RUN 4
12 $/BBL
RUN 4a
5 $/BBL

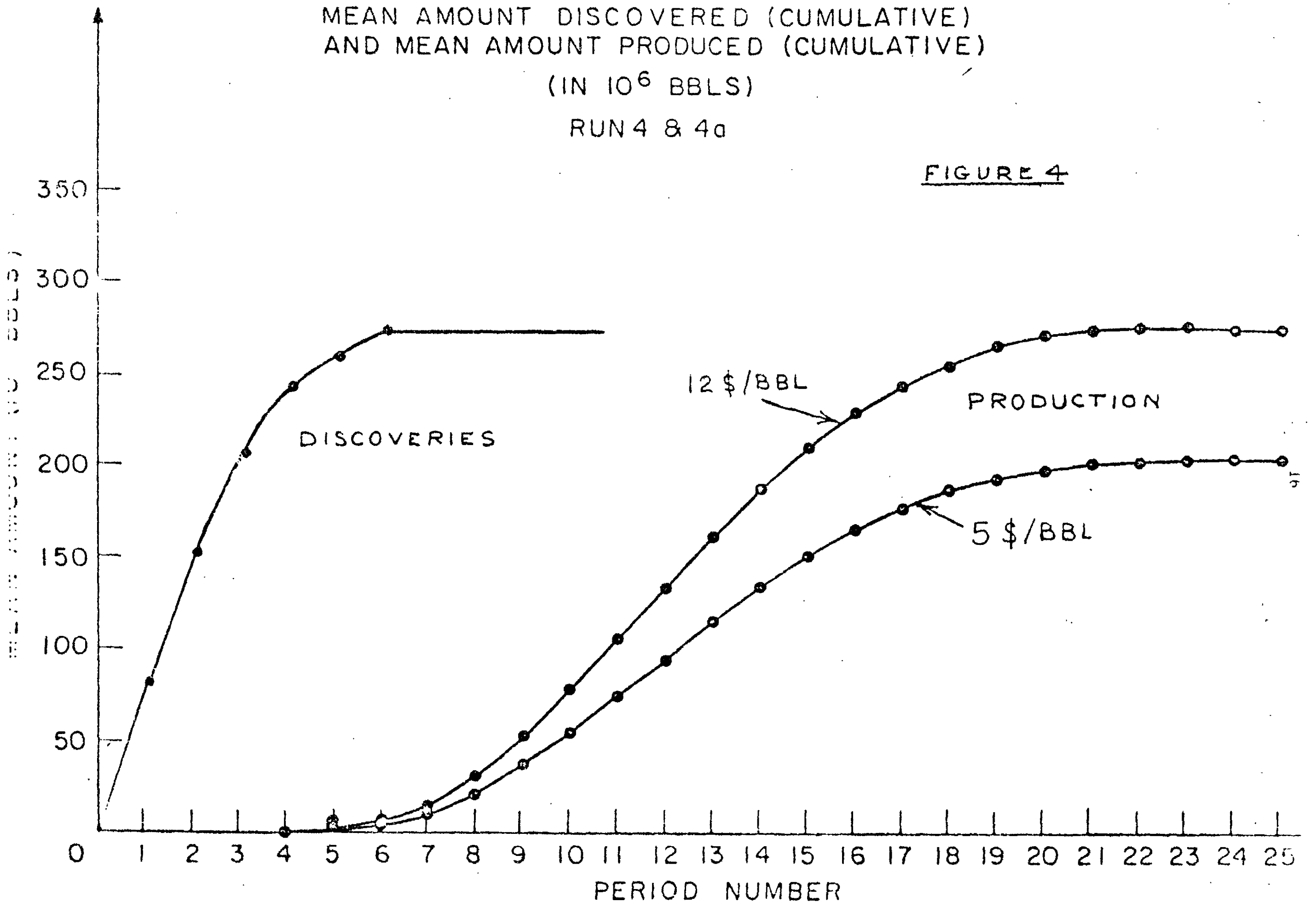
FIGURE 3

MEAN AMOUNT DISCOVERED (CUMULATIVE) AND MEAN AMOUNT PRODUCED (CUMULATIVE)

(IN 10^6 BBLS)

RUN 4 & 4a

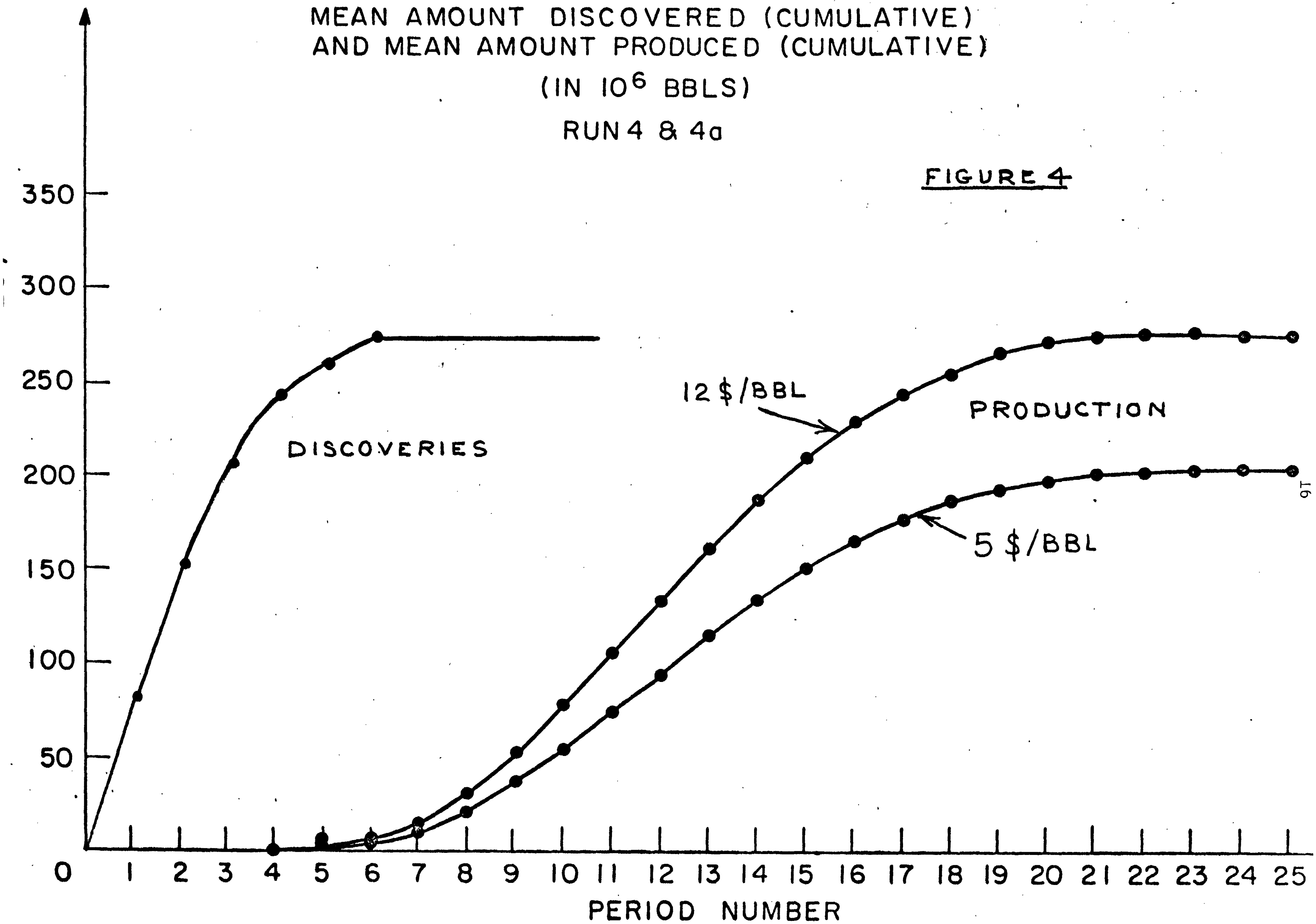
FIGURE 4



MEAN AMOUNT DISCOVERED (CUMULATIVE) AND MEAN AMOUNT PRODUCED (CUMULATIVE) (IN 10^6 BBLS)

RUN 4 & 4a

FIGURE 4



2. Modelling Discovery

A petroleum prospect is a geologic anomaly conceived of as containing hydrocarbons that forms a target for drilling. A petroleum play is a collection of prospects within a geographic region, all of which share certain common geologic attributes; i.e. there is lateral persistence of these attributes across the area in which the play is located. The petroleum play is a "natural" unit for analysis of the evolution of discovery effort over time and of petroleum supply, since it is a conceptual template used by oil and gas explorationists to plan exploration programs.

Predrilling exploratory effort applied to a play generates a collection of prospects, each of which is appraised for its economic viability. Since it is never a priori certain (a) whether or not a prospect contains hydrocarbons, (b) if it does, how much petroleum is in place in it, and (c) how much petroleum can be recovered from it, uncertainty plays an important role in the explorationist's perception of the economic viability of exploratory drilling. Consequently, a model of exploratory drilling should reflect its essential geologic and technological uncertainties. We call a probabilistic model that does this a discovery process model. The particular structure of such a model depends on the level of informational detail about geologic and depositional attributes of prospects in the play assumed to be observable.

At one extreme are models for a small set of observable attributes; e.g. the number of dry exploratory wells drilled and the number that made discoveries; the sizes (BOE in place) of discoveries. At the other extreme are models for descriptively rich sets of attributes, some of which bear on the presence or absence of hydrocarbons in a generic geologic anomaly and others that determine the amount of hydrocarbons in it. Presence or absence of source beds, favorable timing, favorable migration beds, adequacy of seal are typical of the latter; area of closure, porosity, connate water saturation, are examples of the latter. These may be supplemented by maps which describe the spatial disposition of various combinations of attributes. This type of information can be used to construct a spatial probability law incorporating "lateral persistence" or probabilistic dependencies among attribute values across the area of the play. Probabilities for drilling successes and failures and for sizes of discoveries then become "spatial" in character.

Implementation of a play model incorporating this level of detail requires data generally available only to exploration companies or government agencies actively involved in the planning and execution of exploration of the play being modelled. The essential features of the evolution of exploration in a play can be captured by models more modest in their demand for input data. Prediction of the temporal flow of supply from the play can be done using a discovery model built from two key empirical features of petroleum plays: first, the size distribution of deposits is generally positively skewed with a very few large deposits and many small ones; second, on the average, the large deposits are found in the early stages of exploration of the play.

Our model for discovery is composed of three assumptions. The first two together compose a probability law for the sizes of discoveries in the order in which they occur. The third describes a probability law for drilling successes and failures. This latter assumption is somewhat simplistic and may be elaborated in several ways without severe analytical computation. However, in order to divide difficulties we keep it simple; later on we discuss possible extensions of it.

I. (Lognormal size distribution) Let A_i be the size of the ith pool among N pools deposed by nature in the geological zone within which the play takes place. The A_is are values of mutually independent identically distributed lognormal random variables.

II. (Sampling without replacement and proportional to random size) Given $A_1, \ldots, A_N$ the probability of observing $A_1, \ldots A_n$ in that order is

$$\prod_{j=1}^{n} A_j/(A_j + \ldots + A_N).$$

III. There are M prospects in the play of which N are known to be deposits. Drilling successes and failures take place via hypergeometric sampling of these M prospects.

The model for discovery sizes composed of I and II is imbedded in III in the sense that it describes a sequence of $n \leq N$ discovery sizes in order of occurrence conditional on n discoveries having been made. A discrete version of it will be used here. Assumptions I, II, and III together constitute a model for outcomes of a decision to drill a given number of wells independently of when these wells are made. That is, the model describes the physical consequences of a drilling program consisting of drilling a pre-specified number of exploratory wells in the play.

The economic consequences of drilling will be attached to physical outcomes once models of exploration and production costs, prices and fiscal regimes over time are specified.

The process generating sizes of discovered deposits in order of occurrence is imbedded in the drilling process; i.e. deposit discovery occurs only when a well making a discovery occurs. Letting Z_i be barrels of oil in place discovered by the i^{th} exploratory well, and defining

$$x_i = \begin{cases} 1 & \text{if the } i^{th} \text{ well discovers a deposit} \\ 0 & \text{if the } i^{th} \text{ well is dry} \end{cases}$$

the ordered pair (x_i, Z_i) describes the outcome of the i^{th} well drilled. If $x_i = 0$ then $Z_i = 0$ and if $x_i = 1$ then $Z_i > 0$. Consequently, if $x_i = 1$ and $\sum_{\ell=1}^{i} x_\ell = j$, then the j^{th} discovery of size $Y_j \equiv Z_i > 0$ occurs at the i^{th} well. The sequence $\tilde{Y}_j$, $j=1,2,\ldots,N$ is assumed to be uninfluenced by the number of dry holes between discovery wells:

IV. The sequence $\tilde{Y}_1,\ldots,\tilde{Y}_N$ of sizes of deposits discovered in order of occurrence is independent of the sequence $\tilde{x}_1,\ldots,\tilde{x}_i,\ldots$ of drilling successes and failures.

Values $Y_1,\ldots,Y_n$ of the first $n \le N$ discovery sizes may be interpreted as observations produced by sampling without replacement and proportional to size from a finite population $\{Y_1,\ldots,Y_N\}$ of deposit sizes whose elements were generated by independent sampling from a (lognormal) superpopulation. An exact computation of the probability of observing $\tilde{Y}_1 \in dY_1,\ldots,\tilde{Y}_n \in dY_n$ requires numerical evaluation of a rather complicated integral (cf. [2] for discussion). In order to keep computational costs within reasonable bounds we discretize the submodel for discovery sizes in the following way: let $A(k)$, $0 \le k \le 1$ denote the k^{th} fractile for $\tilde{A}_\ell$, $\ell=1,2,\ldots,N$, the sizes of deposits as deposed by nature (Assumption I). Divide [0,1] into K intervals of equal length and define S_m as the geometric mean of a generic $\tilde{A}_\ell$ conditional on $A_\ell \in [A(m-1/K), A(m/K)]$, $m = 1,2,\ldots,K$. These intervals are intervals for $\tilde{A}_\ell$ of equal probability 1/K, and we shall interpret any $\tilde{A}_\ell \in [A(m-1/K), A(m/K)]$ as having "size" S_m. This form of discretization enormously simplifies computation of probabilities for discovery sizes, replacing a density for $\tilde{A}_\ell$ concentrated on $[0,\infty)$ with a multinomial probability function attributing probability 1/K to each size $S_1,S_2,\ldots,S_K$;

$$P\{\tilde{A}_\ell = S_m\} \overset{\text{def}}{=} P\{\tilde{A}_\ell \in [A(m-1/K), A(m/K)]\} = 1/K.$$

Letting N denote the number of deposits in the play, defining N_m as the number of deposits of size S_m, and $\underline{N} = (N_1,\ldots,N_K)$, the probability that nature deposes N_1 deposits of size S_1, N_2 of size S_2, etc. is

$$P\{\tilde{\underline{N}} = \underline{N}\} = \binom{N}{N_1,\ldots,N_K} K^{-N} \equiv \binom{N}{\underline{N}} K^{-N} .$$

Given that nature has generated $\tilde{\underline{N}} = \underline{N}$, the probability $P\{S_{j_1},\ldots,S_{j_N} | \underline{N}\}$ of observing $S_{j_1},S_{j_2},\ldots,S_{j_N}$ in that order follows from assumption II, sampling without replacement and proportional to size: letting $k(n,m)$ denote the number of discoveries of size S_m among the first n discoveries,

$$P\{S_{j_1},S_{j_2},\ldots,S_{j_N} | \underline{N}\} =$$

$$\frac{N_{j_1} S_{j_1}}{\sum_{m=1}^{K} N_m S_m} \cdot \frac{[N_{j_2} - k(1,j_2)] S_{j_2}}{\sum_{m=1}^{K} [N_m - k(1,m)] S_m} \cdots \frac{[N_{j_N} - k(N-1,j_N)] S_{j_N}}{\sum_{m=1}^{K} [N_m - k(N-1,m)] S_m} .$$

Putting these discretized versions of I and II together, we have the probability of observing $S_{j_1},\ldots,S_{j_N}$ in that order as

$$P\{S_{j_1},\ldots,S_{j_N}\} = K^{-N} \sum_{\underline{N}} \binom{N}{\underline{N}} P\{S_{j_1},\ldots,S_{j_N} | \underline{N}\}$$

where summation is over $\{\underline{N} | N_m \geq 0,\ m=1,2,\ldots,K \text{ and } \sum_{m=1}^{N} N_m = N\}$.

Marginal and conditional probabilities for any ordered sequence of sizes $(S_{j_m},\ldots,S_{j_n}), 1 \leq m \leq n \leq N$ are calculable using the above probability function. Fortunately, the multinomial coefficients $\binom{N}{\underline{N}}$ need not be computed in the course of computing these probabilities, for after some algebra we find that

$$P\{S_{j_1},\ldots,S_{j_N} | \underline{N}\} = \frac{N!}{K^N} \times \frac{S_1^{N_1} S_2^{N_2} \cdots S_K^{N_K}}{S[S-S_{j_1}][S-S_{j_1}-S_{j_2}]\cdots[S-S_{j_1}-S_{j_2}-\cdots-S_{j_{N-1}}]} \tag{2.1}$$

where $S = \sum_{m=1}^{K} N_m S_m$.

The above probabilities are probabilities for events that may obtain when <u>all N</u> fields in the play are discovered. The probability of observing discovery sizes $\tilde{Y}_1 = S_{j_1}$, $\tilde{Y}_2 = S_{j_2},\ldots,\tilde{Y}_n = S_{j_n}$ in that order when $n < N$ may in principle be directly computed from (2.1). An alternative is to compute these "forward" probabilities as follows: suppose that among $S_{j_1},\ldots,S_{j_n}$, n_i are of size S_i and define $r_i = N_i - n_i$. Then letting

$$b_k = \left(\sum_{i=1}^{K} n_i S_i\right) - S_{j_1} - S_{j_2} - \cdots - S_{j_k},$$

$$P\{S_{j_1},\ldots,S_{j_n}\} = \frac{N!}{K^N (N-n)!} S_1^{n_1} S_2^{n_2} \cdots S_K^{n_K} \sum{}' \binom{N-n}{r_1,\ldots,r_K} \prod_{k=1}^{n} \left[\sum_{i=1}^{K} r_i S_i + b_k\right]^{-1}$$

where Σ' is summation over $\{r_1,\ldots,r_K | \sum_{i=1}^{K} r_i = N-n,\ r_i \geq 0\}$.

Upon decomposing the products in the above sum by partial fractions and using the relation

$$\frac{1}{r_i S_i + b_k} = \int_0^\infty e^{-\lambda(r_i S_i + b_k)} d\lambda,$$

the above expression for $P\{S_{j_1},\ldots,S_{j_n}\}$ becomes

$$K^{-n} \frac{N!}{N-n!} S_1^{n_1} \ldots S_K^{n_K} \int_0^\infty \left[\sum_{j=1}^{n} c_j e^{-\lambda b_j}\right] \left[\frac{1}{K}\sum_{i=1}^{K} e^{-\lambda S_i}\right]^{N-n} d\lambda, \qquad (2.2)$$

where $c_j = \prod_{\substack{\ell=1 \\ \ell \neq j}}^{n} [b_\ell - b_j]^{-1}$. Notice that $\frac{1}{K}\sum_{i=1}^{K} e^{-\lambda S_i}$ is the Laplace transform of a probability function assigning probability 1/K to each of K values $S_1,\ldots,S_K$ of S. Each S_i is a function $S_i(K)$ of K and is chosen in such a way that

$$\lim_{K\to\infty} \frac{1}{K}\sum_{i=1}^{K} e^{-\lambda S_i(K)} = \int_0^\infty e^{-\lambda x} f(x)dx,$$

f the density for a generic $\tilde{A}_\ell$. In place of (2.3) $P\{S_{j_1},\ldots,S_{j_n}\}$ may be represented as

$$K^{-n} \frac{N!}{N-n!} S_1^{n_1} \ldots S_K^{n_K} \int_0^1 \left[\sum_{j=1}^{n} c_j y^{b_j - 1}\right] \left[\frac{1}{K}\sum_{i=1}^{K} y^{S_i}\right]^{N-n} dy\,.$$

When exploratory drilling is modelled as taking place at discrete points in time and more than one well may be drilled at each time point, there is no natural temporal ordering of discoveries made at a given point in time. Hence we are led to consider sampling as taking place without replacement and proportional to size but for which the natural ordering of observations is partially lost. In the ensuing analysis of intertemporal rates of drilling we need explicit formulae for probabilities of two types of events generated by such a sampling process: the probability that the first k discoveries are of sizes $Y_{1_j},\ldots,Y_k$ and the probability that the next n-k discoveries are of sizes $Y_{k+1},\ldots, Y_n$ given that the first k discoveries are of sizes $Y_1, \ldots, Y_k$. To distinguish probabilities for events composed of ordered discovery sizes from events composed of discovery sizes without regard for order we define $\sigma(Y_1,\ldots, Y_k)$ as the compound event composed of the union of all k! orderings of $Y_1,\ldots, Y_k$, and present integral representations for $P\{\sigma(Y_1,\ldots,Y_n)\}$ and $P\{\sigma(Y_{k+1},\ldots,Y_n) | \sigma(Y_1,\ldots,Y_n)\}$.

If observed sizes are generated by sampling proportional to size and without replacement, but the <u>order of observations is lost</u> the probability of observing $S_{i_1},\ldots,S_{i_n}$ possesses a simple integral representation that follows from the law of total probabilities and the following nice combinatoric identity. Given $Y_1 = S_{j_1}$, $Y_2 = S_{j_2},\ldots,Y_n = S_{j_n}$ and a number S, it can be shown that

$$\left[\prod_{\ell=1}^{n} Y_\ell\right]\left\{\sum_{\sigma}\prod_{i=1}^{n}[S+b_{j_i}]^{-1}\right\} = S\int_0^\infty e^{-\lambda S}\prod_{\ell=1}^{n}\left[1-e^{-\lambda Y_\ell}\right]d\lambda$$

where for a fixed ordering $(Y_{j_1},\ldots,Y_{j_n})$ of $Y_1,\ldots,Y_n$, $b_{j_i} = Y_{j_i} + Y_{j_{i+1}} + \ldots + Y_{j_n}$ for i=1,2,...,n and $\sum_\sigma$ denotes summation over the n! permutations of (1,2,...,n).

Using this combinatoric identity, the probability of discovering sizes $S_{i_1},\ldots,S_{i_n}$ in the first n discoveries without regard to order is

$$\binom{N}{n} K^{-N} \sum{}' \binom{N-n}{r_1,\ldots,r_K} \left[\prod_{\ell=1}^{n} S_{i_\ell}\right] \left[\sum_{\sigma}\prod_{i=1}^{n}(r_i S_i + b_{j_i})^{-1}\right] =$$

$$(N-n)\binom{N}{n}K^{-n}\int_0^\infty \left[\frac{1}{K}\sum_{i=1}^{K} S_i e^{-\lambda S_i}\right]\left[\frac{1}{K}\sum_{j=1}^{K} e^{-\lambda S_i}\right]^{N-n-1}\prod_{k=1}^{K}\left[1-e^{-\lambda S_k}\right]^{n_k} d\lambda\} \tag{2.3}$$

Letting $L(\lambda)$ denote the Laplace transform of f and L' the derivative of L, as $K\to\infty$ with N and the parameters of f fixed, (2.3) approaches

$$(N-n)\binom{N}{n}\left[\prod_{j=1}^{n} f(Y_j)\right]\int_0^\infty [-L'(\lambda)]\,[L(\lambda)]^{N-n-1}\prod_{k=1}^{n}(1-e^{-\lambda Y_k})\,d\lambda, \tag{2.4}$$

or in terms of the inverse function $U(L) = \lambda$ of L,

$$(N-n)\binom{N}{n}\left[\prod_{j=1}^{n} f(Y_j)\right]\int_0^1 x^{N-n-1}\prod_{k=1}^{n}(1-e^{-u(x)Y_k})\,dx\ .$$

The marginal density for the size of the j^{th} $(1\le j\le n)$ discovery when n discoveries have been made is

$$(N-n)\binom{N}{n} f(Y_j)\int_0^\infty [-L'(\lambda)]\,[L(\lambda)]^{N-n-1}[1-L(\lambda)]^{n-1}(1-e^{-\lambda Y_j})\,d\lambda \tag{2.5}$$

if the order in which the first n sizes discovered is "lost". If f has mean M_1, the marginal mean of $\tilde{Y}_n$ in this case is

$$(N-n)\binom{N}{n}\int_0^\infty [-L'(\lambda)]\,[M_1+L'(\lambda)]\,[1-L(\lambda)]^{n-1}[L(\lambda)]^{N-n-1}d\lambda \tag{2.6}$$

when the order of $\tilde{Y}_1,\ldots,\tilde{Y}_n$ is "Lost", while if the order of the $\tilde{Y}_j$s is not "Lost", it is

$$n\binom{N}{n}\int_0^\infty L''(\lambda)[L(\lambda)]^{N-n}[1-L(\lambda)]^{n-1}d\lambda \tag{2.7}$$

Letting $E(\tilde{Y}_j)$ denote the expectation of the j^{th} discovery when order is kept, it can be shown that (2.6) is equal to $\frac{1}{n}\sum_{j=1}^{n} E(\tilde{Y}_j)$.

Using the combinatoric identity presented earlier twice, the joint probability of $\sigma(Y_1,\ldots,Y_k)$ and $\sigma(Y_{k+1},\ldots,Y_n)$ is

$$P\{\sigma(Y_{k+1},\ldots,Y_n),\ \sigma(Y_1,\ldots,Y_k)\}$$

$$= \prod_{j=1}^{n} f(Y_j)\binom{N}{n}\binom{n}{k}\int_0^\infty\int_0^\infty d\lambda d\theta\ \{\frac{d^2}{d^2\lambda}[L(\lambda+\theta)]^{N-n} + (Y_{k+1}+\ldots+Y_n)\frac{d}{d\lambda}[L(\lambda+\theta)]^{N-n}\}$$

$$\times e^{-\lambda(Y_{k+1}+\ldots+Y_n)}\prod_{j=1}^{k}(1-e^{-\lambda Y_j})\prod_{\ell=k+1}^{n}(1-e^{-\theta Y_\ell}) \qquad (2.8)$$

and $P\{\sigma(Y_{k+1},\ldots,Y_n)|\sigma(Y_1,\ldots,Y_k)\}$ can be computed as

$$P\{\sigma(Y_1,\ldots,Y_k),\ \sigma(Y_{k+1},\ldots,Y_n)\}/P\{\sigma(Y_1,\ldots,Y_k)\}.$$

Since the numerical computations presented in section 7 are done by discretizing the size distribution, the above theoretical results are not directly employed. However, they provide a connection between the discrete version and the continuous version of the discovery process that can be exploited when adaptive updating of the parameters of the discovery process is considered, cf. Barouch [5].

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†Integrating (2.8) with respect to $Y_{k+1},\ldots,Y_n$ and then with respect to θ, (2.7) reduces to $P\{\sigma(Y_1,\ldots,Y_k)\}$ and is formally identical to (2.4).

3. Modelling Development and Production for Individual Fields

Once a discovery has been made, the field operator does an economic and engineering analysis to determine whether or not the field should be put into production and if so, he specifies a development and production strategy for the field. At the individual field level, the problem of determining -- relative to some prespecified normative criterion -- the optimal development and production strategy for the field is in fact elaborately complicated and worthy of study in its own right. (Uhler's [8] deterministic optimal control formulation of the individual field problem is a recent example of a long line of studies of the individual field problem.)

Our principal concern here is the temporal evolution of supply at the play rather than individual field level. Consequently we shall not consider dynamic optimization of production from individual fields. Rather, we propose a static model of individual field production over time, one which mirrors practical production experience: each field of size S has an associated production profile

$$\underline{\delta}(S) \equiv (\delta_0(S),\ \delta_1(S),\ldots,\delta_t(S),\ldots.)$$

where $\delta_t(S)$ is the proportion of S produced at the t^{th} period of time after the field's discovery; $\underline{\delta}(S)$ will be assumed to be independent of the time period at which the field is discovered. It will be convenient in subsequent analysis to define $\delta_t(S) = 0$ for $t<0$. In the numerical examples discussed later we choose elements of $\underline{\delta}(S)$ to match actual North Sea production experience as recorded by Eckbo [5].

Once the minimum economic reservoir sizes (MERS) for periods $t=0,1,2,\ldots,T$ are computed we can determine time periods at which a field of a particular size exceeds the MERS for that period.

In general the MERS at t may depend on the past history of exploration prior to t. If, however, (a) prices and costs are known with certainty at $t=0,1,2,\ldots,T$, (b) individual field production profiles are static over time, (c) maximization of expected net present value is the criterion for decision-making, (d) pipeline network decisions -- requiring joint consideration at each time period of the sizes of all discovered fields, of potential future discoveries, and of past pipeline decisions -- are ignored, and (e) the decision when, if at all, to put a discovered field into production is made in light of its economic desireability as a marginal decision-making unit independent of past history, then the MERS at each time period is also independent of past history. We divide our difficulties by adopting these assumptions in subsequent analysis. If at t a field of size S_ℓ exceeds the MERS for at least one time period subsequent to t, a reasonable but not necessarily optimal rule is to put it into production at the first such time-period. More formally, let T_ℓ be the set of time-periods among 0, 1, 2, ..., T at which size S_ℓ exceeds the MERS, and define

$$T_\ell(t) = \{\tau \mid \tau \varepsilon T_\ell \text{ and } t-1 < \tau \leq T\},$$

and

$$\tau_\ell(t) = \begin{cases} \text{smallest element of } T_\ell(t) & \text{if } T_\ell(t) \text{ is non-empty,} \\ +\infty & \text{otherwise.} \end{cases}$$

Then $\tau_\ell(t)$ is a time period at which a field of size S_ℓ discovered at period t and put into production at $T_\ell(t)$ has positive net present value. If $T_\ell(t)$ is empty, then fields of size S_ℓ discovered at time period t and subsequently will not be put into production at any time period within the planning horizon.

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*Throughout, reference to the time period at which a field is "put into production" means that the decision to produce is made at that time period. Physical production may, because of time lags, occur subsequent to this time period.

When future prices and costs are known with certainty, and the decision-making criterion is maximization of expected present value, the optimal time to start production of a discovered field regarded as a production unit operating independently from other discovered fields and of its optimal production profile may be found by deterministic optimization. The optimal production profile for a field of given size will be a function of the time period subsequent to its discovery at which production begins; i.e., a discovery of size S_ℓ at period t will possess a physical production profile $\underline{\delta}_\ell(t+\Delta^*_\ell(t))$ where $\Delta^*_\ell(t)$ is the number of time periods subsequent to period t at which physical production begins. The components of $\underline{\delta}_\ell(t+\Delta^*_\ell(t))$ depend on <u>both</u> S_ℓ and $t+\Delta^*_\ell(t)$. With the above assumptions in force, a discovery of size S_ℓ at period t will physically begin production at period $t+\Delta^*_\ell(t)$.

4. Drilling, Amounts Discovered, Amounts Produced

We assume that drilling is done and outcomes are observed at discrete points 1,2,...,t,.... in time and call the i^{th} point in time "period t". As defined in section 2, the ordered pair (x_i, Z_i) describes the outcome of drilling the i^{th} well: $x_i = 1$ if the i^{th} well is a discovery and $x_i = 0$ if it is a dry hole. The size of deposit discovered by the i^{th} well is $Z_i = 0$ if $x_i = 0$ and is $Z_i > 0$ if $x_i = 1$.

At the outset the numbers $\tilde{d}(t)$ of wells drilled at periods t=0,.,2,...,T are rvs whose joint probability law is determined by dynamic optimization given a normative criterion for decision-making by the operator, a specification of the economic structure of the operator's optimization problem, and the joint probability law for the $\tilde{x}_i$s and $\tilde{Z}_i$s as described in section 2. Once the joint probability law for numbers $\tilde{d}(t)$, t=0,1,2,.... is computed, all essential properties of amounts discovered and amounts produced are computable using it and the joint probability law for $(\tilde{x}_i, \tilde{Z}_i)$, i=1,2,....; i.e. the probability law for the total amount of hydrocarbons discovered at periods 0,1,...,t,...,T and that for amounts produced at periods 0,1,...,t,...,T. The amounts produced at t depends on both the production profiles of deposits discovered at $\tau \leq t$ and on whether or not a deposit of a given size discovered at $\tau_0 \leq t$ is "profitably" put into production at some τ, $\tau_0 \leq \tau \leq t$.

The basic idea is simple: let $\omega(t) = d(0)+d(1)+\ldots+d(t)$ denote the total number of wells drilled at 0,1,...,t and compute the probability distribution of the waiting time ω_n to the n^{th} discovery, measured in number of wells drilled. Then compute the probability that $\omega(t-1) < \omega_n \leq \omega(t)$ -- that the n^{th} discovery is made at period t. The probability distribution for sizes $\tilde{Y}_1,\ldots,\tilde{Y}_n,\ldots.$ of discoveries is as described in section 2 and is independent of times of discoveries. Consequently, if the n^{th} discovery is made at period t, the amount discovered $\tilde{Y}_n$ has a probability distribution given $Y_1,\ldots,Y_{n-1}$ as shown in (2.2); the marginal probability distribution for $\tilde{Y}_n$ is as given in (2.5). Letting $h_{n,\ell}$ denote the marginal probability that the n^{th} discovery is of size S_ℓ, the marginal probability that a discovery of size S_ℓ is made at period t is $P\{\tilde{\omega}(t-1) < \tilde{\omega}_n \leq \tilde{\omega}(t)\}h_{n,\ell}$, so the marginal expectation of amount discovered at period t is $E(\tilde{Z}_{\tilde{\omega}(t-1)+1}+\ldots+\tilde{Z}_{\tilde{\omega}(t)}) =$

$$\sum_{n=1}^{N} P\{\tilde{\omega}(t-1) < \tilde{\omega}_n \leq \tilde{\omega}(t)\}E(\tilde{Y}_n) \tag{4.1}$$

with $E(\tilde{Y}_n)$ as given in (2.6). The probability $P\{\tilde{\omega}(t-1) \leq \tilde{\omega}_n < \tilde{\omega}(t)\}$ can be computed in two stages. First compute $P\{\tilde{\omega}_n = \omega_n\}$; then compute $P\{\tilde{\omega}(t-1) < \tilde{\omega}_n \leq \tilde{\omega}(t)\}$ using the joint probability law for $\tilde{d}(0),\tilde{d}(1),\ldots,\tilde{d}(t)$.

Among simple possible characterizations of $\tilde{\omega}_n$ are:

(i) $\tilde{x}_1,\ldots,\tilde{x}_i,\ldots.$ is a Bernoulli process with known parameter p. Then $P\{\tilde{\omega}_n=\omega_n\} = \binom{\omega_n}{n-1} p^n(1-p)^{\omega_n}$

ii) $\tilde{x}_1,\ldots,\tilde{x}_i,\ldots,x_{\omega_n}$ is a sub-sequence of an infinite sequence of exchangeable rvs. Then the probability that n-1 1s appear among the first ω_n-1 x_is and $\tilde{x}_n=1$ possesses a representation of the form

$$\binom{\omega_n-1}{n-1} \int_0^1 \xi^n(1-\xi)^{\omega_n-n}\, dF(\xi)$$

where F is some cd$f$ concentrated on [0,1]. If F is beta with parameter (α,β),

$$P\{\tilde{\omega}_n=\omega_n\} = \binom{\omega_n-1}{n-1} \frac{B(\alpha+n,\omega_n+\beta-n)}{B(\alpha,\beta)}$$

(iii) $\tilde{x}_1,\ldots,\tilde{x}_{N+M}$ are hypergeometric as in Assumption III. Then

$$P\{\tilde{\omega}_n=\omega_n\} = [\binom{N}{n-1}\binom{M}{\omega_n-n}/\binom{N+M}{N}]\ [N-n+1/N+M-\omega_n+1].$$

Explicit calculation of the time period at which the n^{th} discovery is made may be done by use of

$$P\{\tilde{\omega}_n\le\tilde{\omega}(0)\} = \sum_{j=n}^{\infty} P\{\tilde{\omega}_n\le j\}P\{\tilde{\omega}(0)=j\} \tag{4.2}$$

for $t = 0$, and by use of

$$P\{\tilde{\omega}(t-1)<\tilde{\omega}_n\le\tilde{\omega}(t)\} = \sum_{k=0}^{\infty}\sum_{j=n}^{\infty} P\{\tilde{\omega}(t-1)=j,\ \tilde{d}(t)=k\}[P\{\tilde{\omega}_n\le j+k\} - P\{\tilde{\omega}_n\le j\}] \tag{4.3}$$

for $t > 0$. Formula (4.2) is the probability that the n^{th} discovery is made at period 0 and (4.3) is the probability that the n^{th} discovery is made at period $t > 0$.

The marginal probability distribution for the starting time of production from the n^{th} discovery follows directly from (4.2) and (4.3). Letting $\tau_\ell(t) \equiv t+\Delta_\ell(t)$, the marginal probability that the n^{th} discovery is of size S_ℓ and discovered at t is the marginal probability that a production profile $S_\ell\underline{\delta}_\ell$ commences at $t+\Delta_\ell(t)$. Hence the marginal probability that an amount $S_\ell\delta_{\ell,t-\tau-\Delta_\ell(\tau)}$ is produced at t from the n^{th} discovery is the probability that the n^{th} discovery is made at τ and is of size S_ℓ.†

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†Recall that $\delta_{\ell,\tau}\equiv 0$ for $\tau < 0$, so that if the n^{th} discovery is made at $\tau'>t-\Delta_\ell(\tau')$ the amount produced from it at period t is zero.

Defining $h_{n,\ell}$ as the marginal probability that the n^{th} discovery is of size S_ℓ, the marginal expectation $E(\tilde{\rho}_n(t))$ of the amount $\tilde{\rho}_n(t)$ produced at period t by the n^{th} discovery is

$$\sum_{\ell=1}^{K} h_{n,\ell} \sum_{\tau\in\{j\,|\,j+\Delta_\ell(j)\le t\}} S_\ell\, \delta_{\ell,t-\tau-\Delta_\ell(\tau)} P\{\tilde{\omega}(\tau-1)<\tilde{\omega}_n\le\tilde{\omega}(\tau)\} \tag{4.4}$$

and so the marginal expectation of the amount produced at period t is $\sum_{n=1}^{N} E(\tilde{\rho}_n(t))$.

To sum up, we have given explicit formulae for computation of the expectation of amount discovered at t and for the amount produced at t (4.4). First moments of these quantities are relatively easy to compute, but higher moments of $\tilde{\rho}_1(t)+\ldots+\tilde{\rho}_n(t)+\ldots$ are quite complicated so we defer presentation here.

5. Computation of Probability Law for $\tilde{d}(t)$, t=0,1,2,...,T by Dynamic Optimization

We determine the joint probability law describing the evolution of an optimal sequential drilling policy by use of dynamic programming. Let $\underline{\sigma}_t$ denote the history of exploration at period t: $\underline{\sigma}_t = (Z_{\omega(t-1)+1},\ldots,Z_{\omega(t)})$. The number d(t) of wells drilled at t is the number of components of $\underline{\sigma}_t$, and $Z_{\omega(t-1)+k}$ denotes the size of field discovered by the k^{th} well drilled at period t ($Z_{\omega(t-1)+k} > 0$ is the size of a discovery and $Z_{\omega(t-1)+k} = 0$ denotes a dry hole). The history of exploration up to and including period t is $H_t \equiv (\underline{\sigma}_0,\underline{\sigma}_1,\ldots,\underline{\sigma}_t)$ and H_t in terms of H_{t-1} is $(H_{t-1},\underline{\sigma}_t)$.

Adopting maximization of expected net present value as a normative criterion for decision-making, let $v_t(\cdot)$ be the net present value at t over t,t+1,.... of discovery of sizes $Z_{\omega(t-1)+1},\ldots,Z_{\omega(t)}$ made at t and let $E_{\underline{\sigma}_t|H_{t-1}}$ denote expectation with respect to the distribution of $(\tilde{Z}_{\omega(t-1)+1},\ldots,\tilde{Z}_{\omega(t)})$ given H_{t-1} and d(t). If $V_t(H_{t-1})$ is the expectation of an optimal policy over t,t+1,...,T given H_{t-1}, then given a constant discount rate α, by the principle of optimality

$$V_t(H_{t-1}) = \max_{d(t)\varepsilon D(t)} \{E_{\underline{\sigma}_t|H_{t-1}} v_t(\tilde{\underline{\sigma}}_t) + \alpha E_{\underline{\sigma}_t|H_{t-1}} V_{t+1}((H_{t-1},\underline{\sigma}_t))\}$$

where D(t) is a set whose elements are numbers of wells that are allowable at t.[†] At t = 0, there is no history of drilling, H_{-1} is empty, and so we define

$$V_0(H_{-1}) = \max_{d(0)\varepsilon D(0)} \{E_{\underline{\sigma}_0} v_0(\tilde{\sigma}_0) + \alpha E_{\underline{\sigma}_0} V_1(\tilde{\underline{\sigma}}_0)\}.$$

[†] D(t) may depend on past history H_{t-1}.

No exploratory drilling takes place at $t > T$ so we define

$$V_{T+1}(H_T) = \max_{d(\tau)\varepsilon D(\tau)} E_{\underline{\sigma}_T|H_{T-1}} v_T(\underline{\tilde{\sigma}}_T).$$

6. Implementation of the algorithm and the computer program.

6.1 General

The model and corresponding computational regimes presented in the preceding sections have been programmed for computation. The structure of the resulting software system is discussed here. Inputs to the program are: (1) the number of prospects (denoted by M), (2) the number of fields (NF), (3) the number of discretized sizes (KD), (4) the parameters of the lognormal distribution of sizes, (5) a discount rate employed by the decision maker (ALPHA), and (6) streams of projected costs and prices ($\underline{C}$ and $\underline{P}$). The output consists of an optimal drilling policy, the expectation of the rate of discovery per time period, the expectation of the rate of production per time period, and probabilities attached to other events which may be of interest to the decision maker. Among these conditional probabilities of a specified number of successes in each time period, probabilities of the ith discovery occurring in period t for all (i,t) pairs. The program allows computation of probabilities for other types of events (cf. subsection 6.3).

The obvious way to solve a dynamic programming problem like that posed in Section 4 is by straightforward backward induction. This is done by the first part of the main program, which computes an optimal drilling policy. The second part of the main program employs a specially designed forward looking algorithm designed to exploit features of the optimal drilling strategy computed by the first part of the main program. This algorithm scans a large decision tree displaying possible future outcomes and computes probabilities needed for the next stage of computation: computation of the output quantities -- expectation of rate of discovery per period, etc. -- listed above. This is accomplished with a subprogram in which success and failure probabilities are computed, along with probabilities for discovery of fields in each size class given the current state history.

A description of subprograms is given in the following order: (1) main program, part I and part II, (2) subroutines PP, PROB, and SP, in which needed probabilities are computed, and (3) other subroutines. The following block diagram may help understanding of program structure. An arrow A ⟶ B indicates that block A calls and uses computations done by block A

6.2 The MAIN Program

Suppose the decision maker is at the beginning of some period t . Past history at t consists of the number of wells drilled up to and including $t-1$ and the number of discoveries of each size $S_1, S_2, \ldots, S_{KD}$ made by these wells. At the beginning of period t , no prospects are assumed to be in the process of drilling. A decision as to how many wells to drill is taken immediately subsequent to the beginning of period t. As with most dynamic programming algorithms, all possible past histories must be scanned at each decision point. Because of rapid escalation of computational cost with an increase in dimensions of the problem, it is important that an efficient procedure for scanning past histories be designed. After some trial runs and after rough estimation of the number of computation operations involved, the following procedure was found to be reasonably efficient: loop first on the number of wells drilled in past periods (KKK), then on the number of discoveries (NPS), then on the number of discoveries in each size class (NS1, NS2, NS3, ...), and last on the time period number (JJJ). At each decision point the expected profit (in NPV$ subsequent to production for that period) of drilling 1 to ℓ (LIMIT) wells is computed; ℓ is the capacity constraint on number of wells per period. The upper limit ℓ may be reviewed as a technological or as a budget constraint. Choosing ℓ small substantially reduces computational cost.

Output is generated by MAIN I and includes the optimal number of wells to drill in each period (NND) conditional on each possible state history and the expected profit (EX) in NPV$ of following an optimal strategy at each future decision point.

Methods for scanning past histories that decrease computational cost were tested. For some particular combinations of cost-price vectors, it is not necessary to scan all feasible drilling decisions at each time period. For example, if costs are rising faster than prices, and it is found that for some past history of successes and failures and sizes discovered j wells should be drilled at period t, then the decision at period $t-1$ given the same past history is certainly to drill no less than j wells. By exploiting this fact, computation time can be reduced.

MAIN II moves forward in time, scanning first by period number (JJJ), then by number of wells drilled (KKK), and last by sizes (NS1, NS2, NS3, ...). For example, MAIN II computes the joint probability of a history of drilling successes and failures and discovery sizes occurring at the termination time period t. Denote this probability by F. After computing F, joint probabilities (PT) of j discoveries at period t are computed for all (t,j) pairs. Then the expectation of j discovered (EL) at future time periods and the expectation of amounts produced (AMPRO) at future time periods are computed.

6.3 Computing Probabilities

Suppose NF fields have been found. What is the probability that a particular sequence of sizes $(Y_1,\dots,Y_{NF})$ is discovered in that order? These probabilities, as described in Section 2, are computed by subroutine PP and stored.

Subroutine PROB, called by MAIN, computes the joint probability of N(i) discoveries of size S_i (i=1,...,KD) and NND -- $\sum_{i=1}^{KD} N(i)$ failures for NND wells drilled. This probability is computed as the product of two terms, P_1 and P_2, where $P_1 = \text{Prob}\{N(i) \text{ discoveries of size } S_i,\ i=1,2,\dots,KD \mid \text{past history and } \sum_{i=1}^{KD} N(i) \text{ discoveries}\}$, and where $P_2 = \text{Prob}\{N(i) \text{ discoveries of size } S_i \mid \text{past history; } \sum_{i=1}^{KD} N(i) \text{ discoveries}\}$. The term P_1 is computed by the subroutine PROB. The term P_2 is computed by subroutine SP, called by PROB, using the stored probabilties computed by PP.

The computation of P_2 in SP poses the following problem. If the order of past discoveries is known, and the order of sizes discovered by the next $\sum_{i=1}^{KD} N(i)$ discoveries is also known, then probabilities P_2 can be easily computed as marginal probabilities of the joint probabilites computed in PP. Both orders are, however, unknown. We overcome this difficulty by assuming that the current past history, denoted by H, can arise from many ordered sequences of discovered sizes having the same numbers of discoveries in each size class. Call a generic sequence h_i. The probability of h_i given H is computable from the joint probabilities generated by PP. Calling a generic ordered sequence of $\sum_{i=1}^{KD} N(i)$ sizes of discoveries subsequent to H, k_j, gives

$$P_2 = \sum_{h_i \varepsilon B} \sum_{k_j \varepsilon J} P(h_i|H)\ P(k_j|h_i, \sum_{i=1}^{KD} N(i))$$

where J is the set of ordered sequences of length $\sum_{i=1}^{KD} N(i)$ containing $N(i)$ sizes S_i, $i=1,2,...,KD$ and B is the set containing all ordered sequences of length equal to the number of discoveries in H with the same number of discoveries of each size as in H.

Probability routines incur small cost per one decision point and they yield exact results. In large size problems, however, the total cost may be large because the number of decision points grows rapidly with increasing dimensionality.

6.4 Other Subroutines

SIZE - this subroutine computes discretized sizes from the lognormal distribution as explained in Section 2.

PRICES - this subroutine supplies the program with a price sequence for the entire planning horizon.

TABL - this subroutine computes the profit in NPV$ for each possible number of wells that can be drilled at each time period for each possible past history.

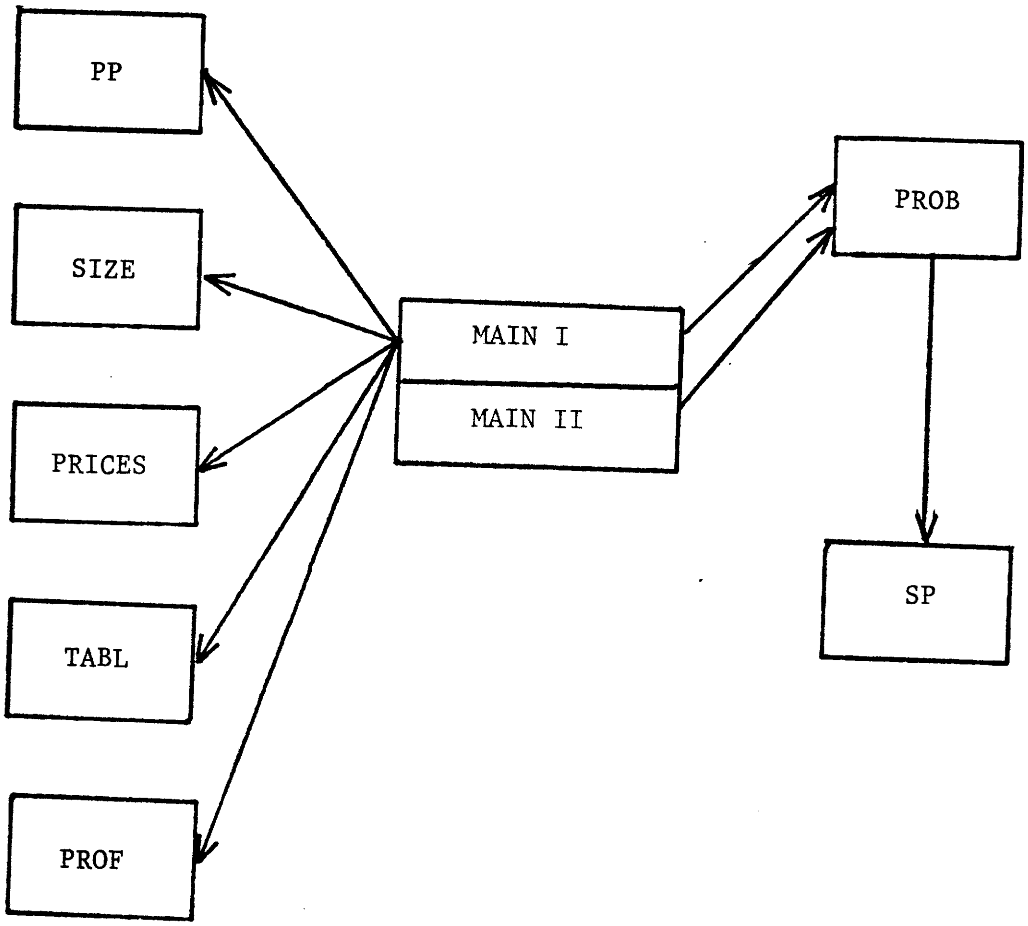
(Cost and revenues depend on the period in which production starts; for a given discovery at t, production does not necessarily start at t).

PROF - this subroutine computes the optimal production start up period for each discovery.

Some discovered fields may not be produced at all. If costs rise faster than prices, a field starts production immediately or never. In other cases production can start in later periods within the planning horizon.

6.5 Computation Time

We used IBM/370 of the M.I.T. Computation Center. Later versions may use the PRIME system. CPU time for run number 1 (see Section 6) is 4 minutes.



The Computer Program - A Block Diagram

7. Results

We present results from the following input parameters:

--Number of sizes=KD=3

--Stream of prices=P(I)=P(O)*GAMMA^{I-1}

--A constant cost inflation factor=BETA

--A constant discounting rate=ALPHA

--Pairs of prospects and fields (M,WF)=(10,5),(12,4),(20,4)

--Upper bound on the number of drilling-LIMIT(=4 in all the runs)

--Size categories are (100x10^5, 450x10^5, 1500x10^5)

--Lognormal distribution $\mu = 5.78$, $\sigma^2 = 1.38$

To facilitate presentation of results we number runs. The run numbers appearing on the graphs and in the tables are:

Run 1: 10 Prospects, 5 Fields. The price starts from $12 in the first period, and increases each period by GAMMA=1.066 . The cost inflation factor BETA=1.066 , and the NPV discounting rate ALPHA=.858.

Run 2: 10 Prospects, 5 Fields. Here GAMMA=1.015 , BETA=.066 and ALPHA-.896.

Run 3: 12 Prospects, 4 Fields. Parameters are identical to those of Run 1.

Run 4: 20 Prospects, 4 Fields. Otherwise the parameters are identical to those of Run 1.

Run 4a: Identical to Run 4, but the present price P(0)=$5. The difference between Runs 4 and 4a are only in the rates of production.

Table 1: Mean Amount and Cumulative Amount Produced in Each Period (in 10^6 BBLS)

	1		2		3		4		4a	
	in period	cum.	in period	cum.	in period	cum.	in period	cum.	in period	cum.
1	--	--	--	--	--	--	--	--	--	--
2	--	--	--	--	--	--	--	--	--	--
3	--	--	--	--	--	--	--	--	--	--
4	--	--	--	--	--	--	--	--	--	--
5	15.64	15.64	15.64	15.64	10.33	10.33	1.11	1.11	.69	.69
6	30.37	46.13	30.38	46.02	21.42	31.74	4.84	5.95	3.29	3.98
7	41.53	87.54	41.66	87.68	29.32	61.06	9.81	15.76	6.70	10.68
8	42.63	130.17	42.74	130.42	32.29	93.35	10.07	31.83	11.28	21.96
9	42.51	172.28	42.55	172.97	33.37	126.73	21.51	52.92	15.18	37.14
10	38.71	210.99	38.68	211.64	31.59	158.32	25.12	78.03	17.71	54.85
11	32.46	243.45	32.45	244.09	26.81	185.12	27.05	105.08	19.22	74.07
12	28.27	271.72	28.24	272.33	23.58	208.71	27.68	132.75	19.87	93.95
13	24.50	296.22	24.46	296.79	20.51	224.22	27.56	160.32	20.20	114.14
14	20.76	316.98	20.74	317.53	17.24	246.46	25.56	185.88	18.91	133.05
15	10.30	327.27	10.25	327.78	10.05	256.50	22.89	208.77	17.13	150.18
16	5.62	332.90	5.58	333.36	5.87	262.37	19.21	227.97	14.42	164.60
17	3.32	336.22	3.26	336.62	3.87	266.24	14.92	242.89	11.09	175.69
18	2.75	338.97	2.73	339.35	2.37	268.61	11.77	254.66	8.80	184.49
19	1.60	340.57	1.50	340.93	1.49	270.10	8.62	263.29	6.61	191.10
20	0.79	341.36	0.79	341.72	0.76	270.86	5.94	269.23	4.66	195.76
21	0.48	341.84	0.00	341.72	0.45	271.32	4.15	273.38	3.39	199.15
22	--	--	--	--	0.27	271.59	2.28	275.66	1.91	201.06
23	--	--	--	--	--	--	1.20	276.85	1.04	202.10
24	--	--	--	--	--	--	0.48	277.35	0.05	202.50
25	--	--	--	--	--	--	0.16	277.49	0.02	202.60
26	--	--	--	--	--	--	0.08	277.57	0.01	202.61
27	--	--	--	--	--	--	0.00	277.57	0.00	202.61

Table 2: Mean Amount Discovered (Cumulative in 10^6 BBLS)

Period \ Run	1	2	3	4
1	196.04	196.04	129.64	81.05
2	292.77	292.86	210.05	149.79
3	341.72	341.67	242.26	205.12
4	341.72	341.67	273.45	243.82
5	--	--	273.45	259.15
6	--	--	--	273.32
7	--	--	--	273.36
8	--	--	--	273.36

Table 3: Mean and Standard Deviation of Number of Wells Drilled

Run	1		2		3		4	
Period	Mean	St.Dev.	Mean	St.Dev.	Mean	St.Dev.	Mean	St.Dev.
1	4.000	0.0	4.0000	0.0	4.0000	0.0	4.0000	0.0
2	3.0126	0.1540	3.0229	0.1541	3.5697	0.5208	3.9988	0.0639
3	2.3009	0.8503	2.7096	0.8539	1.8694	1.0341	3.9385	0.4845
4	0.1669	0.3779	0.0061	0.0779	1.4198	1.2331	3.4822	1.2175
5	0.0	0.0	0.0	0.0	0.0833	0.2771	1.3842	1.2171
6	--	--	--	--	0.0	0.0	0.7412	1.0789
7	--	--	--	--	--	--	0.0328	0.1867
8	--	--	--	--	--	--	0.0007	0.0366

Table 4: Probabilities for n Wells Drilled in Period t

Run 1:

t \ n	0	1	2	3	4
1	0.0	0.0	0.0	0.0	1.0000
2	0.0	0.0	0.0056	0.9762	0.0182
3	0.0854	0.0	0.4428	0.4717	0.0
4	0.8350	0.1632	0.0019	0.0	0.0
5	1.0000	0.0	0.0	0.0	0.0

Run 2:

t \ n	0	1	2	3	4
1	0.0	0.0	0.0	0.0	1.0000
2	0.0001	0.0	0.0	0.9766	0.0233
3	0.0872	0.0	0.0289	0.8839	0.0
4	0.9939	0.0061	0.0	0.0	0.0
5	1.0000	0.0	0.0	0.0	0.0

Run 3:

t \ n	0	1	2	3	4
1	0.0	0.0	0.0	0.0	1.0000
2	0.0020	0.0	0.0009	0.4204	0.5767
3	0.1242	0.1595	0.5169	0.1225	0.0769
4	0.3909	0.0460	0.3155	0.2476	0.0
5	0.9169	0.0829	0.0002	0.0	0.0

Run 4:

t \ n	0	1	2	3	4
1	0.0	0.0	0.0	0.0	1.0000
2	0.0002	0.0	0.0002	0.0	0.9996
3	0.0144	0.0	0.0019	0.0	0.9837
4	0.1018	0.0	0.0052	0.1000	0.7930
5	0.3693	0.1083	0.3133	0.1872	0.0220
6	0.6508	0.0512	0.2039	0.0940	0.0
7	0.9687	0.0297	0.0016	0.0	0.0
8	0.9997	0.0	0.0003	0.0	0.0
9	1.0000	0.0	0.0	0.0	0.0

Table 5: The Joint Probabilities for n Wells in Period t and m Wells Drilled Prior to t

	t	m	n = 0	1	2	3	4
Run 1:	2	4	---	---	---	.9762	.0182
	3	6	.0019	---	.0037	---	---
	3	7	.0714	---	.4330	.4717	---
	3	8	.0121	---	.0061	---	---
	4	6	.0019	---	---	---	---
	4	7	.0714	---	---	---	---
	4	8	.0140	---	.0019	---	---
	4	9	.2698	0.1632	---	---	---
	5	6	.0019	---	---	---	---
	5	7	.0714	---	---	---	---
	5	8	.0140	---	---	---	---
	5	9	.2698	---	---	---	---
Run 2:	2	4	.0001	---	---	.9766	.0233
	3	4	.0001	---	---	---	---
	3	5	---	---	---	---	---
	3	6	---	---	---	---	---
	3	7	.0744	---	.0183	.8839	---
	3	8	.0127	---	.0106	---	---
	4	4	.0001	---	---	---	---
	4	5	---	---	---	---	---
	4	6	---	---	---	---	---
	4	7	.0744	---	---	---	---
	4	8	.0127	---	---	---	---
	4	9	.0122	.0061	---	---	---
	5	4	.0001	---	---	---	---
	5	5	---	---	---	---	---
	5	6	---	---	---	---	---
	5	7	.0744	---	---	---	---
	5	8	.0127	---	---	---	---
	5	9	.0122	---	---	---	---

Table 5 cont.

t	m	n = 0	1	2	3	4
Run 3:						
2	4	.0020	---	.0009	.4204	.5767
3	4	.0020	---	---	---	---
3	5	---	---		---	---
3	6	.0020	---	.0007	---	---
3	7	.0050	---	.3385	---	.0769
3	8	.1170	.1595	.1777	.1225	---
4	4	.0020	---	---	---	---
4	5	---	---	---	---	---
4	6	.0002	---	---	---	---
4	7	.0050	---	---	---	---
4	8	.1172	---	.0004	---	---
4	9	.0242	---	.2262	.2476	---
4	10	.0889	---	.0889	---	---
4	11	.1534	.0460	---	---	---
5	4	.0020	---	---	---	---
5	5	---	---	---	---	---
5	6	.0002	---	---	---	---
5	7	.0050	---	---	---	---
5	8	.1172	---	---	---	---
5	9	.0242	---	---	---	---
5	10	.0891	---	.0002	---	---
5	11	.2967	.0829	---	---	---
6	4	.0020	---	---	---	---
6	5	---	---	---	---	---
6	6	.0002	---	---	---	---
6	7	.0050	---	---	---	---
6	8	.1172	---	---	---	---
6	9	.0242	---	---	---	---
6	10	.0891	---	---	---	---
6	11	.2967	---	---	---	---

Table 5 cont.

	t	m	n 0	1	2	3	4
Run 4:	2	4	.0002	---	.0002	---	.9996
	3	4	.0002	---	---	---	---
	3	5	---	---	---	---	---
	3	6	.0000	---	.0002	---	---
	3	7	---	---	---	---	---
	3	8	.0142	---	.0017	---	.9837
	4	4	.0002	---	---	---	---
	4	5	---	---	---	---	---
	4	6	.0000	---	---	---	---
	4	7	---	---	---	---	---
	4	8	.0142	---	.0001	---	---
	4	9	---	---	---	---	---
	4	10	.0003	---	.0015	---	---
	4	11	---	---	---	---	---
	4	12	.0871	---	.0037	.1000	.7930
	5	4	.0002	---	---	---	---
	5	5	---	---	---	---	---
	5	6	.0000	---	---	---	---
	5	7	---	---	---	---	---
	5	8	.0142	---	---	---	---
	5	9	---	---	---	---	---
	5	10	.0003	---	.0001	---	---
	5	11	---	---	---	---	---
	5	12	.0874	---	.0012	---	---
	5	13	---	---	---	---	---
	5	14	.0009	---	.0027	---	---
	5	15	.0015	---	.0765	---	.0220
	5	16	.2647	.1083	.2328	.1872	---
	6	4	.0002	---	---	---	---
	6	5	---	---	---	---	---
	6	6	.0000	---	---	---	---
	6	7	---	---	---	---	---
	6	8	.0142	---	---	---	---
	6	9	---	---	---	---	---
	6	10	.0003	---	---	---	---
	6	11	---	---	---	---	---
	6	12	.0874	---	.0001	---	---
	6	13	---	---	---	---	---
	6	14	.0012	---	.0009	---	---
	6	15	.0015	---	---	---	---
	6	16	.2656	---	.0018	---	---
	6	17	.0060	---	.0848	.0940	---
	6	18	.1164	---	.1164	---	---
	6	19	.1579	.0512	---	---	---

Table 5 cont.
Run 4 cont.

t	m	n: 0	1	2	3	4
7	4	.0002	---	---	---	---
7	5	---	---	---	---	---
7	6	.0000	---	---	---	---
7	7	---	---	---	---	---
7	8	.0142	---	---	---	---
7	9	---	---	---	---	---
7	10	.0003	---	---	---	---
7	11	---	---	---	---	---
7	12	.0874	---	---	---	---
7	13	---	---	---	---	---
7	14	.0012	---	.0001	---	---
7	15	.0015	---	---	---	---
7	16	.2659	---	.0006	---	---
7	17	.0060	---	---	---	---
7	18	.1173	---	.0009	---	---
7	19	.2130	.0297	---	---	---
8	4	.0002	---	---	---	---
8	5	---	---	---	---	---
8	6	.0000	---	---	---	---
8	7	---	---	---	---	---
8	8	.0142	---	---	---	---
8	9	---	---	---	---	---
8	10	.0003	---	---	---	---
8	11	---	---	---	---	---
8	12	.0874	---	---	---	---
8	13	---	---	---	---	---
8	14	.0012	---	---	---	---
8	15	.0015	---	---	---	---
8	16	.2659	---	.0000	---	---
8	17	.0060	---	---	---	---
8	18	.1176	---	.0003	---	---
8	19	.2130	---	---	---	---
9	4	.0002	---	---	---	---
9	5	---	---	---	---	---
9	6	.0000	---	---	---	---
9	7	---	---	---	---	---
9	8	.0142	---	---	---	---
9	9	---	---	---	---	---
9	10	.0003	---	---	---	---
9	11	---	---	---	---	---
9	12	.0874	---	---	---	---
9	13	---	---	---	---	---
9	14	.0012	---	---	---	---
9	15	.0015	---	---	---	---
9	16	.2659	---	---	---	---
9	17	.0060	---	---	---	---
9	18	.1176	---	.0000	---	---
9	19	.2130	---	---	---	---

Table 6: The Probability of n Discoveries Prior to or at Period t

Run 1:

t \ n	1	2	3	4	5
1	.9762	.7381	.2619	.0238	0.0
2	1.0	1.0	.9167	.5000	.0854
3	1.0	1.0	1.0	1.0	.8350
4	1.0	1.0	1.0	1.0	1.0

Run 2:

t \ n	1	2	3	4	5
1	.9762	.7381	.2619	.0238	0.0
2	1.0	1.0	.9170	.5018	.0871
3	1.0	1.0	1.0	1.0	.9938
4	1.0	1.0	1.0	1.0	1.0

Run 3:

t \ n	1	2	3	4
1	.8586	.4061	.0667	.0020
2	.9899	.8671	.5099	.1242
3	1.0	.9818	.7636	.3909
4	1.0	1.0	1.0	.9169
5	1.0	1.0	1.0	1.0

Run 4:

t \ n	1	2	3	4
1	.6244	.1620	.0134	.0002
2	.8978	.5346	.1531	.0144
3	.9855	.8468	.4654	.1018
4	.9990	.7737	.8172	.3693
5	1.0	.9965	.9123	.6508
6	1.0	1.0	1.0	.9687
7	1.0	1.0	1.0	.9997
8	1.0	1.0	1.0	1.0

MEAN AND STANDARD DEVIATION OF THE NUMBER OF WELLS DRILLED

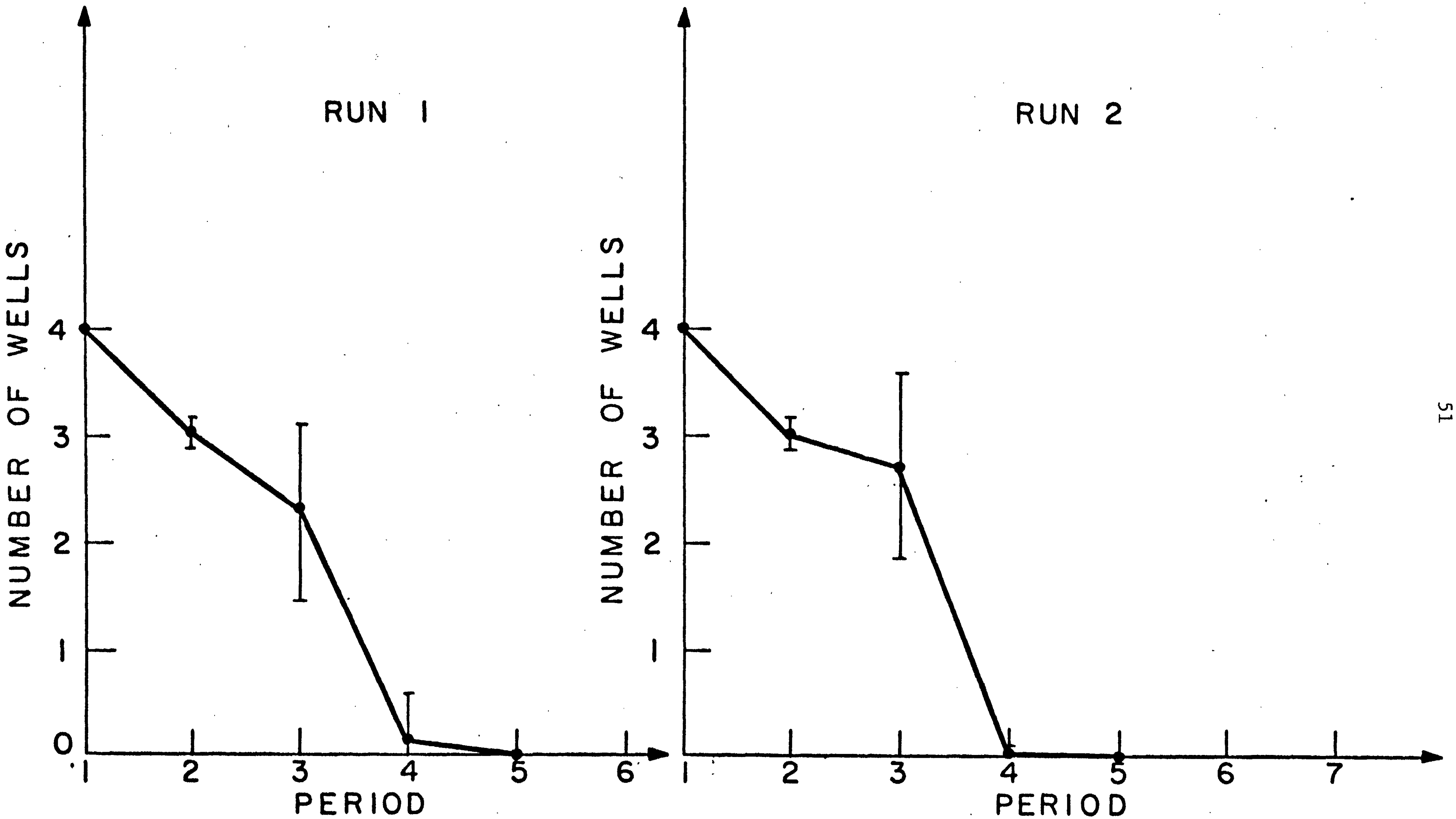


FIGURE 5

MEAN AND STANDARD DEVIATION OF THE NUMBER OF WELLS DRILLED

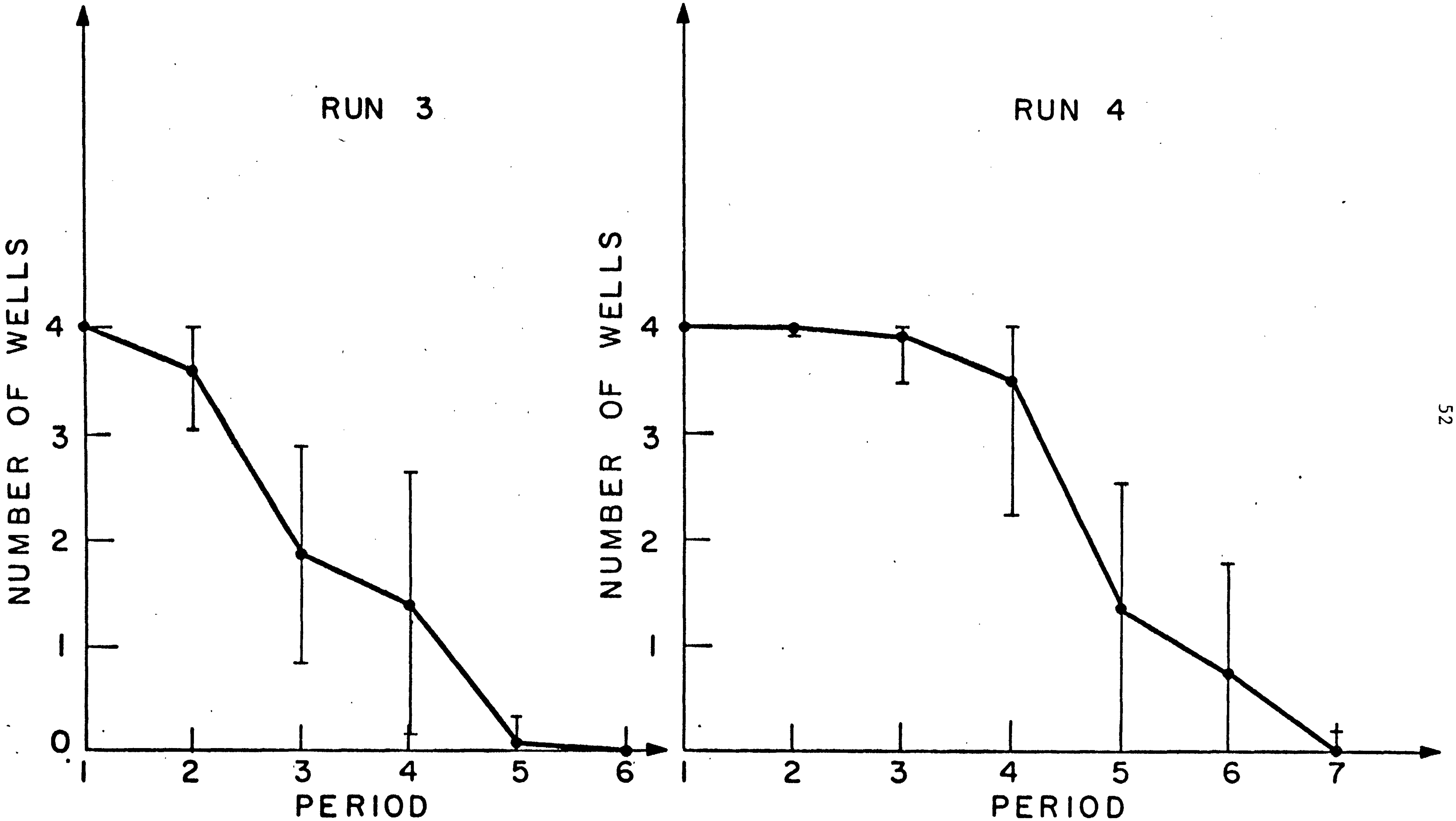
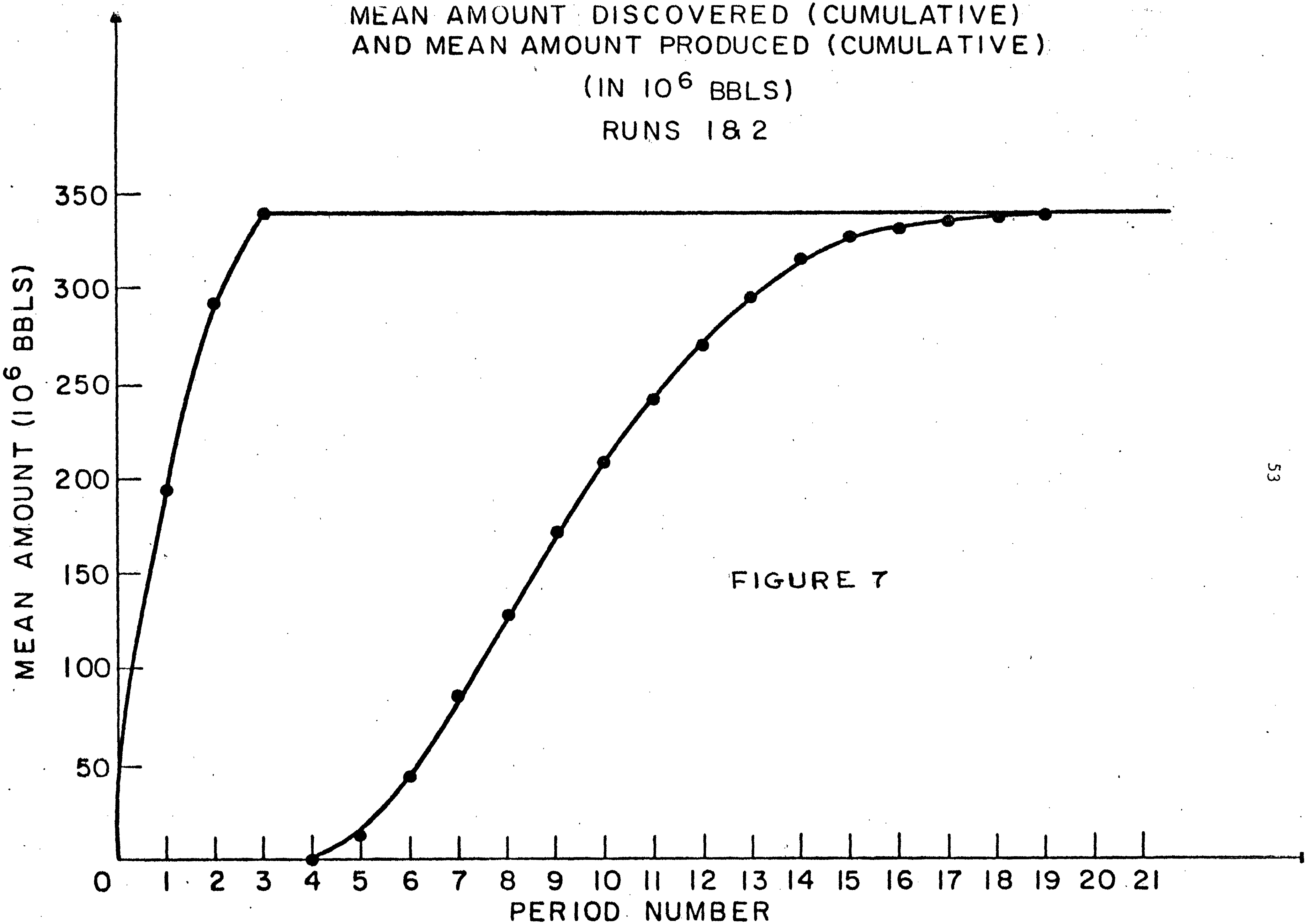


FIGURE 6

MEAN AMOUNT DISCOVERED (CUMULATIVE)
AND MEAN AMOUNT PRODUCED (CUMULATIVE)
(IN 10^6 BBLS)
RUNS 1 & 2



MEAN AMOUNT DISCOVERED (CUMULATIVE)
AND MEAN AMOUNT PRODUCED (CUMULATIVE)
(IN 10^6 BBLS)
RUN 3

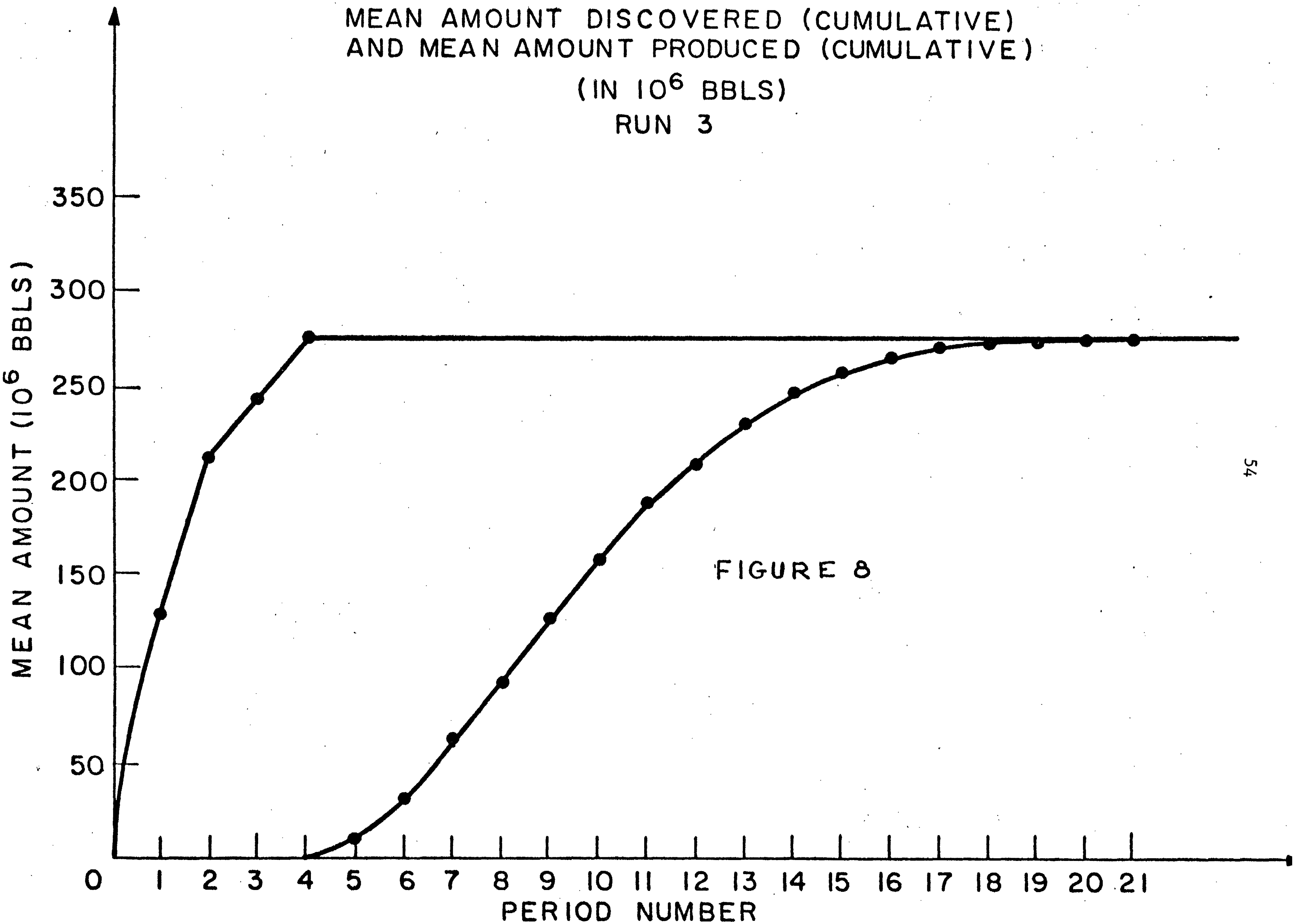


FIGURE 8

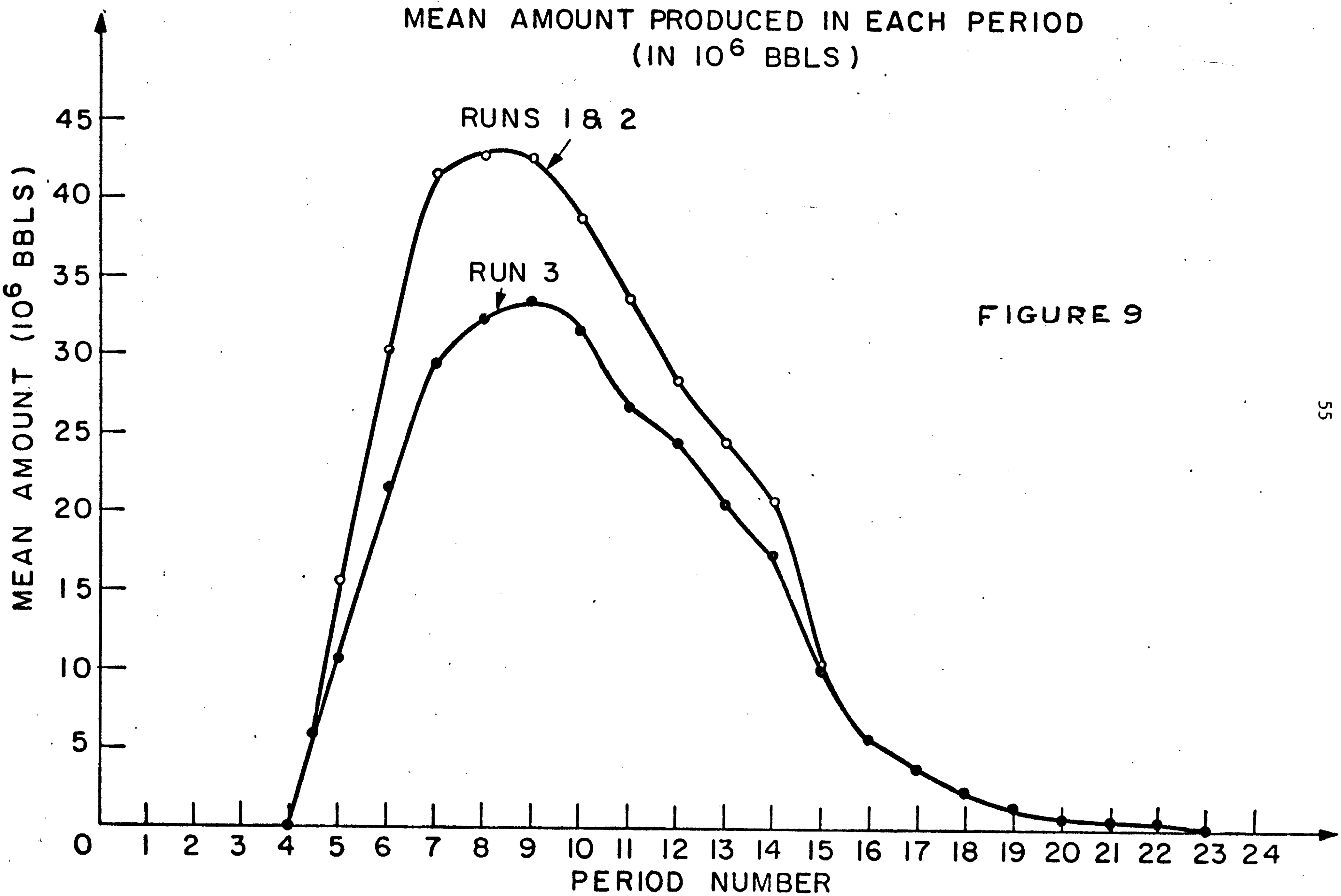

MEAN AMOUNT PRODUCED IN EACH PERIOD
(IN 10^6 BBLS)
RUNS 1 & 2
RUN 3
MEAN AMOUNT (10^6 BBLS)
45
40
35
30
25
20
15
10
5
0
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24
PERIOD NUMBER

FIGURE 9