Constructing Design Concepts
A Computational Approach to the Synthesis of Architectural Form

by

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ABSTRACT
Architects use concepts about space to solve problems and to form designs. A design concept is the manifestation of the basic instability of our mental performance: it is a makeshift that provides general direction for exploration. In synthesis architects explore concepts by inventing transitions that conclude to the description of artifacts.

This study suggests that the process of synthesis, which is an act of human imagination, can be approached by the means of a calculus, as calculation. Taking into account the nature of design concepts and practices, as well as the developments in the field of shape computation, the study explains why and how computational methods can be applied in the process of synthesis of architectural form. Through a theoretical analysis, and actual design paradigms, it shows that shape computation can undertake conceptual and execution tasks in the studio.

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To my parents

Dimitrios Kotsopoulos and Maria Kotsopoulos-Fournaraki

TABLE OF CONTENTS

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Preface

The scope of this study is limited to architectural design. The motivation is practical: My view is that the integration of computation in the architectural studio remains unsatisfactory. Computation is approached as a peripheral task related to representation, or entirely replaces the traditional studio techniques. Overall, computational and studio techniques remain segregated.

Beginning from this problem, the thesis outlines a computational framework for design synthesis, aiming to achieve better integration of computation in the studio.

Central in the thesis is the use of shape computation theory in devising descriptions for artifacts from design concepts. Using my architectural experience, I expose existing studio techniques, I make them explicit, and I model them with shape computational methods.

The contributions are two: The integration of computational means such as shape algebras and rule schemata in design thinking, and the introduction of new computational paradigms for studio practice.

The results and experiments of this study are presented in two parts. The first part outlines concepts that are used in the second part.

The first part, including chapters I, II, III, IV, shows *what is* the possible association between studio techniques and computation and explains *why*. It also presents the relationship between proposed ideas and existing ideas.

Chapter I introduces preliminary questions and terms. The proposition that synthesis can be approached as calculation is also briefly discussed.

Chapter II introduces basic notions of shape computation theory. The core idea of the dissertation, the construction of design concepts through computation is set: When a design concept is proposed, some framework of action is selected. This framework can be informal, or formal following compositional methods or principles. It can be personal, or driven by convention. But it is characterized by some degree of internal coherence.

Chapter III deals with architectural description. Sketches, diagrams, and 3d models are used in the development of spatial concepts. The spatial calculations used in this process involve areas, volumes and their boundaries. Areas and volumes correspond to the "content" of rooms and spaces, in 2d and 3d respectively. Their boundaries are used to describe their "form". Content and form are constantly interrelated in synthesis. But, the properties of the spatial elements expressing content and the elements expressing form are not identical in calculations.

Chapter IV presents the properties of a calculating device suitable for the construction of design concepts: the overlaying of multiple or paper sheets in a stack, to produce a single description. The device allows heterogeneous fragments to be synthesized. Each design description (plan, section, or elevation) is approached as a synthesis of many partial descriptions.

The second part of the thesis, namely chapters V, VI and VII shows *how* shape computation produces designs based on some design concept. Three applications in composition serve as paradigms.

Chapter V examines the compositional concept of the "*domino house*". The project is based on the building program of an actual competition for low cost housing. The design process starts from the definition of a vocabulary of rooms, and a number of spatial relations, which describe how the rooms relate. Then, the possibilities of constructing designs from these are examined systematically. The search evolves from the definition of the "parts" (spatial vocabulary) to the construction of possible "wholes" (designs). The project is introduced as an exercise aiming to integrate rules, analogue, and digital tools in the studio. The described process can be characterized introspective and prescriptive: Introspective because each potential designer can choose to develop different design alternatives; Prescriptive because each designer produces a prescriptive system of rules, which provides a norm for exploration.

Chapter VI presents the making of the plans for an office building. Starting from a specific site and building program, the designer proposes a design concept. This is gradually developed into a design with the aid of rule schemata and rules. Forms and relations are defined gradually on the basis of the design concept. The construction proceeds from a potential "whole" (design concept) to the definition of the "parts" (rooms and spaces). The described process can be characterized retrospective and descriptive: Retrospective, because an original design concept is available; Descriptive, because a computational process is used to derive the consequences of the initial concept, without being a replication of the exact actions of the designer.

Chapter VII presents a computational interpretation of Steven's Holl design concept for Simmons Hall undergraduate dormitory at MIT. The case study includes aspects from both the previous two examples. The search begins from a "whole", conceptually framed by the design concept of "porosity". And also, the building program allows the specification of a spatial unit that is used in the composition. The described process is retrospective and descriptive. It presents how the concept of porosity was used in the composition but it is not a replication of the design steps that Holl and his team followed. The educational interest of this case study is to examine how rule schemata and rules can express the conceptual part of the process. Further, since Simmons Hall is the only implemented design from all three case studies, its educational importance extents to the comparison between "conceptual" and "actual" implementation. As it turns out certain decisions taken in the studio require revision at the stage of the actual implementation. The comparison between conceptual and actual is approached here as a methodological tool of design criticism.

The dissertation ends with the summary of results, the general conclusions, and suggestions for further research.

I. PRELIMINARIES

Abstract. Three terms are introduced: design problem, design concept and synthesis. The proposition that synthesis can be seen as calculation is also briefly discussed.

1. Introduction

Greek thought captures vividly the essence of human thinking in the myth of creation of man by two brothers: Prometheus and Epimetheus (Pausanias 10.4.4). Prometheus is one who offers a description or explanation (μύθος) regarding a state of affairs beforehand (πρό). His name means "he-who-knows-before". Under the light of a priori knowledge Prometheus is able to estimate the consequences and foresee the course of an action-plan (πρόγνωσις). His brother Epimetheus, is quite opposite in nature and habit of thought. Epimetheus can offer a description or explanation (μύθος) only after (επί) the experience. His name means "he-who-knows-after". He is unable to determine the principles of action beforehand.

The myth presents Prometheus challenging Zeus and championing the cause of man. The Western tradition promptly pronounces him a hero, the prototypical scientist, or engineer. Vitruvius (*De Architectura* IX) characterizes Pythagoras as a great "Prometheus", with brilliant contributions to the art of building and engineering. Epimetheus on the other hand is not thought to be very intelligent. He is characterized as sluggish or too impulsive, acting first and only later grasping, by reflection, the significance of what he had done. Prometheus thinks ahead and explains a priori, while Epimetheus perceives first and explains a posteriori. It is here that we should seek the difference between the two brothers whose attitude with regard to the view of the world has so strong an influence on us.

Since then, a great chasm continues to exist between those who relate everything to a universal pre-organizing principle in terms of which all that they do obtains significance, and those who pursue many ends, often unrelated and contradictory.

In the light of the myth of Prometheus and Epimetheus, I claim that design does not involve prognosis (*πρόγνωσις*) but involves diagnosis (διάγνωσις). It is process of pinpointing, or spotting problems and their associations out of a mesh of muddled and intertwined conditions. This study suggests that a universal pre-organizing principle does not exist in design. But architects and practitioners base their actions on their own imperfect concepts, hypotheses, and techniques, and proceed to the construction of self-consistent design systems.

The outcome of this process is the formation of an order, one part of which, or certain conditions of which, are objective and another is subjective. To achieve that, a designer, just like a craftsman depends on a personal technique that has proved effective in the past. Overall, a designer starts out with contradictory hopes: One must act, and simultaneously persist in an attitude, in order to give the elements of one's thought the time to create affinities, and to construct. The hypotheses and restrictions that one imposes on oneself reveal by their randomness that they are only a small part of what one is capable to imagine. New kinds of structure can emerge from new hypotheses. But, it is also by the technique of construction, or the "craftsmanship" that one achieves original goals, and not just by surrender to impulse.

The scope of this study develops around the question how designs are processed when people design new artifacts. In this section the terms design problem, design concept, and synthesis are introduced in the way that are put into use in the rest of the thesis.

2. Design Problem

Newell and Simon (1972) describe a problem as follows: "*A person is confronted with a problem when he wants something and does not know immediately what series of actions he can perform to get it*". This definition is true for all problems. But one can distinguish different kinds of problems.

For example, the problems of Euclidean geometry, or problems within any calculus, are either intuitively or demonstratively certain. They are solvable without direct reference to what exists in the physical world. For problems defined within games, one develops prompt understanding of the permissible actions and objectives and searches for the appropriate combination of actions that leads to the end. But there are problems where the permissible actions are not determined, and the objects of reasoning cannot be ascertained. The contrary of every assertion is possible without implied contradiction. Hume (1775) suggests that all reasoning concerning "facts" is of this type. And although it is often supposed that there is an evident connection between some fact X and another Y inferred from it, there is nothing to bind them together. Design problems belong to this last type of problems.

Simon (1988) believes that "*everyone designs who devises courses of action aimed at changing situations into preferred ones*". In architectural design a preferred situation can be understood in different ways. For example, certain arrangements of forms in physical space may be preferable to other because they cause a visual-intellectual, or psychological, response. Or, some arrangements succeed to accommodate particular activities more effectively than others, and this makes them preferable for some people. And, some other designs turn out to be more economical, in terms of space, time, materials etc.

Architectural design includes problems that cannot be organized deterministically. One has to determine both the rules and the objectives of the search.

A typical statement of a design problem is based on the analysis of the common practice. It mirrors the conventions, the habits and the optimum patterns of this practice. Statistical information, charts, diagrams, text regarding the building code, the program, or the site prohibit certain options while allowing others. But they do not determine the objective of designing. This demands interpretation and evaluation of the given information, which is not a trivial thing to produce.

Further, designers approach the available information regarding a design problem without specific method. And, as they are different in their way of thinking, in culture, in their needs and their capacity for observation, their readings differ.

Compounded with several elements demanding effort, attention, and often the most specialized knowledge, the process of designing has to remain finite. But, it cannot be reduced to a system of uniformly determined acts.

At a particular state s there is a certain understanding of the given conditions, sometimes a simple disposition, which becomes of great value, and impulse. It is the moment that one starts to act. A design concept is a hypothesis that corresponds to the intent-towards-action of the designer, produced at this stage, in response to the transitory understanding of the conditions. By accepting a minimum number of attitudes, and assuming certain definitions, the designer limits the space of search and sets a basis upon which further decisions can be made.

3. Design Concept

Design problems demand that some objectives be defined by an observer. A design concept is a working hypothesis. It indicates the intention regarding the making of an artifact of a certain kind, and establishes an interrelationship among the factors that one considers crucial for the design.

A design concept does not evolve by simple analysis of the provided information. It involves judgment and synthesis. It is not a classification of the given facts for the object under consideration. It suggests a possible new meaning for it.

Minsky (1974) reflecting on the first glimpses of the thinking process notes: "*Thinking always begins with suggestive but imperfect plans and images; these are progressively replaced by better–but usually still imperfect–ideas*".

The generation of artistic concepts, including images, analogies, motifs, and rhythms, is characterized by facility, fragility and incoherence. Design concepts mirror the transitory understanding of the conditions, and disclose a series of potentials and contradictions. They cannot be described in fixed detail. And on the other hand, they can be astonishingly precise on certain details, and relationships. They set the mind into a particular mode of action that leads to thoughts and actions.

A design concept can take the form of a simple narrative, which usually ends up into a sketch. It is a tentative statement, produced in one's mind in response to a problem, and it is unjustifiable with rational criteria.

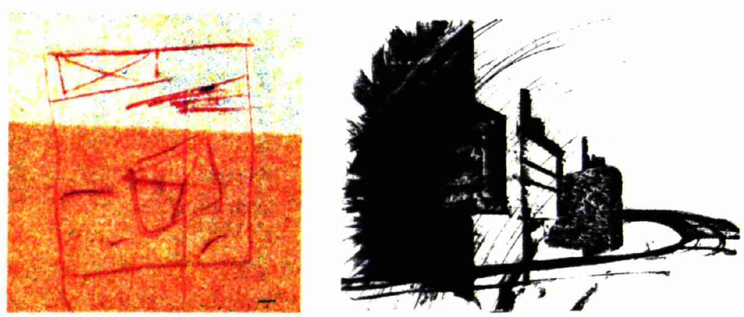


Figure 3. A design concept for an office building in Los Angeles (MPAN, 2003)

March (1976) points out that statements of this kind cannot be evaluated as true or false, because they do not provide quantitative information. The inability to frame the initial concept in a definite way is not due to the complexity of the provided information, or the observed conditions. It belongs to the very nature of interpretation.

To avoid the ambiguity of concepts, Aristotle proposed: "*In order to formulate the appropriate propositions to be proved, one must pick out the divisions of the subject matter*". Both Plato (Πολιτεία, 261^{α}) and Aristotle (Μεταφυσικά, $1038^{\alpha}28$) suggested the operation of *τομή* or *ανατομή* (division). That is, analysis of a subject, to elicit its properties. Aristotle recommended to keep in mind a tree of the genera and the species and to discover the widest class of the whole of which a certain attribute can be predicated.

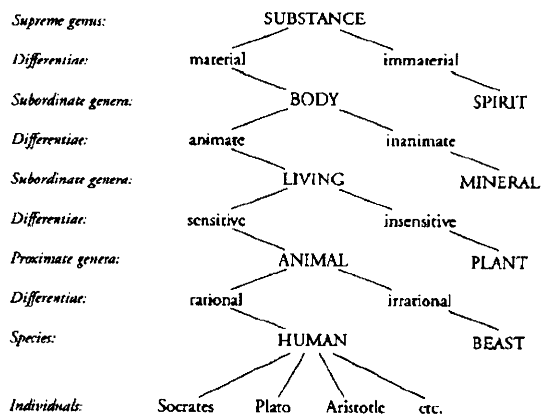


Figure 1. Hierarchical categories: Tree of Aristotle's Substance translated by Peter of Spain (1239), from Eero Hyvonen, *Ontology perspectives* (2003).

The Aristotelian analysis provided a "semantic net" for concepts. It was evolving a hierarchy of the cosmos, including man and his aims. The objectivity of the structure, and not just man and his purposes was to set the standards for the individual thoughts and actions. The characteristic of the structure is the analyzability of everything into separate parts that "work" together. A similar hierarchical analysis was proposed in contemporary design theory by Alexander (1967).

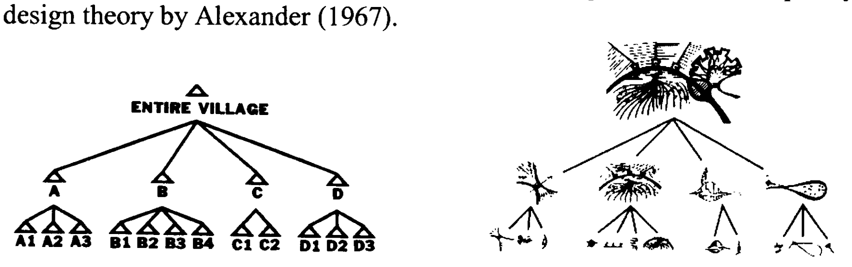


Figure 2. The hierarchical approach of Alexander from the *Notes in the Synthesis of Form* (1967).

Hierarchies of the previous kind are determined by regular connections between their defining characteristics. The relationships between their nodes are predefined. They are based on the assumption that they frame the essential character (physical, functional, or other) of the thing they describe. But, the notion of an essential characteristic is too obscure to become the criterion for any classification. In design, no examination of an object could objectively establish any of its characteristics as more essential than another. Definition is a matter of identity. But it also involves speculation, imagination and theorizing. The ideas by means of which a designer seeks to establish a design solution are chosen with a view to establish something novel and extraordinary. Therefore, the best description is the one that enables us to make further suppositions, and to produce unexpected results. This becomes possible in descriptions that are characterized by absence of standard connections between them.

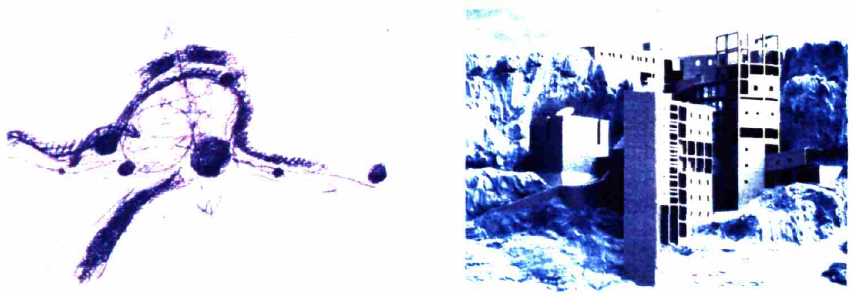


Figure 3. The relationships described in the conceptual schema (left) are not simply hierarchical. Undergraduate Thesis, NTUA, M. Panagopoulou, S. Kotsopoulos, instructor: T. Biris. In Biris, *Signs and Precepts of Architecture* (1996).

Sometimes design concepts include imaginary elements, with no direct correspondence to the experienced facts. Such concepts cannot be eliminated in favor of the existing standards. It is often the fictitious concepts, rather than those that are fully definable, that enable designers to interpret and organize novel designs. Instead of excluding them on the ground that they are vague, one must admit those for empirical interpretation. The discovery of such concepts can revolutionize understanding and ones' way of looking at things.

Finally, design concepts emerging out of specific empirical facts, such as: the precise structural behavior of some material, or component, the movement of the sun, or the requirements of particular light and sound conditions, etc., can also be easily defined at great numbers. Such concepts can be operationally useful and they are usually unambiguous in their definition. However, most of them end up of no great use if they do not provide the principles that connect them with the rest of the characteristics.

In summary, design is an empirical inquiry that involves hypothesis and imagination, deduction and observation. Guided by previous knowledge a designer has to invent a concept, or a set of concepts. These concepts may lack immediate experiential meaning. The designer invents a system of actions implied in terms of them, and an interpretation for the resulting network of relationships. All these are finally implemented in a manner that retains some link with the existing standards.

4. Synthesis

Humans possess a strong ability to assemble and to use compositions made out of words, sounds, physical forms, colors, etc. This ability has been exercised during all the historic phases and socio-economic, cultural or other circumstances.

Assumptions about the process of architectural synthesis must pay attention to the heterogeneous diversity of conditions and problems that impose themselves on the designer and are involved in his work. Designs emerge from the treatment of heterogeneous fragments that evolve, mingle, enter into relations, and produce effects.

Synthesis is a technique of assembling, on the basis of a design concept. Biris (1996) notes: "*The design concept lies constantly at the beginning of a series of phases that evolve incrementally. It is the center of multiple stratums, retaining constant relationship and affinity with all the phases of synthesis*".

In this study, design synthesis, which is an activity of the human imagination, is approached as a process of calculation. The study is conducted within the context of shape computation theory, introduced by Stiny and Gipps (1972), where algebras of spatial elements are used as an abstraction of our interaction with shapes.

It is suggested that the design process consists of posing a design concept, or hypothesis, deriving its consequences, and then testing the results against the known empirical facts. The outcome of this process, as Schon (1987) points out: "*is objective, in the sense that one can discover error to it, but it still remains personal*". It is objective because the testing reveals any inconsistencies against the known standards. And, it is personal because actions are relative to the initial concept and the commitment to a particular system of values.

Synthesis requires the invention of a sequence of thoughts and actions in combination with some initial hypothesis. The possibilities even within the limits of a single hypothesis remain immense. The method of the designer consists of effectively narrowing the space of search by developing a particular way of looking at things: allowing certain procedures, while excluding others.

First, the general consequences of a design hypothesis are sketched out in a broad manner. Abstract schemata of action are used for this purpose. They suggest particular relationships without determining the exact identities of the participating elements. These can be expressed computationally as rule schemata including predicates and variables.

Second, from the action schemata, specific actions can be introduced and several alternative courses of action can be proposed. The possible actions can be expressed as rules. The rules are defined by substitution of the variables in the rule schemata. A rule specifies that given some condition x, a conclusion y can be produced: An objective can be accomplished provided that some conditions are satisfied. But the application of any rule does not guarantee the accomplishment of a desirable broader objective. For this reason, the design concept is necessary to provide general direction.

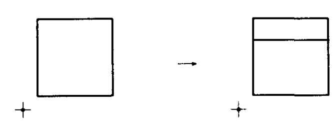
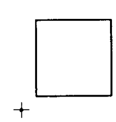
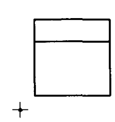
Third, an ordering of the rules into an effective system (*grammar*) can be produced retrospectively.

5. Discussion

A fine work of architecture suggests that it is a manifestation of thought. For this thought some kind of processing is required. This processing is manifested as a sequence of actions.

For example, the dissection of a quadrilateral space by a wall parallel to any of its boundaries can be a possible desirable action. The anticipation of this action consists in an ability to know that a particular result can be produced whenever such a space is found anywhere in the drawing.

The proposed action is expressed by the following computational rule:

rule	
condition	
conclusion	

The application of a sequence of thoughts can proceed in steps, where each thought is expressed by a rule, and becomes a step in a calculation. New steps can be introduced by inserting the appropriate rules, while the design concept serves a general framework of action, on the basis of which computational rules can be proposed.

The next chapter II introduces some basic notions of shape computation theory, and discusses in further detail the proposition that design concepts can be approached through shape computation.

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II. SHAPE COMPUTATION THEORY

Abstract. The proposed view is that in design we can develop appropriate formal means for the treatment of spatial elements, and of design concepts.

1. Introduction

The study of phenomena aims to establish minimum principles by means of which we can describe and explain them. A computational theory includes some abstract deductive part and some syntactic-interpretive part. The deductive part is an environment where calculations of some kind take place (calculus). The syntactic-interpretive part includes statements that assign empirical-practical meaning to calculations. The choice of the appropriate calculating device is important in the formation of such a theory because it may rule out certain interpretations.

Designs have multiple attributes that can be distinguished and described: typological, semantic, psychological, sociological, and more. Shape computation theory deals mainly with form, the elements of space, and their possible ways of interaction. To capture the interaction of shapes, shape computation theory uses a shape calculus, and syntactic-interpretive statements. The shape calculus is an algebraic framework where shapes of 0, 1, 2 and 3 dimensions are used to calculate. The syntactic-interpretive part, uses production rules, and deals with the construction and interpretation of design languages.

This study is conducted along the lines of shape computation theory. It is based on the underlying assumption that in design, apart from anything else, spatial elements are put together to form spatial compositions.

2. Computational Theory

The examination of a domain of phenomena is often supported by a calculus. The phenomena in view can be expressive, or natural. For example, natural language is the expressive medium that we use to communicate our thoughts through speech. Our understanding of the logical structure of natural languages relies on the formal models of logic. And, the examination of their syntactic properties relies on the models of formalized grammars. A calculus is the formal environment where calculations of some kind can take place. The propositional calculus is a calculating device designed to reveal certain characteristics of the logical structure of arguments. And a formal grammar is a calculating device that reveals the syntactic properties and relations of empirically given expressions.

The modern origins of this kind of investigation can be found in Descartes, Leibniz, Boole, Frege, Russell and Whitehead, Carnap, Lesnievski, Tarski, and the logical empiricists. Analogous studies concerning the syntax of languages were performed in the early 20s by Jespersen, and latter in the 50s and the 60s by Chomsky and the researchers of Artificial Intelligence. In the preface of his otherwise technical work *Der logische Aufbau der Welt*, Carnap (1928) places this approach into context with the arts: "*We feel an inner affinity between the attitude that lies at the bottom of philosophical work and the spiritual attitude which expresses itself at present in entirely different areas of life: we sense that attitude in currents of contemporary art, especially in architecture, and in movements that seek to give a meaningful shape to human life*". Carnap took interest in art and architecture by giving lectures at the Bauhaus, and so did other logical empiricists at the Chicago School of Design in the 40s. During the entire period from the 20s to the 50s Klee and Kandinsky attempted to introduce methodic thought in their teaching and practice of painting.

The common aspect in the course of all computational theories is the use of calculating systems and the effort to map empirical data on them. A theory Θ of this kind includes some calculus C and some set of rules of syntax and interpretation R.

For example, in logic and in formal grammars the expressions of natural language are first reduced to strings, to become expressions of the calculus. Then, they are treated according to techniques that originate in set theory. Logicians examine how the words fit together so as to preclude the possibility that the premises are true and the conclusion false. And, grammarians examine the set of conventions that allow the mechanical generation of a corpus of expressions. In logic an argument is shown valid by providing a translation into a demonstrably valid argument in the formal language. Atomic sentences and "connectives" $\wedge$, $\vee$, $\neg$, $\rightarrow$, $\leftrightarrow$, $\forall$, $\exists$ are used to reveal the structure of the argument. In syntax production rules and transformations are used to generate sentences from finite sets of atomic phonemes.

The most common objection to the computational approach is that a computational theory Θ fails to reflect the way in which one acts and thinks, and, that a calculus can represent, at best, only moments in a system that is continually changing. I think that this objection is reasonable, but it is missing the point. The issue is not how to mirror "all" the heuristic and pragmatic aspects of a real process but "some" aspects and features of it. The question is what are these aspects and features in each case, and what is an appropriate calculus to express them? Calculating systems are also constructions. And however ingenious many of them may be, they can have little or no interest from an empirical viewpoint. One cannot just pick any calculus and squeeze the empirical content in, because this may rule out certain interpretations. Therefore, the choice of the appropriate calculus becomes an issue of central importance in the development of a theory, just like choosing the appropriate tool for a task.

Computational design theory was introduced by a group of researchers in the 60's and 70's. The aim of computational design theory was the use of computational methods in design. Some of the proposed computational methods included, set theory (Alexander 1964), graph theory (Steadman 1973), Boolean algebra (March 1972), computer generated design (Eastman 1970; Mitchell 1974), formal syntax (Hillier et al. 1976), and shape grammars (Stiny and Gips 1972). Computation was used either as a prescriptive instrument, or as a descriptive device of the behavior of the designers. In the prescriptive case, computation was applied as a prescriptive system of rules providing a norm for empirical study; in the second, as a descriptive affirmation that the claims of a hypothesis produce analogous results.

3. Computational Design Theory

The expressive performance of a visual artist can be compared to that of a speaker: They both conform to a pre-existent mode of communication, but also they build up means to go beyond the conventions. The speaker uses conversation to produce a response in an audience. The visual artist uses visual-spatial arguments to produce a response in a group of spectators. They both have the authority on the subject of their intention, and a motive for making that intention clear, and being acknowledged for it. The speaker deals with speech, sound and meaning. The designer deals with form, color, and meaning. On these general grounds, an effort to establish a computational theory for design can be initiated.

What could be the general desired attributes of a computational design theory? First, computational design must treat spatial elements and their properties. Symbols have different properties and convey different kinds of information from higher dimensional elements. A computational device for design must acknowledge these differences. Second, a computational device for design must visualize the interaction of spatial elements as they are used in composition. Third, in design the outcome cannot be a single '*correct solution*'. Computational design must allow alternative potentials. Fourth, some generative capacity is required. Devices that are productive are preferred from ones that are simply suggestive. Fifth, a computational design approach must fulfill some minimum explanatory criteria regarding the design process and the produced artifacts.

The goal of computation in design is not to reduce design into prescriptive formulas that eliminate thinking, or to impose restrictions, but to arrive at conventions that propel creativity. One should be able to build one's own system of conventions, or to alter the established, general, conventions of an existent system.

A computational approach can include other attributes of design, such as meaning. And because artifacts express perception they can also produce psychological explanations. Further, art and architecture as mediums of communication and influence among people can be looked from a historical or sociological viewpoint. A comprehensive examination of design can include several perspectives that are not conflicting: formal, semantic, psychological, sociological, and more.

Shape computation theory focuses mainly on shape, the elements of space and their possible relationships. The basic assumption that is adopted in this study is that in design, at the very least, the elements of space are put together to form spatial compositions. To capture the interaction of spatial elements shape computation theory uses shape algebras. Points, lines, planes and solids are used in spatial calculations, while syntactical and interpretive statements are used to assign empirical meaning to calculations.

4. Shape Computation

Shape computation theory examines the applications of shape calculation. The prospect of calculating with shapes, instead of numbers, was set out by Wittgenstein (1956). But the examination of the consequences of shape calculation was left as an open question. Shape computation was examined in depth for the first time in Stiny and Gips 1972.

The original contribution of Stiny and Gips was the questioning of the nature of calculating. A new type of calculation not only with 0-dimensional elements but also with 1 and 2-dimensional elements was proposed, and put into use. Further empirical basis for the attempt was the observation that a designer producing design-descriptions performs calculations with points, lines, planes, and solids. A "design" is a finite description consisting of finite parts, and produced in finite time.

Shape computation was motivated by the desire to provide an intuitive framework for the development of generative design systems. The systems were named *shape grammars*. This view implied that each finite description occurring in space takes the place of an "expression" within a spatial language. Spatial languages include compositions with certain spatial properties. A *shape grammar* is a system of syntactical-interpretative rules that governs the construction of the language.

Therefore, shape computation theory can be roughly summarized by two interrelated parts.

First is the shape calculus, or the algebraic part. The algebraic part deals with the spatial attributes of shapes and the things that happen when we use them to calculate: each time we add, or subtract shapes, or when we break a shape into parts, or when we manipulate a shape by using transformations, like rotations, reflections etc. The standard mathematical tools used in this part are, Boolean algebra, topology, set theory and lattice theory.

Second is the syntactic-interpretive part. The syntactic part deals with the analysis and synthesis of design languages. It provides the formal means for the construction and interpretation of a number of compositions with certain attributes that are named *design languages*. Design languages can find use in industrial or architectural design, civic engineering, painting and sculpture, etc. Sequences of production rules are employed for the description of these languages.

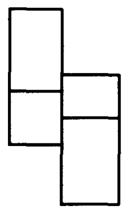
4.1. SHAPE CALCULUS

A shape calculus is a computational framework where shapes of 0, 1, 2 and 3 dimensions are used in calculations that take place in 0, 1, 2 or 3 dimensions. Shape algebras offer a formal account of the spatial properties of shapes and the ways in which they interact.

The construction of shape algebras by Stiny (1991) follows the empirical observation that zero dimensional points interact differently from shapes of dimension greater than zero. Points remain always undivided and discreet. Higher dimensional elements like lines, planes or solids can be divided and embedded on one another in infinite ways. This has some interesting computational and visual consequences.

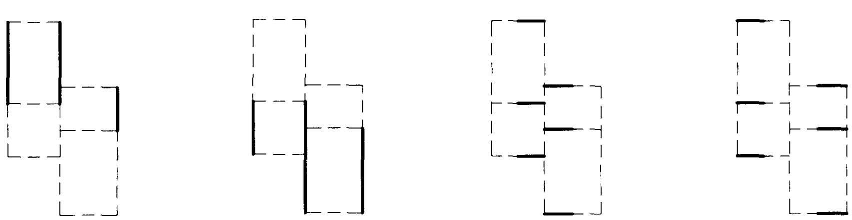
Shapes made out of lines, planes or solids can be decomposed in infinite sets of lines, planes or solids respectively. This allows shapes that look the same to be described by different sets of 0-dimensional points. To treat this ambiguity in the description of shapes, Stiny (1975) proposed to describe shapes of higher than zero dimensions by their maximal elements.

A shape made out of lines,

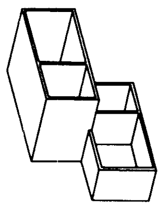


can be described uniquely by a set containing nine maximal lines. The maximal elements of a shape are the larger parts that describe the shape without having common parts. In the example the maximal elements are three vertical, and six horizontal lines.

But the initial shape can also be analyzed in alternative ways. The following option contains six vertical lines, and twelve horizontal lines.

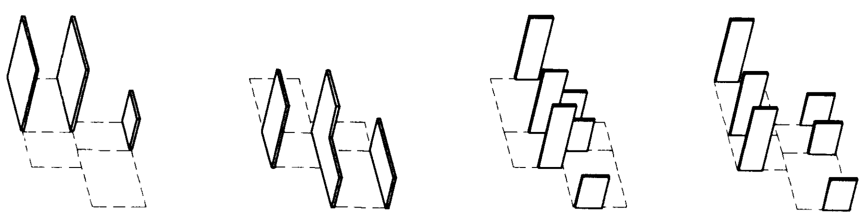


Similarly, the next arrangement made out of solid walls,



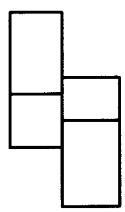
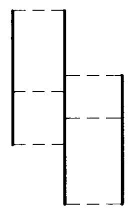
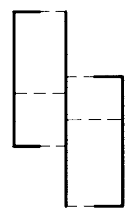
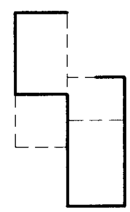
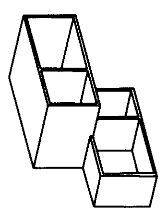
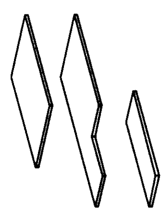
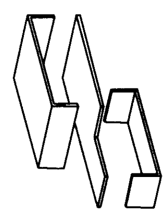
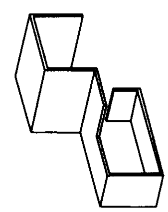
can be analyzed in a set containing nine solid elements: three are extended in length (vertical) and six in width (horizontal) of the object.

And, the initial solid shape can be decomposed in alternative ways to provide different sets of parts: six vertical, and twelve horizontal solid parts.



At a perceptual level Stiny (1996) emphasized that the attribution of structure is not intrinsic in shapes. Names, values and meanings used by convention as a means of identification are the result of retrospective analysis. The interplay between form and meaning opens a field for creative exploration. The next diagram presents some alternative structures that one may retrieve from the initial shapes of the example: I-shaped structure, C-shaped, or W-shaped structure.

TABLE 1. Alternative structures retrieved from (α): I-shaped structure in (β), a C-shaped structure in (γ), a W-shaped structure in (δ)

(α)	(β)	(γ)	(δ)
			
			

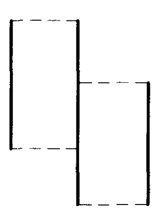
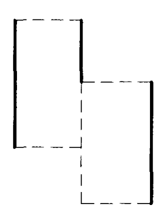
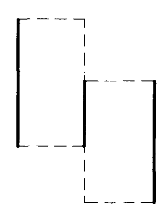
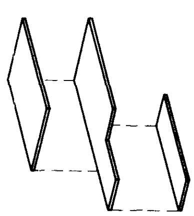
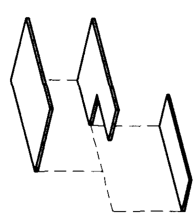
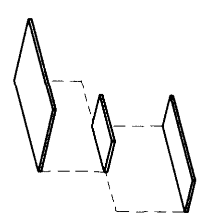
Following these observations about the nature of shapes, Stiny (1980) organized them in algebras. Algebras are sets whose members are closed under a set of operations. In the construction of shape algebras the spatial elements are classified in the Euclidean fashion in four sets containing points, lines, planes and solids respectively. Each algebra U_{ij} contains elements of dimension i = 0, 1, 2 or 3, that are manipulated in dimension j = 1, 2, or 3, so that $j \geq i$. Each set U_{ij} is closed under the operations of sum and product. Each shape-algebra does three things: First, it allows the execution of operations with shapes, second it allows shape-manipulation with the Euclidean transformations, and third it provides a formal ground for the study of the relationship between shape and structure. Due to Stiny 1991 the shape-algebras are classified in the following table,

$$\begin{array}{cccc} U_{00} & U_{01} & U_{02} & U_{03} \\ & U_{11} & U_{12} & U_{13} \\ & & U_{22} & U_{23} \\ & & & U_{33} \end{array}$$

For i = 0, the algebras contain points. For example the algebra U_{00} is formed by a single point. For i = 1, 2 and 3 the algebras contain lines, planes and solids. Shapes made out of lines belong to the U_{1j} row of algebras. Each shape is defined as a finite set of lines of finite and possibly zero length, maximal with respect to one another, manipulated on a line (U_{11}), a plane (U_{12}), or, in space (U_{13}). Shapes made out of planes can be found in the U_{2j} row of algebras. Each shape is defined as a finite set of planes of finite and possibly zero area, maximal with respect to one another, manipulated on a plane (U_{22}), or in space (U_{23}). Shapes made out of solids belong to U_{33} algebra: Each shape is defined as a finite set of solids of finite and possibly zero volume, maximal with respect to one another, manipulated in space.

The following example with lines and solids shows how non-atomic elements interact in space. The elements α and β can be added to produce an element $\alpha+\beta$. Or, the element β can be subtracted from α to produce the difference $\alpha - \beta$. The product $\alpha \cdot \beta$ denotes the common part of α, β.

TABLE 2. Examples of operations with non-atomic spatial elements: lines and solids representing walls. In each of the three examples, α appears on the left, and β on the right. The produced shapes $\alpha+\beta$, α-β, $\alpha\cdot\beta$ appear translated, between α and β

addition	subtraction	product
		
		
α $\alpha+\beta$ β	α $\alpha-\beta$ β	α $\alpha \cdot \beta$ β

In the previous examples two shapes α and β produce some new shape γ. It is expected that shapes made out of lines, planes and solids fuse or exchange parts without preconditions. The "part of" relation $\leq$ is a formal relation that succeeds to express the empirical fact that spatial elements of dimension greater than zero, belonging to the same dimension can be embedded on one another. When the maximal elements of a shape α are embedded on the maximal elements of another shape β the first shape becomes part of the second ($\alpha \leq \beta$).

The relation $\leq$ is an order relation and renders the sets U_{ij} of shapes, into relatively complemented lattices. That is: the relation $\leq$ is *reflexive*, because every shape $\alpha \in U_{ij}$, is part of itself, $\alpha \leq \alpha$; it is *antisymmetric*, because for any two shapes α, $\beta \in U_{ij}$ if shape α is part of shape β, ($\alpha \leq \beta$) and shape β is part of shape α, ($\beta \leq \alpha$) then $\alpha = \beta$; and also $\leq$ is *transitive*, because for any three shapes α, β, $\gamma \in U_{ij}$, if $\alpha \leq \beta$ and $\beta \leq \gamma$ then $\alpha \leq \gamma$.

Further, each U_{ij} lattice is distributive, because any three shapes α, β, $\gamma \in U_{ij}$ satisfy the identities:

$$(\alpha \cdot \beta) + (\beta \cdot \gamma) + (\gamma \cdot \alpha) = (\alpha + \beta) \cdot (\beta + \gamma) \cdot (\gamma + \alpha)$$

$$\alpha \cdot (\beta + \gamma) = (\alpha \cdot \beta) + (\alpha \cdot \gamma)$$

$$\alpha + (\beta \cdot \gamma) = (\alpha + \beta) \cdot (\beta + \gamma)$$

where the shape operations $\cdot$ and + substitute the lattice operations $\cap$ and $\cup$.

For any two shapes α, $\beta \in U_{ij}$ there is a least element denoted by the empty shape, but in all algebras, except from U_{00}, there is no upper element, because there is no shape containing all shapes. Although there is no upper element for shapes, complements are defined in relative manner. Therefore, each U_{ij} lattice turns is a relatively complemented one. For any three shapes α, β, $\gamma \in U_{ij}$ such that $\alpha \leq \beta \leq \gamma$, a shape β' exists such that $\beta \cdot \beta' = \alpha$, and $\beta + \beta' = \gamma$. The shape β' is denoted as the relative complement of β within $[\alpha, \gamma]$. And because the lattice is distributive, all relative complements are uniquely determined in it. That is, if $\alpha \leq \beta \leq \gamma$ at most one β' exists satisfying both $\beta \cdot \beta' = \alpha$, and $\beta + \beta' = \gamma$.

The lattice-theoretic operations of join $\cap$, meet $\cup$, and complement substituted with the operations of sum, product, and complement can form a Boolean algebra. The algebra U_{00} containing a single point is an example. The sextuple $< U_{00}, +, \cdot, ', 1, 0 >$ forms a Boolean algebra with two binary operations +, $\cdot$, and a unary operation of complementation ′, together with two special elements: the zero 0, and the unit 1. The commutative and distributive laws hold for the single point $x \in U_{00}$, and also:

$$x + 0 = x, \quad x + x' = 1, \quad 0 \neq 1, \quad x \cdot 1 = x, \quad x \cdot x' = 0$$

The rest of U_{0j}, U_{1j}, U_{2j}, and U_{33}, algebras are not Boolean algebras, because they are missing the unit element. Similar algebraic structures with two binary operations, product, symmetric difference and 0, without unit, are generally designated as Boolean rings (Mendelson 1970). Birkoff (1948) suggests a one-to-one correspondence between Boolean algebras and Boolean rings with unit. He calls a relatively complemented distributive lattice with 0, *generalized Boolean algebra*. Tarski (1956) examines the generalized Boolean algebras as Boolean rings. And since for every shape y $\in U_{ij}$, $(i, j \neq 0)$ distinct from the empty shape, there are potentially infinitely many elements x divisible by y, the ring is *atomless*. The quadruple $< U_{ij}, +, \cdot, 0 >$ forms a commutative, *atomless* Boolean ring such that for any three shapes $\alpha, \beta, \gamma \in U_{ij}$, the following seven relations hold

$$(\alpha + \beta) + \gamma = \alpha + (\beta + \gamma)$$

$$(\alpha \cdot \beta) \cdot \gamma = \alpha \cdot (\beta \cdot \gamma)$$

$$\alpha + \beta = \beta + \alpha$$

$$\alpha \cdot (\beta + \gamma) = (\alpha \cdot \beta) + (\alpha \cdot \gamma)$$

$$\alpha + 0 = \alpha$$

$$\forall\alpha \text{ there is a unique } \alpha' \text{ such that } \alpha + \alpha' = 0$$

$$\alpha \cdot \alpha = \alpha \quad \text{and} \quad \alpha \cdot \beta = \beta \cdot \alpha$$

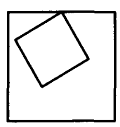
And, because all relative complements can be uniquely defined the shape algebras U_{ij} can be augmented with the operation of *difference*. If $\alpha, \beta \in U_{ij}$ then the difference $\alpha - \beta$ can be uniquely defined as the relative complement of β within the closed interval $[0, \alpha+\beta]$, since $0 \leq \beta \leq \alpha+\beta$, or the relative complement of $\alpha \cdot \beta$ within $[0, \alpha]$, since $0 \leq \alpha \cdot \beta \leq \alpha$.

Finally, the practical usefulness of spatial transformations such as translations, rotations, reflections, and scaling, in the manipulation of shapes, calls for an extension of shape algebras to include such transformations. Stiny (1992) defines the algebras closed to the Euclidean transformations. A Euclidean transformation t acting on a shape s is denoted by t(s). Two shapes are geometrically similar when there is a transformation t that makes the first identical to the second. Krstic (1996) describes two alternatives for including the transformations in the algebras U_{ij}. The first, includes transformations $t(\ )$ as operators in the set of operations acting on the set $\{U_{ij}\}$ of shapes. This turns algebras U_{ij}, into generalized Boolean algebras with infinite operators. The second option is to include the transformations T_j, in the set $\{U_{ij}\}$. This turns shape algebras into two-sorted algebras $\{U_{ij}, T_j\}$, with a Boolean part that handles structure and a group part that handles symmetry. A detailed account on the importance of symmetry transformations exists in Economou 1999. The initial definition of Stiny (1992) regarding transformations is followed in this study.

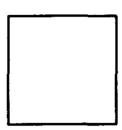
4.2. SHAPE GRAMMAR

The general algebraic framework defined in Stiny 1975 1980 1991 1992 allows shapes and their arrangements to be added, and taken apart with the aid of sum, product, difference, and the embedding relation, and to be moved or transformed with the Euclidean transformations. Within this spatial calculus the medium of *shape rules* is employed in the production and interpretation of spatial compositions.

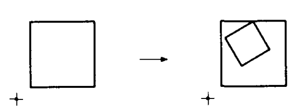
A shape rule is a production of the form $\alpha \rightarrow \beta$ with α, β shapes in some algebra U_{ij}, in this example U_{12}. A shape rule allows shapes to be placed together to produce some desired *spatial relation*, like the following:



A shape rule applies to some initial shape C like the next square, involving a transformation *t*.



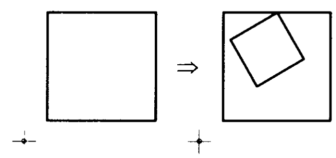
The initial state and the concluding state are connected by a shape rule in the following way,



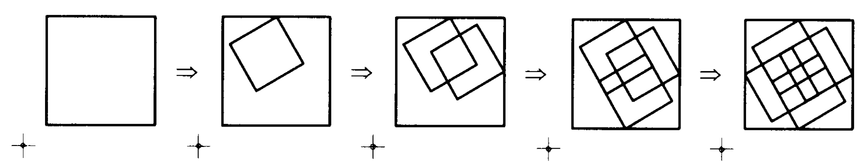
α β

The rule applies in two-steps: First, the transformation *t* is used with the part relation ≤ to distinguish some part of any shape C geometrically similar to the shape α, which appears on the left side of the rule. Second, the same transformation *t* is used with (–) to subtract α from C, and with (+) to add the shape β, which appears on the right side of the rule, in its place.

$$C' = [\, C - t(\alpha)\,] + t(\beta)$$



The rule can apply recursively to produce a series of shapes C, C_1, C_2, … C_n in a sequence that is called a *derivation*.



Rules can be organized in systems called *shape grammars*. Shape grammars generate sets of designs with particular spatial or other properties called design languages. A terminated computation that yields shapes C, C_1, C_2,…C_n within a specific *shape grammar* is a proof within that grammar.

A parametric version of a rule expression takes the form,

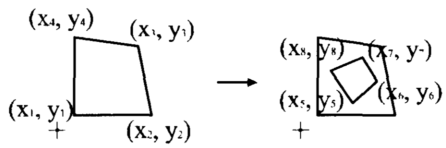
$$C' = [\, C - t(g(\alpha))\,] + t(g(\beta))$$

where g determines an assignment of values that allows the shapes α and β to vary.

When the rules are infinite it is impossible to be written out in full. They are simply indicated by one or more statements in the syntax of the language to introduce a class of rules. Such statements may be seen as rules with an empty class of premises. In this sense the distinction between infinite and finite number of rules is illusory. The actual distinction is between formulations that rely more or less restrictively on syntactical statements, to take the place of rules in a language. A rule schema represents indefinite number of rules by means of an expression containing syntactical variables. The formula, $g(x) \rightarrow g(y)$ is used to denote a rule schema of the form

$$(\forall x)\,(\forall y)\; g(x) \rightarrow g(y)$$

Parametric rule schemata describe the interaction of shapes in a general manner. Rule schemata do not require an explicit shape vocabulary. They contain variables, and they determine rules when all the shape-variables are substituted by some actual shape x, y, z, etc., that belongs to some shape algebra U_{ij}. A shape rule is a rule schema that contains no free variables. A predicate g indicates the attributes of x, y z. Each rule schema applies to the members of a class of shapes with the specified attributes, to produce other shapes. In the next rule schema, of the form x → x + z that places one convex shape z inside another x, the predicate g determines that: "*x, z are convex shapes*".

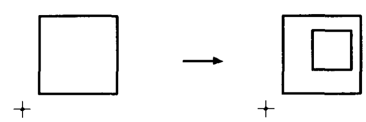


The rule schema can be restricted to apply to specific classes of shapes, like squares: "*x, z are squares*". The restriction is introduced by a predicate g′.

$$g': \forall(x_i)\ \forall(y_i), ((x_1 = x_4) \wedge (x_2 = x_3) \wedge (y_1 = y_2) \wedge (y_3 = y_4)) \wedge$$

$$((x_5 = x_8) \wedge (x_6 = x_7) \wedge (y_5 = y_6) \wedge (y_7 = y_8))$$

For simplicity, all coordinates and symbolic expressions, like the above, are customarily omitted from rule schemata. Restrictions are given verbally.



A *shape grammar* is a syntactic and interpretive collection of rules that determines a particular corpus of designs. It proves arrangements of shapes, by deriving them. Shape grammars belong to the same family of devices as *phrase structure grammars* (Chomsky 1957) and *production systems* (Post 1943). The use of grammars in the analysis and synthesis of 2-d architectural descriptions was first discussed in Stiny and Mitchell 1978 in the generation of Palladian villa plans. Examples of the action of parametric rules in the generation of designs can be found in Stiny 1977, and in Knight 1980, while Stiny 1981 is a grammar application while taking into account architectural functionality. The possible ways of using grammars in architectural morphology were exposed in Knight 1981a; 1981b; 1990; 1994. Numerous papers describing the generation of architectural styles

5. Constructing Design Concepts

Shape computation theory examines formal methods and tools that can be used in design. This study focuses on shape computational methods that can be used in the studio, in designing from scratch.

The suggested design process consists of making a hypothesis (design concept), in response to a problem, deriving its consequences, and then testing them against the available empirical standards. It is proposed that a design concept is not arrived at by an analysis of the provided information, but it is the result of synthesis and interpretation. It does not only express programmatic facts for the object under consideration, but also suggests a possible new meaning for it. A design concept cannot be qualified as either true or false. The role of the design concept is to establish a particular interrelationship among the elements that a designer identifies as crucial for his design. Design concepts can include spatial as well as other parts: semantic, functional, etc.

In the design process one examines the consequences of one's initial hypothesis. Deductive steps with varying degrees of explicitness and extensiveness are used for this purpose. The general consequences of a design hypothesis can be sketched out by rule schemata $S_1, S_2, \ldots S_n$ established from previous experience or invented.

$$S = \{S_1, S_2 \ldots S_n\}$$

Rule schemata are general statements containing at least one free variable. They include predicates and transformations. The formulation of a set of transformations T under which the same rule schema may apply becomes a parameter of great importance in the development of a design.

$$T = \{T_1, T_2 \ldots T_k\}$$

A first approach can be established by organizing rule schemata according to their general consequences. The consequence C_j of a sequence of $j < n$ rule schemata is what is implied by their conjunction

$$S_1 \wedge S_2 \wedge S_3 \wedge \ldots \wedge S_j \rightarrow C_j$$

This does not guarantee the value of the consequence C_j which can be a matter of several interdependent parameters. But it underlines the conditional character of the system: some effect C_j is accomplished provided that some schemata $S_1, S_2, S_3, \ldots S_j$ are satisfied.

While a part of the design activity consists of formulating rule-schemata and transformations, another part is dedicated to the specification of the particular actions. These are expressed as shape rule instances R.

A shape rule of the form R_1: $A_1 \rightarrow F_1$ determines a logical condition: if the shape A_1 is found in a derivation it can be substituted with the shape F_1.

$$\frac{\begin{array}{l} A_1 \rightarrow F_1 \\ A_1 \end{array}}{F_1}$$

This development is applicable to both spatial and non spatial attributes of a design concept. Rules like the above correspond to sufficient but not necessary conditions.

As a sequence of additions or subtractions would never lead by itself to the discovery of a theorem, mechanical rule application would not lead to a design. Unless a hypothesis has been put forward, such application will lack direction. Formal rules are not rules of discovery, leading mechanically to solutions. They only provide criteria for checking the results of proposed actions with respect to a hypothesis. Alternative rule instances R can be produced by substituting the free variables in a rule schema. On the base of a sequence of rule schemata $S_1, S_2, \ldots S_n$ rules $R_1, R_2, \ldots R_n$ can be introduced as instances. The application of the rules has some outcome G.

$$\left.\begin{array}{c|c} S_1 & R_1 \\ S_2 & R_2 \\ \vdots & \vdots \\ S_n & R_n \end{array}\right| \Rightarrow G$$

The question of constructing a system, or a grammar, arises as soon as a number of general rule schemata, and rules, are established. It then becomes possible to arrange them with a better sense of economy and efficiency. Provided that the search has been carefully done, the ordering of rules does not itself create new information. Assuming that some number n of rules is to be organized the question becomes under what ordering relation? The final ordering of rule instances into a system is subjective and happens according to their ability to achieve specific goals. The ordering assures that all the desired goals will be accomplished at the end. Therefore the shape rules R may take the form:

$$R: \{\{[A_1 \rightarrow F_1], \ldots [A_i \rightarrow F_i]\} \Leftrightarrow \{[G_1 \rightarrow M_1], \ldots [G_k \rightarrow M_k]\} \Leftrightarrow$$

$$\Leftrightarrow \{[N_1 \rightarrow W_1], \ldots [N_r \rightarrow W_r]\}\}$$

The set Σ of elements such that $\{[A_1,\ldots A_i,]\ldots,[F_1,\ldots F_i,]\}$, $\{[G_1,\ldots G_k,]\ldots[M_1,\ldots M_k,]\}$, $\{[N_1,\ldots N_r]\ldots,[W_1,\ldots W_r]\}$ are in Σ, is defined retrospectively.

6. Discussion

A computational theory includes some abstract deductive part and some syntactic-interpretive part. The deductive part is an environment where calculations of some kind may take place. The syntactic-interpretive part includes statements that associate empirical meaning to the calculations.

Shape computation theory deals mainly with form, the elements of space, and their possible interactions. The interaction of forms is captured by a calculus for shapes of 0, 1, 2 and 3 dimension and by syntactic-interpretive rule statements. Shape computation theory offers the means to understand and to treat design formally.

This study examines how designers can construct step-by-step processes to develop their spatial concepts. It is proposed that the design process begins with a design hypothesis, (or design concept) in response to a problem. The designer derives the spatial and other consequences of the hypothesis, and performs tests with programmatic and other criteria.

The design concept provides a general schema and establishes a flexible interrelationship among the elements that one considers crucial for the design. It is not repetition or analysis of the building program, but an act of synthesis that involves previous experience, and inspiration. Design concepts may have several interrelated parts: spatial, semantic, functional, etc. This study examines the spatial part of such concepts. In the design process one examines the consequences of a hypothesis. These can be approached as calculations with varying degrees of explicitness and extensiveness. The character of the process is conditional. An effect is accomplished provided that some conditions are satisfied. The sequence of actions becomes clear after testing. When rule schemata and rules are established, then it becomes possible to arrange them in a grammar. Provided that the search has been carefully done, the ordering of rules does not create new information. Moreover, the rules can be organized in different ways. In order to understand the use of rules in the studio we must draw our attention on the process of the development of design concepts. Also, it needs to be shown how designers can produce design descriptions, while taking into account a variety of interconnected problems.

The next two chapters examine the properties of architectural description and its role in the construction of design concepts. Chapter III presents the properties of spatial elements in architectural descriptions. Chapter IV shows how architectural description is put into use in the generation of designs from a design concept.

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III. CONTENT AND FORM

Abstract. Architects use descriptions to develop design concepts. Descriptions deal with the arrangement of area, volume, and their boundaries. Calculations with content (area, volume) and form (boundaries) are constantly interrelated in architectural design.

1. Introduction

Architects develop spatial concepts through descriptions: sketches, diagrams, plans, sections, elevations and 3d models. Descriptions involve calculations with forms and symbols in 2d and 3d space. The required spatial elements are expressed in shape computation theory with product algebras that are formed in two ways. First, each of the participating graphic elements of a single description, (points, lines planes or solids) belongs to some algebra. The description is a shape in the product of the participating algebras. Second, several distinct descriptions (plans, sections, elevations and 3d models) represent the same object from different points of view. The set of these parallel descriptions forms a shape in their product algebra.

This process, involves calculations with areas, and volumes, and their boundaries. The available area and the overall volume of a room, correspond to the "content" of that room, while its "form" is determined by the shape of its boundaries. Areas and their line boundaries are used to describe the content and form on the plane. Volumes and their plane boundaries are used to describe it in 3d space. Calculations with areas and volumes, and calculations with their boundaries remain interconnected expressing the constant interrelationship between content and form.

Architects deal mostly with 3d objects: chairs, tables, buildings, etc. These objects occupy finite space, and they are placed within bounded space (room, site, etc). Since the available volume and area are usually limited, calculations with areas and volumes are bounded. And since, objects can take any form, the calculations with their boundaries, remain unbounded.

2. Shape Computation in Architectural Description

Graphic description plays important role in architecture. Sketches, diagrams, plans, sections, elevations and 3d models serve both explorative and expressive purposes. Architectural description is useful in the studio, and in the construction site. Goodman (1976) points out: "*A drawing initially counts as a sketch, but the final selection of lines, areas and numeric values counts as a diagram, or a score purposed for execution*". Drawings and models of buildings are not simply representational. They convey information about a variety of issues, and obey to the notational language of architecture. This graphic language was developed as a means of coordination and description of the parameters that affect the construction of an object.

Architectural design deals mainly with the construction of 3d objects that occupy physical space. Description in 2d is used for a variety of reasons. It is economical, easy to produce, and to read. In all the phases of the design process, from the initial conception to implementation, 2d and 3d descriptions are interconnected: every 3d element obtains some 2d description, and every 2d description corresponds to some 3d component.

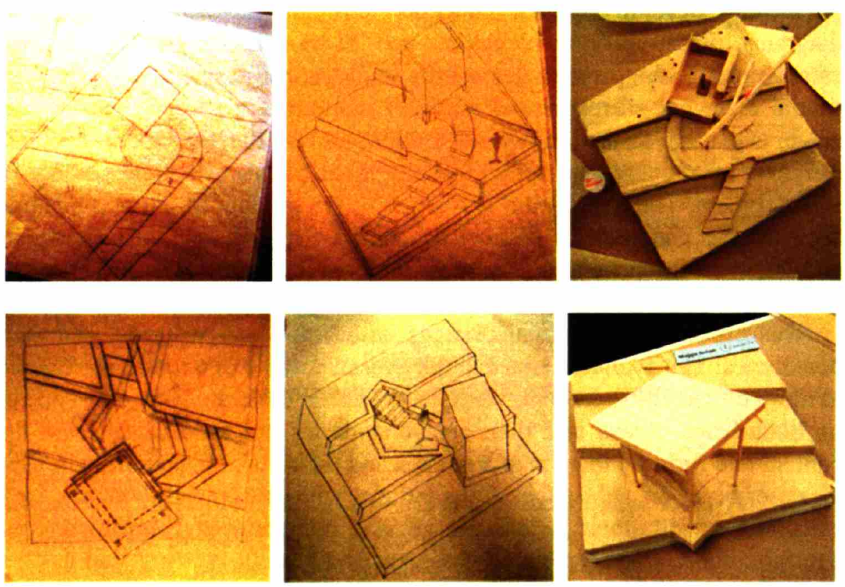
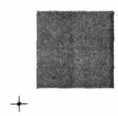
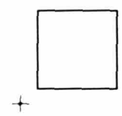
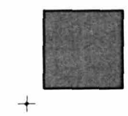
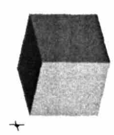
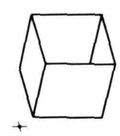
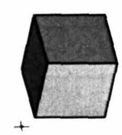


Figure 1. Examples of 2d and 3d descriptions from a student exercise in composition: plans, axonometrics, models. Students: Akari Kameyama (up), Maggie Nelson (down), 4.101 Experiencing Architecture Studio, Instr.: B Hubbard, MIT

Architectural description falls within the expressive range of U_{i2}, or U_{i3} algebras, with i = 0, 1, or 2. In the production of plans, sections, and elevations we are mainly concerned with areas, and their boundaries. In 3d models we use solids, and their boundaries. If necessary, letters and numbers are introduced to assign names and values.

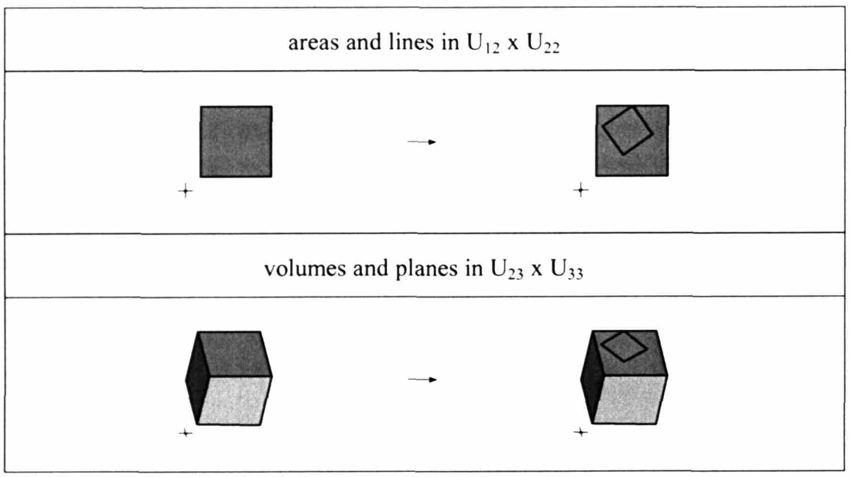
TABLE 3. Non-symbolic elements in architectural drawings: area and its boundaries (U_{12} x U_{22}). In models: volume and its boundaries (U_{23} x U_{33})

 area (U_{22})	 boundary (U_{12})	 area + boundary (U_{22} U_{12})
 volume (U_{33})	 boundary (U_{23})	 volume + boundary (U_{33} U_{23})

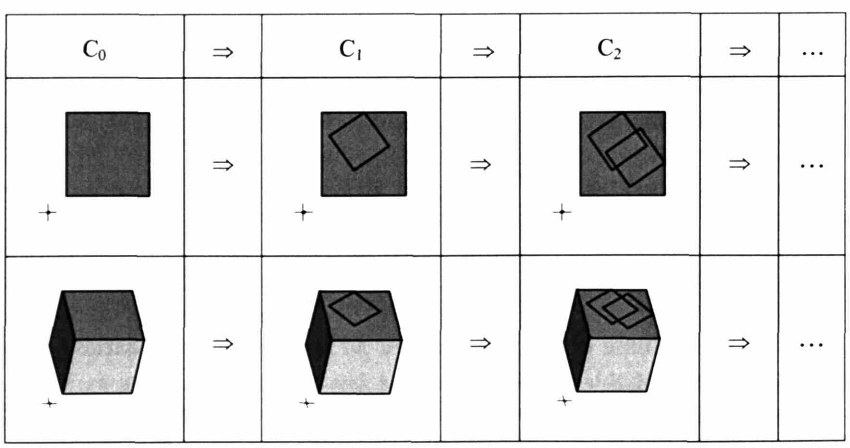
The standard graphic description of architecture involves the above spatial dimensions. Each spatial element conveys a different kind of information. Together, they work well for exploring the "content", and "form" of things, on the plane and in physical space. The "content" of a room in 2d is equal to the area it occupies. It can be represented on the plane in algebra U_{22}. In 3d, the same room occupies volume represented by a solid in algebra U_{33}. The "form" of a room is represented by the form of its outmost boundaries. In 2d, lines represent these boundaries in algebra U_{12}. And in physical 3d space, planes represent the boundaries of the room in algebra U_{23}. An area and its line boundaries represent the area of a room and its outline in the product algebra U_{12} x U_{22}. A solid and its plane boundaries represent the volume of a room and the 3d form of its boundaries in the product algebra U_{23} x U_{33}.

Points, letters and numbers can be used to inform about value, distance, quantity etc. In this study symbols are usually omitted from the descriptions.

In shape computation theory, shapes can be composed with addition, subtraction, product, or with shape rules. A pair (l, p) with $l \in U_{12}$ and $p \in U_{22}$, represents area and its boundaries. A pair (p, s) with $l \in U_{23}$ and $s \in U_{33}$ represents volume and its boundaries. A shape rule schema $x \rightarrow y$ is expressed as $(l, p) \rightarrow (l', p')$ for areas and lines, or, $(s, p) \rightarrow (s', p')$ for planes and solids. For areas and their boundaries, a rule determines a relationship from $(U_{12} \times U_{22})$ to $(U_{12} \times U_{22})$. For solids and their boundaries, a rule determines a relationship from $(U_{23} \times U_{33})$ to $(U_{23} \times U_{33})$.

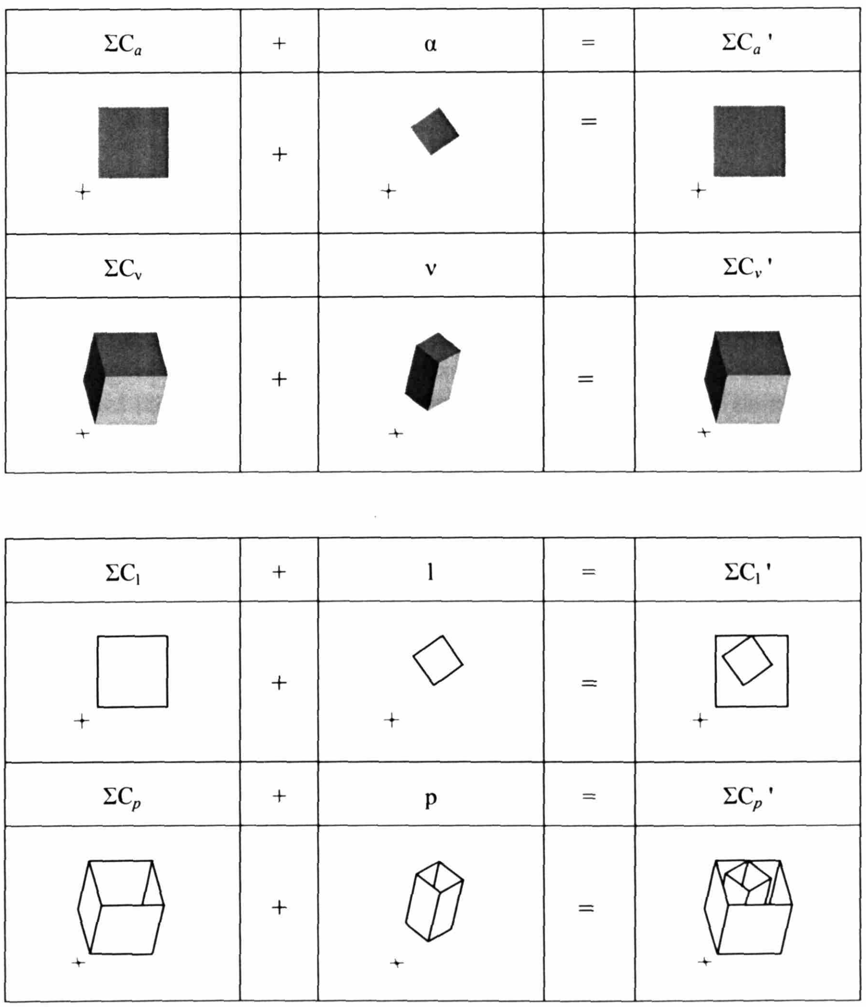


The next example shows the two rules applying in a product $(U_{12} \times U_{22}) \times (U_{23} \times U_{33})$ algebra. A sequence of shapes $C_0, C_1, C_2, \ldots, C_n$ is produced for some finite $n \geq 0$, such that $C_0 \Rightarrow C_1 \Rightarrow C_2 \Rightarrow \ldots \Rightarrow C_n$.

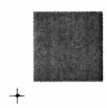
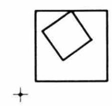
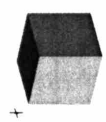
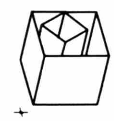


Shape computation theory provides a framework for the execution of operations and the application of rules with shapes (Stiny 1980; 1991). Within this framework the properties of shapes made out of areas, or volumes and those of their boundaries develop differently in calculations.

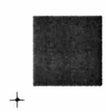
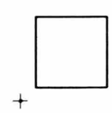
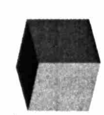
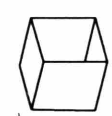
First, the sums of their parts evolve differently at each step. In the shapes of the example, the sums of areas, and those of volumes, render always the same shape: $\Sigma C_a = \Sigma C_a'$ and $\Sigma C_v = \Sigma C_v'$. The sums ΣC_l of boundary lines and ΣC_p of boundary planes change unpredictably. In general, $\Sigma C_l \neq \Sigma C_l'$ and $\Sigma C_p \neq \Sigma C_p'$.



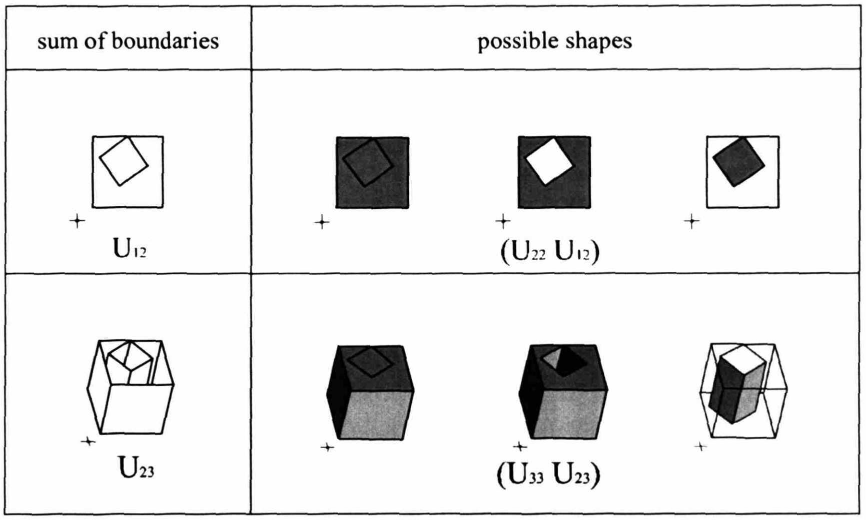
Second, each new added area and volume is embedded in the overall shape. The boundaries of the added shape can be disjoint.

embedded	may be disjoint
 U_{22}	 U_{12}
 U_{33}	 U_{23}

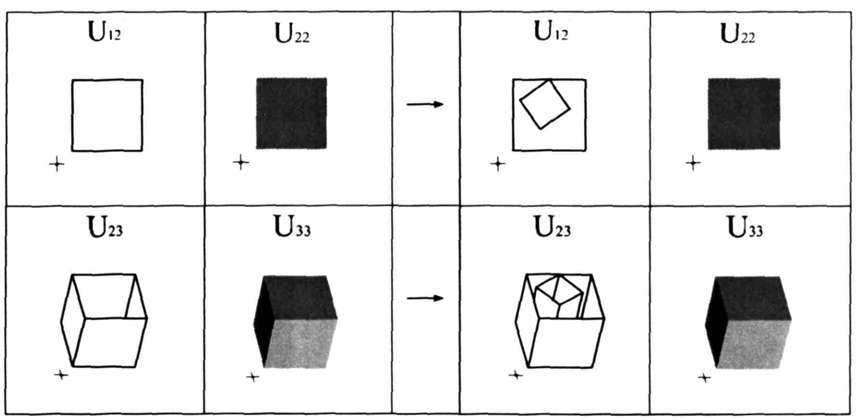
Third, as an immediate consequence of the previous two, it is not possible to determine the boundaries of the derived shapes from the sums of their areas or volumes. The boundaries of the sums of areas in U_{22} algebra render a square and the volumes in U_{33} algebra, a cube.

sum of areas / volume	boundaries
 U_{22}	 U_{12}
 U_{33}	 U_{23}

It is also impossible to construct the overall shapes from the sums of their boundaries, as these may correspond to several different shapes.



To handle the interaction of shapes and their boundaries without any loss we describe them separately. The next diagram shows two additive rules containing two graphic components apiece: one for areas and their boundaries ($U_{12} \times U_{22}$) and one for volumes and their boundaries ($U_{23} \times U_{33}$)



A framework of parallel computation with multiple algebras is formally defined in Stiny 1992, where two components, one for shapes U_{ij} and one for their boundaries $U_{i\text{-}1j}$, are used in a direct product $U_{ij} \times U_{i\text{-}1j}$ algebra.

3. The Mathematics of Plans, Sections, Elevations and Models

In architectural design the existence of a predetermined site frames all spatial calculations within a bounded context of area, and volume. The existence of a site affects the calculation, and the composition. If no predetermined site exists, usually some relative upper bound is provided.

In the next example from a studio exercise in composition, a student begins from an initial arrangement (left), within a given square site (exterior square). The student uses a sheet of tracing paper to apply a rotation (center) and change the relationship between the design and the site. Finally, a third option, slightly different from the previous two is applied, on a third tracing paper (right).

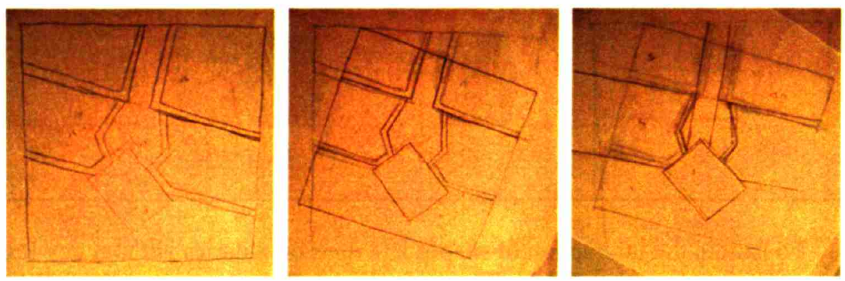


Figure 2. Plan-sketches from student exercise in composition. Student: Maggie. Nelson, 4.101 Experiencing Architecture Studio, Instructor: Bill Hubbard, MIT

In 3d, the site has inclination, area (left), and some overall volume, which can be calculated. The total volume of the design can be depicted abstractly as an "empty box" of specific volume (center). The relationship between the design and the site is exposed by placing the two descriptions together (right).

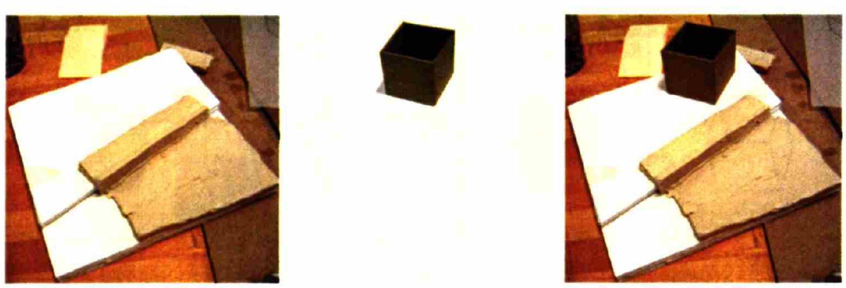


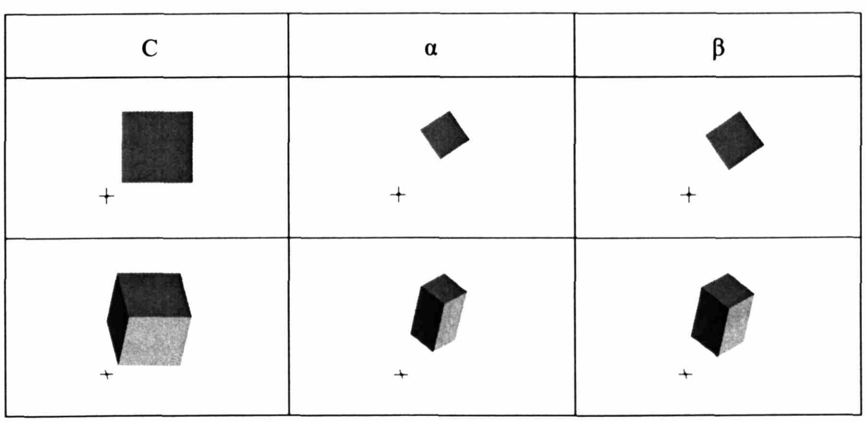
Figure 3. The site (left), the volume of the design (center), and the two descriptions together (right). Student: Maggie Nelson, Instructor: Bill Hubbard, MIT

In architectural design, composition usually evolves within a specific spatial context. The development of relationships among forms that express the design and forms that represent the context is inevitable. The interaction between 2d and 3d descriptions is continuous. And so is the relation between content and form. Content and form are usually treated simultaneously. The content of things is always bounded and finite, while their possible forms inexhaustible. This reaffirms our normal expectation that a finite chunk of space can be treated in infinite ways. In architectural design, the designers try to establish "sensible" relationships both for content and form. We can further refine our views regarding these remarks. For presentation purposes content and form are shown separately.

3.1. CONTENT

Architects deal with tangible constructions and their placement in space. These objects occupy space, and they are usually placed, within limited space. A standard issue in design is how the parts of a design will fit within the limits of a given site, area, or a room. Therefore calculations with areas, solid entities are usually bounded. Since a unit shape can be defined, areas and volumes form complete Boolean algebras. Designers calculate with areas and volumes not arbitrarily, but within a bounded relational system.

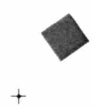
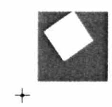
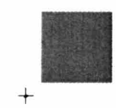
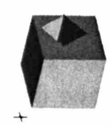
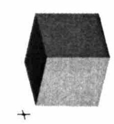
At each step of a computation all shapes in the Boolean algebras of areas, or volumes, form some finite decomposition and finite topology for some shape C, which serves as a unit. Shape C and the empty shape belong to this topology, which is closed under sum and product. Sum and product can be defined in terms of the part relation $\leq$ within the topology. For two shapes α, β the shape $\alpha + \beta$ is the least element within the algebra such that t α and β are its sub-elements. The product $\alpha \cdot \beta$ is the greatest common sub-element of the shapes α, β. In the example $\alpha + \beta = \beta$, and $\alpha \cdot \beta = \alpha$

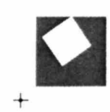
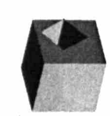


Sum and product play symmetrical role. In every relation we can replace $+$ with $\cdot$ and the unit shape C with the empty shape, or the opposite, and obtain a valid relation. Therefore, $\alpha + C = C$, and $\alpha \cdot C = \alpha$, and dually, $\alpha \cdot \varnothing = \varnothing$ and $\alpha + \varnothing = \alpha$. The complement $-\alpha$ of the shape α has the property:

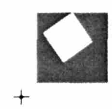
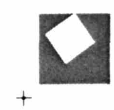
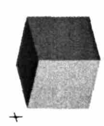
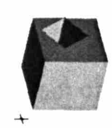
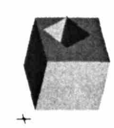
if $\alpha + C_k = C$ and $\alpha \cdot C_k = \varnothing$ then $C_k = -\alpha$

There is only one shape C_k fulfilling both the above conditions,

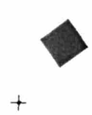
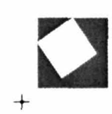
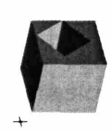
α	$+$	C_k	$=$	C
	$+$		$=$	
	$+$		$=$	

α	$\cdot$	C_k	$=$	C
	$\cdot$		$=$	$\varnothing$
	$\cdot$		$=$	$\varnothing$

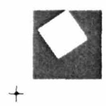
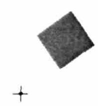
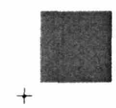
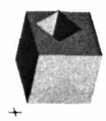
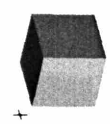
For any two areas, or volumes α, β the shape $\alpha \cdot (-\beta)$ forms the difference $\alpha - \beta$. Also, $C - \alpha = -\alpha$,

C	·	$-\alpha$	=	$-\alpha$
	·		=	
	·		=	

For two shapes α, β it is easy to see that $\alpha \leq \beta$ if and only if $\alpha - \beta = \varnothing$

α	·	$-\beta$	=	$\varnothing$
	·		=	$\varnothing$
	·		=	$\varnothing$

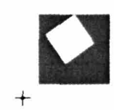
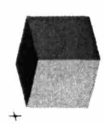
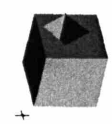
Also, for the shapes α, β it is, $\alpha \leq \beta$ if and only if: $-\alpha + \beta = C$

$-\alpha$	$+$	β	$=$	C
	$+$		$=$	
	$+$		$=$	

The operation $-\alpha + \beta$ is dual to the operation $\beta - \alpha$. The binary operation of symmetric difference $\oplus$ is defined for two shapes α, β:

$$\beta \oplus \alpha = (\beta - \alpha) + (\alpha - \beta) = (\beta + \alpha) - (\beta \cdot \alpha)$$

The symmetric difference of C and α, is the complement of α,

$(C + \alpha)$	$-$	$(C \cdot \alpha)$	$=$	$-\alpha$
	$-$		$=$	
	$-$		$=$	

Every finite non-zero sub-shape x_i, with $i = (1, 2, \ldots n)$ and n finite, forms a sub-algebra X_i of the Boolean algebra X. The sub-algebra is closed under the operations of X. The shape x_i forms a Boolean algebra in its own right, and, the inclusion relation of X remains the same in X_i, but it is restricted to x_i. The empty shape and the unit form the least Boolean sub-algebra of X. The products of any number of sub-shapes of the unit C of X form also a sub-shape of C. For every part Ξ of C, there is a least sub-shape C_0 such that Ξ is part of C_0. The shape C_0 is defined as the intersection of all sub-shapes of C containing Ξ. If Ξ is part of a shape C_1 then C_0 is also part of C_1. The sub-algebra of C_0 can be described as follows: If Ξ is the empty shape, then C_0 is composed by the empty shape and the unit shape.

If the shape Ξ is not empty, a shape $\alpha \leq C$ is also part of C_0 if and only if,

$$\alpha = (\alpha_{1,1} \cdot \alpha_{2,2} \cdot \ldots \cdot \alpha_{1,r1}) + (\alpha_{2,1} \cdot \alpha_{2,2} \cdot \ldots \cdot \alpha_{2,r2}) +$$

$$+ (\alpha_{s,1} \cdot \alpha_{s,2} \cdot \ldots \cdot \alpha_{S,rs})$$

where for any k, v, either $\alpha_{k,v} \leq \Xi$ or $-\alpha_{k,v} \leq \Xi$.

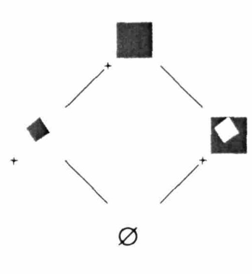
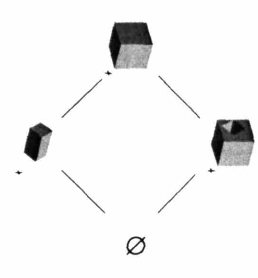
Dually, the sub-algebra generated by the non-empty shape Ξ is the shape that has parts all elements $\alpha \leq C$ such that

$$\alpha = (\alpha_{1,1} + \alpha_{2,2} + \ldots + \alpha_{1,r1}) \cdot (\alpha_{2,1} + \alpha_{2,2} + \ldots + \alpha_{2,r2}) \cdot$$

$$\cdot (\alpha_{s,1} + \alpha_{s,2} + \ldots + \alpha_{S,rs})$$

where for any k, v, either $\alpha_{k,v} \leq \Xi$ or $-\alpha_{k,v} \leq \Xi$.

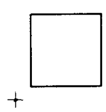
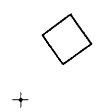
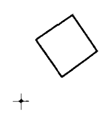
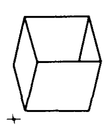
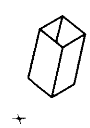
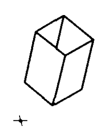
In each step $C_0, C_1, C_2, \ldots, C_n$ of a calculation a finite sub-algebra can be formed corresponding one-to-one to the partitions of the shape C. The characteristic of these k partitions, except from the fact that they are finite, is that their topology forms a 0-dimensional space that can be decomposed into disjoint parts the union of which forms a 0-dimensional set. That is the shape parts are reduced into atoms. They are indivisible and their topology is bounded by least upper bound the shape C. For the shapes $C_0, C_1, C_2, \ldots, C_n$ the lattice of parts is isomorphic with the lattice of all partitions of k elements in the n steps. The finite sub-algebra generated in a particular step of the computation, by a sub-shape α of C, contains only the empty shape, the shape α, the shape $-\alpha$, and C.

U_{22}	U_{33}
	

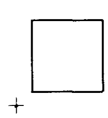
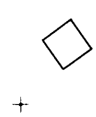
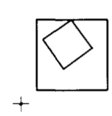
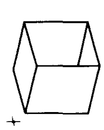
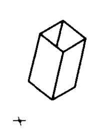
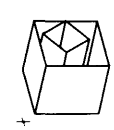
3.2. FORM

The shape of the boundaries of things determines their form. In design a lot of effort is dedicated for determining the form of things as well as the form of their parts, and the ways these are assembled. Objects can take any possible form within finite area, or space. Since no predetermined least upper bound can exist for form, calculations happen within Boolean rings.

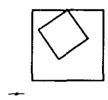
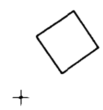
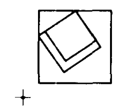
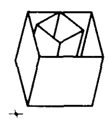
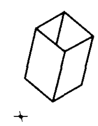
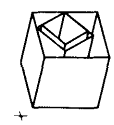
At each step of a calculation different forms may or may not be produced. Retrospectively, when the form of an object is determined, the previous divisions and modifications of parts obtain different meaning, and become members of a Boolean algebra. In the examples the boundaries of areas and solids are represented by the shapes C, α, β in U_{12} or U_{23} algebras.

C	α	β
		
		

In the example of an addition, the added non-empty shape α is not part of the initial shape C_0, and the sum $\alpha + C_0$ produces some shape $C_1 \neq C_0$.

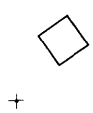
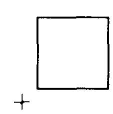
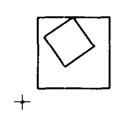
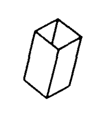
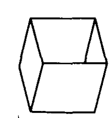
C_0	+	α	=	C_1
	+		=	
	+		=	

Next, the added shape β is not part of C_1, and therefore it is $C_2 \neq C_1$, etc.

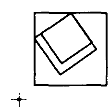
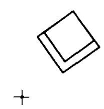
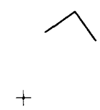
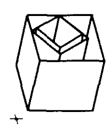
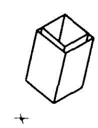
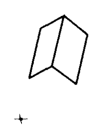
C_1	+	β		C_2
	+		=	
	+		=	

Retrospectively, each shape C_x can serve as a relative upper bound. Therefore, $\alpha + C_1 = C_1$ and $\alpha \cdot C_1 = \alpha$. For each of the possible n steps the produced shapes form some finite decomposition and topology for some shape C_x. The shape C_x and the empty shape belong to this topology, which is closed under sum, and product. The shape C_x and its parts form a Boolean algebra. In the step $C_0 \Rightarrow C_1$ we can define for α, the shape $-\alpha$, such that:

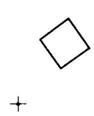
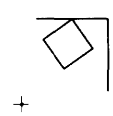
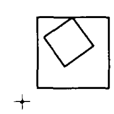
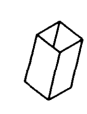
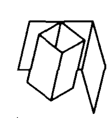
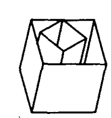
if $\alpha + C_k = C_1$ and $\alpha \cdot C_k = \varnothing$ then $C_k = -\alpha$

α	+	C_k	=	C_1
	+		=	
α	·	C_k	=	$\varnothing$
	·		=	$\varnothing$

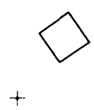
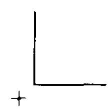
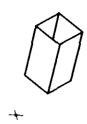
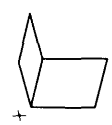
The sum between two shapes α, $\beta \leq C_x$ can be defined with the aid of the $\leq$ relation. For the shape C_2, the shape $\alpha+\beta$ is the least element within the algebra such that the shapes α and β are its sub-elements. The product $\alpha \cdot \beta$ is the greatest common sub-element of the shapes α, β.

C_2	$\alpha + \beta$	$\alpha \cdot \beta$
		
		

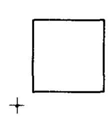
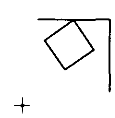
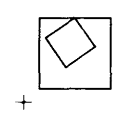
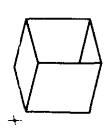
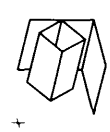
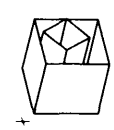
In general $\alpha \leq \beta$ if and only if $\alpha \cdot (-\beta) = \varnothing$. For the shapes α, β of the example, the relationship $\alpha \leq \beta$ holds for areas and solids (see example p. 57). It does not hold for their boundaries, which just have some overlapping parts, as it is shown above. However, for every shape C_x and its parts α, x

α	x	C_x
		
		

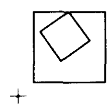
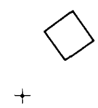
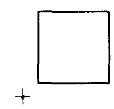
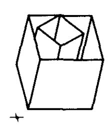
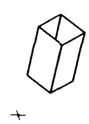
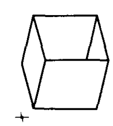
it is true that $\alpha \leq x$ if and only if $\alpha - x = \varnothing$.

α	$\cdot$	$-x$	$=$	$\varnothing$
	$\cdot$		$=$	$\varnothing$
	$\cdot$		$=$	$\varnothing$

Also, it is $\alpha \leq x$ if and only if $-\alpha + x = C_x$. The operation $-\alpha + x$ is dual to the operation $x - \alpha$.

$-\alpha$	$+$	x	$=$	C_x
	$+$		$=$	
	$+$		$=$	

The symmetric difference $\oplus$ is defined for the two shapes as follows:

$(C + \alpha)$	$-$	$(C \cdot \alpha)$	$=$	$-\alpha$
	$-$		$=$	
	$-$		$=$	

Every finite non-zero sub-shape x_i, with $i = (1, 2, \ldots n)$ and n finite, forms a Boolean sub-algebra X_i of the Boolean algebra X, closed under sum. The empty shape and the unit x_i form the least sub-algebra of X. For every shape Ξ of C_x, there is a least sub-shape C_0 such that Ξ is part of C_0. The shape C_0 is defined as the intersection of all sub-shapes of C_x containing Ξ, and if Ξ is part of a shape C_1 then C_0 is also part of C_1. If Ξ is the empty shape, then C_0 is composed by the empty shape and the unit shape. If Ξ is not empty, then a shape $\alpha \leq C_x$ is also part of C_0 if and only if,

$$\alpha = (\alpha_{1,1} \cdot \alpha_{2,2} \cdot \ldots \cdot \alpha_{1,r1}) + (\alpha_{2,1} \cdot \alpha_{2,2} \cdot \ldots \cdot \alpha_{2,r2}) +$$

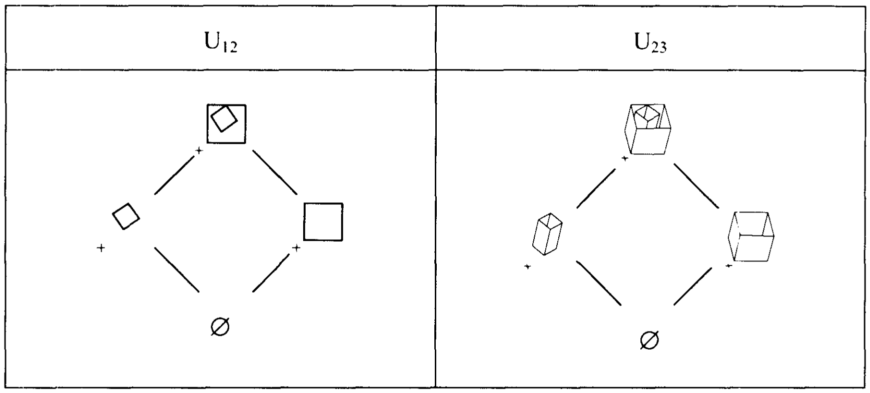
$$+ (\alpha_{s,1} \cdot \alpha_{s,2} \cdot \ldots \cdot \alpha_{S,rs})$$

where for any k, v, either $\alpha_{k,v} \leq \Xi$ or $-\alpha_{k,v} \leq \Xi$.

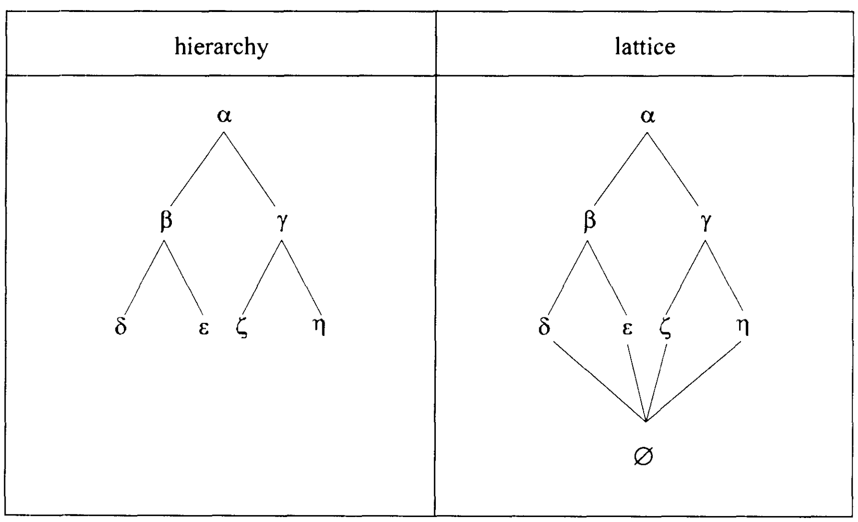
Dually, the sub-algebra generated by the non-empty shape Ξ is the shape that has parts all elements $\alpha \leq C_x$ such that $\alpha = (\alpha_{1,1} + \alpha_{2,2} + \ldots + \alpha_{1,r1}) \cdot (\alpha_{2,1} + \alpha_{2,2} + \ldots + \alpha_{2,r2}) \cdot (\alpha_{s,1} + \alpha_{s,2} + \ldots + \alpha_{S,rs})$, where for any k, v, either $\alpha_{k,v} \leq \Xi$ or $-\alpha_{k,v} \leq \Xi$.

In each step C_0, C_1, C_2,…C_x…, C_n of the calculation a finite sub-algebra can be formed corresponding one-to-one to the partitions of some produced shape C_x. The k partitions of this shape are finite, and their topology forms a 0-dimensional space that can be decomposed into disjoint elements the union of which forms a set, where lines and planes are reduced into points. The space is bounded by the shape C_x, which serves as least upper bound.

The lattice of parts, formed retrospectively, is isomorphic with the lattice of all partitions of k elements in the n steps. The finite sub-algebra generated in a single step by C_x and its finite parts contains the empty shape, the shape α, the shape $-\alpha$ and the shape C_x.



Finally, when the way the parts combine becomes important, these can be organized in a hierarchy. Hierarchical decompositions can be formed for various purposes i.e. function, detailing etc. to clarify how things are assembled. The root contains the whole, and the leaves contain the atoms. Tree decompositions can turn into lattices, after they are augmented with $\varnothing$.



4. Discussion

This chapter examined the kinds of spatial elements architects use in their descriptions, and their association with the development of design concepts. Architectural design is mostly about organizing physical space, and the parts of 3d objects. However, 2d description is also used because is economical, easy to produce, and to read. From the initial conception to the implementation of a design, 2d and 3d descriptions are interrelated.

The same is true for the interaction between "content" and "form". In 2d, the notion of spatial "content" corresponds to area, and in 3d to volume. "Form" is expressed on the plane through the outline of shapes, and in 3d space through forms made out of their outmost plane surfaces. Buildings and artifacts are placed within finite physical space. But their possible forms are inexhaustible. Designers try to establish sensible relationships for the content and the form of artifacts. The required spatial elements are expressed in shape computation theory with product algebras. Product algebras are formed in two ways.

First, each of the participating graphic elements of a single description, (points, lines planes or solids) belongs to some algebra. The description is a shape in the product of the participating algebras. For example, in 2d, a plan is formed in a direct product of U_{i2} algebras, while a 3d model is constructed in a product of U_{i3} algebras.

Second, several distinct descriptions are usually necessary to represent the same object from different points of view. Plans, sections, elevations and 3d models evolve in coordination, according to predetermined relationships. For example, the modification of an element in a plan implies modifications in the corresponding components of the 3d model. The set of these parallel descriptions forms a shape in their product algebra.

The next chapter examines the process of formation of descriptions from scratch. It presents the shape computational properties of a "working" graphic environment. It is proposed that in the exploration phase of any description the elements that form the description do not lie on a single descriptive space. The creation of any description is not the result of combination of parallel descriptions that relate in some predefined manner. It emerges from the superimposition of descriptive fragments with no predefined relationships.

5. References

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IV. OVERLAYING PARTIAL DESCRIPTIONS

Abstract. A calculating device that involves multiple descriptive fragments in the production of a single description is modeled with shape computational means.

1. Introduction

A rule-based approach to the synthesis of form should acknowledge in the most profound way the constructive role of failure in the development of any “successful” solution. Unlike combinatorial devices that use readily available sets of components to generate compositions within a pre-confined domain, a device intended for explorative purpose must allow any particular set of actions to change direction, or to be recalled, and its consequences to be partially or fully erased.

A familiar device of the graphic table exhibiting this property employs multiple sheets of tracing paper to accommodate any finite number of descriptive layers, or sheets, that one may need during a graphic calculation. For example, in a line drawing aiming to determine the second floor plan of a building, a description including the ground plan serves as a reference, while extra blank sheets are superimposed to accommodate consecutive developments. The superimposed sheets may or may not carry complementary descriptions, or descriptions that relate to each other in certain ways. Sheets can contain grids, floor plans, wall outlines, details, circulation diagrams, *partis* etc., or combinations of all these. Each sheet informs the process in a twofold manner. First, shapes belonging to a specific sheet relate to each other to form a spatial arrangement of local importance. And second, shapes of one sheet combined to the shapes of other sheets synthesize the "global" picture. Sheets that are no longer useful can be withdrawn, and so their content. Or, they can be replaced by some finite decomposition of their elements. At each step, the "global" description emerges from the collapse of all the active sheets into a single description. In this way, "local" decisions described at a specific sheet obtain their consequences at the "global" level. The multiplicity of sheets allows different problems to be addressed separately in fragments that evolve independently. The overall view emerges from the superimposition of the fragments that form partial descriptions.

This brings into mind the properties of a *palimpsest.* A *palimpsest* is a manuscript that has been re-used by writing over the original writing, often at right angles to it, and sometimes more than once. Frequently it's impossible to say which layer was first inscribed, and any "development" from layer to layer would be sheer accident. The key property of a palimpsest is that the associations between layers are not only *in sequence*, in time, but also *in juxtaposition*, in space. Letters of layer x might blot out letters in layer y, or vice versa, or might leave blank areas with no markings. One cannot say if layer x was developed into layer y. And yet, the juxtapositions are not meaningless, they provide possible associations and readings. From there, a sequence can be hypothesized and a history can be composed.

The difference between a text-palimpsest and a spatial-palimpsest is that the latter remains totally unfixed. It can be infinitely re-interpreted because the inscriptions contain forms instead of symbols. And forms can be decomposed, transformed and recomposed in infinite many ways. As the consecutive superimposed sheets are semi-transparent in a line drawing, at any stage their content is present on the surface. The order of development remains meaningless. The presence of an overall picture on the surface allows "objective" interpretation. And yet makes it clear that structure is only skin deep. This is the appropriate medium for navigation in a search-space that is developing spatially, *in juxtaposition.* Where we grope our way through an elaborate net of passages, areas, boundaries, letters, clouds…

2. What is a Description?

As proposed in Stiny 1990, "*A design is an element in an n-ary relation among drawings, other kinds of descriptions, and correlative devices as needed*". A design is described by a finite set of interrelated descriptions that are finite in themselves. This definition associates descriptions like plans, sections, elevations, 3-d models, etc. that are developed in parallel, or complementarily to one another, to describe a design.

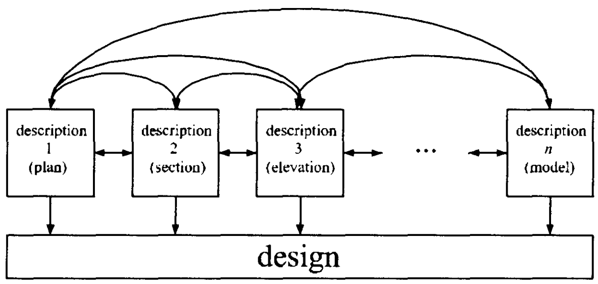


Figure 1. A set of descriptions containing plans sections, elevations and the design

The making of each of the descriptions entails the arrangement of its participating elements in an algebra U_{ij}, where $j \geq i$. Drawings, such as plans sections and elevations, are limited to the arrangement of points, lines, and planes, on the plane. Drawings involve the arrangement of lines and areas on the plane, and 3d models the manipulation of solid forms in physical space. This approach allows designs to be defined recursively in an algebra that is the direct product of the algebras employed in the descriptions.

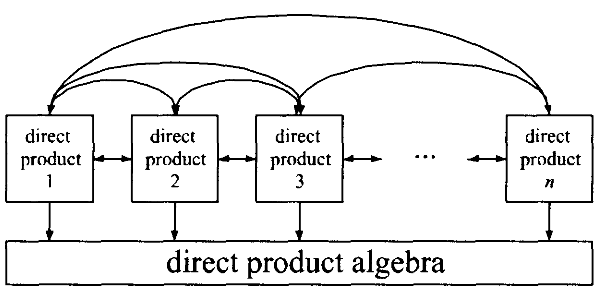


Figure 2. The algebras of descriptions and the algebras of design

The properties of this algebra depend on the algebras that combine to make it. For example a plan description in (U_{12} xU_{22}) algebra influences a 3d model in (U_{23}xU_{33}). The two descriptions form in combination a new description in a direct product algebra (U_{12} xU_{22}) x (U_{23} xU_{33}). The overall description of architectural design is usually formed within a direct product containing some number of U_{i2} and U_{i3} algebras

Descriptions like diagrams (0-dimensional), drawings (2-dimensional) and models (3-dimensional), are identified in connection to some future artifact that these descriptions explain and expose. This view can be extended to explain the making of each design description from scratch.

The proposed idea is that during the exploration process the elements that form the description do not lie on a single descriptive space. Further, a description does not result from the coordination of parallel standalone descriptions that inform each other in some predefined manner. Each description emerges from the interaction of partial descriptions that include incomplete fragments of limited interest, if seen independently. To generalize, in developing a 2d drawing or a 3d model, the elements that form a description, exist in several overlapping "partial" descriptions. These may contain objects that have no meaning in themselves. In order to become meaningful they require being superimposed, and develop certain relationships, exchange parts etc.

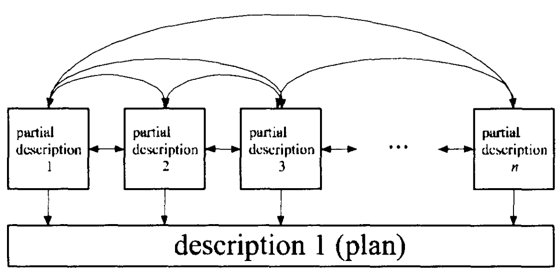


Figure 3. A set of partial descriptions 1, 2, 3, ...n, and a description. The algebra of the description is the direct product algebra of its partial descriptions

Four points illustrate the difference between partial descriptions and standalone descriptions, such as plans, sections and elevations. First, within the conventions of graphic notation of architecture, a partial description appears as a fragment. It may have no meaning on its own, and usually cannot be evaluated for what it describes. On the contrary parallel standalone descriptions like plans sections and elevations have independent meaning. For example, a plan is a standalone description because it can be evaluated by the architect for the information it contains. The evaluation of a plan can happen independently from other sources of information, plans, sections, elevations and models.

Second, unlike parallel standalone descriptions, modifications occurring in a partial description do not imply alterations in other partial descriptions. On the contrary, parallel standalone descriptions such as plans, sections,

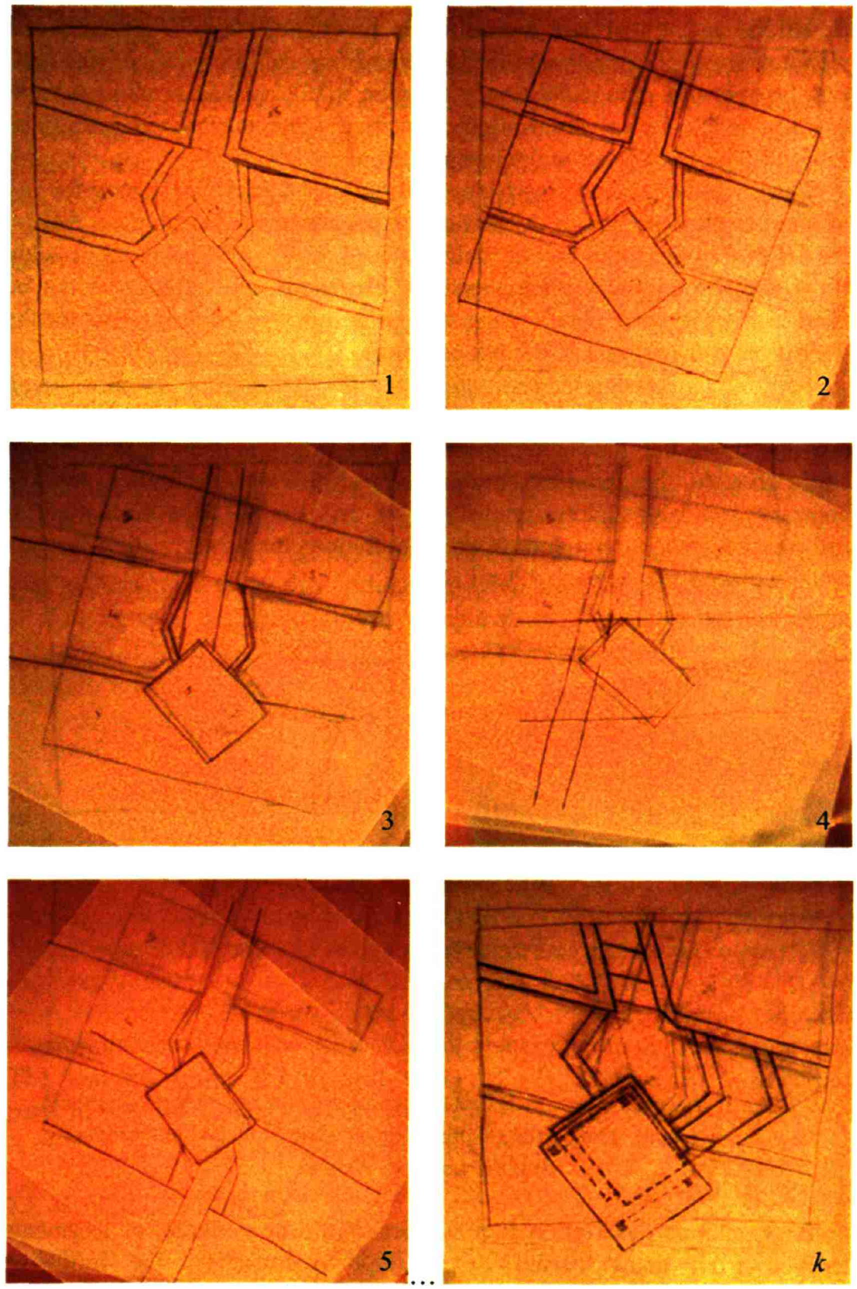


Figure 4. A sequence of partial descriptions 1, 2, 3, 4, 5... *k*. Student work: M. Nelson, Course: 4.101 Experiencing Architecture Studio, Instructor: B. Hubbard

elevations and models are associated in predefined ways. For example, a plan contains information that influences a 3d model in predefined manners, and the opposite. A modification in the plan entails specific modifications in the 3d model. Partial descriptions contain fragments with no predefined associations. They develop their relationships *ad hoc*.

Third, unlike parallel standalone descriptions, it is not necessary for any of the partial descriptions to participate in whole in describing a design. Partial descriptions can be cut, union, erased, and combined in all possible ways. On the contrary, plans, sections, elevations, and models cannot be used in fragments. A plan is a descriptive unit that becomes useless for the definition of the artifact if it is not complete.

Fourth, to become meaningful, partial descriptions require being superimposed. They are placed one over the top of the other. The order does not play role, but the superimposition gives the fragmented elements of one's thought the freedom to mingle, and to create affinities. On the contrary standalone descriptions evolve in parallel. The exchange of information among them does not require their elements to mingle. They are usually placed next to each other.

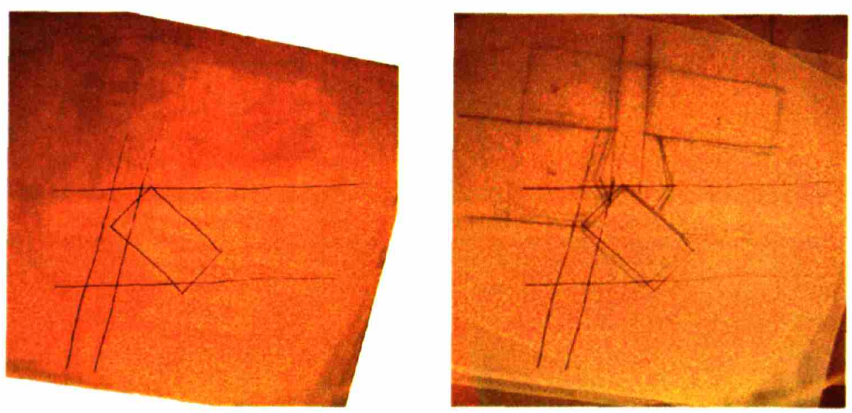


Figure 5. A partial description (left) superimposed on an undergoing graphic calculation (right). The partial description cannot be evaluated independently. Student work: M. Nelson.

A standalone description is a synthesis, in a new whole, of fragments derived from many partial descriptions. Accordingly, the following definition for descriptions can be formed: *A description is an element in an n-ary relation among its partial descriptions.*

3. Partial Descriptions

The computational properties of descriptions with solids, planes, lines and symbols can be extended for *n* partial descriptions. The proposed setting covers the plane with finitely many sheets, or occupies the space with finitely many 3d spaces each one of which contains shapes or the empty shape. Since there is no method for classifying in advance the content of these partial descriptions, there is no way to specify their finite number *n* that renders a single description.

In 2d each partial description contains lines that can be expressed in U_{12} algebra, and areas that can be expressed in U_{22} algebra. Their direct product forms shapes that contain areas and their boundaries in a product U_{12} x U_{22} algebra. In 3d, partial descriptions contain solids and volumes, and their boundary planes that can be depicted by a product U_{23} x U_{33} algebra. Symbols can be included in 2d and 3d descriptions, if necessary. Each partial description contains shapes that are finite sets of maximal elements, and any of them can be empty. A shape can be augmented, transformed or erased. Shapes or finite decompositions of these can be transferred from one partial description to another without pre-fixed order, or objective.

The *n* partial descriptions form a single shape. Whenever we look at the top of the stack of $X_1, X_2, X_3, \dots X_n$, sheets or spaces, the description X is a shape in a direct product algebra $X = X_1 \times X_2 \times X_3 \times \dots X_n$. This shape does not belong on any actual sheet or space! Its space is the space whose elements are $x = x_1, x_2, x_3, \dots x_n$ with $x_1 \in X_1 \dots x_n \in X_n$. The indexing of the i = 1,..., n sheets is arbitrary and the concept of a top sheet has no practical importance in the calculation. Nothing prohibits blank sheets to be added, while other sheets can be retrieved, or withdrawn from the stack. The union $X = X_1 \cup X_2 \cup X_3 \cup \dots X_n$ of the sheets $X_1, X_2, X_3, \dots X_n$, emerges if we finalize what we perceive, by tracing out a snapshot of the participating shapes in a "final" sheet. This becomes known as the final "presentation" drawing.

A series of partial descriptions X_i are used to form a description that remains undetermined and yet capable of determination in some future step. The final description $X = X_1 \cup X_2 \cup X_3 \cup \dots X_n$ is consistent with certain parts of what is depicted in the sub-descriptions. The product algebra $X = X_1 \times X_2 \times X_3 \times \dots X_n$ is used in the calculations.

3.1. MULTIPLE ALGEBRAS

A general way of making a new calculating device by combining two, or more, existing ones, is to form their product. Provided that their operations are similar, (that is, they have the same sequence of ranks: unary, binary, ternary...etc.) the requisite operations are defined accordingly in the product.

With any two similar algebras X'= (X', +', O_0', O_1',... O_ξ'...), and X"= (X", +", O_0", O_1",... O_ξ"...), we can create a new algebra

X= (X, +, O_0, O_1,... O_ξ...)

with X = X' x X". The set X consists of all ordered pairs (x', x") with x'∈X' and x"∈X". The operations of X are defined accordingly: (x', x") + (y', y") = (x' +' y', x" +" y"), and O_ξ ((x_0', x_0"), (x_1', x_1"),...) = (O_ξ' (x_0', x_1',...), O_ξ" (x_0", x_1",...)). This construction is extended to an arbitrary, finite (or infinite) family {X_i} of algebras.

The notion of product shape algebras is introduced in Stiny 1992 to show how spatial elements of different dimensions can be combined to form compound algebras. Compound algebras, including lines of various line-thickness, or lines and planes form n-tuples for the description of their elements.

Direct products in 2d partial descriptions can include lines, areas and possibly symbols. They can be determined by triples of the form (*l*, *p*, *s*) with $l \in U_{12}$, $p \in U_{22}$, and $s \in V_{02}$. Therefore, an addition can take the form:

((l_{X1}', p_{X1}', s_{X1}'), (l_{X2}', p_{X2}', s_{X2}')...(l_{Xn}', p_{Xn}', s_{Xn}')) + ((l_{X1}", p_{X1}", s_{X1}"), (l_{X2}", p_{X2}", s_{X2}")...(l_{Xn}", p_{Xn}", s_{Xn}")) = (((l_{X1}'+ l_{X1}"), (l_{X2}'+ l_{X2}"),... (l_{Xn}'+ l_{Xn}")), ((p_{X1}'+ p_{X1}"), (p_{X2}'+ p_{X2}"),...(p_{Xn}'+ p_{Xn}")), ((s_{X1}'+ s_{X1}"), (s_{X2}'+ s_{X2}"), ...(s_{Xn}'+ s_{Xn}")))

where any member of the triples (l_{Xi}, p_{Xi}, s_{Xi}), can be the empty shape.

Direct products in 3d partial descriptions, including volumes, their plane boundaries and possibly symbols can be determined by triples of the form (*v*, *p*, *s*) with $v \in U_{33}$, $p \in U_{22}$, and $s \in V_{02}$. Accordingly, an addition among shapes containing volumes, planes and symbols (x_1', x_2', x_3',...,x_n') + (x_1", x_2", x_3",...,x_n") obtains the form,

((v_{X1}', p_{X1}', s_{X1}'), (v_{X2}', p_{X2}', s_{X2}')...(v_{Xn}', p_{Xn}', s_{Xn}')) + ((v_{X1}", p_{X1}", s_{X1}"), (v_{X2}", p_{X2}", s_{X2}")...(v_{Xn}", p_{Xn}", s_{Xn}")) = (((v_{X1}'+ v_{X1}"), (v_{X2}'+ v_{X2}"),... (v_{Xn}'+ v_{Xn}")), ((p_{X1}'+ p_{X1}"), (p_{X2}'+ p_{X2}"), ...(p_{Xn}'+ p_{Xn}")), ((s_{X1}'+ s_{X1}"), (s_{X2}'+ s_{X2}"), ...(s_{Xn}'+ s_{Xn}")))

Dual to the space of the product $X = X_1 \times X_2 \times X_3 \times \dots X_n$ is the space $X = X_1 \cup X_2 \cup X_3 \cup \dots X_n$ of the union, or sum, of the spaces $X_1, X_2, X_3, \dots X_n$. The details of the difference between union and product algebras are not subject of this study, but the union of algebras is complementary to the concept of their product (disjoint union) of algebras. The space $X = X_1 \cup X_2 \cup X_3 \cup \dots X_n$ of the union of $X_1, X_2, X_3, \dots X_n$, emerges unruly, and makes the union $X_1 \cup X_2 \cup X_3 \cup \dots X_n$ not a good space for calculation.

In this study, the product of shape algebras is used for calculation, while the union of specific descriptions is often used for presentation purposes.

There are ways to map a given algebra X "homomorphically", or "isomorphically", etc. to another similar algebra X', through a function f. Mappings are of central importance in the study of abstract algebras. The general mathematical notions of such mappings will be considered known. They are used in the format defined in Birkhoff; MacLane 1965, and Halmos 1963. Mappings are used in Stiny 1994, to describe the continuous action of shape rules, and in Krstic 1996, in decompositions of shapes. In this section they are used to show the interaction of shapes in calculations within product algebras (for example, areas and their boundaries), in calculations with partial descriptions.

In the product $X = X_1 \times X_2 \times X_3 \times \ldots X_n$ of a family $\{X_i\}$ of algebras, for each $i = 1,\ldots,$ n, there is a natural epimorphism from X to X_i. That is, there is a projection f_i, defined by f_i (p) = p_i, where every element of X_i is equal to f_i (p) for some p_i in X. Moreover, if X_b is an arbitrary Boolean algebra, and if for each $i = 1, \ldots, n$ there is a homomorphism g_i from X_b to X_i, then there is a unique homomorphism g from X_b to X, such that is, f_i (g(p)) = g_i (p).

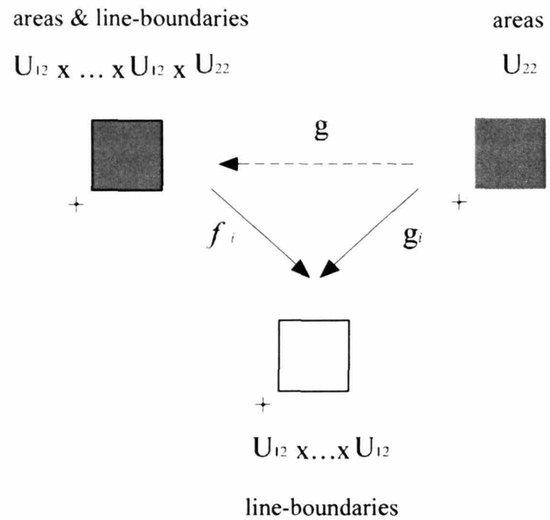
$$\begin{tikzcd}
X \arrow[dr, "f_i"'] & & X_b \arrow[ll, dashed, "g"'] \arrow[dl, "g_i"] \\
& X_i &
\end{tikzcd}$$

The mapping f_i maps the product $X = X_1 \times X_2 \times X_3 \times \ldots X_n$ to X_i. The mapping g maps X_b to X, and the value of the product f_i x g, for each element p, denoted as f_i (g(p)) is given by g_i (p), which is a mapping from X_b to X_i.

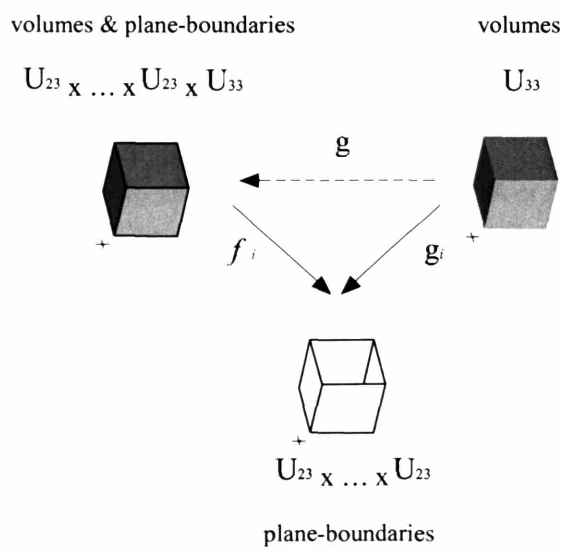
Dually, if $\{A_i\}$ is a family of algebras then there is their sum algebra $A = A_1 \cup A_2 \cup A_3 \cup \ldots A_n$. And for each i, there is a continuous one-to-one mapping d_i from A_i to A. If A_b is any Boolean algebra and if for each i there is a continuous mapping j_i from A to A_b, then there is a unique continuous mapping j from A to A_b such that j (d_i (p)) = j_i (p) for every i.

$$\begin{tikzcd}
A \arrow[rr, dashed, "j"] & & A_b \\
& A_i \arrow[ul, "d_i"] \arrow[ur, "j_i"'] &
\end{tikzcd}$$

These mappings can map the interaction among product shape algebras, in partial descriptions. For example among *content* (area, or volume) and *form* (their line or plane boundaries). The next mapping describes the interaction between an algebra U_{22} representing *area* (content), and a product of any number of U_{12} algebras representing the *boundaries* of the areas (form) on the plane.



Also, the same mapping describes the interaction between an algebra U_{33} representing *volume* (content), and a product of any number of U_{23} algebras representing the *boundaries* of volumes (form) in physical space:



3.2. SHAPE RULE SCHEMATA WITH N VARIABLES

Given a direct product algebra $X_1 \times X_2 \times X_3 \times \ldots X_n$ of possibly empty partial descriptions X_i, with $i = 1, \ldots, n$, a formula

$$g(x_1, x_2, x_3, \ldots x_n) \rightarrow g(y_1, y_2, y_3, \ldots y_n)$$

is a rule schema of n variables $x_1, x_2, x_3, \ldots x_n$ over $X_1, X_2, X_3, \ldots X_n$. The schema becomes a rule if any element of X_i is substituted for each variable x_i, $i = 1, \ldots, n$. That is,

$$g(x_1, x_2, x_3, \ldots x_n) \rightarrow g(y_1, y_2, y_3, \ldots y_n), \qquad x_1 \in X_1, x_2 \in X_2, x_3 \in X_3, \ldots x_n \in X_n$$

Rule schemata in 2d partial descriptions are determined by triples (l, p, s) with $l \in U_{12}$, $p \in U_{22}$, and $s \in V_{02}$ (or $s \in V_{12}$, $s \in V_{22}$). A rule schema $g(x_1, x_2, x_3, \ldots, x_n) \rightarrow g(y_1, y_2, y_3, \ldots, y_n)$ obtains the form,

$$g(\,(l_{X1}, p_{X1}, s_{X1}), (l_{X2}, p_{X2}, s_{X2})\ldots(l_{Xn}, p_{Xn}, s_{Xn})\,) \rightarrow g(\,(l_{y1}, p_{y1}, s_{y1}), (l_{y2}, p_{y2}, s_{y2})\ldots(l_{yn}, p_{yn}, s_{yn})\,)$$

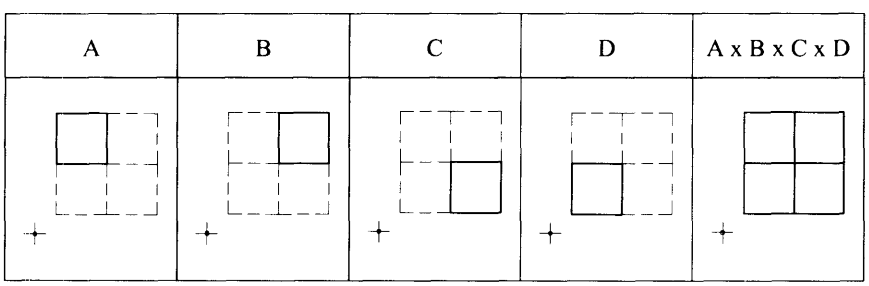
where any member of the triples (l_{Xi}, p_{Xi}, s_{Xi}), can be the empty shape.

Rule schemata in 3d partial descriptions are determined by triples (v, p, s) with $v \in U_{33}$, $p \in U_{22}$, $s \in V_{03}$ (or $s \in V_{13}$, $s \in V_{23}$, $s \in V_{33}$). A rule schema $g(x_1, x_2, x_3, \ldots, x_n) \rightarrow g(y_1, y_2, y_3, \ldots, y_n)$ obtains the form,

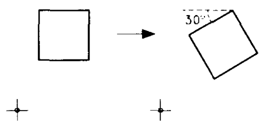
$$g(\,(v_{X1}, p_{X1}, s_{X1}), (v_{X2}, p_{X2}, s_{X2})\ldots(v_{Xn}, p_{Xn}, s_{Xn})\,) \rightarrow g\,(\,(v_{y1}, p_{y1}, s_{y1}), (v_{y2}, p_{y2}, s_{y2})\ldots(v_{yn}, p_{yn}, s_{yn})\,)$$

where any member of the triples (v_{Xi}, p_{Xi}, s_{Xi}), can be the empty shape.

In the next example, a shape is described in $n = 4$ sheets, A, B, C, D that contain lines. A product algebra $U_{12} \times U_{12} \times U_{12} \times U_{12}$ is used in the calculations that follow.

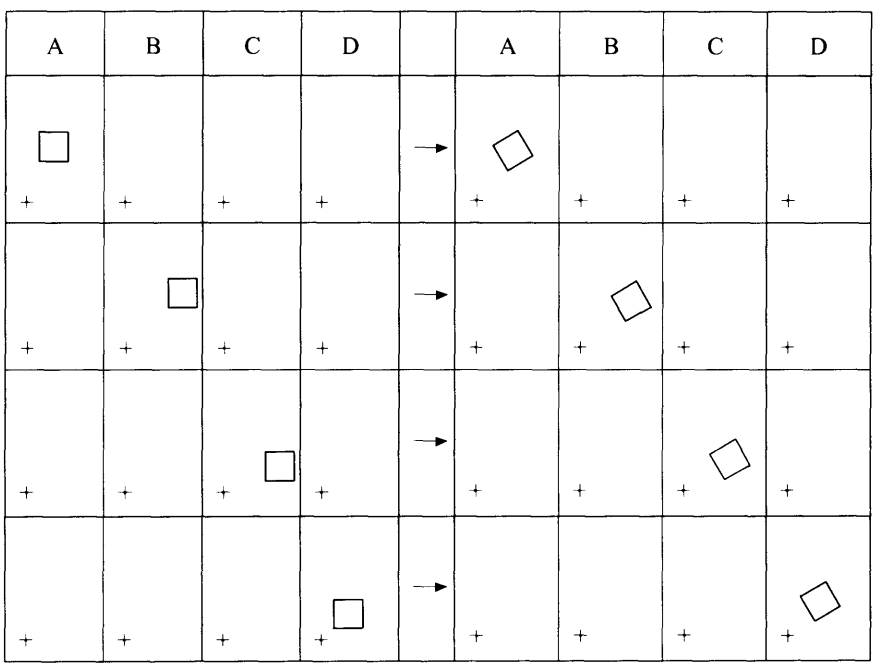


Consider a 30° rotation.

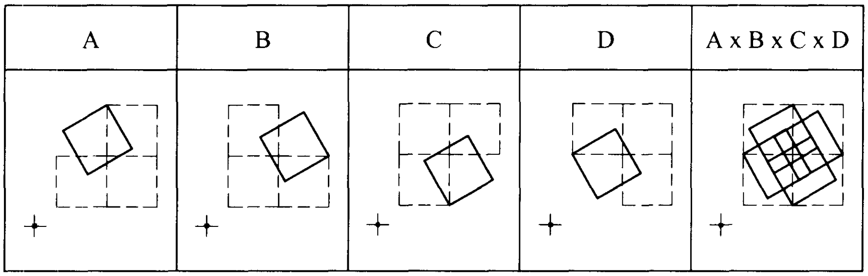


In order to access the squares in all four layers A, B, C, D, the rule takes the form,

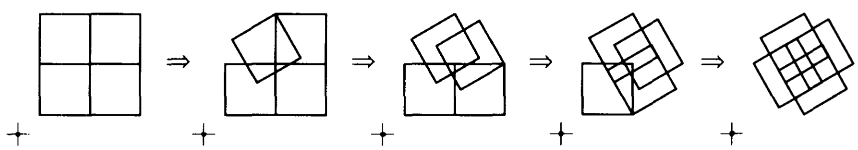
$< x_A, \varnothing, \varnothing, \varnothing >$	$\rightarrow$	$< y_A, \varnothing, \varnothing, \varnothing >$
$< \varnothing, x_B, \varnothing, \varnothing >$	$\rightarrow$	$< \varnothing, y_B, \varnothing, \varnothing >$
$< \varnothing, \varnothing, x_C, \varnothing >$	$\rightarrow$	$< \varnothing, \varnothing, y_C, \varnothing >$
$< \varnothing, \varnothing, \varnothing, x_D >$	$\rightarrow$	$< \varnothing, \varnothing, \varnothing, y_D >$



The rotation can apply on the shapes of the layers A, B, C, D, as next.



One possible derivation is,



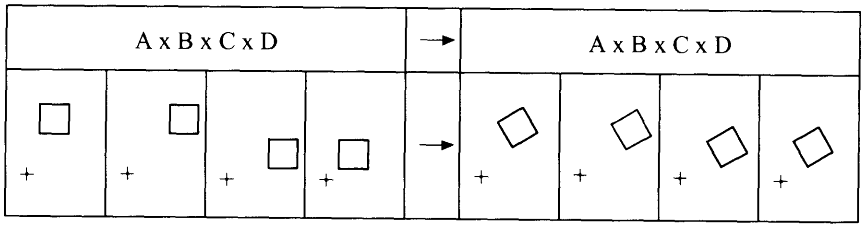
When a rule applies on shapes $C_1, C_2, C_3, \ldots C_n$ with $C_1 \in X_1, \ldots C_n \in X_n$ searches for the shapes $x_1, x_2, x_3, \ldots x_n$, and turns them into $y_1, y_2, y_3, \ldots y_n$ respectively, producing $C_1', C_2', C_3', \ldots C_n'$ according to the formula: $C_1', C_2', C_3', \ldots C_n' = [C_1, C_2, C_3, \ldots C_n - t(x_1, x_2, x_3, \ldots x_n)] + t(y_1, y_2, y_3, \ldots y_n)$.

C_1'	=	$C_1 - t_1(x_1)$	+	$t_1(y_1)$
C_2'	=	$C_2 - t_2(x_2)$	+	$t_2(y_2)$
C_3'	=	$C_3 - t_3(x_3)$	+	$t_3(y_3)$
…	…	…	…	…
C_n'	=	$C_n - t_n(x_n)$	+	$t_n(y_n)$

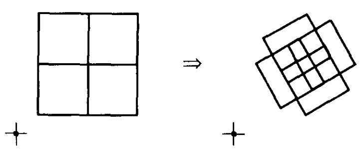
Shape rule schemata of n variables, $x_1, x_2, x_3, \ldots x_n$ over $X_1, X_2, X_3, \ldots X_n$ can be treated as shape rule schemata of one shape variable $z = (x_1, x_2, x_3, \ldots x_n)$ over the direct product $X_1 \times X_2 \times X_3 \times \ldots X_n$. Therefore, a rule schema of n variables over $X_1, X_2, X_3, \ldots X_n$ and a rule schema of one variable ranging over the product $X_1 \times X_2 \times X_3 \times \ldots X_n$ are the same thing.

The four rotations of the example can take the form:

$$< x_A, x_B, x_C, x_D > \rightarrow < y_A, y_B, y_C, y_D >$$



And the initial shape can be treated accordingly.



Given a rule schema $g(x_1, x_2, x_3, \ldots x_n) \rightarrow g(y_1, y_2, y_3, \ldots y_n)$, where $x_1 \in X_1, \ldots x_n \in X_n$. The shapes $(x_1, x_2, x_3, \ldots x_n)$ are members of the product $X_1 \times X_2 \times X_3 \times \ldots X_n$ for which the rule $g(x_1, x_2, x_3, \ldots x_n) \rightarrow g(y_1, y_2, y_3, \ldots y_n)$ can be formed:

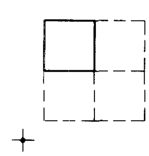
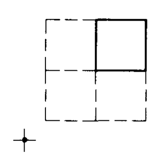
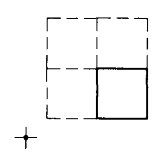
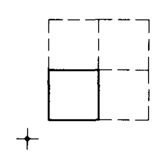
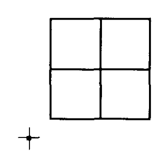
$$\{(x_1, x_2, x_3, \ldots x_n) \in X_1 \times X_2 \times X_3 \times \ldots X_n : g(x_1, x_2, x_3, \ldots x_n) \rightarrow g(y_1, y_2, y_3, \ldots y_n)\}$$

A shape $(\alpha_1, \alpha_2, \ldots \alpha_n)$ in the product $X_1 \times X_2 \times X_3 \times \ldots X_n$ belongs to the shapes $(x_1, x_2, x_3, \ldots x_n)$ if and only if the rule $g(\alpha_1, \alpha_2, \ldots \alpha_n) \rightarrow g(\alpha_1', \alpha_2', \ldots \alpha_n')$ is formable.

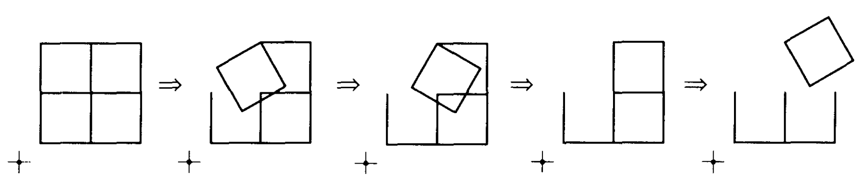
Every rule schema $g(x_1, x_2, x_3, \ldots x_n) \rightarrow g(y_1, y_2, y_3, \ldots y_n)$, where $x_1 \in X_1, \ldots x_n \in X_n$ determines a set of shapes $\{(x_1, x_2, x_3, \ldots x_n) \in X_1 \times X_2 \times X_3 \times \ldots X_n : g(x_1, x_2, x_3, \ldots x_n) \rightarrow g(y_1, y_2, y_3, \ldots y_n)\}$, within the product $X_1 \times X_2 \times X_3 \times \ldots X_n$ and therefore a relation R in $X_1 \times X_2 \times X_3 \times \ldots X_n$. Finally, the following relationship holds for every shape $(\alpha_1, \alpha_2, \ldots \alpha_n)$ of $X_1 \times X_2 \times X_3 \times \ldots X_n$

$$R(\alpha_1, \alpha_2, \ldots \alpha_n) \Leftrightarrow g(\alpha_1, \alpha_2, \ldots \alpha_n) \rightarrow g(\alpha_1', \alpha_2', \ldots \alpha_n')$$

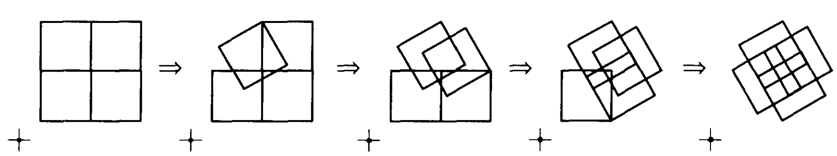
The number of all possible partial descriptions emerging from a set of spaces S = {1, 2,...n}, containing n finite number of 2d sheets, or spaces, is equal to the power set of this set, denoted as P(S) or 2^S. Since S is a set of order n the order of P (S) is 2^n, including S and the empty set. In the previous example with a shape described in n = 4 sheets A, B, C, D. The number 2^n of possible partial descriptions is $2^4 = 16$.

A	B	C	D	A x B x C x D
				

The application of a 30° rotation on the sum A∪B∪C∪D generates shapes like the following,



and stops when no more squares can be found in the description. The application of the rotation in A x B x C x D generates shapes like the next,



and may continue for ever.

Calculations with $n \geq 3$ sheets, or spaces involve the production of a number $p = 2^n - (n+2)$ partial descriptions produced as products or sums among k participating sheets, or spaces, with $1 \leq k \leq n$. That is, except from

4. Designing with Partial Descriptions

Alternative partial descriptions can be used in design. These can be arrangements that are produced on the basis of diverse or interdependent criteria (structure, function etc.)

A rule applying on the X_1, X_2, X_3, … X_n partial descriptions, makes several x_1, x_2, x_3, … x_n distinct fragments of shapes to correspond, as a result of different continuations, to one and the same shape, on X.

The shapes in partial descriptions may end up developing complicated structures, or interconnections that could not be initially anticipated. Each of the X_1, X_2, X_3, … X_n partial descriptions complies to commutativity, associativity, distributivity, identity, and inverse-ness and it is closed for addition and multiplication. The *n*-spaces create a continuous *n-to-one* mapping to *X*.

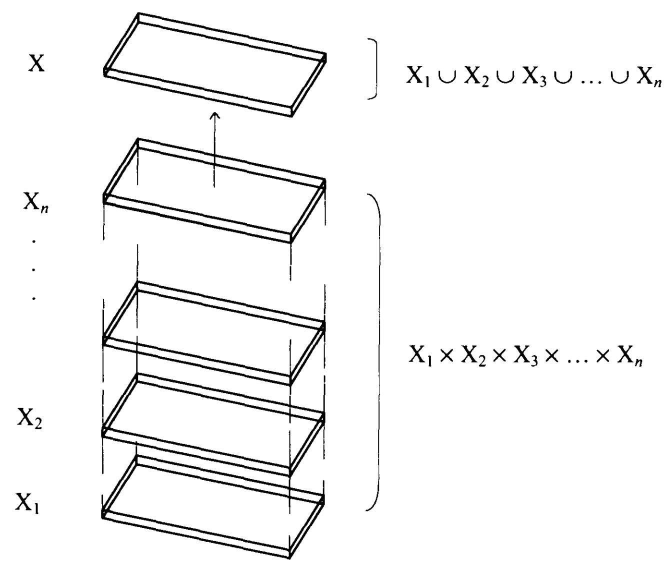
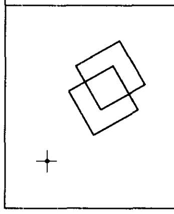
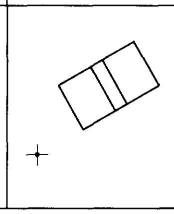
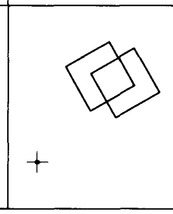
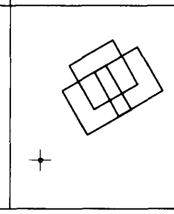
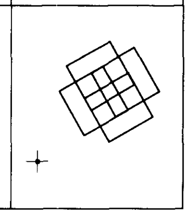


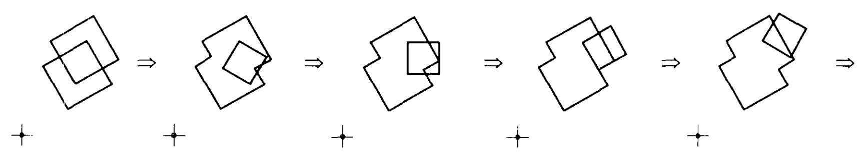
Figure 4. A set of partial descriptions X_1, X_2, X_3, …X_n, creates a space for calculation equal to X_1 x X_2 x X_3 x …x X_n. The overall, final description can emerge as the union $X_1 \cup X_2 \cup X_3 \cup \ldots \cup X_n$

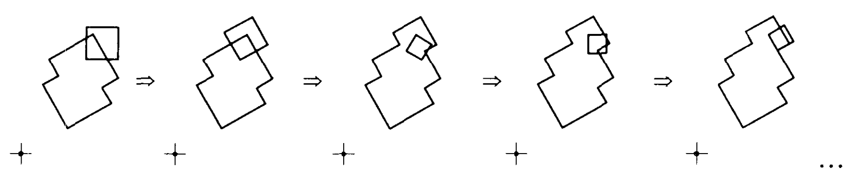
The shapes x_1, x_2, x_3,… x_n are distinct, and each of the *n-to-one* mappings $h_i(x) = x - t(A)$ are continuous: On each X_i a mapping $\{(x, h_i(x))\}$, can be defined, where $x \in X$. And in the mapping $X_i \rightarrow X$, an inverse mapping $X \rightarrow X_i$ can always be defined.

the singletons {A}, {B}, {C}, {D}, which represent the initial sheets, the empty set, and the final description {A, B, C, D}, we can form $p = 2^4 - (4+2) = 10$ partial descriptions, including: {A, B}, {A, B, C}, etc. These, pairs, triples, quadruples etc., build partial sums, or products that become useful in unpredictable ways. For example,

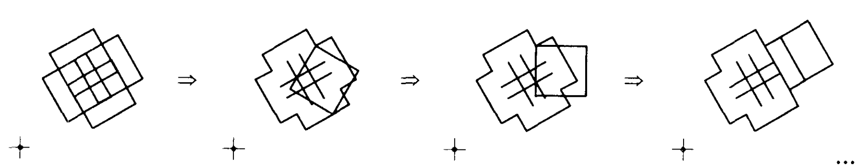
A ∪ D	B ∪ D	A ∪ B	A ∪ B ∪ D	A∪B∪C∪D
				

Products or sums can be combinations of the initial squares, or of their parts. In the example the only permissible rule is a 30° rotation of squares, and therefore, here is a possible derivation from the shape A ∪ D.





And, because the produced arrangements remain non-atomic further elaboration is always possible. For example, the union A ∪ B ∪ C ∪ D,



The trivial case in which all spatial elements of a description lie on a single sheet (n=1) corresponds to a "presentation" drawing. But "working" drawings require always more than one sheet. In this study, they are categorized into two general classes: For $n = 2$, and $n \geq 3$. By convention, in the following examples, it is assumed that for every finite number n, of sheets or spaces, the "presentation" description X can be rendered as a union, on the $(n+1)^{th}$ sheet, whenever is necessary. Therefore, for $n = 2$, the "presentation" description S is formed on the third (2+1=3) sheet, and for $n \geq 3$ and n finite on the $(n+1)^{th}$ sheet. The direct product {A x B x…x n} corresponds with what one perceives by looking at the top of the stack.

4.1. FOR N = 2

In this simple case two spaces containing two or more graphic components apiece are combined. In the example two sheets are used, both containing lines and areas. For $n = 2$, the complete "presentation" description can be formed as a sum on a 3rd sheet. The set of possible partial descriptions emerging from a set of two sheets A and B, is equal to the power set 2^S of the set S = {A, B}. Since P(S) = {{A}, {B}, {A, B}, ∅}, it is evident that except from the singletons {A}, {B}, only one description can be formed, namely {A, B}. This is the expected, because for a set of order $n = 2$ the number $p = 2^n - (n+2)$ of partial descriptions that can be produced as products or sums is $p = 2^2 - (2+2) = 0$.

Therefore, calculations in two sheets containing lines and planes involve three descriptions: sheet A, sheet B, and S that renders {A, B}.

TABLE 1. Table of partial descriptions emerging for n =2 graphic layers

Layer	{A}	{B}	{A, B}	∅
Algebra	U_{12} U_{22}	U_{12} U_{22}	$U_{12}U_{22}$ x $U_{12}U_{22}$	∅
Elements	lines, area	lines, area	lines, area	∅
Description	partial	partial	whole	∅

Since each of the sheets A and B contains only lines and areas a rule is determined by a pair of the form (l, p) with $l \in U_{12}$ and $p \in U_{22}$.

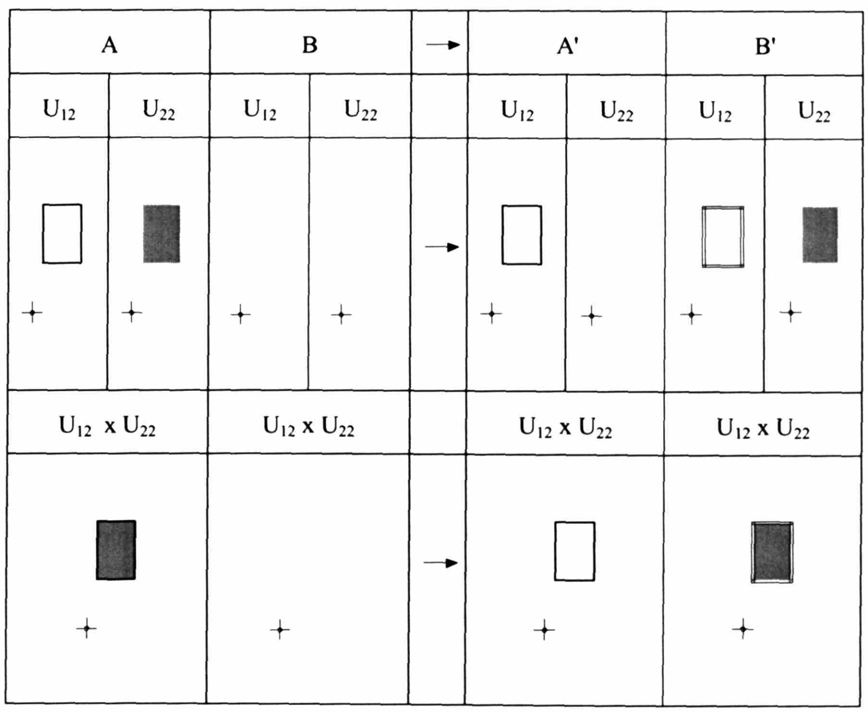
A rule (A, B) $\rightarrow$ (A′, B′) has the form,

$$(l_{A1}, p_{A1}) \; (l_{B1}, p_{B1}) \quad \rightarrow \quad (l_{A2}, p_{A2}) \; (l_{B2}, p_{B2})$$

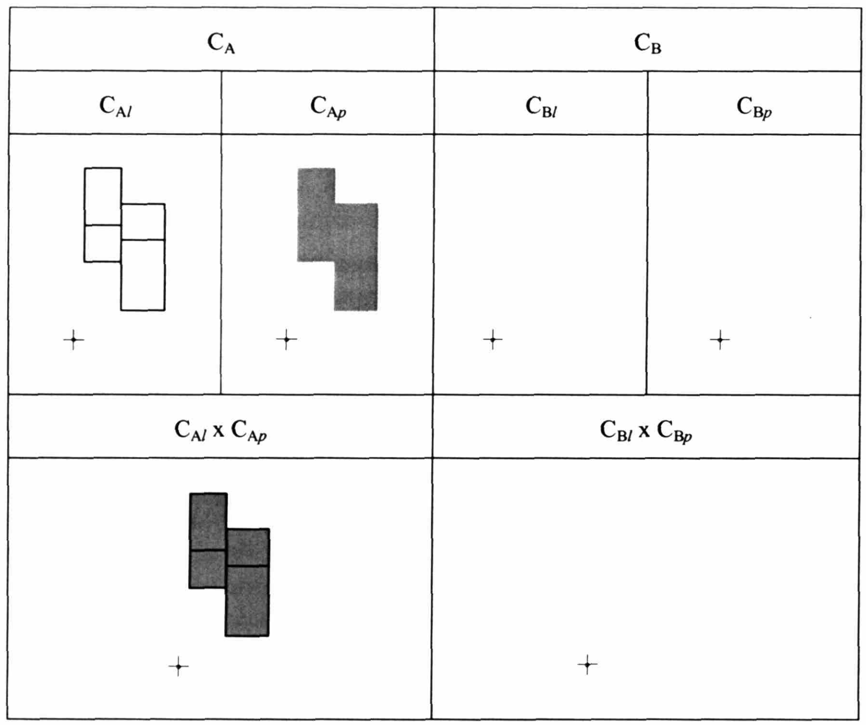
The rule applies on a shape C. The shape includes some linear part and some area $C_{A\,l}$, $C_{A\,p}$ and $C_{B\,l}$, $C_{B\,p}$ to produce a new shape C ′, with C_{Al}', C_{Ap}' and $C_{B\,l}'$, C_{Bp}' respectively. The set of shapes generated by the rule is $\{C' \in U_{12}U_{22} : C \Rightarrow C'\}$.

The parametric rule of the next example performs a very common graphic calculation: Wall-layouts for rooms are placed, on some schematic layout (*parti*). The rule scans the *parti* in sheet A looking for "rooms" of rectangular shape, and draws walls in sheet B, for the corresponding room scanned in A. It also draws the available, remaining room area.

TABLE 2. Example rule for n =2 graphic layers A, B



The initial shape C_A in sheet A is the input arrangement: a schematic layout containing a linear description C_{Al}, and its corresponding footprint C_{Ap}. The initial shape C_B in sheet B is the empty shape: $C_{Bl} = \varnothing$, $C_{Bp} = \varnothing$



The shape $C_A = C_{Al} + C_{Ap}$ of all shapes in sheet A renders an arrangement representing an outline and its area. The shape in sheet B, is $C_B = \varnothing$.

Given the *parti*, the rule can be viewed as a generator of a particular family of wall layouts. A verbal description of how to generate an arrangement in the family would be the following:

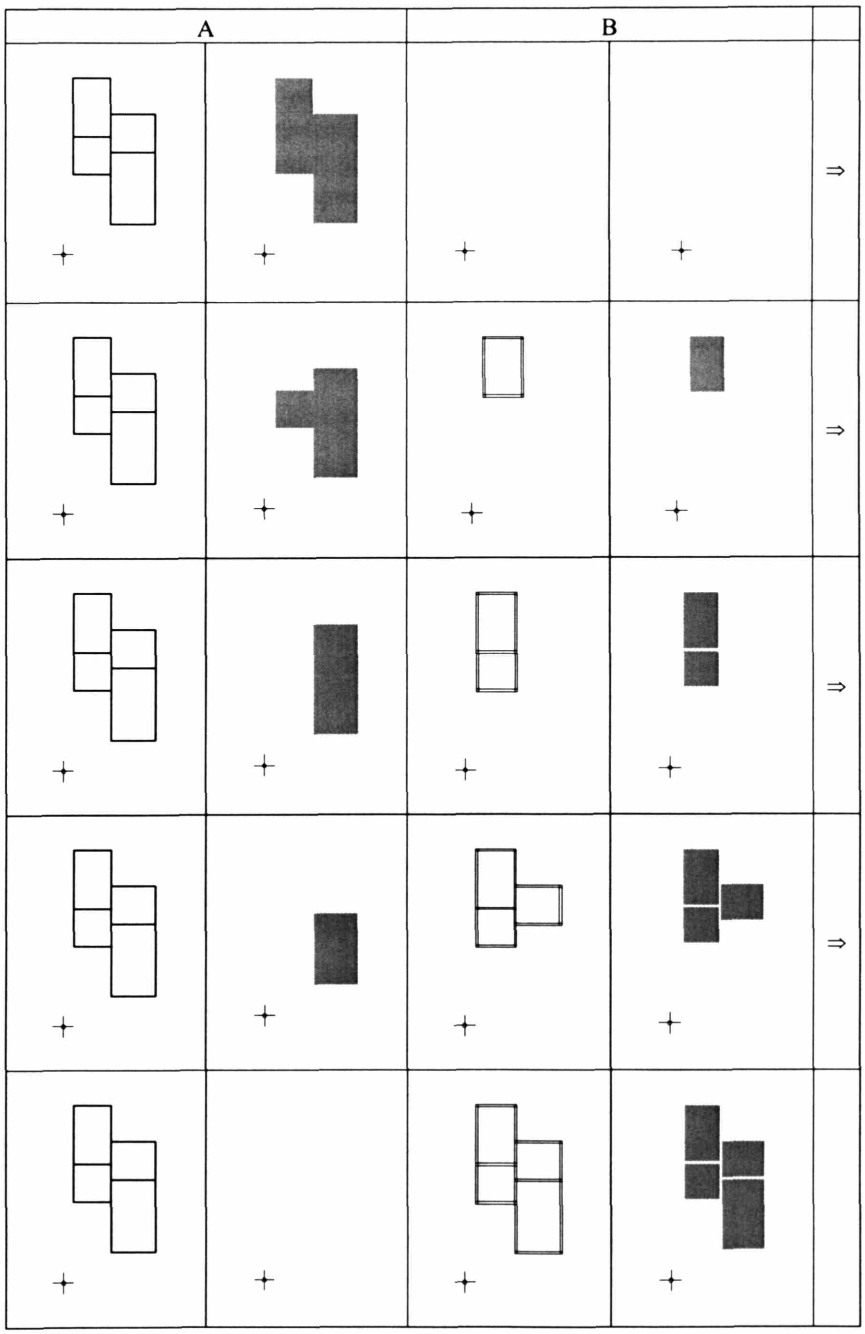
Step (i) For all shapes C_{Al} and C_{Ap} in sheet A,

find $t(l_{A1}) \leq C_{Al}$ and $t(p_{A1}) \leq C_{Ap}$

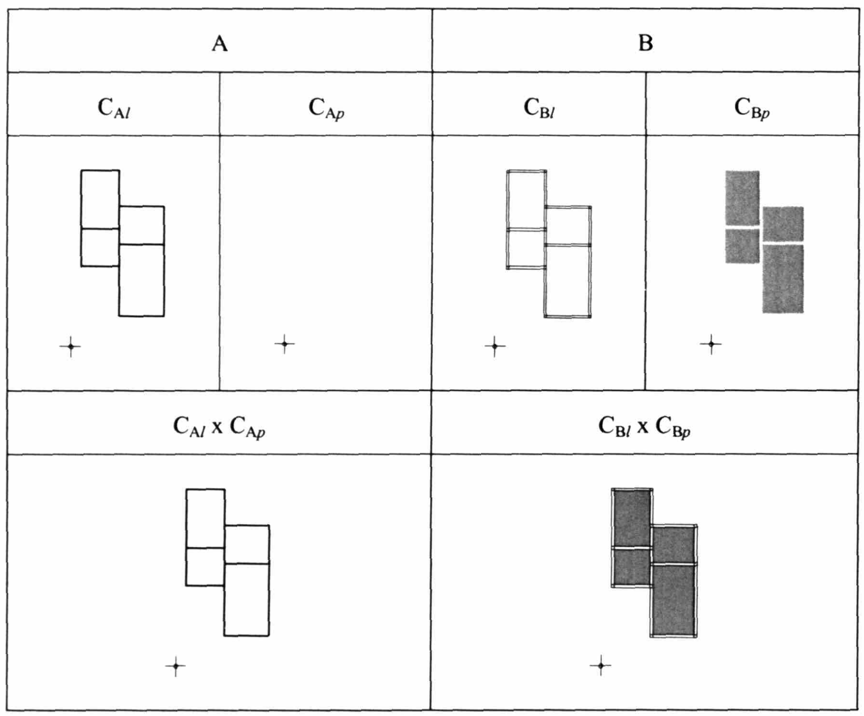
Step (ii) Let $C_{Al}' = C_{Al}$ and $C_{Ap}' = C_{Ap}$.- $t(p_{A1})$ in sheet A, and

let $C_{Bl}' = t(l_{B2})$ and $C_{Bp}' = t(p_{B2})$ in sheet B

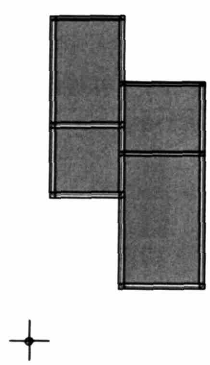
An example appears in the next derivation,



At the end, the sheet A contains the lines of the *parti* while its footprint is erased. In sheet B, two new shapes are formed: the wall-layout C_{Bl}, and the corresponding footprint C_{Bp}. The shape C_{Al} x C_{Ap} renders a shape made out of lines, while C_{Bl} x C_{Bp} renders the wall-layout and the available area.



The *parti* presented in sheet A is transformed in sheet B, and new information is introduced to it. The product A x B provides three descriptive essentials: the *parti*, the wall-layout, and the net room area.



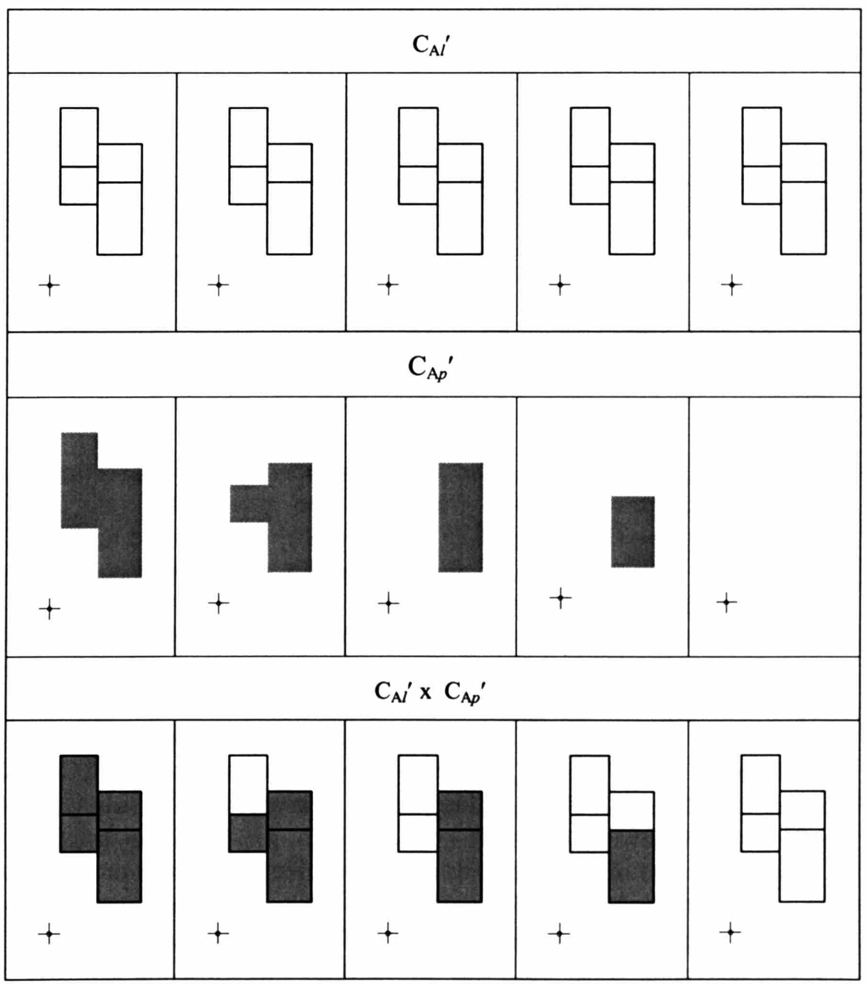
4.1.1. Sheet A

Arrangements of lines C_{Al}' and areas C_{Ap}' evolve in A in the following way:

$$C_{Al}' = [\, C_{A\,l} - t(l_{A1})\,] + t(l_{A2}) \text{ and } C_{Ap}' = [\, C_{Ap} - t(p_{A1})\,] + t(p_{A2})$$

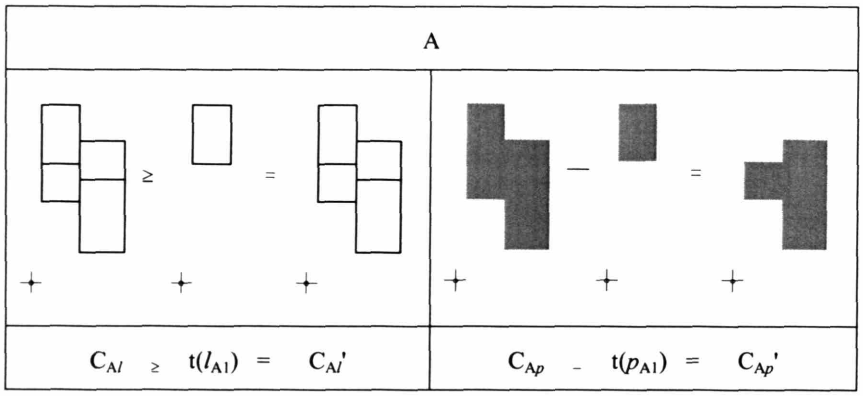
The consecutive shapes made out of lines in C_{Al}' do not change while the areas C_{Ap}' are reduced by one shape at each step, concluding to the empty shape at the end. In the next table, the direct product C_{Al}'x C_{Ap}' of lines and areas is presented in the third row.

TABLE 3. Sequence of shapes for lines C_{Al}, areas C_{Ap} and their product, in sheet A



For lines, at each step, a transformation $t(l_{Al})$ representing the schematic outline of a room is embedded on the input shape C_{Al}. The shapes C_{Al}' and C_{Al} remain identical. This first half of the rule is an identity. The mapping $h(x) = x - [t(l_{A1}) - t(l_{A2})]$ for every part x of C_{Al} leaves the shape intact, because $t(l_{A1}) \leq t(l_{A2})$ and $h(x) = x$.

The closure operations for C_{Al} and C_{Al}' satisfy the conditions $\Gamma(t(l_{Al})) = t(l_{Al})$ and $\Gamma(x) = \Gamma'(x)$, which means that the shape $t(l_{Al})$ is closed, and C_{Al} retains its topology before and after the rule is applied. Essentially, the application of this identity makes $t(l_{A1})$ part of C_{Al} thus dividing it in a particular way: $C_{Al}' = C_{Al} \geq t(l_{Al})$



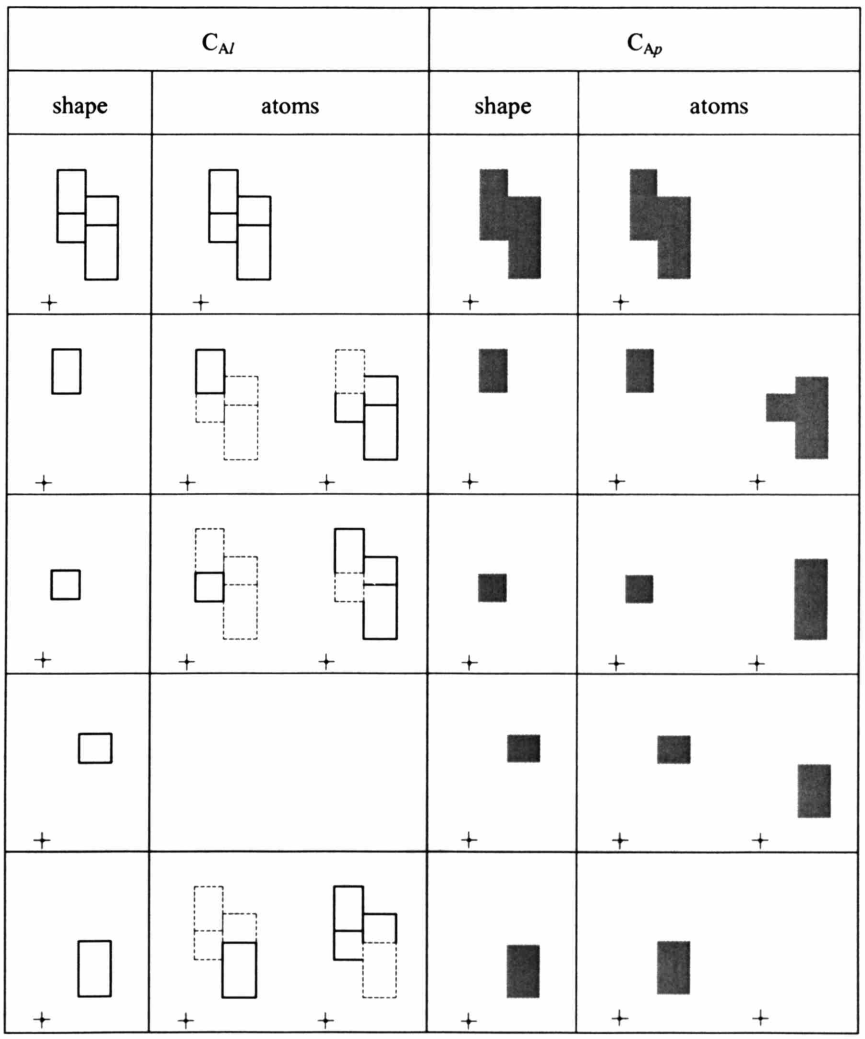
For areas, at each step, the transformation $t(p_{Al})$ representing the footprint of a room is erased from C_{Ap}. This part of the rule causes consecutive subtractions. It is $C_{Ap}' = [C_{Ap} - t(p_{Al})] + t(p_{A2})$, and $t(p_{A2}) = \varnothing$. The mapping $h(x) = x - t(p_{A1})$ for every part x of C_{Ap} produces C_{Ap}' by substituting $t(p_{A1})$ with the empty shape.

The rule changes every part x of C_{Ap}, before erasing $t(p_{A1})$, by associating C_{Ap} with some part of C_{Ap}'. The mapping coordinates the divisions on both shapes, so continuity is preserved. The closure operations for C_{Ap} and C_{Ap}' satisfy the conditions $\Gamma(t(p_{Al})) = t(p_{Al})$ which means that the shape $t(p_{Al})$ is closed, and $\Gamma(x) = \Gamma'[x - t(p_{A1})]$. For every non-empty part x of C_{Ap} the closure operations Γ and Γ' obtained by the topologies for C_{Ap} and C_{Ap}' are related in the following way: $\Gamma(x) - t(p_{Al}) = [C_{Ap} - t(p_{Al})] \cdot \Gamma'[x - t(p_{A1})]$.

The distinction of parts depends on how the identities apply on the *parti* in C_{Al}. The interpretation of the *parti* can be performed in alternative ways. Instead of rectangles one could pick Γ, H or Π structures. The subtraction of the corresponding areas from C_{Ap} serves counting: when the area C_{Ap} is erased, the rule can no longer apply. Counting depends on two things: how structure is attributed and what is the correspondence between the parts of the *parti*, and the area.

Overall, two important things are accomplished: some structure is retrieved from the *parti* and some counting is performed, which determines when the interpretation of the *parti* is completed. The parts of C_{Al} an their complements, can be seen as Boolean algebras. The parts for lines and areas can be organized in equivalence classes, as in the next diagram.

TABLE 4. Equivalence classes for the parts made out of lines and areas, in sheet A



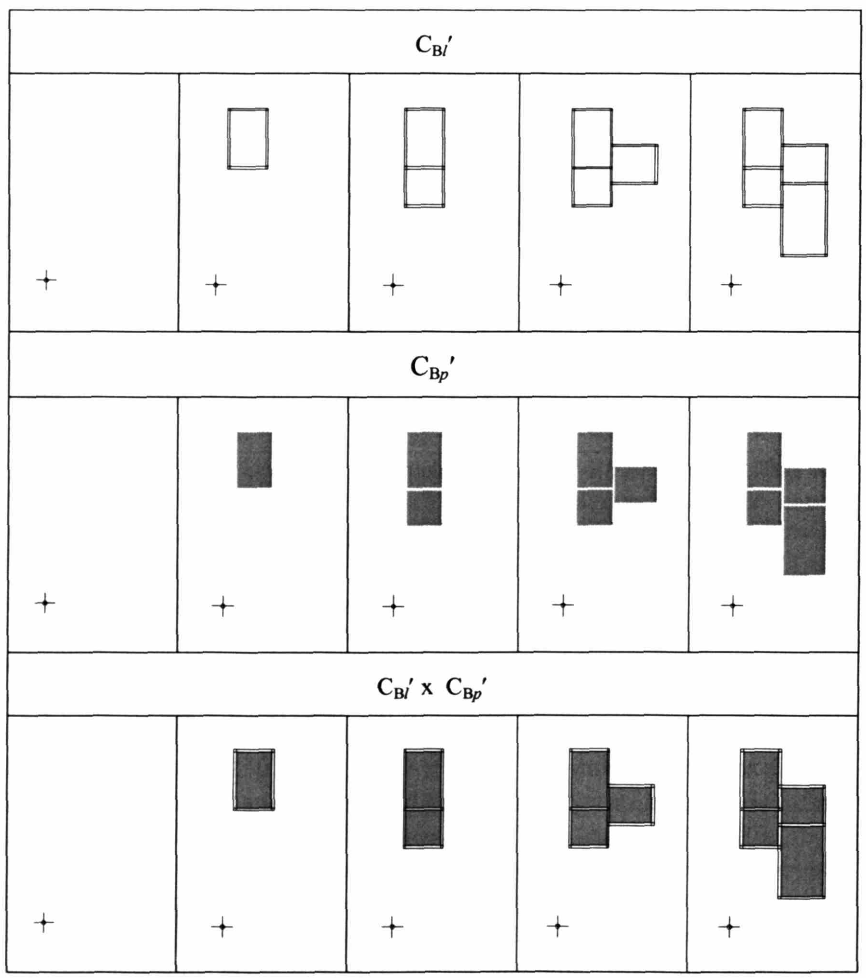
4.1.2. Sheet B

In sheet B, the rule applies on the shape C_B, which includes a linear C_{Bl} and a plane component C_{Bp} for areas and produces the shapes C_{Bl}' and C_{Bp}' according to the double relationship

$$C_{Bl}' = [\, C_{Bl} - t(l_{B1})\,] + t(l_{B2}) \text{ and } C_{Bp}' = [\, C_{Bp} - t(p_{B1})\,] + t(p_{B2})$$

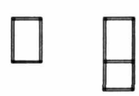
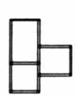
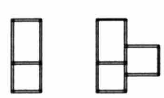
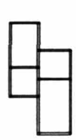
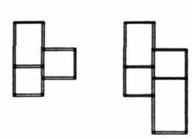
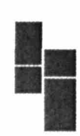
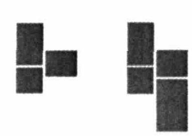
The arrangements for lines C_{Bl}' and areas C_{Bp}' change as new shapes are added. The product C_{Bl}'x C_{Bp}' of lines and areas appears in the third row.

TABLE 5. Sequence of shapes for lines C_{Bl}, areas C_{Bp} and their product, in sheet B



In sheet B, for lines, the shape C_{Bl} is modified to C_{Bl}' according to the relation: $C_{Bl}' = [C_{Bl} - t(l_{B1})] + t(l_{B2})$. The shape $t(l_{B1})$ is the empty shape. And the mapping, h_1: $C_{Bl} \rightarrow C_{Bl}^*$ from the parts of the shape C_{Bl} to some part C_{Bl}^* of the produced shape C_{Bl}', where $h(C_{Bl}) = C_{Bl}^*$, reveals that all the parts of C_{Bl} remain intact in C_{Bl}'. The mapping between the closure algebras for C_{Bl} and C_{Bl}^* is defined by the relation $\Gamma^*(x) = C^* \cdot \Gamma'(x)$, for every x of C^*. Since the shape $t(l_{B1})$ is the empty shape, the shape C_{Bl} retains its topology before and after the rule is applied. For areas, the shape C_{Bp} is modified to $C_{Bp}' = [C_{Bp} - t(p_{B1})] + t(p_{B2})$. Every part of C_{Bp} is associated with some part of the shape C_{Bp} - $t(p_{Bp})$, through the mapping h_1: $C_{Bp} \rightarrow C_{Bp}$ - $t(p_{B1})$, with the shape $t(l_{B1})$ equal to the empty shape. The mapping between the closure algebras for C_{Bp} and C_{Bp}^* is defined accordingly $\Gamma^*(x) = C^* \cdot \Gamma'(x)$, for every x of C^*, and the shape $t(p_{B1})$ C_{Bp} retains its topology.

TABLE 6. Topologies for lines and areas, in sheet B

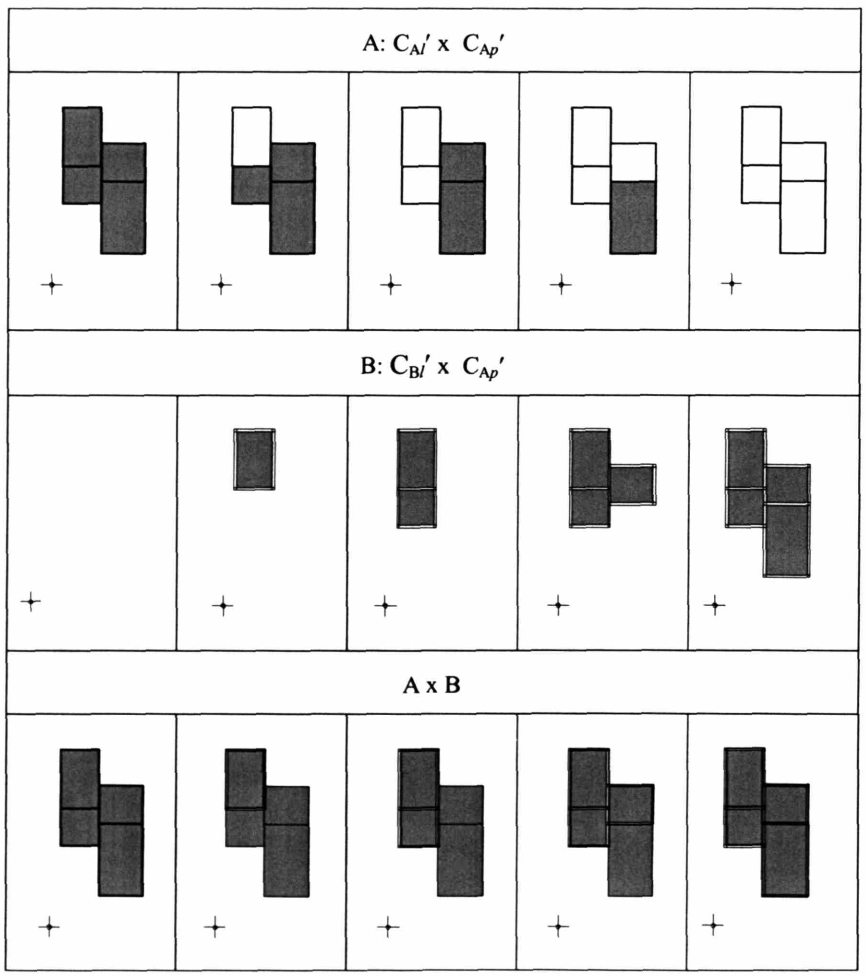
C_{Bl}'	topologies	C_{Bp}'	topologies
∅	∅	∅	∅
	∅ 		∅ 
	∅ 		∅ 
	∅ 		∅ 
	∅ 		∅ 

4.1.3. The description A X B

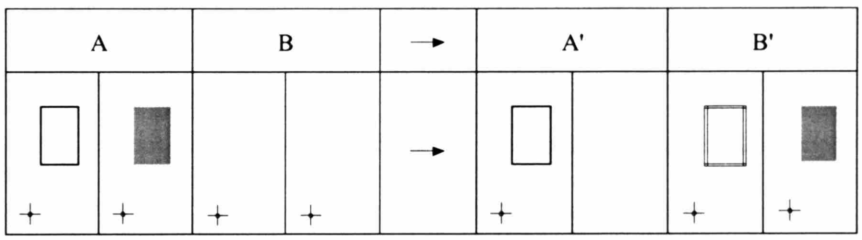
The developments in sheet B are not irrelevant to how things change in sheet A. To make the interrelationship between the sheets A and B visible one has to superimpose the two sheets. One may think that the rule acts on the shape depicted in A x B. This overall shape has a linear part $C_l = C_{Al}$ x C_{Bl} and an area $C_l = C_{Ap}$ x C_{Bp}. The full description Ax B is depicted in the product C = C_l x C_p.

First, an overview of the sequence of shapes produced in each sheet and the shapes resulting from their superimposition

TABLE 7. Sequence of shapes for lines and planes in sheet A, in sheet B, and their product A x B



The sheets A, B are related according to the provisions of the given rule. The left-hand side of the rule sets a single provision to let the rule apply. The provision refers to the content of the sheet A. There are no provisions for sheet B. The rule associates the sheets A and B in the following manner: each time the shapes $t(l_{Al})$ and $t(p_{Al})$ are scanned in A (left-hand side of the rule) some shapes are drawn in B (right-hand side of the rule).



The sequence of rectangles $t(l_{Al})$, and the evolution of the shapes C_{Bl} and C_{Al} are presented in the next Table 7. The sequence of areas $t(p_{Al})$, and the evolution of the shapes C_{Bp} and C_{Ap} are presented in Table 8.

For lines, the development of the wall layout C_{Bl}' in sheet B, can be seen in association to the sequence of shapes $t(l_{Al})$ selected in C_{Al}: the line arrangements C_{Bl} in B, evolve in reference to the rectangles $t(l_{Al})$ selected in sheet A. The relationship becomes visible if we superimpose the linear parts of the sheets A and B: $C_l = C_{Al} \times C_{Bl}$. Each rule application corresponds to a continuous mapping h: $\Sigma C_l \rightarrow \Sigma C_l{*}$ from the parts of C_l to some part of $C_l{*}$ of $C_l{}'$. First, in sheet A the identity on C_{Al} makes the mapping $h(x) = x$ to leave the shape intact. And, the same shape C_{Al} is mapped to some part of C_l. And second, in sheet B the rule on C_{Bl} makes the mapping $C_{Bl} \rightarrow C_{Bl} - t(l_{B1})$ continuous and maps C_{Bl} to some part of C_l'. The parts C_{Al} (from layer A) and C_{Bl} (from layer B) are disjoint in C_l (in the direct product A x B).

Similarly, for the areas depicted in C_{Bp}' in sheet B, the changes happen in association to the sequence of planes $t(p_{Al})$ that are erased from C_{Ap}. The consecutive changes for the overall shape of areas C_p, in both sheets A and B, becomes visible after the superimposition of A and B: $C_p = C_{Ap} \times C_{Bp}$. As it happens with lines, rule application corresponds to a mapping h: $\Sigma C_p \rightarrow \Sigma C_p{*}$ from the parts of C_p to some part of $C_p{*}$ of $C_p{}'$. First, in sheet A, the erasing rule on C_{Ap} makes the mapping $C_{Ap} \rightarrow C_{Ap} - t(p_{A1})$ continuous. And the shape C_{Ap} is mapped to some part of C_p. Second, in sheet B, the rule makes the mapping $C_{Bp} \rightarrow C_{Bp} - t(p_{B1})$ continuous and maps C_{Bp} to some part of C_p. The parts C_{Ap} (from layer A) and C_{Bp} (from layer B) are disjoint in C_p (in the direct product A x B).

TABLE 7. Lines as they evolve in B, in A, and AxB, each time $t(l_{A1})$ is picked in A

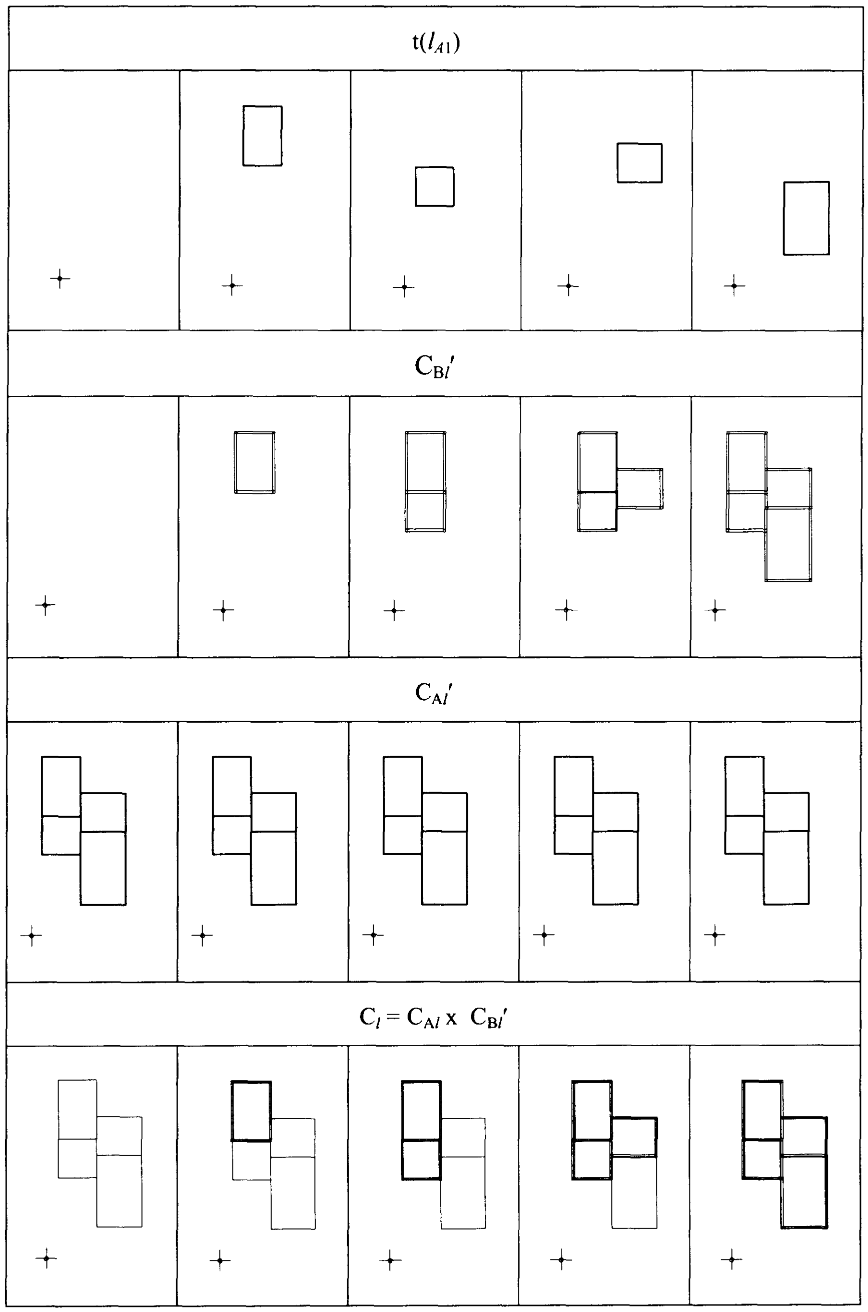
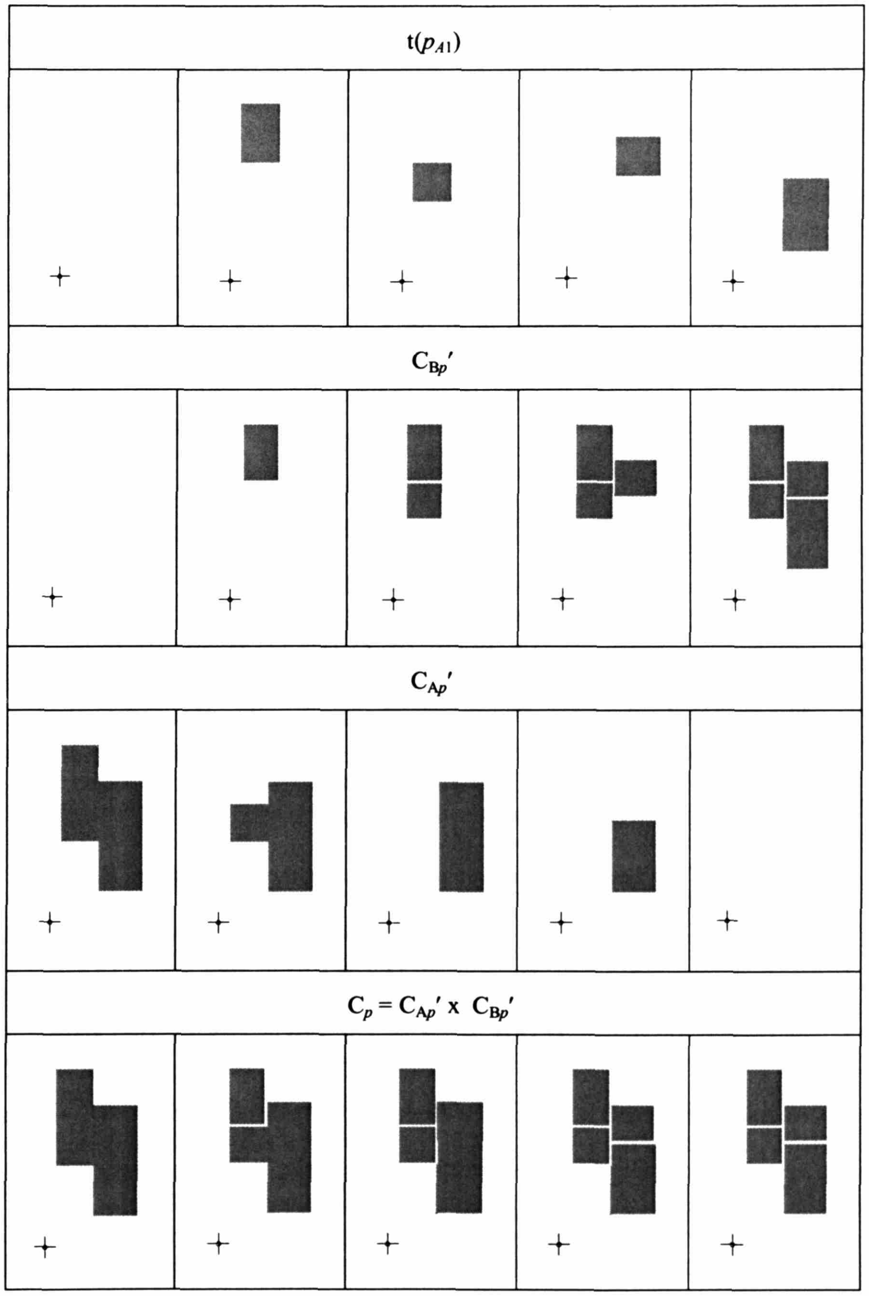


TABLE 8. Areas in B, in A, and in A x B, each time $t(p_{A1})$ is selected in A



4.2. FOR N ≥ 3

For $n \geq 3$, three or more partial descriptions are combined to produce one description. The way of interaction among shapes is not different from when $n = 2$. But for $n \geq 3$ a number $p = 2^n - (n+2)$ sums or products can be formed by combining the existing partial descriptions. The main difference is that a greater number of shapes can be modified in each rule application.

An application of $n = 4$ graphic layers in the production of plans is presented in detail in chapter VI. Some introductory information on the way four graphic layers are combined in the production of plans is provided here.

In chapter IV, four sheets A, B, C, D are superimposed to construct a complete plan description. At the end of the process, the presentation drawing is produced by the union A∪B∪C∪D on the n+1= 5th sheet.

The set of the possible partial descriptions emerging from a set S = {A, B, C, D}, is $2^4 = 16$. Except from the singletons {A}, {B}, {C}, {D} representing the initial sheets, the empty set, and the full description {A, B, C, D}, the set S can form $p = 2^4 - (4+2) = 10$ sums or products among the sheets. More precisely, six partial descriptions contain a pair of sheets apiece, and four partial descriptions contain a triple of sheets.

P(S) = {

(singetons)	{A}, {B}, {C}, {D}, ∅,
(pairs)	{A, B}, {A, C}, {A, D}, {B, C}, {B, D}, {C, D},
(triples)	{A, B, C}, {A, C, D}, {A, B, D}, {B, C, D}
(complete description)	{A, B, C, D} }

The required calculations happen in the product A x B x C x D algebra. The distribution of graphic elements in each of the four sheets A, B, C, D is the following. Sheets A, B, C contain lines and symbols in a product $< U_{12}$ $V_{02} >$, while the sheet D contains areas and symbols in $<U_{22}\ V_{02}>$ algebra.

TABLE 9. The distribution of graphic elements in the four sheets A, B, C, D

A	B	C	D
$U_{12}V_{02}$	$U_{12}V_{02}$	$U_{12}V_{02}$	$U_{22}\ V_{02}$

Accordingly A x B x C x D is a member of the direct product $<U_{12}V_{02}>$ x $<U_{12}\ V_{02}>$ x $<U_{12}\ V_{02}>$ x $<U_{22}\ V_{02}>$. In A, B, C, a shape is determined by a pair (l, s) with $l \in U_{12}$ and $s \in V_{02}$. In D a shape is expressed by a pair (p, s) where $p \in U_{22}$ and $s \in V_{02}$. A rule (A, B, C, D)→(A′, B′, C′, D′) has the form:

$$\{(l_{A1}, s_{A1})\ (l_{B1}, s_{B1})\ (l_{C1}, s_{C1})\ (p_{D1}, s_{D1})\} \rightarrow (l_{A2}, s_{A2})\ (l_{B2}, s_{B2})\ (l_{C2}, s_{C2})\ (p_{D2}, s_{D2})\}$$

A rule is a relation R from $<U_{12}V_{02}>$ x $<U_{12}\ V_{02}>$ x $<U_{12}\ V_{02}>$ x $<U_{22}\ V_{02}>$ to $<U_{12}V_{02}>$ x $<U_{12}\ V_{02}>$ x $<U_{12}\ V_{02}>$ x $<U_{22}\ V_{02}>$. A rule-instance:

$$(A_1, B_1, C_1, D_1) \rightarrow (A_2, B_2, C_2, D_2)$$

applies on a shape C to produce C′, in a step $C \Rightarrow C'$ involving the shapes,

$$((C_{A\,l}, C_{A\,s}), (C_{B\,l}, C_{B\,s}), (C_{C\,l}, C_{C\,s}), (C_{D\,p}, C_{D\,s})) \Rightarrow$$

$$\Rightarrow ((C_{A\,l}', C_{A\,s}'), (C_{B\,l}', C_{B\,s}'), (C_{C\,l}', C_{C\,s}'), (C_{D\,p}', C_{D\,s}'))$$

Any of the shapes that constitute C or C′ can be possibly empty. Rule instances can be defined so that one or more of the participating components are substituted by the empty shape, on both sides of the rule.

$$(A_{\varnothing}, B_1, C_{\varnothing}, D_1) \rightarrow (A_{\varnothing}, B_2, C_{\varnothing}, D_2)$$

This allows shapes in sheets A and C to remain unaffected, while the shapes in B, D may or may not be affected. The areas C_{Dp} in D form a Boolean algebra since an overall area for the design (least upper bound) can be defined. All the other calculations with shapes C_{Al}, C_{Bl}, C_{Cl} in sheets A, B, C happen as usual without unit. A rule instance applies as follows:

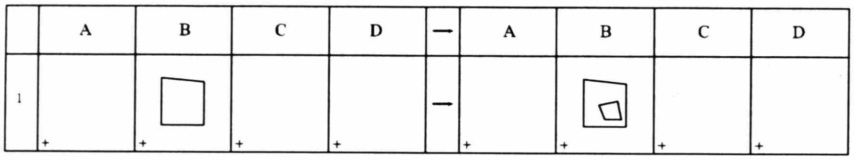
$C_{A\,l}' = [\,C_{A\,l} - t(l_{A1})\,] + t(l_{A2})$	$\wedge$	$C_{A\,s}' = [\,C_{A\,s} - t(s_{A1})\,] + t(s_{A2})$
$C_{B\,l}' = [\,C_{B\,l} - t(l_{B1})\,] + t(l_{B2})$	$\wedge$	$C_{B\,s}' = [\,C_{B\,s} - t(s_{B1})\,] + t(s_{B2})$
$C_{C\,l}' = [\,C_{C\,l} - t(l_{C1})\,] + t(l_{C2})$	$\wedge$	$C_{C\,s}' = [\,C_{C\,s} - t(s_{C1})\,] + t(s_{C2})$
$C_{D\,p}' = [\,C_{D\,p} - t(p_{D1})\,] + t(p_{D2})$	$\wedge$	$C_{D\,s}' = [\,C_{D\,s} - t(s_{D1})\,] + t(s_{D2})$

We write $C \Rightarrow C'$ if and only if there is a transformation t of (l_{A1}, s_{A1}), (l_{B1}, s_{B1}), (l_{C1}, s_{C1}), (p_{D1}, s_{D1}) such that
$t(l_{A1}) \leq C_{Al}$ and $t(s_{A1}) \leq C_{A\,s}$
$t(l_{B1}) \leq C_{B\,l}$ and $t(s_{B1}) \leq C_{B\,s}$
$t(l_{C1}) \leq C_{C\,l}$ and $t(s_{C1}) \leq C_{C\,s}$
$t(p_{D1}) \leq C_{D\,p}$ and $t(s_{D1}) \leq C_{D\,s}$

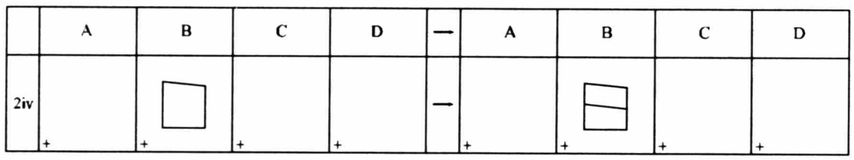
The set of shapes generated by a rule is $\{C' \in <U_{12}V_{02}> \text{ x } <U_{12}\ V_{02}> \text{ x } <U_{12}\ V_{02}> \text{ x } <U_{22}\ V_{02}> : C \Rightarrow C'\}$.

The topologies for C and C′ provide the closure operations for the two shapes Γ and Γ'. Each rule application corresponds to a mapping h: $C \rightarrow C^*$ from the parts of C to some part of C^* of C′ where $h(C) = C^*$. The mapping between the closure algebras for C and C^* is defined by the relation $\Gamma^*(x) = C^* \cdot \Gamma'(x)$, for every x of C^*.

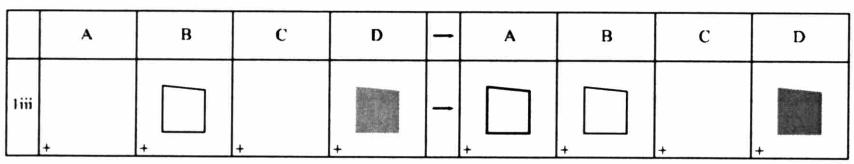
The next rule schemata are examples taken from the chapter VI. Rule schema 1 draws a quadrilateral in a quadrilateral, in sheet B.



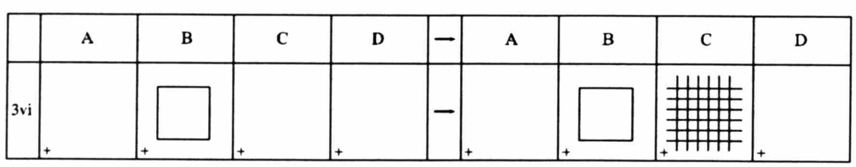
Rule schema 2iv dissects a quadrilateral by drawing a line, in sheet B.



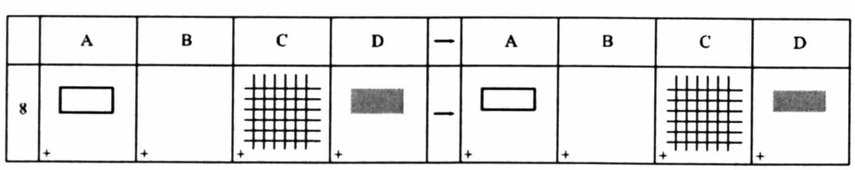
Rule schema 1iii transfers quadrilateral from sheet B to sheet A, and changes its color in sheet D.



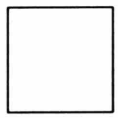
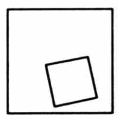
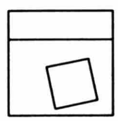
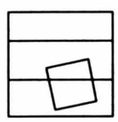
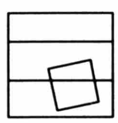
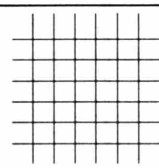
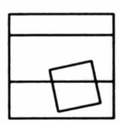
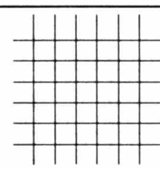
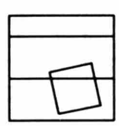
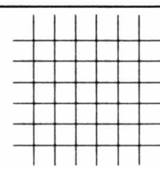
Rule 3vi draws a 5 x 5 grid on sheet C, while taken into account an existing quadrilateral in sheet B

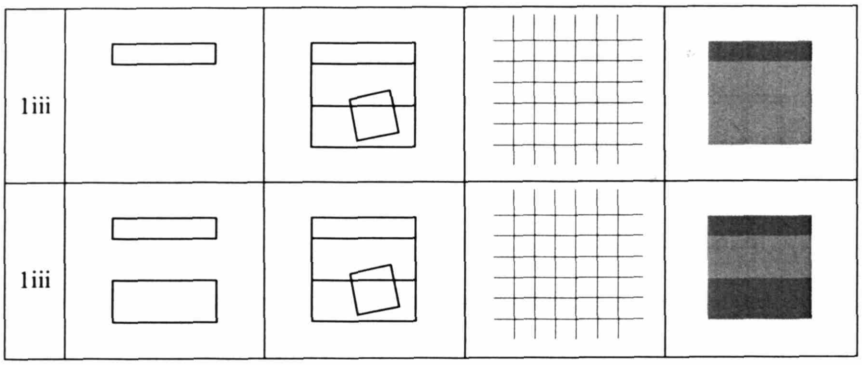


Rule 8 the fifth rule schema aligns a side of a quadrilateral in sheets A and D, to meet an existing grid line in sheet C.



The initial shape is the overall area in sheet D, and its boundary in B. A derivation is presented next,

	A	B	C	D
				
1				
2iv				
2iv				
3vi				
8				
8				

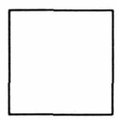
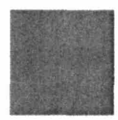
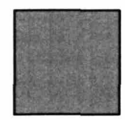
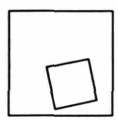
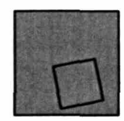
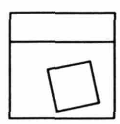
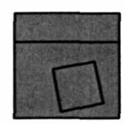
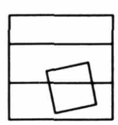
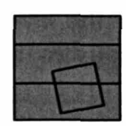
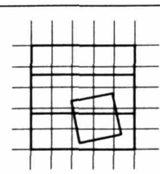
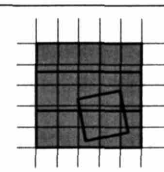
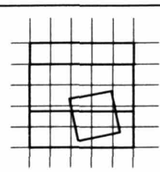
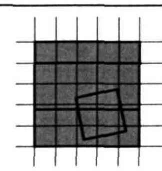


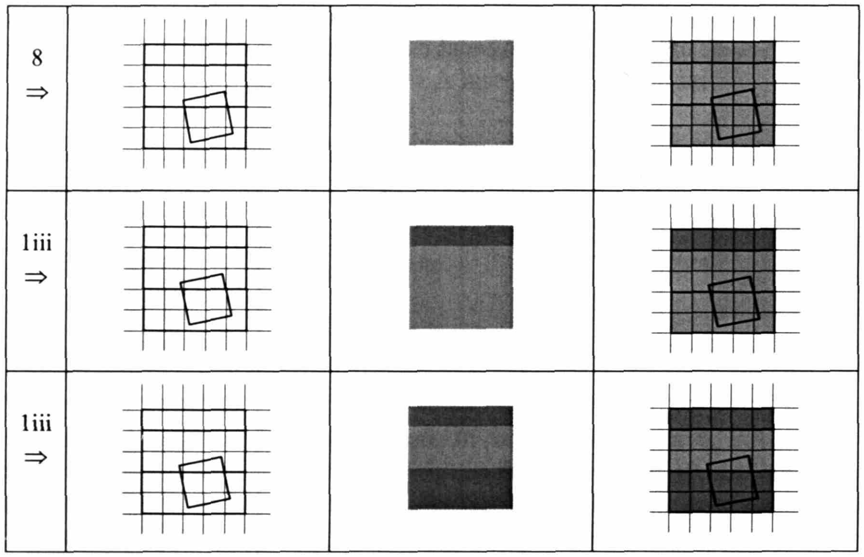
The modification of spatial elements in each of the sheets A, B, C, D, at each step of the derivation, is presented in the next Table 10. The sum of any areas a, b, c in layer D, is always equal to unit shape, denoted with 1. The sum of lines in layers A, B, C is changing in ways that are defined by the rules applied at each step.

TABLE 10. Evolution of the maximal elements of shapes in the A, B, C, D

R	A	B	C	D
	∅	4 lines	∅	1
1	∅	8 lines	∅	1
2iv	∅	9 lines	∅	1
2iv	∅	10 lines	∅	1
3vi	∅	10 lines	12 lines	1
8	∅	10 lines	12 lines	1
8	∅	10 lines	12 lines	1
1iii	4 lines	10 lines	12 lines	$a + b = 1$
1iii	8 lines	10 lines	12 lines	$a + b + c = 1$

In the next Table 11, a product description A x B x C for lines is presented for the same steps. Also at each step of the derivation, is provided the layer D and the full description A x B x C x D.

R	A x B x C	D	A x B x C x D
			
1 ⇒			
2iv ⇒			
2iv ⇒			
3vi ⇒			
8 ⇒			



5. Discussion

The application of rule schemata on multiple layers as a thinking-graphic device with great potential is introduced in this study. Multiple layers allow the execution of multiple tasks.

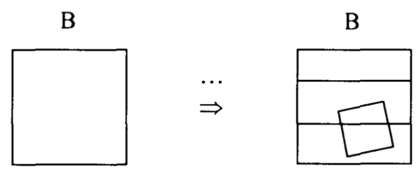
For the n = 2 layers of the presented example each rule application accomplishes the following objectives.

A		B		→	A'		B'	
search	count			→	record		draw walls	draw area

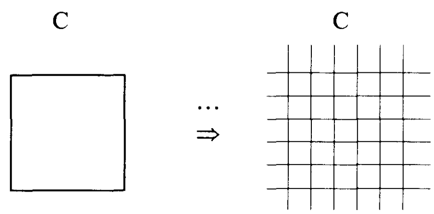
And, for the n = 4 each rule application accomplishes the next objectives.

A	B	C	D	→	A'	B'	C'	D'
search lines	search lines	search lines	search areas	→	transfer lines	draw lines	align	count areas

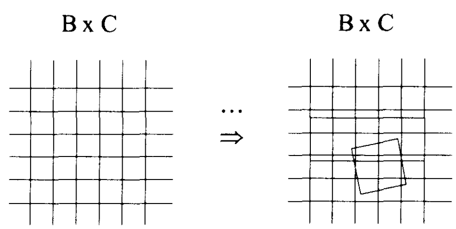
The produced shapes can develop associations. A creative process consists of searching for elements of a specific kind, while having a specific objective in mind. This process can develop as a logic process, in sequence. The interaction of the results of each independent process creates affinities and develops dialectically, in juxtaposition. A task that involves a specific way of thinking in layer B,



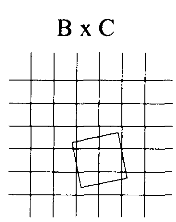
can be seen in juxtaposition with another task in layer C.



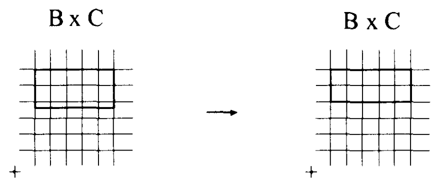
Together they provide a new result: the description B x C.



The actions in B and C may express thoughts short in length, unrelated, or trivial. But their interaction reveals new possibilities. For example, the product B x C shows that none of the partitions coincides with the structural grid. A possible reaction is to translate the partitions to meet the grid.



The previous action suggests a new rule schema: the walls of a room are aligned to an underlying grid structure.



The new rule schema can be used in the design process. Rule schemata derive the general consequences of a hypothesis and organize descriptions. The above rule schema organizes the description with respect to a structural grid, for example. Knowledge represented in a stack is gradually composed into a new whole, according to some general concept. Superimposition becomes critical in the development of possible affiliations. The new object emerges like a fabric made out of threads deriving from the different layers.

The second part of the thesis, presents applications of the concepts that were outlined in the chapters I, II, III and IV. The chapters V and VI present two projects that serve as paradigms.

Chapter V is based on a competition for low cost housing. The design process starts from the definition of a vocabulary of rooms, and a number of spatial relations, which describe their adjacencies. The possibilities of constructing designs from these are examined systematically. The search evolves from the definition of the "parts" (spatial vocabulary) to the construction of possible "wholes" (designs).

Chapter VI presents the production of plans for an office building. Starting from a specific site and building program, a design concept is gradually developed into a design with the aid of rule schemata and rules. The construction proceeds from a potential "whole" (design concept) to the definition of the "parts" (rooms and spaces).

6. References

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V. A STUDIO EXERCISE IN RULE BASED COMPOSITION

Abstract. A studio teaching method aiming to incorporate rule-based systems in design synthesis, using both analogue and digital means, is introduced. An example studio exercise based on a housing competition, is described.

1. Introduction

One of the challenges in using a rule-based system is determining its appropriateness and applicability in the synthesis of form. In this study I develop some elementary rule-based concepts to explain and support the activity of the studio. How can rules be employed to perform goal-driven design tasks? And how can analogue and digital means coexist as part of the studio teaching activity? In order to examine these questions a studio exercise was developed on the basis of a design competition for low cost housing. The exercise aims to become a starting point for the introduction of rule-based methods, in synthesis.

The general strategy, given the building program, is to construct a method of producing a variety of 2-dimentional plan arrangements in response to a variety of functional demands and conditions. The objective is to gradually establish spatial elements, relationships, and rules for the generation of designs.

First, candidate sets of spatial elements and rules are formulated with by hand (analogue), as a hypothesis. Then, they are tested using a digital parametric shape grammar interpreter. The interpreter requires conversion of the rules into LISP scripting format and provides computer aid in clarifying the ramifications of a hypothesis. Using the interpreter the designer determines if a rule-set had any desired outcomes. If not, the rule-set is modified and re-tested. The decision process involves a selection among alternative rule-sets, where the designer explores possible results. The digital interpreter offers fast broad exploration of the products of the rules. Further, the digital 2-d representations generated by the rules can readily be used in solid modeling allowing alternative representations to participate in the evaluation. The heuristics of the process are organized in three interdependent levels of abstraction. The first is dedicated to the *formation* of *partis*, the second to the *transformation* of a *parti* and the generation of layouts, and the third to the description of the *tectonic* details. Each level begins by forming candidate rule-sets and has an analogue and a digital part. Through an iterative process of *formation*, *transformation* and *refinement* the rules are evaluated and redefined according to their compliance to programmatic, intuitive, and construction criteria.

2. The Studio Exercise

The examination of housing projects is standard part of the architecture curriculum. Therefore, a studio exercise on housing is ideal for the introduction of rule-based compositional techniques to architecture students. The exercise is based on a housing competition sponsored by the *Habitat For Humanity* (HFH) in the summer of 2002 in Boston, Massachusetts. The HFH describes the goal of the competition as: "*the building of simple, decent, affordable houses*". The program calls for adaptable types of 2, 3 and 4-bedroom houses without determining the square-footage of rooms or house types. All houses include: primary covered entrance, circulation, dining area, living area, at least one full bathroom, kitchen, and bedrooms.

Figure 1. Examples of HFH housing in East Boston, Dorchester, and Roxbury

A minimum living space limit for all house types is suggested: 900 s. f. for 2-bedroom apartments, 1050 s. f. for 3-bedroom apartments, and 1150 s. f. for 4-bedroom apartments. Further, the organizers do not designate specific sites, but offer several possible ones. Small, quadrilateral lots less than 5000 s. f. are an option, but lots larger than 20000 s. f. with complex shapes are also typical.

The students identify sets of rooms and spaces, and relate them through spatial relations. Then, shape rules are used to generate *partis* for possible designs. From each selected *parti* the students develop several plan layouts and 3d mockups. The last part of the exercise is dedicated to the refinement of selected layouts and the addition of certain tectonic details (wall-thicknesses, windows, doors, stairs, etc.)

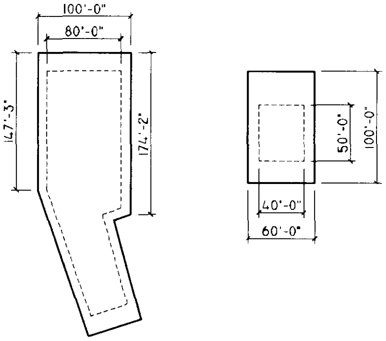


Figure 2. Two examples of typical HFH sites

3. Design Concept and Method

The design approach is influenced by three factors: a) the absence of a predetermined site, b) the specifications of the building program, and c) the provision for low construction-cost. The design concept follows the general principles of the *domino house* concept: Starting from an initial number of spatial entities (rooms), selected by the designer, the objective is to develop rules to generate house arrangements of variable size and morphology. The systematization of the plan is the method used for the attainment of this objective. In order, to achieve certain room adjacencies some spatial relationships and general rule schemata must be proposed.

The process evolves from the definition of the "parts" (spatial vocabulary) to the construction of "wholes" (designs). The presentation of this process can be characterized introspective and prescriptive: Introspective because each potential designer could possibly develop different design alternatives; Prescriptive because each designer proposes a prescriptive system of rules that provide a norm for empirical exploration.

The computational framework defined in Stiny and Gips 1972, and in Stiny 1976; 1980; 1991 is employed. Shapes that belong in some algebra U_{ij} are composed with the aid of rule schemata, and rules. Design descriptions are produced by rules and spatial entities (rooms, spaces, etc). Similar models, referring to the construction of rule systems for 0-dimentional languages exist in Carnap 1937, and Chomsky 1957.

In architecture, in the generation of plans for minimum cost dwellings, Mitchell (1974) reduces the plans to a minimal representation. The representation involves a rectangular frame, and a minimum rectangular grating. This description becomes the *parti*, on the basis of which, several variations for dwellings are generated. The plans are generated with the aid of dimensioning vectors and adjacency relations, while optimization procedures are used in the enumeration and selection of alternatives.

The possibility of establishing more visual methods in the analysis and synthesis of 2d architectural descriptions was first discussed in Stiny and Mitchell 1978 in the production of Palladian villa plans. The grammar of Stiny and Mitchell captures the generation of Palladian villa plans in eight stages. Numerous papers have followed describing the generation of Frank Lloyd Wright's prairie houses (Koning and Eizenberg 1981), Japanese tea-room designs (Knight 1981), Queen Ann houses (Flemming 1987), Taiwanese houses (Chiou and Krishnamurti 1995), Yingzao fashi houses (Li 2000), and Alvaro Siza's houses (Duarte 2001).

The novelty of the proposed approach is that it captures the exploratory effort of an intuitive design process without dealing with analysis of existing designs. It involves testing and selection of rules, as the designer explores their possible outcomes using analogue and digital means. First, the spatial entities and rules are described by hand, and then, are tested digitally.

The heuristics of the process are organized in three levels. Each level contains rules that achieve some objective. At the top level of *formation*, rules produce *parti* diagrams. At the middle level, of *transformation*, a chosen *parti* is transformed to a boundary-layout. The transformations may or may not convert the input *parti* into a new derived one. At the level of *refinement*, the rules apply on the boundary-layout to determine its *tectonic* details (doors, windows etc). The *transformation* and *refinement* levels require some input shape from the preceding levels in order to apply, while the *formation* proceeds from the empty shape. The process is not linear, as the output of each level may influence both the preceding and the subsequent levels. The framework is sketched out as follows:

$$\Sigma : \{ \text{ finite set of spatial elements (rooms...) } \}$$

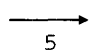
$$\begin{array}{lccccccl}
R: \{ & \textit{Formation} & & \textit{Transformation} & & \textit{Refinement} & & \\
 & A_1 \rightarrow F_1 & \Leftrightarrow & G_1 \rightarrow M_1 & \Leftrightarrow & N_1 \rightarrow W_1 & & \\
 & \vdots & & \vdots & & \vdots & & \\
 & A_n \rightarrow F_n & & G_k \rightarrow M_k & & N_r \rightarrow W_r & & \}
\end{array}$$

where $A_1,\ldots, A_n$, $W_r,\ldots, W_r$ are elements in Σ.

The three levels make use of analogue and digital means. The shape grammar interpreter is used only on the first two levels of *formation* and *transformation*. The analogue part of the process includes the articulation of candidate rules, while the digital part the rule-testing. Through an iterative process of *formation*, *transformation* and *refinement* similar to the *see-move-see* concept (Schon and Higgins 1992) the rules are evaluated and redefined according to their compliance to programmatic, intuitive, and construction criteria. Finally, the rules are grouped in *grammars*.

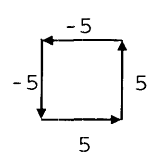
3.1 THE DIGITAL INTERPRETER

A shape grammar interpreter (Liew 2003) is used for the digital part of the exercise. The interpreter, written in VisualLISP, uses a scripting language based on LISP to describe a rule. Each rule has four parts: left-hand schema, right-hand schema, transformation mapping, and variable mapping. A vector description format (Nagakura 1995) is used to describe the geometry and variables of a schema. The transformation mapping determines any transformation changes between the left-hand schema and the right-hand schema. The variable mappings define a relationship between the parameters of both schemata. A schema is composed of two parts, the geometry and the constraints on the geometry variables. The geometry of a schema is described using a series of vector displacements. Each vector has 3 components: action, vector and label. The action component determines if the shape is a line or a point. The vector component describes the x and y displacement of the shape. The label component determines the name of the label. For example, a horizontal *parti* line that is 5 units long is described as:




```
((action "line") (vector 5 0) (label "parti"))
```

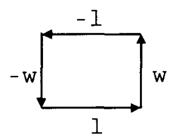
A shape is described as a series of vector displacements that are connected from end to end. For example, the following describes a "*parti*" square that is 5 units by 5 units in size.




```
(((action "line") (vector 5 0) (label "parti"))
 ((action "line") (vector 0 5) (label "parti"))
 ((action "line") (vector -5 0) (label "parti"))
 ((action "line") (vector 0 -5) (label "parti")))
```

To describe a parametric shape, the numbers in the vector displacement description are substituted with variables.

The following describes a schema that finds all *parti* rectangles.

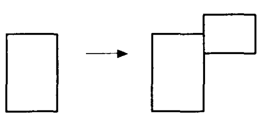



```
(((action "line") (vector l 0) (label "parti"))
 ((action "line") (vector 0 w) (label "parti"))
 ((action "line") (vector (- l) 0) (label "parti"))
 ((action "line") (vector 0 (- w)) (label "parti")))
```

Restrictions can be set on the geometry variables to limit the type of sub-shapes found. Theses restrictions are added in the binding-constraints component of the schema. The following example restricts the size of the square to be less than 10 units.

```
((binding-constraints
  (l (< l 10))
  (w (< w 10)))
```

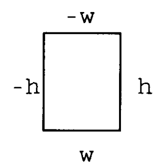
To apply a rule of the form x → x + y, like the ones that are used in the studio exercise,



the interpreter recursively searches the input shape for all instances of the left-hand schema and presents the possibilities through an interactive menu that highlights the embedded schemata. Once the user selects an embedded schema, the rule application is completed by subtracting the selected schema from the input shape and adding the right-hand schema of the rule.

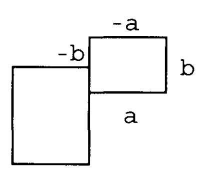
The above additive rule of the form x → x + y, applying on *parti* rectangles x and y, can be expressed in the symbolic meta-language. The left-hand schema describes, in symbols, the left-hand shape of the rule. The right-hand schema describes the right-hand shape of the rule. There is a part where the transformation and the parameter relationships between left and right-hand shapes are set, and finally, a fourth part where all the previous three parts are linked. In the illustrations of the example, for simplicity, the vector-arrows are omitted from the shapes.

First, the left-hand schema of the rule,




```
(setq schema-left-rule
  '((geometry
   ((action "line") (vector w 0) (label "parti"))
    ((action "line") (vector 0 h) (label "parti"))
    ((action "line") (vector (- w) 0) (label "parti"))
    ((action "line") (vector 0 (- h)) (label "parti"))
  )
  (parameter-constraints
   (w (> w 0))
   (h (> h w))
  )
 )
)
```

Second, the right-hand schema of the rule,



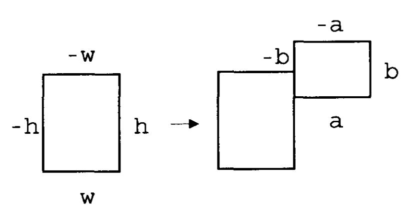

```
(setq schema-right-rule
      '((geometry
    ((action "line") (vector w 0) (label "parti"))
    ((action "line") (vector 0 h) (label "parti"))
    ((action "line") (vector (- w) 0) (label "parti"))
    ((action "line") (vector 0 (- h)) (label "parti"))
    ((action "move") (vector w (- h (* 0.375 w))))
    ((action "line") (vector a 0) (label "parti"))
    ((action "line") (vector 0 b) (label "parti"))
    ((action "line") (vector (- a) 0) (label "parti"))
    ((action "line") (vector 0 (- b)) (label "parti"))
   )
```

```
    (parameter-constraints
     (w (> w 0))
     (h (> h w))
     (a (> a 0))
     (b (> b 0))
    )
   )
 )
```

Defining the transformation and parameter mapping,

```
(setq tmap-rule
    '((delta-xo . 0)
      (delta-yo . 0)
      (delta-ro . 0)
       (delta-za . 0)
   )
)
 (setq pmap-rule
  '((w w)
    (h h)
    (a w)
    (b (* 0.75 w))
   )
)
```

And, the connection of the left and the right-hand schemata of the rule,



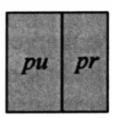

```
 (setq housing-rule
  '((left . schema-left-rule)
    (right . schema-right-rule)
    (tmap . tmap-rule)
    (pmap . pmap-rule)
    (success . nil)
    (failure . nil)
    (applymode . "single")
    (rulename . "housing-rule")
   )
)
```

4. Spatial Elements

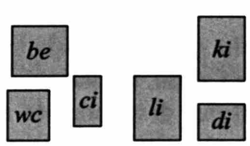
In the development of 2d descriptions, planes and lines are used to represent the area of the spatial entities, in the algebra $<U_{22}>$, and their outline, in the algebra $<U_{12}>$. Sometimes, symbols, such as letters and numbers in the algebra $<V_{02}>$ are used to assign names and values.

First, let a labeled, bounded area (*h*) emulate the schematic plan (*parti*) of a house unit. This can be represented by a labeled plane and its linear boundaries in the product algebra $<V_{02} \times U_{12} \times U_{22}>$

The parametric shape (*h*) remains abstract. The total area, the form of the boundaries, and the parts of (*h*) are not depicted in this representation. It is still possible to imagine that (*h*) includes two general functional zones. The first is represented by a parametric shape, labeled "public" (*pu*) and the second by a parametric shape labeled "private" (*pr*). The distinction "public-private" does not designate any permanent or fundamental distinction applicable to houses, in general. It is a functional distinction that was found useful in the context of solving the present design problem.

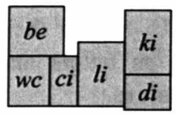


Second, the house (*h*) can be seen as an aggregate of parts. These are indicated in the building program: circulation "*ci*", dining area "*di*", living area "*li*", one full bathroom "*ba*", kitchen "*ki*" and bedroom(s) "*be*". The six parts can be represented by parametric shapes, in the product algebra $<V_{02} \times U_{12} \times U_{22}>$. The choice of rectangles, the most conventional room geometry, can be easily debated. It is one among the many possible.

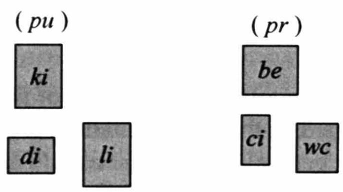


Abstractly, all the arbitrary arrangements that can be formed as aggregates of the above six labeled shapes may serve as *partis* for house designs.

For example,

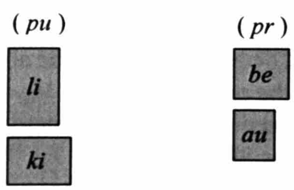


The distinction "public – private" permits the classification of the six shapes. For example, the living "*li*", dining "*di*" and kitchen "*ki*" can be labeled public (*pu*), while the bedroom "*be*", the circulation "*ci*" and the bathroom "*wc*" can be labeled private (*pr*).



This classification follows previous design experience. It is one among several alternatives. For example, one can label the circulation "public", the kitchen "private" etc.

Further, it is not necessary for the six parametric shapes to correspond to six distinct rooms. It is possible to merge any two, or more of them within the same category. For example, dining "*di*" and living "*li*" can be merged into a single shape, labeled "*li*". Similarly, circulation "*ci*" and bathroom "*wc*" are merged into one shape, labeled "*au*", for auxiliary. The parametric shapes are therefore reduced to four

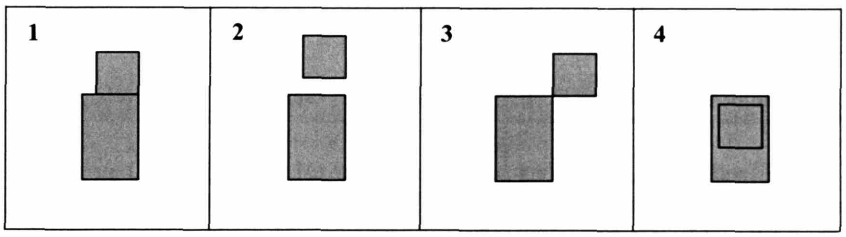


To summarize: First, there is no clear picture for the design as a whole. Second, there were sketched out two possible classes of areas, the members of which need to be related to each other. To convert this description to a *parti*, a process of rule application must be employed, according to a set of relationships that the areas satisfy. The conditions that the relationships must fulfill cannot be deduced from any prefixed set of premises. In the sections that follow it is shown how the set of relationships, and the rules, are gradually invented on the basis of further hypotheses, and tests.

5. Spatial Relations and Shape Rules

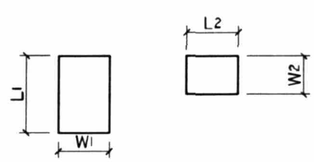
In general, two parametric planes and their boundaries form four general spatial relations: (1) they can be placed next to each other to share one common boundary (share a line, in the algebra U_{12}), (2) they can be placed so that they do not touch (discreet, share nothing), (3) they can meet in a corner (share a point, in the algebra U_{02}), or, (4) they can be placed one inside the other or they can overlap (share an area, in the algebra U_{22}).

TABLE 1. Generic spatial relations

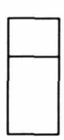


One can imagine that all four spatial relationships between two bounded areas represent rooms. The relation **1**, of Table 1 depicts two adjacent areas having one common boundary. The specific spatial relation will be used extensively in this case study.

From this point, only the boundaries of the parametric shapes are used, to show how the shapes relate to each other. The algebra $<U_{12}>$, which includes lines manipulated on the plane, is adequate for the task. For each parametric rectangle made out of lines two parameters L_j and W_j are defined: L_j represents the length and W_j the width. It is $L_j \geq W_j$.



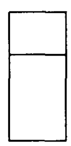
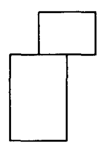
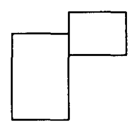
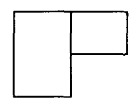
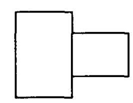
For $L_2 = W_1$ the relation **1** of Table 1 forms the shape,



relation A

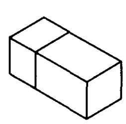
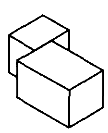
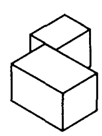
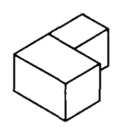
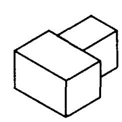
From the relation A, four spatial relations, between the same pair of rectangles, in which the participating rectangles retain some common boundary, are distinguished in the algebra $<U_{12}>$. The five relations are presented in the following table

TABLE 2. The five spatial relations A, B, C, D, E

				
relation A	relation B	relation C	relation D	relation E

The relation A produces an I-shape, the relations B, C, D produce L-shape and the relation E a T-shape arrangement. All five relations A, B, C, D and E can be visualized as relations among solids with equal heights *H*.

TABLE 2b. Axonometric representation of the five spatial relations A, B, C, D, E

				
relation A	relation B	relation C	relation D	relation E

The five spatial relations among pairs of rectangles (or solids) that retain a common line (or plane) can be expressed by two general parametric relations, and the instantiation of the constraints D_W and D_L.

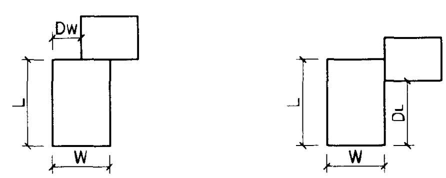
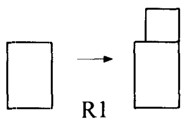


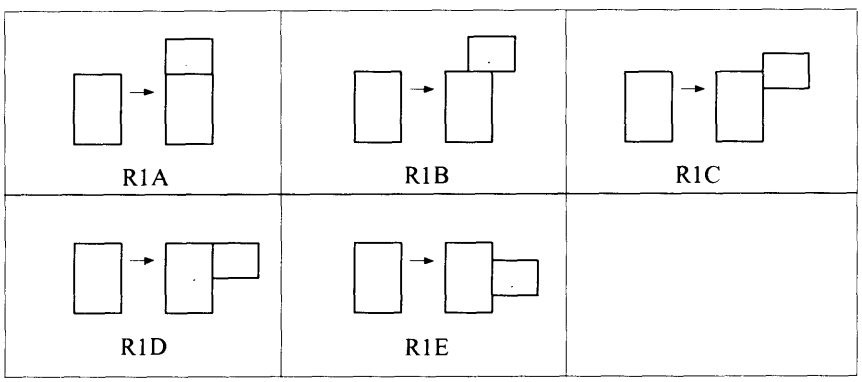
Figure 3. Parametric relation AB (left), and parametric relation CDE (right)

If $D_W < W$, then the parametric relation AB and, the spatial relations A and B are formed. If $D_L < L$ then the parametric relation CDE the relations C, D and E are formed. For $D_W = 0$, the spatial relation A is formed, and for $D_L = 0$, the spatial relation D is formed. The five spatial relations are generated by a parametric rule schema of the form $x \rightarrow x+y$



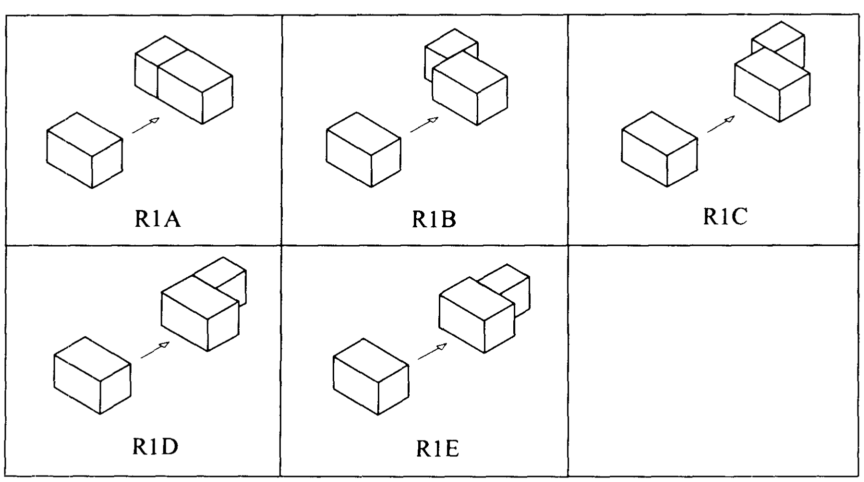
The five spatial relations of Table 2 are produced by five instances of R1

TABLE 3. Five instances of the rule schema R1: R1A, R1B, R1C, R1D, R1E



In parallel we can introduce rules for solids

TABLE 3b. The instances of the rule schema R1 in axonometric



The five parametric rule schemata R1A, R1B, R1C, R1D and R1E can be expressed abstractly by two parametric rule schemata. First, the rule schema R1AB indicates that for $D_w < W_1$, a rectangle (or rectangular solid) is added on the short side of an initial shape.

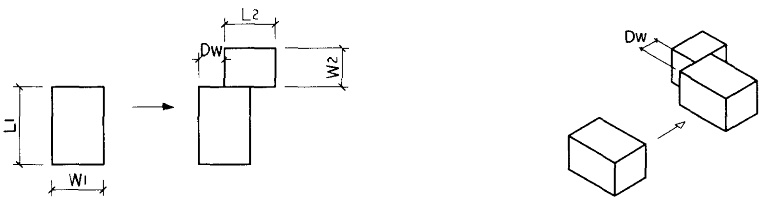


Figure 4. Parametric rule schema R1AB

Second, the parametric rule schema R1CDE indicates that for $D_L < L_1$ a rectangle (or rectangular solid) is added on the long side of an initial shape.

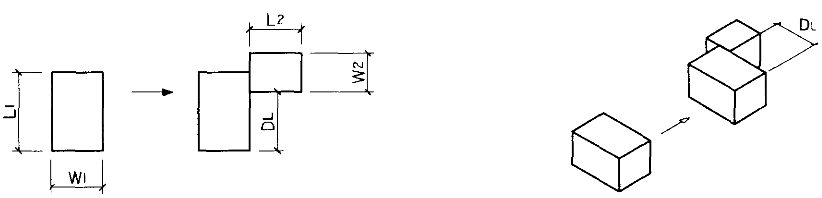


Figure 5. Parametric rule schema R1CDE

To recapitulate: a rule schema of the general form $x \rightarrow x + y$, adds a rectangle to a given rectangle made out of lines, so that the two rectangles share a common line. The position of the added shape is determined by instantiation of two parameters D_L and D_W. The lengths L_j and widths W_j are determined so that the length L_1 is always greater than the width W_1, ($L_1 \geq W_1$), and, the length L_2 is always equal to the width W_1, ($L_2 = W_1$). Parallel rule schemata of the same form for 3d solids can be expressed by instantiating a parameter H for height, with $H \leq W_2$

6. Derivation

In this section I examine how can we specify and "put together" rules that generate descriptions for designs. In the *formation* stage labeled rectangles made out of lines and labels in the product algebra $< V_{02} \times U_{12} >$, and labeled solids in the product algebra $< U_{03} \times U_{23} >$ are used in 2d and 3d descriptions, respectively. In the *transformation* and *refinement* stages labels are omitted. Descriptions in 2d use lines manipulated on the plane in the algebra $< U_{12} >$. Representations in 3d use planes manipulated in space, in the algebra $< U_{23} >$. The planes represent the boundaries of the solids.

6.1. FORMATION

The design concept that is used in this exercise suggests that: *Starting from an initial number of rooms, one can create devices that use rules to compose house arrangements of variable size and morphology.* The absence of a predetermined site is a significant factor in choosing this approach. The design concept establishes a specific interrelationship among the rooms: In order, to achieve certain room adjacencies spatial relationships, rule schemata and room-proportions must be selected accordingly.

The building program provides information for the required areas of each house. Each designer could possibly develop different design alternatives. For example, the choice of rectangular forms for the rooms is a design decision. Further, each designer proposes a prescriptive system of rules that provide a norm for exploration. One has to define the possible rooms (parts), spatial relationships, and rule schemata, and then test them by constructing possible house designs (wholes).

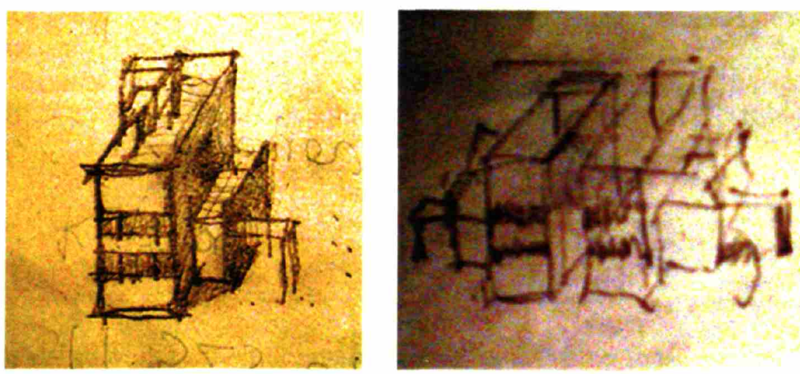


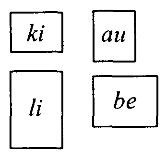
Figure 6. Examples of early sketches, depicting a possible house unit (left), and use of symmetry transformations for its placement in a given lot (right).

Each potential set of rooms and rule schemata becomes a hypothesis. The designer determines if a particular set of choices has any desired outcomes. If not, the set is modified and re-tested. The decision process involves a selection among alternative sets of choices, where the designer explores possible results.

The general consequences of each design hypothesis are initially sketched out by parametric rule schemata. Their further specification becomes a factor of great importance in the development of designs. A great part of the design activity consists of formulating and testing rule instances and transformations on the basis of some set of initial rule schemata.

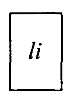
At the more abstract level the rule schemata produce *partis*. Each *parti* serves as a higher-level representation for possibly infinite designs. Given a finite set of labeled rectangles Σ,

Σ:

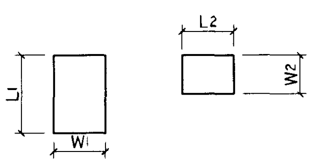


an initial shape I∈Σ is also designated. In the working example the shape representing the "living" room initiates the process

I:



Further, in every derivation all shapes (rooms) must be used at least once. The distinction "public-private" allows additional provisions to be applied. For example, the public rooms can be wider than the private rooms ($W_{pr} \leq W_{pu}$), and rooms labeled "public" can be used only once, while rooms labeled "private" may be used more than once, as needed



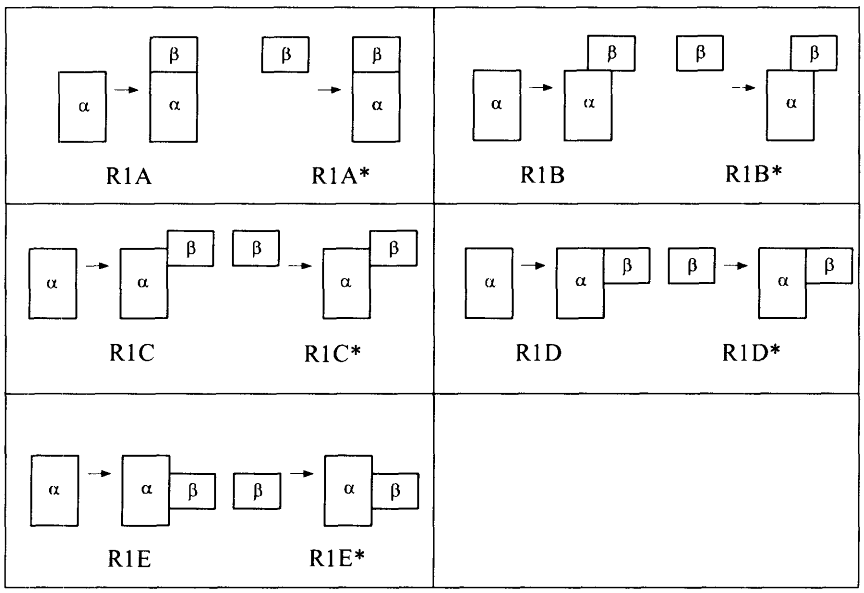
The full list of general constraints for rooms forms the following table

TABLE 4. List of general provisions

(i)	the length L_j of each room is greater than, or equal to, the width W_j, ($L_j \geq W_j$). Let $L_j = \tau \cdot W_j$, and $\tau \leq 1$
(ii)	the length L_2 is always equal to the width W_1, ($L_2 = W_1$)
(iii)	private rooms are not wider than public rooms ($W_{pr} \leq W_{pu}$), Let $W_{au} = k \cdot W_{pu}$, with $k \leq 1$
(iv)	public rooms must be used once
(v)	private rooms can be used more than once, as needed
(vi)	for 3d representations, height $H \leq W_2$

The parametric rule schemata of Table 3 can be augmented with labels in the product algebra $<V_{02} \times U_{12}>$. The next rule schemata (Table 5) are instances of the two parametric rule schemata R1AB and R1CDE (p. 120). The symbols α and β stand as variables for labels. For every rule schema $x \rightarrow x + y$ the rule schema (*) forms $y \rightarrow y + x$.

TABLE 5. The set R of labeled parametric rule schemata



A labeled version of R1AB: for $D_W = 0$ forms R1A or 1RA*
» : for $D_W = W_1 / 2$ forms R1B or 1RB*
A labeled version of R1CDE: for $D_L = L_1 - W_2 / 2$ forms R1C or 1RC*
» : for $D_L = L_1 - W_2$ forms R1D or 1RD*
» : for $D_L = (L_1 - W_2) / 2$ forms R1E or 1RE*

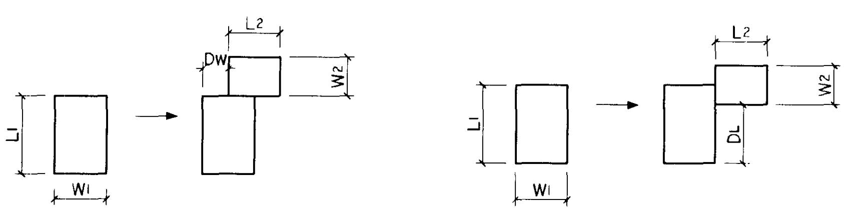
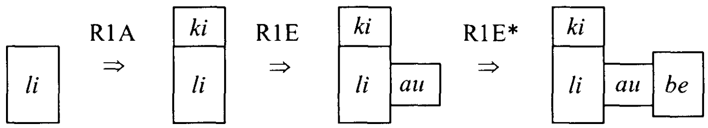


Figure 7. Parametric rule schema R1AB (left) , and R1CDE (right)

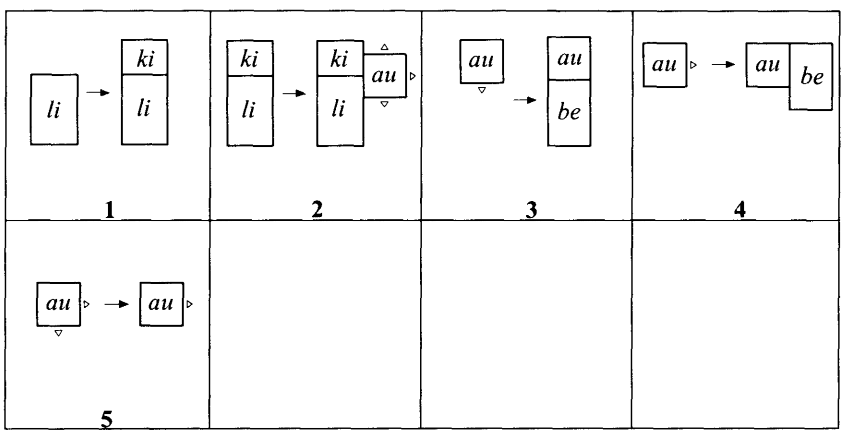
The sets, Σ (p. 122), and R (Table 5, p. 123) and the constraints of Table 4 (p. 122) allow the derivation of *partis*. A sample derivation is the next,



The described elementary framework generates an infinite number of *partis,* some of them with potentials for further implementation, but also many irrelevant. It is useful to develop a method to control the attributes of the generated arrangements. In design practice this is achieved by imposing restrictions, mirroring limitations, preferences and dislikes. A similar result can be achieved by restricting the ways the rule schemata apply to produce the arrangements. Restrictions of the proposed kind can apply on the basis of a variety of criteria. In this case study, the adjacencies between rooms serve as criterion for restricting the generation of *partis.*

An example of more restricted rules is a set of four rules, in the product algebra $<V_{02} \times U_{12}>$

TABLE 6. Set of restricted shape rules



The restricted rules can be examined in relation to the unrestricted ones (Table 5). Rule **1** is an instance of the rule schema R1A: it generates an arrangement between the "*li*" and "*ki*" components. Rule **2** is an instance of rule schema R1C: it adds the "*au*" compound. The parametric shape for "*au*" is specified here as a square (four equal sides), while the ▷ symbol specifies the direction of room-addition and prevents undesirable overlapping. Rule **3** is an instance of rule schema R1A, specifying how "bedrooms" can be added. Rule **4** is an instance of rule schema 1RD* that also adds bedrooms. Rule **5** allows the addition of one, two or three bedrooms and prevents the generation of *partis* without bedrooms.

The rules apply to specifically labeled shapes. In buildings like hospitals, airports, etc. the specification of room-adjacencies can be deduced from objective performance criteria. But typically, relies on the earlier suppositions, and requires further testing.

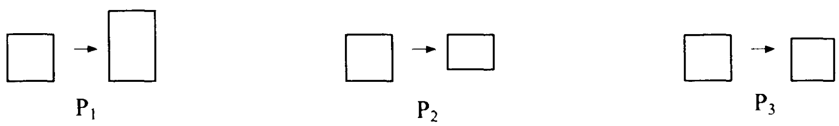
To code the rules digitally, provisions of proportion are necessary. In the example the proportions are based on the square (or, the cube), its golden section $\phi = 1{:}1.618$, and the half-square (or, half-cube). The designer begins by setting the width of the "living" room to $W_{li} = \alpha$ units. Then, without violating the general provisions of Table 4, (p. 122) the widths and lengths W_j and L_j of the rest of the shapes are set to the following

TABLE 7. Relationships of proportion

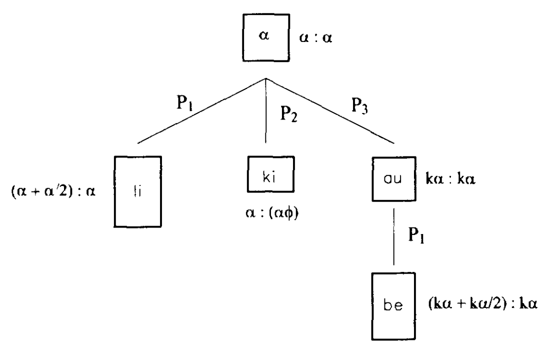
PUBLIC "*pu*" :	living "*li*":
	From provision (i), $L_{li} \geq W_{li}$
	Therefore, if $W_{li} = \alpha$ units, let $L_{li} = \alpha + \alpha/2$ units
	Kitchen "*ki*":
	From provision (ii), $L_{ki} = W_{li} = \alpha$ units
	From provision (i), $W_{ki} \leq L_{ki}$
	Therefore, since $L_{ki} = \alpha$, let $W_{ki} = \alpha \cdot \phi$ units
PRIVATE "*pr*":	auxiliary "*au*":
	From provision (iii), $W_{au} \leq W_{li}$
	Since $W_{li} = \alpha$, let $W_{au} = k \cdot \alpha$ units, with $k \leq 1$
	$L_{au} = W_{au}$, and therefore, $L_{au} = k \cdot \alpha$ units
	bedroom "*be*":
	From provision (ii), $L_{au} = W_{be} = k \cdot \alpha$ units
	And since $L_{be} \geq W_{be}$, let $L_{be} = (k \cdot \alpha) + (k \cdot \alpha)/2$ units

The relationship between public "*pu*" and private "*pr*" rooms is controlled by the ratio $k \leq 1$. After setting $W_{li} = \alpha$, the designer has to specify the ratio k. This choice does not lack spatial-functional meaning: First, the designer specifies the width of the living room "*li*", which is also the initial shape. Then, the width of bedrooms "*be*" and auxiliary spaces "*au*" can be adjusted to be smaller or equal to the main living room.

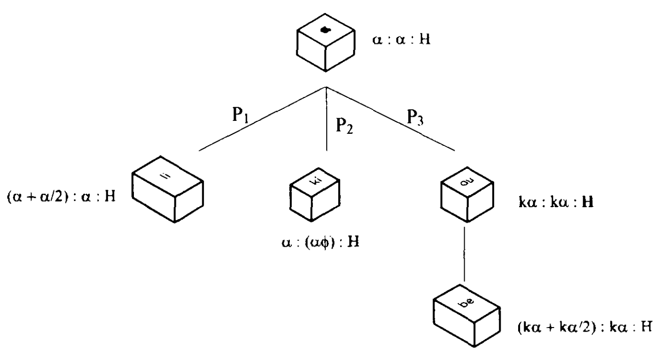
The instantiation of the parameters W_j and L_j is expressed visually by three proportion-rules P_1, P_2, and P_3. Rule P_1 takes an [x : x] square on the left, into an [(x + x /2) : α] rectangle on the right: [x : x]→[(x + x /2) : x]. Rule P_2 takes an [α : α] square into an [α : (α · ϕ)] rectangle: [x : x]→[x : (x · ϕ)]. And, rule P_3 scales an [x : x] square on the left, into an [(k · x) : (k · x)] square on the right: [x : x]→ [(k · x) : (k · x)]



By defining W_{li} = α and the ratio k, and applying the rules P_1, P_2, P_3 the system allows the formation of the rooms. At the example, k = 10/12

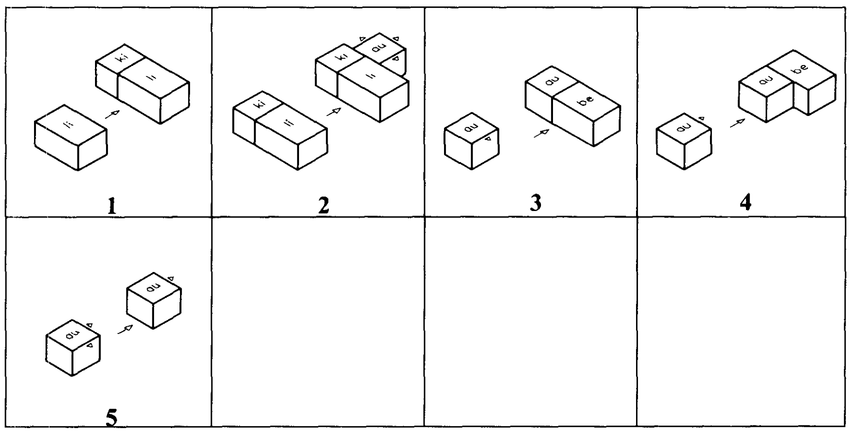


The instantiation of the height H is required in order to express the rules in the product algebra <V_{03} x U_{23}>. It is set $H = h_u \cdot t$, where h_u is an average human height and t is a constant initially set to t = 1.618. This permits the immediate instantiation of 3d rules relative to the human scale, and allows H to be redefined more precisely later, at the *refinement* stage

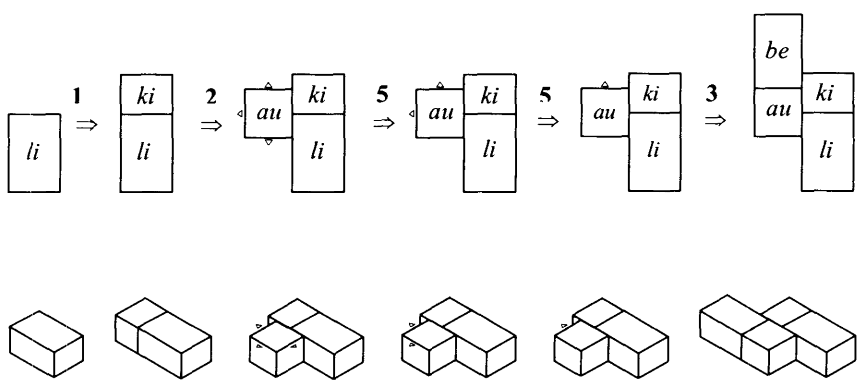


The rules form a parallel rule-set in the product algebra $<V_{03} \times U_{23}>$

TABLE 8. Restricted 3d shape rules

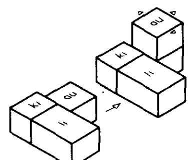


The derivation of a *parti* in 2d and 3d format is presented below. For simplicity name labels are omitted in 3d derivations

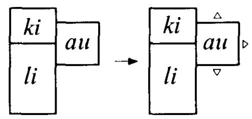


The above elementary rule-set generates house *partis* with specific attributes, in the following discreet steps: starts from the living room "*li*" and the addition of the kitchen "*ki*" according to spatial relation A (Table 2 and 2b, p. 118), then, follows the addition of the auxiliary room "*au*" according to spatial relation C, and the *parti* is completed with the addition of bedrooms (spatial relations A, or D).

A sixth rule is included to allow the addition of a 2^{nd} floor

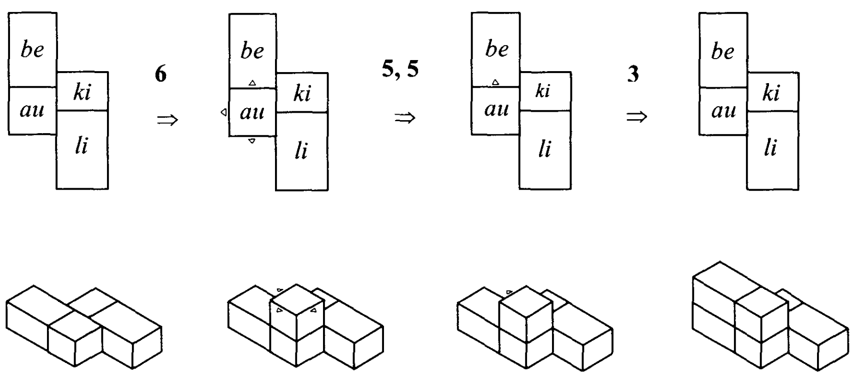


6



6

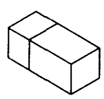
Rule **6** permits the reapplication of rules **5, 4** and **3**. The re-application of any of the rules **3**, **4**, or **5** generates acceptable arrangements.



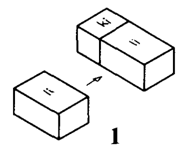
The derived *parti* is a schema that can be used for the specification of designs. The abstract character of a *parti* allows several designs to be derived from it. But also, the generation of different *partis* may produce variation. The spatial properties of the *partis* are expressed in the rules that generate them. The rules project some selected spatial properties to a large set of compositions. Different sets of rules can be constructed to produce several different kinds of *partis*. After a rule-set is tested, it can be organized into a grammar. Therefore, grammars serve as memory devices, where the rules are gradually classified.

In the present exercise, the rules are evaluated on the basis of the room-adjacencies they produce. For this purpose, the rules that form different room-adjacencies are encoded into the scripting language to facilitate the quick visualization of the produced *partis*. The preferred ones are saved for further use.

A way to achieve variation is by redefining the relationship between two rooms: for example, between living "*li*" and kitchen "*ki*". The particular relationship follows initially, the spatial relationship A (see Tables, 2 and 2b, in page 118). Accordingly, the spatial relationship is generated by rule 1



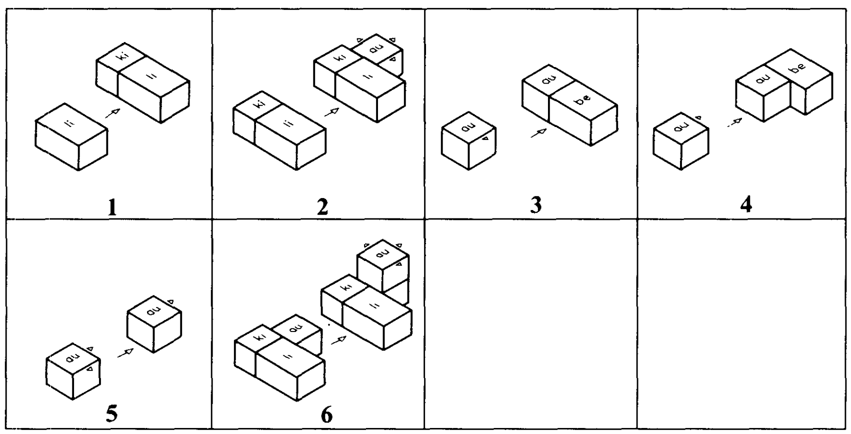
relation A



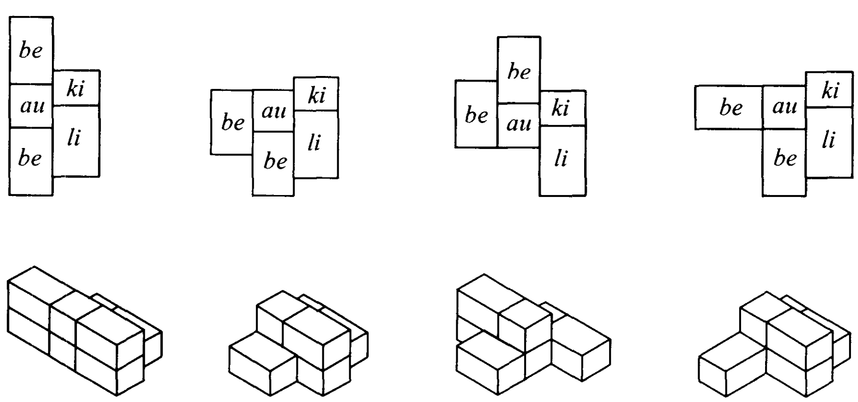
1

The initial set of the six restricted rules (as appear in Table 6, p. 124, and Table 8, p. 127) is recapitulated in the following Table 8a, in the product algebra $<V_{03} \times U_{23}>$. The 2d representation of the rules, in the product algebra $<V_{02}, U_{12}>$ is omitted for brevity

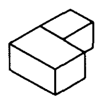
TABLE 8a. The initial set of six restricted shape rules, in 3d



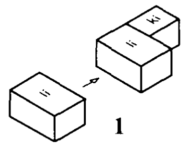
Sample *partis* (in 2d and 3d) generated by the rules are presented bellow



If the spatial relationship between living "*li*" and kitchen "*ki*" switch from A to D (Tables, 2, 2b, p. 118), then the rule 1 is modified accordingly



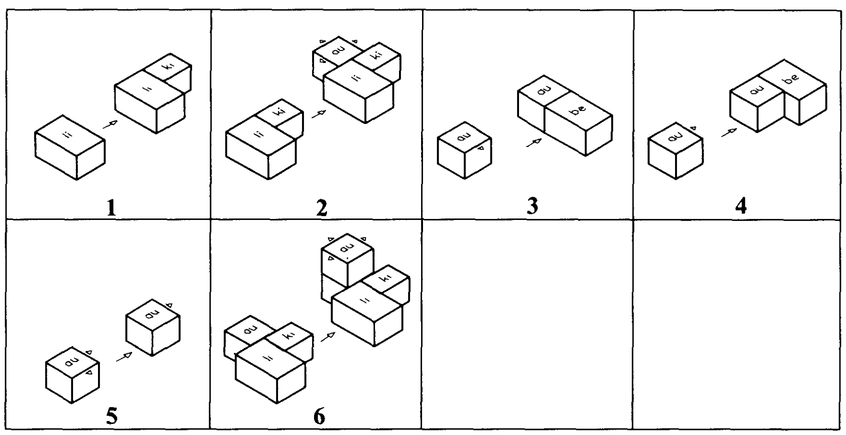
relation D



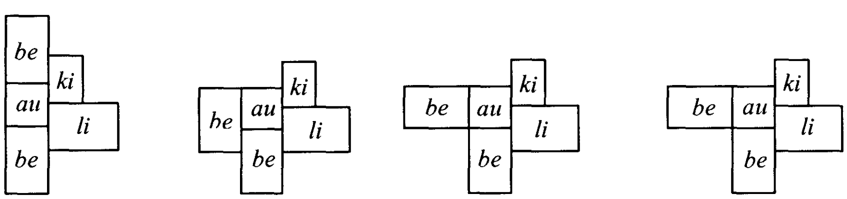
1

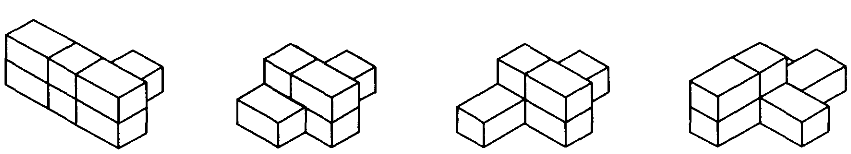
The set of six rules requires some further modifications that are depicted in the following Table 8b

TABLE 8b. An alternative version of the restricted, shape rules, in 3d

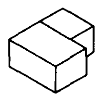


Partis (in 2d and 3d) derived by this new set of rules are presented below

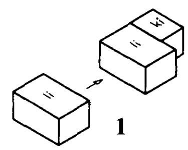




Finally, in the spatial relationship D, a simple rotation of 90° of the added "*ki*" unit, can be the basis of a third set of alternatives

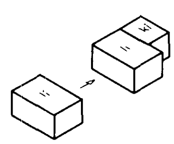
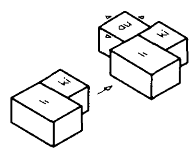
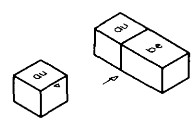
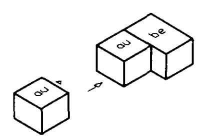
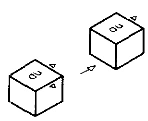
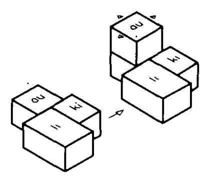


relation D′

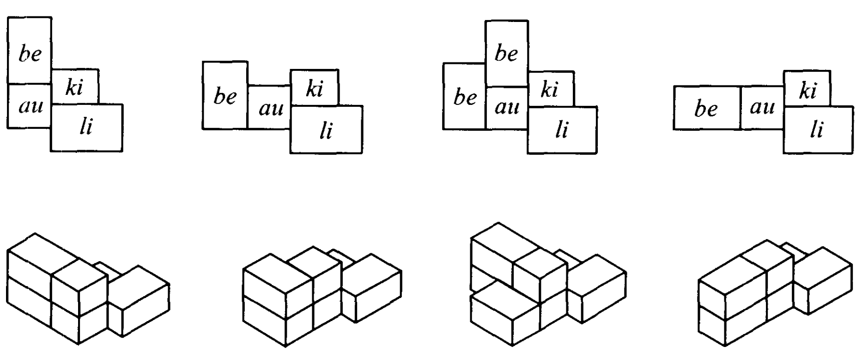


The new alternative, modified set of six rules, is depicted in the following Table 8c

TABLE 8c. Third alternative version of the six restricted shape rules, in 3d

 1	 2	 3	 4
 5	 6		

Some of the produced *partis* (in 2d and 3d) by the new set of rules are depicted bellow



To summarize, the *formation* produces *partis*. The *parti* is a generic description that is used in the generation of designs. The *partis* are produced by rules that describe how to compose them. During the process of *formation* several spatial relationships and rules are tested. Gradually, they are restricted to generate the preferred *partis*. The criteria of evaluation vary. In the example the evaluating basis was the adjacency and distribution of rooms. Other criteria can serve as well the formation and the identification of shapes that can be used as *partis*.

6.2. TRANSFORMATION

In this section, an input *parti* is used to produce boundary-layouts for designs. Some of the basic transformations are exposed: The introduction of boundaries and the distribution of solid and void are two of them. A single *parti* yields several boundary-layouts. One is selected for further implementation.

There are three interesting points to be made:

First, an input *parti* is required. The application of transformation rules may convert this input into a new arrangement with a new derived *parti*. Or, it can conclude to the production of alternative boundary-layouts, without changing the *parti*.

In the example, the labels are omitted from the *parti*. The plan description contains lines manipulated on the plane, in the algebra $<U_{12}>$. The 3d description of the *parti* contains planes manipulated in space, in the algebra $<U_{23}>$.

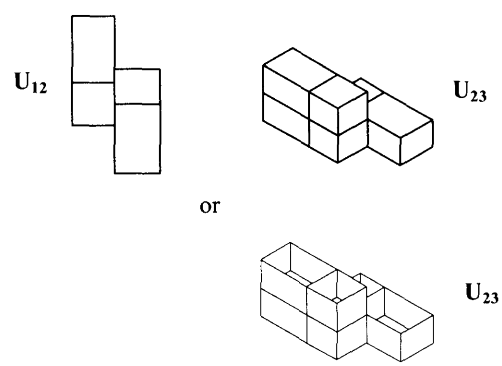


Figure 3. The *parti* in 2d and 3d representation

Second, the generation of alternative boundary-layouts usually requires multiple 2d graphic layers, and 3d models. In the example, a first graphic layer A contains the *parti*, and a second layer B contains the boundary-layout. Superimposed sheets or AutoCAD layers can model this setting.

In a 2d plan the *parti* is denoted by a continuous line and the boundaries (walls) by parallel continuous lines indicating the boundary thickness

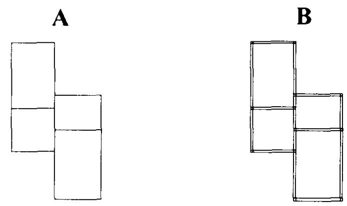
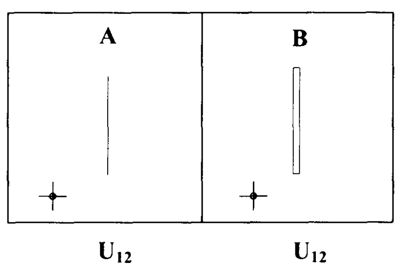
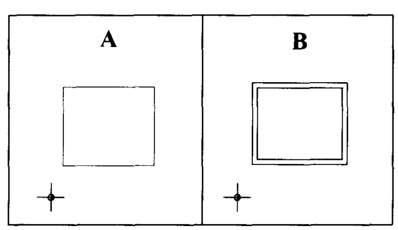


Figure 4. The parti (left) in U_{12} and the boundary-layout (right) in U_{12}

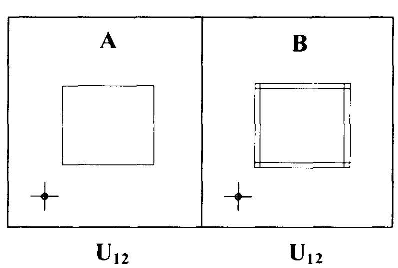
In 2d, an element described in layer A is a line in algebra U_{12}. The same element described in layer B can be a rectangle, in the algebra U_{12}.



In layer A, a room can be described as a rectangle in the algebra U_{12}. In layer B, the same room can be described as a rectangle made out of parallel lines.



Or,



In 3d representation, the *parti* in layer A is described by planes, manipulated in 3d space, in algebra U_{23}. In layer B, the boundary-layout is composed by solids, in the algebra U_{33}.

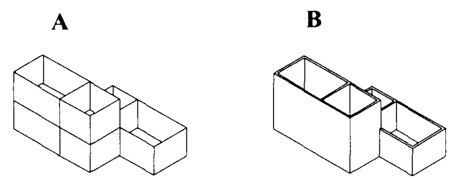
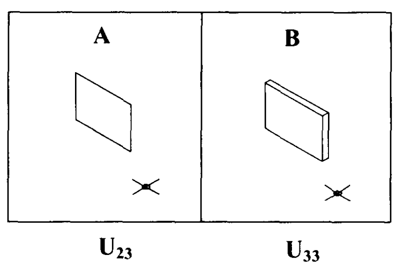
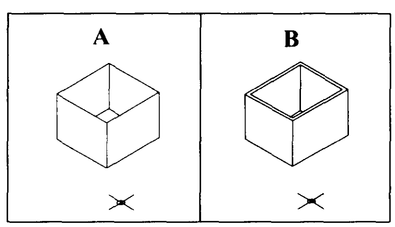


Figure 5. The parti (left) in U_{23} and the boundary layout (right) in U_{33}

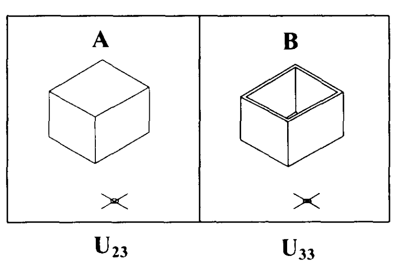
An element in layer A is a plane in algebra U_{23}. The same element in layer B is a solid, in the algebra U_{33}.



In layer A, a room can be described by planes as an empty volume, in the algebra U_{23}. In layer B, the same room can be described as a shape made of solids, in the algebra U_{33}.

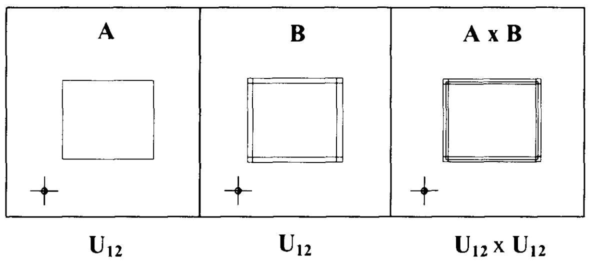


Or, sometimes alternatively,

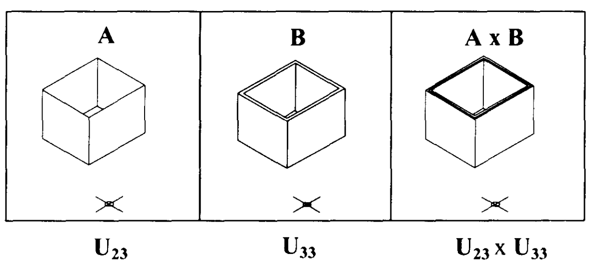


Third, the participating 2d layers or 3d models contain descriptions. The product $X_1 \text{ x } X_2 \text{ x } X_3 \text{ x} \ldots X_n$ corresponds to the stack of n descriptions X_1, X_2, X_3, ... X with each individual description superimposed to the other descriptions. A description can be withdrawn, or replaced by some finite decomposition of its elements.

In our example, a room described in layer A, and B, in the algebra U_{12}, forms a Cartesian product A x B, in the product algebra $U_{12} \text{ x } U_{12}$.



In 3d, the same room described in layer A and B, forms a Cartesian product A x B in the product algebra $U_{23} \text{ x } U_{33}$.



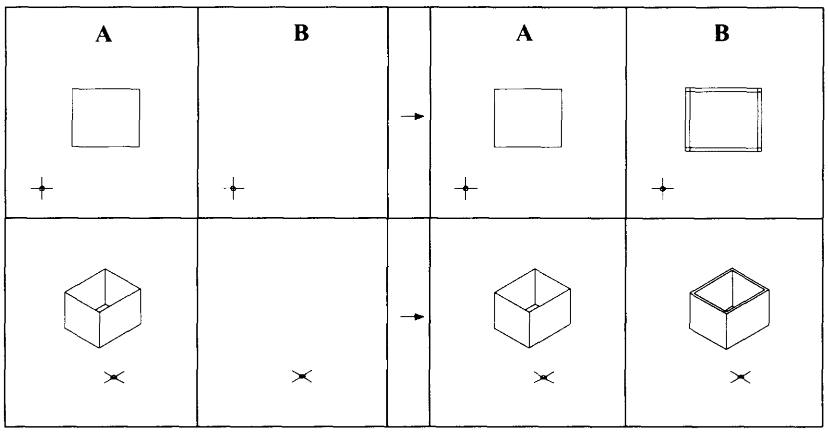
The rules obtain the form:

$$< x_{11}, x_{12}, x_{13}, \ldots x_{1n} > \rightarrow < y_{11}, y_{12}, y_{13}, \ldots y_{1n} >$$
$$< x_{21}, x_{22}, x_{23}, \ldots x_{2n} > \rightarrow < y_{21}, y_{22}, y_{23}, \ldots y_{2n} >$$
$$\ldots$$
$$< x_{r1}, x_{r2}, x_{r3}, \ldots x_{rn} > \rightarrow < y_{r1}, y_{r2}, y_{r3}, \ldots y_{rn} >$$

The shapes x_{ij}, y_{ij} are made out of lines, if $x_{ij}, y_{ij} \in <U_{12}>$ or solids if $x_{ij}, y_{ij} \in< U_{33} >$, or the empty shape. A rule applies on multiple shapes C_1, C_2, C_3, ... C_n looking for shapes x_1, x_2, x_3, ... x_n, and turns them into shapes y_1, y_2, y_3, ... y_n respectively. The application of a rule generates multiple shapes:

$$C_1', C_2', C_3', \ldots C_n' = [C_1, C_2, C_3, \ldots C_n - t(x_1, x_2, x_3, \ldots x_n)] + t(y_1, y_2, y_3, \ldots y_n)$$

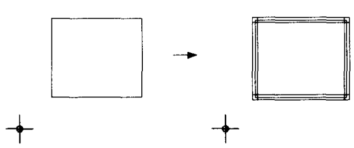
A rule can take the form: $<x_1, \varnothing, x_3, \ldots\varnothing, x_n> \rightarrow <y_1, \varnothing, y_3, \ldots\varnothing, y_n>$. A parametric rule of this type – acting on two layers A and B – draws boundary layouts. In 2d, the following rule (a) scans a *parti*-rectangle in layer A and adds the corresponding 2d shape of the boundaries, in layer B, while the *parti*-rectangle remains intact in layer A. In 3d, the rule (a) takes a volume scanned in A, to a 3d boundary-shape in layer B, while the *parti*-shape remains intact in A.



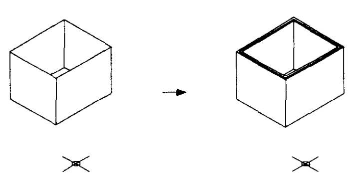
rule (a)

Rule (a) can be fully understood if we place layer B over layer A, so that the coordinate systems coincide. The product A x B depicts the superimposition of the layers. The rule can be expressed A x B → A x B

$<U_{12} \text{ x } U_{12}> \rightarrow <U_{12} \text{ x } U_{12}>$

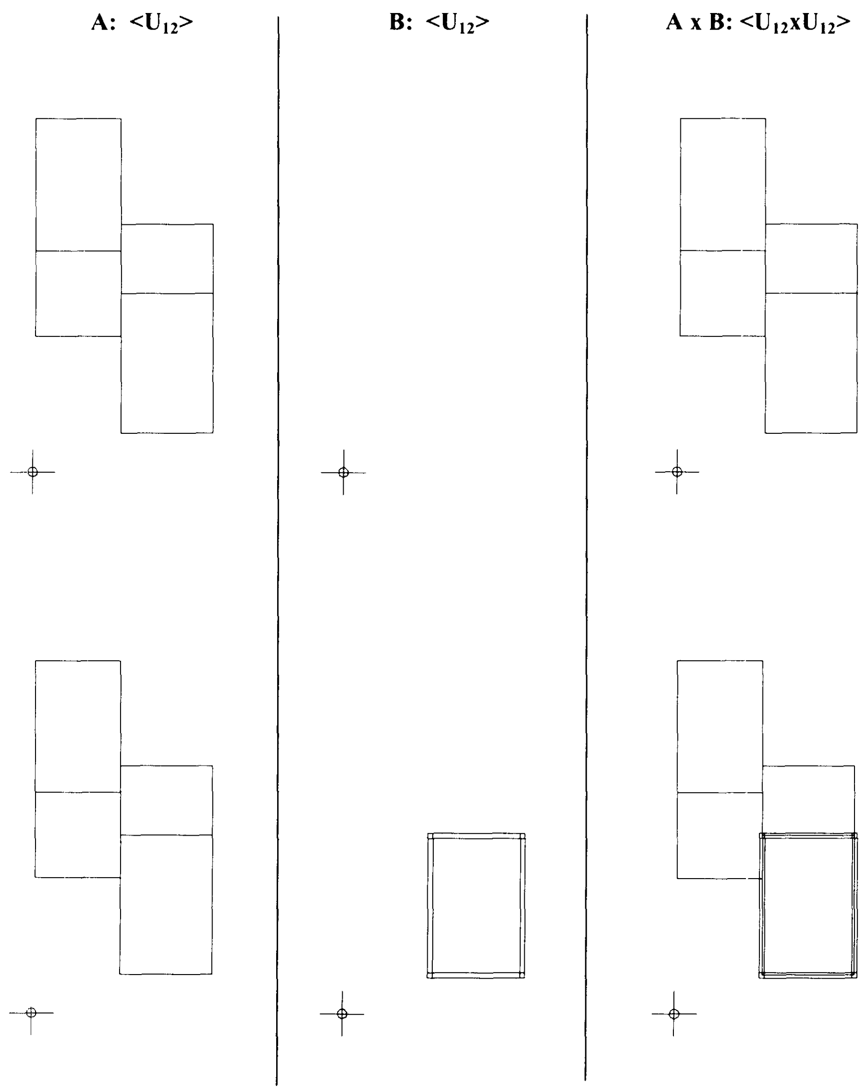


$<U_{23} \text{ x } U_{33}> \rightarrow <U_{23} \text{ x } U_{33}>$

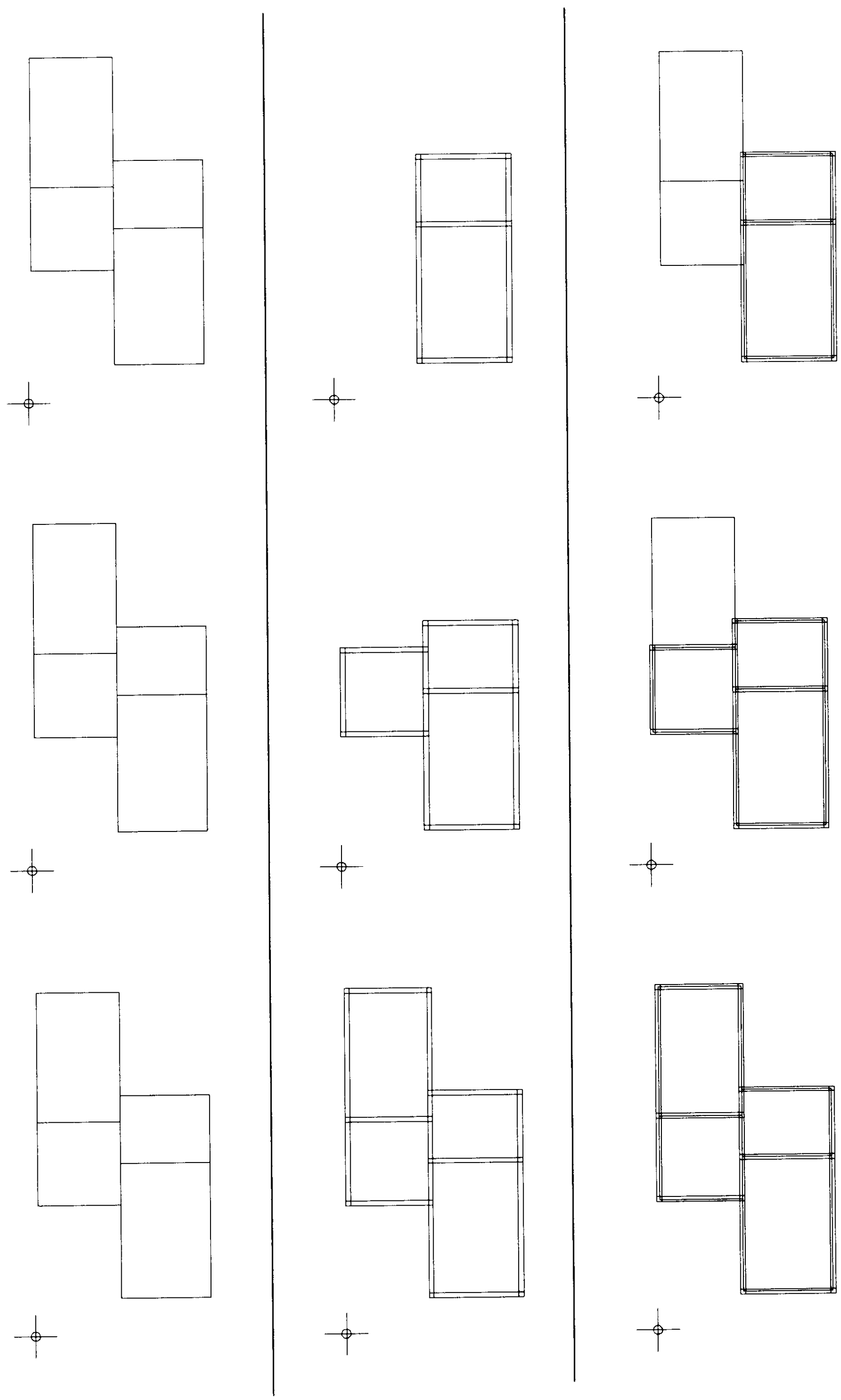


The rule (a) acts after the schema $< x, \varnothing > \rightarrow < x, y >$ with $x, y \in <U_{12}>$. At the end of a derivation the shape of the *parti* remains intact in layer A, and a new shape is created in layer B.

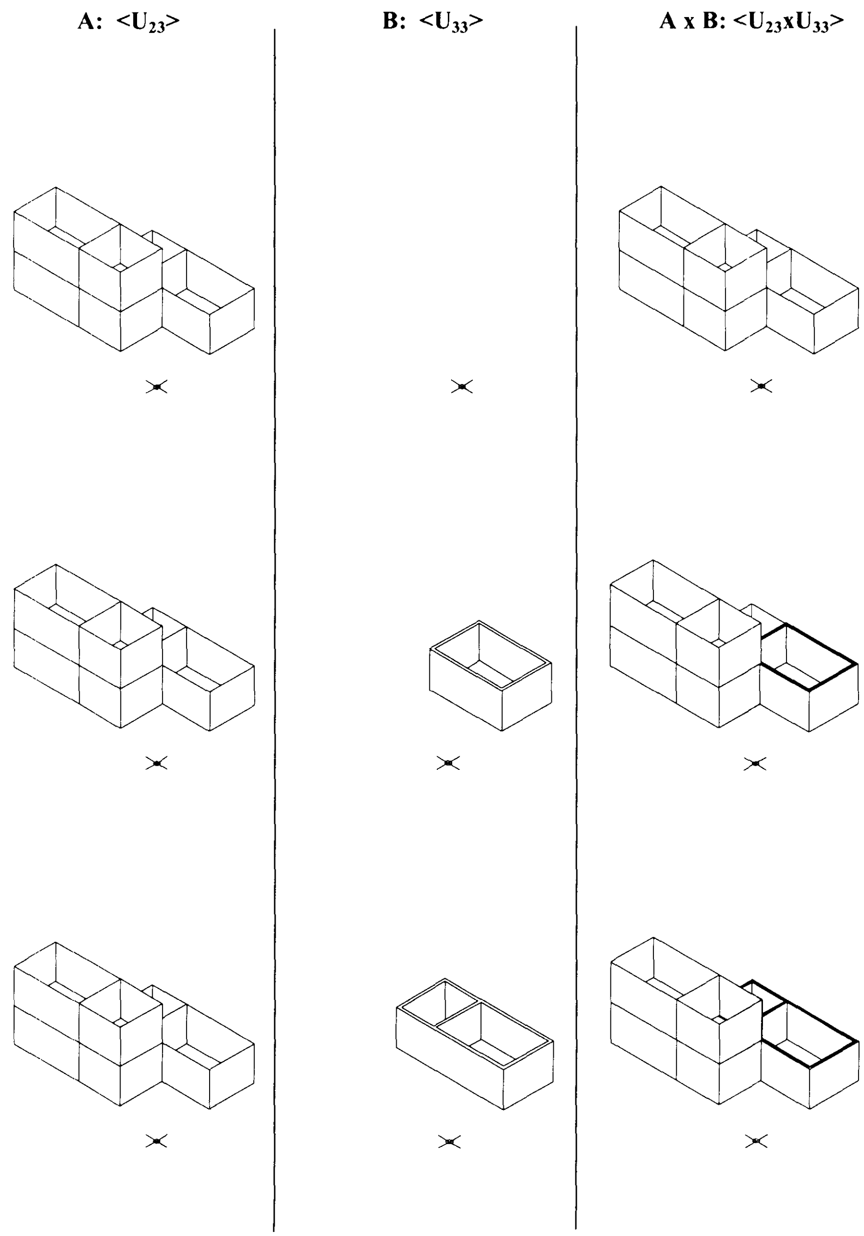
The following example of a derivation presents the shapes in layers A and B, and their product A x B

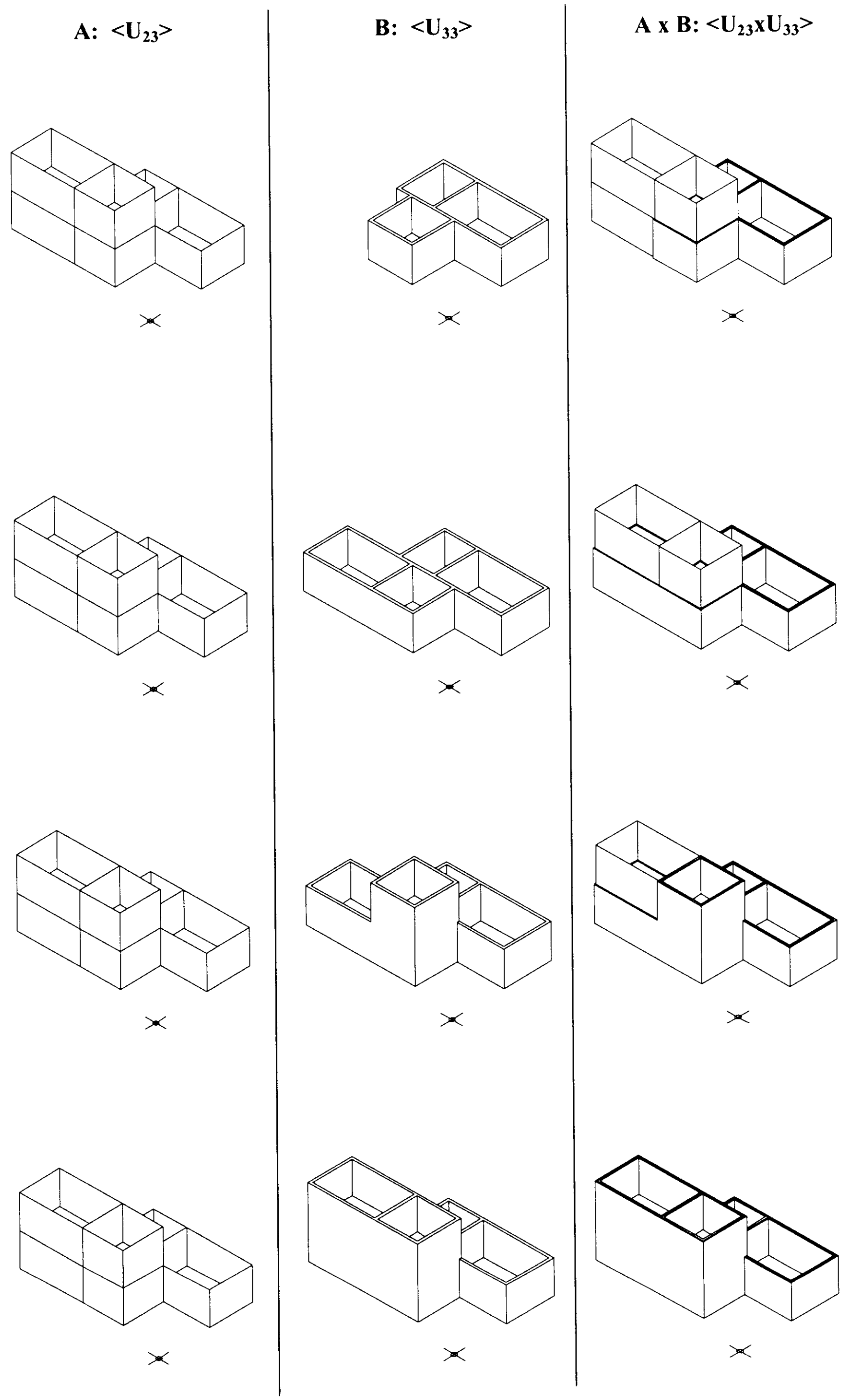


A: $<U_{12}>$ **B: $<U_{12}>$** **A x B: $<U_{12}xU_{12}>$**



In 3d, the rule (a) adds for each shape x scanned in layer A, a solid y in layer B. The shape in layer A remains intact.

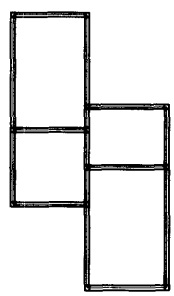



A: $\langle U_{23} \rangle$
B: $\langle U_{33} \rangle$
A x B: $\langle U_{23} \mathrm{x} U_{33} \rangle$

In layer A the rule (a) identifies shapes that compose the *parti*. The rule substitutes those shapes with new shapes in layer B without changing the shapes in layer A.

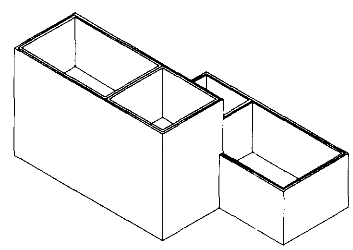
The superimposition of the two layers A and B gives the complete picture: a boundary-layout (in layer B), and its underlying *parti* shape (in layer A). The product A x B of the layers A and B, forms a single description, in the product algebras $<U_{12} x\ U_{12}>$ and $<U_{23} x\ U_{33}>$ respectively

A x B



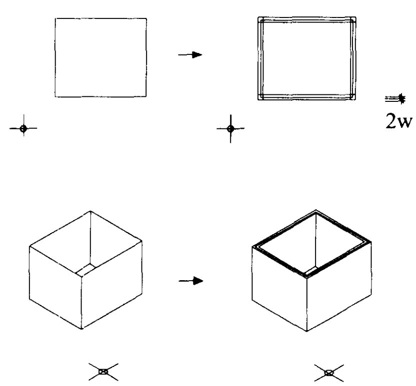
U_{12} x U_{12}

A x B



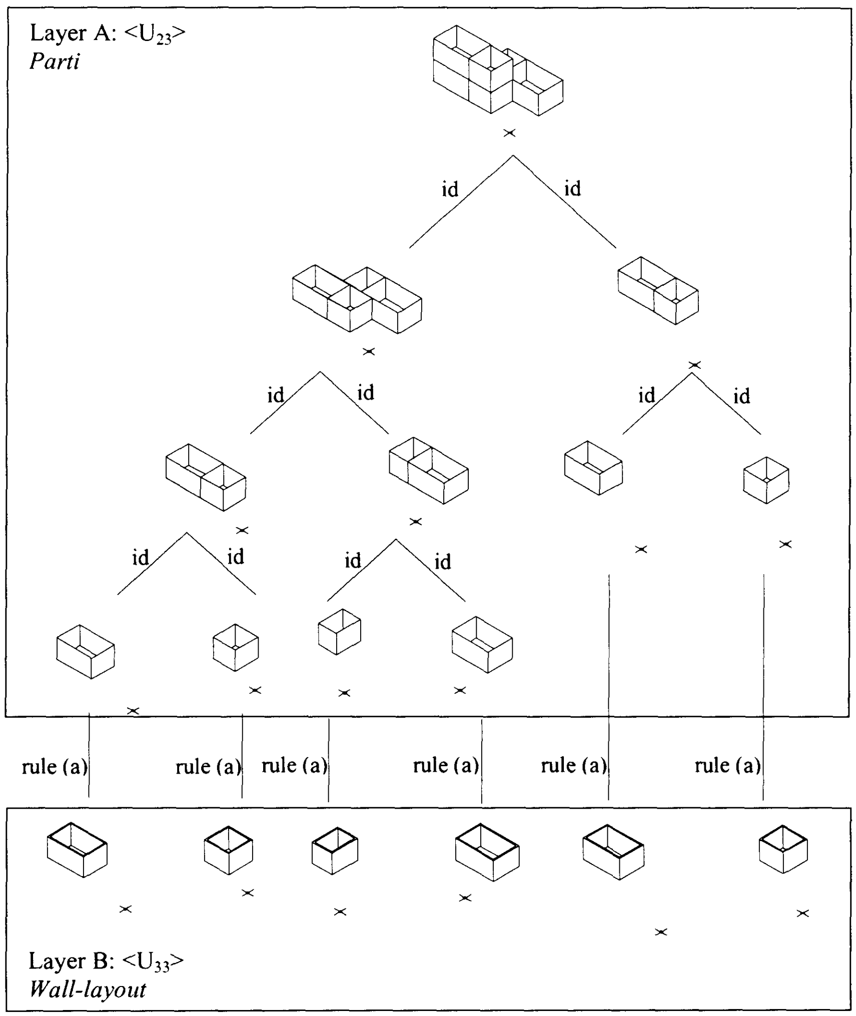
U_{23} x U_{33}

The rule (a) obeys the schema $< x, \emptyset > \rightarrow < x, y >$ with $x, y \in <U_{12}>$. The relationship between the shapes $x \in A$ and $y \in B$ of the layers A and B is established by the rule (a), and a predicate G: "*Each parti element in layer A, lies axially in space, in relation to a boundary element, in layer B*". Therefore, the rule (a) obtains the form: $<x, \emptyset > \rightarrow < x, Gxy>$.



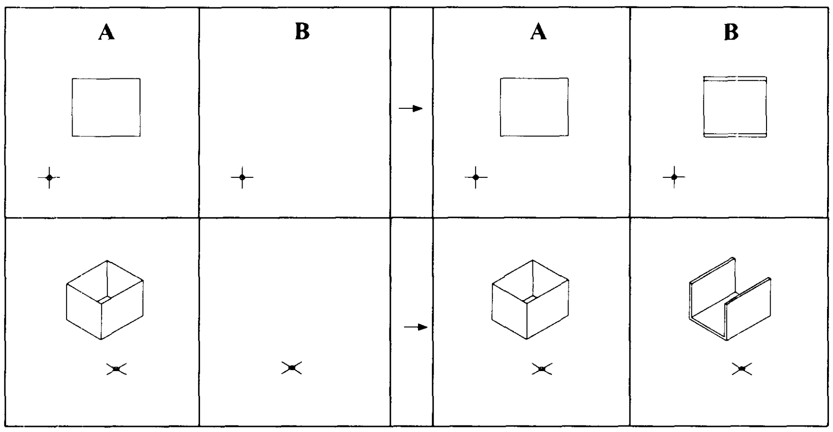
where the thickness of the boundary is equal to $2w$.

The next diagram is an overview of the derivation, not an actual working model. The tree depicts the substitution of *parti* shapes with boundary-shapes. At the root of the tree the *parti* is divided, with the aid of identities in first and second floor. The rooms of each floor are substituted by forms that appear at the bottom leaves. These leaves can be substituted in alternative ways.



A boundary layout can be derived according to a different rule (a′) after the schema $<x_1, \varnothing> \rightarrow <x_1, G'x_1y_2>$ with a predicate G′ stating: "*The parti elements lie axially, in relationship to the boundaries of the two long* *parallel sides of a room, while the remaining two short sides remain open (without double boundary-lines)* ".

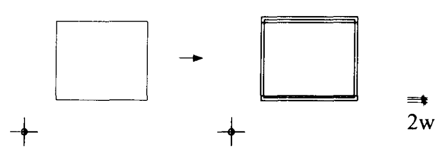
In 2d, the rule (a′) scans a *parti*-rectangle in layer A and substitutes it with the new 2d shape in layer B, while in layer A the *parti*-rectangle remains intact. In 3d, the rule (a) takes a volume scanned in A, to a 3d shape in layer B, while the shape in A remains intact.



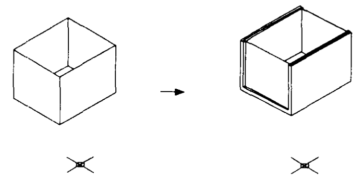
rule (a′)

Rule (a′) is described in the form A x B → A x B. The product A x B depicts the superimposition of the layers A, B

$\langle U_{12} \text{ x } U_{12} \rangle \rightarrow \langle U_{12} \text{ x } U_{12} \rangle$

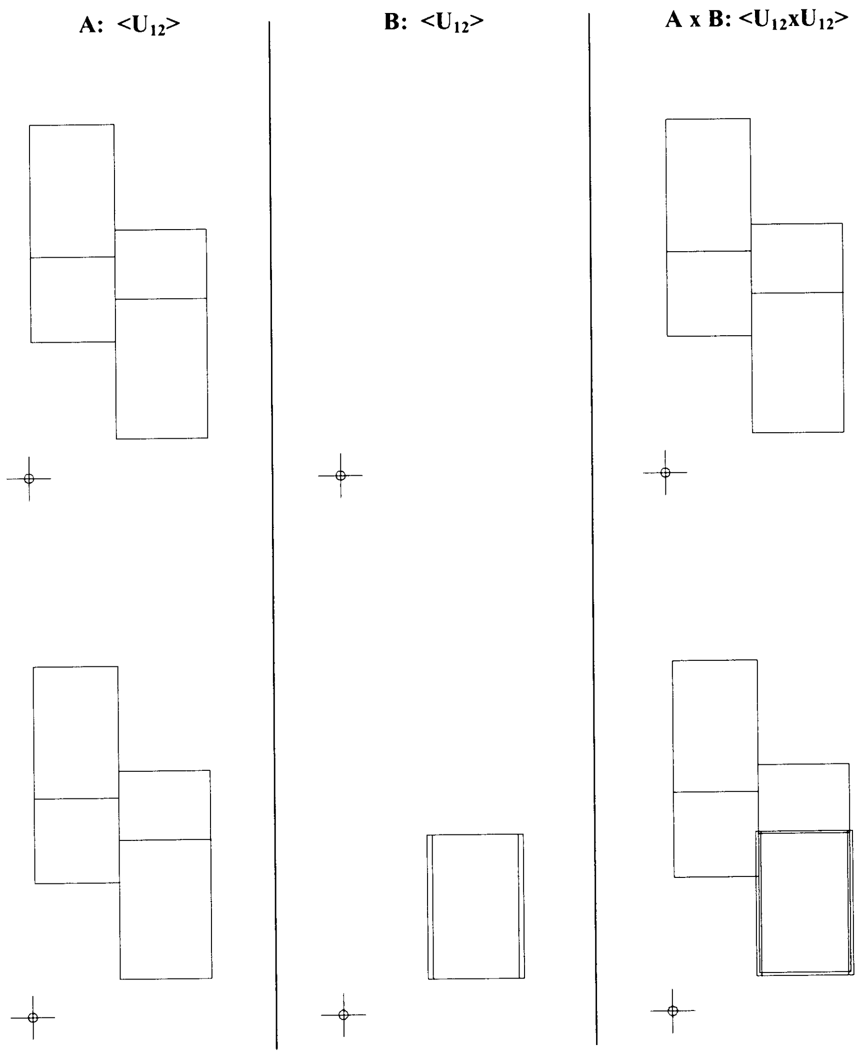


$\langle U_{23} \text{ x } U_{33} \rangle \rightarrow \langle U_{33} \text{ x } U_{33} \rangle$

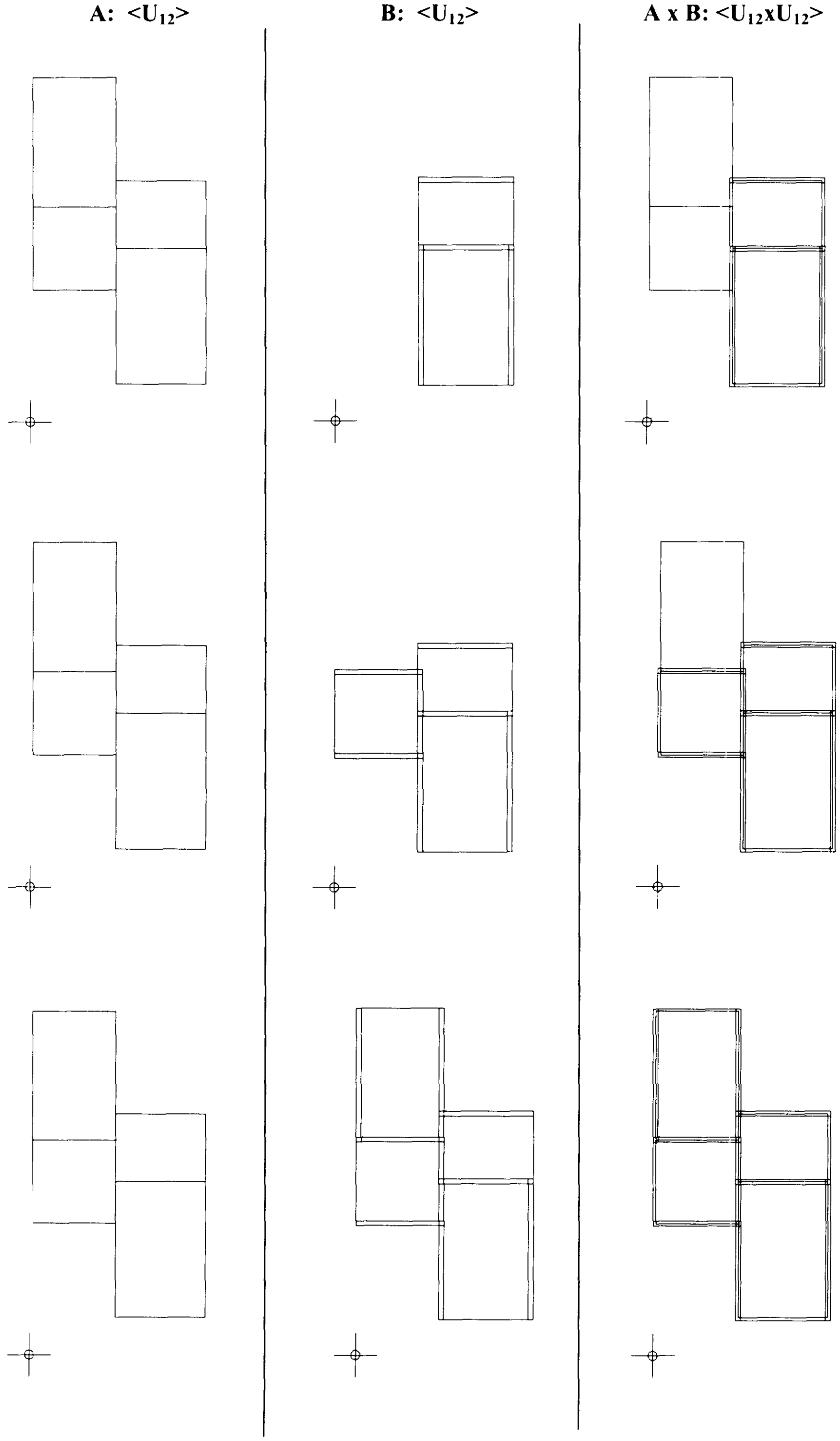


The new rule (a′) acts after the schema $< x, \varnothing > \rightarrow < x, y >$ with $x, y \in$ $<U_{12}>$. The shape of the *parti* remains intact in layer A and a new shape is generated in layer B.

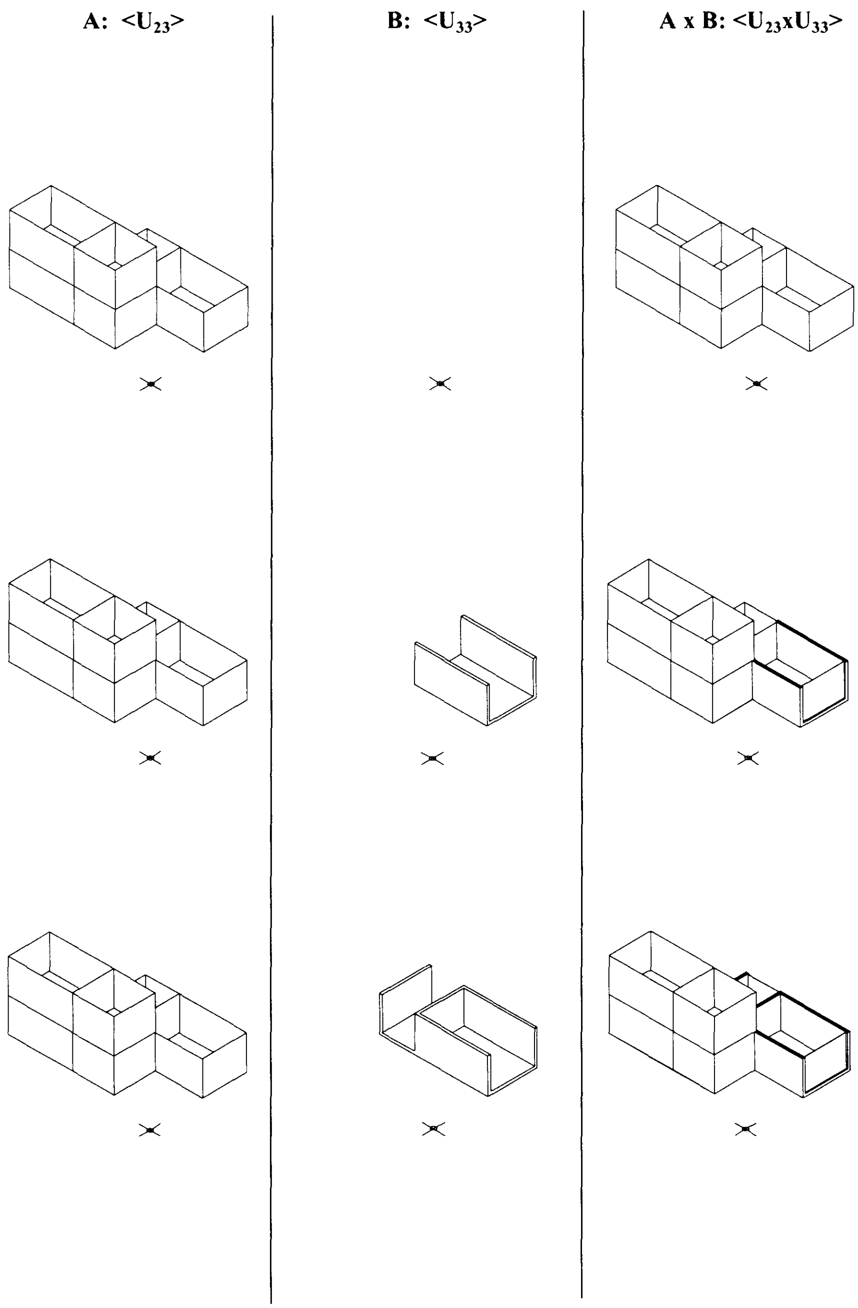
The derivation presents the shapes in the layers A and B, and their product A x B

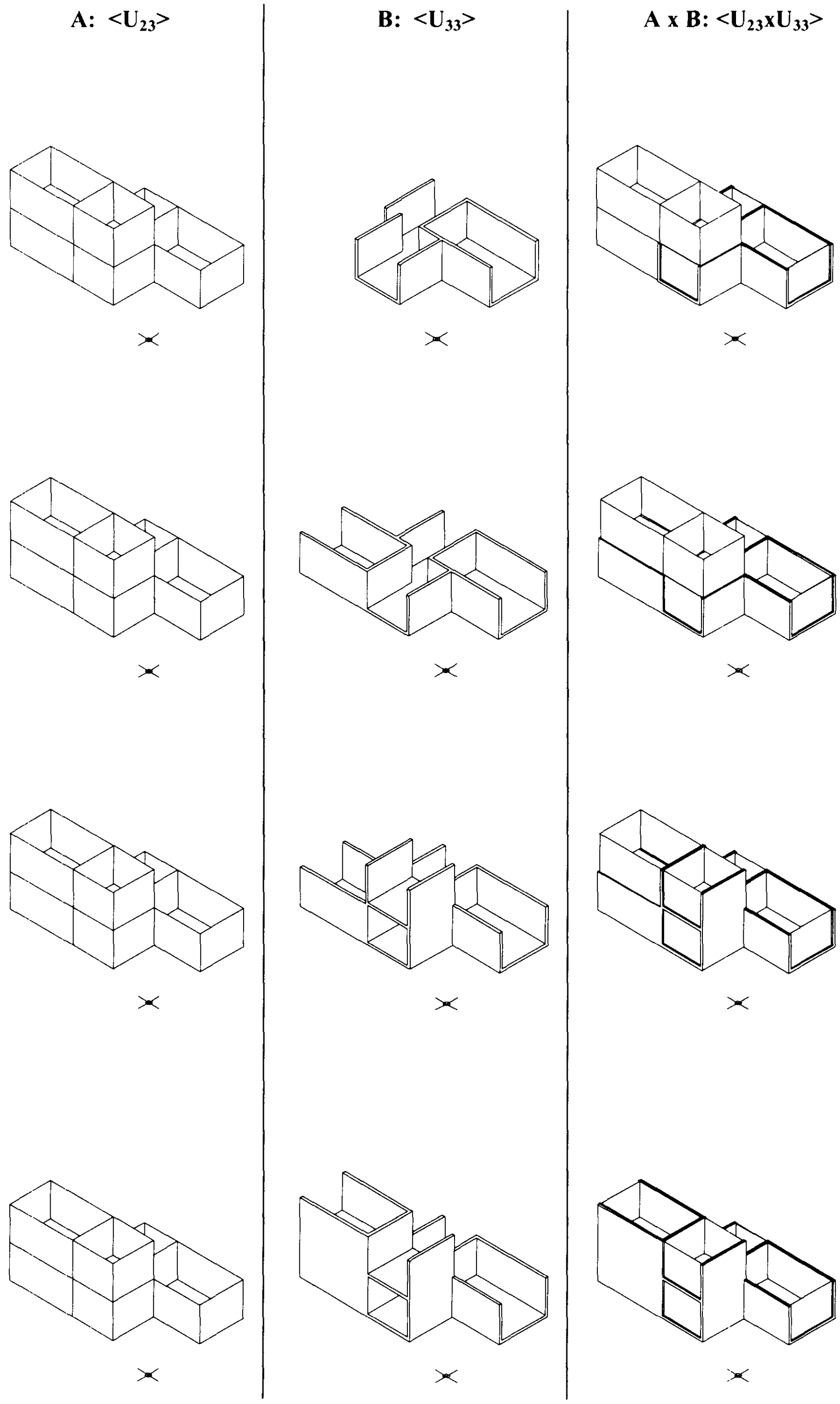


A: $<U_{12}>$ **B: $<U_{12}>$** **A x B: $<U_{12} x U_{12}>$**

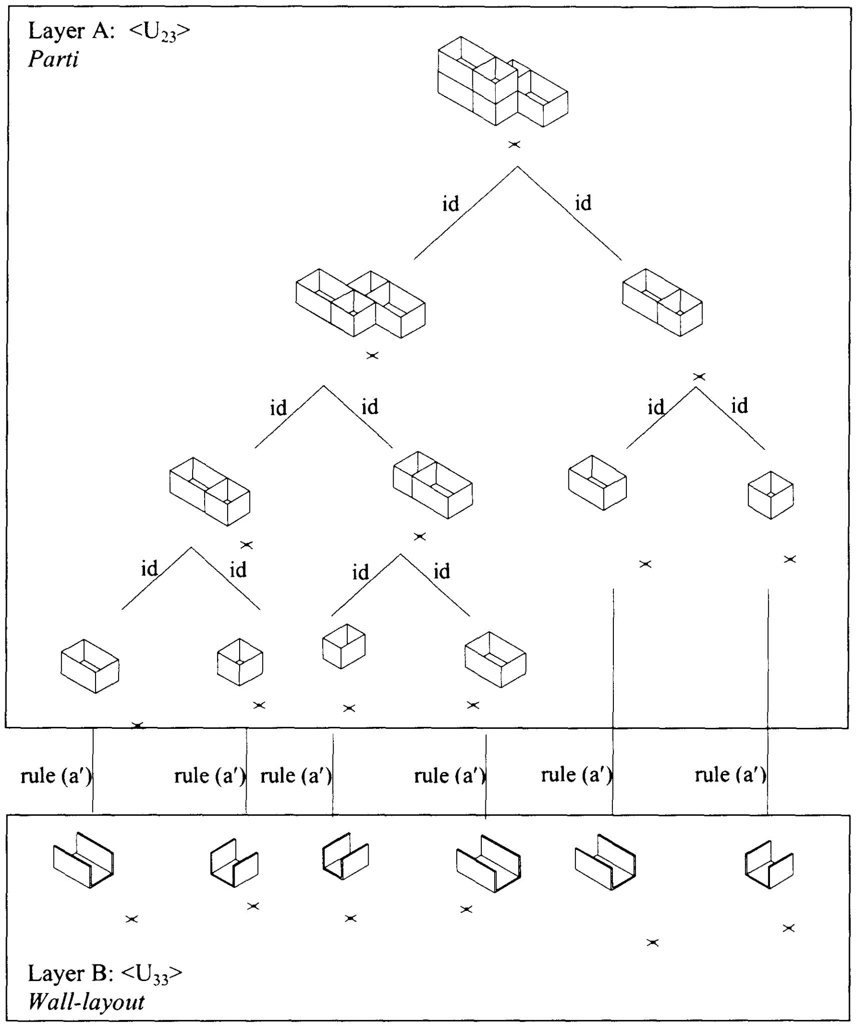


In 3d, rule (a′) takes a shape $x \in <U_{23}>$ scanned in layer A, to a new shape $y \in <U_{33}>$ in layer B, while the shape in A remains intact

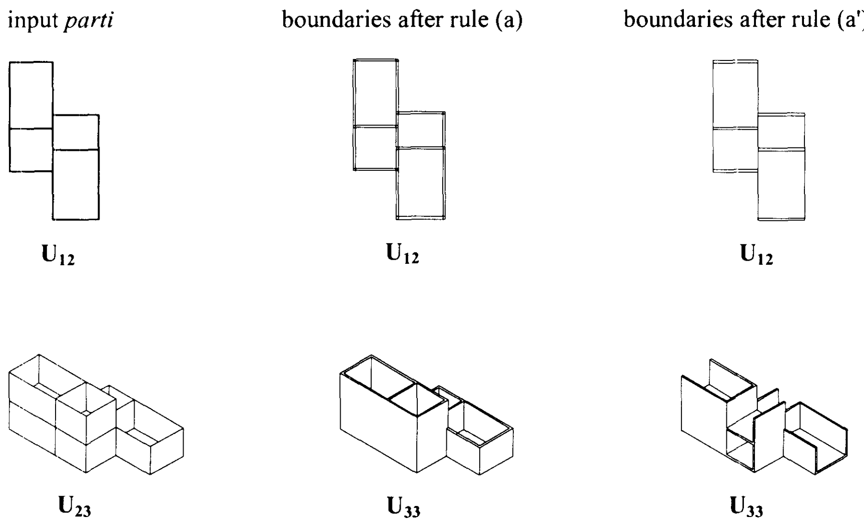



A: $\langle U_{23} \rangle$
B: $\langle U_{33} \rangle$
A x B: $\langle U_{23} x U_{33} \rangle$

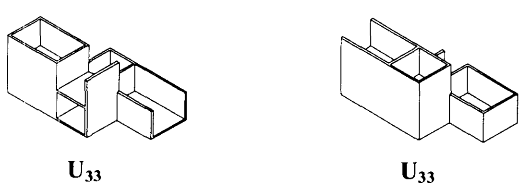
The following tree offers an overview of the derivation. At the root of the tree we see the 3d *parti*. The *parti* is divided, by identities in first and second floor, and then in rooms. The forms that compose the new boundary-layout appear at the bottom leaves of the tree.



A comparative view of the derived boundary-layouts in 2d and 3d, after the rules (a) and (a′), is presented below. The *parti* of the working example appears on the left



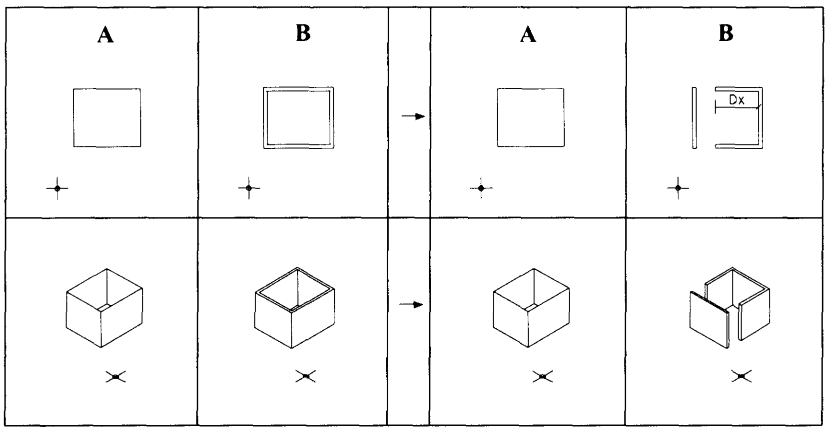
The rules (a) and (a′), generate different spatial arrangements. Other arrangements can be produced by alternating the use of the rules (a) and (a′).



Additional rules can be used to generate geometrically diverse results including curves, and complex shapes. The transformation of a *parti* to a boundary-layout remains open to interpretation. It can be performed independently from the geometry of the *parti*.

The next five parametric rules are examples of five of the most common transformations. The rules (b), (c) and (d) concern the distribution of solid and void. Rule (e) modifies the *parti* by adding rooms and inserting new *parti* lines. Rule (f) can be used for the discrimination of interior-exterior walls.

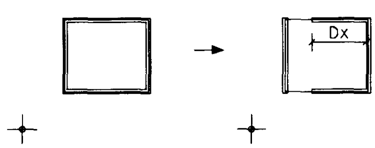
Parametric rule (b) creates a circulation zone in a given room, at a distance D_x from a boundary element (i.e. a wall), by cutting openings to the intersecting boundaries. Rule (b) does not transform the *parti*



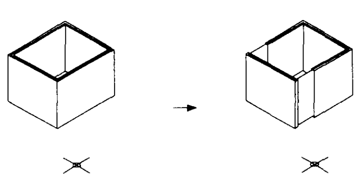
Rule (b)

Rule (b) is also presented as a product rule <A x B> → <A x B>. It depicts the superimposition of the layers A, B

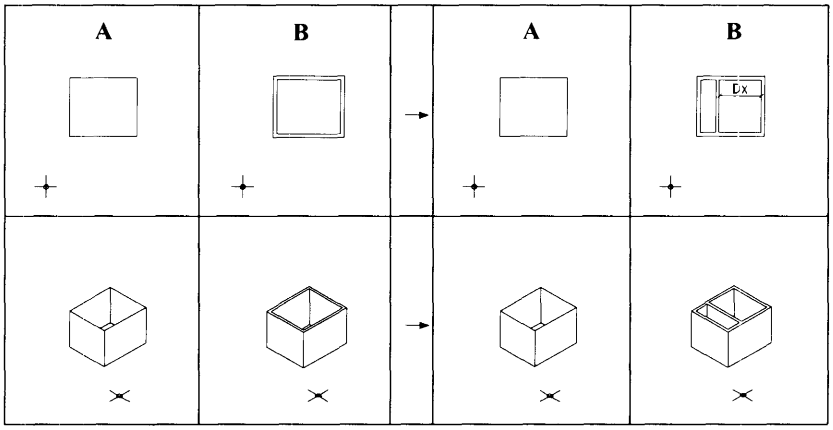
$\langle U_{12} \text{ x } U_{12} \rangle \rightarrow \langle U_{12} \text{ x } U_{12} \rangle$



$\langle U_{23} \text{ x } U_{33} \rangle \rightarrow \langle U_{23} \text{ x } U_{33} \rangle$



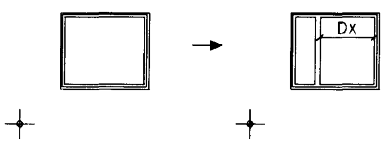
The next parametric rule (c) creates partitions in an existent room. The new partitions do not add new *parti* lines.



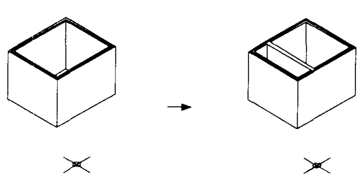
rule (c)

Rule (c) can be described in <A x B> → <A x B> form, as follows:

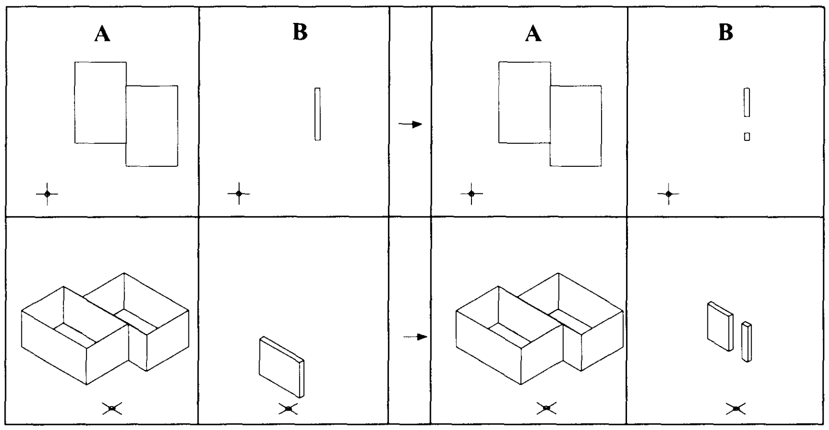
$<U_{12}\ x\ U_{12}> \rightarrow <U_{12}\ x\ U_{12}>$



$<U_{23}\ x\ U_{33}> \rightarrow <U_{23}\ x\ U_{33}>$



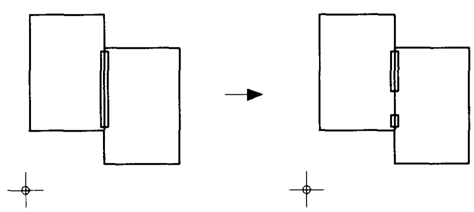
Rule (d) creates an opening between two interior spaces. Rule (d) does not transform the *parti*



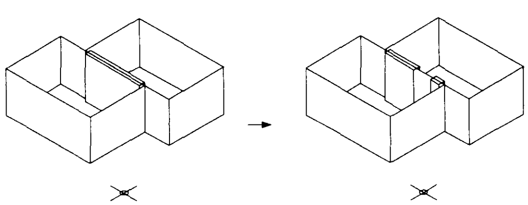
rule (d)

Rule (d) can also be described as follows: <A x B> → <A x B>

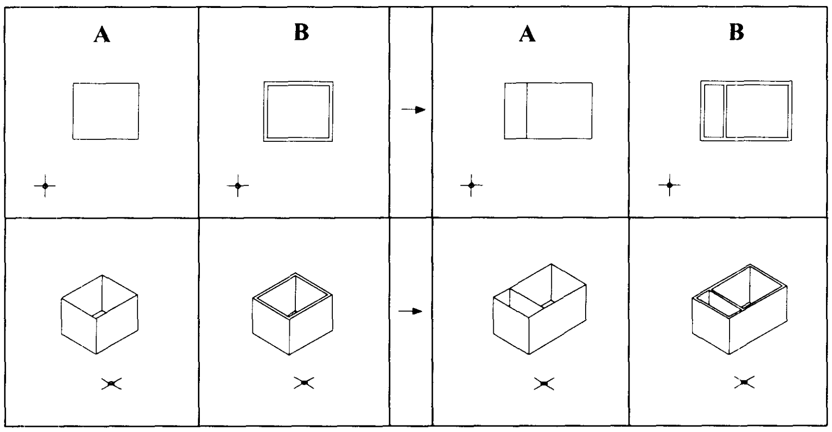
$<U_{12} \times U_{12}> \rightarrow <U_{12} \times U_{12}>$



$<U_{23} \times U_{33}> \rightarrow <U_{23} \times U_{33}>$



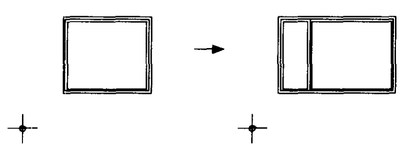
Rule (e), adds a new room. The rule translates an existent room from an initial position to a new position, adds new walls, and a new *parti* line. Unlike any other rule, the rule (e) changes the *parti*



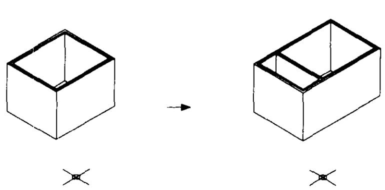
rule (e)

The rule (e) is described in <A x B> → <A x B> form as follows

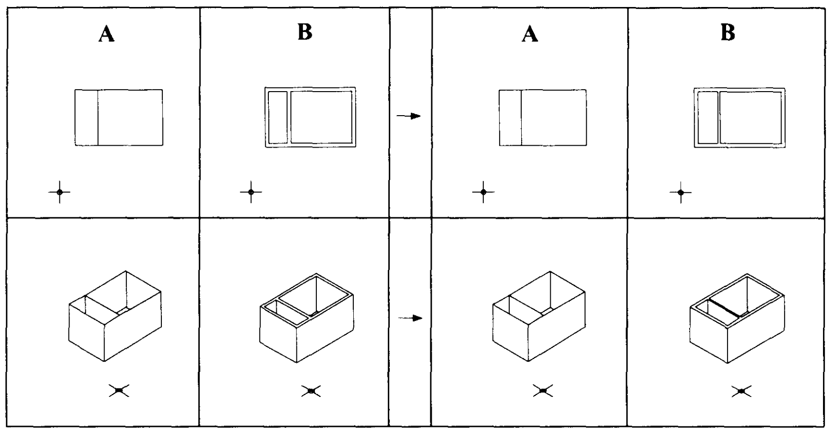
$<U_{12}\ x\ U_{12}> \rightarrow <U_{12}\ x\ U_{12}>$



$<U_{23}\ x\ U_{33}> \rightarrow <U_{23}\ x\ U_{33}>$



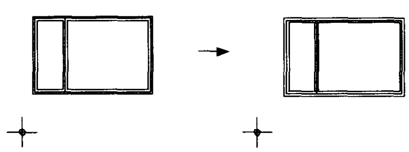
Finally, the parametric rule (f) can be used for the discrimination between interior and exterior walls. It makes an interior wall thinner than an exterior one. Rule (f) does not change the *parti*



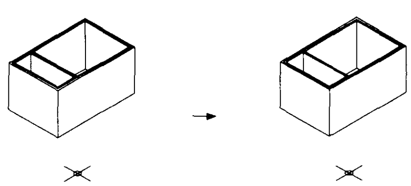
rule (f)

Rule (f) is described in the form <A x B> → <A x B> as follows:

$<U_{12}\ \mathrm{x}\ U_{12}> \rightarrow <U_{12}\ \mathrm{x}\ U_{12}>$

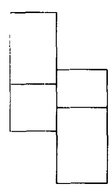


$<U_{23}\ \mathrm{x}\ U_{33}> \rightarrow <U_{23}\ \mathrm{x}\ U_{33}>$

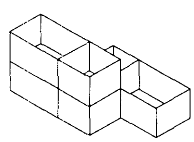


At this stage, the rules were encoded into the scripting language and 2d variations were derived from a single *parti*. The produced arrangements were exploded manually in AutoCAD to generate 3d forms. The original *parti* from which all boundary-layouts were derived, is

A



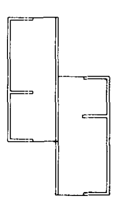
U_{12}



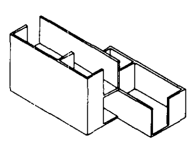
U_{23}

One of the produced boundary-layouts was chosen for implementation,

B



U_{12}

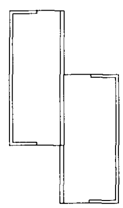
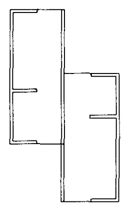
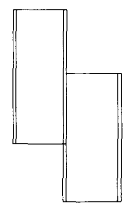
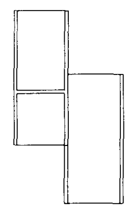
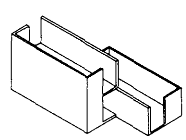
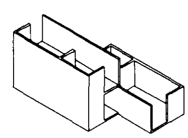
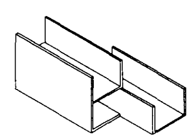
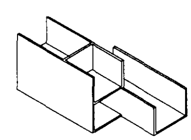


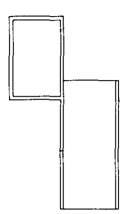
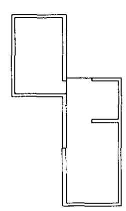
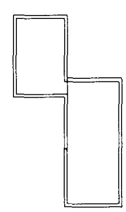
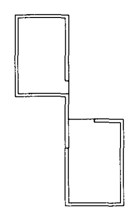
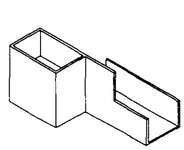
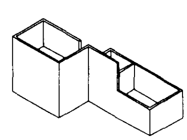
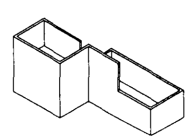
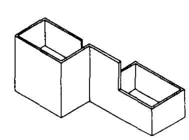
U_{33}

More boundary-layouts deriving from the original *parti*, according to the rules (a), (b), (c), (d), (e), and (f) exists in the next page.

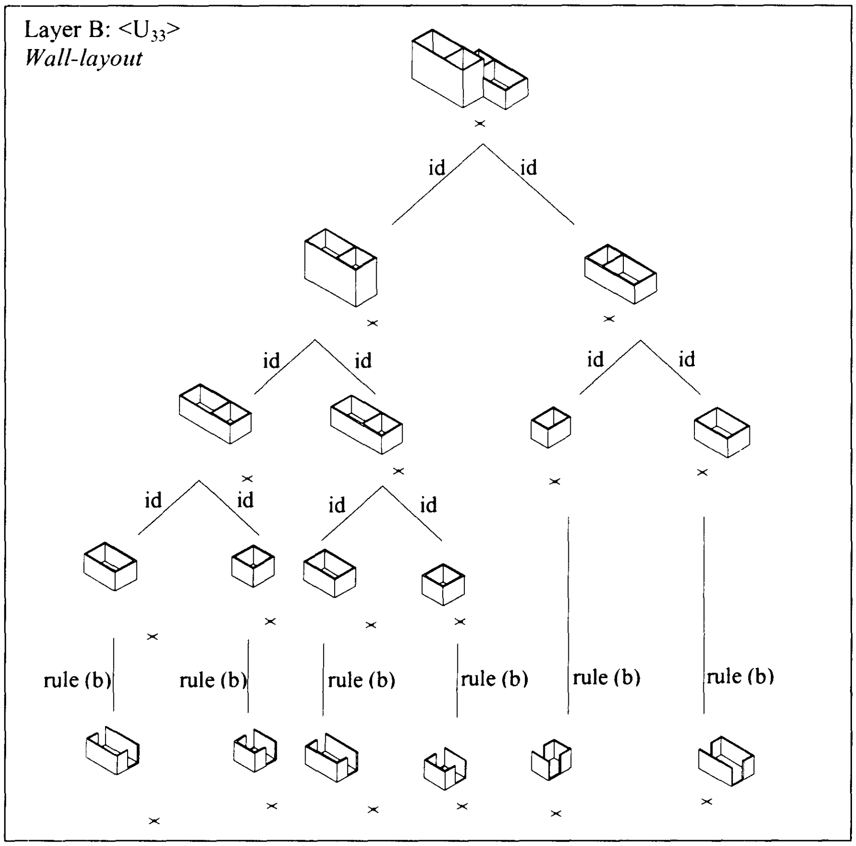
The variations B **i-viii** contain lines expressed in the algebra $<U_{12}>$ or solids representing alternative boundary-layouts, in the algebra $<U_{33}>$

TABLE 9. Samples Bi-viii of derived boundary-layouts from the parti

B_i	B_{ii}	B_{iii}	B_{iv}
			
			

B_v	B_{vi}	B_{vii}	B_{viii}
			
			

A diagrammatic representation of the derivation of the chosen boundary-layout is depicted in the following tree. At the root of the tree we see the layout before the application of the transformation rule (b). The layout is divided in rooms, where rule (b) applies. The transformed shapes that compose the new boundary-layout appear at the bottom leaves of the tree.



To summarize, the *parti* acts as a generic schema in the generation of designs. The *transformation* rules apply on an input *parti* to produce boundary-layouts. Rules for the definition of boundaries and the distribution of solid-void are some of the most common transformations. These rules require the superimposition of multiple graphic layers, or 3d-models. In the first layer we find the input parti, while other layers include alternative derivations. The participating layers and models contain descriptions X_1, X_2, X_3, ... X_n. The product X_1 x X_2 x X_3 x ... X_n forms a sub-algebra that corresponds to the stack of descriptions 1, 2, ... *n*.

6.3. REFINEMENT

In the refinement of boundary layouts we gradually determine their *tectonics*. The refinement process requires an input boundary layout from the preceding level on which the refinement rules apply. The refinement rules do not convert this input shape into one with a new *parti*.

In the working example, the input shapes exist in two layers: layer A and layer B. The *parti* lies in layer A, and the boundary-layout in layer B. Both inputs are represented in 2d and in 3d.

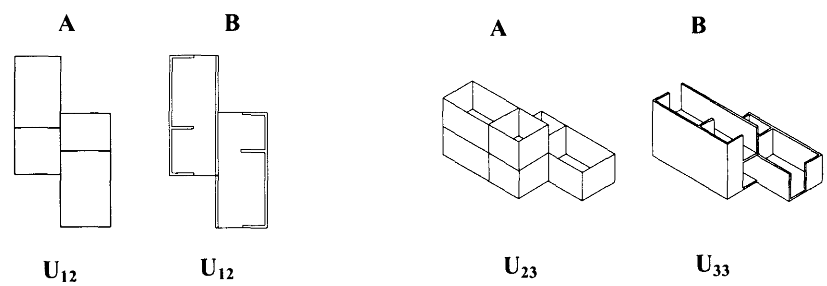


Figure 5. The input parti and boundary-layout in 2d (left) and 3d (right)

The general pattern of application of the refinement rules is the following: First, identity rules distinguish new sets of parts. Second, the rules of refinement apply on the distinguished parts. Third, the parts are recomposed for evaluation.

The input boundary layout is a shape that is decomposed and recomposed several times, with different purposes at view (proportion, structure, function). These decompositions are in continuous interdependence and contribute to a single final description.

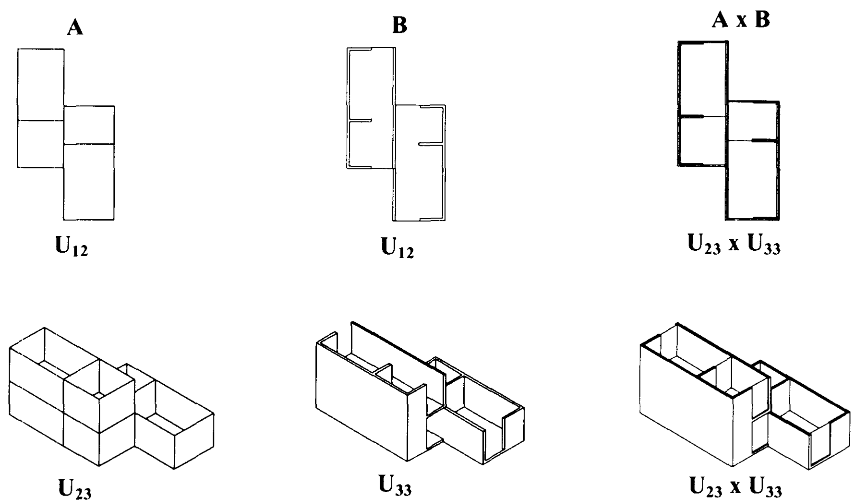
It is possible to introduce infinite sub-levels of refinement, and infinite descriptions, the exact ordering of which remains unattainable. The most elementary refinement process includes dimensioning, structural and functional refinements, and secondary, the specification of energy, lighting, and sound performance of the building.

The present study examines only the dimensioning, structural, and functional refinements. The next parametric rules attempt to capture some of the more characteristic steps in the process of those refinements. The rules are organized in three interdependent categories corresponding to dimensioning, structural and functional refinements. The developments in each general category are parallel. Their sequencing serves presentation purposes and does not show how the refinements occur in the process.

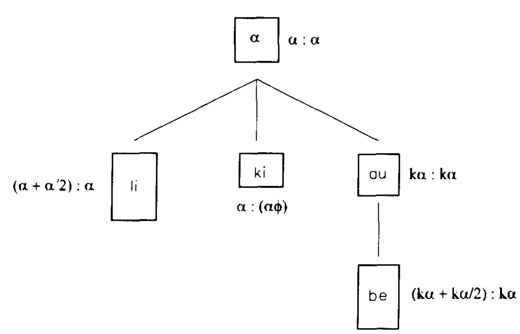
6.3.1. Dimensions

Part of the refinement process is the dimensioning of the rooms and of their boundaries (walls). The dimensioning rules control the dimensions of widths, lengths and the thickness of the building elements. The human scale and the choice of materials play significant role in dimensioning. Stylized treatments of proportion can be used as well. Changes in dimensions can be dictated by changes in other descriptions (i.e. structural, functional etc.). And conversely, changes in dimensions can impact the layout of boundaries, the distribution of openings etc.

From the previous phase, the two descriptive layers A and B include the *parti* and the boundary-layout, while the product A x B shows the association of the *parti* with the boundary-layout.



The general proportions of rooms are organized abstractly at the initial stage of *formation* (see Table 7, p. 125). In the working example these room-proportions follow the general schema



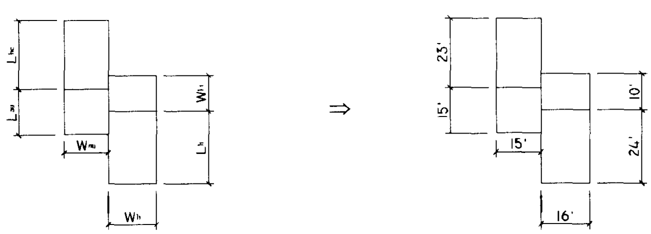
The specific numeric values for most dimensions are usually determined by standards, experience, or other criteria. Accordingly, the dimensions of the example are set to the following numeric values:

TABLE 10. Room-dimensions

PUBLIC "*pu*":	**living "*li*":** $W_{li} = \alpha = 16'$, and $L_{li} = \alpha + \alpha/2 = 16' + 8' = 24$
	kitchen "*ki*": $L_{ki} = W_{li} = 16'$, and $W_{ki} = 16' \cdot \phi \approx 10'$
PRIVATE "*pr*":	**auxiliary "*au*":** $W_{au} = \alpha \cdot \mathrm{k}$, ($\mathrm{k} \leq 1$), then $W_{au} = 15'$, and $L_{au} = W_{au} = 15'$
	bedroom "*be*": $L_{au} = W_{be} = 15'$, and $L_{be} = 15' + 15'/2 = 22\,½' \approx 23'$

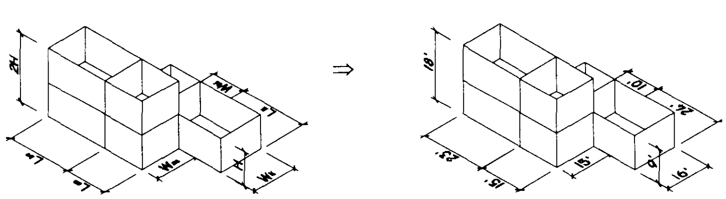
The numeric values of room widths, lengths and heights are first instantiated in the *parti*, at layer A, by substituting the appropriate numeric values in the place of the variables L_{li}, W_{li}, L_{ki}, W_{ki}, L_{au}, W_{au}, L_{be}, W_{be}.

A A

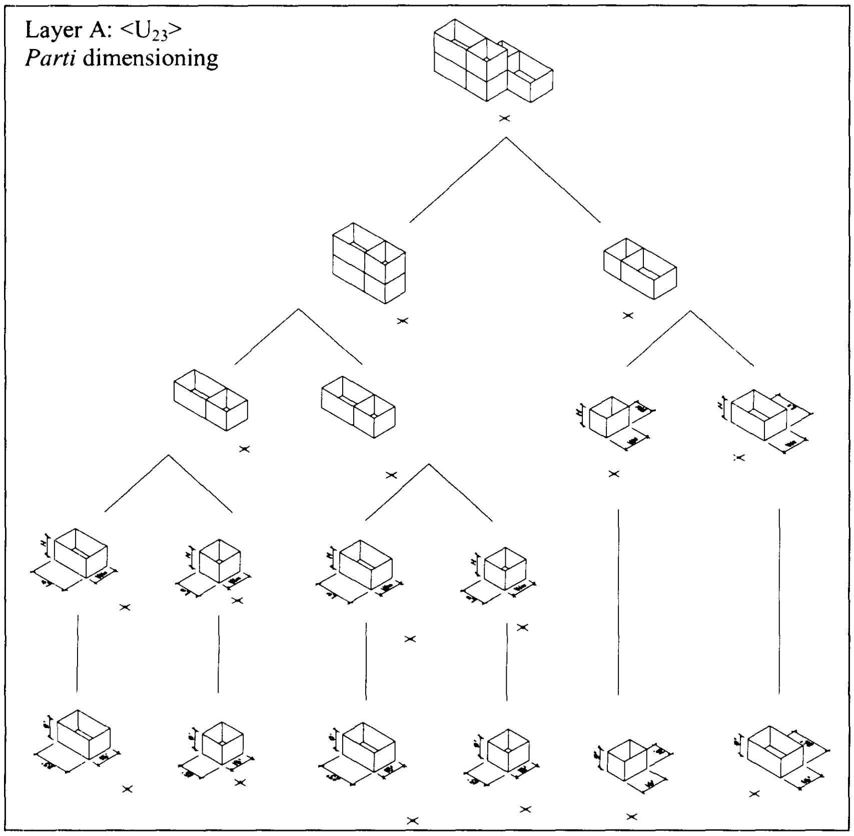


Also, in 3d descriptions the height H is set equal to 9′ (H = 9′)

A A

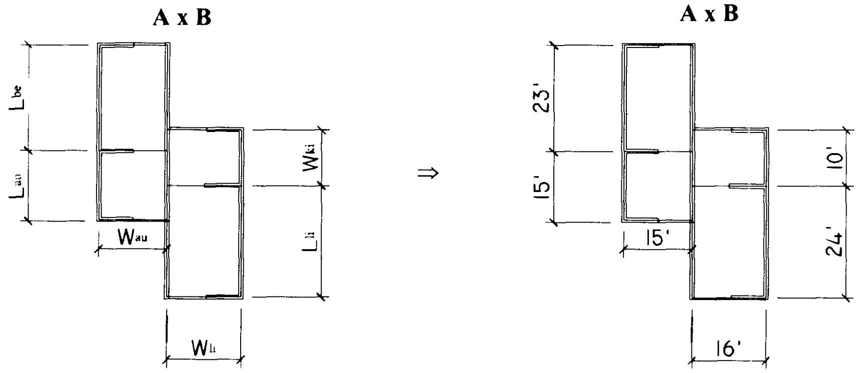


An overview of the dimensioning of the 3d *parti* is shown in the next tree



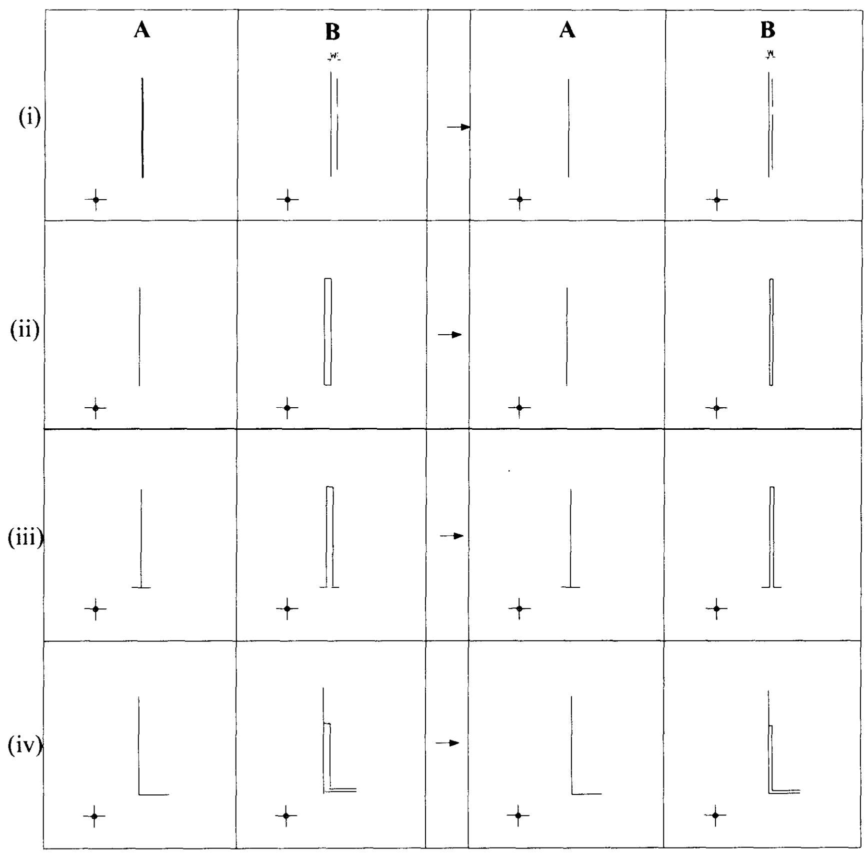
In general, the *parti* elements are always axial in relation to the walls. In 2d plans, this becomes obvious in the product A x B of the descriptions A and B, in the product algebra $\langle U_{12} \times U_{12} \rangle$

A x B **A x B**



The exact thickness of the boundaries (walls) is set after the general room-dimensions are given to the *parti*. The next example presents rules that re-adjust the width of a given boundary element in relation to a given *parti* element.

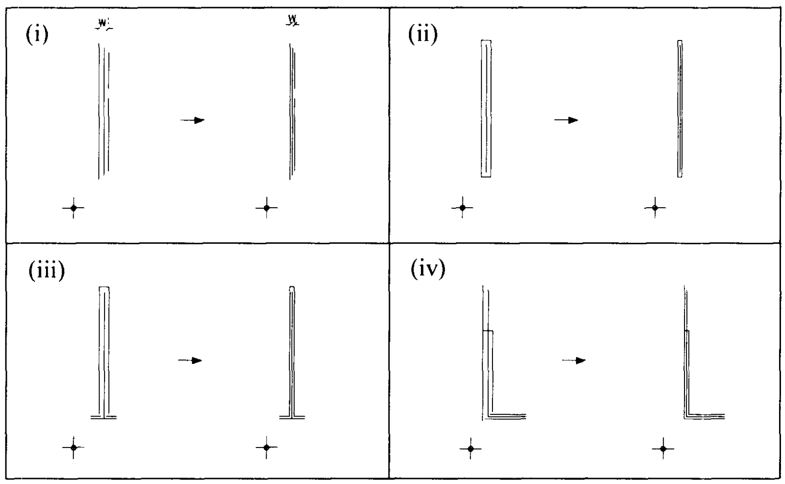
In 2d descriptions, the rule scans a *parti* line in layer A and the boundary lines in B, and sets the exact thickness of this boundary to the preferred width. The four rules presented bellow, apply deterministically in specific sequence, as one rule in four steps



rule (1) – steps i, ii, iii, iv

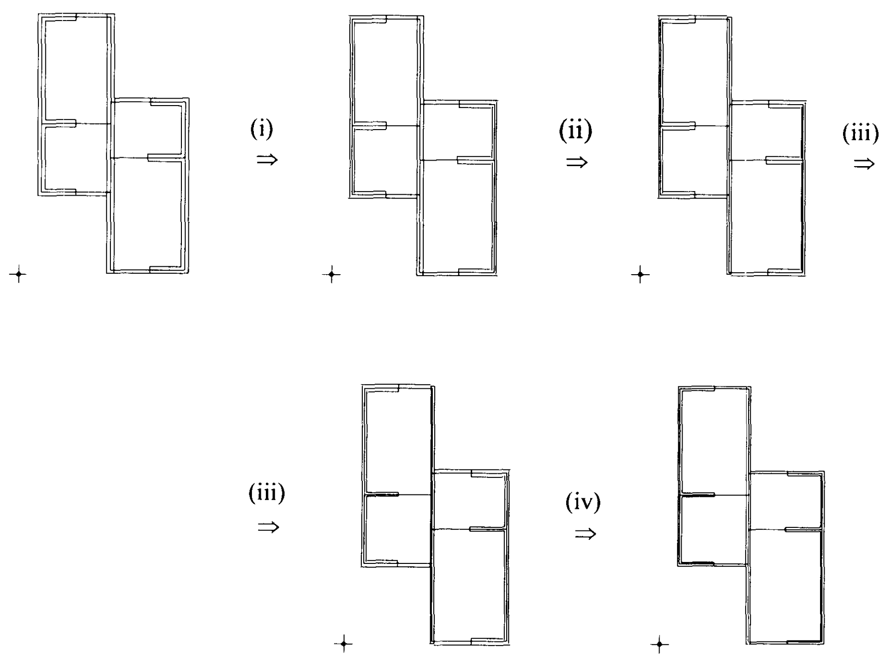
This adjustment of the wall thickness does not make distinction between exterior-interior walls. And, apparently rules like the above can be set in alternative ways. In the working example, the thickness w of walls is set equal to 6″ ($w = 6''$).

The rule sequence is shown as a product $\langle U_{12}xU_{12}\rangle \rightarrow \langle U_{12}xU_{12}\rangle$

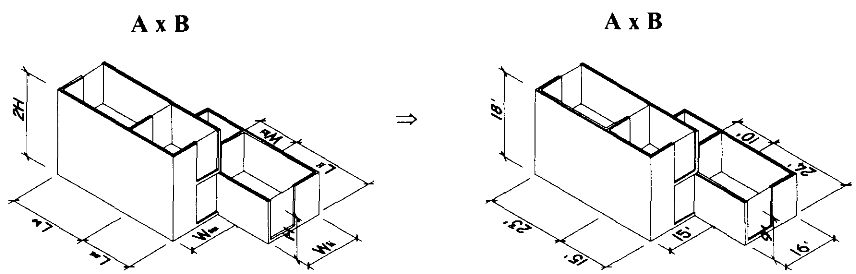


rule (1) – steps i, ii, iii, iv

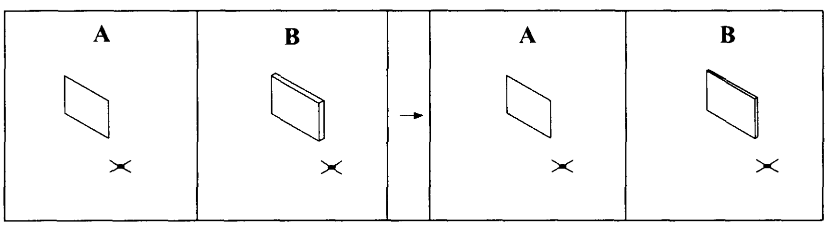
A derivation, of the working example, is shown below



In 3d models, the *parti* planes also lie axially in relation to the boundary elements. This becomes more obvious in the product A x B of the descriptions A, B in the product algebra $<U_{23} \text{ x } U_{23}>$

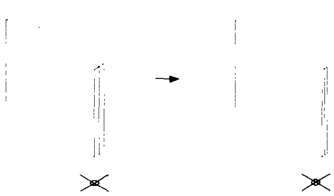


A general example of a 3d rule, for the adjustment of boundary widths, can be formed in the product algebra $<U_{23} \text{ x } U_{33}>$. The rule scans a *parti* plane in layer A and the existent boundary in layer B, and sets the exact thickness of the solid boundary to the preferred width



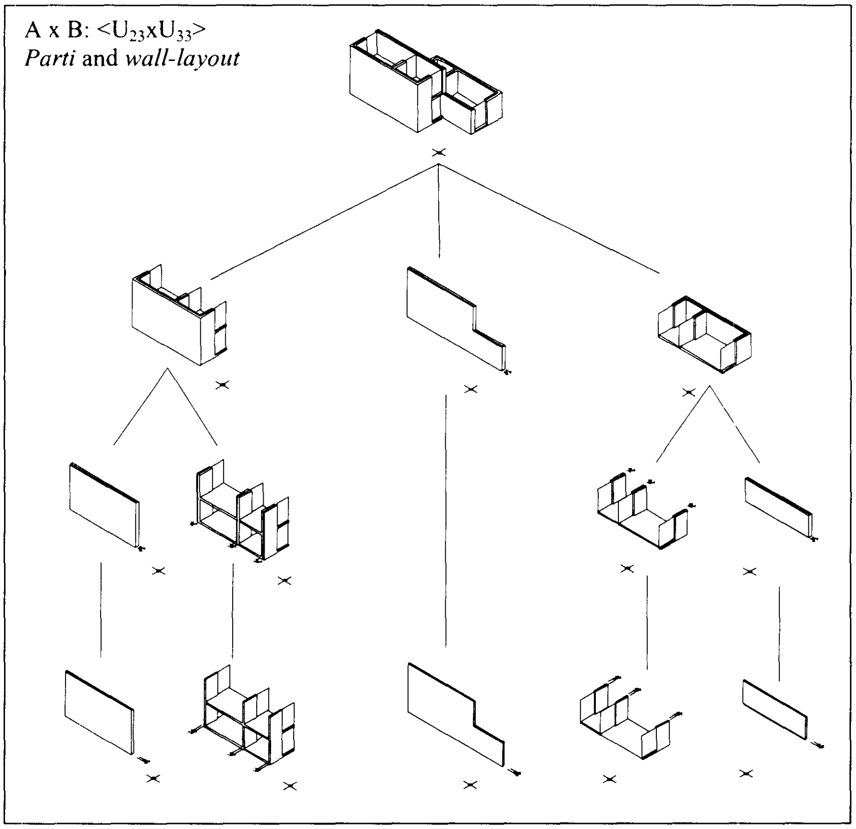
rule (1)

The above rule can be expressed in the product $<U_{23} \text{ x } U_{33}> \rightarrow <U_{23} \text{ x } U_{33}>$ of the descriptions A, B

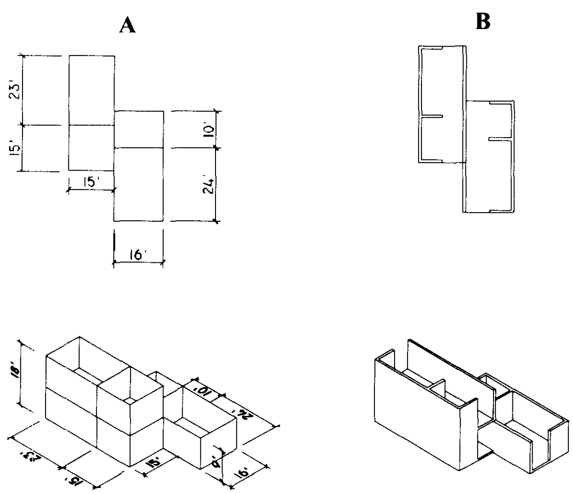


A diagram, depicting the process of width adjustment of boundaries in the 3d model, is presented next,

A x B: <U_{23}xU_{33}>
Parti and *wall-layout*



The output 2d and 3d descriptions in the layers A, B, after the insertion of the dimensions, are

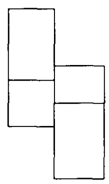


6.3.2. Structure

The structural refinement examines the structural behavior of an input arrangement. The structural behavior depends on the weights and the strength of the selected building components, but also, on the design: the forms, the openings, and the density of the elements. And, design decisions rely on construction decisions. This examination of structural refinement focuses on the interaction of the different descriptive layers.

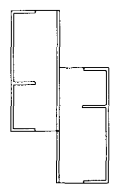
In the example layer A includes the *parti* and the layer B the wall-layout.

A

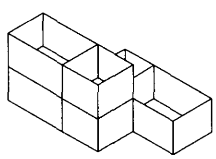


U_{12}

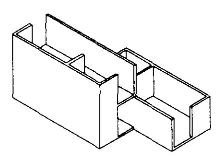
B



U_{12}



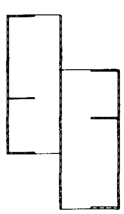
U_{23}



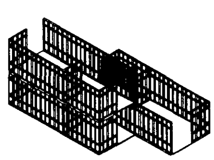
U_{33}

Layer C is used for the structural description. The example shows a structural frame in plan and axonometric. The frame includes horizontal and vertical structural elements.

C



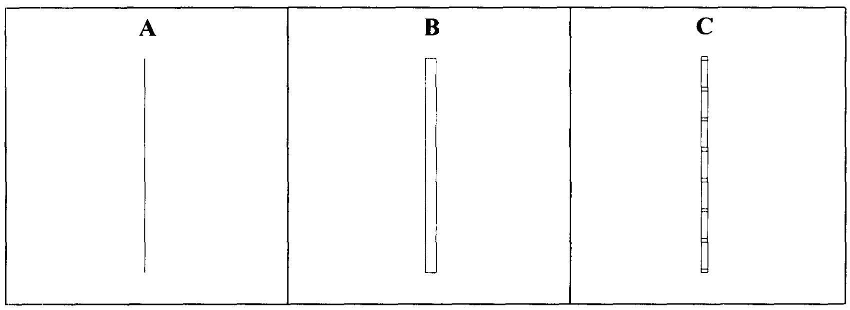
U_{12}



U_{33}

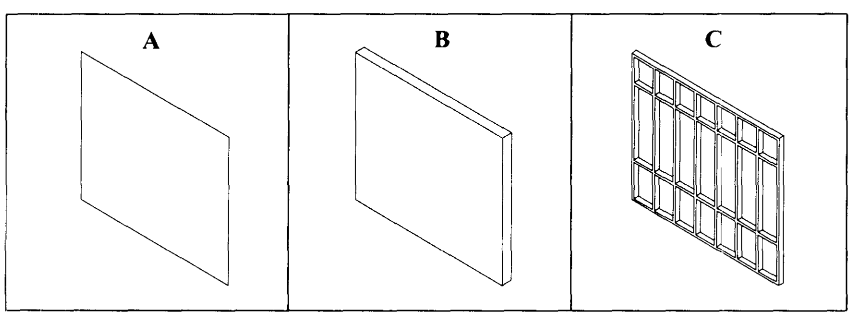
The overall description now includes three descriptive layers: *parti* (A), boundary-layout (B), structural-layout (C). Let us examine the correspondence of a single element in each of the descriptions A, B, and C.

In 2d plan: In layer A we see a *parti* line in the algebra U_{12}. The description B shows the corresponding wall element described as a rectangle in the algebra U_{12}. The description C shows the corresponding frame component as a shape in the algebra U_{12}.



U_{12} U_{12} U_{12}

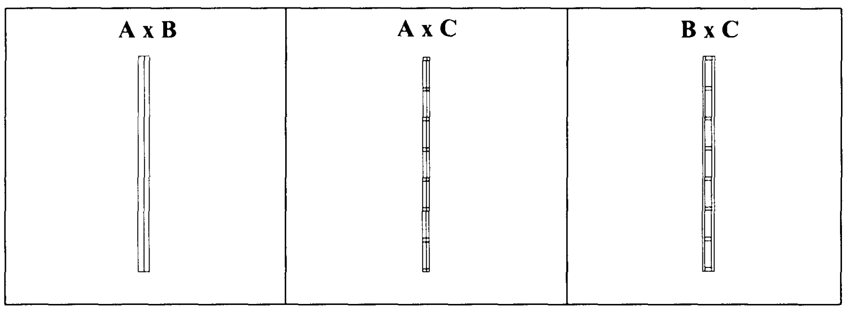
The above descriptions are presented in 3d axonometric: The description A shows a *parti* plane in the algebra U_{23}. The description B shows a wall element, described as a solid in the algebra U_{33}. The description C shows in axonometric a possible structure for a wood-frame, described as a shape in the algebra U_{33}.



U_{23} U_{33} U_{33}

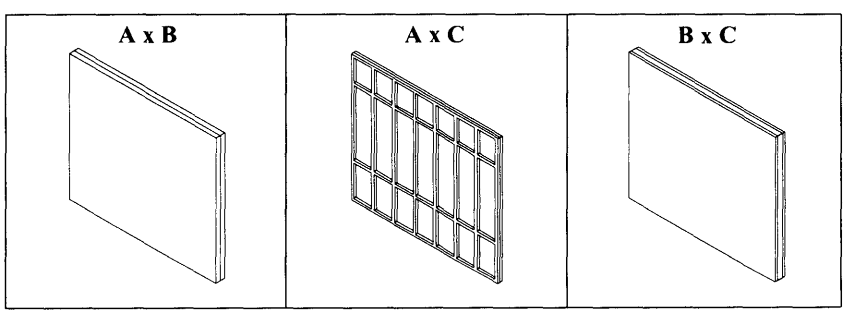
The descriptions A, B, C superimposed on one another reveal the relationships between the *parti* and the walls <A x B>, the *parti* and the structural frame <A x C>, and the walls and the structural frame <B x C>.

In 2d (in the product algebra U_{12} x U_{12}), the description A x B shows that a *parti* line lies axially in a wall element. The description A x C shows that a *parti* line lies axially in a frame component. Last, in the product B x C a structural frame component lies axially in a wall.



U_{12} x U_{12} U_{12} x U_{12} U_{12} x U_{12}

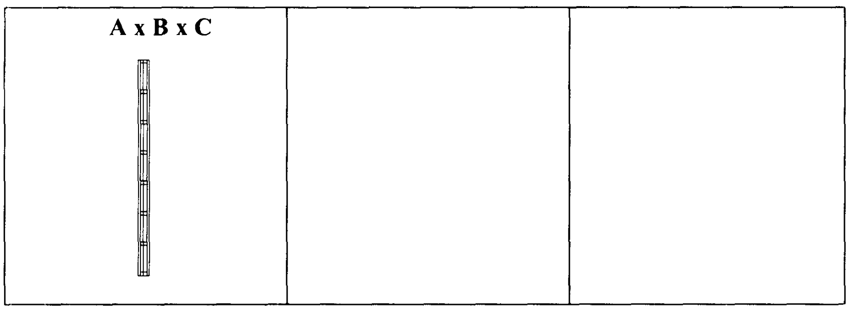
Next the descriptions are presented in 3d axonometric: The description A x B shows (in the product algebra U_{23} x U_{33}) a *parti* plane, in the algebra U_{23} dissecting axially a wall, described as a solid in the algebra U_{33}. In the description A x C a *parti* plane in the algebra U_{23} dissects axially a frame component, described as a solid shape in the algebra U_{33}. Last, the description B x C in the product algebra U_{33} x U_{33} shows a structural frame lying axially within a wall.



U_{23} x U_{33} U_{23} x U_{33} U_{33} x U_{33}

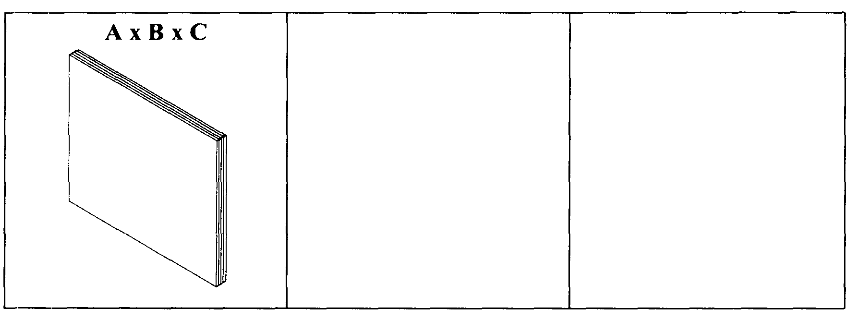
The product A x B x C describes the superimposition of the descriptive layers A, B and C.

In 2d, this description is the product algebra emerging from the participating U_{12} algebras $<U_{12} \text{ x } U_{12} \text{ x } U_{12}>$. The description reveals the relationship between the *parti*, the wall, and the structural frame: the *parti* line serves as the common axis, for both the structural frame and the wall layout.



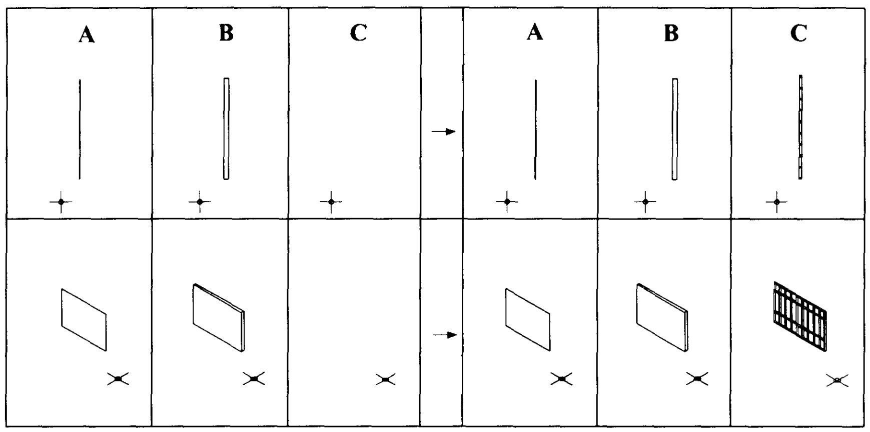
U_{12} x U_{12} x U_{12}

The above description presented in 3d axonometric is expressed in the product algebra $<U_{23} \text{ x } U_{33} \text{ x } U_{33}>$. The *parti* plane in the algebra U_{23} dissects axially the frame and the wall that are described as solids in the algebra U_{33}.



U_{23} x U_{33} x U_{33}

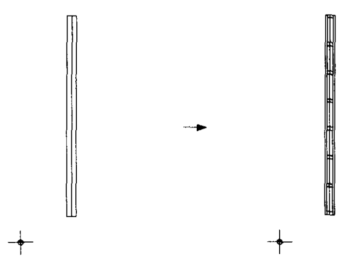
The next example of a rule shows how a wall obtains structural representation. The rule acts on the layers A, B and C: scans a *parti* element in layer A and a wall in layer B, and specifies a structural frame in layer C.



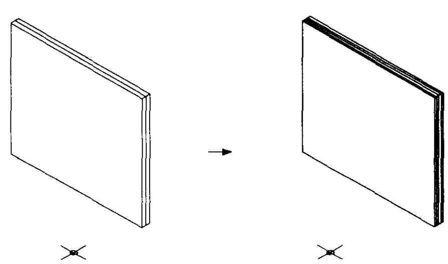
rule (2)

The above rule can also take the form $\langle A\ x\ B\ x\ C\rangle \rightarrow \langle A\ x\ B\ x\ C\rangle$:

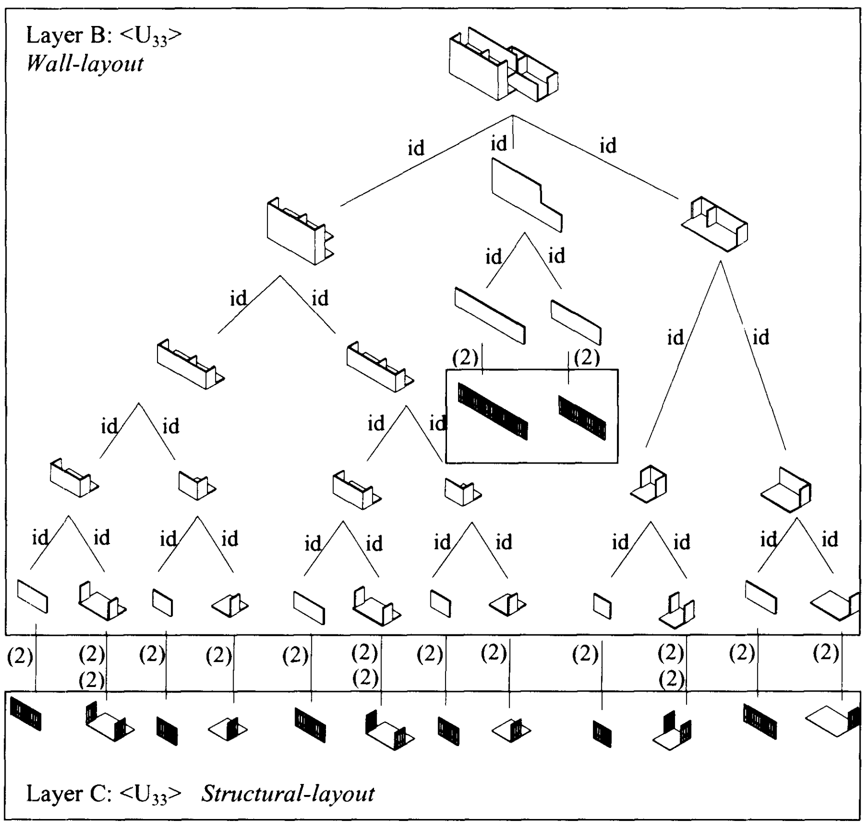
$\langle U_{12}\ x\ U_{12}\ x\ U_{12}\rangle \rightarrow \langle U_{12}\ x\ U_{12}\ x\ U_{12}\rangle$



$\langle U_{23}\ x\ U_{33}\ x\ U_{33}\rangle \rightarrow \langle U_{23}\ x\ U_{33}\ x\ U_{33}\rangle$

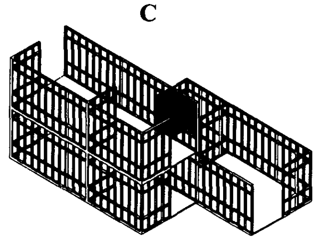


The arrangement is first decomposed with the aid of identities that distinguish parts. This decomposition may happen in alternative ways that can change the structural behavior of the design. The details of this decomposition are not presented here. The distinguished wall-parts obtain their structural frame by application of the rule (2). An overview of this process is appears in the next diagram,



The structural parts are recomposed to form a structural description

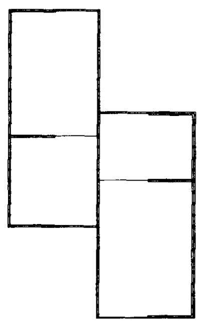
C



U_{33}

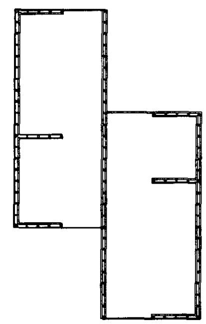
The products A x C and B x C, resulting from the superimposition of the descriptive layers A, C and B, C respectively, are presented next for the entire design. We see the relationship between *parti* and structure (A x C), and between walls and structure (B x C).

A x C

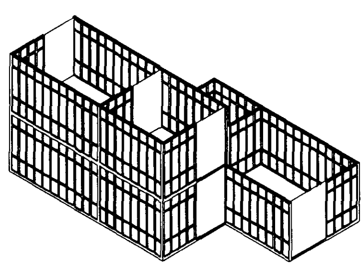


U_{12} x U_{12}

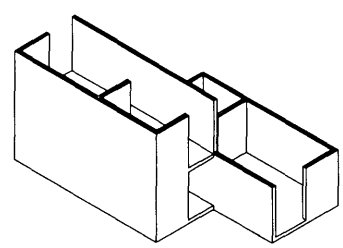
B x C



U_{12} x U_{12}



U_{23} x U_{33}

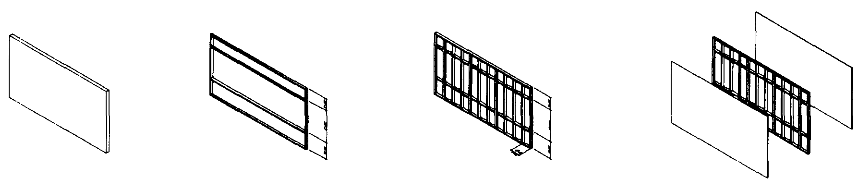


U_{33} x U_{33}

The selection of a structural system instead of another depends on a wide variety of criteria: The geometry, the forces that affect the building components, the materials, etc. A comprehensive structural description requires the calculation of forces, widths, lengths, thicknesses, and strengths for every structural element. For example, the specification of a thickness for a component made of a specific material prohibits its use beyond a certain length. And, the placement of windows and openings requires their coordination with the underlying structural frame.

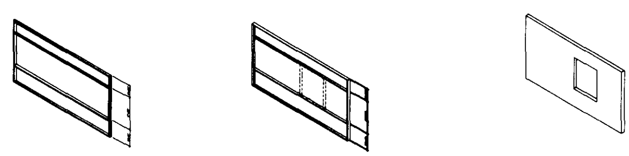
The specification of the structural details becomes the task of engineers. But construction details influence the design, and designers consider the implications of different structural solutions while designing. Although designers do not deal with the construction in full depth, they use structural descriptions and construction details to design. There is a constant interaction between structural descriptions and design descriptions.

In our simple working example, the selection of wood-frame structure implies that every wall (i) is a composite element. The structure includes a wood-frame made out of horizontal elements (ii) and vertical elements (iii), and two dry-walls (iv). This detail is depicted in the following sequence



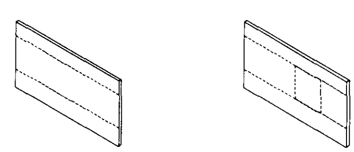
(i) (ii) (iii) (iv)

A possible way in which a structural description is used in design is the following: The horizontal beams determine a "zone" suitable for the placement of windows. Therefore, the structural grid of beams (ii) becomes a grid for windows (ii-a, ii-b)



(ii) (ii-a) (ii-b)

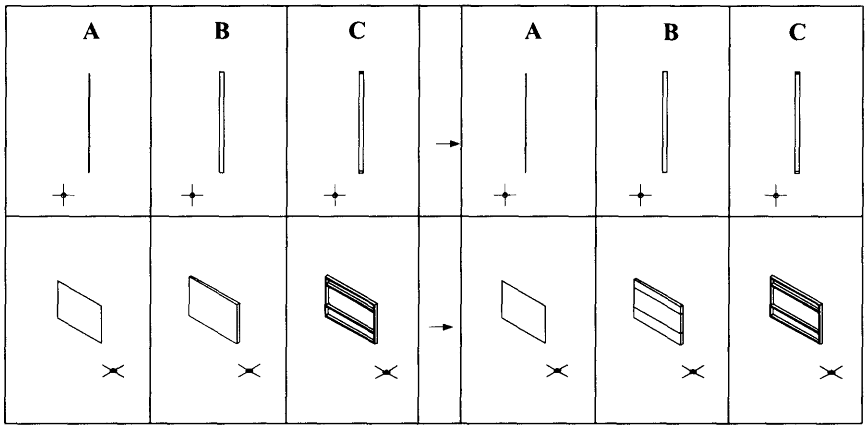
The next description includes the information relevant to the placement of windows. Two parallel axes, indicating the existence of the underlying horizontal beams (ii), are used to define the wall-part that windows or openings can be placed (ii-c, ii-d)



(ii-c) (ii-d)

The next rule-example shows how the above simple idea can be expressed in a shape rule.

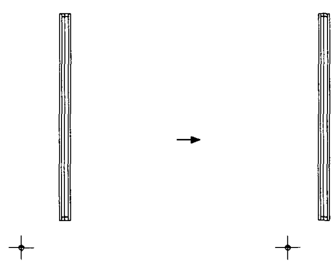
The rule scans a *parti* element in layer A, a wall in layer B and its corresponding structural frame in layer C. The elements in layers A and C remain intact. The wall in layer B is dissected in three horizontal stripes.



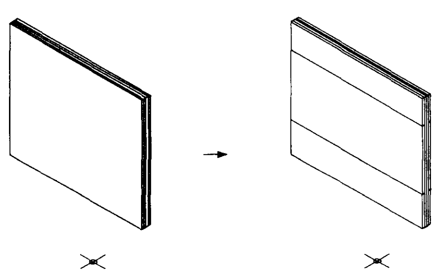
rule (3)

The rule can take the form $<A \text{ x } B \text{ x } C> \rightarrow <A \text{ x } B \text{ x } C>$:

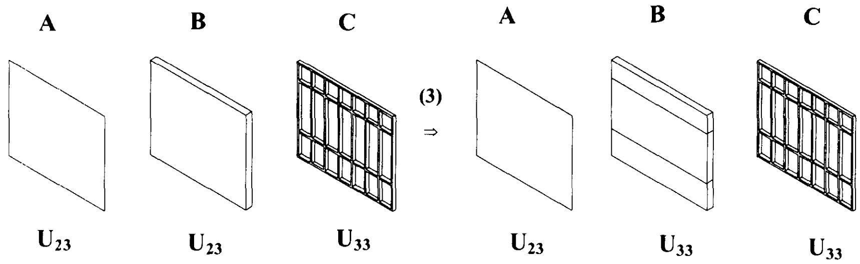
$< U_{12} \text{ x } U_{12} \text{ x } U_{12}> \rightarrow <U_{12} \text{ x } U_{12} \text{ x } U_{12}>$



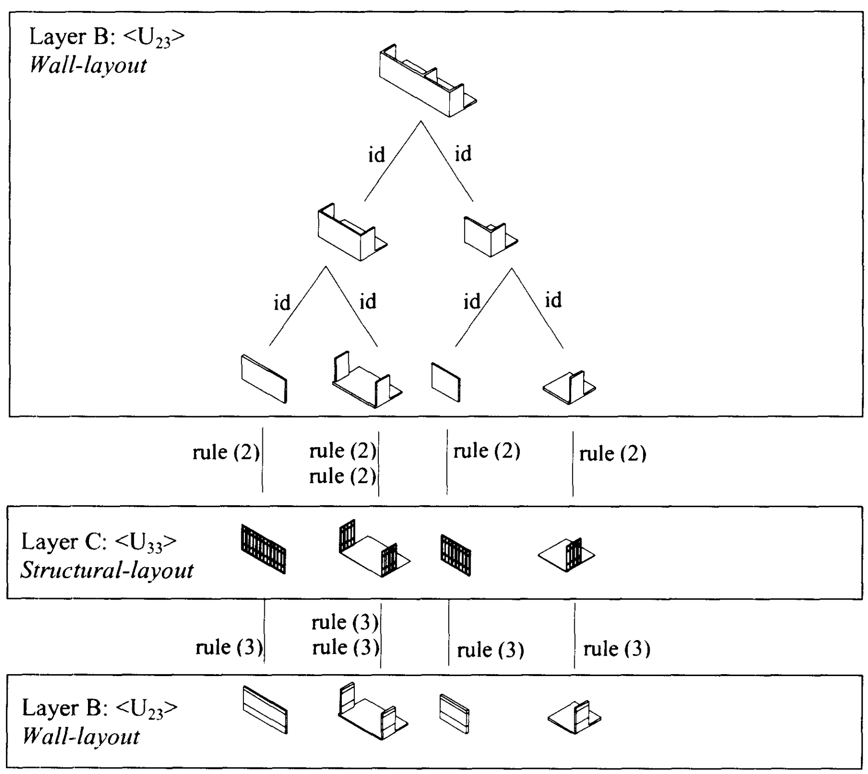
$< U_{23} \text{ x } U_{33} \text{ x } U_{33}> \rightarrow < U_{23} \text{ x } U_{33} \text{ x } U_{33}>$



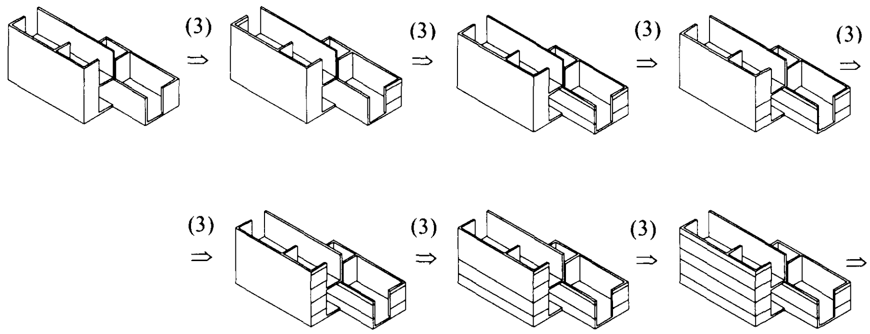
The application of rule (3) causes the division of a wall into three solid parts. The next derivation presents the result of applying rule (3)



Rule (3) determines where windows can be placed. The next diagram is a sub-part of the tree diagram of p. 171. It shows how rule (2) applies on every wall and determines its underlying structure in layer C. Further, rule (3) applies to determine the window-grid, in layer B



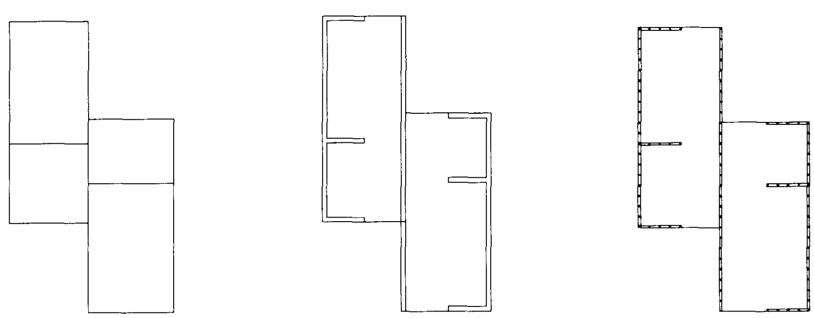
A sample derivation of applying rule (3) is offered below. Since the layers A and C remain unaffected by rule (3), the derivation includes only the wall-layout, in layer B, in the algebra <U_{33}>,



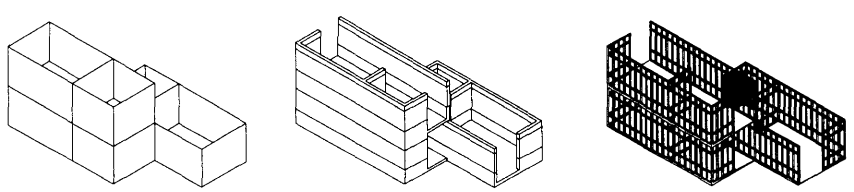
...etc.

Finally, after the application of the rules (2) and (3) is completed the resulting 2d and 3d descriptions in the layers A, B and C are the following

A **B** **C**



$\mathbf{U_{12}}$ $\mathbf{U_{12}}$ $\mathbf{U_{12}}$

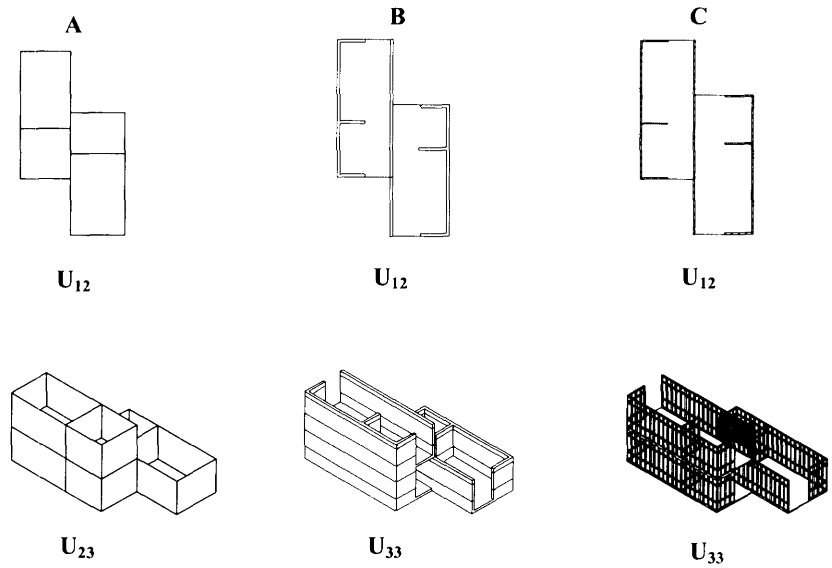


$\mathbf{U_{23}}$ $\mathbf{U_{33}}$ $\mathbf{U_{33}}$

6.3.3. Function

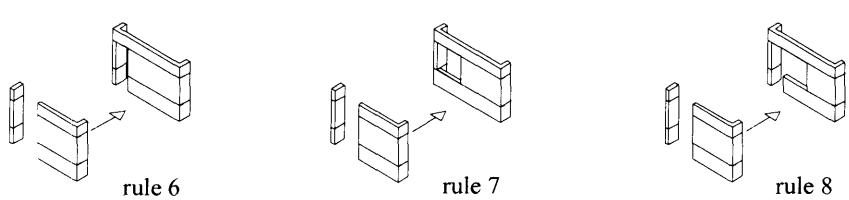
The next section attempts to capture characteristic steps in the process of functional refinement of spatial arrangements. It employs rules that deal with the coordination of details such as stairs, windows, doors etc. and the fulfillment of functional standards. The brief presentation offers some general examples of rules for doors and windows.

In the working example, the description includes three descriptive layers: *parti* (A), boundary-layout (B), structural-layout (C),

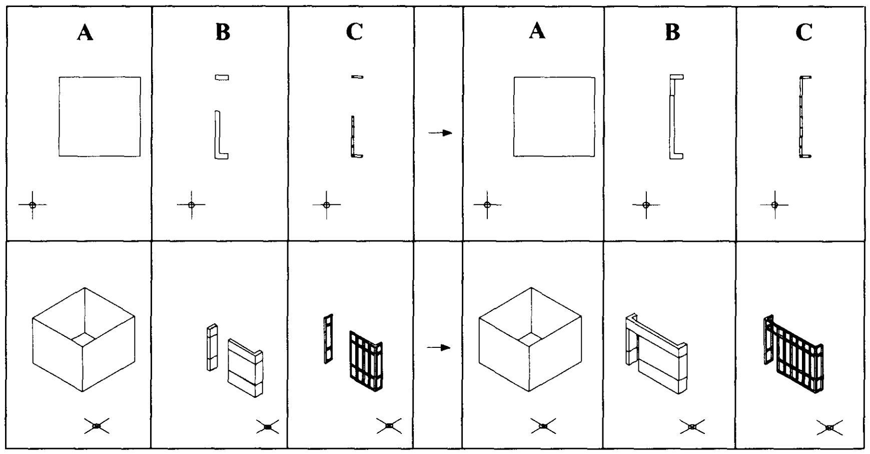


However, the use of all the three layers is not obligatory. The descriptive layer that is mainly used in the next examples is layer B. The rules are presented first diagrammatically in the algebra U_{33}, as they would apply in layer B. Then, they are presented for all three layers A, B, and C. The ordering of the rules does not indicate their sequencing in the actual process, where the developments can be interchangeable, or simultaneous.

The next three parametric rules (6-8) specify what kind of opening is applied on an existing void. Three opening-types are applied: door, window, and door-window. The three rules are presented here diagrammatically in the algebra U_{33}, as they apply in layer B

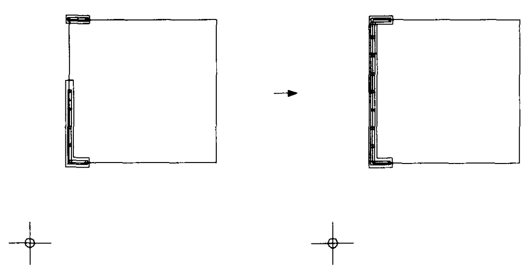


The three parametric rules are presented next as they would apply in all the layers A, B and C. Parametric rule (6) applies a door to an existing void,

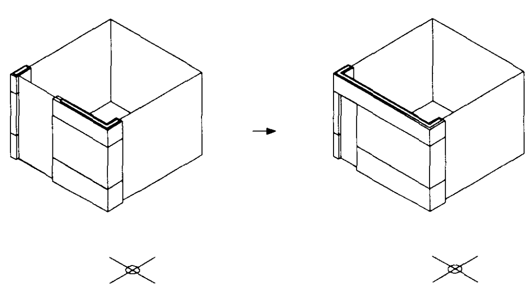


rule (6)

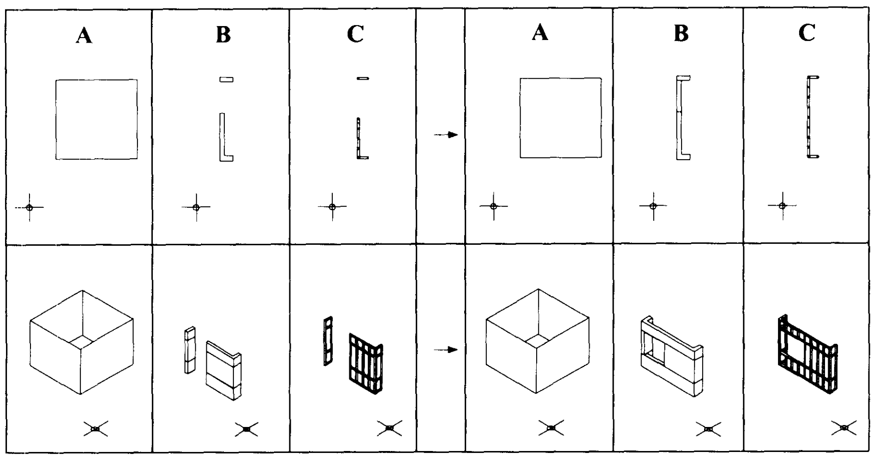
$\langle U_{12}\ \mathrm{x}\ U_{12}\ \mathrm{x}\ U_{12}\rangle \rightarrow \langle U_{12}\ \mathrm{x}\ U_{12}\ \mathrm{x}\ U_{12}\rangle$



$\langle U_{23}\ \mathrm{x}\ U_{33}\ \mathrm{x}\ U_{33}\rangle \rightarrow \langle U_{23}\ \mathrm{x}\ U_{33}\ \mathrm{x}\ U_{33}\rangle$

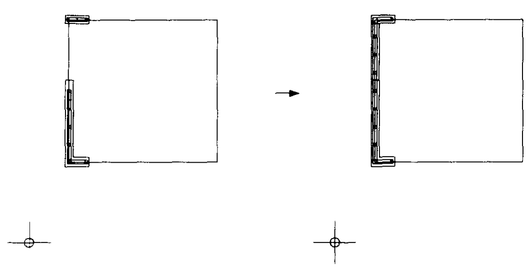


Parametric rule (7) applies a window to an existing void,

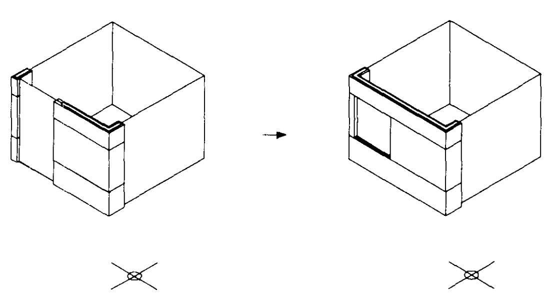


rule (7)

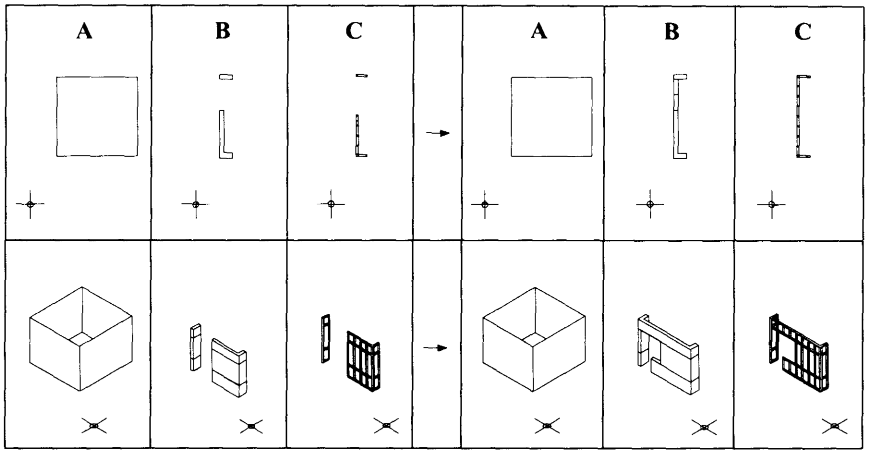
$<U_{12} \text{ x } U_{12} \text{ x } U_{12}> \rightarrow <U_{12} \text{ x } U_{12} \text{ x } U_{12}>$



$<U_{23} \text{ x } U_{33} \text{ x } U_{33}> \rightarrow <U_{23} \text{ x } U_{33} \text{ x } U_{33}>$

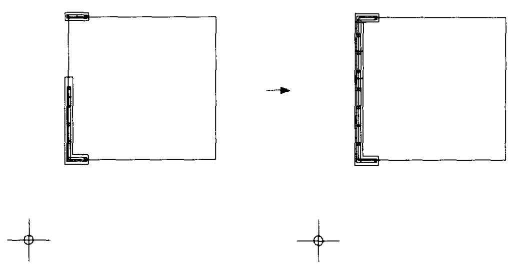


Parametric rule (8) applies a door and window, to an existing void,

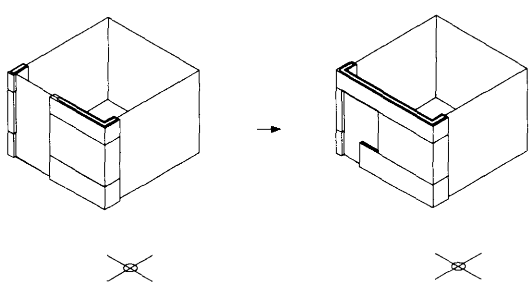


rule (8)

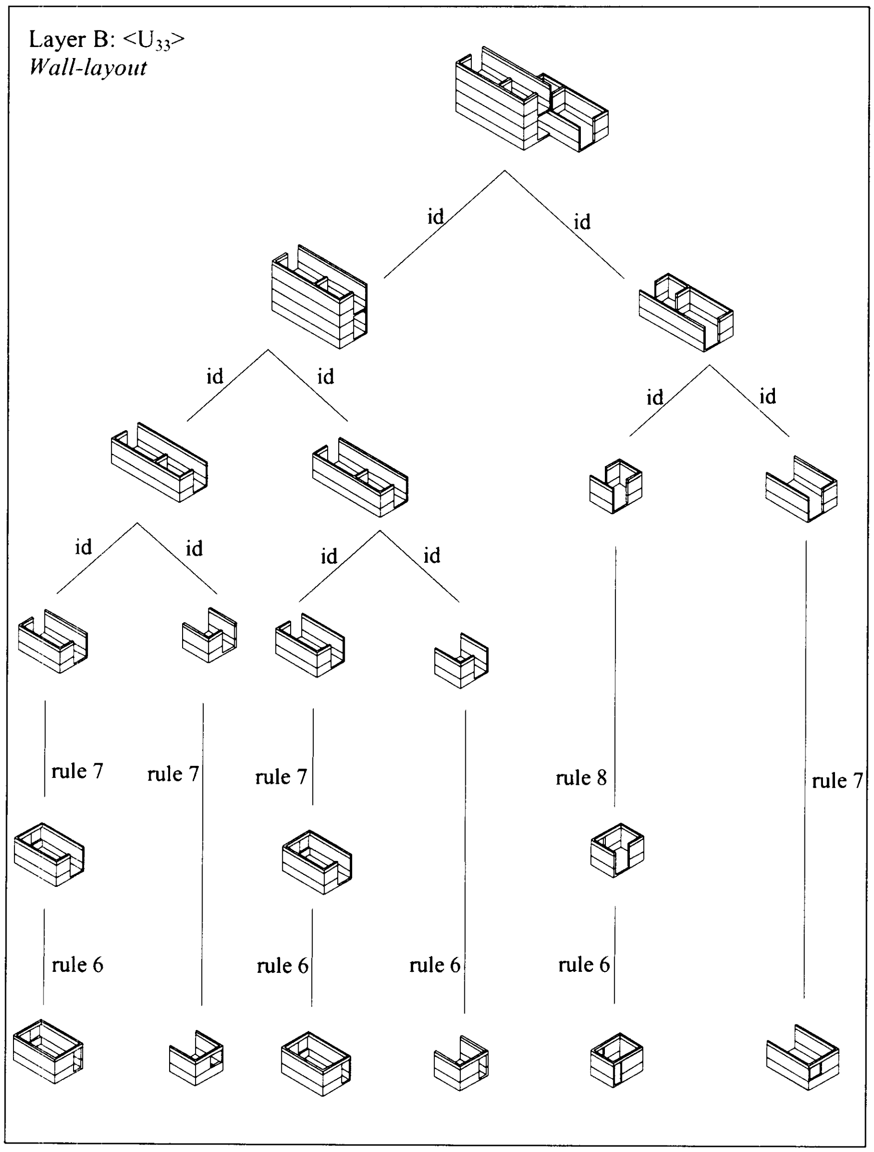
$<U_{12}$ x U_{12} x $U_{12}> \rightarrow <U_{12}$ x U_{12} x $U_{12}>$



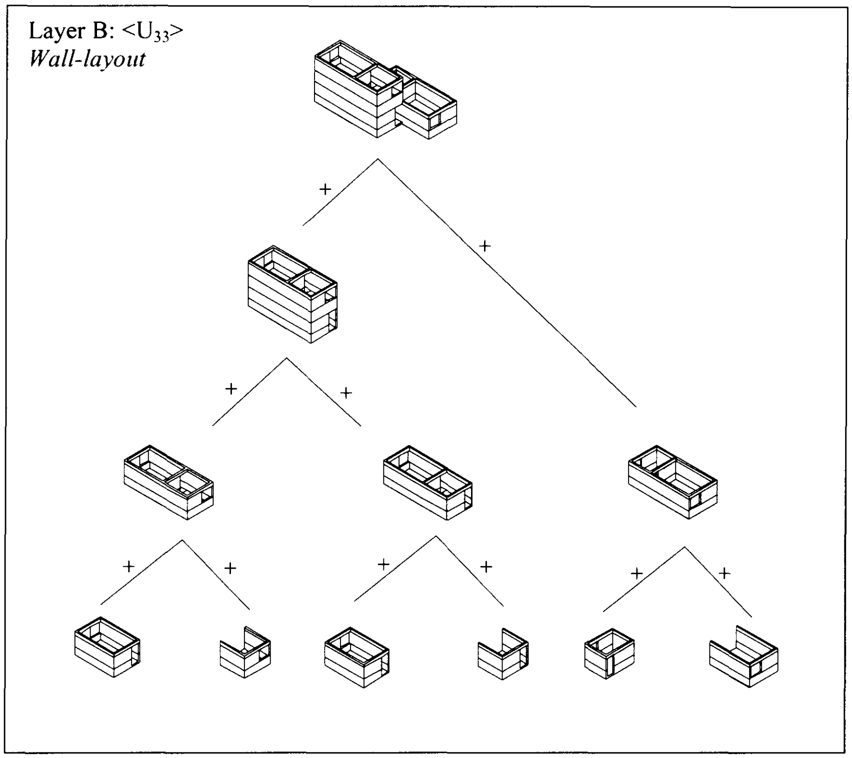
$<U_{23}$ x U_{33} x $U_{33}> \rightarrow <U_{23}$ x U_{33} x $U_{33}>$



An overview of the developments caused by the rules (6-8) appears in the following diagram. The diagram describes only the layer B: The input spatial arrangement appears at the root. The arrangement is divided and decomposed in parts. This decomposition is not permanent. It is a description that serves the particular stage of the process. The rules (6), (7) and (8) apply to modify the distinguished parts, and determine the openings.

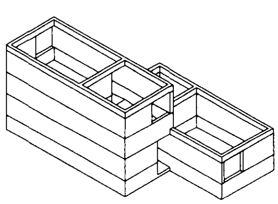


The modified building parts are reassembled for evaluation of the results. The re-assembling is substitution of the modified parts in their original positions. This action can be illustrated as addition of the parts in their original positions, with respect to the same coordinate system. At the root of the tree see the derived arrangement.



The output arrangement in layer B, is

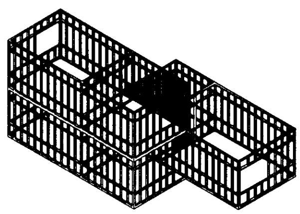
B



U_{33}

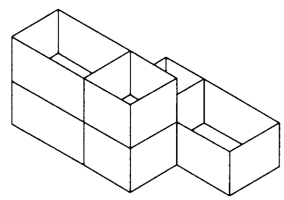
The produced arrangements in the layers C and A, after the application of the rules (6), (7) and (8) are

C



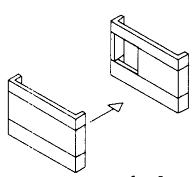
U_{33}

A

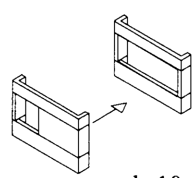


U_{23}

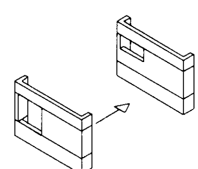
Five more parametric rules (9-12), for openings are presented next in the algebra U_{33}. The parametric rule 9 creates a new window-opening. Rules 10 and 11 are rules of adjustment of length and height of window-openings



rule 9

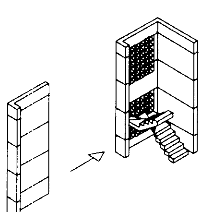


rule 10

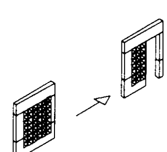


rule 11

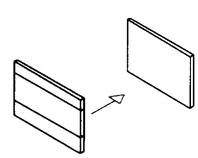
Rule 12 determines the position of a vertical circulation element (stair), in combination with a particular type of window that signifies it. Rule 13 creates a door embedded within the preexistent opening. And, rule 14 eliminates the window grid form the description



rule 12



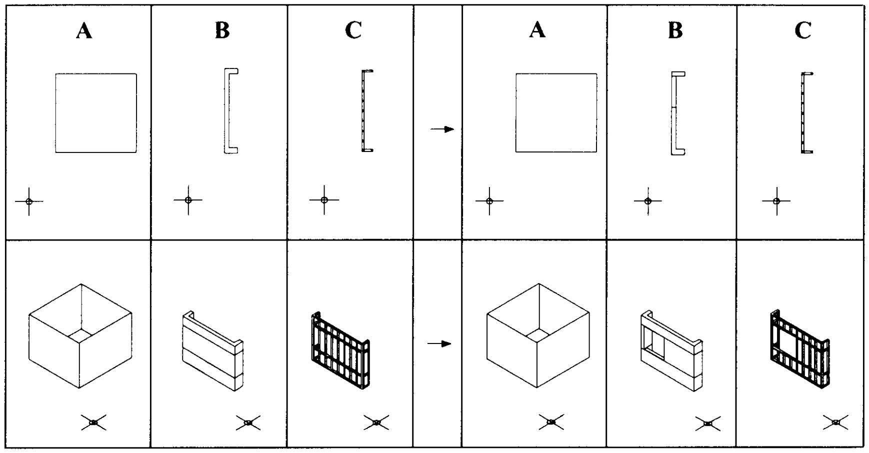
rule 13



rule 14

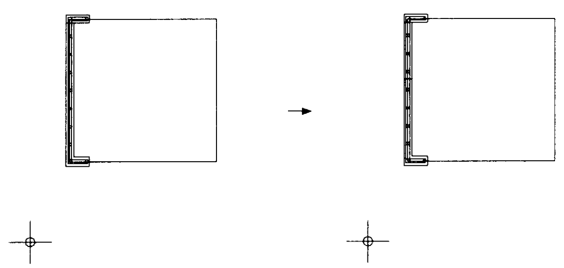
The five parametric rules (9-14) are presented next in detail, as they apply in all three descriptive layers A, B, C.

Parametric rule (9) applies a new window-opening to a wall

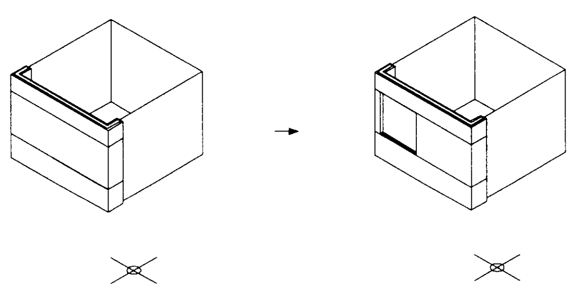


rule (9)

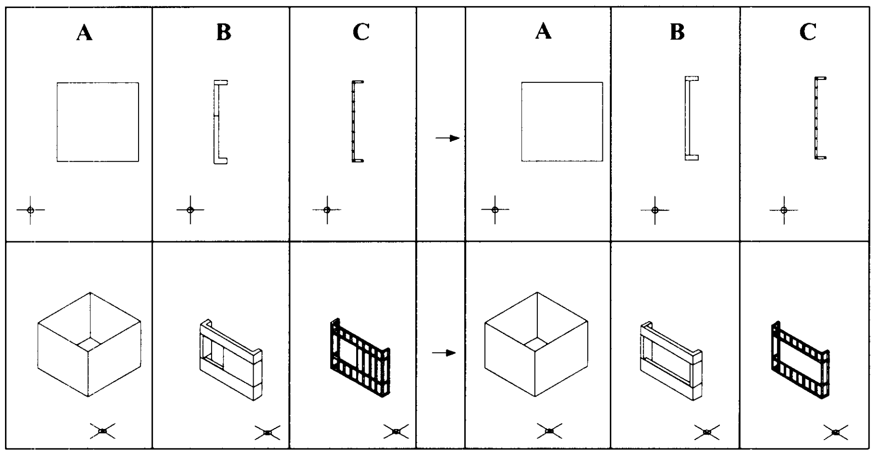
$\langle U_{12}\ \mathrm{x}\ U_{12}\ \mathrm{x}\ U_{12} \rangle \rightarrow \langle U_{12}\ \mathrm{x}\ U_{12}\ \mathrm{x}\ U_{12} \rangle$



$\langle U_{23}\ \mathrm{x}\ U_{33}\ \mathrm{x}\ U_{33} \rangle \rightarrow \langle U_{23}\ \mathrm{x}\ U_{33}\ \mathrm{x}\ U_{33} \rangle$

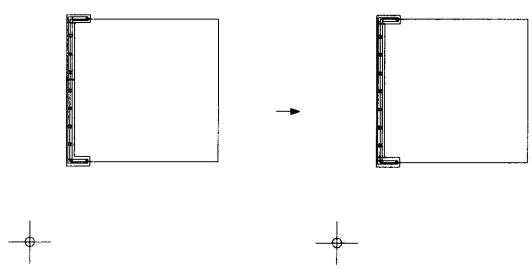


Parametric rule (10) extends the width of an existing window-opening,

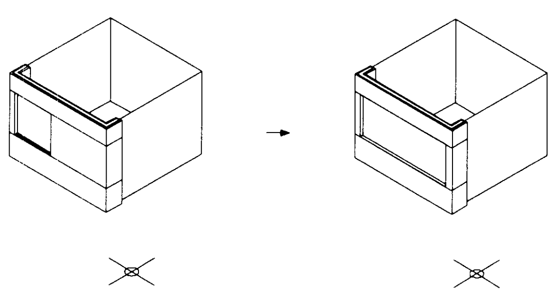


rule (10)

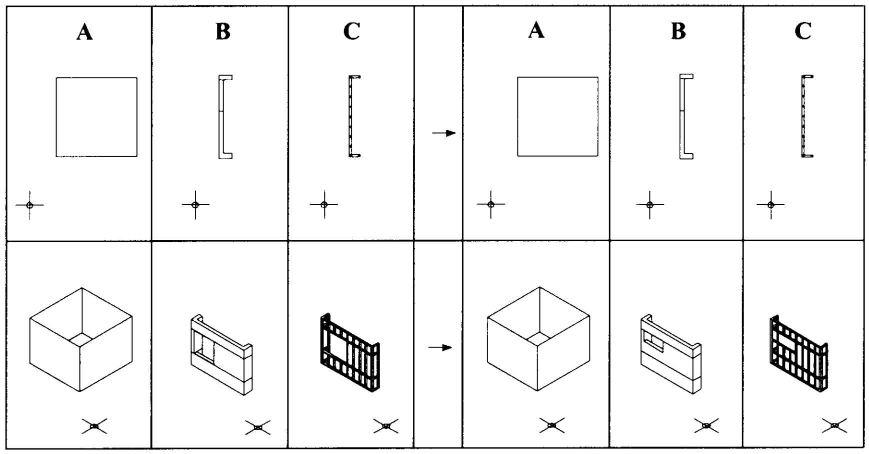
$<U_{12} \mathrm{~x~} U_{12} \mathrm{~x~} U_{12}> \rightarrow <U_{12} \mathrm{~x~} U_{12} \mathrm{~x~} U_{12}>$



$<U_{23} \mathrm{~x~} U_{33} \mathrm{~x~} U_{33}> \rightarrow <U_{23} \mathrm{~x~} U_{33} \mathrm{~x~} U_{33}>$

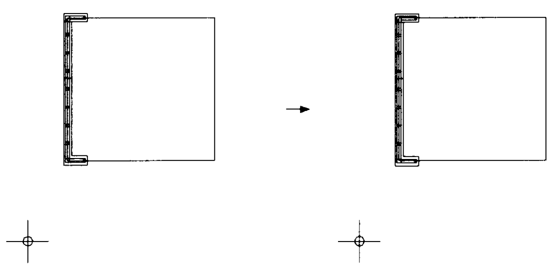


Parametric rule (11) modifies the height of an existing window-opening

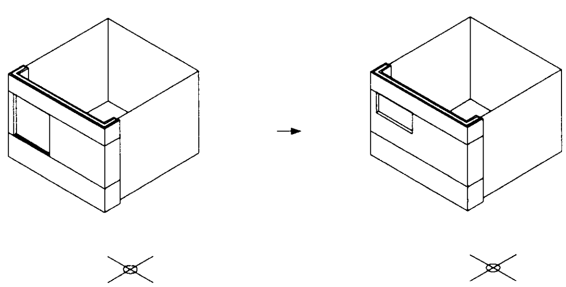


rule (11)

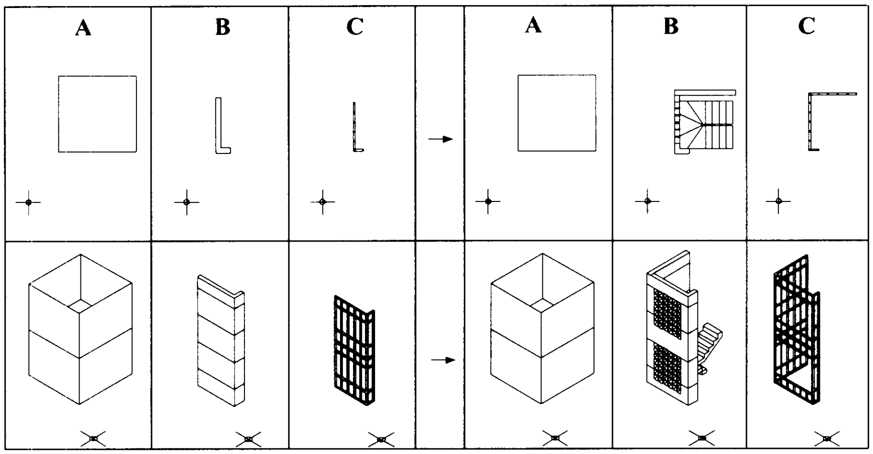
$\langle U_{12}\ \mathrm{x}\ U_{12}\ \mathrm{x}\ U_{12}\rangle \rightarrow \langle U_{12}\ \mathrm{x}\ U_{12}\ \mathrm{x}\ U_{12}\rangle$



$\langle U_{23}\ \mathrm{x}\ U_{33}\ \mathrm{x}\ U_{33}\rangle \rightarrow \langle U_{23}\ \mathrm{x}\ U_{33}\ \mathrm{x}\ U_{33}\rangle$

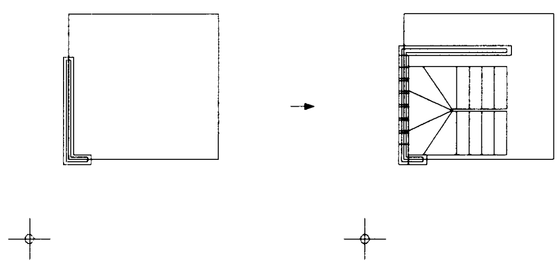


Parametric rule (12) applies a new type of opening to a wall. The rule determines the position of a stair and the opening in relation to it

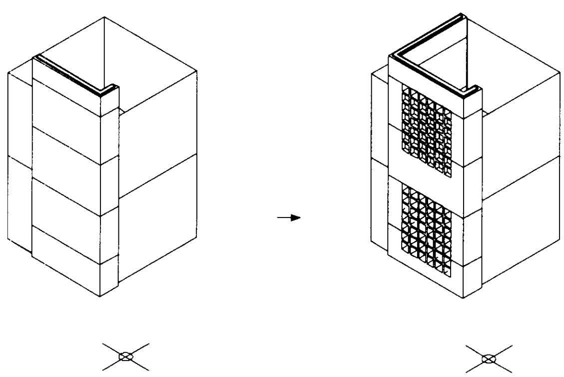


rule (12)

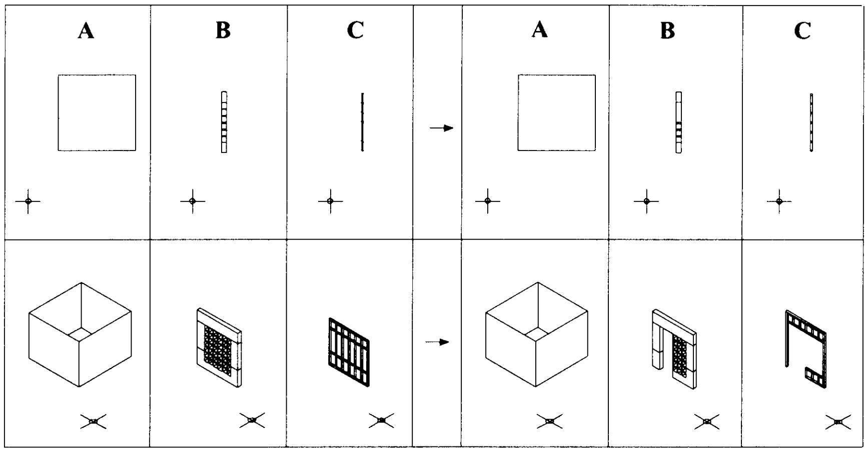
$<U_{12} \text{ x } U_{12} \text{ x } U_{12}> \rightarrow <U_{12} \text{ x } U_{12} \text{ x } U_{12}>$



$<U_{23} \text{ x } U_{33} \text{ x } U_{33}> \rightarrow <U_{23} \text{ x } U_{33} \text{ x } U_{33}>$

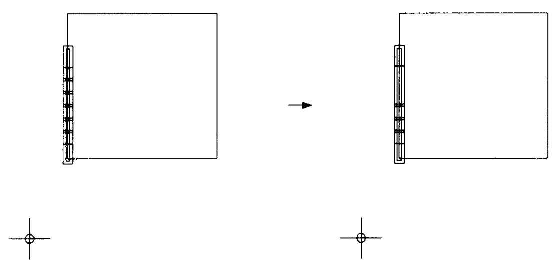


Parametric rule (13) creates a door within the existing previous opening,

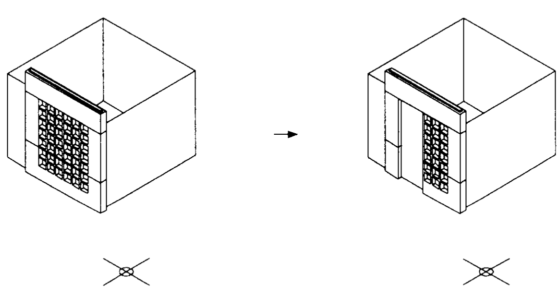


rule (13)

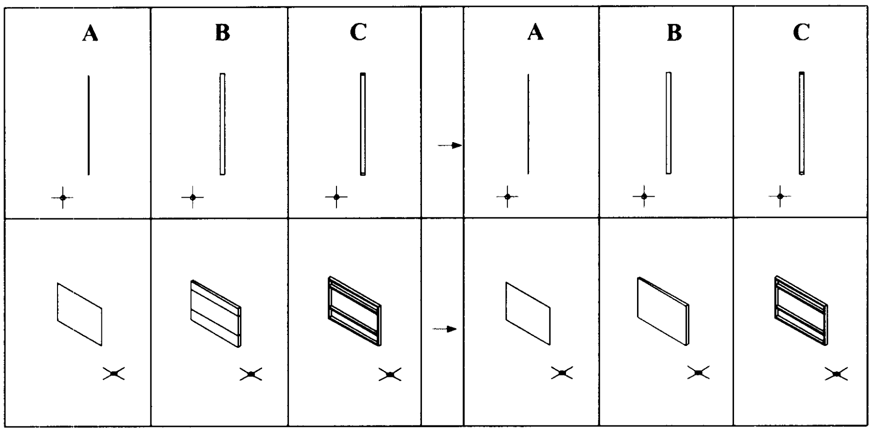
$<U_{12} \times U_{12} \times U_{12}> \rightarrow <U_{12} \times U_{12} \times U_{12}>$



$<U_{23} \times U_{33} \times U_{33}> \rightarrow <U_{23} \times U_{33} \times U_{33}>$

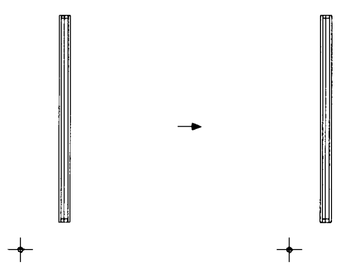


Rule (14) has the opposite action from rule (3) [see p. 174]. It scans a *parti* shape in layer A, a wall in layer B, and its structural frame in layer C. Shapes in A and C remain intact. The wall in layer B is union in one piece.

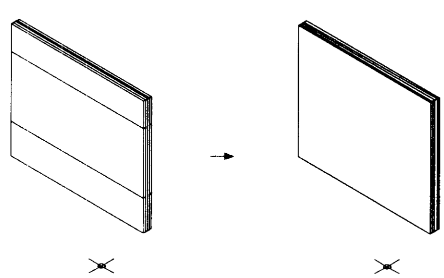


rule (14)

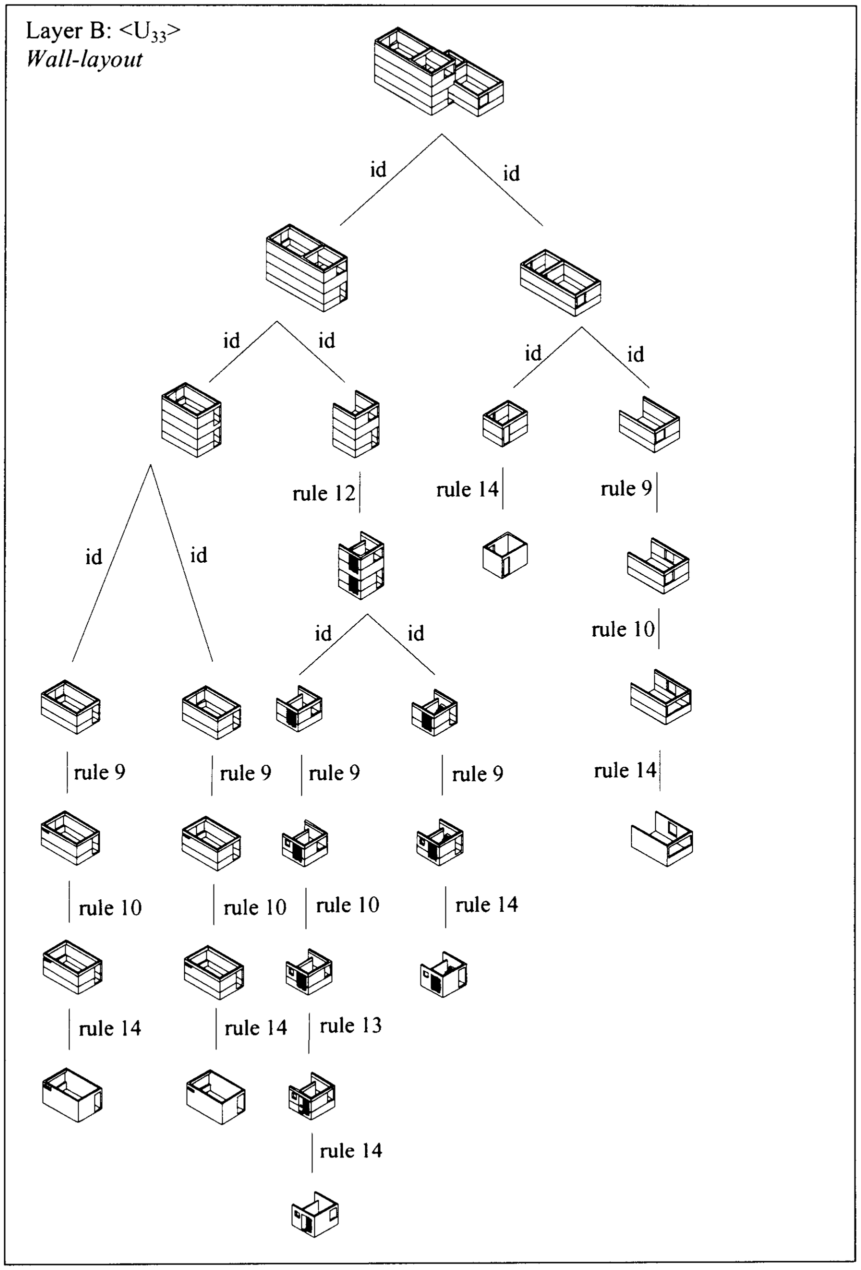
$< U_{12}\ x\ U_{12}\ x\ U_{12}> \rightarrow <U_{12}\ x\ U_{12}\ x\ U_{12}>$



$< U_{23}\ x\ U_{33}\ x\ U_{33}> \rightarrow < U_{23}\ x\ U_{33}\ x\ U_{33}>$

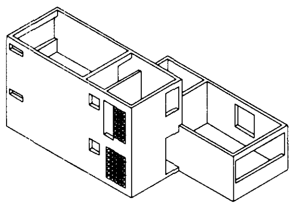


An overview of the developments caused by the application of the parametric rules 9-14 is captured in the next tree. The diagram presents only the shapes in layer B,



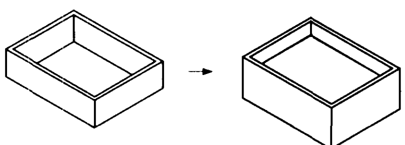
The reassembling of the modified parts, after the application of the rules (9-14) produces a new arrangement, in layer B,

B

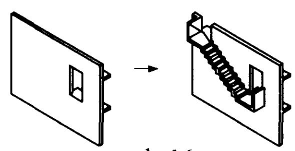


U_{33}

Rule 15 adds the top of the building, rule 16 adds an exterior staircase

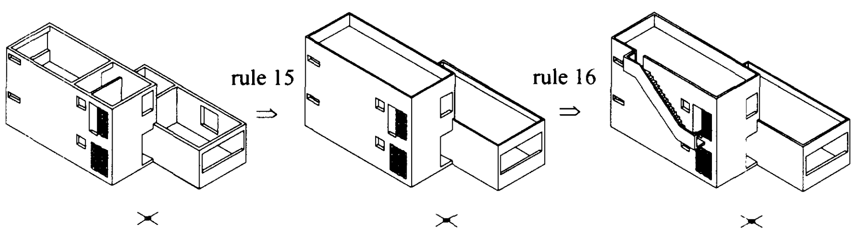


rule 15



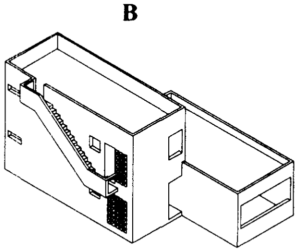
rule 16

The developments caused by the three rules are depicted in the derivation



The formed design is

B



U_{33}

The detailing of components with repetitive character, like window frames can be studied with the aid of rules. The rules for windows create variation. In the example, five rules (presented in the algebra U_{12}) serve the exploration of window-frame configurations. The frame vocabulary, based on the square and the half-square, produces frames that occupy the existent openings (from the previous stages), or extends them

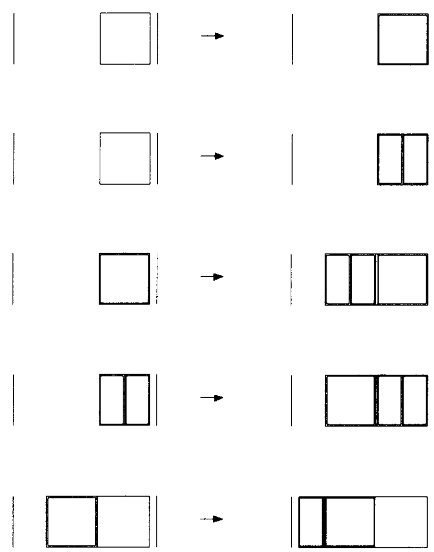
rule 17: (i)

(ii)

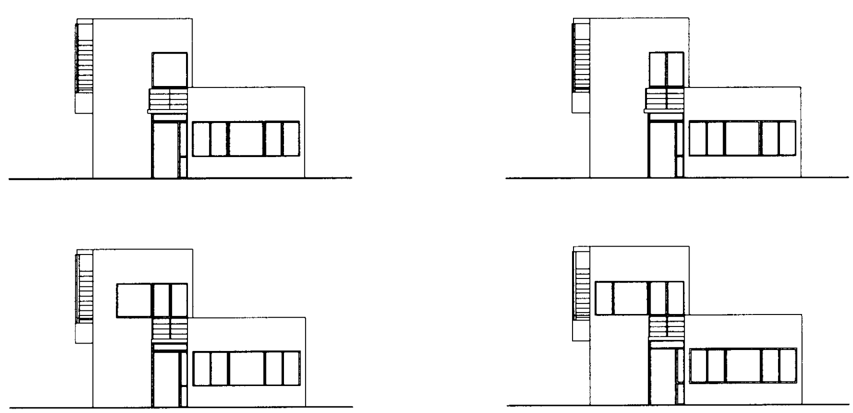
(iii)

(iv)

(v)

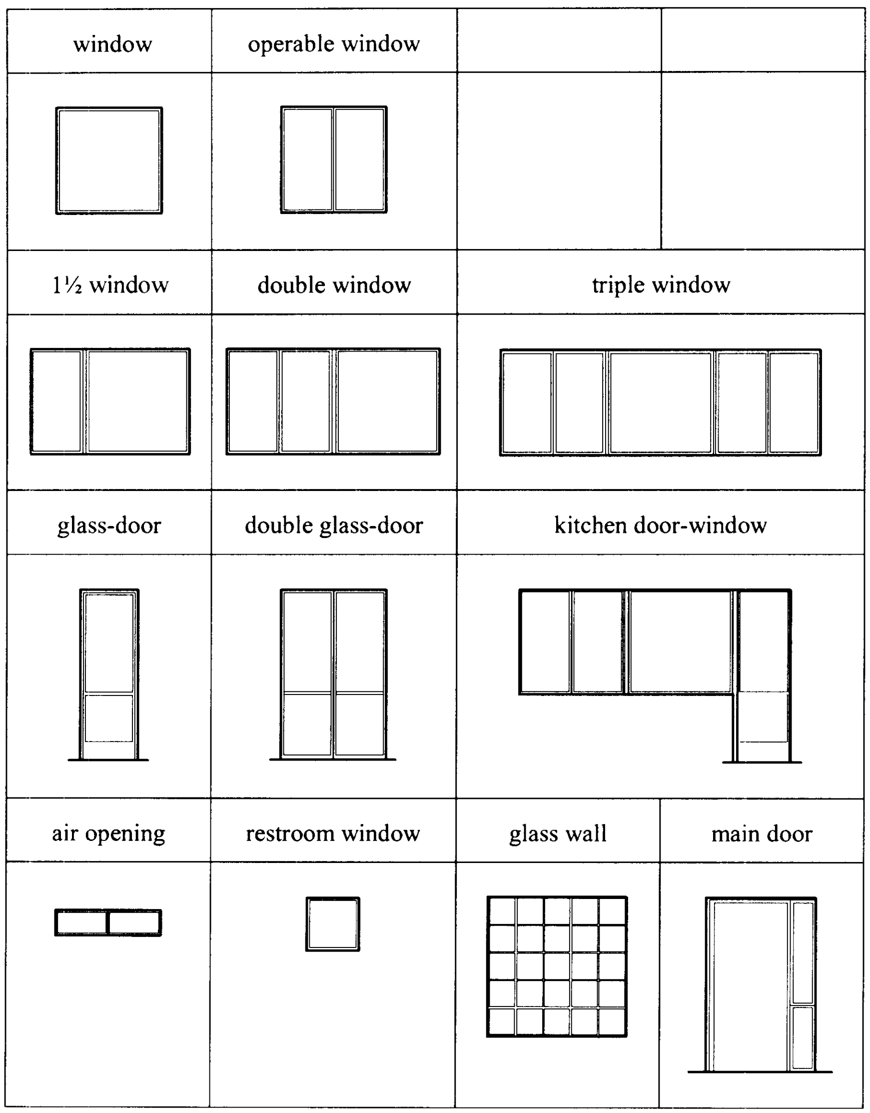


The rules are used in the exploration of alternative configurations of openings, in elevations. A sample is presented bellow in the algebra $\langle U_{12} \rangle$.



A vocabulary of openings based on the square and the half square was developed for the designs, after several tests. Similar openings can be found in the Modulor or, at the houses at Pessac, by Le Corbusier. The vocabulary of openings includes: single window, double, and triple window, glass-door, double glass-door, door-window, air-opening, restroom window, glass-wall, and main entrance door. These items appear in the next Table 10.

TABLE 10. The vocabulary of openings that was developed for the designs



To summarize, the process of refinement determines the tectonic details of spatial arrangements. During the refinement an input spatial arrangement is decomposed, transformed, and recomposed many times, with different objectives, proportion, structure, function, etc. It is possible to introduce infinite parallel sub-levels of refinement. This presentation emphasizes the interaction among descriptions, and shows examples of rules of three kinds: dimensioning, structure, and function.

The dimensioning of rooms and walls deals with the specification of widths, lengths and thickness of structural components. Changes in dimensions can be dictated by changes in other parallel descriptions (i.e. structural, functional etc.). And conversely, changes in dimensions can impact the boundary-layout, the distribution of openings etc.

The structural refinement deals with the structural behavior of a design. The structural behavior depends on the weights and the strength of the selected building components, but also, on the forms, the openings, and the density of the elements. Construction details can also influence the form of a design. Although designers do not deal with the construction in full depth, there is a constant interaction between structural and design descriptions.

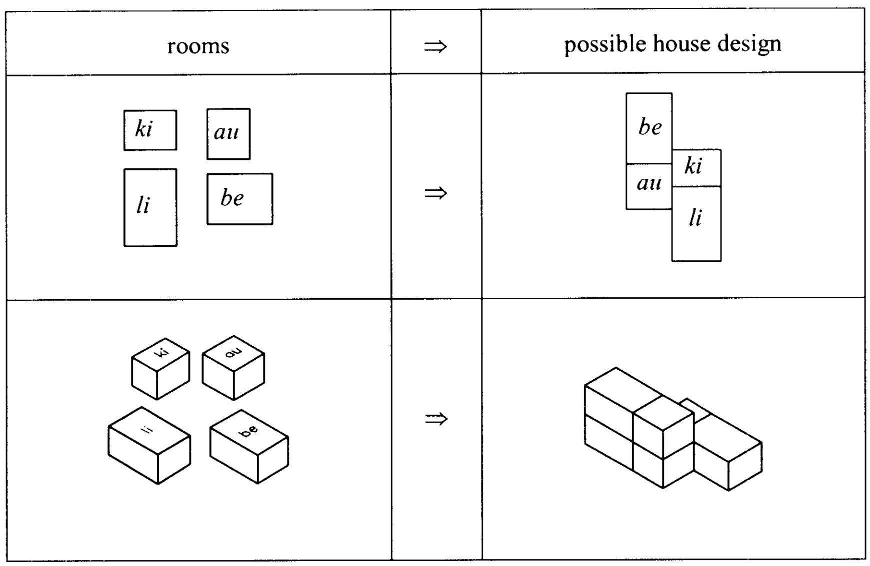
Finally, the process of functional refinement deals with the coordination of details such as stairs, windows, doors etc. according to the existing functional standards. The examination of functional details requires the use of multiple descriptions. It offers a great opportunity for variation, through the alternative treatment of stairs, windows, doors, etc.

7. Discussion

This chapter outlined an educational framework of rule based composition for the architectural studio. A design competition for low cost housing was used as example. The aim was to combine theoretical devices such as rule schemata, and rules, and digital tools such as scripting and modeling, to deal with a common studio project. Another, parallel objective was to show how some known studio techniques can be approached computationally without loosing their expressive subtlety.

The exercise was conducted along the lines of the formal design theory introduced by Stiny (1976, 1980, 1991). The novel aspect of the approach is that composition is based on a design concept, and not on analysis of pre-existing designs. The described process is characterized as introspective and prescriptive: Introspective because it explores a particular class of design alternatives, and each designer can possibly choose to develop different ones; Prescriptive because it involves a prescriptive system of rules that provide a norm for the exploration.

A typical compositional concept is used: the "*domino house*". The absence of a predetermined site is a significant factor in choosing this approach. Starting from a finite spatial vocabulary and a small number of general spatial relations, the designer examines systematic ways of constructing designs. The construction proceeds from the definition of the "parts" (spatial vocabulary of rooms) to the definition of the possible "wholes" (house designs). The building program provides general information for the required areas of each house type.



The design concept suggests that starting from any initial number of rooms one can create rule based routines, to compose houses of variable size and morphology. Rooms and their adjacencies are expressed by shapes, spatial relationships, and rule schemata. After defining possible rooms (parts), spatial relationships, and general rule schemata, the testing involves the construction of possible house designs (wholes).

The decision process involves a selection among alternative choices of rule schemata and rules, where the designer explores possible results. The specification of the rule schemata becomes the objective of this process. A great part of the design activity consists of formulating rule instances and transformations, in order to determine if a particular set of choices has any desired outcomes. If not, the set is modified and re-tested. As the parametric rules are tested, the values that determine parameters and dimensions are gradually established. The need to develop focused methods to control the generation of the preferable solutions leads to the restriction of the rules and the production of designs with specific attributes.

The heuristics of the search are organized to achieve three general objectives: First, rule schemata of *formation* sketch out diagrammatic arrangements (*partis*). Second, rule schemata and rules of *transformation* generate variations of wall-layouts from chosen *partis* and organize the general distribution of solid and void. Third, *refinement* rules determine tectonic details (stairs, windows and doors) in chosen wall-layouts. Sample descriptions from two working examples derived at the three levels of *formation*, *transformation* and *refinement*, are given below in 2d and 3d.

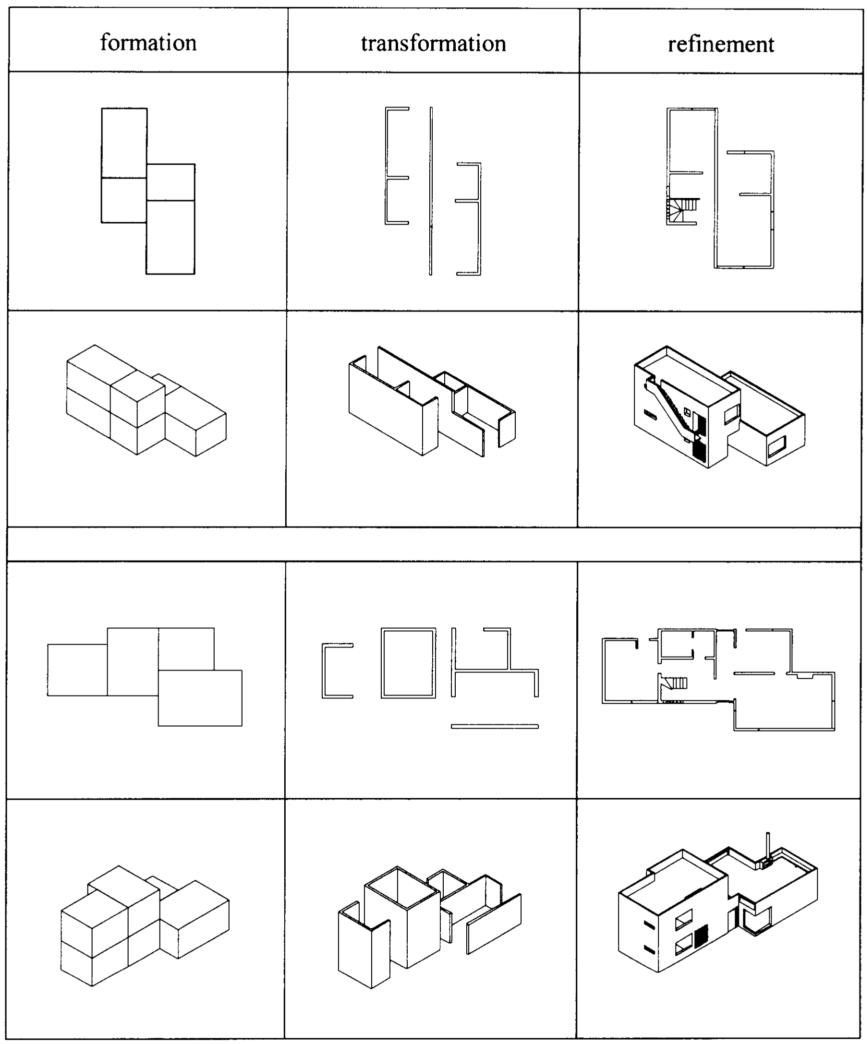
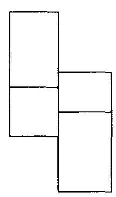
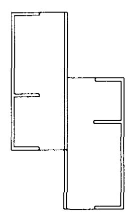
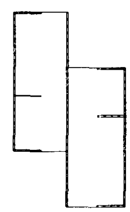
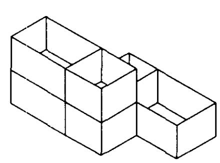
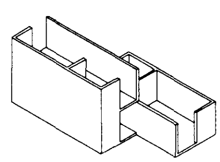
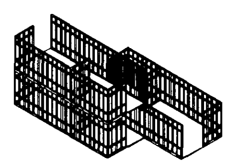


Figure 6. Two working examples at the three levels of abstraction

Further, the design process involves different kinds of partial descriptions, each contributing a different view of the design. Several superimposed layers of information are composed within the framework of a chosen *parti*, to produce a design description. In the example, three superimposed layers A, B, C are used to produce 2d plans and 3d models. The layers A, B, C model a simplified version of an actual process of composition, where greater number of alterative descriptions is used.

parti	wall-layout	structural-layout
A	B	C
 U_{12}	 U_{12}	 U_{12}
 U_{23}	 U_{33}	 U_{33}

In 2d, the layers A, B, C include lines in the algebra $\langle U_{12} \rangle$. The product description A x B x C is formed in the product algebra $\langle U_{12} \rangle \times \langle U_{12} \rangle \times \langle U_{12} \rangle$. In 3d, the layer A include planes in the algebra $\langle U_{23} \rangle$, and the layers B, C solids in the algebra $\langle U_{33} \rangle$. The product description A x B x C is formed in the product algebra $\langle U_{23} \rangle \times \langle U_{33} \rangle \times \langle U_{33} \rangle$.

Analogue and digital means are used in the design process. For the superimposition of multiple 2d and 3d descriptions, the analogue representation uses multiple sheets of tracing paper, while the digital tool uses multiple AutoCAD layers. Analogue representation (paper and pencil) is employed at the initial phase of the process when the relationships among the values of the parametric rules are unknown. The digital tool is more efficient in clarifying the ramifications of a rule-set, by allowing the mechanical execution of large number of tests. The digital description of the shape rules requires their translation in symbolic form. Therefore, the values of variables within the rules require clarification. The digital tool is particularly useful in the exploration of 2d *partis*, at the stage of *formation*. In *transformation*, calculations with multiple layers make the symbolic description of the rules increasingly complex. The process of refinement is executed manually in AutoCAD, without the digital interpreter.

The aim of rules in composition is to project a finite set of properties to a large set of compositions. The problem in using them in the synthesis of original designs is to define the basis on which we evaluate them. In actual languages a test of adequacy for a grammar (Chomsky 1957) is to have native speakers accept the produced sentences and to identify the false ones. Chomsky assumes intuitive knowledge of the English sentences, and asks "*what sort of grammar is able to produce these sentences in some effective and illuminating way?*" Similarly, in the analysis of a corpus of designs, the rules can be extracted from some original, previously analyzed, instance (i.e. Palladian Villas, or Queen Ann houses, or Frank Lloyd Wright houses etc.)

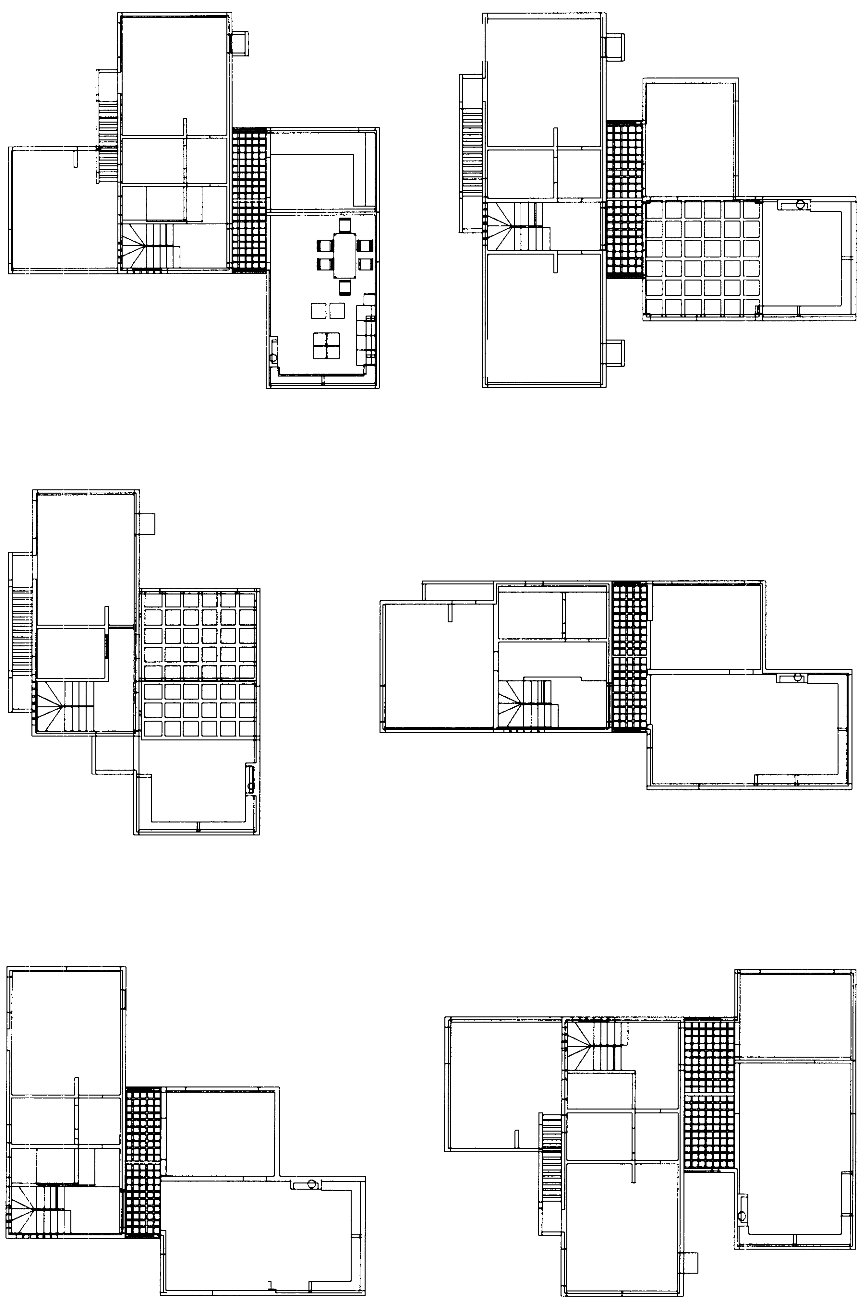
But in synthesis from scratch; there is no predetermined criterion of evaluation. The designer has to set an objective, which remains open to re-evaluation. Based on previous experience, and some finite set of observations, this objective is expressed in the form of a design concept. The design concept provides a general principle for forming rule schemata that are gradually better specified. Rules are tested and then classified to generate compositions with desired properties. The outcome is the construction of a system governed by rule schemata and rules, in which the thoughts and the actions of the designer are expressed.

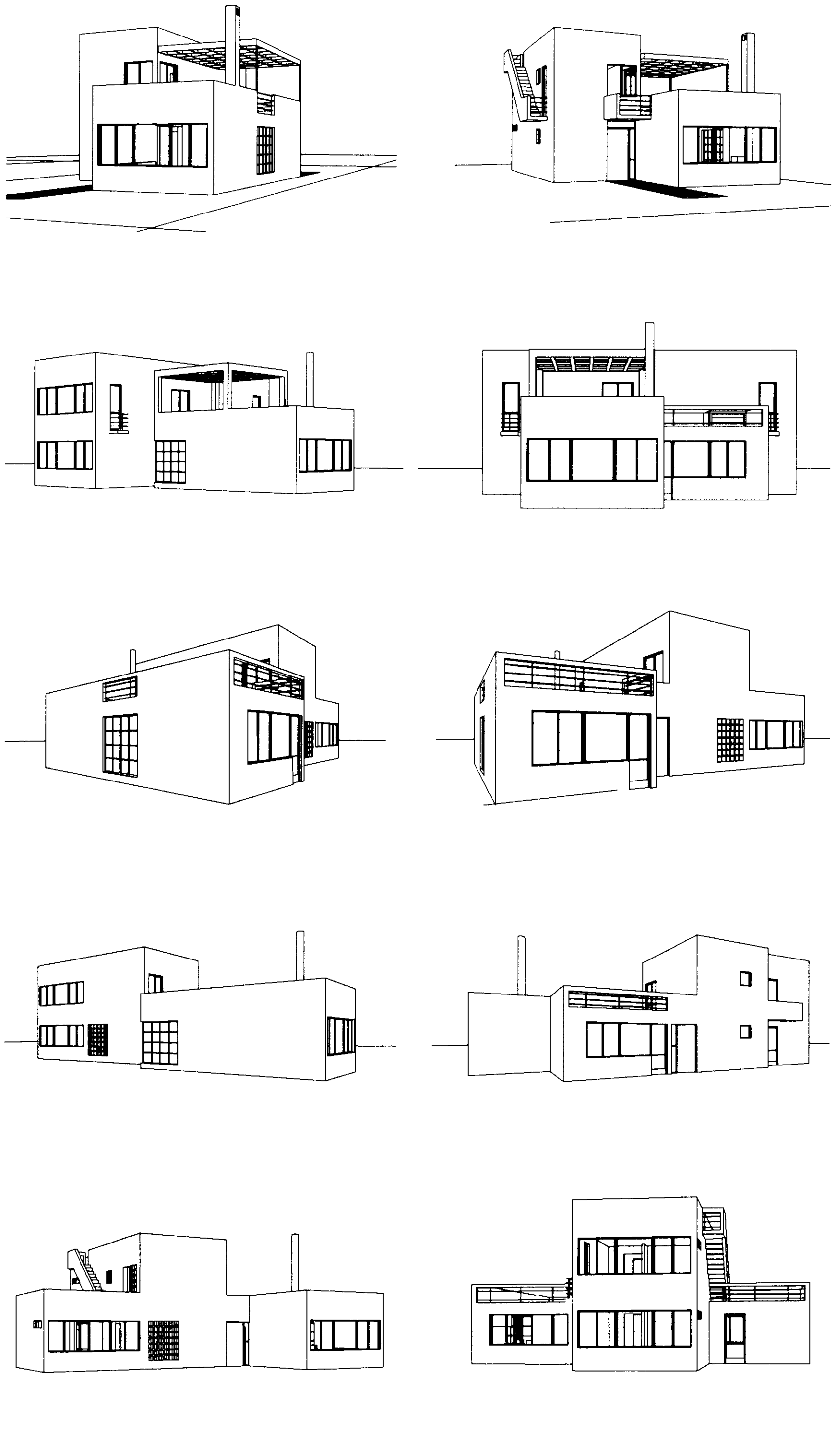
A grammar is a retrospective construction that mirrors, in an abstract way, ones behavior. It is produced on the basis of finite familiarity with a problem, if one wants to preserve one's findings, in order to address similar problems in the future. Therefore, a grammar is like a memory device, were the rules are categorized and organized to generate certain "languages". In this sense, the grammar is the result of a process of construction, rather than discovery.

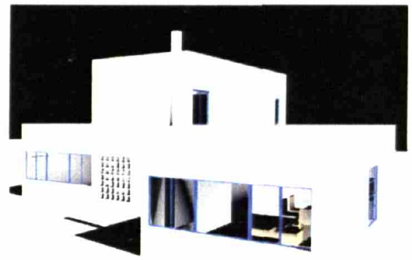
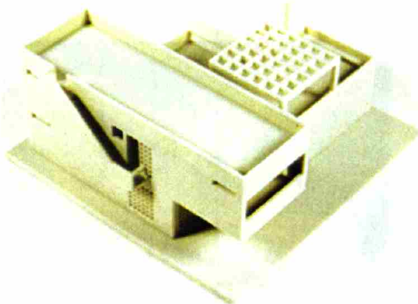
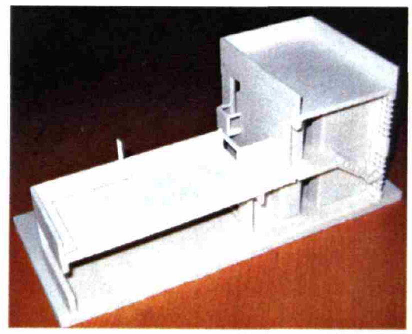
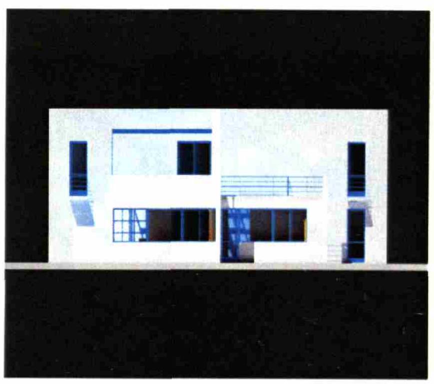
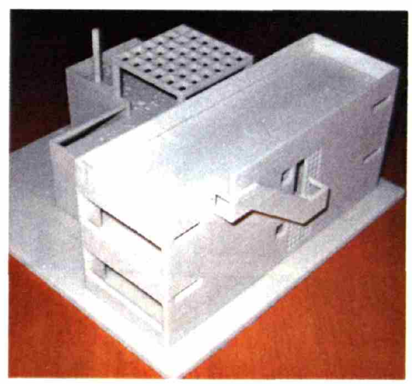
7.1. RESULTS – ILLUSTRATIONS

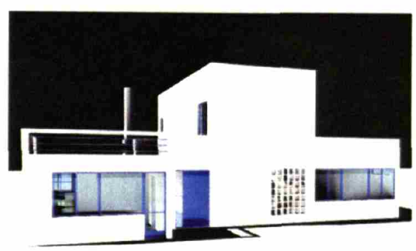
The next descriptions (plans, perspectives, 3d models) present sample designs derived from a particular collection of rules. The designs are variations developed on the basis of the example presented in this section. The selected rules were only a small fraction of the tested rules. These rules can be eventually organized into a grammar, to generate the particular house types. The following Table 11 presents plan arrangements.

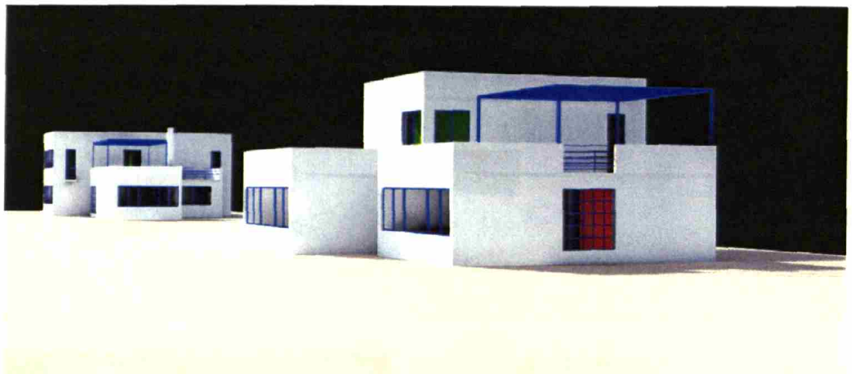
TABLE 11. Sample designs in plan belonging to the same sublanguage.

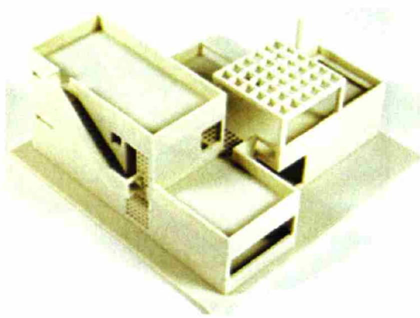


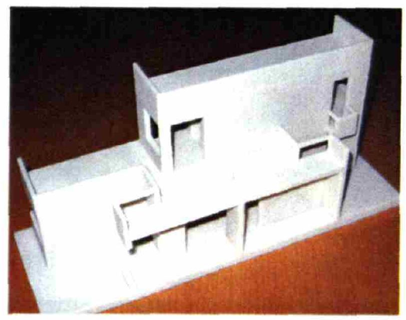


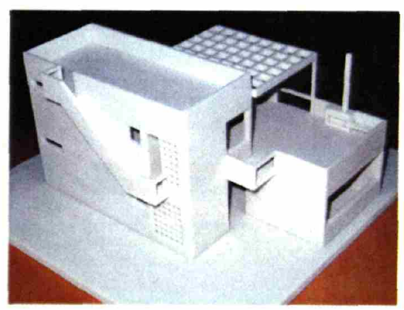







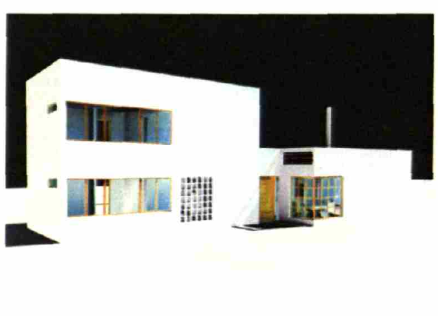












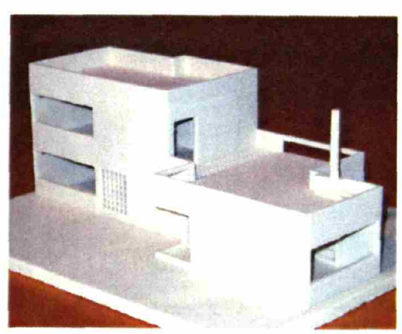
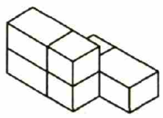
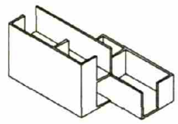
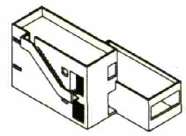
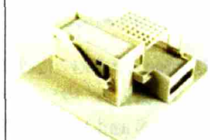
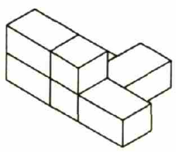
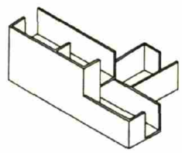
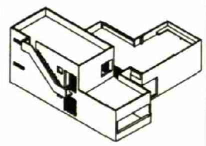
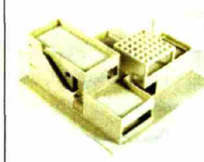
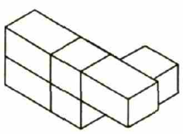
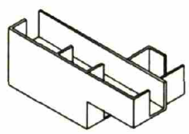
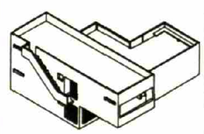
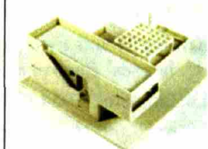
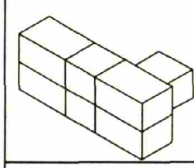
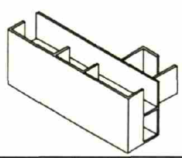
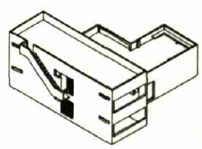
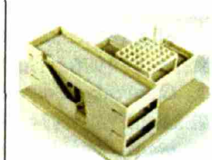
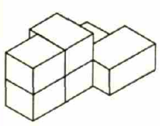
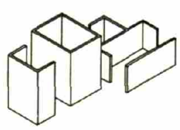
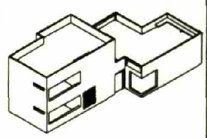
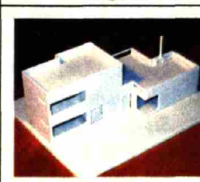
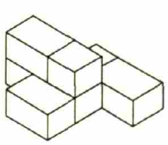
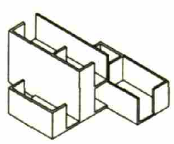
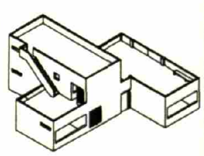


TABLE 12. Sample designs that belong to the same sublanguage, at the three levels, formation, transformation and refinement

Formation	Transformation	Refinement	Model
			
			
			
			
			
			

The next Chapter VI presents the construction process of a rule based device for making the plans for an office building. The process begins from a specific site, the building program, and a design concept. The construction proceeds in opposite fashion from the process described in this chapter: From a potential "whole" (design concept) to the definition of the "parts" (rooms and spaces).

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VI. A RULE BASED APPROACH TO THE GENERATION OF PLANS

Abstract. A system of parametric shape rule schemata that generates the plans for an office building is developed as a model appropriate for the composition of plans from scratch.

1. Introduction

A rule based approach to the generation of 2d architectural plans from scratch is presented in this section. The approach was based on the view that designers develop spatial concepts because they want to express their intentions about space, to solve design problems, and to form designs. A "design concept" is a narrative that provides ground for exploration. The pursuit of designers is to elaborate their design concepts by inventing the appropriate transitions that result in the formation of feasible artifacts.

The making of the plans for an office building is used as an example for this case study. The designer of the project[1] proposes a conceptual schema of spatial organization (*parti*) for the design. The *parti* is gradually developed into a design, with the aid of general shape rule schemata.

The outcome is the construction of a design system governed by rule schemata, in which the actions of the designer can be expressed. The study does not aim to show how an entire universe of interdependent design problems (such as function, structure, materials, etc.) can be represented in a single device. It only considers some fundamental aspects of the generation of plan-descriptions that can be conceived as shape calculations and can be described by rule schemata.

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[1] The original design concept for this project was proposed by Maria Pan., Architect.

2. The Project

The project of the example is the design for an office building of a publishing firm in downtown Los Angeles. The site of the project is located at the junction of freeway 10 and freeway 110. The site belongs to the block located on the intersection of Venice Blvd and Wright Street. The specific lot is circumscribed by the South ramp of freeway 110.

Figure 1. General aerial view of the site area, map, and exact location of the site

The site has a trapezoid shape. The side towards Wright Street is 118 ft (or, 36 m approx.) The square-footage required by the program fits tightly the site's available area.

Figure 2. Diagram of the site, and relation to the freeway ramp

The building program attempts to balance the public and the creative character of the publishing firm. It includes several private, working, and public, spaces: studios, private rooms and offices with their facilities, but also, spaces aiming to accommodate public events. Entrances, exhibition spaces and cafés must be accessible to the public. Offices, design studios, study rooms, private lounges, facilities, lobbies, kitchens, storage rooms and their circulation areas are intended to accommodate the working activities of the firm. The program indicates that the spaces of primary importance are the creative studios, and the main lobby of the firm. The first represents the creative core of the firm. The second is envisioned to serve as a central public space for exhibitions, receptions and public events.

3. Design Concept and Method

The design approach was influenced by three factors: a) the space limitations of the given site, b) the programmatically defined public/private character of the building, and c) the proximity of the building to the massive freeway ramp. Starting from a specific site and building program the designer proposed the concept for the design. The designer proposed to exhaust the usable site area and height to construct the maximum building envelope, and envisioned this envelope as a "box"; a protected container for the public and private activities of the program. The designer reasoned that the conceptual framework was motivated by the factors (a) the space limitations of the site, and (c) the proximity to the freeway ramp.

The search proceeded from a potential "whole" (design concept) to the definition of the "parts" (rooms and spaces). The presentation of this process in the present study is characterized retrospective and descriptive: Retrospective, because an original design concept is provided by the actual designer of the project; Descriptive, because a computational process is proposed to examine the consequences of the concept. The computational process does not replicate the exact actions of the designer.

The concept was developed into a design with the aid of rule schemata and rules. A vocabulary of forms and relations was established gradually. The computational framework defined in Stiny; Gips 1972 and elaborated in Stiny 1976; 1980; 1991 was put into use. Lines, symbols and areas are used in the formation of 2d plan-descriptions. "Compound algebras" (Stiny 1992) are used for calculations with lines and symbols $<U_{12}V_{02}>$ and for calculations with planes and symbols $<U_{22}\ V_{02}>$, on the plane. The design process was executed by hand, mainly due to the time constraints of the project. But the same process could be transferable to a digital environment.

Initially, rule schemata are used to generate a *parti*. Historic examples in the use of *partis* in architecture can be found in Durand's (1801) systematic generation of building-plans. Recent examples in the use of *partis* in the computational generation and transformation of plans exist in Eastman 1970; March 1972; Mitchell 1974; 1977; Steadman 1983.

In this case study the arrangement of the *parti*, corresponds to a design concept proposed by the designer. The design concept is expressed with spatial relations and rule schemata. The forms are gradually composed to produce plan arrangements, within the general conceptual frame.

Multiple layers of description are used in the construction of a single description. They serve the practical need for sketching out possibilities and addressing multiple issues. The layers contain families of shapes with or without specific descriptive objective. During composition the layers develop unpredictable associations and produce emergent arrangements.

The formal idea of rules that apply in parallel to form parallel descriptions appears in Stiny 1992; 2000, and in Knight 2003. Stiny and Knight propose parallel grammars as a formal device to link the generation of plans, layouts, details etc. Knight's example presents a parallel grammar that links different grammars: one generates plans, another, layouts, while others are used for storing. According to this approach, a number n of individual descriptions evolve in coordination. These descriptions exchange information in more or less predetermined ways (i.e. an elevation relates in a predetermined way with the plans, etc.) The same number of n individual descriptions is the output of the process at the end ($n \Rightarrow n$)

$$\text{A, B, C, } \ldots\, n \;\Rightarrow\; \text{A, B, C, } \ldots\, n$$

Applications of these ideas can be found in Li's grammar for the generation of Yingzao fashi houses (2000), and in Duarte's grammar for the generation of Alvaro Siza's houses (2001).

In this study n interacting descriptions A, B, C,... n are used to produce a single description. The novelty of the approach is that the process does not involve coordination of individual descriptions that exchange information in fixed ways. The descriptions mingle in ways that are not predetermined and a single description is the output of the process ($n \Rightarrow 1$).

$$\text{A, B, C, } \ldots\, n \;\Rightarrow\; \text{X}$$

More specifically, $n = 4$ descriptions A, B, C, D are composed and none of them is preserved at the end as an individual description. The "partial" descriptions A, B, C, D, develop their associations at rote, and the output X is an emergent description. The four descriptions occupy the same position in space (com-position). A practical way to achieve this is with four superimposed tracing sheets. The four layers A, B, C, D are a simplification of an actual design process, where more tracing sheets are used to produce a description. The characteristics of the layers A, B, C, D appear in Table 1.

TABLE 1. The four graphic layers A, B, C, D

layer	*name*	*algebra*
A	intersecting	$<U_{12}\ V_{02}>$
B	projecting	$<U_{12}\ V_{02}>$
C	grids	$<U_{12}\ V_{02}>$
D	areas	$<U_{22}\ V_{02}>$

Layers can be withdrawn, or erased, partially or totally. The layers A, B, C, are dedicated to the calculation with "form". They include lines and labeled points. The algebra $<U_{12}>$ facilitates the representation of rooms as shapes made out of lines, and the algebra $<V_{02}>$ provides labeled points. The layer D is dedicated to the calculation with "content". It includes planes and labeled points. The algebra $<U_{22}>$ facilitates the calculations with the areas of rooms and spaces. And the algebra $<V_{02}>$ provides labeled points.

The *name* of each layer is related to the kind of lines, or planes it contains. The naming, and ordering of layers into A, B, C, D is a convention serving presentation purposes. The layer A contains lines, named *intersecting*. Shapes in layer A can be affected by rules or can be the result of decomposition of shapes from the layers B or C. The layer B contains lines named *projecting*. Projecting lines can be drawn in relationship to shapes that exist in layer C or A. Shapes belonging to layer B, or finite decompositions of those, can be transferred to any of the layers A, or C. The layer C contains lines that form *grids*. It also includes lines that extend, or refine a grid. The layer D contains colored planes that are used in the calculations with *areas*. A unit shape for areas is determined at the beginning, equal with the available area of the site.

The heuristics of the search are organized in three parts: a) *Formation* of the conceptual schema (*parti*), b) *Transformation* of the *parti* to a plan layout, c) *Refinement* of the plan layout. The *formation* rules apply on the shape of the site to produce the *parti*. After the *formation* of the *parti*, the examination of its possible consequences, against functional, programmatic, and other criteria follows. In this process, the designer elaborates the *parti* by adding shapes, and interprets the produced arrangements by selecting parts. Elaboration and interpretation are performed repeatedly during the *transformation* of a *parti*. The *refinement* adds details to the descriptions. This framework is described for the four layers A, B, C, D as follows:

Σ : { set of maximal elements }

R: {

Formation

$$\begin{array}{l} D_{1A}\, D_{1B}\, D_{1C}\, D_{1D} \rightarrow F_{1A}\, F_{1B}\, F_{1C}\, F_{1D} \\ \qquad\qquad\vdots \qquad\qquad\qquad\qquad \Leftrightarrow \\ D_{nA}\, D_{nB}\, D_{nC}\, D_{nD} \rightarrow F_{nA}\, F_{nB}\, F_{nC}\, F_{nD} \end{array}$$

Transformation

$$\begin{array}{l} G_{1A}\, G_{1B}\, G_{1C}\, G_{1D} \rightarrow M_{1A}\, M_{1B}\, M_{1C}\, M_{1D} \\ \qquad\qquad\vdots \qquad\qquad\qquad\qquad \Leftrightarrow \\ G_{kA}\, G_{kB}\, G_{kC}\, G_{kD} \rightarrow M_{kA}\, M_{kB}\, M_{kC}\, M_{kD} \end{array}$$

Refinement

$$\begin{array}{l} N_{1A}\, N_{1B}\, N_{1C}\, N_{1D} \rightarrow W_{1A} W_{1B} W_{1C} W_{1D} \\ \qquad\qquad\vdots \\ N_{rA}\, N_{rB}\, N_{rC}\, N_{rD} \rightarrow W_{rA}\, W_{rB} W_{rC} W_{rD} \end{array}$$

}

This study focuses mainly on the stage of *transformation*.

4. Spatial Elements

In two-dimensional plan descriptions lines can be used to represent the boundaries of rooms and spaces, in the algebra <U_{12}>. Planes can be used for the calculation of their areas, in the algebra <U_{22}>. Symbols like letters and numbers are used specifically in this section to assign names, in <V_{02}>.

The first known spatial element for the project is the site. The site *s* is specific, in shape and area. The usable area of the site is a fraction of its total area. In the following diagram, the usable area is represented by a square *a* x *a*. The usable area and the free area form the following decomposition of the site-area

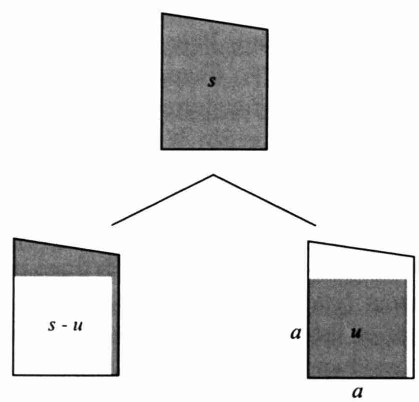
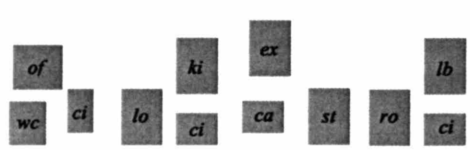


Figure 3. Total area (top node), non-usable area (left node), usable area (right node)

Further, a building can be decomposed in a collection of rooms and spaces that are listed in the program: circulation *ci*, studios *st*, offices *of* restroom facilities *wc*, study rooms *ro*, lounges *lo*, lobbies *lb*, kitchens *ki*, exhibition spaces *ex*, cafés *ca* etc. These rooms are represented by parametric shapes in the algebra <V_{02} U_{22} >.



...etc.

The sum of these spaces produces a quantitative description for the building, but not a design description, unless there is a set of relationships that describes how the parts are put together.

The forms of the rooms are not defined, nor their potential relationships. At this point, the designer distinguished three more general programmatic entities: *public*, *private* and *creative*. Each one corresponds to a parametric labeled shape, *pu*, *pr* and *cr* respectively. Their union is equal to the total usable area *us* of the site.

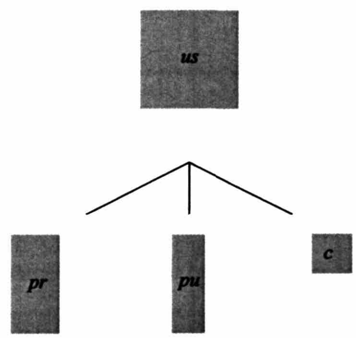
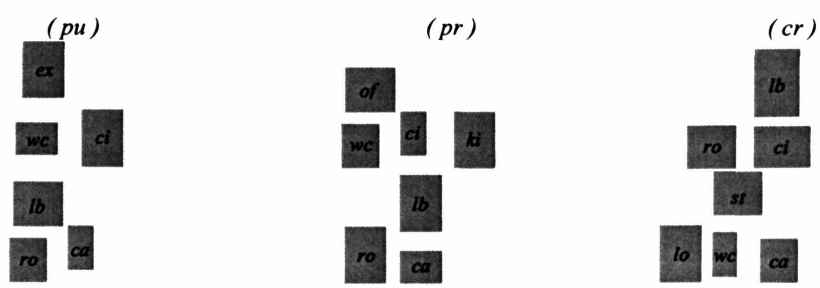


Figure 4. Usable area of the site (root) and the three areas: private, public, creative

The above decomposition is not applicable to all office buildings or publishing firms in general. It is used to frame the existing rooms and spaces of the program into three categories.

For example: the exhibition spaces *ex* can be labeled public *pu*; the offices *of*, can be labeled private *pr*; and the studios *st*, can be labeled creative *cr*. Lounges *lo*, lobbies *lb*, circulation areas *ci*, cafés *ca*, restroom facilities *wc* and study rooms *ro* may belong to all three categories.



The proposed categorization of the rooms permits the calculation of the area of each of the three parametric spatial entities *pu*, *pr* and *cr*. The forms and the distribution of rooms in each, remain unknown.

The three spatial entities *pu*, *pr* and *cr* correspond to three parametric labeled solids. Their sum is equal to the total usable volume of the site

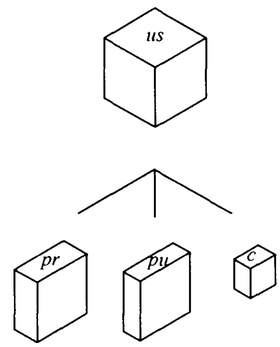


Figure 5. The usable volume (root) and the three volumes: private, public, creative

To summarize: The available site-area is calculated first. Then, three groups of spatial entities *pu*, *pr*, *cr* are distinguished. The relationships among the three parametric entities *pu*, *pr*, *cr* are explored in the next section. It is shown that their forms and their relationships cannot be deduced from the program or from a specific previous experience. They are determined by the designer, on the grounds of a certain design hypothesis.

5. Spatial Relations and Rule Schemata

This section examines how spatial relations and rules can be formed. The interaction of lines and symbols on the plane finds its formal expression within the shape-algebra $<U_{12}\ V_{02}>$. The $<U_{12}>$ component contains shapes that are finite arrangements of lines and occupy specific positions in the Cartesian system of coordinates. The $<V_{02}>$ component contains symbols.

At this preliminary stage parametric rule schemata are proposed instead of rules. As it was pointed out earlier, rule schemata describe spatial relationships in a general manner. And since they do not require the specification of a shape vocabulary, they are suitable for the expression of relationships at the early stages of the process.

The overview of the 20 basic rule schemata is presented in the next Tables 3a and 3b, in the algebra $<U_{12}\ V_{02}>$ that contains lines and labeled points. The rule schemata describe the addition of two convex shapes, the dissection of a convex shape by a line, the generation of a grid, and the addition of a line parallel to a side of a convex shape. They include transformations such as translation, scaling, etc. and also, rule schemata that permit the selection of shapes, or deal with details such as stairs and columns etc.

TABLE 3a. Basic rule schemata: additions, grid generation, transformations etc.

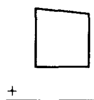
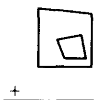
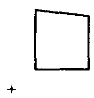
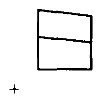
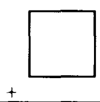
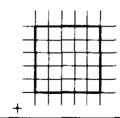
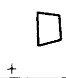
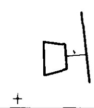
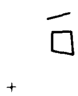
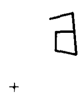
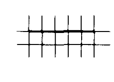
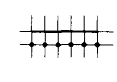
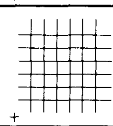
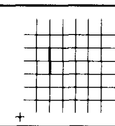
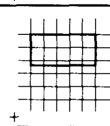
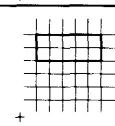
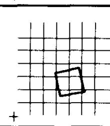
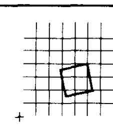
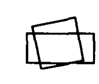
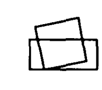
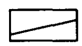
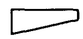
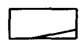
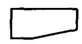
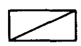
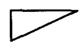
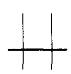
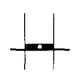
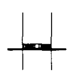
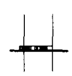
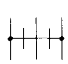
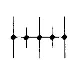
	g(x)	→	g(y)	
RULE SCHEMA 1		→		draw n-sided convex shape inside n-sided convex shape
RULE SCHEMA 2		→		draw dissecting line, inside an n-sided convex shape
RULE SCHEMA 3		→		draw grid n X n inside quadrilateral
RULE SCHEMA 4		→		draw line parallel to n-sided convex at distance v
RULE SCHEMA 5		→		extend side of an n-sided convex shape to meet a line
RULE SCHEMA 6		→		add columns to unoccupied grid intersections
RULE SCHEMA 7		→		draw line on the top of grid lines
RULE SCHEMA 8		→		translate side of n-sided convex shape to meet grid line
RULE SCHEMA 9		→		scale n-sided convex shape to meet a grid intersection
RULE SCHEMA 10		→		given two convex shapes translate one to meet a corner of the other

TABLE 3b. Basic rule schemata: shape selection, addition of stairs and columns.

	g(x)	→	g(y)	
RULE SCHEMA 11		→		select from an n-sided convex an n-sided convex
RULE SCHEMA 12		→		select from an n-sided convex an (n+1)-sided convex
RULE SCHEMA 13		→		select from an n-sided convex an (n-1)-sided convex
RULE SCHEMA 14		→		modify angle between two sides of shape
RULE SCHEMA 15		→		draw stair within two limits
RULE SCHEMA 16		→		translate stair to create landing
RULE SCHEMA 17		→		merge column into wall
RULE SCHEMA 18		→		add columns between two columns
RULE SCHEMA 19		→		change shape of columns from round to square
RULE SCHEMA 20		→		add columns at the ends, and intersections of shape

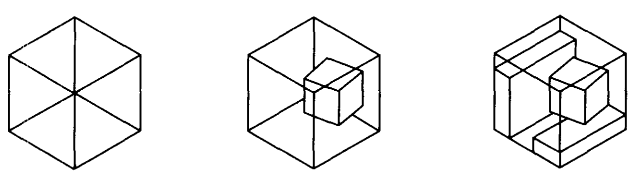
The 20 rule schemata (Tables 3a, 3b) are explained and presented next within the context of the project. They are divided in three general groups, *formation*, *transformation*, *refinement*, and they are reformulated to apply in four layers A, B, C, D, in algebras <U_{12} V_{02}> and <U_{22} V_{02}>.

5.1. FORMATION

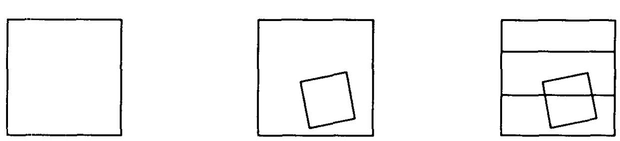
The area limitations of the site and the program are decisive parameters for every design. The former defines the availability of space and the latter the basic functions of a building. But, the most important for the design process is a design concept that establishes a particular interrelationship among all the above. This is not arrived at by an analysis of the provided information. It is expressed through a narrative, and requires judgment and synthesis, rather than analysis. The narrative does not simply repeat common facts for the object under consideration. It suggests a new meaning for it.

The designer of the project proposed the next conceptual narrative: *The building is a cubic box occupying the entire usable site-area. A second small cubic box placed within the first accommodates the core, creative activities of the firm. Administrative rooms are placed on the perimeter of the large cubic container. A public lobby occupies the central, ground floor area.*

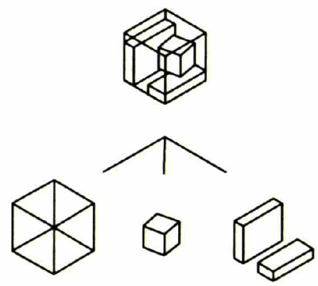
Proposals of this kind cannot be qualified as either true or false. The schema is represented next, in a <$U_{13}U_{33}$> algebra. The exterior cubic envelope is represented on the left only by its vertices. In the middle illustration a smaller cube is added within the large cube. The full conceptual schema appears on the right.



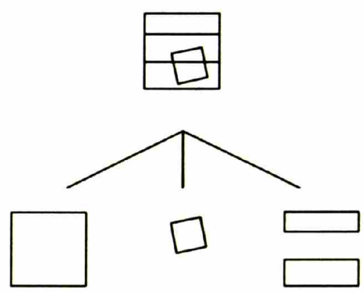
The description of the conceptual schema in plan is described in the algebra <U_{12}> in the following manner



The parts of the conceptual schema form the following decomposition in the algebra $\langle U_{13}U_{33} \rangle$. The full conceptual schema appears on the root of the tree. The exterior cubic envelope occupies the left, the studios the center and the offices the right leaves.



In plan, the conceptual schema forms the following decomposition of parts, in the algebra $\langle U_{12} \rangle$



The full schema appears at the root. The parts include the exterior envelope represented by a large square (left), the studios represented by a small square (center), and the offices represented by two rectangles (right).

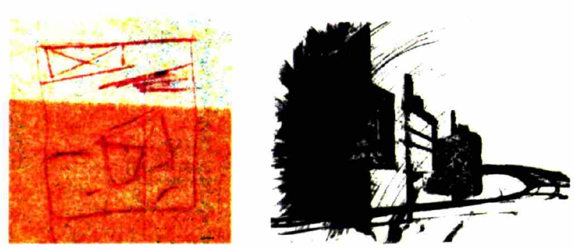
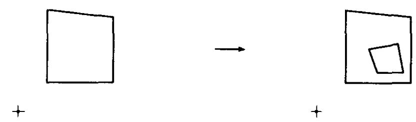


Figure 6. A preliminary sketch of the conceptual schema, in plan and in 3d

Developments of conceptual character, like the previous, can be sketched out by rule schemata without loosing their subtlety:

Rule schema 1: The action of placing one space within another is a compositional decision with several spatial consequences. It can be expressed in plan by drawing one convex shape z within another x. The produced arrangement y = x + z results from the application of the following rule schema g(x) $\rightarrow$ g(y),

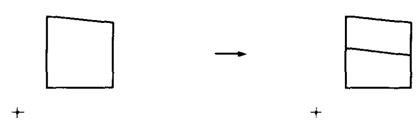
RULE
SCHEMA 1



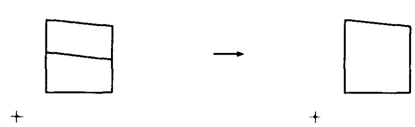
with g: "*x, y are convex shapes*". Following the above action, an erasing rule schema allows shapes to be erased, and several variations to be tested. Therefore, the next rule schema allows erasing an *n*-sided convex shape.

Rule schema 2: The partitioning of a room or a space is an action of compositional character that most designers perform repeatedly in design. It can be expressed by a rule schema. The rule schema applies to a convex shape x, representing a room or a space in plan, and produces a shape x + z. The shape z is a boundary element (wall) represented by a line

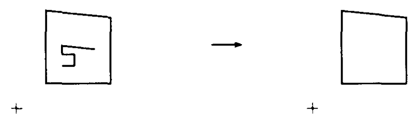
RULE
SCHEMA 2



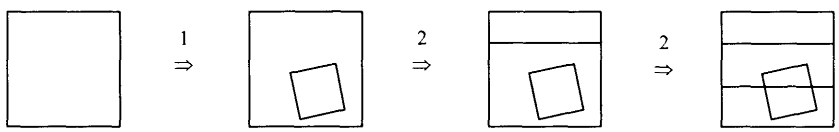
A complementary rule schema allows erasing a line that lies within an *n*-sided convex shape. The spatial importance of this action is to union two rooms or spaces by erasing their common boundary.



A useful variation of the previous rule schema erases a parametric concave shape that lies within an *n*-sided convex shape. The significance of this action is that eliminates all the spatial elements from the interior of a room, or a space.



Instances of the two parametric rule schemata 1 and 2 are enough to generate the proposed conceptual schema, in rough terms. This is shown in the next derivation



To summarize, the conceptual schema (*parti*) becomes useful in three ways: a) it is a hypothesis that links program and form b) it makes some of the spatial properties of the participating elements explicit by pointing to corresponding rule schemata, and c) it provides an initial form, and frames the search-space of the exploration.

5.2. TRANSFORMATION

The examination of the spatial consequences of the *parti* is conducted with the aid of additional parametric rule schemata. This process, which results in the transformation of the *parti*, relies on functional, programmatic, stylistic, analytic and other criteria. A great part of this activity consists of formulating and testing rule schemata. At the beginning, the rule schemata are tested and placed in sequences to attain certain general goals. The placement of shape-rule schemata in general groups serves to make the information they contain more comprehensive. This categorization will result to the ordering of rules into grammars. This ordering is a retrospective issue. It does not of itself create any new information.

In this section the rule schemata are examined in two complementary, general groups. The key argument for the first group is "*draw* _____". This group of rule schemata also includes transformations: translations, reflections, rotations, scaling etc. The key argument for the second group is "*select* _____ *from* _____". The empty spaces in the two expressions are occupied by names of shapes. The first group results into the *elaboration* of the *parti*: new shapes are added and/or transformed in the description. The second group assists in the *interpretation* of the produced arrangements: some of the existing shapes are selected and assigned certain attributes. Elaboration and interpretation are performed repeatedly at this stage.

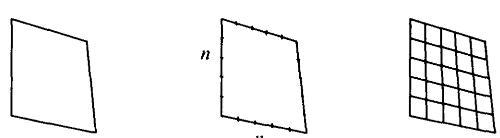
In the next presentation of the rule schemata the thicknesses of walls are ignored. Walls are represented by single lines and columns by points. Also, window and door openings are ignored, and symbols are omitted from the computation. All descriptions are shapes in the $<U_{12}V_{02}>$ algebra.

5.2.1. Elaboration

What are the spatial consequences of a conceptual schema? This is a question difficult to answer before performing several tests. In this section the rule schemata make the intentions of the designer and the ways in which forms are treated in the composition increasingly evident. Specific rules are defined from the shape rule schemata in later stages. When a rule schema becomes specific it is converted to a rule. As the shapes that a rule schema can apply to become explicit, the rule schema looses its generality. It is reduced from a general action of compositional character to a drafting routine that applies only under the precisely defined conditions. The definition of rules from general rule schemata is a task of *refinement*.

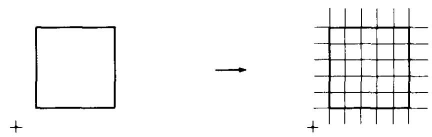
Rule schema 3: The generation of a spatial grid introduces proportion and scale to descriptions, while the descriptions remain 'dimensionless'. In this project, the boundaries of existing rooms are used as axes for the generation of grids. Several overlapping grids can be produced from the boundaries of different rooms in space.

The adjacent sides of a quadrilateral room are divided into n and k number of segments on the basis of some preferable ratio. The same divisions are applied on their opposite sides. For $n = k$, the grid is n x n.



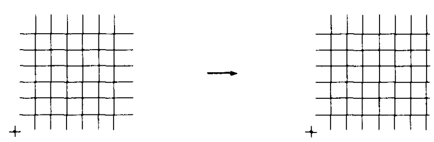
The next rule schema specifies the construction of an orthogonal 5 x 5 grid. The spacing ratio of the grid is 1:1.

RULE
SCHEMA 3

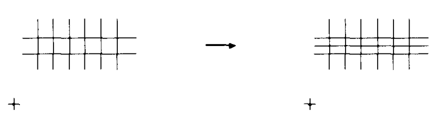


Examples of the creative use of grids in the generation and transformation of plans exist in Eastman 1970; March 1972; Mitchell 1974; 1977; and Steadman 1983. A historic example of the use of grids in architecture is Durand's (1801) combinatorial generation of building-plans. In art, Durer's (1528) drawings of human heads exhibit how variation is produced from a single drawing by altering the spacing ratio of the grid-lines. Finally, Thompson's (1917) *On Growth and Form*, shows how rules of 'deformation' of rectangular grids depict the evolution of animal forms.

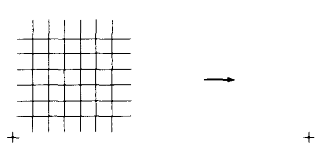
The extension of an existing grid through the addition of an extra grid-axis is relevant to the generation of a grid



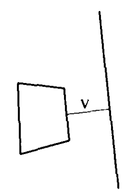
Also, a local subdivision of the grid, expressed by a local grid-axis, can be drawn at some preferred distance between any two consecutive grid-lines



Finally, the next rule schema allows for erasing a grid,

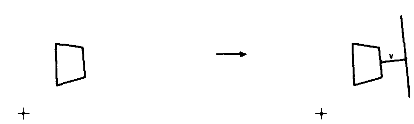


Rule schema 4: The next rule schema formalizes one simple action: a new line is drawn parallel to the boundary line (wall) of an existing shape (room). The new line is drawn at some distance v from the boundary of an n-sided convex shape.

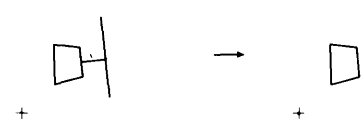


The complete rule schema is,

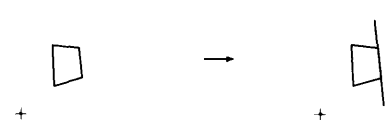
RULE
SCHEMA 4



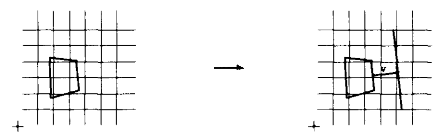
The added line permits the placement of spatial elements (rooms, columns, walls) in position relative to the specific room. The next rule schema allows the erasing of a line constructed by the previous rule,



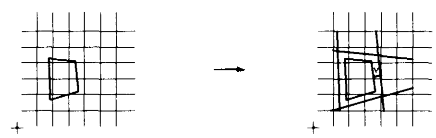
Some characteristic instances of the particular rule schema are presented next. For example, if $v = 0$, part of the newly drawn line is embedded on the boundary of the existing shape.



Also, in the rule schema 4, the distance v can be restricted so that each new line passes from an intersection of an underlying grid. In this way one can take advantage of an underlying grid-structure to develop a different structure.

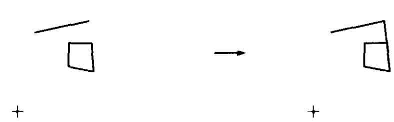


The consecutive application of a rule schema that draws all the parallels to the sides of a room generates a mesh relative to the particular room. The new mesh-lines have the property to be parallel to the sides of the room at some distance *kv*. This rule schema is not used in the design.



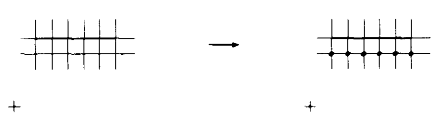
Rule schema 5: A room-wall can be extended towards a specific direction to meet another existent wall, or boundary element. This is expressed in a rule schema that extends a boundary line of an *n*-sided convex shape, until it meets another line. The extension of a wall towards another wall unites the two boundaries and the separates the two areas.

RULE
SCHEMA 5



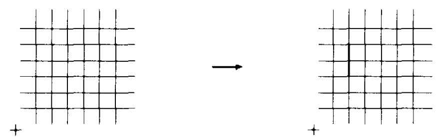
Rule schema 6: This rule schema allows the placement of a series of points on the intersections of a grid. The points may correspond to columns, or to indications of structural, or spatial elements. They are drawn to occupy only the free intersections of the grid.

RULE
SCHEMA 6



Rule schema 7: This rule schema draws a line segment embedded on an existent grid-line. To make this new line segment visible, the line may have different color, or line-weight. The purpose of the rule schema is to draw new walls in alignment to an underlying grid-mesh.

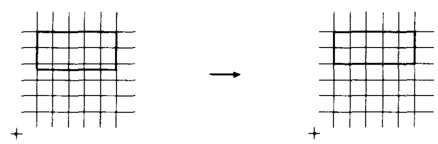
RULE
SCHEMA 7



The next three rule schemata, 8, 9, 10 perform alignments of existing rooms and their boundaries, to an underlying structure. They involve translation and scaling, and they can be characterized rule schemata of alignment.

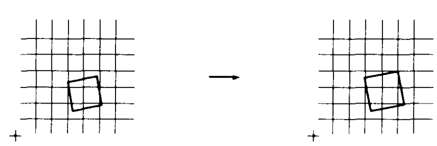
Rule schema 8: This rule schema is a translation. One side of an n-sided convex shape lying in an arbitrary position with respect to a grid is translated to meet a grid division. The rule schema can align the walls of a room to the underlying grid structure.

RULE
SCHEMA 8



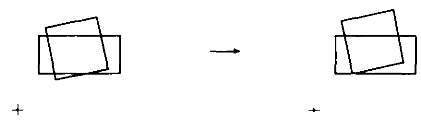
Rule schema 9: This rule schema is a scaling transformation. It scales an n-sided convex shape that lies in an arbitrary position within a grid-mesh, until one of its corners meet a grid intersection. In this way, the rule schema relates a room with an existing grid.

RULE
SCHEMA 9



Rule schema 10: This rule schema is also a translation. In any pair of rectangles that overlap arbitrarily, one can be translated to meet the nearest corner of the second. In this way, the rule schema aligns two spaces that have an arbitrary relation.

RULE
SCHEMA 10



5.2.2. Interpretation

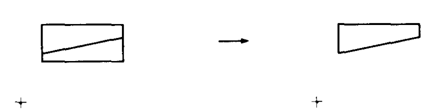
The next parametric rule schemata of interpretation serve the decomposition of existing arrangements in parts. Shapes are naturally undivided and meaningless, unless we assign some structure to them. This assignment is not permanent but related to a specific purpose at view. Decompositions serve the distinction of parts, and their properties, depending on what is to be emphasized. The parts can be depicted retrospectively by symbolic devices, like lattices, or hierarchies, and sets.

This section presents three rule schemata of interpretation, without setting any specific functional, programmatic, stylistic, structural or other criteria for the decomposition of arrangements. Of course, the above criteria may be involved in the performed decompositions. But the power of the proposed parametric rule schemata relies on their ability to select shapes in any given spatial arrangement, and context. The shapes are selected on the basis of their spatial properties. The way of selection becomes explicit in the rule schemata.

The rule schemata of interpretation are more useful when they are abstract. In this way they permit the selection of parts from all participating arrangements, and graphic layers. In this section the parametric rule schemata are presented in the algebra $<U_{12}\ V_{02}>$. Better specified shape rules, which apply in shapes that belong into four graphic layers A, B, C, D in the algebra $<U_{12}\ V_{02}>$ x $<U_{12}\ V_{02}>$ x $<U_{12}\ V_{02}>$ x $<U_{22}\ V_{02}>$, are presented in the section of derivation.

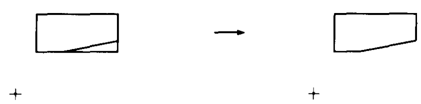
Rule schema 11: This rule schema allows the selection of an *n*-sided convex shape, from an *n*-sided convex shape. In the following illustration of a rule instance, from a given rectangle the rule selects a second rectangle, while the remaining part of the shape is erased from the description.

RULE
SCHEMA 11



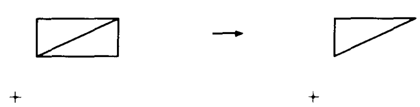
Rule schema 12: This rule schema allows the selection of an $(n+1)$-sided shape from an n-sided shape. In the next illustration the rule selects a 5-sided shape, from a rectangle, while the remaining part of the shape is erased.

RULE
SCHEMA 12



Rule schema 13: The last rule schema of interpretation allows the selection of an $(n-1)$-sided shape from an n-sided shape. In the illustration the rule selects a triangle from a rectangle. The remaining part of the shape is erased.

RULE
SCHEMA 13



5.3. REFINEMENT

Refinement serves the definition of certain details in the produced spatial arrangements. It determines details relative to the tectonic properties of the parts, and the ways these are put together. Designs and their parts are characterized for their structural, technical, functional, and other specifications. And because the shapes can be decomposed in infinitely many ways, and with infinite different objectives, it is possible to introduce infinite sub-levels of refinement. These decompositions may or may not conflict with each other. But they always converge in describing the same object from a different viewpoint.

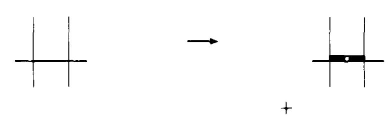
An overview of rule schemata of refinement is presented briefly, in this section, in continuation to the previous section of transformation. Refinements such as the specification of the wall-thicknesses, or window and door openings are deliberately ignored although they would be part of every common design process. Walls are represented by single lines in the algebra $\langle U_{12} \rangle$. Columns are represented by labeled points in the algebra $\langle V_{02} \rangle$. The rule schemata are presented in the product algebra $\langle U_{12}\ V_{02} \rangle$.

Rule schema 14: The next rule schema modifies the meeting angle, between two sides of a convex shape. The lines represent walls, or boundaries. In this way the rule schema changes the angles among the boundaries of rooms or spaces.

RULE SCHEMA 14

Rule schema 15: This rule schema draws a stair in parallel to an existing wall. The length of the stair fits within a given length.

RULE SCHEMA 15

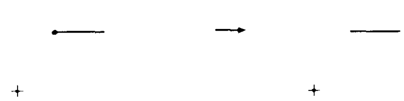


Rule schema 16: The next rule schema translates a stair in direction parallel to its length, and creates the necessary space for the landing area of the stair.

RULE SCHEMA 16

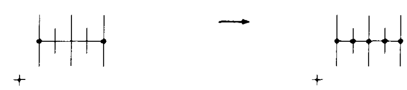
Rule schema 17: This rule allows a column to be erased whenever a wall-line passes through. The practical meaning of this action is that a column which is represented by a point is merged within the wall, which is represented by a line.

RULE SCHEMA 17



Rule schema 18: This rule schema adds columns in the intermediate distance between two existent columns. All the consecutive columns must have equal distances among them.

RULE
SCHEMA 18

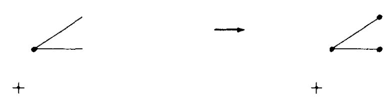


Rule schema 19: This rule schema transforms the shape of a sequence of columns, from round, to square.

RULE
SCHEMA 19

Rule schema 20: This rule adds columns at the intersections, and ends, of a shape made out of lines

RULE
SCHEMA 20



A recapitulation of the 20 basic compositional rule schemata is presented in the next three Tables 4a, 4b, 4c, 4d, 4e, 4f, 4g, 4h, in the algebra $<U_{12}V_{02}>$. The 20 basic rule schemata are numbered by Arabic numerals (1,…,20). Each rule schema is accompanied by its corresponding erasing rule schema, numbered by Latin numerals. The erasing rule schema of "Rule Schema 1" is "Rule Schema 1ii".

Other related rule schemata, or extensions of the 20 basic rule schemata, are grouped together with the specific rule schema they extend and with criterion the accomplishment some objective, i.e. generation and manipulation of stairs, columns, etc. These rule schemata are also numbered by Latin numerals. For example, the "Rule Schema 3" is accompanied by three rule schemata numbered respectively: "Rule Schema 3ii", "Rule Schema 3iii", and "Rule Schema 3iv". This grouping of rule schemata still remains flexible and general.

TABLE 4a. Basic rule schemata for the generation of the conceptual schema (*parti*), and for erasing

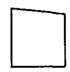
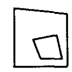
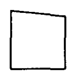
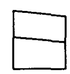
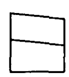
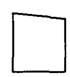
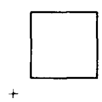
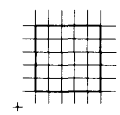
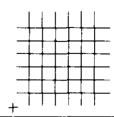
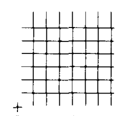
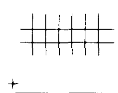
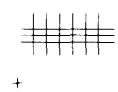
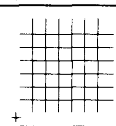
	g (x)	→	g (y)	
RULE SCHEMA 1	 +	→	 +	draw n-sided convex shape inside n-sided convex shape
RULE SCHEMA 1ii	 +	→	+	erase n-sided shape
RULE SCHEMA 2	 +	→	 +	draw dissecting line, inside an n-sided convex shape
RULE SCHEMA 2ii	 +	→	 +	union two n-sided convex shapes by erasing their common boundary
RULE SCHEMATA GENERATING THE INITIAL CONCEPT				

TABLE 4b. Basic rule schemata for the generation, and modification of grids, and for erasing

	g (x)	→	g (y)	
RULE SCHEMA 3		→		draw grid n X n inside quadrilateral
RULE SCHEMA 3ii		→		draw grid line to extend grid
RULE SCHEMA 3iii		→		draw local axis between two grid lines
RULE SCHEMA 3iv		→	+	erase grid n X n

RULE SCHEMATA FOR GENERATION AND MODIFICATION OF GRIDS

TABLE 4c. Basic rule schemata for the generation of lines in relationship to existing convex shapes (rooms), and for erasing

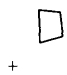
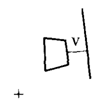
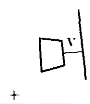
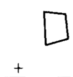
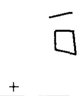
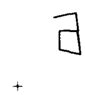
	g(x)	→	g(y)	
RULE SCHEMA 4		→		draw line parallel to n-sided convex in distance v
RULE SCHEMA 4ii		→		erase line parallel to n-sided convex in distance v
RULE SCHEMA 5		→		extend side of an n-sided convex shape to meet a line
RULE SCHEMATA GENERATING LINES IN RELATIONSHIP TO A SIDE OF A CONVEX SHAPE				

TABLE 4d. Basic rule schemata for the generation of columns and lines in relationship to an existing grid

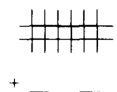
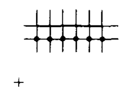
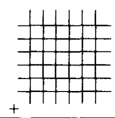
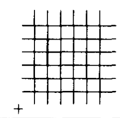
	g (x)	→	g (y)	
RULE SCHEMA 6		→		add columns to unoccupied grid intersections
RULE SCHEMA 7		→		draw line on the top of grid lines
RULE SCHEMATA GENERATING SHAPES IN RELATIONSHIP TO A GRID				

TABLE 4e. Basic rule schemata for the translation and scaling of convex shapes (rooms)

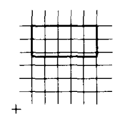
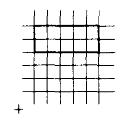
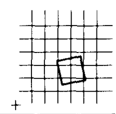
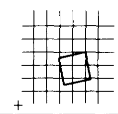
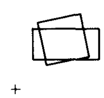
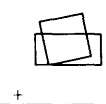
	g (x)	→	g (y)	
RULE SCHEMA 8		→		translate side of n-sided convex shape to meet grid line
RULE SCHEMA 9		→		scale n-sided convex shape to meet a grid intersection
RULE SCHEMA 10		→		given two n-sided convex shapes translate one to meet the other
RULE SCHEMATA FOR TRANSLATION AND SCALING OF CONVEX SHAPES				

TABLE 4f. Basic rule schemata for selecting convex shapes from existing convex shapes

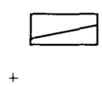
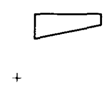
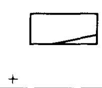
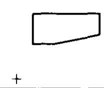
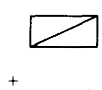
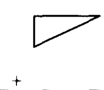
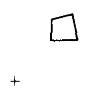
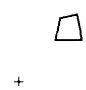
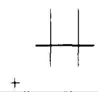
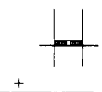
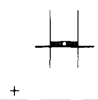
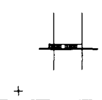
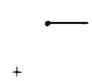
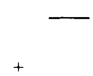
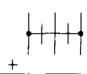
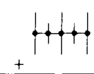
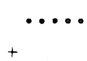
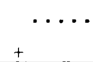
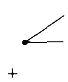
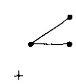
	g(x)	→	g(y)	
RULE SCHEMA 11		→		select from an n-sided convex shape an n-sided convex shape
RULE SCHEMA 12		→		select from an n-sided convex an (n+1)-sided convex shape
RULE SCHEMA 13		→		select from an n-sided convex an (n-1)-sided convex shape
RULE SCHEMATA OF SELECTION (INTERPRETATION)				

TABLE 4g. Basic rule schemata for changing the angles of lines in existing convex shapes (rooms), and for the placement and modification of stairs

	g(x)	→	g(y)	
RULE SCHEMA 14		→		modify angle between two sides of a shape
RULE SCHEMA 15		→		draw stair within two limits
RULE SCHEMA 16		→		translate stair to create landing

RULE SCHEMATA OF REFINEMENT
FOR CHANGING ANGLES OF WALLS, AND STAIRS

TABLE 4h. Basic rule schemata for the modification and refinement of columns, and for their erasing

	g(x)	→	g(y)	
RULE SCHEMA 17		→		merge column into wall
RULE SCHEMA 18		→		add columns between two columns
RULE SCHEMA 19		→		change shape of columns from round to square
RULE SCHEMA 20		→		add columns at the ends, and intersections of shape
RULE SCHEMATA OF REFINEMENT FOR COLUMNS				

6. Derivation

This section examines how spatial relationships and rule schemata are put into use to create plan descriptions. In the previous sections, it was suggested that the role of a design concept is to interrelate certain elements of the provided programmatic and other information in a particular way. Following the general frame of the design concept, a group of general rule schemata that can be used in the process of composition was outlined.

In this section it is shown how the rule schemata apply on multiple graphic layers. The use of multiple graphic layers attempts to model an intuitive design process, where different fragments of information develop associations and produce a single description. The layers of information are treated with the initially proposed rule schemata that are appropriately modified to apply on four layers.

Finally, the three distinct plan-types that are necessary for the description of the building are derived with the aid of the rule schemata.

6.1. THE FOUR GRAPHIC LAYERS

In the derivations of descriptions the rule schemata are put into use, one at the time thus forming a succession. The classification of the rule schemata in *formation*, *transformation*, and *refinement*, is a heuristic distinction, which does not add anything new to their ability to produce a description.

A description is derived in a process that attempts to accomplish several diverse objectives. In the broader sense, several layers of information participate to contribute diverse views regarding the produced artifact. Multiple descriptive fragments are composed with the aid of rule schemata, rules, and the Euclidean transformations to produce a single description that contains elements from all the participating layers.

Unlike existing examples, where parallel grammars link individual descriptions that evolve in coordination: plans, sections, layouts, etc. the proposed approach uses multiple descriptions in order to compose them in a single output.

Therefore, while individual descriptions, evolving in coordination, exchange information in predetermined ways, in the proposed model fragments of descriptions mingle in ways that are not predetermined. And, while in parallel grammars a number n of individual descriptions evolve in parallel to produce n individual descriptions ($n \Rightarrow n$), in the proposed model n descriptions are composed into a new description. None of the participating descriptions is used as an individual, and a single description is the output of the process ($n \Rightarrow 1$).

The present example uses four interacting layers A, B, C and D are superimposed to produce a single description. The layers A, B, C, include lines and symbols. Lines in $<U_{12}>$ algebra represents boundaries of spaces (i.e. walls). The component $<V_{02}>$ provides labeled points and symbols. The layer D includes planes in algebra $<U_{22}>$ that are used in the calculation of areas, and symbols $<V_{02}>$. The whole description is expressed in the product algebra $<U_{12}\ V_{02}> \times <U_{12}\ V_{02}> \times <U_{12}\ V_{02}> \times <U_{22}V_{02}>$. A color-distinction among the graphic elements of the layers A, B, C, D is used as a labeling device that makes the reading of the derivations easier.

The layers A, B, C, contain black, red, and blue lines or points, respectively. The black lines or labeled points, in layer A, are used to form the main description. The red lines, or labeled points, in layer B participate in the composition as auxiliary elements. The blue lines or labeled points, in layer C are used for the construction of grids. Four different kinds of colored planes are used for areas in layer D. The different colors signify functional distinctions indicated by the program: pink signifies empty area, yellow signifies "*public*" area, brown signifies "*private*" office area, and blue signifies "*studio*" area. A summary of the properties of layers A, B, C, D, as described above appears in the next Table 5.

TABLE 5. The three graphic layers A, B, C, D

Layer	*Content*	*Color*	*Algebra*
A	main lines/points	black	$<U_{12}\ V_{02}>$
B	auxiliary lines/points	red	$<U_{12}\ V_{02}>$
C	grids	blue	$<U_{12}\ V_{02}>$
D	areas	pink, yellow, brown, blue	$<U_{22}\ V_{02}>$

For simplicity, in the expressions of rule schemata the colors of planes in layer D are reduced into two color-variables: light and dark grey. In the derivations light grey and dark-gray may correspond to any of the following colors: pink, yellow, brown and blue, as it is shown in the next diagram.

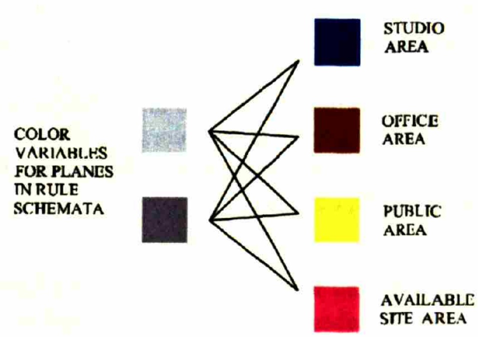
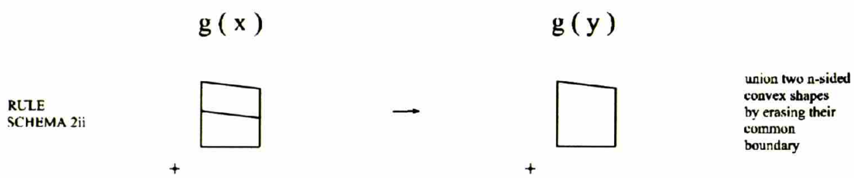


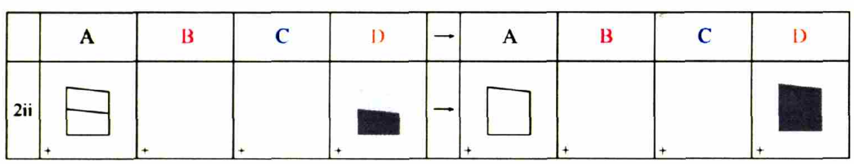
Figure 7. Variable colors in rule-schemata (left), and colors in derivations (right)

Three examples of rule schemata from the Tables 4a-4h, (pp. 228-235), are presented next. The schemata are modified to apply on the four graphic layers A, B, C, and D. The used color-variables and colors follow the conventions presented in the previous diagram, of Figure 7.

The rule schema 2ii, (Table 4a, p. 228) unites two rooms by erasing their common boundary.



The same rule schema 2ii is expressed differently, to apply on the four layers A, B, C, D,



In the example, the expression $x \rightarrow y$ becomes $(x_A, x_B, x_C, x_D) \rightarrow (y_A, y_B, y_C, y_D)$ where any of the x_i, y_i can be a dissected convex shape or the empty shape. The example involves only the two layers A and D, while the layers B, C remain intact. The shapes x_B, y_B and x_C, y_C are equal to the empty shape. The boundaries of two adjacent rooms are represented by lines in layer A. Their areas are represented by two planes of different color, in layer D. When their common boundary line is erased, in layer A, the areas are also modified in layer D so that the overall area obtains a single color. The new color can be any of the two existing colors.

This is represented in the following sample derivation. The initial state in each of the four layers A, B, C, D is,

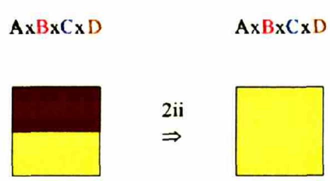
The concluding state, after the application of rule 2ii, is

The modification of color in layer D has the meaning that the newly emerged room is assigned a single function (*yellow* = *public*, according to the diagram of *Figure 7*).

A condensed version of the rule schema 2ii as a product A x B x C x D is presented next in the algebra $<U_{12}V_{02}>$ x $<U_{12}V_{02}>$ x $<U_{12}V_{02}>$ x $<U_{22}\ V_{02}>$

	A x B x C x D	→	A x B x C x D
2ii		→	

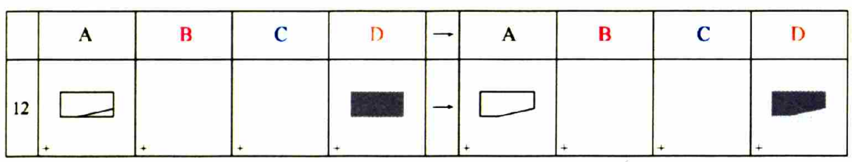
The next derivation shows the result from the application of the rule schema 2ii, after the superimposition of the layers A, B, C, D, also in the algebra $<U_{12}V_{02}>$ x $<U_{12}V_{02}>$ x $<U_{12}V_{02}>$ x $<U_{22}\ V_{02}>$



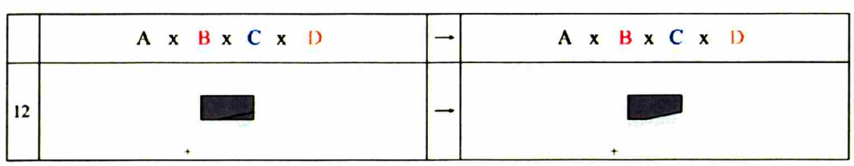
The overall results after the application of the rule 2ii are that the area of the two rooms is unified, their common boundary is erased, and the overall area obtains a single functionality, which can be any of the two.

In the next example, the rule schema 12, (Table 4f, p. 233) selects an (n+1)-sided convex shape, from an existing n-sided convex shape.

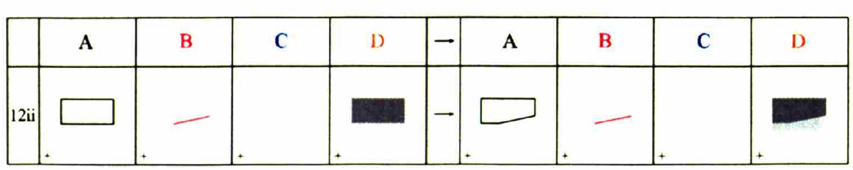
The same rule schema 12 can be expressed so that it can apply on the four graphic layers A, B, C, D,



The previous rule schema involves only two of the four layers, namely the layers A and D. A condensed description of rule schema 12, as a product A x B x C x D is presented next

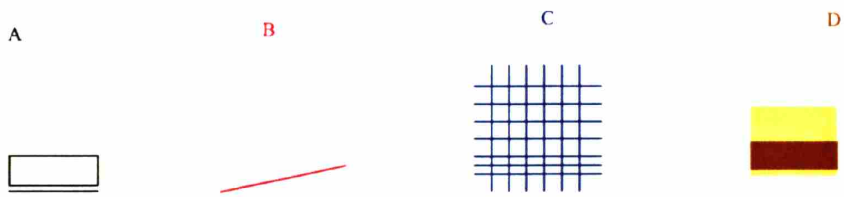


The next rule is similar. The two layers A and D are used, while the layers B, C remain intact. The boundaries of two adjacent rooms are represented by lines in layer A, and their areas by two colored planes in layer D. When one of the two rooms is erased, in layer A, the areas are also modified in layer D. This is expressed in the next rule schema 12ii,

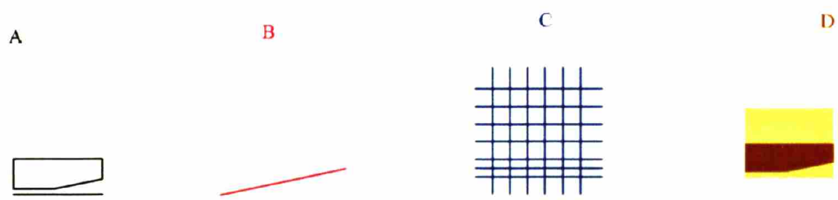


In the rule schema 12ii the layers A and D are modified and the layers B and C remain intact. The linear parts of the shapes are represented in the layers A and D, and their areas are represented by colored areas in layer D. A sample of the application of the rule schema 12ii follows next.

The initial state, in each of the layers A, B, C, D is,



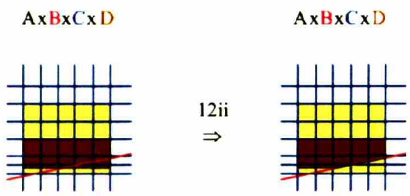
The concluding state, after the application of the rule schema 12ii is,



The example shows the interaction of descriptions that belong to different layers. This interaction is specified in the rule schema 12ii: A line in layer B "interacts" with the shapes in layers A and D: The red line in B determines that some part of the shapes in the layers A and D is selected and some other part is erased, or modified. A condensed description of the rule schema 12ii is formed as a Cartesian product of the layers A x B x C x D, in the algebra $<U_{12}V_{02}>$ x $<U_{12}V_{02}>$ x $<U_{12}V_{02}>$ x $<U_{22}\ V_{02}>$

	A x B x C x D	→	A x B x C x D
12ii		→	

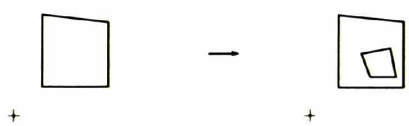
The derivation of the previous page is presented in the next illustration as a product of the layers A x B x C x D. The description results from the application of rule schema 12ii and the superimposition of A, B, C, and D



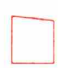
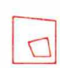
Examples of rule schemata that are immediate extensions of rules presented in the Tables 4a-4h, (pp. 228-235), are presented next. The schemata are specifically formed for the application on four graphic layers.

The rule schema 1 (Table 4a, p. 228) draws an n-sided convex shape (representing a room) inside another convex n-sided shape (representing another room).

RULE
SCHEMA 1



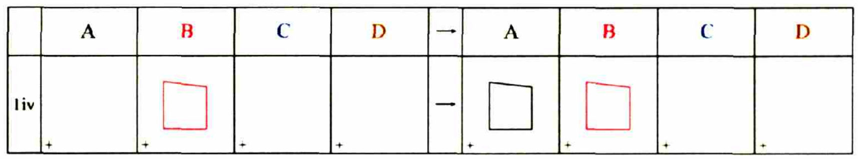
The same general rule schema 1 is used to apply on four graphic layers,

	A	B	C	D	→	A	B	C	D
1					→				

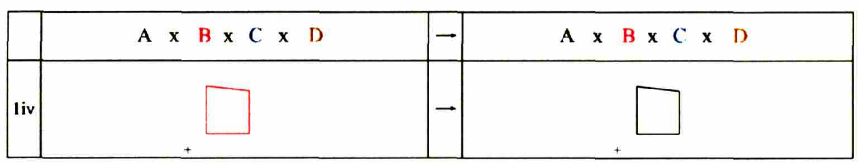
The above rule schema 1 draws one shape inside another, in layer B. The rule schema leaves the layers A, C, and D intact. The condensed version of the above rule schema, is

	A x B x C x D	→	A x B x C x D
1		→	

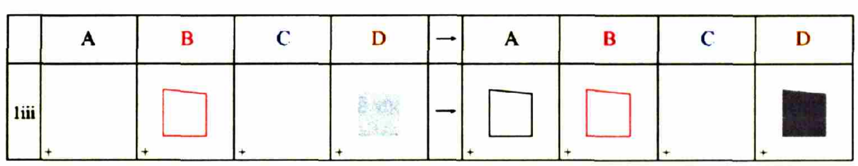
The next two rule schemata are presented as useful extensions of the rule schema 1. The rule schema 1iv,



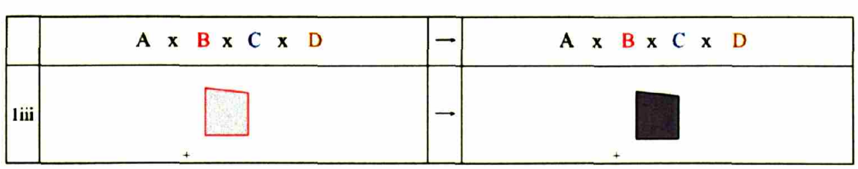
draws a convex shape in layer A over a convex shape found in layer B. The layers C, D remain intact by this change. The layer B participates in the identification of the shape, while the layer A is modified. A condensed version of the above rule schema 1iv, is



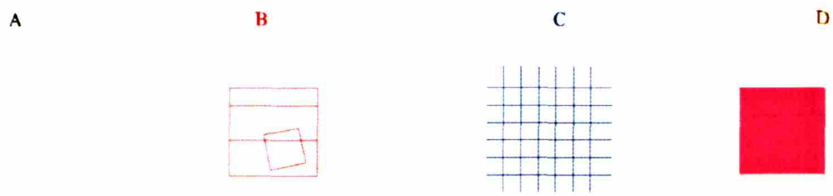
The shape in layer A is drawn over the red shape in layer B, which remains unchanged. The next rule schema 1iii draws a convex shape in layer A over a convex shape found in layer B, but it also modifies the color of its area, in layer D. Three layers A, B, and D, participate in this development.



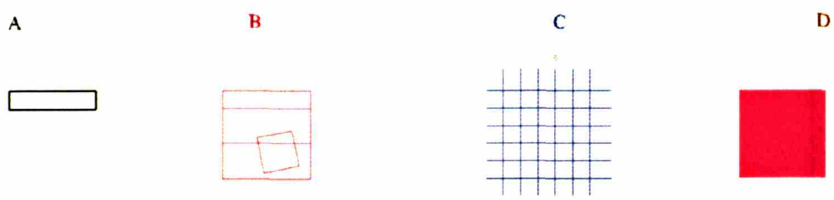
The condensed version of rule schema 1iii, is



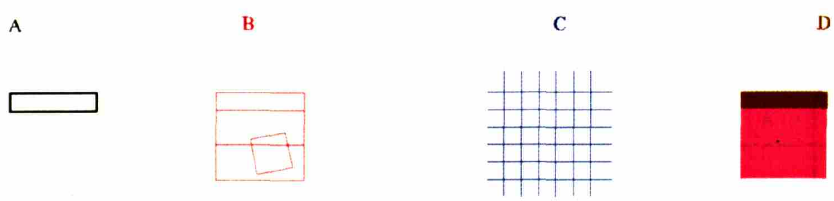
The similarities and the differences between the two rule schemata 1iv and 1iii become apparent in the next example of a derivation. The initial state, in each of the layers A, B, C, D is,



The concluding state, after the application of the rule schema 1iv is,

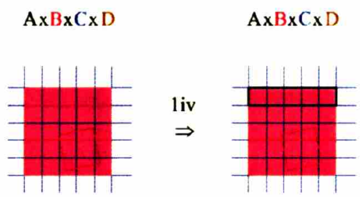


The concluding state, after the application of the rule schema 1iii is,

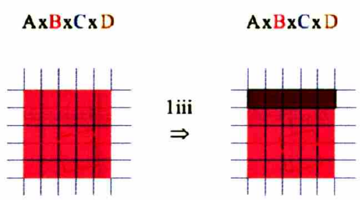


In both rule schemata 1iv and 1iii, a convex shape made out of lines is drawn in layer A over a similar shape in layer B. The difference is that the rule 1iii modifies the color of the corresponding area, while 1iv leaves the color of the area intact. Notice that both rule schemata leave the shape in layer B unchanged.

The condensed version of the previous derivation as product A x B x C x D, after the application of rule schema 1iv, is



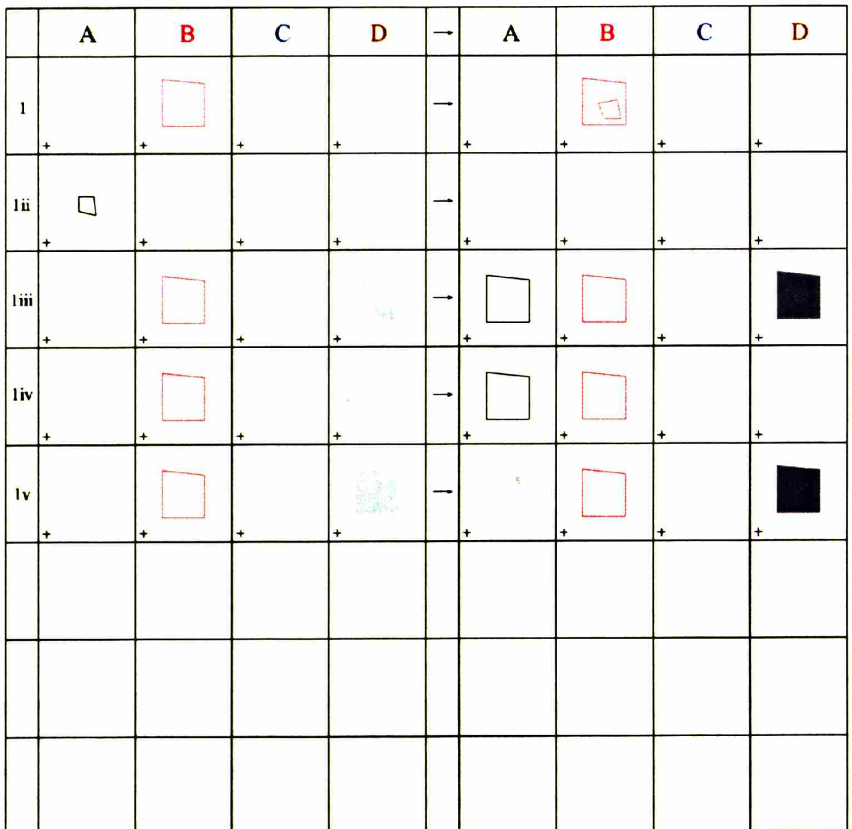
The condensed version of the derivation as product A x B x C x D after the application of rule schema 1iii, is



The extended list of compositional rule schemata of the Tables 4a-4h, (pp. 228-235), and their corresponding erasing, and extension rules, appears in the next series of twelve Tables 6a-6l. The new Tables contain the complete modified versions of the rule schemata so that these can apply in four layers A, B, C, D in the algebra $<U_{12}V_{02}>$ x $<U_{12}V_{02}>$ x $<U_{12}V_{02}>$ x $<U_{22}\,V_{02}>$.

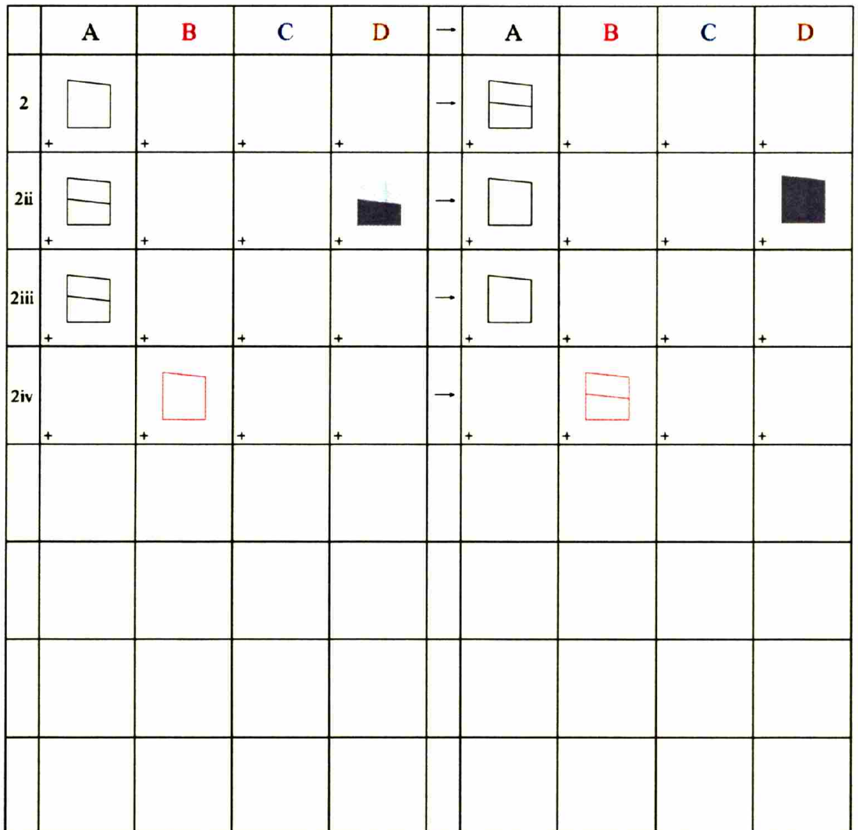
The ordering and the variations under which the same rule schema can apply in shapes that belong to the four descriptions A, B, C, D, to cause some desirable development, are explored in more detail. The rule schemata are still not ordered into a grammar, or strictly restricted to apply on specific shapes. This can be the subject of a subsequent refinement of the preferred dimensions of rooms, ratios of spaces, etc., and it is not part of this study. But the parametric instances of rule schemata can be grouped together according to the more general compositional objectives they accomplish.

TABLE 6a.



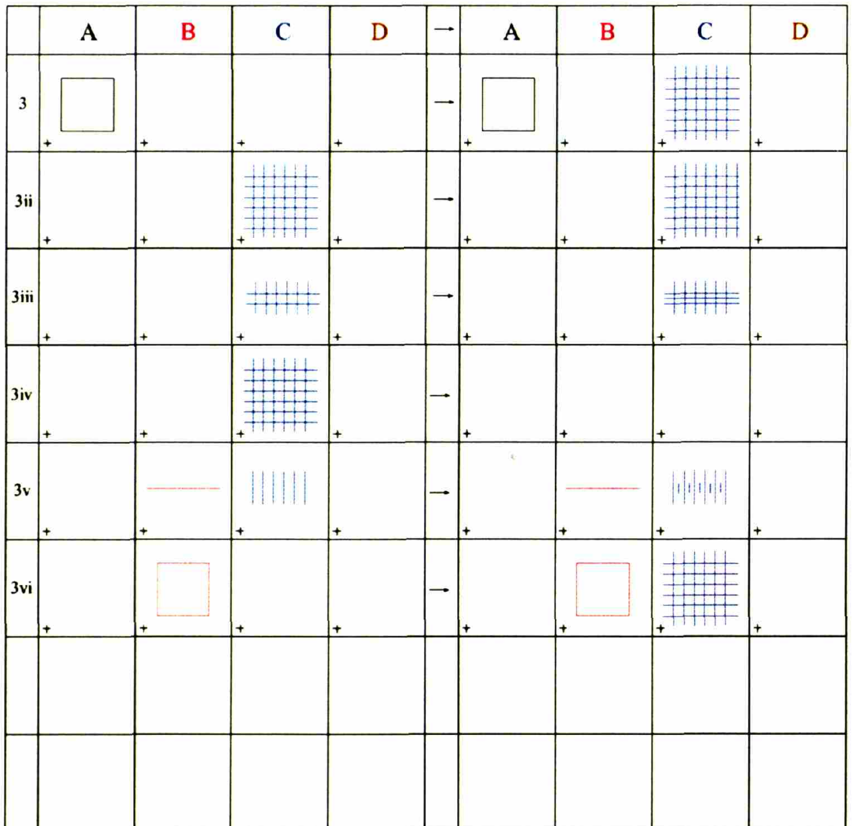
RULE SCHEMATA GENERATING THE INITIAL CONCEPT

TABLE 6b.



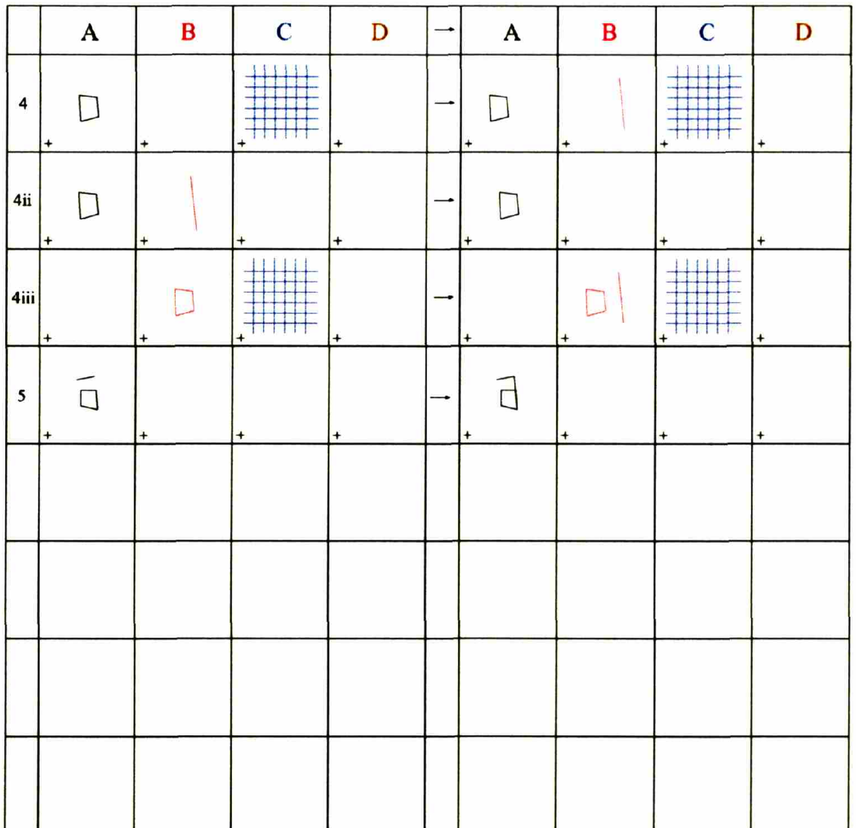
RULE SCHEMATA GENERATING THE INITIAL CONCEPT

TABLE 6c.



RULE SCHEMATA FOR GENERATION AND MODIFICATION OF GRIDS

TABLE 6d.



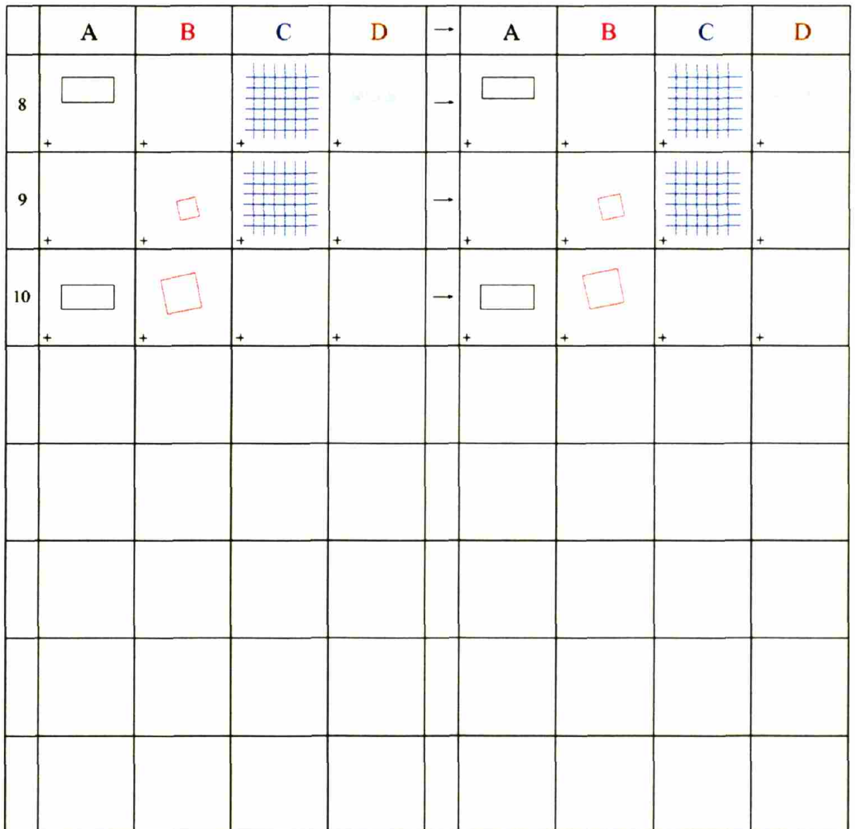
RULE SCHEMATA GENERATING LINES IN RELATIONSHIP TO A SIDE OF A CONVEX SHAPE

TABLE 6e.

	A	B	C	D	→	A	B	C	D
6					→				
6ii					→				
7					→				
7ii					→				

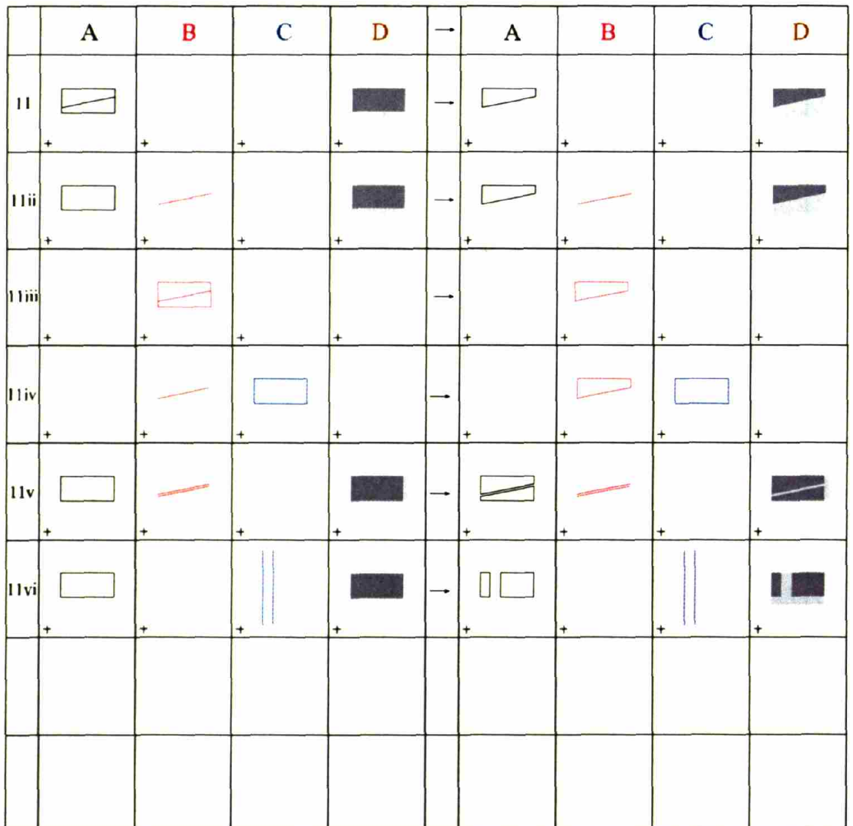
RULE SCHEMATA GENERATING LINES IN RELATIONSHIP TO A SIDE OF A CONVEX SHAPE

TABLE 6f.



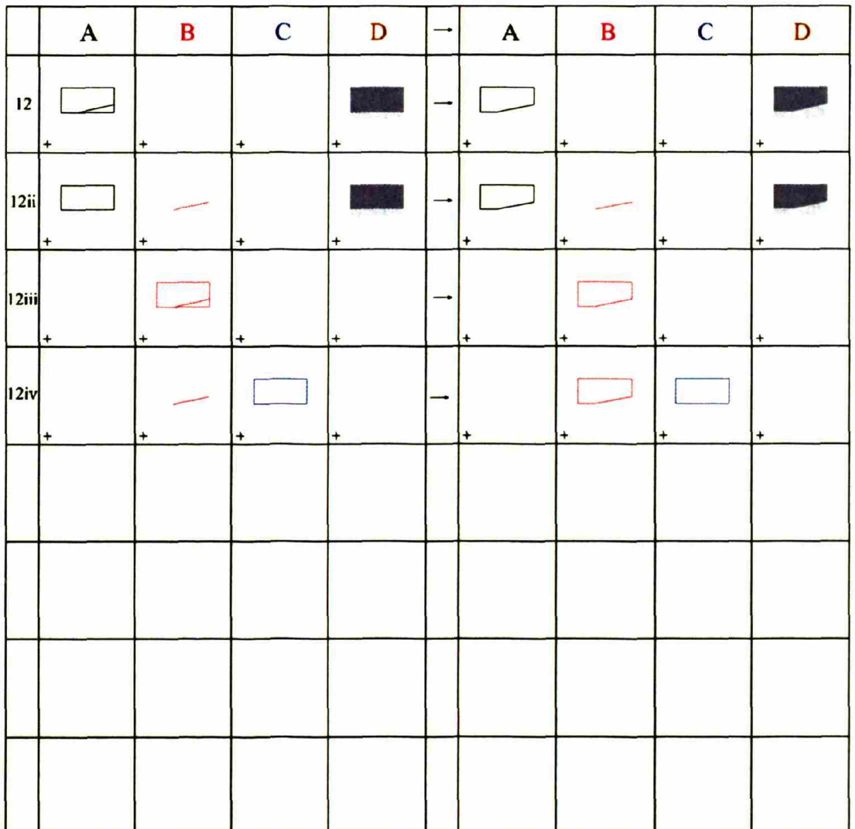
RULE SCHEMATA FOR TRANSLATION AND SCALING

TABLE 6g.



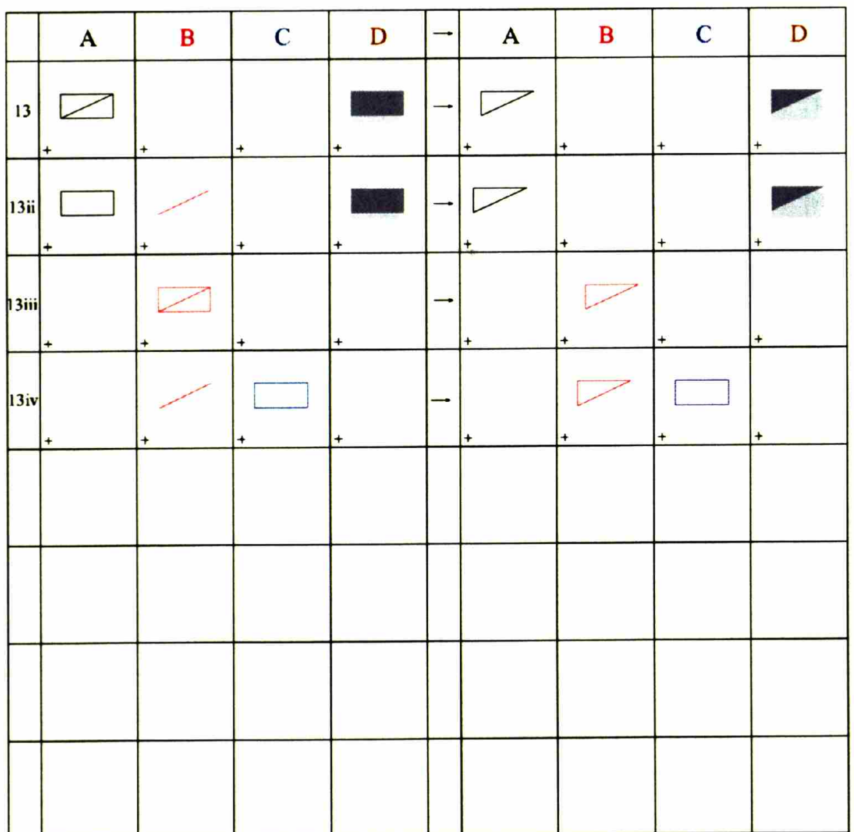
RULE SCHEMATA FROM n-SIDED CONVEX SHAPE TO n-SIDED CONVEX SHAPE

TABLE 6h.



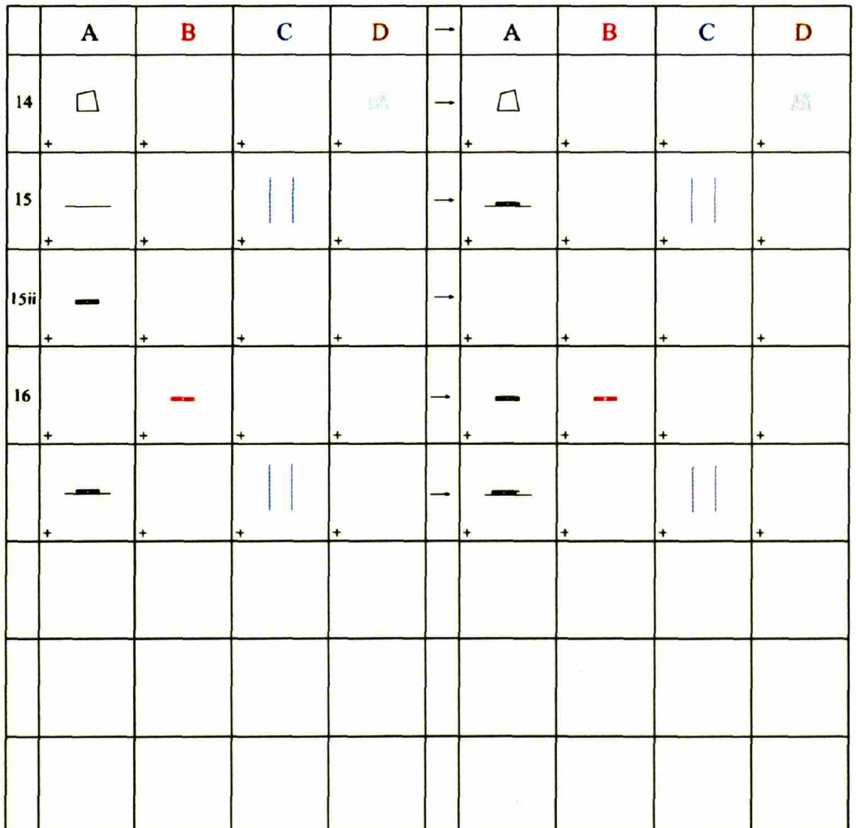
RULE SCHEMATA FROM n-SIDED CONVEX SHAPE TO (n+1)-SIDED CONVEX SHAPE

TABLE 6i.



RULE SCHEMATA FROM n-SIDED CONVEX SHAPE TO (n-1)-SIDED CONVEX SHAPE

TABLE 6j.



RULE SCHEMATA OF REFINEMENT

TABLE 6k.

	A	B	C	D	→	A	B	C	D
17					→				
18					→				
19					→				
20					→				
21					→				
21ii					→				
21iii					→				

RULE SCHEMATA OF REFINEMENT

TABLE 61.

	A	B	C	D	→	A	B	C	D
22					→				
23					→				

RULE SCHEMATA FOR ERASING AND CHANGING GARPHIC LAYERS

6.2. THE THREE PLANS

Three plans I, II, III, necessary for the complete description of the building are specified in this section.

In the preceding sections it was suggested that the conceptual schema (*parti*) is a hypothesis that links building program and form, by expressing some of the basic spatial properties of the design. More importantly, the conceptual schema provides an initial representation, and frames the search-space. In this project, the building was described by the designer as a "box" of cubic form (105ft x 105ft x 105ft) with a second small cubic box (approximately 50ft x 50ft x 50ft) floating within the first, containing the creative studios. Offices are placed at the sides of the exterior cubic envelope. The public lobby occupies the central ground floor area.

The careful calculation of the programmatically required square-footage made the addition of extra office-space necessary. The conceptual schema was modified. The initial and modified versions of the conceptual schema are presented next in the algebra $<U_{13}U_{33}>$. The full schema occupies the top node of each tree, and the parts appear on the leaves. The cubic envelope, represented by its vertices, appears at the left node. Studios and offices represented by solids, occupy the center and right nodes. The extra office-space, appears on the right

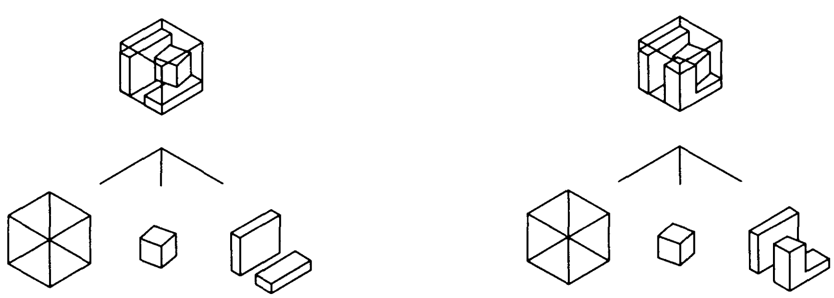


Figure 8. The conceptual schema: on the left the initial version, and on the right the modified version of the schema with extra office space

The plan of a physical object is an artificial representation. It is a view not obtainable in reality. It is produced by dissecting the object horizontally and projecting the view on the graphic plane. Several consecutive dissections may be necessary for the comprehensive description of an artifact. The positioning of the dissections is critical.

The next sequence of diagrammatic perspective views of the conceptual schema helps us to distinguish the plan-section heights, and to determine the necessary number of plans.

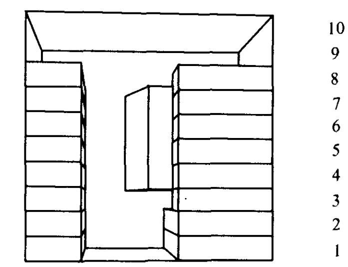


Figure 9. Perspective view of the full conceptual schema

The building is described as a cube 105ft x 105ft x 105ft, divided in ten 10.5 ft high floors A second small cubic box, approximately 50ft x 50ft x 50ft, is placed between the floors 4-8. The offices occupy the floors 1 through 8, while the floors 9 and 10 remain empty.

Three distinct plans I, II and III are necessary for the comprehensive description of the building. The floors 1 and 2 have the same footprint. The next perspective diagram presents the section-line of the plan I, which describes the floors 1 and 2

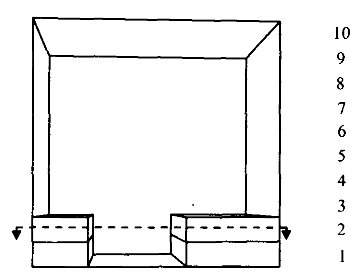


Figure 10. Section line of plan I

Next, floor 3 differs from the floors 1 and 2, but also from the floors above. The perspective diagram presents the section-line of plan II. The plan II describes only floor 3,

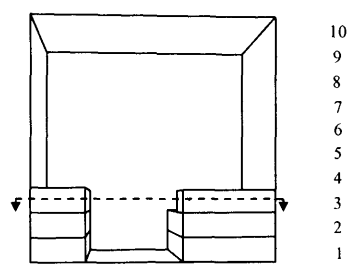


Figure 11. Section line of plan II

The floors 4 through 8 have the same footprint, and they are described by the same plan III. The perspective diagram presents the section-level of a typical plan III.

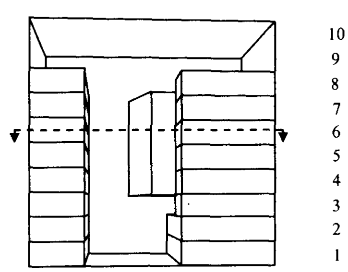


Figure 12. Section line of plan III

The next axonometric recapitulates the distribution of the three plans I, II, and III.

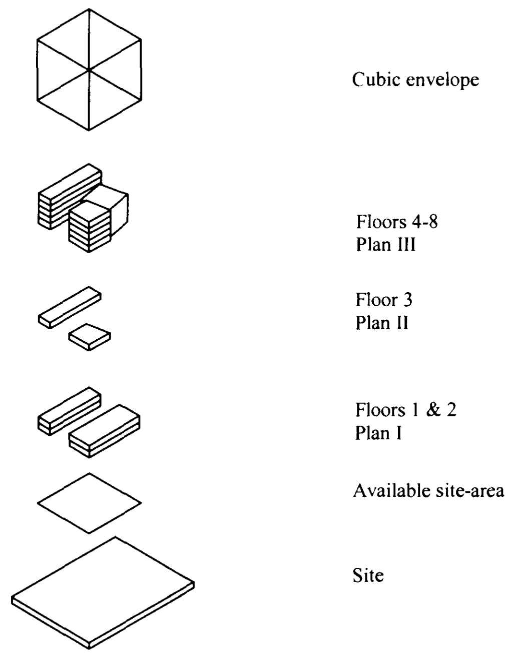
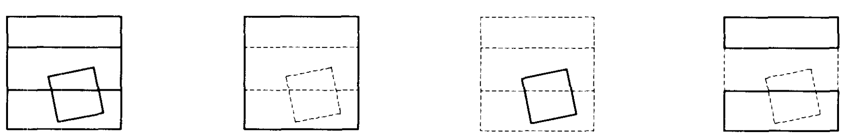


Figure 13. Distribution of the plans I, II, III

All plans are derived from a single diagrammatic plan, composed of: a square, representing the exterior envelope, a smaller square, representing the studios, and two rectangles representing the office-spaces.



In the derivations of the plans I, II, and III the wall-thicknesses are ignored, walls are represented by single lines, and window or door openings are disregarded.

The four layers A, B, C, D correspond to four sheets of paper. The layers A, B, C are represented by transparent pages, while the layer D is an opaque sheet. Each sheet contains nine descriptions of the design that form a sequence in the derivation and are read from left to right.

For example, at each step of the derivation, in layer A, one can see the following nine descriptions A_1, A_2, A_3,...A_9. In layer B: B_1, B_2, B_3,...B_9. Similarly, in each of the layers C and D, one can see nine descriptions. In order to understand a change in the design, apart from this sequence of descriptions that are contained in each layer, one has to perceive all the four layers superimposed. That is,

$$(A_1 \times B_1 \times C_1 \times D_1) \Rightarrow (A_2 \times B_2 \times C_2 \times D_2) \Rightarrow \ldots \Rightarrow (A_9 \times B_9 \times C_9 \times D_9)$$

On each page, brief text-descriptions indicate which rule schema applies to a specific design description.

In the beginning of the derivation of plan I, the initial shape is the shape of the available site-area and its boundaries. It consists of a square, in made out of red lines, in layer B, and a pink area, in layer D, indicating the available area of the site (unit shape). The layers A and C contain no shapes.

Figure 14. Initial shapes in the layers A, B, C, D in the derivation of plan I.

After the completion of plan I, the produced descriptions in all layers A, B, C, D, become the initial shapes in deriving plan II. Layer A contains some arrangement of black lines, layer B some arrangement of red lines, layer C some blue grid lines and layer D some colored areas.

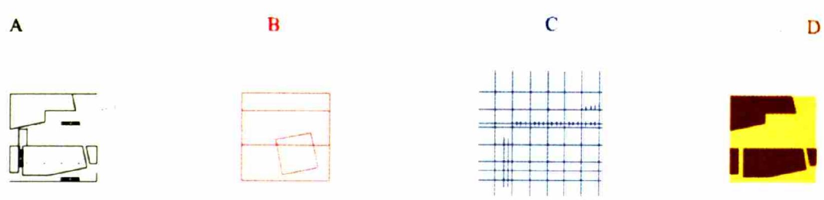


Figure 15. Initial shapes in the layers A, B, C, D in the derivation of plan II. The shapes are those of the terminating state of plan I

At the first step of the derivation of plan II, the arrangement in layer A is transferred to layer B. The produced shapes are $C_A' = \varnothing$ and $C_B' = C_A \cup C_B$. In layer D, the colored areas are substituted by the unit shape of the overall available area. The layer C remains as it is ($C_C' = C_C$).

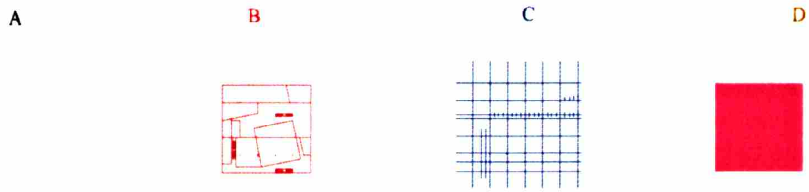


Figure 16. Shapes in the derivation of plan II, immediately after the first step.

After the completion of plan II, the produced descriptions in all layers A, B, C, D become the initial shapes in deriving plan III. Layer A contains some arrangement of black lines, layer B some arrangement of red lines, layer C some blue grid lines, and layer D some colored areas.

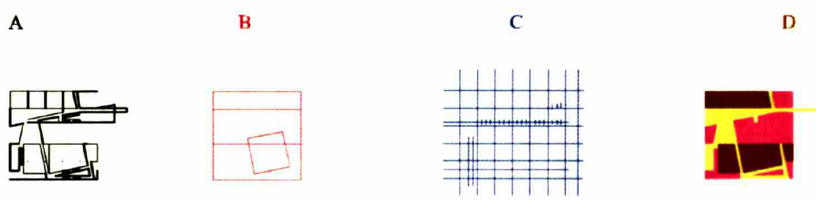


Figure 17. Initial shapes in the layers A, B, C, D in the derivation of plan III. The shapes are those of the terminating state of plan II

In the first step of the derivation of plan III, the arrangement in layer A is transferred to layer B. The produced shapes are $C_A' = \varnothing$ and $C_B' = C_A \cup C_B$. In layer D, the colored areas are substituted by the unit shape of the overall available area. The layer C remains as it is ($C_C' = C_C$).

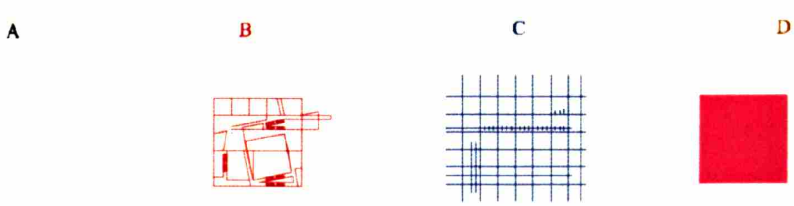


Figure 18. Shapes in the derivation of plan III, immediately after the first step.

A recapitulation of the initial shapes in all four layers A, B, C, D, at the initial stage of the derivation of the plans I, II, III is presented in Table 7.

TABLE 7. Initial shapes in each of the layers A, B, C, D, for the plans *I*, *II*, *III*

four layers	*plan I*	*plan II*	*plan III*
A	∅	layer A from plan I	layer A from plan II
B	site boundaries	layer B from plan I	layer B from plan II
C	∅	layer C from plan I	layer C from plan II
D	site area	layer D from plan I	layer D from plan II

The interaction of the three plans I, II, III as it is presented in this study, is shown in the next table,

TABLE 8. The interaction among the plans I, II, III: The terminated plan II is transferred at the initial state the derivation of plan II. The terminated plan II is transferred at the initial state of the derivation of plan III.

start *plan I* $(A_I \times B_I \times C_I \times D_I)$		
. . .		
end *plan I* $(A_I \times B_I \times C_I \times D_I)$ →	start *plan II* $(A_{II} \times B_{II} \times C_{II} \times D_{II})$	
	. . .	
	end *plan II* $(A_{II} \times B_{II} \times C_{II} \times D_{II})$ →	start *plan III* $(A_{III} \times B_{III} \times C_{III} \times D_{III})$
		. . .
		end *plan III* $(A_{III} \times B_{III} \times C_{III} \times D_{III})$

The previous Table 8 shows each of the plans I and II to be used in their subsequent derivation. But, in practice, if a description is used in the subsequent derivation, then it is not preserved. A copy of it must be produced, so that the description is preserved and the copy is passed to the next stage of the derivation. A possible solution[2] to this problem is described next: At the start of a derivation one can create duplicates A', B', C', D' of the layers A, B, C, D. The duplicates follow the same developments like the main description in every step of the derivation. When the derivation A x B x C x D is terminated, the duplicate representation A' x B' x C' x D' is used for the derivation of the next plan. This possible process, including duplicate representations, is captured in the next Table 9,

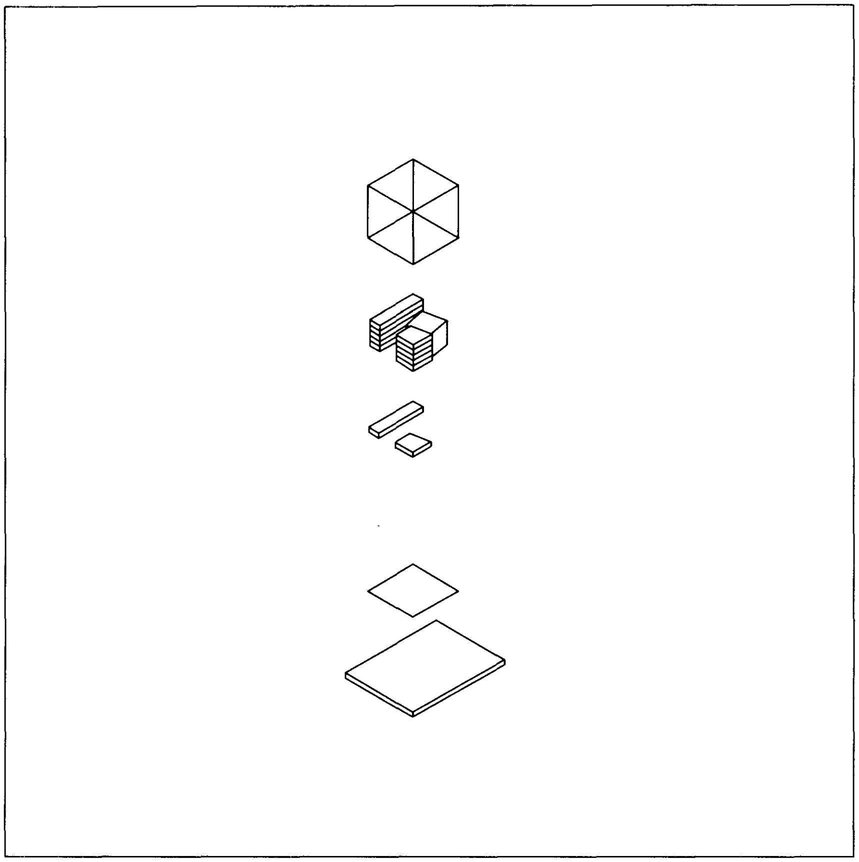
TABLE 9. A duplicate description A' x B' x C' x D' of plan I is transferred at the initial state of deriving plan II. The same happens between plan II and plan III

start *plan I* (A_I x B_I x C_I x D_I)	start duplicate *plan I* (A_I' x B_I' x C_I' x D_I')	
. . .	. . .	
end *plan I* (A_I x B_I x C_I x D_I)	end duplicate *plan I* (A_I' x B_I' x C_I' x D_I')	
	start *plan II* (A_{II} x B_{II} x C_{II} x D_{II})	start duplicate *plan II* (A_{II}' x B_{II}' x C_{II}' x D_{II}')
	. . .	. . .
	end *plan II* (A_{II} x B_{II} x C_{II} x D_{II})	end duplicate *plan II* (A_{II}' x B_{II}' x C_{II}' x D_{II}')
		start *plan III* (A_{III} x B_{III} x C_{III} x D_{III})
		. . .
		end *plan III* (A_{III} x B_{III} x C_{III} x D_{III})

The process of derivation that is presented in this study is the one of Table 8, without duplicate representations.

[2] This approach was suggested by Prof. Terry Knight.

6.2.1. Derivation of Plan I

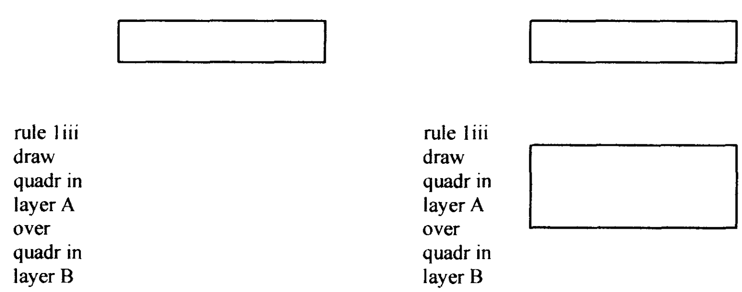


The next derivation of plan I, (pp. 269-324) in four sheets A x B x C x D, generates the plan type corresponding to the floors 1 and 2, as shown in the above diagram with green color.

The initial shape is the available site area, represented by a pink square plane (in sheet D) and its boundary, represented by a square made out of red lines (in sheet B). The pink square in sheet D serves as unit shape for areas. In the derivation, the sum of all areas of any color (pink, yellow, brown or blue) is always equal to the area of this initial pink square, which represents the available free area of the site.

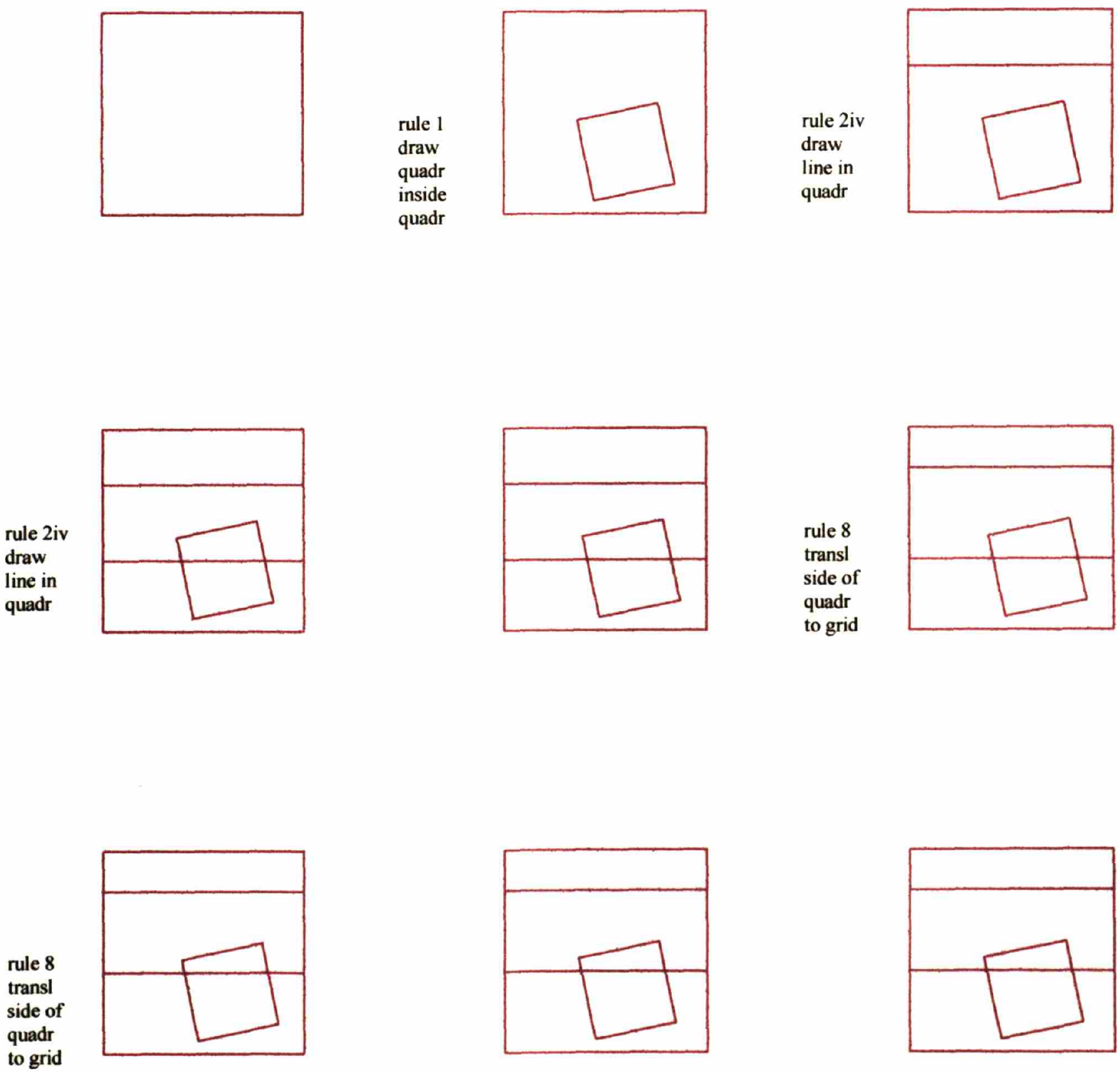
The overall derivation unfolds in the product algebra $<U_{12}\ V_{02}>$ x $<U_{12}\ V_{02}>$ x $<U_{12}\ V_{02}>$ x $<U_{22}V_{02}>$.

A


rule 1iii
draw
quadr in
layer A
over
quadr in
layer B
rule 1iii
draw
quadr in
layer A
over
quadr in
layer B

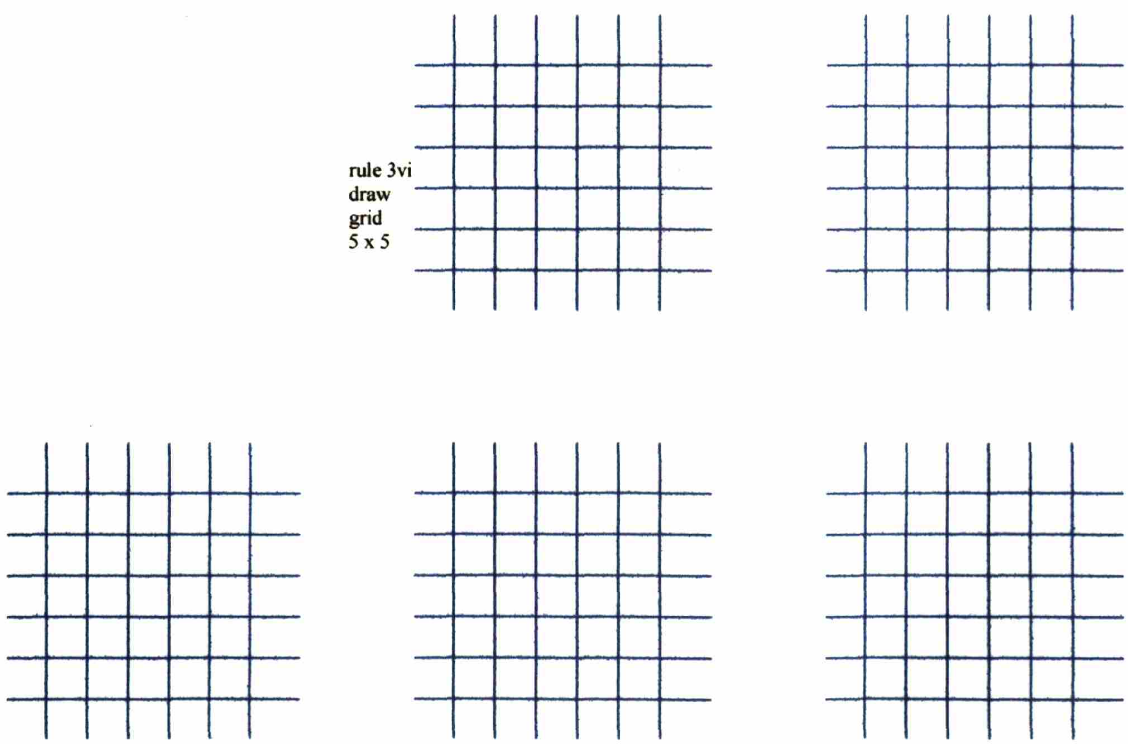
PLAN I

B


rule 1
draw
quadr
inside
quadr
rule 2iv
draw
line in
quadr
rule 2iv
draw
line in
quadr
rule 8
transl
side of
quadr
to grid
rule 8
transl
side of
quadr
to grid

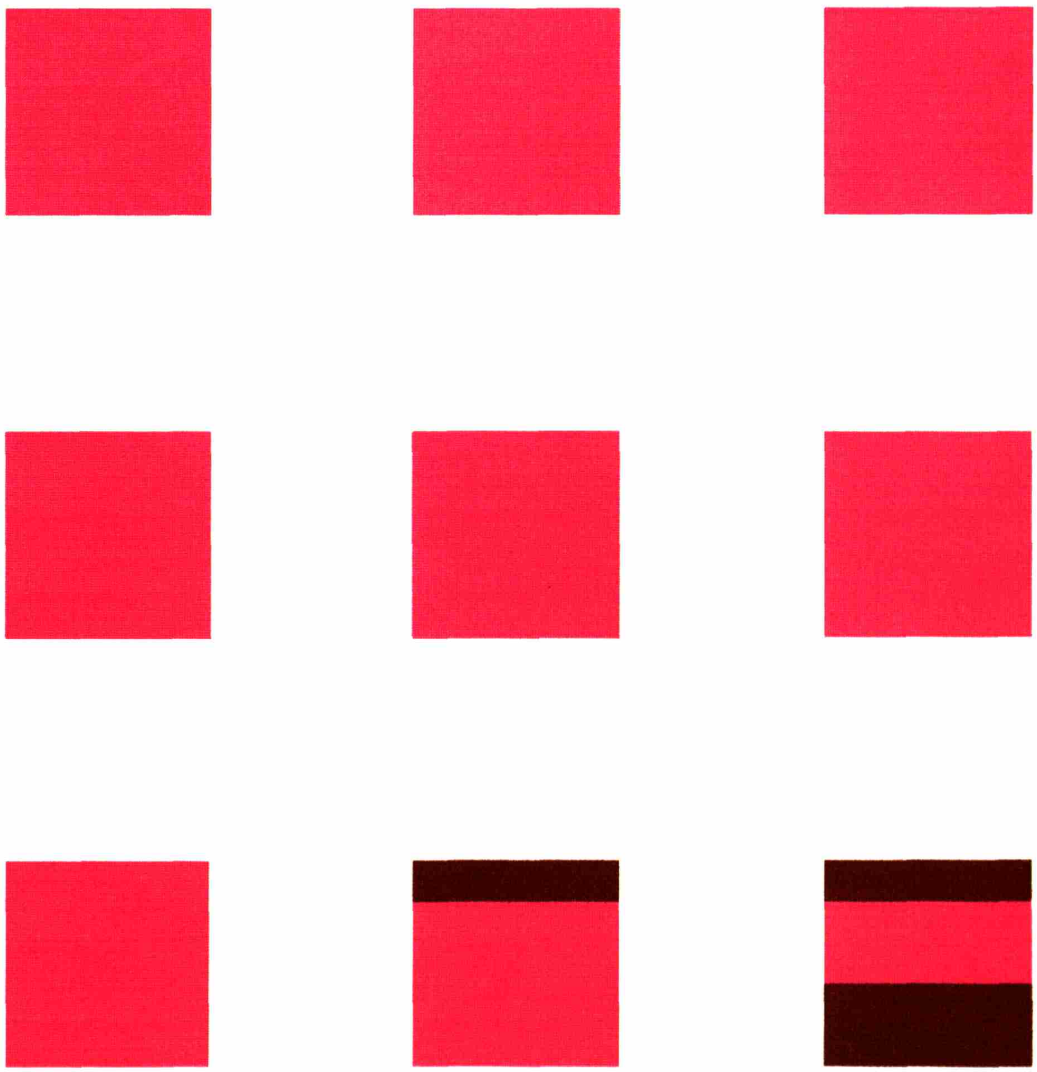
PLAN I

C



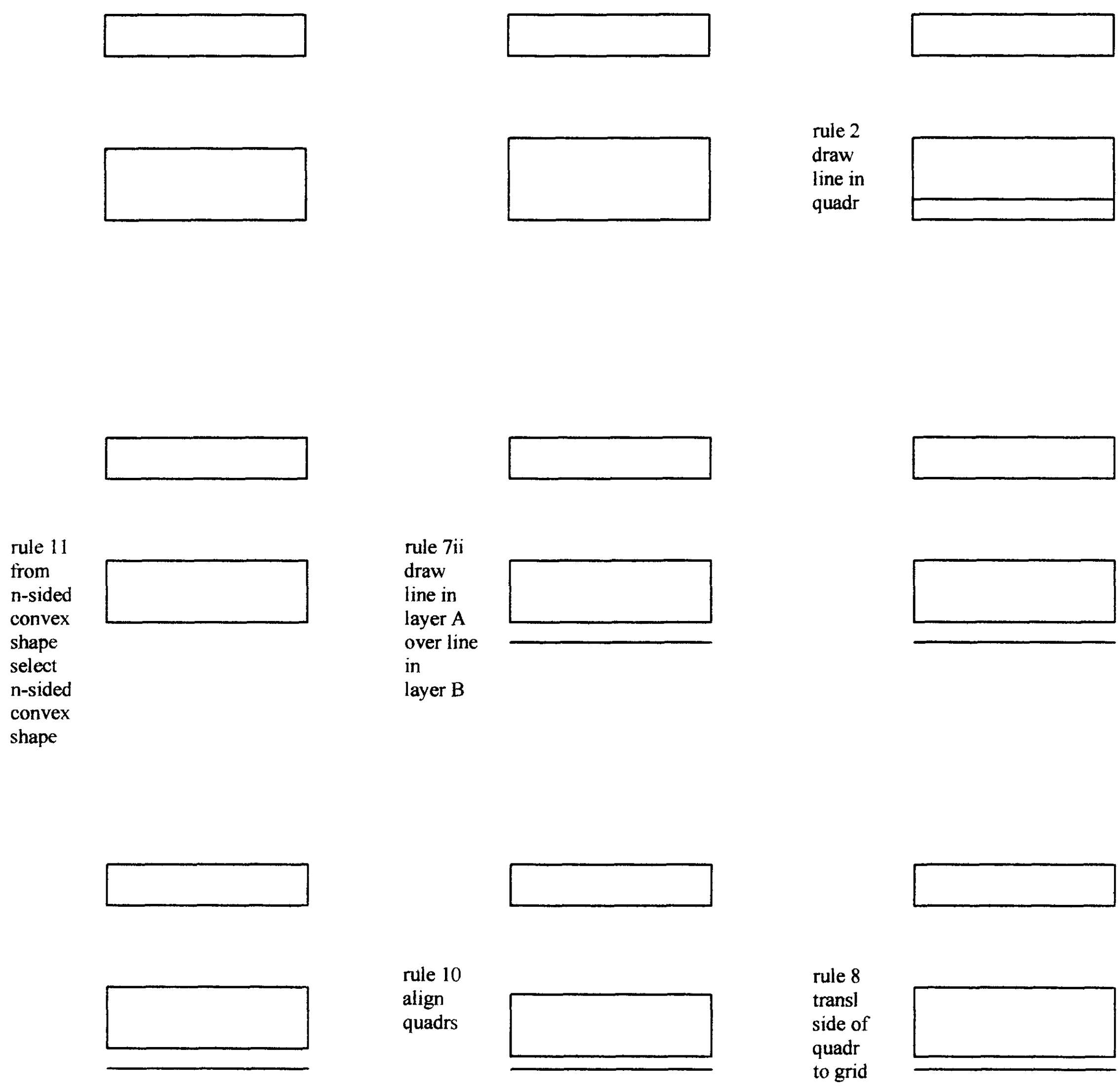
PLAN I

D



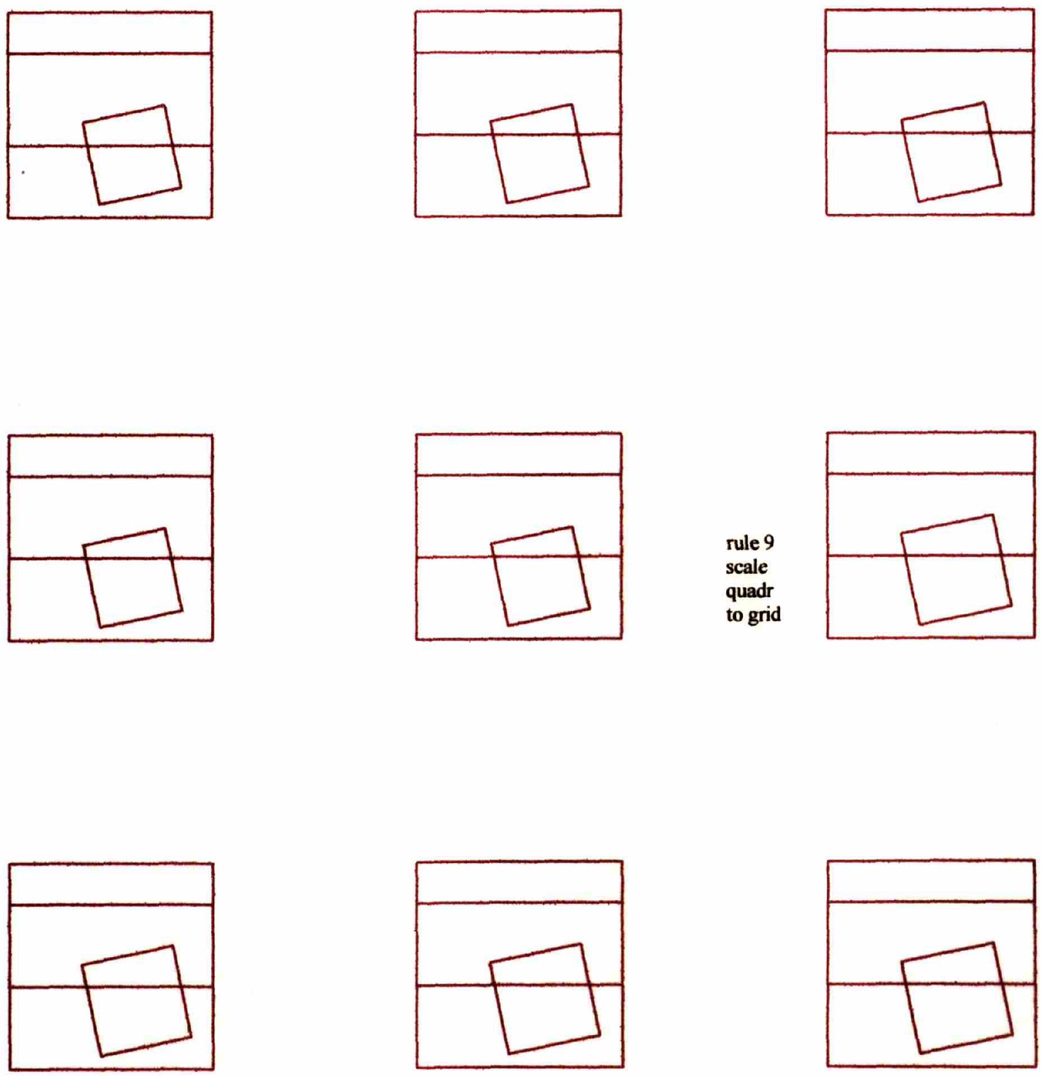
PLAN I

A


rule 2
draw
line in
quadr
rule 11
from
n-sided
convex
shape
select
n-sided
convex
shape
rule 7ii
draw
line in
layer A
over line
in
layer B
rule 10
align
quadrs
rule 8
transl
side of
quadr
to grid

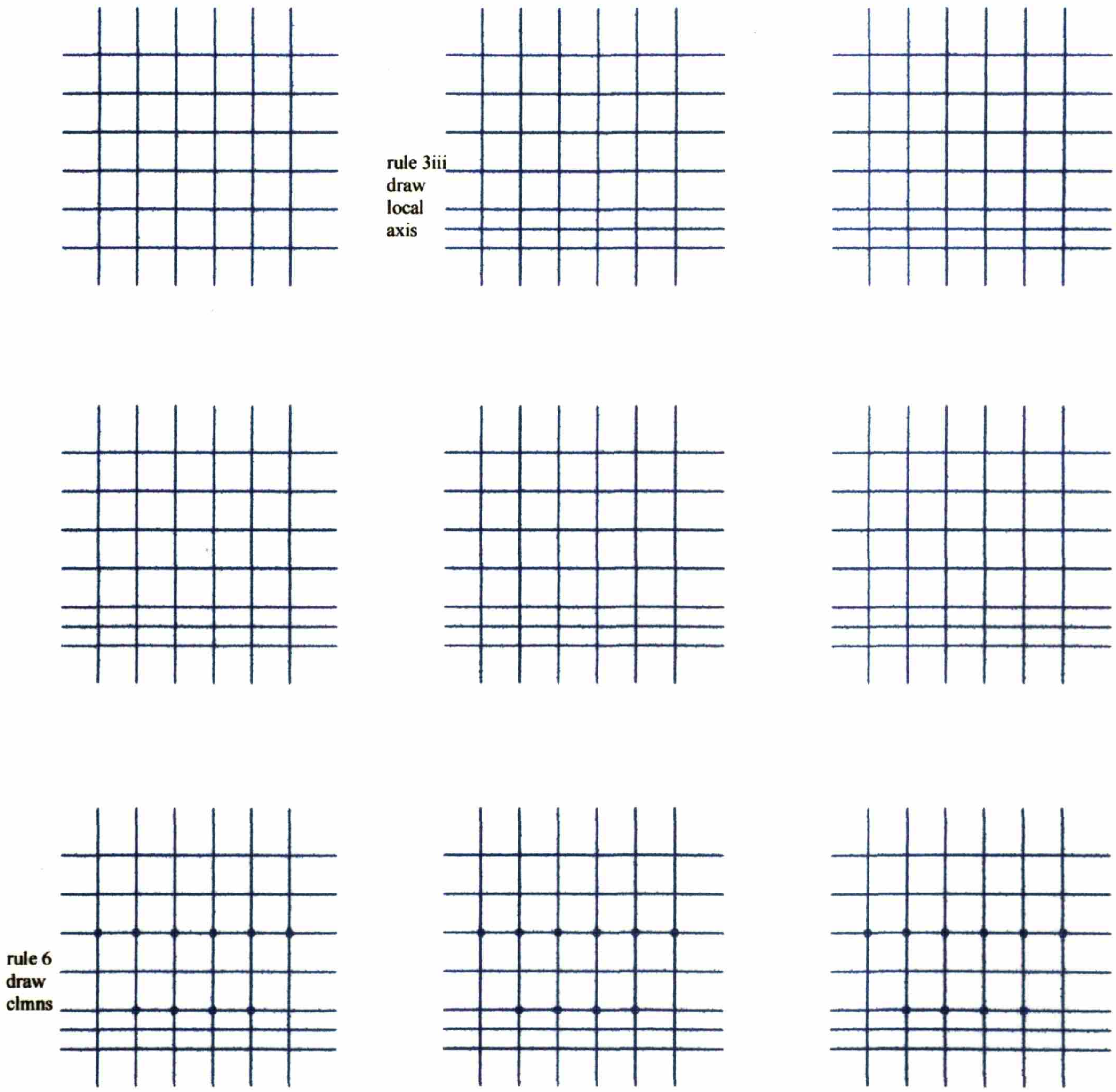
PLAN I

B



PLAN I

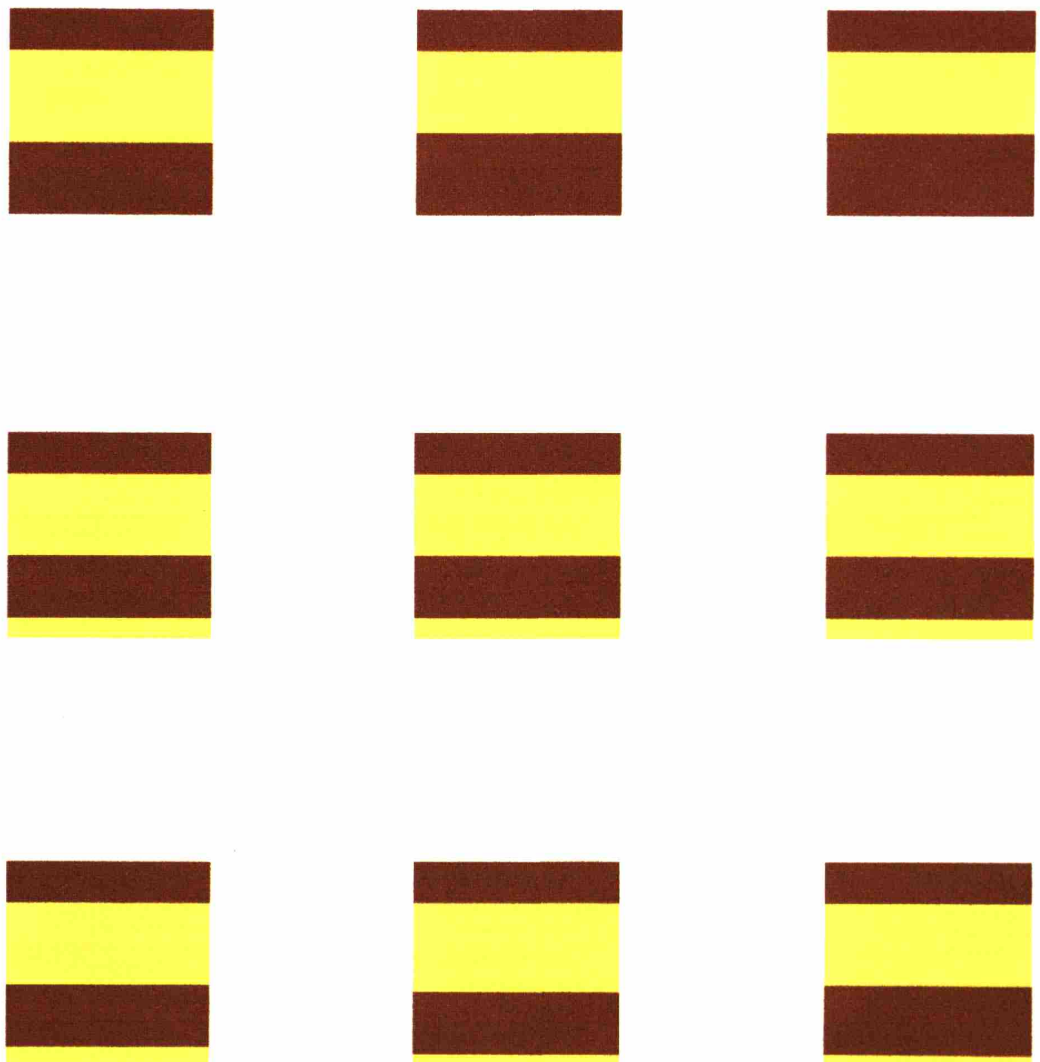
C



PLAN I

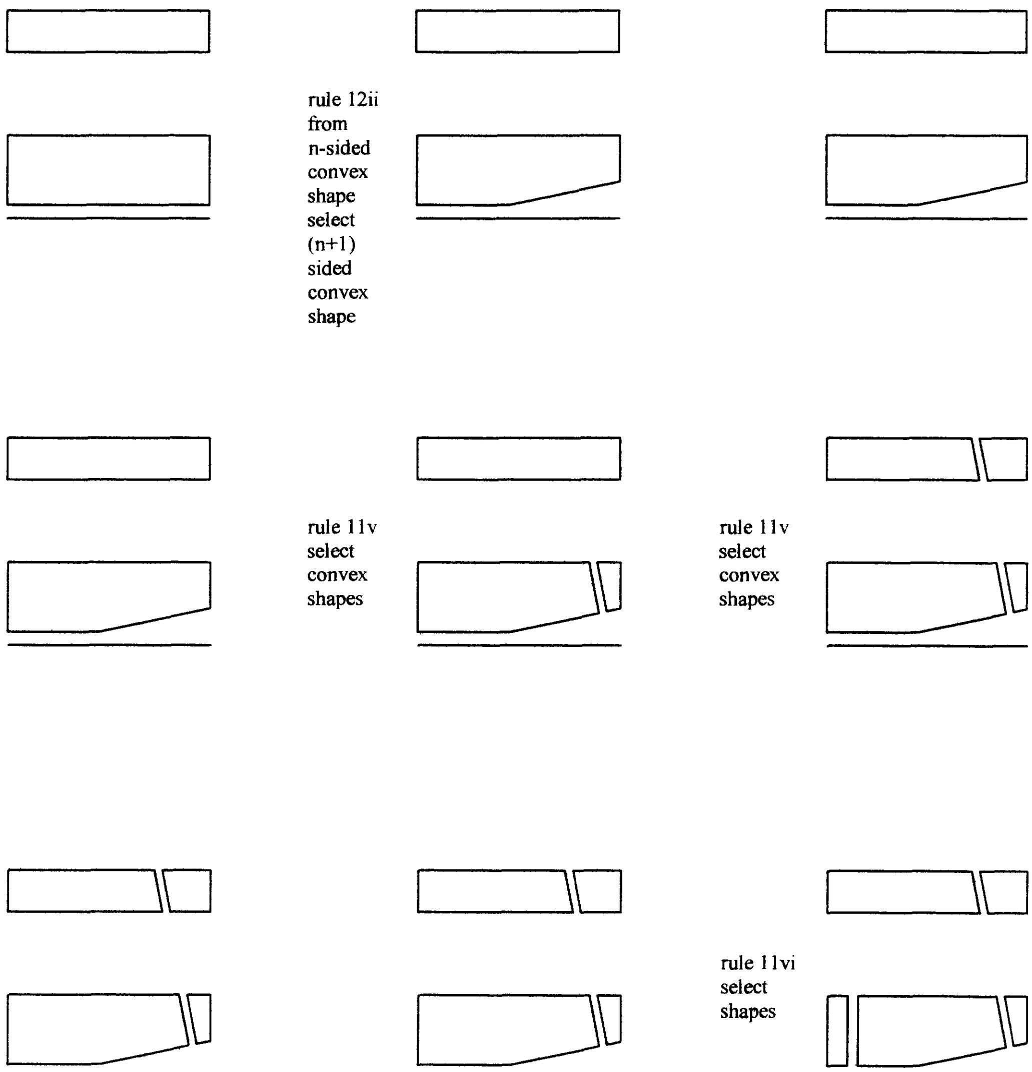
D

rule 1v
assign
area
public



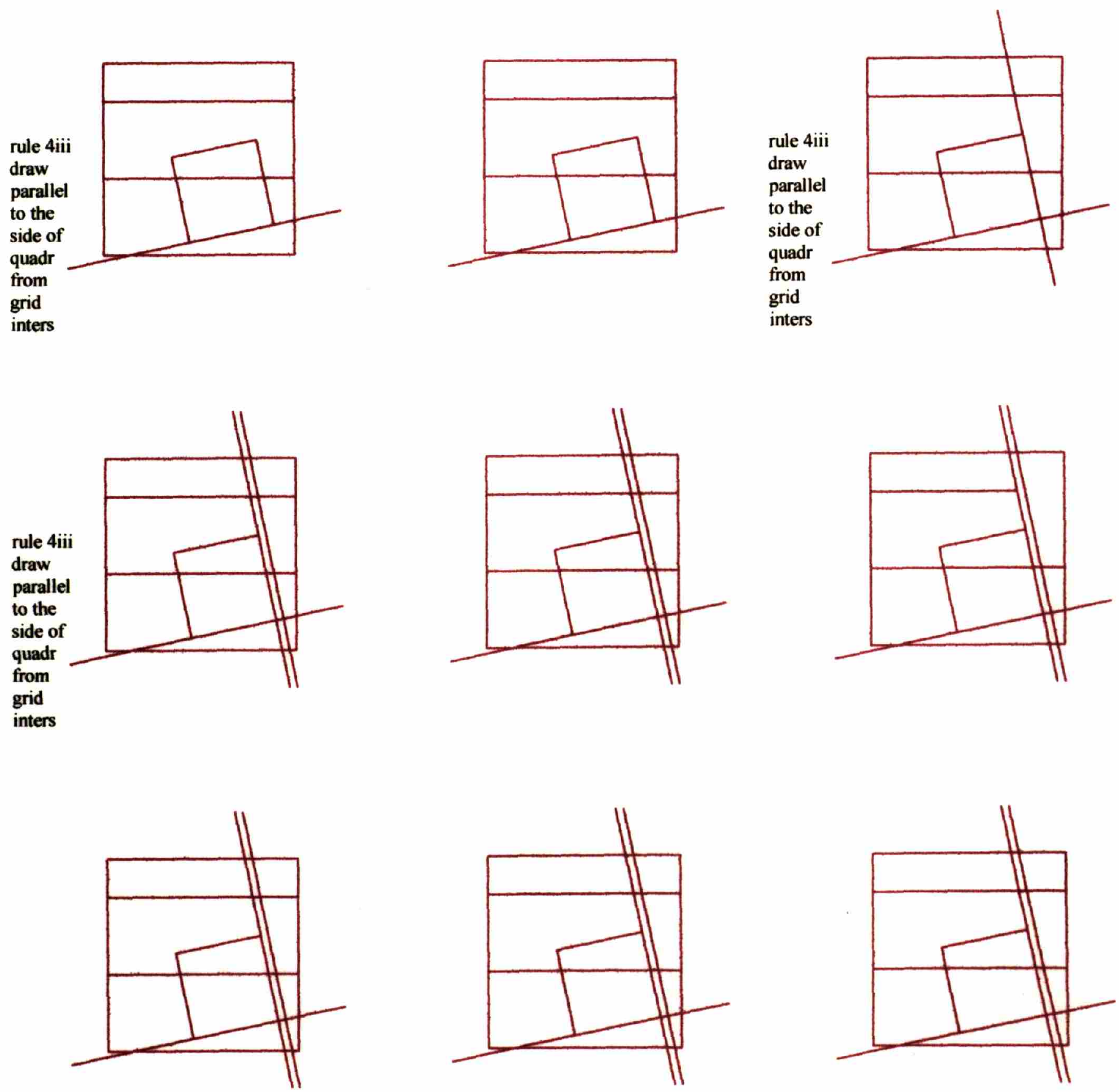
PLAN I

A


rule 12ii
from
n-sided
convex
shape
select
(n+1)
sided
convex
shape
rule 11v
select
convex
shapes
rule 11v
select
convex
shapes
rule 11vi
select
shapes

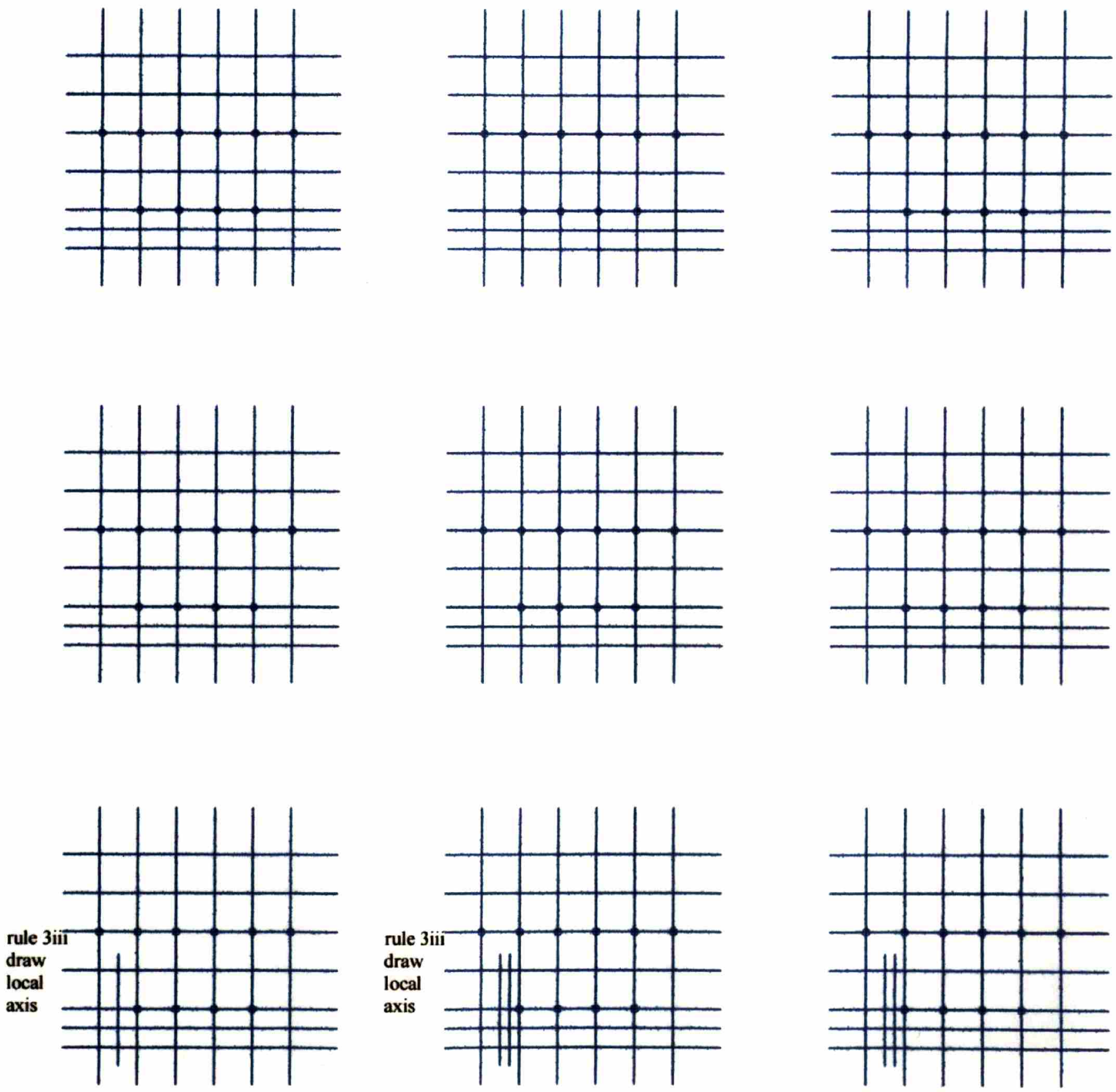
PLAN I

B


rule 4iii
draw
parallel
to the
side of
quadr
from
grid
inters
rule 4iii
draw
parallel
to the
side of
quadr
from
grid
inters
rule 4iii
draw
parallel
to the
side of
quadr
from
grid
inters

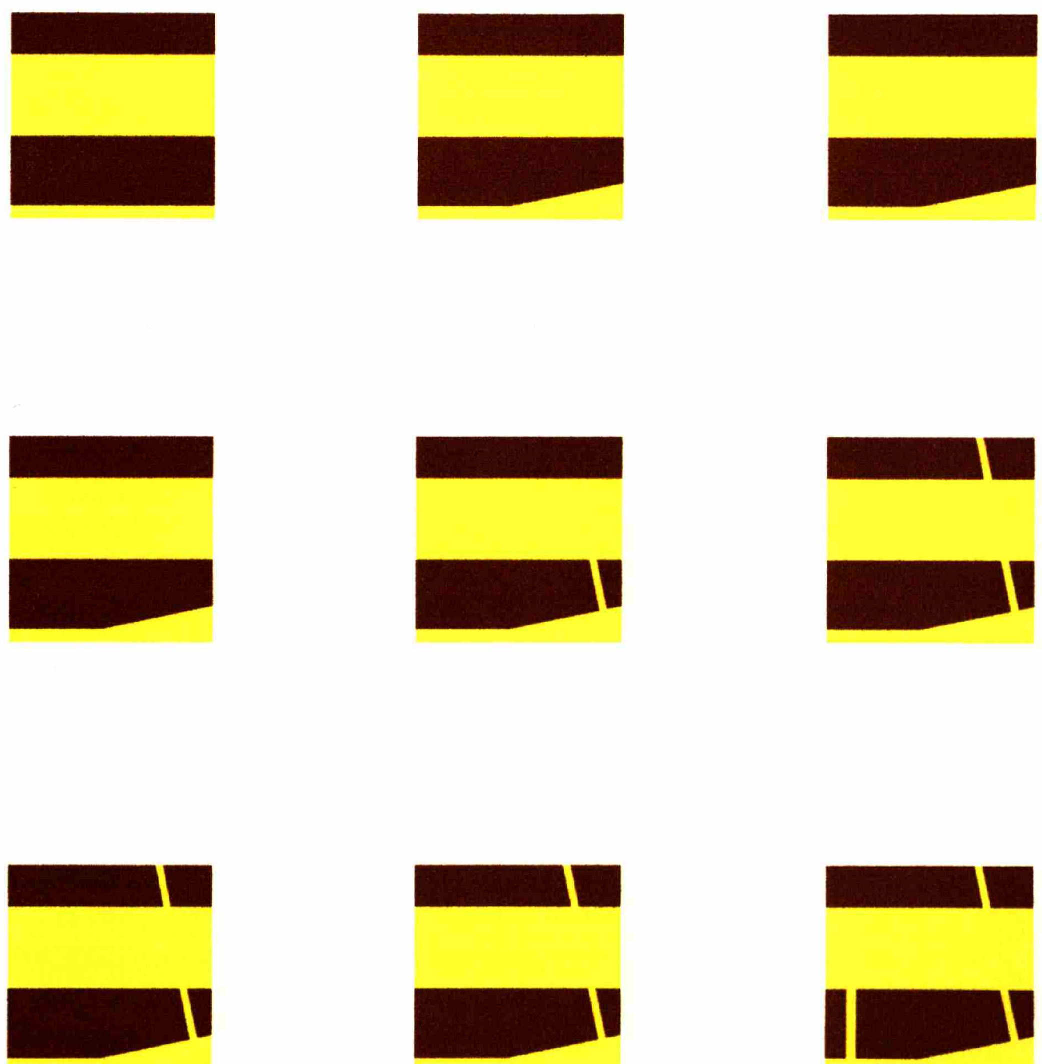
PLAN I

C

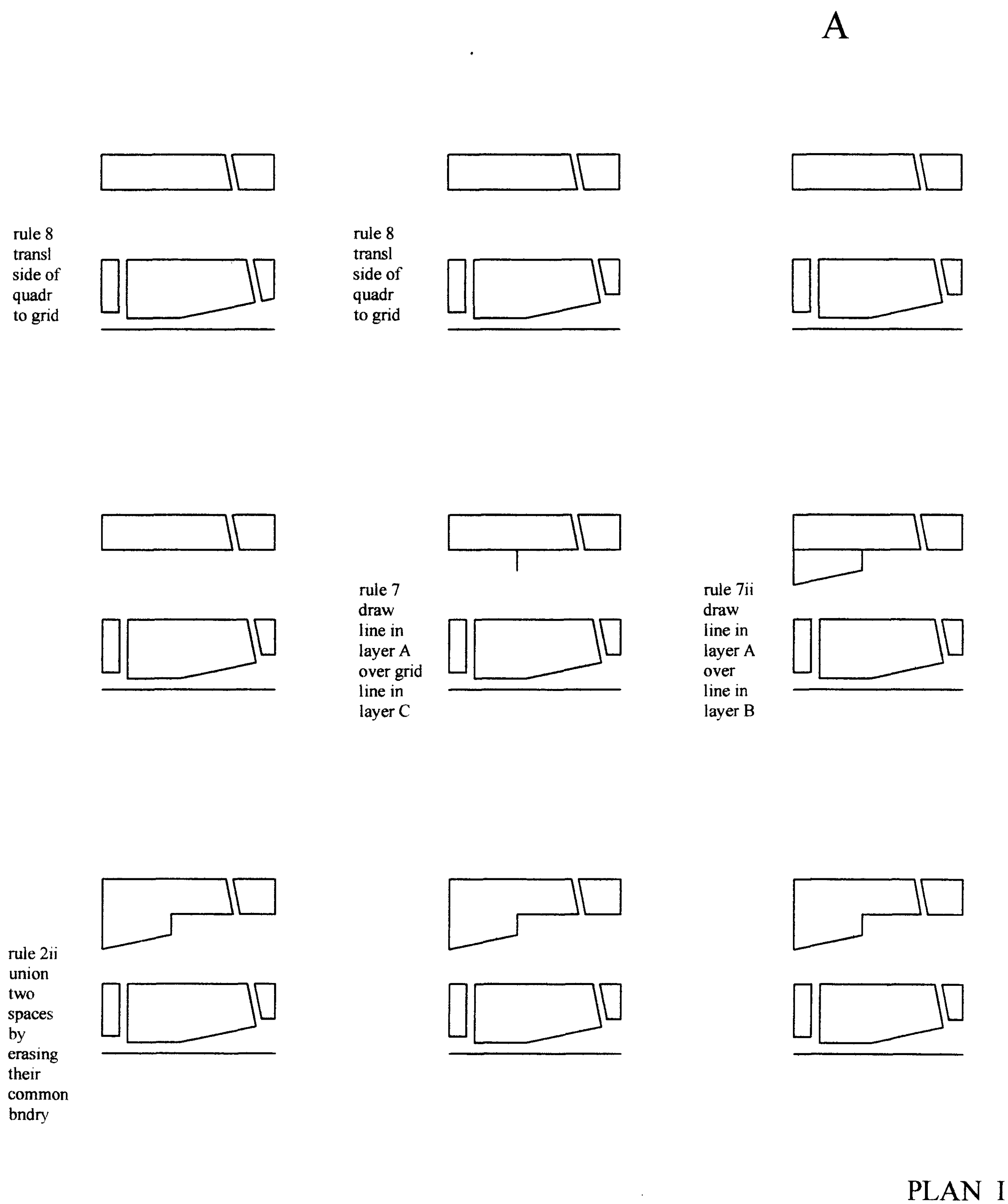


PLAN I

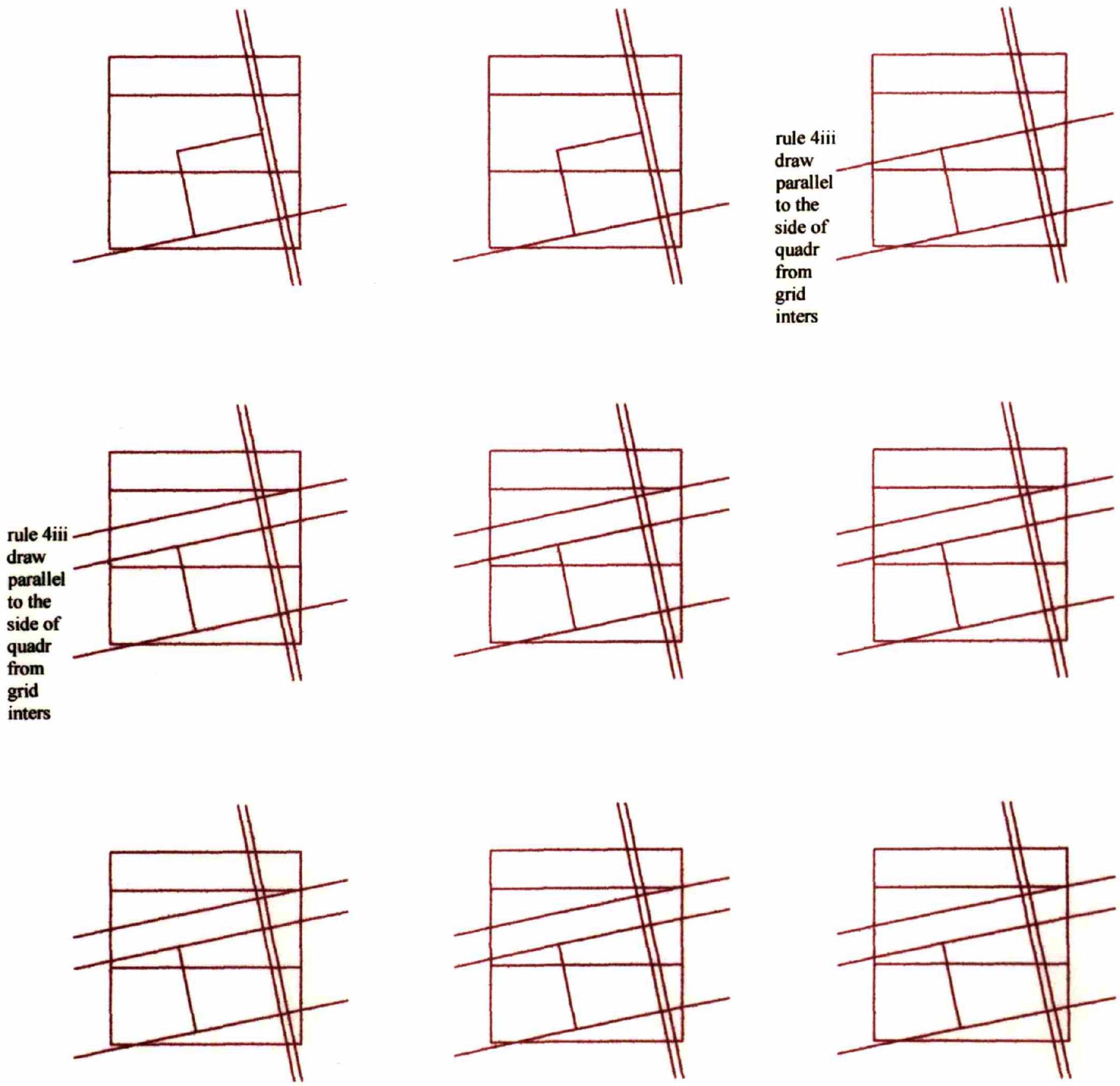
D



PLAN I

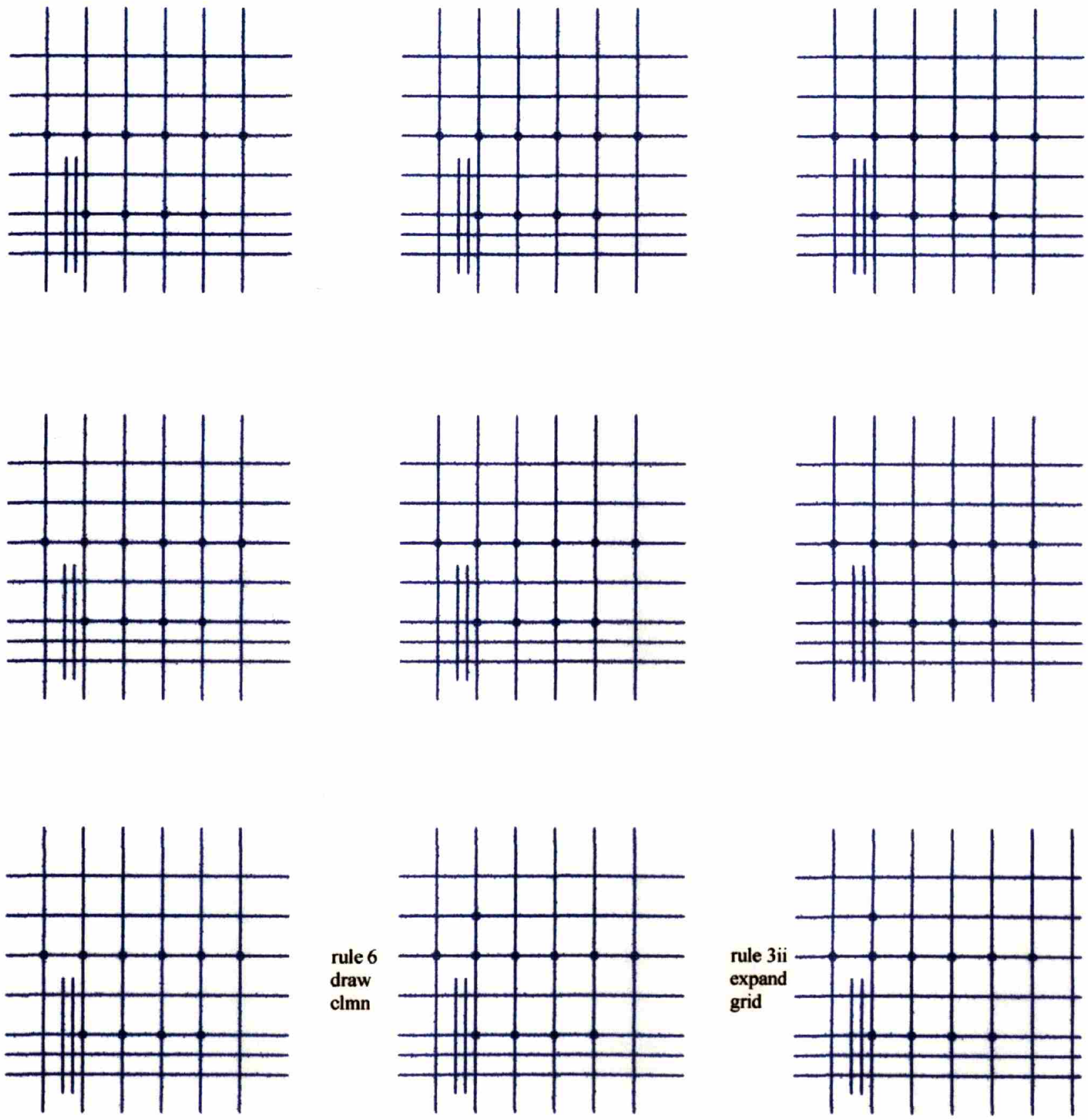

A
rule 8
transl
side of
quadr
to grid
rule 8
transl
side of
quadr
to grid
rule 7
draw
line in
layer A
over grid
line in
layer C
rule 7ii
draw
line in
layer A
over
line in
layer B
rule 2ii
union
two
spaces
by
erasing
their
common
bndry
PLAN I

B


rule 4iii
draw
parallel
to the
side of
quadr
from
grid
inters
rule 4iii
draw
parallel
to the
side of
quadr
from
grid
inters

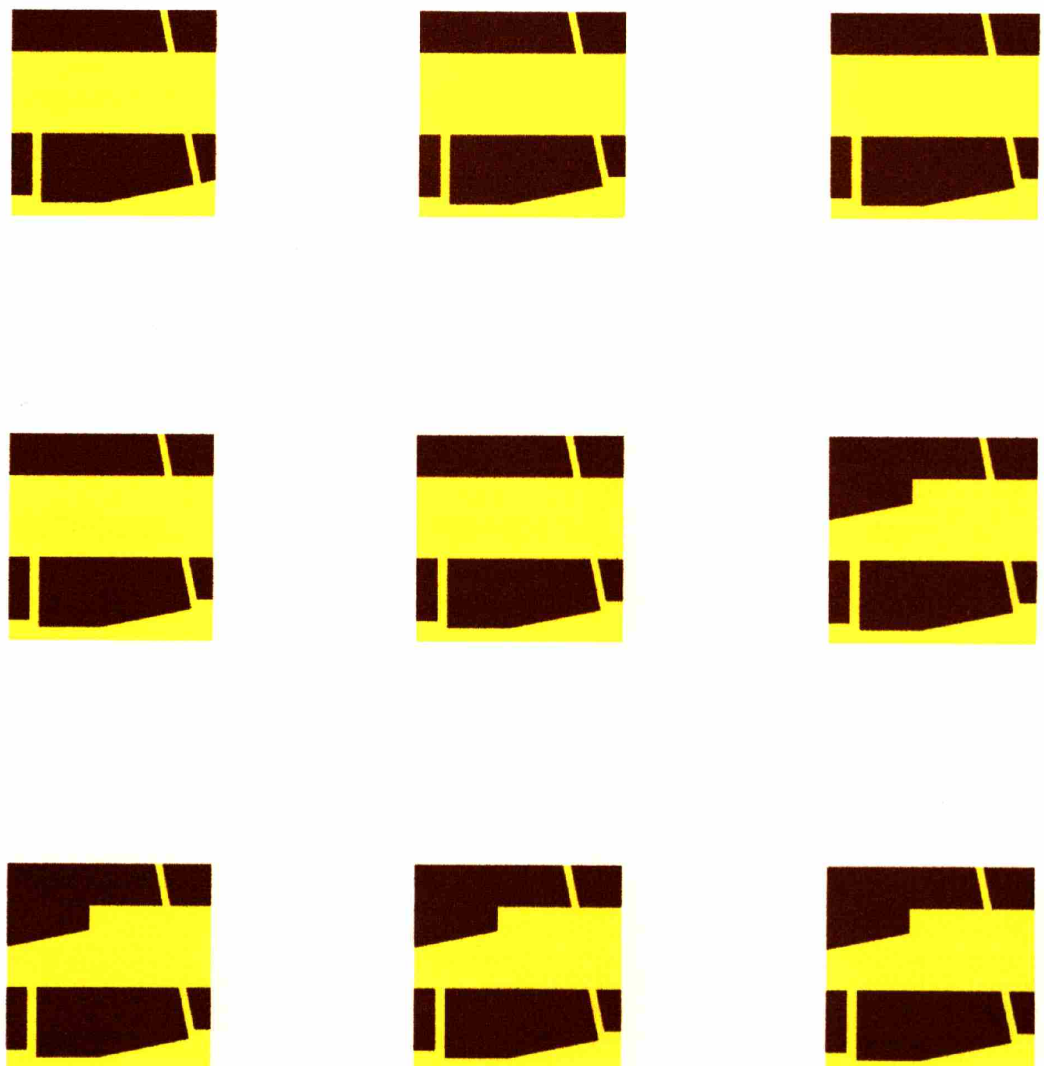
PLAN I

C



PLAN I

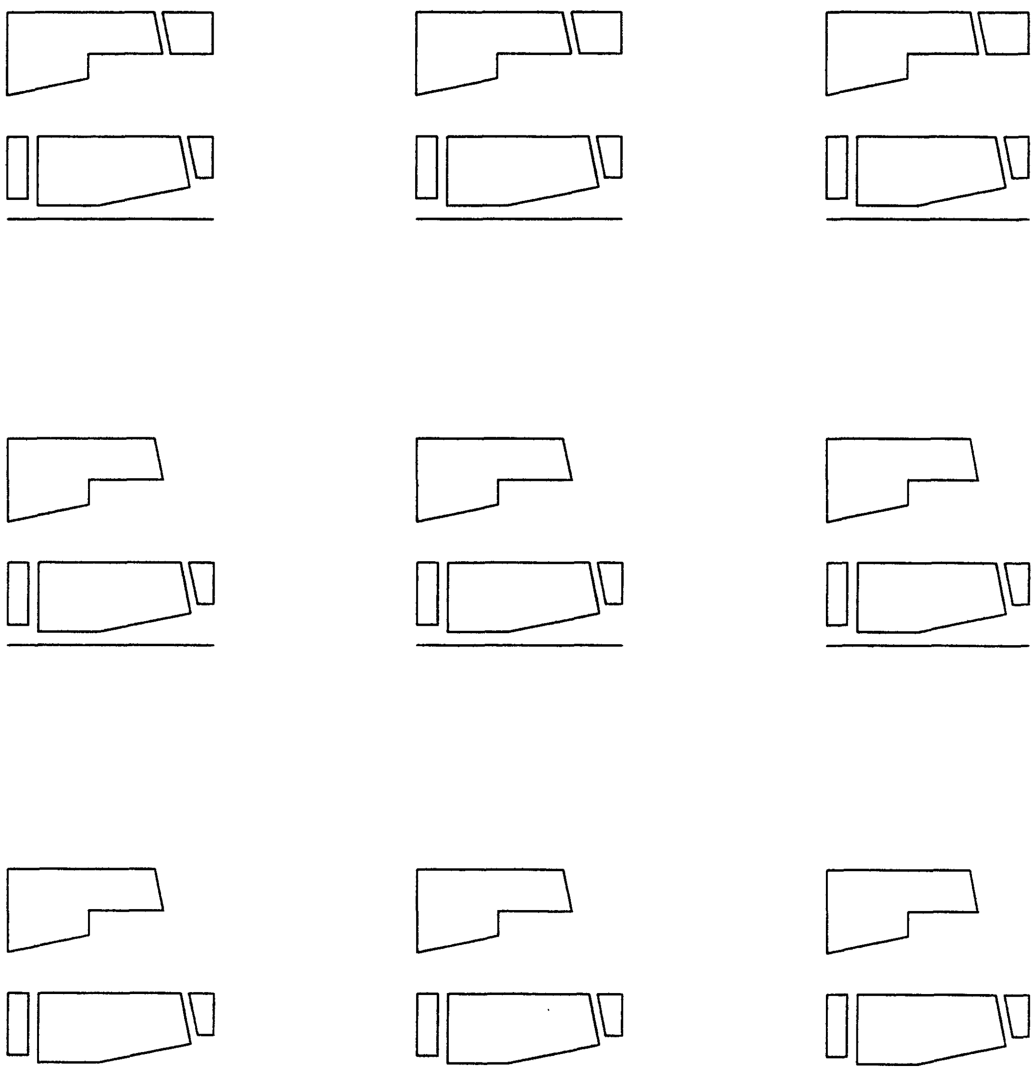
D



PLAN I

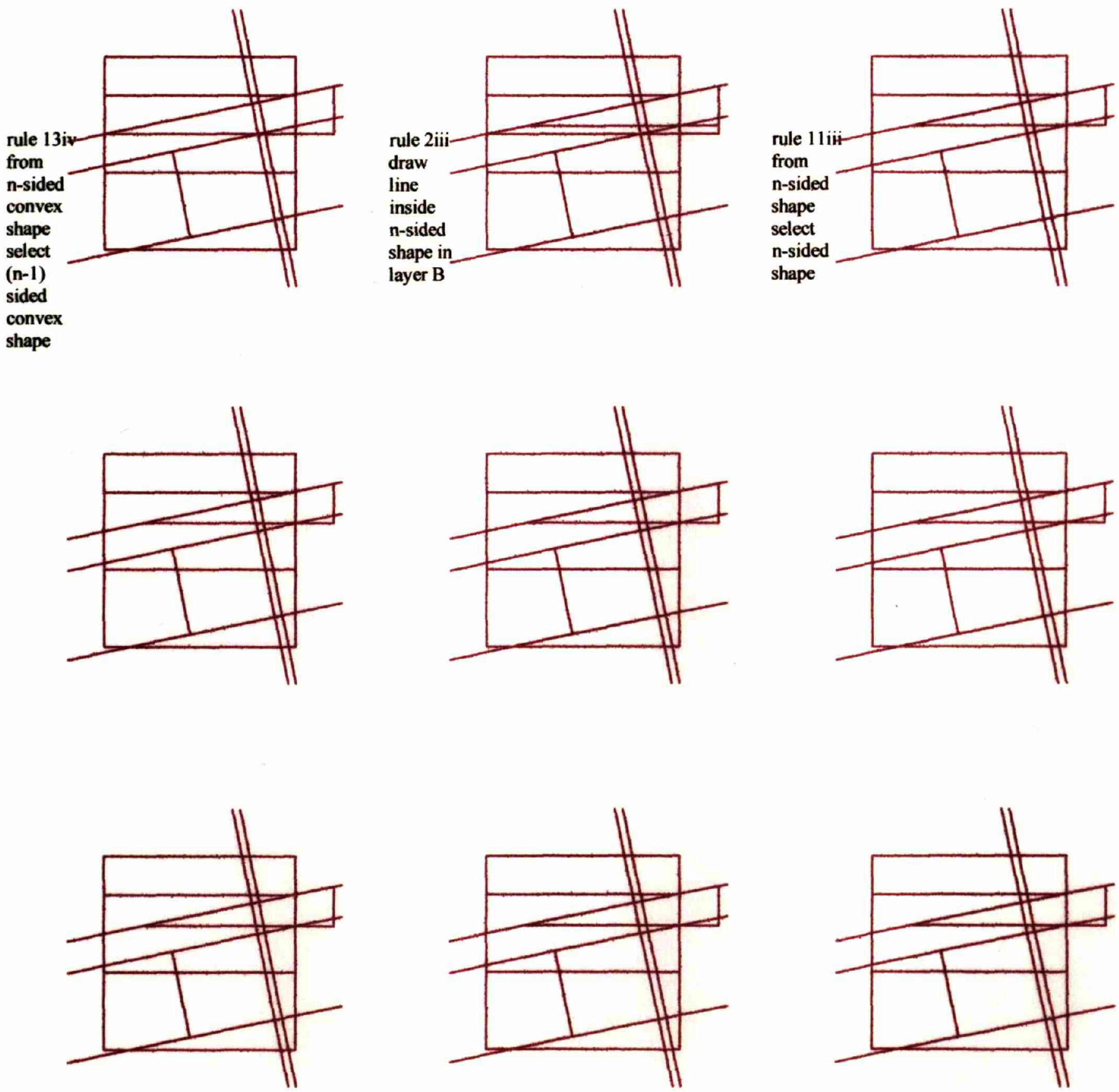
A

rule 1ii
erase
n-sided
shape



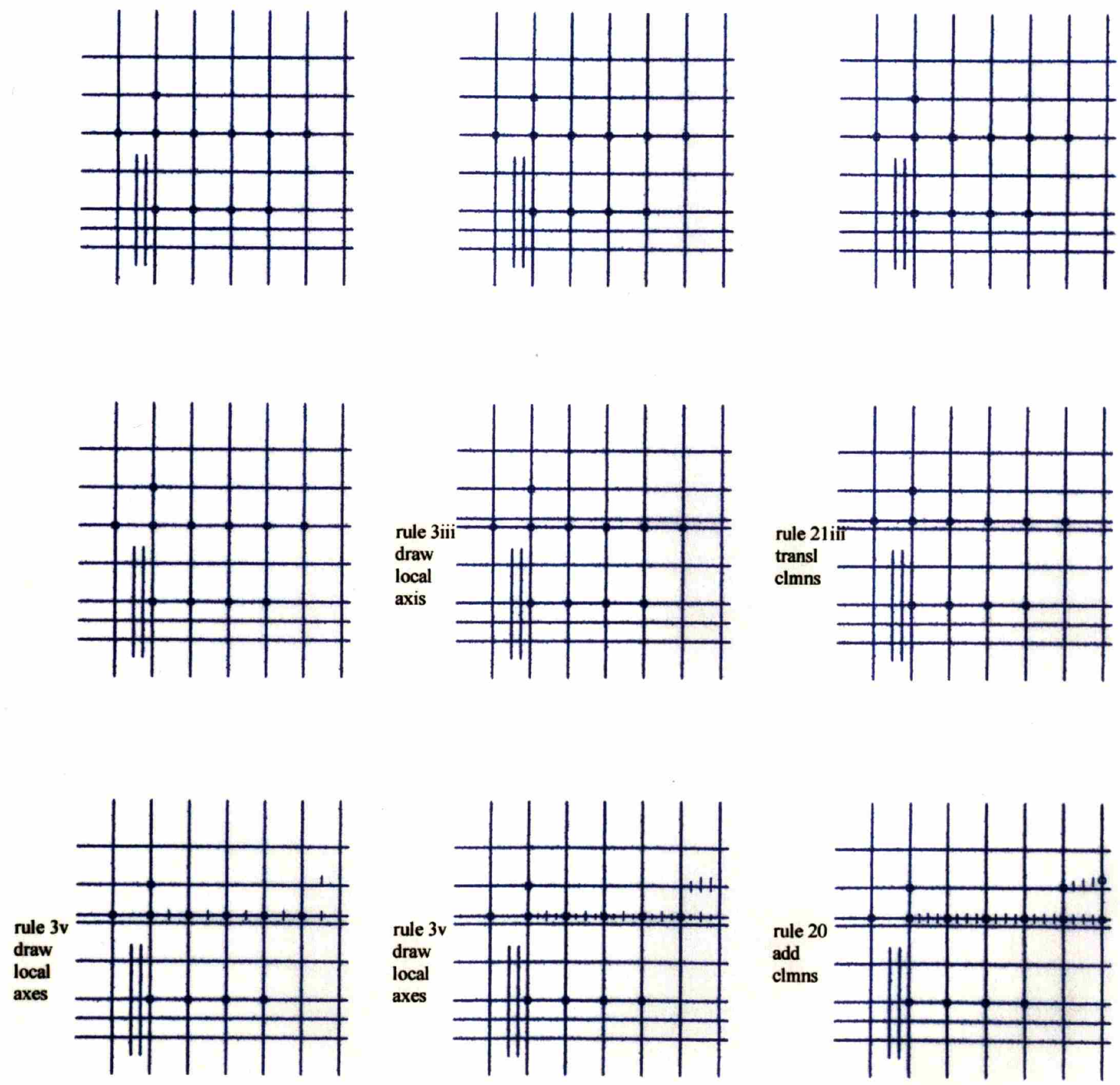
PLAN I

B


rule 13iv
from
n-sided
convex
shape
select
(n-1)
sided
convex
shape
rule 2iii
draw
line
inside
n-sided
shape in
layer B
rule 11iii
from
n-sided
shape
select
n-sided
shape

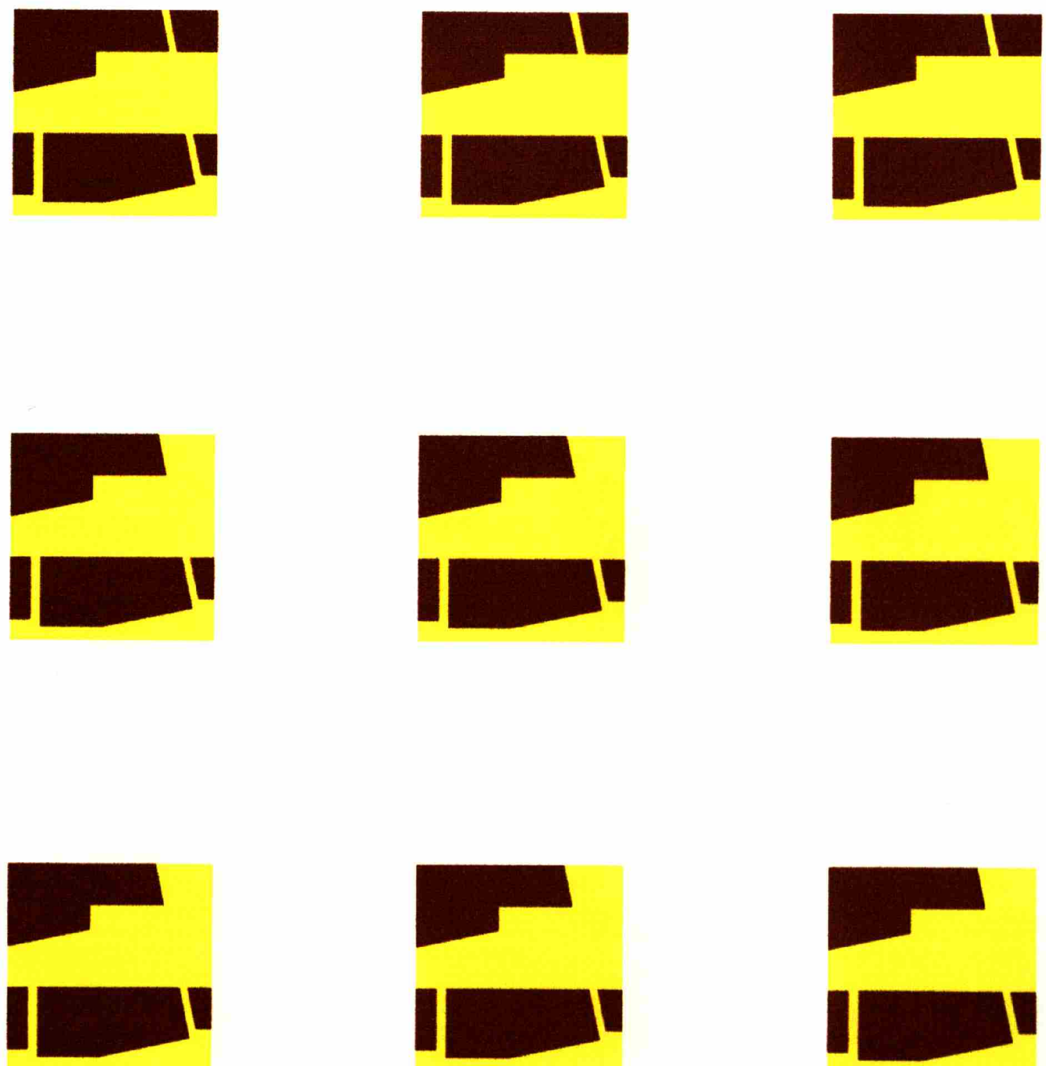
PLAN I

C

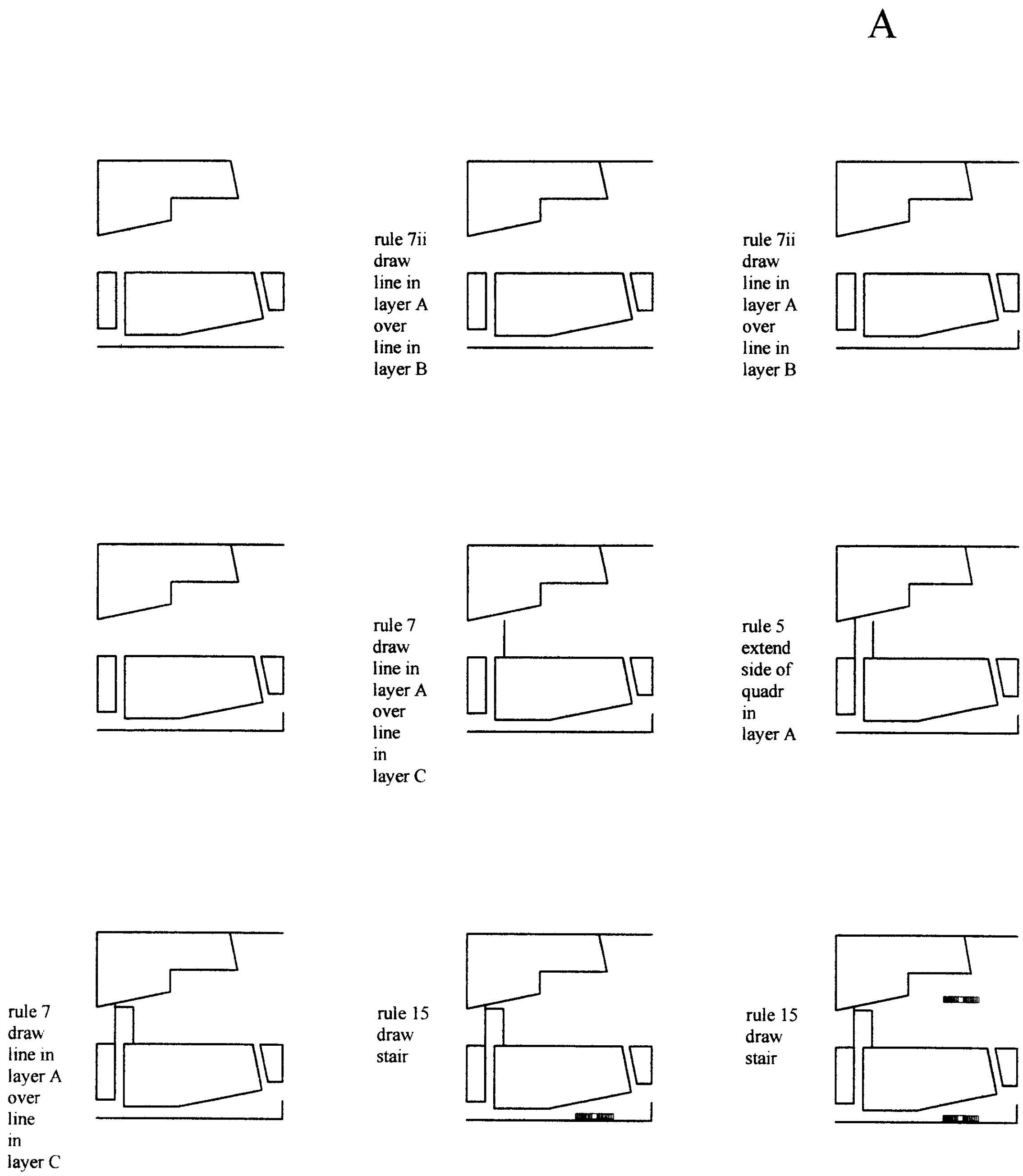

rule 3iii
draw
local
axis
rule 21iii
transl
clmns
rule 3v
draw
local
axes
rule 3v
draw
local
axes
rule 20
add
clmns

PLAN I

D

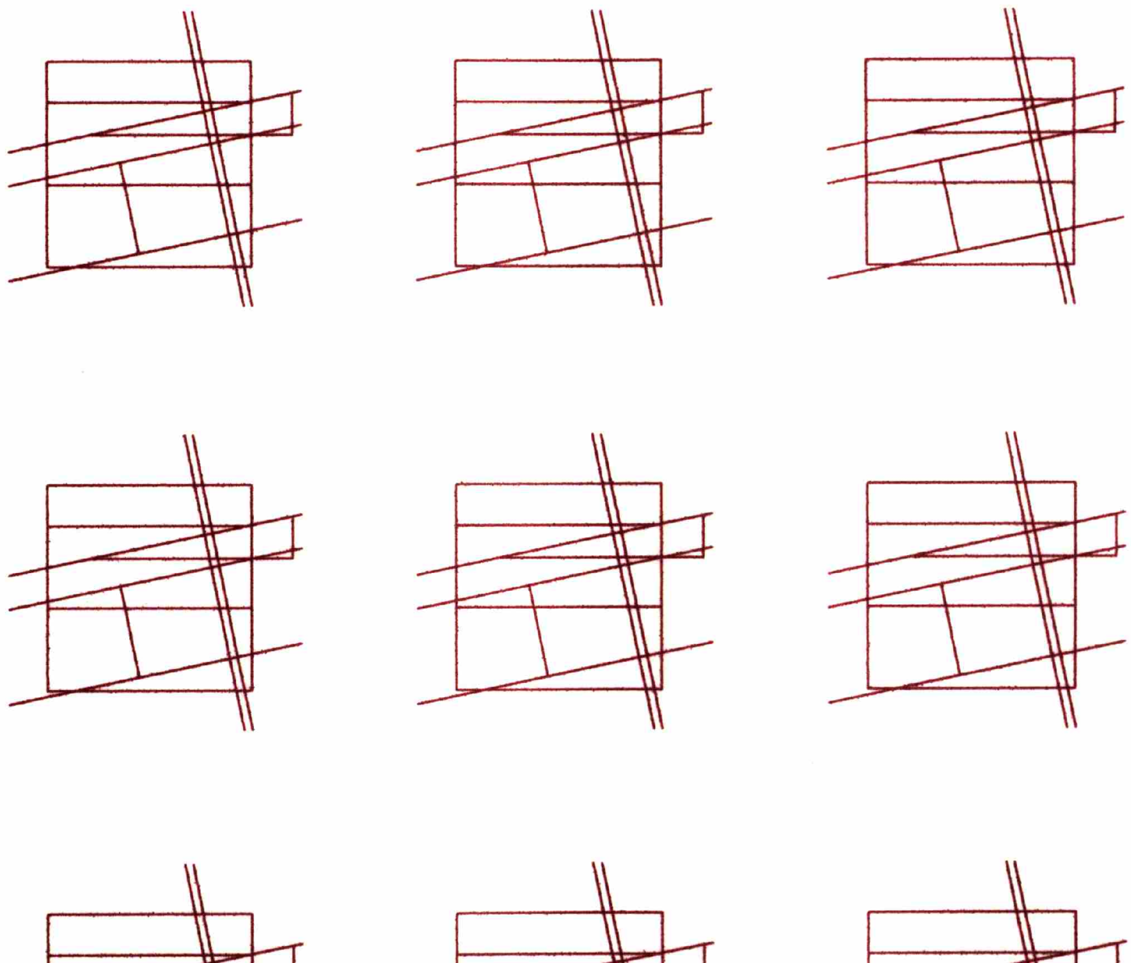


PLAN I


A
rule 7ii draw line in layer A over line in layer B
rule 7ii draw line in layer A over line in layer B
rule 7 draw line in layer A over line in layer C
rule 5 extend side of quadr in layer A
rule 7 draw line in layer A over line in layer C
rule 15 draw stair
rule 15 draw stair

PLAN I

B

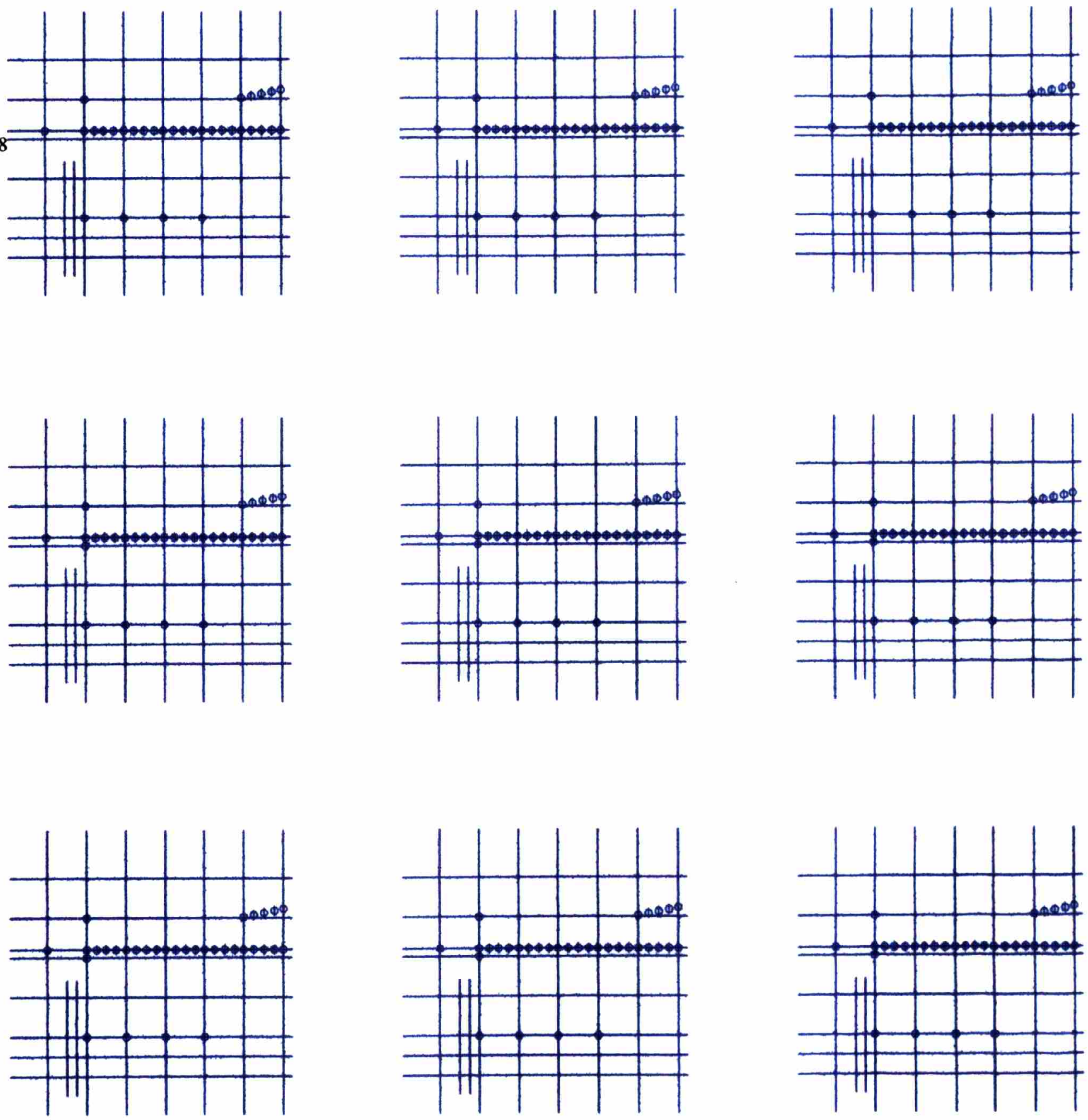


PLAN I

C

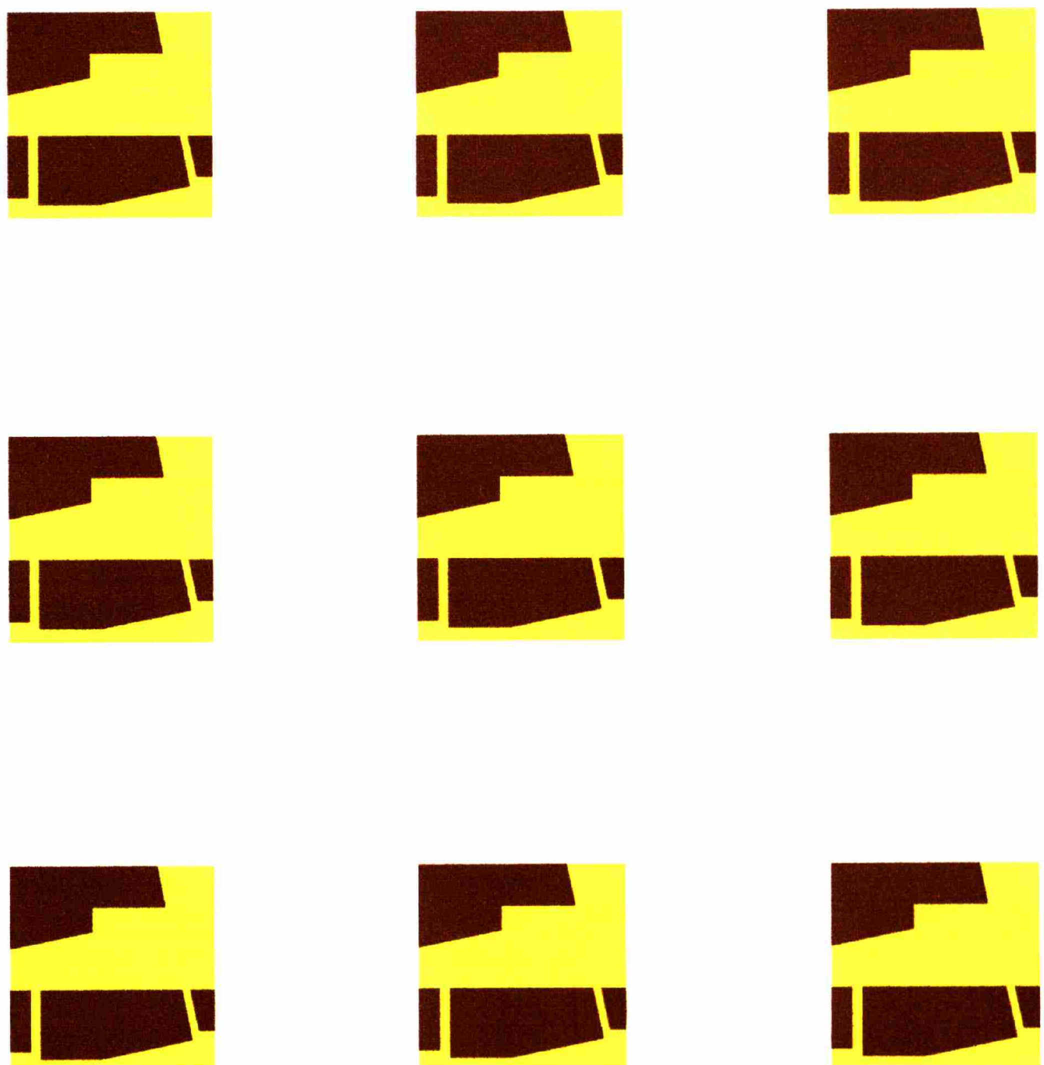
rule 18
add
clmns

rule 6
draw
clmn



PLAN I

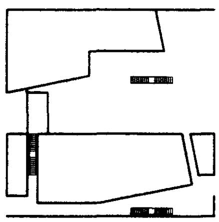
D



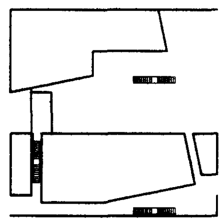
PLAN I

A

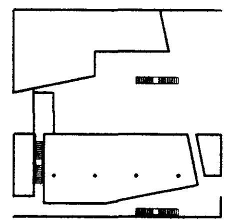
rule 15
draw
stair



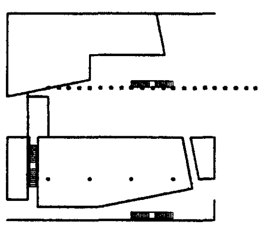
rule 15ii
move
stair



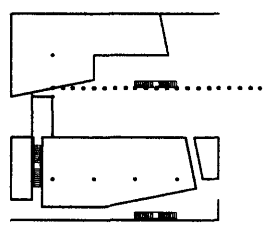
rule 21ii
draw
clmns
in
layer A



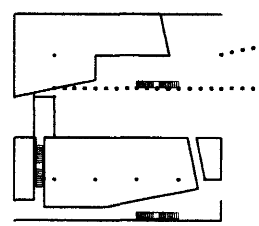
rule 19
draw
clmns
in
layer A

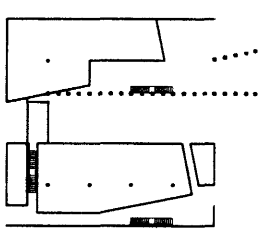


rule 21ii
draw
clmns
in
layer A



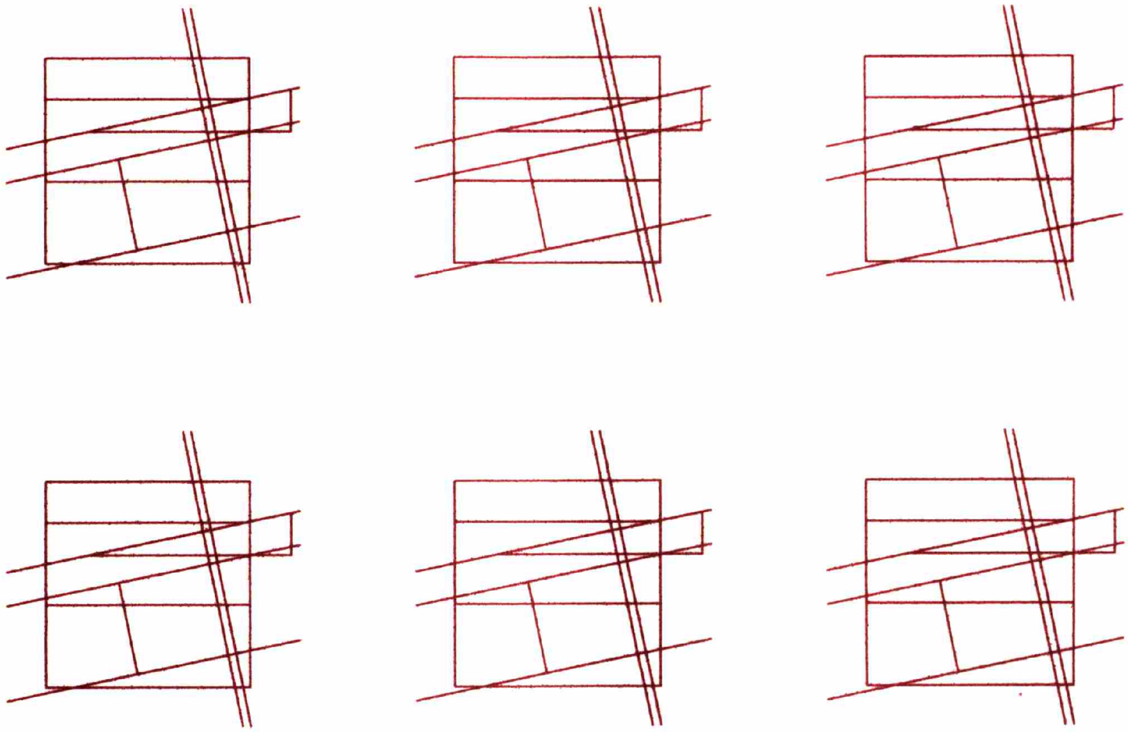
rule 19
draw
clmns
in
layer A



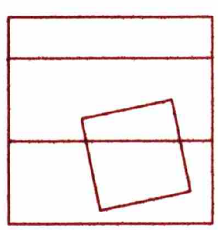


PLAN I

B

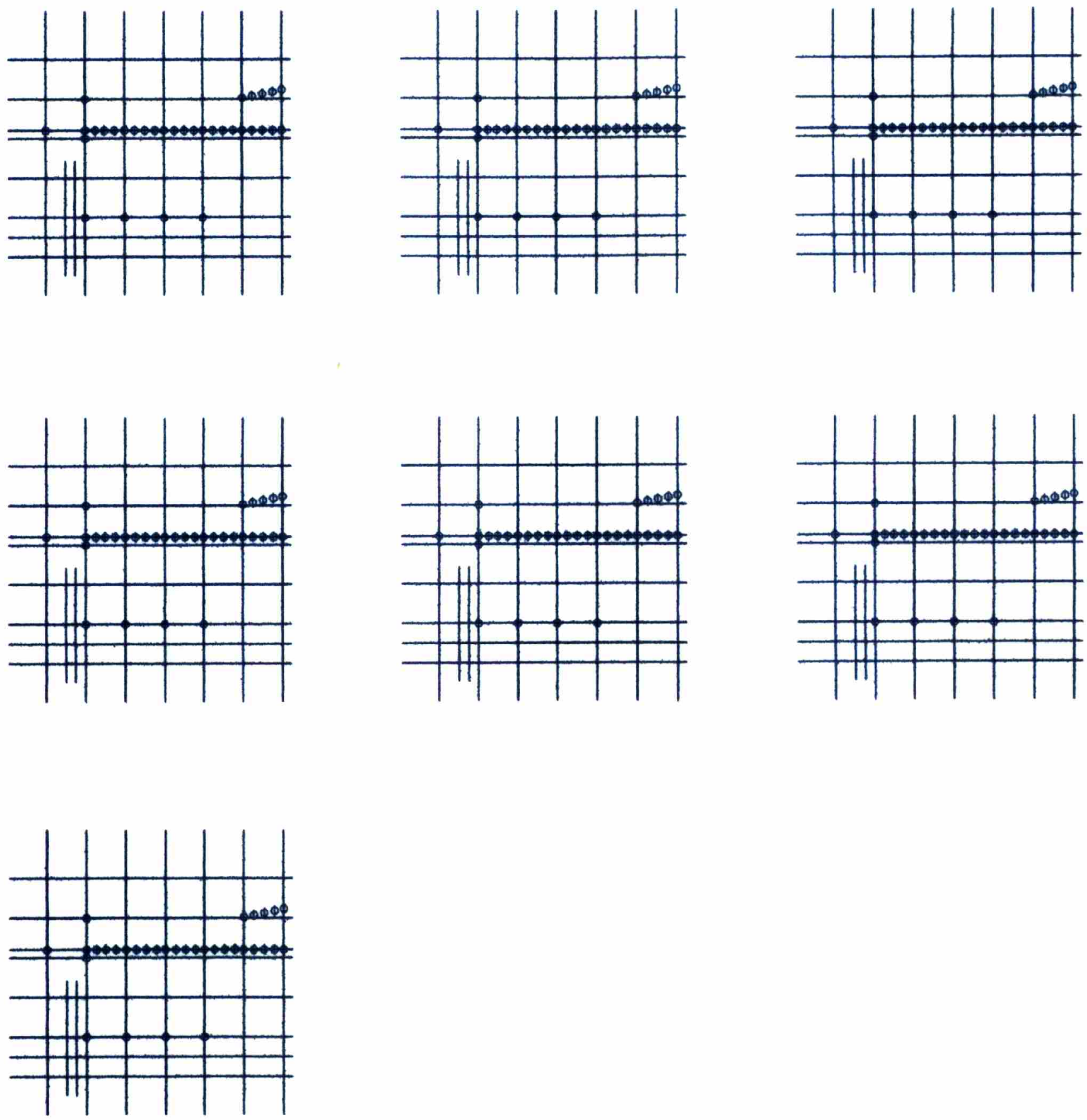


rule 23
erase
line
extens
in
layer B



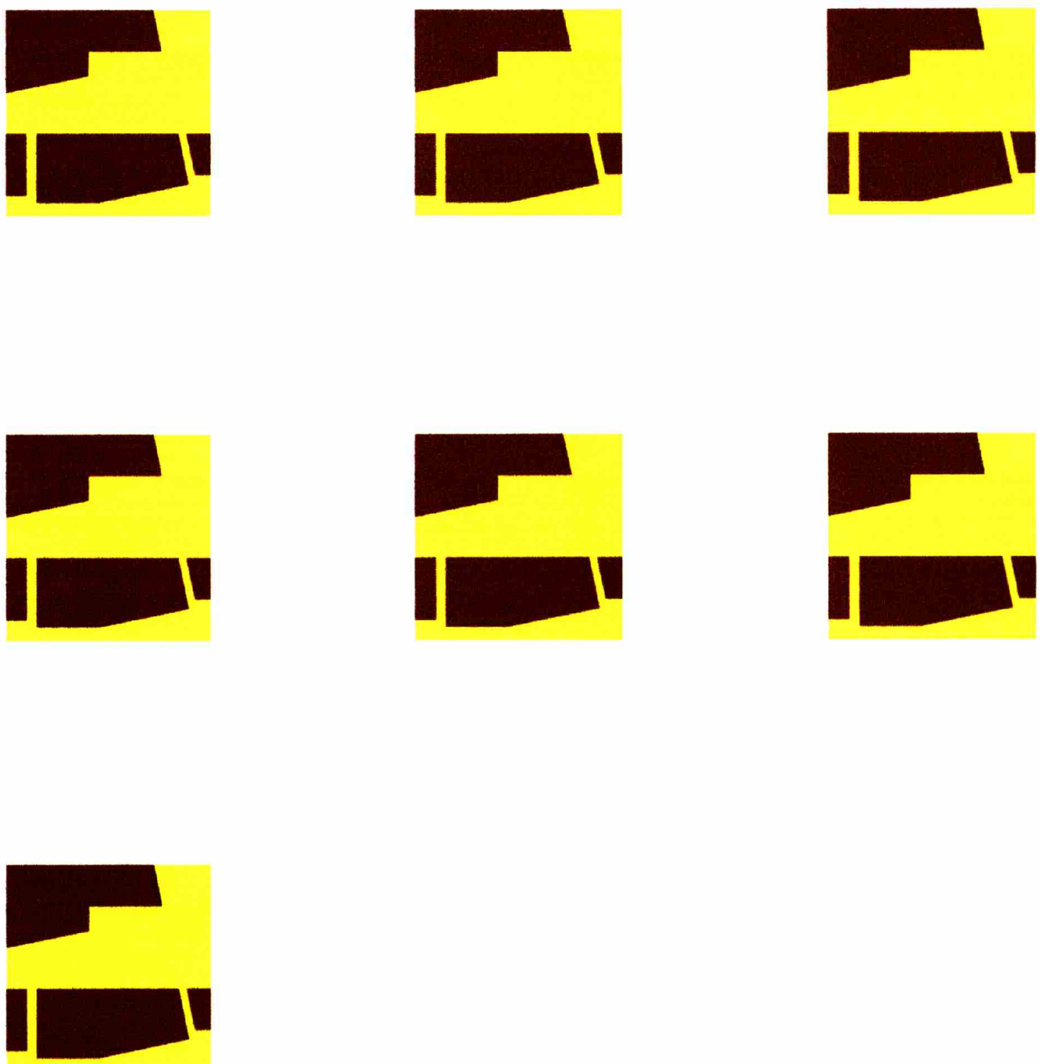
PLAN I

C



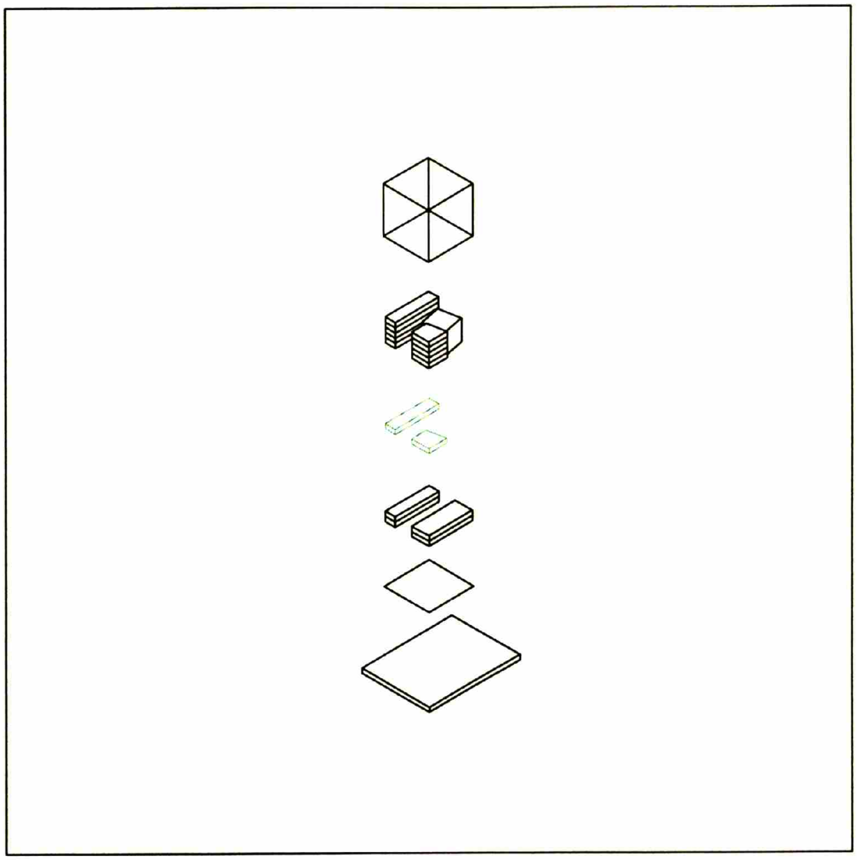
PLAN I

D



PLAN I

6.2.2. Derivation of Plan II

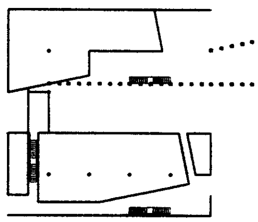


The next derivation of plan II (pp. 327-382), in four sheets A x B x C x D, generates the plan type corresponding to floor 3, as shown in the above diagram with green color.

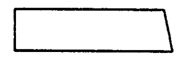
The layers A x B x C x D of the derived plan I are passed, providing the initial information for plan II. Layer A contains some arrangement of black lines, layer B some arrangement of red lines, layer C some grid, and layer D some colored areas. In layer D, the sum of all areas of any color (pink, yellow, brown or blue) is always equal to the area of the initial pink square (plan I), which represents the available free area of the site.

The overall derivation unfolds in the product algebra $<U_{12}\ V_{02}>$ x $<U_{12}\ V_{02}>$ x $<U_{12}\ V_{02}>$ x $<U_{22}V_{02}>$.

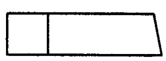
A



shape
from
previous
stage

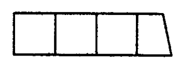


rule 1iii
draw
quadr in
layer A
over
quadr in
layer B

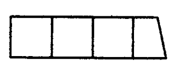


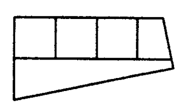
rule 2
draw
line
in
quadr

rule 2
draw
line
in
quadr



rule 2
draw
line
in
quadr

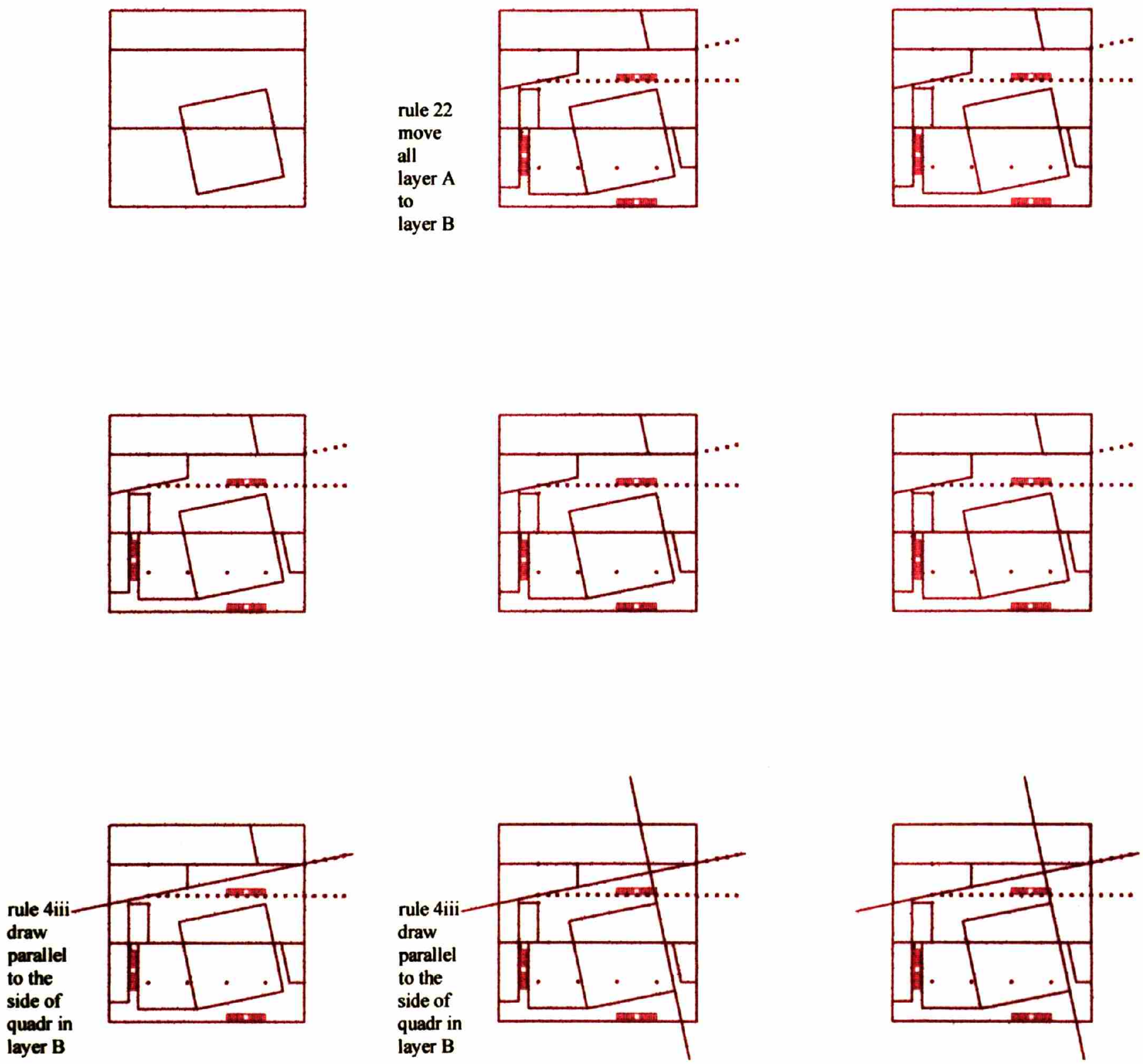




rule 1iii
draw
quadr in
layer A
over
quadr in
layer B

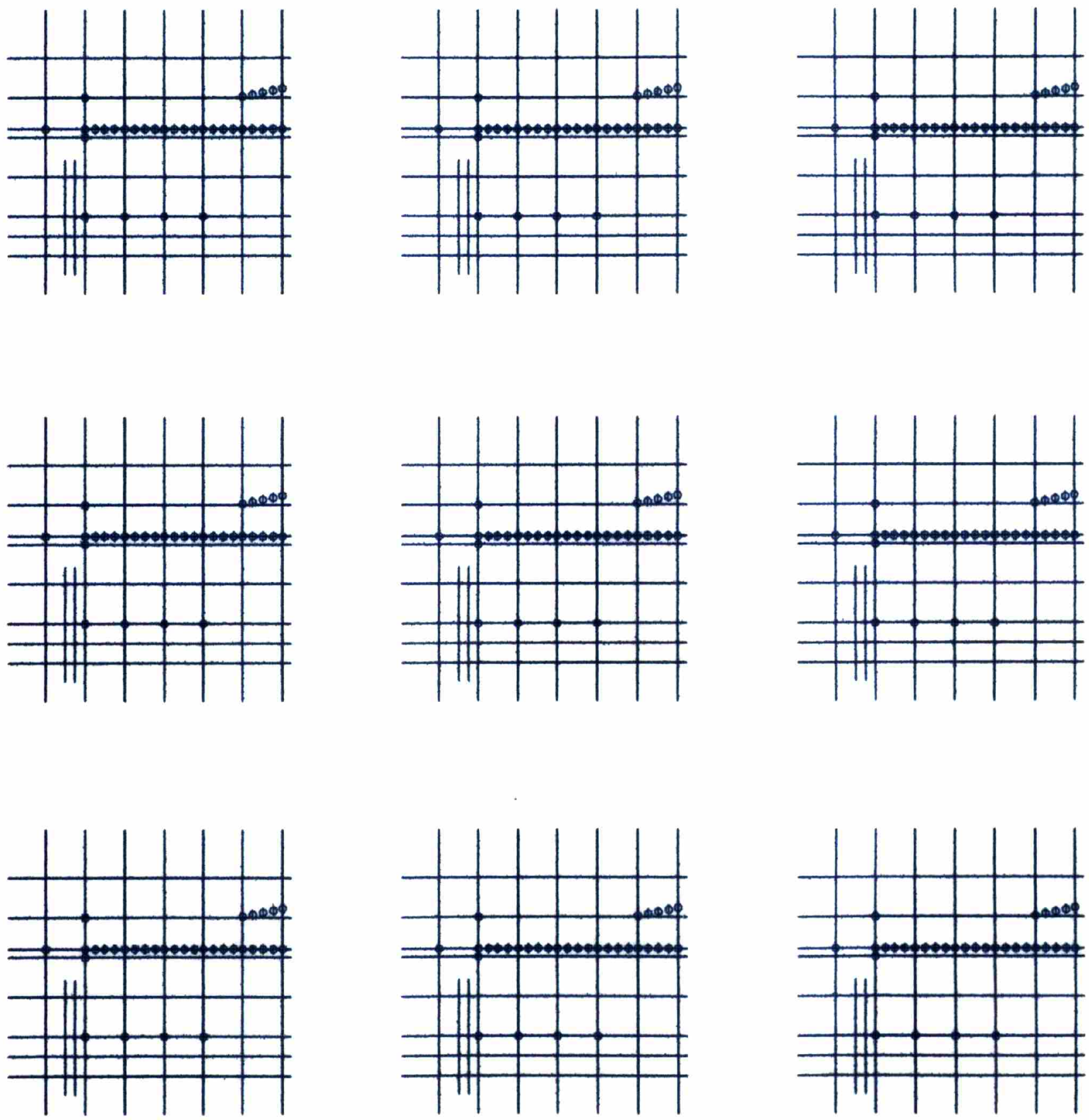
PLAN II

B


rule 22
move
all
layer A
to
layer B
rule 4iii
draw
parallel
to the
side of
quadr in
layer B
rule 4iii
draw
parallel
to the
side of
quadr in
layer B

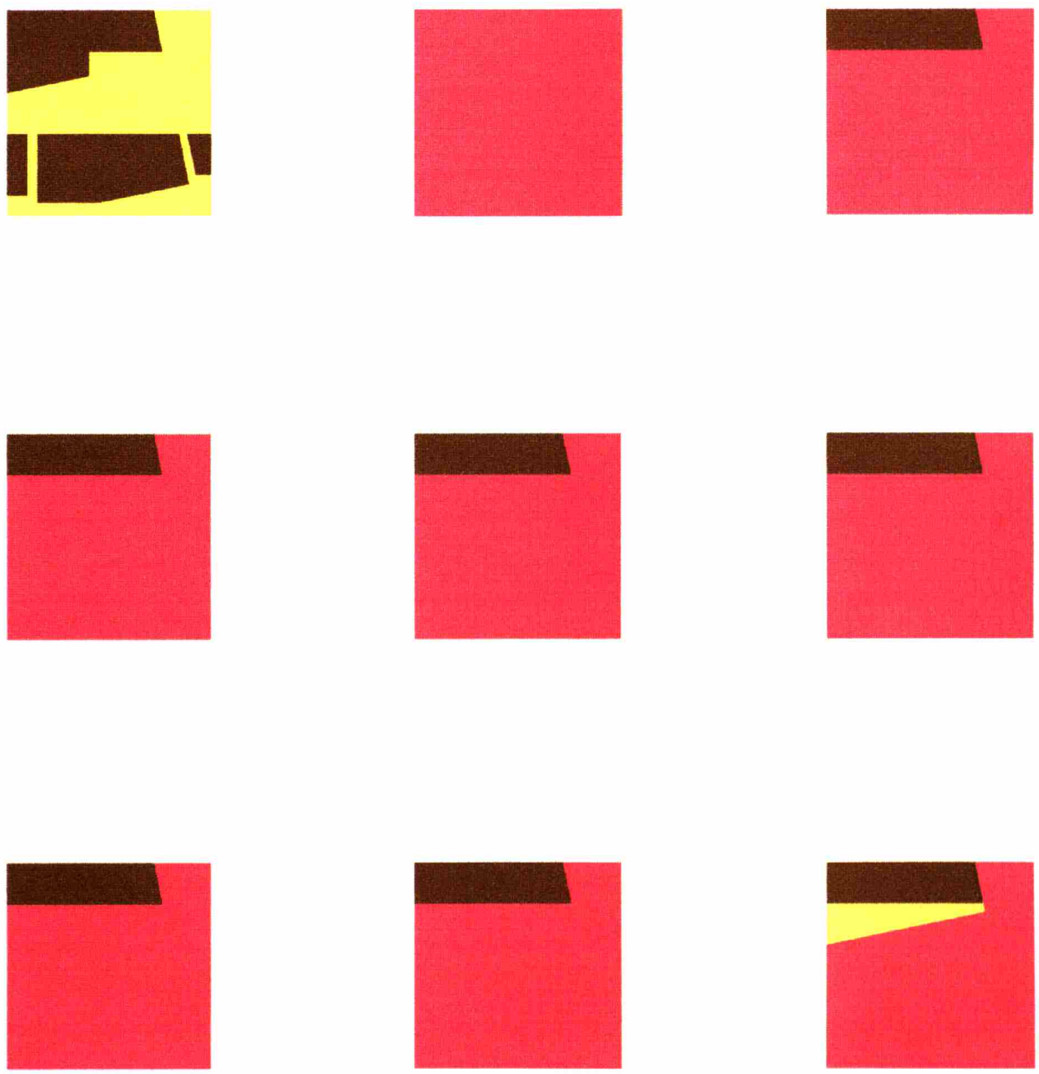
PLAN II

C



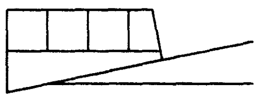
PLAN II

D

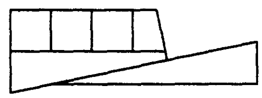


PLAN II

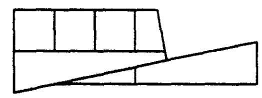
A



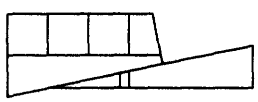
rule 7
draw
line in
layer A
over a
line in
layer C



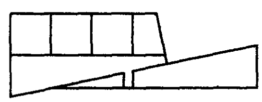
rule 7
draw
line in
layer A
over a
line in
layer C



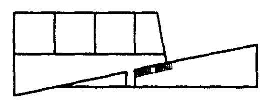
rule 2
dissect
convex
in
layer A



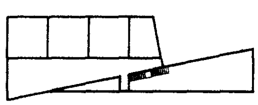
rule 2
dissect
convex
in
layer A

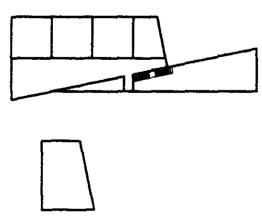


rule 2ii
union
two
shapes
by
erasing
their
common
bndry

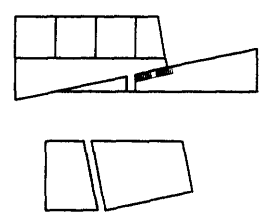


rule 15
draw
stair





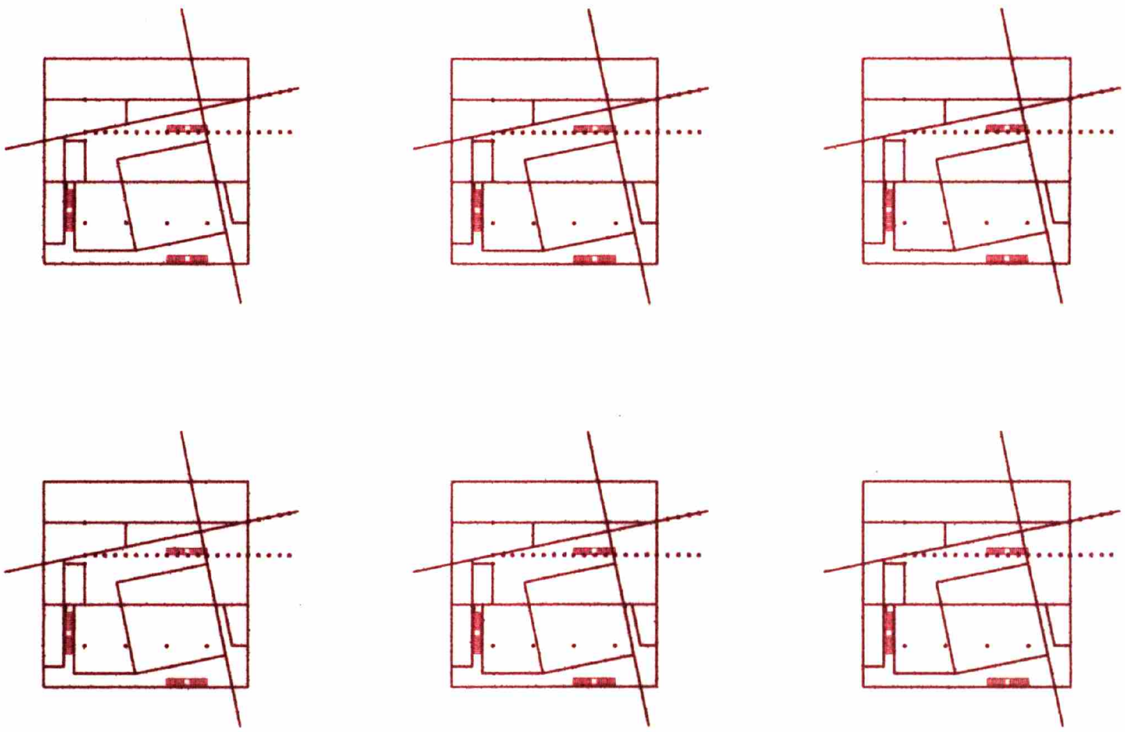
rule 1iii
draw
quadr in
layer A
over
quadr in
layer B



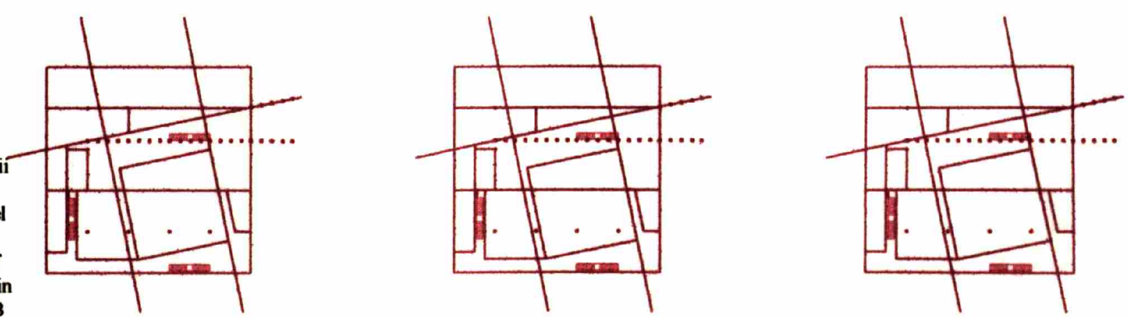
rule 1iii
draw
quadr in
layer A
over
quadr in
layer B

PLAN II

B

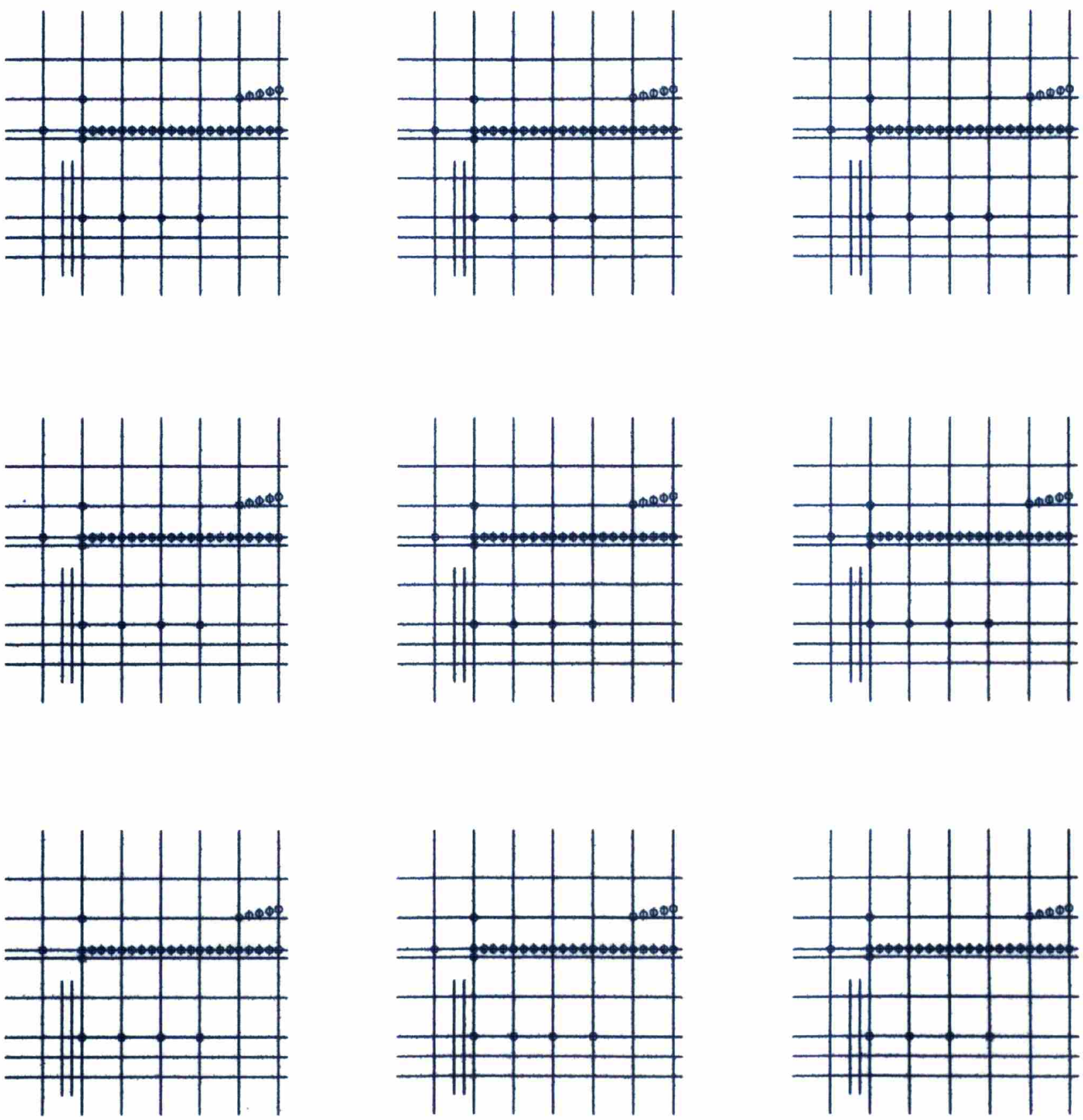


rule 4iii
draw
parallel
to the
side of
quadr in
layer B



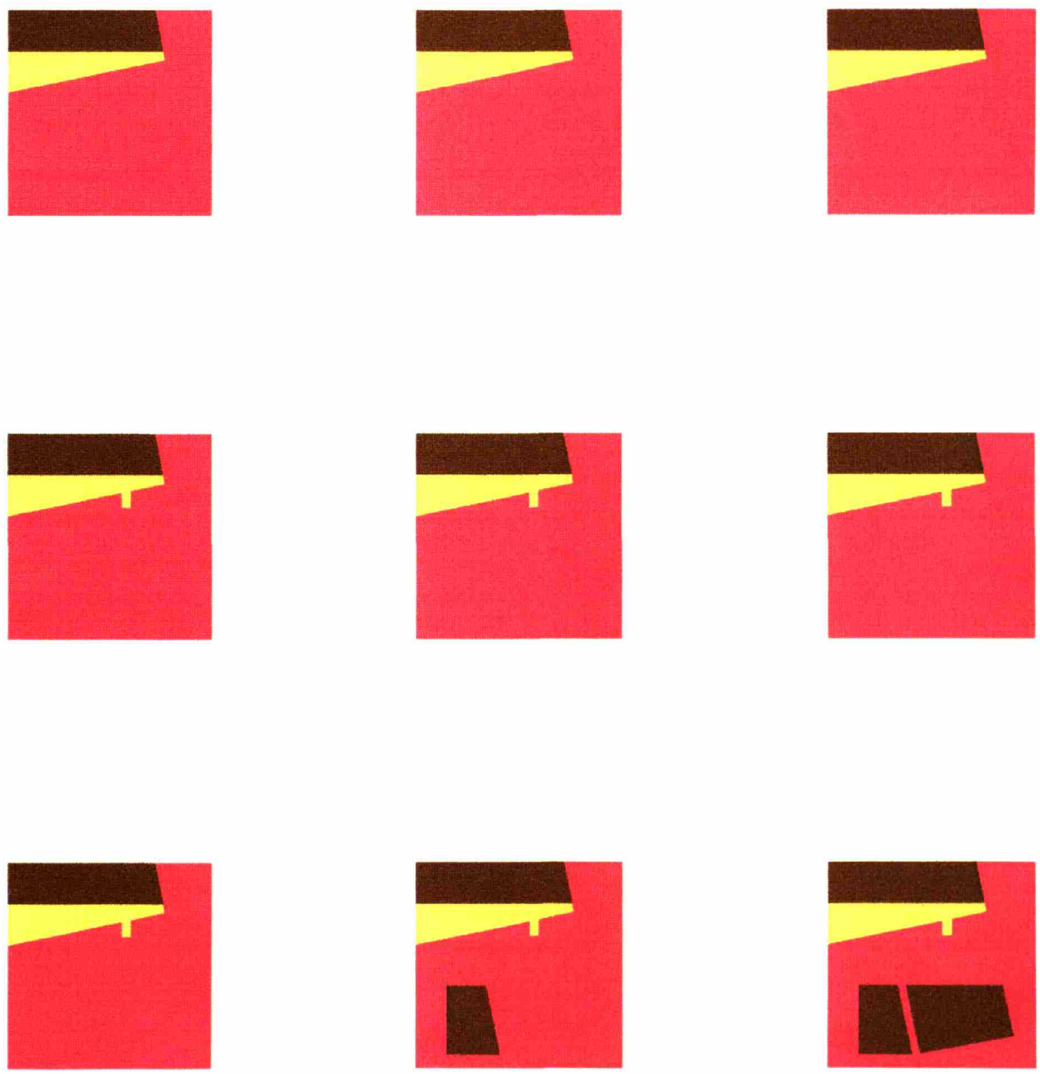
PLAN II

C



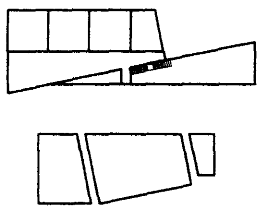
PLAN II

D

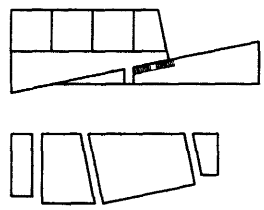


PLAN II

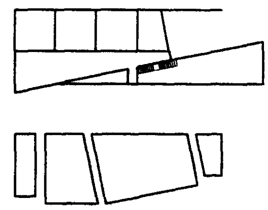
A



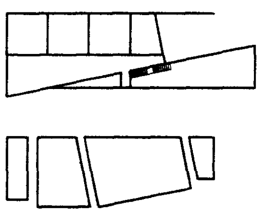
rule 1iii
draw
quadr in
layer A
over
quadr in
layer B



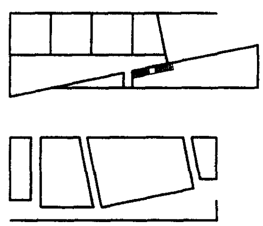
rule 1iii
draw
quadr in
layer A
over
quadr in
layer B



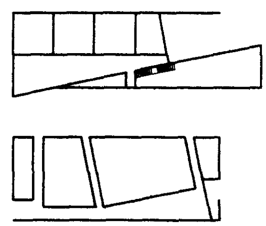
rule 7ii
draw
line in
layer A
over
line in
layer B



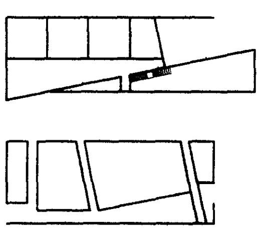
rule 7ii
draw
line in
layer A
over
line in
layer B



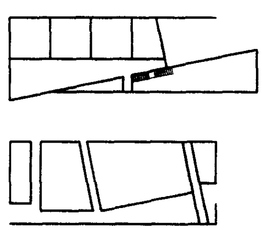
rule 7ii
draw
line in
layer A
over
line in
layer B



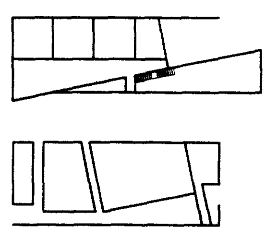
rule 5
extend
side of
quadr in
layer A



rule 7ii
draw
line in
layer A
over a
line in
layer B



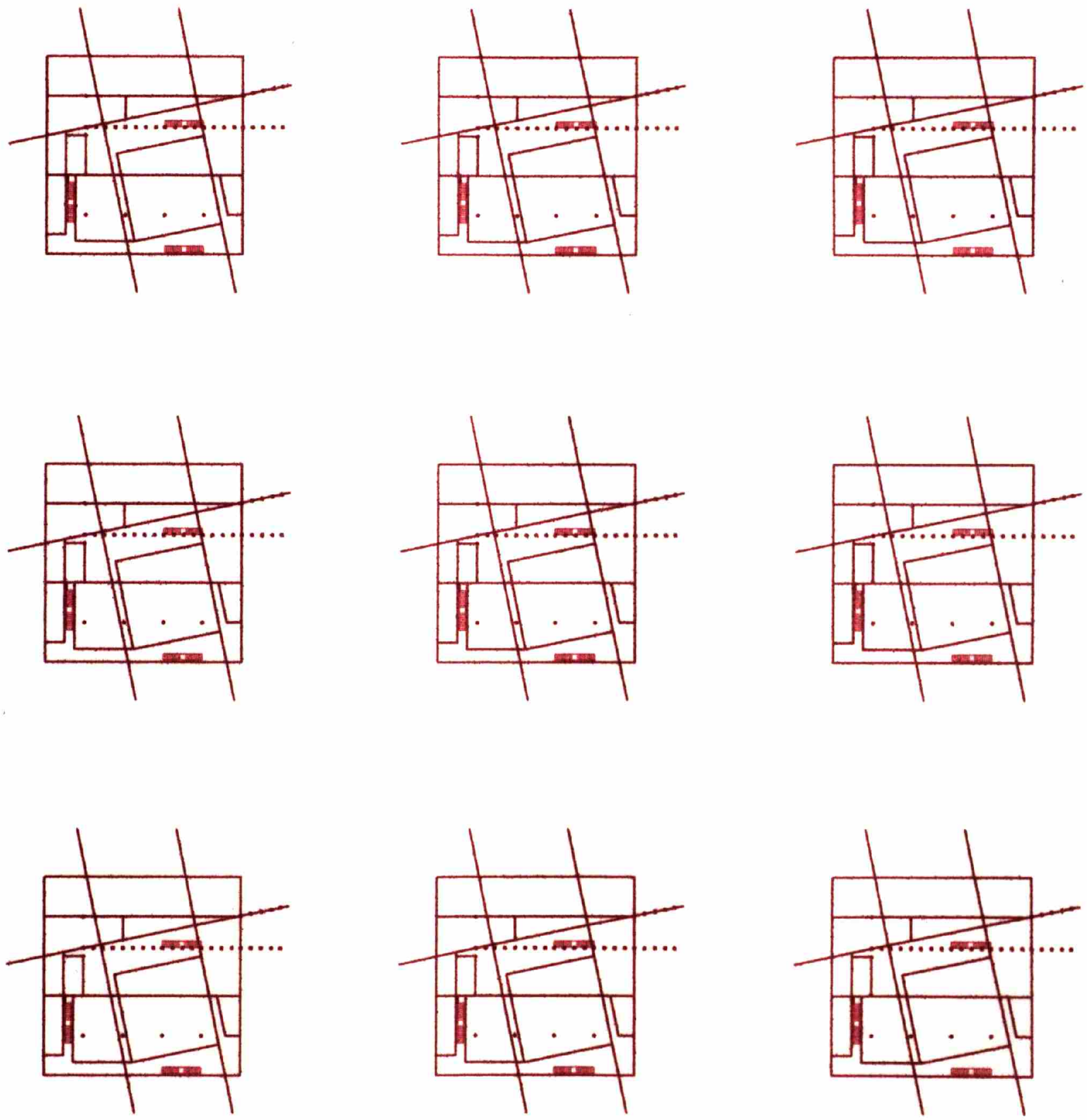
rule 7ii
draw
line in
layer A
over a
line in
layer B



rule 2ii
union
shapes
by
erasing
their
common
bndry

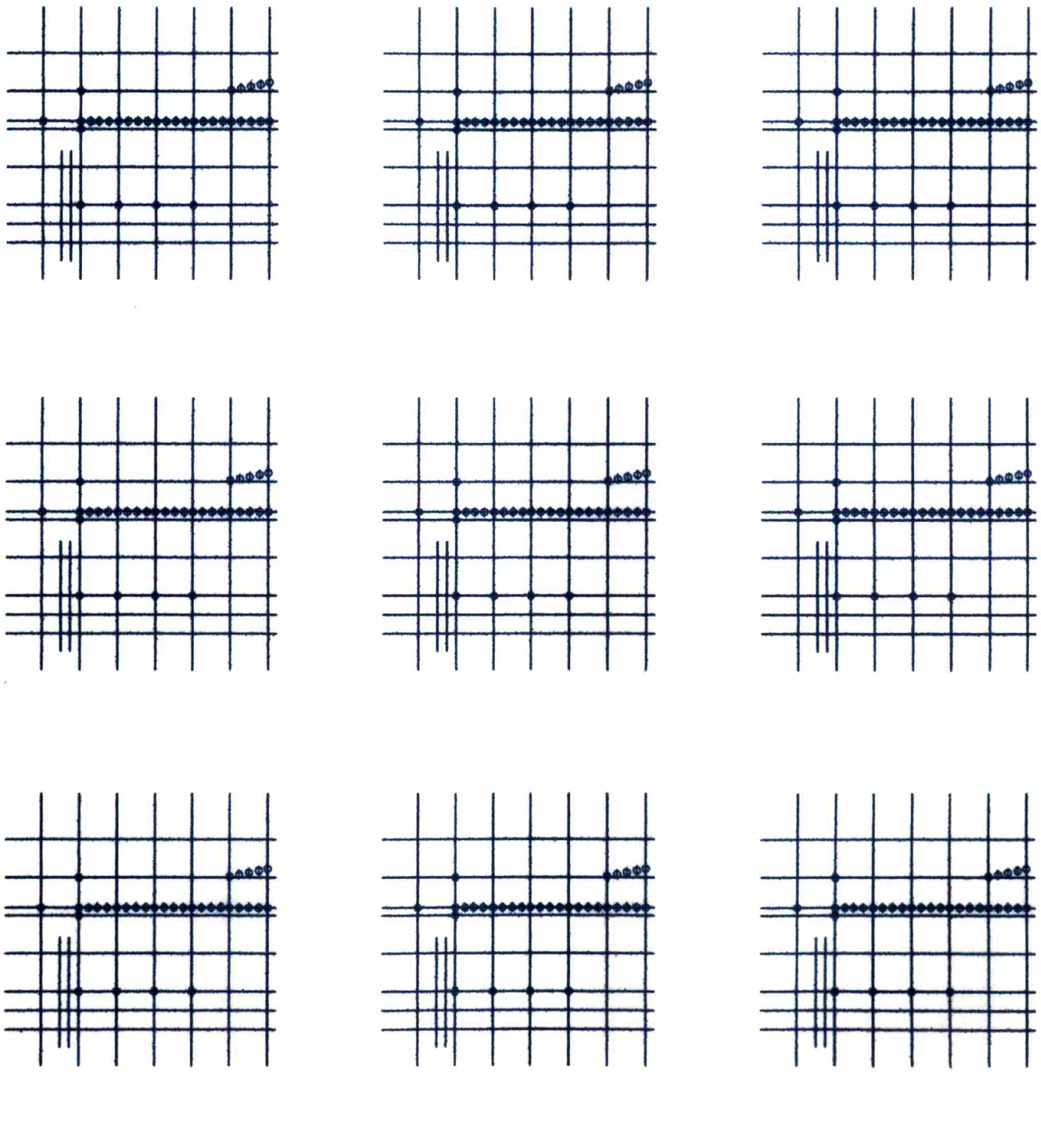
PLAN II

B



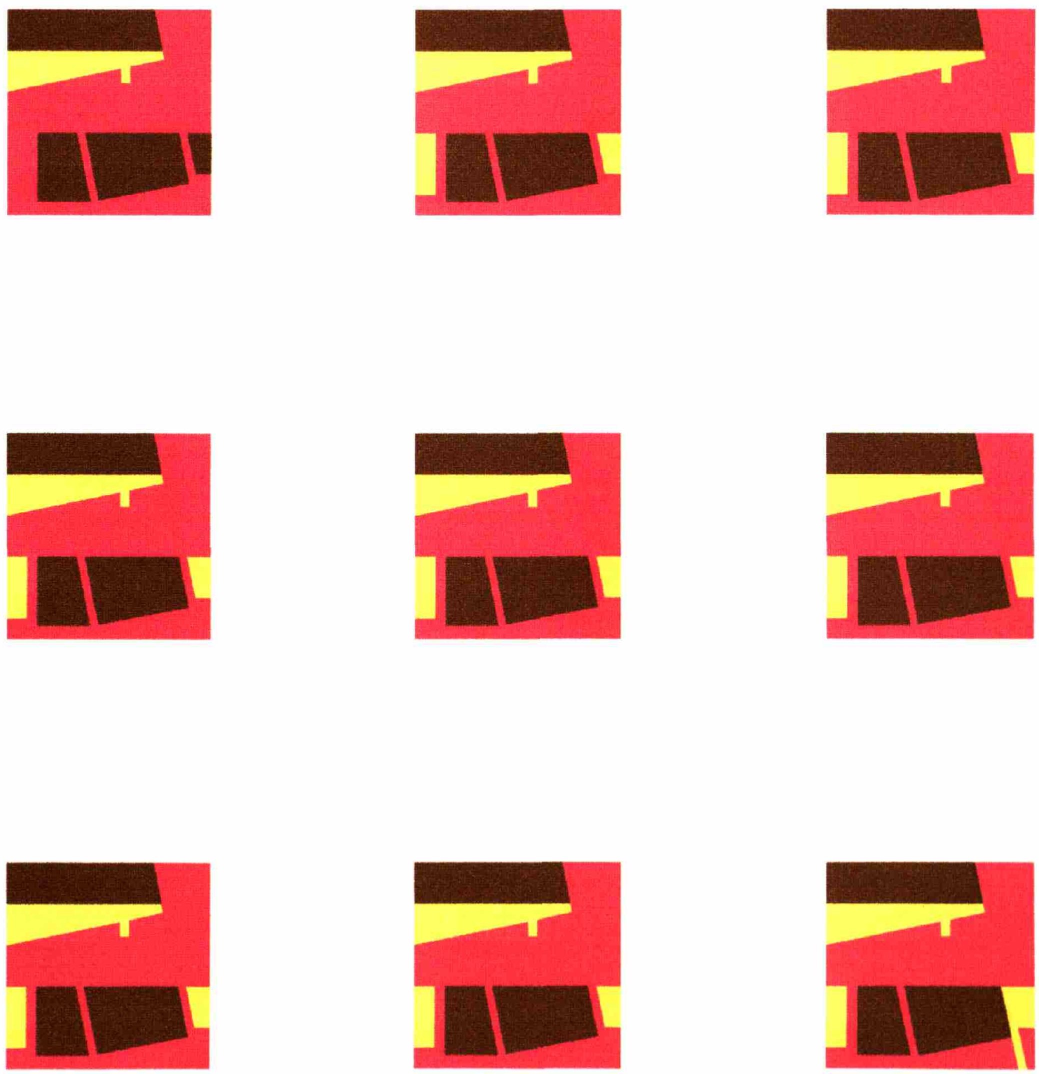
PLAN II

C



PLAN II

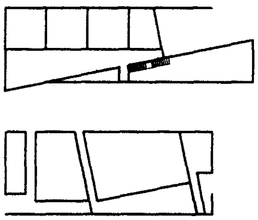
D



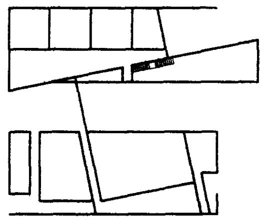
PLAN II

A

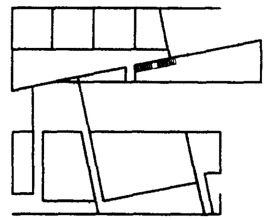
rule 7ii
draw
line in
layer A
over
line in
layer B



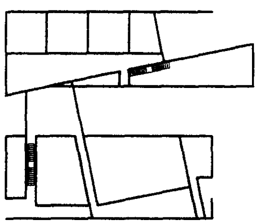
rule 5
extend
side of
quadr in
layer A



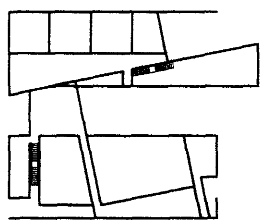
rule 7ii
draw
line in
layer A
over
line in
layer B



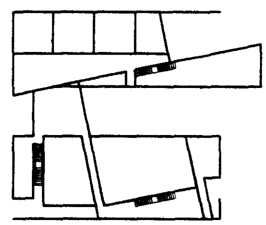
rule 15ii
draw
stair in
layer A
over
stair in
layer B

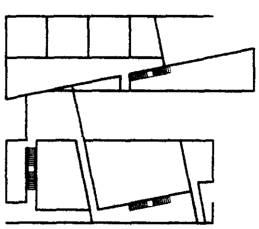


rule 2ii
union
shapes
by
erasing
their
common
bndry

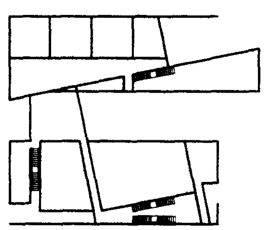


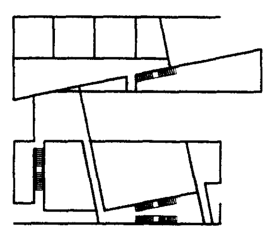
rule 15
draw
stair in
layer A





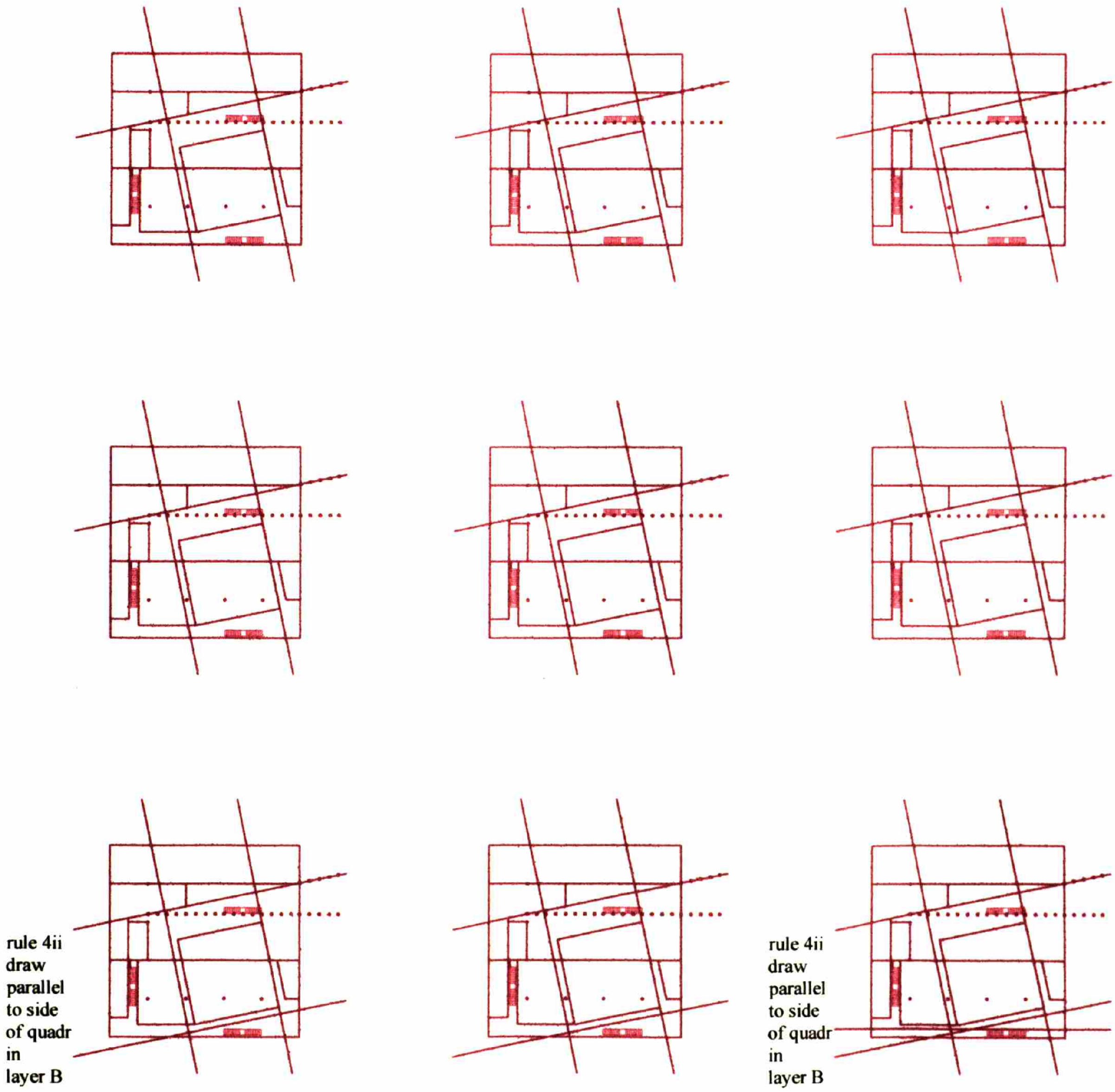
rule 15ii
draw
stair in
layer A
over
stair in
layer B





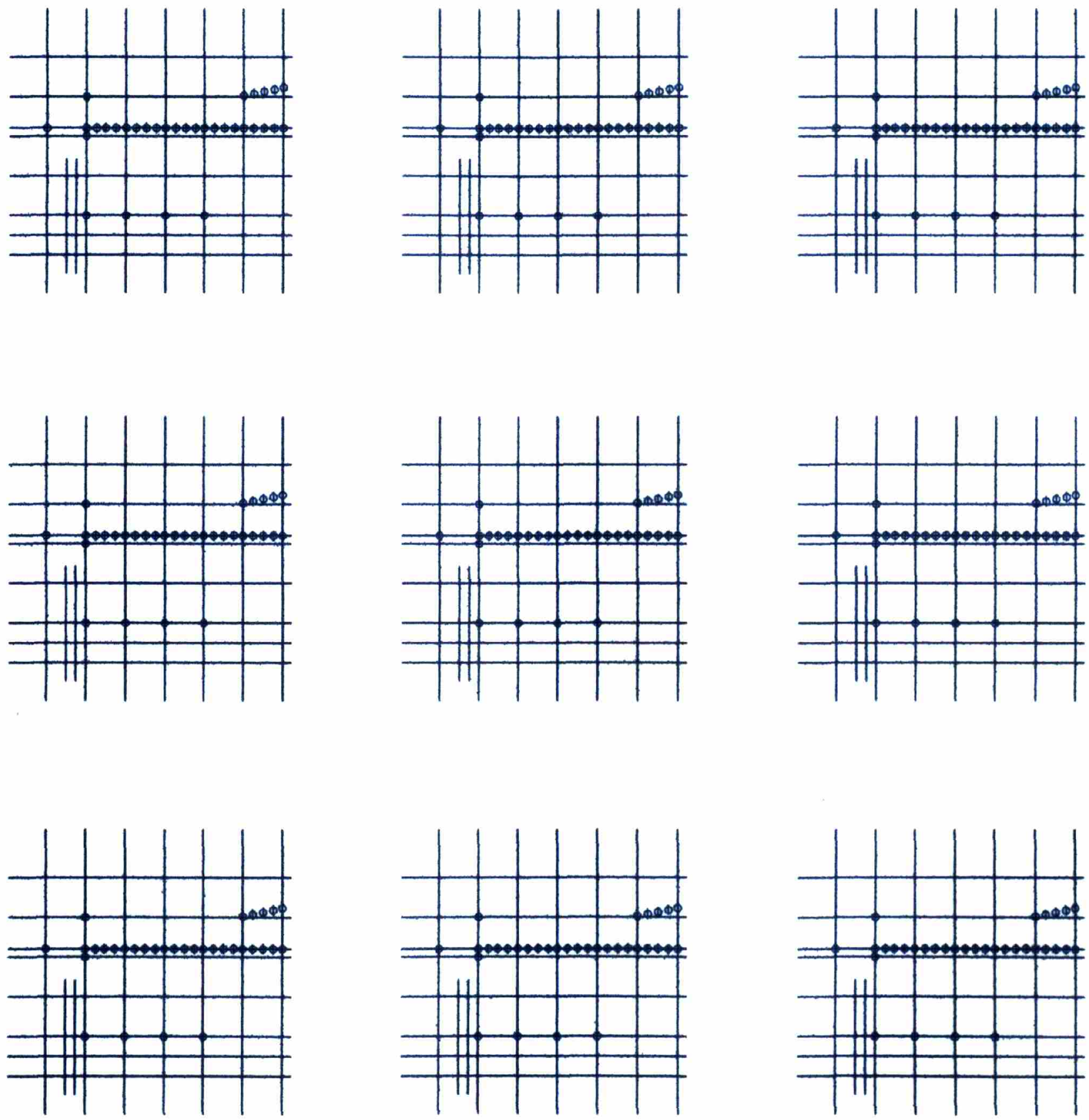
PLAN II

B


rule 4ii
draw
parallel
to side
of quadr
in
layer B
rule 4ii
draw
parallel
to side
of quadr
in
layer B

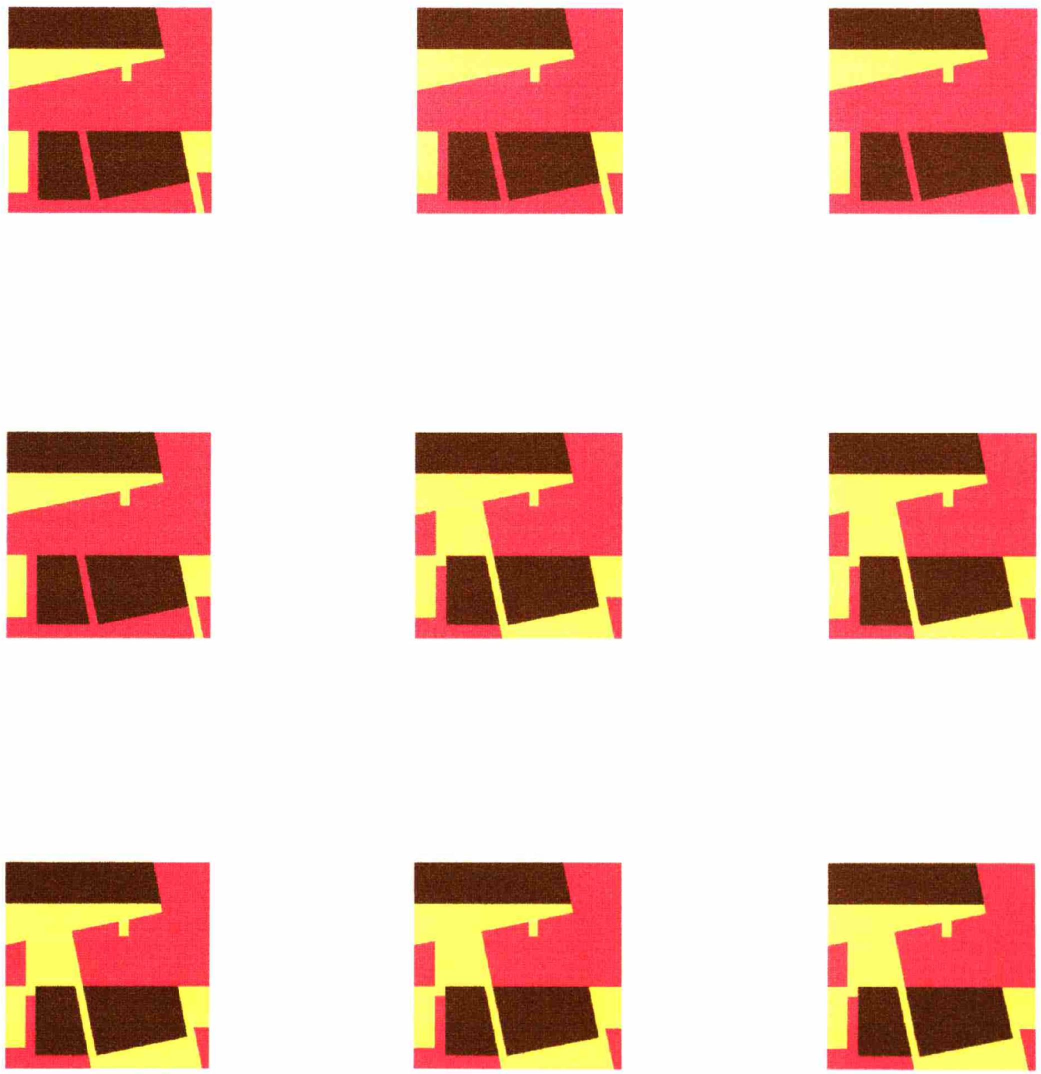
PLAN II

C



PLAN II

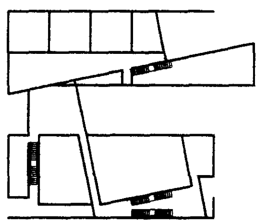
D



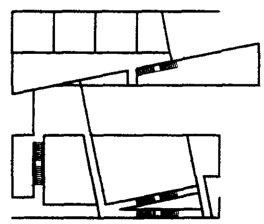
PLAN II

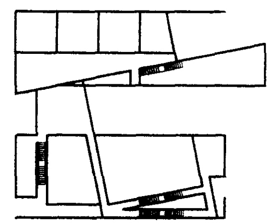
A

rule 2ii
union
shapes
by
erasing
their
common
bndry

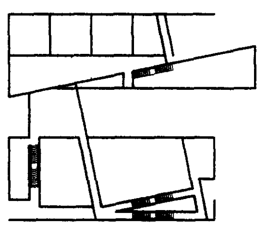


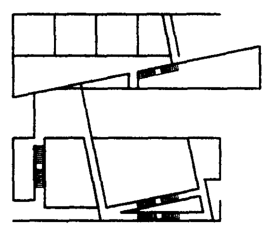
rule 1iii
draw
triangle
in
layer A
over
triangle
in
layer B

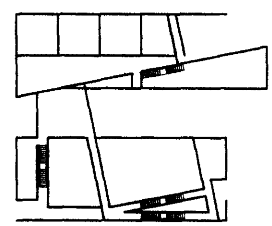




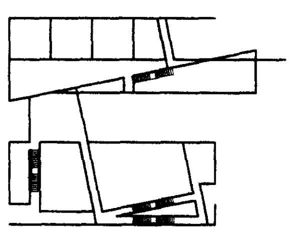
rule 7ii
draw
line in
layer A
over
line in
layer B



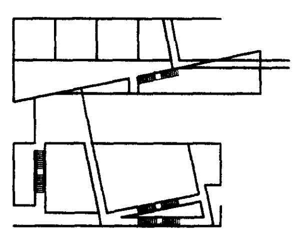




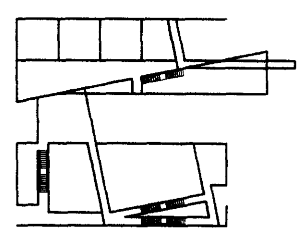
rule 7
draw
line in
layer A
over
line in
layer C



rule 7ii
draw
line in
layer A
over
line in
layer B

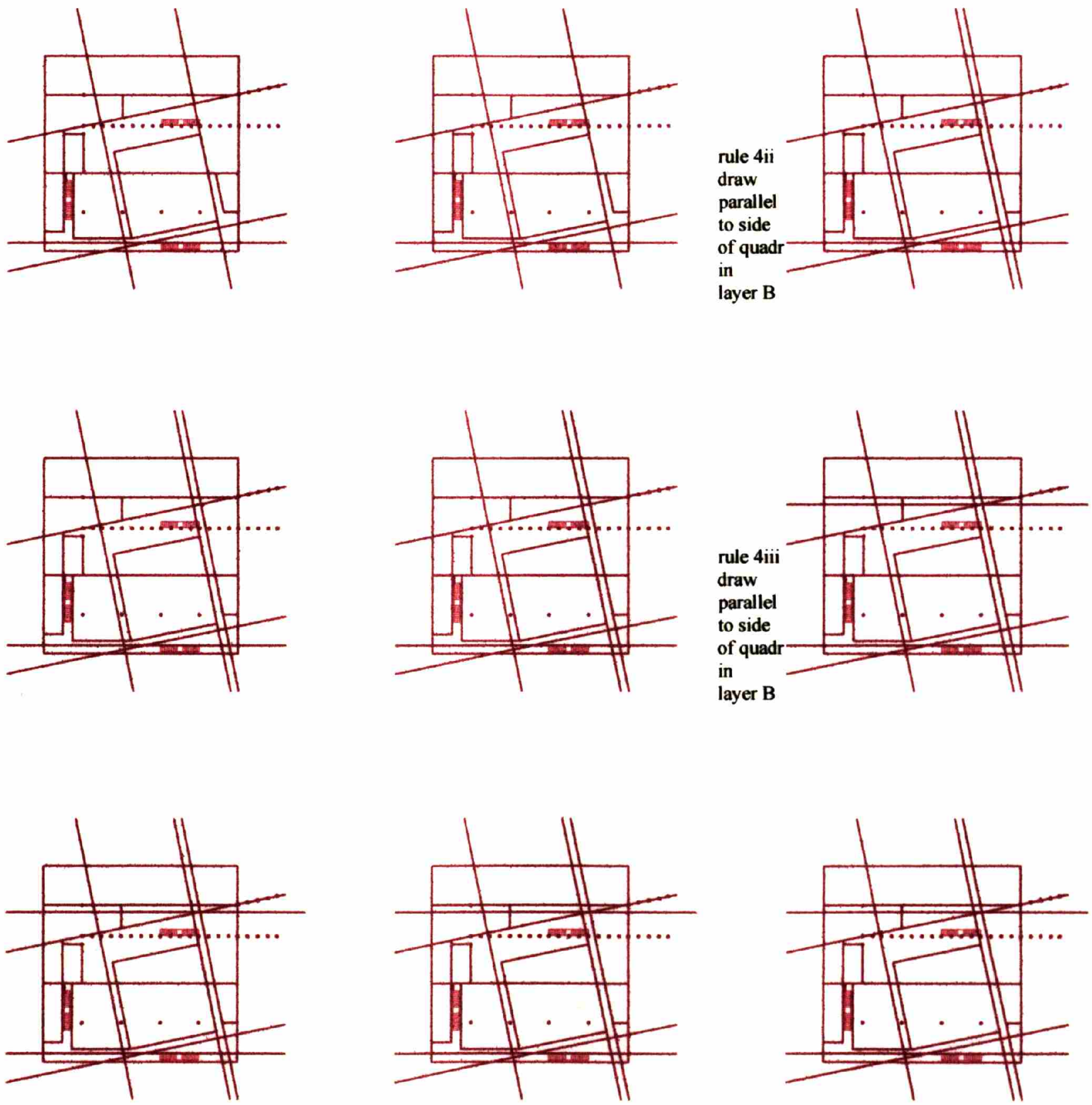


rule 7
draw
line in
layer A
over
line in
layer C



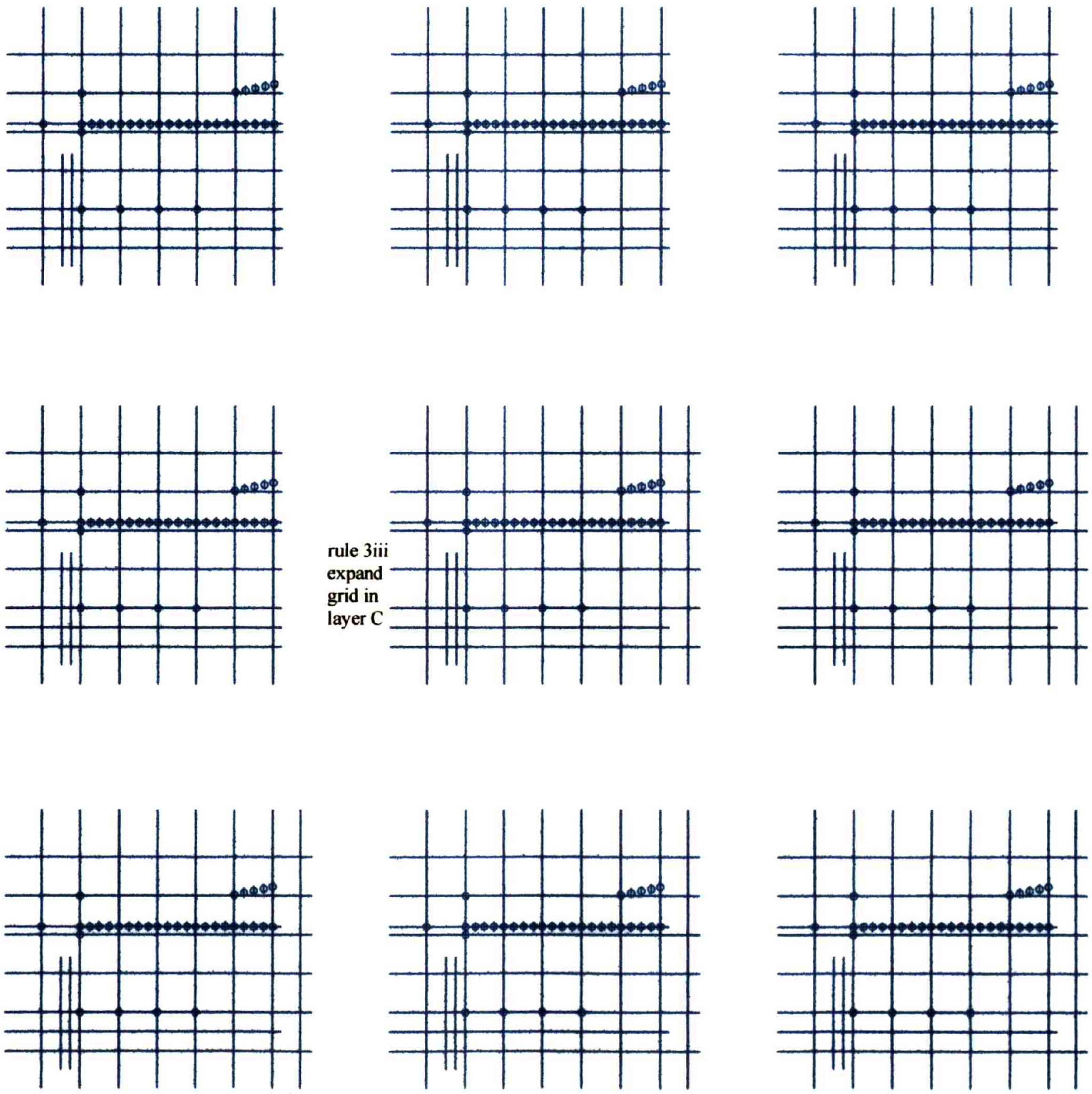
PLAN II

B


rule 4ii
draw
parallel
to side
of quadr
in
layer B
rule 4iii
draw
parallel
to side
of quadr
in
layer B

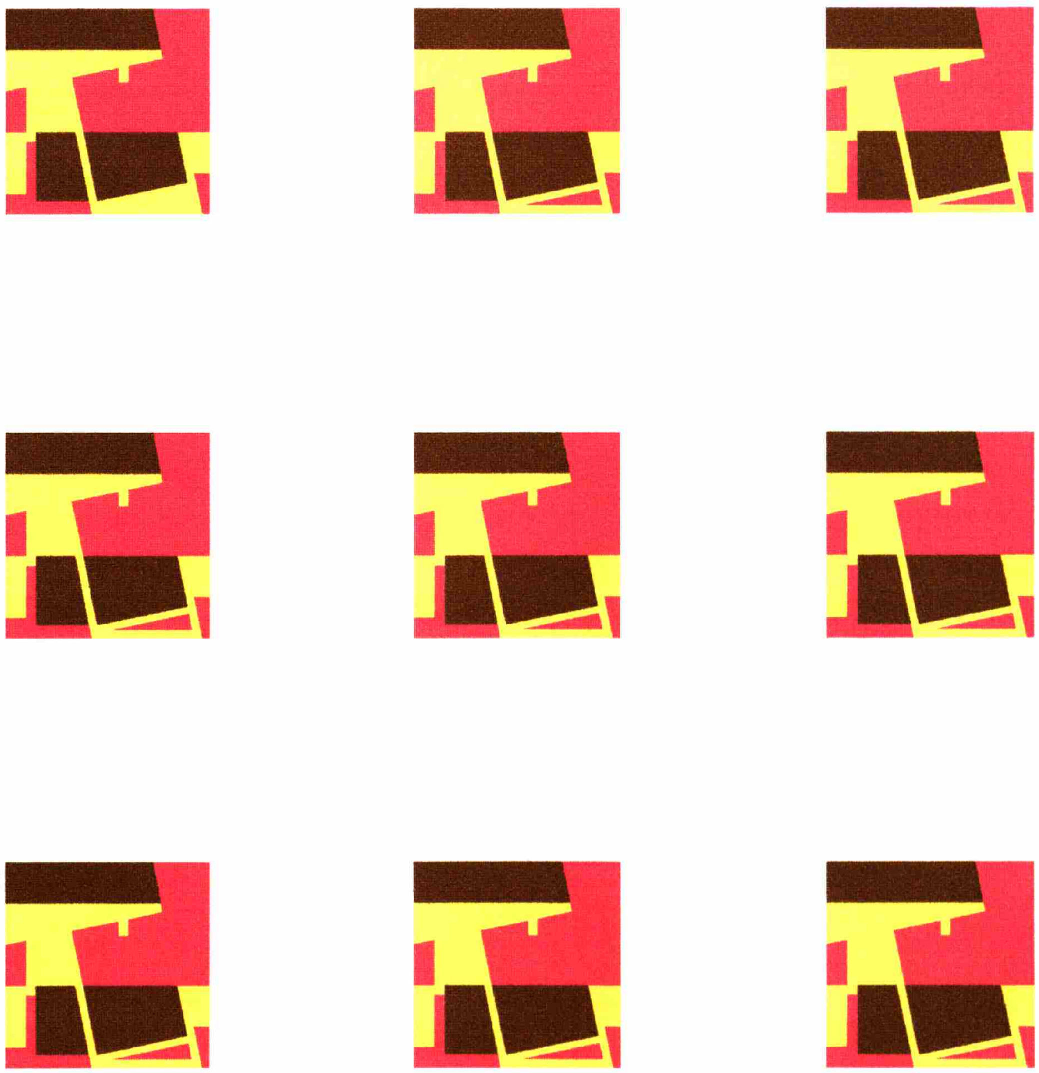
PLAN II

C



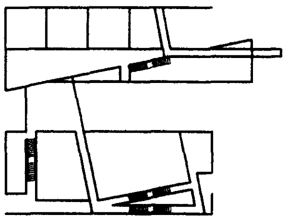
PLAN II

D

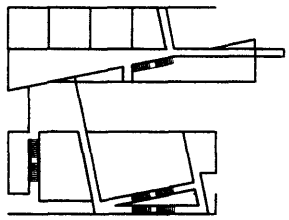


PLAN II

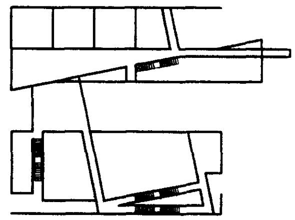
A

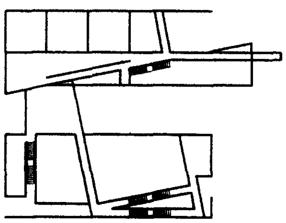


rule 2ii
union
shapes
by
erasing
their
common
bndry

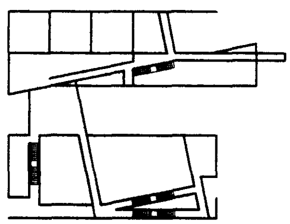


rule 2ii
union
shapes
by
erasing
their
common
bndry

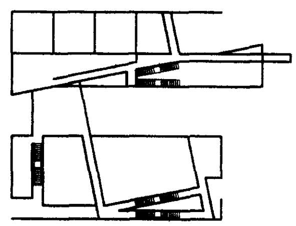




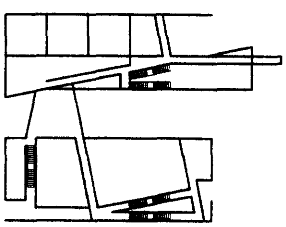
rule 7ii
draw
line in
layer A
over
line in
layer B



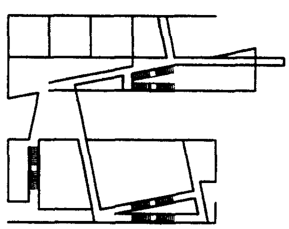
rule 5
extend
side of
quadr



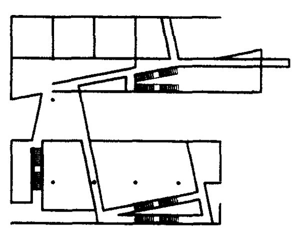
rule 15ii
draw
stair in
layer A
over
stair in
layer B



rule 14
change
angle
of line
in
layer A



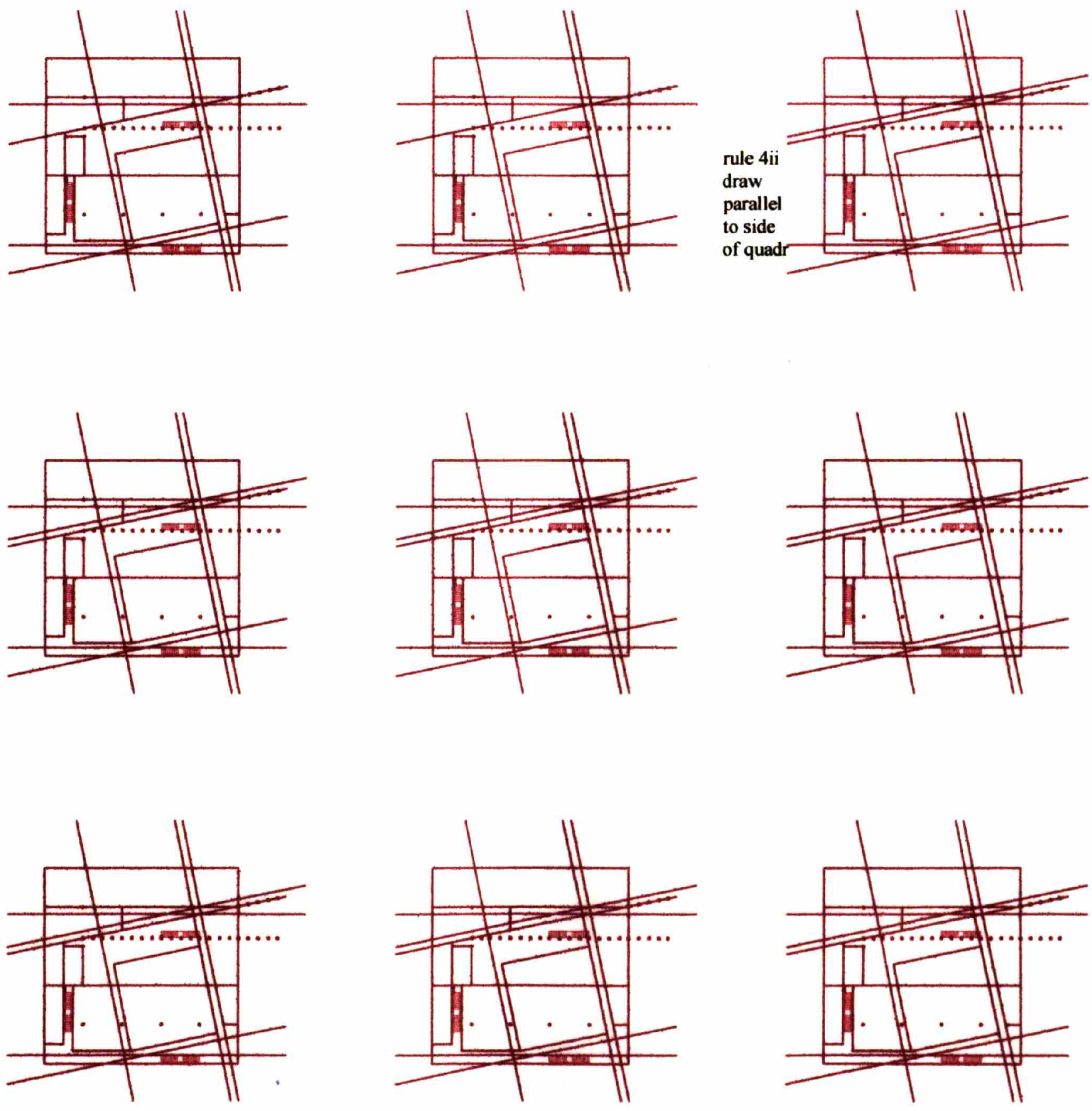
rule 2ii
union
shapes
by
erasing
their
common
bndry



rule 21
draw
clmns in
layer A
over
clmns in
layer B

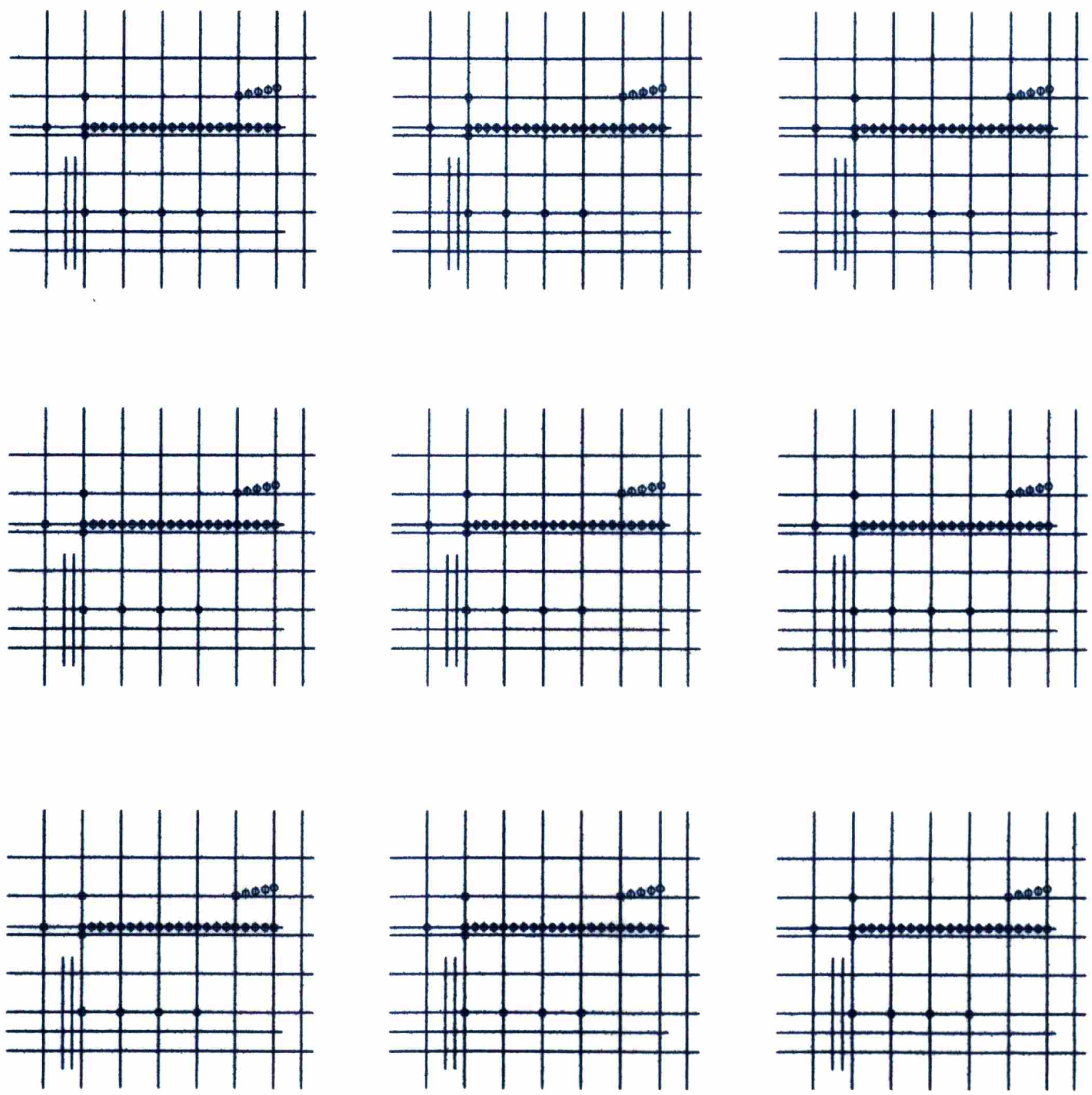
PLAN II

B



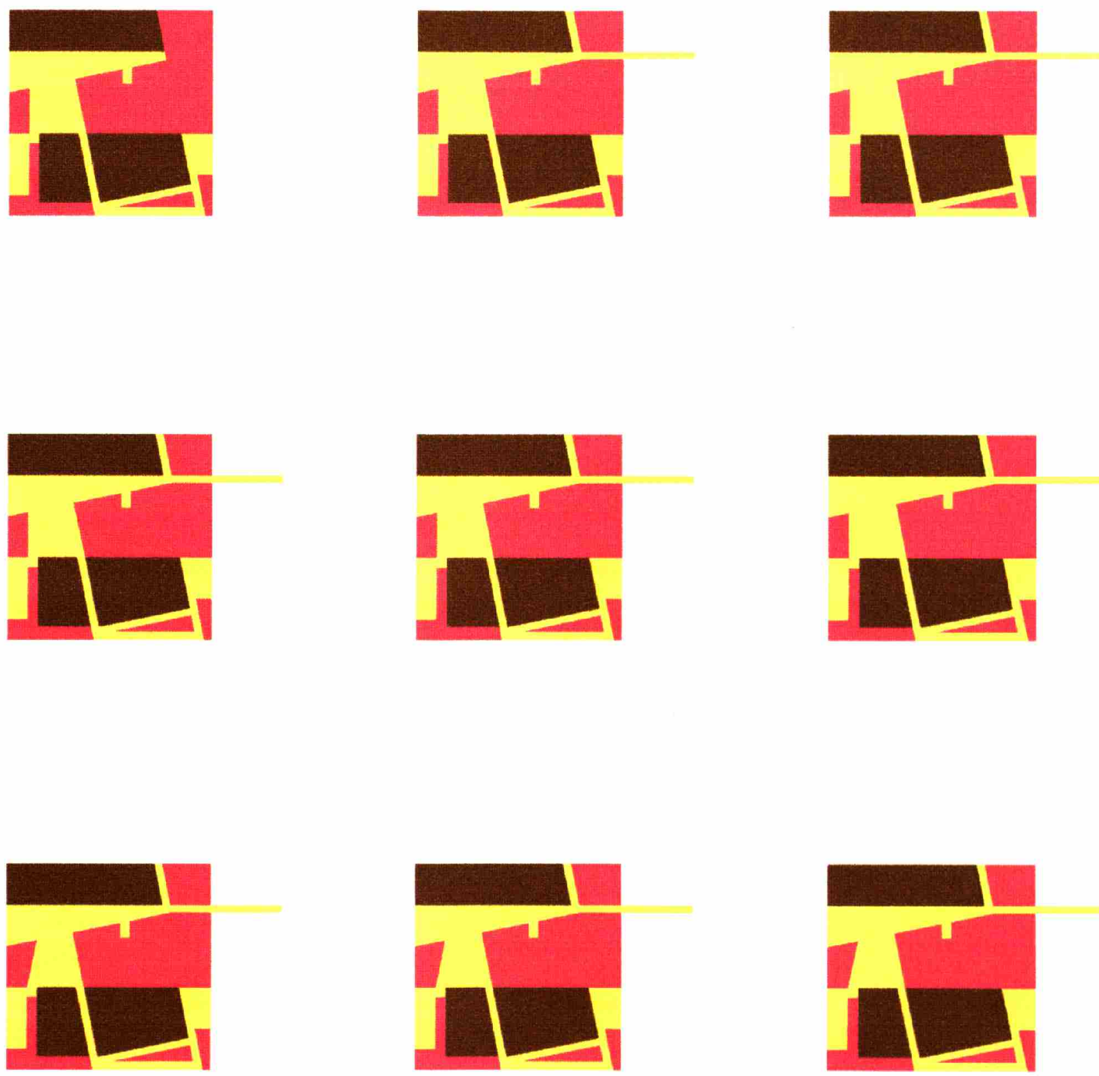
PLAN II

C



PLAN II

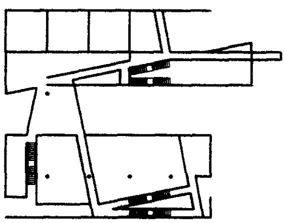
D



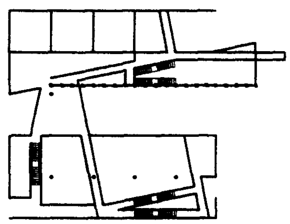
PLAN II

A

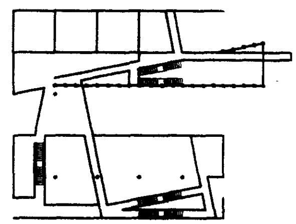
rule 17
merge
clmn
in wall



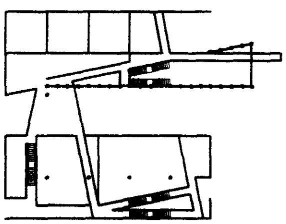
rule 19
draw
clmns



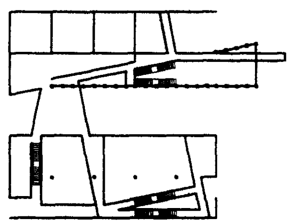
rule 19
draw
clmns

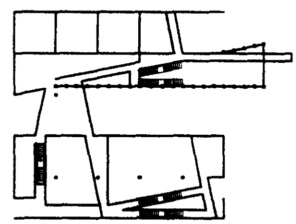


rule 2
dissect
convex
shape



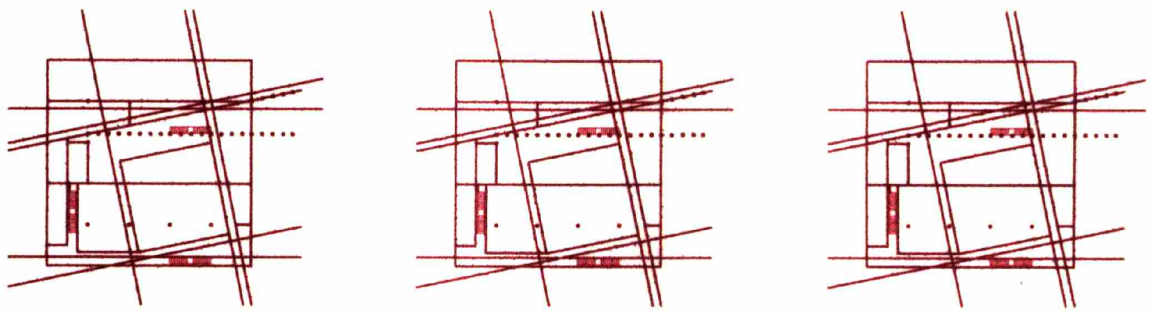
rule 2ii
union
shapes
by
erasing
their
common
bndry

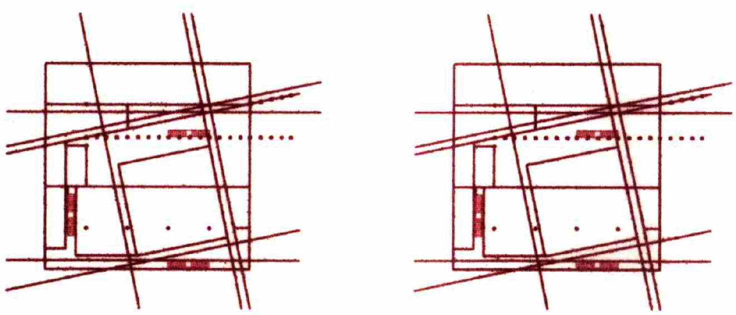




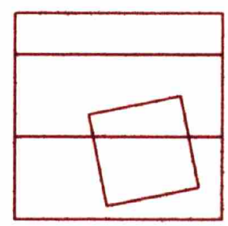
PLAN II

B



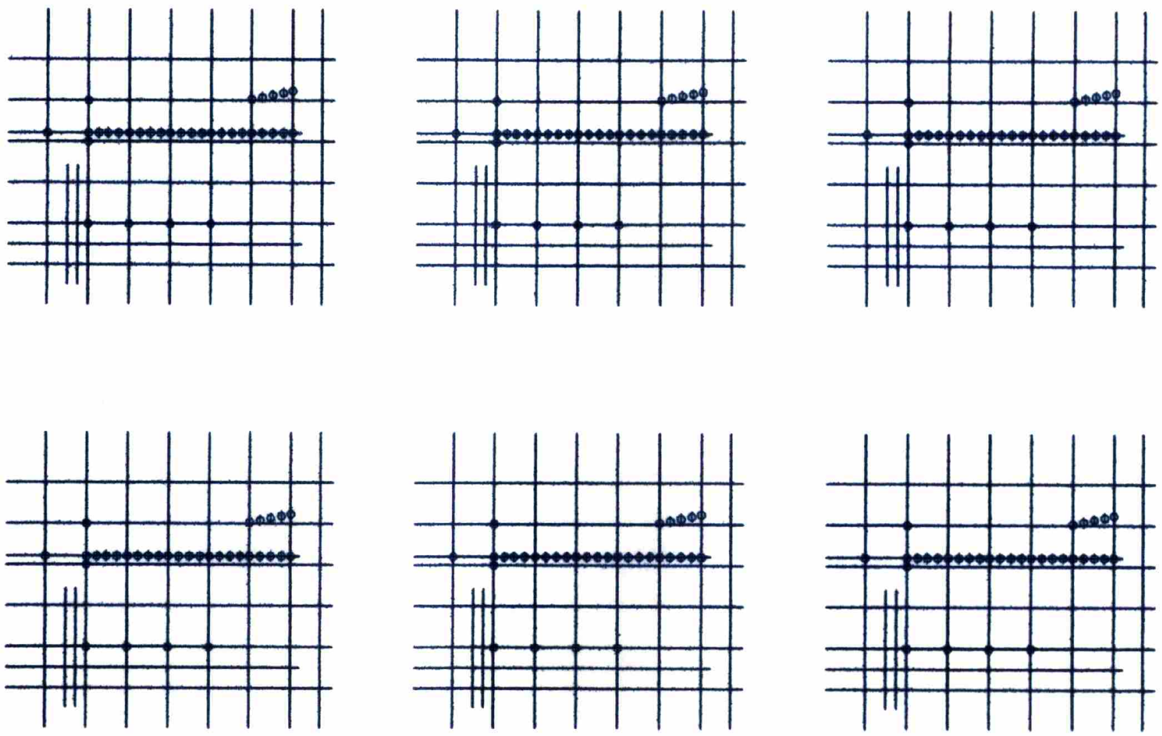


rule 23
erase
line
extens
in
layer B



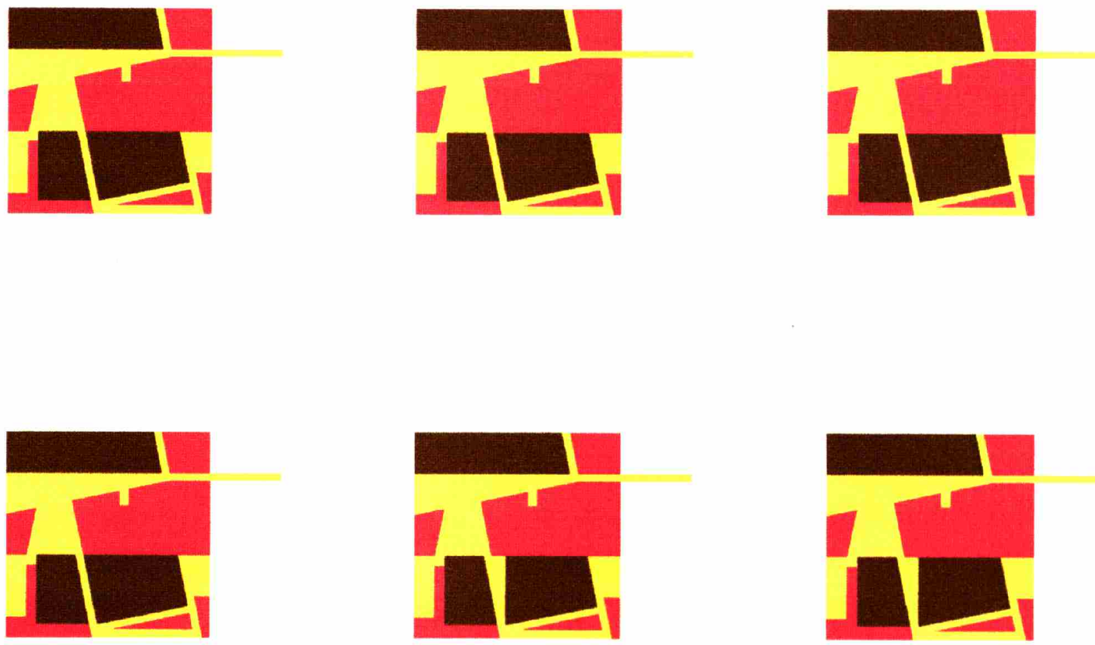
PLAN II

C



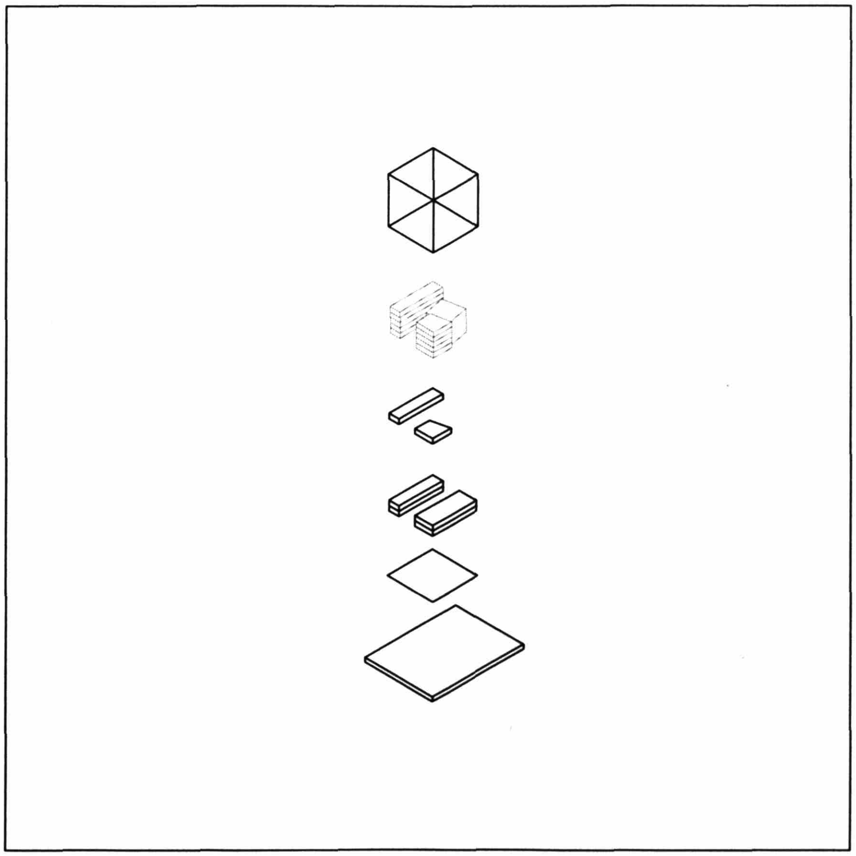
PLAN II

D



PLAN II

6.2.3. Derivation of Plan III



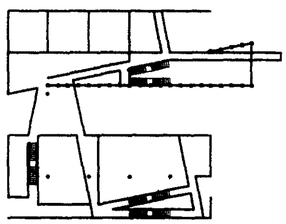
The next derivation of plan III, (pp. 385– 440) in four sheets A x B x C x D, generates the plan type corresponding to the floors 4-8, as shown in the above diagram with green color.

The layers A x B x C x D of the derived plan II are passed, providing the initial shape for plan III. Therefore, layer A contains black lines, layer B contains a shape made out of red lines, layer C a grid, and layer D contains colored areas. The sum of all areas of any color (pink, yellow, brown or blue) is always equal to the area of the initial pink square (plan I), which represents the available free area of the site.

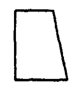
The overall derivation unfolds in the product algebra $<U_{12}\ V_{02}> \times <U_{12}\ V_{02}> \times <U_{12}\ V_{02}> \times <U_{22}V_{02}>$.

A

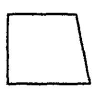
shape
from
previous
stage



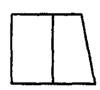
rule 1iii
draw
quadr in
layer A
over
quadr in
layer B



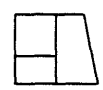
rule 8
transl
side of
quadr
to grid

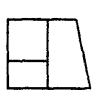


rule 2
dissect
quadr
in
layer A

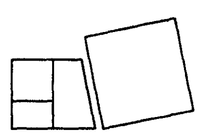


rule 2
dissect
quadr
in
layer A

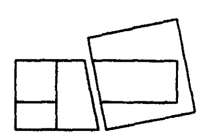




rule 1iii
draw
quadr in
layer A
over
quadr in
layer B

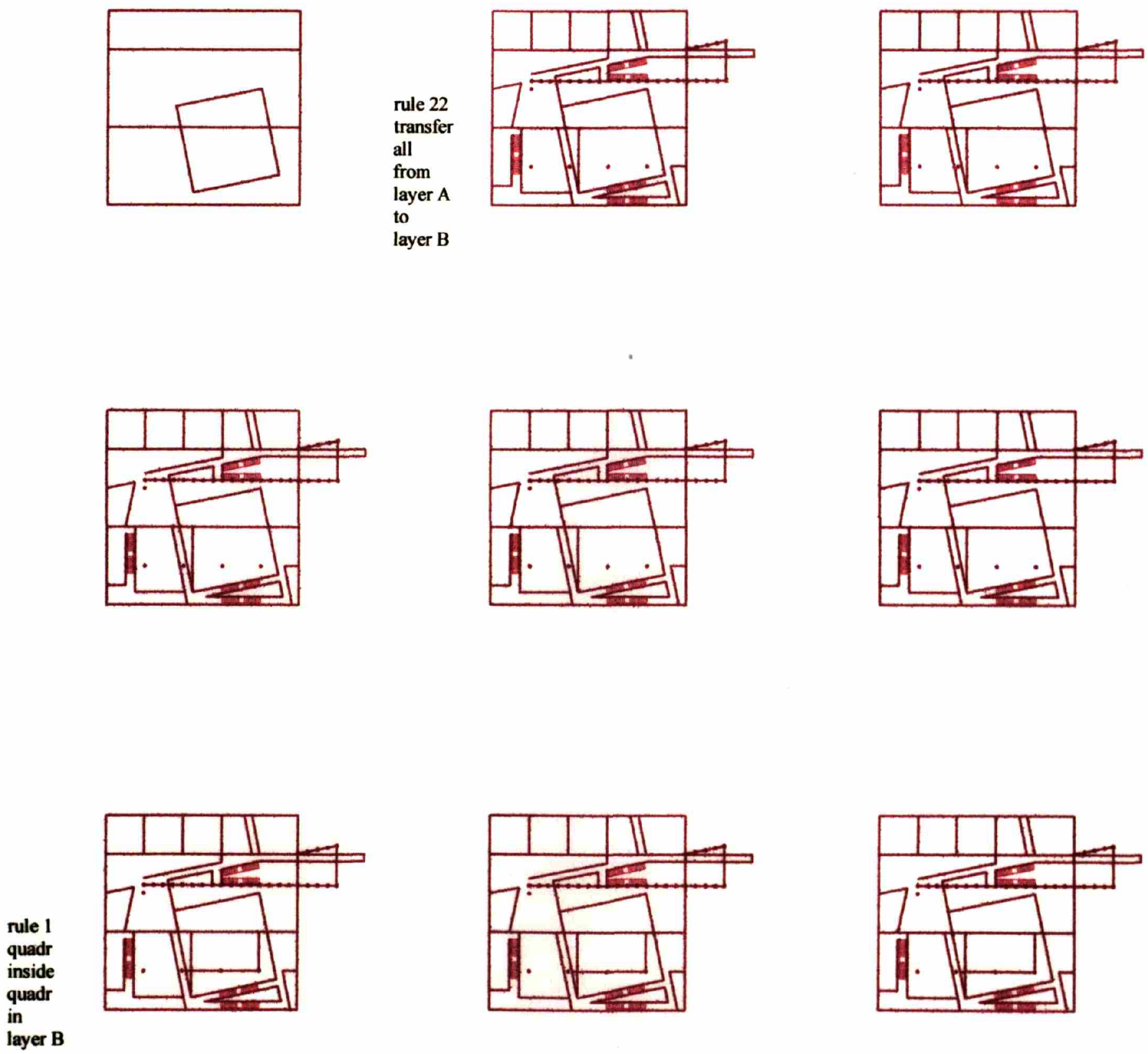


rule 1iii
draw
quadr in
layer A
over
quadr in
layer B



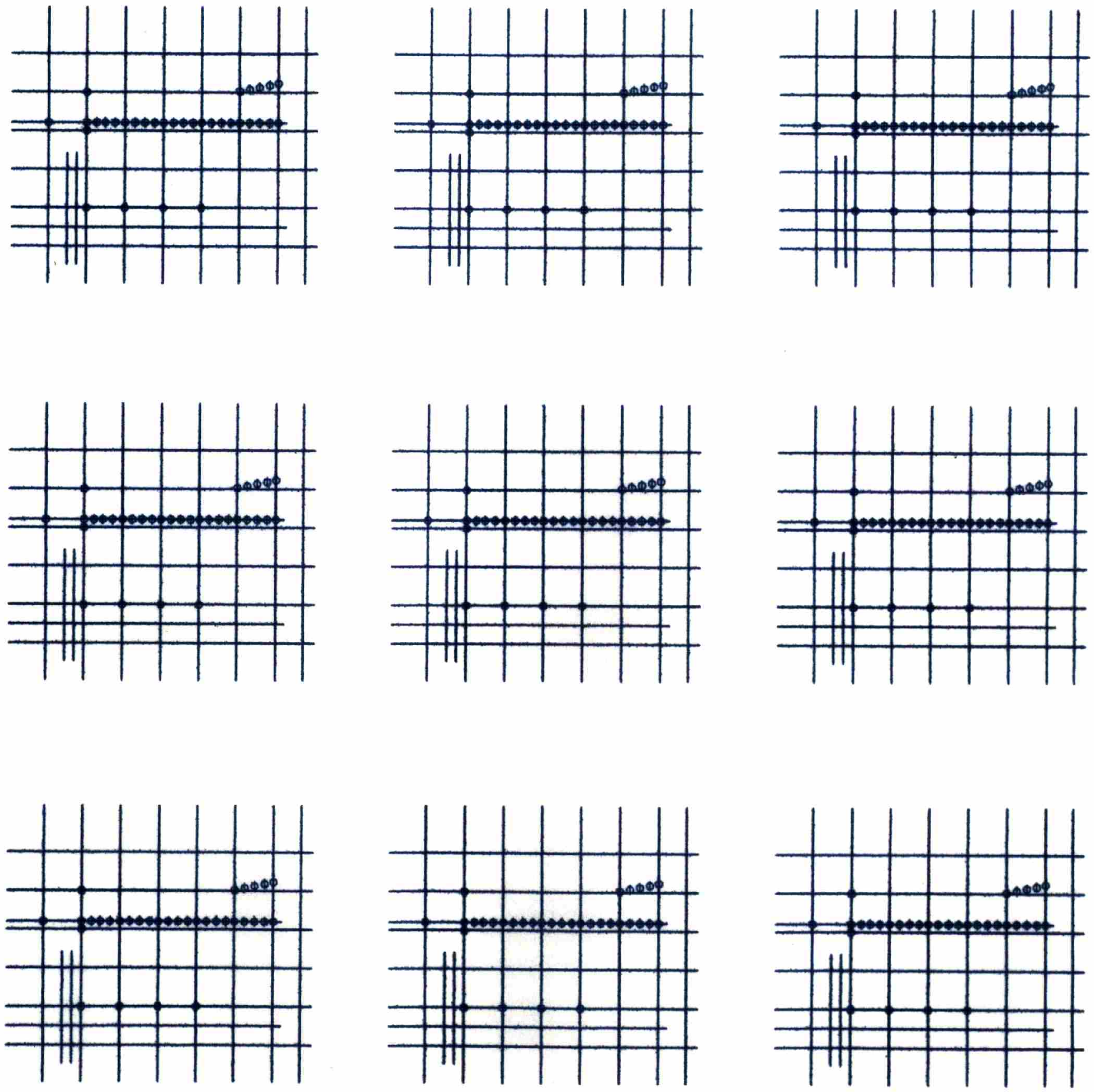
PLAN III

B


rule 22
transfer
all
from
layer A
to
layer B
rule 1
quadr
inside
quadr
in
layer B

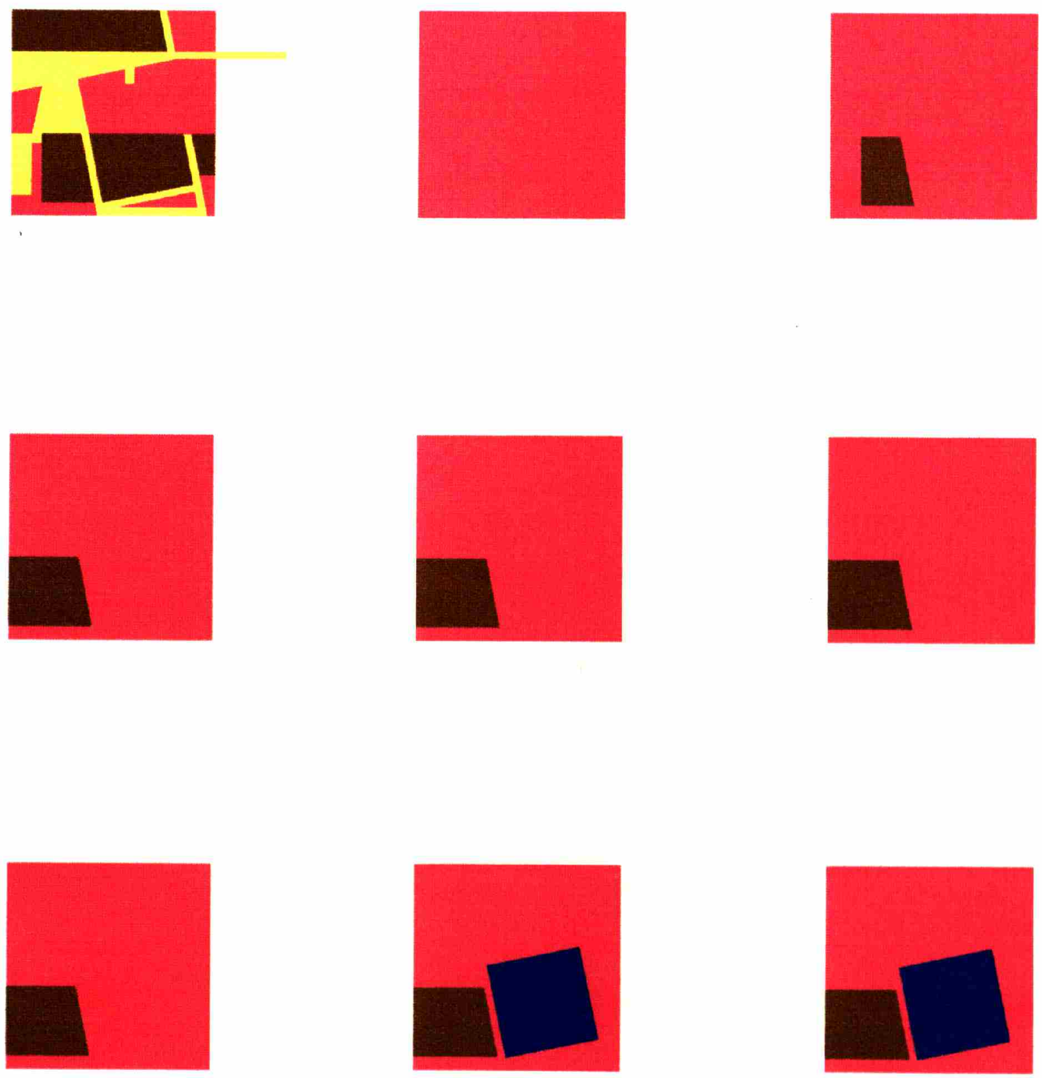
PLAN III

C



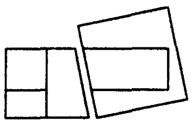
PLAN III

D

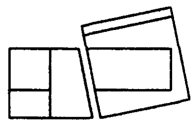


PLAN III

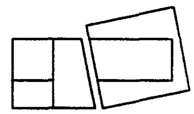
A

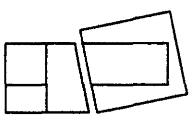


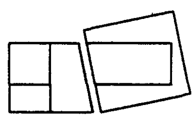
rule 7
draw
line in
layer A
over a
line in
layer C

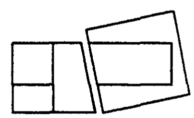


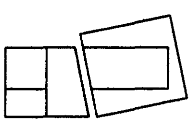
rule 11
from
n-sided
convex
shape
select
n-sided
convex
shape

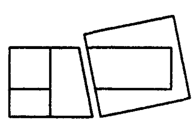




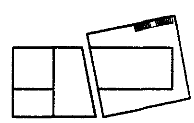






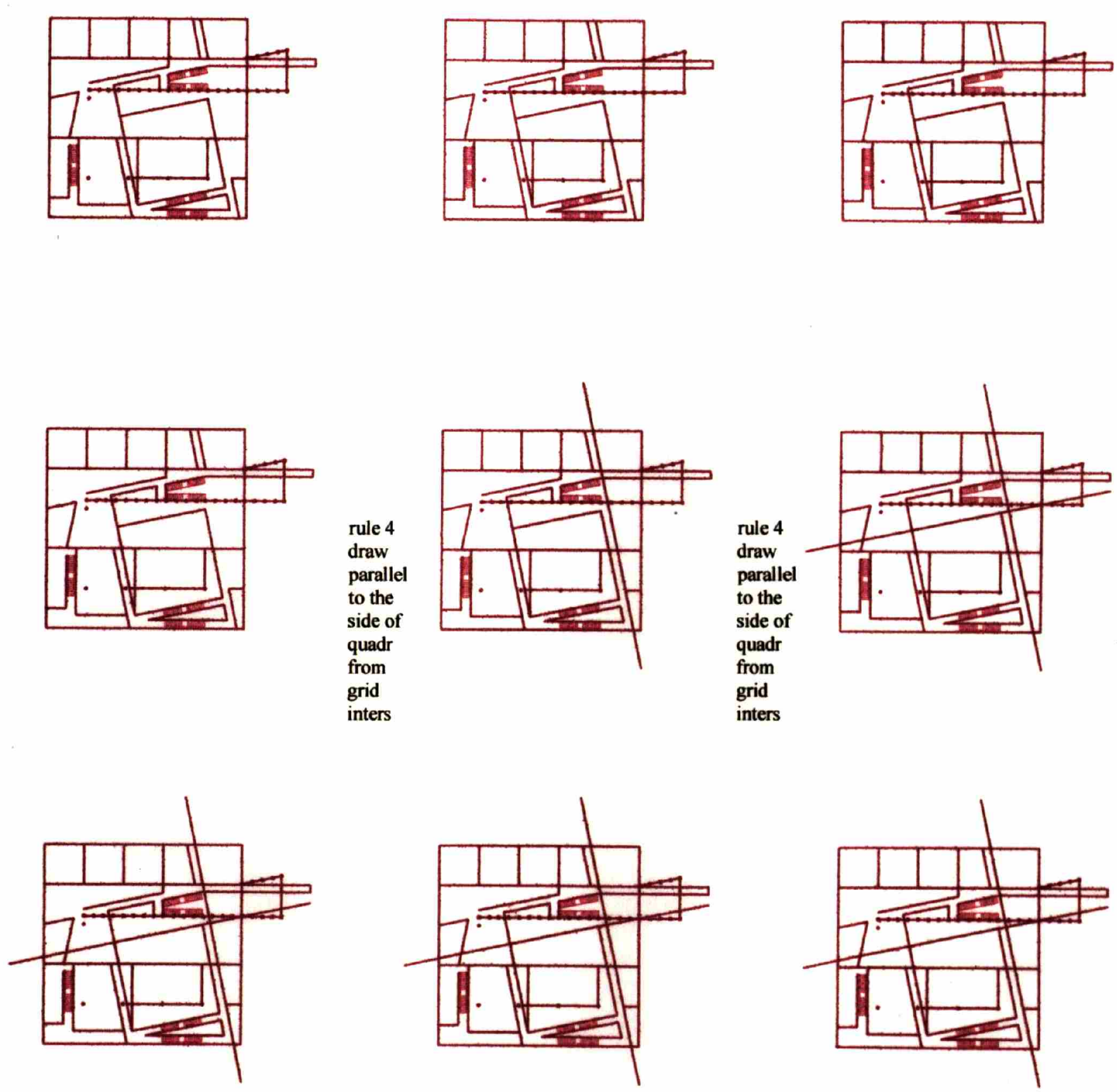


rule 15
draw
stair



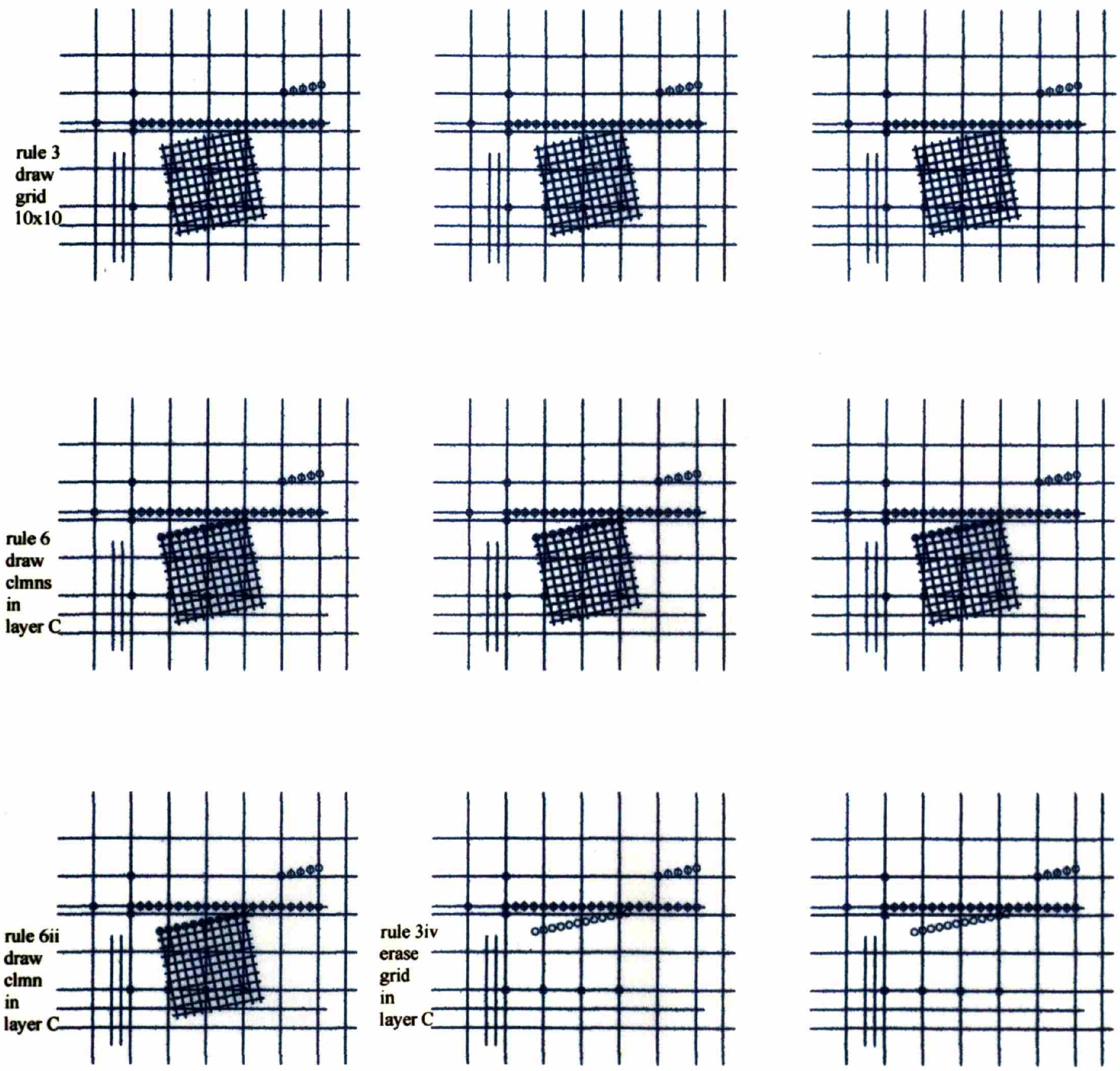
PLAN III

B


rule 4
draw
parallel
to the
side of
quadr
from
grid
inters
rule 4
draw
parallel
to the
side of
quadr
from
grid
inters

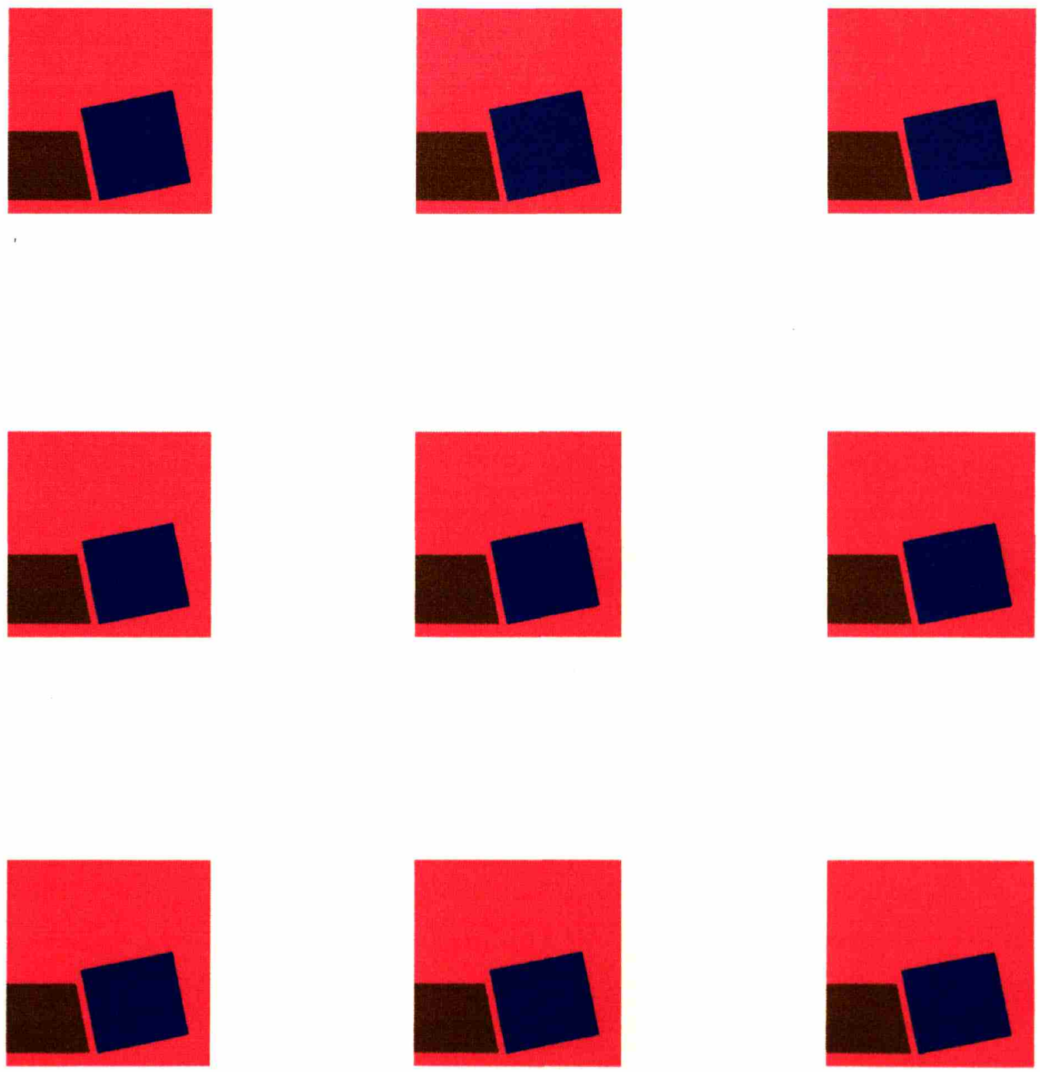
PLAN III

C

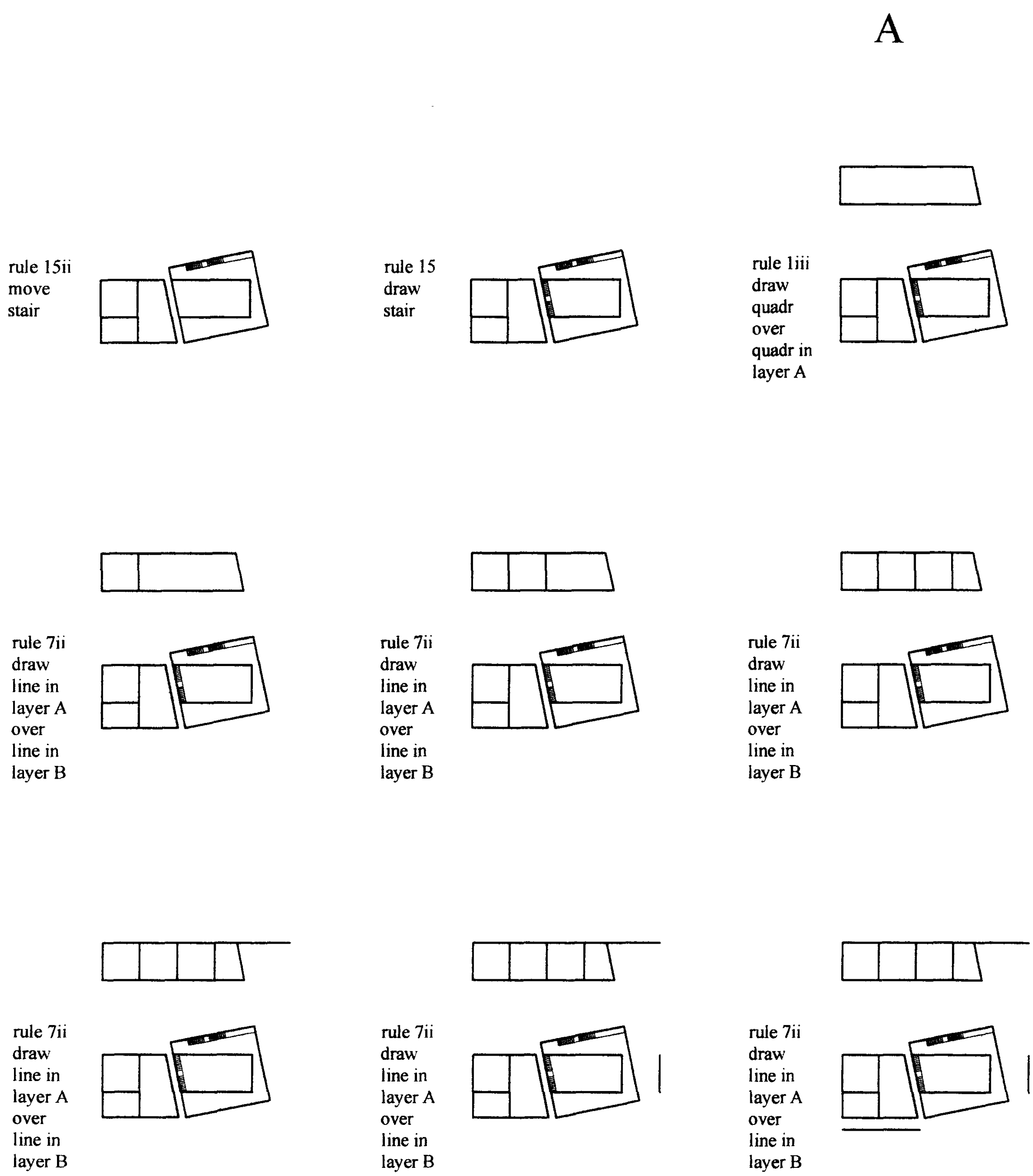

rule 3
draw
grid
10x10
rule 6
draw
clmns
in
layer C
rule 6ii
draw
clmn
in
layer C
rule 3iv
erase
grid
in
layer C

PLAN III

D

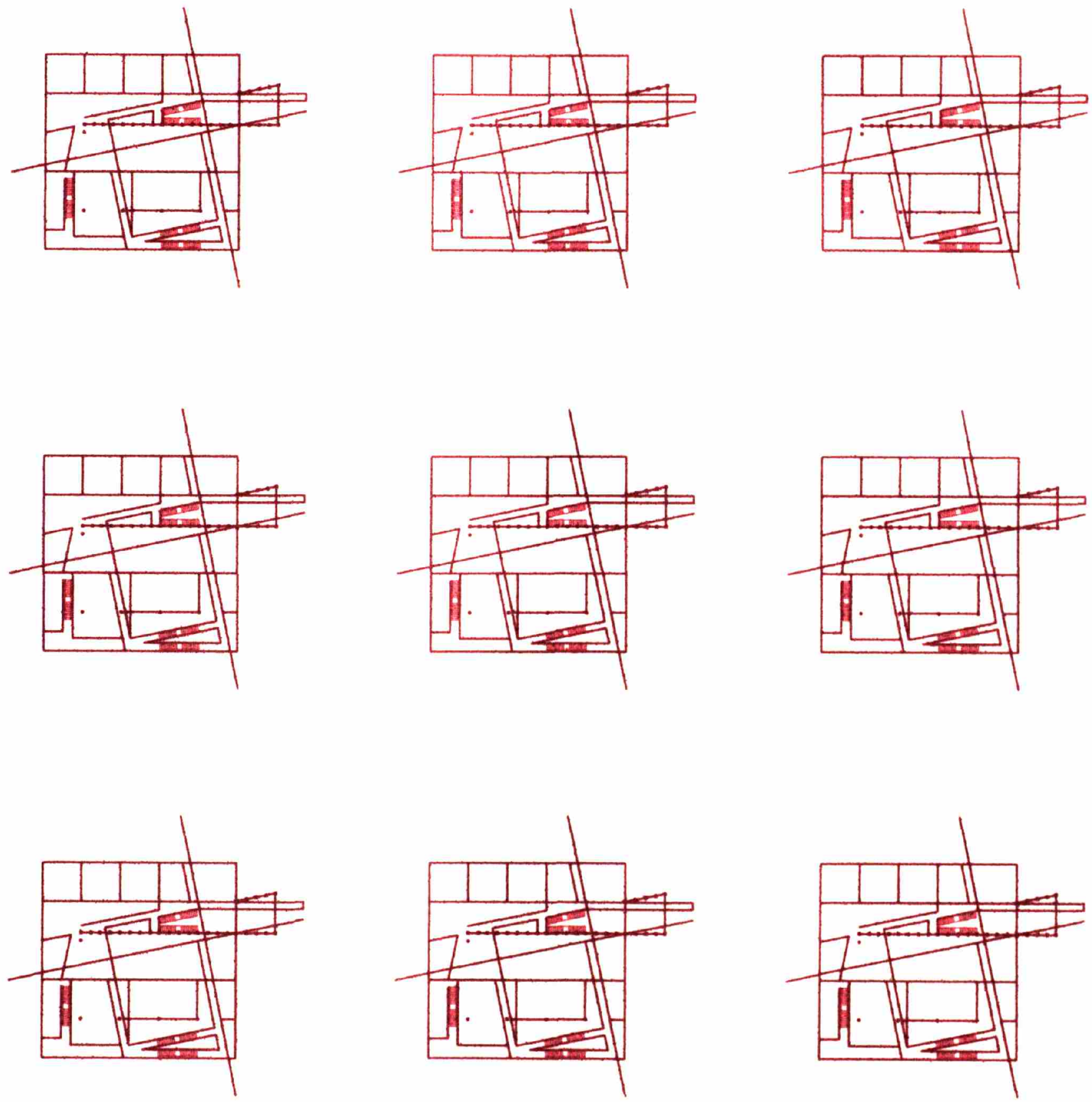


PLAN III


A
rule 15ii move stair
rule 15 draw stair
rule 1iii draw quadr over quadr in layer A
rule 7ii draw line in layer A over line in layer B
rule 7ii draw line in layer A over line in layer B
rule 7ii draw line in layer A over line in layer B
rule 7ii draw line in layer A over line in layer B
rule 7ii draw line in layer A over line in layer B
rule 7ii draw line in layer A over line in layer B

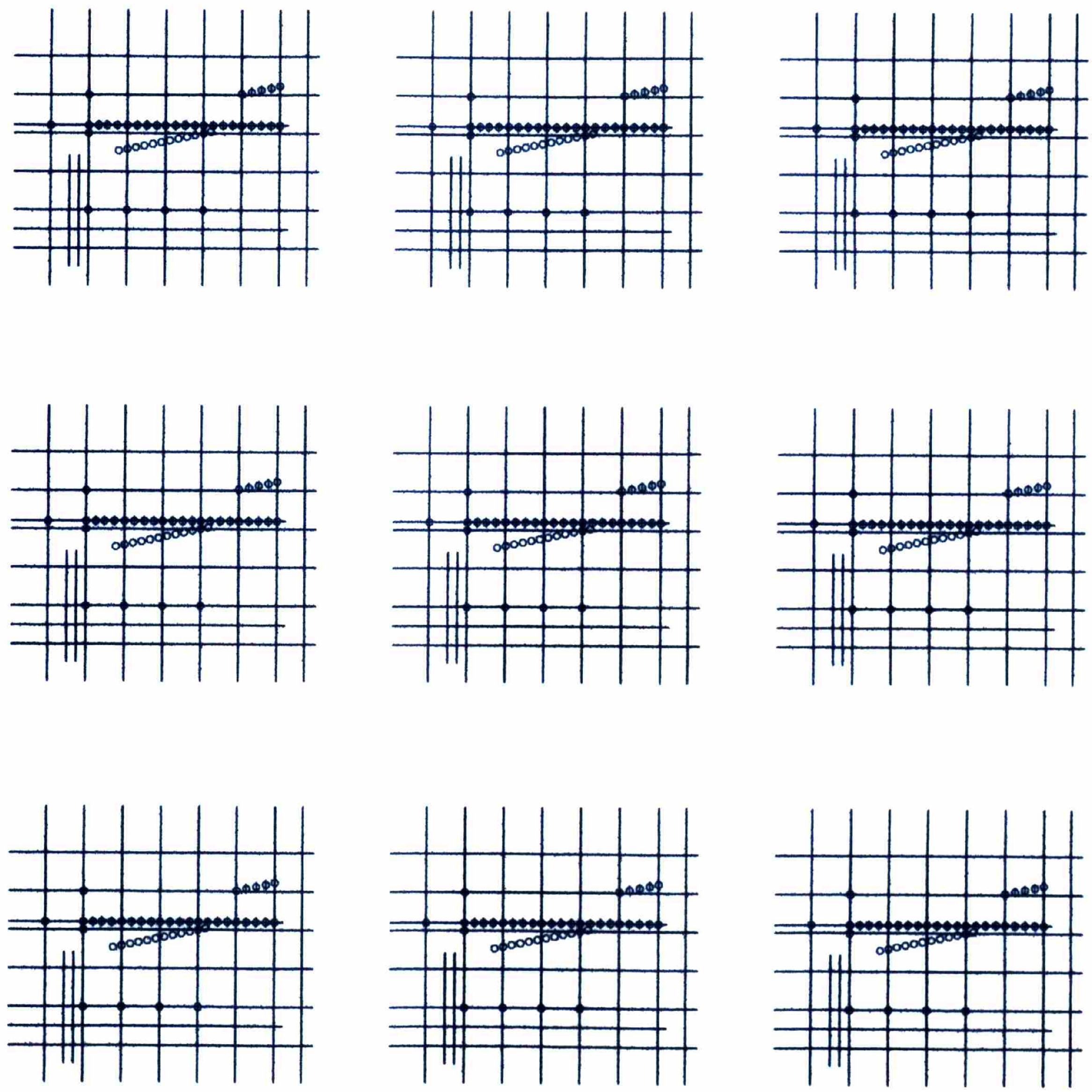
PLAN III

B



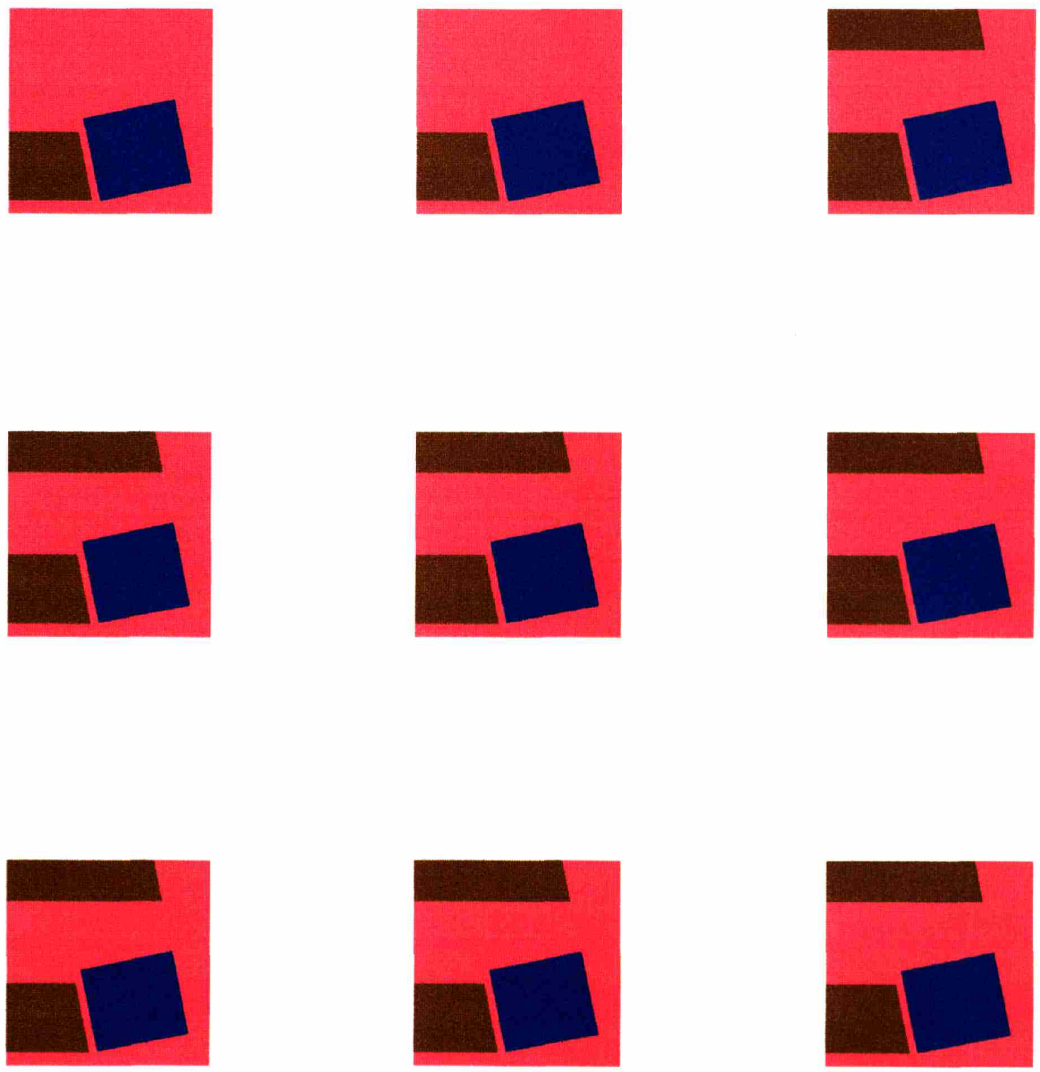
PLAN III

C



PLAN III

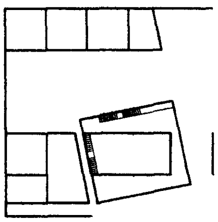
D

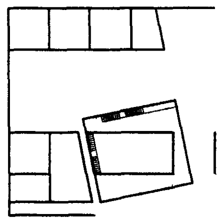


PLAN III

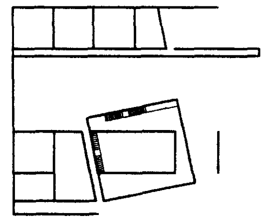
A

rule 7ii
draw
line in
layer A
over a
line in
layer B

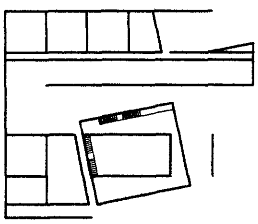




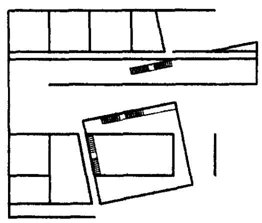
rule 7ii
draw
lines in
layer A
over
lines in
layer B



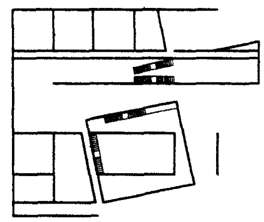
rule 7ii
draw
lines in
layer A
over
lines in
layer B

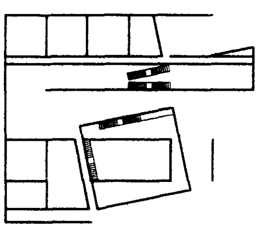


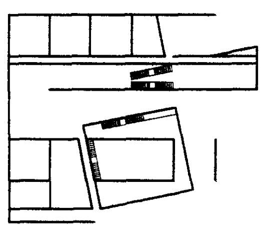
rule 15ii
draw
stair in
layer A
over
stair in
layer B



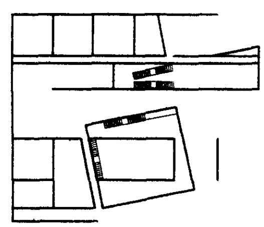
rule 15ii
draw
stair in
layer A
over
stair in
layer B





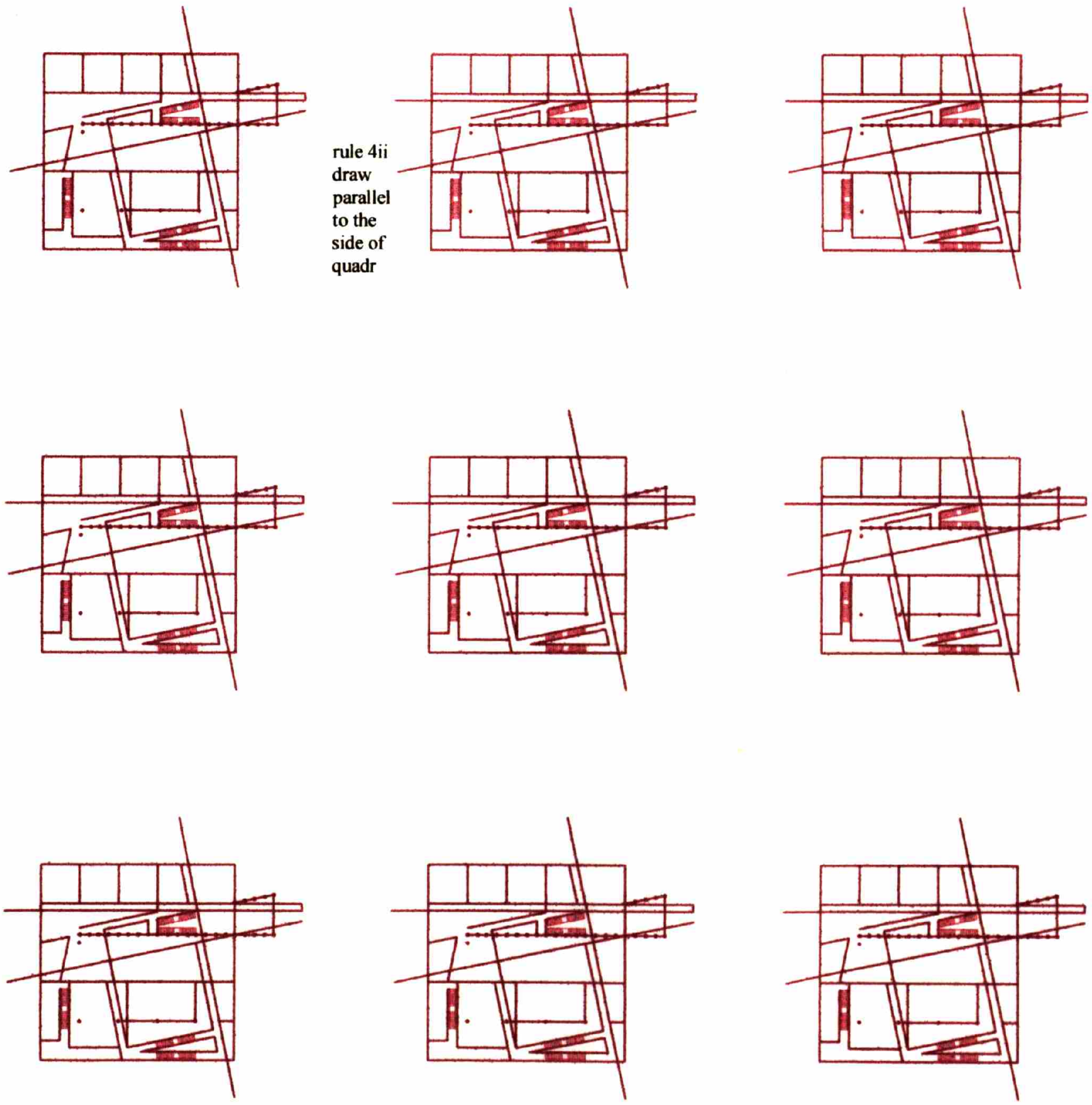


rule 7
draw
line in
layer A
over
axis
line in
layer C



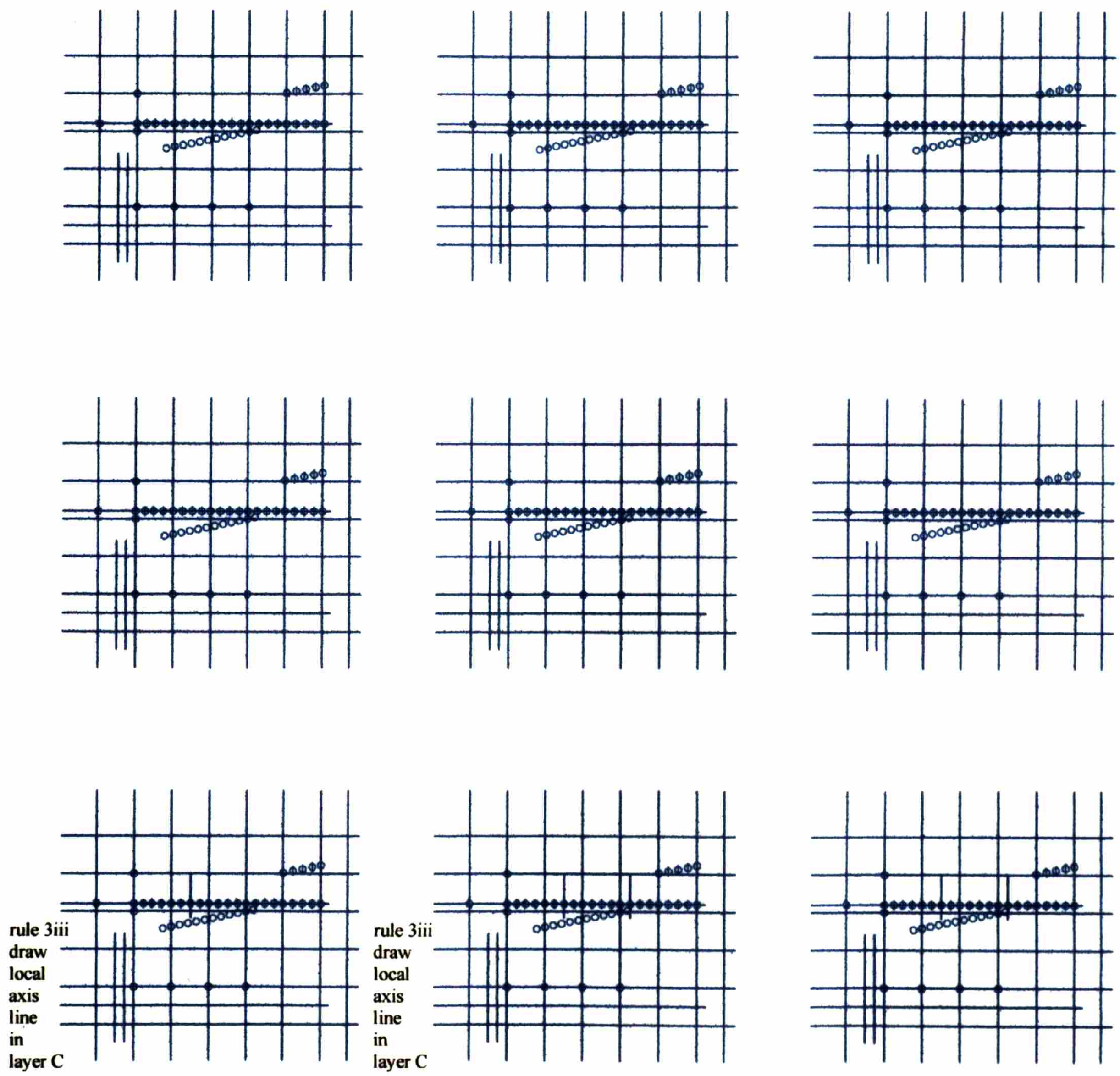
PLAN III

B



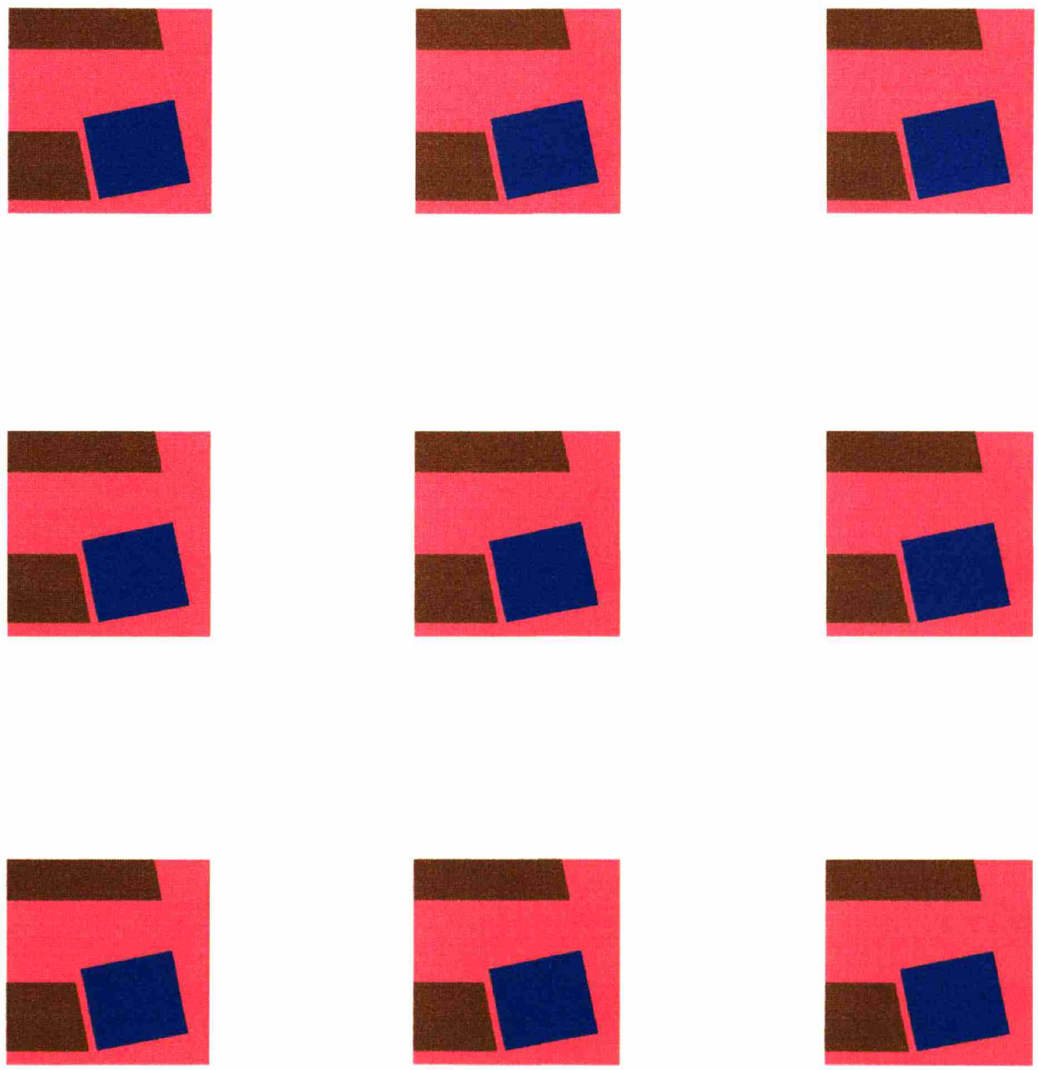
PLAN III

C


rule 3iii
draw
local
axis
line
in
layer C
rule 3iii
draw
local
axis
line
in
layer C

PLAN III

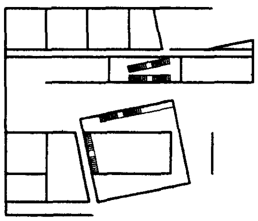
D



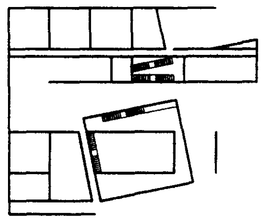
PLAN III

A

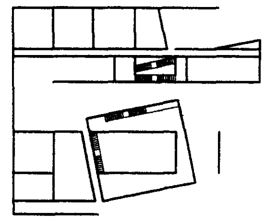
rule 7
draw
line in
layer A
over
axis in
layer C



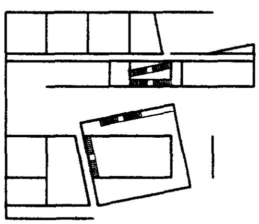
rule 5
extend
side of
quadr
in
layer A



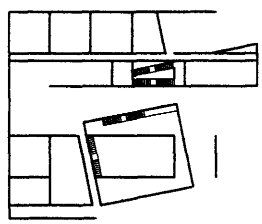
rule 5
extend
side of
quadr
in
layer A



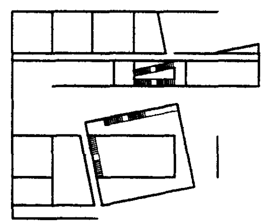
rule 5
extend
side of
quadr
in
layer A



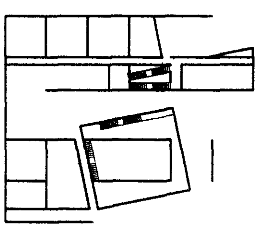
rule 2ii
union
shapes
by
erasing
their
common
bndry



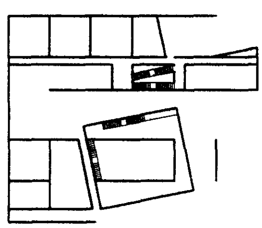
rule 2ii
union
shapes
by
erasing
their
common
bndry

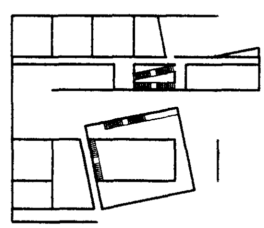


rule 2ii
union
shapes
by
erasing
their
common
bndry



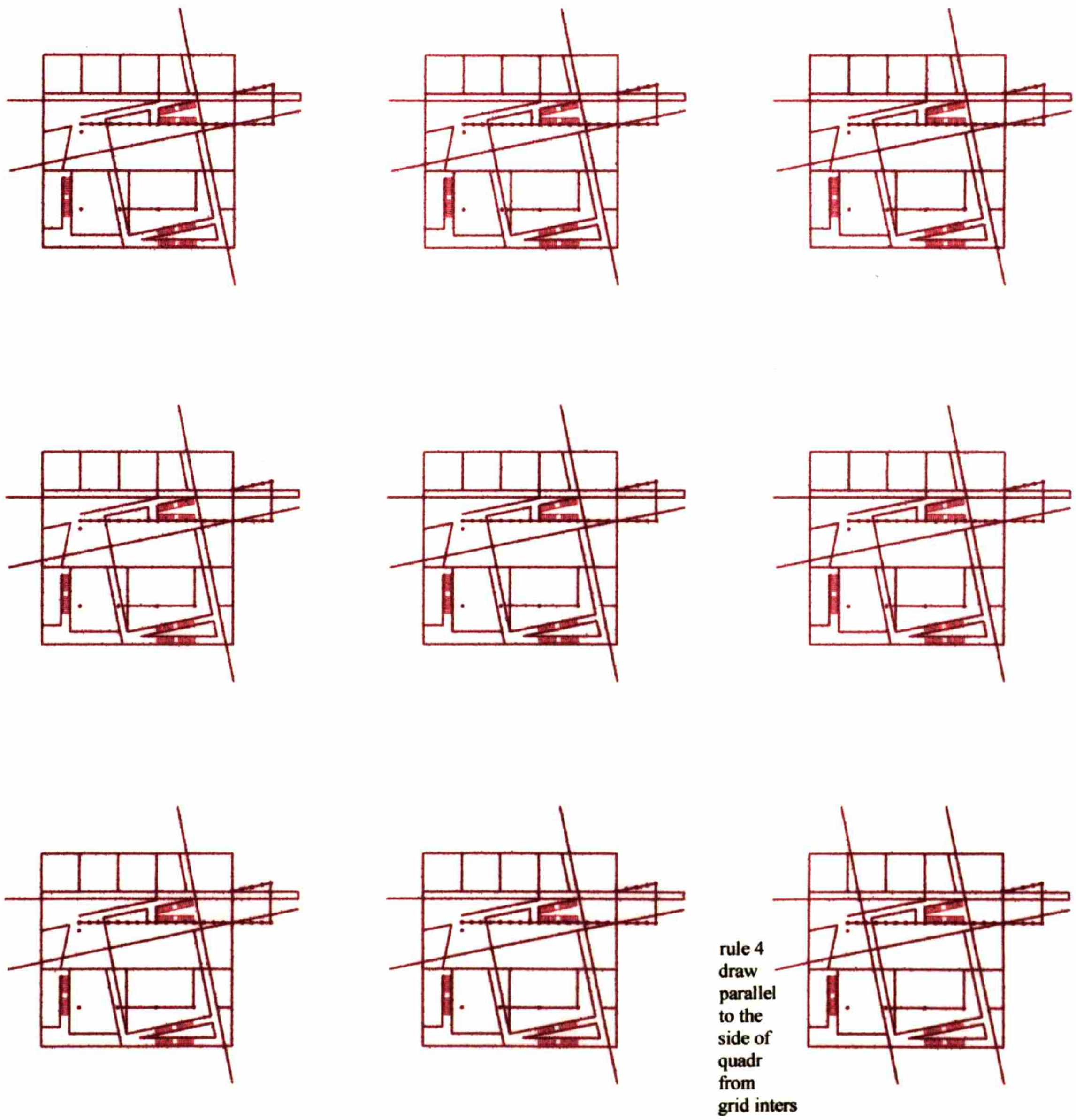
rule 2ii
union
shapes
by
erasing
their
common
bndry





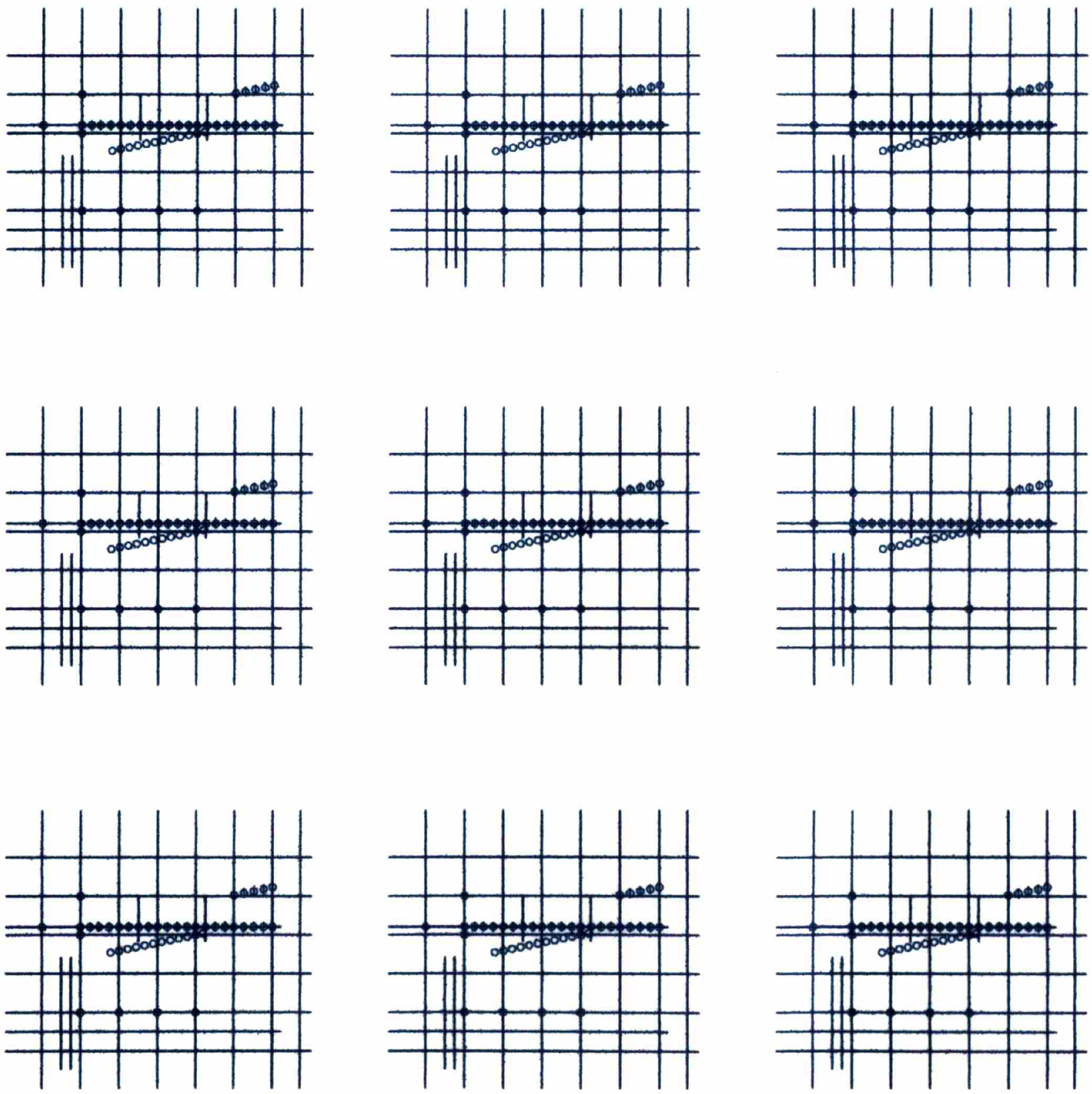
PLAN III

B



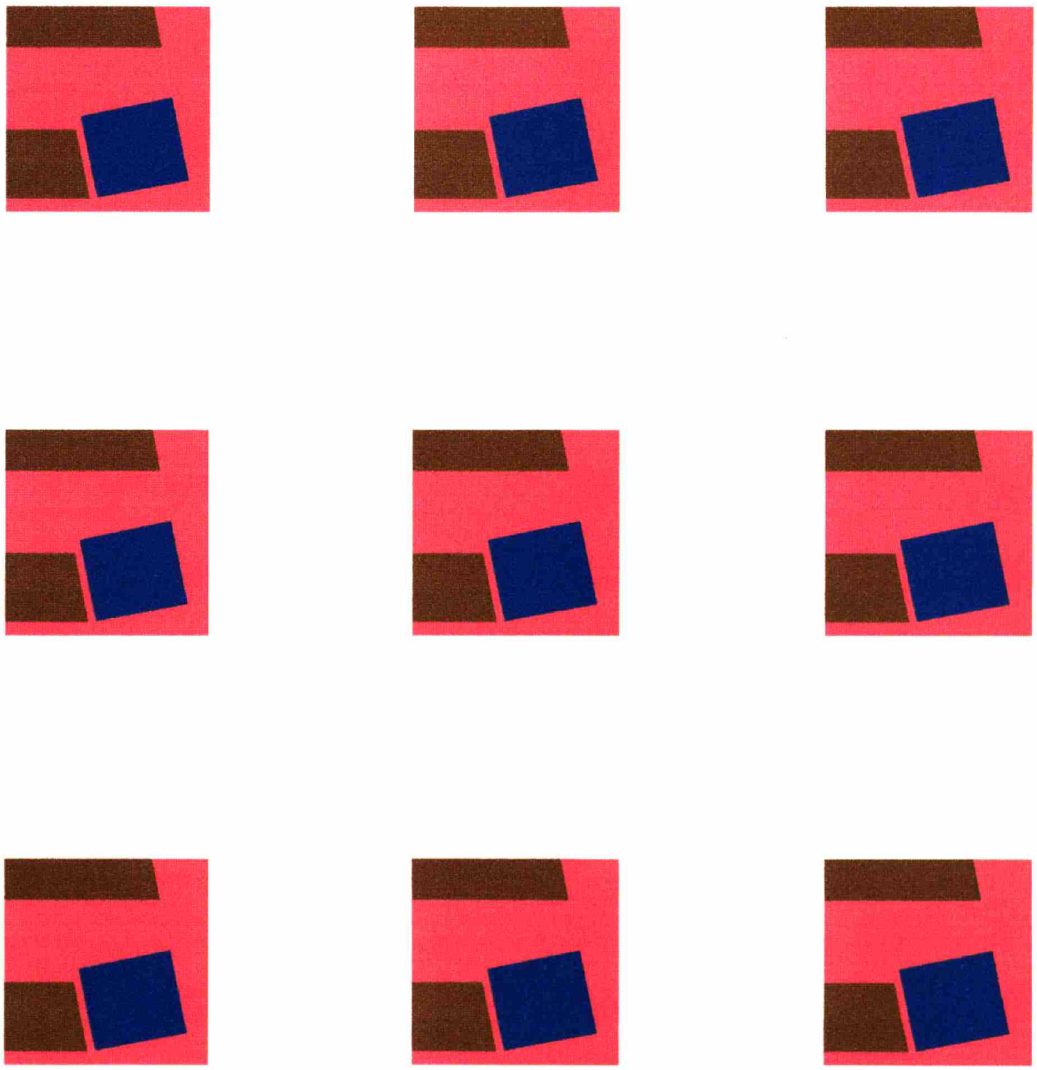
PLAN III

C



PLAN III

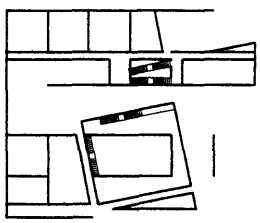
D



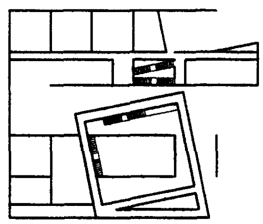
PLAN III

A

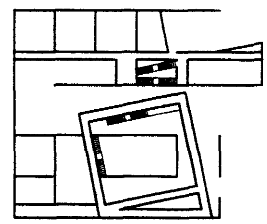
rule 1iii
draw
triangle
in A
over
triangle
in B



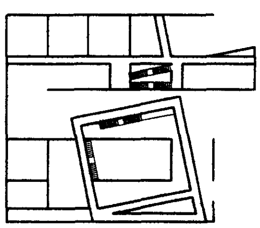
rule 1iii
draw
quadr
in A
over
quadr
in B



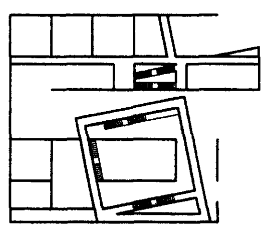
rule 7
draw
lines in
layer A
over
lines in
layer B



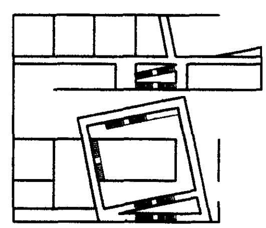
rule 7
draw
lines in
layer A
over
lines in
layer B

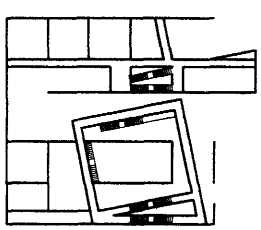


rule 15ii
draw
stair in
layer A
over
stair in
layer B

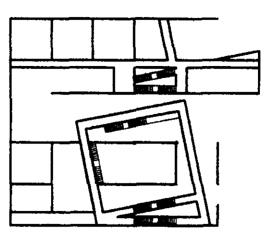


rule 15ii
draw
stair in
layer A
over
stair in
layer B

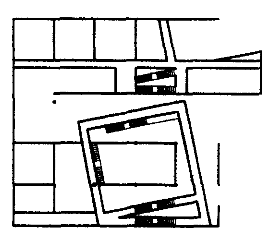




rule 21ii
draw
clmns
in
layer A

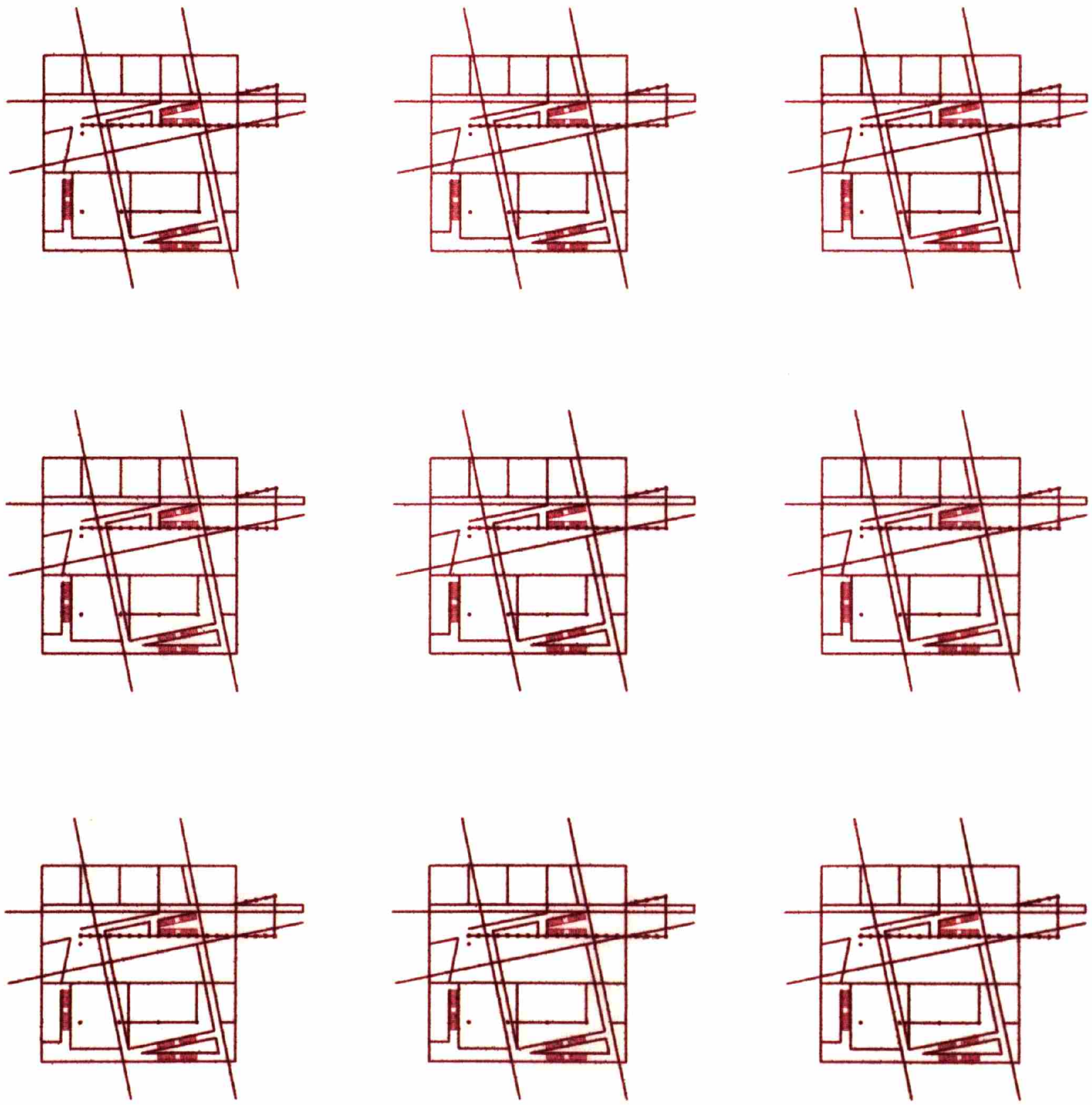


rule 21
draw
clmns in
layer A
over
clmns in
layer B



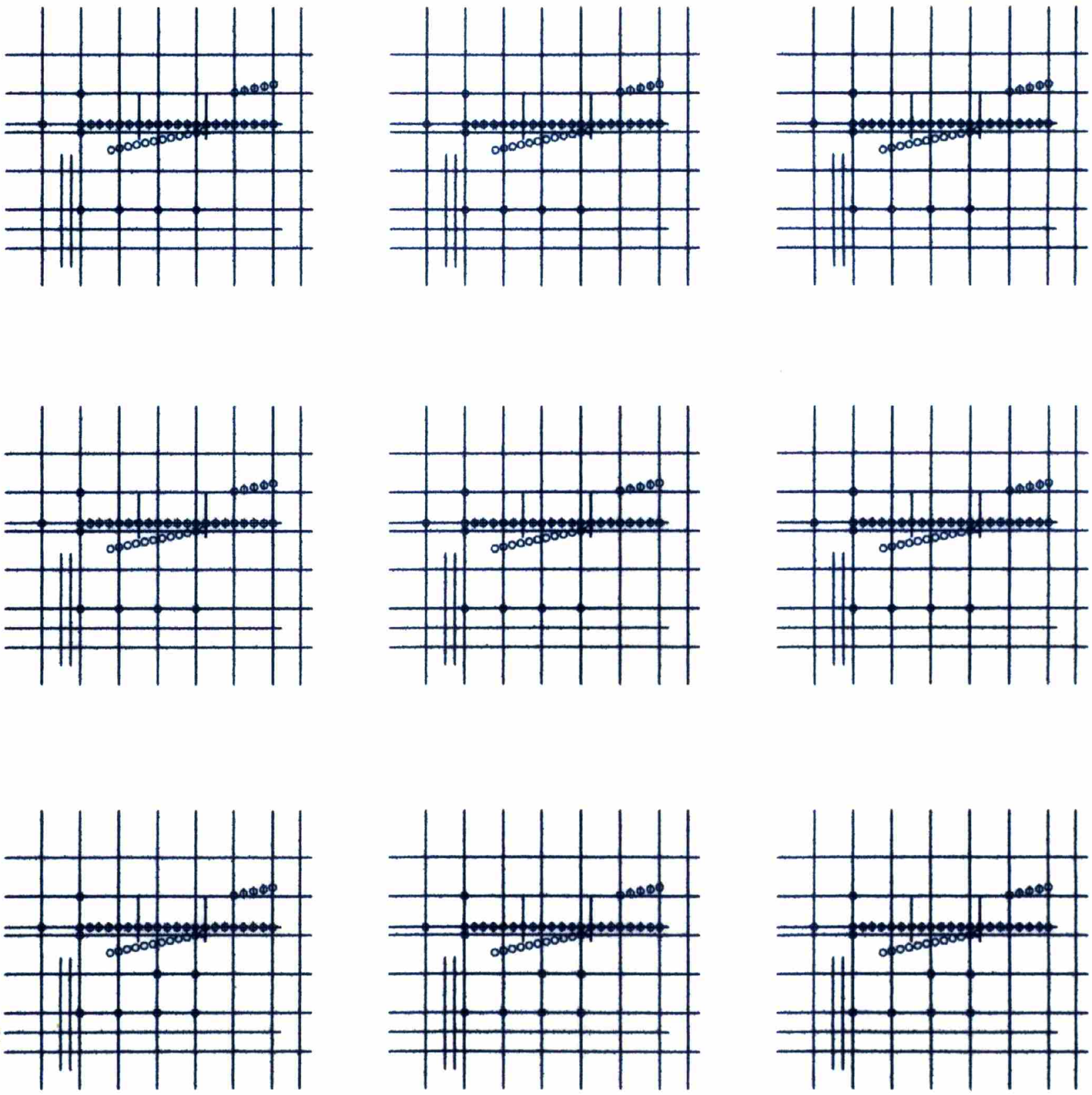
PLAN III

B



PLAN III

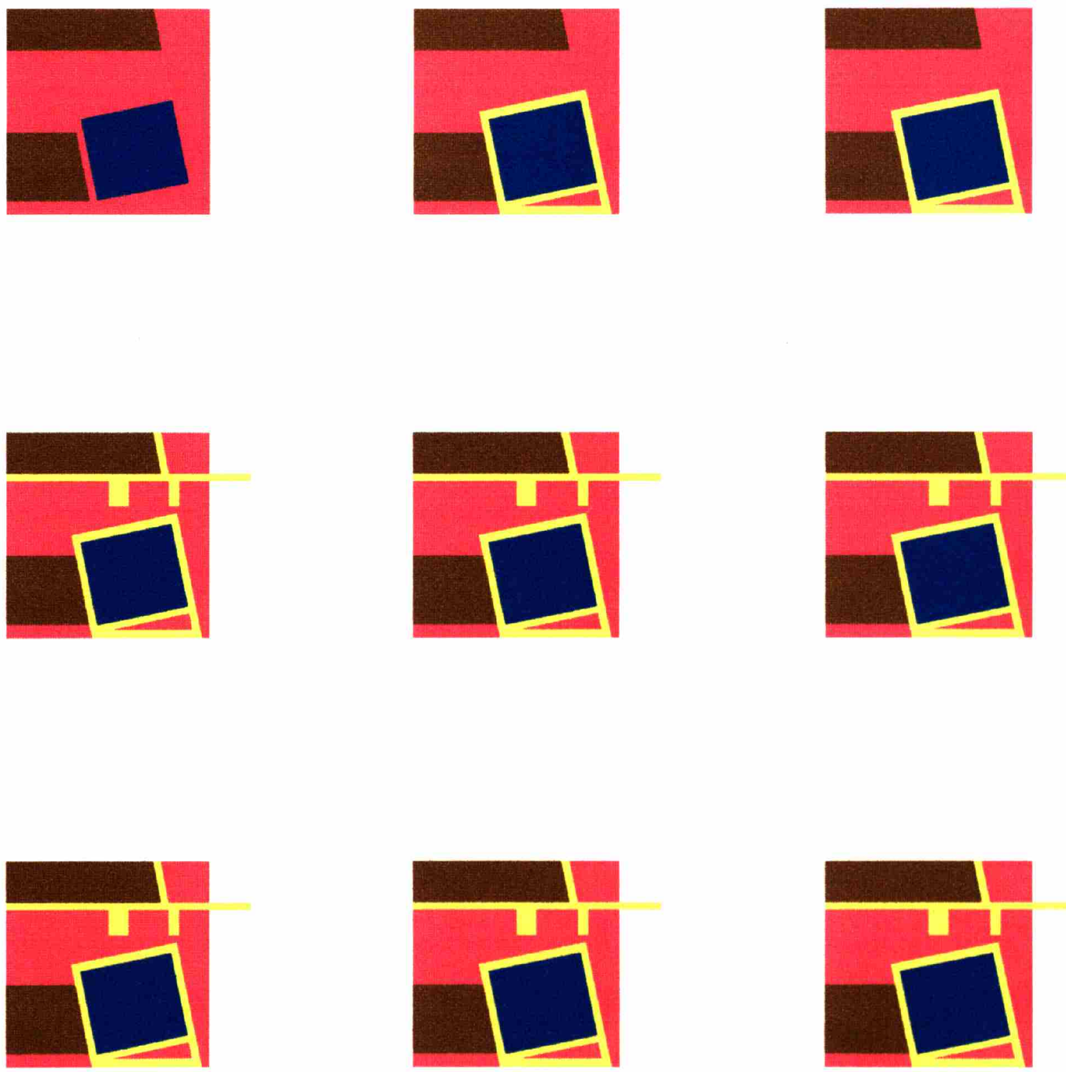
C



rule 6
draw
clmns
in
layer C

PLAN III

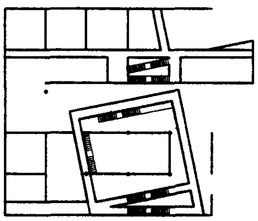
D



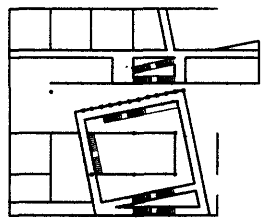
PLAN III

A

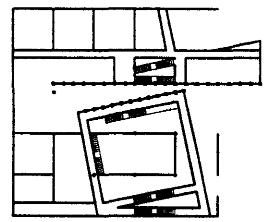
rule 17
merge
clmn
in wall



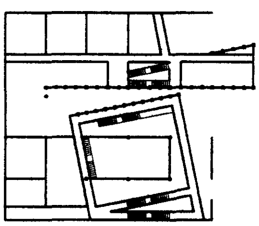
rule 19
draw
clmns
in
layer A

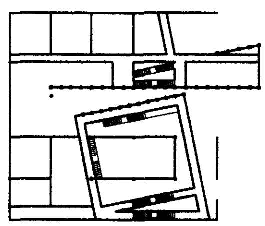


rule 19
draw
clmns
in
layer A



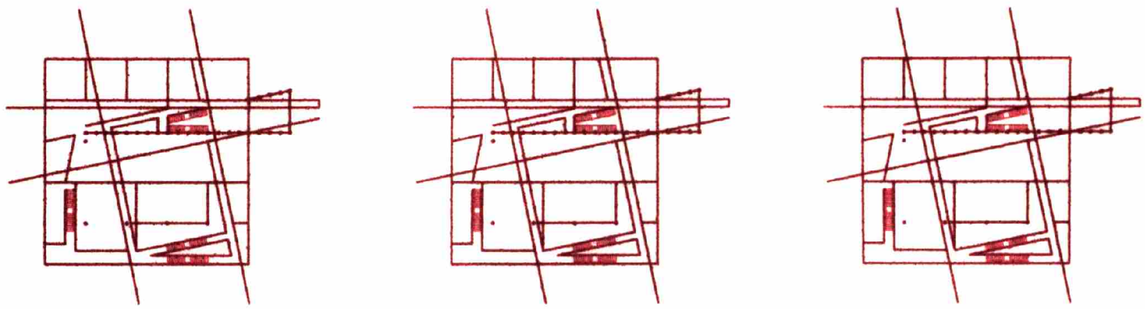
rule 19
draw
clmns
in
layer A

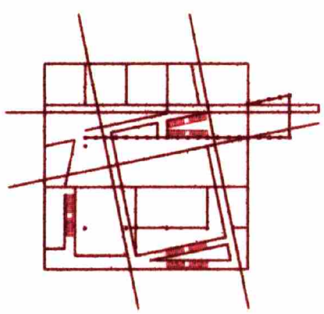




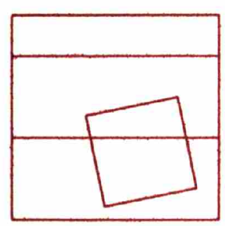
PLAN III

B



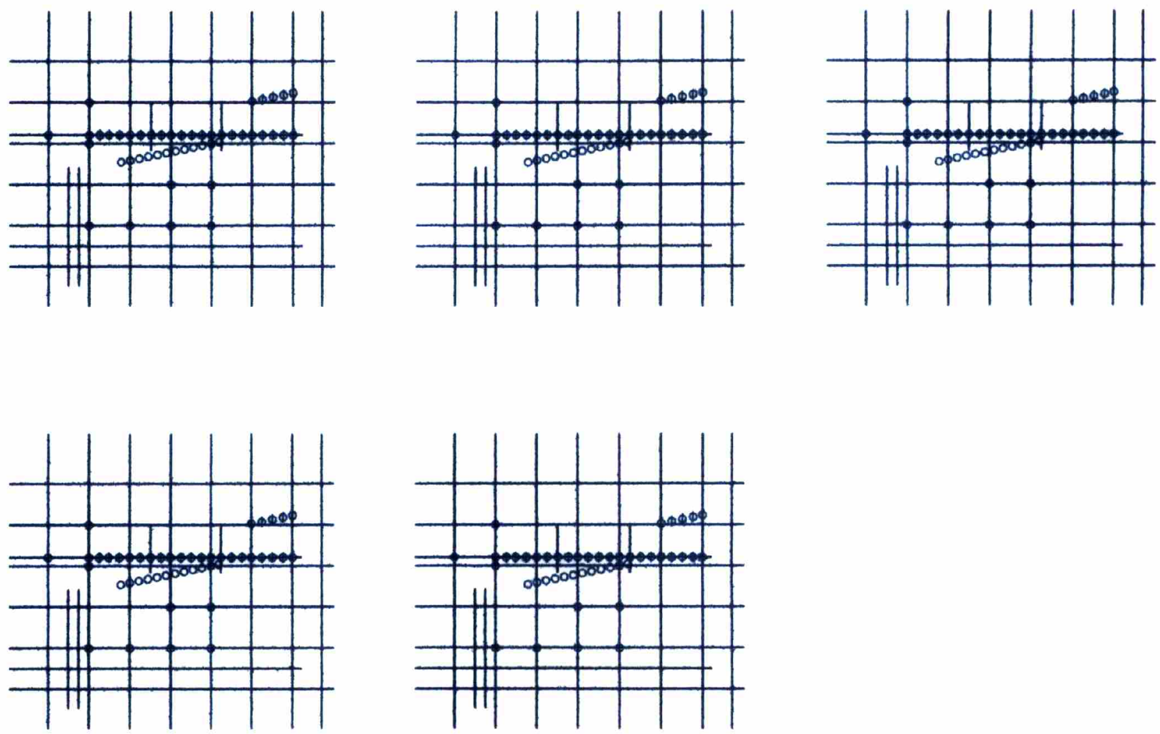


rule 23
erase
line
extens
in
layer B



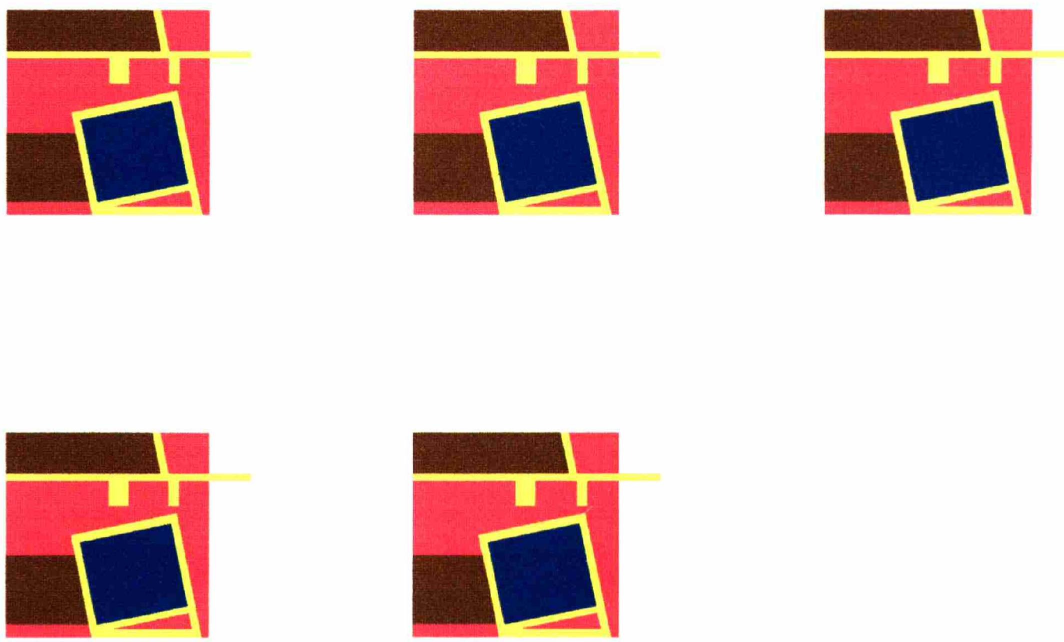
PLAN III

C



PLAN III

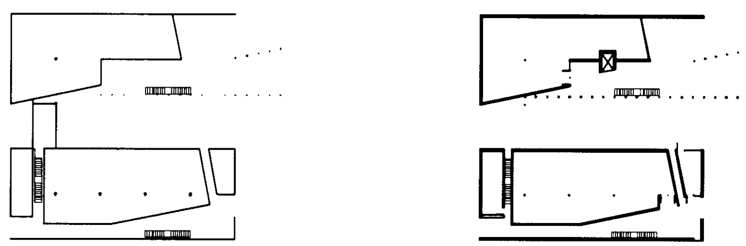
D



PLAN III

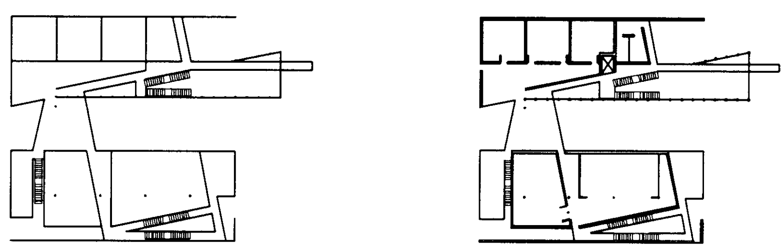
6.3. RESULTS – ILLUSTRATIONS

The next three pairs of illustrations present the derived plans I, II, III in two different stages of their development: as derived by the rules (left), and after the addition of wall-thicknesses, door openings, and details (right). Plan I, corresponds to floors 1 and 2,



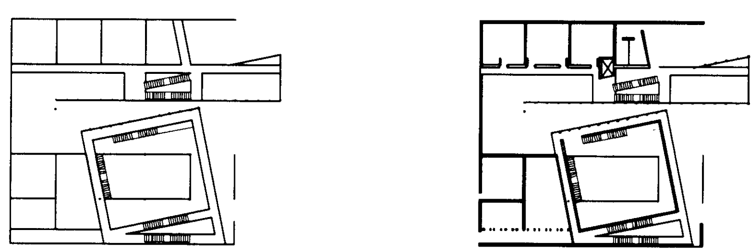
PLAN I

Plan II, corresponds to floor 3,



PLAN II

Plan III, corresponds to the floors 4-8



PLAN III

The three illustrations (*Figure 19*) present the derived plans I, II, III in axonometric. Some tectonic details such as columns, stairs, and wall-thicknesses, are included. The next general axonometric (*Figure 20*) shows the distribution of floors in the overall composition.

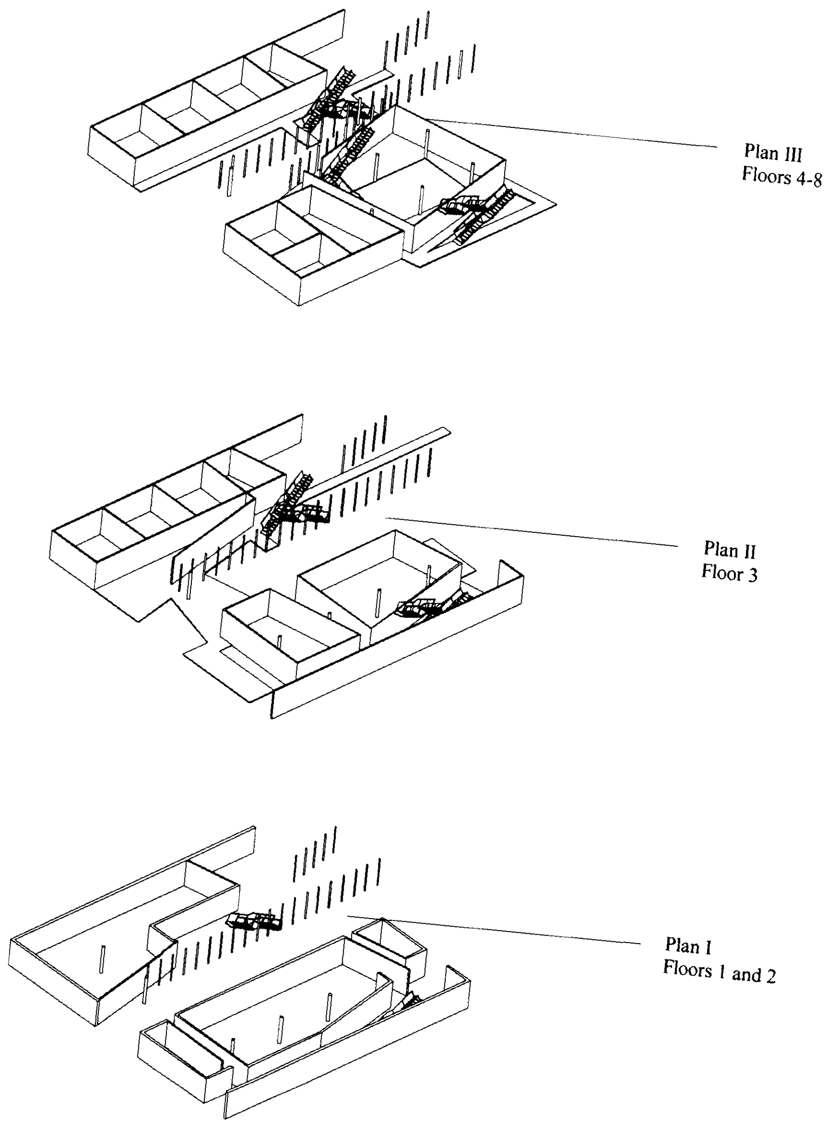


Figure 19. The output descriptions of the plans I, II, III in axonometric

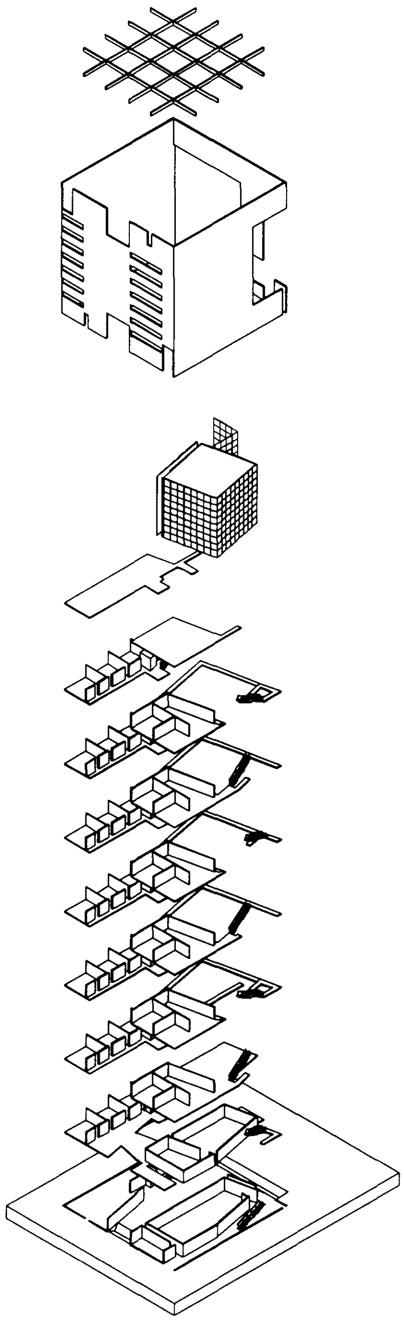
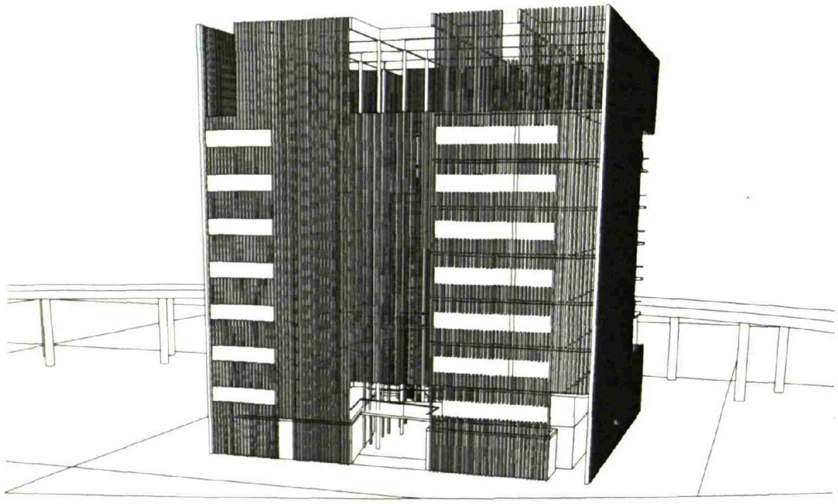
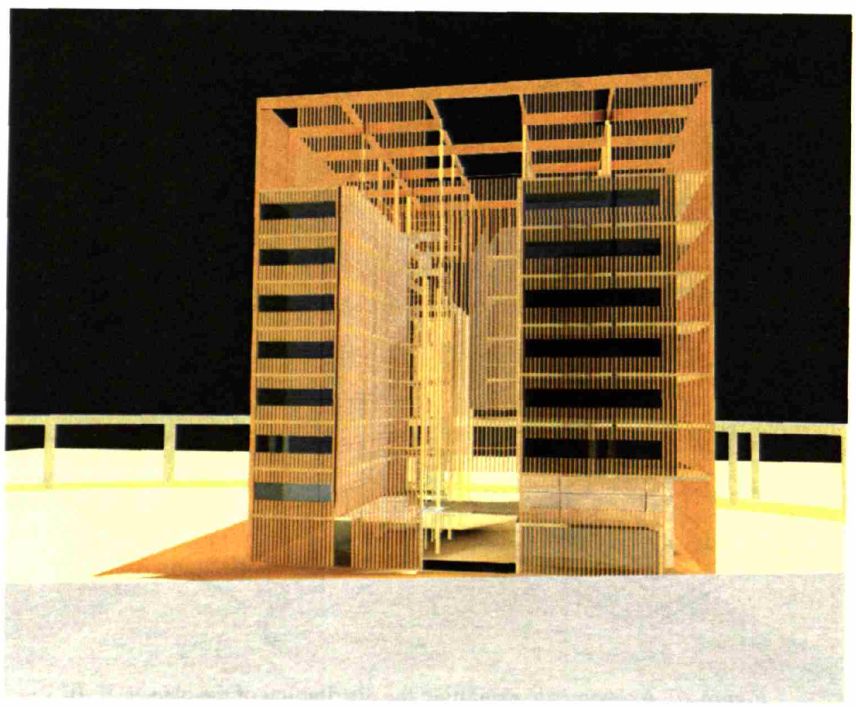
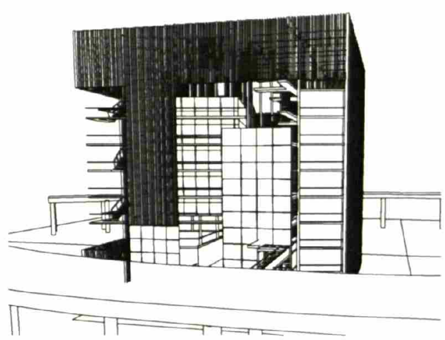
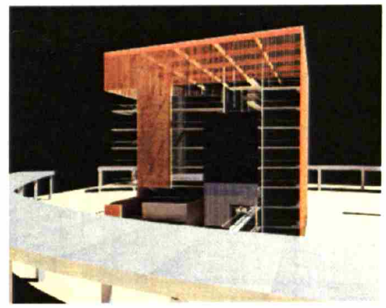
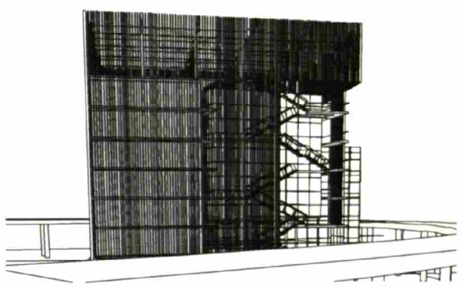
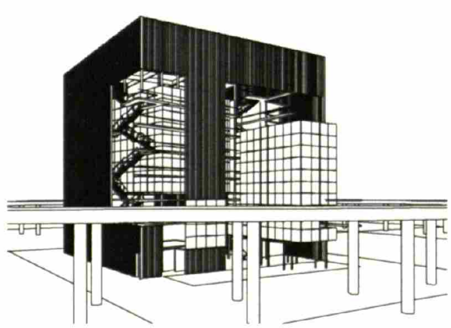
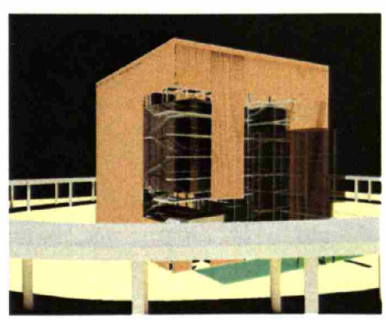


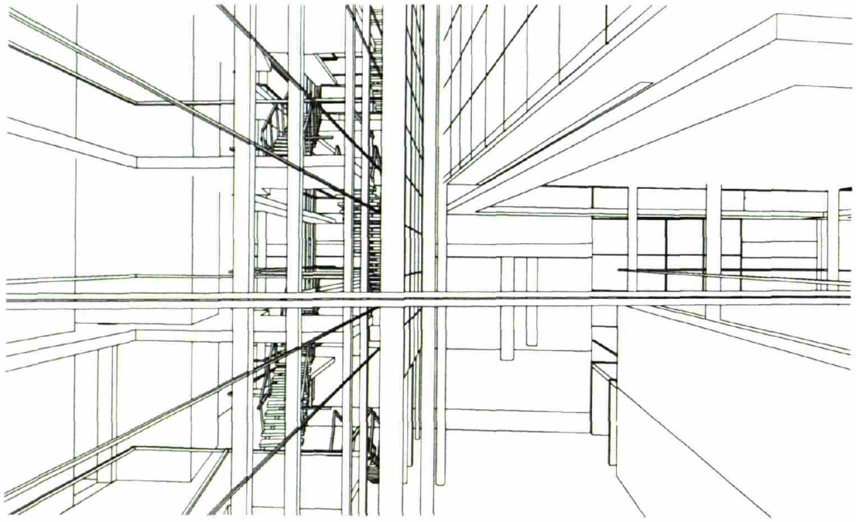
Figure 20. Axonometric exhibiting the distribution of the plans I, II, III

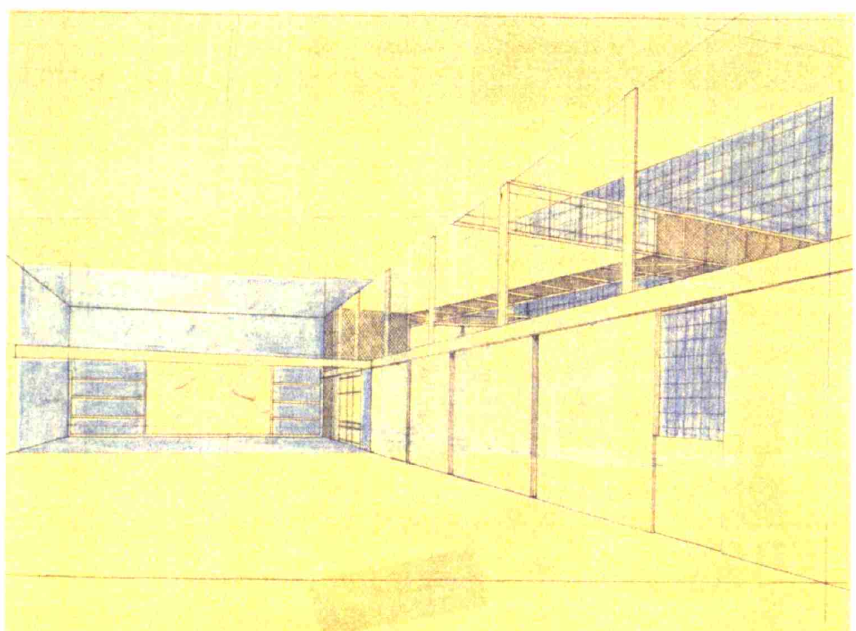


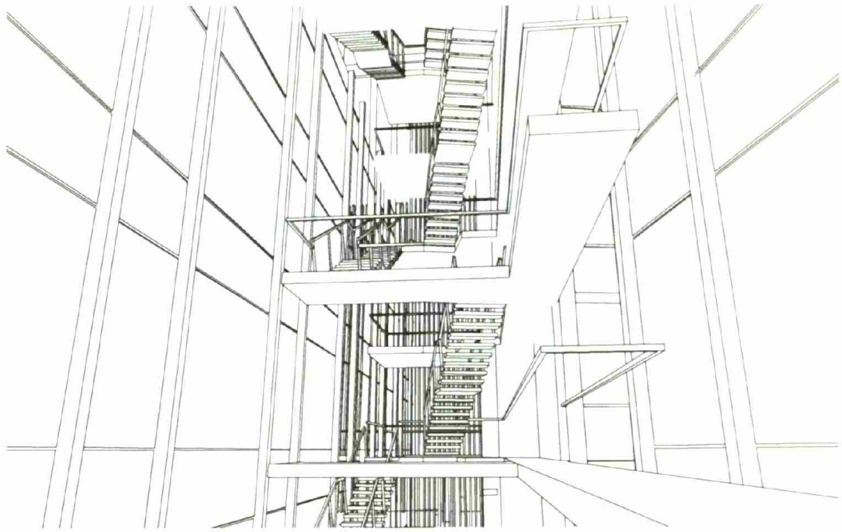


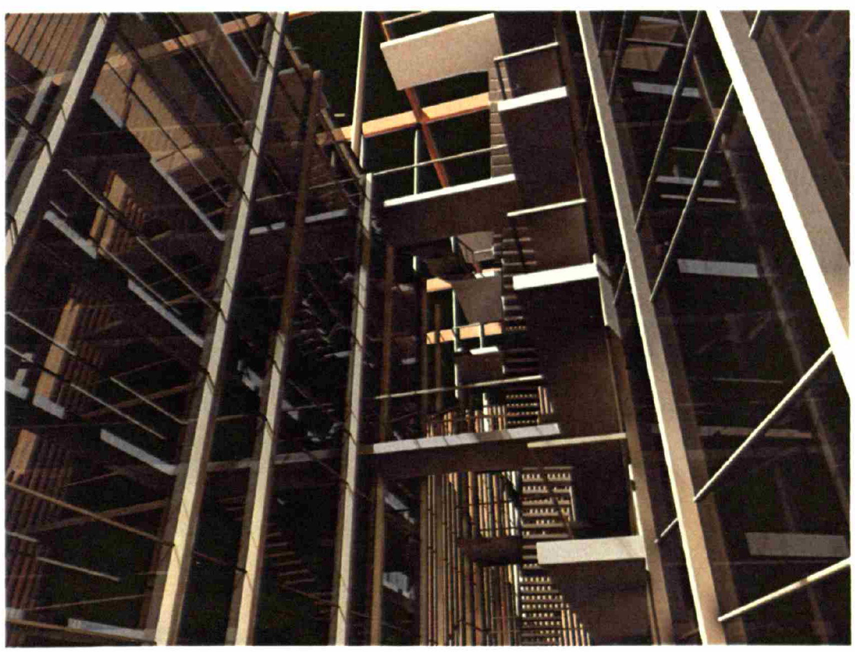




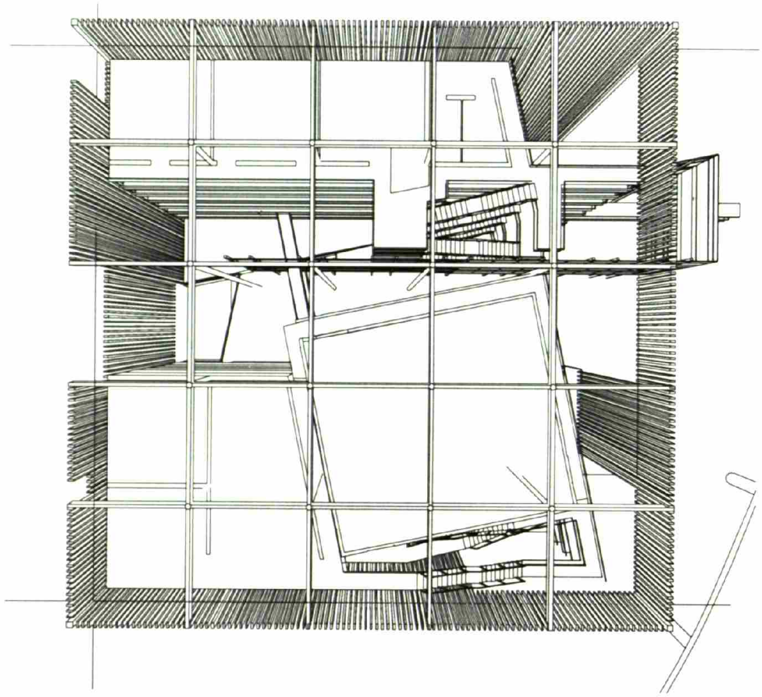



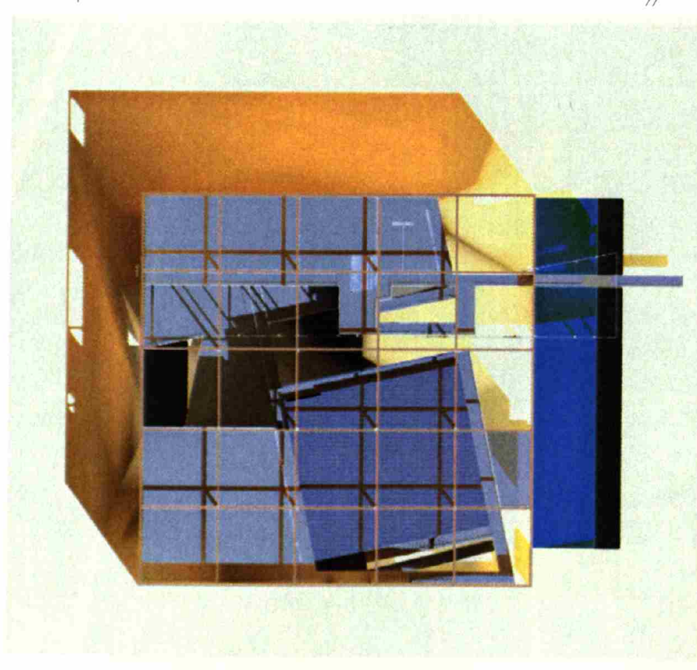


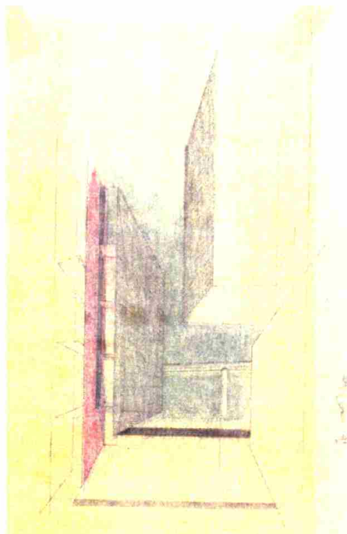


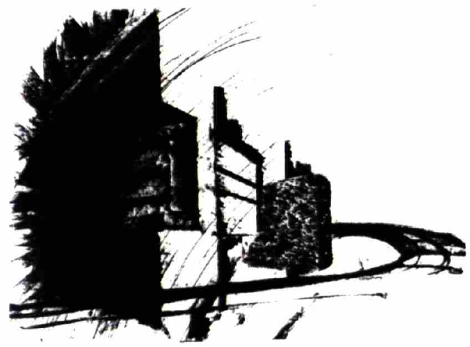


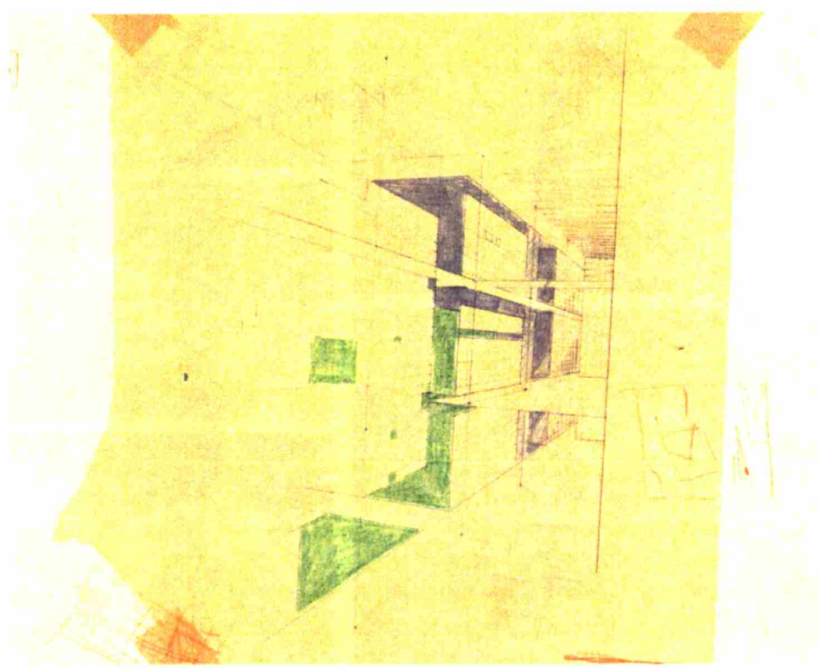


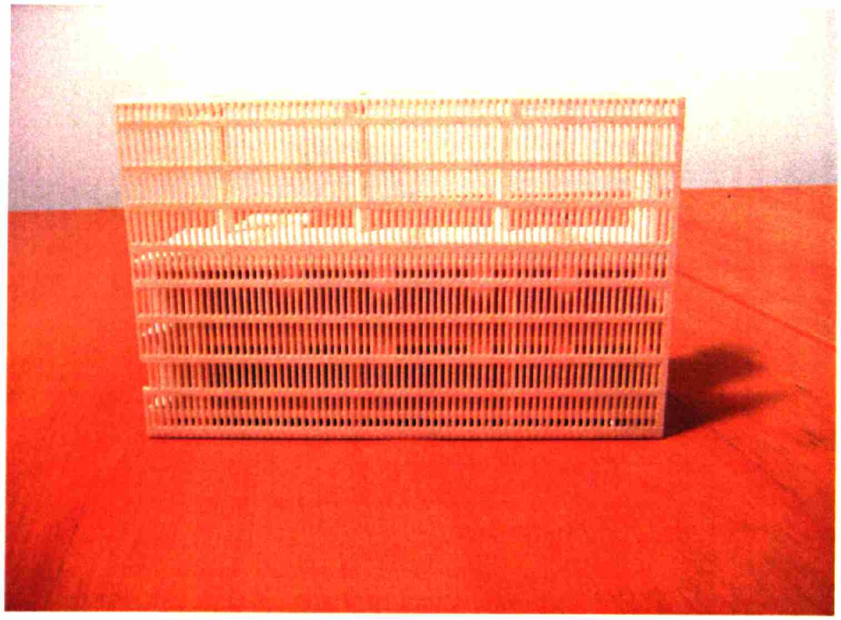


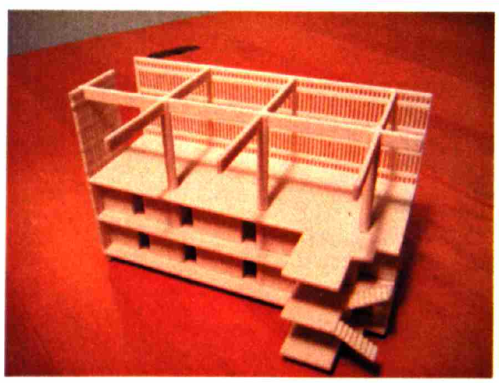


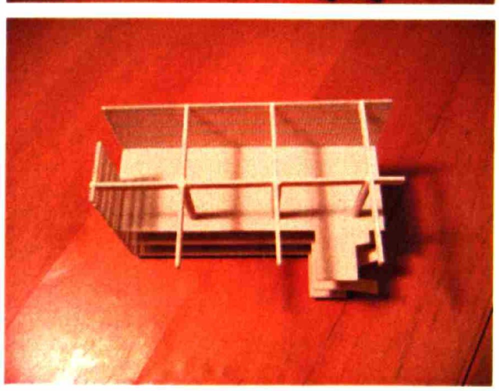


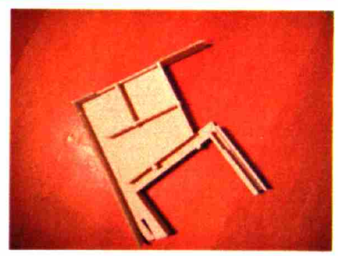


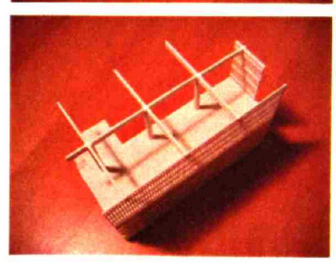


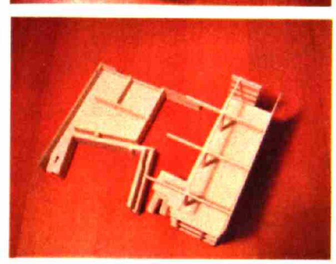












7. Discussion

A rule-based approach to the generation of plans from scratch was presented in this study. The making of the plans for an office building was used as an example. The motive was to create a framework that captures an intuitive design process. The approach was based on the view that designers use design concepts to express their intentions about space, to solve design problems and to form designs.

In the presented example, the design search proceeded from a potential "whole" (design concept) to the definition of the "parts" (rooms and spaces). The presentation was retrospective, because an original design concept was provided by the designer of the project, and, descriptive, because the computational process was used to examine the consequences of the concept, without replicating the exact actions of the designer.

It was suggested that the analysis of the programmatic and other information in a design problem relies on relative criteria, and provides certain facts for the design. Designers establish relationships among these facts intuitively, at the early stage of the first contact with the problem. The relationships are expressed in the form of design concepts, or hypotheses. A design concept (or design hypothesis) is produced through synthesis of the provided information. It is not repetition of the given facts for the object under consideration, but it suggests a new meaning for it.

In the example, the designer proposed the next conceptual narrative: *The building is a cubic box occupying the entire usable area of the site. A second small cubic box placed within the first accommodates the core creative activities of the firm. Administrative rooms are placed on the perimeter of the large cubic container. A public lobby occupies the central, ground floor area.*

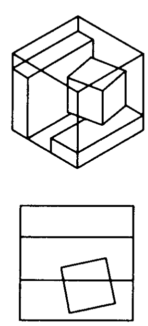


Figure 21. A depiction of the design concept in axonometric and plan

The design concept provided a general law for forming rule schemata. The outcome was the construction of a system governed by rule schemata, in which the actions of the designer were expressed.

Further, it was suggested that in practice each design description is the result of synthesis of many partial descriptions. The design process involves a "pile" of information, where each stratum may contribute something to the design. These superimposed descriptive layers are composed into a new whole, within the framework of the design concept.

In this case study, four superimposed sheets of tracing paper A, B, C, D, were used to produce plans. The sheets A, B, C, D represented a simplified version of the design process, where a larger (finite) number of tracing sheets are used in the production of a single description.

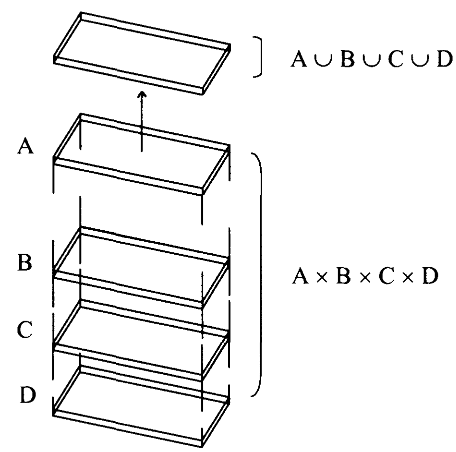


Figure 22. A pile including four partial descriptions A, B, C, D was used in the production of a single plan.

The layers A, B, C included lines and labeled points, and layer D areas and labeled points. The description A x B x C x D was formed in the product $<U_{12}\ V_{02}>$ x $<U_{12}\ V_{02}>$ x $<U_{12}\ V_{02}>$ x $<U_{22}V_{02}>$.

The novelty of the approach is that the four partial descriptions A, B, C, D do not evolve as individual, parallel descriptions that simply inform each other. Further, the associations among A, B, C, D are not predetermined. And, none of the A, B, C, and D is individually preserved at the end. The four partial descriptions develop unpredictable associations that conclude to a single composition and comprise fragments from all four.

Three plans P_I, P_{II}, P_{III}, were generated. Blueprints of the descriptions A_I x B_I x C_I x D_I, and A_{II} x B_{II} x C_{II} x D_{II} were used as initial shapes in the generation of the plans P_{II} and P_{III}, respectively, as it is shown in Table 10.

TABLE 10. Blueprints of the descriptions A_I x B_I x C_I x D_I, and A_{II} x B_{II} x C_{II} x D_{II} were used in the generation of the plans P_{II}, P_{III}, respectively.

DERIVATION I	DERIVATION II	DERIVATION III
$(A_I \times B_I \times C_I \times D_I) \Rightarrow P_I$		
copy $(A_I \times B_I \times C_I \times D_I) \Rightarrow$	$(A_{II} \times B_{II} \times C_{II} \times D_{II}) \Rightarrow P_{II}$	
	copy $(A_{II} \times B_{II} \times C_{II} \times D_{II}) \Rightarrow$	$(A_{III} \times B_{III} \times C_{III} \times D_{III}) \Rightarrow P_{III}$

The proposed rule schemata were general compositional decisions. They described general modes of action that emerged from the conceptual schema, and originated in previous experience. Gradually the rule schemata became more specific, and were organized to achieve better specified goals. The heuristics of the search process were organized in three categories: *formation*, *transformation*, and *refinement*.

Rule schemata of *formation*, generated the *parti*. The *transformation* of the *parti* was a composite process. The further consequences of the *parti* were examined with the aid of additional rule schemata, against functional, programmatic, stylistic, and other criteria. First, it involved elaboration through addition of new forms, (or scaling, reflection, translation, etc). The key argument for this set of actions was "*draw* _____", where _____ was substituted by a shape. Second, a designer's ability to interpret and choose among infinite parts was expressed through another set of rule schemata. Key argument for this set of actions was "*select* _____ *from* _____". Both these general classes of actions of elaboration and interpretation were used repeatedly. Finally, the rules of *refinement* added some details.

TABLE 11. The heuristics of the process involved three general categories of rule schemata and rules: formation, transformation, and refinement.

PROCESS		
formation	transformation (*elaboration* $\Leftrightarrow$ *interpretation*)	refinement

Retrospectively, the available rule schemata could be organized in alternative ways. For example, devices Grammar I, Grammar II, Grammar III, could be responsible for generating the plans P_I, P_{II}, P_{III}, respectively.

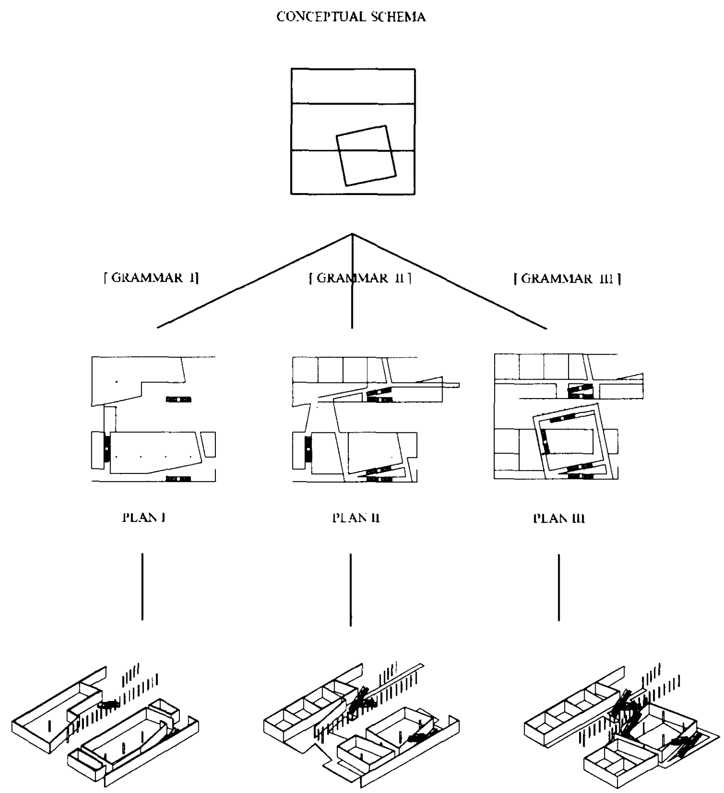


Figure 23. The plans I, II, III emerge from the same conceptual schema. The used rule schemata can be retrospectively organized in three grammars I, II, III

Overall, the example presented the construction of a rule based approach for the generation of plans. Rules and rule schemata comprised the conditions that required to be attended to in the specific problem, by a specific designer. The rule schemata derived from an initial design concept, or hypothesis, and from observation, and previous experience.

In general, the consequences of the design concept need to conform to the existent conditions. Therefore, the designer has to determine the appropriate rule instances for each purpose. The complete description of the design process consists of rules together with the propositions that justify their selection. These justifications were not provided in this study.

A design process involves of the definition of the rules for the production of the conditions and the effects that a designer aims at creating. This process brings together the most diverse and remote parts, and facts, related to the production of different and heterogeneous conditions.

Practitioners consider rules of art as provisional, as opposed to logical or mathematical rules, which are considered permanent. In this study it is shown that rules, if nothing else, provide a way to bring ourselves in the position to "see" more successfully, by making particular actions explicit, and by connecting them in logical chains. No calculus can decide a design problem.

The selection of rules, in relation to a design concept requires observation and judgment. It is related to the visual and general education of a specific observer. Rightly Mill (1872) identifies observation with invention: "*The rules of observation do not teach how to do the thing, but how to make ourselves capable of doing it*". Good observation consists of the ability to identify in the given situation the parts that serve the accomplishment of an objective. And judgment is the ability to decide intelligently among the several possible alternatives.

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VII. IDEA AND PHENOMENA

A rule based interpretation of Simmons Hall undergraduate residence hall

Abstract. A computational interpretation of the design concept of porosity as it was implemented for Simmons Hall undergraduate residence at MIT is presented.

1. Introduction

A rule based design approach must be able to capture a spatial concept with general rule schemata, and eventually with rules. This chapter presents a computational interpretation of Steven's Holl design concept for Simmons Hall undergraduate dormitory at MIT. The objective of the chapter is to test the ability of rule schemata in expressing the design concept of a mature, recognized architect, like Steven Holl.

The material for this chapter emerged out of three interviews with the project architect Timothy Bade and two meetings with the architect Steven Holl. I am indebted to both of them. Without their contribution this chapter would not be possible. The illustrations include original sketches, working drawings and models from all the stages of the design process of Simmons Hall. This material was also presented publicly in an exhibition, at the opening of Simmons Hall residence, at MIT, in February 2003.

The computational approach of this chapter follows the design developments in the drawings and the models. The presentation is retrospective. The proposed rule schemata capture the evolution of the design concept and not the actual steps of the design process.

The motivation for this chapter was that Steven Holl's design approach emphasizes the conceptual basis of architecture. Holl resists conforming to the known building typologies, and proposes a shift from the typological to the topological investigation. He proceeds through the development of open-ended theoretical frames that move independently of the existent morphologies, or typologies. Holl (2000, p. 174) explains: "*An absolute* *exists in the specific. Site, geometry, program, circumstance, and materials are forged into spaces by an idea. A unique site and circumstance requires a specific idea, a "limited concept". More than just a verbally expressed idea, a limited concept sets a manifold relation. It refers to a non-hegemonic local stability. It is semi-hierarchical. A limited concept states an ideal. An ideal aspiration in architecture is not eclectic. In the mind, an ideal is seen; it is a kind of perfection. It is closer to classical than eclecticism. However, a limited concept thrives on going forward into the unknown, embracing doubt*".

The notion of a "*limited concept*" suggested by Holl coincides to that of a "*design concept*" that was presented in the previous chapters. Both notions involve imagination and point to a possible scenario of action. In both cases, a "*concept*" results from synthesis and not analysis of the provided information: site, program, circumstance etc. A "*concept*" works as a hypothesis that establishes novel relationships among the given facts. The authority of a "*concept*" is limited to a specific designer and a specific problem. It provides direction for action without claiming general validity.

For Holl, the investigation of a concept becomes a vehicle for the elaboration of novel solutions to given problems. Extending Merleau-Ponty's philosophical thoughts, Holl accepts that "*human environments include patterns, 'lines of force' and – if we can read them – meanings*". One may succeed in developing novel solutions if one succeeds in focusing on some characteristic that reveals the uniqueness of a given situation. The built "phenomenon" occurs as the manifestation of this investigation that deals partially with the circumstantial and partially with the absolute character of things. Holl continues: *"The essence of a work of architecture is an organic link between concept and form. Pieces cannot be subtracted or added without upsetting fundamental properties. The concept – whether a rationally explicit statement, or subjective demonstration – establishes an order, a field of inquiry, a limited principle. Within the phenomena of experience in a built construction the organizing Idea as a hidden thread connecting dispersed parts with exact intention, although the experience of a semi-transparent plane of glass defining a space with a glow of light presents a sensory experience irreducible to a state concept this in-expression is not a gap between concept and phenomena, but the range or field of various conclusions intersect. The intertwining of Idea and Phenomena occurs when a building is realized".*

This chapter examines the concept of "porosity" that Holl and his team developed for Simmons Hall. Porosity, which is an attribute of biological or organic bodies, is treated by Holl and his team as a compositional principle that generates architectural results. The concept of porosity is approached here computationally. It is seen as a basis for the invention of spatial relationships and rules able to produce compositions with certain properties.

2. The Program

The program of Simmons Hall at MIT called for an undergraduate residence for 350 students located on a rectangular site on the 2100-foot strip of Vassar Street. The requirements of the program included single and double rooms for upper-class and freshmen MIT students but also a variety of public spaces for common activities such as dinning hall, kitchens, study rooms, computer cluster and amphitheater.

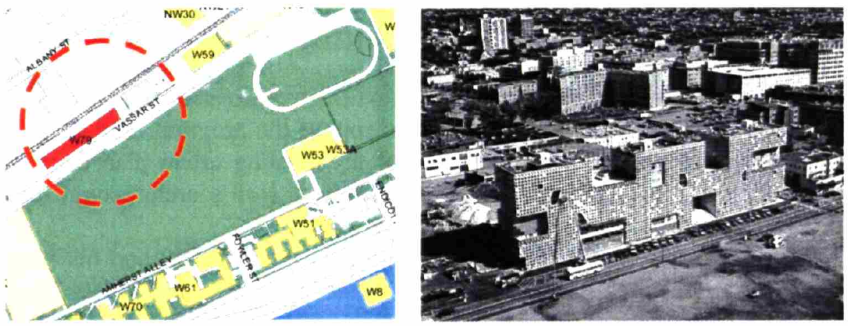


Figure 1. Simmons Hall undergraduate residence: Map and aerial view of the site

Apart from 350 beds (95 double rooms, and 155 single rooms) the proposed dormitory included amenities such as a 125-seat theater, a night café, a large dining room at the street level, a country kitchen, 5 atrium student lounges, 5 atrium student study rooms, small group study rooms, a computer room, a game room, photography lab, 2 music rooms, exercise rooms, meeting rooms, 7 outdoor terraces and a vending area, while its hallway was envisioned as a public place.

Figure 2. Simmons Hall undergraduate residence, MIT: View form Vassar Street, and exterior view of the "*perfcon*" structure during construction

3. Design Concept and Method

The building was approached by the architectural team as a "*vertical slice of a city*". The architectural team compared its corridors to streets that connect rooms, providing a variety of urban-like experiences. "Individuation" was achieved through a variety of room types, while mixed functionalities and alternate circulation paths foster social interaction.

The design approach was influenced by four factors: a) the limitations of the site area, b) the rejection of the classic brick building type, c) the need to develop an interior "urban" environment for social interaction d) The air and light circulation.

The economy of space was an important factor because the given lot on Vassar Street was small for the requirements of the program. The study of the building began from the rejection of the existing urban plan that was suggesting a wall "Boston-type" brick building. Holl's architectural team proposed the counter concept of "urban porosity".

From the early stage of the design process the architect and his team, including fifteen architects, developed a series of building alternatives. Each of these case-study-buildings was a demonstration of different ways of implementing the same design concept: "porosity". The variations of the schematic designs include "horizontal", "vertical", "diagonal" and "overall" porosity alternatives. These variations were characterized by their various types and degrees of "permeability", and by the different kinds of architectural space they created.

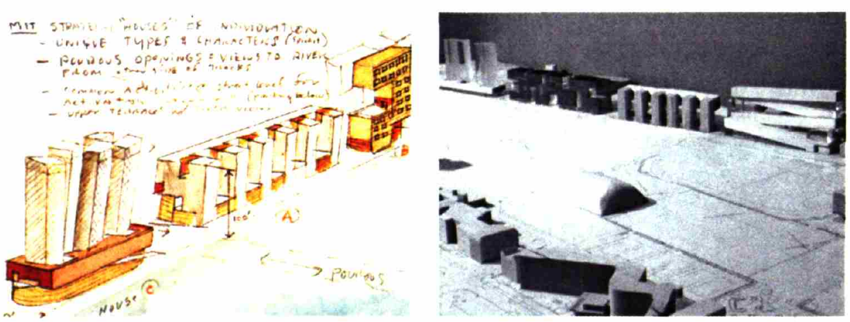


Figure 3. Schematic arrangements implementing the concept of horizontal, vertical, diagonal and overall porosity (right) and an early sketch depicting three of them. Illustrations by Steven Holl Architects, NY

The completion of the schematic phase of the design ended with the adoption of the "overall porous" schematic arrangement as most appropriate. A schematic building section of the adopted schema of overall porosity is presented on the left, and an early conceptual sketch on the right.

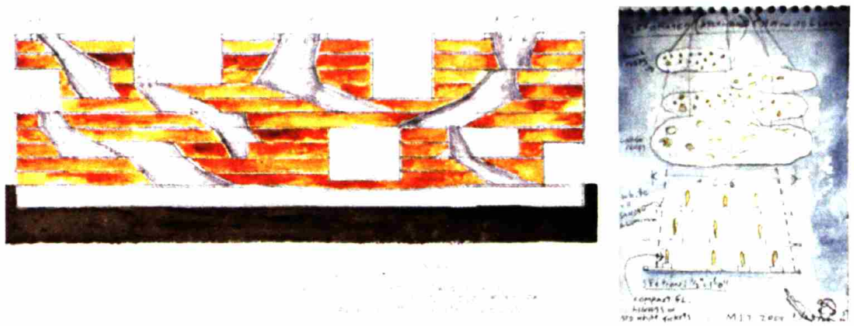


Figure 4. Schematic section of the adopted proposal (left). The definition of the principle of the "overall permeability" begins from the observation of sponge sections (right). Illustrations by Steven Holl Architects, NY.

The air and light circulation was a great design concern. It became an additional factor for the development of the design and the implementation of the concept of "porosity".

The mass of this concrete building is perforated to include a large variety of openings and windows. It includes 5 large-scale openings that signify the main entrances and the main outdoor activity terraces of the dormitory. The elevations have 5538 windows, nested in a structural component the "*perfcon*". The "*perfcon*" is a concrete wall 18" thick that fuses window, wall and structure in prefabricated pieces. Each individual room has nine operable windows. The 18" of the "*perfcon*" thickness shades the rooms naturally during the summer, and allows the low angled sun to contribute to the heating of the building during the winter. Additional vertical cavities, roughly corresponding to the different fraternity houses of the dorm, organize a ruled system of additional large openings that play important role in the circulation of air and natural light.

Holl accepts that the uniqueness of a design concept relies on the intertwining between absolute and circumstantial elements, in respect to a given design problem and its unique parameters. The designer uses the design idea as a 'limited concept' that sets out a group of possible relations among the given elements.

In relation to the original problem – which always remains a unique amalgam of diverse constituents – a design concept becomes for Holl (2000): "*a way of local stability among factors, and not a universal one*". The term "*semi-hierachical*" attempts to capture a heuristic that allows the possibility for dynamic reconfiguration of the relations among the elements of the building. At the same time, the concept, based on the consideration of both the circumstantial and the objective facts and limitations, forces things forward, in the process of investigation.

Holl (2000) accepts the possibility of expressing design concepts with analytic tools. He notes: "*A concept can be in some case mathematically precise. Mathematicians follow four laws in approaching a problem. They express it verbally, numerically, algebraically, and visually, likewise, architecture has a verbal concept, numeric size and proportion, algebraic integration of structure and material dynamics, and form. These four aspects forge an integrated connection for each site and program*".

Therefore, according to Holl, a design concept can be captured with descriptions that include words, numbers, symbols, parameters indicating proportion and finally form. This chapter deals with the computational interpretation of this last aspect of design concepts: form.

But how can one deal with a concept in visual and tactile terms? And how can one pull "actions" out of static words, (like the word "porosity")? From a shape computational point of view this study suggests that to describe a concept is to express it with general rule schemata (Stiny 1980).

In the example of Simmons Hall, the word "porosity" is used to indicate a particular intention towards action. This chapter shows that the concept of porosity can be treated computationally, and architecturally. Computationally, the depiction of a future action through a rule schema provides a way towards a particular order that one is willing to apply to things. Architecturally, the testing of a rule schema allows the evaluation of the produced spatial arrangement against the program and the known building standards. Proposing alternative ways of action (rule schemata) within the same general concept becomes itself the vehicle of one's thought.

Therefore, thinking (i.e. proposing rule schemata) and doing (i.e. testing) proceed step by step. The retrospective making of a grammar, is like the invention of an instrument for a purpose, and it is analogous to the invention of a process. The grammar is a calculating device for a particular kind of design activity organized to achieve some desired objective.

The general set of design actions that were suggested and tested by the architect Steven Holl and his team in the effort to implement the concept of porosity architecturally, include, roughly, the organization of a building body and the invention of operations and methods of developing multiple pores: openings that open the interior of the building towards the exterior, and internal channels and cavities that allow the circulation of air and light.

4. Spatial Elements

The initial overall available area and volume of the site provide the limits within which the architect has to arrange the building. In a total building site area 143,640 s.f. Holl developed a design for a 10-floor building of overall footstep 143,430 s.f. 53' deep 385' long, 105' high, thus exhausting the available volume and area of the site.

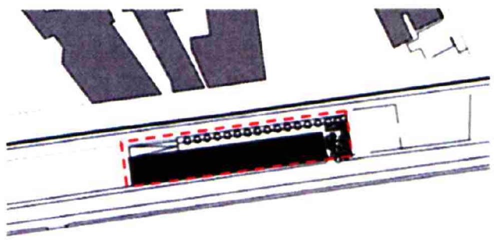
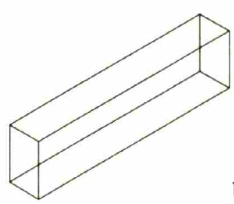
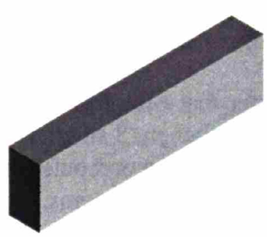
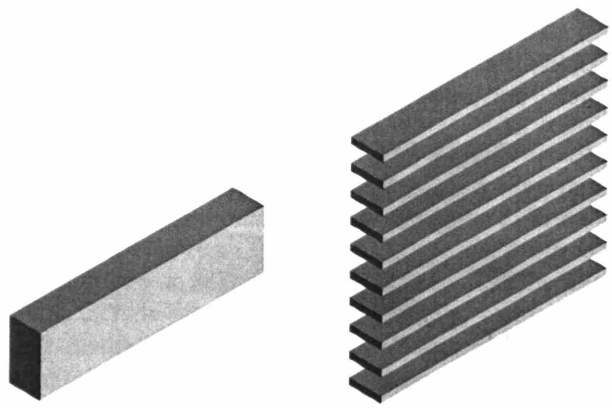


Figure 5. The footprint of Simmons Hall (black rectangle), and the site (red line).

In this presentation the shape of the overall available area and volume for the building is considered the initial shape. Alternative representations of this volume and area are provided next. In 3d, the representation in the algebra U_{13} shows the overall volume with lines, as an empty transparent box, and in the algebra U_{33} as solid. In 2d the shapes in the algebras U_{12} and U_{22} represent the outmost boundaries and the overall building area.

3d	2d
 U_{13}	 U_{12}
 U_{33}	 U_{22}

The next illustration shows the division of the overall volume into ten floors. Each floor is 10' height (100' from the ground to the top of the structure).



The basic "unit" of space in the program of a student dormitory, is the student-room. In designing a dormitory, one must pay attention to the characteristics of each room and the possibilities of creating variation. Following the directions of the program the building was organized to include 155 single rooms, and 95 double rooms. A typical student room has dimensions: 15' 6" length, 9' 6" width, and 10' height. It occupies 135 square feet of area and 1350 cubic feet of volume. Initially, a larger spatial unit 20' x 20' x 10' was set, corresponding roughly to a double room.

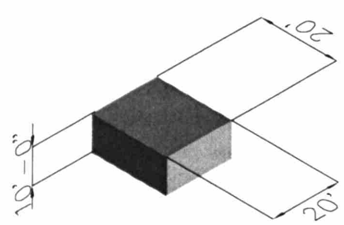
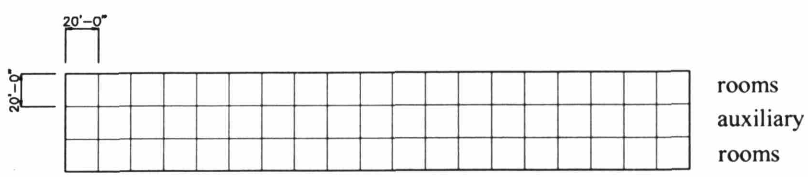


Figure 6. The basic special unit in the algebra U_{33}, represents approximately the double student room of Simmons Hall

The dormitory also includes a 125-seat theater that occupies a large part of the ground floor, a café, a large dining room, a kitchen, 5 smaller student lounges, 5 study rooms, and smaller group study rooms, a computer room, a game room, photography lab, 2 music rooms, exercise rooms, meeting rooms that were distributed in the main volume of the building. All these spaces were treated as multiples or subdivisions of the spatial unit.

The hallway of Simmons Hall was envisioned to operate as a public place. It is a protected public space of circulation and interaction, which is necessary during the winter. On each floor, the corridor works as a central node around which rooms and spaces are organized into two flanks.

In plan, the division of the initial area of the building into flanks and a corridor was made with a grid that includes three rows of squares 20' x 20'. The initial grid of the building gives a total width of 60'. In the final implementation of the building the building width was cut down to 53'.



The above setting corresponds to a common building arrangement. The two side grid-rows are occupied by two parallel flanks of rooms. The middle row accommodates the corridor, and the auxiliary spaces (restrooms, elevators, etc.). The next illustration shows a 3d representation of a typical floor, in axonometric, in the algebra U_{33}. The floor is divided into solid entities according to the divisions of the 20' x 20'. This particular way of illustrating 3d solids is going to be used in the rest of this chapter.

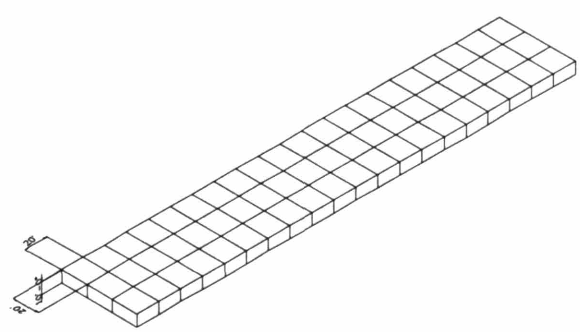


Figure 7. Typical floor in the algebra U_{33} divided by the 20' x 20' x 10' grid

Finally, there was intent to organize the programmatic entities of the building in three fraternity houses. This implied that the three houses corresponded roughly to three distinct volumes with autonomous presence.

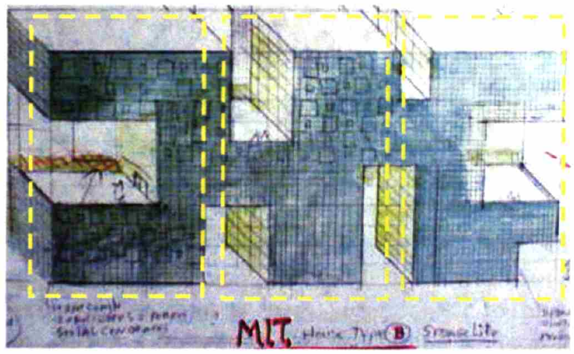


Figure 8. Simmons Hall was envisioned to be composed out of three visual distinct volumes. Illustration by Steven Holl Architects, NY

Due to the excessive requirements of the program and the limitations of the site this intention was implemented only partially in the building.

In sum, the basic spatial entities of the building, starting from the smaller are: the room, the flank, the floor and the house. The overall building has 10 floors, and accommodates three fraternity houses. Other spaces, like student lounges, study rooms, and smaller study rooms, music rooms, exercise rooms, meeting rooms etc. were distributed equally in the three houses.

5. Spatial Relations and Rule Schemata

The previous general division of the program into spatial entities is the result of analysis. The proposed spatial entities provide a general context for the composition. And the design concept activates specific design processes and determines a way of interrelationship among the entities of the program.

Steven Holl describes the concept as follows: "*The Sponge concept for the Undergraduate Residence Hall transforms a porous building morphology via a series of programmatic and biotechnical functions*".

The Greek word *πόρος* means a minute opening. The word is used in organic chemistry, medicine and the study of plants and animals to indicate opening configuration of analogous character. In biology and chemistry porosity is defined as the attribute of an organic body to have a large number of small openings and passages that allow matter to pass through. In nature, the shapes of pores are usually arbitrary, and the measurement of pore sizes and pore distribution is difficult. However, the functionality of a pore is always associated with circulation and filtration, with respect to an "external" environment.

Steven Holl (2000), and his team, approach the concept of porosity, within an architectural context: "*What if one aspect of a site – porosity – becomes a [design] concept? Porosity can be a new type of being. Its potentiality of consciousness indicates an opening where the horizon is included within it. We hope to develop the possibility of a collection of things held together in a new way where the 'horizon' is open and merges with both exterior and interior*". The concept of "porosity" becomes like the making of a drawing that describes what one is to do.

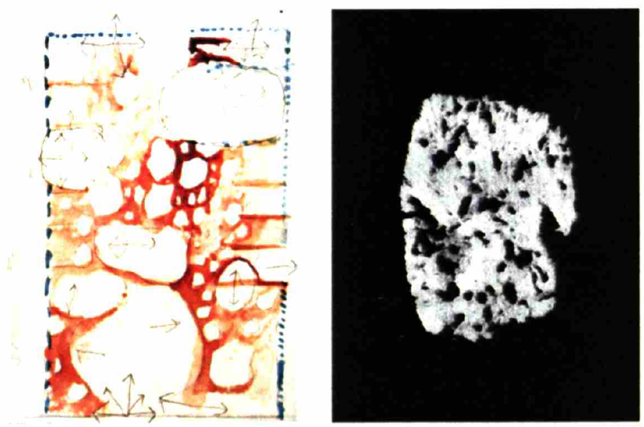


Figure 9. Porosity indicates an order that one wants to apply.
Illustrations by Steven Holl Architects, NY

The properties outlining the design concept of "porosity" were described by the architectural team. The natural qualities of sponge or any porous material were approached as tectonic possibilities.

TABLE 1. Qualities of porosity as a basis for the development of tectonic qualities

QUALITIES	
porous, permeable	honeycomb
screen, net	riddle, sponge
porosity, pore	opening, hole
aperture, passageway	cribiformity
sieve-like, sieve	pervious
unrestricted	

The rule schemata and rules expressing the concept of porosity are described in this presentation, by four rule schemata. Only the three of them were finally implemented in the actual building. The first two rule schemata correspond to two different ways of applying this concept to the overall building mass. The second group includes two rule schemata that add openings of various kind and scale.

5.1. POROSITY

The air and light circulation and the communication of interior and exterior are the main accomplishments of the use of the concept of porosity in the particular building.

The concept of porosity is implemented by bringing in contact as much of the building interior with the exterior as possible. This is accomplished in four general ways: First, by creating recesses in the overall building mass; Second by creating protrusions of building mass; Third, by distributing a large number of windows in the elevations of the building through the placement of prefabricated *perfcon* panels; And fourth, by distributing a limited number of vertical openings of free form, that create cavities in the interior of the building.

Porosity is achieved by applying instances of four rule schemata. The rule schemata can be described roughly as follows: a) prismatic voids are created on the building mass through subtraction, b) protrusions are created by translating half of the building along the direction of the central corridor, c) sieve-like openings are applied on the surfaces of the building, in elevations, through subtraction d) vertical sponge-like openings are created from top to bottom through subtraction and generation of curves.

Evidence of the application of the above operations is found in the sketches, drawings and 3D models produced throughout the design process of Simmons Hall. But many of the results of the conceptual rule schemata were reversed in part, or totally eliminated in later stages of the process. This became necessary for programmatic, functional, or other reasons. More specifically, many of the recesses created by the first rule schema were reversed in part by adding volumes. The results of the second rule schema were totally eliminated in the final implementation of the building, and in the case of the third rule schema, many windows were blocked by concrete pieces due to construction requirements. Finally, the creation of sponge like openings and cavities was limited to three basic cavities distributed in the three student houses. Due to fire-safety restrictions the cavities do not penetrate the building from top to bottom.

Rule schema 1: The first operation allows the creation of recesses of prismatic shape, in the building solid mass. This exposes more building surfaces towards the exterior and creates outdoor terraces. The desired result can be achieved by subtraction of shapes from the solid mass of the building. The subtracted shapes in this case are oblongs and prisms. The option to add volumes is also provided, if the results of subtraction need to be reversed. The application of rule schema 1 affects both the overall building volume and area (content), and the form of the building.

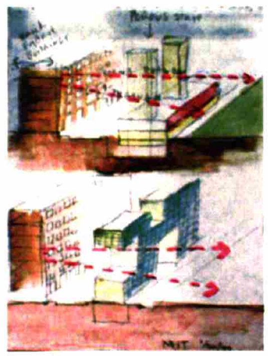
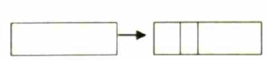
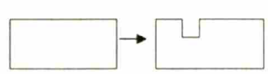
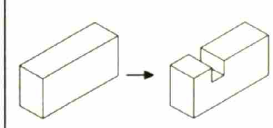
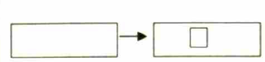
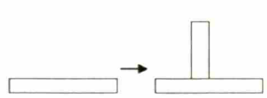
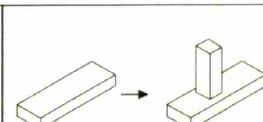


Figure 10. Addition (above) and subtraction (below) as depicted in an early sketch by Steven Holl.

Two rule schemata for subtraction and addition respectively, express these developments. The two rule schemata are depicted next in their general form, plan and elevation, in the algebra U_{12}, and in axonometric, in the algebra U_{33}.

plan (U_{12})	elevation (U_{12})	axonometric (U_{33})
subtraction		
		
addition		
		

Rule schema 2: A second rule schema that can express the concept of porosity, as this applies in the overall building mass, is the translation of each of the two building halves along the axis line of the main corridor. This transformation, it is named by the architectural team "diagonal porosity". The corresponding rule schema divides the building mass vertically into two volumes and translates one, along the building axis. In this way more of the interior of the building is exposed towards the exterior. The application of rule schema 2 affects only the form of the building, without altering the overall square-footage or the volume (content).

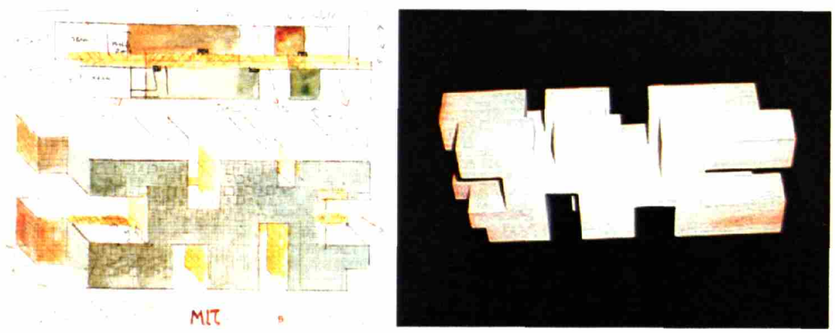
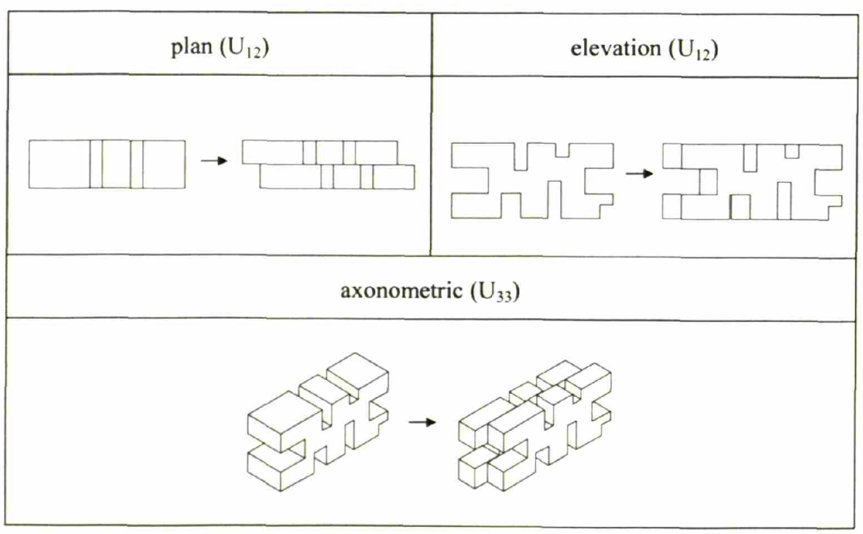
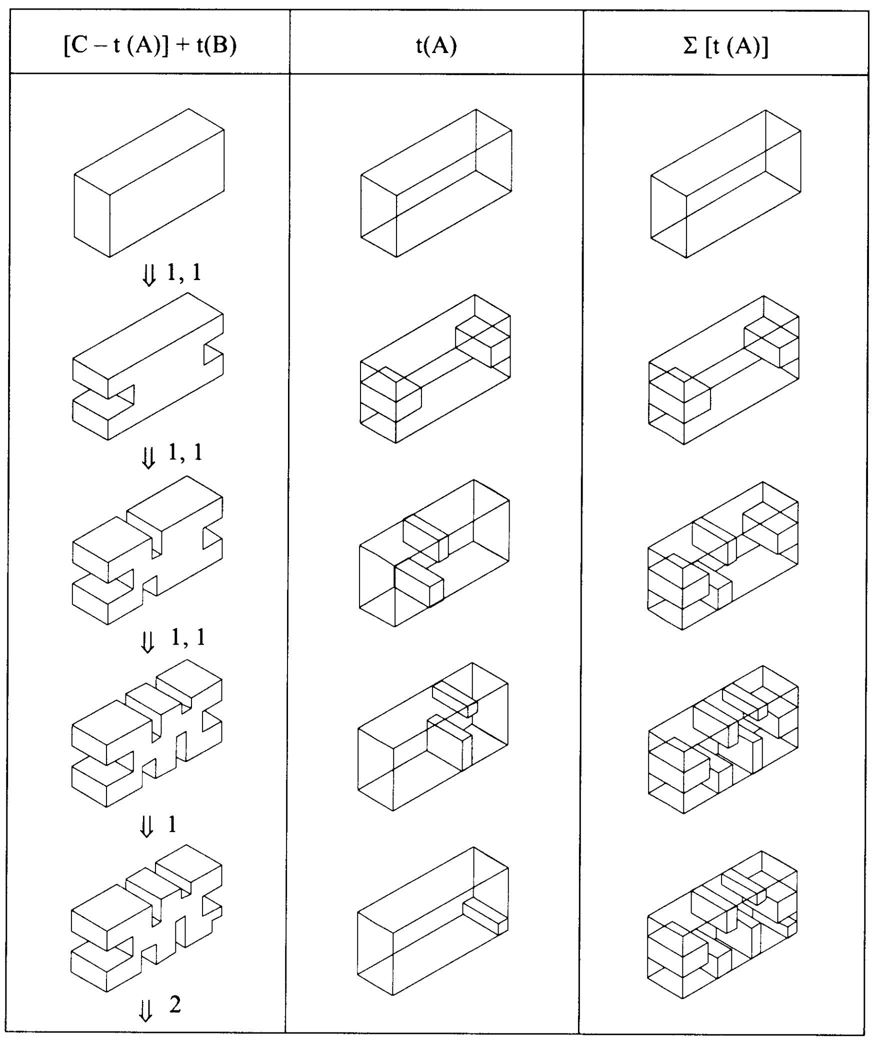


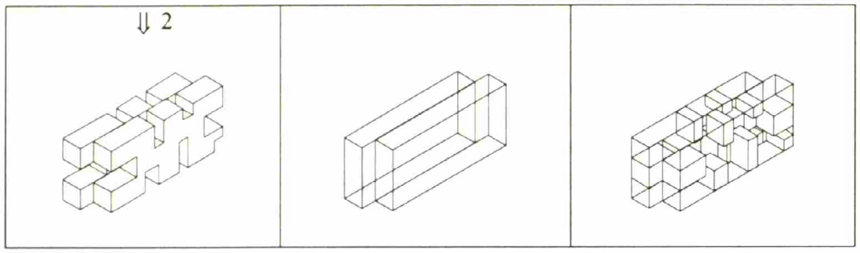
Figure 11. The translation along the corridor axis was present in early sketches (left) and models (right). Sketch and model by Steven Holl Architects NY

A rule schema expressing the above development is depicted next in plan and elevation, in the algebra U_{12}, and in axonometric, in the algebra U_{33}.



A possible derivation using the first two rule schemata is presented next. Initial shape is the overall building volume. The derivation appears in the first column in the algebra U_{33}. The second column shows the subtracted or added shapes at each step, in the algebra $<U_{13}xU_{33}>$. The subtracted or added shapes are represented by solids in U_{33} and the overall volume is represented with the boundaries of its boundaries: lines in the algebra $<U_{13}>$. The third column presents the overall initial volume and the sum of the subtracted or added shapes, in the algebra $<U_{33}>$. For brevity, the rule schema 1 is applied twice in the first three steps of the derivation.



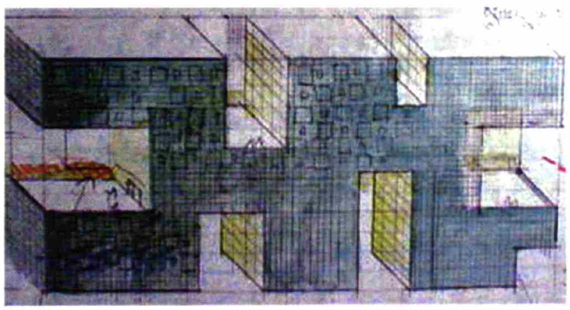


Although the design intentions expressed through the rule schemata 1 and 2, are present in several of the early drawings and 3d models, it seems that for various reasons (probably related to the need for extra space) they are not visible in later models and illustrations, or in the actual implementation of the design.

Figure 12. The results of rule schemata 1 and 2 are less visible in later models (left) and illustrations (right). Model and illustration by Steven Holl Architects, NY

The study of the early conceptual drawings and 3d models shows that the results of the application of rule schema 1 were partially reversed during the design process. More specifically, it seems that parts of the subtracted masses were added back, most likely due to the programmatic requirements. Therefore, the recesses and the "voids" in the mass of the building that appear clearly in the early models and drawings are not as intense and visible in the building. A possible explanation is that both rule schemata 1 and 2 were used in the design process. But certain of the results of rule schema 1 had to become less intense in later stages because they collided with programmatic requirements, while the results of rule schema 2 were entirely lost.

Rule schema 3: A third variation of porosity is used for the treatment of the elevations of the building: sieve-like openings are applied on all the facades of the building. A similar approach appears in a design proposal for an office pavilion for the social housing company Het Ooosten, in Amsterdam (2000). Holl uses multiple layers of porous and geometrically perforated materials (Menger sponge), which allow the bouncing of natural light between the different layers of its interior. The application of rule schema 3 affects only the exterior form of the building, without changing the square-footage or the volume.



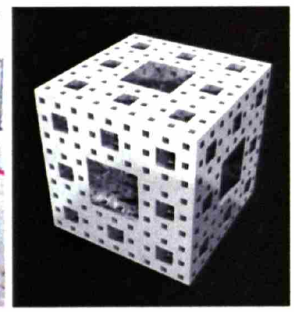
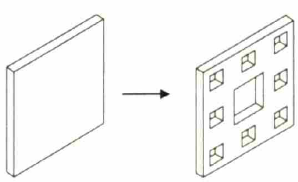


Figure 13. The porosity schema describing a configuration of perforated exterior surfaces, in an early drawing of Simmons Hall (left), and a Menger sponge (right). Both illustrations by Steven Holl Architects, NY

The above general idea of perforated panels for the elevations of Simmons Hall that is depicted in the early sketches can be expressed by a rule schema in axonometric, in the algebra U_{33}.



However, in Simmons Hall the exterior concrete construction of the building becomes the main load bearing grid. The construction requirements of this decision dictated that the majority of openings have a standard size.

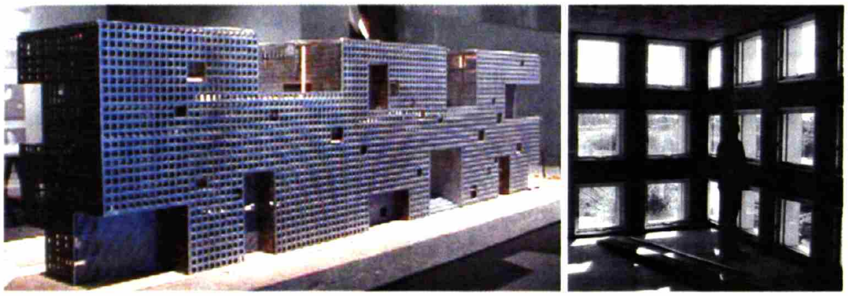
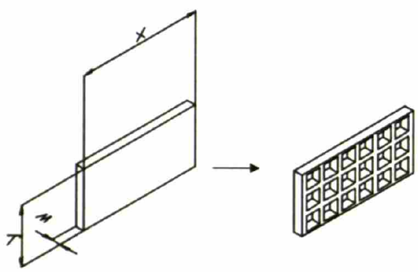
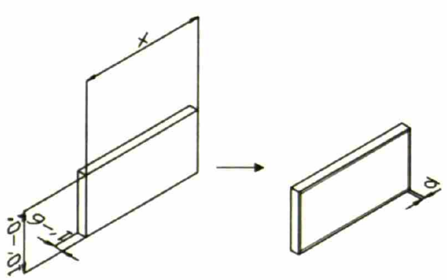


Figure 14. The perforated exterior of Simmons Hall (left), in an early 3d model, and as it was implemented (interior-right). Illustrations by Steven Holl Architects, NY

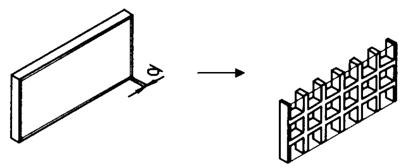
The elevations have a total of 5538 windows. These 2' x 2' windows are nested in a structural component, the "*perfcon*". The "*perfcon*" is a concrete prefabricated wall 18" thick that fuses windows and structure. The formation of *perfcon* panels, for the elevations of Simmons Hall is depicted next, in a general way by a rule schema, in the algebra U_{33}.



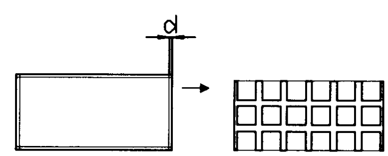
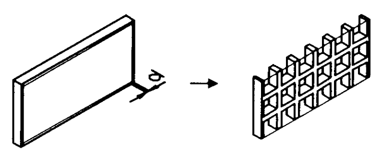
The schema applies on a piece of exterior wall of length x, height y and width w. The height of each wall-piece is equal to the height of a building floor (10'). The thickness w of each wall-piece is 18". A second version of this rule schema can apply in two steps. In the first step a grid is constructed. The grid rectangles correspond to the outmost limit of each panel. The grid applies on the façade to specify the position of the panels.



In the second step, the rule schema applies the openings according to the grid.

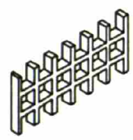
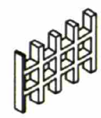
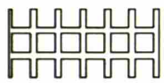
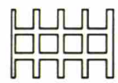
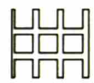
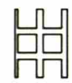


A general expression for the rule schema is depicted next in plan and elevation, in the algebra U_{12}, and in axonometric, in the algebra U_{33}

plan (U_{12})	elevation (U_{12})
	
axonometric (U_{33})	
	

The majority of the *perfcon* panels used in Simmon's Hall have three windows in height and six windows in width (3 x 6). Each individual room has nine operable windows in total: it occupies three windows in height and three in width (3 x 3). Each perforated panel covers two adjacent rooms.

Corner *perfcon* panels are different. They have three windows in height and four windows in width (3 x 4), while a limited number of 3 x 6, 3 x 3, and 3 x 2 corner panels have also been used.

3 x 6 corner	3 x 4 corner	3 x 3 corner	3 x 2 corner
			
			

The placement of corner panels happens with the appropriate instances of the previous general rule schema, not shown here. The discrimination of the corner panels can be done with labels, or by creating a deterministic process for the application of the panels: common panels are applied first, corner panels next etc. Further, the implementation of the perforated elevations includes a limited number of larger openings corresponding to lounges, or public spaces. And, the structural requirements of the *perfcon* dictated that some of the openings be occupied by concrete blocks.

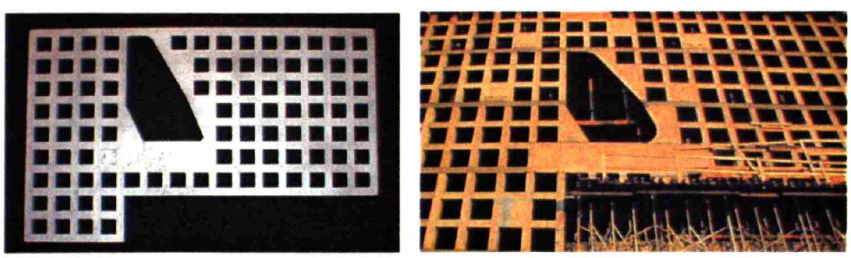
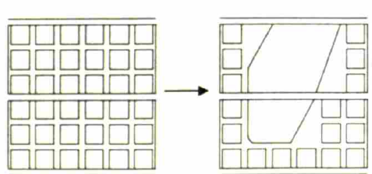
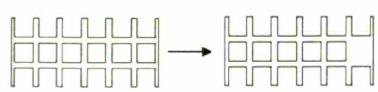


Figure 15. Perforated surface from Simmons Hall, as it was modeled (left) in an early 3d model, and as it was implemented (right). Steven Holl Architects, NY

For the treatment of these cases two additional rule schemata are formed. The first selects a number of *perfcon* panels and unites some of their windows into a single large window. The second selects a *perfcon* panel and substitutes one of its windows with a concrete surface.

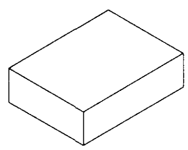
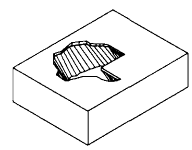
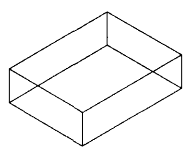
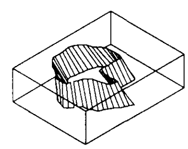




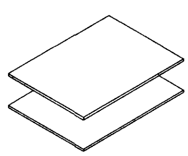
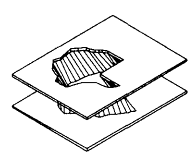
Rule schema 4: A fourth variation of porosity is what Holl indicates as "vertical porosity". It involves the creation of vertical sponge-like openings penetrating the building from top to bottom. The position of the vertical cavities corresponds roughly to the three fraternity houses. Vertical porosity is achieved through the creation of free form cavities. The vertical cavities contribute in the circulation of air and light. Rule schema 4 affects both the area and the volume of the building, but also the form of its interior.

Figure 16. The idea of vertical cavities, as it was presented in an early sketches (1) and modeled in early (2, 3, 4, 5) and later (6) 3d models. Sketch and models by Steven Holl Architects, NY.

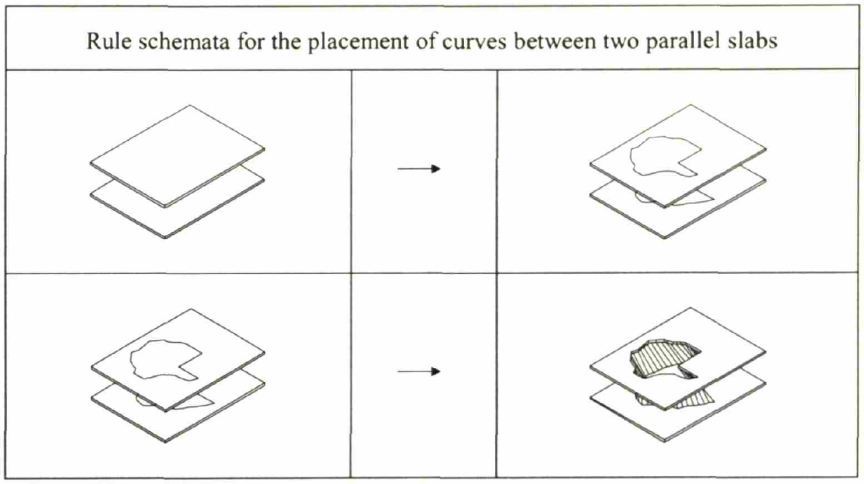
The original intention of creating a free form cavity penetrating a solid mass is expressed by the next rule schema in the algebra U_{33}. An additional depiction of the same rule schema is provided in the algebra U_{13} representing the solid by linear boundaries

Rule schema generating cavities		
	→	
	→	

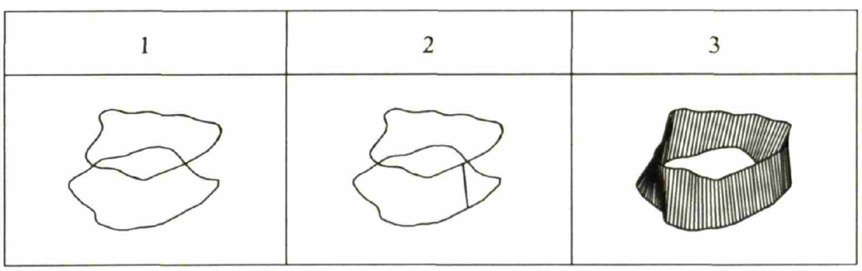
Because the generation of cavities happens between two parallel slabs a slightly different rule schema can be provided, also in the algebra U_{33}.

Rule schema for the generation of cavities between two parallel slabs		
	→	

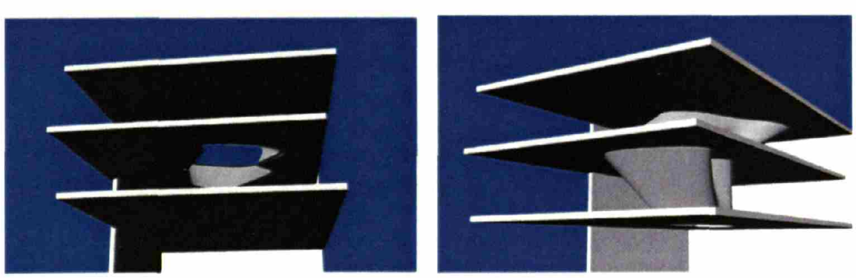
The generation of cavities between two parallel slabs can also be described in two steps in the algebra $U_{13} \times U_{33}$. The first step allows the positioning of the linear outmost boundaries of the curves on the slab. The second step deals with the generation of the curved surface, and the opening.



The construction of each cavity can be expressed by a simple series of actions. It results from the translation of a line on two closed curves lying in parallel (1). The single line that connects the two parallel curves in (2) serves as the generator of the surface (3).



The use of such vertical openings is to bring light and air, and allow the visual contact among different floors. A curve can be repeated on the slab that lies above, or below two initial slabs.



This allows the vertical openings to combine and penetrate the building from top to bottom. The curves can have doors or staircases and allow the vertical circulation among floors.

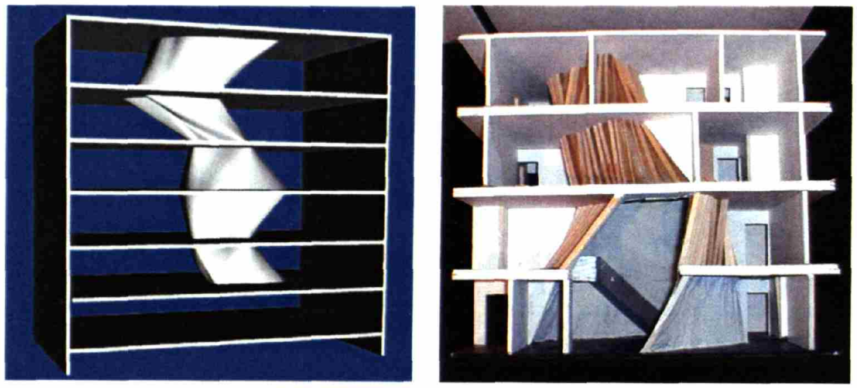


Figure 17. Vertical cavities combine to penetrate the building from top to bottom. Digital representation by the author and model by Steven Holl Architects, NY.

Aesthetically, the above combination causes an interesting interplay between the overall orthogonal geometry of the building and the forms of the cavities.

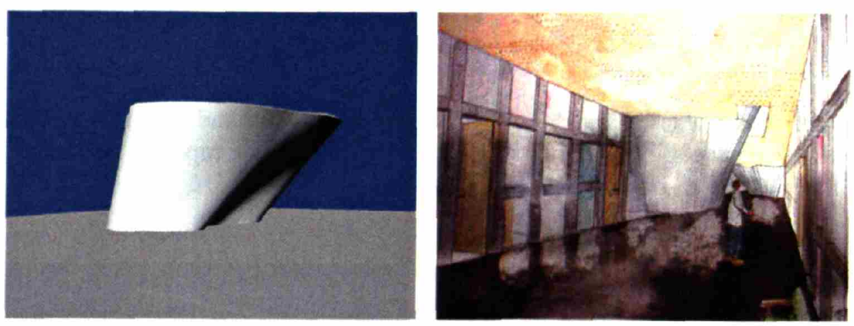


Figure 18. Vertical cavities (left) coexist with the orthogonal *perfcon* geometry. Digital representation by the author and watercolor drawing by Steven Holl.

The forms of the curves that construct the vertical cavities at each floor were explored by tracing out shapes produced with sponge on a piece of paper. This required several experiments. In order to keep constant reference to the actual scale and sizes of the building the sponge shapes were superimposed on the 20' x 20' building grid. In later stages the sponge-like curves were superimposed on the actual floor plans for arrangement with the other spatial elements (rooms, corridors, elevators, beams, etc).

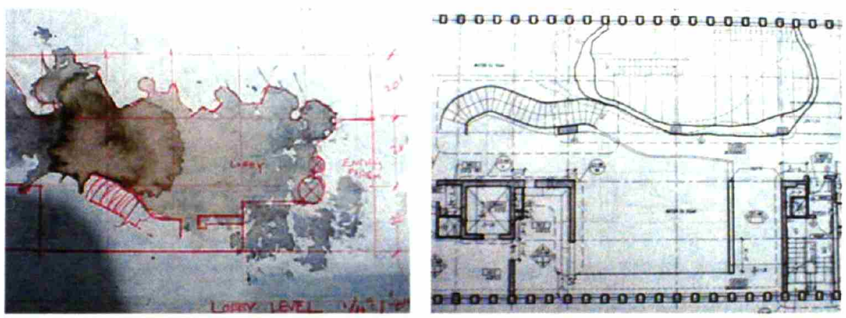


Figure 19. The shape of the curves was first explored, on paper with sponge and colors. Then, the possible shapes were used in the formation of the actual building plans. Illustrations by Steven Holl Architects, NY.

The arrangement of "curved" forms and "straight" forms remained a constant compositional issue at all the stages of the design process. The interaction between the two is also evident in next final sections of the building: North-South (left) and South-North (right).

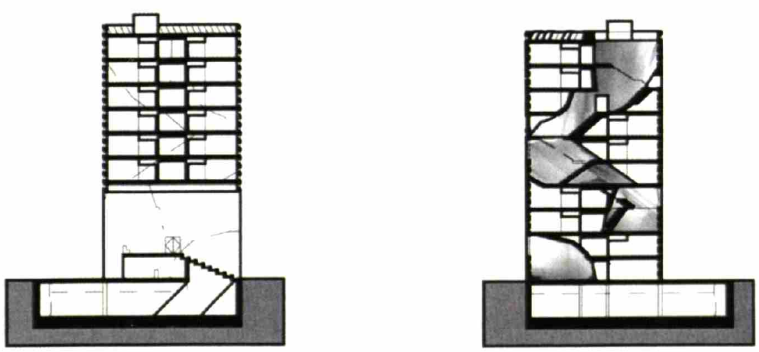
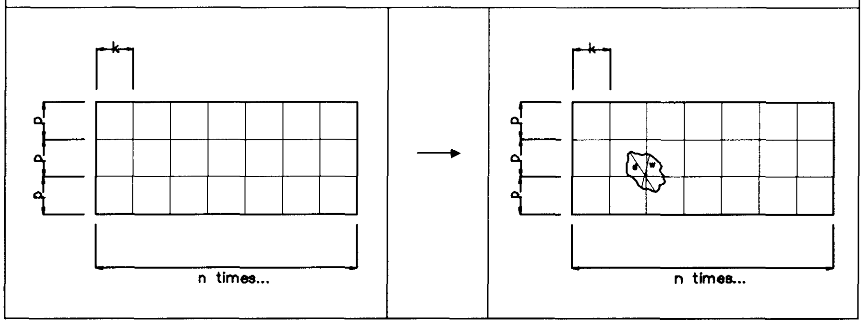


Figure 20. Sections that exhibit the coexistence of a typical building geometry and sponge-like cavities. Illustrations by Steven Holl Architects, NY.

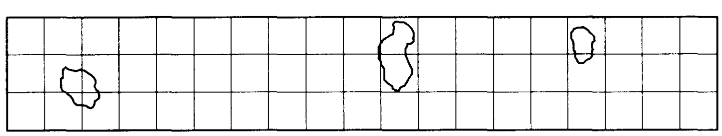
The positioning of the curved linear boundaries of the cavities in plan can be done by following a parametric rule schema that places curves for potential cavities on the building grid.

Rule schema for the placement of cavities in plan



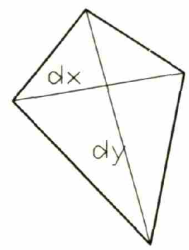
The rule schema places a curve on a grid p x k feet, and n divisions. The initial grid of the building was set to $p = k = 20'$, which gives a 20' x 20' grid.

The distribution of the vertical openings on the building corresponded to the approximate placement of the three different fraternity houses. But the final distribution of the vertical cavities was developed while taking into account the plans of each floor.



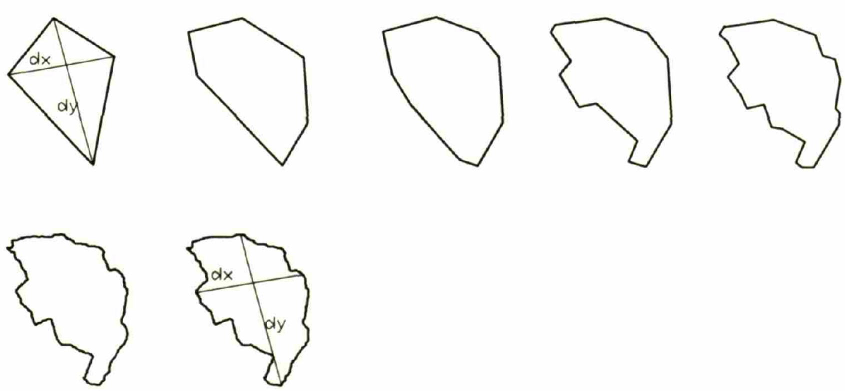
The initial placement of the curves on the building grid helps to visualize their potential positions and gives information about their scale. It also informs about the potential areas that will not be occupied by student rooms (since they will be occupied by the curves).

An alternative way to produce the shapes of sponge-like curves computationally would be to treat them as fractals. The process would begin with the placement of a quadrilateral of maximum width dx and length dy ($dy \geq dx$) on the building grid

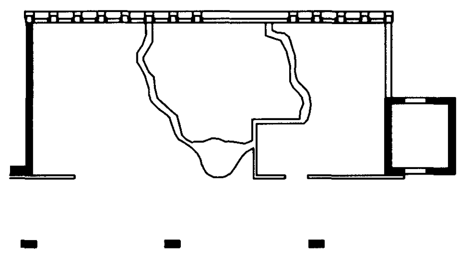


The application of two rule schemata can produce shapes very similar to sponge like shapes. These interactions remained

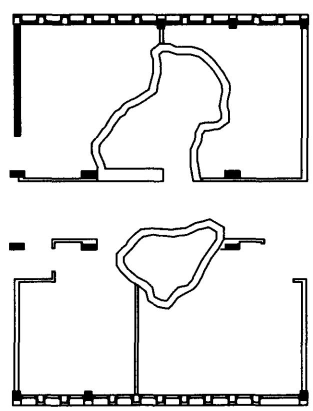
The two rules can apply under the general provision that always $dy \geq dx$. The provision can assure that the proportions an the direction of the opening remain unaltered as the two rules apply. In an alternative case a specific proportion can apply to dy and dx (for example $dx = 3/5\ dy$).



The arrangement of the cavities on each of the 10 floor-plans requires further refinement of their forms. This can be achieved with the aid of additional rule schemata. The main characteristic of the forms of the vertical cavities is that they do not have any structural significance: they bear only their own weight. This gives vertical cavities the flexibility to develop freely in relationship to the existent structural system of beams, and the perforated concrete surface of the *perfcon*. Their curves behave like walls of interior subdivision that often extend to more than two floors, and bring light and air from the roof skylights. Two general types of cavity-curves are distinguished. The first includes curves attached on the *perfcon* structure:

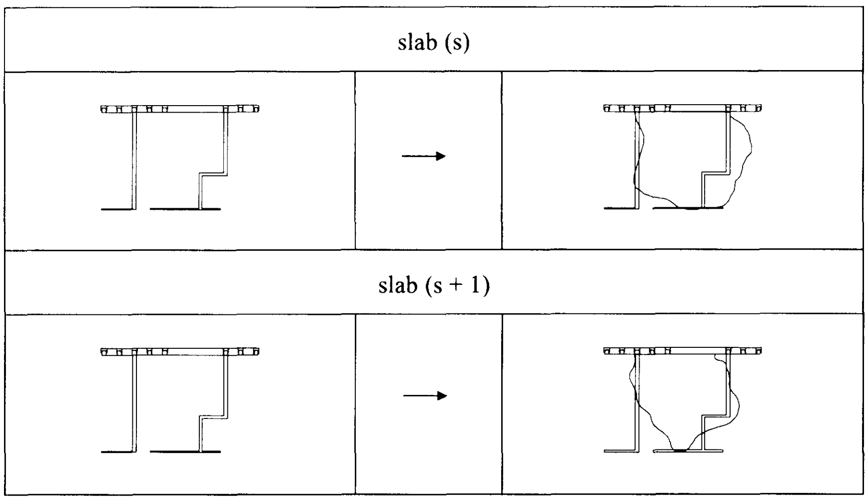


The second type includes cavity-curves that lie independent from the *perfcon* structure.

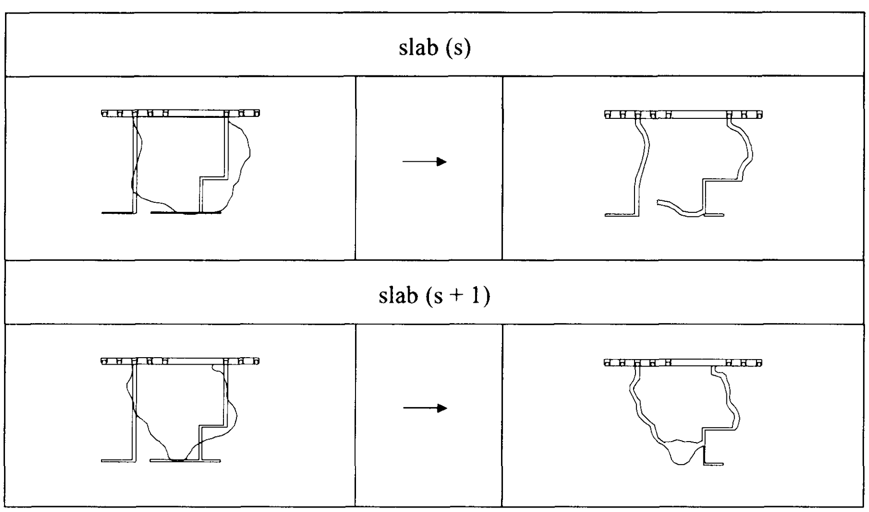


A cavity that extends on several floors can begin as cavity of the first type and end up as cavity of the second type, or the opposite.

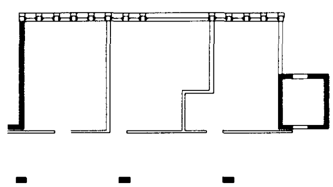
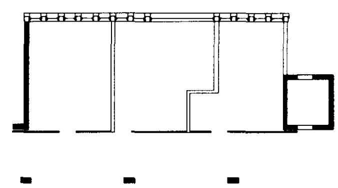
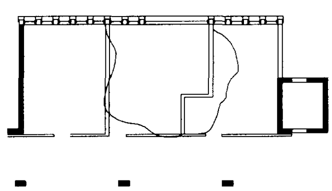
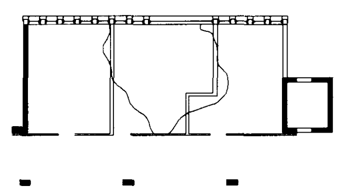
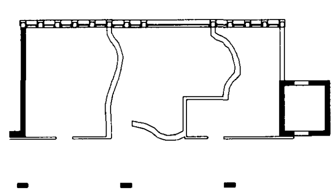
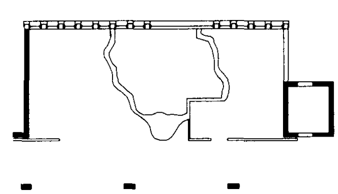
The first curve type includes curves adjacent to the *perfcon*. The second type includes curves that are penetrated by the main building corridor, and are divided into two parts. It is possible to provide a parametric rule schema for the treatment of each curve type. In the next rule schemata the cavity-curves are first drawn on top of some room arrangement. Then, they are arranged with the existent walls corridors, beams, etc. Each cavity requires at least one pair of curves. The rule schema applies on two consecutive slabs (s, s+1), to create two 2d curves respectively. Then, the curves can generate a curved surface in 3d. Each curve retains a relationship to the existent context. In plan, the rule schema that creates curves adjacent to the *perfcon*, applies in two steps. First it draws the outmost boundary of the constructed curve, on each of the two slabs.



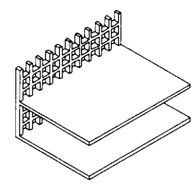
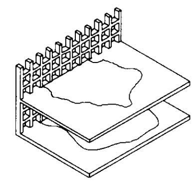
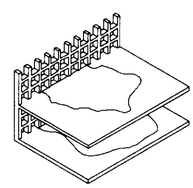
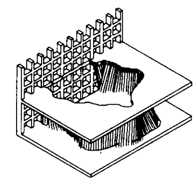
Second it draws the innermost boundary of the constructed curve, on each of the two slabs and transforms the existent walls.



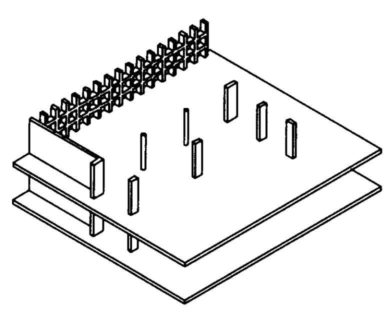
In the next sample of a possible derivation, the thick black line corresponds to structural parts of the building.

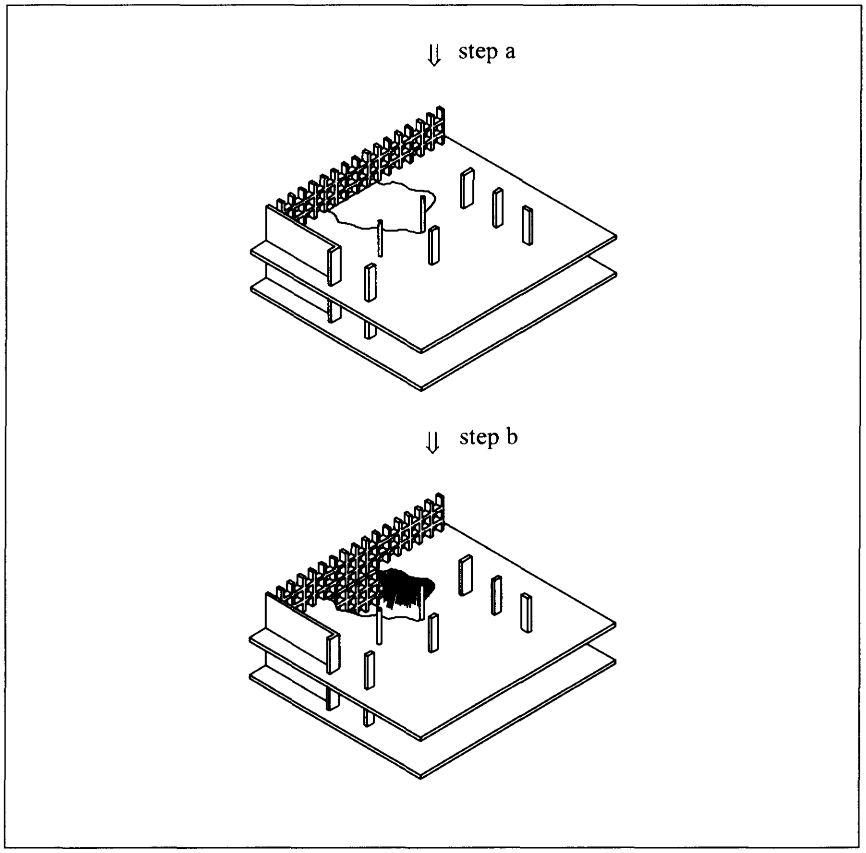
slab (s)	slab (s +1)
	
⇓	⇓
	
⇓	⇓
	

In axonometric, for the same class of curves the rule schema applies also in two steps, in the product algebra $U_{13} \times U_{33}$: a) draws the outmost boundary of the curve, on each of the two consecutive slabs, and b) it constructs the cavity.

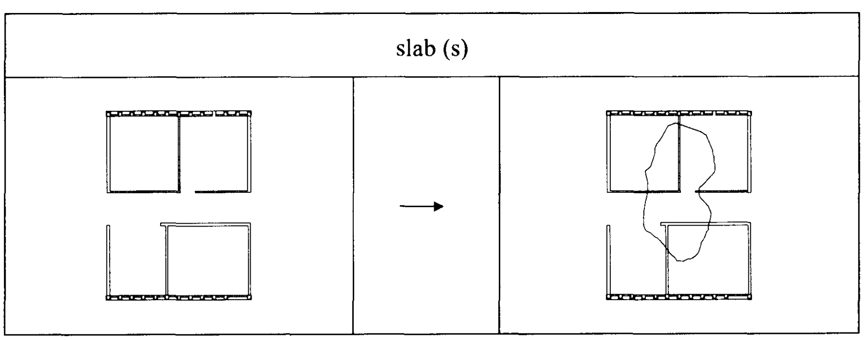
step a		
	→	
step b		
	→	

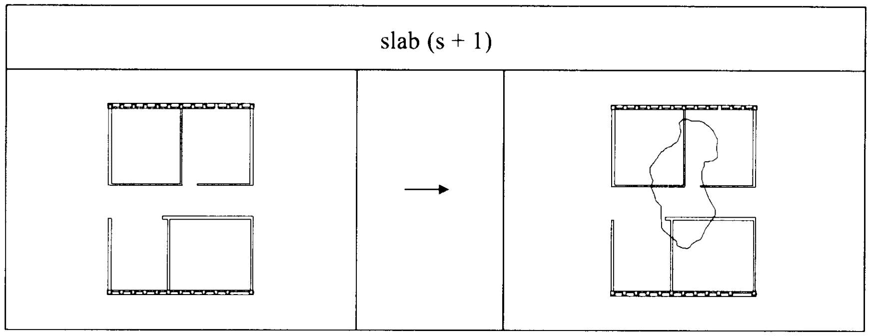
The next illustration of a possible derivation shows how the closed 2d curves are first applied on two subsequent, parallel slabs, and then how a curved surface is generated.

Vertical opening derivation

⇓ step a

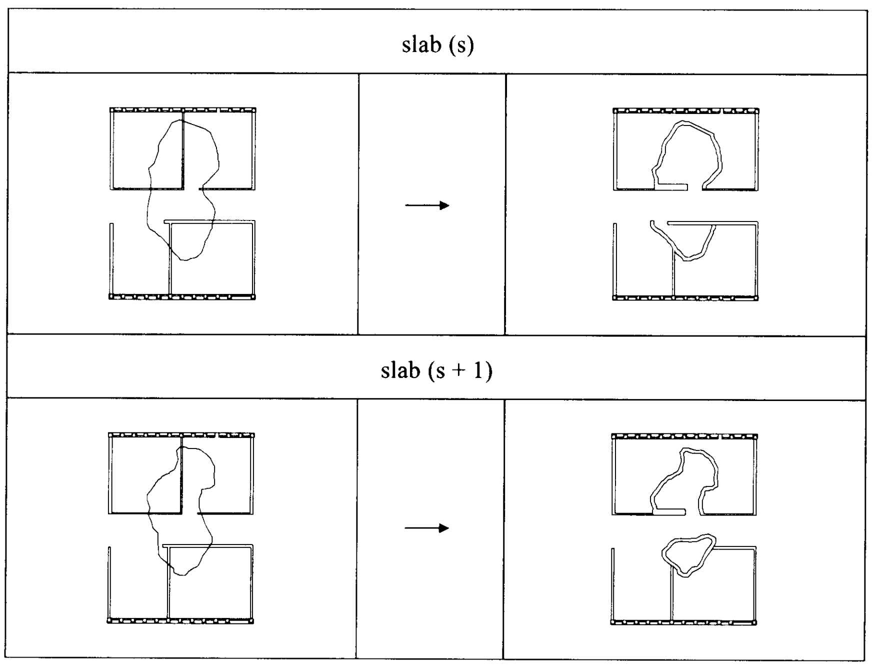


A second curve type, not adjacent to the *perfcon* structure, is applied also in two steps. First the outmost boundary of the constructed curve, on each of the two slabs, is drawn on the plan

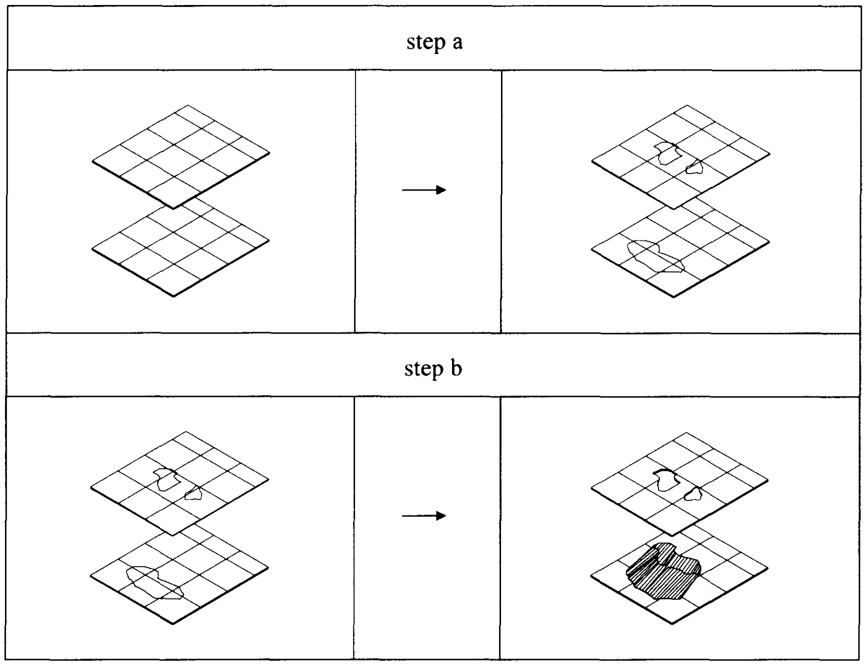




Second, the rule schema draws the innermost boundary of the curve, on each of the two slabs and combines the existing and the new lines.



The construction of the curves in axonometric, in the algebra $U_{13} \times U_{33}$, is similar to the previous one. The distance between slabs is exaggerated for presentation purposes, and the use of the grid helps specifying the position of the curves.



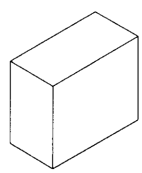
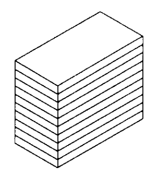
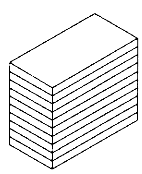
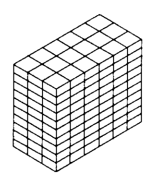
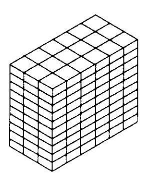
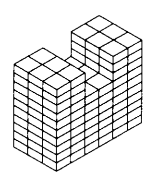
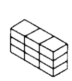
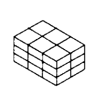
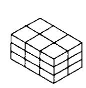
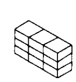
6. Derivation

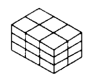
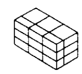
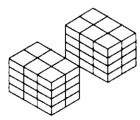
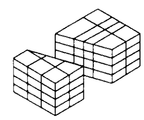
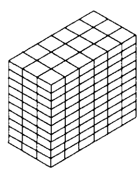
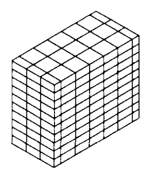
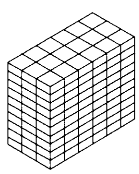
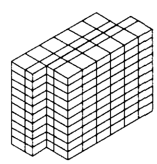
The next derivation examples show a possible sequence of design decisions and actions that are expressed with the aid of the previous rule schemata. The objective of the derivation examples is not to simulate the actual design process, but to offer an example on how a design concept can be expressed with the aid of shape rule schemata, and how sequences of actions can be implemented by computational processes.

Three examples of derivations are presented. Each corresponds to one of the three types of porosity. The order of their presentation may not coincide with their ordering in the design process. The first derivation presents rules and transformations that change the distribution of volumes and voids. It corresponds to the application of the concept of porosity in the overall building mass (rule schemata 1 and 2). The second derivation is an example of deriving a floor elevation by creating and arranging the *perfcon* panels (rule schema 3). The third derivation is an example of distributing free form cavities. It corresponds to the application of vertical porosity (rule schema 4). The rule schemata 1, 2, 3 and 4 are refined in each derivation in order to apply to specific examples. The rules that are proposed in each derivation are instances of the rule schemata 1, 2, 3 and 4 but may also include additional auxiliary rules that serve a derivation.

6.1. BUILDING MASS

The first part of the derivation begins from the overall volume. It concerns the manipulation of this volume according to the rule schemata 1 and 2. A series of additions, subtractions, and a translation change the initial volume. The changes have both quantitative and qualitative importance as they affect both the content (area and volume) and the form of the building. The entities that serve as units are the student rooms represented by solids 20' x 20' x 10', in the algebra U_{33}. As it is shown in the derivation, the solid units are produced by division of the overall height in ten floors of approximately 10' height and the division of each floor according to the 20' x 20' building grid.

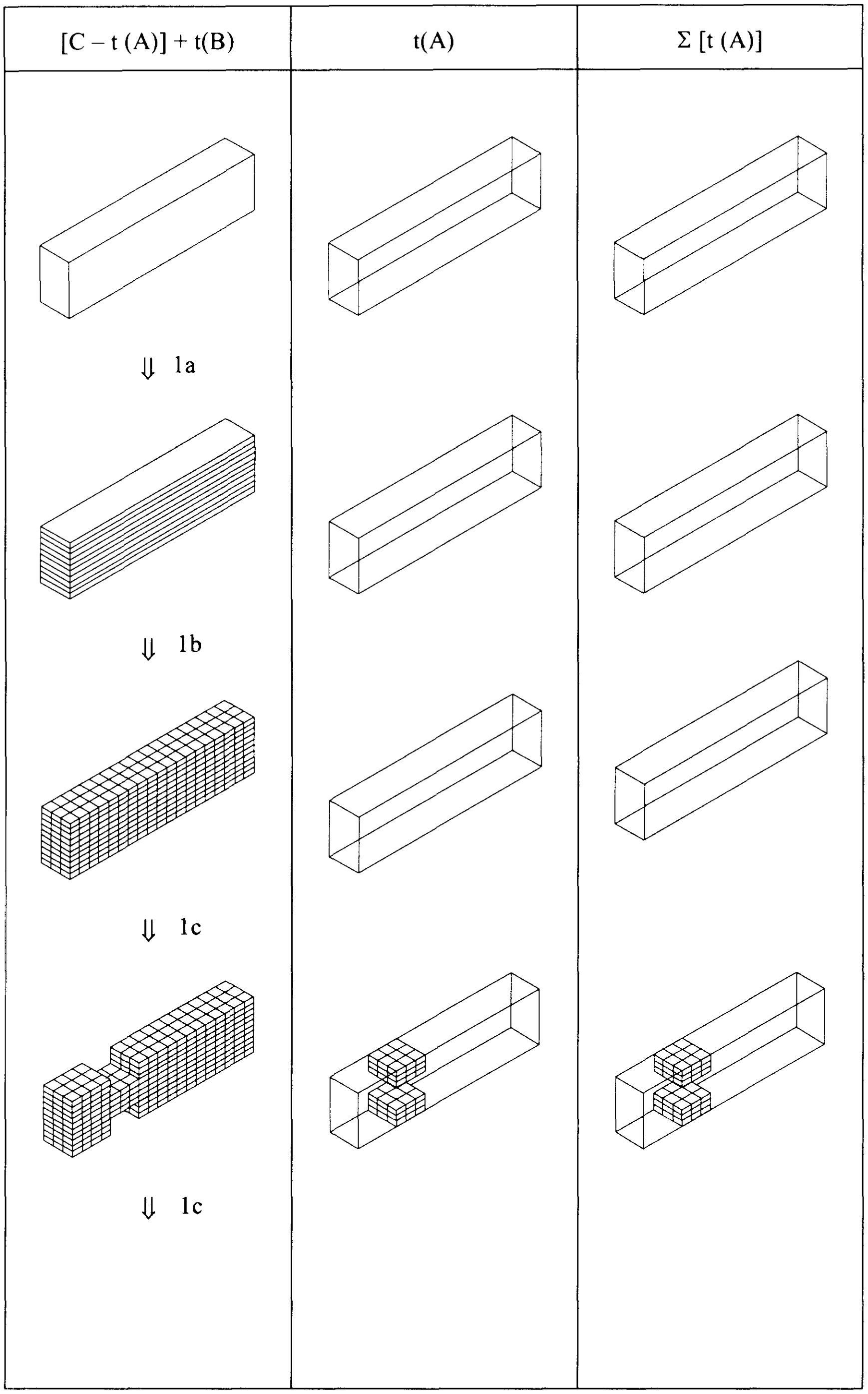
Rules for the treatment of the overall building mass			
1a		→	
1b		→	
1c		→	
1d		→	
1e		→	

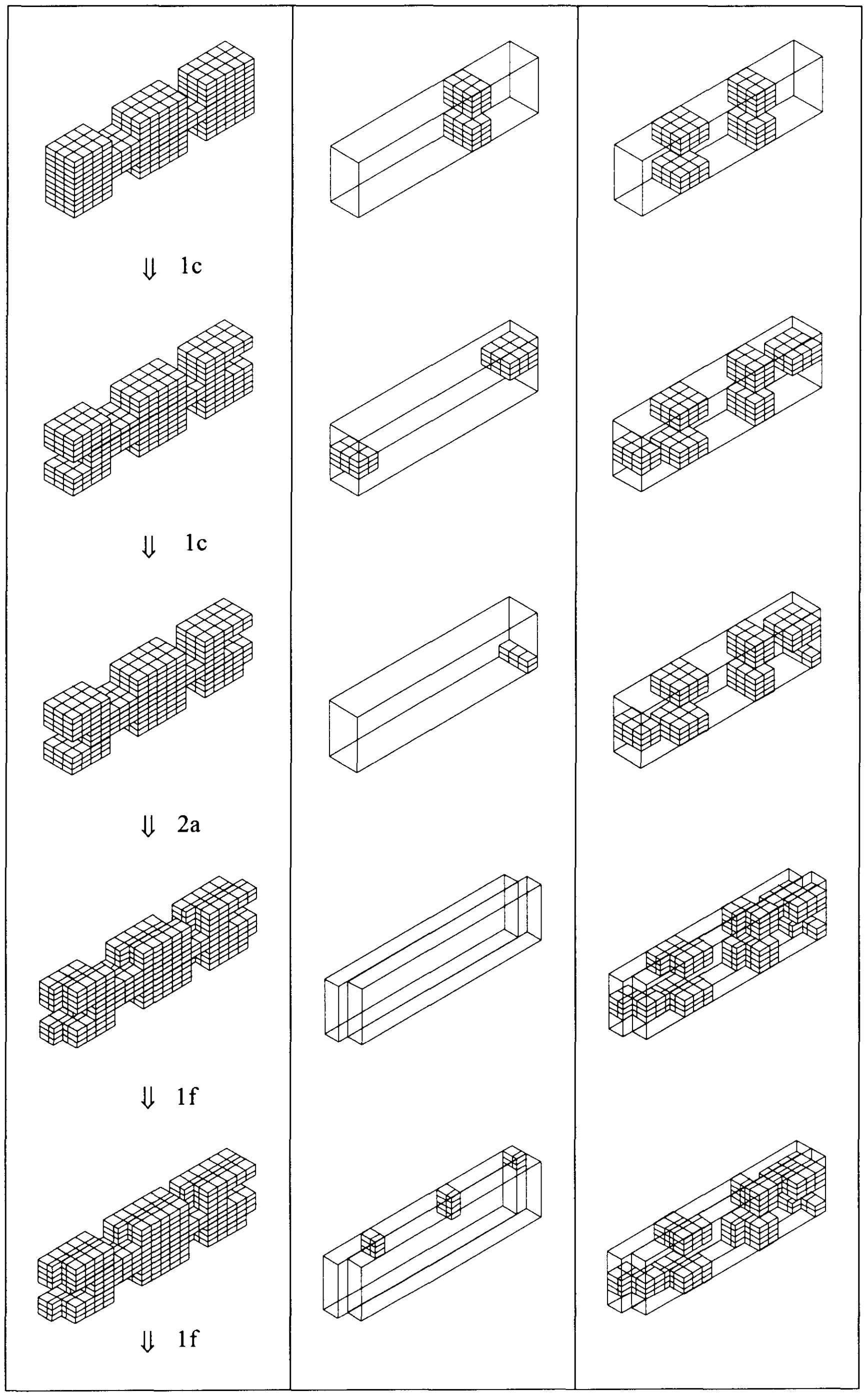
1f		→	
1h		→	
1k		→	
2a		→	

The derivation is presented in three columns. The main derivation appears always in the first column, where all shapes are represented as solids, in the algebra U_{33}. It involves the subtraction, addition, or translation of units of space, which correspond roughly to the student rooms.

In the second column the overall initial volume is represented as an empty container with the boundaries of its boundaries: lines in the algebra $<U_{13}>$. The subtracted or added shapes are represented by solids. The second column presents shapes in the algebra $<U_{13} \times U_{33}>$. The second column serves mainly for presentation purposes, to allow visibility of the subtracted or added shapes.

The third column presents the overall initial volume with lines in the algebra $<U_{13}>$, together with the sum of the subtracted or added shapes. The third column presents shapes in the algebra $<U_{13} \times U_{33}>$. The first column shows the calculations with the rooms of the building the third column shows the corresponding calculations with the voids of the building. It visualizes the relationship between solid and void, at each step of the derivation and makes possible the counting of the void units.


[C – t (A)] + t(B)
t(A)
Σ [t (A)]
⇓ 1a
⇓ 1b
⇓ 1c
⇓ 1c


⇓ 1c
⇓ 1c
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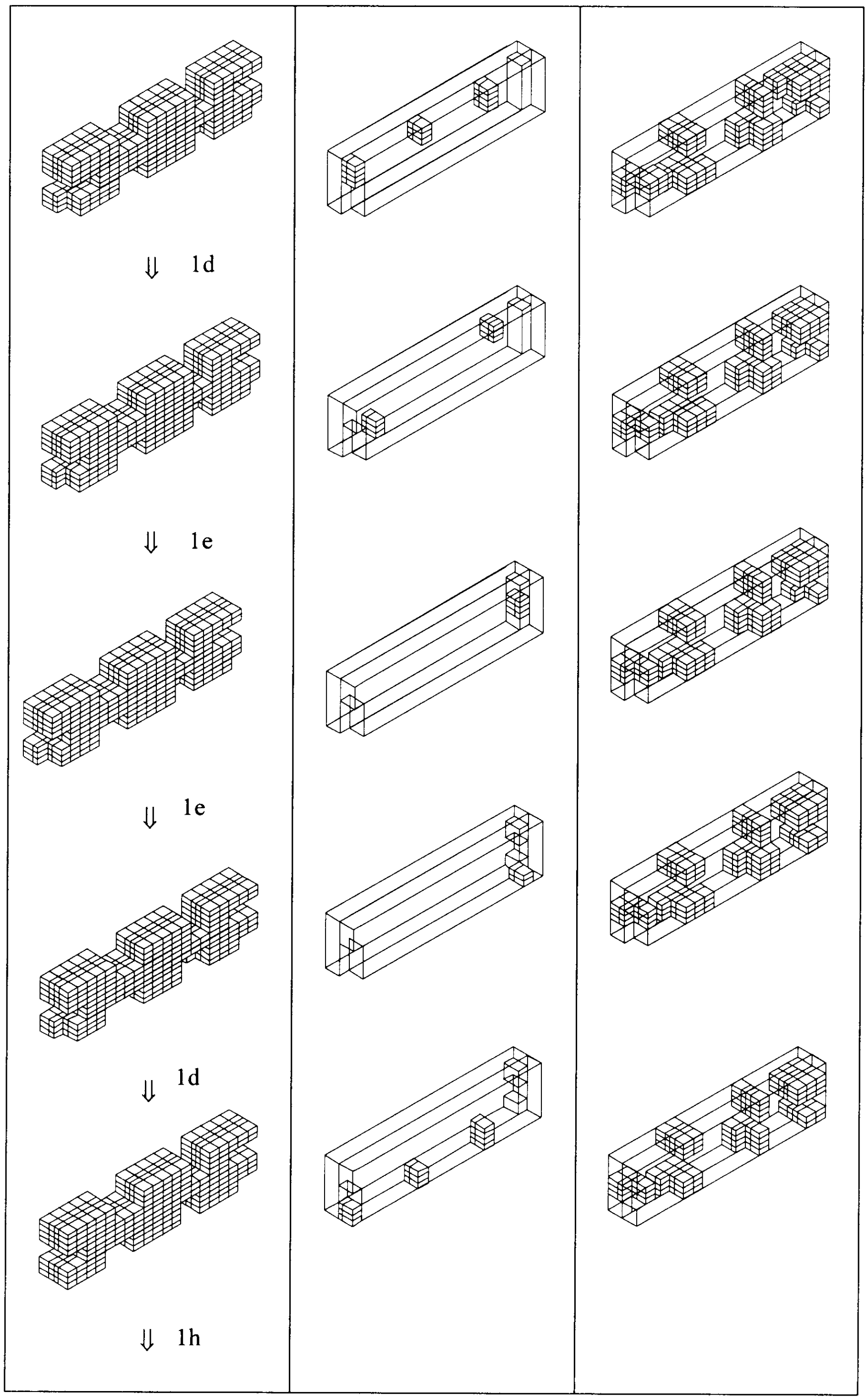
⇓ 1f
⇓ 1f


⇓ 1d
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⇓ 1e
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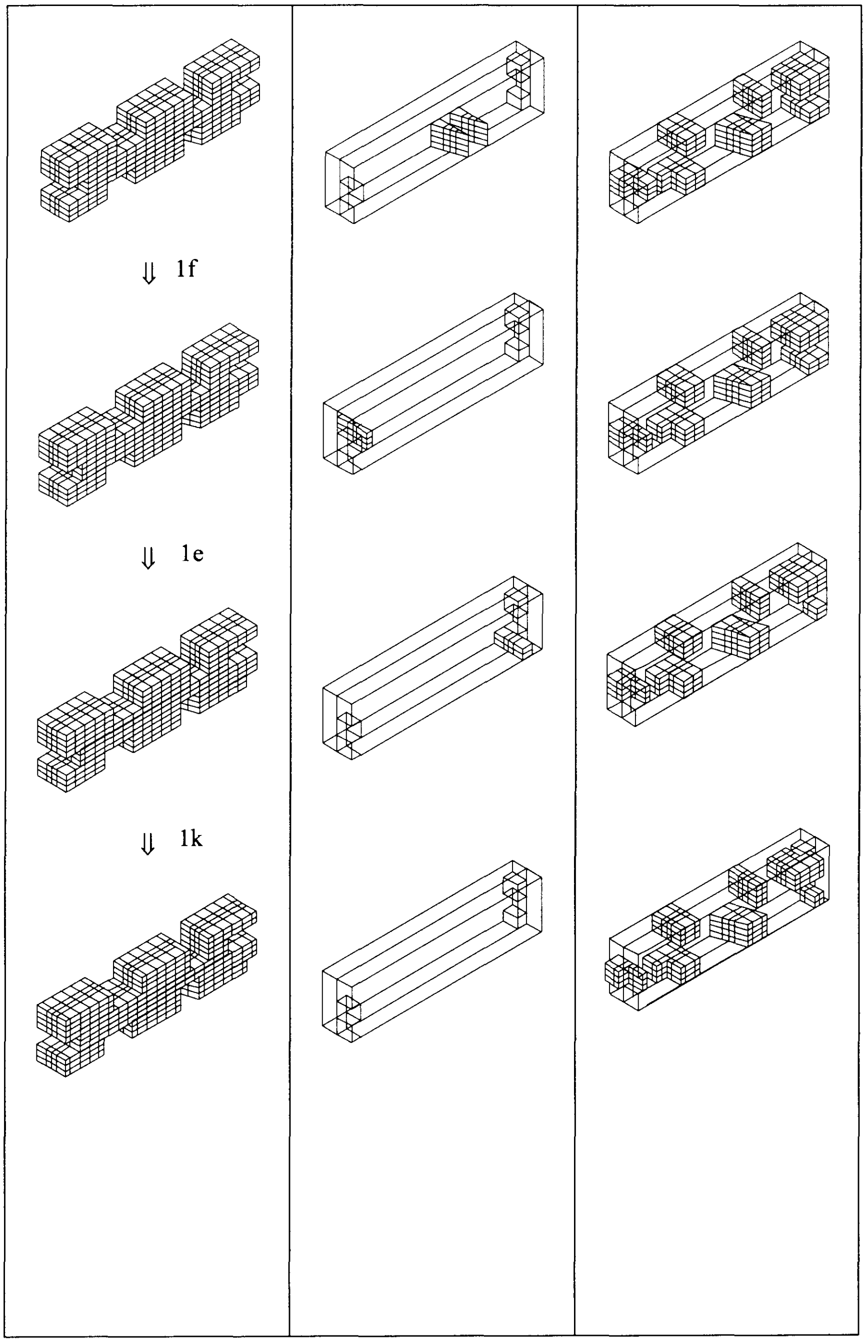
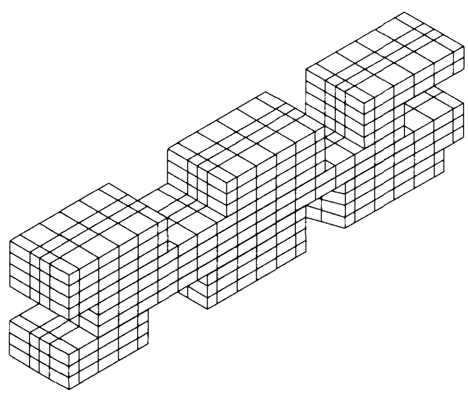
⇓ 1h

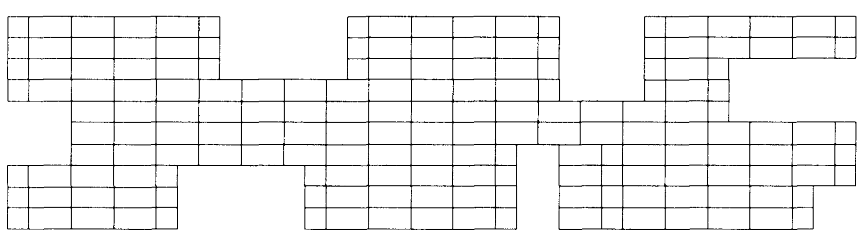

⇓ 1f
⇓ 1e
⇓ 1k

6.2. PERFCON STRUCTURE

The second derivation is an example of deriving an elevation of Simmons Hall. The presented elevation is the one on Vassar Street. The derivation proceeds by arranging the *perfcon* panels at each floor. It is based on rule schema 2, and the descriptions involve lines that are manipulated on the plane. The final shape of the previous stage of the derivation is necessary. It shows the arrangement of the rooms in 3d, in the algebra $U_{33.}$ Accordingly, one specific façade of the derived solid arrangement is going to be the subject of the derivation.

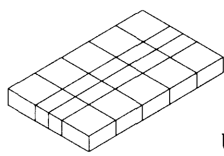
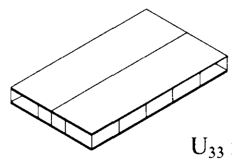
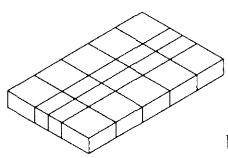
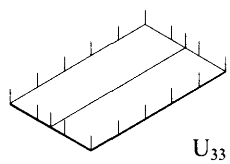


The Vassar Street façade, the derivation of which is presented here in the algebra U_{12} appears in the next diagram,

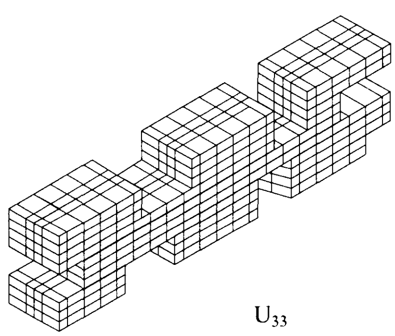


The same divisions of the building mass in units serves the placement of *perfcon* panels in the façade. In this elevation, the divisions are represented by a grid, also in the algebra U_{12}. The representation of the overall building arrangement is slightly modified for the needs of the derivation.

The next two rules transform the solid representation of spatial units, into a representation that includes slabs and a grid. The initial division of the building in spatial units is preserved in the grid that is represented by lines in the algebra U_{13}. And, also each floor includes a slab, which is a solid in the algebra U_{33}. The top floor is substituted by two slabs, while all other floors are substituted by a single slab, and by the necessary grid lines that indicate the divisions in units.

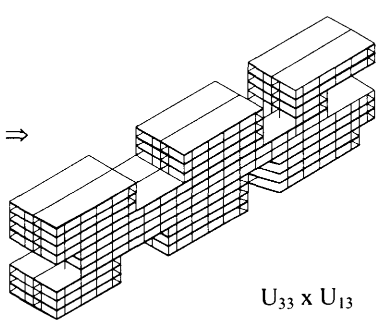
top (10nth) floor		
 U_{33}	→	 $U_{33} \times U_{13}$
all subsequent floors		
 U_{33}	→	 $U_{33} \times U_{13}$

The application of the above two rules changes the representation of the building as it is shown next. The produced representation is in the product algebra ($U_{33} \times U_{13}$).



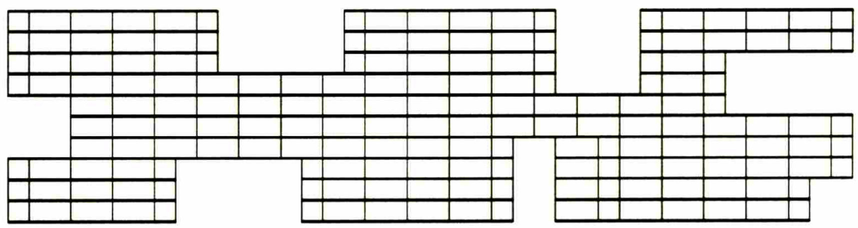
U_{33}

$\Rightarrow$



$U_{33} \times U_{13}$

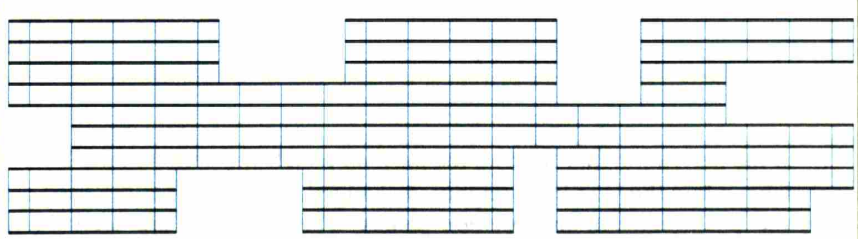
Accordingly, the elevation of our interest obtains the following description, in the algebra (U_{12})



The above description includes the slabs and the building grid. First, the horizontal double lines at each floor represent the building slabs, in elevation. And second, horizontal and vertical grid lines correspond to the division of the floors into units. Both the slabs and the grid are used in the placement of the *perfcon* panels, in the façade.

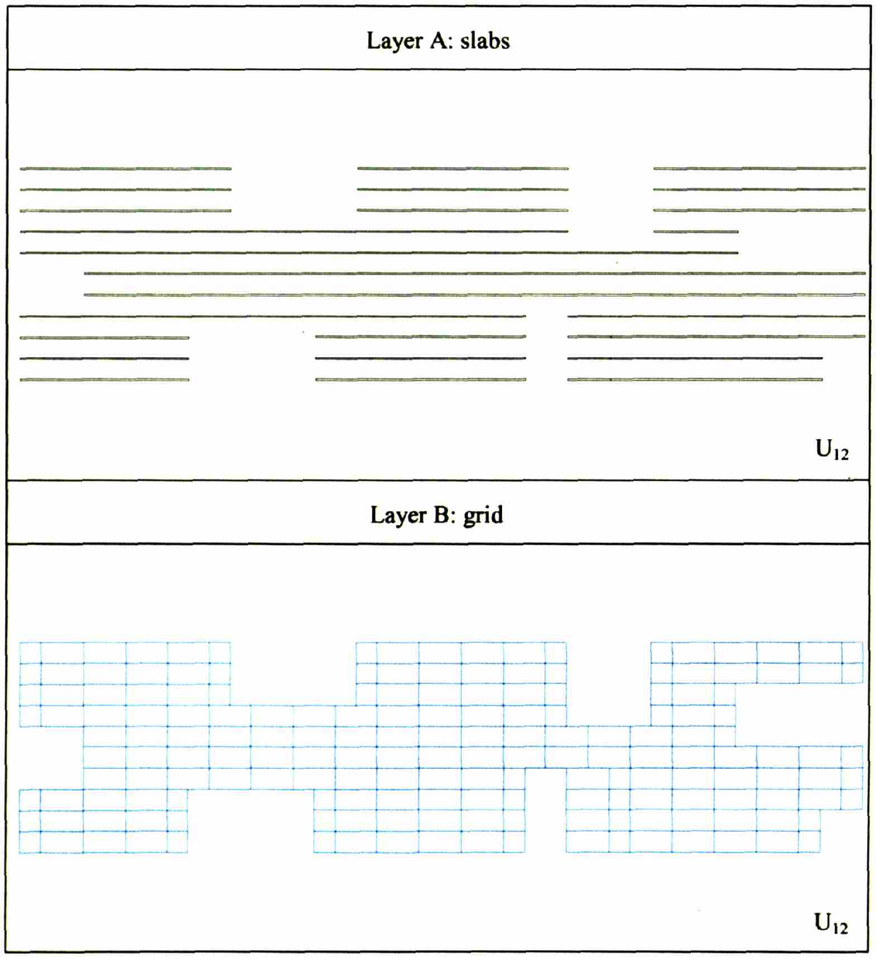
In order to make the presence of these two graphic entities more visible, the slabs and the grid are placed into two different layers that also have different colors. The slabs are in layer A and are represented by black lines, in the algebra U_{12}. The grid lines, also in the algebra U_{12} are moved into a second layer B and become blue. The overall description becomes a shape in the $U_{12} \times U_{12}$ algebra.

Description A: slab and grid lines



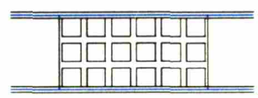
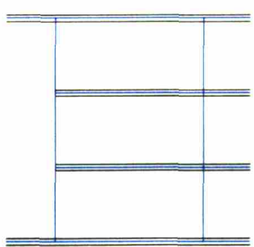
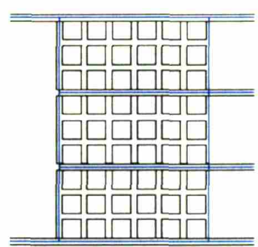
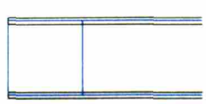
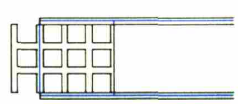
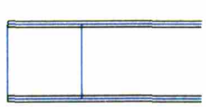
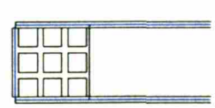
$U_{12} \times U_{12}$

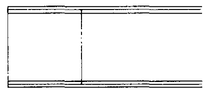
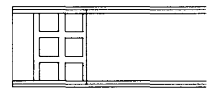
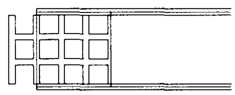
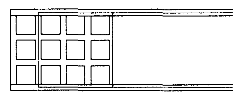
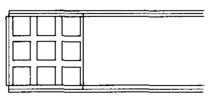
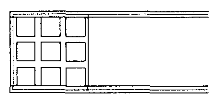
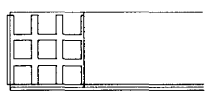
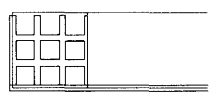
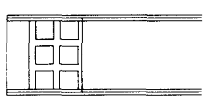
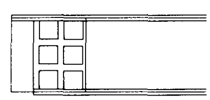
The slabs and the grid lines as they appear on the two layers are presented next. The distinction in two layers and two different colors, black for the slabs and blue for the grid lines, serves only presentation purposes.



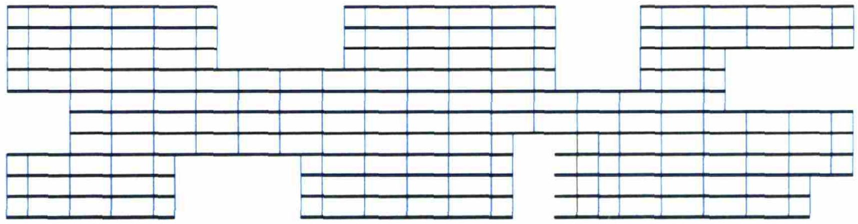
In the derivation that follows the *perfcon* panels are added in the layer A, of slabs, while the grid lines remain unchanged. The grid lines are used only for the positioning of the *perfcon* panels. The rules that are used for the placement and the modification of the *perfcon* panels are instances, or extensions of the parametric rule schema 2. They are presented next in two groups.

The first group of nine rules 2a-2i contains rules for the placement of *perfcon* panels, and the extension of slabs when this becomes necessary. Rule 2a places all the 3 x 6 *perfcon* panels except from the corner panels. Rule 2b is used only once, to place three 3 x 6 corner panels, in three subsequent floors. The rules 2c, 2d and 2e place corner panels, which are (3 x 4), (3 x 3) and (3 x 2), respectively. The rules 2f, 2g, and 2h extend the slabs in the corners of the building and align them with the existent panels. Finally, the rule 2i aligns a slab by cutting some part of it.

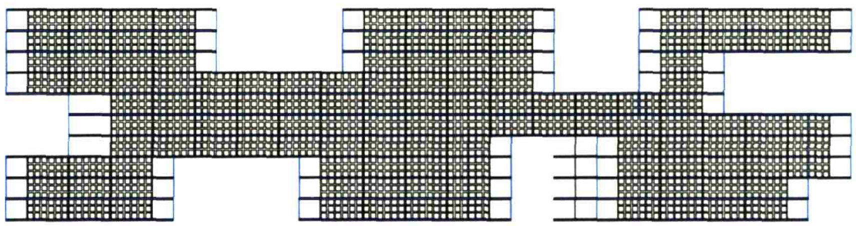
Rules for the placement of *perfcon* panels			
2a		→	
2b		→	
2c		→	
2d		→	

2e		→	
2f		→	
2g		→	
2h		→	
2i		→	

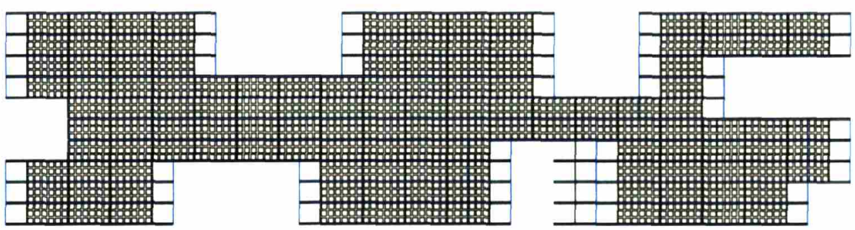
The nine rules are applied deterministically in the given order. This order does not reflect the actual steps of the design process but captures the different kinds of actions that are related to the arrangement of the *perfcon* panels. The derivation begins with the placement of all the (3 x 6) *perfcon* panels, continues with the placement of the three subsequent corner (3 x 6) panels, and ends with the alignment of the slabs.



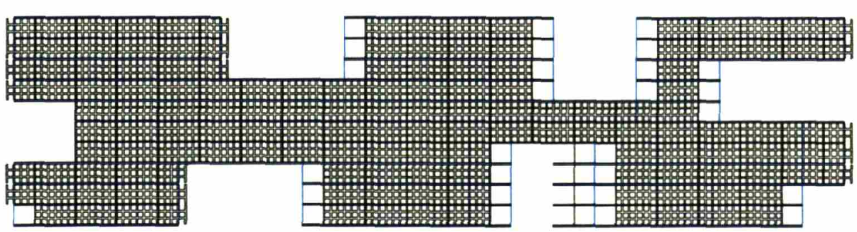
⇓ 2a



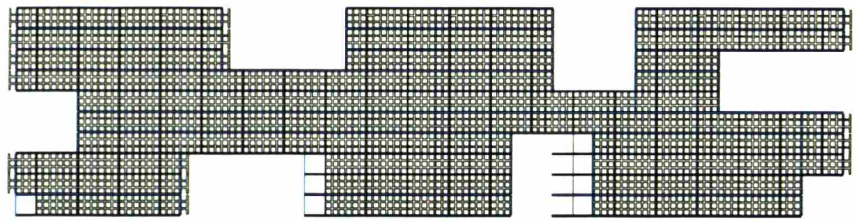
⇓ 2b



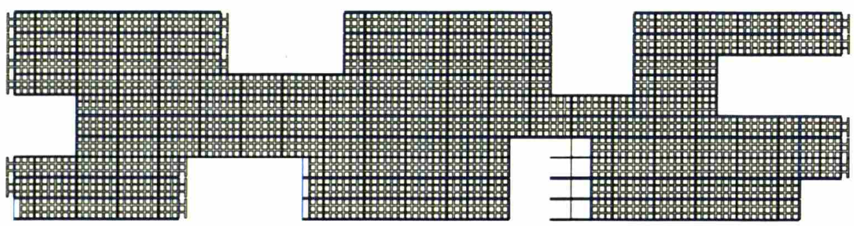
⇓ 2c



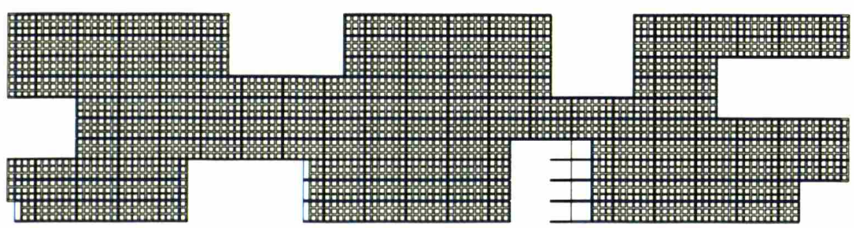
⇓ 2d



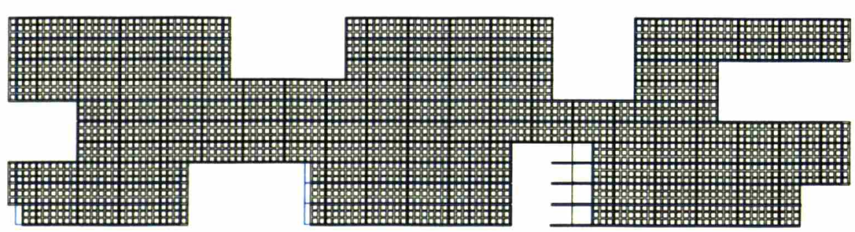
$\Downarrow$ 2e



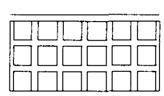
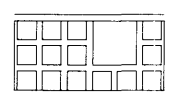
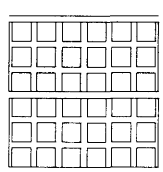
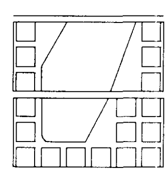
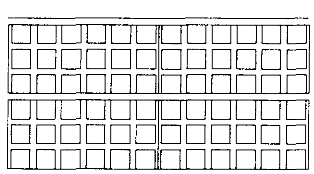
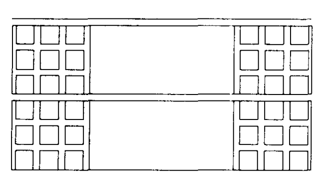
$\Downarrow$ 2f

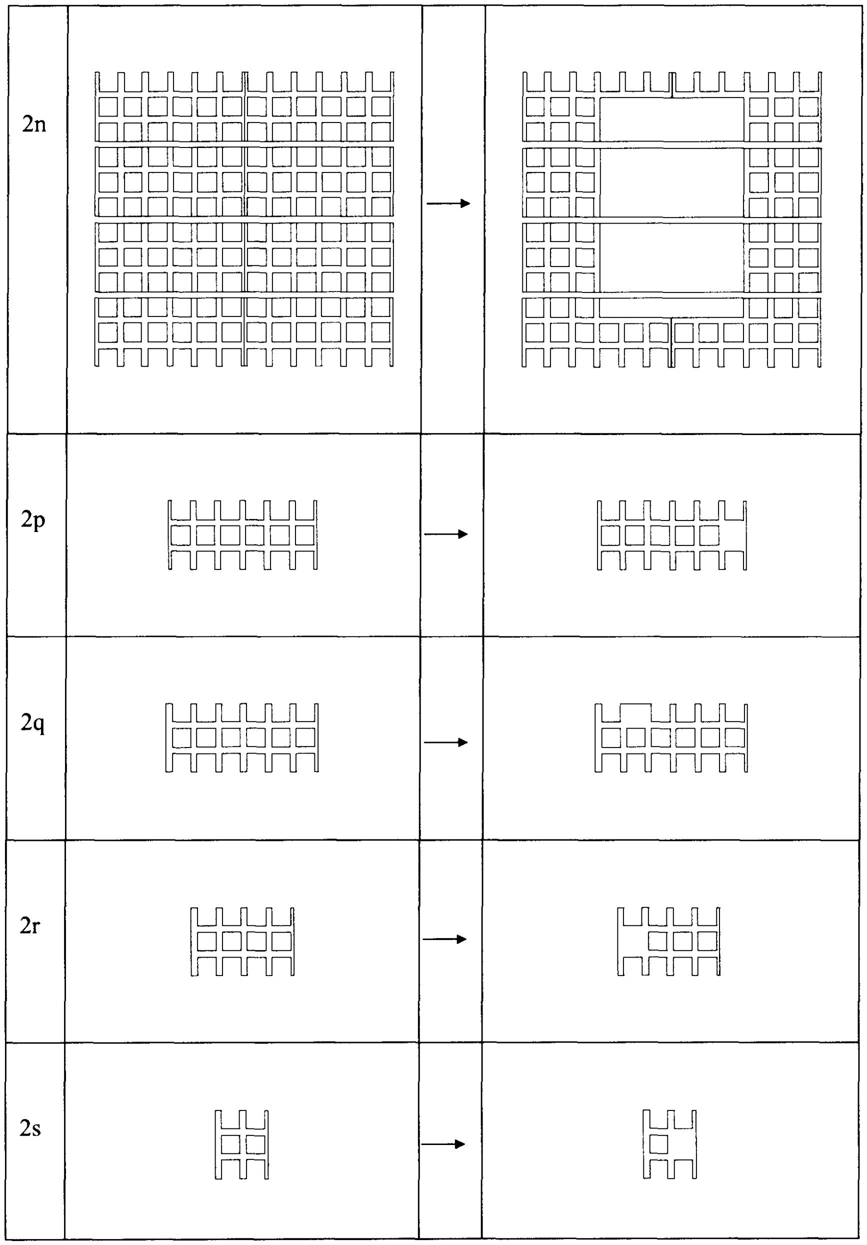


$\Downarrow$ 2g, 2h, 2i



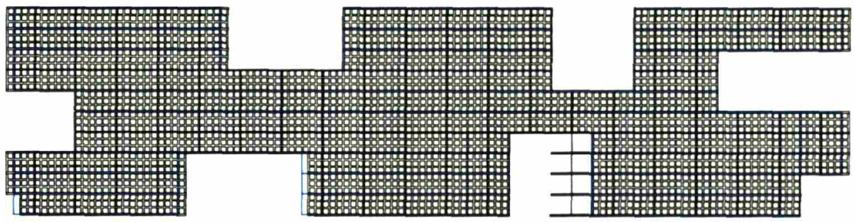
The second group of nine rules 2a-2i contains rules that apply after the placement of the *perfcon* panels in the façade. The rules are used for the modification of the *perfcon* panels: the creation of larger openings, within the *perfcon* structure, or the creation of solid surfaces, instead of windows, when this is required for structural support. Rule 2j creates a large square window within a common 3 x 6 *perfcon* panel. Rule 2k is used to open a curved-shape window that involves two 3 x 6 *perfcon* panels placed one over the top of the other. The parametric rule 2m creates a large opening of rectangular shape that is extended in two floors. The parametric rule 2n creates even larger rectangle openings that may extend to more than two floors. The rules 2p, 2q, 2r and 2s are used to create solid concrete surfaces instead of windows. The parametric rule 2p places a concrete patch inside a common 3 x 6 *perfcon* panel. The parametric rule 2q places a concrete patch inside a corner 3 x 6 *perfcon* panel. Finally, the parametric rules 2r and 2s eliminate windows in 3 x 4 and 3 x 2 panels, respectively.

Rules for the modification of *perfcon* panels			
2j		→	
2k		→	
2m		→	

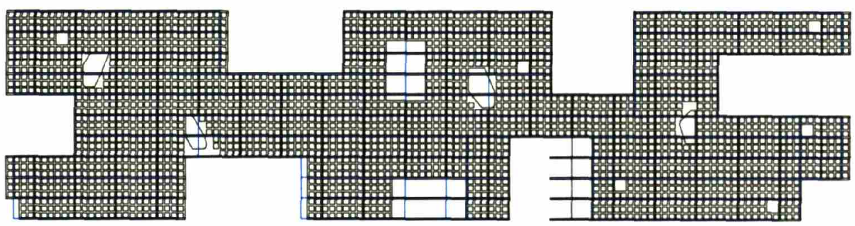


The eight rules are applied non-deterministically independently of order. In the example, the derivation begins with the placement of all the large openings in the elevation, according to the parametric rules 2j, 2k, 2m, 2n.

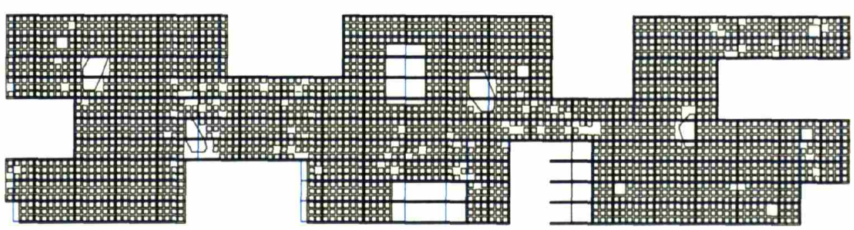
Then, it continues with the placement of concrete blocks that eliminate a number of windows, by applying the rules 2p, 2q, 2r, 2s.



⇓ 2j, 2k, 2m, 2n

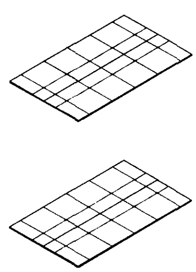
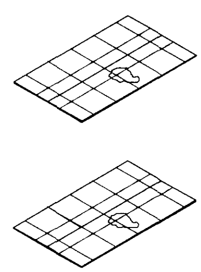
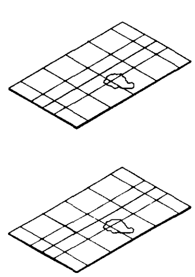
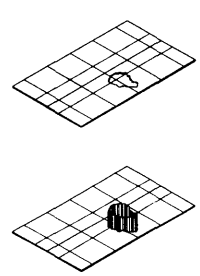


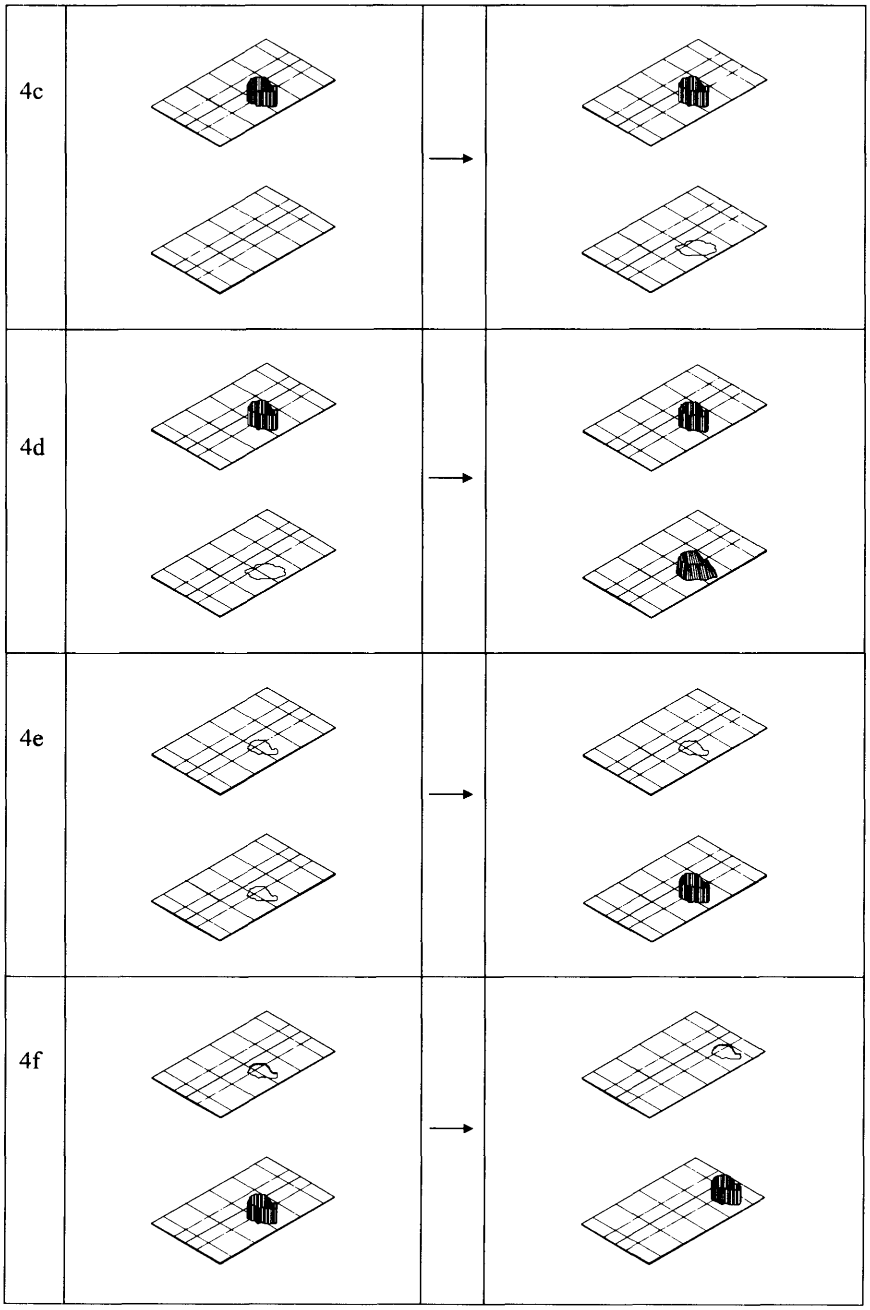
⇓ 2p, 2q, 2r, 2s



6.3. VERTICAL POROSITY

Third is an example of generating vertical cavities. It corresponds to the application of vertical porosity (rule schema 4). The computations affect the composition quantitatively and qualitatively: They change the area, volume and the form of the building. Four rules 4a-4d are provided as a way to describe the actions producing vertical cavities, in the algebra $<U_{13}xU_{33}>$, in axonometric. Rule 4a deals with the placement of the boundary curves of the cavities, which are lines in the algebra U_{13}. These curves are placed on two parallel slabs of two subsequent floors while taking into account the underlying grid. Rule 4b is used to generate the solid surface that connects the two boundary curves and, also to create an opening on the slab that lies above. Rules 4c and 4d are used to extent an existent cavity. Rule 4e is like rule 4b, but it does not create an opening on the upper slab. Finally, the rule schema 4f is used to change the position of a cavity. It is not used in the next derivation, which derives the cavities in their final positions, but it would be necessary in an actual design process. Each cavity is a free element with no structural significance. The alternative forms are placed on the building grid, under the provision that each of the three fraternity houses would have at least one vertical cavity.

Rules for cavities			
4a		→	
4b		→	


4c
4d
4e
4f

A basic assumption is that the desired goal is to have cavities penetrating the building from top to bottom. This allows more air and light circulation in the interior of the building, which is the main reason for introducing the vertical cavities. But the "top-to-bottom" penetration was not finally implemented for reasons related to the fire-safety regulations of Massachusetts. In many cases the cavities end, and begin in a different position, without retaining their continuity from the building top. These cavities may have common sides with the *perfcon* structure, or may be "blind" openings of interior circulation.

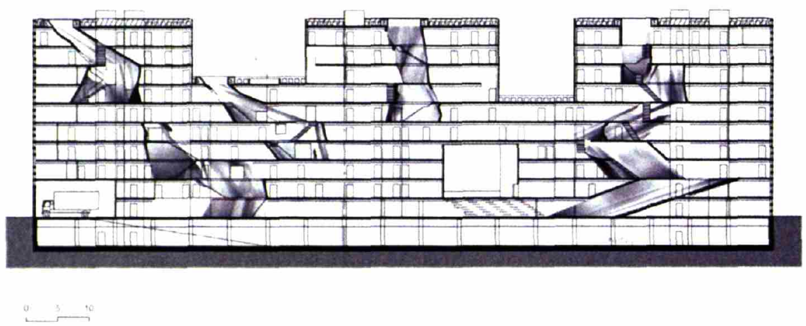


Figure 21. In the final implementation cavities are not continuous (building section by Steven Holl Architects, NY.).

The presented derivation does not show the actual steps of the design process. It captures abstractly the actions involved in the generation of cavities in their final positions.

The derivation begins from the top slabs and proceeds to the lower ones by adding all the cavities floor by floor, in their final positions. It involves shapes in the algebra $<U_{13}\ U_{33}>$. The initial shape includes the slabs represented as solids in the algebra U_{33}, and the grid in the algebra U_{13}.

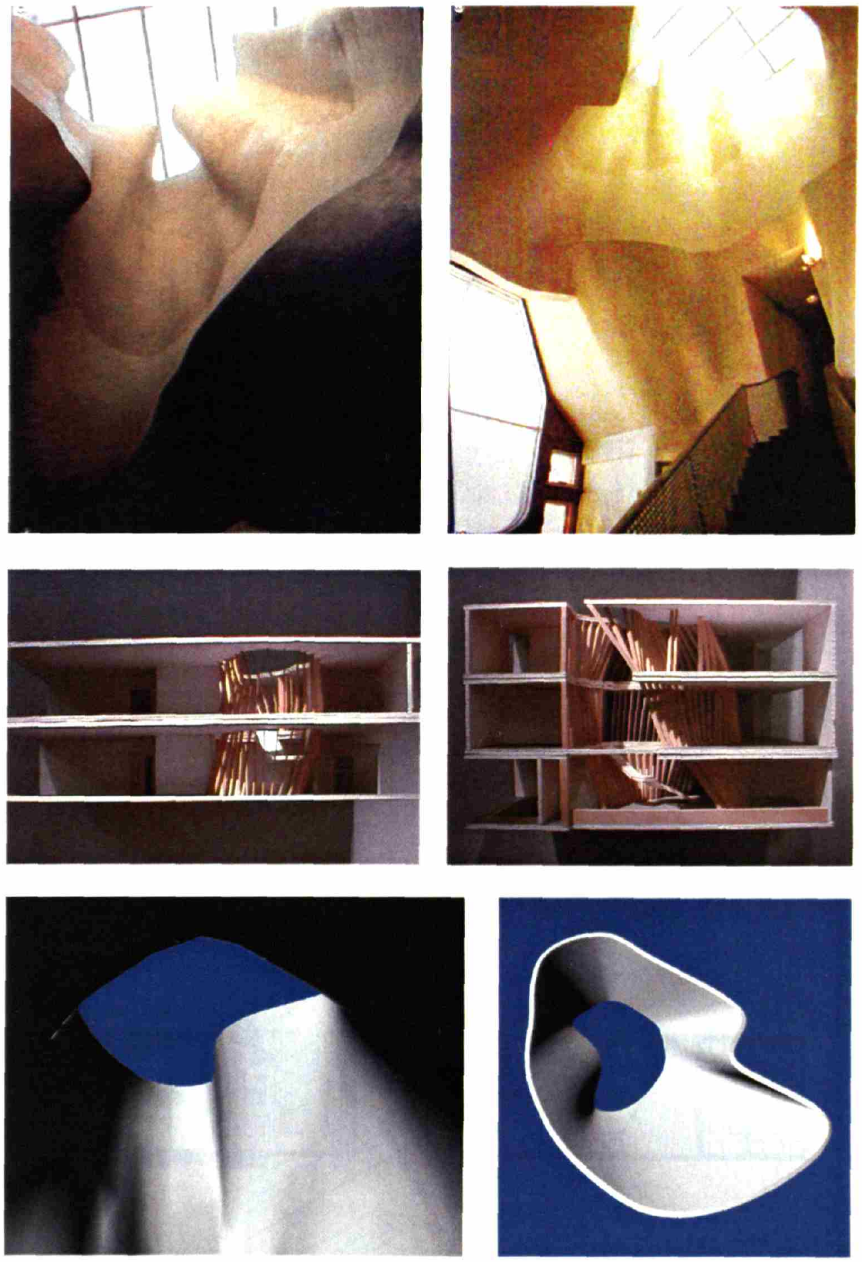


Figure 22. Examples of cavities extending from the building top to several floors (above, center, models and photos by Steven Holl Architects, NY). A computer generated cavity-shape (below).

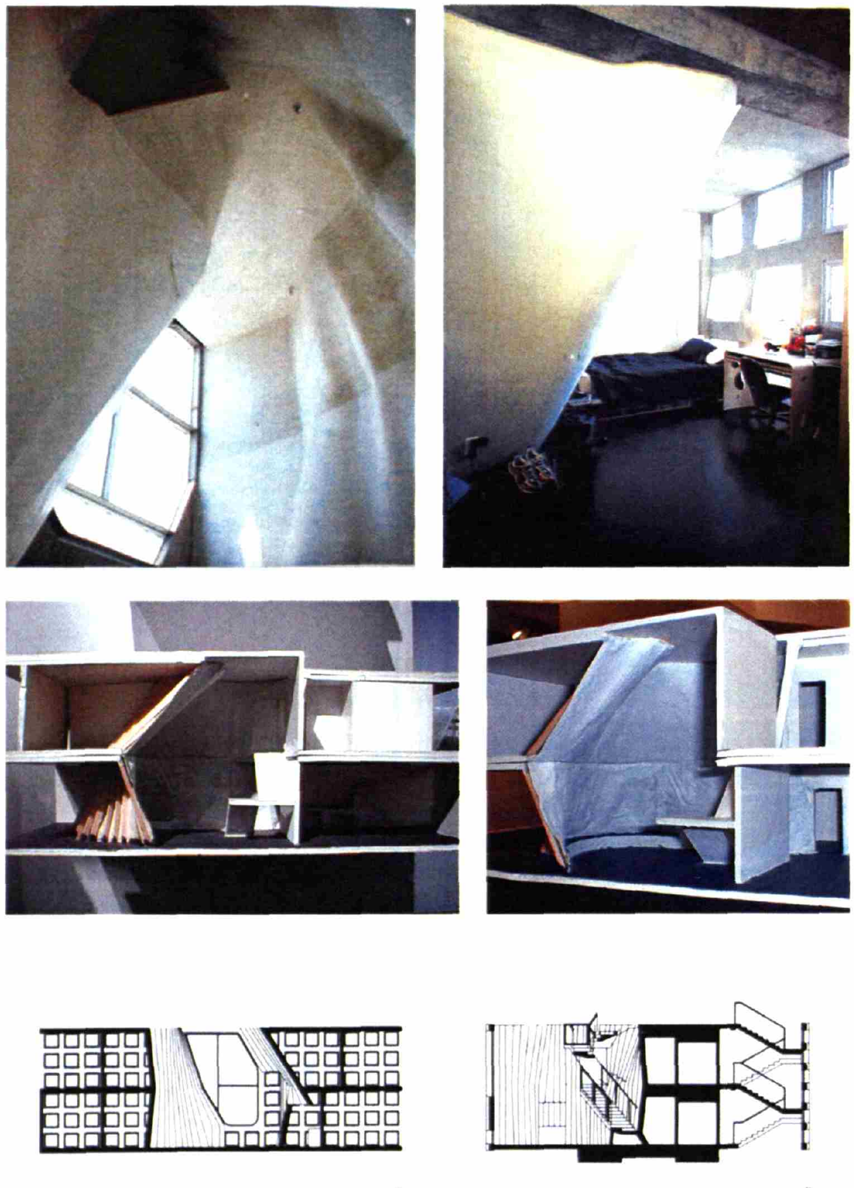
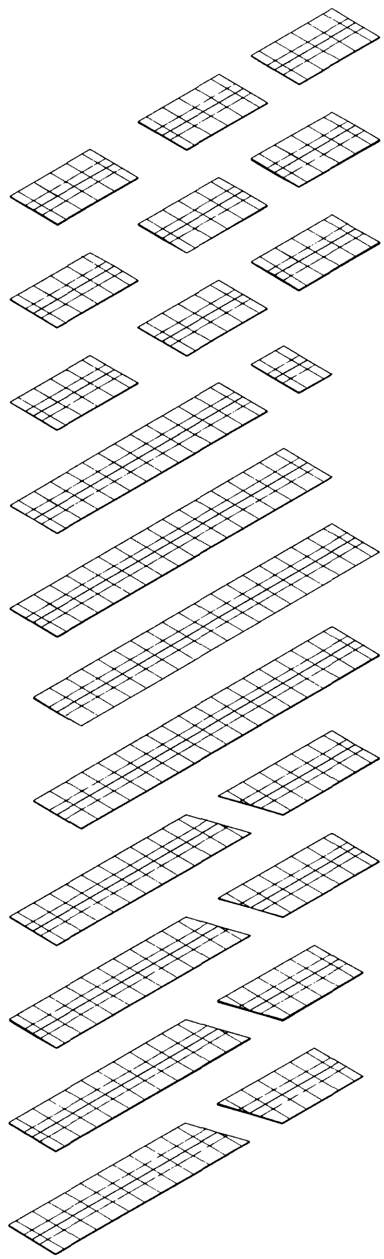
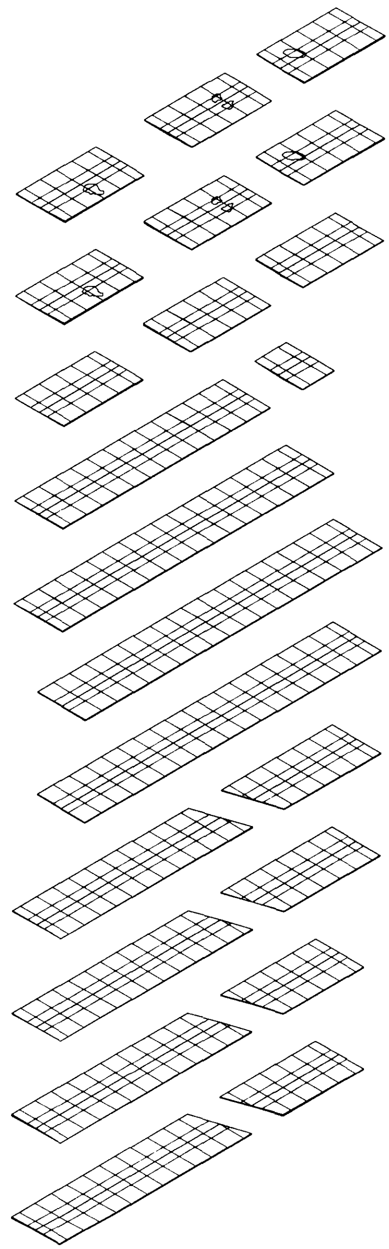


Figure 23. A blind cavity extending in two floors, attached to the *perfcon*, with a large window. Models – photos, Steven Holl Architects, NY.

derivation	rules
	

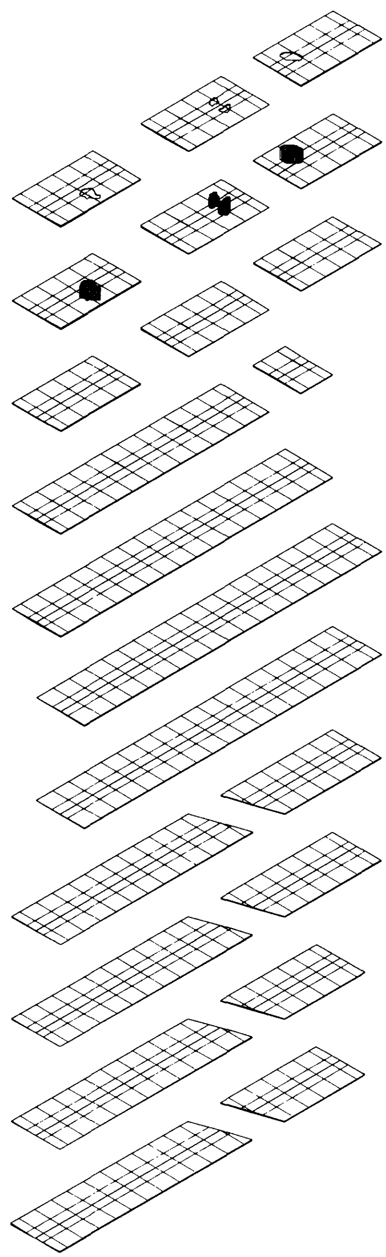


4a

4a

4a

4a



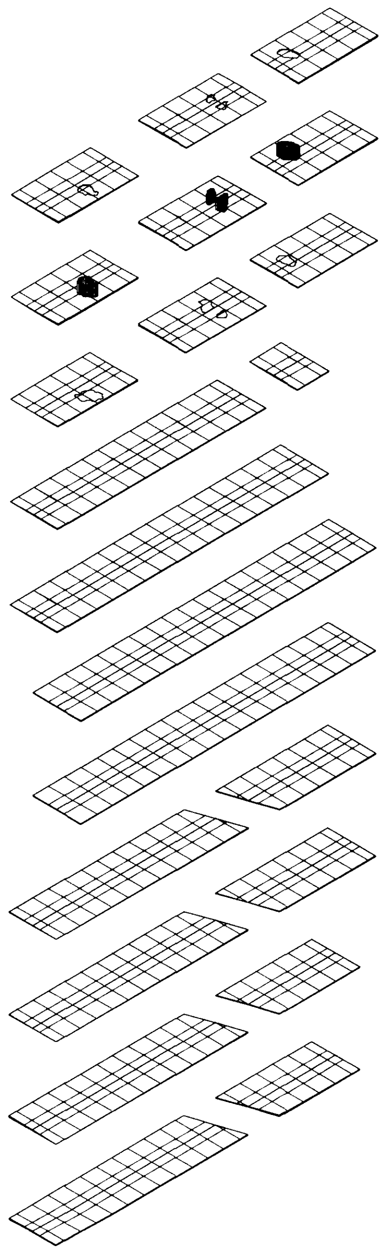
4b

4b

4b

4b

4a



4c

4c

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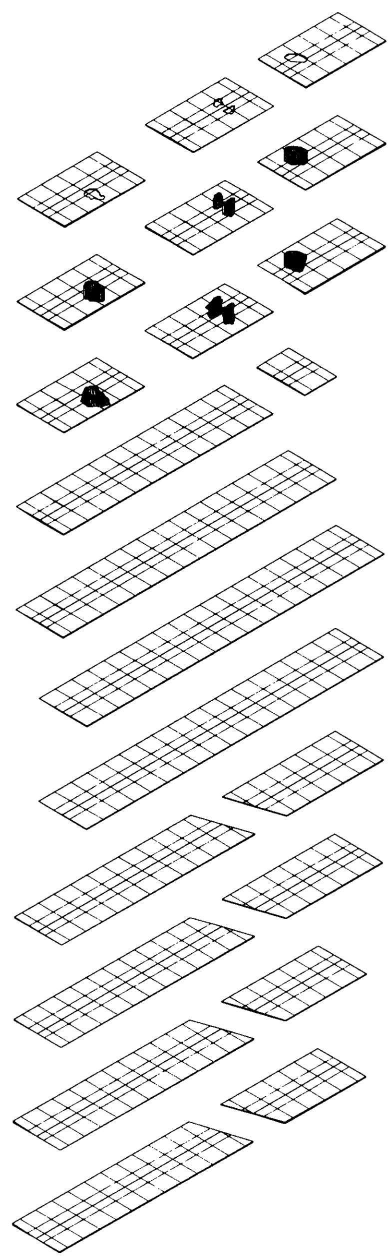

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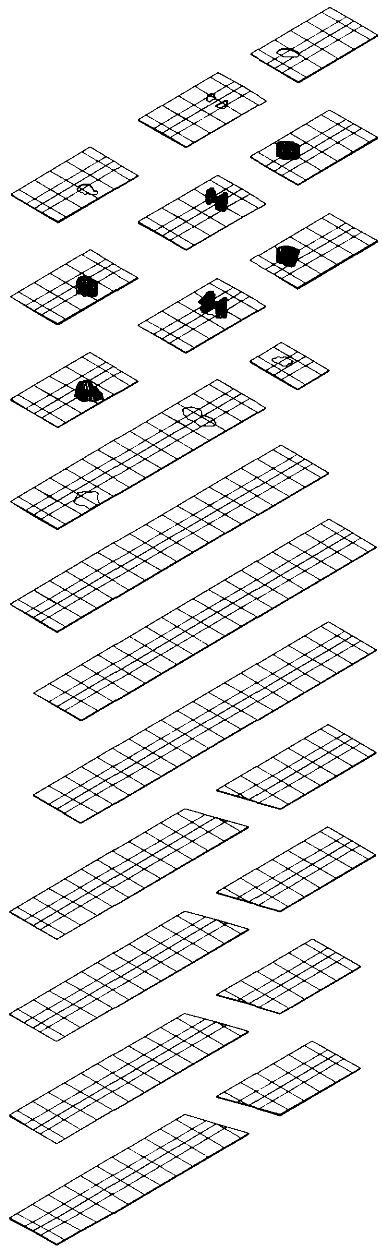

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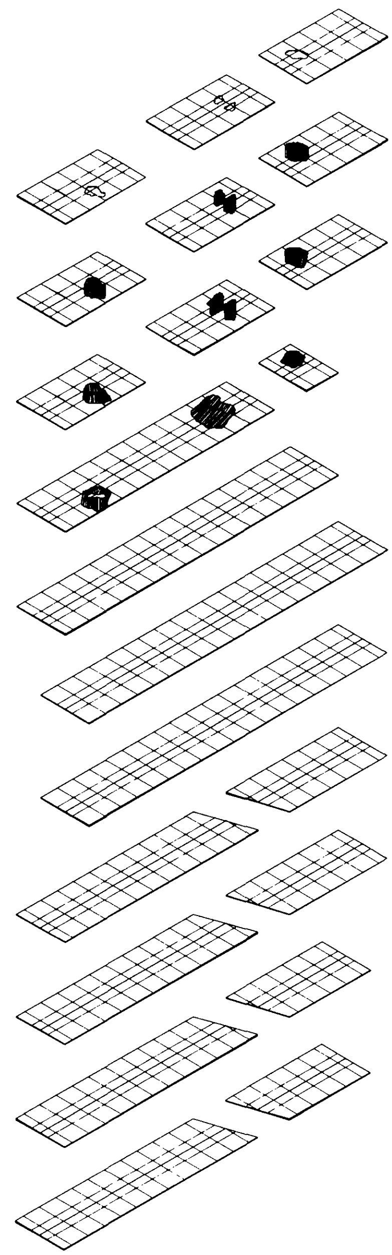

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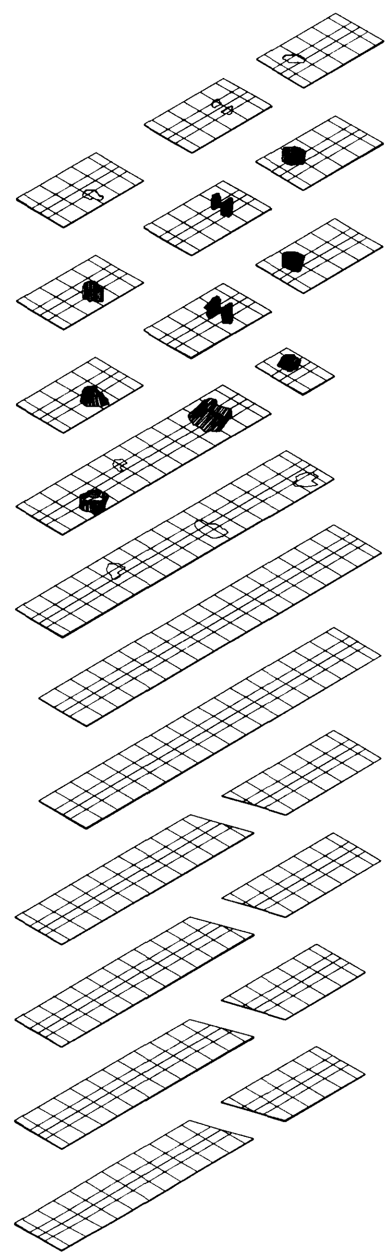

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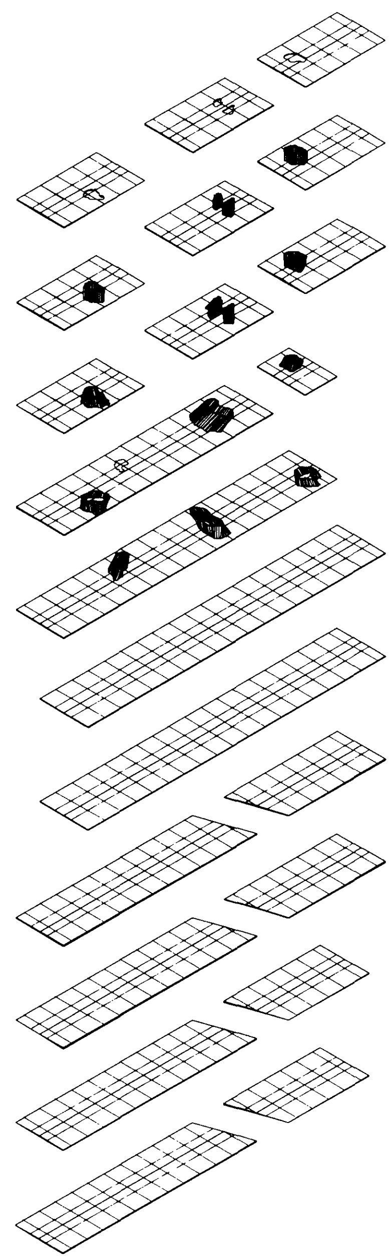

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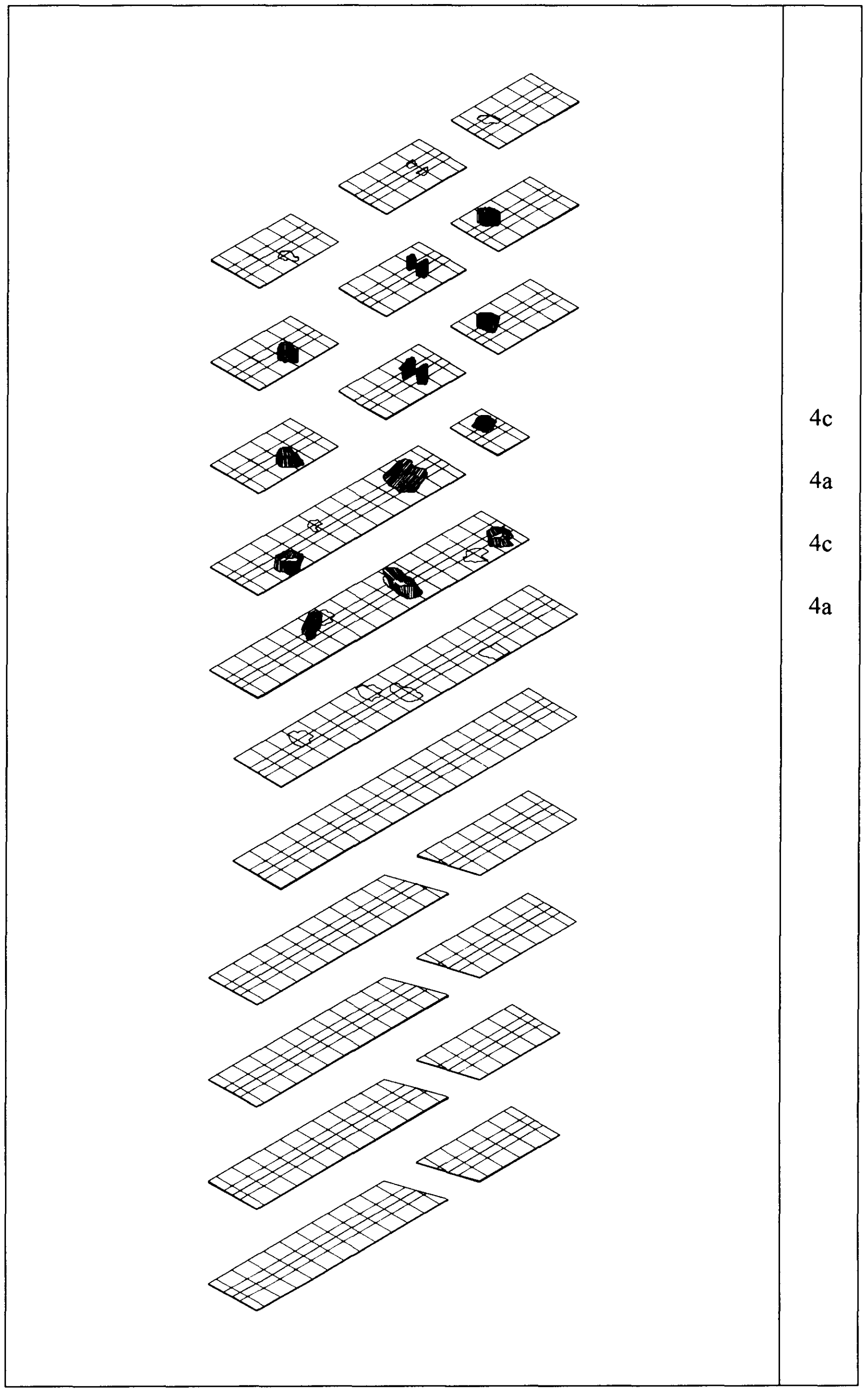
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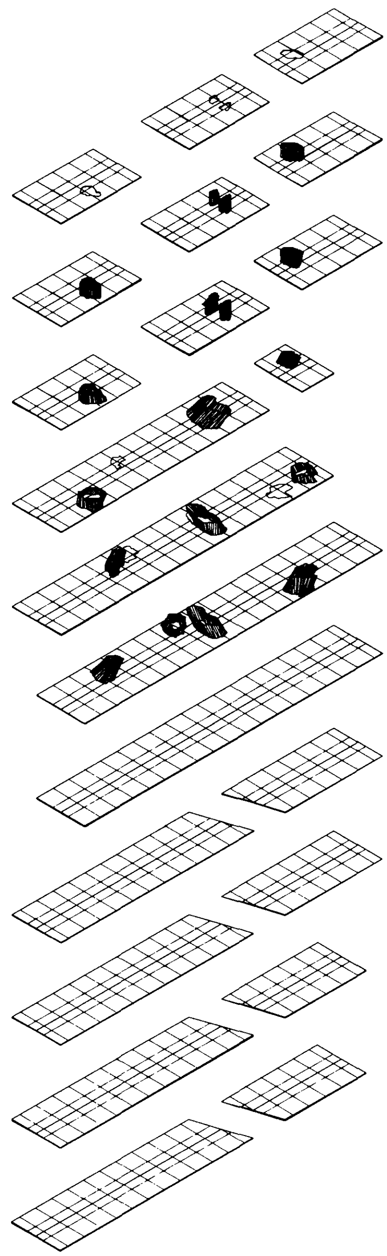

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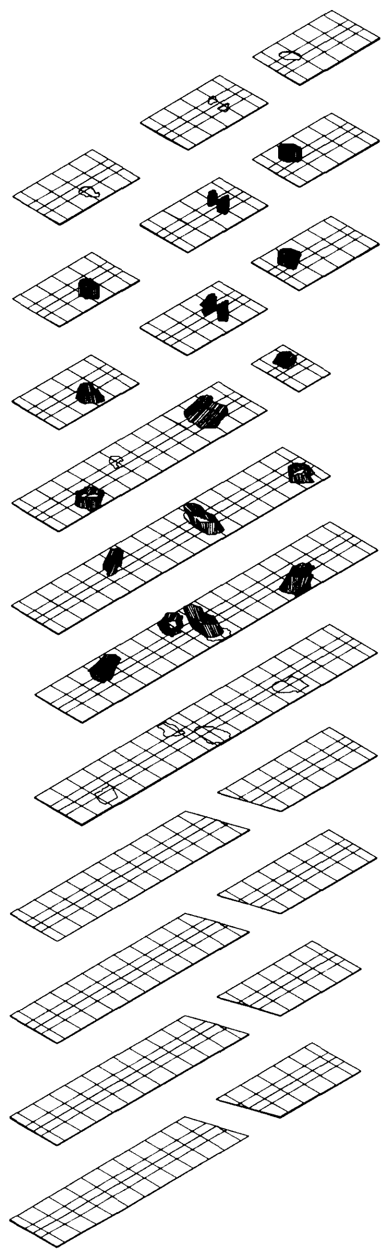

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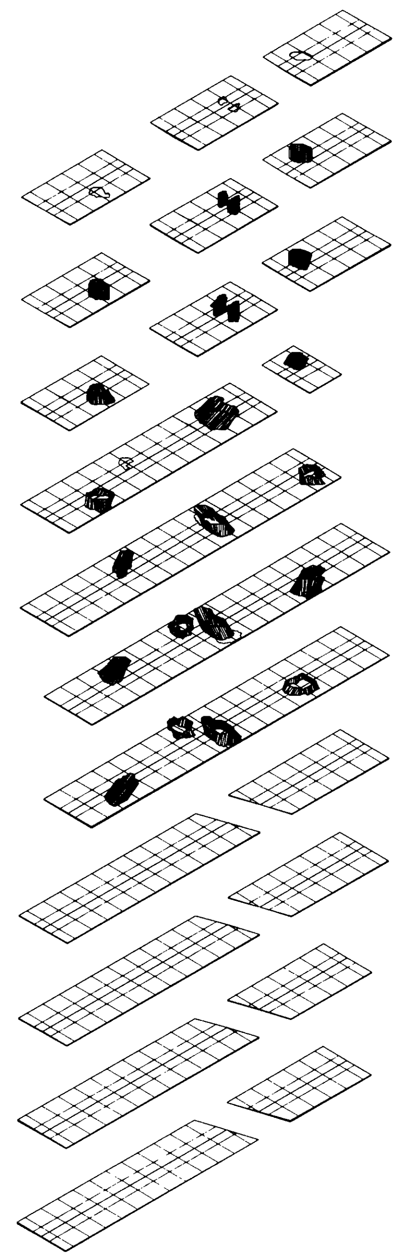

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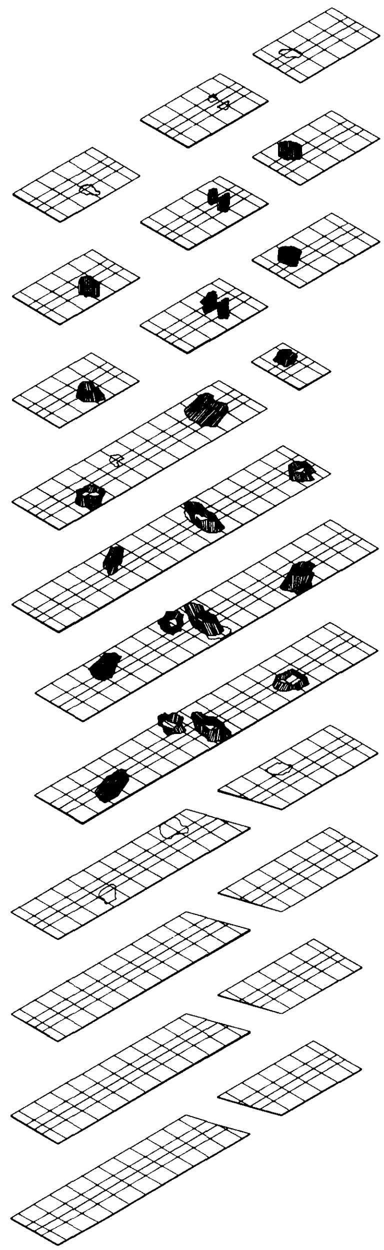

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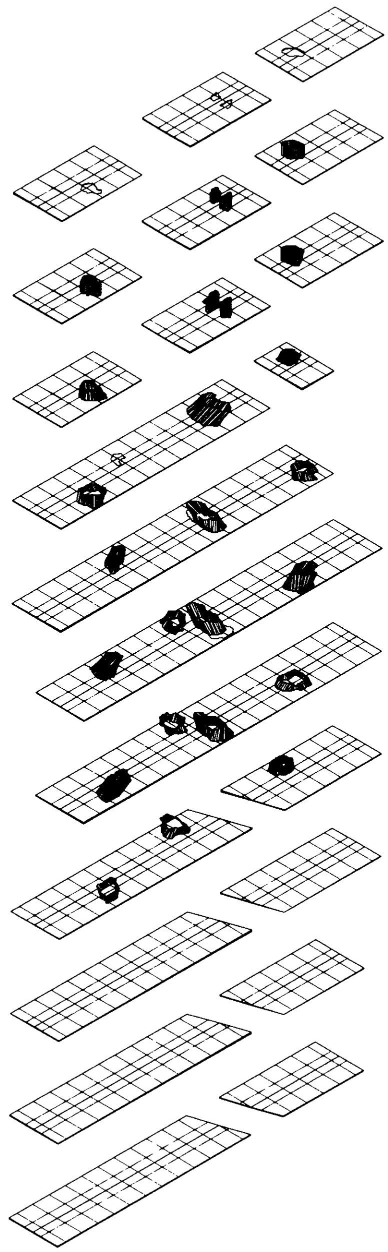

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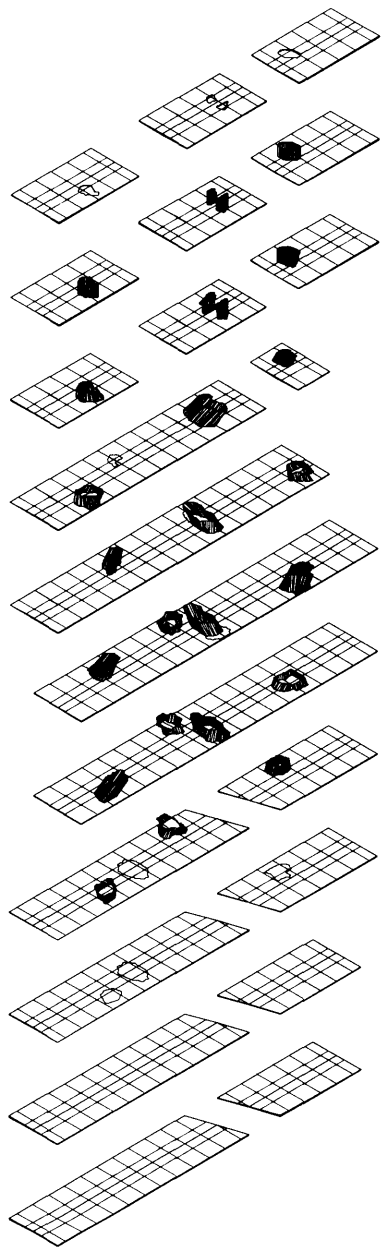

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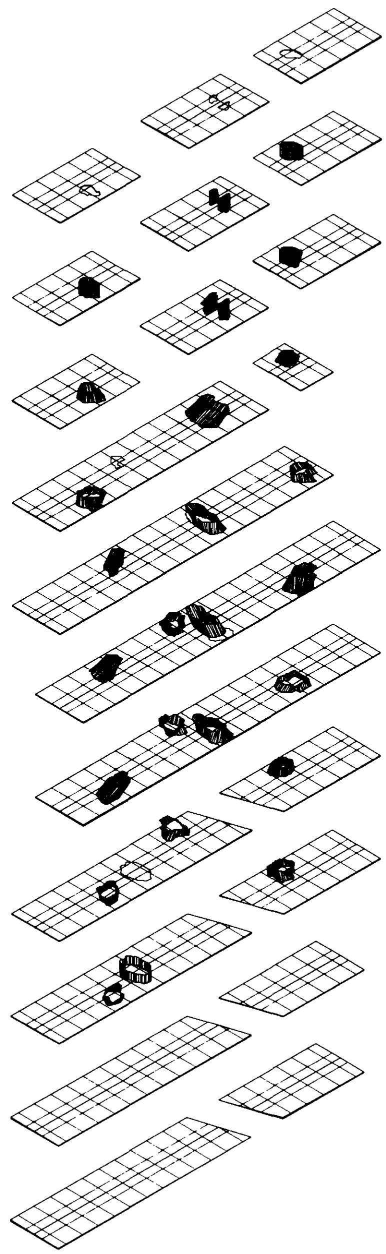

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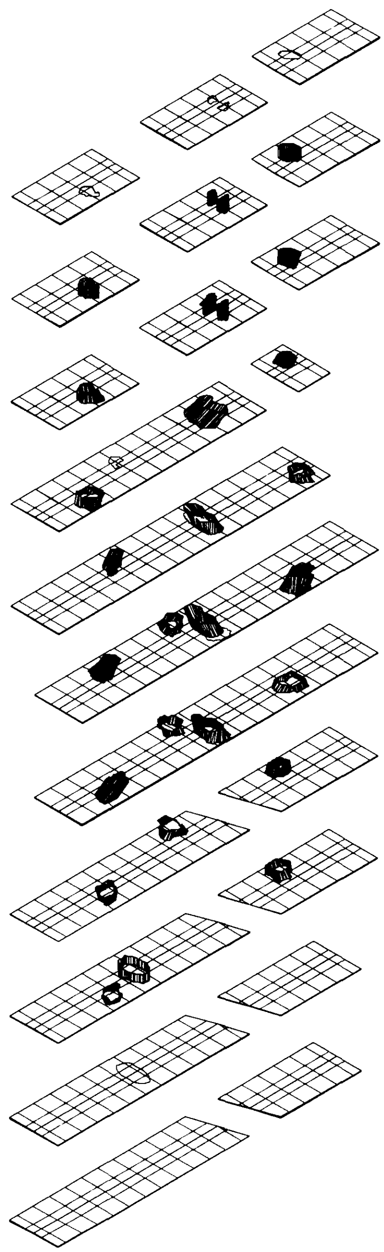

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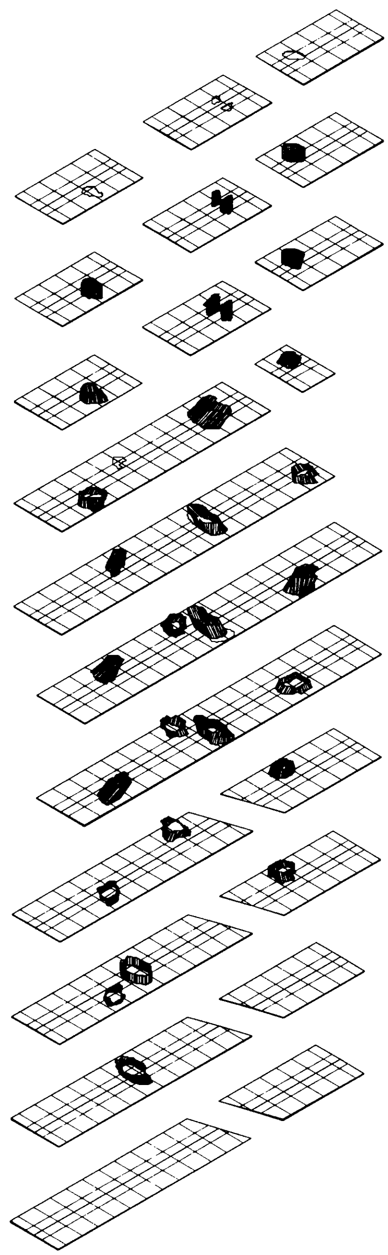

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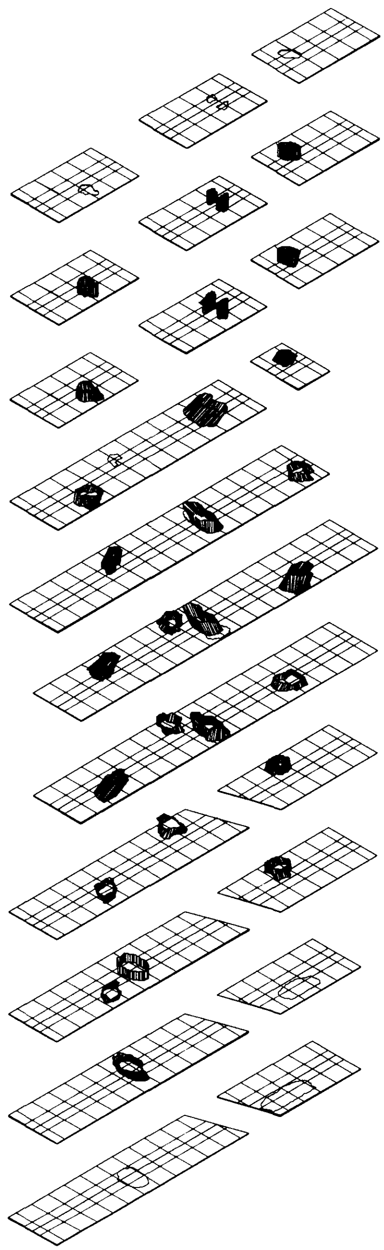

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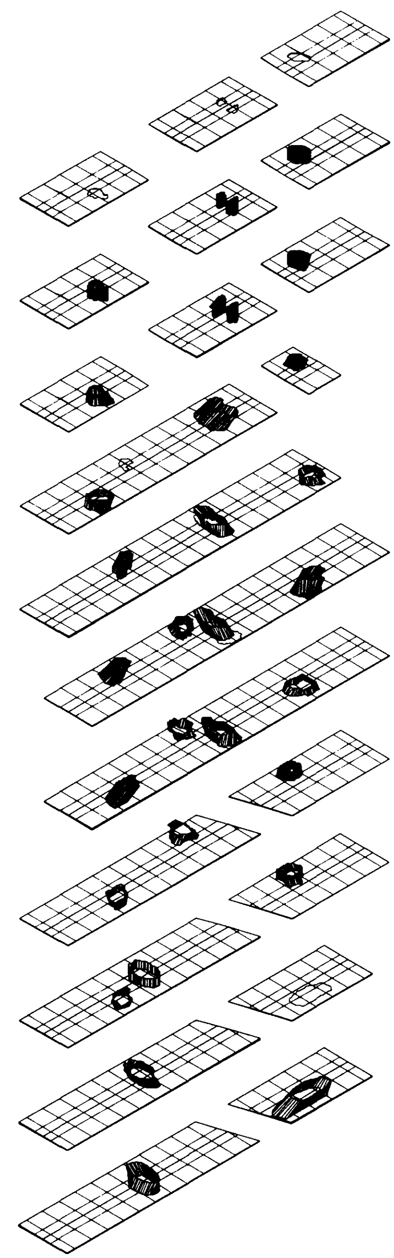

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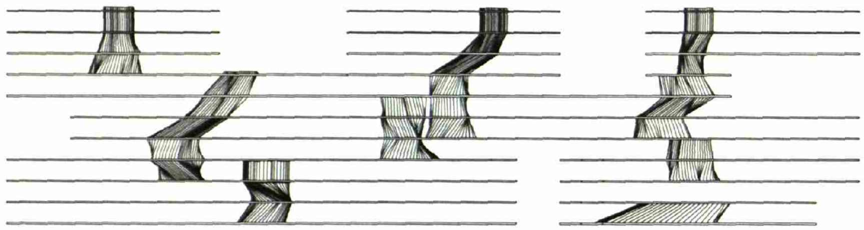
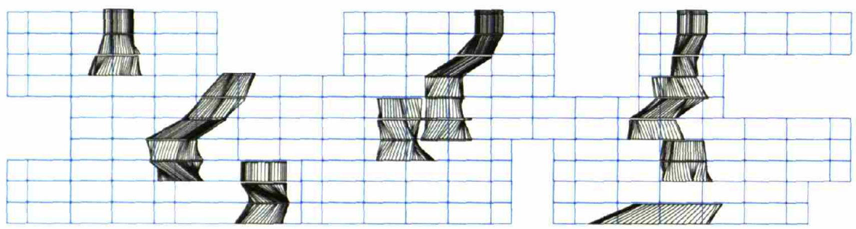
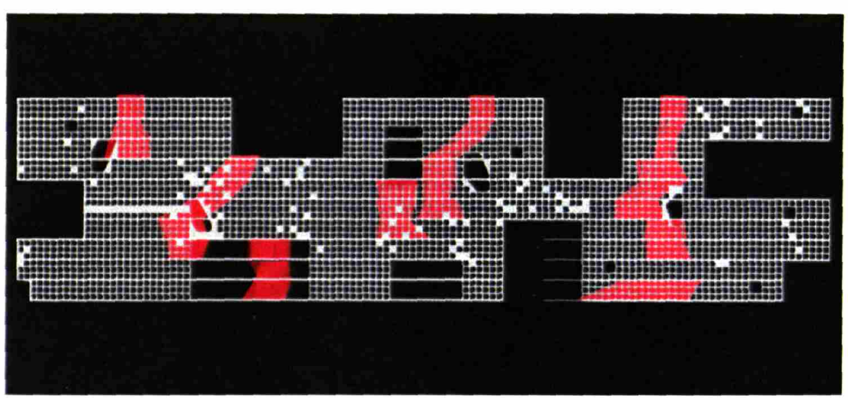
The next illustration presents the distribution of vertical cavities in section, in relation to the horizontal slabs of the building.



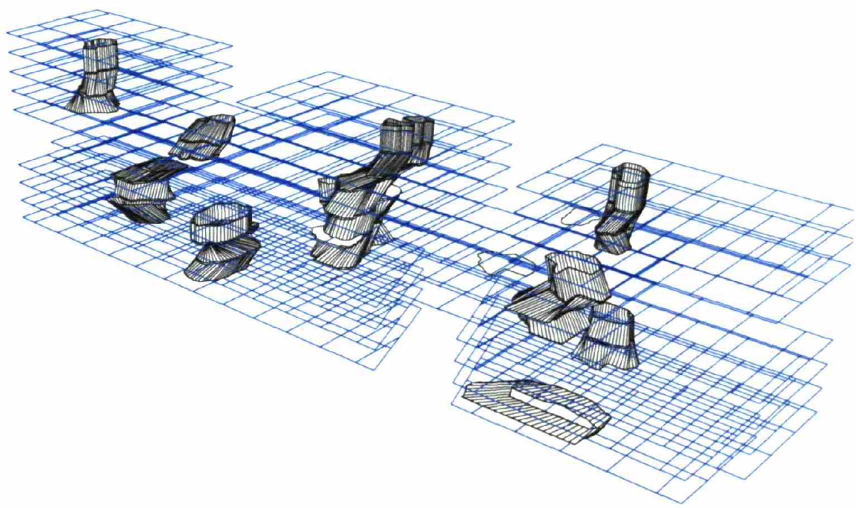
The next illustration presents in elevation the distribution of vertical cavities in relation to the underlying grid of the building



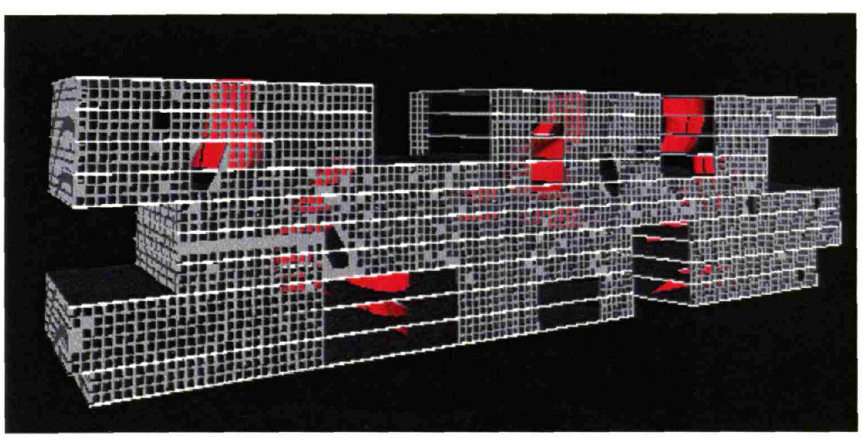
The next illustration is the result of the combination of the three implemented kinds of porosity: subtraction of volumes, perforated façade, and vertical porosity. The illustration presents a "hypothetical" Vassar Street elevation.



The next view presents how the free geometry of the vertical cavities is embedded within the grid geometry of the floor plans, in 3d.



The next conceptual illustration presents how the vertical cavities penetrate the building slabs and how the orthogonal building grid, implemented by the *perfcon* structure coexists with the vertical free form cavities.



6.4. RESULTS

The four kinds of porosity, described in early sketches and models, lead to four parametric rule schemata that express the concept of porosity as this is applied at Simmons Hall. In the later phases of the design and in the construction phase, several revisions became necessary due to imposed programmatic, construction, and other limitations. The results of the four rule schemata of porosity can be compared with the implemented design, and the final results can be examined in comparison to the design intentions.

The large recesses of building mass were implemented with less intensity from what was originally intended. Building masses were added in the places of the openings. However, the approach remained consistent with the initial conceptual framework and can still be expressed by rule schema 1.

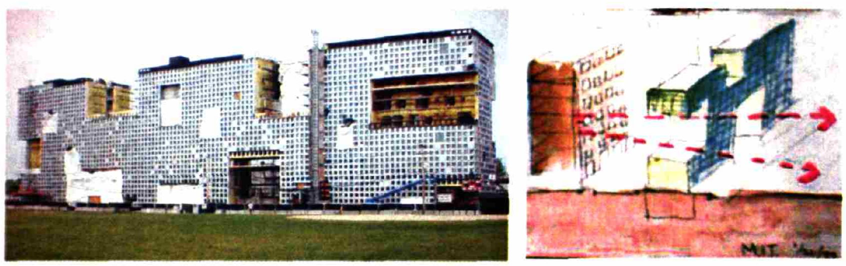


Figure 24. Implementation of Simmons Hall (left), and conceptual schema (right), as it is depicted in an early sketch by Steven Holl.

Also, the results of "diagonal porosity", expressed by rule schema 2, were entirely erased from the results of the process, as extra rooms occupied the produced recesses. Diagonal porosity intended to translate half of the building mass along the long axis of the building.

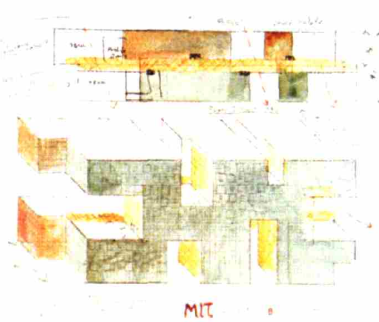


Figure 25. Diagonal porosity, as depicted in an early sketch by Steven Holl.

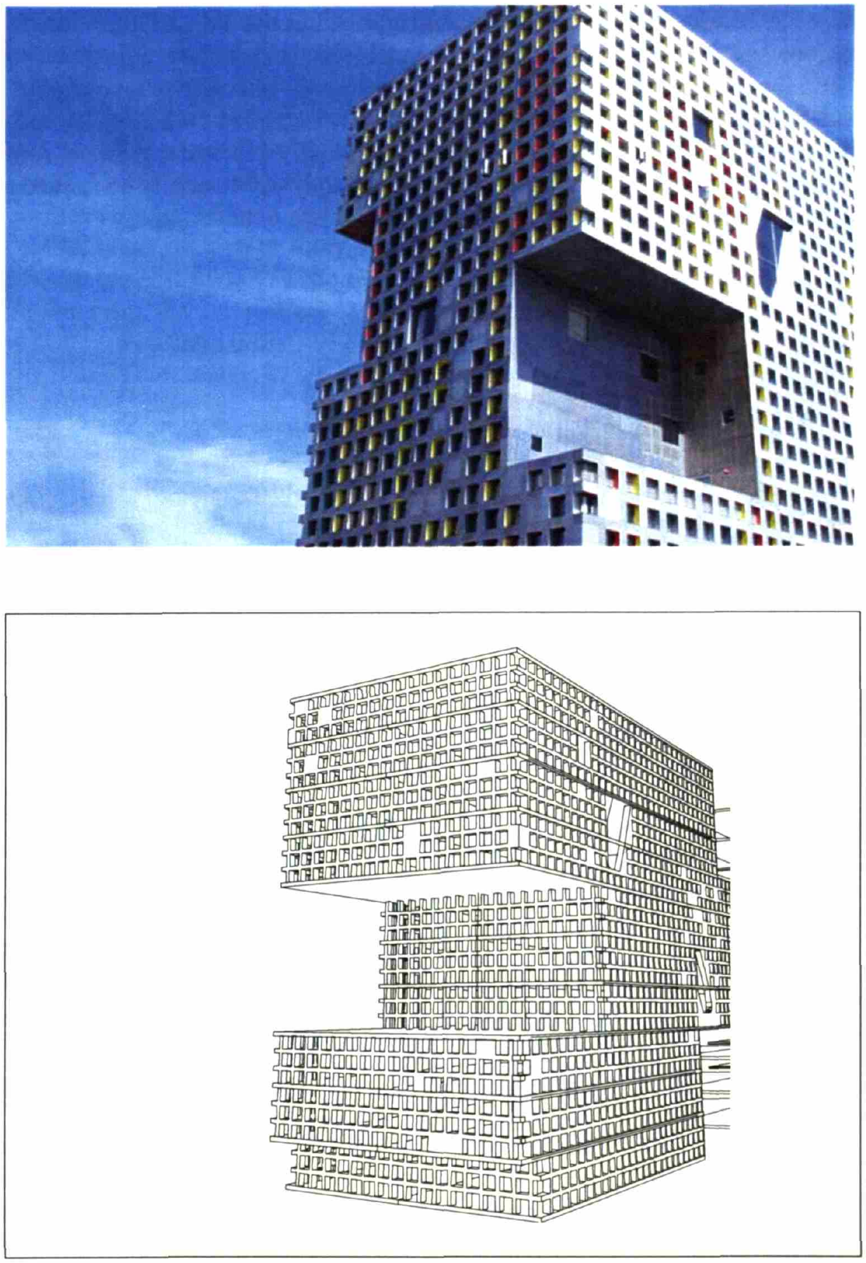


Figure 26. In the implementation of the building the large recesses of building mass are less intense (up) from what was intended (a derived representation of the same view according to the conceptual rule schema appears in the illustration below)

The results of these revisions can be traced in the Vassar Street façade . More specifically, the large scale openings A, and F were substituted by building surfaces. The large scale opening B was also occupied by a restaurant, while the opening D was partially occupied by a corridor and a concrete structure. Opening G was also partially blocked by the building entrance. The openings C and E remained as they were initially conceived to signify the existence of the three fraternity houses in the building.

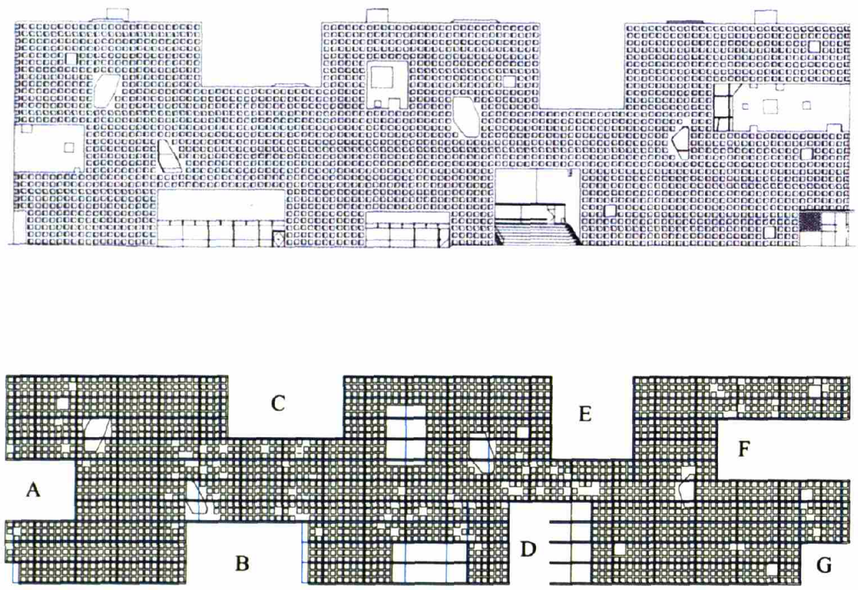


Figure 27. In the implementation of the building many of the large scale recesses of the building were blocked by spaces (up). A derived representation of the same view according to the conceptual rules appears below.

The general rule schema 3 of making perforated panels and the rules for their positioning succeed to generate the Vassar street façade. The differences between the derived elevation and the one that was finally implemented were caused by the addition of extra building mass. The changes remain consistent with the general conceptual approach, and can be expressed by the existent framework of rule schemata. The rules 2a-2s can perform the changes, which obey to the underlying grid.

Finally, the general rule schema 4 describes the generation and placement of vertical sponge-like cavities. These cavities were initially intended to penetrate the building from top to bottom, thus allowing free circulation of light and air. In the implementation of the design this intention was modified, due to fire safety regulations. The design concept of vertical porosity was applied under slight variation: discontinuous sponge-like cavities were distributed in the three fraternity houses of the building. The cavities penetrate two or more floors, and bring air and light to the interior through large windows in the *perfcon* structure or through openings on the building top. But the vertical cavities do not penetrate the building from top to bottom as it was initially intended. The set of shape rules 4a-4e succeed to generate the cavities in their final positions.

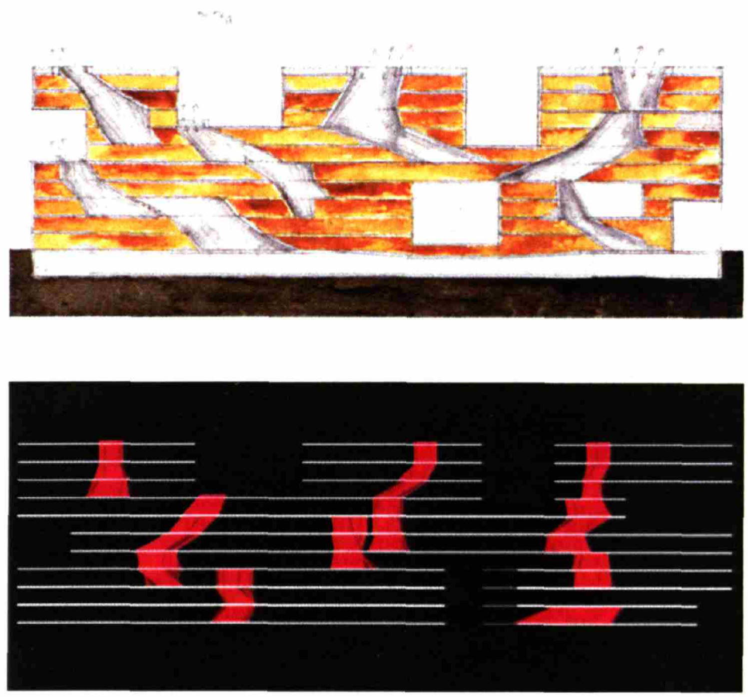


Figure 28. In the implementation of vertical porosity the cavities do not penetrate the building from top to bottom as it was intended (early sketch by Steven Holl, up). The cavities are discontinuous as shown in a derived representation (bellow)

7. Discussion

The chapter presented a computational interpretation of Steven's Holl design concept for Simmons Hall dormitory at MIT. The objective of this presentation was to show that computation can express a design concept of a well-known architect, like Steven Holl.

The material for this chapter emerged out of three interviews with the architect in charge of the project Timothy Bade and two meetings with Steven Holl. The presented original visual material, including sketches, drawings and photographs of 3D working models belongs to the different stages of the design process, executed by Steven Holl and his team of fifteen architects and designers. This same visual material was presented publicly in an exhibition in February 2003 at MIT.

The rule schemata and the computational process proposed in this study are a retrospective attempt to capture the conceptual and not the actual steps of the design process. Motivation for this chapter was that Holl emphasizes the conceptual basis of his architecture, and proposes the use of concepts as a method to achieve originality and to move away from the known to the unknown, and to new kinds of investigation. Holl proceeds in the development of open-ended theoretical frames that move independently of the existent morphologies, or typologies. Holl suggests that a "*limited concept*" is an expression of an ideal. The design investigation begins as the designer invents different kinds of actions in order to achieve this ideal.

The notion of a "*limited concept*" suggested by Holl coincides to that of a "*design concept*" that was presented in the previous chapters of this thesis. Both notions involve imagination and point to a possible set of actions that can be expressed with the aid of general rule schemata. It should be emphasized that a "*concept*" is the result of synthesis and not analysis of the provided information: site, program, circumstance etc. Further a "*concept*" becomes for the designer a working hypothesis that establishes novel relationships among the given facts. The authority of a "*concept*" remains limited to a specific designer and a specific problem.

For Holl, the concept becomes a vehicle in elaborating novel solutions. Novel solutions are produced when one succeeds to reveal some unique characteristic in a given situation. The built "phenomenon" occurs as the materialization of an investigation that deals equally with the "circumstantial" and the "absolute" character of things. Holl approaches the essence of a work of architecture as an organic link between concept and form, where parts cannot be substituted without upsetting the process of composition. The concept establishes an order, and a field of inquiry. Given the facts of experience, the organizing *Idea* becomes a hidden thread that connects dispersed parts with exact intention. The intertwining of *Idea* and *Phenomena* occurs when a building is realized".

This chapter examines the concept of "porosity" that Holl and his team developed for Simmons Hall. Porosity, which is an attribute of biological or organic bodies, is treated by Holl and his team as a compositional principle. The concept of porosity is approached in this presentation computationally. It is seen as a basis for the invention of spatial relationships and rules able to produce compositions with certain properties.

The concept of porosity is implemented by bringing in contact as much of the interior of the building with the exterior as possible. This is accomplished in the design in four general ways: First, by creating recesses in the overall building mass; Second by creating protrusions of building mass; Third, by distributing a large number of windows in the elevations of the building through the placement of prefabricated *perfcon* panels; And fourth, by distributing a limited number of vertical openings of free form, that create cavities in the interior of the building.

Holl believes that a design concept can be captured with descriptions that include words, numbers, symbols, parameters indicating proportion and finally form. This chapter deals with the computational interpretation of this last aspect of design concepts: form.

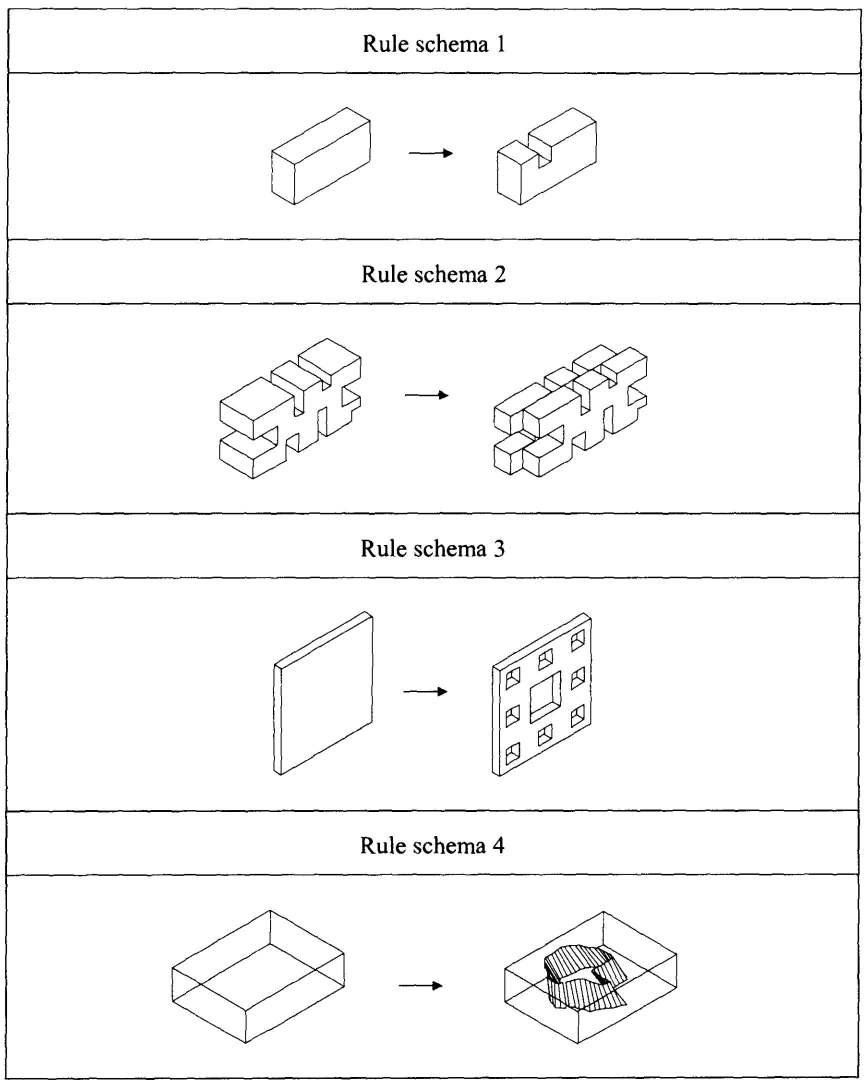
But how can one deal with a concept in visual and tactile terms? And how can one pull "actions" out of static words, (like the word "porosity")? From a shape computational point of view this study suggests that to describe a concept is to express it with some set of general rule schemata.

In the example of Simmons Hall, the word "porosity" is used to indicate a particular intention towards action. This chapter shows that the concept of porosity can be treated computationally, and architecturally. Computationally, the depiction of a future action through a rule schema provides a way towards a particular order that one is willing to apply to things. Architecturally, the testing of a rule schema allows the evaluation of the produced spatial arrangement against the program and all kinds of building standards. Proposing alternative ways of action (rule schemata) within the same general concept becomes itself the vehicle of one's thought.

Thinking (i.e. proposing rule schemata) and doing (i.e. testing) proceed step by step. The retrospective making of a grammar is analogous to the specification of a process. The grammar is a calculating device for a particular kind of design activity organized to achieve some desired objective.

The general set of design actions that were suggested and tested by Holl and his team in the effort to implement the concept of porosity, include, roughly, the organization of a building body and the invention of operations and methods of developing multiple pores: openings that open the interior of the building towards the exterior, and internal channels and cavities that allow the circulation of air and light.

"Porosity" is implemented by applying four rule schemata: a) prismatic voids are created on the building mass through subtraction, b) protrusions are created by translating half of the building along the direction of the central corridor, c) sieve-like openings are applied on the surfaces of the building, in elevations, through subtraction d) vertical sponge-like openings are embedded within the orthogonal building grid.



Evidence of the application of the above operations is found in the sketches, drawings and 3D models produced throughout the design process of Simmons Hall. But many of the results of the conceptual rule schemata were reversed in part, or totally eliminated in later stages of the process. This became necessary for programmatic, functional, structural and other reasons. More specifically, many of the recesses created by the first rule schema were reversed in part by adding volumes. The results of the second rule schema were totally eliminated in the final implementation of the building, and in the case of the third rule schema, many windows were blocked by concrete pieces due to construction requirements. Finally, the creation of sponge like openings and cavities was limited to three basic cavities distributed in the three student houses. These cavities due to fire-safety restrictions do not penetrate the building from top to bottom as it was initial intended.

Simmons Hall is the only actually implemented design from the three projects that are included in this study. Therefore, the importance of this discussion goes beyond the "studio implementation" to the "actual implementation" of the building. Studio implementation, in the schools of architecture, deals with hypothetical constraints, and it is always theoretical. Actual implementation deals with restrictions and problems that become visible in the course of action. The comparison between "theoretical" and "actual" implementation reveals that many of the conceptual decisions of the studio require revision during the actual implementation of the design. This is commonly the fate of many creative ideas, when they come to the point of the actual implementation.

The conflicts between design decisions taken in the studio and factual constraints, imposed by standards and conditions, arise by the existence of a general conceptual framework. The mere categorization of standards and constraints is unable to suggest any particular design approach, or action. A design approach is always conceptual. It is the result of synthesis and interpretation of the provided programmatic and other information, by a designer, or a design team. Every conceptual framework inevitably causes conflicts with the empirical standards, but also allows the necessary revisions and compromises to happen within a general frame of compositional principles.

The comparison between what is intended and what is implemented in a design process opens the ground for broader discussion and criticism of educational value. The contribution of computational theory in this discussion can be highly useful and productive. Rule schemata become efficient devices expressing the intentions of the designer and depicting the course of revisions and compromises that a general concept goes through, in the process of its implementation.

Acknowledgement

I am indebted to the architect Steven Holl and his project architect Timothy Bade for dedicating several hours of their time in discussing the design and implementation process of Simmons Hall, and for giving me access to the original design material. Without their contribution this chapter would not be possible. Special thanks are due to the architect Christian Wassmann, member of the architectural team, for the discussions we had during the preparation of the Simmons Hall exhibition at MIT.

8. References

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VIII. CONCLUSIONS

1. Summary of Results

The aim of this study is to contribute in two areas: The integration of shape computation with the ways architects think, and the development of new shape computational paradigms for designing from scratch.

Central in this work was the association of shape computation theory and studio practice. Shape computation theory offers a formal way to approach design, without establishing any particular design method. Based on shape computational precedents in analysis and synthesis of designs, this study attempted to extend and establish new concrete examples for the application of shape computation in studio teaching.

The first idea of the dissertation was that in designing from scratch, designers formulate design concepts. Design concepts are suggested for a practical and expressive purpose, with a view to develop unique design solutions. A design concept involves speculation, imagination and theorizing, and provides a general framework of action. This framework can be informal, or formal. It can be original, or driven by convention. In the course of the design process one examines the consequences of a concept and provides an interpretation for the network of the relationships it creates. The design concept is finally implemented in a manner that does not contradict the existing standards (aesthetical, functional, structural, etc.).

In computational terms, a design concept is a working hypothesis. It takes the position of a general law that provides direction for proposing rule schemata and rules. The general consequences of a design concept can first be sketched out by general rule schemata. Further design activity consists of determining and testing specific rule instances for the accomplishment of certain results. This search process involves several computational steps.

The character of the steps is conditional. At each step, an effect is accomplished provided that some condition is satisfied.

When the preferred rule schemata and rules are established, it is possible to order them. Careful ordering can lead to the construction of a generative design system (*grammar*). Such a system uses computational rules to produce designs with some desired properties. The rules can be ordered in alternative ways, and, provided that the rule-search has been carefully done, the final selection and ordering of the rules does not create new information.

The heuristics of the rule-search process were organized into: *formation*, *transformation* and *refinement*. Formation rules construct *partis*. Transformation rules develop variations on the basis of a chosen parti. And, refinement rules add tectonic details to the designs.

The second idea of the dissertation was that synthesis from scratch involves, on one hand, a series of calculations that have clear objectives and evolve like short logical processes, and on the other hand, the dialectical interaction among the results of these possibly independent processes. The study proposed a computational equivalent of a manual calculating device with great creative potential: the overlaying of multiple layers (2d sheets, or 3d spaces) in order, to produce a single description. The overlaying allows heterogeneous and incomplete descriptions to be synthesized. A description (plan, section, elevation, model etc.) becomes the result of the composition of several "partial descriptions" (i.e. sketches).

Descriptions in 2d, involve spatial calculations with areas, and their boundary lines. Descriptions in 3d, involve spatial calculations with volumes, and their boundary planes. Spatial calculations with areas and volumes express the "content" of things. Spatial calculations with boundaries, lines or planes, express the "form" of things. Content and form are constantly interrelated in architectural design. And also, a continuous interrelation exists between 2d and 3d descriptions.

During spatial calculations, the properties of the elements expressing content and the elements expressing form are not identical. Content, represented as area (in 2d) or volume (in 3d) is usually bounded by a shape that serves as a least upper bound. Form, on the other hand, represented with lines (in 2d) or planes (in 3d) remains always unbounded.

A set of results comes from the three case studies, which aim to serve as design paradigms, for use in the studio teaching.

1.1. THE CASE STUDIES

The first study offers an example of a design process progressing from the definition of the "parts" (spatial vocabulary) to the construction of possible "wholes" (house-designs). The design process applies the well known concept of the "domino" (or "polyomino") house. The absence of a predetermined site was a significant factor in choosing the particular approach. Instead of determining a single design solution, the search starts from the definition of a vocabulary of rooms, and a number of spatial relations among them. In the testing phase one examines different alternatives of constructing a rule-based system that produces more than one design. Computation by hand is suggested for outlining the spatial units and the rules, and digital computation is proposed for the testing phase.

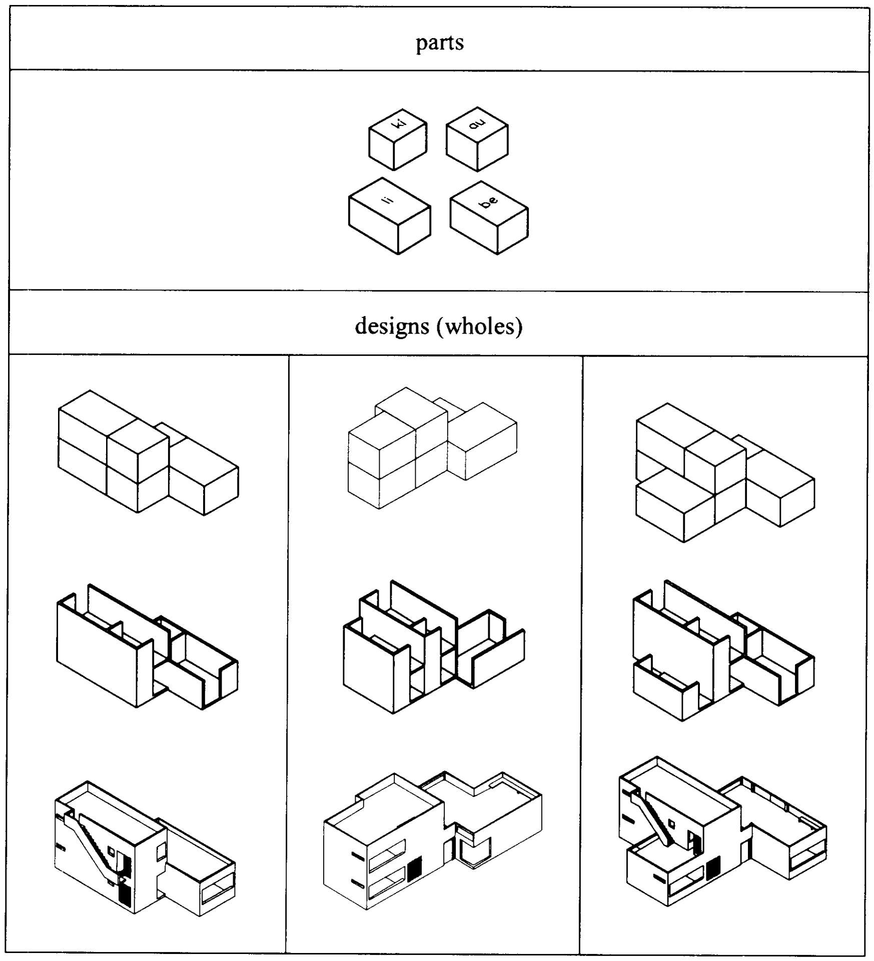


Figure 1. In the first case study the search proceeds from the "parts" (above) to the configuration of possible "wholes" (below)

The second study is an example of a design process that develops in the opposite direction: from a potential "whole" framed by a design concept, to the definition of the "parts" that compose it (floors, rooms, etc.). The design concept is that of a cubic "container". The absence of programmatic entities, which could serve as spatial units, was a factor in choosing this approach.

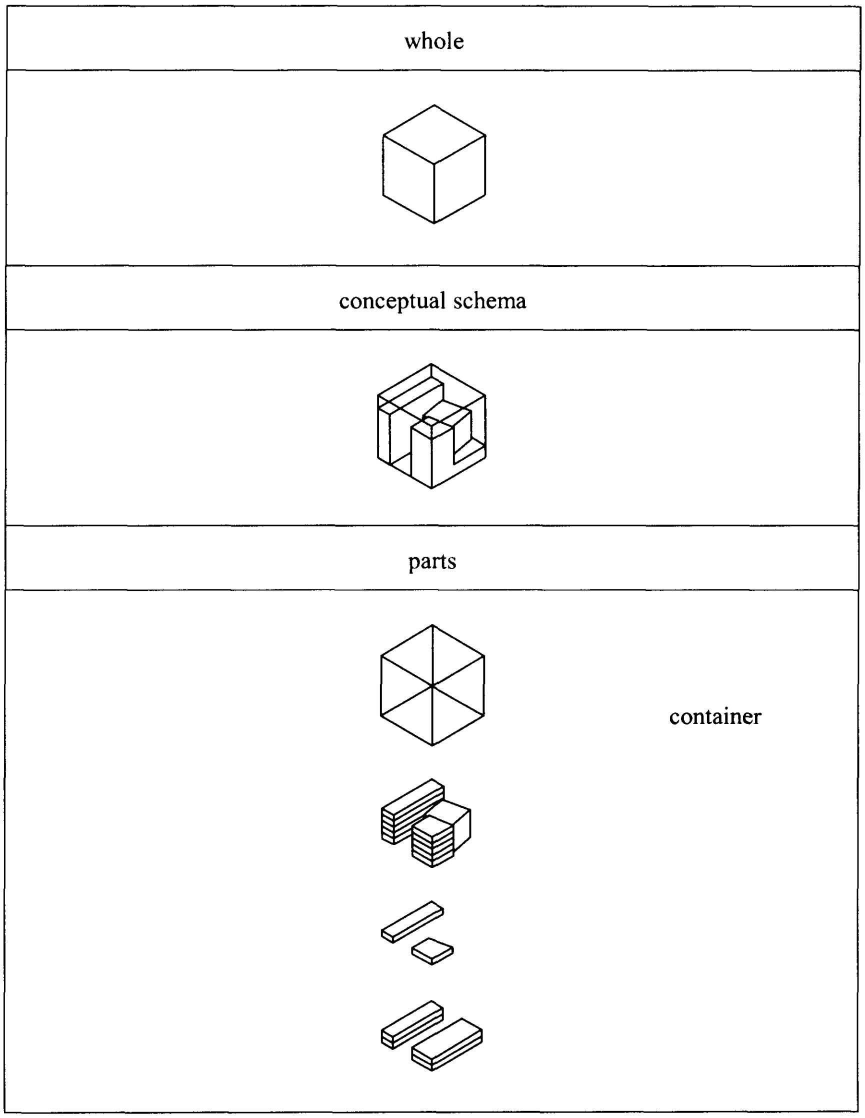


Figure 2. In the second case study the search proceeds from the "whole" (above) to the possible "parts" (below)

In the second case study, the rule schemata express the design concept and produce a general conceptual schema. They are gradually converted to more detailed rule sequences that generate the building plans. The rule schemata and the rules become educationally useful in two ways: The rule schemata can possibly be used in their more general form, in approaching other design problems. And, the rules can be ordered in a grammar to produce variations of the same general concept.

The third study is a computational interpretation of the conceptual part of Simmons Hall dormitory, by Steven Holl. The case study includes aspects from both the previous examples. The search proceeds from a "whole", determined by the space limitations of the given site, and it is conceptually framed by the design concept of "porosity". The building program also allows the specification of a spatial unit: the student room.

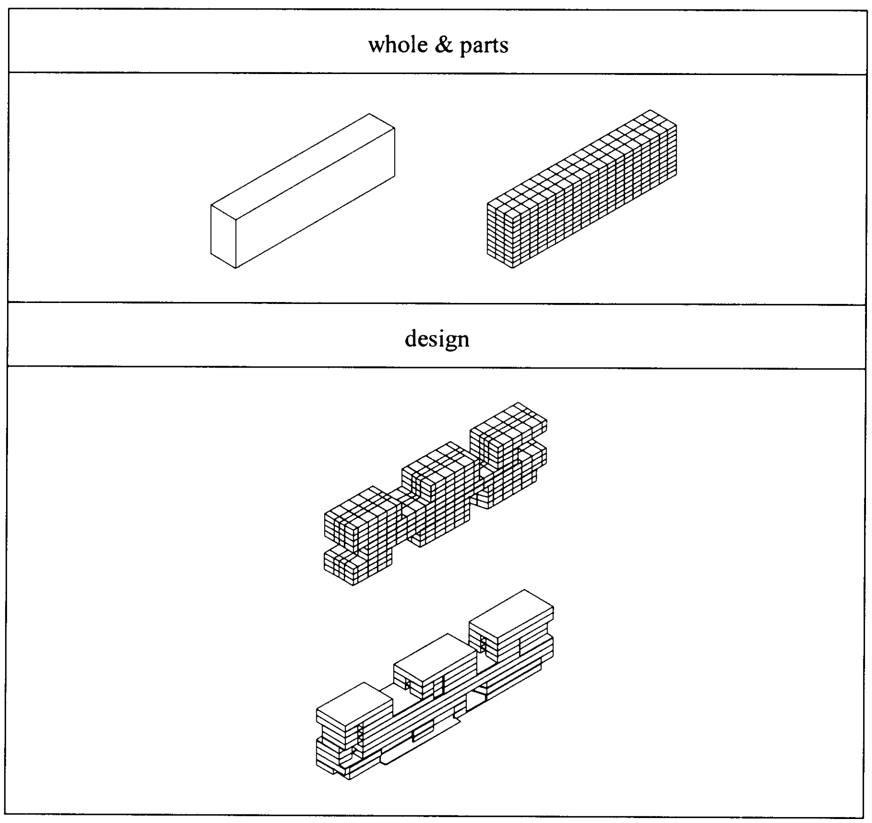


Figure 3. In the third case study the search involves both a given "whole" (above) and a spatial unit that serves for determining the "parts" (below).

Four general rule schemata are initially proposed to express the design concept of porosity. The four rule schemata are gradually converted to more detailed sequences of rules that generate characteristic elements of the building. The educational interest of the third case study is to see how the rule schemata and rules express the concept of "porosity" as this was defined by the architect Steven Holl. Further, since Simmons Hall is the only implemented design from all three case studies, the educational importance of the case study extend to the comparison between "conceptual" and "actual" implementation. The comparison becomes a tool for critique. It shows that many of the conceptual decisions require revision during the actual implementation of the design. The design concept allows the necessary revisions to happen within a framework of compositional principles without altering the general character of the design.

1.1.1. Concepts, Rule Schemata, Rules

What kinds of general rule schemata and rules do architects apply in the studio? How do they emerge? And how can we develop the appropriate computational framework to express them? How can shape calculation and design become identical in practice?

Designers, and design students in the studio, usually begin from general descriptions, of abstract character. First sketches and verbal descriptions are valuable because they depict key relationships, and words. For example, in the case studies the first abstract sketches suggest particular kinds of interaction among their elements.

Figure 4. The first sketches reveal fundamental relationships

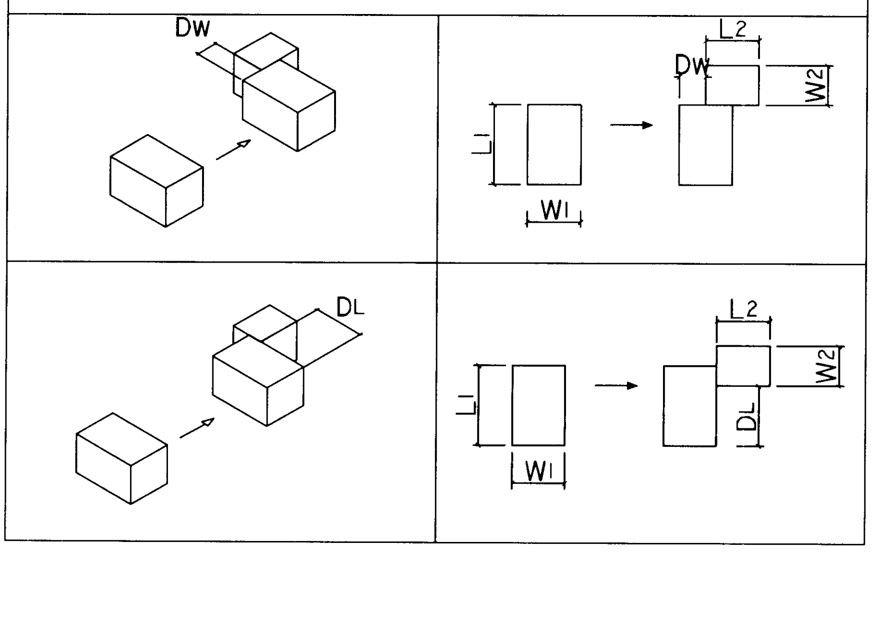
Common words like "domino" (in case study A), "container" (in case study B), and "porosity" (in case study C), obtain a new meaning as they try to suggest something fundamental for the design. These words are re-invented to suggest a particular order, and a series of actions.

The nature of the first descriptions is qualitative and "conceptual". It provides a general direction for exploration and implementation. The conceptual descriptions by means of which a designer seeks to establish a design solution are chosen with a view to establish something novel and extraordinary. General descriptions and actions of the previous kind can be expressed computationally with the aid of parametric rule schemata.

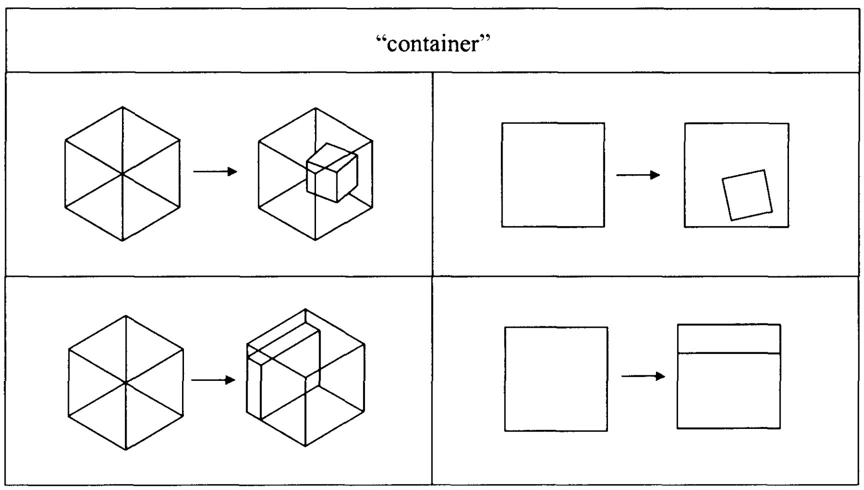
A parametric shape rule schema $\alpha \rightarrow \beta$ consisting of parameterized shapes α and β including an assignment g that gives values to the variables according to some predicate, defines a shape rule of the form $g(\alpha) \rightarrow g(\beta)$. The parametric rule schema starts from a shape C, and produces new arrangements C′, involving a Euclidean transformation τ, according to the relationship $C' = \{C - \tau[g(\alpha)]\} + \tau[g(\beta)]$.

The role of predicates and parameters in the conceptual use of rule schemata is to allow flexible description of spatial elements and their relationships. In the first case study, the concept of the "domino" house determines that the room adjacencies will be treated in particular ways. A predicate g indicates that the rooms will be rectangles (or rectangular solids). The rule schema shows that rooms are arranged to have some common boundary part. The participating rooms are parametric in size (length L, width W) and the length of their adjacent boundary is determined by the parameter D. The two parametric rule schemata presented below, distinguish between two general cases of adjacency: A new room is added on the short, or the long side of an existent room.

"domino"

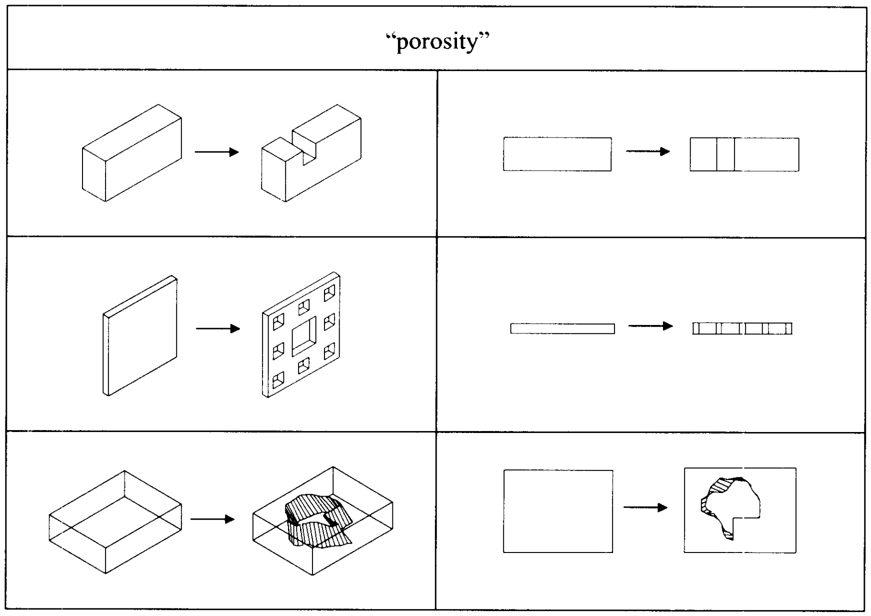


In the second case study, the concept of the “container” for an office building is described in a general way by determining that the building entities will be contained within a larger entity. A predicate g indicates that the shapes will be four-sided convex shapes. The rule schema parameters determine that the lengths, widths, and heights of the participating parametric shapes must create the described relationship: the added shape must always be contained within the existent one. The examples of the two parametric rule schemata presented below describe two variations of the same rule schema: A spatial entity is added in an independent angle from the existent shape, or the added shape is added at right angle with respect to the existent shape.



In the third case study, the concept of the “porosity” for a student dormitory, defined by Steven Holl and his working team, can be described in a general way by three rule schemata. The general intention for the building is to be “porous”, and perforated. This is achieved through orthogonal openings, and free form cavities. Orthogonal openings can be produced through subtraction of orthogonal forms, from the overall building mass and surfaces. Free form cavities can be also constructed and placed in the interior.

A first shape rule schema treats the overall shape of the building mass as a rectangular solid. Orthogonal voids are created in it through subtraction of orthogonal solids. A second subtractive rule schema creates perforated, sieve like surfaces that are applied in the elevations of the building. And a third rule schema penetrates with free sponge-like forms the orthogonal building. The examples of the three parametric rule schemata presented below describe are essentially variations of the same more general rule schema: orthogonal and free forms penetrate an orthogonal solid.



In all the above three case studies the conceptual rule schemata are put into use to create spatial arrangements with certain general properties and characteristics. In all three examples it becomes apparent that the proposed concepts are not determined simply by objective analysis of programmatic or other information, but are the result of personal diagnosis, and synthesis.

The produced arrangements are evaluated on the basis of existing general standards and circumstantial limitations and are developed further. In the course of their development from generalities to specific instances the concepts gradually acquire more detailed descriptions. These are generated by instances, or variations of the initial general rule schemata, and often by new rule schemata that operate complementary to the initial ones. In this way the small initial number of rule schemata starts to grow, and rule instances are put into use. As the focus of the rule instances moves gradually from the general to the detailed, the rules become more restricted in order to depict personal preferences and objective limitations.

Many of the rules that are used in the elaboration of a design concept can become specific and can start to act as drafting devices. But much of the success of the entire design process relies on how the general principles of the conceptual rule schemata are put into use by rule instances. The rule instances specify how the principles apply in particular cases. The choices and alternatives remain multiple even within the limits of the same conceptual schema.

In the first case study labeled rule instances can generate specific arrangements of rooms and avoid others. Also, proportions can be inserted for the lengths and widths of the participating room, as soon as its function is decided. In this way, the initial rule schemata are restricted but begin to articulate a desired set of results.

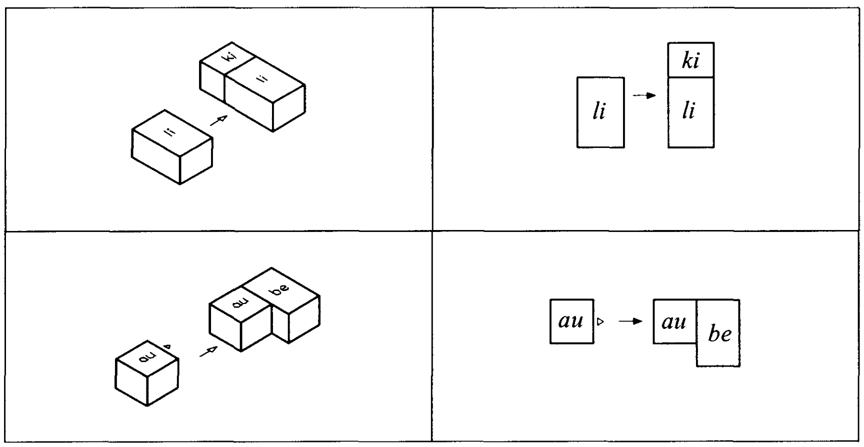


Figure 5. In the first case study restricted rule instances are gradually defined to generate only the preferable arrangements

In the second case study, where the vocabulary of rooms is not initially specified a different strategy is introduced: rule instances are organized to apply on four superimposed graphic layers. The superimposition of the generated arrangements on each layer permits the selection of room-shapes within a rich mesh of alternatives. In order to be able to operate in the graphic environment of the four graphic layers the initial rule schemata are modified. For example, the first rule schema that places a four-sided convex shape inside another is expressed so that it can apply on four graphic layers, A, B, C including colored lines and D including colored areas. An extension of the same rule schema, the rule schema 1iii, draws a convex shape in layer A over a convex shape found in layer B, but it also modifies the color of its area in layer D, to denote a change in the function of the space.

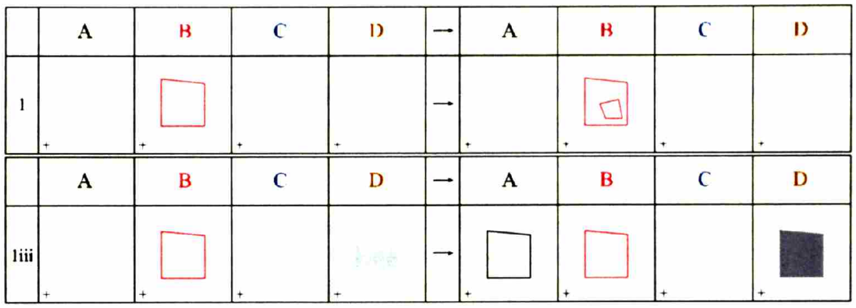


Figure 6. In the second case study parametric rule instances are gradually defined to apply in four superimposed graphic layers, including lines and areas

In the third case study, the concept of "porosity" is further determined by rules. For example, an instance of the first rule schema which creates orthogonal voids through subtraction of orthogonal solids can be further specified so that each application subtracts some desired number of spatial units. Moreover, a parametric variation of the same initial rule schema can be formed to create perforated panels for the façades of the building. A variation of the second general rule schema (that creates free form cavities in the conceptual solid of the building), generates a free form cavity between two parallel slabs.

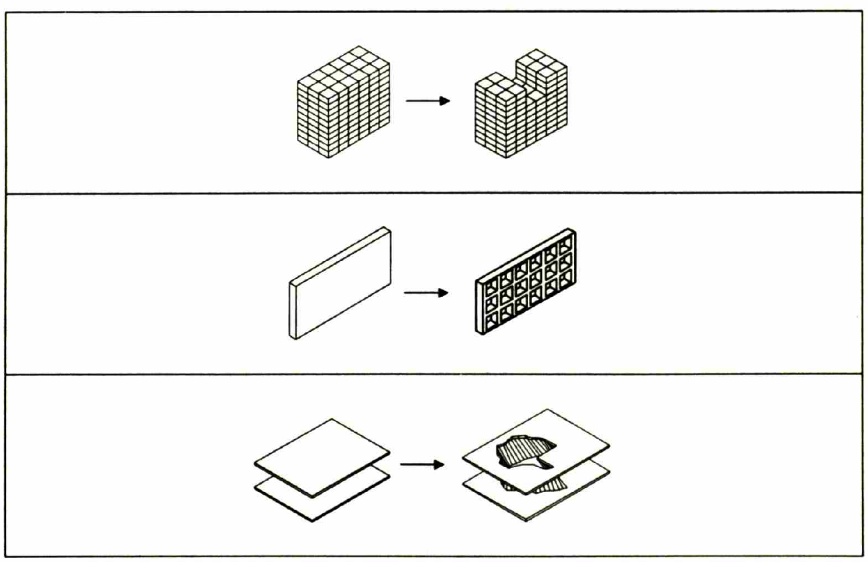


Figure 7. In the third case study the concept of porosity is applied by rule instances specifying the generation of pores

1.1.2. Design Process

In synthesis from scratch, architects begin with a general view about a problem, and develop methods to achieve more specific goals. This process of evolution from something open ended to something specific is analogous to moving from general rule schemata to the specific rules of a grammar.

In the studio, the students of architecture use verbal descriptions, text, sketching, photographs and all sorts of experiments and constructions to express a general design intention, in response to a problem. Analysis of the given information is always part of this initial stage. The articulation of a working hypothesis (design concept) is the result of synthesis. It involves only a subpart of the provided information: the elements that one estimates as crucial for his/her design.

In the next diagram, this is expressed with two co-centric circles. If seen as areas, the exterior circle represents the wider design problem and the interior circle represents the concept.

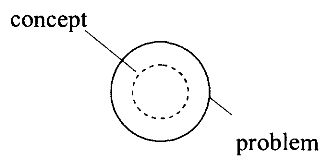


Figure 8. A design problem is a wider issue from what a designer can initially conceive

A design concept is stated in response to a specific problem, and mirrors the approach of an individual. The relationship between a problem and a concept is dialectic. It represents two states in a closed system of understanding. This system may not lead to any further action. For example, in conceptual projects, students or professional designers are called to propose only general design concepts, without dealing with the details of the implementation. The next diagram represents a conceptual system of this kind. Detecting a problem and elaborating a conceptual answer is a thinking process that can simply terminate to itself after several loops of evolution.

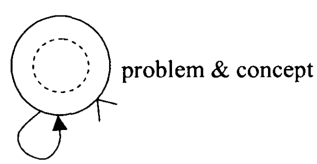


Figure 9. The conceptual proposal can stand independently from its detailed implementation

The role of a design concept in the studio implementation is to link the different complementary search-activities of the process, by providing the "big picture" for the design. These activities explore diverse domains of interest: function, construction, style, proportion,...etc, which without the design concept appear disconnect, independent and indifferent. The design concept brings them in a particular interrelationship, organizes priorities among them, and reveals the possible conflicts.

The next diagram shows that the design problem and the concept become the center in a system with several nodes. The existence of double arrows to and from each node indicates that every activity, or calculation of any kind, begins from the central node (concept-problem), and concludes at it.

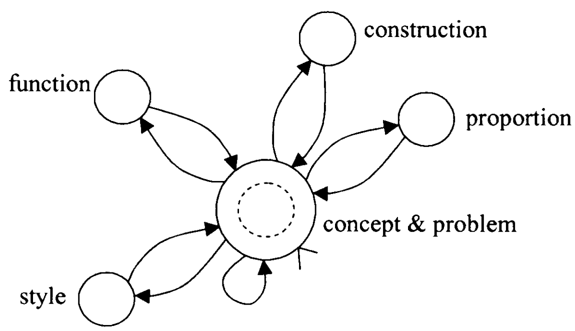


Figure 10. Design implementation involves the interaction of several diverse domains of inquiry. These interact under the "authority" of the design concept

The diagram implies the interaction of diverse domains of inquiry and the use of diverse descriptive components towards the elaboration of a concept. The interaction of the different components is dictated by the general concept. But the concept is also evolving as a result of this interaction.

In the design studio, students practice their acuity to "diagnose" a given design problem, and their ability to make productive hypotheses. Their hypotheses are expressed as "concepts". Although these concepts may often be imaginary and may lack immediate experiential meaning, students or designers invent a system of actions implied in terms of them, and an interpretation for the resulting network of relationships. Then, they proceed to their implementation usually in a manner that does not contradict the existing building standards and conventions of experience.

The use of general rule schemata allows the expression of concepts and of the actions implied by them without restricting the imagination. The use of rule schemata and rules in the studio, for educational purposes, offers fertile ground for discourse between design instructors and students. The evaluation of the results of a studio project becomes an issue of evaluating the conceptual schemata proposed by the students and their possible architectural consequences. Further, it involves exploring their expressive potentials and inventing ways for their implementation.

The suggested studio process is a process of inventing, proposing, testing, and modifying rule schemata and rules. This process attempts to capture the flow of design ideas, and the ways these evolve. Unsurprisingly the computational approach advocated here, is not entirely foreign to the processes of studio instruction as it happens today. The difference is that today most of the design activities remain implicit, or situated within particular design movements, and trends and their generative implications are entirely ignored. The educational merit of an educational process that makes the relationship between design concepts and rule schemata explicit is to emphasize this generative side. This would result to a much more self-consistent approach to design, independent of what particular preferences and conventions one adopts.

Outside the studio, in design practice, in the actual implementation of designs as buildings the use of conceptual rule schemata and rules, allows the evaluation of our design decisions, and gives the opportunity to designers to make possible revisions without defying the desired conceptual framework.

2. Further Research

The extensions for further research that one can draw beyond this point are multiple.

From a shape computational point of view, the examination of the relationship between 3d and 2d descriptions, and the relationship between bounded volume, or area, and their boundaries (content and form), are issues that deserve further exploration. Further, it would be useful to determine which design processes are better executed in 3d, and which in 2d. Because, although today most representations happen in 3d digital models, 2d information is always required in the implementation phase of a design, and the extraction of 2d information from 3d descriptions is usually not easy thing to produce.

Further, the different descriptions required in the development of a design solution are usually made by different groups of specialists who do not participate in the early stages of the design process. Because the existence of a "single model" is not achievable, each participating group of specialists involved in a design re-models the same object for their own purposes. A key issue in this collaborative design process is the need for communicating the design concept, which expresses the intent of the designer. Do all the participating specialists "get" the big picture? The role of the architect as general coordinator becomes difficult without an efficient way to communicate the concept of a design.

Shape calculation, and grammars could be used for general communication purposes. One could devise a system through which designers, and all kinds of engineers, and their descriptions communicate using shapes, shape rule schemata, and derivations. The development of a shape computational system for communication among the different specialists who are involved in the implementation of a design could be a meaningful and practical objective for experimentation.

The development of the theoretical basis upon which one would be able to identify the role of computation in the elaboration of specific design ideas, in specific projects, could lead to a new level of architectural criticism. This remains an entirely unexplored area of inquiry.

And finally, in design education, the development of experimental studios that bring into practice methods originating in computational design theory, would be an excellent research project for educators in the field. Along the lines of this study, the emphasis would be placed on the generative character of design concepts and the use of rule schemata in the process of implementing them.