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OPTICAL DPSK WITH GENERALIZED PHASE NOISE MODEL AND NARROWBAND RECEPTION

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Abstract

The performance of a narrowband DPSK receiver is analyzed in the case where the phase of the received signal is impaired by laser phase noise. This receiver is commonly used for the reception of DPSK modulated signals corrupted by additive white Gaussian noise. Unlike conventional analyses which consider the asymptotic performance in the infinite signal-to-noise ratio regime, the effects of both additive and phase noises are fully taken into account. A general phase noise model is employed which includes the well-known Brownian motion model as a special case. This treatment enables a performance evaluation under feedback control of frequency noise. Numerical results indicate a superior performance due to both the narrowband nature of the receiver and the phase noise stabilization mechanism.

1 Introduction

Phase modulation is an attractive modulation scheme for digital communication systems that are impaired by additive white Gaussian noise. In particular, binary Phase Shift Keying (PSK) has a 3 dB performance gain over Frequency and Amplitude Shift Keying modulation formats with average power limited transmitters and coherent receivers. This performance advantage has not been realized in optical communication systems, primarily due to the random nature of the phase of the output field in a semiconductor laser. This randomness is a result of spontaneous photon emissions during the laser operation, and is commonly called *phase noise.*

Phase modulation is particularly vulnerable to phase noise, as the information and the noise are both embedded in the phase. Since Differential Phase Shift Keying (DPSK) conveys the information in the change in the phase instead of its instantaneous value, the effective phase distortion is limited only to the current and previous bit intervals instead of the entire history of the phase noise process. This makes DPSK more robust against phase noise than PSK.

The conventional model for phase noise in a semiconductor laser is that of a Brownian motion process. This corresponds to a white frequency noise, and accurately describes the power spectral density of the laser output. In this paper, we use a general phase noise model, which includes this standard model as a special case, in order to be able to evaluate the effect of frequency feedback on the system performance.

We consider a conventional electronic receiver. This receiver consists of a filter matched to the phase noise free sinusoid followed by a delay-multiply circuitry. Since phase noise broadens the emitted spectrum with respect to the case of a signal with stable phase, receivers with wideband filters are usually considered to remedy the effect of phase noise at the expense of strengthening the effect of additive noise. We will show that the simple matched filter receiver performs satisfactorily when used in conjunction with feedback control, and that the effect of phase noise can be minimized with appropriately chosen system parameters, enabling the use of phase modulation in phase noisy optical systems.

2 Mathematical Model

Optical DPSK can be implemented in a direct detection system with an optical filter and Mach-Zehnder interferometer which jointly implements the filtering and delay-correlation operations. Homodyne detection may also be used. We will assume that heterodyne detection is used although the analysis is valid for the other schemes as well [1]. In heterodyne detection, the received optical signal is combined with a local oscillator signal and the sum is photodetected. This processing results in a downshift of the carrier frequency from optical range to electrical range as well as the introduction of an additive noise process. The intermediate frequency (IF) signal that is input to the electronic receiver is of the form

$$r(t) = A\cos(2\pi f_c t + \theta(t) + b_k\pi) + n(t) \qquad (1)$$

where $\theta(t)$ is the combined phase noise process of the transmitter and receiver lasers, and $b_k = 0, 1$ is the differentially encoded data bit for the kth bit duration. The encoding is such that when the kth data bit a_k is 0, we have $b_k = b_{k-1}$, and similarly when $a_k = 1$, $b_k \neq b_{k-1}$. The additive noise $n(t)$ is a white Gaussian noise process with two-sided spectral height $N_0/2$; it is a consequence of random nature of electron emissions in a photodetector when the local oscillator power is assumed to be large.

The phase noise process $\theta(t)$ is the integral of the frequency noise $\mu(t)$:

$$\theta(t) = 2\pi \int_0^t \mu(\tau)\, d\tau \,.$$

The frequency noise is commonly modeled as a white Gaussian process with spectral height $\beta/2\pi$. This results in a Brownian motion phase noise process where β is the combined linewidth of the transmitter and local oscillator lasers. In this model $\theta(t)$ is a zero-mean Gaussian process with variance $2\pi\beta t$.

In this work we allow the frequency noise $\mu(t)$ to have an arbitrary spectral density $\beta S(f)/2\pi$. The parameter β corresponds to the spectral intensity while $S(f)$ models the spectral shape of the frequency noise. Later, we will specify $S(f)$ to investigate the effect of frequency feedback.

It is convenient to normalize the time with respect to the bit duration T and to scale the phase noise process. We define the normalized phase noise process

$$\psi(t) = \frac{1}{\sqrt{\gamma}}\theta(tT) \tag{2}$$

where $\gamma = 2\pi\beta T$. This process has the correlation function

$$K_\psi(t,s) = \frac{1}{T}\int_0^{Tt}\int_0^{Ts} K(u-v)\, du\, dv \tag{3}$$

where $K(\tau)$ is the inverse Fourier transform of $S(f)$. In the case of Brownian motion phase noise model, we have $S(f) = 1$ which implies $K_\psi(t,s) = \min(t,s)$. This justifies the normalization since $\psi(t)$ has variance t. The parameter γ in that model corresponds to the increase in the variance of $\theta(t)$ within a bit duration, and will be called the ***phase noise strength.*** In the rest of this paper we will express the phase noise process in terms of $\psi(t)$ when convenient.

It is also useful to define the differential phase noise process

$$\Delta\psi(t) \triangleq \psi(t) - \psi(t-1)\,. \tag{4}$$

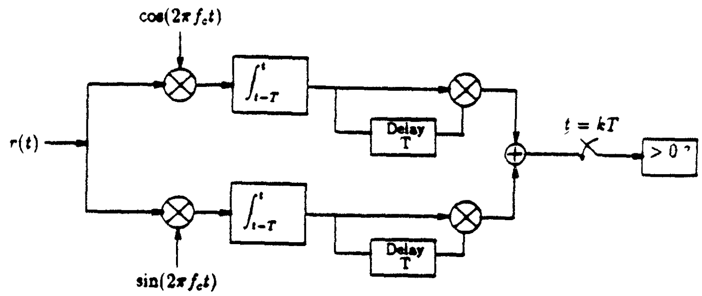
We assume that the frequency noise $\mu(t)$ is a stationary Gaussian process. Then $\Delta\psi(t)$ will also be stationary since the integral of a stationary process has stationary increments. This process has the correlation function

$$K_{\Delta\psi}(t,s) = K_\psi(t,s) + K_\psi(t-1,s-1) - K_\psi(t,s-1) - K_\psi(t-1,s)\,.$$

Its spectral density is given by

$$S_{\Delta\psi}(f) = S(f/T)\left(\frac{\sin(\pi f)}{\pi f}\right)^2 . \tag{5}$$

The receiver structure that will be considered in this paper is shown in Figure 1. This receiver first performs quadrature demodulation by multiplying the IF signal $r(t)$ with in-phase and quadrature sinusoids, passes the mixer outputs through integrators of duration T, correlates the two filter outputs with their delayed versions, and finally samples the sum of the two correlations. The sign of the sampled value is used to reach a decoding decision. This structure is optimal in the absence of phase noise. Since phase noise broadens the spectrum of the received signal, some signal power will be lost due to the relatively narrow filter bandwidth.



3 Error Floor

Error floor is the residual error probability when the signal-to-noise ratio tends to ∞. It is a measure of the effect of phase noise on the performance. Since the error floor is easier to obtain than the actual error probability, most of the work on phase noisy DPSK is concerned with floor computation. Before we present a complete probability of error evaluation in the next section, we first consider the error floor for a general frequency noise spectrum.

When the additive noise $n(t)$ is neglected, it is easy to see that the decision variable for the receiver in Figure 1, within a multiplicative constant, is

$$D = \frac{1}{T^2}\int_{(k-1)T}^{kT}\int_{(k-2)T}^{(k-1)T} \cos[\theta(t) - \theta(s) + a_k\pi]\, ds\, dt\,. \tag{6}$$

Then the error floor can be written as

$$P_{ef} = \Pr\left[\int_{(k-1)T}^{kT}\int_{(k-2)T}^{(k-1)T} \cos[\theta(t)-\theta(s)]\, ds\, dt \leq 0\right].$$

It is difficult to obtain this quantity exactly. Therefore we derive an upper bound by using the inequality $\cos x \geq \pi/4 - x^2/\pi$. (The better known inequality $\cos x \geq 1 - x^2/2$ can also be used, but yields a looser upper bound.) We then obtain

$$P_{ef} \leq \Pr\left[\frac{1}{T^2}\int_{(k-1)T}^{kT}\int_{(k-2)T}^{(k-1)T} [\theta(t)-\theta(s)]^2\, ds\, dt \geq \frac{\pi^2}{4}\right].$$

After normalization the error floor bound can be expressed as

$$P_{ef} \leq \Pr\left[Z \geq \frac{\pi^2}{4\gamma}\right]$$

with the random variable Z defined in terms of $\psi(t)$ as

$$Z = \int_{k-1}^{k}\int_{k-2}^{k-1} [\psi(t)-\psi(s)]^2\, ds\, dt\,. \tag{7}$$

Note that the statistics of Z does not depend on k. It is shown in [1] that the classical Karhunen-Loève analysis can be modified to express Z as

$$Z = \sum_{i=1}^{\infty} z_i^2$$

where z_i are independent zero-mean Gaussian random variables with variance λ_i. λ_i are the solutions to the integral equation

$$\int\int G(s_1,s_2)K_\psi(t,s_1)\left[\phi(s_1)-\phi(s_2)\right]ds_1\,ds_2=\lambda\phi(t) \quad (8)$$

where $G(t,s)=1$ when one of the pair (t,s) lies in the interval $(k-2,k-1)$ and the other in $(k-1,k)$ and $G(t,s)=0$ otherwise.

Once the eigenvalues $\{\lambda_i\}$ are found, the moment generating function of Z can be obtained as

$$M_Z(s)=E(e^{-sZ})=\prod_{i=1}^{\infty}(1+2s\lambda_i)^{-1/2}$$

which can be inverse Laplace transformed to obtain the density function $p(z)$ of Z. The error floor bound can now be obtained as

$$P_{ef}\le\int_{\pi^2/4\gamma}^{\infty}p(z)\,dz\;.$$

The results of this procedure for different phase noise processes will be given in Section 6.

4 Effect of Additive Noise

In order to fully characterize the system performance, we now include the effect of additive noise, or equivalently a finite signal-to-noise ratio (SNR).

When the additive noise $n(t)$ is not neglected in the received IF signal of Equation 1, the decision variable of the narrowband single filter receiver will not only contain the signal component D given in (6), but also signal cross noise and noise cross noise components. Let c_1 be the signal component of the in-phase branch integrator at the sampling time kT, and let c_2 be its delayed version. Also let s_1 and s_2 be similarly defined for the quadrature branch of the receiver. With appropriate scaling and assuming a data bit of 0, we have

$$\begin{aligned}c_1&=\frac{1}{T}\int_{(k-1)T}^{kT}\cos(\theta(t))\,dt\\ c_2&=\frac{1}{T}\int_{(k-2)T}^{(k-1)T}\cos(\theta(s))\,ds\;,\end{aligned}$$

and s_1, s_2 are identical except cosines are replaced by sines. Then the decision variable D' is given by

$$D'=(c_1+n_1)(c_2+n_2)+(s_1+n_3)(s_2+n_4)$$

where n_i are independent, zero-mean, Gaussian random variables with variance $\sigma^2=(2\xi)^{-1}$. Here $\xi=A^2T/2N_0$ is the SNR.

The signal component $D=c_1c_2+s_1s_2$ is given by (6) while the total noise component is $N=c_1n_2+c_2n_1+s_1n_4+s_2n_3+n_1n_2+n_3n_4$. The moment generating function of N conditioned on D can be found via direct integration as

$$E_c(e^{-sN})=(1-\sigma^4s^2)^{-1}\exp\left[\frac{\sigma^2s^2}{2(1-\sigma^4s^2)}(R_1^2+R_2^2-2\sigma^2sD)\right]$$

where $R_i^2=c_i^2+s_i^2$ for $i=1,2$. Using standard results of communication theory [2, 3], one obtains the conditional error probability as

$$P_c=\frac{1}{2}\left[1-Q(\sqrt{x},\sqrt{y})+Q(\sqrt{y},\sqrt{x})\right] \quad (9)$$

where $Q(x,y)$ is the Marcum's Q function, and

$$\begin{aligned}x&=\frac{\xi}{2}(R_1^2+R_2^2+2D)\\ y&=\frac{\xi}{2}(R_1^2+R_2^2-2D)\;.\end{aligned}$$

For the special case without phase noise, $R_1=R_2=D=1$, and the error probability reduces to the well known result $P_e=\frac{1}{2}e^{-\xi}$. In the presence of phase noise, however, the evaluation of P_e requires a joint statistical characterization of $R_1^2+R_2^2$ and D. Obtaining the marginal distribution of the former random variable alone is a formidable task [4, 5, 6]. Therefore, we will provide a tight upper bound to the error probability.

It is easy to show that for a given $R_1^2+R_2^2$, P_c is increased by lower bounding D. On the other hand, the variance of the total noise N is increasing with $R_1^2+R_2^2$. It can be seen that increasing the value of this random variable increases the Chernoff bound to the error probability as well. Furthermore, we have numerically observed that maximizing $R_1^2+R_2^2$ also maximizes P_c. Thus, since $0<R_i<1$, setting $R_1^2+R_2^2=2$ and using a random variable that is smaller than D will result in an upper bound to the error probability. For the latter purpose, we will use either of the bounds

$$\begin{aligned}D&\ge1-\frac{\gamma}{2}Z\\ D&\ge\frac{\pi}{4}-\frac{1}{\pi}Z\end{aligned}$$

where Z is the random variable introduced in Equation 7. These result from the two lower bounds to $\cos(x)$ discussed in Section 3. Any other parabolic bound to $\cos(x)$ can also be used, but the two we employ are the extremal ones being tightest at $x=0$ and $x=\pi/2$ respectively. The first bound yields tighter results for small ξ where the additive noise is the more dominant cause of bit errors. Conversely, the second bound is tighter in the high SNR regime where phase noise is more dominant. In obtaining the numerical results we have chosen the minimum of these two bounds. The first bound is given by

$$P_1=\Pr(X_1\ge1)$$

where the random variable X_1 is characterized by the moment generating function

$$E(e^{-sX_1})=\left[1-(s/2\xi)^2\right]^{-1}\exp\left(\frac{s^2/2\xi}{1+s/2\xi}\right)M_Z\left(\frac{s\gamma/2}{1-(s/2\xi)^2}\right)$$

where $M_Z(s) = E(e^{-sZ})$ is the moment generating function of Z. The second bound is

$$P_2 = \Pr(X_2 \geq \pi/4)$$

where $E(e^{-sX_2})$ is the same as $E(e^{-sX_1})$ except $\gamma/2$ in the argument of M_Z is now replaced by γ/π.

5 Frequency Feedback Model

In the last two sections we provided a general framework to evaluate the error floor and the error probability. Now we will specify the statistical characterization of the phase noise process that results from the application of a frequency feedback stabilization scheme considered in [7].

In the absence of an external electronic feedback, the frequency noise $\mu(t)$ at the output of a semiconductor laser is well approximated by a white Gaussian process. Hence, $S(f) = 1$ in our general formulation and β corresponds to the combined 3 dB linewidth of the transmitter and local oscillator lasers.

White frequency noise models result in plausible performances although the noise has infinite mean power. Therefore the low frequency components of the frequency noise degrade the performance most. This argument can be made precise by an examination of the spectral density of the differential phase noise process $\Delta\psi(t)$. It is readily seen from (5) that the high frequency components of $S(f)$ are attenuated by a factor with the envelope $1/(\pi f)^2$, while the low frequency components are preserved. Thus, a highpass filtering operation on the frequency noise process is beneficial in reducing the effect of phase noise on the performance. A feedback system that uses part of the laser output to provide electronic feedback to the laser, via a frequency discriminator, has been proposed in this context. The use of feedback to achieve frequency stabilization has been suggested by many authors [7, 8, 9, 10].

A linearized form of the generic frequency feedback loop is shown in Figure 2. The open loop frequency noise $\mu_0(t)$ has the spectral density $\beta/2\pi$. For analytical convenience we use a simple integrator for the feedback filter, that is $H(f) = b/j2\pi f$, where b is a gain that determines the system bandwidth. Then the frequency noise output $\mu(t)$ is a zero-mean Gaussian process with spectral density

$$S_\mu(f) = \frac{\beta}{2\pi}\frac{(2\pi f)^2}{(2\pi f)^2 + b^2}.$$

It is seen that $\mu(t)$ has a highpass spectral density with a 3 dB cutoff frequency $b/2\pi$. The low frequency components of the input noise process have been attenuated by the feedback loop.

In the notation of Section 2, we have $S(f) = (2\pi f)^2/[(2\pi f)^2+b^2]$ and its inverse Fourier transform $K(\tau) = \delta(\tau) - 0.5be^{-b|\tau|}$. The correlation function of the normalized phase noise process $\psi(t)$ is then obtained from (3) as

$$K_\psi(t,s) = \frac{1}{2r}\left(1 + e^{-r|t-s|} - e^{-rt} - e^{-rs}\right) \qquad (10)$$

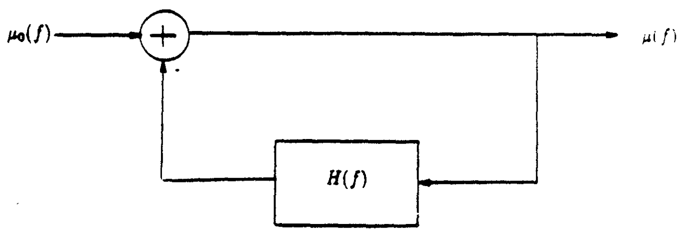


Figure 2: The linear time-invariant model for the frequency control system.

where we have defined $r \triangleq bT$. The feedback disappears in the limit of $r = 0$. In this limit, the correlation function becomes $K_\psi(t,s) = \min(t,s)$.

The correlation function of the differential phase process $\Delta\psi(t)$ is easily found from (10) as

$$K_{\Delta\psi}(t-s) = \frac{1}{2r}\left[2e^{-r|t-s|} - e^{-r|t-s-1|} - e^{-r|t-s+1|}\right]. \qquad (11)$$

In the absence of feedback this reduces to $K_{\Delta\psi}(t-s) = 1 - |t-s|$ for $|t-s| < 1$.

Having obtained the complete statistics of the phase noise process under this feedback scheme, we can now obtain the explicit performances as outlined previously.

6 Numerical Results

To obtain the error floor, we solve the integral equation in (8) with $K_\psi()$ as given in (10). This equation reduces to a third order differential equation from which the eigenvalues are found [1]. The resulting performance is shown in Figure 3. It is seen that the feedback reduces the error floor dramatically. In the absence of feedback, the phase noise strength must be as small as 0.05 for an error floor of 10^{-12}, while with $r = 10$ (feedback filter having a bandwidth that is 1.6 times the bit rate) $\gamma \leq 2.5$ is required.

The performance in the presence of additive noise is illustrated in Figure 4 where the upper bound[1] to P_e is shown as a function of the SNR for two values of γ. The dramatic improvement with increasing feedback is clearly seen. For $\gamma = 0.1$, the floor is about 10^{-9} without feedback, while with $r = 4$ (i.e. a feedback filter bandwidth that is 64% of the bit rate) a bit error rate of 10^{-16} is attained with an SNR penalty of only 0.4 dB. This penalty has almost vanished when r is increased to 10. For $\gamma = 1$ and $r = 10$, the SNR penalty is about 3 dB at $P_e = 10^{-16}$ and 1.8 dB at $P_e = 10^{-9}$, while without feedback the error floor is at 7.5×10^{-2}. Using 100 Mbps DPSK with a feedback bandwidth of 160 MHz, one

[1]The visible kinks in some of the curves are due to switching from one bound to the other.

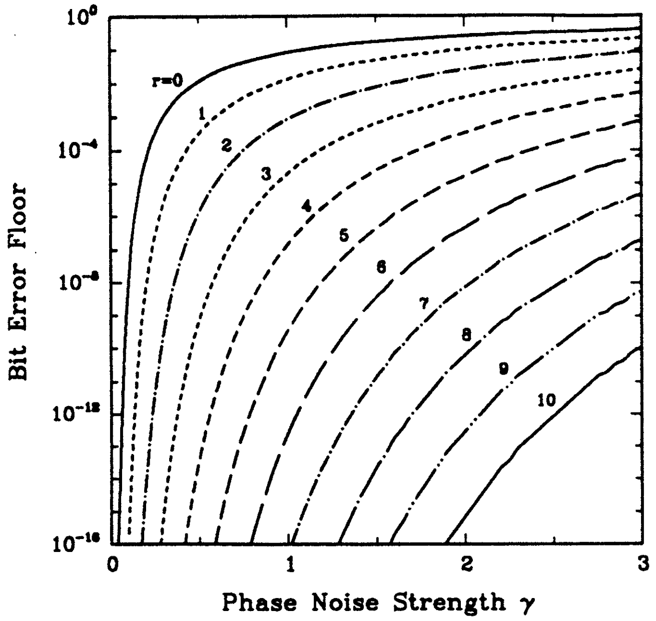


Figure 3: Bit error floor as a function of phase noise strength for different feedback parameters.

can achieve a 10^{-9} bit error rate with 30 photons/bit, and a 10^{-16} bit error rate with 72 photons/bit while having individual laser linewidths of up to 8 MHz. Higher bit rates will tolerate higher linewidths, but will also require faster feedback electronics.

7 Conclusion

In this paper we have provided a study of the performance of DPSK in the presence of phase noise. We obtained upper bounds to the error floor and to the probability of error for a narrowband receiver structure. The analytical framework developed in this paper is significant for a number of reasons. First, it does not use any approximations, e.g. a Gaussian approximation for the statistics of the decision variable, which may render the accuracy of the results questionable. Second, the analysis is valid for an arbitrary phase noise model. This may make it useful in situations where phase randomness assumes a different statistical character. Third, the effect of additive noise on the performance is fully taken into account. Finally, the performance improvement due to frequency feedback has been evaluated.

Our numerical results show that in the absence of frequency feedback, the Brownian motion nature of the phase noise noise process causes very high error floors. On the other hand, when frequency feedback is used to control the phase noise, the error performance is considerably better. Optical DPSK with a simple frequency control loop and a conventional IF receiver is a promising alternative for optical communication systems.

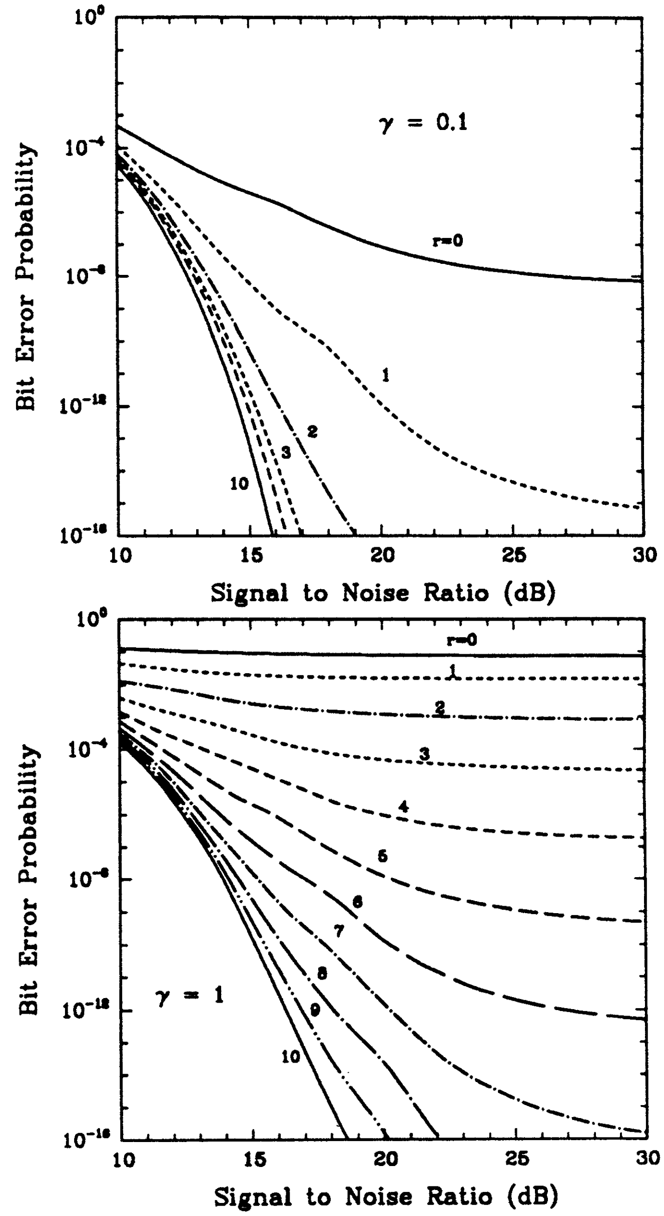


Figure 4: Bit error probability as a function of SNR for different feedback parameters: a) $\gamma = 0.1$, b) $\gamma = 1$.

Acknowledgement

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