Surface Self-Compensated Hydrostatic Bearings

by

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ABSTRACT

It has long been known in the machine tool industry that hydrostatic bearing technology has several unique advantages over rolling element bearings. The thin fluid film between the bearing pads and the rail provides virtually infinite motion resolution due to lack of static friction, very low straightness ripple, high squeeze film damping, potentially infinite bearing life, immunity to fretting, tolerance to ceramic swarf, and superior shock load capacity. However, a major impediment to the use of hydrostatic bearings is that there are no standard, pre-engineered designs that are commercially available, and custom designing a bearing is often prohibitively expensive and time consuming.

In light of the opportunity just mentioned, the goal of this thesis is to present and demonstrate the feasibility of a family of novel modular hydrostatic bearings which are well suited for mass production and are designed to be bolt-for-bolt compatible with modular rolling element bearings. A size 35 prototype of one of the novel designs is presented along with measured and predicted performance (load verses deflection, flow rate, pumping power). The novel design that is tested uses a set of auxiliary restricting surfaces on a profile rail and form fitting truck that make an acute angle relative to each load bearing pocket they supply, thus allowing the truck to be machined and ground as one piece, and eliminating the need for capillaries, diaphragms, or other unmachinable features. In addition to the first prototype work, a second engineered embodiment of the novel design is presented which, via a sophisticated mathematical model, is designed to have an acceptable stiffness and load capacity variation given realistic production manufacturing tolerances.

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Stepped out off at the wrong floor
Used the wrong key in my door
And only yesterday I swore
I wouldn’t do that any more

An hour staring at my screen
I haven’t done a single thing
Where else could that file be
Damn its right in front of me

Dialed a number while half asleep
Heard a whistle and a beep
Man that’s so embarrassing
I just called a fax machine

Woe is my poor tortured brain
Why is it in so much pain
Even very simple things
Are never easy

People tell me
To just be happy
But that advice
Doesn’t work for me

Its not that simple
Its not that easy
Guess that’s another thing
That’s wrong with me

And then a friend said something
That I think
Is pretty cool

She said now close your eyes
And stop everything
And think of someone you love unconditionally

She said now don’t just try
To be happy
And don’t keep thinking
About the way you should be
So now I'm closing my eyes
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Unconditionally
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Chapter 1

INTRODUCTION

The purpose of this chapter is to motivate the development of a modular linear hydrostatic bearing that is bolt-for-bolt compatible with rolling element linear guides. After the motivation is provided, the fundamental contributions of this thesis are outlined.

1.1 Overview of Modular Linear Guides

Briefly stated, modular linear guides are rolling element linear bearings whose rail and truck are designed with industry-wide standard dimensions, so that a linear guide manufactured by one company is bolt-for-bolt interchangeable with a linear guide made by another. The popularity of linear guides is indicated by the numerous manufactures of them, including the STAR, Thomson Bay Company, Schneeberger, INA, THK, SKF, IKO, and others. A major application of linear guides is for precision machine tools, such as machining centers, CNC lathes, surface and cylindrical grinders, centerless grinders, jig grinders, etc.. Modular linear guides have tremendous market appeal because they are easy and fast to design with, and their use greatly reduces assembly time and labor costs of a machine. For companies designing new machines on a tight schedule, modular linear guides are rightly or wrongly often the default choice. Since all currently available modular linear guides have rolling elements, however, which physically contact the truck and the rail, they have inherent performance limitations which can give disappointing machine performance for demanding applications. These limitations include limited motion resolu-
tion due to mechanical contact effects, straightness ripple due to ball or roller entry and exit, low to moderate damping, and premature failure due to particle infiltration.

To provide superior performance in the categories just mentioned, hydrostatic bearing technology is ideal. The thin fluid film between the bearing pads and the rail provide virtually infinite motion resolution, very low straightness ripple (when pump pressure variations are properly attenuated), high squeeze film damping, and potentially infinite bearing life. Despite the superior performance of hydrostatic bearings, they are seldom used because no standard, pre-engineered designs are commercially available, and custom designing a bearing is often an arduous task. In addition, conventional hydrostatic bearings typically have small orifices and capillaries which can clog, and their stiffness and load capacity are highly sensitive to manufacturing errors. Since designers do not have time, nor do they wish to take the risk of designing a conventional hydrostatic bearing themselves, they and the machine tool buyers they sell to choose to live with the limitations of existing rolling element technology.

In light of the opportunity just outlined, the goal of this thesis is the invention, design, modeling and testing of an alpha prototype size 35 modular hydrostatic bearing, and design of a beta prototype hydrostatic bearing suitable for mass production.

If such a product became available on the market, builders who use modular linear guides could offer high performance hydrostatic machines to users. Or, the users themselves could afford to retrofit an existing machine. Due to the performance advantages, modular hydrostatic bearings could become a commonly used bearing for top-of-the-line machine tools.

1.2 Specifications and Room for Improvement

In this section, the following specifications for linear guides are explained, and aspects of rolling element linear guides that could be improved upon are discussed.
Important Specifications for Linear Guides

- Bearing Life
- Load which Causes Permanent Damage
- Static Stiffness
- Dynamic Stiffness*
- Preload
- Straightness Accuracy
- Straightness Repeatability and Smoothness*
- Static Friction
- Dynamic Friction
- Motion Resolution*
- Maximum Speed and Acceleration
- Support Equipment Needed
- Maintenance
- Mounting Requirements
- Availability
- Cost

*Not normally provided by a bearing manufacturer.

Bearing Life

For rolling element guides not exposed to excessive vibration while standing still or abrasive particles, bearing life depends only on the applied load. All major bearing manufacturers provide the following empirically based formula for bearing life $L$, which is defined to be the distance that 90% of all bearings will travel under load $F$ without failure

$$L = \left(\frac{C}{F}\right)^{z} \frac{L_0}{\text{Servicefactor}}$$

(1.1)

where $C$ is the rated dynamic load capacity provided in the product catalogue, $F$ is the actual applied load, $L_0$ is the rated life (typically 100km), $z$ is an exponent, equal to $10/3$ for roller elements, and $3$ ball elements, and Servicefactor is recommended by some manu-
Rolling element bearings can fail before Equation 1.1 predicts by either fretting or by particle contamination. These two modes of failure are discussed below.

**Premature Failure by Fretting**

Fretting failure occurs when a rolling element sits on a raceway at one place while being exposed to prolonged vibration. In industry, premature failure due to fretting is a problem on cam grinding machines and hard turning machines. Figure 1.1 shows a photo of a size 35 roller rail product that failed after 9 months, when it was supposed to last for 3 years, on a high speed cam grinding machine made by Weldon Machine Tool.

![Fret marks from rollers on rail surface](image)

**Figure 1.1** Size 35 roller rail damaged by fretting on a cam grinder after 9 months; the rail was supposed to last 3 years. (photo from Weldon Machine Tool, York, PA)
Premature Failure by Particle Contamination

Due to mechanical contact, rolling element bearings are very sensitive to particle contamination. Any slight damage is aggravated quickly as an element rolls over the damaged area over and over again. In industry contamination is a problem on machines that mill and grind ceramics and graphite. Figure 1.2 and Figure 1.3 shows typical damage to a rail that occurs after only six months of grinding aluminum oxide on a machine at Wilbanks International. These machines are overhauled every six months.

Figure 1.2 A roller bearing rail which has been worn away by ceramic dust in between the roller and the rail. The lighter gray line is the region where the roller travels and has been worn away over a period of about 6 months. (photo from Paul A. Scagnetti, Ph.D., MIT Dissertation)

Figure 1.3 Close up of a rail surface damaged by ceramic particles (photo from Paul A. Scagnetti, Ph.D., MIT Dissertation)
Load which Causes Permanent Damage

For rolling element bearings, the maximum allowed load capacity $C_o$ is between 40% and 80% larger than the rated dynamic load capacity $C$. When $C_o$ is exceeded, the rolling elements permanently brinzel the rail and truck races, and the damaged area quickly deteriorates. In practice $C_o$ is exceeded by the impact of a crash as opposed to being exceeding during normal use. As a result, in practice linear guides are often chosen one or two sizes larger than needed to avoid damage if a crash should occur.

Static Stiffness

Rolling element linear guides provide excellent static stiffness, and since a preload is present, there is never any lost motion, as there can be with sliding bearings. In practice, the static stiffness of linear guides more than adequate, compared to other compliances in the structural loop of a machine tool. For grinding, it should be noted that the cutting forces at sparkout are very low, and as such the error caused by roller recirculation, (about 1 micrometer), thermal drift, and straightness errors, will be much more than caused by bearing deflection.

Dynamic Stiffness

Dynamic stiffness is an important characteristic that is not normally not quoted in product literature. Dynamic stiffness refers to stiffness as a function of the frequency of an applied oscillatory force. Rolling element bearings are always highly under damped, and hence they do not help to reduce vibration due to cutting forces. In practice, the poor damping of linear guides is problem on most types of grinding machines, especially machines that grind ceramics, on hard turning machines, and on some high speed machine tools. At the same time, it is well know in industry that the squeeze film damping effect of hydrostatic and hydrodynamic bearings provide excellent damping and such bearings provide the smoothest possible surface finish.
Specifications and Room for Improvement

Straightness Accuracy

The highest accuracy class specified in product catalogues is typically 3 micrometers of error motion of one truck relative to its rail per meter of rail length. However, it should be noted that in practice the actual bearing straightness is dictated by the straightness error of the machine bed the rails are bolted to. Another important effect is that an assembly of four trucks on two rails results in a good deal of error averaging which improves straightness. The net result is that in practice ultra precision linear guides provide a straightness that is usually no worse, if not better than, the straightness of the machine bed the rails are bolted to.

Straightness Repeatability and Smoothness

Straightness repeatability is more important than straightness accuracy for machines that can compensate for errors. For rolling element linear guides, ball entry and exit zones at the truck races can create a non-repeatable straightness ripple of up to 2 micrometer in amplitude. Appendix B shows 0.6 and 0.4 micron straightness ripple that was measured for lightly preloaded ultra precision AccuGlide and AccuMax trucks, respectively. The results are shown again in Figure 1.4. As with all types of linear bearings, heat generated by the motion can introduce non-repeatable thermal drift.

Static And Dynamic Friction

Static friction is due to the rolling elements, and dynamic friction is due to both the wipers and the rolling elements.

In practice, static friction of linear guides is not a problem on most machine tools of today, but there are notable cases where there is unwanted reversal error on machines that do high speed CNC contour milling. An important example is high speed milling of scroll
compressors, where even a nipple due to reversal of 2 micrometers (0.0008 inch) can cause premature failure of mating scrolls.

In practice, dynamic friction of linear guides is not a problem on most machine tools used today, which move slower than 0.3 m/s. However, for machines that move 1 m/s or faster, the heat generated can cause significant thermal error, and for such cases an effective means for cooling would be a useful attribute.
Motion Resolution

Motion resolution refers to the smallest increment of motion the bearing can be moved in the axial direction repeatable, using an actuator with much higher motion resolution capability. Motion resolution is not quoted in product literature, and it is difficult to quote reliably because it depends on the stiffness and resolution of the actuator that is used. In one study [Futami] one on rolling element bearings, it was found that in a range from 0 to 1 micrometer, nanoscale resolution was possible, but in the range from 1 to 10 micrometers resolution of about 1 micrometer was possible. The explanation given was that in the former range, the rolling elements act like flexural bearings and thus provide smooth motion, whereas in the latter range, the elements begin to roll, but because of the finite Hertzian contact areas, they exhibit some stick-slip like behavior.

In practice, motion resolution is a major concern on grinding machines and high accuracy milling machines.

Maximum Speed and Acceleration

The maximum speed and acceleration of a rolling element bearing is limited by jamming or excessive slipping of the elements. Typical values quoted by manufacturers are 2 m/s, and 50 m/s², although higher values are possible (the manufacturer must be contacted for a specific application). In practice, for most high speed machines, jamming is not a problem unless lubrication is poor.

Support Equipment Needed

To achieve the longest life, automatic lubing systems are required for rolling element linear guides. This involves a pump, lubrication lines, and drainage gutters.

Maintenance

For bearings without an automatic lubrication system, grease is required regularly, and periodic inspection of the rails and bellows is required to insure contaminants are not leaking in.
Mounting Requirements

While mounting methods for linear guides are similar, the required tolerances of the mating surfaces may differ between designs. All linear guide trucks are designed to allow bolting from above and below. Standard rails allow bolting from above, while custom rails can be ordered to allow bolting from below. A critical requirement for mounting is that jibs or push plates be used to push at least one rail up against a precision shoulder, before tightening the rail bolts.

Availability

While modular linear guides are always available, they can have lead times of 6 weeks or more, depending on the rail length and accuracy class.

Cost

In general, bearings that use ball elements cost less than bearings that use cylindrical elements. For the rails, the cost ranges from $20 per foot for small, low precision rails to $200 per foot for large, high precision rails. For trucks, the cost ranges from $50 to $600 per truck, depending on size and accuracy class.

1.3 Applications for a Linear Hydrostatic Guide

At a fundamental level, because there is no mechanical contact, a hydrostatic bearing offers several advantages over rolling element bearings. These advantages are listed in Table 1.1, along with applications that could most benefit from the advantage.
Applications for a Linear Hydrostatic Guide

TABLE 1.1 Advantages and Applications for a Linear Hydrostatic Guide.

<table>
<thead>
<tr>
<th>Advantages of a Hydrostatic Linear Guide</th>
<th>Promising Applications in Industry</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 No wear</td>
<td>Hard turning, high speed machining</td>
</tr>
<tr>
<td>2 Not vulnerable to fretting</td>
<td>Hard turning, high speed grinding and machining</td>
</tr>
<tr>
<td>3 Tolerant to particles</td>
<td>Grinding ceramics, graphite</td>
</tr>
<tr>
<td>4 Crash resistant</td>
<td>Some high speed machines</td>
</tr>
<tr>
<td>5 High dynamic stiffness</td>
<td>Grinding, hard turning, high speed machining</td>
</tr>
<tr>
<td>6 Straightness repeatability</td>
<td>Grinding</td>
</tr>
<tr>
<td>7 Smoothness of motion</td>
<td>Grinding</td>
</tr>
<tr>
<td>8 Zero static friction</td>
<td>Grinding, contour milling</td>
</tr>
<tr>
<td>9 Low dynamic friction</td>
<td>high speed machining</td>
</tr>
</tbody>
</table>

From this list it can be seen that grinding processes can benefit in the greatest number of ways from hydrostatic bearing technology. Other important applications are high speed machining (more than 1 m/s traverse rate) and hard turning.

1.3.1 Immediate Markets for a Linear Hydrostatic Guide

At this time, several companies are interested in a being a beta sight for testing a modular linear hydrostatic bearing. Table 1.2 lists these companies, along with the reasons they are interested, from most important to least important, and the estimated number of machines equipped with the hydrostatics that they could sell a year.

TABLE 1.2 Immediate Markets for a Linear Hydrostatic Guide

<table>
<thead>
<tr>
<th>Companies to date that want to beta test a linear hydrostatic bearing</th>
<th>Desired advantage (most important to least)$^1$</th>
<th>Estimated market, machines per year$^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hardinge</td>
<td>1,2,5</td>
<td>100</td>
</tr>
<tr>
<td>Weldon Machine Tool</td>
<td>1,2,5,7,8,6,3</td>
<td>2</td>
</tr>
<tr>
<td>Jung</td>
<td>3,5,7,8,6</td>
<td>5</td>
</tr>
<tr>
<td>Moore Tool</td>
<td>5,7,8,6</td>
<td>10</td>
</tr>
<tr>
<td>Star Cutter</td>
<td>5,7,8,6</td>
<td>10</td>
</tr>
</tbody>
</table>

$^1$Advantages listed in previous table

$^2$Cost of hydrostatic system will be about $5,000 more per machine tool than a rolling element system

This preliminary market data reveals some important insights.
• The hard turning market is an order of magnitude larger than any individual grinding market, and its primary need is no wear. The no wear and related features appears at this time to be the most valuable asset of a hydrostatic modular guide.

• Grinding is the only other known immediate market, and the dominant need is dynamic stiffness. Dynamic stiffness appears to be the second most valuable asset of a hydrostatic modular guide.

1.3.2 Motto for the Future: "No Contact - Forever Perfect"

Looking at the longer term market of high speed machining, a modular linear hydrostatic bearing is ideal for use with linear motors. By using all hydrostatic bearings actuated by direct drive motors, all the precision motion elements of the machine tool could be non contacting and hence free of wear.

Such a non contact Hydrostatic / Direct Drive machine would have the following very significant advantages over existing machines:

Advantages of a Hydrostatic / Direct Drive Machine

• Accuracy of axes will not degrade over time
• No need to overhaul the bearing axes
• One axis can stay in place indefinitely and not be damaged by vibration
• Tolerant to ceramic swarf
• Bearings will not be damaged by a high speed crash
• The best possible damping to accommodate high speed motion
• All bearings are temperature controlled via their fluid
• No need to tune and retune the controllers to eliminate reversal errors
• Zero static friction makes all aspects of control very deterministic

An affordable off-the-shelf hydrostatic guide will be an important enabling component of the machine tool of the future with all of the features described above.
1.4 Fundamental Contributions of Thesis

The fundamental contribution of this thesis is the invention, design and testing of a radically new modular linear hydrostatic bearing (see Figure 1.5).

Figure 1.5 Fundamental contribution of thesis: Angled Surface Self compensation, and its application to a modular linear bearing rail.
The core innovative feature, called angled surface self-compensation, is a set of auxiliary restricting surfaces on a profile rail and truck that make an acute angle relative to each load bearing pocket they supply.

This deceptively simple innovation, as will be explained in Chapter 2, leads to several very significant manufacturing advantages over prior art hydrostatic bearing designs that can be applied to a modular bearing rail. The fundamental advantages of the novel design are summarized below.

**Fundamental Manufacturing Advantages over Standard Hydrostatic Concepts**

1. Only design known by author to be economical to manufacture using existing manufacturing equipment of linear bearing manufacturers.
2. Truck is monolithic without intricate internal passages.
3. All critical precision features (restrictor and bearing pads) can be profile ground in one set-up.

**Fundamental Performance Advantages over Standard Hydrostatic Concepts**

1. Feedback is the most efficient for vertical loading, where it is needed most.
2. More squeeze film area is present, providing better dynamic stiffness.

**Fundamental Robustness Advantages over Standard Hydrostatic Concepts**

1. Tolerant to dirt and "goo" because all gaps are subjected to shearing.
2. No passage between restrictor and pocket is present, so stiffness degrading air bubbles can't be trapped between.

**Fundamental Marketing Advantage over Standard Hydrostatic Products**

1. Bolt-for-bolt interchangeable with modular rolling element bearings (35, 45, 55 etc.).
Fundamental Contributions of Thesis
Chapter 2

COMPARISON OF DIFFERENT HYDROSTATIC DESIGNS

In this chapter, criteria are presented for a hydrostatic linear guide to be a viable commercial product, and then seven possible hydrostatic bearing designs are compared in a detailed design study.

2.1 Criteria for a Hydrostatic Guide to be a Viable Commercial Product

Based on Alexander Slocum’s and my experience working with several machine tool companies and two major linear guide manufacturers, we developed the following criteria for making a hydrostatic bearing into a viable commercial product:

Size

Must be bolt-for-bolt compatible with rolling element linear guides. Size 35, Size 45, and Size 55 normal width and narrow width are most commonly desired. To gain extra load capacity, we can use the long truck option as it will fit in most machines that use a standard truck length.

Manufacturing

1. Must be mass producable using profile grinding technology, which is already in place and has proven to be economical for rolling element bearings.

2. Minimum number features to be machined and parts to be assembled. A one piece rail and a one piece truck with a few holes and pockets is the ideal.
Simplicity should be comparable to a rolling element linear guide minus the rolling elements themselves.

3. "Grind and go" design - no hand tuning or delicate hand assembly of parts required.

**Performance**

1. Infinite life load capacities of 1000 lb, 1800 lb, and 2600 lb for sizes 35, 45 and 55, respectively. These load capacities are adequate for most precision machine tool applications. While over a short travel life of 100 km, rolling element guides can provide many times these load capacities, over 10,000 km of travel, generally regarded as an "infinite" life, the load capacities of rolling element guides are roughly comparable under ideal conditions. However, if grit or excessive vibration is present, rolling element guides can fail well below 10,000 km, regardless of loading.

2. Static stiffnesses of about 3 lb/uin, 5 lb/uin, and 7 lb/uin for sizes 35, 45, and 55 respectively. These are comparable to typical rolling element guides with a medium preload.

3. A 1.5 hp, 4 gallon per minute pump required to power two linear axes with 8 trucks (regardless of bearing size). It is desirable for the pressure to be kept below 700 psi so that less expensive hoses can be used.

**Fluid**

Depending on the application, water or oil can be used. The main motivation for using water is for compatibility with a water based cutting fluid. However, since water is about 10 to 60 times less viscous than light and medium hydraulic oil, respectively, the pump power required when using water will be 10 to 60 times greater at the same supply pressure. Table 2.1 summarizes the advantages and disadvantages of using water versus using hydraulic oil.
TABLE 2.1 Advantages and Disadvantages of Using Water Versus Using a Hydraulic Oil

<table>
<thead>
<tr>
<th>Advantages</th>
<th>Disadvantages</th>
</tr>
</thead>
<tbody>
<tr>
<td>Not vulnerable to contamination of a water</td>
<td>10 to 60 times more pump power,</td>
</tr>
<tr>
<td>based cutting fluid</td>
<td>causing heat which can reduce accuracy</td>
</tr>
<tr>
<td>10 to 60 times less viscous drag</td>
<td>Fluid system is at least 50% more</td>
</tr>
<tr>
<td></td>
<td>expensive than comparable oil system</td>
</tr>
<tr>
<td></td>
<td>Particles are more difficult to filter</td>
</tr>
<tr>
<td></td>
<td>Pump life is less in some cases</td>
</tr>
<tr>
<td></td>
<td>Can gum up small orifices</td>
</tr>
<tr>
<td></td>
<td>Erodes small pasageways faster</td>
</tr>
</tbody>
</table>

Because of the many difficulties of using water, for most machine tool applications it will probably be easier to isolate the hydraulic oil using drain grooves and bellows.

**Dirt Tolerance**

1. Highly desirable if all restricting surfaces move so that any goopy blobs which can cause clogging, (common in the water based machine tool coolants that may be used) can be sheared away. This feature would preclude using orifices and capillaries for the final bearing.

2. Highly desirable if there is a means for large dirt chunks and chips, which are inevitably in a newly plumbed system, to be flushed out of all the trucks when they are first turned on by opening a valve. We want this feature because several of our older bearings used on machines clogged with chips and dirt when they were first turned on, and they had to be taken apart at least once to be cleaned. After the initial cleaning the bearings ran with no problems. While meticulous flushing of the plumbing before assembly could eliminate this problem, we would prefer to eliminate the need for such meticulousness.

3. Desirable is there is a means for small dirt particles that are nearly the size of the bearing gap to leave the bearing pockets through an escape path, without allowing excessive leakage out of the bearing. While we have found that with good filtration, bearings without this feature have run without problems for over 3 years, we would like extra insurance against a bad filter or a sloppy filter change.

**2.2 Novel Contributions Studied for Thesis**

Figure 2.1 illustrates a novel feature and a novel design that will be studied in this thesis.

The significance of these innovations are discussed below.
Novel Feature to Reduce Flow:
No drain groove between bearing pockets (NGBP)

Prior Art Applied to a Modular Rail:
Planar bearing pads

Novel Design to Simplify Manufacturing:
Angled Surface Self Compensation

Figure 2.1 Illustration of a novel feature and novel hydrostatic bearing design studied in this thesis, applied to a modular linear rail.

Novel Feature: No Drain Groove Between Bearing Pockets (NGBP)

The idea is to have a continuous profile between the upper and lower bearing pockets on a modular bearing carriage, rather than having planar bearing pads that are separated by a drain groove. The benefit is that flow rate to the atmosphere is reduced by 40%, but the penalty is that the vertical stiffness is reduced because a short exists between the upper and lower pockets. The net benefits will be discussed in Section 2.3.
Novel Hydrostatic Bearing (Angled Surface Self Compensation)

The idea is to eliminate the need for capillaries, diaphragms, or complex internal passages by using a special profile that has a set of angled restricting surfaces that feed each load bearing pocket. To illustrate the concept, Figure 2.2 shows qualitatively how vertical and lateral stiffness is achieved.

![Diagram of vertical and lateral displacement](image)

Figure 2.2 Illustration of how Angled Surface Self Compensation achieves vertical and lateral stiffness.

The major advantage of this design is that, unlike prior art concepts that are known to the author, the angled surface design is the only one that can be manufactured using well known processes and equipment that are already used by makers of modular rolling element bearings (i.e. the angled surface design will not require investment and R&D into unfamiliar manufacturing equipment or processes). Performance wise, the design provides relatively high vertical stiffness but low lateral stiffness when compared to other designs. These trade-offs will be discussed in more detail in the Comparison section.


2.3 Design Concepts Considered

The purpose of this section is to present the compensation principle that underlies the prior art and novel bearing designs that are studied and compared in this thesis. In simple language, compensation is what makes a pocket pressure go up when the bearing gap closes, and equivalently what makes a pocket pressure go down when the bearing gap opens. Hence compensation provides static stiffness to a hydrostatic bearing. It is the way compensation is achieved that distinguishes one hydrostatic bearing design from another.

2.3.1 Constant Flow Compensation

This method uses a constant flow source connected to each pocket to achieve compensation. The principle is as follows: as a bearing gap gets smaller, its resistance to flow goes up, and because flow is forced to be constant, the pocket pressure must go up. A schematic of a constant flow system is shown in Figure 2.3, along with the resistance circuit used to model the bearing.

![Fluid circuit for a four pocket system using constant flow compensation](image)

As bearing gap decreases, pocket pressure increases because resistance goes up while flow is forced to be constant.

Each pocket requires a separate constant flow source. This is a major practical disadvantage because of the special pump and numerous hoses required.

**Figure 2.3** Constant flow compensation.
The constant flow method is not studied in this thesis because it has a major practical drawback. A standard constant pressure hydraulic system cannot be used to power it. Instead, each load bearing pocket must have its own constant flow source. This results in several major problems:

**Major Problems With Constant Flow System**

- Each pocket requires a constant flow pump and a separate hose (4 bearing trucks must have at least 16 pockets, requiring 16 pumps and hoses).
- Compliance in long hoses will reduce static and dynamic stiffness.
- Flow ripple of pumps will make bearing vibrate, reducing accuracy, and accumulators can not be used else static stiffness will be effected.

Since the author cannot think of a remedy to these problems, the constant flow system was not studied in this thesis.

### 2.3.2 Capillary or Orifice Compensation

This method is the most commonly used in industry, and it is readily adapted to a modular bearing profile. It uses a fluid resisting device (either a capillary or an orifice) placed in series with each bearing pocket. The principle of operation is as follows: when a bearing gap closes, its resistance goes up, dropping the flow rate through the capillary or orifice, and in turn reducing the pressure drop that occurs across the capillary or orifice, thus increasing the pressure in the bearing pocket. Figure 2.4 shows an implementation of capillary compensation in a size 35 bearing, with and without a drain groove placed in between the bearing pockets. The fluid circuit used to analyze the bearing is also shown.
2.3.3 Diaphragm Compensation

This method uses an elastic restricting device to amplify compensation beyond what capillary compensation provides. Figure 2.5 shows a size 35 bearing embodiment that uses diaphragm compensation, along with a fluid circuit schematic. The circuit shows that each pocket pressure modulates its own and its partner's restrictor resistance.

The principle of operation can be easily understood by first imagining that the diaphragm does not flex, thus making the system behave the same as a capillary based system. Now if
Design Concepts Considered

Figure 2.5 Diaphragm compensation implemented in a Size 35 modular bearing, with and without the NGBP feature.

...you imagine the diaphragm is allowed to flex, the low pressure pocket will be further choked and the high pressure pocket will be further fed due to the gap change as the diaphragm flexes, hence increasing any difference in pocket pressure that would be present if the diaphragm were rigid.
2.3.4 Self Compensation with Internal Passages

This method uses miniature bearing lands that act as the restrictors [Slocum]. The fluid flowing out of each restrictor is routed to an opposed bearing pocket via internal passages or external hoses. Using this method, feedback is twice as effective as with a capillary system, because each restrictor resistance changes in the opposite sense as each bearing resistance. Figure 2.6 shows a size 35 bearing embodiment that uses self-compensation with internal passages, along with a fluid circuit model.

Figure 2.6 Self Compensation with Internal Passages implemented in a Size 35 modular bearing, with and without the NGBP feature.
2.3.5 Angled Surface Self Compensation

Like self compensation with internal passages, this method also uses miniature bearing lands that act as restrictors, except that rather than using internal passages to carry fluid from each restrictor to each pocket, each restrictor is placed right next to each bearing pocket on an auxiliary surface that makes an acute angle relative to each pocket. Figure 2.7 shows a size 35 bearing embodiment that uses self-compensation with angled auxiliary surfaces, along with a fluid circuit model.

![Diagram of angled surface self compensation](image)

**Figure 2.7** Angled Surface Self Compensation implemented in a Size 35 modular bearing.
2.4 Overview of Analysis Issues

Prior to the comparison sections, the definitions of load capacity, minimum static stiffness, and optimization scenarios are discussed.

2.4.1 Desired Load Capacity and Static Stiffness are Linked

In general, a designer must make a trade-off between a hydrostatic bearing’s desired load capacity and the minimum stiffness that it will provide when it is loaded with a force that is less than or equal to the desired load capacity. Having said this, load capacity must be clearly defined for a hydrostatic bearing system.

Definition of Load Capacity for a Hydrostatic Bearing: Load which can be applied to the bearing in all directions that causes no more than an x % closure of the bearing gap.

Factors Effecting Choice of Max. % Gap Closure:

1. Trade-off between load capacity and stiffness. At a given supply pressure, or at a given pumping power, to achieve a relatively high static stiffness with a penalty of a relatively low allowed load capacity, the bearing should be optimized using a relatively low max. gap closure (20% is relatively low in practice). To achieve a relatively high load capacity with a penalty of relatively low minimum stiffness that occurs using allowed loads, the bearing should be optimized using a relatively high max. gap closure (60% is relatively high in practice).

2. Max. tilt error expected for bearing pads. The chosen Max. gap closure should be less than or equal to an amount that will prevent a corner of a mis-aligned bearing pad from touching down.

2.4.2 Checking for Minimum Static Stiffness

For optimization one must check the minimum static stiffness $K_s(F_{apt}, \theta_{apt}, \theta_{\Delta F})$ of the 4 pad hydrostatic bearing system that occurs for all allowed loading scenarios. Given a maximum allowed force $F_{l_c}$, one needs to check the minimum stiffness that occurs in the zone $F_{apt} \leq F_{l_c}$, $\theta_{apt} = [0 \text{ to } 2\pi]$, and $\theta_{\Delta F} = [0 \text{ to } \pi]$. Figure 2.8 illustrates the parameters $F_{apt}$, $\theta_{apt}$, and $\theta_{\Delta F}$. 
2.4.3 Two Optimization Scenarios

This section provides a discussion of two standard types of optimization normally suggested for hydrostatic bearings. For ease of understanding, these scenarios are presented in a Given, Find format.

Scenario 1: Pumping Power is Primary Concern, Hydraulic System can Supply Most any Supply Pressure Required.

Given: External Dimensions $D_o$, Max. Allowed Load $F_{ic}$, Pump Power $W_p$

Find: Internal Dimensions $D_1$ which maximize the min. stiffness $K_s$ that occurs in the zone $F_{apl} \leq F_{ic}$, $\theta_{apl} = [0 \text{ to } 2\pi]$, and $\theta_{AF} = [0 \text{ to } \pi]$
Scenario 2: Practical Limit on Available Supply Pressure Due to Nature of Pump System, Pump Power is of Secondary Importance.

**Given:** External Dimensions $D_o$, Max. Allowed Load $F_{lc}$, Supply Pressure $F_s$, and Max. Allowed Pump Power $W_{p_{\text{max}}}$

**Find:** Internal Dimensions $D_1$ which maximize the min. stiffness $K_s$ that occurs in the zone $F_{apl} \leq F_{lc}$, $\theta_{apl} = [0 \text{ to } 2\pi]$, and $\theta_{\Delta F} = [0 \text{ to } \pi]$

### 2.4.4 Thermal Control of Fluid: Area for Future Work

For use in machine tools, it should be noted that the reason one wants to keep pump power low is not primarily because of pump system cost, it is because one wants as little heat as possible transferred to the machine bed. **It should be noted that what really matters is keeping thermal drift of the machine tool to a minimum.** A non optimal pump power may not matter, so long as the fluid temperature can be kept as constant as possible using a chiller system to keep the thermal drift of the machine to a minimum. Based on this insight, the author poses the following questions and hypothesizes answers which point the way to future work.

**Question 1:** For accurate temperature control of the machine bed, at a given pump power and viscosity which is better: Low $P_s$ and High $Q$, or High $P_s$ and Low $Q$?

The author hypothesizes that Low $P_s$, High $Q$ is better because the temperature rise of the oil will be less for the latter scenario, and it is likely a smaller temperature rise is more desirable from a controls standpoint.

**Question 2:** Is it worthwhile to design for low $P_s$ without regard to pump power to allow for accurate temperature control of the bearing fluid?

The author hypothesizes that in the end designing for low $P_s$ at a given load capacity requirement and viscosity will make little difference over using a design optimized for minimum pumping power. However he welcomes proof to the contrary. The author believes that proper choice of viscosity will make a much bigger difference on thermal controllability and stability.
Question 3: Given the average travel velocity of the carriage, what fluid viscosity should be chosen to allow for temperature control which ultimately optimizes thermal stability of the machine?

For brute force accurate control of fluid temperature, one would choose a low viscosity because this will reduce shear power, which is variable, at the expense of pump power, which is steady and hence controllable. The unanswered question is how low should the viscosity choice be before the large pump power significantly effects thermal drift of the machine.

2.4.5 Approximations Made for Analysis of Hydrostatic Designs

The following approximations were made in predicting load capacity and stiffness of the different hydrostatic designs. Most of these assumptions tend to make the predicted performance better than what an actual production bearing will provide, so the resulting analysis is an optimistic estimate of bearing performance.

1. No bearing gap errors present.

2. Effect on gamma due to initial elastic deflection caused by initial pocket pressures not accounted for. However, gaps could be ground so that they open to what they should be.

3. Effect of elastic deflection on static stiffness was accounted for in a rough fashion: a hydrostatic compliance $C_{hy}$ (1/stiffness) was first computed in a given direction $\theta_{AF}$, assuming a perfectly rigid truck, and then it was added to an effective elastic compliance $C_{eff}$ in direction $\theta_{AF}$. For the size 35 truck with L=164mm, $C_{eff}(\theta_{AF})$ was computed by making elliptical quadrants from a lateral elastic compliance reference of $C_{eff\text{x}} = 1/1400$ (\mu m/N), a tensile elastic compliance reference of $C_{eff\text{y}(+)} = 1/2500$ (\mu m/N), and a compressive elastic compliance reference of $C_{eff\text{y}(-)} = 1/3000$ (\mu m/N).

4. For the diaphragm system, it is assumed the diaphragm is tuned so that it gives the same static feedback as a self compensating bearing. It should be noted that the elastic compliance of the truck will result in a diminishing return in increasing hydrostatic stiffness, and there is not a great benefit to designing for extremely high hydrostatic stiffness.

5. The effect that the (NGBP) feature has on diminishing pocket pressures was accounted for roughly by reducing the difference between the upper and lower pocket pressures computed with a groove by 20%. This was deter-
mined to be reasonable based on an approximate analysis for a capillary sys-
tem.

2.5 Comparison of Hydrostatic and Rolling Element Systems

In this section, the size 35 embodiments shown in the Design Concepts section are com-
pared. A size 35 was chosen for this study because its sells the most to machine tool com-
panies. While other common sizes (30, 45, and 55) are not compared, their proportions are
within a few percent and hence the relative performance will be similar.

To allow the reader to quickly assimilate the current state of knowledge of the author, key
information predicted for the size 35 hydrostatic embodiments and size 35 ball and roller
bearings currently on the market have been tabulated for quick comparison in three cate-
gories: Load capacity and stiffness, manufacturability, and robustness. Each matrix is pre-
seated first, then plots are presented to supplement the matrices.
### 2.5.1 Load Capacity and Stiffness Comparison Matrix

#### TABLE 2.2  Load Capacity and Stiffness Comparison Matrix of Size 35 Hydrostatic and Rolling Element Systems

<table>
<thead>
<tr>
<th>Size 35 Hydrostatic Systems (Predicted)</th>
<th>Gamma factor</th>
<th>bo (flow rate)</th>
<th>Ps (supply pressure)</th>
<th>Flow Rate per truck</th>
<th>P (working load capacity)</th>
<th>R (relative working load capacity)</th>
<th>Kmin (min. static stiffness)</th>
<th>Q (approximate Q at resonance)</th>
<th>Kdyn (estimated minimum dynamic stiffness)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(L=164mm long truck)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Capillary (D = 0.2 mm, L = 10mm)</td>
<td>?? ??</td>
<td>2.9 4.5</td>
<td>112 0.54</td>
<td>9.9 100%</td>
<td>439 100%</td>
<td>2.2 200</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>*Capillary NGBP</td>
<td>?? ??</td>
<td>2.7 4.7</td>
<td>151 0.40</td>
<td>12.0 121%</td>
<td>491 107%</td>
<td>2.2 214</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Diaphram</td>
<td>?? ??</td>
<td>8.4 4.4</td>
<td>172 0.35</td>
<td>17.6 178%</td>
<td>602 142%</td>
<td>2.2 285</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>*Diaphram NGBP</td>
<td>?? ??</td>
<td>6.4 5.2</td>
<td>225 0.27</td>
<td>20.5 207%</td>
<td>636 145%</td>
<td>2.2 291</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>*SC, Internal passages</td>
<td>?? ??</td>
<td>8.5 4.0</td>
<td>129 0.47</td>
<td>11.0 111%</td>
<td>419 95%</td>
<td>2.2 191</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>*SC, Internal passages NGBP</td>
<td>?? ??</td>
<td>7.5 4.0</td>
<td>142 0.42</td>
<td>11.3 114%</td>
<td>400 91%</td>
<td>2.2 183</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>*SC, Angled surface</td>
<td>?? ??</td>
<td>9.5 2.3</td>
<td>114 0.52</td>
<td>9.5 96%</td>
<td>315 72%</td>
<td>2.2 144</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Size 35 Ball Systems (From Catalogue)</th>
<th>Load values shown for rolling elements based on DIN ISO 281 with 10^4 km Life and 1% Failure Rate</th>
<th>KN %</th>
<th>N / μm %</th>
<th>Q</th>
<th>Kdyn</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thomson, L= 109mm</td>
<td></td>
<td>3%</td>
<td>20%</td>
<td>500 114%</td>
<td>5.5 91</td>
</tr>
<tr>
<td>Thomson, L= 134mm</td>
<td></td>
<td>3%</td>
<td>0%</td>
<td>500 114%</td>
<td>5.5 91</td>
</tr>
<tr>
<td>Thomson, L= 134mm</td>
<td></td>
<td>8%</td>
<td>0%</td>
<td>500 114%</td>
<td>5.5 91</td>
</tr>
<tr>
<td>Thomson, L= 134mm</td>
<td></td>
<td>13%</td>
<td>0%</td>
<td>500 114%</td>
<td>5.5 91</td>
</tr>
<tr>
<td>Star, L= 105mm</td>
<td></td>
<td>8%</td>
<td>0%</td>
<td>500 114%</td>
<td>5.5 91</td>
</tr>
<tr>
<td>Star, L= 133mm</td>
<td></td>
<td>8%</td>
<td>0%</td>
<td>500 114%</td>
<td>5.5 91</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Size 35 Roller Systems (From Catalogue)</th>
<th>%</th>
<th>N / μm %</th>
<th>Q</th>
<th>Kdyn</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thomson, L= 109mm</td>
<td>3%</td>
<td>80 80</td>
<td>7.3 74%</td>
<td>667 152%</td>
</tr>
<tr>
<td>Thomson, L= 109mm</td>
<td>8%</td>
<td>80 80</td>
<td>5.4 55%</td>
<td>714 163%</td>
</tr>
<tr>
<td>Thomson, L= 134mm</td>
<td>13%</td>
<td>80 80</td>
<td>1.9 19%</td>
<td>769 175%</td>
</tr>
<tr>
<td>Star, L= 114mm</td>
<td>13%</td>
<td>114 114</td>
<td>2.3 23%</td>
<td>1000 228%</td>
</tr>
<tr>
<td>Star, L= 138mm</td>
<td>13%</td>
<td>149 149</td>
<td>2.9 29%</td>
<td>1261 287%</td>
</tr>
<tr>
<td>Schneeberger, L= 109mm</td>
<td>3%</td>
<td>93 93</td>
<td>8.2 83%</td>
<td>541 123%</td>
</tr>
<tr>
<td>Schneeberger, L= 109mm</td>
<td>8%</td>
<td>93 93</td>
<td>6.0 61%</td>
<td>833 190%</td>
</tr>
<tr>
<td>Schneeberger, L= 136mm</td>
<td>13%</td>
<td>93 93</td>
<td>2.1 21%</td>
<td>1111 253%</td>
</tr>
<tr>
<td>Schneeberger, L= 136mm</td>
<td>3%</td>
<td>129 129</td>
<td>11.2 114%</td>
<td>706 161%</td>
</tr>
<tr>
<td>Schneeberger, L= 136mm</td>
<td>8%</td>
<td>129 129</td>
<td>8.3 84%</td>
<td>1089 248%</td>
</tr>
<tr>
<td>Schneeberger, L= 136mm</td>
<td>13%</td>
<td>129 129</td>
<td>2.9 30%</td>
<td>1452 331%</td>
</tr>
</tbody>
</table>

*Designs with features unique to thesis

?? Shock load capacity will be superior to rolling elements, however prediction is beyond the scope of this thesis.
Table 2.2 shows the predicted performance of different Size 35 hydrostatic designs operating with 100W per truck, oil viscosity of 60 cSt (60 times more viscous than water; a medium weight oil) and with a 20 micron nominal bearing gap, compared to existing ball and roller bearing systems. The 20 micron gap was determined to be practical given production tolerances and mounting tolerances allowed for rolling element bearings. The 60 cSt was chosen because it is typical for machine tool hydraulic systems. The Wp=100W per truck was chosen as a reasonable value for machine tools. The load capacity scales with Wp^0.5 and the static stiffness scales roughly with Wp^0.4 (if the truck and rail were perfectly rigid, scaling would precisely be Wp^0.5).

2.5.2 Static and Shock Load Capacity Discussion

These two categories refer to the maximum static load and sudden shock load a bearing can support before permanent damage occurs. As stated in most product catalogues, the static and shock load capacities of ball and roller systems are the same and are a result of permanent deformation of the races due to hertzian stress. The failure quantities shown were obtained from the product catalogues.

Due to squeeze film damping, those experienced with fluid film bearings know that fluid bearings have a much greater shock load capacity than rolling element bearings of the same size. Static load capacity will also be much higher, but if touch down occurs while the bearing is moving very slowly, galling could result. Predicting galling and shock load capacity is beyond the scope of this thesis and has been left as an area for further research.

2.5.3 Load Capacity Versus Life Comparison Plot

To fully understand how hydrostatic systems compare with rolling element systems in terms of load capacity, one must look at the load versus life curve shown in Figure 2.9.
Conclusions from Load Versus Life Plot:

1. For a typical bearing load on a size 35 truck, under normal conditions and assuming perfect mounting, 99% of lightly preloaded ball systems and medium and a lightly preloaded roller system will essentially last forever.
2. For a heavy bearing load on a size 35 truck, under normal conditions and assuming perfect mounting, 99% of lightly preloaded roller systems will essentially last forever.

3. In a real machine, mounting errors and thermal growth of the machine carriage can cause higher forces than those shown. The force can be estimated using the static stiffness of a truck and the estimated error displacement.

4. All hydrostatic designs provide more than enough load capacity at 100 W. The diaphragm systems can offer the best load capacity when optimized for 60% gap closure, while the other designs are fairly close.

5. The NGBP feature boosts load capacity by about 20% for the Capillary and Diaphragm systems, but provides little benefit for Self Compensation with Internal Passages.

6. Angled Surface Self Compensation provides a load capacity close to Capillary and Internal Passage designs.

### 2.5.4 Static Stiffness Comparison Plots

To have a more complete understanding of how hydrostatic systems compare with rolling element systems in terms of static stiffness, it is helpful to look at the initial static stiffness versus direction $\theta_{AF}$ of the perturbing load. Figure 2.10 shows such a plot.

**Conclusions from Initial Static Stiffness Plot:**

1. Hydrostatic designs operating at $W_p = 100W$ will have a static stiffness comparable to a medium to heavy preloaded ball system. Hydrostatic stiffness scales roughly with $W_p^{0.4}$.

2. A heavily preloaded roller system will be 2 to 3 times more statically stiff than the hydrostatic designs operating at 100W per truck.

For the hydrostatic designs, the minimum static stiffness $K_{smin}$ was computed at $F_{apl} = F_{lc}$ for several directions $\theta_{apl}$. The result is shown in Figure 2.11.

**Conclusions from Final Static Stiffness Plot:**

1. The diaphragm system can provide the best all around static stiffness, while the other designs are comparable in terms of stiffness magnitude.

2. The angled surface design differs from the others fundamentally in that its minimum stiffness occurs laterally, while for the others it occurs vertically. The advantage of this attribute depends on the application.
Figure 2.10 Initial static stiffness (based on catalogue data) of rolling element bearings versus $\theta_{AF}$ shown with hydrostatic bearing designs operating at 100 W per truck.
Min. Static Stiffness at $F_{c, \theta}$: $K_{s\text{min}}(D, F_{c, \theta}, \theta_{\text{ aperture}}, \theta_{\text{ direction}}=[0, \pi])$ of Optimal Designs Vs. $\theta_{\text{ aperture}}$ Size 35, $W = 100W$, $u = 60$ cSt, $h_{\text{min}} = 20 \mu m$

Figure 2.11 Minimum final static stiffness versus $\theta_{\text{ aperture}}$ shown for hydrostatic bearing designs operating at 100 W per truck.
2.5.5 Dynamic Friction Comparison Plots

Figure 2.12 shows a plot of dynamic friction of a size 35 roller bearing system versus a hydrostatic system operating with different oil viscosities.

![Friction Force Vs. Velocity for Size 35 Rolling Element and Hydrostatic Trucks](image)

Figure 2.12  Friction force versus travel velocity for rolling element systems and hydrostatic systems operating at different oil viscosities.
In order to draw meaningful conclusions, it is helpful to look at friction power per truck versus the travel velocity.

Conclusions from Friction Power Versus Travel Velocity Plot:

1. The friction power of a rolling bearing becomes significant beyond 1 m/s. It is known that roller systems overheat severely if they are not actively cooled.

2. A hydrostatic system produces lower friction power at high speeds and the heat can be actively removed by a chiller system, along with pumping power. The cooling means coupled with low friction power is a major advantage of a hydrostatic bearing for high speed machining.

2.5.6 Manufacturing Comparison Matrix

Table 2.3 compares the manufacturability of the different hydrostatic designs. Each criteria is discussed below.
Figure 2.13 Friction power versus travel velocity for rolling element systems and hydrostatic systems operating at different oil viscosities.
### TABLE 2.3 Manufacturing Comparison Matrix

<table>
<thead>
<tr>
<th>Size 35 Hydrostatic systems (Predicted)</th>
<th>Can be made right now, in house, with current manufacturing equipment and processes</th>
<th>Can be Monolithic</th>
<th>All precision features ground in one set up</th>
<th>Insensitive to spread error</th>
<th>Insensitive to skew error</th>
<th>Insensitive to tilt error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capillary ((D = 0.2 \text{ mm}, L = 10 \text{ mm}))</td>
<td>No</td>
<td>??</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>*Capillary NGBP</td>
<td>No</td>
<td>??</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Diaphragm</td>
<td>No</td>
<td>??</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>*Diaphragm NGBP</td>
<td>No</td>
<td>??</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>*SC, Internal passages</td>
<td>No</td>
<td>??</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>*SC, Internal passages NGBP</td>
<td>No</td>
<td>??</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>*SC, Angled surface</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
</tbody>
</table>

**Can Be Made "Right Now"**

Of all the hydrostatic designs studied for this thesis, the angled surface design is the only one that can be made economically using the existing equipment used by makers of rolling element linear bearings. To make any of the other designs, further investigation will be required into manufacturing methods to find an economical means for production. The problem with each of the other designs is discussed below.

**Capillary Systems:** One issue is the capillary needed will have dimensions \(D = 0.2 \pm 0.005 \text{ mm}, L = 10 \text{ mm}\). In small quantities the cost for such a capillary is about $75 each, although research may uncover a cheaper source. The target manufacturing cost for an entire bearing truck is $300. Another issue is assembling the capillaries into the bearing truck.

**Diaphragm Systems:** These are much more complex than a capillary system, so cost is even more of an issue.
Self Compensation With Internal Passages: The difficulty is making the internal passages. A lost wax or 3-D printing method could be used, but neither are economical enough.

Can Be Monolithic

Of all the designs considered, only the angled surface design can be made in one piece. This relates to its economy.

All Precision Features Can Be Ground in One Set Up

By definition, all self compensating designs have this attribute, and other designs do not. This attribute relates to the number of steps required for manufacturing, and hence relates to the bearing’s economy.

Sensitivity to Manufacturing Errors

Only self compensation with internal passages is insensitive to spread error. All the other designs are sensitive to all types of error. The relative sensitivity of each design will likely be similar, but further analysis will be required.

2.5.7 Robustness Comparison Matrix

Table 2.3 compares the manufacturability of the different hydrostatic designs. Each criteria is discussed below.
TABLE 2.4  Robustness Comparison Matrix

<table>
<thead>
<tr>
<th>Size 35 Hydrostatic systems (Predicted)</th>
<th>Shearing at all restricting surfaces (Self Cleaning)</th>
<th>Can be hydraulically clamped for alignment during assembly</th>
<th>5 DOF stability feasible</th>
<th>Lateral displacement that reduces load cap. to 5000N, 100 W per truck, ( v = 60\text{cm/s}, h = 20\text{mm} )</th>
<th>Temperature change of a steel carriage 500mm wide which causes Max. Lateral displacement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capillary (D = 0.2 mm, L = 10mm)</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
<td>7.9</td>
<td>3.0</td>
</tr>
<tr>
<td>*Capillary NGBP</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
<td>8.6</td>
<td>3.2</td>
</tr>
<tr>
<td>Diaphragm</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>12.2</td>
<td>4.6</td>
</tr>
<tr>
<td>*Diaphragm NGBP</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>12.0</td>
<td>4.5</td>
</tr>
<tr>
<td>*SC, Internal passages</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>7.7</td>
<td>2.9</td>
</tr>
<tr>
<td>*SC, Internal passages NGBP</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>8.2</td>
<td>3.1</td>
</tr>
<tr>
<td>*SC, Angled surface</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>8.7</td>
<td>3.3</td>
</tr>
</tbody>
</table>

Shearing at all Restricting Surfaces (Self Cleaning)

A major advantage of the self compensating designs is that shearing occurs at all restricting surfaces, thus shearing away "goo" that may be present in the hydraulic oil. The other designs are vulnerable to clogging over time.

Can be Hydraulically Clamped to One Side of Rail

This means that oil can be supplied to one half of the bearing truck, thus locking it to one side of the rail. This can be useful for aligning the trucks to the rail when the machine tool is being assembled.

5 DOF Stability Feasible

All designs presented are not stable in tilt; they must be bolted to a carriage plate with at least two other trucks. However, if one would want a single truck to have tilt stability, this
is only practical for the capillary and the angled surface designs. The other designs would be prohibitively complex and would have a greatly reduced bearing area.

**Lateral Displacement that Reduces Load Capacity to 5000N (for Benchmark operating conditions)**

A major concern for some applications is lateral displacement imposed on the trucks due to thermal expansion of a spindle carriage. For the benchmark conditions, the diaphragm designs are most tolerant to such an expansion, while the other designs are comparable.

### 2.6 Conclusions of Design Study

The following conclusions are drawn from the preceding design study:

1. For ordinary operating conditions, a medium or lightly preloaded roller bearing will work well. For long life, a hydrostatic bearing will only be useful when severe vibration is present.

2. The static stiffness of hydrostatic designs will be about the same as a medium to lightly preload ball system.

3. For high speed machining at greater than 0.3 m/s, the inherent cooling attribute of the hydrostatic systems is a major advantage over rolling element bearings.

4. The angled surface hydrostatic design has strong manufacturing advantages, and its load capacity and all around stiffness are comparable to all designs except the diaphragm systems. Therefore, it is a good choice for further development.
Chapter 3

LUMPED PARAMETER BEARING MODEL

The goal of this chapter is to create a model which can predict bearing load capacity, stiffness, friction, flow rate, and pump power of a bearing truck given the external dimensions, rail profile, bearing gaps, and bearing gap errors. Each section shows the steps involved.

An accurate bearing model is needed so that the best trade-off between load capacity in the presence of gap errors and pumping power can be made. In production, the bearing gaps along a typical profile can vary by about +/-5 micrometers (each profile can vary +/- 2.5 microns), and unfortunately, this is a significant portion of a typical bearing gap of 20 micrometers or so that will give a reasonable pump power. If we choose a larger gap to boost load capacity in the presence of errors, pump power goes up with the cube of the gap, and thus the pumping power quickly goes up. If we choose a smaller gap to lower pump power, load capacity can be severely hampered given the worst case profile errors. In order to be sure we are making a good compromise between load capacity in the presence of errors and pumping power, an accurate model is needed.

3.1 Profile Geometry

For this thesis, a profile for either the truck or rail is defined as being a chain of flats connected by rounds. This section shows how a such a profile is represented in arrays, how unknowns in such a profile can be computed, how gaps are represented, and how a
slave profile can be computed from a master profile and the gaps between the slave and the master profiles.

### 3.1.1 Representing a Profile of Flats and Rounds

For the purpose of modeling, each flat or round on a right or left profile is assigned a number i, an angle \(T(i, rl)\), and a length or radius (as the case may be) dimension \(S(i, rl)\), where \(rl=1\) is for a parts' right profile, and \(rl=2\) is for a parts left profile. Figure 3.1 shows a generalized back to back profile used to model the Hydrorail, with some sample \(T(i, rl)\) and \(S(i, rl)\) values to show what they mean.

![Profile Arrays](image)

**Figure 3.1** A profile of flats and rounds is represented in an angle array \(T(i, rl)\) and a length array \(S(i, rl)\).

### 3.1.2 Computing x and y Distances on a Profile

Given \(T(i, rl)\) and \(S(i, rl)\) of a profile, it is often necessary to compute the x and y coordinates of one point relative to another point on the profile. Figure 3.2 describes the four types of distances which are routinely computed from \(T(i, rl)\) and \(S(i, rl)\) from a distance finding function \(FnDpr[cs, i, j, T, S, rl, xy]\).
Distance Finding Function $\text{FnDpr}[cs, i, j, T, S, rl, xy]$

- $cs = 1$, $\text{FnDpr} =$ distance from lead of segment $i$ to trail of segment $j$
- $cs = 2$, $\text{FnDpr} =$ distance from lead of segment $i$ to midpoint of segment behind $j$
- $cs = 3$, $\text{FnDpr} =$ distance from lead of segment $i$ to center of radius $j$
- $cs = 4$, $\text{FnDpr} =$ distance from center of radius $i$ to center of radius $j$

- $rl = 1$ for right profile, $rl = 2$ for left profile
- $xy = 1$ for $x$ direction, $xy = 2$ for $y$ direction

Examples of the 4 types of distances described above:

- $\text{FnDpr}[1,0,10, T, S, 1, 2]$
- $\text{FnDpr}[2,0,10, T, S, 1, 2]$
- $\text{FnDpr}[3,0,12, T, S, 1, 2]$
- $\text{FnDpr}[4,2,12, T, S, 1, 2]$

Figure 3.2 Four types of distances frequently computed from a profile of flats and rounds.

The distance finding function $\text{FnDpr}[cs, i, j, T, S, rl, xy]$ was coded using the geometric relationships shown in Figure 3.3 for a flat segment "i" and a round segment "i".

**dx and dy for a flat**

Tangent point

Midpoint of flat $i$

$dx1 = S(i)*\cos[T(i)]$
$dx2 = S(i)*\sin[T(i)]$
$dy1 = -S(i)*\cos[T(i)]$
$dy2 = -S(i)*\sin[T(i)]$

**dx and dy for a round**

When "i" is a round

$T(i) = (T(i-1) + T(i+1))/2$

$S(i) =$ concave radius (as shown)
$S(i) =$ convex radius

$dx1a = S(i)*\cos[T(i)]$
$dx2a = S(i)*\sin[T(i)]$
$dy1a = S(i)*\cos[T(i)]$
$dy2a = S(i)*\sin[T(i)]$

Figure 3.3 The dx and dy increments for a flat and a round segment "i", used to code the function $\text{FnDpr}[cs, i, j, T, S, rl, xy]$. 
3.1.3 Profile Constraints and Unknowns

For the design study later in this thesis, a face to face and a back to back design is considered. The two designs can both be fully represented by the $T(i,rl)$ and $S(i,rl)$ arrays. An important operation is to compute unknown $T(i,rl)$ and $S(i,rl)$ values given dimension constraints on the rail and truck. Figure 3.4 lists the constraints on a Size 35 modular bearing that sets up the geometry problem to be solved for both the face to face and back to back designs.

3.1.4 Representing Bearing Gaps

At the midpoint of segment $i$ (for $i$ up to $i=21$ only), a bearing gap $H(i,rl)$ is present. Figure 3.5 shows some example $H(i,rl)$ values. As will be shown in later sections, the midpoint values $H(i,rl)$ will be used to compute the fluid resistance and effective area of various gap regions.

3.1.5 Computing a Slave Radius from Three Gaps

For the most general design case, the design gap at the midpoint of a round can be set independently of the design gaps at either of the neighboring flats. In order to compute a complete slave profile that will provide the desired gaps relative to a master profile, a means is needed to compute the magnitude of a slave radius, given the magnitude of a master radius and the three desired design gaps associated with it. Figure 3.6 shows how a step by step procedure for how a slave radius and the x and y coordinates of its center have been computed from a master profile.

3.1.6 Profile Dimensions Sent to ProE Solid Model

For the sake of efficiency and to reduce data transfer hassles, the code written to model bearing performance computes all of the profile dimensions needed to drive a ProE solid model. To avoid any stack up errors (ProE regenerates to no better than 0.1 of a micron, which may cause a stack-up problem if serial the dimensions $S(i,rl)$ are used) the magni-
Profile Geometry

Constraints for Size 35 Face to Face Profile:
- \( h_{total} = 48 \text{ mm} \)
- \( h_{above\rail} \geq 0.5 \text{ mm} \)
- \( h_{plate} \geq 11.5 \text{ mm} \) Therefore \( h_{rail} \leq 36 \text{ mm} \)
- \( h_{top} = 3 \text{ mm with cover strip, 0 mm without} \)
- \( hu \geq 5.5 \text{ mm} \)
- \( w_{rail} = 34 \text{ mm} \)
- \( dh = 13 \text{ mm (Head DIA of M8 screw)} \)
- \( c1 \geq 1 \text{ mm} \)
- \( c2 \geq 3 \text{ mm} \)

Setting above to minimum allowed values to maximize pad area:
- \( h_{active} = 27.5 \text{ mm with cover strip, 30.5 mm without} \)
- \( w_{min} = 10.5 \text{ mm} \)

Geometry Problem that is Solved:
- \( \text{Given: } h_{total}, hu, h_{top}, w_{min}, w_{rail}, ws \)
- All \( T(i,rl) \) except \( i = \text{even} \)
- All \( S(i) \) except \( i = 4, 18, 23, 25, 27 \)
- \( \text{Find: All unknown } T(i) \text{ and } S(i) \) values

Constraints for Size 35 Back to Back Profile:
- \( h_{total} = 48 \text{ mm} \)
- \( h_{above\rail} \geq 0.5 \text{ mm} \)
- \( h_{plate} \geq 11.5 \text{ mm} \) Therefore \( h_{rail} \leq 36 \text{ mm} \)
- \( h_{top} = 3 \text{ mm with cover strip, 0 mm without} \)
- \( hu \geq 5.5 \text{ mm} \)
- \( w_{rail} = 34 \text{ mm} \)
- \( dh = 13 \text{ mm (Head DIA of M8 screw)} \)
- \( c1 \geq 1 \text{ mm} \)
- \( c2 \geq 1.5 \text{ mm} \)

Setting above to minimum allowed values to maximize pad area:
- \( h_{active} = 27.5 \text{ mm with cover strip, 30.5 mm without} \)
- \( w_{min} = 9 \text{ mm} \)

Geometry Problem that is Solved:
- \( \text{Given: } h_{total}, hu, h_{top}, w_{min}, w_{rail}, dw, Tave=(T(3)+T(17))/2 \)
- All \( T(i) \) except \( i = \text{even,3,5,17,19} \)
- All \( S(i) \) except \( i = 4,18,25,27 \)
- \( \text{Find: All unknown } T(i) \text{ and } S(i) \) values

Figure 3.4 Constraints used on Size 35 Face to Face and Back to Back designs.

tude and absolute x and y center coordinates of each profile radius were sent to the ProE models of both the rail and truck. For the rail (the master profile), \( \text{FnDpr[cs=4, i, j, T, S, rl, xy]} \) function was used to compute the x and y center coordinates of each round, using \( T(i,rl) \) and \( S(i,rl) \). For the truck (the slave profile), a slave radius function is used to com-
Figure 3.5  Representing bearing gaps in the array $H(i,r_l)$.

pute each slave radius magnitude and its x and y center coordinates, using x and y center coordinates of the rail profile.
Computing Fluid Resistance and Effective Area

Problem:
Given: Design gaps $h_1, h_2, h_3$, Angles $t_1, t_2, t_3$, Master radius $r_a$, Master center $(x_a, y_a)$
Find: Slave radius $r_b$, Slave center $(x_b, y_b)$
Solution: First solve for $h_2$ given $r_b$, and then iterate to find $r_b$ given $h_2$

Step 1: solve for $d_{s1}$ and $d_{s3}$ from the following two equations:

$$r_b = r_a + h_1 - \frac{d_{s1}}{\tan(t_1)} - \frac{d_{s3}}{\sin(t_1)}$$

$$r_b = r_a + h_3 - \frac{d_{s1}}{\sin(t_1)} - \frac{d_{s3}}{\tan(t_1)}$$

where $d_t = t_3 - t_1$

Step 2: compute $dx$, $dy$ and Slave Center point $(x_b, y_b)$

$$dx = (r_a+h_1-r_b)\cos(t_1) - d_{s1}\sin(t_1)$$

$$xb = xa + dx$$

$$dy = (r_a+h_1-r_b)\sin(t_1) + d_{s1}\cos(t_1)$$

$$yb = ya + dy$$

Step 3: solve for $(x_{a1}, y_{a1})$ and $(x_{b1}, x_{b2})$ from the following three equations

Master Circle: $(x-xa)^2 + (y-ya)^2 = r_a^2$

Slave Circle: $(x-xb)^2 + (y-yb)^2 = r_b^2$

$t_2$ Line: $y-ya = \tan(t_2)(x-xa)$

By inspection we can write:

$xa1 = rb^\cos(t_2)$

$ya1 = rb^\sin(t_2)$

Do algebra to find point $xb1$, $yb1$ where Slave circle and $t_2$ line intersect as shown

(reject the second solution $xb2$, $yb2$)

Step 4: compute $f_{nh2}$

$$f_{nh2} = (xb1-xa1)^\cos(t_2) + (yb1-ya1)^\sin(t_2)$$

Step 5: iterate to find $r_b$ such that $f_{nh2} = h_2$, using Newton Raphson

Figure 3.6 Computing a slave radius from three gaps.

### 3.2 Computing Fluid Resistance and Effective Area

Before a lumped resistance model can be created, an algorithm is needed to compute the fluid resistance and effective area of a chain of gaps formed by a master and a slave profile. In this section, such an algorithm is derived, using the equations for fully developed laminar flow between parallel plates.
3.2.1 Validity of Fully Developed Laminar Flow

This section shows that the assumption of fully developed laminar flow is valid for practical bearings studied in this thesis.

According to Fox and McDonald, flow between parallel plates is laminar when Reynolds number $Re_h = (\rho u_m h) / \mu < 1400$, where $\rho$ is fluid density, $u_m$ is mean flow velocity, $h$ is the bearing gap, and $\mu$ is the fluid viscosity. According to Shah, the length of undeveloped flow between parallel plates $L_{hy}$ is

$$L_{hy} = 0.011 D_h Re_{Dh}$$  \hspace{1cm} (3.1)$$

where hydraulic diameter $D_h$ and Reynolds number $Re_{Dh}$ are

$$D_h \equiv (4Area)/(Perimeter)$$  \hspace{1cm} (3.2)$$

$$Re_{Dh} \equiv \frac{\rho u_m D_h}{\mu}$$  \hspace{1cm} (3.3)$$

In order to study how $Re_{Dh}$ and $L_{hy}$ vary with typical bearing design conditions, it is convenient to express $Re_{Dh}$ and $L_{hy}$ in terms of a pressure difference $(P_a - P_b)$ which forces a flow of fluid with properties $\rho$ and $\nu$ through a bearing restrictor with a gap $h$ and a land length $L$. Assuming fully developed flow occurs the mean flow velocity is

$$u_m = \frac{h^2}{12\mu} (P_a - P_b)$$  \hspace{1cm} (3.4)$$

The above expression, along with the definition

$$\mu \equiv \rho \nu$$ \hspace{1cm} (3.5)$$

and the hydraulic diameter for a small gap between parallel plates

$$D_h = 2h$$ \hspace{1cm} (3.6)$$
yields the following convenient expressions for the Reynolds number and undeveloped length for a parallel plate system with a driving pressure \((P_a - P_b)\).

\[
Re_{Dh} = \frac{h^3}{6\rho v^2 L}(P_a - P_b)
\]

\[
\frac{L_{hy}}{L} = \frac{0.022hRe_{Dh}}{L}
\]

Since the restrictor land of the bearings being studied here have both the largest gap \(h\) and the smallest length \(L\) of all lands, \(Re_{Dh}\) will be highest for the restrictor and hence only the restrictor need be investigated for laminar flow. Table 3.1 shows \(Re_{Dh}\) and \(L_{hy} / L\) for a restrictor whose land is \(L=1.5\) mm long, whose bearing gap is 30 micrometers, and which has a pressure drop of 50 Bars (all of these are practical design values).

**TABLE 3.1** Reynolds number and undeveloped length of flow through a practical bearing restrictor.

<table>
<thead>
<tr>
<th>Values For a Practical Bearing Restrictor Land</th>
<th>Medium Oil</th>
<th>Light Oil</th>
<th>Water</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dynamic Viscosity (\mu) cSt</td>
<td>60</td>
<td>10</td>
<td>1</td>
</tr>
<tr>
<td>Density (\rho) kg/m(^3)</td>
<td>870</td>
<td>870</td>
<td>1000</td>
</tr>
<tr>
<td>Bearing Gap (h) mm</td>
<td>30</td>
<td>30</td>
<td>30</td>
</tr>
<tr>
<td>Land Length (L) mm</td>
<td>1.5</td>
<td>1.5</td>
<td>1.5</td>
</tr>
<tr>
<td>Pressure Drop (P_a - P_b) Bar</td>
<td>50</td>
<td>50</td>
<td>50</td>
</tr>
</tbody>
</table>

All above units are converted to SI units for calcs. When \(Re_{Dh} < 2800\) flow is laminar. \(L_{hy}\) is length of undeveloped flow.

<table>
<thead>
<tr>
<th>Reynolds Number (Re_{Dh})</th>
<th>Medium Oil</th>
<th>Light Oil</th>
<th>Water</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>4.8</td>
<td>172</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Normalized Undeveloped Flow Length (L_{hy} / L) %</th>
<th>Medium Oil</th>
<th>Light Oil</th>
<th>Water</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>0.21%</td>
<td>7.59%</td>
</tr>
</tbody>
</table>

This study shows that

1. For both oils laminar flow occurs.
2. Using medium oil fully developed flow is a very good approximation whereas for light oil it is fair.
3. If water were used the flow would not be laminar and hence the fluid resistance would be significantly higher than laminar theory predicts. For good accuracy using water a turbulence model would have to be used.
3.2.2 Effect of Viscous Heating on Viscosity and Gamma

In this section the amount fluid viscosity and the gamma ratio changes due to Poiseuille flow friction is investigated, and the expected error is predicted for the present model (which currently does not include a variable viscosity).

Temperature Rise of Fluid Due to Poiseuille Flow

Assuming all friction energy goes to heating the fluid, the energy equation applied to a pure Poiseuille flow scenario yields

\[ (P_a - P_b)Q = \rho Q c_p (\Delta T_p) \]  

(3.9)

The resulting temperature rise \( \Delta T_p \) is

\[ \Delta T_p = \frac{(P_a - P_b)}{\rho c_p} \]  

(3.10)

As can be seen, the energy equation shows that for pure Poiseuille flow, when no heat leaves the fluid, the fluid temperature rise due to viscous heating only depends on the driving pressure, the density, and the heat capacity.

Fluid Viscosity Versus Temperature

The viscosity \( \mu \) of both oil and water can be accurately represented by the empirical equation [Fox]

\[ \nu = \frac{A}{\rho} e^{B/(T + 273.15)} \]  

(3.11)

where \( A \) and \( B \) are constants and \( T \) is the fluid temperature in degrees C. For practical calculations it is convenient to compute \( A \) and \( B \) given two viscosities at two different reference temperatures.
$$B = \frac{\ln\left(\frac{\nu_{\text{REF}1}}{\nu_{\text{REF}2}}\right)}{\left(\frac{1}{(T_{\text{REF}1} + 273.15)} - \frac{1}{(T_{\text{REF}2} + 273.15)}\right)}$$ \hspace{1cm} (3.12)

$$A = (\nu_{\text{REF}1})e^{-\frac{B}{T_{\text{REF}1}}}$$ \hspace{1cm} (3.13)

Calculating the Change in Viscosity and Gamma

As an example, Table 3.2 shows that at a supply pressure of 70 Bar, which is expected for most bearing applications, the exiting oil viscosity (for both medium and light oils) due to the driving pressure will be around 13% for real oils. The increase in overall flow rate will be somewhere around half the 13% value associated with the exit viscosity.

The resulting change in gamma, which is equal to a bearing’s restrictor resistance divided by a bearing’s land resistance, has been estimated by starting with a typical gamma value, computing the mean viscosity over a restrictor land and a bearing land, and then computing the expected change in gamma. Table 3.3 shows such a calculation for an operating pressure of 70 Bar.
TABLE 3.3 Estimate of how much viscous heating will effect the gamma ratio.

<table>
<thead>
<tr>
<th>Pressure</th>
<th>No Viscous Heating</th>
<th>With Viscous Heating</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fluid In Supply Groove Bar 70</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>Average Viscosity Across Restrictor: 21</td>
<td>1.00</td>
<td>0.95</td>
</tr>
<tr>
<td>Fluid In Pocket Bar 21</td>
<td>1.00</td>
<td>0.91</td>
</tr>
<tr>
<td>Average Viscosity Across Bearing Land: 0</td>
<td>1.00</td>
<td>0.89</td>
</tr>
<tr>
<td>Fluid In Drain Bar 0</td>
<td>1.00</td>
<td>0.87</td>
</tr>
</tbody>
</table>

Gamma: 2.33 2.50

Estimated Relative Change in Gamma = 0.95/0.89-1 = 7%

The resulting change in gamma is estimated from the relative mean viscosities as 0.95/0.89-1 = 7%. While viscosity is assumed to be constant for the present bearing model, heating can be accounted for by choosing a gamma that is about 7% or so lower than what the model says is optimum, if 70 Bar is the target operating pressure.

3.2.3 Single Rectangular Element

This section shows the basic equations for the fluid resistance and effective area of a single rectangular gap element. These element equations will subsequently be used to compute the resistance and effective area of a chain of elements which form a profile.

The previous two sections in this chapter show that using a medium or light oil the following assumptions are reasonable conditions for the actual bearings being studied for this thesis.

1. Fluid is Newtonian
2. Fully developed laminar flow between infinite parallel plates applies
3. Viscosity is constant (temperature effect is neglected)

Figure 3.7 shows a single rectangular gap element and the parameters associated with it.

Using the basic equations for fully developed laminar flow between parallel plates, the flow $Q_i$ and resistance constant $R_i$ are given by
Computing Fluid Resistance and Effective Area

Fully developed laminar flow through a parallel plate element

Figure 3.7 A single parallel plate element "i".

\[ Q_i = \frac{(P_a - P_b)}{R_i} \quad (3.14) \]

\[ R_i = \frac{12\mu L_i}{w_i h_i^3} \quad (3.15) \]

The viscous skin drag \( F_{di} \) and friction coefficient \( C_{di} \) associated with a parallel plate element "i" are given by

\[ F_{di} = v_{rel} C_d \quad (3.16) \]

\[ C_{di} = \frac{\mu w_i L_i}{h_i} \quad (3.17) \]

Where \( v_{rel} \) is the relative velocity between the plate elements.

In a 2D plane, the net force on one of the plates of element "i", due to pressures \( P_a \) and \( P_b \), is given by

\[ F_{Xi} = A_{Xai} P_a + A_{Xbi} P_b \quad (3.18) \]

\[ F_{Yi} = A_{Yai} P_a + A_{Ybi} P_b \quad (3.19) \]

Figure 3.8 visually shows the components of \( F_{Xi} \) and \( F_{Yi} \).
The proportionality constant associated with $P_a$ will be called effective area "a", and the proportionality constant associated with $P_b$ will be called effective area "b". For a simple parallel gap element, effective area "a" is always equal to effective area "b", and both are equal to half the element’s projected area.

\[
A_{Xai} = A_{Xbi} = 0.5L_iw_i\cos(T_i) \tag{3.20}
\]

\[
A_{Yai} = A_{Ybi} = 0.5L_iw_i\sin(T_i) \tag{3.21}
\]

It should be noted at this point that for a chain of parallel plates with different gaps that all share the same w dimension, effective areas "a" and "b" are not necessarily equal, and hence it is important to not think of the two quantities as being the same, even though they happen to be equal for an individual parallel plate element.

### 3.2.4 Effect of Pad Tilting on Resistance

This section shows the amount of error in resistance values that will result by neglecting a tilt error of the truck along its length. Figure 3.9 shows a wide short land as is present down the length of a truck.

Approximating the resistance using the average gap yields the expression

\[
R_{hve} = \frac{C}{W_{have}^3} \tag{3.22}
\]
The actual resistance in terms of $h_1$ and $h_2$ is closely approximated for $W>>L$ by the integral

$$R_{h_1h_2} = C \left[ \int_0^W \left( \frac{h_2 - h_1}{W} x + h_1 \right)^3 dx \right]^{-1} \quad (3.23)$$

A relative resistance error can be computed by evaluating the above integral, simplifying, and then dividing by $R_{hve}$ and subtracting 1, yielding

$$\frac{R_{h_1h_2}}{R_{hve}} - 1 = \frac{(h_1 + h_2)^2}{2(h_1^2 + h_2^2)} - 1 \quad (3.24)$$

Table 3.4 shows the resistance error given a realistic gap error of 6 micrometers for progressively smaller values of $h_{ave}$, starting at 20 micrometers and going down to 5 micrometers.
TABLE 3.4 The relative resistance error versus $h_{\text{ave}}$, when a tilt of 6 micrometers is present.

<table>
<thead>
<tr>
<th>$h_1$ (um)</th>
<th>$h_{\text{ave}}$ (um)</th>
<th>$h_2$ (um)</th>
<th>% Error $\left(\frac{R_{112}/R_{\text{ave}}}{R_{\text{ave}}-1}\right)$*</th>
</tr>
</thead>
<tbody>
<tr>
<td>23</td>
<td>20</td>
<td>17</td>
<td>-2%</td>
</tr>
<tr>
<td>18</td>
<td>15</td>
<td>12</td>
<td>-4%</td>
</tr>
<tr>
<td>13</td>
<td>10</td>
<td>7</td>
<td>-8%</td>
</tr>
<tr>
<td>8</td>
<td>5</td>
<td>2</td>
<td>-26%</td>
</tr>
</tbody>
</table>

*(Resistance using $h_1$, $h_2$ shown) / (Resistance using $h_{\text{ave}}$ shown) - 1

This analysis shows that resistance drops with tilt, and when the average gap is less than 10 micrometers, the resistance drops by less than 10%, and hence neglecting tilt is reasonable. To compensate for the lower resistance due to tilting, gamma should be made smaller than a non tilted design suggests.

### 3.2.5 Chain of Parallel Plate Elements with Common Width

In this section, the total resistance and "a" and "b" effective areas are derived for a chain of parallel plates that share the same width dimension. This problem is stated below in a Given, Find format.

**Given:** Common "w" Chain with $i = 1$ to $n$ elements with $R_i$, $C_{di}$, $A_{Xai}$, $A_{Yai}$, $A_{Xbi}$, $A_{Ybi}$

**Find:** $R$, $C_d$, $A_{Xa}$, $A_{Xb}$, $A_{Ya}$, $A_{Yb}$ for the chain

Figure 3.10 shows a side view of a chain of parallel plate elements that share the same width.

Because the resistances $R_i$ are in series, the total resistance $R$ is simply

$$R = \sum_{i=1}^{n} R_i$$ (3.25)

The viscous drag coefficients $C_{di}$ also add as well
Computing Fluid Resistance and Effective Area

Each Flat comprises one Parallel Plate Element $S(1)$

Parallel Plate Elements

Each Round comprises several Parallel Plate Elements $S(2)$

Each Flat comprises one Parallel Plate Element $S(3)$

Note: the index for $T(i), S(i), H(i)$ is not the same as the index for Parallel Plate Elements

Figure 3.10 Chain of parallel plate elements with a common width.

![Diagram of parallel plate elements](image-url)

For the total effective area derivation, only the derivation for the $x$ effective area component is shown because the $y$ component derivation is identical. To begin, by the definition of an "a" and "b" effective area, the total force $F_x$ on the chain is

$$ F_x = P_1 A_{xa} + P_{n+1} A_{xb} $$

(3.27)

The total force $F_X$ is also equal to the sum of the force that each element exerts.

$$ F_X = \sum_{i=1}^{n} \left( P_i A_{xa} + P_{i+1} A_{xb} \right) $$

(3.28)

The total effective areas $A_{xa}$ and $A_{xb}$ can be derived by making use of the fact that the flow through each element is the same because the elements are in series. Therefore we can write

$$ C_d = \sum_{i=1}^{n} C_{di} $$

(3.26)
Performing some algebra on the above relation, the pressure \( P_i \) can be solved for in terms of \( P_1 \) and \( P_{n+1} \)

\[
P_i = P_1 (1 - Z_{i-1}) + P_{n+1} Z_{i-1}
\]

(3.30)

\[
Z_i = \frac{\sum_{j=1}^{i} R_j}{R}, \quad Z_0 = 0
\]

(3.31)

Using Equation 3.29 we can eliminate \( P_i \) and \( P_{i+1} \) in Equation 3.28, and then factor out \( P_1 \) and \( P_{n+1} \). After the factoring operation, the coefficient associated with \( P_1 \) is the total "a" effective area, and the coefficient associated with \( P_{n+1} \) is the total "b" effective area of the entire chain. Doing the algebra yields the following result:

\[
A_{Xa} = \sum_{i=1}^{n} [A_{Xa_i}(Z_{i-1} - 1) + A_{Xb_i}(Z_i - 1)]
\]

(3.32)

\[
A_{Xb} = \sum_{i=1}^{n} [A_{Xa_i}(Z_{i-1}) + A_{Xb_i}(Z_i)]
\]

(3.33)

The analysis for the total y effective area is exactly the same, and hence yields:

\[
A_{Ya} = \sum_{i=1}^{n} [A_{Ya_i}(Z_{i-1} - 1) + A_{Yb_i}(Z_i - 1)]
\]

(3.34)
3.2.6 Chain of Parallel Plate Elements with Common Length

In this section, the total resistance and "a" and "b" effective areas are derived for a chain of parallel plates that share the same length dimension. This problem is stated below in a Given, Find format.

**Given:** Common "L" Chain with \( i = 1 \) to \( n \) elements with \( R_i, A_{Xai}, A_{Xbi}, A_{Yai}, A_{Ybi} \)

**Find:** \( R, A_{Xa}, A_{Xb}, A_{Ya}, A_{Yb} \) for the chain

Figure 3.11 shows a side view of a chain of parallel plate elements that share the same length.

![Figure 3.11 Chain of Parallel Plate Elements with Common Length.](image)

Making the approximation that no cross flow occurs between the plate elements, the resistances \( R_i \) are all in parallel, and the total resistance \( R \) is thus

\[
A_{Yb} = \sum_{i=1}^{n} [A_{Yai}(Z_{i-1}) + A_{Ybi}(Z_i)]
\] (3.35)
\[ R = \left( \sum_{i=1}^{n} \frac{1}{R_i} \right)^{-1} \]  

(3.36)

The viscous drag coefficients \( C_{di} \) add, yielding

\[ C_d = \sum_{i=1}^{n} C_{di} \]  

(3.37)

Making the approximation that no cross flow occurs between the plate elements, the effective areas simply add.

\[ A_{xa} = \sum_{i=1}^{n} [A_{xai}] \]  

(3.38)

\[ A_{xb} = \sum_{i=1}^{n} [A_{xbi}] \]  

(3.39)

\[ A_{ya} = \sum_{i=1}^{n} [A_{yai}] \]  

(3.40)

\[ A_{yb} = \sum_{i=1}^{n} [A_{ybi}] \]  

(3.41)

### 3.2.7 General Function Used in Code

The formulas presented in the previous sections were coded into a single function \( \text{FnEpr}[cs, i, j, LL, T, S, H, hpk] \) that computes \( R \), \( C_d \), and the effective areas of a chain of parallel plate elements. Exploiting symmetry, \( \text{FnEpr} \) computes the properties of four chains that are mirrors of one another and stores the output in the array \( \text{Epr}(tb, rl, xyaf, ab) \). The inputs and outputs are summarized below.
Function that finds $R$, $C_d$, and effective areas for a chain of parallel plate elements:

$FnEpr[cs, i, j, LL, T, S, H, hpk] \text{ Inputs}$

$cs = 1$ to 3, 1 for common "l" chain, 2 for common "w" chain, 3 for a pocket

$i = 1$ to 11, marks the first element of a chain on the top half of a profile

$j = 1$ to 11, marks the last element of a chain on the top half of a profile

$LL(rl, tb) =$ common "L" or common "W" dimension for a chain on right profile, left profile, top half of the profile, and bottom half of the profile

$T(i, rl) =$ angle of segment "i" on right and left profiles

$S(i, rl) =$ length or radius of segment "i" on right and left profiles

$H(i, rl) =$ gap at midpoint of segment "i" on right and left profiles

$hpk =$ depth of pocket (only applies when $cs = 3$)

$\text{Output array } Epr(tb, rl, xyaf, ab).$

$tb = 1$ to 2, 1 for top half of profile, 2 for bottom half of profile

$rl = 1$ to 2, 1 for right profile, 2 for left profile

$xyaf = 1$ to 4, 1 for $A_X$, 2 for $A_Y$, 3 for admittance ($1/R$), 4 for viscous drag coefficient ($C_d$)

$ab = 1$ to 2, 1 for "a" effective area, 2 for "b" effective area

3.3 Lumped Resistance Circuit

Figure 3.12 shows the anatomy of the lumped resistance circuit used to predict pocket pressures. Taking advantage of symmetry, a resistance circuit need only be created for one end of one side of a truck.

3.3.1 Assumptions

To model this bearing as a lumped resistance system, several approximations were made. These are described below, one point at a time:
The Neck Resistor Patches

For a practical bearing, the total force contribution due to the neck region is only about 20%. Therefore, there is not a great incentive to model this region exactly. Although not exact, the overlapping Neck Resistor Patches shown are intended to simulate the leakage that occurs from the neck region to the upper and lower bearing pockets. The Neck Resistor Patches are computed using a function that uses the Common "w" and Common "L" equations for a chain of flats and rounds.

Figure 3.12  Lumped resistance circuit used to predict pocket pressures.
Uniform Gaps Along Truck Length

The present model computes performance using a uniform gap along the length of the truck. The analysis in Section 3.2.4 showed this to be a reasonable approximation.

The Supply Slot and Bearing Pockets

These are treated as nodes, because the are at least 10 times deeper than the bearing gaps and hence their resistance is about $10^3$ or 1000 times lower.

The Restrictor Lands

These are modeled as a single rectangular element. The flow out of the sides is accounted for via the neck resistor patches.

The Bearing Lands

Each bearing land comprises a long Common "w" flat/round/flat chain and a short Common "L" flat/flat chain at the end. The flow out of the corners is not specifically accounted for, but region 3 on the end bearing land accounts for some flow out the corners.

3.3.2 Computing the Lumped Resistances

The above assumptions allow each resistance $R_{pq}$ (where p and q are adjacent nodes) in the circuit shown in Figure 3.12 to be systematically computed using the Common "w" and Common "L" chain algorithms. As an example, resistance $R_{41}$ is computed using the Common "L" chain algorithm, which in turn uses the profile's angle array $T(i,rl)$, the flat/round array $S(i,rl)$, the gap array $H(i,rl)$, and the common width $X_e$, where the index "i" goes from 10 to 5.

3.3.3 Solving for Node Pressures and Flow

This section explains the algorithm used for solving for node pressures and the total flow rate given the circuit shown in Figure 3.12 (which applies to one end of one side of a truck). The problem at hand is presented in a given, find format.
**Given:** Circuit in Figure 3.12 with nn nodes, \( R_{pq} \) (p, q are adjacent nodes), \( P_0, P_6 = 0 \)

**Find:** \( P_1 \) to \( P_{nn-1} \), \( Q_{TrSide} = 2(Q_{01} + Q_{02} + Q_{02}) \)

At this point it is convenient to convert all resistances to admittances

\[
A_{pq} = \frac{1}{R_{pq}} \quad (3.42)
\]

\[
Q_{pq} = A_{pq}(P_p - P_q) \quad (3.43)
\]

A set of \( nn-1 \) linear equations with the unknown node pressures can systematically be created by applying continuity at each node. In general, for any resistance circuit, an efficient way to express continuity at each node \( i \) is

\[
\sum Q_{IntoNode_i} = \sum A_{iAdj}P_{Adj} - (\sum A_{iAdj})P_i = 0 \quad (3.44)
\]

Where \( A_{iAdj} \) is the admittance between node \( i \) and an adjacent node, \( P_{Adj} \) is the pressure at the corresponding adjacent node, and \( P_i \) is the pressure at node \( i \). As an example of using this equation, continuity for node 1 yields

\[
\sum Q_{IntoNode_1} = A_{10}P_0 + \sum_{i=3}^{nn-1} A_{1i}P_i - \left( A_{10} + \sum_{i=3}^{nn} A_{1i} \right)P_1 = 0 \quad (3.45)
\]

Applying the continuity equation to nodes 1 through \( nn-1 \) yields a total of \( nn-1 \) linear equations with \( nn-1 \) \( P_i \) values, which are put into the matrix form

\[
[A]P = B \quad (3.46)
\]

where the admittance matrix \([A]\) comprises the coefficients in front of \( P_1 \) through \( P_{nn-1} \), \( P \) is the unknown vector of \( P_1 \) through \( P_{nn-1} \) pressures, and \( B \) is the negative of all terms that
have a $P_0$. The unknown vector $P$ is readily solved for by multiplying $B$ by the inverse of $[A]$.

$$P = [A]^{-1}B$$

(3.47)

The flow rate out of one truck side (meaning a pair of full length pockets on one side) is equal to twice the total flow leaving node 0 (because the circuit in Figure 3.12 represents only half length pockets, because end to end symmetry is present).

$$Q_{TrSide} = 2 \sum_{i=1}^{3} A_0i(P_0 - P_i)$$

(3.48)

### 3.3.4 Lumped Effective Areas

All of the resistance patches except for the Common "L" patches in the neck region are used in the model to compute the effective area of the land regions. To compute force on each patch region, the node pressures $P_0$ through $P_{nn}$ are used as either pressure $P_a$ or pressure $P_b$ for each patch region, as appropriate. To compute force on each pocket region, the entire area each pocket region was multiplied by the node pressure associated with it.

### 3.4 Incorporating Elastic Deflection from 2D FEA Models

Preliminary analysis shows that elastic deflection of both the truck and rail can reduce the lateral stiffness by up to 50%. Hence, an extensive effort was made to include the effect in the present bearing model. Including elastic deflection is not trivial, because in order to solve for a set of final hydrostatic forces, iteration is required, because any change in gaps due to elastic deflection will change the resistances and hence change the hydrostatic forces, and visa versa. To allow for fast iteration for the present model, the linear superposition principle was exploited. In summary, a set of reference deflections obtained from FEA simulations are used as input values for the present model, and these values are scaled according to actual hydrostatic forces present to get actual gap changes for the iteration process. The process is explained in detail in the following sections.
3.4.1 Problems that are Addressed in this Section

For this section, only the problem of how to compute gap changes caused by hydrostatic forces is addressed. As mentioned earlier, to solve for a final set of hydrostatic forces, iteration is required because any change in gaps due to elastic deflection will change the resistances and hence change the hydrostatic forces, and visa versa.

Three subproblems are solved in this section that lead to the ultimate goal of finding gap changes due to elastic deflections caused by a set of hydrostatic forces. The subproblems are explained below.

Index symbols used for data arrays in the three subproblems:

- \( p = 1 \) to \( 10 \), \# of point "p"
- \( i = 1 \) to \( 21 \), \# of point "i"
- \( r_l = 1 \) to \( 2 \), 1 for deflection on right profile, 2 for deflection on left profile
- \( xy = 1 \) to \( 2 \), 1 for x direction, 2 for y direction
- \( ld = 1 \) to \( 3 \), loads 1 to 3
- \( lrtl = 1 \) to \( 2 \), 1 for load on right profile, 2 for load on left profile

Subproblem 1: FEA Simulations

**Given:** Truck and rail solid models

**Pref(ld, lrtl=1)** Array of three reference load pressures acting on the right profile of both the truck and rail.

**Find:** ExyRR1(p, rl, xy, ld, tr) Array of x and y elastic deflections. This array consists of the deflection at each point "p" at the right and left profiles, in the x and y direction, caused by reference load pressures Pref(ld,1), acting on both the truck or the rail.

Subproblem 2: Computing array of reference gap changes at the midpoint of each segment "i"

**Given:** ExyRR1(p, rl, xy, ld, tr)

**Find:** HEref(i, rl, ld, lrtl) Array of reference gap change values at the midpoint of each segment "i" at the right and left profiles caused by the reference pressures Pref(ld, lrtl) acting on the right and left profiles.
Subproblem 3: Computing array of gap changes at the midpoint of each segment "i"

**Given:** Pref(ld, lrl)

Aref(ld, lrl) Area over which each reference pressure acts.

HEref(i, rl, ld, lrl)

Fhy(ld, lrl) Array of three hydrostatic forces acting on right and left profiles.

**Find:** HEE(i, rl) Array of gap changes due to elastic deflection at the midpoint of each segment "i" at the right and left profiles.

### 3.4.2 Subproblem 1: FEA Simulations

The goal of this section is explain how the array ExyRR1(p, rl, xy, ld, tr) is filled via FEA simulations. Figure 3.13 shows 10 "p" points on the right and left side of both the truck and rail which were marked in ProMechanica. The x and y deflection at each point "p" was marked as a "measure" in ProMechanica. Each point "p" shown is easy to mark on a meshed ProMechanica model because it is a point of tangency between an adjacent flat and round.

Figure 3.14 shows the six FEA simulations that are run to generate all of the deflections present in the array ExyRR1(p, rl, xy, ld, tr). For the simulations, Pref(1,1) = Pref(2,1) = Pref(3,1) = 1 MPa.

### 3.4.3 Subproblem 2: Computing Array of Reference Gap Changes

The goal of this section is to explain how the array HEref(i, rl, ld, lrl) is filled. Figure 3.15 shows the midpoint of each segment "i" on the right and left profile of both the truck and rail, at which we seek a compute gap change at "i" in the array HEref(i, rl, ld, lrl).

To fill HEref(i, rl, ld, lrl) starting with ExyRR1(p, rl, xy, ld, tr), a linear deformation formula was used to compute the deflections at the "i" points which are in between the "p" points, yielding a new array ExyRR2(i, rl, xy, ld, tr). Using ExyRR2(i, rl, xy, ld, tr), the
Figure 3.13  Shows each point "p" marked on the truck and rail FEA models, and the regions over which the three reference pressures act. Each point "p" is a tangent point between a flat and a round, and hence is a natural vertex generated by ProMechanica.

reference gap change array $HE_{ref}(i, rl, ld, 1)$ is easily computed using the angle array $T(i, rl)$ and $ExyRR2(i, rl, xy, ld, tr)$.

$$dx = ExyRR2(i, rl, 1, ld, 1) - ExyRR2(i, rl, 1, ld, 2) \quad (3.49)$$

$$dy = ExyRR2(i, rl, 2, ld, 1) - ExyRR2(i, rl, 2, ld, 2) \quad (3.50)$$

$$HE_{ref}(i, rl, ld, 1) = dx \times \cos[T(i, rl)] + dy \times \sin[T(i, rl)] \quad (3.51)$$

The sign convention for $HE_{ref}(i, rl, ld, lrl)$ is that a positive value indicates a gap increase, and a negative value indicates a gap decrease. Symmetry dictates that all gap changes due to reference loads acting on the left side (i.e. when $lrl=2$) are the mirror of all gap changes due to reference loads acting on the right side (i.e. when $lrl=1$). This symmetry relationship is expressed as follows

$$HE_{ref}(i, 1, ld, 2) = HE_{ref}(i, 2, ld, 1) \quad (3.52)$$

$$HE_{ref}(i, 2, ld, 2) = HE_{ref}(i, 1, ld, 1) \quad (3.53)$$
The array \( \text{HEref}(i, rl, ld, lrl) \) is now complete. Now the gap change at each point "i" can be computed efficiently given any arbitrary set of hydrostatic forces \( \text{Fhy}(ld, lrl) \).

### 3.4.4 Subproblem 3: Computing Array of Gap Changes

The elastic gap change \( \text{HEE}(i, rl) \) at midpoint "i" can be computed by linearly combining the reference gap changes stored in \( \text{HEref}(i, rl, ld, lrl) \). In order to perform the scaling, an equivalent uniform pressure \( \text{Peq}(ld, lrl) \) must be computed for each hydrostatic force \( \text{Fhy}(ld, lrl) \), using the area over which the reference pressures act \( \text{Aref}(ld, lrl) \).

\[
\text{Peq}(ld, lrl) = \frac{\text{Fhy}(ld, lrl)}{\text{Aref}(ld, lrl)} \tag{3.54}
\]

The gap change at each midpoint "i" is equal to the sum of the gap changes caused each of the six equivalent pressures \( \text{Peq}(ld,lrl) \).
Figure 3.15 Shows the midpoint of each segment "i", at which a gap change is stored in the array HErrf(i, rl, ld, lrl). The reference pressures Pref(ld, lrl) are uniform and they act on both the truck and rail over the same area as is specified in Figure 3.13.

\[
\text{HEE}(i, rl) = \sum \left[ \text{HEref}(i, rl, ld, lrl) \times \text{Peq}(ld, lrl) / \text{Pref}(ld, lrl) \right] \quad (3.55)
\]

The principle of linear superposition is valid because the part size is much larger than the deflections being studied (all deflections are less than 10 micrometers, whereas the rail width is 35mm, or 350 times larger).
Chapter 4

ANGLED SURFACE SELF COMPENSATION PROTOTYPE

In this chapter, a prototype angled surface self compensation design is presented along with test results.

4.1 Size 35 Angled Surface Self Compensation Prototype

The first prototype of the hydrorail was designed to prove the concept presented in Chapter 2 and to allow a comparison with the detailed mathematical model presented in Chapter 3. To lower the cost of the first prototype, a three piece design was chosen. The design is shown in Figure 4.1. The bearing gaps used are shown in Figure 4.2.

The reasoning behind the design choices made for the alpha prototype are listed below.

**Design Choices Made for Alpha Prototype**

1. To lower the cost of prototyping a three piece truck design was chosen. The two identical halves can be ground with a large diameter grinding wheel, hence making grinding much easier.

2. To lower costs and to make load capacity high, no seal or drain groove system was designed.

3. Nominal gaps at bearing pads were chosen to be 12.7 micrometers, which is relatively small but makes using water feasible (about 1.5 hp for 4 trucks), and hence provided us the option of using water.
Figure 4.1 Basic features of the Size 35 alpha prototype design.

Figure 4.2 Design bearing gaps and some key profile dimensions used for Size 35 alpha prototype.

The profile grinding use to make the truck and rail is shown in Figure 4.3 (the prototype was manufactured by profile grinding at Jung, GmbH).
4.2 Deflection Versus Load

In this section, the measured and predicted deflection versus load is presented for lateral, compressive, and tensile loading cases.

4.2.1 Test Set-Up for Deflection Vs. Load

Figure 4.4 shows the rig that was used to load a truck in compression and in tension, and to measure the resulting vertical deflection and the pressure in the four bearing pockets.

To prevent bending of the steel carriage plate, which would cause undesired tilting of the bearing truck being measured, thin strips were placed above both trucks, under a "whiffle tree" block, upon which the force was applied.
Figure 4.4 Test rig used to load a truck in compression and tension. Only the deflection of the left truck was measured. Thin spacers were placed under the whiffle tree block, above each truck, to prevent the steel carriage plate from bending when a force is applied on the transducer.

Figure 4.5 shows the rig that was used to load a truck laterally and to measure the resulting lateral deflection. There are two important points concerning this lateral set-up.

1. Unlike for the vertical loading case, the lateral loads on the right and left trucks are not statically determined using the set-up shown. For expediency, the lateral loads were assumed to be equal.

2. The reaction force on each truck is not purely lateral. Using the equal load assumption, from a free body diagram, the vertical component was computed and input in to the bearing model along with the lateral component.

Figure 4.6 shows a free body diagram of the lateral set-up and the problem that was solved.

Force, displacement, and pressure data were recorded digitally (using a National Instruments DAQ board) while a load was being applied to the measured truck. The load on the force transducer was applied by manually tightening the nut on its threaded rod, pushing (or pulling) the transducer against the steel box beam behind it.
Figure 4.5 Test rig used to load both trucks laterally. Unlike for the vertical loading set-up, the load on the right and left trucks is not statically determined. For expediency, the lateral load component on the truck being measured was assumed to be half the force transducer load. The small vertical component was computed using a FBD of the set-up.

Figure 4.6 Free body diagram of the lateral load test set-up.
4.2.2 Matching Pocket Pressures and Flow to Account for Gap Errors

A major difficulty in trying to predict performance is how to measure, or at least account for, gap errors that are due to manufacturing and mounting errors of the parts (Using part tolerances from Jung I expect the 12.7 and 17.8 micron bearing and restrictor gaps to vary by +/- 2 microns, and mounting errors could account for another +/- 2 microns). The brute force method of measuring the truck and rail via a CMM is very difficult because of the accuracy that would be required (at least 0.5 micron) and the fact that the rail must be measured while it is mounted, because it is not straight in its free state. The more feasible approach of measuring lateral and vertical clearance of the truck relative to the rail is actually quite difficult for two reasons 1) oil creates a sluggish response it is difficult to tell where touch down actually occurs and 2) if any tilt is present one corner touches down first, giving a falsely low clearance value for the purpose of hydrostatic resistance.

To provide a realistic error to input into the bearing model, a matching approach was used. A set of restrictor and bearing gap errors were found (via iteration) that match predicted pocket pressures and flow rate at zero load with the experimentally measured initial pocket pressures and flow rate. The errors used for matching and the resulting flow rate are shown in Figure 4.7.
4.2.3 Load Versus Displacement Plots

Compression

Figure 4.8 and Figure 4.9 shows the measured and predicted deflection under compressive, lateral, and tensile loading.
Figure 4.8  Measured and predicted downward and lateral deflection versus force of prototype bearing truck.
Figure 4.9 Measured and predicted tensile deflection versus force of prototype bearing truck.
### TABLE 4.1 Summary of Load Versus Deflection Test Results

<table>
<thead>
<tr>
<th>(Bar)</th>
<th>(N)</th>
<th>(µm)</th>
<th>(N)</th>
<th>(µm)</th>
<th>(N/µm)</th>
<th>(N/µm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compression</td>
<td>17.2</td>
<td>1170</td>
<td>8.1</td>
<td>2878</td>
<td>20.8</td>
<td>144</td>
</tr>
<tr>
<td></td>
<td>17.2</td>
<td>300</td>
<td>2.9</td>
<td>2324</td>
<td>16.0</td>
<td>103</td>
</tr>
<tr>
<td></td>
<td>17.2</td>
<td>1375</td>
<td>12.6</td>
<td>1852</td>
<td>14.1</td>
<td>109</td>
</tr>
<tr>
<td>Tension</td>
<td>34.5</td>
<td>2338</td>
<td>10.1</td>
<td>5429</td>
<td>19.6</td>
<td>231</td>
</tr>
<tr>
<td></td>
<td>34.5</td>
<td>800</td>
<td>4.2</td>
<td>4925</td>
<td>18.4</td>
<td>190</td>
</tr>
<tr>
<td></td>
<td>34.5</td>
<td>2251</td>
<td>14.1</td>
<td>3651</td>
<td>15.0</td>
<td>160</td>
</tr>
<tr>
<td>Lateral</td>
<td>68.9</td>
<td>4700</td>
<td>10.7</td>
<td>10406</td>
<td>19.2</td>
<td>439</td>
</tr>
<tr>
<td></td>
<td>68.9</td>
<td>N.A.</td>
<td>N.A.</td>
<td>10085</td>
<td>21.1</td>
<td>331</td>
</tr>
<tr>
<td></td>
<td>68.9</td>
<td>N.A.</td>
<td>N.A.</td>
<td>7200</td>
<td>16.8</td>
<td>302</td>
</tr>
</tbody>
</table>

**Conclusions Drawn from Each Load Vs. Deflection Test:**

1. In Compression correlation was good. By stacking weights contact was observed at 68.9 Bar at about 4700 N, however contact was not clearly visible in the tests.

2. In Tension premature touchdown was observed. Because the bearing is symmetrical this is likely due to the bolted joints causing excessive deflection. At 68.9 Bar, however, touchdown is not clearly observable, most likely because the initial pocket pressures spread the truck halves open enough to delay touchdown significantly.

3. Laterally the correlation is good except at 34.5 Bar. It may be that the bearing is touching prematurely at 68.9 Bar, but further testing is required to verify this.

**Overall Conclusions Drawn from Load Vs. Deflection Tests**

1. Model correlates reasonably well after matching pocket pressure and flow rate, even though a very rough assumption was made about the profile errors to enable matching.

2. 12.7 um nominal bearing gap is likely too tight given the accuracy of the parts and set-up, because premature touch down occurred.
3. The bolted joints may have caused some of the problems too. A one piece design would eliminate issues associated with bolted joints.

4.3 Dynamic Stiffness Tests

In this section, dynamic stiffness measured for the alpha prototype and for a Thomson ball and roller system, tested under the same conditions, are presented. A more detailed discussion is presented in Appendix B.

4.3.1 Test Set-Up for Measuring Dynamic Stiffness

Figure 4.10 Dynamic stiffness testing using a hammer and accelerometer. The force and accelerometer data are fed to a signal analyzer to obtain the Acceleration/Force transfer function. Dividing by frequency squared and inverting yields Force/Displacement, i.e. the dynamic stiffness versus frequency of the scenario.

Figure 4.10 shows the hammer tests that were performed to obtain dynamic stiffness data for the alpha prototype, a ball system, and a roller system (as mounted in the test set-up in our lab), for the purpose of comparison. The bearings were mounted and tested under the same conditions.

The dynamic stiffness experiments presented here are only intended to be an indicator of the relative damping provided by different bearing systems. Actually quantifying the
damping of each bearing system, and then predicting dynamic stiffness of a structure that a given bearing is mounted to, is beyond the scope of this thesis. From both a marketing and engineering standpoint, dynamic stiffness is a difficult beast because it may well be important for preventing chatter, but one really won’t be able to convince a skeptic of the true benefit of a better damped bearing unless the bearing is tried on the machine in question under the actual cutting conditions.

4.3.2 Measured Dynamic Stiffness (Hydro, Ball, & Roller Systems)

Figure 4.11 shows a plot of the dynamic stiffness measured for the alpha prototype, a lightly preloaded Thomson Accuglide(TM) bearing (ball system), and a lightly preloaded Thomson AccuMax(TM) bearing (roller system). The method of testing is shown in Figure Figure 4.10 (for more details, consult Appendix B). The conclusions drawn from the dynamic stiffness tests are summarized below.

![Figure 4.11](image-url)
Conclusions Drawn from Dynamic Stiffness Tests

1. The hydrostatic prototype shows improved damping because it did not add any resonances to those exhibited by the on rail test, unlike the roller systems.

2. In fairness to the ball and roller systems, a light preload, standard length truck (L=109mm) is the worst case scenario in terms of static stiffness (and most likely for dynamic stiffness). For further research, tests should be performed on medium and heavily preloaded systems, to see if a significant difference exists. However, it should be noted that a heavier preload will result in greater straightness ripple (more than the 0.6 and 0.4 microns for the lightly preloaded AccuGlide and AccuMax, respectively) so the best trade-off must be made.
Chapter 5

CONCLUSIONS AND FUTURE WORK

5.1 Conclusions

1) A modular hydrostatic bearing is promising for the following applications:
   - Cam Grinding
   - Ceramics Grinding
   - General Precision Grinding
   - Hard Turning
   - Very High Speed Machining

2) For ordinary machining applications rolling element bearings are still the best choice.

3) A major benefit of hydrostatic systems is for speeds of 1 m/s or greater, where the inherent cooling that can be provided by chilling the hydraulic fluid is a very important advantage over standard rolling element guides.

4) Initial damping tests show that the hydrostatic system does not introduce resonances. The direct benefit this provides to a customer needs to be proven by run-off tests.
5) The best possible hydrostatic system studied for this thesis (a diaphragm based system), operating at 100W, 60 cSt viscosity and using a bearing gap of 20 µm (reasonable for production tolerances and mounting errors), will provide roughly a static stiffness that is comparable to a heavy preloaded ball system or a lightly preloaded roller system.

6) The novel angled surface design presented, operating at 100W, 60 cSt viscosity and using a bearing gap of 20 µm (reasonable for production tolerances and mounting errors), will provide a static stiffness that is between a lightly preloaded and medium preloaded ball system.

7) Because it is the only design of the ones studied that can be manufactured using existing manufacturing processes at linear bearing companies, the novel angled surface design presented is highly preferable over other designs in terms of cost.

5.2 Future Work

It is important to realize that a key advantage of a hydrostatic system is better damping, but the direct benefit to the customer of damping must be established by experiments done on real machines. To address this key question, the following plan will be carried out in a partnership with a major linear bearing manufacturer (whose name must be kept confidential at this point in the research).

**Finish Design of Beta Prototype For Production Tolerances**

According to a preliminary error study a bearing gap of 20 µm (as opposed to 12.7 µm for the prototype) is viable for production. A full design review must be done with production engineers to finalize the design bearing gap, or perhaps two different gaps will be selected and tested.

**Make and Test Beta Prototype on In-House Grinder at the Linear Guide Factory**

A promising first application is on an in-house bearing grinder at the linear guide factory that grinds linear races for very small bearings. For the application, surface finish (and hence smoothness of motion and dynamic stiffness) is critical.
If Significant Improvement is Observed
  • Then implement on other in-house grinders, and assess markets.

If No Significant Improvement is Observed
  • Then assess cam grinding and ceramics grinding markets, where a hydrostatic bearing has definitive advantages.
REFERENCES


[Schneeberger Catalogue] Schneeberger, Inc., 11 DeAngelo Drive, Bedford, MA, 01730.

[Star Linear Catalogue] A division of Mannesmann Rexroth, 14001 South Lakes Drive, Charlotte, NC 28273.
Appendix A

TEST BED

In this appendix, the test bed used to measure vertical straightness and dynamic stiffness of size 35 linear guides is explained in detail. In the sections that follow, the bed design, truck and rail mounting methods, and the laser interferometer system used for straightness measurements are all described.

A.1 Overall View of Test Bed System

Figure A.1 shows a side view, and Figure A.2 shows a top view of the system used for measuring the straightness of size 35 modular linear guides. In summary, the system consists of a carriage plate mounted on four size 35 trucks, which travel along two rails bolted to a bed. The bed is supported by three feet which rest on a granite surface plate, which is in turn supported by four pneumatic feet. A Zygo ZMI-1000 laser interferometer system is used to measure the vertical straightness over one of the bearing trucks, and the x-travel position of the carriage. A Trilogy linear motor is used to drive the carriage back and forth. A PC is used with software provided by Zygo to acquire straightness data from the ZMI-1000, and another PC is used to send command signals to the linear motor, using Delta-Tau P-Mac hardware and software.
Figure A.1 Side view of test bed system designed for measuring straightness.
Figure A.2 Top view of test bed system designed for measuring straightness.
A.2 Carriage System

Figure A.1 shows the side view of a carriage plate mounted on four size 35 trucks, which ride on two rails. The rails are mounted to a 2 foot wide, 4 foot long, by 1 foot tall bed, which rests on a granite surface plate. The carriage plate has blanchard ground top and bottom surfaces, and is 2 inches thick, 24 inches wide, and 30 inches long. In Figure A.1 is shown, between the carriage plate and each truck, a 1 inch thick pillow block, whose faces are surface ground. The carriage plate and the four pillow blocks together load the four trucks with a weight of 409 lbs. Measured from truck center to center, the trucks under the carriage plate are 17.76 inches apart in the transverse direction, and 23.62 inches apart in the travel direction. With rubber mechanical stops installed, the maximum working travel of the carriage on the bed is 15.45 inches.

A.3 Bed Design and Manufacture

An end view of the bed on which the rails are mounted to, which incorporates Shear DampersTM for vibration damping, is shown in Figure A.3.

The bed is a weldement consisting of two 8 x 8 by 0.5 inch thick wall square steel tubes, welded between a couple of two inch thick steel plates. The resulting bed structure measures 2 feet wide, by 1 foot high, by 4 feet long.

In order achieve maximum accuracy when grinding the rail grooves, and to damp vibrations during testing, Shear DampersTM where installed inside the three cavities inside the bed, as shown in Figure A.3. The Shear DampersTM consisted of six 3 X 6 inch shear tubes, wrapped with a viscoelastic tape, which were cast inside the three cavities of the bed structure, using grout mixed with an expanding agent. The principle behind shear dampers is as follows. Thinking of the structure as a beam protruding into the page, when the structure bends as a beam, shearing motion must occur at each viscoelastic tape surface. This is because the neutral axis of the shear tubes is offset from the neutral axis of the bed structure. This shearing motion dissipates energy when the structure vibrates as a
Figure A.3  End view of bed, showing shear dampers(TM).

beam. In order to amplify the shearing action, and hence the damping, the neutral axis of each shear tube was displaced further away from the structure’s neutral axis by plug welding a 3.5 inch wide by 1.25 inch thick steel slab along the entire inside length of each shear tube. The resulting structure was highly damped using readily available materials.

The manufacturing steps and methods for the bed were as follows. First, the bed and the shear tubes were cut and welded using stock low carbon structural steel. After welding, the bed and shear tube members were annealed for a period of 24 hours, after which the shear tubes were wrapped with viscoelastic tape and cast inside the bed structure using the expanding grout. Next, both sides of the bed were machined, and the rail, gib grooves, and threaded holes were machined. Next, the bed was shipped to Hoffacker, in Cranston, RI, where the horizontal and vertical rail groove surfaces were ground to within 0.00028 inch straightness over the entire 4 foot length of the bed. To see the autocollimator straightness data of the rail grooves, consult Figure A.7 and Figure A.8.
A.4 Component Mounting

Both the AccuMax™ and AccuGlide™ rails and carriages were mounted according to their respective Linear Roller Bearing System Installation Procedure pamphlet, provided by Thomson. In the following sections, drawings of the rail and carriage mounting methods are shown and discussed.

A.4.1 Rail Mounting

To properly mount linear guides, mechanical means must be present to push the rail against a reference shoulder, before the rail is bolted down. On the test bed, tapered gibbs are used to push both rails against a reference shoulder. A tapered gib is present at each rail bolt location. A scale view of an AccuMax™ rail bolted in place with a gib on the test bed is shown in Figure A.4.

![Figure A.4 Scale drawing of rail mounted with tapered gib.](image)

On both rail grooves, the vertical shoulders face in the same direction. It was felt that this would ensure maximum parallelism between the two shoulders, because when they are ground, the same side of the wheel is used on each surface.

Prior to mounting the rails, the precision surfaces were cleaned and degreased, and each threaded hole was vacuumed clean, to ensure no liquids would ooze up onto the rail sur-
face when tightening the rail bolts. A small amount of grease was dabbed onto each rail bolt, gib, and gib bolt. After cleaning the rail surface with lint free optical tissue and acetone, the mating surfaces were stoned.

The rails were bolted using the maximum torque value for class 12.9 M8 bolts, which is 39 Nm. The bolts and gibs were tightened sequentially by the following procedure. Starting at one end of the rail, a rail bolt was tightened to 4 Nm, to insure the rail laid flat against the bottom rail groove surface, and then the adjacent tapered gib was tightened to 7 Nm, which was enough to overcome static friction and push the rail sideways up against the shoulder. This tightening sequence was repeated for each rail bolt and adjacent gib bolt, down the entire length of the rail. After the first pass, the rail bolts were tightened in three more passes, using a torque of 13 Nm, then 26 Nm, and finally 39 Nm.

### A.4.2 Carriage Plate Mounting

![Figure A.5 Scale drawing of a truck mounted to the carriage plate.](image)

To provide the flattest possible surface for the trucks to bolt against, a surface ground pillow block was placed between each truck and the carriage plate. The surfaces of the pillow
blocks were match ground on an Okomoto surface grinder, to achieve maximum uniformity in thickness. Each truck was bolted to each pillow block via the two middle bolts, and then each truck and pillow block assembly was bolted to the carriage plate via the four corner bolts. This arrangement assured that the center region of each truck was bolted against the very flat surface of each pillow block, before the truck and pillow block assemblies were secured to the carriage plate. All of the carriage bolts, which were class 12.9 M10's, were tightened using a torque of 77 Nm, as stated in the Linear Roller Bearing System Installation Procedure pamphlet.

A.5 Rail Groove Straightness

The vertical straightness of each rail groove was measured using a Nikon Model 6D manual autocollimator, and using a sled moved along the rail groove in increments equal to the center-to-center distance between the rear sled pad and the front sled pad (1.024 inches). The autocollimator had a resolution of 0.5 seconds (2.42 E-6 radians). Each sled pad was 0.208 inches (5.28 mm) wide in the travel direction, and 1.629 inches (41.38 mm) long transverse to the travel. Using the pad center-to-center length, the resolution using the sled for measuring vertical straightness was thus 2.5 microinches (0.063 microns). The accuracy of the autocollimator was quoted to be 1 arcsecond, corresponding to a straightness measuring accuracy of 5 microinches (0.127 microns). The actual accuracy is less due to mechanical contact non-repeatabilities.

The test procedure using the autocollimator was as follows. First, marks were placed beside the rail groove, spaced 1.024 inches apart. Next, the groove and sled surfaces were stoned, the sled was placed at the first location, and the autocollimator was zeroed. The sled was moved forward to subsequent marks and readings were taken. After every seven readings were recorded, the groove and sled pad surfaces were stoned again, and the sled was placed on the first location again to verify that the reading at the first location was still zero, insuring thermal drift was not occurring. For each rail groove, the measurements were repeated in the described fashion three times. For each rail groove, after the three
tests, the sled was placed at the first location once again, and the reading was found to be zero, verifying no thermal drift occurred.

Figure A.6 shows rail groove #1 and rail groove #2 on the bed, and Figure A.7 and Figure A.8 show the vertical straightness of each groove vs. \( X \), generated using the autocollimator angular data. Each plot shows data from the three repeated tests.

![Figure A.6 Rail groove names.](image)

From Figure A.7, in the range from \( X=0 \) to \( X=393 \) mm, rail groove #1 is straight to within 2.2 \( \mu \mathrm{m} \) (87 \( \mu \mathrm{in} \)), and the profile changes gradually. In the range from \( X=783 \) mm to \( X=1200 \) mm, rail groove #1 is straight to within 1.5 \( \mu \mathrm{m} \) (59 \( \mu \mathrm{in} \)), and the profile changes gradually as well. In the range from \( X=393 \) mm to \( X=783 \) mm, however, the rail groove is straight to within 7 \( \mu \mathrm{m} \) (276 \( \mu \mathrm{in} \)), and the profile has alternating high and low points. Looking at Figure A.7, one can see that every high point is near a bolt hole in the rail groove. Numerical investigation showed that every high point was within 6.6 \( \mu \mathrm{m} \) from a bolt hole center, which meant the sled foot was overlapping the hole rim. However, it was also found that in the non-oscillatory regions, several points within 6.6 \( \mu \mathrm{m} \) of hole centers were not high spots. One cause for the high points could be that the autocollimator tests were done after two sets of rails had been previously bolted to the bed (using the recommended bolt torque of 39 Nm for class 12.9 M8 bolts), and some rim deformation...
occurred. Why high points occur over a sequence of some bolt hole rims and not others, is unknown at this time.

Figure A.7  Autocollimator straightness of rail groove #1.

Figure A.8  Autocollimator straightness of rail groove #2.
For the highest precision applications, the source of this problem should be further investigated, to determine if it is specific to this test bed, or could be a problem with production machines sold in the marketplace.

From Figure A.8, it can be seen that in the range from X=211 mm to 965 mm, rail groove #2 is straight to within 2.5 µm (98 µin). However, oscillating surfaces were present at either end of the rail, in the ranges from X=0 to X=211 mm, and from X=965 mm to X=1200 mm. As with rail groove #1, further investigation showed that each high point was within 6.6 µm of a bolt hole center. However, in the non-oscillatory region, it was also found that several points within 6.6 µm of a hole center were not high points.

A.6 Linear Motor Drive System

In order to drive the carriage continuously for straightness testing, a Trilogy linear motor stage was used, in conjunction with Delta Tau P-mac hardware and software to send command signals to the motor. The linear motor stage includes its own carriage, supported by its own set of bearings. To prevent the linear motor carriage from "fighting" the test carriage motion, thereby tainting the straightness measurements, a flexural coupling was designed to connect the two. The flexural coupling is shown in Figure A.9.

The coupling was designed with two flexure regions, so that it could accommodate a lateral translation error. The part was machined from a single piece of 6061-T6 aluminum. To insure an absolute minimum of flexing required by the coupling, the linear motor stage was mounted parallel to one of the rails to within 0.001 inch, by using spacer blocks for precise alignment. To prevent axial vibrations from being transmitted to the carriage, a damping rubber was placed between the flexure and the carriage, and between the mounting bolts and the carriage.
A.7 Interferometer System Discussion

A Zygo ZMI-1000 laser interferometer was used to measure both the vertical straightness, and X position of the carriage. Figure A.2 shows most of the optical components, and the mounting horses and mirror boat.

The laser rests on a laser horse, which is a weldement fabricated from 8 x 8 square steel tubes. In front of the laser is a beam splitter. The undiverted beam is used to measure vertical straightness; the diverted beam is used to measure the X position of the carriage.

The undiverted beam measures vertical straightness via a differential interferometer, in conjunction with an 18" long Zerodur optical mirror, straight to within 1 \( \mu \text{in} \) \( (0.0254 \ \mu \text{m}) \), which is supported at its quarter points via pins which pass through the mirror boat, located above the differential interferometer. The mirror boat has three feet which rest on
the two mirror horses. The mirror horses are steel weldments which each have three rounded foot pods.

The diverted beam measures straightness via a linear interferometer mounted to the laser horse in conjunction with a retroreflector mounted on the moving carriage.

The two main sources of error in the interferometer measurements are due to changes in the air's index of refraction, and thermal deflections of the mirror boat, horses, and/or bed. For the preliminary straightness tests, each straightness reading was taken over a period of 79 seconds, so significant thermal changes over this time span are not of major concern. When the carriage is stationary, noise in the signal was 0.067 μm. When the carriage is moved from X=0 forward and then back to X=0, the Z reading always returns to zero within the Z repeatability of the bearing itself, which is as great at 1 micron in the case of the AccuGlide™.

The straightness data is sent to a DOS program provided by Zygo. The program was set to acquire 1050 points over a period of 79 seconds, with the carriage traveling at 5 mm/s.
Appendix B

STRAIGHTNESS AND DYNAMIC STIFFNESS MEASUREMENTS

In this appendix, the results of straightness and dynamic stiffness tests done on an AccuGlide™ (ball system, L=109mm, lightly preloaded) and an AccuMax™ (roller system, L=109mm, lightly preloaded) bearing are presented and discussed.

B.1 Straightness

The vertical straightness of both an AccuGlide™ and an AccuMax™ truck was measured by placing the interferometer on top of the carriage plate over the center of a truck. Figure B.1 shows the bearing truck under test. Figure B.2 and Figure B.3 show the laser interferometer data (with the best fit straight line subtracted) taken for the AccuGlide™ and AccuMax™ bearings, respectively, along with the autocollimator data taken for the rail groove, and the rail bolt locations. For the interferometer data, the linear motor velocity was 5 mm/s, and, a point was taken about every 0.37 mm (0.015 in) of travel. Table B.1 shows a summary of the results.
Figure B.1 The bearing truck under test.

**TABLE B.1** Measured straightness and straightness ripple.

<table>
<thead>
<tr>
<th>Vertical Straightness 400mm travel</th>
<th>AccuGlide (ball system)</th>
<th>AccuMax (roller system)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.07 μm noise in measurements</td>
<td>1.9 μm</td>
<td>2.4 μm</td>
</tr>
<tr>
<td>Truck specs: Lightly Preloaded Ultra Precision</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Length = 109mm (standard)</td>
<td>1 μm</td>
<td>1 μm</td>
</tr>
<tr>
<td>Absolute</td>
<td>0.6 μm</td>
<td>0.4 μm</td>
</tr>
<tr>
<td>Estimated relative to autocollim-</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ator data for rail groove</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average straightness ripple (due to</td>
<td></td>
<td></td>
</tr>
<tr>
<td>circulating rolling elements)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average wavelength of ripple from</td>
<td></td>
<td></td>
</tr>
<tr>
<td>data</td>
<td>13.1 mm (close to 2DIA)</td>
<td>8.4 mm (close to 2DIA)</td>
</tr>
<tr>
<td>Ball or Roller diameter</td>
<td>6.35 mm</td>
<td>4 mm</td>
</tr>
</tbody>
</table>
Points measured using an autocolimnator sled on bed rail was mounted to. (a bad point sometimes occurs when a sled foot overlaps a small lip present around some rail screw.

Note: raised lip around this screw hole in bed did not cause noticeable straightness error of truck motion.

Vertical straightness of truck motion measured using interferometer system.

0.6 micron average straightness ripple
Thomson AccuGlide Truck (Ball system)
Length=109 mm, Lightly preloaded (3\%C)

Figure B.2 Measured straightness of an AccuGlide\textsuperscript{TM} truck (ball system, ultra precision, lightly preloaded at 0.03\%C, Standard length of L=109mm).

Points measured using an autocolimnator sled on bed rail was mounted to. (a bad point sometimes occurs when a sled foot overlaps a small lip present around some rail screw.

Note: raised lip around this screw hole in bed did not cause noticeable straightness error of truck motion.

Vertical straightness of truck motion measured using interferometer system.

0.4 micron average straightness ripple
Thomson AccuMax Truck (Roller system)
Length=109 mm, Lightly preloaded (3\%C)

Figure B.3 Measured straightness of an AccuMax\textsuperscript{TM} truck (roller system, ultra precision, lightly preloaded at 0.03\%C, Standard length of L=109mm).
It is informative to note that the spike (which, as explained in Section A.5, occurs locally around the rim of some rail mounting holes) has almost no effect on the AccuMaxTM truck straightness, for which every hole in the rail groove is used. For the AccuGlideTM, however, the spike, which is 4.5 μm high, causes only about a 1 μm rise in the truck motion.

The ripple observed is caused by circulating elements. As expected, the AccuMaxTM had a smaller wave amplitude than the AccuGlideTM because its rolling elements are smaller (the more elements in contact at one time, the less effect one element entering or exiting has). The amplitude of the ripple is close to twice an element diameter, as expected (because the truck moves twice as fast relative to the rail as do the elements relative to the truck.)

**B.2 Dynamic Stiffness**

The dynamic stiffness measurements on a bearing truck were performed via a hammer test, using an HP35670A Signal Analyzer, and a PCB modally tuned hammer and accelerometer. A hammer test utilizes a modally tuned hammer with a force transducer on its tip, and an accelerometer, both sending signals to the signal analyzer. Upon striking the device under test, the signal analyzer records the force impulse signal from the hammer, and the acceleration, and from these calculates the frequency response of the device under test. The frequency response data, which has units of acceleration per unit force, is input into a spreadsheet where it is converted to stiffness verses frequency by dividing each acceleration by frequency squared.

For both an AccuGlideTM and an AccuMaxTM truck, two test scenarios were performed. For the first scenario, only a pillow block was bolted to a truck (using all six holes), and for the second, the carriage plate was bolted in place according to the original test set up. Figure B.4 shows the two test scenarios.
Figure B.4 The two test scenarios for dynamic stiffness testing.

Figure B.5 shows the results with the pillow block only for the AccuGlide and AccuMax, and Figure B.6 shows the results with the hydrostatic prototype included (the hydrostatic truck could not be tested objectively with pillow block only because it is not stable in tilt).

Table B.2 summarizes the minimum stiffnesses observed, along with the reduction in stiffness Q that occurs at resonance.
Figure B.5 Compressive dynamic stiffness of AccuGlide™ (ball system) and AccuMax (roller system) trucks, lightly preloaded (0.03% C), standard length of L=109 mm, shown with the response measured on top of the rail alone. The hydrostatic prototype could not be tested under these conditions because it is not stable in tilt.

Figure B.6 Compressive dynamic stiffness of AccuGlide™ (ball system) and AccuMax (roller system) trucks, lightly preloaded (0.03% C), standard length of L=109 mm, and the hydrostatic prototype, L=134 mm, shown with the response measured on top of the rail alone.
TABLE B.2 Summary of compressive static and dynamic stiffnesses.

<table>
<thead>
<tr>
<th>Truck specs: L=109mm, Light preload (0.03% C) Ultra Precision</th>
<th>Static stiffness from catalogue (N/μm)</th>
<th>Min. Dynamic (N/μm)</th>
<th>Frequency (Hz)</th>
<th>Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>AccuGlide, pillow block only</td>
<td>260</td>
<td>34</td>
<td>1464</td>
<td>7.6</td>
</tr>
<tr>
<td>AccuMax, pillow block only</td>
<td>850</td>
<td>66</td>
<td>1720</td>
<td>12.9</td>
</tr>
<tr>
<td>AccuGlide, with carriage plate</td>
<td>260</td>
<td>47</td>
<td>928</td>
<td>5.5</td>
</tr>
<tr>
<td>AccuMax, with carriage plate</td>
<td>850</td>
<td>80</td>
<td>232</td>
<td>10.6</td>
</tr>
<tr>
<td>Hydrostatic prototype, Ps =70 Bar, with carriage plate</td>
<td>420</td>
<td>192</td>
<td>220</td>
<td>2.2</td>
</tr>
</tbody>
</table>

B.3 Friction Force

TABLE B.3 Measured static friction per truck.

<table>
<thead>
<tr>
<th>F accuracy = +/- 0.25N</th>
<th>Static Friction Per Truck</th>
</tr>
</thead>
<tbody>
<tr>
<td>AccuGlide, L=109mm, 0.03%C</td>
<td>7N</td>
</tr>
<tr>
<td>AccuMax, L=109mm, 0.03%C</td>
<td>10N-12N</td>
</tr>
<tr>
<td>Hydrostatic prototype</td>
<td>0N</td>
</tr>
</tbody>
</table>

Table B.3 shows the force per truck required to initiate movement, with the carriage plate bolted in place. As a note, when pushing the carriage plate by hand, the AccuGlideTM bearings felt smooth, while the AccuMaxTM bearings felt as if the rollers partially jammed and then let go in the races. The variable friction force may be a problem in high precision applications.

For the future, it is proposed that the force required to drive the carriage at different speeds be measured with a force transducer.

B.4 Conclusions

The conclusions from this study are summarized below:
Conclusions Drawn from Straightness Tests

1. The straightness of both rolling element systems would be about 1 micron if they were bolted to a perfectly flat rail groove.

2. The straightness ripple of the lightly preloaded AccuGlide and AccuMax was about 0.6 and 0.4 microns, respectively. With a higher preload we would expect more ripple. The AccuMax has better ripple in part because its rollers are have a smaller diameter (6.35 mm for AccuGlide, 4 mm for AccuMax).

3. The period of ripple was close to twice the element diameter, as expected. If a part surface had ripple with these periods, ripple from the linear guides would be the culprit.

Conclusions Drawn from Dynamic Stiffness Tests

1. The hydrostatic prototype shows improved damping because it did not add any resonances to those exhibited by the on rail test, unlike the roller systems.

2. In fairness to the ball and roller systems, a light preload, standard length truck (L=109mm) is the worst case scenario in terms of static stiffness (and most likely for dynamic stiffness). For further research, tests should be performed on medium and heavily preloaded systems, to see if a significant difference exists.

3. The rolling elements showed better damping with the carriage plate bolted in place. This could be due to the effects of being over constrained.
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