Wavelength Routing for All-Optical Networks[1]

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Abstract

We consider passive all-optical networks using wavelength division multiplexing and wavelength routing, i.e. the path of a signal is determined by the wavelength of the signal and the signal origin. We present upper and lower bounds on the required number of wavelengths to achieve a given blocking probability. Specifically we show that between $\Omega\left((\sqrt{\rho M})^{(1-P_b)}\right)$ and $O\left(\sqrt{\rho M}\right)$ wavelengths are required to support ρM active session requests in a network with M users and blocking probability P_b. The lower bound holds for all networks with passive wavelength routing and fixed wavelength changing devices. The upper bound is a passive construction without wavelength changers.

Keywords: Wavelength Routing, All-Optical Networks, Connectors.

1 Introduction

In a wavelength routing all-optical network (λ-routing AON), the path of a signal is determined by the wavelength of the signal, the location of the signal transmitter, and the state of the network nodes. If the signal paths are under the control of the network, e.g. through the use of switches or dynamic wavelength routing nodes, we say that the network is *configurable*, a.k.a. reconfigurable, a.k.a. adaptive. Otherwise, we say the network is *fixed* or *passive*, a.k.a. non-reconfigurable, a.k.a. non-adaptive. In a passive network, the signal paths are only a function of the signal wavelength and origin.

Since we are allowing the use of wavelength conversion within a fixed or configurable network, a signal launched from a transmitter may arrive at a receiver on a different wavelength. In fact, a signal launched from a transmitter may arrive at a variety of receivers on many different wavelengths and/or arrive at a single receiver on several different wavelengths.

In networks where the number of active sessions far exceeds the number of available wavelengths, it will be necessary to simultaneously assign many transmitters the same wavelength. Since two signals using the same wavelength cannot travel over the same fiber simultaneously, certain collisions need to be prevented. In particular, we must insure that signals do not collide at any intended receiver. That is, if receiver m is listening to wavelength λ at time t, we must insure that only one signal arrives at receiver m on wavelength λ and time t. If two or more arrive, we say there is *contention.*

[1]Research supported by the Army Research Office under contracts DAAL03-86-K-0171 and DAAL03-92-G-0115 and by NSF grant NCR-9206379 and under DARPA grant MDA972-92-J-1038.

Contention is avoided by isolating signals of the same wavelength onto different fibers using λ-routing and λ-changing. Previously, it has been shown that there is a limit to the possible amount of isolation, or equivalently a limit on the wavelength re-use, in non-blocking networks [1, 2]. This limit depends on the number of wavelengths, the number of devices, the functionality of the devices, and the requirements of the users. In section 5, we generalize this result and show that the same limit applies to networks where a small probability of blocking P_b is tolerated. We then present a passive wavelength efficient network with arbitrarily small blocking probability [Section 6]. The limit on wavelength re-use allows the possibility of wavelength changing; the construction does not use any wavelength changing.

First in sections 2 and 3 we formalize the network model and problem statement, respectively. Then a survey of previous relevant results are discussed [section 4]. More comprehensive surveys on wavelength routing can be found in [3, 4, 5].

2 Network Model

We consider networks with M users and F wavelengths. Each user has one fully tunable transceiver[2]; for results on AONs where either the transmitter or the receiver are fixed tuned, see [8]. Each transmitter (receiver) is connected to one outgoing (incoming) fiber.[3] To model wavelength changing, we define an *origin-destination* channel, or OD channel, as an ordered pair of wavelengths $f : f'$. We say that transmitter n is connected to receiver m on OD channel $f\!:\!f'$ if a signal launched from n on wavelength f is received at m on wavelength f'. If a transmitter or receiver is tuned to wavelength f, we say that it is *assigned* f. Note that there is no assumed relationship between the OD channels connecting transmitter n to receiver m and the OD channels connecting transmitter m to receiver n.

Using the OD channel terminology, the connectivity of a λ-routing network can be fully described by the set $\mathcal{H} = \{H_\psi \mid \psi \in \Psi\}$, where $H_\psi(n, m)$ is the set of OD channels connecting transmitter n to receiver m in *switching state* ψ and Ψ are the switching states of the network. A switching state represents the joint state of all devices in the network, i.e. switches, wavelength routers, and wavelength changers.

In networks without wavelength changing, $f\!:\!f' \in H_\psi(n, m)$ implies that $f = f'$. In this case, we use the obvious short hand notation of f for $f\!:\!f'$.

If $|\Psi| = 1$, the network is passive, $\mathcal{H} = \{H\}$ and we use the notation of H for $\mathcal{H}$. A broadcast nework is the simplest example of a passive network where $H(i, j) = \{1, 2, ..., F\}$ for all (i, j).

Also, if $F = 1$, then the network is a conventional circuit switched network. In this case, the number of switches $\log |\Psi|$ required for non-blocking operation is $\Theta(M \log M)$ [10, 11].

[2]Since we are not considering multi-point connections, we view the alternate assumption of multiple transceivers per user as a single transceiver and a switch. For results on optical networks with multiple transceivers, see [6, 7].

[3]If transceivers have access to multiple fibers, then some switching mechanism is required to isolate or select the signal on only one of the fibers. Therefore this option is equivalent to a transceiver and a switch. For results on optical networks of this type, see [9].

3 Problem Statement and Previous Results

Define a *session* (n, m) as an ordered pairing of a transmitter to a receiver. We assume that each session requires one full wavelength of bandwidth; for results when sessions require less bandwidth, see [12, 6, 7]. A *traffic* is a set of sessions; we only consider traffics without multi-point connections.

A non-blocking network is one which can support all possible traffics. Non-blocking networks require at least $\sqrt{M/e}$ wavelengths [1].

Most previously work has focused on the asymptotic growth of the required number of wavelengths for non-blocking operation. Rearrangeably and wide-sense non-blocking networks with and without fixed wavelength changers have been considered. Currently there is no known benefit of using wavelength changers and only a small benefit of being rearrangeable. Specifically, it has been shown that no more than $\Theta(\sqrt{M \log M})$ wavelengths are required for non-blocking operation; see [2] for a proof using wavelength changers and [4, 13] for two different proofs without wavelength changers. These results apply to rearrangeable and wide-sense non-blocking networks, but the constant in front of the $\sqrt{M \log M}$ term can be made slightly smaller for the rearrangeable case [14].

Although it is known that there exist networks requiring only $\Theta(\sqrt{M \log M})$ wavelengths, no explicit constructions with this efficiency are known. The best construction without wavelength changing is due to [4] and requires $O\left(2^{(\log M)^{.8+o(1)}}\sqrt{M}\right)$ wavelengths. However $2^{(\log M)^{.8+o(1)}}$ decays so slowly that more wavelengths are needed for this construction than in a broadcast network unless $M \geq 2^{32}$. For reasonable M, the best constructions with and without wavelength changing are $M^{2/3}$ and $\left\lceil \frac{M}{2} + 2 \right\rceil$ respectively [2]. The $M^{2/3}$ construction was obtained by using an analogy to 2-stage switching networks and then using a construction in [15]. The $\left\lceil \frac{M}{2} + 2 \right\rceil$ wavelength network is also the best known strict sense non-blocking network. There is currently no lower bound for the number of wavelengths required in a strict sense non-blocking network, however for the special case of *Light Tree AONs* [16], the last construction is the optimal strict sense non-blocking network [14].

Therefore, although non-blocking passive networks are unscalable, i.e. $M \leq eF^2$, wavelength routing can significantly increase a network's capacity over a broadcast star. This is one reason passive λ-routing networks are being considered as part of a larger configurable AON [16].

4 Problem Statement and New Results

Since the construction of wavelength efficient non-blocking networks has been so elusive and since a small amount of blocking in a network is usually tolerable, we turn our attention and effort to networks with blocking. In this case, we have essentially solved the problem. First we show in section 5 that passive blocking networks are still unscalable. Then in section 6, we show that we can meet the theoretical lower bound with an explicit construction, beating the best existence proof by a factor of $\sqrt{\log M}$.

Consider a one-shot routing problem with ρM random session *requests* $s_1, \ldots, s_{\rho M}$ where all lists of ρM requests without multi-point connections are equally likely. We call ρ the *utilization* of and ρM the *load* on the network, respectively.

A non-blocking network can always *honor* all requests. A blocking network *blocks* some of the requests. Define the blocking probability P_b to be the expected fraction of blocked requests.

P_b depends on the network and the strategy used for honoring or blocking. We distinguish between two types of strategies: *sequential* and *non-sequential*. In a sequential strategy, the network must honor or block request i without knowledge of requests $i+1, ..., \rho M$. If request i is honored, it must be assigned an OD channel which does not contend with any previously honored requests; we do not allow the possibility of re-arranging OD channels on previously honored requests. The *greedy* strategy is a particularly simple sequential strategy which honors request i iff there exists a feasible OD channel.

In a non-sequential strategy, the network waits until the last request before honoring or blocking any request. A special case of a non-sequential strategy is an *optimal* strategy. An optimal strategy always honors the maximum possible number of requests from any list of requests. Note that the optimal strategy depends on the network and *may* be difficult to determine and/or implement; however the concept is useful for lower bounding the required number of wavelengths. In addition, we will see that for the efficient construction presented in section 6, the optimal strategy is trivial to determine and implement; in fact in this case, the optimal strategy is the greedy strategy.

For example, recall that in a broadcast AON each transmitter is connected to each receiver on all wavelengths. Since there can be at most F honored requests, the optimal and greedy blocking probabilities of a broadcast AON are both $P_b = 1 - \frac{F}{\rho M}$. Therefore $F = (1-P_b)\rho M = \Theta(\rho M)$ wavelengths are required for a broadcast AON.[4] We will see that far fewer wavelengths are required if wavelength routing is used to spatially isolate signals of the same wavelength on different fibers and that λ-changing is not required to realize this savings in the number of wavelengths.

In the next two sections we present a lower bound on the number of required wavelengths for any passive network possibly using wavelength changing and then explicitly construct passive networks without wavelength changing which come very close to the bound. An optimal strategy is assumed for the lower bounds, a greedy strategy for the constructions. Specifically if $F(M, \rho, P_b)$ is the minimum number of wavelengths required for any passive λ-routing AON with M users, blocking probability no more than P_b, and load at least $\rho M \geq L(P_b)$, we show that

$$\left(\sqrt{\frac{\rho M}{e}}\right)^{1-P_b} \left(1 + O\left(\frac{\ln \rho M}{\rho M}\right)\right) - 1 \;\leq\; F(M, \rho, P_b) \;\leq\; c(P_b)\sqrt{\rho M} \qquad (1)$$

where $c(P_b)$ is a constant which depends only on P_b and is between 5 and 9 for P_b between 10^{-3} and 10^{-6}. The upper bound in eqn. (1) is valid for $\rho M \geq L(P_b)$ where L is a constant which depends only on P_b and increases as P_b decreases. For $P_b = 10^{-6}$, $L \leq 721$ so the fact that $\rho M \geq L$ is a relatively minor restriction.

Notice that except for excessively large P_b, the lower bound is approximately $\sqrt{\rho M/e}$, the same as the non-blocking case; however, unlike the non-blocking case, we have essentially met the theoretical lower bound, i.e. $\Omega\left(\left(\sqrt{\rho M}\right)^{1-P_b}\right) \leq F(M, \rho, P_b) \leq O\left(\sqrt{\rho M}\right)$. Also unlike the non-blocking case, the networks which achieve this wavelength efficiency are explicit constructions (the constructions use a simple generalization of the well known WDM cross-connect [17, 18]).

[4]Recall that we are assuming that each session requires a full wavelength of bandwidth.

5 Lower Bound for Blocking Networks

In this section, we prove the lower bound in eqn. (1). A few preliminaries will simplify the discussion.

Recall that a traffic ϕ is a set of sessions. If $(n,m) \in \phi$, we say that (n,m),n, and m are *active* in ϕ. We assume that each transmitter and each receiver are active in at most one session in any traffic, that is we do not consider any multi-point connections. If two sessions are active in the same traffic ϕ, they are said to be *concurrent* in ϕ. We say that a network *supports* ϕ if all the sessions in ϕ can be connected without contention.

Consider any passive AON with connectivity matrix H and let $\mathcal{T}(H)$ be the set of traffics H can support.[5] Also recall that $s_1, s_2, ..., s_{\rho M}$ is a list of session requests and that an optimal strategy is one which always honors the maximal number of requests possible. Formally, an optimal strategy is one which always honors

$$q_\phi = \max\{|\phi'| \;\; : \phi' \subseteq \phi \;\; \text{and} \;\; \phi' \in \mathcal{T}(H)\} \tag{2}$$

sessions where $\phi = \{s_1, ..., s_{\rho M}\}$. Note again that the optimal strategy depends on the network.

The expected number of honored requests under an optimal strategy is

$$E[q] \;\; = \;\; \frac{1}{\binom{M}{\rho M}^2 (\rho M)!} \sum_{\phi} q_\phi \tag{3}$$

where the sum is taken over all traffics with ρM sessions. The optimal blocking probability is

$$P_b \;\; = \;\; \frac{\rho M - E[q]}{\rho M} \tag{4}$$

In this section, we will lower bound P_b by first proving the bound for the special case of $\rho = 1$. Then using the following lemma, we can simply substitute ρM for M in the general case of $\rho < 1$.

Lemma 1 *Let $F(M, \rho, P_b)$ be the minimum number of wavelengths for a network with M users, ρM requests, and blocking probability no more than P_b. Then $F(M, \rho, P_b) \geq F(\rho M, 1, P_b)$.*

> **Proof.** *Consider an arbitrary passive AON operating with an optimal strategy and a blocking probability P_b. Conditioning on the set of transmitters and set of receivers requesting sessions, the blocking probability can be written as the expected blocking probability given a set of transmitters Tran and a set of receivers Rec, i.e.*
>
> $$P_b \;\; = \;\; \frac{1}{\binom{M}{\rho M}^2} \sum_{Tran,Rec} P_b(Tran, Rec) \tag{5}$$
>
> *where the sum is taken over all sets of ρM transmitters and all sets of ρM receivers and where $P_b(Tran, Rec)$ is the blocking probability given Tran and Rec.*

[5] Since the traffics supported by a network are determined by H, we will informally refer to H as the network itself.

Pick a $(Tran, Rec)$ with $P_b(Tran, Rec) \le P_b$; there must be at least one. Now form a new network with the ρM transmitters in $Tran$ and the ρM receivers in Rec with the same wavelength connectivity between these users as the original network. Then at least $F(\rho M, 1, P_b(Tran, Rec))$ wavelengths are required for this new network. Since $P_b(Tran, Rec) \le P_b$, the result follows. $\square$

We need the following lemma the proof of which can be found in [2, 14].

Lemma 2 *A passive λ-routing network can support at most $(F+1)^{2M}$ different traffics, i.e. $|\mathcal{T}(H)| \le (F+1)^{2M}$ for any H.*

We are now in a position to prove the bound.

Theorem 3 Lower Bound for Networks with Blocking
At least

$$F(M, \rho, P_b) \ge \left(\sqrt{\frac{\rho M}{e}}\right)^{1-P_b} \left(1 + O\left(\frac{\ln \rho M}{\rho M}\right)\right) - 1 \tag{6}$$

wavelengths are needed for any passive AON with blocking probability P_b.

Proof. *Let H be any connectivity matrix and $\mathcal{T} = \mathcal{T}(H)$ be the traffic set the network supports. We use a slight abuse of notation and set $|H| = |\mathcal{T}(H)|$ to be the number of traffics supported by the network. By the lemma, it is sufficient to prove the theorem for $\rho = 1$.*

The first step of the proof is to show that if $E[q]$ is large, then $|H|$ must also be large. Specifically, we first show that for any integer $0 \le k \le M-1$,

$$E[q] \le k + |H| \left(\frac{M}{M-1}\right) \frac{(k+1)}{(M)_{k+1}} \tag{7}$$

where $(n)_i = \binom{n}{i} i!$ is the lower factorial function.

The expected number of honored requests routed under an optimal strategy is

$$E[q] = \sum_{q=1}^{M} qP(q) \tag{8}$$

where $P(q)$ is defined to be the probability that the maximum number of sessions that can be honored is q. Define $P'(q) \ge P(q)$ to be the probability of being able to honor q sessions, i.e. the probability that there exists a q-subset of ϕ (a subset of ϕ with size q) that is supported by the network. So for any k between 1 and $M-1$,

$$E[q] \ \le k + \textstyle\sum_{q=k+1}^{M} qP'(q) \tag{9}$$

Now let $\mathcal{T}_k$ be the set of traffics in $\mathcal{T}(H)$ with exactly k sessions. There are $\binom{M}{q}$ q-subsets of ϕ and if any of these subsets is in $\mathcal{T}_q$, we can route q sessions. The probability that one of these subsets picked at random is in $\mathcal{T}_q$ is

$$\frac{|\mathcal{T}_q|}{\binom{M}{q}^2 q!} \tag{10}$$

since there are a total of $\binom{M}{q}^2 q!$ possible q-traffics, i.e. traffics of size q, and if we pick a q-subset of ϕ any q-traffic is equally likely. Using the union bound, the probability of being able to honor q requests is no more than

$$P'(q) \leq \frac{\binom{M}{q}|\mathcal{T}_q|}{\binom{M}{q}^2 q!} = \frac{|\mathcal{T}_q|}{(M)_q}. \tag{11}$$

Therefore, the expected number of honored requests is upper bounded by

$$E[q] \leq k + |H| \sum_{q=k+1}^{M} q \left(\frac{\alpha_q}{(M)_q}\right) \tag{12}$$

where $\alpha_q = |\mathcal{T}_q|/|H|$ and $\sum_{q=k+1}^{M} \alpha_q \leq 1$. It is not difficult to upper bound the sum, from which it follows that

$$E[q] \leq k + |H| \left(\frac{M}{M-1}\right) \frac{k+1}{(M)_{k+1}} \tag{13}$$

which proves eqn. (7).

For the second part of the proof, pick $k = \lfloor E[q] \rfloor - 1 \leq M - 1$. For this k, the second term in eqn. (7) above must be at least 1, and solving for $|H|$ this gives

$$|H| \geq \left(\frac{M-1}{M}\right) \left(\frac{1}{\lfloor E[q] \rfloor}\right) (M)_{\lfloor E[q] \rfloor} \tag{14}$$

Now $(M)_n \geq \frac{\sqrt{2\pi}}{e}(M/e)^n$ which follows from $\sqrt{2\pi n}(n/e)^n \leq n! \leq e\sqrt{n}(n/e)^n$ [19]. So,

$$|H| \geq \left(\frac{M-1}{M}\right) \left(\frac{1}{\lfloor E[q] \rfloor}\right) \left(\frac{\sqrt{2\pi}}{e}\right) \left(\frac{M}{e}\right)^{\lfloor E[q] \rfloor} \tag{15}$$

and since $E[q] \geq \lfloor E[q] \rfloor \geq E[q] - 1$, since $E[q] = M(1 - P_b)$, and since the number of supported traffics is no more than $(F+1)^{2M}$,

$$(F+1) \geq \epsilon \left(\sqrt{\frac{M}{e}}\right)^{1-P_b} \tag{16}$$

where

$$\epsilon \stackrel{\text{def}}{=} \left[\left(1 - \frac{1}{M}\right) \cdot \frac{1}{M(1-P_b)} \cdot \frac{\sqrt{2\pi}}{e} \cdot \frac{e}{M}\right]^{\frac{1}{2M}} \tag{17}$$

Now $\ln \epsilon = O\left(\frac{\ln M}{M}\right)$ so that $\epsilon = 1 + O\left(\frac{\ln M}{M}\right)$. In fact, $\epsilon \approx 1$ even for moderate M. For instance, for all $P_b \leq .5$ and $M \geq 1000$, $\epsilon \geq .99$. $\square$

6 Efficient Passive Constructions

In the previous section, we essentially showed that at least $\sqrt{\rho M/e}$ wavelengths are required for any network with a small blocking probability. Here we will construct a network with a very small blocking probability using less than $c\sqrt{\rho M}$ wavelengths, where c is a small constant which depends on P_b.

We call the construction a LAN-LR since it consists of N local area networks (LANs) interconnected by a Latin Router (LR) [18]. Specifically, there are N transmitting LANs (*T-LANs*) with $\frac{M}{N}$ transmitters each and N receiving LANs (*R-LANs*), each with $\frac{M}{N}$ receivers.

Numbering the T-LANs from 0 to $N-1$, let t_n be the T-LAN of transmitter $n = 1, 2, \ldots M$. Similarly number the R-LANs and let r_m be the R-LAN of receiver $m = 1, 2, \ldots M$. The connectivity matrix of the LAN-LR is specified by

$$H(n,m) \;=\; \{f \mid \left\lfloor \frac{f}{k} \right\rfloor = t_n - r_m\} \tag{18}$$

k is called the *coarseness* of the network and represents the number of wavelengths connecting any T-LAN to any R-LAN. Using the terminology in [4], a [T-LAN,R-LAN] pair is called a *block*. Note that the total number of wavelengths is $F = Nk$.

Before deriving the blocking probability of a LAN-LR, note that for all $\rho \geq \frac{1}{N}$, the LAN-LR requires M wavelengths to be non-blocking since all transmitters in a T-LAN could request all receivers in an R-LAN. Therefore $k = \frac{M}{N}$ for non-blocking operation and $F = Nk = M$ wavelengths are required. This makes the LAN-LR a very wavelength **inefficient** non-blocking connector even for very small ρ.[6] However, we will see below that for proper choice of N and k, the LAN-LR is a very wavelength **efficient** blocking connector.

To see that, let's derive P_b, the expected fraction of blocked requests. Since there are N^2 blocks and since up to k requests can be honored in any block,

$$P_b \;=\; \frac{1}{\lambda} \sum_{i=k+1}^{M/N} (i-k)P(i) \tag{19}$$

where $P(i)$ is the probability of i requests in a block and $\lambda \stackrel{\text{def}}{=} \rho M/N^2$ is the expected number of requests per block. It is not too difficult to show that

$$P(i) \;\leq\; P'(i) \stackrel{\text{def}}{=} \frac{1}{\sqrt{2\pi i}} \left(\frac{\lambda e \gamma^2}{i}\right)^i \tag{20}$$

where $\gamma = exp(k/F)$. Since $P'(i+1) \leq \frac{\gamma^2 \lambda}{k} P'(i)$, we have that

$$P_b \;\leq\; \frac{P'(k+1)}{\lambda} \sum_{j=0}^{b-1} (j+1) \left(\frac{\lambda\gamma^2}{k}\right)^j \tag{21}$$

$$\;\leq\; \frac{P'(k+1)}{\lambda} \left(1 - \frac{\lambda\gamma^2}{k}\right)^{-2}. \tag{22}$$

Finally, plugging in for $P'(k+1)$ and simplifying,

$$P_b \;\leq\; \left(1 - \frac{\lambda\gamma^2}{k}\right)^{-2} \frac{\gamma^{2k}}{k\sqrt{2\pi k}} \left(\frac{\rho M k e}{F^2}\right)^k \tag{23}$$

[6] Recall that $F = \Theta(\sqrt{M \log M})$ wavelength suffice for a non-blocking connector.

We first use this bound to choose a good value of k holding the number of wavelengths F and the total number of users M fixed but allowing the number of LANs N and the number of users per LAN $\frac{M}{N}$ to vary with k. Then using this choice of k, we bound the number of wavelengths required to achieve a desired blocking probability. We will see that for proper choice of k, the blocking probability can be made very small. On the other hand, if care is not taken in choosing k, many more wavelengths may be required [14].

For large F, i.e. $F \geq 2k^2$, the bound on P_b is mainly dependent on the last term. Therefore, we choose k to minimizes that term: $k = k_o \stackrel{\text{def}}{=} F^2/e^2\rho M$. For this value of k,

$$P_b \leq \left(\frac{e^2}{e^2 - \gamma_o}\right)^2 \cdot \frac{\gamma_o^{2k_o}}{\sqrt{2\pi}} \cdot e^{-\{\frac{F^2}{e^2\rho M} + 3\ln\frac{F}{e\sqrt{\rho M}}\}} \tag{24}$$

where $\gamma_o \stackrel{\text{def}}{=} exp\{k_o/F\}$. Now if $F \geq 2k_o^2$ or equivalently $\rho M \geq 4k^3/e^2$, $\gamma_o^{2k_o} \leq e$ and $\gamma_o^2 \approx 1$. Assuming that this is so, the following table lists various values of this bound as a function of F. Also listed are the minimum ρM for which the bounds are valid. For smaller ρM, eqn. (23) should be optimized over k considering the effect of γ^{2k}.

F	k	$min\{\rho M\}$	$P_b \leq$
$(e\sqrt{5})\sqrt{\rho M}$	5	68	$.9 * 10^{-3}$
$(e\sqrt{7})\sqrt{\rho M}$	7	185	$.7 * 10^{-4}$
$(e\sqrt{9})\sqrt{\rho M}$	9	395	$.7 * 10^{-5}$
$(e\sqrt{11})\sqrt{\rho M}$	11	721	$.7 * 10^{-6}$

Notice that we did not relax the integer constraints on k. However, we did relax the integer constraints on N and $\frac{M}{N}$. Let's quickly address the validity of these two approximations. Since $N = F/k$, we should have restricted ourselves to k which divide F. But since k divides $F + a$ for some $a < k$, the number of wavelengths can be increased by at most k. Now imagine that we did not relax the integer constraints on $\frac{M}{N} = \frac{Mk}{F}$, but wished to keep the number of users M fixed. We could still use a network with a LR backbone and N LANs, but some LANs will have $\lfloor Mk/F \rfloor$ users and some $\lceil Mk/F \rceil$ users. Since in either case, the number of users is $(Mk/F)(1 + O(1/\sqrt{M}))$, the approximation is sufficient for our purposes.

7 Conclusions

Unlike the non-blocking case, we have been able to construct wavelength efficient blocking networks. Specifically for a blocking probability of P_b, the minimum number of wavelengths is between $\Omega((\sqrt{\rho M})^{1-P_b})$ and $O(\sqrt{\rho M})$. The lower bound allows the possibility of fixed wavelength conversion; the upper bound is achieved by the LAN-LR network which does not use wavelength changing and which has recently been proposed as part of a larger Wide Area AON [16].

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